Damage Identification in Reinforced Concrete Beams using Digital Image Correlation

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Damage Identification in Reinforced Concrete Beams using Digital Image Correlation

by

Patricia C. Wilbur

A Thesis
Submitted to the Faculty of Graduate Studies through Civil and Environmental Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor

Windsor, Ontario, Canada

2011

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DECLARATION OF ORIGINALITY

I hereby certify that I am the sole author of this thesis and that no part of this thesis has been published or submitted for publication.

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ABSTRACT

The following thesis investigates the use of Digital Image Correlation (DIC) for the measurement of the vertical displacement profile of reinforced concrete beams. Furthermore, this study aims to utilize the DIC-measured displacement profiles of test specimens obtained at given load levels in an inverse analysis procedure for the identification of the flexural stiffness distribution and damage profile of that member.

The DIC measurement study consisted of four simply supported reinforced concrete beams subjected to four-point bending. The results of the study show a $10^{-2}$ $mm$ accuracy in comparison to potentiometer measurements. The enclosed inverse analysis procedure was validated using numerical simulations of steel I-beams subjected to four-point bending with induced stiffness reductions along their central span lengths, and then used for the identification of the stiffness of the central span length of the control specimen from the DIC-measurement study.
DEDICATION

I would like to dedicate this thesis to my family. To my parents, as well as to my brother and sister – thank you for always believing in me and encouraging me to follow my dreams. And to Ben – thank you for being there for me during the hardest months of this process. The completion of this thesis would not have been possible without any of you.
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NOTATION

\( P(x, y) \) - coordinate in reference digital image for which correlation analysis is being performed on
\( P'(x', y') \) – coordinate in deformed digital image that corresponds to reference coordinate
\( \alpha(x_i, y_i) \) – displacement mapping shape function
\( \beta(x_i, y_i) \) – displacement mapping shape function
\( l \) – DIC pixel subset dimension
\( C_{NCC} \) – normalized cross-correlation coefficient
\( f(x_i, y_i) \) – intensity value of pixel in reference image
\( \bar{f} \) – average intensity value of reference pixel subset
\( g'(x'_i, y'_i) \) – intensity value of pixel in deformed image
\( \bar{g} \) – average intensity value of deformed pixel subset
\( f'_c \) – measured concrete compressive strength
\( A_s \) – area of steel tension reinforcement
\( A'_s \) – area of steel compression reinforcement
\( f_y \) – yield strength of steel reinforcement
\( P_{cr} \) – calculated cracking load of beam
\( P_{ult} \) – ultimate load resistance
\( E_c \) – concrete elastic modulus
\( I \) – moment of inertia
\( P \) – applied load
\( v(x) \) – transverse deflection profile
\( \rho \) – reinforcement ratio
\( E1(x) \) – distributed flexural stiffness
\( q(x) \) – distributed lateral applied load
\( M(x) \) – bending moment distribution
\( \delta(x) \) - delta function
\( L \) – beam length
\( B(x) \) – flexural stiffness distribution
\( G(x,s) \) – Green function
\( d(x) \) – beam damage distribution
\( \chi(x) \) – beam curvature distribution
\( \alpha \) – Tikhonov regularization parameter
Chapter 1. INTRODUCTION

1.1. GENERAL

Due to its wide availability, its versatility and its adaptability to many different construction projects, concrete is one of the most commonly used building materials in existence today. It is durable and economical, and throughout its lifetime it will require substantially less maintenance than other building materials (Brzev et al., 2006). Inspection and maintenance is crucial for ensuring the safety of any structure, as the effects of environmental conditions and various loading scenarios can cause material degradation to occur. It is common that repairs will need to be performed throughout the life of a structure to fix any areas that are exhibiting signs of deterioration, in order for the structure to remain safe and to prevent existing damage from worsening (Saadatmanesh, 1997).

Much of the existing infrastructure in Canada and the United States was built many years ago, and was designed for loads much lower than those that are now being applied. Although a variety of inspection techniques do exist for damage detection in concrete structures, such as visual inspection and sounding, the demand caused by the aging of existing infrastructure leads to an inability to monitor every structure thoroughly and in a timely fashion (Chang et al., 2003). Additionally, the majority of current health monitoring techniques require contact with a structure in order for data to be obtained.

Recent infrastructure failures, such as the de la Concorde overpass in Quebec (Johnson et al., 2007) and the I-35W Bridge in Minnesota (NTSB, 2008), have highlighted the importance of structural inspections and increased awareness of the need for efficient,
objective damage detection techniques to avoid the occurrence of any future catastrophic failures. The development of a non-contact health monitoring technique that is efficient and requires little preparation to implement would greatly benefit the civil engineering world, as it would lead to the ability to monitor a greater number of structures each year, as well as allow for the early detection of major areas of existing material deterioration. This thesis aims to present a damage detection technique which utilizes the deflection profile of reinforced concrete beams measured by digital image correlation to reconstruct the flexural stiffness of the member and detect the level of damage existing in each member.

1.2. RESEARCH SIGNIFICANCE AND OBJECTIVE

In order to be classified as safe, a reinforced concrete structure must meet both strength and serviceability requirements (Brzev et al., 2006). Therefore, a structure must be designed to withstand its expected loadings, as well as to prevent the occurrence of excessive deflections and crack width openings, throughout its entire service life. Inspection techniques in use today are unable to assess strength and serviceability simultaneously, and so multiple inspection methods must be employed to fully assess the integrity of a structure.

Digital image correlation is a non-contact measurement technique that can be used to monitor a specimen’s deformation as it undergoes loading (Pan et al., 2009). Recent research on the use of static test data to characterize a material’s mechanical properties has proven successful, and there exists an opportunity to extend this for the assessment of reinforced concrete structures (Roux et al., 2008). In this way, displacement data gathered during the load testing of a structure can be used to verify adherence to both
strength and serviceability. This would greatly increase the efficiency of a structural inspection, and provide an opportunity to monitor structures on a more regular basis.

For horizontal flexural members subjected to in-plane loading, significant deformation is expected to occur only in the direction of the applied loading. The research study enclosed herein investigates the use of static test data gathered during the flexural load testing of reinforced concrete beams for the determination of the flexural stiffness distribution and the quantification of damage existing in each specimen. The specific objectives of this study are to:

- Utilize digital image correlation to measure the complete vertical displacement profile of each specimen at various load steps throughout testing
- Investigate a variety of damage scenarios to determine their effect on the static response of a specimen at a given load
- Develop an inverse analysis procedure that is capable of determining the flexural strength distribution of a structural member, as well as the corresponding damage level existing in the member

1.3. ORGANIZATION

In Chapter 2 of this thesis, a detailed literature review is presented that details the state of existing infrastructure in Canada and the United States, evaluates current health monitoring techniques, and reviews the capabilities of various methods of obtaining displacement measurements. The theoretical basis of image matching techniques and the development of the digital image correlation algorithm for the measurement of vertical displacement is presented in Chapter 3, followed by the details of the experimental
program and the results of the displacement measurement study that was undertaken. Chapter 4 will detail the inverse analysis procedure used for the determination of the distributed flexural stiffness profile and the level of existing damage in a flexural member, followed by numerical simulations for the validation of the developed model, and the application of the technique to selected test specimens from the experimental study. Major conclusions of this study and recommendations for future work will be provided in Chapter 5.
Chapter 2. REVIEW OF LITERATURE

In order to assess the relevance of the proposed research study, a variety of literature must first be reviewed. It is necessary to understand the current health of existing infrastructure to determine whether the development of a new structural damage detection technique is worthwhile. A review of existing inspection techniques must also be undertaken in order to assess their performance and determine the advantages and disadvantages of their implementation. Finally, various displacement measurement techniques must be investigated so that the benefits of using digital image correlation can be understood.

The following chapter is divided into three main sections: a review of the current condition of existing infrastructure will be given in Section 2.1, and existing damage detection techniques will be presented in Section 2.2. Finally, Section 2.3 will present a thorough review of the many existing object deformation measurement techniques, including both contact and non-contact data acquisition methods.

2.1. CURRENT HEALTH OF CANADIAN AND AMERICAN INFRASTRUCTURE

Much of the infrastructure existing in Canada and the United States today was built between the 1950s and 1970s, during a time of economic expansion. Since most civil infrastructure is designed for a 50 year service life, many existing structures are now reaching the end of their useful lives. With proper maintenance and rehabilitation, the service life of a structure can be extended; however, if damage exists in a structure and it is left unrepaired, the deterioration of the surrounding material accelerates and may lead to catastrophic failure. In the early 1980s the economy faced a recession, and all levels of
government faced major budgetary deficits. Consequently, spending on infrastructure maintenance was decreased until the national debt was reduced. Although the spending cuts were successful at relieving some of the accumulated deficit, the effects on the infrastructure network were less positive. Combining the advanced age of many structures with years of neglect and a lack of rehabilitation work, the state of infrastructure in North America has reached a breaking point (Group, T.B.F., 2004). The recent collapses of the de la Concorde overpass in Laval, Quebec in September 2006 (Johnson et al., 2007) and the I-35W Bridge in Minneapolis, Minnesota in August 2007 (NTSB, 2008), shown in Figures 2.1a and 2.1b, respectively, have brought to light the need for extensive infrastructure repairs and more appropriate inspection techniques in order to prevent the occurrence of future catastrophic failures.

In 2007, the Federation of Canadian Municipalities commissioned a study to investigate the health of Canada’s municipal infrastructure (Mirza, 2007). Surveys were sent to 166 municipalities in Canada with populations ranging from less than 10,000 to greater than 1,000,000. Responses were gathered from municipalities in every province and territory in order to accurately assess the cost of rehabilitating Canada’s infrastructure. With the information gathered from the surveys, it was determined that an investment of $123
billion dollars was needed in order to repair existing municipal infrastructure. The deficit was further broken down into five main categories of municipal infrastructure: water and wastewater, transportation, transit systems, waste management and community buildings. Shown in Figure 2.2 is the proportion of the total investment needed for each of these categories.

Figure 2-2 - Canadian Municipal Infrastructure Deficit in terms of the five main infrastructure categories

In 2009, the American Society for Civil Engineers released a report card which assessed the health of various sectors of America’s infrastructure (ASCE, 2009). Of the 15 categories in the study - including water distribution, transportation, energy and public facilities – not one area received a grade higher than a C, and the average grade for all existing infrastructure was a D. In total, it was estimated that a trillion investment was needed over five years in order to rehabilitate and maintain America’s existing infrastructure. The study revealed that approximately one-quarter of the bridges existing in America were either structurally deficient or functionally obsolete, meaning that these bridges are insufficient to handle existing traffic volumes and vehicle weights. An annual
investment of $17 billion is necessary to rehabilitate these bridges in order for them to be satisfactory for current loading conditions.

From the above studies, it is clear that North American infrastructure needs substantial investments in order to maintain and/or improve the quality of life of its citizens and ensure their safety. Water supply systems are failing to meet their necessary demands, and roads are unable to accommodate current traffic loads. The recent failures of bridges and overpasses have underlined the need for substantial investments to rehabilitate existing infrastructure in order to reduce the potential for catastrophic failure and loss of lives.

As a first step to rehabilitation work, inspections must be performed to locate areas in a structure needing repair. However, current inspection techniques are often inaccurate and subjective, resulting in unnecessary repairs or missing critical flaws. In order to maximize the effectiveness of any rehabilitation project, an inspection technique that is capable of accurately and efficiently locating damage in a structure must be developed so that proper repairs can be performed and future collapses can be avoided.

2.2. NON-DESTRUCTIVE EVALUATION OF CONCRETE STRUCTURES

A variety of different techniques exist for the health monitoring of structures which rely on a wide range of parameters for the identification of structural damage. Some techniques rely on the skill and the experience of test operators, while others monitor specific material properties to quantify the amount of damage existing in a structure. Other methods utilize the mechanical properties of a structure measured during load testing to quantify the extent of any existing damage. In the following sections, a review
of commonly used damage detection techniques for concrete structures will be given, including examples of their use and the advantages and disadvantages of each technique.

2.2.1. VISUAL INSPECTION

Visual inspection is the most commonly used damage evaluation technique for civil infrastructure, due to its straightforward application and ease of use. This technique consists of looking at a structure to assess the extent and severity of any existing visible damage. Concrete cracking, honeycombing, spalling and efflorescence are all types of damage that can be detected using visual inspection (Woodson, 2009). Examples of honeycombing and spalling are shown in Figures 2.3a and 2.3b, respectively.

![Figure 2-3 - Visual inspection is used to detect damage in reinforced concrete structures, including a) honeycombing (Thomas, 2008), and b) spalling (Martens and Francis, 2008)](image)

In order to assess the reliability of visual inspection for the detection of damage in bridges, a study was performed by the Federal Highway Administration for the U.S. Department of Transportation (Moore et al., 2001). To assess the structural integrity of bridges, inspectors are required to utilize the Condition Rating system and assign a rating from 0 to 9 to describe the extent of any detected deficiencies, with zero indicating a failed condition and 9 indicating excellent condition. In the study, 49 bridge inspectors were each asked to carry out 10 bridge inspections. The results showed a wide variation
in the quality of the inspections performed and the associated reports. It was shown that while each inspector spent roughly the same amount of time for each inspection on the different bridges, the time per inspection varied for each inspector, with individual inspections lasting from just a few minutes to several hours. Additionally, inspectors were hesitant to assign rankings at either extreme of the rating scale, specifically for elements exhibiting poor health, which led to a majority of severe flaws being classified as healthier than they were.

The consequences of poor visual inspection can be seen in the example of the de la Concorde overpass collapse in Laval, Quebec which occurred in September 2006. This collapse resulted in the death of five individuals and injuries to six others. A Commission of Inquiry was called to investigate the collapse and determine the causes of the event (Johnson et al., 2007). It was determined that a major factor contributing to the collapse of the overpass was a consistent lack of appropriate inspections throughout the entire life of the structure. Although visual inspections were conducted regularly, and severe deterioration was evident between 1992 and 2005, it was repeatedly decided that the bridge was in satisfactory condition and that no repairs were necessary. Shown in Figure 2.4 is an image of the observed deterioration of the east cantilever of the bridge deck at the time of a special inspection performed in 2004. Despite the presence of excessive efflorescence and shear cracks that are visible in this photograph, the inspector stated that the concrete was in good condition and that there was no need for further testing to assess the safety of the structure. Upon investigation of the inspections, the Commission determined that if the special inspection of 2004 had been performed properly, a hands-on inspection would have been executed following the visual
inspection, and the damage leading to the collapse would have been detected early enough to prevent catastrophic failure.

Although the occurrence of a failure of this calibre is rare, its event does bring to light the need for more accurate inspection techniques that have less reliance on the subjective interpretation of any existing deterioration, and instead focus on quantitative evaluation techniques.

2.2.2. **SOUNDING**

Sounding is performed by striking the surface of a structure with a small hammer or utilizing chain dragging to listen for variations in the audible response created upon impact. This test is useful for detecting delaminations of the concrete-reinforcement interface; a sharp “ping” indicates no delamination, while a hollow sound typically indicates that the bond between the concrete and reinforcement has deteriorated (Woodson, 2009). Although the successful application of sounding is reliant on the skill...
of the inspector, the interpretation of the survey is less subjective than with visual inspection due to the distinct difference in the audible response of the structure when delamination does or does not exist. An example of sounding is shown in Figure 2.5, with the implementation of a chain drag test to detect delaminations in a retail store floor.

