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Space-time block coding with imperfect channel estimation and synchronization

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SPACE-TIME BLOCK CODING WITH
IMPERFECT CHANNEL ESTIMATION AND SYNCHRONIZATION

by

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AUTHOR'S DECLARATION OF ORIGINALITY

I hereby certify that I am the sole author of this thesis and that no part of this thesis has been published or submitted for publication.

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ABSTRACT

Two major challenges of applying Alamouti's space-time block coding (STBC) [1] to a practical system are the imperfect channel estimation and rough synchronization. Without the full knowledge of channel state information (CSI), the receiver is highly likely to make wrong decisions; on the other hand, without the time alignment of the transmit antennas, the system will suffer from the inter-symbol interference (ISI) [32].

The subject of this thesis is to propose a novel receiver to improve the overall system performance. In the first part of this thesis, we focus on the performance analysis of STBC with imperfect channel estimation and synchronization. In the next part, we investigate the L-MMSE estimator [16] and derive its general solutions. Finally, a novel receiver based on the L-MMSE estimator and a modified parallel interference cancellation (PIC) detector [37] is proposed.

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To all my friends, thank you for your friendship, you made my journey in Windsor a much enjoyable and easier one.

As a final note, I would like to thank my parents in China, without their endless support and unconditional love, it would have been impossible for me to study abroad and to finish my MASc. This thesis is dedicated to them.

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Abbreviations

AS	Antenna selection
AWGN	Additive white Gaussian noise
A&F	Amplify-and-forward
BLAST	Bell laboratories layered space-time
BER	Bit error rate
BE-STBC	Block-based equalization STBC
CSI	Channel state information
DC	Frequency down conversion
D&F	Decode-and-forward
EP	Error propagation
FEC	Forward error correction
ISI	Inter-symbol interference
LS	Least-squares
MSE	Mean square error
MIMO	Multiple-input multiple-output
ML	Maximum likelihood

MRRC	Maximal ratio receiver combining
OFDM	Orthogonal frequency-division multiplexing
pdf	Probability distribution function
PEP	Pair-wise error probability
PIC	Parallel interference cancellation
PQO-STBC	Power allocated quasi-orthogonal STBC
PSK	Phase shift keying
QAM	Quadrature amplitude modulation
QoS	Quality of service
SED	Squared Euclidean distance
SFC	Space-frequency code
SISO	Single-input single-output
SNR	Signal-to-noise ratio
STBC	Space-time block code
STC	Space-time code
STTC	Space-time trellis code
UC	Frequency up conversion

Notations

$\mathbf{a}$	A vector
$\mathbf{A}$	A matrix
$\lvert A \rvert$	Magnitude/ absolute value of $\mathbf{A}$
$[\cdot]^T$	Transpose of a vector or a matrix
$[\cdot]^H$	Hermitian transpose of a vector or a matrix
$[\cdot]^*$	Complex conjugate
$(\mathbf{A})^{-1}$	Inverse of matrix $\mathbf{A}$
$\lVert \mathbf{a} \rVert^2$	$\lVert \mathbf{a} \rVert^2 = \mathbf{a}^H \mathbf{a}$
$E[x]$	Statistic average of a random variable x
$\mathrm{var}\{x\}$	$\mathrm{var}\{x\} = E\left[x^2\right] - \left(E[x]\right)^2$
$\mathbf{C}_{\mathbf{rr}}$	Covariance matrix of $\mathbf{r}$
$\mathbf{C}_{\mathbf{sr}}$	Cross-covariance matrix of $\mathbf{s}$ and $\mathbf{r}$
$p[x]$	Probability density function of a random variable x
$P[X]$	Probability of an event X
E_s	Average symbol energy
$S_{K \times N}$	A STBC matrix of size $K \times N$

Chapter 1

Introduction

Since 1897, when Guglielmo Marconi first used radio to contact with ships sailing the English channel, new wireless communications methods and services have been evolved remarkably and adopted by people enthusiastically throughout the world. Driven by the transformation of demand from voice telephony service into other services, such as transmission of images, video and data, the telecommunication industry has shifted towards 3G and 4G services. These new services require the wireless systems to have higher data rates, better quality of service (QoS) and coverage, and be deployed in diverse environments. However, unlike wired systems, such as fiber or coaxial cable, whose demands for additional capacity can be fulfilled largely with the addition of new private infrastructure, such as additional optical fiber, cable, routers, and so on, additional wireless capacity cannot be derived from the addition of two major wireless resources: radio bandwidth and transmitter power. Since these two resources are among the most severely limited in the development of modern wireless networks: radio bandwidth because of the very tight situation with regard to useful radio spectrum, and transmitter power because the battery must remain small since the wireless devices must remain simple and portable.

Another reason why it is impractical to improve the wireless capacity by increasing the transmitter power is the multipath fading effect [10]. In wireless systems, signals are transmitted by diverse ways of electromagnetic wave propagation, such as reflection, diffraction, scattering, and so on. Since most mobile wireless systems operate in urban area, the transmission path between the transmitter and the receiver can vary from simple line-of sight to one that is severely obstructed by buildings and foliage. Due to multipath reflections from various objects, the electromagnetic waves travel along different paths of varying lengths. The interaction between these waves causes multipath fading effect. In additive white Gaussian noise (AWGN) channel, 1-dB improvement in signal to noise ratio (SNR) may reduce the bit error rate (BER) by 90%. In a multipath fading environment, however, 10 dB higher SNR may be needed to achieve the similar amount of reduction of BER.

Given these circumstances, higher data rates can be achieved by the mitigation of multipath fading effect at both the transmitter and the receiver, without additional transmitter power or bandwidth. In recent years, there has been considerable research effort aimed at developing more efficient wireless signaling techniques to combat the multipath fading effect, among them are the multi-input multi-output (MIMO) systems [10], which demonstrate a remarkable increase in wireless capacity due to the application of multiple antennas at both ends of the wireless link.

1.1 The MIMO System

Compared to single-input single-output (SISO) wireless systems, MIMO systems are more power and bandwidth efficient, as the capacity limit of MIMO systems increases approximately linearly with the number of antennas [32]. In other words, the performance of a MIMO system can be considerably enhanced without raising the transmitter power and expanding the bandwidth.

Figure 1.1 shows the block diagram of a MIMO wireless system that has N_T transmit and N_R receive antennas. The source data stream is fed to the transmitter block, after a series of data processing including data compression and channel coding, the data stream is encoded and divided into separate symbol streams, which can be independent, partially redundant or fully redundant. Each symbol stream is then sent to one of the transmit antennas and transmitted over the wireless channel after frequency up conversion and amplification.

At the receiver, the signal received by each receive antenna is a linear combination of the signals transmitted from all N_T transmit antennas plus noise. After amplification and frequency down conversion, the decoder combines the received signals from all N_R receive antennas into one data stream and detects the transmitted data streams.

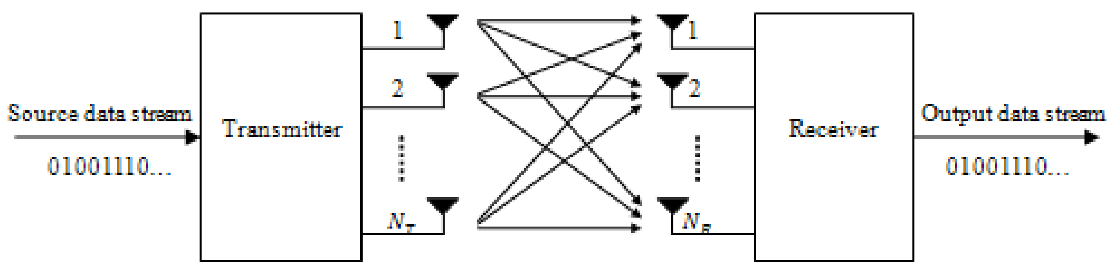


Figure 1.1: Block Diagram of MIMO System

1.2 Diversity Techniques

Diversity techniques are widely applied in wireless MIMO systems to combat deep fading in the path. By increasing the diversity order of the transmitted signals, same information will be carried by signals through multiple independent fading channels, and thus the probability that all signals will encounter the same deep fading will be minimized [24]. Three of the conventional diversity techniques are time diversity, frequency diversity and space diversity.

Time diversity techniques involve transmitting signals with the same information in diverse time slots [24]. Since the transmitted signals are independent with each other, the received signals in each time slot will experience independent fading. An example of time diversity techniques in practical wireless systems is the forward error control (FEC) coding in conjunction with time interleaving.

In frequency diversity techniques, signals carrying the same information are transmitted over different carrier frequencies [32]. To guarantee that different frequencies experience different fading, the carrier frequencies must be separated with each other by more than one coherent bandwidth of the channel. The RAKE receiver is generally considered as on form of frequency diversity.

Space diversity techniques employ multiple antennas in the transmitter and/or the receiver [32]. The primary requirement for space diversity techniques is that the signals transmitted from different antennas be uncorrelated and hence experience independent fading. To ensure this, the transmit/receive antennas must be separated far enough.

1.3 Space-Time Coding Background

As discussed in previous sections, the MIMO system cooperated with various diversity techniques can provide the wireless communication system higher resistance to multipath fading effect. Another fact is that the capacity of the MIMO channel increases linearly with $\min\left(N_T, N_R\right)$. In other words, the capacity of the wireless system can be improved by increasing the spatial diversity order without extra power and bandwidth consumption. This leads to the development of space-time and space-frequency codes (STC & SFC).

By applying a well designed STC or SFC to the MIMO system, the spatial diversity order can be maximized and so does the system capacity. STC is accomplished in space and time domain, while SFC is done in space and frequency domain. In this thesis, we will concentrate on the study of STC.

In 1996, Gerard Foschini proposed the laboratories layered space-time (BLAST) architecture at Lucent Technologies' Bell Laboratories. This is the first STC architecture in the world that exploits the concept of spatial multiplexing and provides high data rate transmission. The problem of this technique, however, is that it only provides some diversity gain at the receiver and does not provide any transmit diversity.

The elementary trade-off between spatial multiplexing gain and diversity gain in the MIMO system, which can be translated to the trade-off between speed and reliability, has obsessed researchers for a long time until 1998, when Siavash Alamouti developed a novel but simple two-branch STC scheme called Alamouti’s space-time block coding or STBC [1]. The key feature of this scheme is that it achieves a full diversity gain and data rate with a low decoding complexity order.

