

University of Windsor

Scholarship at UWindsor

Industrial and Manufacturing Systems
Engineering Publications

Department of Industrial and Manufacturing
Systems Engineering

7-15-2016

Optimal disposition decisions for a remanufacturing system considering time value of products

Mostafa Pazoki
University of Windsor

Walid Abdul-Kader
University of Windsor

Follow this and additional works at: <https://scholar.uwindsor.ca/industrialengpub>



Part of the [Industrial Engineering Commons](#)

Recommended Citation

Pazoki, Mostafa and Abdul-Kader, Walid. (2016). Optimal disposition decisions for a remanufacturing system considering time value of products. *Computers and Industrial Engineering*, 99, 124-136.
<https://scholar.uwindsor.ca/industrialengpub/12>

This Article is brought to you for free and open access by the Department of Industrial and Manufacturing Systems Engineering at Scholarship at UWindsor. It has been accepted for inclusion in Industrial and Manufacturing Systems Engineering Publications by an authorized administrator of Scholarship at UWindsor. For more information, please contact scholarship@uwindsor.ca.

Optimal disposition decisions for a remanufacturing system considering time value of products

Mostafa [Pazoki](#)^a

pazoki@uwindsor.ca

Walid [Abdul-Kader](#)^{b,*}

kader@uwindsor.ca

^aIndustrial and Manufacturing Systems Engineering, University of Windsor, Canada

^bMechanical, Automotive and Materials Engineering, University of Windsor, Canada

*Corresponding author.

Abstract

This paper studies disposition decisions of cores where the value of returns deteriorates over time. Mainly in disposition decisions, a remanufacturer is interested to determine how many units to remanufacture and to salvage. To address this research problem along with value deterioration of returns, a rough-cut mathematical model is developed by considering various parameters of interest such as selling price, salvage value and remanufacturing rate with the aim to maximize total profit. Due to uncertainty limitations, the model can provide decision-makers with relevant insights about disposition decisions. Simulation modeling techniques are used to validate the proposed model. Numerical examples are presented to demonstrate the applicability of the model and to show the negative relation between the deterioration rate and the total profit. However, the above-indicated parameters of selling price, salvage value and remanufacturing rate work the opposite way.

Keywords: Remanufacturing; Time value of products; Mathematical model; Simulation

1 Introduction

Reverse logistics triggers the backward flow of materials in order to preserve the value of the used products and protect the environment. Potential profitability, environmental consideration and federal laws are the three main driving forces of remanufacturing. The total value of the remanufactured items in United States till 1998 was reported to worth \$53 billion at the time of the study ([Lund, 1998](#)). In 2011 the total value of the remanufacture production went up to \$43 billion, expecting to grow each year.¹

Decision making levels in reverse logistics have been expanded from strategic to operational. In the literature of reverse logistics and remanufacturing, a variety of operational level assumptions and considerations are taken into account. One of the considerations studied in the literature is the deteriorated value of the returned product over the delay time. High-tech products such as computers are prone to lose their value due to delays from the time they are received till the time they are resold to the secondary market. The addressed delay, which is the time between receiving a returned product until it is remanufactured and resold in the secondary market, has several consequences. It deteriorates customers' willingness to pay, incurs holding cost and reduces the selling price of the products. The loss incurred by the delay may be as high as 1% per week for some products ([Blackburn, Guide, Souza, & Van Wassenhove, 2004](#)). Thus, the time value of products should be taken into account ([Guide, Gunes, Souza, & Wassenhove, 2008](#); [Voutsinas & Pappis, 2002, 2010](#)). The IBM case addressed in [Ferguson, Fleischmann, and Souza \(2011\)](#) and the HP case studied in [Guide et al. \(2008\)](#) are two examples of this consideration. Generally, electronic devices such as laptops and cellphones can be categorized in this group of high-tech and short lifecycle products ([Guide et al., 2008](#)). The second approach is to consider work in process (WIP) cost for the remanufacturing line. The time interval in which the returned product is kept unfinished can be translated to WIP cost ([Pazoki & Abdul-Kader, 2014](#)). This approach has been considered in the scheduling context ([Alidaee & Womer, 1999](#)) where the value of products depends on the completion time ([Janiak & Krysiak, 2007](#)).

The remanufacturing process is not absolutely continuous for many industries. The machines have to be serviced on a regular basis to increase the utilization rate and decrease the defect rate. In addition, many remanufacturing systems are involved in manufacturing. Therefore, after manufacturing brand new products, the machines need to be cleaned and set up to start remanufacturing. Thus in the addressed firm, not only does the remanufacturing process have to be done in discrete periods of time, but also it must be started when the production line is available for such activities. This situation, which to the best of our

knowledge has been overlooked in the literature, is considered in this article.

The aim of this research work is to address the optimal disposition decision. The addressed assumptions of deteriorating value of returns and the discrete period assigned to remanufacturing are the two key considerations of this study. The quality conditions of returns are not identical and all unclassified arrivals are delivered to the system in bulk. This assumption is commonly considered in the scheduling context where the value of the product depends on the completion time and the goal is to maximize the total profit (Voutsinas & Pappis, 2002). A threshold is considered in this research as a boundary to accept or reject the categorized returns on the basis of the expected delay and the exponential value deterioration function. Considering the salvage value of a returned product and its expected deteriorated value, decisions would be made whether to remanufacture the product or to sell it immediately at its salvage value. This problem is classified as disposition problem in the literature. To tackle the addressed problem, a mathematical model is presented. The schematic view of the disposition decision is illustrated in Fig. 1.

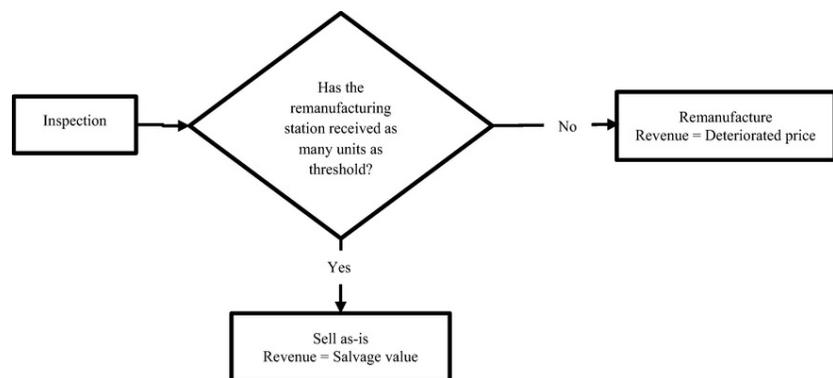


Fig. 1 Disposition decision and flow of materials.

As explained earlier, the remanufacturing process is performed in discrete periods of time. Throughout this article, the total available length of time for manufacturing and remanufacturing is called *production period*, and the time assigned for remanufacturing within a production period is called *operating period*. The portion of the production period in which remanufacturing is not performed is referred to as *idle time*. Returns are inspected, sorted and put in *classes* or *categories*. Note that the terms “class” and “category” are used interchangeably throughout this paper. *Remanufacturing rate* refers to the rate at which returns are remanufactured, and *remanufacturing time* is the time it takes to remanufacture a unit of product (inverse of remanufacturing rate).

This proposed work contributes to the literature in several terms. Firstly, to the best of our knowledge, no previous or published research work has considered the availability of unclassified returns at time zero together with disposition decisions and time value of the products. This consideration makes it possible to incorporate disposition decisions with tactical level decisions about inventory and pricing. Secondly, the operating periods and idle times are considered in this model. Dividing the total production period into operating period and idle time is adapted from real situations. If the company is fully involved in remanufacturing, the operating period is the time assigned to remanufacturing and the idle time is the time devoted to secondary tasks such as maintenance (Su & Xu, 2014). Otherwise if it is a hybrid (re)manufacturing system, the operating periods can be interpreted as the time assigned to remanufacturing task and the idle time would be the times assigned to manufacturing and maintenance tasks.

Thus, as a summary, this proposed research contributes to the literature by considering (in addition to the quantity to remanufacture and to salvage) the following points:

- Deterioration value for returns and time value of the products.
- Assuming availability of returns at time zero, and.
- Availability of discrete time intervals for remanufacturing.

Moreover, it is shown that although the mathematical model is generated on the basis of average process times, it could give good total revenue estimation for stochastic systems. Consequently, it is called a rough-cut model.

The remaining of the paper is organized as follows. The related research studies are discussed in Section 2. The research problem is defined and the mathematical model is presented in Section 3. A numerical

example is given to show the relevance of the model. Given the rough-cut nature of the model in the absence of uncertainty considerations, the model is validated using simulation modeling techniques where a variety of probability distributions is used to represent the processing times. Sensitivity analysis together with accuracy of the model in general conditions is performed in Section 4. Finally, conclusion and recommendations for future research are presented.