![Chain dragging to detect delaminations](River Valley Testing, 2010)

In the same study assessing the reliability of visual inspection for highway bridges, an in-depth delamination survey was executed to investigate the performance of sounding for detecting bridge deck deterioration (Moore et al., 2001). Twenty-two teams were asked to perform this task and sketch a scaled drawing of their findings so that their results could be compared with the results of a detailed delamination survey performed by the Non-Destructive Evaluation Validation Center (NDEVC). The survey performed by the NDEVC determined that the bridge deck was 19% delaminated; only five of the 22 surveys performed during the study estimated the percentage of existing delaminations to fall within an acceptable error range of ±5 percentage points of the NDEVC survey. Shown in Figure 2.6 are three sketches of the delaminations of the surveyed bridge deck identified in the study, which illustrate the wide range of variability in the performed
inspections. The outlined white areas indicate delaminations detected in the NDEVC survey, while the grey areas illustrate regions of delamination detected by the inspection teams in the delamination study. The team to provide the first sketch estimated the lowest amount of delamination at 2%, the second was closest to the NDEVC survey with an estimation of 21%, and the third was the highest estimation at 35%. With each team being given the same tools and amount of time to complete the task, the large variability in the results of each survey show the inadequacy of sounding for damage detection in bridge decks.

Figure 2-6 - Results of three delamination surveys compared to reference NDEVC delamination survey (Moore et al., 2001)

Although sounding can be performed with relative ease and by using non-specialized equipment, the subjective interpretation of the test data leads to many discrepancies in the
final reports of the surveys. In the above example the inspectors were given only two hours to complete the survey of the entire bridge deck, while the completion of the NDEVc survey took two days. It is clear that the accuracy of this test method is highly time-dependent; however, with over 600,000 bridges existing in the United States, it would be uneconomical and impractical to perform a sounding survey on every bridge with the frequency necessary to ensure they remain structurally sound throughout their entire useful life.

2.2.3. ULTRASONICS

The use of ultrasonics for material characterization began in the 1940s. Since sound travels through a material at a specific velocity, by propagating sound waves through a material and recording the distance between the source and receiver, as well as the travel time of the wave, useful information on the properties of that material can be determined. Since ultrasonic waves cannot travel across an air-material interface, any imperfections inducing voids in the specimen will cause the path of the ultrasonic wave to differ and lead to a decrease in the measured wave velocity. Knowing the wave velocity of the material used for a structure, an estimation of the degree of deterioration existing in a structure can be achieved (Malhotra and Carino, 2006).

Illustrated in Figure 2.7 below are two common methods of operation: a) direct transmission, and b) indirect transmission. For the utilization of the direct transmission method, transmitting and receiving transducers are placed on opposite faces of the test specimen; for indirect transmission both transducers are placed on the same face of the specimen, and wave reflections are recorded by the receiving transducer. A comparison of the two techniques was performed in (Sutan and Meganathan, 2003), where it was
determined that direct transmission offers a more well-defined path length and the least signal attenuation of the two techniques; however, access to both sides of the test specimen is necessary, which is often not possible during field testing. Although data processing for indirect transmission is more time consuming than with direct transmission, the advantage of this technique is that access to only one surface of a specimen is sufficient for performing measurements. The indirect transmission method allows for the determination of the depth of test specimens and leads to a wider use for field testing.

There have been attempts to utilize ultrasonic test data for the determination of concrete strength, leading to the development of various empirical formulas relating measured wave velocities to concrete strength. However, a study investigating these computational models (Popovics, 2001) found that the calculated strength has an accuracy of ±20% in laboratory tests, with a much higher error in the field. The concluding reason for such high error was that factors such as concrete age, porosity, and aggregate size affect the strength and the wave velocity differently; thus, a clearer understanding of these effects must be gained before a more precise model for estimating concrete strength can be developed.

Many limitations exist with the application of ultrasonic testing to concrete structures. As mentioned earlier, sound waves cannot propagate through an air-concrete interface. Therefore, it is important to ensure that complete contact exists between the transducers and the concrete surface. However, due to the rough nature of concrete, a coupling agent such as glycerin must be used to ensure that accurate measurements are recorded. Additionally, the detection of defects is directionally dependent, and linear flaws oriented
parallel to the wave path may not be detected (NDT Resource Center, 2010). Due to the
difficulties in obtaining accurate measurements, ultrasonic testing is most often applied
for assessing the overall quality of concrete in an entire structure rather than for detecting
the presence of internal defects (Gilmour et al., 2002).

2.2.4. GROUND PENETRATING RADAR

Ground penetrating radar uses electromagnetic energy to detect damages in a structure by
propagating radio waves into a structure with a transmitting antenna and measuring the
time for the wave to be detected by a receiving antenna. Areas of the structure exhibiting
defects will cause the radio waves to be reflected back to the receiving antenna in a
shorter time than areas with no damage, and once the collected data is processed, the
location of flaws can be determined (Chang and Liu, 2003).

In (Hugenschmidt, 2002), the use of ground penetrating radar for the inspection of bridge
decks is presented with the results of numerous case studies. The technique was used to
measure the reinforcement cover depth of four bridges as well as the depth of asphalt
overlay on a concrete overpass undergoing rehabilitation work. The average
reinforcement cover depth for the first case study was determined without the need for
any corresponding destructive testing being performed; however, radar wave velocity is dependent on the material it is traveling through. Therefore, in order to determine the thickness of the asphalt overlay during the second case study, 40 core samples were necessary to calibrate the collected radar data.

There are many disadvantages when using ground penetrating radar, including the need for expensive equipment and expert interpretation of the collected data in order to produce meaningful results (Chong et al., 2003). Additionally, the necessary equipment for data collection is often large and bulky, leading to lane closures in order to perform any investigatory surveys (Hugenschmidt, 2002).

2.2.5. RADIOPHraphy

Radiography for structural health monitoring offers the ability to capture images of the complete internal structure of a specimen. As radiation travels through an object, it will be absorbed by the material it is passing through. Any anomalies in the test specimen will affect the absorption of the radiation and be visible on the recorded radiographic image. Using this principle, it is possible to apply radiography to locate internal defects in a specimen, and assess the amount of deterioration that has occurred (Chang and Liu, 2003).

The setup of a typical test is illustrated below in Figure 2.8. A radiation source is placed on one side of the test specimen, while the film is placed on the opposing side. Any flaws or internal features, such as reinforcement or electrical conduits, will affect the radiation absorption of the specimen and be detectable in the recorded images. The recorded images can then be interpreted to determine the depth of reinforcement or assess
the degree of deterioration of the test specimen. Radiographic testing can be performed prior to coring to prevent any accidental damage to reinforcement and electrical wires. More advanced calculations can be performed to determine the reinforcement depth and the size of the reinforcement if needed (Forbis, 2001).

Radiography was used to assess the condition of the prestressing tendons of the Tsing Yi bridge in Hong Kong during an extensive inspection performed to determine the amount of deterioration existing in the bridge deck (Beard and Tung, 1987). Due to excessive traffic loading and modifications to the existing structure, the bridge exhibited excessive deflection, and a variety of inspection techniques were utilized to determine methods of rehabilitation that could be implemented to increase the bridge’s lifetime. The radiographic study determined that approximately 10% of the prestressing ducts in the bridge deck were not fully grouted, which may have been a major contributor to the excessive measured deflections.

In order to employ radiography, care must be taken to ensure no civilians are in the vicinity of the test, and security must be posted at the entrances of restricted areas to prevent unauthorized access. Although complete images of the internal components of a
concrete member can be obtained with radiography, the operators of the test must be specially trained to work with radiation sources and must wear protective clothing to remain safe during testing. Additionally, large amounts of power are necessary to generate enough radiation to penetrate the entire thickness of a test specimen, and the equipment for testing is typically large and difficult to transport. Depending on the scope of the test, exclusion zones of up to 1 km must be cleared of civilians and livestock to protect public safety (McCann and Forde, 2001). For these reasons, the use of radiography for structural health monitoring is limited.

2.2.6. Damage Identification Using Dynamic Structural Response

The dynamic response of a structure is dependent on its mass, damping and stiffness properties. Therefore, any damage that causes changes to these properties should be reflected in variations of the modal properties of the structure. Typically, dynamic analysis focuses on the measurement of the natural frequencies or mode shapes of a structure, which are then compared with the expected response of the same structure in its healthy condition. Finite element models of the structure are developed, and the model parameters are updated until an agreement between the measured and calculated responses is reached (Friswell, 2007).

(Maeck et al., 2000) developed a model to detect damage in reinforced concrete beams subjected to varying degrees of loading. Two 6.0 m long reinforced concrete beams were subjected to static load testing, and vibrated by hammer excitation following each load step. The vibration response was recorded by accelerometers placed at 20 cm intervals along the length of the beams. A finite element model of each beam was developed, and a damage function, shown in Figure 2.9, was developed in which the bending stiffness of
each element could be updated to reflect the amount of damage existing in each element. The damage function was dependent on three parameters: the first to represent the length of the damage zone, $\beta$; the second to represent the severity of the damage, $\alpha$; and the third to represent the variation of the elastic modulus from the center of the damage zone to the end of the damage zone, $n$. This model assumes the damage to be symmetric about the centerline of the beam.

![Damage function to describe the extent of existing damage in a concrete beam (Maeck et al., 2000)](image)

Frequency measurements are relatively easy to perform, thus the use of frequencies in damage detection algorithms is quite common. However, in some cases, significant amounts of damage are needed to effect small changes in the frequencies of a structure, especially if the damage occurs at a location of low stress (Salawu, 1997). Therefore, damage must occur at areas of high stress, as damage located close to the modal nodes will have minimal effect on the natural frequencies of a structure. Additionally, it has been shown by (Cornwell et al., 1999) that temperature variations can induce changes in
A structure’s frequency of up to 5% over a 24 hour period; therefore, unless the damage occurring in a structure is severe enough to effect frequency changes greater than those caused by environmental factors, the presence of damage may go undetected. For these reasons, methods of detecting structural damage through the use of mode shape data have also been investigated.

A model was developed by (Lauwagie et al., 2002) to detect damage in a cement beam using measured mode shapes of the beam subjected to dynamic excitation. A scanning laser Doppler vibrometer was used to measure the vibration response of the beam, and the collected data was used in an FEM-updating routine to identify the stiffness of the beam in both the damaged and undamaged state. The collected mode shape data was very noisy; thus, it could not be used in its raw state to calculate the modal curvature values. A tenth order Lagrange polynomial was utilized to smooth the mode shape data in order to obtain useful modal curvatures. Calculations made using the direct stiffness method compared well with the results obtained through inverse analysis; however, the amount of data smoothing necessary to obtain useable data could also have eliminated any changes in the mode shape that could be indicative of damage.

Both of the above reviewed models, and indeed, most attempts to quantify damage in structures using dynamic test data, do not account for the variations in mass and damping that also occur with damage, and aim to quantify existing damage based on a decrease in the stiffness of a structural element. It is also very difficult to include the effects of temperature and other environmental factors into a damage identification model, as the effects of these factors on the dynamic response of a structure are not known with
certainty. For these reasons, it is difficult to determine whether variations in the dynamic response of a structure are caused by damage or by other, unaccounted for, phenomena.

2.2.7. DAMAGE IDENTIFICATION USING STATIC STRUCTURAL RESPONSE

The static response of a structure is much easier to measure than its dynamic response, and for this reason, investigation into the use of static test data to monitor the health of a structure has gained interest in recent years. Although the effects of damage may be concealed due to the limitation of load paths and less data is gathered through static testing than with dynamic testing, the static equilibrium of a structure is only dependent on its stiffness. Thus, the static response of a structure is less susceptible to variations caused by environmental changes than its dynamic response. Additionally, it is much easier to obtain precise measurements of the static response of a structure than to measure frequencies and mode shapes.

In a study by (Choi et al., 2004), an elastic damage load theorem was developed in which it was stated that the shape of the displacement variation in a beam due to existing damage would be equal to the influence line of the moment on a conjugate beam where the damage occurs. A numerical study was performed to assess the viability of the proposed model, followed by an experimental study investigating the accuracy of the model at detecting several different damage scenarios. For the numerical analysis, a 9.9 m long steel I-beam was discretized into eleven elements and damage was induced to the fifth element, as shown in Figure 2.10. A load of 475 N was applied successively at multiple points along the span of the beam, and the displacement was measured by displacement transducers. Comparison of the deflections of the damaged beams to the deflection of a healthy beam showed that the largest variation of the deflection occurred
at the location of the damage site, regardless of the load location, and as the amount of damage increased, the variation from the deflection of the healthy beam also increased.

(Bakhtiar-Nejad et al., 2005) developed a model that utilized static test data to locate and quantify any damage existing in a structure by comparing the response of the structure in its undamaged and damaged state using an iterative optimization procedure. A numerical analysis was performed on a two-dimensional truss, in which one to four of the members were simulated to be damaged. Using the change in the measured static displacement that occurs as a result of the damaged truss members, the stiffness parameters of each truss member were updated until the error between the measured and calculated responses was minimized. It was shown that in all cases the model correctly identified the damaged member and also accurately assessed the extent of damage existing in each damaged member. Experimental validation of the model was performed through testing of a steel frame structure, where damage was introduced into one member of the structure, reducing the stiffness of the frame at that section by 62%. The model correctly identified the damaged member, and successfully quantified the extent of the existing damage.
A model was developed by Li and Ghrib (2010) to fully reconstruct the stiffness profile of a statically determinate beam by using the static displacement profile of the beam as input into a finite element model updating procedure. Using only the vertical displacement as input into the model, it was possible to identify the full stiffness distribution along the entire length of the beam. The model was tested through numerical simulations of a simply supported beam subjected to a concentrated load at the mid-span. The model was able to accurately predict the stiffness profile of two beams: one with a constant stiffness along the length of the beam, and one with a parabolic stiffness distribution. An experimental study was then performed on an aluminum cantilever beam for model validation. Damage was simulated by introducing a stiffness reduction of 17% to a section 20 cm in length located near the center of the beam. The deflection profile was obtained using digital image correlation and used as input for the developed model. Results of the experiment show that the developed model was successful at identifying the damaged section of the beam to within ±5% of the actual stiffness reduction.

2.2.8. SUMMARY OF NONDESTRUCTIVE EVALUATION TECHNIQUES

The preceding sections outline many of the damage detection techniques currently used for health assessment of existing infrastructure, as well as their advantages and limitations. It can be seen that although sounding and visual inspection offer a low operating cost and simple application procedures, the subjectivity of these techniques leaves much opportunity for incorrect assessment of the extent of any existing damage. These techniques are beneficial for detecting severe damage in a structure, but are unable to detect flaws located deep within a structural member.
Techniques based on utilizing the physical properties of a structure to assess damage are useful for the health monitoring of structures made from homogeneous materials, such as steel, but the variability of the material properties of concrete make accurate assessment of damage difficult. Radiographic methods offer an advantage over ultrasonic testing due to the fact that images of the internal components of a structure can be obtained; however, the safety risks inherent in the application of radiographic techniques makes their use difficult in many situations.