1.4 Research Objective and Contributions

As stated in section 1.3, STBC has proved to be an effective technique to combat multipath fading effect and achieve transmit diversity, due to its high diversity order and low decoding complexity. So far, most research on STBC has assumed that cooperative transmit antennas are perfectly synchronized and the receiver has full knowledge of the channel state information (CSI). Such assumptions, unfortunately, is difficult or even impossible to be satisfied in many practical systems: imperfect synchronization because of the drifting of parameters of electronic components and the lack of common clock oscillator in low-cost cooperative systems, and partial knowledge of CSI because of the channel estimation can never be perfect and fading factors derived from pilot symbols cannot represent the channels for data symbols in fast fading environment [32].

This research is motivated by problems listed above, and the goal of this thesis is to propose a simple and novel receiving scheme for the basic Alamouti's two-branch STBC system when both perfect channel estimation and synchronization are unavailable.

The main contributions of this thesis are:

- Evaluated the performance of Alamouti's 2-branch STBC system under imperfect channel estimation and synchronization.
- Established system models for imperfect channel estimation and synchronization.
- Proposed a low complexity linear receiver for STBC systems under imperfect channel estimation and synchronization.

1.5 Organization of the Thesis

The organization of this thesis is as follows:

In Chapter 2, STC system is discussed in detail with emphasis on the two-branch STBC scheme. A comparison of STBC and maximal-ratio receiver combining (MRRC) is also presented in this chapter.

Chapter 3 presents performance analysis for STBC under imperfect synchronization and channel estimation, and introduces existing techniques addressing STBC under imperfect conditions with emphasis on four state of the art techniques, including the block-based equalization (BE-STBC), the parallel interference cancellation, the antenna selection technique (AS) and the power allocated quasi-orthogonal STBC (PQO-STBC).

In Chapter 4, the minimum mean square error (MMSE) estimator is introduced systematically with emphasis on the linear minimum mean square error (L-MMSE) estimator.

Chapter 5 develops the proposed receiver. System models of imperfect channel estimation and rough synchronization are established first, followed by the deduction of proposed receiver based on these models.

Chapter 6 presents the simulation results, which include the BER performance of the proposed receiver and conventional designs for comparison.

Finally the conclusion to this thesis is presented in Chapter 7, along with future work directions.

Chapter 2

Space-Time Codes

STC is a coding technique used in Wireless communication systems to combat channel fading effect. Using multiple transmit and receive antennas the technique provides high diversity order and spatial multiplexing gain. Appling STC to a MIMO system maximizes power and bandwidth efficiency, as well as the system capacity. There exist two major classes of STC, the space-time block codes (STBC) and space-time trellis codes (STTC). Both satisfy the Rank Criterion and achieve full diversity order. In this chapter, structures of both STC classes are described with emphasis on Alamouti's two-branch STBC.

2.1 Space-Time Trellis Codes

STTC was first introduced by Tarokh et al. in 1998 [30]. Transmitting a trellis codes over multiple transmit antennas and time slots this scheme provides high transmit diversity and coding gain at the price of higher decoding complexity. An example of a four state STTC trellis diagram is shown in Figure 2.1. In this example, the number of transmit antennas $N_T = 2$ and the number of receive antennas can be any integer greater than zero. The initial state is S_0, the next transition state is determined by the next information symbol. Two adjacent encoded symbols are then transmitted simultaneously by two transmit antennas. Table 2.1 shows an example of this transmission sequence. The information symbols after 4-PSK modulation shown in Figure 2.2 is 1,3,1,2,0,1,0,0,3,⋯, then after the initial state, the second state is S_1 because the first symbol is 1. In the first time slot, two symbols 0 and 1 will be transmitted by antenna 0 and 1, respectively. The encoding process keeps on going like this until all of the information symbols are encoded. The bandwidth efficiency of this scheme is 2 bits/sec/Hz.

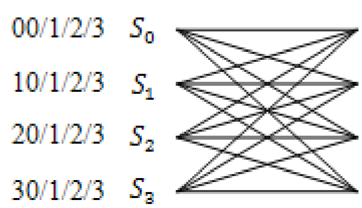


Figure 2.1: Four-State Space-Time Trellis Diagram.

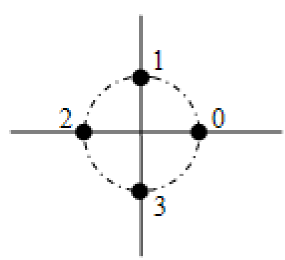


Figure 2.2: 4-PSK Modulation Constellation.

Information symbol sequence: 1, 3, 1, 2, 0, 1, 0, 0, 3 ⋯						
Transmit antenna	Time slot 1	Time slot 2	Time slot 3	Time slot 4	Time slot 5	Time slot 6
0	0	1	3	1	2	0
1	1	3	1	2	0	1

Table 2.1: Transmission Sequence of Four-State STTC.

The major challenge of implementing STTC in practice is that the decoding complexity of STTC increases exponentially with transmission rate and number of transmit antennas [29]. In this scenario, space-time block coding is more appropriate to use due to its low decoding complexity.

2.2 Space-Time Block Codes

Late in 1998, Siavash M. Alamouti proposed a simple two-branch transmit diversity scheme known as STBC [1]. At the transmitter, STBC applies orthogonal encoding to the information symbols generated by the modulator, and then transmit the encoded signal with two transmit antennas. At the receiver, maximum likelihood (ML) detector is implemented with linear processing. The STBC scheme takes advantage of orthogonal design and multiple antennas to allow the use of simple linear combiner and ML detector at the receiver. Because of this new feature, STBC guarantees high diversity order and low decoding complexity at the same time, and thus it tackles the complexity problem in STTC and becomes a promising solution to channel fading problem in wireless communication systems.

2.2.1 Alamouti's Scheme

In Alamouti's STBC model, the encoder encodes a block of two modulated symbols s_0 and s_1 at a time both in space and time domain, which is why it is called space-time block codes. The code matrix for two-branch STBC is specified as

$$\mathbf{S} = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}. \tag{2.1}$$

Row 1 and 2 represent transmit antenna 0 and 1, respectively. Column 1 and 2 represent time slot 1 and 2, respectively. The encoding and transmission sequence is shown in Table 2.2. In time slot 1, antenna 0 transmits s_0 and antenna 1 transmits s_1. In time slot 2, antenna 0 transmits $-s_1^*$ and antenna 1 transmits s_0^*, where $[\cdot]^*$ denotes "complex conjugate". Since two symbols are transmitted in two symbol time slots, Alamouti's two-branch STBC is the first and only STBC scheme that achieves full data rate.

	Transmit antenna 0	Transmit antenna 1
Time t	s_0	s_1
Time t + T	$-s_1^*$	s_0^*

Table 2.2: Encoding and Transmission Sequence of Alamouti's STBC.

After the encoder, coded signals will be transmitted from the transmitter to the receiver through a quasi-static flat fading channel. Since each transmit antenna goes through a different path to reach the receiver, the channel fading coefficients vector may be represented as

$$\mathbf{h} = [h_0, h_1]^T . \tag{2.2}$$

where $[\cdot]^T$ denotes "transpose" and $h_m = \alpha_m e^{j\theta_m}$, $m = 1, 2$, is the channel fading gain from transmit antenna m to the receiver. These fading factors are assumed to be independent and have Rayleigh distributed amplitudes. At the receiver, if we assume that both transmitters are perfectly synchronized with the receiver, the received signals may be represented as

$$\mathbf{r} = \mathbf{S}^T \mathbf{h} + \mathbf{n} = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}^T \begin{bmatrix} h_0 \\ h_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \end{bmatrix} = \begin{bmatrix} h_0 s_0 + h_1 s_1 + n_0 \\ -h_0 s_1^* + h_1 s_0^* + n_1 \end{bmatrix}, \tag{2.3}$$

where $\mathbf{n}$ is the additive white Gaussian noise vector which is composed of $\mathcal{CN}\left(0, \sigma_n^2\right)$ distributed noise samples.

The decoder for Alamouti's two-branch STBC consists of three major parts including channel estimator, linear combiner and maximum likelihood (ML) detector. If the receiver has full knowledge of the CSI, then the channel estimations derived from channel estimator are the same as the real channel factors. The linear combiner is an estimator of the transmitted symbols. It combines the received signals and the channel fading factors with a simple linear combination rule. The combination rule is given by

$$\hat{\mathbf{s}} = \begin{bmatrix} \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = \begin{bmatrix} h_0^* r_0 + h_1 r_1^* \\ h_1^* r_0 - h_0 r_1^* \end{bmatrix}. \tag{2.4}$$

Substitute (2.3) into (2.4), the estimations of transmitted symbols would be

$$\hat{\mathbf{s}} = \begin{bmatrix} \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = \begin{bmatrix} h_0^* r_0 + h_1 r_1^* \\ h_1^* r_0 - h_0 r_1^* \end{bmatrix} = \begin{bmatrix} \left(\alpha_0^2 + \alpha_1^2\right) s_0 + h_0^* n_0 + h_1 n_1^* \\ \left(\alpha_0^2 + \alpha_1^2\right) s_1 + h_1^* n_0 - h_0 n_1^* \end{bmatrix}. \tag{2.5}$$

The estimated symbols then pass to the ML detector where hard decisions are made. The hard decision criteria used in the ML detector is the squared Euclidean distance (SED). The SED between x and y is defined as

$$d^2(x, y) = (x - y)\left(x^* - y^*\right). \tag{2.6}$$

The decision rule:

choose s_i if and only if

$$\left(\alpha_0^2 + \alpha_1^2 - 1\right)\left|s_i\right|^2 + d^2(\hat{s}_0, s_i) \le \left(\alpha_0^2 + \alpha_1^2 - 1\right)\left|s_k\right|^2 + d^2(\hat{s}_0, s_k), \quad \forall\, i \ne k \tag{2.7}$$

is used to decode s_0 and

choose s_i if and only if

$$\left(\alpha_0^2 + \alpha_1^2 - 1\right)\left|s_i\right|^2 + d^2(\hat{s}_1, s_i) \le \left(\alpha_0^2 + \alpha_1^2 - 1\right)\left|s_k\right|^2 + d^2(\hat{s}_1, s_k), \quad \forall\, i \ne k \tag{2.8}$$

is used to decode s_1.

Since this STBC scheme is orthogonal, there is no cross product of s_0 and s_1 in the decision metric (the estimation of s_0 and s_1). This property makes the combiner very simple since symbols s_0 and s_1 can be decoded individually. Figure 2.3 demonstrates the block diagram of Alamouti's two-branch STBC model.

The Alamouti's STBC can also accommodate multiple receive antennas. A generalized STBC model with an arbitrary number of transmit and receive antennas is given in next section. The discussion is brief and introductory since it is not the subject of this thesis. To readers who are interested in STBC with multiple transmit and receive antennas, investigations and discussions can be found in detail in [32] and the references therein.