2 Literature review

2.1 On the quality variability, marginal value of time and arrival/service times

Quality variability complicates remanufacturing planning (Guide, 2000). Many research studies have taken quality variability into account either as a continuous function (Denizel, Ferguson, & Souza, 2010; Galbreth & Blackburn, 2010) or a discrete function (Teunter & Flapper, 2011). Admission decision belongs to remanufacturing planning problem presented in 1970s where the remanufacturer decides on the quantity to remanufacture and quantity to salvage. Generally, remanufacturing has greater profit than recycling. However, since the value of high-tech products deteriorates over time and considering holding cost, it should be decided whether to salvage the product immediately or remanufacture it. Harrison (1975) studies the problem of admission control for a single server queue with linear cost functions. Souza, Ketzenberg, and Guide (2002) study a system where the returned products are required to be processed in parallel GI/G/1 stages. Later, Guide et al. (2008) modify the assumption of positive salvage value in Harrison (1975) and present a mathematical model under the assumption of Poisson arrival rates and general service rates (M/G/1). Their work is followed by Karamouzian, Teimoury, and Modarres (2011) who study the multistage version of the problem with stochastic remanufacturing routing, but limit their mathematical model by considering Exponential service rates. Karamouzian, Naini, and Mazdeh (2014) extend Guide et al. (2008) by applying shortest expected processing times (SEPT) as the dispatching rule for a single-stage remanufacturing process (M/G/1/SEPT). Fathi, Zandi, and Jouini (2014) study the disposition decision problem further in two directions: (1) considering two streams of returns where one of the inter-arrival times follows Hyperexponential distribution (H/M/1) and the other follows Exponential distribution (M/M/1), and (2) investigating the impact of merging the remanufacturing stages for the two streams (H + M/M/1). All of the papers introduced above take quality uncertainty of the returns, exponentially deteriorated values and net present value into account.

Ferrer and Swaminathan (2006) analyze two-period and multi-period scenarios where the manufacturer only produces the new product in the first period, but has the option of making new and remanufactured products in subsequent periods. Pricing decisions impact the dynamics across periods in such cases. In their paper, they characterize the production quantities associated with self-selection and explore the effect of various parameters in the Nash equilibrium. Naeem, Dias, Tiberwal, Chang, and Tiwari (2013) consider a single item dynamic lot sizing problem with manufacturing and remanufacturing provisions and present two models: (1) deterministic demands and returns model, and (2) probabilistic demands and returns models. In the former the demands and returns are discrete and deterministic over a full time horizon i.e. exact future demands can be predicted. A dynamic programming based model is also developed with the objective to determine the quantities that have to be manufactured or remanufactured at each period to minimize the total cost, including production cost, holding cost for returns and finished goods, and backlog cost. Geyer, Van Wassenhove, and Atasu (2007) model and quantify the cost-savings potential of production systems that collect, remanufacture, and remarket end-of-use products as perfect substitutes while facing the fundamental supply-loop constraints of limited component durability and finite product life cycles. They investigate the profitability of remanufacturing under basic supply-loop constraints such as accessibility of end-of-use products (collection rate), technical feasibility of remanufacturing (durability), and market demand for remanufactured products (life cycle).

Although a couple of recent research studies have been conducted to address different versions of this problem, we are not aware of a research study in which availability of returns at time zero incorporated with disposition decision has been considered. All of the reviewed papers in this context consider the stochastic arrival times which makes it difficult to integrate their models with tactical decisions such as pricing models. Considering the availability of all the jobs at time zero is common in scheduling context (Alidaee & Womer, 1999). Thus, this paper contributes to the current literature on disposition decision in remanufacturing problem.

2.2 On the modeling methodology

The disposition problem is approached with two main methods: Mathematical modeling and Discrete-Event Simulation (DES). Since the arrival and process rates are stochastic, the mathematical formulations represent the average expected values in steady-state period (Fathi et al., 2014; Guide et al., 2008; Karamouzian et al., 2011, 2014). Otherwise, even the average expected values have to be approximated (Souza et al., 2002). To validate the mathematical models presented in research studies, DES is employed frequently (Fathi et al., 2014; Guide et al., 2008). However, DES can also be used to approximate the best average expected value of the objective function (Guide et al., 2008). In spite of the research problem intricacy, a mathematical model facilitates analyzing the system by clearly flagging the parameters that impact the objective function and the extent to which they impact it. Therefore, a mathematical model is an appropriate method to tackle this problem.

3 Problem definition and mathematical model development

3.1 Problem definition

Unclassified returns are received in batches. Inspection is the first process a returned product must undergo. From the inspection stage and with multinomial distribution, returns are directed to different remanufacturing stations on the basis of the class they are assigned to. Discrete classes of returns with multinomial distribution function are considered in [Teunter and Flapper \(2011\)](#). After inspection, two options are available for the classified products: to be sold immediately on their salvage values or to be remanufactured. The remanufactured products are sold and the deteriorated profit is calculated with an exponential function like $r e^{-\beta t}$, where t is the delay time, r represents the selling price at time zero and β incorporates holding cost, remanufacturing cost and revenue decay ([Guide et al., 2008](#)). Note that in [Guide et al. \(2008\)](#) and the research studies conducted after, r is considered as a decreasing function of time to capture the remanufacturing cost. However, in this research, we incorporate remanufacturing cost implicitly in the deterioration rate so that it still increases if the delay time increases. The configuration of the system under study is depicted in [Fig. 2](#). [Karamouzian et al. \(2014\)](#) consider a multi-class M/G/1 system where each station is remanufacturing a certain grade of the returns. [Souza et al. \(2002\)](#) also study a remanufacturing system consisting of multi-class GI/G/1 remanufacturing station where each station is devoted to a certain quality category or type of the returns.

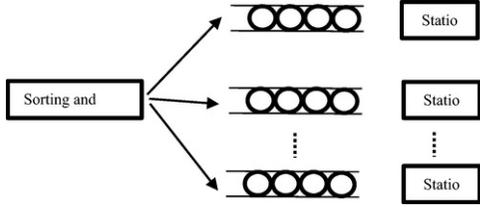


Fig. 2 Material flow in the system under study.

After inspection, returns are transferred to one of the intermediate storage areas according to their quality conditions. It is assumed that the inspection rate is higher than the remanufacturing rates, so that the returns are accumulated in the intermediate storage areas (remanufacturing stages will not starve). For many groups of products, remanufacturing process may be time consuming whereas the inspection process can be performed in a relatively shorter period of time. Therefore, the addressed assumption makes sense for many real world cases.

The associated unit costs and selling prices are unique for each category. In other words, quality condition variability and the diversity of selling prices are taken into account. There is no assumption for the probability distributions of inspection and remanufacturing times; only the expected values are considered.

3.2 Mathematical model

In this paper, mathematical modeling is the main methodology to approximate the desired values (quantity of returns to remanufacture and the total profit). Although mathematical models are difficult to generate, they provide an insight into the role of the model parameters in a reasonable period of time compared to simulation models. But, simulation modeling techniques are commonly used to validate such mathematical models as it is the case with this proposed model.

The notation, parameters and decision variables used in the model are as follows:

I : Set of quality condition classes and remanufacturing stations.

S_i : The number of returns to be remanufactured during one operating period from class $i \in I$.

R_i : The number of operating periods required to remanufacture all of the returns for class $i \in I$.

R_i^j : The operating period in which the j th return belonging to class $i \in I$ would be remanufactured.

W_i^j : The waiting/delay time of the j th item belonging to the i th class.

β : The deterioration rate.

μ_i : The rate at which station $i \in I$ remanufactures the returns (remanufacturing rate).

p_i : The probability that a return belongs to class $i \in I$.

λ : Inspection rate.

r_i : The selling price at time zero for a return which is categorized in class $i \in I$ and remanufactured.

Sal : The salvage value of a return which is sent for recycling.

T : Production period.

U : Operating period.

Ψ : The total number of received returns.

k_i : The quantity of returns to remanufacture from class $i \in I$.

Z_i : The total deteriorated profit gained by remanufacturing or salvaging the returns categorized in class $i \in I$. However, the total profit from all the items is: $Z = \sum_{i \in I} Z_i$.

Each quality condition class is indexed by $i \in I$. In other words, different remanufacturing processes which are required due to quality heterogeneity are represented by set I . R_i is the required number of operating periods to remanufacture the returns in category $i \in I$ and the number of returns which can be remanufactured during one operating period is represented by S_i . To determine R_i and S_i , production period (T), operating period (U), expected proportion of the returns categorized in class $i \in I$ (p_i), inspection rate (λ) and remanufacturing rates (μ_i) should be considered. According to Pazoki and Abdul-Kader (2014), if all of the returns in station $i \in I$ are remanufactured continuously with no breaks or idle times, the delay time of the j th item, W_i^j , would be:

$$W_i^j = \frac{j}{\mu_i} + \frac{1}{\lambda p_i} \quad (1)$$

Thus, the number of returns to remanufacture during each operating period (except the last period), S_i , is (Pazoki & Abdul-Kader, 2014):

$$\begin{aligned} S_i &= \lfloor \mu_i(U - 1/\lambda p_i) \rfloor \\ S_i &= \lfloor \mu_i U \rfloor \end{aligned} \quad (2)$$