The use of the mechanical properties of a structure for health monitoring provides the opportunity to remove dependence from the physical characteristics of a material and instead utilize the response of a structure to determine its strength and integrity. Although more information about a structure is obtained during dynamic testing than during static testing, it is very difficult to accurately measure the mode shapes and frequencies of a structure, especially when it is vibrating in higher modes. Static test data is much easier to obtain, and there exists a wide variety of displacement measurement techniques for its acquisition. Since attempts to quantify damage in homogeneous structural members using static test data have been successful, there exists an opportunity to expand this method of analysis to apply to reinforced concrete members as well.

2.3. **Displacement Measurement Techniques**

The accuracy of the mechanical properties of a test specimen identified through static analysis is directly dependent on the quality of the displacement measurements obtained during testing; therefore, it is important to review the capabilities of existing displacement measurement techniques in order to determine which is best suited to the research at hand. There are two basic categories of displacement measurement
techniques: those which require contact with a test specimen to obtain measurements, and those that do not. A review of common measurement techniques will be provided in the following sections.

2.3.1. DISPLACEMENT TRANSDUCERS

Linear displacement measurements can be obtained by using contacting sensors such as dial gauges, linear variable displacement transducers (LVDTs) and potentiometers. Since these devices have been around for many years, their use remains one of the most common methods for obtaining reliable displacement measurements. Their implementation is straightforward, and their operation is simple to understand; thus, these devices have been implemented in a variety of applications, including dimensional tolerancing (Henzold, 2006), quality control measurements (Singh, 2009), and manufacturing processes (Kazmer, 2009).

Shown below in Figure 2.11 is a typical setup of a displacement transducer. The housing of the device is mounted to a fixed location while the actuator is in contact with the surface of the test specimen. Deformation of the specimen causes the actuator of the measurement device to move relative to the housing, which results in a variation of the output of the device that is recorded by data acquisition software. By performing instrument calibration with gauge blocks prior to testing and properly configuring the software, the variations in the device output can be directly converted to displacements during testing, allowing for real-time monitoring. Measurements obtained using contact sensors typically have an accuracy of up to $10^{-4}$ m (Farago et al., 1994; Nyce, 2004).
The use of contact sensors in structural engineering applications is often limited to laboratory work, and for the validation of newer measurement techniques (Jáuregui et al., 2003; Park et al., 2007). In order to utilize these devices for field applications, direct access to the structure and a stable reference location to which the devices can be attached are both necessary; however, these requirements are often unmet, leading to the necessity that a temporary structure be erected prior to testing so that deformation measurements can be obtained. An example of this is shown in (Merkle and Myers, 2004), where an LVDT array was implemented for use in monitoring the deflections of a bridge subjected to load testing. Since the bridge spanned a river, temporary scaffolding was erected so that the LVDTs were able to contact the bridge deck. This process took several hours and made up a significant portion of the duration of the test. In addition to this, it was noted that a large number of LVDTs were necessary to monitor the displacement profile of the bridge deck, as displacement transducers are only able to obtain discrete measurements at a few select locations on the surface of a test specimen.

One of the greatest drawbacks of using displacement transducers is their ability to measure displacement in only one direction. Out-of-plane motion that occurs during
testing can significantly affect the quality of the recorded measurements. In (Ronnholm et al., 2009), erroneous measurements made by one of the three LVDTs used were explained by the presence of out-of-plane motion occurring during the test, which resulted in the detection of a larger deflection than what actually occurred.

2.3.2. TARGET-BASED MEASUREMENTS

Three instruments have been utilized for target-based measurement applications: total stations, digital cameras, and laser scanners. The differences of each technique become apparent during data processing; however, all three approaches begin with the setup of a target network on the surface of the test specimen. Shown below in Figure 2.12 is a typical target network. The chosen instrument is set up at a stable location from which the entire test specimen can be viewed, and targets are installed on the test specimen, as well as on stable reference locations surrounding the specimen. Since movement of the instrument will result in errors in the measured displacements, reference targets are included so that any movement of the instrument that occurs during testing can be detected and corrected.
(Merkle and Myers, 2004) utilized a total station to monitor the deflection of two in-situ bridges subjected to load testing for serviceability monitoring. The total station was used to measure the target location coordinates prior to loading, as well as for each load epoch, and the displacement of the bridge deck as a result of the loading was determined by calculating the difference in the target locations before and after loading. In order to assess the accuracy of the total station measurements, an LVDT monitoring system was also implemented to measure the deflection of the bridge deck. A comparison of forty sample points showed the variance between the LVDT and total station measurements to be 0.051 $mm$, with a maximum deflection of approximately 0.5 $mm$. The measurement discrepancy was explained by the setup of the LVDT system, as it was not possible to ensure a stable tripod base for every instrument used.

Photogrammetry utilizes digital images of a test specimen to determine object dimensions and displacements caused by loadings. (Whiteman et al., 2002) assessed the performance of photogrammetry for monitoring structural deformations in an extensive laboratory
study in which tested thirty-seven reinforced concrete beams were tested. Each beam was subjected to either shear or flexural loading until failure, and LVDTs and dial gauges were used for verification of the photogrammetric measurements. In a shear test performed on an inverted U-shaped reinforced concrete beam, the photogrammetric targets were located along the same vertical line as the LVDTs so that a direct comparison of the LVDT and photogrammetric measurements could be performed. Although the LVDTs had to be removed at a deflection of $5 \text{mm}$ to avoid damage to the instruments, it was found that the error of the photogrammetric measurements compared to the LVDT measurements was $\pm 0.1 \text{mm}$.

A study comparing target-based photogrammetric, laser scanner and total station measurements was performed by (Ronnholm et al., 2009), in which the three techniques were used to measure the displacement of a reinforced concrete beam subjected to four-point bending. Deformation data of the rectangular beam was collected by all three instruments, and the measured displacements were compared with readings from dial gauges installed at three locations along the length of the beam, as shown in Figure 2.13. At a load of $153 \text{kN}$, the displacement measured at the center of the beam by the dial gauge was $14.94 \text{mm}$, with the photogrammetric, laser scanner and total station measurements varying from this reading by $0.02 \text{mm}$, $0.02 \text{mm}$ and $0.26 \text{mm}$, respectively.
It can be seen that of the three techniques utilized in this study, measurements made by the total station were the least accurate when compared to the dial gauge readings. A disadvantage of using total stations is the fact that each target must be sighted individually, so while set-up time is greatly decreased in comparison to an LVDT system, the measurement time per load epoch increases with each measurement point included on the test specimen (Merkle and Myers, 2004). Photogrammetry offers fast data acquisition, as multiple measurements can be obtained from a single set of photographs (Jauregui et al., 2003). Although it was shown that measurements obtained from laser scanning compared well with dial gauge measurements, this was most likely due to the close proximity of the laser scanner to the tested beam. A field study assessing the performance of a laser scanner in (Lichti et al., 2000) found the point-wise measurement accuracy to be in the range of $\pm 30 - 50$ mm, which is far below the accuracy needed for structural monitoring applications. It can also be seen that the use of a target network increases the test set-up time, and while offering more dense measurement capabilities
than those offered with displacement transducers, target-based measurement techniques are still only capable of obtaining discrete measurements at specific points on the surface of a structure. With the advancement of computing capabilities and improvements made to data analysis techniques, it is possible to use the entire surface of a test specimen to obtain full-field measurements, as will be seen in the following section.

2.3.3. SURFACE-BASED MEASUREMENTS

Instead of installing targets on a structure, surface-based measurement techniques utilize the entire surface of a test specimen in order to monitor its full-field displacements when it is subjected to loading. Therefore, instead of obtaining discrete measurements at specific points of a specimen, the entire deformation profile of the object can be monitored, and any deviation from the expected response of the test specimen can be detected at a much earlier stage than when using other measurement techniques. Additionally, eliminating the need for the installation of a target network on a structure will greatly reduce the set up time during testing and allow for more attention to be paid to the acquisition of accurate measurements. Full-field displacement measurements can be obtained through laser scanning, microwave interferometry or digital imaging; the following three sections will review these techniques.

2.3.3.1. LASER SCANNING

Laser scanners obtain dense, three-dimensional point clouds fully describing the surface of a scanned object. These point clouds can be used to develop models describing the entire surface deformation of an object. In a study performed by (Gordon et al., 2004), researchers gathered laser scanner data during the load testing of a timber beam and
developed a polynomial displacement model through double integration of the elastic curve equation. Following this, the scan cloud was manually processed to extract the data describing the top face of the beam, and used to determine the coefficients of the displacement equations. Data was obtained by two laser scanners at each load epoch, one scanner with a single point precision of $\pm 6 \text{ mm}$ and the other with a single point precision of $\pm 25 \text{ mm}$. Due to the difference in the rate of data acquisition for each scanner, the higher precision scanner obtained one scan per load epoch, while the lower precision scanner obtained three. Data from the higher precision scanner was used to develop one model, while the three scans from the lower precision scanner were averaged together and used to develop a second model. Comparison measurements were taken by photogrammetry to assess the accuracy of the developed models. Using the data from the higher precision scanner, identified displacements fell within $\pm 0.29 \text{ mm}$ of the photogrammetric displacements, while the model developed using the data from the lower precision scanner identified displacements to within $\pm 3.6 \text{ mm}$.

In a similar experiment performed in (Park et al., 2007), a model was developed to describe the deformation of a steel I-beam subjected to loading. The collected scan clouds were first manually edited to extract the points falling on the line of intersection of the flange and the web of the I-beam at each load step, as shown in Figure 2.14 a) and b). Following the data extraction, the vector equations of the lines were determined by using least squares approximation. By subtracting the vector of the loaded beam from the vector of the unloaded beam, the vertical displacement profile of the beam was determined for each load step. Compared to measurements taken by LVDTS, differences in the measurements obtained from the displacement model fell below 1.6%.
From the above-mentioned studies, it can be seen that the accuracy of the displacements measured by laser scanning is dependent on both the quality of the scanner and the quality of the model being used. The data acquisition rate of the scanner being used is inversely proportional to the quality of the obtained measurements. Although the accuracy of lower quality scanners can be improved by using the average of multiple scans, this increases the necessary data acquisition time. In a comparative study of photogrammetry and laser scanning performed in (Lichti et al., 2002), it was shown that during loading of an in-situ bridge, time restraints limited data acquisition to two minutes per load epoch. While this was enough time for a sufficient amount of images to be obtained for the photogrammetric study, it was only possible to obtain a single scan per load epoch with the laser scanner. Additionally, though laser scanning offers the capability of obtaining full-field surface measurements of an object, the relatively recent emergence of this technology results in high equipment costs and the need for specially trained personnel to operate it.

2.3.3.2. MICROWAVE INTERFEROMETRY

Microwave interferometry uses radar waves to determine the amount of displacement occurring in a structure as it is subjected to loading. In practice, an interferometer is placed in a stable location at a sufficient distance from the object to be monitored so that
the entire structure can be seen by the instrument. Radar pulses are then sent towards the object and reflected back to the interferometer. By determining the phase shift between successive pulses, it is possible to determine the magnitude of any occurring displacements. An advantage over laser scanning is that the transmission and reception of the microwaves is very fast, and microwave interferometry can be used to monitor both the static and dynamic response of a structure. Other advantages include its ability to operate under any light and weather condition, and its measurement accuracy of 0.01 – 0.02 mm. Disadvantages of this technique include its high equipment cost and the need for trained personnel for data interpretation.

In (Bernardini et al., 2007), the accuracy of microwave interferometry was investigated by monitoring the ambient vibration of an in-situ bridge caused by normal traffic loading. Data was gathered for 50 minutes, and compared to measurements obtained using piezoelectric sensors located at the same measurement points as the interferometer measured. The velocities and mode shapes determined through the differentiation of the displacement data gathered by the interferometer show excellent agreement with the accelerometer data. While accelerometers can only obtain measurements at the points on the structure where they are located, the interferometer monitored the complete span length of the bridge with a measurement frequency of 100 Hz.

2.3.3.3. Digital Image Correlation

Digital image correlation (DIC) is a measurement technique which extends the principles of photogrammetry to obtain full-field surface displacement measurements of an object. Similar to photogrammetry, digital images of the object undergoing testing are captured at several load steps throughout the experiment. However, instead of tracking the motion
of discrete targets placed on an object, DIC utilizes a random speckle pattern on an object’s surface to evaluate the entire displacement field. Thus, if the surface of the object being monitored has a random grey speckle pattern, no surface preparation is necessary. Like in the case of photogrammetry, the use of a single camera oriented with its viewing direction perpendicular to the specimen’s surface is sufficient to obtain in-plane measurements (Pan et al., 2009), while two or more cameras are necessary for three-dimensional measurements (Sutton, 2008).

The use of DIC for material characterization was originally investigated in the beginning of the 1980s, as a method of obtaining the full-field strains and displacements of test specimens made of composite materials and/or subjected to difficult loading scenarios (Chu et al., 1985); however, the imaging equipment and computing capabilities at that time were major factors in hindering the advancement of the technique. The technological advancements of the past three decades have led to higher resolution cameras and much faster data processing; consequently, a renewed interest in the use of digital image correlation for experimental mechanics has recently been shown. Extensive research into the use of DIC has been performed on a variety of tests on composite materials, including a biaxial tension test on a composite sample composed of a vinylester matrix and reinforced with E-glass fibers (Claire et al., 2004). In this experiment, DIC was able to detect inconsistencies in the displacement field of the specimen, and detected the location of failure prior to critical loading. The measured displacement field was also used in a damage detection algorithm to assess the progression of damage existing in the specimen as it was subjected to loading.
A major benefit of DIC is its ability to adapt to a wide variety of test scenarios and specimen sizes. The accuracy of the measurements obtained is dependent on the resolution of the acquired images and the size of the test subject; thus, interest has been shown in the use of DIC for applications in civil engineering. In (Küntz et al., 2006), DIC was used to monitor the behaviour of a shear crack in a bridge girder of the Saint-Marcel Bridge in Quebec. Five cracks on the bridge were instrumented with potentiometers to monitor crack width openings, and it was found in an earlier study that one of the five instrumented cracks exhibited a larger crack width opening than the rest. Since the potentiometer was not installed perpendicular to the direction of the crack opening, it was decided to utilize DIC to assess the accuracy of the potentiometer measurements. The bridge was loaded with calibrated dump trucks placed at various locations on the bridge deck, and images were obtained for each load step. Each pixel in the gathered images corresponded to 0.19 mm in the object coordinate system, and using a sub-pixel algorithm it was possible to improve the measurement resolution to approximately 0.02 pixels. The calculated displacement fields indicated both horizontal and vertical motion of the crack, verifying that a single potentiometer was insufficient to fully monitor the crack width opening.

In (Yoneyama et al., 2007), DIC was used to monitor the deflection of a bridge as static load testing was performed. The bridge, shown in Figure 2.15, was loaded with a calibrated cargo truck placed in a variety of locations along the bridge deck in order to assess its serviceability. Verification of the DIC measurements was performed with displacement transducers placed at three locations along the length of the girders. Images of the bridge were obtained prior to loading as well as at each load step, and the
displacements were determined by subtracting the pixel coordinates of the beam under loading from the pixel coordinates of the beam while free of loading. The bridge girders were $14.8 \text{ m}$ in length, corresponding to a length of $5.9 \text{ mm}$ per pixel. The measurements obtained by digital image correlation were within $\pm 0.5 \text{ mm}$ of the measurements obtained by the displacement transducers, with a maximum deflection of approximately $2 \text{ mm}$.