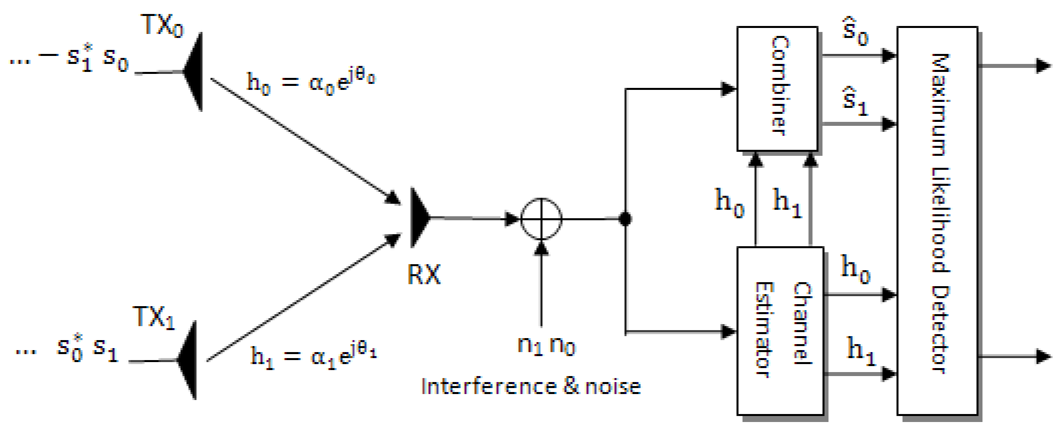


Figure 2.3: Block Diagram of Alamouti's STBC.

2.2.2 Generalization of STBC System Model

The Alamouti's STBC can be extended to a more generalized model with an arbitrary number of transmit and receive antennas. By utilizing the theory of quasi-orthogonal design, extended STBC schemes can achieve partial diversity order and low decoding complexity [32].

A generalized STBC system is considered with N_T transmit antennas and N_R receive antennas. The encoder encodes a block of *p* information symbols at a time and generates *q* encoded symbols for each transmit antenna. The system achieves full data rate for $p = q$ and partial rate for $p < q$. Thus, at an arbitrary symbol time slot *t*, the symbol transmitted by each transmit antenna may be represented as $s_i(t)$, $i = 1, 2, \cdots N_T$. An example of extended STBC schemes is the partial rate STBC with 3 transmitting antennas. The code matrix for this scheme is given by

$$\mathbf{S}_3 = \begin{bmatrix} s_0 & -s_1^* & s_2^* & 0 \\ s_1 & s_0^* & 0 & -s_2^* \\ s_2 & 0 & -s_0^* & s_1^* \end{bmatrix}.$$

The rows represent the symbols transmitted by each antenna. The columns represent different time slots. In this example, $p = 3$ and $q = 4$. The data rate is therefore 3/4.

Assume that the channel between each transmit antenna and receive antenna is quasi-static and flat, and time-invariant in one data frame. The channel fading coefficients matrix may be represented as

$$\mathbf{H} = \begin{bmatrix} h_{0,0} & \cdots & h_{0,N_R} \\ \vdots & \ddots & \vdots \\ h_{N_T,0} & \cdots & h_{N_T,N_R} \end{bmatrix}, \tag{2.9}$$

where $h_{i,j}$ represents the fading factor of the channel between the *i*th transmit antenna and the *j*th receive antenna. The received signal by the *j*th receive antenna at symbol time slot *t* is a linear combination of signals transmitted from all transmit antennas and may be represented as

$$r_j(t) = \sum_{i=1}^{N_T} h_{i,j} s_i(t) + n_{i,j} \ , \tag{2.10}$$

where $n_{i,j}$ is a complex random variable represents the receiver noise and interference in each channel. For additive white Gaussian noise (AWGN) channel, $n_{i,j}$ has the distribution of $\mathcal{CN}\left(0, \sigma^2_{n_{i,j}}\right)$.

2.3 Simulation Results for STBC

In this section, we present error performance simulation for Alamouti's in Rayleigh fading channel. For all simulations, information symbols are BPSK modulated and un-coded by any other channel encoders.

It is assumed that the receiver has full knowledge of the CSI (perfect channel estimation) and transmit antennas are perfectly synchronized with receive antennas (perfect synchronization). We also assume that the channel is quasi-static and flat fading, for example, the fading factors are constants over a data frame (two symbol periods for two-branch STBC) and vary from one data frame to another. Figure 2.5 shows the BER performance of two-branch STBC, compared with two-branch maximal ratio receiver combining (MRRC) and un-coded transmission. In order to ensure that each system radiates the same total energy, we assume that each transmit antenna in STBC system radiates half the energy. Therefore, there is a 3-dB difference between STBC and MRRC.

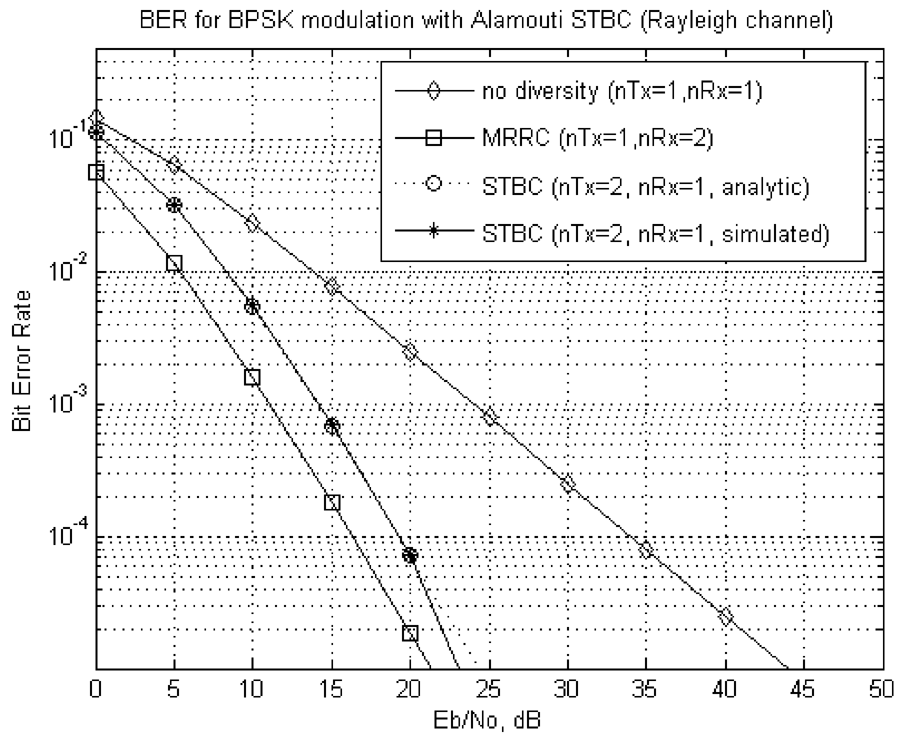


Figure 2.4: The BER Performance of Alamouti's STBC in Rayleigh Fading Channel.

Chapter 3

Performance of STBC with Imperfect Channel Estimation and Synchronization

In this chapter, we study the performance of STBC when both perfect knowledge of CSI and synchronization are unavailable. This chapter is organized as follows. In Section 3.1, we build the noisy CSI model and perform system analysis. Simulations are also given at the end of this section. Approximate models for received STBC signals combined with inter-symbol interference (ISI) have been built in Section 3.2. The impact of imperfect synchronization has been described and simulated in succession. In Section 3.3, we give background introductions to some state of the art techniques addressing STBC under noisy CSI and imperfect synchronization.

3.1 Effect of Imperfect Channel Estimation

By using the orthogonal design, Alamouti's STBC can be decoded by a linear combiner and simple ML detector, and thus it provides the best trade-off between performance and complexity. However, the decoding of Alamouti's STBC heavily depends on the knowledge of CSI, and the conventional decoder was derived assuming that the channel fading coefficients are perfectly known to the receiver although not known to the transmitter [1]. In practical MIMO systems, these coefficients must be estimated, which are usually not accurate, thereby leading to the performance degradation [9]. On the other hand, there is an elementary trade-off between the channel estimation accuracy and system capacity. Since estimation of channel fading factors requires overhead training data sequences, or so called pilot symbols. Increasing the number of pilot symbols may improve the accuracy of the channel estimation [16]. However, this will cause the sacrifice in data rate, and thus leading to the system capacity degradation. It is also shown in [12] that as the number of transmit and receive antennas increases the system becomes more dependent on the channel estimation accuracy.

Although perfect channel fading coefficients are impossible to get in practice, the receiver might have partial knowledge of CSI, with this partial knowledge, variance of the channel estimation error can be derived. It has been proved in [29] that the system performance can be improved by introducing this information into the decision rule. In this section, we consider modeling of estimation error of the CSI and investigate BER performance for Alamouti's STBC with both perfect and estimated channel state information.

3.1.1 System Model of STBC with Imperfect Channel Estimation

Consider a wireless channel with complex fading coefficient *h*. The fading factor *h* can be modeled as a complex Gaussian random variable with zero mean and unity variance. No matter what method is used to estimate this parameter, the estimated channel factor $\hat{h}$ can always be expressed by the following general model [32]:

$$\hat{h} = h + e, \tag{3.1}$$

where e is the channel estimation error. Without loss of generality, we assume that e is a complex Gaussian random variable with zero mean and variance of σ_e^2. We also assume that e is independent of h. Hence the variance of estimation of channel fading factor $\sigma_{\hat{h}}^2$ can be written as

$$\sigma_{\hat{h}}^2 = \sigma_h^2 + \sigma_e^2, \tag{3.2}$$

where σ_h^2 is the variance of the real channel fading factor. The correlation between h and $\hat{h}$ can be expressed by σ_e^2 as

$$\mathrm{C}_{h\hat{h}} = \frac{1}{\sqrt{1+\sigma_e^2}}. \tag{3.3}$$

Since this model of channel estimation is general and widely accepted, we use it in our work. To evaluate the effect of imperfect channel estimation, let us first examine the pair-wise error probability based on this model [29]. Consider a basic model of STBC with N transmitting and M receiving antennas. The information symbol $s(i)$ at symbol time slot i is encoded by the STBC encoder as $c_0(i), c_1(i) \cdots c_{N-1}(i)$. Each code symbol is transmitted simultaneously from a different transmit antenna. Assuming ideal time and frequency synchronization, the base-band signal received by the receive antenna $k = 0,1 \cdots M-1$ can be represented as

$$r_k(i)=\sqrt{2E_s}\sum_{j=0}^{N-1}h_{jk}(i)c_j(i)+n_k(i), \tag{3.4}$$

where $2E_s$ is the average energy of the base-band signal. The coefficient $h_{jk}(i)$ is the channel fading factor between transmit antenna j and receive antenna k at time slot i. The additive noise $n_k(i)$ is an independent sample of the base-band white Gaussian process with $\mathcal{CN}\left(0,\sigma_n^2\right)$ distribution. Let

$$\mathbf{c}(i)=\left[c_0(i),c_1(i)\cdots c_{N-1}(i)\right]^T,$$

$$\mathbf{h}_j(i)=\left[h_{j0}(i),h_{j1}(i)\cdots h_{jM-1}(i)\right]^T,$$

$$\mathbf{H}(i)=\left[\mathbf{h}_1(i),\mathbf{h}_2(i)\cdots \mathbf{h}_{N-1}(i)\right]^T,$$

$$\mathbf{n}(i)=\left[n_0(i),n_1(i)\cdots n_{M-1}(i)\right]^T,$$

$$\mathbf{r}(i)=\left[r_0(i),r_1(i)\cdots r_{M-1}(i)\right]^T.$$

The received signals at time slot i by all receive antennas can therefore be written in a matrix form as

$$\mathbf{r}(i)=\sqrt{2E_s}\,\mathbf{H}(i)\mathbf{c}(i)+\mathbf{n}(i), \tag{3.5}$$