where $\lfloor \mu_i(U - 1/\lambda p_i) \rfloor$ is the quantity to remanufacture in the first operating period and $S_i = \lfloor \mu_i U \rfloor$ is the quantity to remanufacture for all operating periods except the first one and the last one. For the sake of simplicity, we assume that for all operating periods except the last one we have $S_i = \lfloor \mu_i(U - 1/\lambda p_i) \rfloor$. Assuming Ψ as the total number of returns, then Ψp_i would be the expected number of returns assigned to class $i \in I$. Therefore, the number of required operating periods to remanufacture the returns assigned to class $i \in I$ is (if all of the returns are supposed to be remanufactured):

$$R_i = \left\lceil \frac{\Psi p_i}{S_i} \right\rceil \quad (3)$$

In a similar way, the operating period order at which the j th returned item would be remanufactured can be calculated as indicated below:

$$R_i^j = \left\lceil \frac{j}{S_i} \right\rceil \quad (4)$$

From Eq. (1), we know that the completion time of the j th item in the i th category is $j/\mu_i + 1/\lambda p_i$. If the j th item is completed in the first operating period, its total delay consists of the time it spends in queue, and then in inspection and remanufacturing stations. However, if this item is processed in the second operating period, then its total delay time also includes one idle time or $(T - U)$; if this item is to be completed in the third operating period, then it needs to wait two idle times or $2(T - U)$. These idle times are in addition to the time it spends in queue, inspection and remanufacturing stations, and so on. Therefore, if this item is to be processed in operating period k , it will see $(k - 1)$ idle times or $(k - 1)(T - U)$, before it is finished. Thus, incorporating Eqs. (1) and (4), the delay time of the j th returned item considering operating periods and idle times should be revised to:

$$W_i^j = (T - U) \left(R_i^j - 1 \right) + \frac{j}{\mu_i} + \frac{1}{\lambda p_i} \quad (5)$$

The first term of Eq. (5) is the total number of idle times before remanufacturing the j th item, $(R_i^j - 1)$, multiplied by the idle time duration $(T - U)$. Thus, the first term is the total delay time before finishing the j th item. The second and third terms, according to Eq. (1), are the required processing times to inspect and remanufacture the j th item. Therefore, Eq. (5) is the total time elapsed to remanufacture the j th item, which is called "waiting time". Eqs. (1)–(5) are the basic equations upon which the mathematical model is built.

The profit gained from each remanufactured item is deteriorating exponentially, which is presented with $r_i e^{-\beta W_i^j}$ where W_i^j is the total waiting time for the j th item in the i th class, r_i is the current selling price of a

remanufactured item, and β incorporates holding cost and discounted selling price (Guide et al., 2008).

The total deteriorated profit for remanufacturing the returns from class $i \in I$ in the first operating period is calculated in Eq. (6).

$$\sum_{n=1}^{n=S_i} r_i e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{n}{\mu_i}\right)} = r_i e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{1 - e^{-\frac{S_i \beta}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) \quad (6)$$

Detailed explanation of Eqs. (6)–(10) is presented in Appendix A. The equation above is the total profit of the first S_i units. In a similar way, the total profit gained for the second operating period (the second S_i units) is:

$$\sum_{n=S_i+1}^{n=2S_i} r_i e^{-\beta\left(T-U + \frac{1}{\lambda p_i} + \frac{n}{\mu_i}\right)} = r_i e^{-\beta\left(T-U + \frac{1}{\lambda p_i} + \frac{S_i+1}{\mu_i}\right)} \left(\frac{1 - e^{-\frac{S_i \beta}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) \quad (7)$$

Totally, k_i items are remanufactured from category $i \in I$. Therefore, from the required number of operating periods to remanufacture all of the items in category $i \in I$ is $R_i^k = \lceil k_i/S_i \rceil$. In a similar manner, the total profit for the operating period $R_i^k - 1$ is:

$$\sum_{n=(R_i^k-2)S_i+1}^{n=(R_i^k-1)S_i} r_i e^{-\beta\left((R_i^k-2)(T-U) + \frac{1}{\lambda p_i} + \frac{n}{\mu_i}\right)} = r_i e^{-\beta\left((R_i^k-2)(T-U) + \frac{1}{\lambda p_i} + \frac{(R_i^k-2)S_i+1}{\mu_i}\right)} \left(\frac{1 - e^{-\frac{S_i \beta}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) \quad (8)$$

Thus, the total remanufacturing profit gained during first $R_i^k - 1$ operating periods can be obtained by Eq. (9).

$$\sum_{n=1}^{n=R_i^k-1} r_i e^{-\beta\left((n-1)(T-U) + \frac{1}{\lambda p_i} + \frac{(n-1)S_i+1}{\mu_i}\right)} \left(\frac{1 - e^{-\frac{S_i \beta}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) = r_i e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{1 - e^{-\beta \frac{S_i}{\mu_i}}}{1 - e^{-\beta \frac{1}{\mu_i}}} \right) \left(\frac{1 - e^{-\beta(R_i^k-1)\left(T-U + \frac{S_i}{\mu_i}\right)}}{1 - e^{-\beta\left(T-U + \frac{S_i}{\mu_i}\right)}} \right) \quad (9)$$

For all of the operating periods except the last one, the number of remanufactured products from class $i \in I$ is equal to S_i . In the last operating period, however, less than S_i products are remained to be remanufactured. Therefore, the total profit for the returns remanufactured in the last operating period is not the same as per Eqs. (6)–(8). During operating periods 1 through $R_i^k - 1$, the total number of remanufactured products is $S_i (R_i^k - 1)$. Thus, the quantity of remaining products to remanufacture from the λ th class is equal to $k_i - S_i (R_i^k - 1)$. Therefore, the total profit gained from remanufacturing the returns category $i \in I$, which has been delayed to be remanufactured till the last operating period is:

$$\sum_{n=S_i(R_i^k-1)+1}^{n=k_i} r_i e^{-\beta\left(T-U + (R_i^k-1) + \frac{1}{\lambda p_i} + \frac{n}{\mu_i}\right)} = r_i e^{-\beta\left(T-U + (R_i^k-1) + \frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{e^{-\beta \frac{S_i(R_i^k-1)}{\mu_i}} - e^{-\beta \frac{k_i}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) \quad (10)$$

The total remanufacturing profit function of the λ th class (see Eq. (11) below) is obtained by adding (10) and (9).

$$Z_i = r_i \left[e^{-\beta\left(T-U + (R_i^k-1) + \frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{e^{-\beta \frac{S_i(R_i^k-1)}{\mu_i}} - e^{-\beta \frac{k_i}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) + e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{1 - e^{-\beta \frac{S_i}{\mu_i}}}{1 - e^{-\beta \frac{1}{\mu_i}}} \right) \left(\frac{1 - e^{-\beta(R_i^k-1)\left(T-U + \frac{S_i}{\mu_i}\right)}}{1 - e^{-\beta\left(T-U + \frac{S_i}{\mu_i}\right)}} \right) \right] \quad (11)$$

The objective function of the addressed problem is to maximize the total profit. If the remanufacturer receives a certain quantity of returns and decides on remanufacturing or salvaging them, then the salvage value should be included in the total profit function. Hence, the mathematical model would be:

$$\text{Maximize : } Z = \sum_{i \in I} r_i \left[e^{-\beta\left(T-U + (R_i^k-1) + \frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{e^{-\beta \frac{S_i(R_i^k-1)}{\mu_i}} - e^{-\beta \frac{k_i}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) + e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{1 - e^{-\beta \frac{S_i}{\mu_i}}}{1 - e^{-\beta \frac{1}{\mu_i}}} \right) \left(\frac{1 - e^{-\beta(R_i^k-1)\left(T-U + \frac{S_i}{\mu_i}\right)}}{1 - e^{-\beta\left(T-U + \frac{S_i}{\mu_i}\right)}} \right) \right] + Sal \left(\Psi - \sum_{i \in I} k_i \right) \quad (12)$$

Subject to:

$$k_i \leq \Psi p_i, \quad \forall i \in I \quad (13)$$

Objective function (12) is the total of profit gained by remanufacturing products from all categories and salvaging the remaining products. Constraints (13) limit the quantity of returns in each category to the expected quantity

of returns, which follows multinomial distribution function. Note that the value of S_i is determined by Eq. (1) and $R_i^k = \lceil k_i/S_i \rceil$, which is not substituted in the total profit formulation to avoid structural complexity. In the next section, a numerical example is presented and the results are discussed.

3.3 Mathematical model verification

A numerical example is presented to show the accuracy of the model in the presence of uncertainty in inspection time and remanufacturing time. The parameters and values in this numerical example are presented in Table 1.

Table 1 Parameters used in the numerical example.

Parameters and notation	Value
Number of stations (i)	4
Time unit	t
p_i	0.1, 0.2, 0.3, 0.4
r_i	1000, 1250, 1500, 1800
λ	700 per t
μ_i	70, 80, 90, 100 per t
β	0.05 per t
Ψ	10,000
T	24 t
U	8 t

As addressed earlier in this paper, the mathematical model is developed on the basis of Pazoki and Abdul-Kader (2014), in which it is assumed that the remanufacturing stations would never starve. Thus, the arrival rate from the inspection center to a remanufacturing station is always greater than or equal to the process rate of that station (i.e. $\lambda p_i \geq \mu_i$). However, such an increase is not large and can be easily accommodated by the buffer or queue located upstream of the station.