The studies performed above show that DIC can be adapted to many different measurement applications for a large range of test specimen sizes. There are few requirements for specimen preparation, and implementation of the technique is straightforward. All of the examples presented here show a good agreement between the measurements obtained by DIC and the displacement transducers used for validation.

![Image of the bridge studied in (Yoneyama et al., 2007) from the location of image acquisition](image)

**Figure 2-15** – Image of the bridge studied in (Yoneyama et al., 2007) from the location of image acquisition

### 2.3.4. SUMMARY OF DISPLACEMENT MEASUREMENT TECHNIQUES

Measuring the deflection of a structural member can be performed with a variety of techniques, and it is important to determine which method will be most effective in meeting the requirements of the prescribed research. In order to assess the integrity of a
structural member, the chosen measurement technique should be capable of obtaining dense displacement measurements along the entire length of the member to increase the likelihood that damage will be detected. Measurements should be obtained in an efficient manner, causing little increase in testing time. It is desirable that processing of the obtained data remain simple.

Shown below in Table 2.1 is a summary of the major attributes of the above-reviewed measurement techniques. It can be seen that while contacting sensors are capable of obtaining high accuracy measurements, the necessity that these devices be fixed to a secure location in order to obtain reliable measurements greatly restricts their use in many field applications. For that reason, they are typically used in laboratory settings and for verification of alternative measurement techniques. Target-based approaches do offer many advantages over contact sensors in that measurements can be obtained from a remote location, but the need for targets affixed to the specimen surface greatly increases test set-up time. Additionally, since both laser scanning and digital imaging can be utilized for full-field measurement applications, it seems impractical to utilize these instruments for target-based measurement techniques. Laser scanning offers the most dense data acquisition of the three non-contact methods reviewed, but the cost of the necessary equipment is high due to the recent emergence of this technology and there is a need for highly trained operators to make use of the equipment. Additionally, in order to develop useful displacement models with laser scanning data, a large amount of data processing must be performed. Digital image correlation uses off-the-shelf equipment, and is straightforward to implement; thus, it has been used in a wide variety of recent
research projects. Test specimens require little to no surface preparation prior to testing, and the technique can be adapted to a large range of specimen sizes.

Table 2-2-1 - Summary of displacement measurement techniques

<table>
<thead>
<tr>
<th></th>
<th>Mechanical Devices (Farago and Curtis, 1994; Nyce, 2004)</th>
<th>Total Stations (Merkle et al., 2004)</th>
<th>Photogrammetry (Albert et al., 2002)</th>
<th>Laser Scanning (Tsakiri et al., 2006; Park et al., 2007)</th>
<th>Digital Image Correlation (Küntz et al., 2006; Yoneyama et al., 2007; Pan et al., 2009)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>&lt;0.1 mm</td>
<td>0.2-4mm</td>
<td>0.02 pixel</td>
<td>±2-±50mm</td>
<td>0.02 pixel</td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>&lt;1mm - &gt;1m</td>
<td>&gt;500m</td>
<td>&lt;1mm - &gt;500m</td>
<td>&gt;1m - 700m</td>
<td>10cm - &gt;15m</td>
</tr>
<tr>
<td>Instrumentation &amp; Setup Time</td>
<td>Slow</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Fast</td>
<td>Fast</td>
</tr>
<tr>
<td>Data Acquisition Rate</td>
<td>Fast</td>
<td>Slow</td>
<td>Fast</td>
<td>Moderate</td>
<td>Fast</td>
</tr>
<tr>
<td>Cost of Equipment</td>
<td>Moderate</td>
<td>High</td>
<td>Moderate</td>
<td>High</td>
<td>Moderate</td>
</tr>
<tr>
<td>Ease of Data Processing</td>
<td>Easy</td>
<td>Easy</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>Measurement Capability</td>
<td>1D</td>
<td>3D</td>
<td>2D/3D</td>
<td>3D</td>
<td>2D/3D</td>
</tr>
</tbody>
</table>
Chapter 3. DEFLECTION MEASUREMENT USING DIGITAL IMAGE CORRELATION

3.1. INTRODUCTION

As seen in the previous chapter, DIC is a versatile, non-contact displacement measurement technique that can be adapted for use in a wide variety of test scenarios. It can be used to obtain two-dimensional or three-dimensional measurements, the setup for image acquisition is simple, and the necessary specimen preparation is minimal. For horizontal flexural members subjected to in-plane loading, the dominant deformation will occur in the direction of the applied load. Thus, a modified DIC algorithm that calculates only the vertical displacement of a specimen can be developed to simplify the calculation process and decrease the necessary computational effort.

In Section 3.2 of this chapter, the structure of a digital image will be described, so that the basis of image matching techniques can be presented. Section 3.3 will explain how the information stored in an image is used to track an object’s deformation and will present the technique of 2D-DIC. The adaptation of 2D-DIC for vertical displacement measurements will then be presented in Section 3.4. Finally, Section 3.5 will present details of the experimental program performed to validate the DIC algorithm for vertical displacement measurement, as well as the results of the study.

3.2. STRUCTURE OF A DIGITAL IMAGE

Digital images are represented by rectangular arrays of individual picture elements, or pixels, which store information about the colour intensity of a specific point on the surface of an object. Under normal magnification, individual pixels are not visible to the
human eye; as shown in Figure 3.1a, the colour intensity variations existing between neighbouring pixels blend together and appear as continuous, gradual colour changes in an image. Under higher magnification, the discrete nature of the pixel array is revealed, as illustrated in Figure 3.1b.

The colour intensity information contained in each pixel of an image is most commonly stored as an 8-bit integer and can vary between zero and 255 (Reichmann, 2011). A value of zero indicates no colour intensity, while 255 indicates a maximum intensity. Images are typically captured in one of two colour spaces: the red, green, blue (RGB) colour space or the grey-scale colour space. In the RGB colour space, each pixel contains three layers of information, so that the intensity level of each of the three colours can be described. When displayed separately, each colour intensity matrix appears as a grey-scale image; combining the three results in a full-colour image, as shown in Figure 3.2.

Grey-scale images are commonly utilized for DIC, since only one layer of information describing the colour distribution of an object’s surface is needed (Maas and Hempel, 2006). There are two methods of obtaining grey-scale images; either through direct
acquisition by measuring the intensity of light reflecting off the surface of an object, or by scaling the three colour intensity arrays of an RGB image to obtain a single “averaged” intensity matrix (Johnson, 2006). The two alternatives are both suitable for the application of DIC, as long as sufficient variations in the intensity values of each pixel exist in the images.

![RGB Image](image1)

![Red Level Intensity](image2)

![Green Level Intensity](image3)

![Blue Level Intensity](image4)

**Figure 3-2 -** RGB image displayed with all three colour matrices, and separated into each respective colour matrix (Courtesy of: Mary McNamara)

3.3. **TWO-DIMENSIONAL DIGITAL IMAGE CORRELATION**

Since each pixel in an image contains information about the colour intensity of a specific point on an object’s surface, then the deformation of the object as it is loaded can be determined by collecting a series of images throughout testing and matching the colour
intensity values of pixels in two successive images. The process used for obtaining measurements with DIC is shown in Figure 3.3 (Jáuregui et al., 2003).

The first step consists of the specimen preparation and experimental setup. As was mentioned in the previous chapter, the surface of the test specimen must exhibit a random speckle pattern; therefore, if the specimen does not naturally exhibit this pattern, one must be applied to its surface before measurements can be obtained. This can be accomplished by spraying the surface with white and black spray paint until a sufficient colour variation exists.

Once the specimen preparation and experimental setup are complete, testing can begin. Images of the specimen are obtained prior to loading, and at every load for which displacement measurements are desired. Although only one image per load step is necessary, it is preferable to take multiple images to ensure adequate data acquisition at each load step.

Processing of the images begins by defining the region of interest (ROI) in the reference image for which deformation measurements are necessary, as shown in Figure 3.4a.
Since the intensity of each pixel in an image can only vary from 0 to 255, while there are typically millions of pixels in an image, it is likely that there will be many pixels in an image that will have the same intensity value. Therefore, the ROI is divided into a grid of square subsets of pixels of size \((l \times l)\) – as shown in Figure 3.4b – that will be used for the correlation analysis (Hild and Roux, 2006).

![Figure 3-4 - (a) selection of ROI in reference image, and (b) division of ROI into pixel subsets](image)

The displacement of the test specimen at each load step is determined by locating each subset from the reference image in an image of the deformed test specimen through the use of a correlation criterion (Pan et al., 2009). Typically, a cross-correlation (CC) or sum-of-squared differences (SSD) coefficient is calculated for each subset in the deformed image that the reference subset is compared to, and displacements are then determined by finding the maximum of the correlation coefficient matrix and calculating the shift in the location of the subset between the two images. These coefficients assess
the similarity between the two pixels being compared. Illustrated in Figure 3.5a is an example of a pixel subset in its reference position, and Figure 3.5b illustrates the same pixel subset after deformation has occurred.

It is clear that at this stage of image processing the displacement vector calculated for each subset does not fully characterize the occurring deformation of the test specimen. Although the subset shown in the deformed image is skewed, the displacement vector would remain the same regardless of the deformation of the entire subset, as shown below in Figure 3.6. This is because the correlation analysis does not account for the translations and rotations of the subset that occur under loading, and merely determines which subset in the deformed image contains a colour intensity distribution that most closely resembles the reference subset.
In order to obtain the full-field displacement profile of a test specimen, interpolation must be performed on the displacement vectors determined through the correlation analysis. There are a variety of sub-pixel interpolation techniques, including genetic algorithms, intensity interpolation and correlation coefficient curve fitting, which can be used for achieving different levels of accuracy (Bing, 2006).

Alternatively, a method based on finite element discretization was developed that is capable of obtaining a globally continuous displacement profile of a test specimen (Hild and Roux, 2006). Classical continuum mechanics states that neighbouring particles in a solid object at rest remain adjacent when the object undergoes deformation. Therefore, by knowing the displacement vectors of four neighbouring subsets, it is possible to determine the displacement of every pixel in the ROI. If the displacement vector of each subset is referenced to its top left corner, then after the completion of the correlation analysis the displacement of every vertex in the subset will be known. With this information, it will then be possible to map the coordinates of each point, \( P(x_i, y_j), \) in the subset to its corresponding deformed location, \( P'(x'_i, y'_j), \) using the displacement mapping functions shown in Equations (1a) and (1b).

\[
\begin{align*}
x'_i &= x_i + \alpha(x_i, y_j) \\
y'_j &= y_j + \beta(x_i, y_j)
\end{align*}
\]

(3.1)

(3.2)

Where \( \alpha(x_i, y_j) \) and \( \beta(x_i, y_j) \) are shape functions developed by interpolating between the four vertices of the subset. Any shape function order can be used for displacement interpolation.
mapping; however, to incorporate any occurring translations and rotations into the displacement field, it is recommended that at least a first-order shape function be used.

Finally, once the interpolation has been completed, it is necessary to convert the obtained pixel displacement values to the object coordinate system. By using scales marked on the test specimen, as shown in Figure 3.7, scale factors can be calculated and used for the conversion between the two coordinate systems.

Figure 3-7 - Scales marked on the surface of a test specimen, used to develop scale factors for the transformation of measurements from the image to object coordinate system

Most applications of 2D-DIC involve specimens that are subjected to multi-axial loading, where the horizontal and vertical displacement components of the test specimen are of similar magnitude. Additionally, most of the reviewed DIC measurement studies investigate test specimens having similar length and width dimensions. In the case of structural beams, the horizontal dimension of the test specimen is often many times greater than its vertical dimension, and significant displacement will only occur in the direction of the applied load. Therefore, the DIC technique described above can be
adapted to measure only vertical displacements, so that the computational effort needed can be reduced and the algorithm can be simplified.

3.4. DIC FOR VERTICAL DISPLACEMENT MEASUREMENT

If the location of the object in a series of images only moves in a single direction, then it is only necessary to search in the direction of the object’s motion when performing a correlation assessment of the pixel subsets. Since only vertical forces will be applied to the members tested in the enclosed study, it is reasonable to assume that displacement will only occur in the vertical direction. To simplify the computational process and increase the efficiency of the method, a modified DIC algorithm has been developed that detects vertical deflections only.

In 2D-DIC, an area surrounding the original location of a pixel subset is searched in both the horizontal and vertical direction in the deformed image to locate the subset with the highest correlation to the reference subset. If the test specimen is only moving vertically within a series of images, then locating the pixel subsets from the reference image in the deformed image should be possible by searching only in the vertical direction.

Similar to 2D-DIC, the ROI is first defined in the reference image of a test specimen, as shown previously in Figure 3.4a. A single row of pixel subsets of size $(l \times l)$ – in this case, $l = 50$ – is defined in the reference image, and the images of the deformed test specimen are then divided into vertical columns of width $l$, as shown in Figure 3.8. The similarity between the reference subset and each subset in the deformed image is assessed by the normalized cross-correlation coefficient, shown in Equation (3.2):
where $f(x,y)$ is the intensity value of the pixel located at point $(x,y)$ in the reference image, $\bar{f}$ is the average intensity of the reference subset, $g(x',y')$ is the intensity at point $(x',y')$ in the deformed image, and $\bar{g}$ is the average intensity of the subset in the deformed image (Lewis, 1995).

Once the correlation coefficient has been calculated for each subset in a given column, the displacement of the beam is determined by locating the maximum correlation coefficients and then calculating the movement of each subset between the reference and deformed image, as shown in Figure 3.9.

Since only vertical displacement is being measured, the interpolation process becomes somewhat easier than with 2D-DIC. Instead of using shape functions or other interpolation techniques, polynomial curve fitting can be used to interpolate between discrete pixel displacements. Numerical simulations have shown that higher-order polynomials have a higher potential for accuracy, but suffer from higher sensitivity to noise in the collected data. Therefore, in the following study, second and third order
polynomials are used for comparison and to determine which polynomial better describes the data. The Matlab code used for the measurement study is given in Appendix A.

![Diagram of pixel subset from reference image and its location in deformed image](image)

**Figure 3-9** - Pixel subset from reference image and its location in deformed image

### 3.5. Experimental Program for Validation of Vertical Displacement Measurement Using DIC

In order to assess the accuracy of the proposed DIC algorithm for vertical displacement measurements, a series of load tests were performed on four reinforced concrete beams in the Structural Laboratory at the University of Windsor. Each beam had dimensions of 2.4 m in length, 0.35 m in height and 0.25 m in width. The concrete mix for each beam was designed following CSA A23.1/A23.2, and consisted of 170 kg of coarse aggregate, 201 kg of fine aggregate, 128 kg of cement and 51 kg of water, with a target compressive strength of 25 MPa.

The experimental setup is shown in Figure 3.10. Each specimen was simply-supported – with the supports located 75 mm from each end of the beam – and subjected to four-point bending, with each point load being applied to the top face of the beam 400 mm to each side of the centerline. To minimize the variation between the two applied loads, a
loading beam was placed on top of the beam, and the total load was applied by a single hydraulic jack centered on the loading beam. Linear potentiometers were placed at quarter-points of the span length to obtain comparison displacement measurements throughout the duration of each test.