To estimate the channel matrix $\mathbf{H}(i)$, we transmit a sequence of L pilot symbols with each transmit antenna, which forms a $N\times L$ pilot symbol matrix given by

$$\mathbf{P}=\begin{bmatrix} p_{0,0} & \cdots & p_{0,L-1} \\ \vdots & \ddots & \vdots \\ p_{N-1,0} & \cdots & p_{N-1,L-1} \end{bmatrix}, \tag{3.6}$$

where rows represent pilot symbols transmitted from different antennas. Columns represent different index in different pilot symbol sequences. In our study, pilot symbol sequences for all transmit antennas are orthogonal to each other.

Let the received pilot symbols and noise be given by

$$\mathbf{R_P}=\begin{bmatrix} r_{p0,0} & \cdots & r_{p0,L-1} \\ \vdots & \ddots & \vdots \\ r_{pN-1,0} & \cdots & r_{pN-1,L-1} \end{bmatrix}; \tag{3.7}$$

$$\mathbf{n_P}=\begin{bmatrix} n_{p0,0} & \cdots & n_{p0,L-1} \\ \vdots & \ddots & \vdots \\ n_{pN-1,0} & \cdots & n_{pN-1,L-1} \end{bmatrix}. \tag{3.8}$$

Using (3.6), (3.7) and (3.8), (3.5) can be rewritten as

$$\mathbf{R_P}=\sqrt{2E_s}\,\mathbf{HP}+\mathbf{n_P}\,. \tag{3.9}$$

The minimum mean square estimate of **H** can be obtained from (3.9) as

$$\begin{aligned}\hat{\mathbf{H}}&=\frac{1}{\sqrt{2E_s}}\mathbf{R_P}\mathbf{P}^H\left(\mathbf{P}\mathbf{P}^H\right)^{-1}\\ &=\frac{1}{\sqrt{2E_s}}\mathbf{R_P}\mathbf{P}^H/\left\|\mathbf{P}\right\|^2 \end{aligned} \tag{3.10}$$

Combining (3.9) and (3.10) , we have

$$\begin{aligned}\hat{\mathbf{H}} &= \mathbf{H} + \frac{1}{\sqrt{2E_s}}\mathbf{n}_{\mathbf{P}}\mathbf{P}^H / \|\mathbf{P}\|^2 \\ &= \mathbf{H} + \mathbf{e}\end{aligned} \tag{3.11}$$

where **e** is the estimation error matrix given by

$$\mathbf{e} = \frac{1}{\sqrt{2E_s}}\mathbf{n}_{\mathbf{P}}\mathbf{P}^H / \|\mathbf{P}\|^2 \tag{3.12}$$

Assuming a $N \times L$ code matrix

$$\mathbf{C} = \begin{bmatrix} c_{0,0} & \cdots & c_{0,L-1} \\ \vdots & \ddots & \vdots \\ c_{N-1,0} & \cdots & c_{N-1,L-1} \end{bmatrix} \tag{3.13}$$

is transmitted. The probability that the ML detector decides in favor of other code matrix

$$\tilde{\mathbf{C}} = \begin{bmatrix} \tilde{c}_{0,0} & \cdots & \tilde{c}_{0,L-1} \\ \vdots & \ddots & \vdots \\ \tilde{c}_{N-1,0} & \cdots & \tilde{c}_{N-1,L-1} \end{bmatrix} \tag{3.14}$$

based on the imperfect estimation of the channel fading gain is given by [29]

$$P\left(\mathbf{C} \to \tilde{\mathbf{C}} \mid \hat{\mathbf{H}}\right) \leq \exp\left(-\mathrm{C}_{\mathbf{H}\hat{\mathbf{H}}}^2 d^2\left(\mathbf{C}, \tilde{\mathbf{C}}\right) \frac{E_s}{4N_0 + 4N\left(1 - \left|\mathrm{C}_{\mathbf{H}\hat{\mathbf{H}}}\right|^2\right)E_s} \right), \tag{3.15}$$

where $N_0 / 2$ is the noise variance per dimension and

$$d^2\left(\mathbf{C},\tilde{\mathbf{C}}\right)=\sum_{m=0}^{M-1}\sum_{l=0}^{L-1}\left|\sum_{n=0}^{N-1}\left(\hat{h}_{m,n}/\sqrt{2}\sigma_{\hat{h}}\right)\left(\mathbf{c}_l^{n+1}-\hat{\mathbf{c}}_l^{n+1}\right)\right|^2 . \tag{3.16}$$

Intuitively, for Alamouti's STBC, the effect of noisy CSI can be best shown by combining the received signals and estimated channel fading factors with the linear combiner described in (2.5). Assuming the CSI is perfectly known to the receiver, which means $\hat{h}_0 = h_0$ and $\hat{h}_1 = h_1$, the outputs of the linear combiner are given by

$$\hat{s}_0 = h_0^* r_0 + h_1 r_1^* = \left(\left|h_0\right|^2 + \left|h_1\right|^2\right) s_0 + h_0^* n_0 + h_1 n_1^*; \tag{3.17}$$

$$\hat{s}_1 = h_1^* r_0 - h_0 r_1^* = \left(\left|h_0\right|^2 + \left|h_1\right|^2\right) s_1 + h_1^* n_0 - h_0 n_1^* . \tag{3.18}$$

For imperfect channel estimation, we use the model in (3.11) and derive following combined signals:

$$\begin{aligned}\hat{s}_0 &= \hat{h}_0^* r_0 + \hat{h}_1 r_1^* \\ &= \left(\left|h_0\right|^2 + \left|h_1\right|^2\right) s_0 + \left(e_0^* h_0 + e_1 h_1^*\right) s_0 + \left(e_0^* h_1 - e_1 h_0^*\right) s_1 + h_0^* n_0 + h_1 n_1^* + e_0^* n_0 + e_1 n_1^* ;\end{aligned} \tag{3.19}$$

$$\begin{aligned}\hat{s}_1 &= \hat{h}_1^* r_0 - \hat{h}_0 r_1^* \\ &= \left(\left|h_0\right|^2 + \left|h_1\right|^2\right) s_1 + \left(e_1^* h_0 + e_0 h_1^*\right) s_0 + \left(e_1^* h_1 - e_0 h_0^*\right) s_1 + h_0^* n_0 + h_1 n_1^* + e_1^* n_0 + e_0 n_1^* .\end{aligned} \tag{3.20}$$

3.1.2 Simulations

In this section, we analyze the performance of a cooperative STBC system with two relay nodes and one receiver under imperfect channel estimation. BPSK modulation is applied and the signal to noise ratio (SNR) is defined as $\mathrm{SNR} = \sigma_s^2 / \sigma_n^2\,(\mathrm{dB})$. We assume that the signals transmitted from two transmit antennas are perfectly synchronized with each other both in time and frequency. We also assume that the channel is quasi-static and Rayleigh fading. The bit error rates (BER's) of the conventional STBC receiver with imperfect channel estimation under different values of σ_e^2 are shown in Figure 3.1. The performance of perfect channel estimation is also given for comparisons.

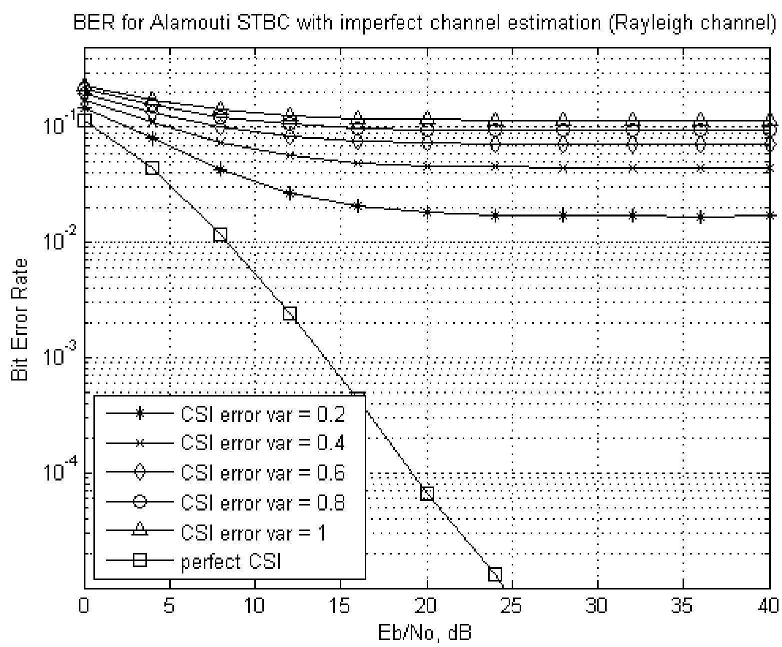


Figure 3.1: Performance of Alamouti's STBC under Imperfect Channel Estimation.

3.2 Effect of Imperfect Synchronization

So far, we have assumed that the transmitters are perfectly synchronized in the STBC system, which means signals from different transmit antennas arrive at the receiver at the same time. However, in many practical STBC systems, this assumption is difficult or even impossible to achieve.