To investigate the accuracy of objective function (12), a simulation model is built in ProModel 8.6 and run for three scenarios. The addressed scenarios consider Exponential, Uniform and Normal distributions for all the remanufacturing and inspection times. The results are reported in Table 2, considering the average value over 30 replications to overcome the variability and to reach the average expected value for the simulation model. The total profit value obtained by the mathematical model is 4,129,022. The relative error is calculated as follows:

$$|Total\ profit\ of\ scenario - Total\ profit\ of\ mathematical\ model| / Total\ profit\ of\ scenario.$$

Table 2 Total profit for the deterministic (4,129,022) and stochastic systems.

Scenarios	Inspection time	Remanufacturing time	Total profit	Relative error compared to the mathematical model
1	$E(1/\lambda)$	$E(1/\mu_i)$	4,065,982	0.016
2	$U(1/2\lambda, 3/2\lambda)$	$U(1/2\mu_i, 3/2\mu_i)$	4,105,114	0.006
3	$N(1/\lambda, 1/2\lambda)$	$N(1/\mu_i, 1/2\mu_i)$	4,086,683	0.010

Scenarios 2 and 3 have the least error values. Note that in the deterministic model the proportions still follow multinomial distribution which creates relative error (i.e. quality conditions are uncertain). Among the three distributions with the given parameters, the smallest variance belongs to the Uniform distribution or scenario 2, followed by scenario 3 and scenario 1. The same order can be observed in the relative error values. Thus, it can be obviously concluded that the amount of error may depend on the distribution variation. These results verify the proposed mathematical model. Thus, although the mathematical model is built upon the assumption of deterministic process times, we expect to see the same average results in the case of stochastic process times.

There are four main reasons why the cumulative deteriorated values (total profit) are almost the same. The first reason is the assumption of greater arrival rate that dictates accumulation of returns before the remanufacturing stations. In this case, whether we consider stochastic or deterministic times, the returns which arrive later have to wait more. The second reason is the structure of the deterioration function which powers e to the negative waiting time. Therefore, the deterioration function scales down the values which, in turn, smooth out the observed variability of the waiting times. The third reason is the deterioration rate which is a very small value and also scales down the waiting time. The fourth reason is that the objective function is the total deteriorated value of the returns. Cumulating the deteriorated values also reduces the variability caused by the stochastic arrival and process times.

In this section, we showed that the mathematical model is relatively acceptable for stochastic systems. In the next section, we further investigate the model performance in the presence of uncertainty.

3.4 Brief notes about model complexity

First, we show that the objective function (12) is concave. For the sake of simplicity and without losing generality, we assume only one category for the returns. The objective function then turns to be the following:

$$\text{Maximize : } Z = r \left(e^{(-\beta(T-U)(R^k-1) + \frac{1}{\lambda p} + \frac{1}{\mu})} \left(\frac{e^{-\beta \frac{S(R^k-1)}{\mu}} - e^{-\beta \frac{k}{\mu}}}{1 - e^{-\beta \frac{1}{\mu}}} \right) + e^{-\beta(\frac{1}{\lambda p} + \frac{1}{\mu})} \left(\frac{1 - e^{-\beta \frac{S}{\mu}}}{1 - e^{-\beta \frac{1}{\mu}}} \right) \left(\frac{1 - e^{-\beta(R^k-1)(T-U + \frac{S}{\mu})}}{1 - e^{-\beta(T-U + \frac{S}{\mu})}} \right) \right) + Sal(\Psi - k) \quad (14)$$

Assuming $R^k = k/S$ and $S = \mu U$, the first order derivative of Z is:

$$\frac{dZ}{dk} = \frac{r\beta T}{\mu U(-1 + e^{-\beta/\mu})(-1 + e^{-\beta T})} \left(e^{-\frac{\beta(\lambda p T k + \mu U + \mu \lambda p)}{\mu \lambda U p}} - e^{-\frac{\beta(\lambda p T k + \mu U + \mu \lambda U^2 p + \mu \lambda p)}{\mu \lambda U p}} \right) - Sal,$$

and the second order derivative is:

$$\frac{d^2 Z}{dk^2} = -\frac{r\beta^2 T^2}{\mu^2 U^2(-1 + e^{-\beta/\mu})(-1 + e^{-\beta T})} \left(e^{-\frac{\beta(\lambda p T k + \mu U + \mu \lambda p)}{\mu \lambda U p}} - e^{-\frac{\beta(\lambda p T k + \mu U + \mu \lambda U^2 p + \mu \lambda p)}{\mu \lambda U p}} \right).$$

The second order derivative is negative which makes the local optimum the global one. However, because of function floor in defining R^k , the objective function is not continuous. Therefore, using the first order condition is not helpful in this case. If the feasible region for k is all real numbers, then the problem may not have the optimal solution. However, since k only accepts integer values, discontinuity has no impact on the existence of the optimal solution. Since in real world problems the daily number of products received in batches is not too high, direct numeration is an appropriate method to deal with this objective function. As the objective function is concave, if we start from $k_i = 0$ and increase it one unit at a time, we can stop the moment the objective function starts deteriorating and be sure that we are at the optimum point. Each step we compare the marginal profit of remanufacturing (the first term of the objective function) with the salvage value. The point beyond which the salvage value is greater than the marginal remanufacturing profit is the optimum point. If there is only one type of returns, obviously the maximum number of calculations would be equal to Ψ . If there are two types of returns or more, since the objective function (12) can be decomposed for each type of return, the maximum number of times we calculate Z would be at most Ψ . Hence, the computational burden is not significant for this model.

4 Numerical experiments and factor analysis

4.1 Investigating the impact of remanufacturing rates

In Section 3.3, we showed the accuracy of the mathematical model to approximate the average expected value when the remanufacturing times are stochastic. This section investigates the impact of uncertain remanufacturing times on the accuracy of the model. The other limiting assumption we made was that the arrival rate in the intermediate buffers (λp_i) should be greater than the remanufacturing rate (μ_i) of its remanufacturing station. First we relax this assumption and calculate the accuracy of the mathematical model. Note that in the presence of uncertainty, the simulation model has been run for 30 replications to obtain the average expected profit.

The input data is the same as what was in Section 3 except for the remanufacturing rates. From the data presented in Section 3.3, the critical values for the four remanufacturing stations (λp_i) would be (for μ_i) 70, 140, 210 and 280, respectively. The remanufacturing time distributions are the same as presented in Table 2 above. The results illustrated in Figs. 3-6 are obtained assuming that all of the returns should be admitted and remanufactured. Note that for each graph, the other inspection and remanufacturing rates are held constant. For instance in Fig. 3 below, only μ_1 is subject to change and all other values are the same as in Table 1. The following notations are used in Figs.

3-6:

- *Exponential* is the objective function values obtained by the simulation model for exponentially distributed inspection and remanufacturing times.
- *Uniform* is the objective function values obtained by the simulation model for uniformly distributed inspection and remanufacturing times.
- *Normal* is the objective function values obtained by the simulation model for normally distributed inspection and remanufacturing times.
- *Mathematical model* is the objective function value obtained by the mathematical model solved in MAPLE 16.

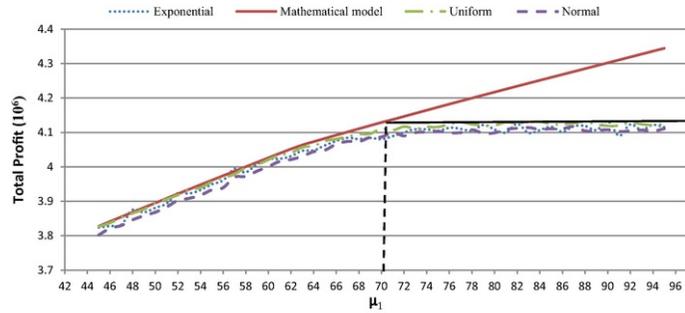


Fig. 3 Impact of the first station's remanufacturing rate, μ_1 , on the objective function for $\mu_2 = 80, \mu_3 = 90, \mu_4 = 100$, and $\lambda = 700$.

The results are reported in [Tables 4-7](#). The graphs below are depicting the impacts.

Table 3 Parameters used in the numerical experiments.

Parameters and notation	Value
Number of stations (j)	1
Time unit	h
p_i	1
r_i	\$8000
λ	3.12 per hour
μ_i	1.04 per hour
β	0.0006875 per hour
Ψ	250 or 500 items
T	24 h
U	24 h

Table 4 Scenario definition.

Scenarios	r_1	r_2	r_3	μ_1	μ_2	μ_3
-----------	-------	-------	-------	---------	---------	---------

1	High	Medium	Low	High	Medium	Low
2	Medium	Medium	Medium	Medium	Medium	Medium
3	High	Medium	Low	Medium	Medium	Medium
4	Low	Medium	High	High	Medium	Low
5	Medium	Medium	Medium	Low	Medium	High

Table 5 Input data for numerical analysis.