![Figure 3-10](image_url)

Figure 3-10 – Elevation view of experimental setup for each specimen

An image of the completed test setup is shown below in Figure 3.11. The camera used for image acquisition was placed approximately 2 m from the beam and oriented with its viewing direction perpendicular to the face of the beam. During testing, three images per load step were acquired in order to ensure adequate data capture.
With the exception of the third beam tested, each beam had the same general reinforcement design, as shown in Figure 3.12. The first two beams were reinforced with 3-15M bars in tension, 2-10M bars in compression, and 10M stirrups placed at 150 mm o.c. The fourth beam was designed with the same reinforcement cage, except with 3-20M bars used for tension reinforcement instead of 15M bars. The third beam was reinforced only in the tensile zone with 3-20M bars. Each of the four test specimens were given a three-character identifier in order to better organize the test data; these identifiers and their explanations are presented in Table 3.1. A design summary of each of the beams is also given in Table 3.2; associated calculations are provided in Appendix B.
Table 3-1 – Test Matrix for experimental study

<table>
<thead>
<tr>
<th>Beam Identifier</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1C</td>
<td>Control Beam</td>
</tr>
<tr>
<td>B2P</td>
<td>Beam with painted surface</td>
</tr>
<tr>
<td>B3T</td>
<td>Beam with only tension reinforcement</td>
</tr>
<tr>
<td>B4D</td>
<td>Beam with induced damage on tension reinforcement</td>
</tr>
<tr>
<td>B5R</td>
<td>Beam with patch repair at midspan</td>
</tr>
</tbody>
</table>

Table 3-2 – Summary of design parameters for each beam tested during the experimental study

<table>
<thead>
<tr>
<th>Beam Identifier</th>
<th>$f_{c,measured}^c (MPa)$</th>
<th>$A_s (mm)$</th>
<th>$A_e (mm)$</th>
<th>$f_y (MPa)$</th>
<th>$P_r (kN)$</th>
<th>$P_{ult} (kN)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1C</td>
<td>58.6</td>
<td>600</td>
<td>200</td>
<td>400</td>
<td>40</td>
<td>190</td>
</tr>
<tr>
<td>B2P</td>
<td>51.4</td>
<td>600</td>
<td>200</td>
<td>400</td>
<td>55</td>
<td>220</td>
</tr>
<tr>
<td>B3T</td>
<td>44.4</td>
<td>900</td>
<td>---</td>
<td>400</td>
<td>40</td>
<td>120</td>
</tr>
<tr>
<td>B4D</td>
<td>49.2</td>
<td>900</td>
<td>200</td>
<td>400</td>
<td>26</td>
<td>50</td>
</tr>
</tbody>
</table>

The measured values of the concrete compressive strength greatly exceed the design compressive strength of the concrete, and as shown in Figure 3.13, the use of these strengths for the calculation of the concrete elastic modulus, $E_c$, results in deflection estimations that are approximately half that of the measured deflections. Since the equations for the determination of $E_c$ are empirically derived, and their accuracy is dependent on the type of aggregates used as well as the value of $f_{c,measured}^c$, it is reasonable to
assume that there will be some discrepancy between the calculated and actual value of $E_c$ (Noguchi et al., 2009).

Calculations of beam deflections are also dependent on the moment of inertia, $I$, of the beam cross-section. It has been shown that the Branson formula for the effective moment of inertia of a cracked reinforced concrete member will underestimate deflections of lightly reinforced concrete beams (having $\rho < 1.0\%$), and for beams with a gross-to-cracked $I$ ratio greater than 4.0 for loads higher than the cracking load (Bischoff et al., 2007).

Although both of these factors can lead to differences between measured and calculated deflections, it is unrealistic to assume that they are responsible for a 100% difference between the deflection values in any experiment, especially due to the fact that such a high error exists even at very low load levels. It is therefore assumed that the error between the calculated and measured deflections is most likely due to the difference in the curing processes used for the cylinders and the beams. The cylinders were
submerged in water during the curing phase, while the beam was cured by applying water with a hose. For this reason, the measured values of $f'_c$ cannot be used for the determination of $E_c$. Using linear regression analysis to model the beginning portion of the load-deflection curve, as shown in Figure 3.14, it is possible to rearrange the mid-span deflection equation to calculate $E_c$. The experimentally determined values of $E_cI_o$ for each beam are displayed in Table 3.3.

![Figure 3-14 - Linear regression analysis applied to beginning portion of experimental load-displacement curve for the determination of $E_c$](image)

**Table 3-3 - Experimentally determined values of $E_c$**

<table>
<thead>
<tr>
<th>Beam</th>
<th>$E_cI_o$ ($10^{12}$N-mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1C</td>
<td>16.97</td>
</tr>
<tr>
<td>B2P</td>
<td>17.27</td>
</tr>
<tr>
<td>B3T</td>
<td>19.11</td>
</tr>
<tr>
<td>B4D</td>
<td>19.07</td>
</tr>
</tbody>
</table>

The results of the experimental study outlined above will be shown in Sections 3.5.1 to 3.5.4.

### 3.5.1. RESULTS OF DIC MEASUREMENTS FOR BEAM B1C – CONTROL BEAM

In order to assess the quality of the concrete surface for DIC, beam B1C was tested while having no surface preparation, as shown in Figure 3.15. Images of the specimen were
acquired at 10 kN load steps throughout testing, reaching a maximum load of 200 kN. In order to prevent damage from occurring, the potentiometers were removed at a load of 190 kN.

The measured load-displacement curve of the control beam is shown below in Figure 3.16, along with the analytical load-displacement curve determined using the experimental value of $E_c$. It can be seen that cracking occurred at approximately 40 kN, which is below the calculated cracking load; however, this was expected due to the doubts in the accuracy of the measured $f'_{	ext{cr}}$. Using the experimentally identified value of $E_c$ dramatically decreases the error between the measured and calculated displacement values; for detailed calculations of the analytical load-displacement curve, please see Appendix C.

Displacement models for each load step were developed using the DIC algorithm for vertical displacement measurement, and the scales marked on the beam were utilized for the coordinate transformation. The results are displayed below in Figures 3.17 through 3.22, along with the measurements obtained by the potentiometers. Figure 3.17 displays
the quadratic interpolation of the displacements for loads below the calculated $P_{cr}$, and Figure 3.18 shows the cubic interpolation of the same load steps. Figures 3.19 and 3.20 are of the quadratic and cubic interpolations, respectively, for loads between 70 kN and 190 kN at 10 kN load steps. Comparisons of the quadratic and cubic interpolations at three load levels are shown in Figure 3.21. Coefficients of the quadratic and cubic displacement models are given in Tables A-1 and A-2 in Appendix A.
Figure 3-18 – Beam B1C – Cubic interpolation for loads below 60 kN

Figure 3-19 – Beam B1C – Quadratic interpolation for loads between 70 kN and 190 kN at 10 kN load steps
From the figures shown above, it can be seen that the accuracy of the DIC measurements improves as the level of the applied load increases. Below 60 kN, the occurring displacements have a low magnitude, and the calculated integer displacements do not have a large enough range for sufficient interpolation. Once the concrete has cracked, it provides little resistance to tensile forces, and the tension reinforcement carries the majority of the tensile load. This results in an increase in the rate of displacement, and,
therefore, an increase in the range of the calculated integer displacement values. With a larger range of values, the interpolation can be performed successfully, and an accurate model describing the displacement profile along the span of the beam is obtained. It can also be seen that the cubic interpolation of the integer displacements is more sensitive to noise in the images, and does not accurately model the deformed shape of the beam for loads below cracking.

The differences between the displacements measured by the potentiometers and DIC are displayed in Table 3.4. It can be seen that the difference between the measurements obtained from the two interpolation schemes do not differ greatly, and in all cases there is sub-millimetre accuracy. For the loads below cracking, the average error of the DIC-identified displacements is 21.4% for the quadratic interpolation and 24.6% for the cubic interpolation. Considering only the loads after cracking, the average error decreases to 2.2% for the quadratic interpolation and 2.3% for the cubic interpolation.

Table 3-4 – Beam B1C – Difference, in millimetres, between the measurements obtained by the potentiometers and DIC for each load level

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>Left</th>
<th>Center</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quad.</td>
<td>Cubic</td>
<td>Quad.</td>
</tr>
<tr>
<td>30</td>
<td>0.21</td>
<td>0.40</td>
<td>0.07</td>
</tr>
<tr>
<td>40</td>
<td>0.05</td>
<td>0.22</td>
<td>0.06</td>
</tr>
<tr>
<td>50</td>
<td>0.44</td>
<td>0.00</td>
<td>0.41</td>
</tr>
<tr>
<td>60</td>
<td>0.17</td>
<td>0.20</td>
<td>0.05</td>
</tr>
<tr>
<td>70</td>
<td>0.00</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>80</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>90</td>
<td>0.08</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>100</td>
<td>0.11</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>110</td>
<td>0.07</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>120</td>
<td>0.15</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>130</td>
<td>0.07</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>140</td>
<td>0.01</td>
<td>0.04</td>
<td>0.14</td>
</tr>
<tr>
<td>150</td>
<td>0.10</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>160</td>
<td>0.07</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>170</td>
<td>0.11</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td>180</td>
<td>0.18</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td>190</td>
<td>0.07</td>
<td>0.14</td>
<td>0.07</td>
</tr>
</tbody>
</table>
3.5.2. RESULTS OF DIC MEASUREMENTS FOR BEAM B2P – PAINTED BEAM

An attempt to improve the DIC-measured displacements was performed by applying a pattern to the face of the second beam prior to testing. White paint was sprayed on the specimen, and horizontal black lines were drawn at three different heights; the specimen is shown below in Figure 3.22.

![Figure 3-22 – Beam B2P – Artificial pattern applied to the face of the second test specimen](image)

The specimen was loaded up to 220 kN, and images were acquired at 20 kN load steps. The load-displacement curve is shown in Figure 3.23. As with the previous specimen, cracking occurred below the calculated cracking load, at approximately 55 kN.

Since the results of the previous test showed that the quadratic and cubic displacement models performed similarly at higher load levels, while the quadratic models were more accurate at low load levels, it was decided to use only quadratic interpolation for the remainder of the experimental study. The quadratic models developed for loads between 40 kN and 220 kN at 20 kN load steps are shown in Figure 3.24, and the displacement model coefficients are given in Table A-3 in Appendix A.
The displacements for low load levels once again have low accuracy, while the displacement models for higher load levels compare well with the displacements measured by the potentiometers. The differences between the potentiometer readings and the DIC-measured displacements, in millimeters, are shown in Table 3.5. The average error of the displacement models for loads below the cracking load was 32%, and for
loads above the cracking load was 8.0%, which is higher than the error of the previous test. Since the surface of concrete already exhibits a random speckle pattern, it is thought that the method used to apply the paint to the face of the beam may have decreased the variation of the grey-level intensity of the test specimen surface, resulting in the decreased measurement accuracy. Due to the results of this test, it was decided that paint would not be applied to the surface of the remainder of the test specimens.

Table 3-5 – Beam B2P – differences, in millimeters, between potentiometer and DIC measurements

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>Left</th>
<th>Center</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.39</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>60</td>
<td>0.35</td>
<td>0.34</td>
<td>0.18</td>
</tr>
<tr>
<td>80</td>
<td>0.39</td>
<td>0.31</td>
<td>0.30</td>
</tr>
<tr>
<td>100</td>
<td>0.33</td>
<td>0.36</td>
<td>0.25</td>
</tr>
<tr>
<td>120</td>
<td>0.28</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>140</td>
<td>0.30</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td>160</td>
<td>0.33</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>180</td>
<td>0.89</td>
<td>0.76</td>
<td>0.59</td>
</tr>
<tr>
<td>200</td>
<td>0.34</td>
<td>0.13</td>
<td>0.06</td>
</tr>
<tr>
<td>220</td>
<td>0.34</td>
<td>0.20</td>
<td>0.08</td>
</tr>
</tbody>
</table>

3.5.3. RESULTS OF DIC MEASUREMENTS FOR BEAM B3T – BEAM WITH TENSION REINFORCEMENT

The third test performed was to assess the ability of the DIC algorithm to measure displacement after a substantial amount of cracking had occurred in the specimen. Since the results of the test with the artificial pattern applied to the test specimen had a higher error than the test with no surface preparation, it was decided that the face of the current test specimen would be left mostly in its natural state, with the exception of a single horizontal line drawn at approximately mid-height on the beam. Since no shear reinforcement was provided in the beam, it was determined that the beam would exhibit shear failure, with a shear resistance equal to 83 $kN$. The specimen is shown below in Figure 3.25.
To increase the crack development, the specimen was subjected to four loading cycles, as shown in the load-displacement curve in Figure 3.26. The maximum load reached in each cycle was increased from $70 \, kN$ for the first cycle to $120 \, kN$ in the fourth cycle. Since the shear resistance of the beam had been greatly exceeded following the fourth load cycle, it was decided to not load the specimen any further to prevent shear failure from occurring.

Images of the test specimen were captured at $20 \, kN$ load steps, and the vertical displacements were measured using the DIC algorithm. The quadratic models of the DIC-measured displacements during the fourth load cycle for loads between $20 \, kN$ and
120 kN are shown in Figure 3.27, along with the corresponding potentiometer measurements.

Table 3.6 shows the difference between the potentiometer and DIC measurements for each of the displayed load steps. The average error of all the displacement measurements for the fourth load cycle was 6.6%, while for loads above 60 kN the average error was 3.23%. Although this value is higher than the average error of the control beam test, it is important to remember that the magnitude of the applied load was much higher in the first test – thus, the magnitude of the occurring displacements was also much higher in the test of the control beam. Comparing the average difference between the two measurement techniques for the control test and the current test for loads between 60 kN and 120 kN shows that the control test measurements varied 0.08 mm, while the current test measurements varied by 0.07 mm.

![Figure 3-27 – Beam B3T – Load cycle four, for loads between 20 kN and 120 kN, at 20 kN load steps](image-url)
Table 3-6 – Beam B3T – Differences between DIC and potentiometer measurements for the fourth load cycle

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>Left</th>
<th>Center</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.12</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>40</td>
<td>0.10</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td>60</td>
<td>0.03</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>80</td>
<td>0.01</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>100</td>
<td>0.01</td>
<td>0.09</td>
<td>0.16</td>
</tr>
<tr>
<td>120</td>
<td>0.07</td>
<td>0.02</td>
<td>0.13</td>
</tr>
</tbody>
</table>

3.5.4. RESULTS OF DIC MEASUREMENTS FOR BEAM B4D – BEAM WITH INDUCED DAMAGE

A fourth test was performed to assess the ability of the DIC algorithm to locate areas of damage in a reinforced concrete beam. In order to simulate damage, the two outer bars used for tension reinforcement were cut to eliminate the ability of load transference. The damage was located 200 mm from the right of the centerline of the beam, as shown in Figure 3.28.