One of the many popular applications of STBC in practice is to combine STBC with a distributed wireless system, such as an ad-hoc or a wireless sensor network [10]. This kind of application is commonly known as cooperative transmission since the distributed transmitters in the network will cooperate with each other and apply STBC to increase the system capacity. In such systems, common local clock oscillator among different transmitters is always unavailable [17]. Furthermore, due to the restriction of the cost and size of the transmitters, the parameters of electronic components may also be drifting. Another fact is that the delay synchronization with respect to two or more receive antennas simultaneously is impossible. Therefore, at the receiver, there will be small time misalignments among the signals from different transmit antennas [17].

The synchronization problem in cooperative transmission has been investigated in [17-20, 34, 37-38]. Imperfect synchronization in time will introduce inter-symbol interference (ISI). For a STBC coded system, this interference will jeopardize the required orthogonal structure and thus makes the conventional STBC linear decoding method fail.

In this section, we consider a two-branch distributed STBC transmission when there is a limited time misalignment between two transmit antennas. We derive the model of the ISI first and then study its impact.

3.2.1 System Model of STBC with Imperfect Synchronization

We consider a 4-node cooperative STBC system depicted in Figure 3.2.

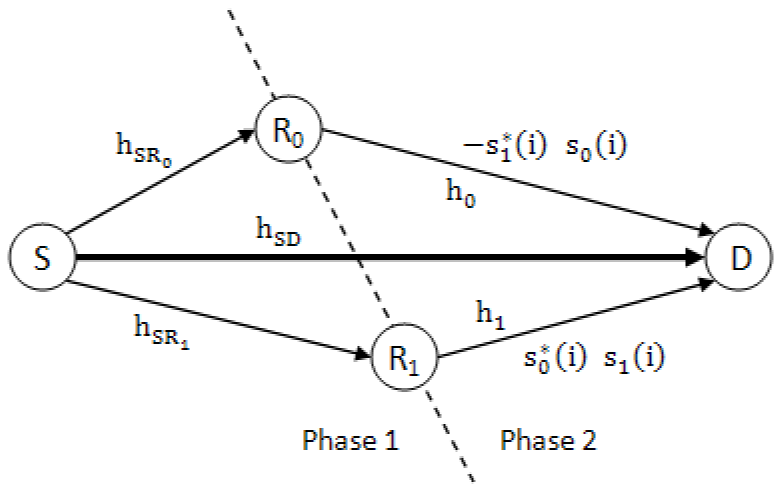


Figure 3.2: Cooperative STBC Model with 2 Relay Nodes.

In Phase I, the source node S broadcast its information to potential relay node (R_0 and R_1) and the destination node D. The coefficient h_{SD} denotes the channel fading gain between S and D, while h_{SR_n} denotes the channel fading gain between S and relay R_n.

In phase II, S stops transmission, R_0 and R_1 cooperate with each other and encode the received data packet by STBC structure, and then transmit the encoded signals to D. The complex coefficient h_n denotes the channel fading factor between R_n and D.

There are two different transmission schemes for each relay: one is amplify-and-forward (A&F), another is decode-and-forward (D&F) [32]. In the A&F scheme, the relays just amplify the received signals and send them to the destination after STBC processing, while in the D&F scheme, each relay detects the source information data first, and only the relays that can successfully detect the source information will be cooperate with each other and perform STBC encoding. In our case, we use the D&F scheme and assume that all relays can detect the source information successfully, and they will be both enrolled in Phase II transmission.

Denoting the *i*th modulated symbol generated by the source as

$$s(i) = Ab(i)\,p(t - iT)\,e^{j\omega_c t}, \tag{3.21}$$

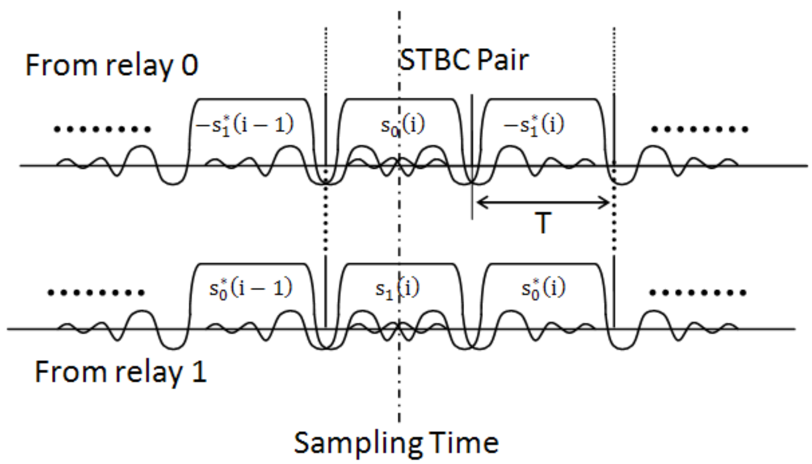
where $b(i)$ is the complex symbol transmitted at symbol interval $\left[iT, (i+1)T\right]$, $p(t - iT)$ is the base-band pulse shaping filter associated with the *i*th symbol. The positive scalar A denotes carrier amplitude and ω_c is the carrier frequency. After a packet of two modulated symbols is received and detected by R_0 and R_1, the two relays will apply STBC encoding to the symbol packet and send the encoded symbols to the destination. The encoded STBC symbols can be expressed by the matrix:

$$\mathbf{S} = \begin{bmatrix} s_0(i) & -s_1^*(i) \\ s_1(i) & s_0^*(i) \end{bmatrix}. \tag{3.22}$$

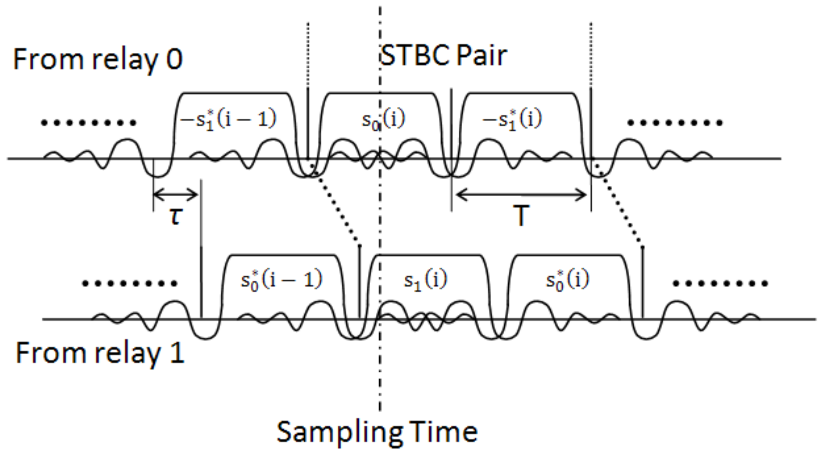
After multipath fading channel and additive thermal noise and other interference, the pass-band signal from relay *n* at symbol time slot *i* can be expressed as

$$s_n(i) = \alpha_n A b_n(i)\, p(t - iT - \tau_n) e^{j(\omega_c t - \theta_n)} + n_{n_p}(i), \tag{3.23}$$

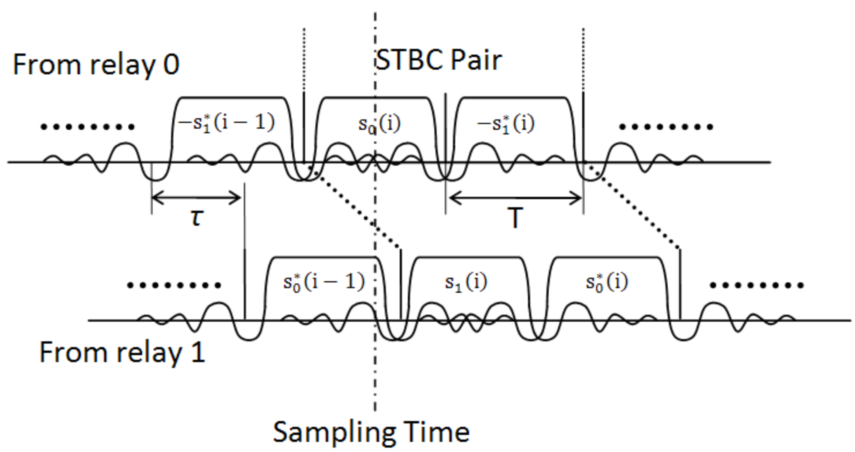
where α_n is the multipath fading gain of the channel between relay *n* and the destination. Term $n_{n_p}(i)$ is the pass-band noise, while τ_n and θ_n denote time delay and phase shift, respectively. The relative delay between R_0 and R_1 is therefore given by $\tau = \tau_1 - \tau_0$. The received signals at the data collector are linear combinations of $s_n(i)$. When the time delay and frequency offset between R_0 and D are different with those between R_1 and D, imperfect synchronization problem occurs. Figure 3.3 (a—c) shows the effect of time errors on sampling process at the receiver.



(a) $\tau = 0$ Perfect Synchronization



(b) $\tau \leq 0.5T$ Imperfect Synchronization



(c) $\tau > 0.5T$ Imperfect Synchronization

Figure 3.3: Impact of Imperfect Synchronization between 2 Relay Nodes.

For many practical STBC applications such as the cooperative transmission, it is impossible to achieve synchronization in time and frequency because signals from different relays have different τ_n and θ_n [23], we demodulate the signal with local carrier $e^{-j\omega_c t}$ and perform sampling at time instances $iT_s + t_0$ and $(i+1)T_s + t_0$ (for arbitrary t_0). Assuming the channel is quasi-static, the base-band samples therefore can be given by

$$r_0^{'}(i) = A[\sum_{n=0}^{1}\alpha_n b_n(i)\, p(t_0 - \tau_n)e^{-j\theta_n} + \sum_{k\neq i} ISI(s_0(k)) + \sum_{k=-\infty}^{+\infty} ISI(-s_1^*(k)) + \sum_{k\neq i} ISI(s_1(k)) + \sum_{k=-\infty}^{+\infty} ISI(s_0^*(k))] + n_0(i) \tag{3.24}$$

$$r_1^{'}(i) = A[-\alpha_0 b_1(i)\, p(t_0 - \tau_1)e^{j\theta_1} + \alpha_1 b_0(i)\, p(t_0 - \tau_0)e^{j\theta_0} + \sum_{k=-\infty}^{+\infty} ISI(s_0(k)) + \sum_{k\neq i} ISI(-s_1^*(k)) + \sum_{k=-\infty}^{+\infty} ISI(s_1(k)) + \sum_{k\neq i} ISI(s_0^*(k))] + n_1(i), \tag{3.25}$$

where $r_m^{'}(i)$ represents the received signal at symbol time slot $(i+m)T_s$ and $n_n(i)$ is the base-band noise. We use the prime variables to indicate that the received signals contain ISI. Equations (3.24) and (3.25) can be further simplified as

$$r_0^{'}(i) = h_0 s_0(i) + h_1 s_1(i) + I_{00} + I_{01} + n_0(i); \tag{3.26}$$

$$r_1^{'}(i) = -h_0 s_1^*(i) + h_1 s_0^*(i) + I_{10} + I_{11} + n_1(i), \tag{3.27}$$

where $h_n = \alpha_n e^{j\theta_n}$ denotes the channel between the relay *n* and the destination D. I_{mn} represents the ISI experienced by $r_m^{'}(i)$ from the symbol transmitted over channel *n*.