Parameters	Levels	Value
r_i	High	1000 (unit)
	Medium	750 (unit)
	Low	500 (unit)
μ_i	High	100 (unit/t)
	Medium	75 (unit/t)
	Low	50 (unit/t)
β	N/A	0.0005 (\$/t)
		0.001 (\$/t)
		0.002 (\$/t)
		0.003 (\$/t)
λ_{p_i}	N/A	200 (unit/t)
Sal	N/A	\$250 (unit)
T	N/A	24 t
U	N/A	8 t
A	N/A	100,000

Table 6 Admission decision for scenario 1.

Category	Optimum decision	$\beta = 0.0005$	$\beta = 0.001$	$\beta = 0.002$	$\beta = 0.003$
1	Quantity	33,333	33,333	23,792	15,690
	Percentage	100%	100%	71.4%	47.1%
2	Quantity	33,333	28,200	14,168	9600
	Percentage	100%	84.6%	42.5%	28.8%

3	Quantity	33,333	12,118	6000	4154
	Percentage	100%	36.6%	18%	12.5%

Table 7 Admission decision for scenario 2.

Category	Optimum decision	$\beta = 0.0005$	$\beta = 0.001$	$\beta = 0.002$	$\beta = 0.003$
1	Quantity	33,333	28,200	14,168	9600
	Percentage	100%	84.6%	42.5%	28.8%
2	Quantity	33,333	28,200	14,168	9600
	Percentage	100%	84.6%	42.5%	28.8%
3	Quantity	33,333	28,200	14,168	9600
	Percentage	100%	84.6%	42.5%	28.8%

Considering Figs. 3-6, the following observations are made: (1) as expected, increasing the remanufacturing rate results in increasing the total profit to some extent; (2) increasing the remanufacturing rate results in increasing the error of the mathematical model, and decreasing the stability of the average expected profit; (3) in all the cases, the uniform distribution which shows the lowest variability is best approximated by the mathematical model; (4) the optimum total profit tends to be stable above a certain value of the remanufacturing rate; and (5) the average expected total profit is stabilized if the remanufacturing rate raises beyond the critical value (λp_i). Obviously, this is due to starvation of the station. After the critical value, the objective function in all of the simulated scenarios shows a negligible or no tendency to increase. The critical value is depicted by vertical black dashed line. In Fig. 3, the critical value is 70. In Figs. 4-6 we identify 140, 210 and 280, respectively, as the critical values. The value of the mathematical model at the critical values is also illustrated by horizontal black solid line on the graphs.

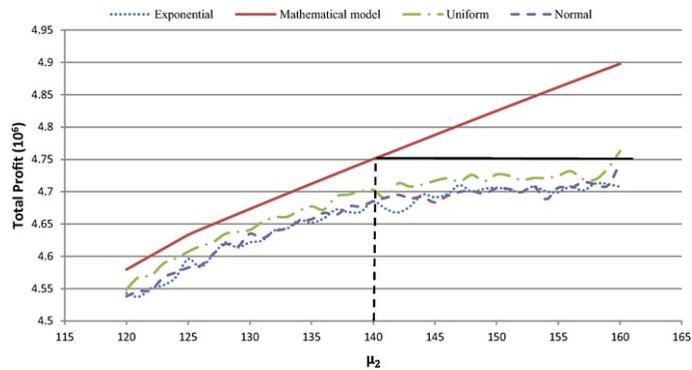


Fig. 4 Impact of the second station's remanufacturing rate, μ_2 , on the objective function for $\mu_1 = 70, \mu_3 = 90, \mu_4 = 100$, and $\lambda = 700$.

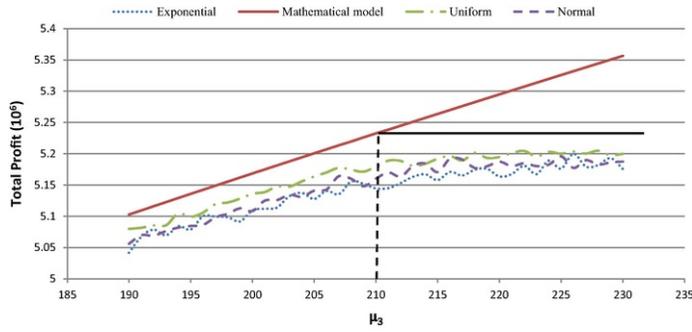


Fig. 5 Impact of the third station's remanufacturing rate, μ_3 , on the objective function $\mu_1 = 70, \mu_2 = 80, \mu_4 = 100$, and $\lambda = 700$.

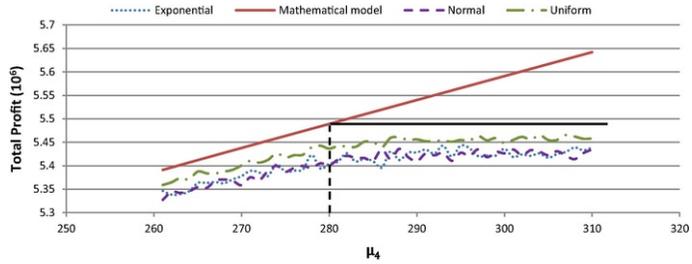


Fig. 6 Impact of the fourth station's remanufacturing rate, μ_4 , on the objective function $\mu_1 = 70, \mu_2 = 80, \mu_3 = 90, \lambda = 700$.

Thus, it can be concluded that if $\lambda p_i \leq \mu_i$ is the case for a remanufacturing firm, the mathematical model should be applied with a reasonable error and can provide insight to decision-makers. Therefore, it is concluded that it is also possible to apply this model where $\lambda p_i \leq \mu_i, \forall i \in I$.

In the next section, the sensitivity analysis is performed to investigate the effect of the model parameters.

4.2 Sensitivity analysis of the key parameters for HP printers' case

The numerical analyses above are made on the basis of remanufacture-all condition. Next step is to see the impact of various values of primary selling prices, remanufacturing rates, salvage values and deterioration rates on the disposition decision and total profit. To investigate the impact of the aforementioned parameters, the case study of HP printers is considered. The addressed case study is presented in [Guide et al. \(2008\)](#). We make assumptions about the required information which is not presented in [Guide et al. \(2008\)](#). For simplicity, we only consider one grade for the returns. The primary selling price of a remanufactured printer is \$8000. We assume that only one shift out of 3 working shifts is assigned to remanufacturing. Considering the estimation of 10 days to remanufacture a truckload of printers (250 units) and assuming that the company is operating 3 shifts a day; one printer on average takes 0.96 of an hour to be remanufactured. We also assume that the inspection process takes one third of the remanufacturing process. The deterioration rate for a printer is the total of 5% yearly discount rate and 2% weekly decay rate. Since the time unit in our example is hours, the hourly deterioration rate is to be considered in the model. Moreover, we consider inventory cost and remanufacturing cost implicitly in the deterioration rate. Therefore, we consider $\beta = 0.0006875$ as the minimum hourly deterioration rate by which the value gained by remanufacturing a printer deteriorates. Salvage value of a used printer is assumed to be half of the primary selling price. In order to assess the impact of the addressed parameters on the optimal disposition decision and total profit, the parameters' values are modified in the range of 50% above and below of the addressed values. As a summary, [Table 3](#) below contains the parameters considered in the numerical experiments.

First, the impact of deterioration rate is investigated. For only one truckload of printers to remanufacture, we conduct numerical experiments and we realize that the deterioration rate has no impact on the optimal quantity to remanufacture. However, assuming that 2 full trucks deliver the used printers to the firm, the deterioration rate does affect the total profit and the optimal remanufacturing quantity, as illustrated in [Fig. 7](#).

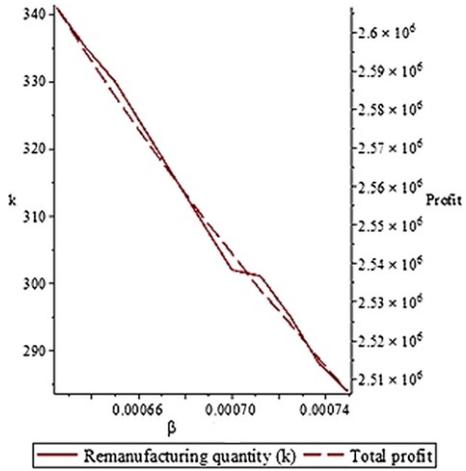


Fig. 7 Impact of the deterioration rate, β , on the optimal remanufacturing quantity and total profit for 2 truckloads of printers.

As expected, increasing the deterioration rate reduces profitability of the remanufacturing process. Furthermore, when the deterioration rate increases, fewer returns should be admitted so that the waiting time for the products decreases. Deterioration rate consists of holding cost rate, interest rate and depreciation rate of used products. Thus, an increase in any of the addressed rates results in less remanufacturing and more salvaging, which reduces the total profit.

In Fig. 8, the primary selling price, r_i is graphed against the optimal remanufacturing quantity and the optimal total profit.

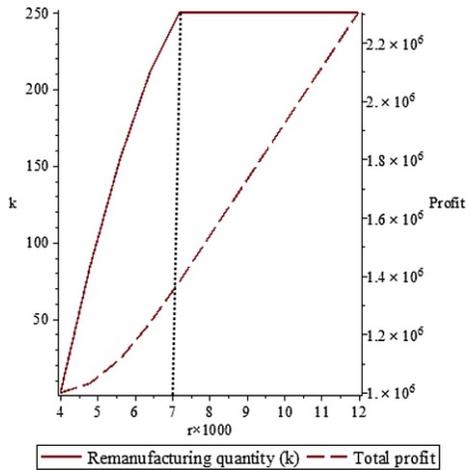


Fig. 8 Impact of the primary selling price, r_i , on the optimal remanufacturing quantity and total profit for one truckload or $k = 250$ printers.