The load-displacement curve for the damaged beam is given in Figure 3.29. It can be seen that beam failed before reaching the cracking load of the intact beam, indicating that the reduced load carrying capacity of the tension reinforcement was substantial enough to cause premature failure. Loading of the beam was continued until a maximum load of 50 kN was reached, to increase the occurring displacements and collect additional images of the specimen.
Images of the test specimen were collected at 10 kN load steps up to 40 kN, as well as at the first indication of failure and at the maximum applied load. Results of the DIC measurements for the 30 kN, 45 kN and 50 kN load steps, along with the potentiometer measurements, are displayed in Figure 3.30, and the differences between the two measurement techniques for these load steps are displayed in Table 3.7. The average difference between the DIC and potentiometer measurements was 0.26 mm, which is far below the accuracy achieved for the previously reported tests. Recalling that the DIC technique did not perform well at low load levels in the previous tests, it is believed that the magnitude of the occurring displacements for the damaged specimen was not large enough to allow for a sufficient interpolation of the calculated integer displacements.
3.6. Discussion of the Results of the DIC Measurement Study

The results of the tests reported above indicate that the proposed vertical displacement measurement technique utilizing DIC performs well in a variety of test scenarios. It was shown that the natural pattern of the concrete surface is suitable for measurements, eliminating the need for surface preparation and reducing the setup time of the test considerably. At higher load levels, the accuracy of the technique increases. Although it is preferable that the technique performs well at all load levels, displacements of reinforced concrete structures are typically of no concern until a structure is being subjected to service loads, which often exceed the cracking load of a member.
The results of the experimental study also show that the displacement is not symmetric about the centerline of the beam; there is a skew in the displacement curve in the direction of the left support. Since the DIC-identified displacements compare well with those identified using potentiometers, it is believed that the increased deflection on the left-hand side of the beam was caused by a combination of the following factors:

- uneven load application caused by uneven surface conditions on the top face of each specimen
- slightly elastic support conditions

In order to symmetrize the displacement curves of the beams for use in the proposed inverse analysis technique, the displacements will be averaged around the centerline of the beam, as shown in Figure 3.31.

![Figure 3-31 – Identified DIC displacement curve and symmetrized displacement curve](image-url)
Chapter 4. BEAM STIFFNESS RECOVERY

In structural analysis, it is common that the occurring displacements and rotations of a member are calculated by utilizing the known stiffness of the member as well as the loads applied to that member. However, when the displacements and applied loads of a structural member are known, it is desirable that the stiffness of the member can be determined, so that the strength of the member can be ascertained. Inverting the common equilibrium equations used for the analysis of structural members often results in ill-posed expressions that do not yield a meaningful solution; thus, destructive testing is commonly used for the determination of structural strength.

In the proceeding chapter, a solution of the inverted equilibrium equation will be developed by utilizing a minimization procedure. The formulation of the direct structural analysis will be presented in Section 4.1, followed by the development of the inverse analysis technique that will be utilized for the identification of the flexural stiffness distribution of a structural member. Section 4.3 will present the validation of the inverse analysis procedure with numerical examples, and the results of the inverse analysis performed to Beam B1C from Chapter 3 will be presented in Section 4.4.

4.1. STIFFNESS IDENTIFICATION IN EULER-BERNOULLI BEAMS

4.1.1. DIRECT PROBLEM

Consider a simply supported beam subjected to lateral in-plane loading, as shown in Figure 4.1. The equilibrium of the beam can be expressed in terms of the transverse deflection, $v(x)$, determined by the classical fourth-order differential equation:
\[
d\frac{d^2}{dx^2} \left( EI(x) \frac{d^2v}{dx^2} \right) = q(x)
\]

(4.1)

where \( EI(x) \) is the flexural stiffness of the beam and \( q(x) \) is the lateral applied load.

When the stiffness of a beam is uniform along its length, its flexural stiffness is constant and is therefore independent of the position, \( x \). The boundary conditions of the beam can then be combined with Equation (4.1) to solve for the transverse deflection that occurs as the beam is loaded.

Figure 4-1 - Simply-supported beam subjected to lateral loading

Equation (4.1) can also be separated into a system of two coupled second-order differential equations, as shown in Equations (4.2a) and (4.2b):

\[
EI(x) \frac{d^2v(x)}{dx^2} = M(x), \quad 0 \leq x \leq L
\]

(4.2a)

\[
\frac{d^2M(x)}{dx^2} = q(x), \quad 0 \leq x \leq L
\]

(4.2b)

where \( M(x) \) is the bending moment developed within the beam.

In engineering applications, it is common that the applied loading on a structure is composed of one or more concentrated forces and/or moments. Without the loss of generality, the delta function, \( \delta(x) \), can be included in \( q(x) \) and \( M(x) \), where \( \delta(x) \) is defined as:

\[
\delta(x) = \begin{cases} 
1 & \text{for } x = 0 \\
0 & \text{otherwise}
\end{cases}
\]
\begin{align*}
\delta(x - x_o) &= 1 \quad \text{when } x = x_o \\
\delta(x - x_o) &= 0 \quad \text{when } x \neq x_o
\end{align*}
(4.3)

In the case of statically determinate structures, Equation (4.2) can be solved, beginning with Equation (4.2b). In fact, by using the boundary conditions of \( v(0) = 0, M(0) = 0, \) \( v(L) = 0, \) and \( M(L) = 0, \) Equation (4.2b) can be solved independently of Equation (4.2a).

The present work will focus on the classical four-point bending problem shown in Figure 4.2. Given the loading, \( q(s) = P\delta\left(s - \frac{L}{3}\right) + P\delta\left(s - \frac{2L}{3}\right), \) the moment distribution is therefore given as:

\[
M(s) = \begin{cases} 
Ps, & 0 \leq s \leq \frac{L}{3} \\
\frac{PL}{3}, & \frac{L}{3} \leq s \leq \frac{2L}{3} \\
\frac{2L}{3} \leq s \leq L 
\end{cases}
(4.4)
\]

One can check that \( M(0) = M(L) = 0. \)

The direct problem consists of solving Equation (4.2a), assuming the knowledge of the distribution of the bending stiffness, \( EI(x). \) To simplify the notation, the bending stiffness distribution will be denoted \( B(x). \) The direct problem then becomes:

\[
\frac{d^2v(x)}{dx^2} = \begin{cases} 
\frac{Px}{B(x)}, & 0 \leq x \leq \frac{L}{3} \\
\frac{PL}{3B(x)}, & \frac{L}{3} \leq x \leq \frac{2L}{3} \\
\frac{P(L - x)}{B(x)}, & \frac{2L}{3} \leq x \leq L 
\end{cases}
(4.5)
\]
For the case when $B(x)$ is constant, the solution can be obtained analytically; however, when the bending stiffness of a structure is not simple, numerical integration, including the Finite Element Method, can be used to solve for $v(x)$.

![Simply supported beam subjected to four-point bending](image)

**Figure 4-2 - Simply supported beam subjected to four-point bending, with associated moment and deflection distribution diagrams**

When the deflection of a structure is available through direct measurements, as illustrated in the previous chapter, the problem of identifying the function $B(x)$ is an inverse problem is associated with Equations (4.2a) and (4.2b). The inverse problem proposes the determination of the bending stiffness distribution based on the knowledge of the deflection curve, $v(x)$.

### 4.1.2. FORMULATION OF BEAM EQUILIBRIUM AS INTEGRAL EQUATION

To prepare for the formulation of the inverse problem, the equilibrium equation of the beam must first be re-formulated into the form of an integral equation. The following treatise is a standard mathematical tool used to convert a differential equation into an
integral equation (Butkov, 1968; Collins, 2006). For the current study, it will be assumed that the beam is statically determinate; therefore, the moment distribution, $M(x)$, of the beam is known.

To begin, Equation (4.2a) will be solved for a concentrated moment, $M(x) = M_o \delta(x - s)$, acting at point $x = s$. Since Equation (4.2a) is linear, we shall seek a solution for Equation (4.6):

$$\frac{d^2 G(x,s)}{dx^2} = \delta(x - s)$$  \hspace{1cm} (4.6)

Where $G(x,s)$ is the Green function associated to Equation (4.2a). The Green function has the boundary conditions of $G(0,s) = G(L,s) = 0$, and it can be seen that $G(x,s)$ satisfies the homogeneous form of Equation (4.2a) for all values of $x$, except when $x = s$.

Thus, $G(x,s)$ takes the form:

$$G(x,s) = ax + \beta, \hspace{1cm} 0 \leq x \leq s$$  \hspace{1cm} (4.7)

where $\beta$ is the integration constant. Knowing that $G(0,s) = 0$, Equation (4.7) can be solved for $B = 0$; however, the value of $A$ is still unknown. Similarly, we know that:

$$G(x,s) = a'x + \beta', \hspace{1cm} s \leq x \leq L$$  \hspace{1cm} (4.8)

From the boundary condition of $G(L,s) = 0$, a solution of $-a'L \pm \beta'$ can be obtained. To represent a possible physical solution, it is known that $G(x,s)$ must be continuous at $x = s$. Therefore, Equation (4.8) yields a solution of:

$$as = a'(s - L)$$  \hspace{1cm} (4.9)

From Equation (4.9), it can be seen that it is necessary to define only one of either $a$ or $a'$. Since the beam is subjected to a concentrated moment at the point $x = s$, it can be
expected that there will be a jump in the slope of the beam at this location. Therefore, one can obtain:

$$\lim_{\varepsilon \to 0} \left[ \frac{dG}{dx}(s + \varepsilon, s) - \frac{dG}{dx}(s - \varepsilon, s) \right] = 1$$

(4.10)

Then, using:

$$G(x, s) = \frac{as}{s - L} (x - L), \quad x \geq \varepsilon$$

(4.11)

yields:

$$\lim_{\varepsilon \to 0} \frac{dG}{dx}(s + \varepsilon, s) = \frac{as}{s - L}$$

(4.12)

Similarly, using $$G(x, s) = \alpha x, \quad x < s$$, one obtains:

$$\lim_{\varepsilon \to 0} \frac{dG}{dx}(s - \varepsilon, s) = \alpha$$

(4.13)

Then, from $$\frac{as}{s - L} - \alpha = 1$$, it is determined that $$\alpha = \frac{s - L}{L}$$.

Finally, the expression of the Green function associated with the second-order differential equation, Equation (4.2a), is given by:

$$G(x, s) = \begin{cases} \frac{x(s - L)}{L}, & 0 \leq x \leq s \\ \frac{s(x - L)}{L}, & s \leq x \leq L \end{cases}$$

(4.14)

It can be noticed that $$G(x, s) = G(s, x)$$, indicating that the Green function is symmetric.

According to the general superposition principle, it is expected that the solution of the general equation, Equation (4.2a), is given by:

$$\int_0^L G(x, s) \frac{M(s)}{B(s)} \, ds = v(x)$$

(4.15)

Since $$M(x)$$ is explicitly known, the full expression of the equilibrium of a beam subjected to four-point bending can be determined. Assuming the length of the beam to
be 1 and using Equation (4.4), without loss of generality, the kernel function associated with Equation (4.2a), \( G(x, s)M(s) \), becomes:

\[
G(x, s) = \begin{cases} 
\frac{(L-x)Fs^2}{L}, & s < x \\
\frac{x(L-s)Fs}{L}, & x < s < \frac{L}{3} \\
\frac{3L}{2}, & \frac{L}{3} < s < \frac{2L}{3} \\
\frac{x(L-s)^2F}{L}, & \frac{2L}{3} < s < L 
\end{cases}
\]

(4.16a)

\[
G(x, s) = \begin{cases} 
\frac{s(L-x)Fs}{L}, & s < \frac{L}{3} \\
\frac{s(1-x)F}{L}, & \frac{L}{3} < s < x \\
\frac{x(1-s)F}{3L}, & x < s < \frac{2L}{3} \\
\frac{x(1-s)^2F}{L}, & \frac{2L}{3} < s < L 
\end{cases}
\]

(4.16b)

\[
G(x, s) = \begin{cases} 
\frac{s(L-x)Fs}{L}, & s < \frac{L}{3} \\
\frac{s(L-x)F}{3L}, & \frac{L}{3} < s < \frac{2L}{3} \\
\frac{s(L-x)(L-s)F}{L}, & \frac{2L}{3} < s < \frac{3L}{2} \\
\frac{x(1-s)^2F}{L}, & x < s < L 
\end{cases}
\]

(4.16c)

With the knowledge of \( G(x, s), M(s), \) and \( B(s) \), it is clear that for the direct problem, the deflection of the beam can be obtained by calculating the integral on the left-hand side of Equation (4.15).

4.1.3. IDENTIFICATION OF BENDING STIFFNESS USING INVERSE PROBLEM FORMULATION
Now that the formulation of the direct problem is complete, the case where the deflection of a beam is known and the distribution of its bending stiffness must be identified can be considered. For the case of concrete beams, it is expected that loading will induce damage in the form of cracking, and consequently, the bending stiffness of the beam decreases. The bending stiffness can be written in the following form:

\[
EI(x) = EI_0 \left(1 - d(x)\right)
\]  

(4.17)

where \(EI_0\) is the reference bending stiffness of the beam, taken as the stiffness of the undamaged cross-section. A reasonable estimate of the reference stiffness can be utilized in the event that the actual value is not known in advance. The variation of the actual stiffness of the beam is encapsulated in the function \(d(x)\). This variation can be caused by either the damage accumulated in the beam during the loading stages, or it can be associated with uncertainty in the developed model. From Equation (4.17), it can be seen that the value of \(d(x)\) must fall between 0 and 1. The lower bound, \(d(x) = 0\), indicates that there is no damage detected at position \(x\), and the upper bound, \(d(x) = 1\), corresponds to a vanishing bending stiffness, or \(EI(x) = 0\), indicating a complete plastic hinge in the beam.

Equation (4.15) is known as the Fredholm integral of the first kind. The Green function, \(G(x,s)\), is also called the kernel function. This equation establishes a linear relationship between the curvature, \(\chi(x) = \frac{M(x)}{EI_0(1-d(x))}\), and the deflection, \(\nu(x)\), of the beam. The kernel function, therefore, is a representation of the equilibrium relationship between the curvature and the deflection of the beam. When the moment and the bending stiffness are known, the deflection is easily calculated by evaluating the left hand side of Equation (4.15). In most cases, a numerical integration would be necessary to obtain the solution.
This problem is known as the forward problem, and the integration would be a forward computation.

When the moment is known, and the deflection is directly measured – using DIC or any other measurement technique discussed in the previous chapter – the evaluation of Equation (4.15) is seen as an inverse problem. This problem consists of computing the curvature, \( \chi(x) \), of the beam through the use of the known deflection distribution, \( v(x) \).