3.2.2 Simulations

In this section, we analyze the performance of a cooperative STBC system with two relay nodes and one receiver under imperfect synchronization. BPSK modulation is applied and the signal to noise ratio (SNR) is defined as $\mathrm{SNR} = \sigma_s^2 / \sigma_n^2 \left(\mathrm{dB}\right)$. We assume that the receiver has perfect knowledge of the carrier frequency of each transmitter, as well as the fading coefficient of each channel. We also assume that the channel is quasi-static and Rayleigh fading. The bit error rates (BER's) of the conventional STBC receiver with imperfect synchronization under different values of τ are shown in Figure 3.4. The performance of conventional STBC under perfect synchronization is also given in the simulation for comparison.

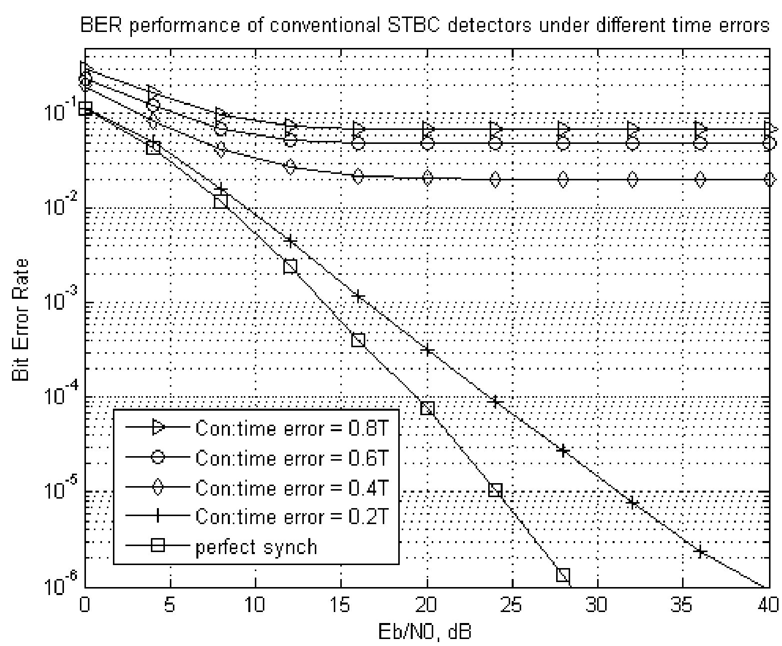


Figure 3.4: Performance of Alamouti's STBC under Imperfect Synchronization.

3.3 Previous Works

A summary of the existing techniques addressing STBC under imperfect channel estimation is as follows. The partial knowledge of CSI was discussed and a modified decision rule was proposed by Tarokh in [29]. The effect of imperfect channel estimation on STBC is analyzed in [9], [12], and [15]. Power-allocated quasi-orthogonal STBC is studied in [13]. Antenna selection technique is discussed in [21]. Analytical evaluation of the diversity combining technique under imperfect channel estimation is studied in [36]. All of above works assume that the transmitters and receivers are synchronized both in timing and frequency.

On the other hand, for STBC with imperfect synchronization, block-based equalization for STBC is studied in [17]. Parallel interference cancellation technique is investigated in [37]. Delay diversity technique is discussed in [35]. Design issues for distributed quasi-orthogonal STBC without perfect synchronization are studied in [20]. Analytical evaluation of error probability without the effect of noisy CSI is studied in [26]. In [23], the authors present solutions to STBC-OFDM system with timing and frequency errors. Performance of space-time trellis coding (STTC) under imperfect synchronization is studied in [39], [40]. None of the above works discusses noisy CSI effects.

Chapter 4

Linear MMSE Estimator

Estimation theory, which can be found at the heart of many modern electronic signal processing systems, is a branch of statistic that deals with estimating the values of a group of parameters based on observed or measured signals [16]. Based on the type of parameters of interest, approaches to statistic estimation can be divided into two major categories: the estimation of deterministic but unknown constants and the estimation of random variables. We consider the latter case only since all signals in our study are random variables. In order to estimate the parameters of interest, it is first necessary to determine a system model in which the parameters, as well as the points of uncertainty and noise, can be described. After deciding upon a model, an estimator needs to be developed or applied if an existing estimator is valid for the model.

In this chapter, we investigate one of the commonly used estimators in random signal processing: the minimum mean square error (MMSE) estimator [16]. As this thesis is based on the linear class of MMSE, the concept of L-MMSE is explained in detail. This chapter is organized as follows. In Section 4.1, we give the background information of the MMSE estimator. In Section 4.2, we introduce the L-MMSE and derive the general L-MMSE estimator for complex random variables.

4.1 Minimum Mean Square Error Estimator

In statistics, the mean square error (MSE) of an estimator is one of many ways to quantify the difference between an estimator and the true value of the quantity been estimated [16]. MSE measures the average of the square of the error. The error occurs because of the randomness or because the estimator dose not account for information that could produce a more accurate estimate. The MSE of an estimator $\hat{s}$ with respect to the estimated parameter *s* is defined as [16]

$$MSE(\hat{s}) = E[\left(s-\hat{s}\right)^2], \tag{4.1}$$

where $E\left[\cdot\right]$ is the expectation operator. Since *s* is a random variable, the expectation operator is with respect to the joint pdf $p\left(\mathbf{r},s\right)$, where **r** is the sequence of observed or measured signals. Thus (4.1) can be rewritten as

$$MSE(\hat{s}) = \iint \left(s-\hat{s}\right)^2 p\left(\mathbf{r},s\right) d\mathbf{r}ds \tag{4.2}$$

Using Bayes' theorem, $p\left(\mathbf{r},s\right)$ can be rewritten as

$$p\left(\mathbf{r},s\right) = p\left(s\,|\,\mathbf{r}\right) p\left(\mathbf{r}\right). \tag{4.3}$$

Substituting (4.3) into (4.2), we have

$$MSE(\hat{s}) = \int\left[\int \left(s-\hat{s}\right)^2 p\left(s\,|\,\mathbf{r}\right) ds\right] p\left(\mathbf{r}\right) d\mathbf{r}. \tag{4.4}$$

A minimum mean square error (MMSE) estimator describes the approach which minimizes the MSE and may be represented as

$$\begin{aligned}\arg\min MSE(\hat{s}) &= \arg\min E\left[(s-\hat{s})^2\right] \\ &= \arg\min \int\left[\int (s-\hat{s})^2\, p(s\,|\,\mathbf{r})\, ds\right] p(\mathbf{r})\, d\mathbf{r}\end{aligned}$$

To minimize (4.4), we fix **r** and derive the partial derivative of the integral in brackets with respect to s as

$$\begin{aligned}\frac{\partial}{\partial_s}\int (s-\hat{s})^2\, p(s\,|\,\mathbf{r})\, ds &= \int \frac{\partial}{\partial_s}(s-\hat{s})^2\, p(s\,|\,\mathbf{r})\, ds \\ &= \int -2(s-\hat{s})\, p(s\,|\,\mathbf{r})\, ds \\ &= -2\int s p(s\,|\,\mathbf{r})\, ds + 2\hat{s}\int p(s\,|\,\mathbf{r})\, ds \end{aligned} \tag{4.5}$$

Set (4.5) to zero results in

$$\begin{aligned}\hat{s} &= \int s p(s\,|\,\mathbf{r})\, ds \\ &= E[s\,|\,\mathbf{r}]\end{aligned} \tag{4.6}$$

Therefore, (4.6) is the MMSE estimator that minimizes $E\left[(s-\hat{s})^2\right]$.

In general, the MMSE estimator depends on the prior knowledge as well as the observed data [16]. If the connection between the prior knowledge and the measured data is weak, then the estimator will ignore the prior knowledge. Otherwise, the estimator will be biased towards the prior mean. Basically, the use of prior information always improves the estimation accuracy. On the other hand, choosing a wrong prior pdf may result in a poor estimator.

4.2 Linear MMSE Estimator

Since the evaluation of mean requires integration, the estimator shown in (4.6) cannot be used in practice. For practical MMSE estimators, we need to be able to express them in a closed form. One of many methods to determine a closed form for a MMSE estimator is to seek the technique minimizing MSE within a particular class, such as the class of linear estimators. The linear MMSE (L-MMSE) estimator is the estimator achieving minimum MSE among all estimators of the form $aX + b$ [16]. In this section, we concentrate on the class of linear estimators and derive the general closed form for the linear L-MMSE estimator.

We begin our discussion by assuming a parameter s is to be estimated based on single received signal r. The parameter s is models as the realization of a random variable. Later on, the solution is extended to multiple received signals. A linear estimation $\hat{s}$ of a transmitted symbol s using the received signal r is

$$\hat{s}(r) = ar + b, \tag{4.7}$$

where $\hat{s}$ and r are random variables. Choosing the weighting coefficients a and b to minimize the Mean Square Error (MSE) based on a measurement of r:

$$MSE(\hat{s}) = E[(s - \hat{s})^2 \mid r]. \tag{4.8}$$

Substituting (4.7) into (4.8) and differentiating with respect to b

$$
\begin{aligned}
\frac{\partial}{\partial b} E[(s-ar-b)^2 \,|r] &= -2E[(s-ar-b)\,|\,r] \\
&= -2E[s\,|\,r] + 2E[ar\,|\,r] + 2E[b] \\
&= -2E[s] + 2aE[r] + 2b \qquad (4.9)
\end{aligned}
$$

Please be noted that for continuous random signals, acknowledgement of single deterministic measurement will not change the mean of the signals. This theorem can be proved by the following equation:

$$
E\left[r\,|\,r=R\right] = \int_{-\infty}^{+\infty} \alpha\, f_{r|r=R}\left(\alpha\right) d\alpha = E\left[r\right],
$$

where R is a single deterministic measurement of r.