According to Fig. 8, increasing the selling price decreases the chance that the salvage value surpasses the deteriorated profit for a unit of product. Thus, unless the selling price is below \$7000, all of the returns should be remanufactured. The point representing $r = \$7000$ is illustrated in Fig. 8 by the dotted line. At $r = \$4000$ where the selling price is equal to the salvage value, apparently there would be no incentive to remanufacture a product. Both the remanufacturing quantity and total profit increase by increasing the selling price. Next, we aim at investigating the impact of Salvage value on the remanufacturing quantity and the total profit.

Contrary to the selling price, increasing the salvage value decreases the chance of remanufacturing profitability (Fig. 9). The selling price in Fig. 9 is \$8000. If the salvage value increases, the incentive to remanufacture obviously decreases. Below \$4500 for the salvage value, which is illustrated with a black dotted line on the graph, no product would be sold at its salvage value. From \$4500 to \$6000, however, the remanufacturing quantity drops from

250 to 130. The salvage value dramatically increases the total profit after \$4500, implying that how effective is the salvage increase on the company's profit. This means that there exists a salvage value threshold below which all of the returns would be remanufactured. As there is no closed form for the objective function, these threshold values are to be obtained numerically.

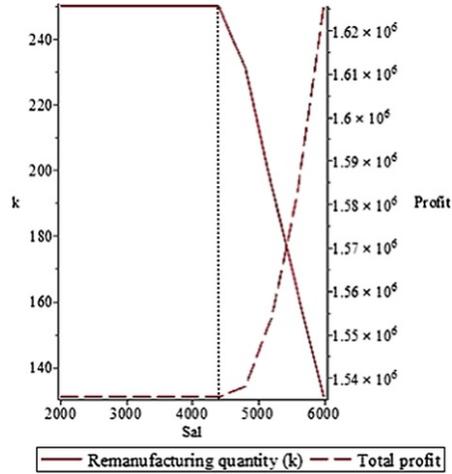


Fig. 9 Impact of the salvage value, Sal , on the optimal remanufacturing quantity and the total profit for one truckload of $k = 250$ printers.

Fig. 10 illustrates the impact of remanufacturing rate, μ , on the company profitability. For a remanufacturing station which remanufactures only one printer in two hours (i.e. $\mu = 0.5$), 44% of the returns have to be sold immediately to preserve their values. If the remanufacturing process speeds up to remanufacturing 4 printers in 5 h (i.e. $\mu = 0.8$), then all of the returns are kept to remanufacture (illustrated by the black dotted line in Fig. 10). Increasing the remanufacturing rate, μ , definitely increases the profit, as all of the returns would be remanufactured and sold in a relatively shorter time, comparing with the salvage value.

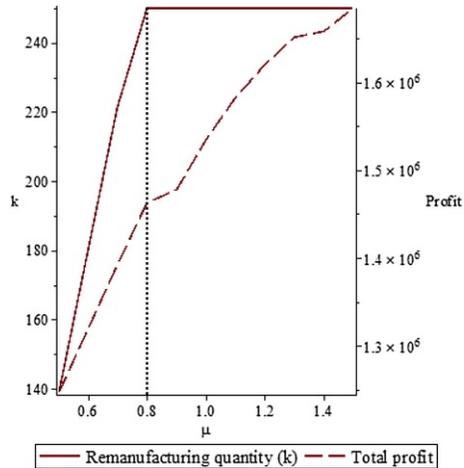


Fig. 10 Impact of the process rate (number of printers to remanufacture per t) on the optimal remanufacturing quantity and total profit.

The factor analysis performed in this section indicates how significant could be the product value assessment on the remanufacturing decision and consequently on the company profitability. The addressed analyses also show the drastic impact of remanufacturing process acceleration on the total profit.

4.3 Additional numerical analysis

Several parameters are involved in the objective function. However, the impact of some of them is known with certainty and is demonstrated by the graphs for HP printer case. We admit the returns as long as the deteriorated

value is higher than the salvage value. Increasing the deterioration rate β yields in lower profitability of remanufacturing process and consequently decreasing the number of admitted returns. The same holds for the salvage value. On the contrary, when the primary selling price is high, more products would be admitted as the difference between the deteriorated value and the salvage value increases. Increasing the arrival rate also impacts the admission decision only if all of the returns are admitted. The production time has the same effect as the remanufacturing rate; it defines the production capacity of the system. However, the interaction between the aforementioned parameters may be of concern. Therefore, we limit our numerical analysis to a set of scenarios defined upon changes in the proportion of selling price to the remanufacturing rate. Then, we investigate the impact of the deterioration rate on the scenarios. Since the arrival rate for each category λp is constraining the production quantity, we assume the arrival rate large enough the way that all remanufacturing stations will never starve. A manufacturing facility with three manufacturing stations is considered. Five different scenarios are taken into account for the combination of primary selling prices and remanufacturing rates. The addressed scenarios are introduced in [Table 4](#) below:

The addressed scenarios sufficiently cover all plausible outcome we may be interested to study. We are interested to see if the interaction between the product value and remanufacturing rates would impact the optimum admission decision. In scenario 1, the category with the highest primary selling price also has the highest remanufacturing rate. In scenario 2, which could be considered as a benchmark for scenarios 3 and 5, there is no superiority for one category over the others. Scenario 3 represents the situation where the categories are only different in terms of the selling price. Inverse situation represented by scenario 1 is addressed in scenario 4. Finally, in scenario 5 the categories are only different in terms of the remanufacturing rate. Comparing the results of these scenarios reveals interesting facts about the optimum policies under different conditions. [Table 5](#) below includes the model parameter values in the numerical study.

For each scenario, the quantity and percentage of admitted returns are provided in [Tables 6-10](#).

Table 8 Admission decision for scenario 3.

Category	Optimum decision	$\beta = 0.0005$	$\beta = 0.001$	$\beta = 0.002$	$\beta = 0.003$
1	Quantity	33,333	33,333	17,739	11,998
	Percentage	100%	100%	53.2%	36.0%
2	Quantity	33,333	28,200	14,168	9600
	Percentage	100%	84.6%	42.5%	28.8%
3	Quantity	33,333	18,000	9000	6108
	Percentage	100%	54.0%	27.0%	18.3%

Table 9 Admission decision for scenario 4.

Category	Optimum decision	$\beta = 0.0005$	$\beta = 0.001$	$\beta = 0.002$	$\beta = 0.003$
1	Quantity	33,333	23,237	12,000	7882
	Percentage	100%	69.7%	36.0%	23.7%
2	Quantity	33,333	28,200	14,168	9600
	Percentage	100%	84.6%	42.5%	28.8%
3	Quantity	33,333	23,600	11,953	8000
	Percentage	100%	70.8%	35.8%	24.0%

Table 10 Admission decision for scenario 5.

Category	Optimum decision	$\beta = 0.0005$	$\beta = 0.001$	$\beta = 0.002$	$\beta = 0.003$
1	Quantity	33,333	33,333	17,739	11,998
1	Percentage	100%	100%	53.2%	36.0%
2	Quantity	33,333	28,200	14,168	9600
2	Percentage	100%	84.6%	42.5%	28.8%
3	Quantity	33,333	18,000	9000	6108
3	Percentage	100%	54.0%	27.0%	18.3%

Category	Optimum decision	$\beta = 0.0005$	$\beta = 0.001$	$\beta = 0.002$	$\beta = 0.003$
1	Quantity	33,333	18,799	9200	6262
	Percentage	100%	56.4%	27.6%	18.8%
2	Quantity	33,333	28,200	14,382	9600
	Percentage	100%	84.6%	43.1%	28.8%
3	Quantity	33,333	33,333	18,890	12,257
	Percentage	100%	100%	56.6%	36.8%

In the first scenario, the category with the highest primary selling price has the highest remanufacturing rate. Not surprisingly, increasing the deterioration rate yields in decreasing the admitted quantity. However, doubling the deterioration rate had more significant effect on the categories with lower selling prices and remanufacturing rates.

In scenario 2, all categories have the same selling price and remanufacturing rate. These results could be considered as benchmark for scenarios 3 and 5.

In scenario 3, the selling prices are different but the remanufacturing rates are the same for all categories. Comparing the results obtained by scenarios 2 and 3 validates the proposed model by showing that the higher the primary selling price and the remanufacturing rate are, the more of the returns should be admitted.

Scenario 4 has the reverse form of scenario 1. In this scenario, the category with lower primary selling price has the highest remanufacturing rate. Results show that in this situation, the quantity of admitted returns for the category with medium primary selling price and remanufacturing rate is more than the two other categories. Moreover, the admitted quantity for categories 1 and 3 is happened to be almost the same.

Finally, scenario 5 is comparable with scenarios 2 and 3 for similarity between the categories in terms of primary selling price or remanufacturing rate. As expected, category 3 with the highest remanufacturing rate experienced the lowest reduction in admission quantity.