The treatment of the inverse problem expressed as the Fredholm integral of the first kind is a classical topic, and we therefore refer to (Vogel, 2002) and (Hansen, 2010) for the algorithmic solution. Finally, the identification of the bending stiffness of the beam based on the measurement of the deflection would be obtained by solving the following problem:

\[
\int_0^L G(x,s) \frac{M(s)}{EI_0(1 - d(s))} \, ds = v(x)
\]

(4.18)

4.2. NUMERICAL SOLUTION OF THE INVERSE PROBLEM

4.2.1. DISCRETIZATION AND QUADRATURE

In order to solve the continuous problem of Equation (4.18), a discrete and finite dimensional problem needs to be formulated. Since the underlying integral is linear, we seek to convert it to a linear algebraic equation, which we shall refer to as the discrete inverse problem. The basic approach to discretize Equation (4.18) is to compute an approximation, \( \tilde{\chi}_j = \frac{M(x)}{EI_0(1 - d(x_j))} \), of the solution \( \chi(x) \) at selected abscissas \( x_1, x_2, \ldots, x_n \), such that:
\[ \tilde{\chi}_j \equiv \tilde{\chi}(x_j) = \frac{M(x)}{EI_o(1 - d(x_j))} \quad (4.19) \]

The easiest approach to discretize (4.18) is to use the quadrature method, which takes its general form as:

\[ \int_0^L \varphi(x) dx = \sum_{j=1}^{n} w_j \varphi(x_i) + E_n \quad (4.20) \]

where \( \varphi \) is the function whose integral needs to be evaluated, \( E_n \) is the quadrature error, and \( w_j, (j = 1, 2, \ldots, n) \) are the quadrature weights. To evaluate Equation (4.20), it was chosen to use the mid-point rule, assuming that the beam length is sub-divided into equidistant points, \( x_j \), where

\[ x_j = \frac{L}{n} \left( j - \frac{1}{2} \right), \quad j = 1, 2, \ldots, n \quad (4.21) \]

and \( w_j = \frac{L}{n} \). The quadrature rule is then formally applied to Equation (4.18) to obtain

\[ \int_0^L G(x, s) \tilde{\chi}(s) ds = \sum_{j=1}^{n} w_j G(x, s_j) \tilde{\chi}(s_j) + E_n(x) \quad (4.22) \]

It can be noticed that both Equation (4.22) and the quadrature error, \( E_n(x) \), are both functions of \( x \). Now, the collocation requirement is enforced between the left and right sides of Equation (4.18) and the \( n \) selected points, leading to:

\[ \sum_{j=1}^{n} w_j G(x_i, s_j) \tilde{\chi}(x_i) = v(x_i) - E(x_i), \quad i = 1, 2, \ldots, n \quad (4.23) \]

Finally, if the error term is neglected, the \( n \) equations of (4.23) for each \( x_i \) form the following linear system:
\[
\sum_{j=1}^{n} w_j G(x_i, s_j) \hat{x}_j \approx v(x_j) \tag{4.24}
\]

and the system can then be written as:

\[
[A][\chi] = [b] \tag{4.25}
\]

Where the matrix \([A] = [a_{ij}]\) is given by \(a_{ij} = w_j G(x_i, s_j)\) and \([b]\) is the vector of measured displacements along the length of the beam, \(b_i = v(x_j)\).

### 4.2.2. Regularization and Numerical Solution

The inverse problem consists of identifying the bending stiffness (or its inverse, \(\frac{1}{B(x)}\)) from the given deflection measurements, \(v(x)\). From a mathematical point of view, (Lesnic et al., 1999) has studied the conditions under which the distributed flexural stiffness can be estimated by using Euler-Bernoulli theory. The authors have demonstrated the uniqueness of the solution for the inverse problem in the case of simply-supported beams. As for obtaining the solution, it is well known that inverse problems written as a Fredholm integral of the first kind are ill-posed, and the discretization does not eliminate the intrinsic ill-posedness of such problems.

Thus, the solution of the discrete form of the problem is formulated as a minimization in regularized form:

\[
\chi_\alpha = \arg \min_{\chi} (\|A\chi - v\|^2 + \alpha \|\chi\|^2) \tag{4.26}
\]

The first term of the minimized functional \(\|A\chi - v\|^2\) can be associated to the least mean square solution of the discrete linear system, Equation (4.25). The added term, \(\alpha \|\chi\|^2\), is the classical Tikhonov regularization known as the penalty functional, which imposes the
regularity of the curvature of the beam. The parameter, $\alpha$, is a scalar that weights the regularized terms in the equation against the least mean square terms. This parameter controls the goodness-of-fit of the solution, i.e. how close the solution $\chi$ predicts the given deflection data, $\nu$.

As the deflection data comes from measurements, they are necessarily contaminated with noise. Intuitively, it should not be expected that the residual $\|A\chi - \nu\|$ will be smaller than the average size of the error in $\nu$. Mathematical analysis of the least squares problem shows the effect of high-frequency components, which produces large amplitude terms in the solution. The role of the regularization term is to suppress most of the large noise components. The goal of the Tikhonov regularization is to find an optimal balance between the goodness-of-fit and a sufficient regularity of the solution, $\chi_j$.

Other penalty functionals can be used to incorporate a priori information for one-dimensional problems. For the present problem, the total variation is an alternative, which is given by:

$$TV(\chi) = \sum_{i=1}^{n-1} |\chi_{i+1} - \chi_i|$$

(4.27)

This functional penalizes highly oscillatory solutions while allowing jumps in the regularized solution. This alternative is referred to as the first-order Tikhonov regularization. In comparison to the original formulation of Equation (4.26), which is considered the zero-order Tikhonov regularization, the generalization of the solution of Equation (4.25) would be:

$$\chi_\alpha = \arg \min_{\chi} \|A\chi - \nu\|^2 + \alpha \|D\chi\|^2$$

(4.28)

where $D = I$, the identity matrix, for the zero-order Tikhonov regularization, and
\[ D = \begin{bmatrix}
1 & -1 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & 1 & -1
\end{bmatrix} \]

for the first-order Tikhonov regularization. Using the following notation:

\[ J(\chi) = \|A\chi - \nu\|^2 + \alpha \|D\chi\|^2 \quad (4.29) \]

and setting the gradient of the functional equal to zero, \( \nabla J(\chi) = 0 \), the following equation is obtained:

\[ \nabla J(\chi) = 2A^T(A\chi - \nu) + 2\alpha D^TD\chi = 0 \quad (4.30) \]

Finally, the solution of the linear system is given by:

\[ (A^TA + \alpha D^TD)\chi = A^T\nu \quad (4.31) \]

The conjugate gradient algorithm can be utilized to obtain the solution of the above linear system.

**4.3. Validation using Numerical Simulation**

As a first step to assessing the performance of the above-outlined algorithm for reconstructing the flexural stiffness profile and detecting the level of existing damage in a beam, numerical simulations were performed for a series of steel I-beams subjected to four-point bending and having a constant stiffness reduction imposed along the inner span length. Shown in Figures 4.3 a) through d), the applied stiffness reduction of the four simulated beams ranged from a maximum of 76% – Figure 4.3a – to a minimum of 22% – Figure 4.3d – in order to characterize the sensitivity and accuracy of the algorithm. Each beam was simply-supported, 2.4 m in length, and subjected to two 100 kN concentrated loads applied at third points of the span length.
The solution of the damaged beam was obtained by finite element analysis. For analysis, each beam was discretized into 96 elements, each having a length of 25 mm, resulting in 97 displacement measurements for each beam. In order to better simulate real-world data collection, the analysis of each beam was performed with three cases: case (1) with no noise applied to the input displacement data, case (2) with a 2% level of noise applied to the input data, and case (3) with a 5% noise level applied to the input data.

![Figure 4-3 - Steel I-beams used for numerical validation of inverse analysis algorithm, having a stiffness reduction along the inner span length of: a) 76%, b) 51%, c) 35%, and d) 22%](image)

The identified flexural stiffness distribution profiles of the four beams as a function of $EI_o$ are given in Figures 4.4 through 4.7, in decreasing severity of the induced stiffness reduction. It can be seen that as the induced stiffness reduction decreases, the accuracy of the model also decreases; however, in all four cases the stiffness reduction identified by inverse analysis falls within 6% of the actual stiffness reduction imposed on the beam.

As the induced noise level increases, it can be seen that the quality of the solution decreases slightly, especially for the identification of the flexural stiffness along the outer span lengths. Also, the symmetry of the identified solution decreases with an increase in...
the induced noise level; however, the identified stiffness for the outer spans of each beam fall within 10% of the actual flexural stiffness of that section of the member. It can also be concluded that the induced noise level has negligible effect on the quality of the identified solution for all four simulations.

Figure 4-4 - Identified flexural stiffness distribution for the simulation with a 76% induced damage level along the inner span length

Figure 4-5 - Identified flexural stiffness distribution for the simulation with a 51% induced damage level along the inner span length
It can be seen that the width of the identified damage zone increases with a decreasing induced damage level. Considering the beam in terms of the discretization utilized for the calculation of the simulated displacement profile, in the case of the 76% induced damage, the damage zone was one element larger than the induced damage zone at each
extreme, corresponding to an increase of 6.2% in the identified damage zone length. Similarly, for the case of the 22% induced stiffness reduction, the inverse analysis program identified an additional six elements at each extreme, corresponding to a 43% increase in the identified length of the reduced stiffness zone. Although the error seems to be high, especially in the case of the lowest induced damage level, it is thought that the error can be reduced by increasing the number of elements used during the discretization process.

The identified damage function for each of the four simulated beams are given in Figures 4.8 through 4.11, in decreasing severity of the induced damage level. Similar to the identified flexural stiffness distributions shown above, it can be seen that the accuracy of the identified damage level decreases as the actual level of existing damage decreases. As well, the length of the identified damage zone increases as the level of existing damage decreases, and the symmetry of the identified damage distribution decreases as the induced noise level increases.
Figure 4-9 - Detected damage level for the beam having an induced stiffness reduction of 51% along the inner span length

Figure 4-10 - Detected damage level for the beam having an induced stiffness reduction of 35% along the inner span length
As shown in the above examples, for the case of a flexural member with a homogeneous modulus of elasticity, $E$, the inverse analysis procedure described in Section 4.2 is able to detect a continuous decrease in the flexural stiffness induced along the inner span length of the member. The difference between the applied and identified reduction of flexural stiffness was below 6% in all cases, and aside from the beam with the 22% reduction in stiffness, the identified damage level existing in each beam was also within 6% of the induced damage level.

4.4. IDENTIFICATION OF FLEXURAL STIFFNESS AND EXISTING DAMAGE FOR BEAM B1C: CONTROL BEAM

The results of the numerical simulations from the preceding section show that the inverse analysis procedure for the identification of the flexural stiffness distribution of a beam and the corresponding level of existing damage performs well for members made of a homogeneous material; thus, in the following section, the technique will be utilized to
analyze the control beam presented in Section 3.5.1, using the DIC-measured displacement profile as input for the algorithm.

Six load levels were studied using the inverse analysis procedure described above: 80 kN, 100 kN, 120 kN, 140 kN, 160 kN and 180 kN. The reference flexural stiffness value was $17.0 \times 10^{13} N \cdot mm^2$, corresponding to a concrete elastic modulus, $E_c$, of 18995 MPa and a gross moment of inertia, $I_o$, of $893 \times 10^6 mm^4$. The identified $EI_o/EI$ ratio of the six load levels are presented below in Figure 4.12, and the identified damage level for each load is shown in Figure 4.13.

It can be seen that the inverse analysis identified a constant flexural stiffness along the mid-span of the member, as well as a constant damage level, for each of the six investigated load levels. It is known that the flexural stiffness of a reinforced concrete member decreases as the applied moment increases. Therefore, it was expected that in the region of a beam subjected to a constant moment – i.e. the inner span length of a beam subjected to four-point loading – the flexural stiffness will remain constant, while for the outer span regions the stiffness will vary from the mid-span stiffness at the location of the applied loads to the un-cracked stiffness when the value of the applied moment on the beam lowers below the cracking moment. Explanations for these results will be discussed in Section 4.5.
Knowing that the Branson equation for the effective moment of inertia, $I_e$, determines an average stiffness for the entire length of the member at a given load level, it is worthwhile to also investigate the similarity between the results obtained using the inverse analysis procedure and the predicted flexural stiffness using the Branson equation. The comparison of the two methods is presented in Figure 4.14. It can be seen
that the results of the inverse analysis are similar to the values of the flexural stiffness calculated using the Branson equation for lower load levels, while for higher load levels the Branson equation predicts a greater flexural stiffness of the member than that which was identified through inverse analysis. Recalling from Figure 3.16 in Section 3.5.1 that the deflections calculated using the Branson equation compared well with the measured deflections for lower load levels and under-predicted the deflections at higher load levels, the results of the inverse analysis confirm that the Branson equation overestimates the flexural stiffness of this member for higher load levels.

![Graph showing comparison of flexural stiffness values for constant moment region of Beam B1C determined using the Branson equation and identified through inverse analysis.](image)

**Figure 4-14** - Comparison of flexural stiffness values for constant moment region of Beam B1C determined using the Branson equation and identified through inverse analysis

### 4.5. Discussion of Results of Inverse Analysis for the Identification of the Distributed Flexural Stiffness Profile

From the results of the numerical simulations presented in Section 4.3, it was seen that the inverse analysis procedure was capable of identifying an induced flexural stiffness reduction along the mid-span of a beam made from a homogeneous material with an high degree of accuracy. The accuracy of the identified length of the damage zone was not as
high as expected, especially for the case of the 22% induced flexural stiffness reduction; however, this can be explained, at least in part, by the discretization process used for the development of the input displacement data. By increasing the number of elements used during analysis, it is expected that the accuracy of the identified damage zone length will improve.

The results of the inverse analysis performed for the control beam presented in Section 4.4 showed that a constant flexural stiffness was identified for the constant moment region of the member at each of the six investigated load levels. Although it would have been preferred if the flexural stiffness distribution of the entire span length could have been identified, there was not enough data describing the outer span lengths of the beam gathered during the experimental study for this to be possible.
Chapter 5. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1. CONCLUSIONS

The research study enclosed in this thesis aimed to develop a non-contact damage detection technique for reinforced concrete beams that could potentially be utilized for applications of structural health monitoring for civil infrastructure. The aim of this study was to present the idea of using the vertical displacement profile of a structural member obtained by a non-contact measurement technique for use as input into an inverse analysis procedure to determine the distributed flexural stiffness profile and associated damage of that member. In order to validate the proposed DIC algorithm, an experimental study consisting of four reinforced concrete beams, and investigating a variety of surface preparations and damage scenarios, was performed and presented in Chapter 3. The development of an inverse analysis procedure for the identification of the flexural stiffness of a structural member was presented in Chapter 4.

The set-up for the experimental study undertaken in this work was designed to obtain symmetric loading, however the results of the study show that there was a skew in the obtained displacement profiles towards the pin support for each test specimen. This was thought to be due to a combination of the following:

- slightly elastic support conditions
- an unevenly applied load caused by the rough finish on the top surface of the beam
- a stress concentration at the pin support
Although the DIC technique presented in Chapter 3 performed well for the identification of the complete displacement profile of the tested members, it was noticed that the identified displacement profiles slightly underestimated the deflections measured by the potentiometers for all of the specimens. This was most likely caused by the fact that the DIC technique detected only the discrete pixel displacements, while the interpolation was performed using a quadratic model.

In Chapter 4, it was seen from the numerical simulations that the developed inverse analysis technique was capable of detecting a constant stiffness reduction imposed along the mid-span of members made from homogeneous materials. It was also capable of detecting a constant flexural stiffness, as well as the associated damage level, along the constant moment region of Beam B1C when the DIC measured displacement profile was used as input. The inverse analysis was unable to identify any information describing the stiffness or damage of the outer span lengths of Beam B1C, due to the fact that there was not enough displacement data gathered for these regions.