Setting (4.9) to zero produces

$$
b = E\left[s\right] - aE\left[r\right]. \qquad (4.10)
$$

Substituting (4.10) into (4.8), the MSE can be rewritten as

$$
\begin{aligned}
MSE(\hat{s}) &= E[s - ar - E[s] + aE[r]^2] \\
&= E[(s-E[s]) - a(r-E[r])^2] \\
&= E[s-E[s]^2] - 2aE[s-E[s]r - E[r]] + a^2 E[r-E[r]^2]. \qquad (4.11)
\end{aligned}
$$

If the means of s and r are zero, then

$$
MSE\left(\hat{s}\right) = C_{ss} - 2aC_{sr} + a^2 C_{rr}, \qquad (4.12)
$$

where C_{ss} is the variance of s, C_{sr} is the cross-covariance of s and r, and C_{rr} is the variance of r. We can minimize (4.12) by taking the gradient to yield

$$\frac{\partial MSE\left(\hat{s}\right)}{\partial a} = 2aC_{rr} - 2C_{sr} \,,$$

which when set to zero results in

$$a = C_{rr}^{-1}C_{sr} \,. \tag{4.13}$$

Substitute (4.10) and (4.13) into (4.7) yields

$$\begin{aligned}\hat{s}(r) &= ar + b \\ &= C_{rr}^{-1}C_{sr}r + E[s] - C_{rr}^{-1}C_{sr}E[r] \\ &= E[s] + C_{rr}^{-1}C_{sr}(r - E[r])\end{aligned} \tag{4.14}$$

This is the L-MMSE estimator for single random variable based on single received signal. If the means for *s* and *r* are zero, then

$$\hat{s}\left(r\right) = C_{rr}^{-1}C_{sr}r \,. \tag{4.15}$$

The minimum MSE is then given by substituting (4.13) into (4.12):

$$\begin{aligned}MSE(\hat{s}) &= E[(s - C_{rr}^{-1}C_{sr}r)^2] \\ &= E[s^2] - E[2sC_{rr}^{-1}C_{sr}r] + E[C_{rr}^{-2}C_{sr}^2 r^2] \\ &= C_{ss} - C_{sr}^2 C_{rr}^{-1}\end{aligned} \tag{4.16}$$

Now we extend the solution to multiple received signals. The L-MMSE estimator for a random variable based on multiple received signals does not entail anything new but only a need of vector calculations.

For multiple received signals $\mathbf{r} = \left[r\left[0\right], r\left[1\right] \ldots r\left[N-1\right]\right]^T$, we have

$$\hat{s}(\mathbf{r}) = \mathbf{a}^T\mathbf{r} + b\,,$$

where $\mathbf{a} = [a_0, a_1 \ldots a_{N-1}]^T$. N is the number of received signals. By applying the same procedure shown above, the solution of L-MMSE estimator for a random variable based on multiple received signals can be derived as

$$\mathbf{a} = \mathbf{C}_{\mathbf{rr}}^{-1}\mathbf{C}_{\mathbf{r}s}\,,$$

$$b = E[s] - \mathbf{a}^T E[\mathbf{r}]\,,$$

$$\hat{s}(\mathbf{r}) = E[s] + \mathbf{C}_{s\mathbf{r}}\mathbf{C}_{\mathbf{rr}}^{-1}(\mathbf{r} - E[\mathbf{r}])\,,$$

where $\mathbf{C}_{\mathbf{rr}}$ is the $N \times N$ covariance matrix of **r**, and $\mathbf{C}_{s\mathbf{r}}$ is the $1 \times N$ cross-covariance vector having the property that $\mathbf{C}_{s\mathbf{r}}^T = \mathbf{C}_{\mathbf{r}s}$.

If the means for *s* and all elements in **r** are zero, then

$$\hat{s}(\mathbf{r}) = \mathbf{C}_{s\mathbf{r}}\mathbf{C}_{\mathbf{rr}}^{-1}\mathbf{r}\,. \tag{4.17}$$

Chapter 5

Proposed Receiver for STBC with Imperfect Channel Estimation and Synchronization

In this chapter, we propose a new simple receiver for the STBC coded system under imperfect channel estimation and synchronization. The proposed receiver is designed for Alamouti's first STBC scheme with two transmit and one receive antennas [1], though it can be further extended to more general cases. We assume that the synchronization between two transmit antennas is imperfect but the relative delay is smaller than one symbol period. We also assume that the receiver has noisy CSI and the variance of the channel estimation error is available. The channel estimation error model and the ISI model derived in Chapter 3 are used in the derivation of the new receiver. The proposed receiver adopts the L-MMSE estimator to estimate the transmitted symbols. It utilizes a PIC detector to pre-cancel the ISI [37] and a modified decision rule to mitigate the impact of noisy CSI. The modified decision rule is derived from the pdf of the observed samples and has identical performance as the one proposed by Tarokh but with lower computational complexity [29]. When perfect channel estimation and synchronization are both unavailable, simulation and numerical results show that the proposed receiver outperforms conventional time error-combat alone techniques and/or CSI error-combat alone techniques.

This chapter is organized as follows. Section 5.1 gives background information of this chapter and the previous works in this area. Section 5.2 describes system models and assumptions used in this chapter. The derivation of the proposed receiver is given in Section 5.3. In Section 5.4 we compare the proposed receiver with conventional STBC detector, and investigate the relationship between the L-MMSE estimator and Alamouti's conventional linear combiner. Section 5.5 studies some advantages and disadvantages of the proposed receiver and concludes the chapter.

5.1 Background and Previous Works

Diversity techniques have been widely utilized in wireless communication systems to improve the system capacity and to combat channel fading [10]. Space-Time block coding (STBC) combines space and time diversity techniques and applies two transmit antennas and one receive antenna to increase the system resistance to multipath fading effect [32]. As stated earlier in Chapter 2, STBC is attractive to researchers because it provides high diversity order with low decoding complexity. Such outstanding features, however, is only achievable under perfect synchronization and channel estimation. In most of practical STBC applications such as the cooperative transmission, the multiple transmitters can never be precisely synchronized with each other. In addition, the receiver never has the perfect knowledge of CSI, since the channel fading factors are random variables. In such cases, the conventional receiver is no longer able to remove the cross-terms due to the mismatch in time and CSI, and thus the entire STBC system fails.

In addition to the previous works summarized in Chapter 3, systematic performance analyses of STBC with imperfect synchronization and channel estimation have been done in papers such as [15] and references therein. Beside of existing works addressing STBC for the joint problems, more works have been for other coded systems such as Turbo codes. For example, joint synchronization and channel estimation problem has been addressed by Sun and Valenti in [27] for Turbo codes; the utilizing of channel estimation error variance in the decision rule appears in Frenger's paper also for Turbo coded systems [7],[8]. Similar approaches can be taken here with different system models.

5.2 System Models and Assumptions

We consider the very first Alamouti's STBC, where 2 transmitting and 1 receiving antennas are used. A simplified system block diagram is shown in Figure 5.1. The source data stream is fed to the transmitter block, after a series of data processing including data compression and channel coding, the data stream is divided into separate data blocks with two symbols in each block. The data block is then encoded with STBC scheme and subdivided into separate symbol streams. Each symbol stream contains two encoded symbols. After the insertion of pilot symbols, each encoded data stream is then sent to one of the transmitting antennas and transmitted over the wireless channel after frequency up conversion and amplification. At the receiver, the signal received by the receiver is a linear combination of the signals transmitted from two transmitters plus noise. After amplification and frequency down conversion, the channel estimator uses the prior information of the pilot symbols and the received pilot sequence to estimate the channel fading factors. The decoder combines the estimated channel fading factors and the received signals from two symbol periods into two symbols and detects the transmitted data streams. The detected data streams are then sent to signal processors such as demodulator and source decoder to recover the source information [32].

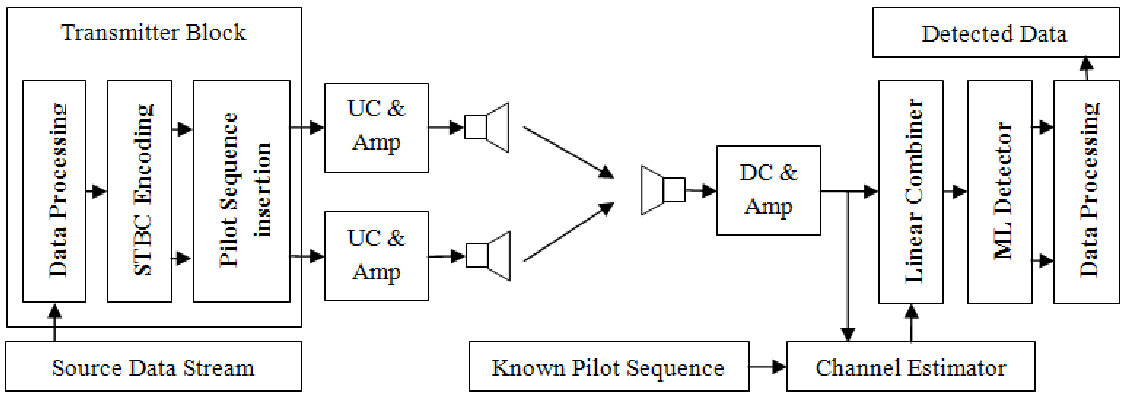


Figure 5.1: Block Diagram of Alamouti's STBC System.

As stated in Chapter 2, the code matrix for Alamouti's two-branch STBC is specified as

$$\mathbf{S} = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}.$$

We assume that the wireless channel is flat fading and quasi-static, so that the channel fading factors are constant over a symbol frame and vary from one frame to another. Since each transmit antenna goes through a different path to reach the receiver, the channel fading coefficients vector may be represented as

$$\mathbf{h} = [h_0, h_1]^T .$$

These fading factors are modeled as samples of zero mean, independent complex

Gaussian random variables with Rayleigh distributed amplitudes. The received signals with perfect synchronization may be represented as

$$\mathbf{r} = \mathbf{S}^T \mathbf{h} + \mathbf{n} = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}^T \begin{bmatrix} h_0 \\ h_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \end{bmatrix} = \begin{bmatrix} h_0 s_0 + h_1 s_1 + n_0 \\ -h_0 s_1^* + h_1 s_0^* + n_1 \end{bmatrix}. \tag{5.1}$$

The linear combiner combines the received signals and the channel fading factors estimated by the channel estimator with a simple linear combination rule given by

$$\hat{s} = \begin{bmatrix} \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = \begin{bmatrix} h_0^* r_0 + h_1 r_1^* \\ h_1^* r_0 - h_0 r_1^* \end{bmatrix}. \tag{5.2}$$

Substitute (5.1) into (5.2), the estimations of transmitted symbols under perfect synchronization and channel estimation would be

$$\hat{s} = \begin{bmatrix} \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = \begin{bmatrix} h_0^* r_0 + h_1 r_1^* \\ h_1^* r_0 - h_0 r_1^* \end{bmatrix} = \begin{bmatrix} \left(\alpha_0^2 + \alpha_1^2\right) s_0 + h_0^* n_0 + h_1 n_1^* \\ \left(\alpha_0^2 + \alpha_1^2\right) s_1 + h_1^* n_0 - h_0 n_1^* \end{bmatrix}. \tag{5.3}$$