The admitted proportions for all scenarios are illustrated in Figs. 11-15 presented below.

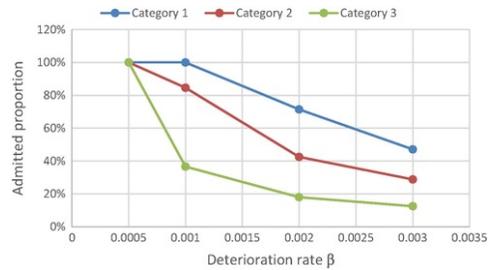


Fig. 11 Admitted proportions for scenario 1.

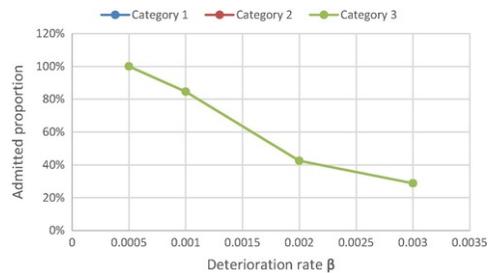


Fig. 12 Admitted proportions for scenario 2.

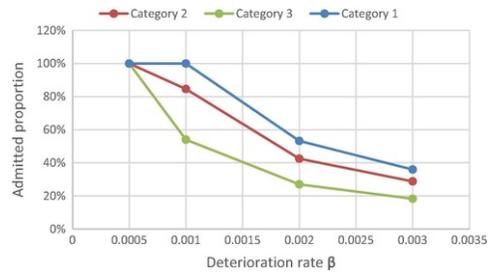


Fig. 13 Admitted proportions for scenario 3.

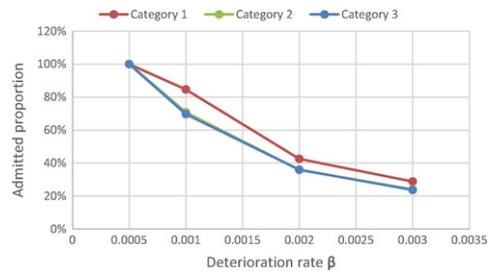


Fig. 14 Admitted proportions for scenario 4.

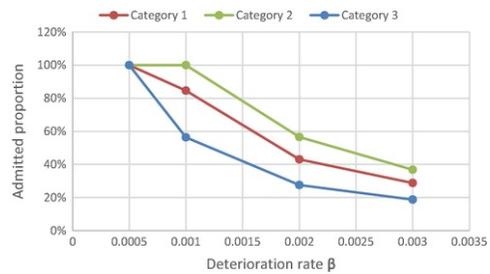


Fig. 15 Admitted proportions for scenario 5.

In Fig. 12, since all of the categories have the same remanufacturing rates and selling price, they must have the same admission decision. This graph acts like a benchmark for the sake of comparison and also to verify the model.

The first reduction in the admission proportions is shown to be less significant than the second reduction, disregarding the remanufacturing rate and the selling price. Furthermore, the third reduction is observed to be less significant than the second one. Therefore, as it can be observed in all of the graphs, the admitted proportions flat out by further increasing the deterioration rates. Thus, the admission proportions has an S-shape form.

5 Conclusion

One of the main decisions in a remanufacturing system is to choose how many units to remanufacture and how many units to salvage (disposing of the returns). In this paper, disposition decision for a remanufacturing system with multi-level quality grades of the returned products is studied. The received returns first undergo inspection and then the decision would be made to whether remanufacture the returns or sell them immediately at their salvage value. A mathematical model is presented to approximate the total profit.

Two key assumptions make this research study different from the current literature on remanufacturing systems. The first one is the assumption of receiving all of the returns at time zero. This assumption is commonly taken into account in scheduling problems with the value of the returns deteriorating exponentially over the delay time. Unlike most research studies, in this paper returns are received in batches but the

deteriorated value is calculated when each unit of product is completely remanufactured. The second assumption is to consider discrete periods of time for the remanufacturing process. A hybrid (re)manufacturing system that is involved in both manufacturing and remanufacturing cannot perform remanufacturing process continuously. In this case, a part of the production period should be assigned to remanufacturing while the rest is devoted to manufacturing new products or any secondary activity such as maintenance. Taking this assumption into account provides managers with the opportunity of investigating the impact of increasing/decreasing the operating period length for remanufacturing activities on the profitability of the company.

The proposed model is concave and also the decision variable is integer. Therefore, the global optimum could be found by comparing the marginal revenue and marginal cost of admitting one more unit. It is emphasized that since the objective function is not continuous and also the decision space is integer, the closed form of the optimum decision is not of interest.

From the numerical example, the results show that while increasing the salvage value, the remanufacturing rate and the primary selling price, increase the profit. However, the deterioration rate has exactly the opposite impact. Moreover, the numerical example showed that although the model is generated on the basis of deterministic remanufacturing times, it still helps the decision-making process by approximating the expected total profit in a system with stochastic arrival and remanufacturing times; the amount of error is relatively low. This capability of the model is confirmed with the aid of a simulation model. Therefore, decision makers may use this model even if the system under investigation is stochastic.

As expected, increasing the deterioration rate and salvage value results in admitting fewer returns. In the contrary, increasing remanufacturing rate and primary selling price increases the admitted quantity. The impact of deterioration rate is higher on the admitted quantity for the returns which are of less value or have been remanufactured with the lower remanufacturing rate. For the categories which are not the slowest to remanufacture and not of the least value, deterioration rate has less impact on the admitted quantity. Finally, we observed that the admission-deterioration rate graph is S-shaped, implying that there is a unique interval where each category undergoes faster decline in the admitted proportion.

In the presented model it is assumed that different classes of returns are remanufactured in the different stations. However, remanufacturing process for all of the classes of returns may be performed in the same station. This limitation of the present study needs to be addressed in a future research. Moreover, dispatching the returns to the station on the basis of remanufacturing costs and times shall be considered as an extension of this paper.

Appendix A

To obtain Eqs. (6)-(10), we have to solve the following series:

$$\sum_{k=0}^{k=n} ar^k = a \frac{1-r^{k+1}}{1-r} \quad (\text{A.1})$$

In Eq. (6) we have:

$$\sum_{n=1}^{n=S_i} r_i e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{n}{\mu_i}\right)} = \sum_{n=1}^{n=S_i} r_i e^{-\beta\left(\frac{1}{\lambda p_i}\right)} \left(e^{-\beta\left(\frac{n}{\mu_i}\right)} \right) = \sum_{n=1}^{n=S_i} r_i e^{-\beta\left(\frac{1}{\lambda p_i}\right)} \left(e^{-\left(\frac{\beta}{\mu_i}\right)} \right)^n \quad (\text{A.2})$$

where $r_i e^{-\beta\left(\frac{1}{\lambda p_i}\right)}$ is the fixed part (a in A.1) and $e^{-\beta\left(\frac{1}{\mu_i}\right)}$ is the terms whose power changes (r in A.1). Note that (A.1) starts from $k = 0$. Starting from $k = 1$ we have:

$$\sum_{k=1}^{k=n} ar^k = a \frac{1-r^{k+1}}{1-r} - a = a \left(\frac{1-r^{k+1}}{1-r} - 1 \right) = a \left(\frac{r-r^{k+1}}{1-r} \right) = ar \left(\frac{1-r^k}{1-r} \right) \quad (\text{A.3})$$

Using (A.3), the Eq. (A.2) would be:

$$\sum_{n=1}^{n=S_i} r_i e^{-\beta\left(\frac{1}{\lambda p_i}\right)} \left(e^{-\left(\frac{\beta}{\mu_i}\right)} \right)^n = r_i e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{1 - \left(e^{-\left(\frac{\beta}{\mu_i}\right)} \right)^{S_i}}{1 - \left(e^{-\left(\frac{\beta}{\mu_i}\right)} \right)} \right) = r_i e^{-\beta\left(\frac{1}{\lambda p_i} + \frac{1}{\mu_i}\right)} \left(\frac{1 - e^{-\frac{S_i \beta}{\mu_i}}}{1 - e^{-\frac{\beta}{\mu_i}}} \right) \quad (\text{A.4})$$

which is the same as (6).

In a similar way, for Eq. (7) we have:

$$\sum_{n=S_i+1}^{n=2S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}}+\frac{n}{\mu_i})} = \sum_{n=S_i+1}^{n=2S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}})} \left(e^{-\beta(\frac{n}{\mu_i})} \right) = \sum_{n=S_i+1}^{n=2S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}})} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^n \quad (\text{A.5})$$

where $r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}})}$ is the fixed part (a in A.1) and $e^{-\beta(\frac{n}{\mu_i})}$ is the terms whose power changes (r in A.1). However, we make the following transformation:

$$\sum_{n=S_i+1}^{n=2S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}})} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^n = \sum_{n=1}^{n=S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}})} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^{n+S_i} = \sum_{n=1}^{n=S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}})} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^{S_i} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^n \quad (\text{A.6})$$

This transformation let us to change the range of the series and also its fixed part (a in A.1). Therefore, now we have $r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}}+\frac{S_i}{\mu_i})}$ as the fixed part (a in A.1) and $e^{-\beta(\frac{n}{\mu_i})}$ is the terms whose power changes (r in A.1).