When the flexural stiffness values determined through inverse analysis were compared to the flexural stiffness values calculated using the Branson equation, the results corresponded well with the load-deflection curves presented in Chapter 3. In Figure 3-16, the deflections calculated using the Branson equation for $l_e$ underestimated the measured deflections for higher load levels, indicating that the Branson equation overestimated the actual flexural stiffness of the beam observed in the study. In Figure 4-14, the inverse analysis confirmed this observation by detecting a flexural stiffness similar to the Branson equation for lower load levels, and less than the Branson equation for higher load levels.
5.2. **Recommendations for Future Work**

The recommendations for future work that can be determined from the results of the study presented in this thesis are as follows:

- Re-design the experimental setup to ensure symmetric loading conditions, correcting the errors mentioned in the previous section
- Consider using two pin supports rather than a pin and a roller support for future experimental work
- Implement a sub-pixel interpolation scheme in the DIC algorithm for the identification of a more accurate displacement profile
- Upon refinement of the DIC algorithm, use of the technique for in-situ structures to assess the accuracy of the measurement technique for field conditions
- Use of the presented inverse analysis technique for the remaining beams in the study
- Analysis of different load cases and different beam designs
- Investigation of the use of the developed techniques with FRP-reinforced concrete members


Gilmour, R. S., A. N. Ibrahim and G. Singh (2002). *Guidebook on non-destructive testing of concrete structures*, INTERNATIONAL ATOMIC ENERGY AGENCY.


Appendix A

**Digital Image Correlation: Matlab Code and Curve Fitting Coefficients**
M-FILE FOR VERTICAL DISPLACEMENT MEASUREMENT USING DIC

% read in images of test specimen in reference and deformed positions

Ref = imread('image1.jpg');  % reference image of test specimen
Loaded = imread('image2.jpg'); % loaded image of test specimen

% crop images to removed unnecessary data and focus only on test specimen

Ref_new = Ref(1825:2025,50:4230,1); % reference image
Loaded_new = Loaded(1825:2025,50:4230,1); % loaded image

% show cropped images

figure(1);
subplot(2,1,1); imshow(Load0top_new); title('Reference Image')
subplot(2,1,2); imshow(Load20top_new); title('Deformed Image')

A = Ref_new;
B = Loaded_new;

size(A)

new = ones(150,2341);

template = Ref_new;
offsetx = 0;  offsety = 4;

new((1:size(template,1))+offsety,(1:size(template,2))+offsetx ) = template;

% calculating normalized cross-correlation coefficients between reference and deformed subsets

corrsize = 50;  % set size of pixel subset

for i = 1:(4181-corrsize)
    AA = Ref_new(50:(50+corrsize),i:(i+corrsize));
    BB = Loaded_new(20:200,i:(i+corrsize));
    cc = normxcorr2(AA,BB);
for i = 1:(4181-corrsize)
    AA = Load0top_new(50:(50+corrsize),i:(i+corrsize));
    BB = Load0top_new(20:200,i:(i+corrsize));
    cc = normxcorr2(AA,BB);
    [max_cc, imax] = max(abs(cc(:))); 
    [ypeak, xpeak] = ind2sub(size(cc),imax(1));
    y_offset1(i) = ypeak-size(A,1) ;
end

% calculation and display of discrete pixel displacement values
disp1 = y_offset1-y_offset; 
figure(2) 
plot(disp1); title('Pixel Value Displacement'); hold on

% polynomial curve fitting of pixel displacements

cubictop=polyfit(1:4131,disp1,3);
cubictop(1),cubictop(2),cubictop(3),cubictop(4)
x=1:4131;
ct=cubictop(1)*x.^3+cubictop(2)*x.^2+cubictop(3)*x+cubictop(4);
plot(ct,'--','color','r'); hold on
quadtop=polyfit(1:4131,disp1,2);
quadtop(1),quadtop(2),quadtop(3)
qt=quadtop(1)*x.^2+quadtop(2)*x+quadtop(3);
plot(qt,'--','color','k'); hold on
legend('data','cubic fit','quadratic fit')
Figure A-1 - Discrete pixel displacement curve; displacement models
POLYNOMIAL DISPLACEMENT MODEL COEFFICIENTS

Cubic Displacement Model:

\[ y_c(x) = a_0 x^3 + a_1 x^2 + a_2 x + a_3 \]

Quadratic Displacement Model:

\[ y_q(x) = a_0 x^2 + a_1 x + a_2 \]

Control Beam – B1C:

Table A-1 - Cubic displacement model coefficients for Beam B1C

<table>
<thead>
<tr>
<th>a0</th>
<th>a1</th>
<th>a2</th>
<th>a3</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 kN</td>
<td>-3.41E-11</td>
<td>3.40E-07</td>
<td>-5.90E-04</td>
</tr>
<tr>
<td>40 kN</td>
<td>5.99E-11</td>
<td>-2.27E-07</td>
<td>2.07E-04</td>
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<tr>
<td>50 kN</td>
<td>2.84E-12</td>
<td>-1.53E-08</td>
<td>2.14E-05</td>
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<td>60 kN</td>
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<td>-8.44E-04</td>
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<td>70 kN</td>
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<tr>
<td>80 kN</td>
<td>-9.15E-12</td>
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<td>-0.0018</td>
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</tr>
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<td>8.49E-07</td>
<td>-0.0035</td>
</tr>
<tr>
<td>120 kN</td>
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<td>2.94E-06</td>
<td>-0.0077</td>
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<td>150 kN</td>
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</tr>
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<td>160 kN</td>
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</tr>
<tr>
<td>180 kN</td>
<td>5.47E-11</td>
<td>1.72E-06</td>
<td>-0.00727</td>
</tr>
<tr>
<td>190 kN</td>
<td>1.92E-10</td>
<td>1.14E-06</td>
<td>-0.00688</td>
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### Table A-2 - Quadratic displacement model coefficients for Beam B1C

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<th>Force (kN)</th>
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<th>$a_1$</th>
<th>$a_2$</th>
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<tbody>
<tr>
<td>30</td>
<td>1.23E-07</td>
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<td>-1.953</td>
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<td>40</td>
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<td>-1.8004</td>
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<td>50</td>
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<td>-9.54E-06</td>
<td>-1.995</td>
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<td>1.39E-07</td>
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<td>-3.9628</td>
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<td>-4.5967</td>
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<td>-5.4339</td>
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<td>-5.9152</td>
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<td>-7.601</td>
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<td>-8.194</td>
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### Painted Beam – B2P:

### Table A-3 - Quadratic Displacement Model coefficients for Beam B2P

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<tr>
<th>Force (kN)</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
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<td>-0.7976</td>
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<td>100</td>
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**Tension Beam – B3T:**

Table A-4 - Quadratic displacement model coefficients for Beam B3T

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</thead>
<tbody>
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<td>120</td>
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**Damaged – B4D:**

Table A-5 - Quadratic displacement model coefficients for beam B4D

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<td>45</td>
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<td>50</td>
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**Repaired – B5R:**

Table A-6 - Quadratic displacement model coefficients for Beam B5R

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</thead>
<tbody>
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<td>30 kN</td>
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<td>-9.38E-04</td>
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<td>60 kN</td>
<td>6.45E-07</td>
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</tr>
<tr>
<td>Max Load</td>
<td>1.24E-06</td>
<td>-0.0045</td>
</tr>
</tbody>
</table>
Appendix B

Calculation of Cracking moment and Ultimate Moment
Sample Calculation for beam B1C – Control Beam:

Definition of Variables:

- $f'_c = 58.6 \text{ MPa}$
- $f_y = 400 \text{ MPa}$
- $A_s = 600 \text{ mm}^2$
- $A'_c = 200 \text{ mm}^2$
- $d = 302.5 \text{ mm}$
- $d' = 45 \text{ mm}$
- $E_s = 200000 \text{ MPa}$
- $E_c = 18995 \text{ MPa}$
Cracking Moment:

\[ M_{cr} = \frac{f_r l_y}{\bar{y}_t} \]

\[ f_r = 0.6\sqrt{f'_c} \]

\[ f_r = 4.57 \text{ MPa} \]

\[ M_{cr} = \frac{4.59 \text{ MPa} \times 1.016 \times 10^9 \text{ mm}^4}{180.1 \text{ mm}} \]

\[ M_{cr} = 24.9 \text{ kN} \cdot \text{m} \]

\[ P_{cr} = 68.7 \text{ kN} \]

Ultimate Moment Resistance:

\[ \alpha_1 = 0.85 - 0.0015f'_c \]

\[ \alpha_1 = 0.762 \]

\[ \beta_1 = 0.97 - 0.0025f'_c \]

\[ \beta_1 = 0.824 \]

\[ T_r = A_s f_y \]

\[ T_r = 240 \text{ kN} \]

Assuming: \( f'_s = 275 \text{ MPa} \) (T).

\[ C'_r = -55 \text{ kN} \]

\[ a = \frac{T_r - C'_r}{\alpha_1 f'_c b} \]

\[ a = 26.7 \text{ mm} \]

\[ a = \beta_1 c \]

\[ c = 32.3 \text{ mm} \]

\[ \varepsilon'_s = -0.00137 \]

\[ f'_s = E_s \varepsilon'_s \]

\[ f'_s = 274.6 \text{ MPa} \]

\[ C'_r = A'_s f'_s \]
\[ C_r' = -54.9 \text{ kN} \]

\[ C_r = \alpha_1 f_c' ab \]

\[ C_r = 295 \text{ kN} \]

\[ M_r = C_r'(d - d') + C_r(d - \frac{a}{2}) \]

\[ M_r = 71.1 \text{ kN} \cdot m \]

\[ P_r = 196 \text{ kN} \]
Appendix C
CALCULATION OF EXPECTED DEFLECTIONS
Use of Branson Equation for Deflection Calculations

Defining Variables

\[ f_c := 58 \quad E_s := 200000 \quad A_s := 600 \]

\[ h := 250 \quad d := 302.5 \quad A_{sc} := 200 \]

\[ h := 350 \quad d_c := 45 \]

Calculating gross transformed Moment of Inertia

\[ E_c := 18995 \]

\[ \eta := \frac{E_s}{E_c} \quad \eta = 10.529 \]

\[ A_{st} := (\eta - 1) \cdot A_s \quad A_{st} = 5.717 \times 10^3 \]

\[ A_{sc} := (\eta - 1) \cdot A_{sc} \quad A_{sc} = 1.906 \times 10^3 \]

\[ y_{bar} := \frac{b \cdot h \cdot \left( \frac{h}{2} \right) + A_{st} \cdot d + A_{sc} \cdot d_c}{b \cdot h + A_{st} + A_{sc}} \]

\[ y_{bar} = 180.059 \]

\[ I_{gt} := \left[ b \cdot \frac{h}{12} + b \cdot h \left( y_{bar} - \frac{h}{2} \right)^2 + A_{st} \cdot (y_{bar} - d)^2 + A_{sc} \cdot (y_{bar} - d_c)^2 \right] \]

\[ I_{gt} = 1.016 \times 10^9 \]
Finding neutral axis for cracked section

\[
Y_{\text{barc}} := \frac{A_{\text{sc}}(d - \eta \cdot A_s \cdot d + b \cdot Y_{\text{barc}} \cdot \frac{Y_{\text{barc}}}{2})}{A_{\text{sc}} + \eta \cdot A_s + b \cdot Y_{\text{barc}}}
\]

\[
Y_{\text{barc}} \cdot (A_{\text{sc}} \cdot \eta \cdot A_s + b \cdot Y_{\text{barc}}) - A_{\text{sc}} \cdot d + \eta \cdot A_s \cdot d + b \cdot Y_{\text{barc}} \cdot (Y_{\text{barc}})^2 = 0
\]

\[
Y_{\text{barc}} \cdot (A_{\text{sc}} \cdot \eta \cdot A_s) + b \cdot Y_{\text{barc}}^2 - b \cdot Y_{\text{barc}}^2 / 2 - (A_{\text{sc}} \cdot d + \eta \cdot A_s \cdot d) = 0
\]

\[
\frac{(b/2) \cdot Y_{\text{barc}}^2 + (A_{\text{sc}} + \eta \cdot A_s) \cdot Y_{\text{barc}} - (A_{\text{sc}} \cdot d + \eta \cdot A_s \cdot d)}{2} = 0
\]

\[
a_1 := \frac{b}{2} \\
b_1 := A_{\text{sc}} + \eta \cdot A_s \\
c_1 := -(A_{\text{sc}} + d + \eta \cdot A_s \cdot d)
\]

\[
a_1 = 125 \\
b_1 = 8.923 \times 10^3 \\
c_1 = -1.913 \times 10^6
\]

\[
Y_{\text{barc}} := \frac{-b_1 - \sqrt{b_1^2 - 4 \cdot a_1 \cdot c_1}}{2 \cdot a_1}
\]

Finding Cracked Moment of Inertia

\[
I_{cr} := b \cdot \frac{Y_{\text{barc}}^3}{12} + b \cdot Y_{\text{barc}} \left( \frac{Y_{\text{barc}}}{2} \right)^2 \cdot \eta \cdot A_s \cdot (d + Y_{\text{barc}}) + (A_{\text{sc}} \cdot d + Y_{\text{barc}})^2
\]

\[
I_{cr} = 3.482 \times 10^8 \text{ mm}^4
\]

Finding Cracking Load

\[
f_{cr} := 0.6 \cdot f_c
\]

\[
f_{cr} = 4.569
\]

\[
M_{cr} := \frac{f_{cr} \cdot l_{gt}}{y_{bar}}
\]

\[
M_{cr} = 2.578 \times 10^7 \text{ (N-mm)}
\]

\[
P_{cr} := \frac{M_{cr}}{725}
\]

\[
P_{cr} = 7.112 \times 10^4 \text{ (N)}
\]
Deflection Calculations

\[ Pa_1 := 50000 \]
\[ Pa_2 := 80000 \]
\[ Ma_2 := \frac{Pa_2 \cdot 725}{2} \]
\[ Ma_2 = 2.9 \times 10^7 \]
\[ Pa_3 := 200000 \]
\[ Ma_3 := \frac{Pa_3 \cdot 725}{2} \]
\[ Ma_3 = 7.25 \times 10^7 \]
\[ \Delta_1 := \left(\frac{Pa_1 \cdot 725}{2 \cdot Ec \cdot Ie_1}\right) \left(3 \cdot 2250^2 - 4 \cdot 725^2\right) \]
\[ \Delta_1 = 0.512 \quad \text{(mm)} \]
\[ \Delta_2 := \left(\frac{725 \cdot Pa_2}{2 \cdot Ec \cdot Ie_2}\right) \left(3 \cdot 2250^2 - 4 \cdot 725^2\right) \]
\[ \Delta_2 = 1.018 \quad \text{(mm)} \]
\[ \Delta_3 := \left(\frac{725 \cdot Pa_3}{2 \cdot Ec \cdot Ie_3}\right) \left(3 \cdot 2250^2 - 4 \cdot 725^2\right) \]
\[ \Delta_3 = 5.502 \quad \text{(mm)} \]
Table C-1 - Calculated displacements for Beam B1C, using Branson’s Equation

<table>
<thead>
<tr>
<th>$P_a$ (kN)</th>
<th>$M_a$ (kN∙m)</th>
<th>$I_{e,bran}$</th>
<th>$\Delta$ (mm)</th>
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Table C-2 - Displacement calculations for beam B2P, using Branson's equation

<table>
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<th>$P_a$ (kN)</th>
<th>$M_a$ (kN·m)</th>
<th>$I_{e,bran}$</th>
<th>$\Delta$ (mm)</th>
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Table C-3 - Calculated displacements for beam B3T, using Branson's equation

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<th>$I_{e,bran}$</th>
<th>$\Delta$ (mm)</th>
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</tr>
</tbody>
</table>
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