Using (A.3) to solve (A.6) we have:

$$\sum_{n=1}^{n=S_i} r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}}+\frac{S_i}{\mu_i})} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^n = r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}}+\frac{S_i}{\mu_i})} \left(e^{-\beta(\frac{1}{\mu_i})} \right) \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) = r_i e^{-\beta(T-U+\frac{1}{\lambda_{p_i}}+\frac{S_i+1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \quad (\text{A.7})$$

which is the same as Eq. (7).

If we go through the same calculations for the third period, we will obtain:

$$\sum_{n=1}^{n=S_i} r_i e^{-\beta(2(T-U)+\frac{1}{\lambda_{p_i}}+\frac{2S_i}{\mu_i})} \left(e^{-\beta(\frac{n}{\mu_i})} \right)^n = r_i e^{-\beta(2(T-U)+\frac{1}{\lambda_{p_i}}+\frac{2S_i}{\mu_i})} \left(e^{-\beta(\frac{1}{\mu_i})} \right) \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) = r_i e^{-\beta(2(T-U)+\frac{1}{\lambda_{p_i}}+\frac{2S_i+1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \quad (\text{A.8})$$

Looking at the trend in (A.4), (A.7) and (A.8), we conclude that for the Rth period we have:

$$r_i e^{-\beta((R-1)(T-U)+\frac{1}{\lambda_{p_i}}+\frac{(R-1)S_i+1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \quad (\text{A.9})$$

Therefore, the total deteriorated profit from the first period to the Rth period is:

$$\sum_{n=1}^{n=R} r_i e^{-\beta((n-1)(T-U)+\frac{1}{\lambda_{p_i}}+\frac{(n-1)S_i+1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \quad (\text{A.10})$$

Eq. (A.10) is transformed to:

$$\sum_{n=1}^{n=R} r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \left(e^{-\beta((n-1)(T-U)+\frac{(n-1)S_i}{\mu_i})} \right) = \sum_{n=1}^{n=R} r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \left(e^{-\beta((T-U)+\frac{S_i}{\mu_i})} \right)^{n-1} = \sum_{n=0}^{n=R-1} r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \left(e^{-\beta((T-U)+\frac{S_i}{\mu_i})} \right)^n \quad (\text{A.11})$$

where $r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right)$ is the fixed part and $\left(e^{-\beta((T-U)+\frac{S_i}{\mu_i})} \right)^n$ is the part whose power changes. Since the range of the series has changed, we cannot use (A.3). Using we have:

$$\sum_{n=0}^{n=R-1} r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \left(e^{-\beta((T-U)+\frac{S_i}{\mu_i})} \right)^n = r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \left(\frac{1 - e^{-\beta((T-U)+\frac{S_i}{\mu_i})} R^{-1}}{1 - e^{-\beta((T-U)+\frac{S_i}{\mu_i})}} \right) = r_i e^{-\beta(\frac{1}{\lambda_{p_i}}+\frac{1}{\mu_i})} \left(\frac{1 - e^{-\beta(\frac{S_i}{\mu_i})}}{1 - e^{-\beta(\frac{1}{\mu_i})}} \right) \left(\frac{1 - e^{-\beta(R-1)(T-U+\frac{S_i}{\mu_i})}}{1 - e^{-\beta(T-U+\frac{S_i}{\mu_i})}} \right) \quad (\text{A.12})$$

which is the same as (9).

Eq. (10) can be obtained in the same way as (6)-(8).

Acknowledgement: The authors would like to acknowledge the financial support received from the Natural Sciences and Engineering Council of Canada (NSERC) under the Discovery Grant Program. References

- Alidaee B. and Womer N.K., Scheduling with time dependent processing times: Review and extensions, *The Journal of the Operational Research Society* **50** (7), 1999, 711-720.
- Blackburn J.D., Guide V.D.R., Souza G.C. and Van Wassenhove L.N., Reverse supply chains for commercial returns, *California Management Review* **46** (2), 2004, 6-22.
- Denizel M., Ferguson M. and Souza G., Multiperiod remanufacturing planning with uncertain quality of inputs, *IEEE Transactions on Engineering Management* **57** (3), 2010, 394-404, <http://dx.doi.org/10.1109/TEM.2009.2024506>.
- Fathi M., Zandi F. and Jouini O., Modeling the merging capacity for two streams of product returns in remanufacturing systems, *Journal of Manufacturing Systems* 2014, <http://dx.doi.org/10.1016/j.jmsy.2014.08.006>.
- Ferguson M.E., Fleischmann M. and Souza G.C., A profit-maximizing approach to disposition decisions for product returns, *Decision Sciences* **42** (3), 2011, 773-798.
- Ferrer G. and Swaminathan J.M., Managing new and remanufactured products, *Management Science* **52** (1), 2006, 15-26.
- Galbreth M.R. and Blackburn J.D., Optimal acquisition quantities in remanufacturing with condition uncertainty, *Production and Operations Management* **19** (1), 2010, 61-69.
- Geyer R., Van Wassenhove L.N. and Atasu A., The economics of remanufacturing under limited component durability and finite product life cycles, *Management Science* **53** (1), 2007, 88-100.
- Guide V.D.R., Jr., Production planning and control for remanufacturing: Industry practice and research needs, *Journal of Operations Management* **18** (4), 2000, 467-483.
- Guide V.D.R., Gunes E.D., Souza G.C. and Wassenhove L.N., The optimal disposition decision for product returns, *Operations Management Research* **1** (1), 2008, 6-14, <http://dx.doi.org/10.1007/s12063-007-0001-8>.
- Harrison J.M., Dynamic scheduling of a multiclass queue: Discount optimality, *Operations Research* **23** (2), 1975, 270-282.
- Janiak A. and Krysiak T., Single processor scheduling with job values depending on their completion times, *Journal of Scheduling* **10** (2), 2007, 129-138, <http://dx.doi.org/10.1007/s10951-006-0004-6>.
- Karamouzian A., Naini S.G.J. and Mazdeh M.M., Management of returned products to a remanufacturing facility considering arrival uncertainty and priority processing, *International Journal of Operational Research* **20** (3) 2014, 331, <http://dx.doi.org/10.1504/IJOR.2014.062454>.
- Karamouzian A., Teimoury E. and Modarres M., A model for admission control of returned products in a remanufacturing facility using queuing theory, *The International Journal of Advanced Manufacturing Technology* **54** (1-4), 2011, 403-412, <http://dx.doi.org/10.1007/s00170-010-2933-7>.
- Lund R., Remanufacturing: An American resource, In: *Proceedings of the fifth international congress environmentally conscious design and manufacturing* **Vol. 16**, 1998, June.
- Naeem M.A., Dias D.J., Tiberwal R., Chang P.C. and Tiwari M.K., Production planning optimization for manufacturing and remanufacturing system in stochastic environment, *Journal of Intelligent Manufacturing* **24**, 2013, 717-728.
- Pazoki M. and Abdul-Kader W., Considering holding cost in optimal core acquisition and remanufacturing policy under deterministic process times, In: *Proceedings of the 2014 international conference on industrial engineering and operations management*, 2014, Bali, Indonesia, 2371-2379.
- Souza G.C., Ketzenberg M.E. and Guide V.D.R., Capacitated remanufacturing with service level constraints, *Production and Operations Management* **11** (2), 2002, 231-248.
- Su C. and Xu A., Buffer allocation for hybrid manufacturing/remanufacturing system considering quality grading, *International Journal of Production Research* **52** (5), 2014, 1269-1284, <http://dx.doi.org/10.1080/00207543.2013.828165>.
- Teunter R.H. and Flapper S.D.P., Optimal core acquisition and remanufacturing policies under uncertain core quality fractions, *European Journal of Operational Research* **210** (2), 2011, 241-248, <http://dx.doi.org/10.1016/j.ejor.2010.06.015>.
- Voutsinas T.G. and Pappis C.P., Scheduling jobs with values exponentially deteriorating over time, *International Journal of Production Economics* **79**, 2002, 163-169.
- Voutsinas T.G. and Pappis C.P., A branch and bound algorithm for single machine scheduling with deteriorating values of jobs, *Mathematical and Computer Modelling* **52** (1-2), 2010, 55-61, <http://dx.doi.org/10.1016/j.mcm.2009.12.024>.

Footnotes

Highlights

- Optimal disposition.
 - Quantity to remanufacture.
 - Quantity to salvage.
 - Deterioration value for returns and time value of the products.
-

Queries and Answers

Query: Your article is registered as a regular item and is being processed for inclusion in a regular issue of the journal. If this is NOT correct and your article belongs to a Special Issue/Collection please contact v.pandey.1@elsevier.com immediately prior to returning your corrections.

Answer: Yes; this article is registered as a regular item and is being processed for inclusion in a regular issue of the journal.

Query: The author names have been tagged as given names and surnames (surnames are highlighted in teal color). Please confirm if they have been identified correctly.

Answer: yes

Query: Please check the edits made in the sentence 'The total remanufacturing profit function...', and correct if necessary.

Answer: Yes