Now, let us model the estimated channel fading coefficients and the estimation error caused by imperfect channel estimation. As described in Chapter 3, the estimated channel factor $\hat{h}$ can be expressed by

$$\hat{h} = h + e\,. \tag{5.4}$$

Consider the basic STBC system with two transmitters and one receiver. The information symbol $s(i)$ at symbol time slot i is encoded by the STBC encoder as STBC code symbols $c_0(i), c_1(i)$. Each code symbol is transmitted simultaneously from a different transmit antenna. Assuming ideal timing and frequency synchronization, the base-band signal received by the receive antenna can be represented as

$$r(i) = \sqrt{2E_s} \sum_{j=0}^{1} h_j(i) c_j(i) + n(i), \tag{5.5}$$

where $2E_s$ is the average energy of the base-band signal. $h_j(i)$ is the channel fading factor between transmit antenna j and receive antenna at time slot i. $n(i)$ is the independent base-band white Gaussian noise with $\mathcal{CN}(0, \sigma_n^2)$ distribution. Let

$$\mathbf{c}(i) = [c_0(i), c_1(i)]^T, \tag{5.6}$$

$$\mathbf{h}(i) = [h_0(i), h_1(i)]^T. \tag{5.7}$$

The received signals at time slot *i* can thus be written as

$$r(i) = \sqrt{2E_s} \mathbf{h}^T(i) \mathbf{c}(i) + n(i). \tag{5.8}$$

To estimate the channel vector $\mathbf{h}(i)$, we transmit a sequence of L pilot symbols with each transmit antenna, which forms a $2 \times L$ pilot symbol matrix given by

$$\mathbf{P} = \begin{bmatrix} p_{0,0} & \cdots & p_{0,L-1} \\ p_{1,0} & \cdots & p_{1,L-1} \end{bmatrix}, \tag{5.9}$$

where rows represent pilot symbols transmitted from different antennas. Columns represent different index in different pilot symbol sequences. Let the received pilot symbol matrix and noise matrix be given by

$$\mathbf{r_P} = \begin{bmatrix} r_{p_0} & \cdots & r_{p_{L-1}} \end{bmatrix}, \tag{5.10}$$

$$\mathbf{n_P} = \begin{bmatrix} n_{p_0} & \cdots & n_{p_{L-1}} \end{bmatrix}, \tag{5.11}$$

Using (5.9), (5.10) and (5.11), equation (5.8) can be rewritten as

$$\mathbf{r_P} = \sqrt{2E_s}\,\mathbf{h}^T(i)\,\mathbf{P} + \mathbf{n_P}. \tag{5.12}$$

The minimum mean square estimate of **h** can be obtained from (5.20) as

$$\begin{aligned}\hat{\mathbf{h}} &= \frac{1}{\sqrt{2E_s}}\mathbf{r_P}\mathbf{P}^H\left(\mathbf{P}\mathbf{P}^H\right)^{-1} \\ &= \frac{1}{\sqrt{2E_s}}\mathbf{r_P}\mathbf{P}^H / \left\|\mathbf{P}\right\|^2\end{aligned} \tag{5.13}$$

Combining (5.12) and (5.13), we have

$$\begin{aligned}\hat{\mathbf{h}} &= \mathbf{h} + \frac{1}{\sqrt{2E_s}}\mathbf{n_P}\mathbf{P}^H / \left\|\mathbf{P}\right\|^2 \\ &= \mathbf{h} + \mathbf{e},\end{aligned} \tag{5.14}$$

where **e** is the estimation error matrix given by

$$\mathbf{e} = \frac{1}{\sqrt{2E_s}}\mathbf{n_P}\mathbf{P}^H / \left\|\mathbf{P}\right\|^2. \tag{5.15}$$

Next, we model the received signal and ISI under imperfect synchronization. As shown in Chapter 3, the base-band received signals can be given by

$$r_0^{'}(i) = h_0 s_0(i) + h_1 s_1(i) + I_{00} + I_{01} + n_0(i); \tag{5.16}$$

$$r_1^{'}(i) = -h_0 s_1^*(i) + h_1 s_0^*(i) + I_{10} + I_{11} + n_1(i), \tag{5.17}$$

where $h_n = \alpha_n e^{j\theta_n}$ denote the channel fading factor between transmitter n and the receiver. I_{mn} represents the inter-symbol interference experienced by $r_m^{'}(i)$ from the symbol transmitted over channel n. For Alamouti's two-branch STBC,

$$I_{00} = \sum_{k \neq i} ISI\left(s_0(k)\right) + \sum_{k=-\infty}^{+\infty} ISI\left(-s_1^*(k)\right);$$

$$I_{01} = \sum_{k \neq i} ISI\left(s_1(k)\right) + \sum_{k=-\infty}^{+\infty} ISI\left(s_0^*(k)\right);$$

$$I_{10} = \sum_{k=-\infty}^{+\infty} ISI\left(s_0(k)\right) + \sum_{k \neq i} ISI\left(-s_1^*(k)\right);$$

$$I_{11} = \sum_{k=-\infty}^{+\infty} ISI\left(s_1(k)\right) + \sum_{k \neq i} ISI\left(s_0^*(k)\right).$$

Here, we assume that there is only one path between each transmit antenna and the receive antenna. The ISI is incurred only because of the imperfect synchronization between two paths, and the ISI from the same symbol over multipath is not considered. We also assume that the relative delay between two transmitters is no more than one symbol period *T*. This assumption is easy to meet in practice since coarse synchronization is always available.

5.3 Proposed Receiver for STBC with Imperfect Channel Estimation and Synchronization

In this section, we derive the proposed receiver for joint CSI and synchronization problems. The derivation is divided into four steps. Firstly, ideal system model is considered and the L-MMSE estimator for Alamouti's STBC is derived based on the structure discussed in Chapter 4. Secondly, we keep the assumption of synchronization unchanged and extend the L-MMSE estimator to imperfect channel estimation. A modified decision rule based on the channel estimation error variance is also given in this step. Next, imperfect synchronization is considered. A PIC-LS estimator is introduced for the estimation of the ideal received signals. Eventually, we combine the results from previous steps and give the complete expression of the proposed receiver.

5.3.1 L-MMSE Estimator for Ideal Cases

For ideal cases where full knowledge of CSI and synchronization are available, transmission sequence for Alamouti's two-branch STBC is given by

$$\mathbf{s}_t = [s_0, s_1];$$

$$\mathbf{s}_{t+T} = [-s_1^*, s_0^*].$$

After transmitting over different paths with channel fading factors $\mathbf{h} = [h_0, h_1]^T$ and additive white Gaussian noise (AWGN), the received signals will be

$$\mathbf{r} = [r_0, r_1]^T = \left[h_0 s_0 + h_1 s_1 + n_0, -h_0 s_1^* + h_1 s_0^* + n_1\right]^T,$$

where n_0 and n_1 are independent samples of the white Gaussian random process with $\mathcal{CN}\left(0, \sigma_n^2\right)$ distributions. By extending the solution in Chapter 4 to multiple transmitted signals, the linear estimation of $\mathbf{s} = [s_0, s_1]^T$ can be written as

$$\hat{\mathbf{s}}(\mathbf{r}) = \mathbf{A}^H \mathbf{r} + \mathbf{b}, \tag{5.18}$$

where $\hat{\mathbf{s}} = [\hat{s}_0, \hat{s}_1]^T$, $\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix}$, $\mathbf{b} = [b_0, b_1]^T$. The mean square error based on the measurements of received signals and estimated channel fading gains is given by:

$$MSE(\hat{\mathbf{s}}) = E\left[(\mathbf{s} - \hat{\mathbf{s}})^H (\mathbf{s} - \hat{\mathbf{s}}) \mid \mathbf{r}, \mathbf{h}\right] = E\left[|\mathbf{s} - \hat{\mathbf{s}}|^2 \mid \mathbf{r}, \mathbf{h}\right] \tag{5.19}$$

Using the results derived in Chapter 4, after minimizing this error, the coefficients matrix can be derived as:

$$\mathbf{A} = E\left[\mathbf{r}\mathbf{r}^H\right]^{-1} E\left[\mathbf{s}^* \mathbf{r}^T\right] = \mathbf{C}_{\mathbf{rr}}^{-1} \mathbf{C}_{\mathbf{rs}} \tag{5.20}$$

$$\mathbf{b} = E(\mathbf{s}) - \mathbf{A}^H E(\mathbf{r}) \tag{5.21}$$

Substituting (5.20) and (5.21) into (5.18), the linear estimation of the transmitted symbols can be rewritten as

$$\hat{\mathbf{s}}(\mathbf{r}) = E(\mathbf{s}) + \mathbf{C}_{\mathbf{sr}} \mathbf{C}_{\mathbf{rr}}^{-1} \left(\mathbf{r} - E(\mathbf{r})\right). \tag{5.22}$$

For $E(\mathbf{s}) = E(\mathbf{r}) = 0$, (5.22) can be further simplified as

$$\hat{\mathbf{s}}(\mathbf{r}) = \left(\mathbf{C}_{\mathbf{rr}}^{-1}\mathbf{C}_{\mathbf{rs}}\right)^{T} \mathbf{r} = \mathbf{C}_{\mathbf{sr}}\mathbf{C}_{\mathbf{rr}}^{-1}\mathbf{r}\ . \tag{5.23}$$

Here, $\mathbf{C}_{\mathbf{rr}}$ is the 2×2 covariance matrix of the complex random vector $\mathbf{r}$ which can be written as

$$\mathbf{C}_{\mathbf{rr}} = \begin{bmatrix} C_{r_0 r_0} & C_{r_0 r_1} \\ C_{r_1 r_0} & C_{r_1 r_1} \end{bmatrix}. \tag{5.24}$$

For r_0, r_1 complex, we have

$$C_{r_0 r_0} = E\left[\left(r_0 - E[r_0]\right)\left(r_0 - E[r_0]\right)^{*}\right]$$

$$C_{r_0 r_1} = E\left[\left(r_0 - E[r_0]\right)\left(r_1 - E[r_1]\right)^{*}\right]$$

In our case, $E[s_0] = E[s_1] = E[r_0] = E[r_1] = 0$ and $C_{s_0 s_0} = C_{s_1 s_1} = \sigma_s^2 = 1$. Elements in the covariance matrix $\mathbf{C}_{\mathbf{rr}}$ therefore can be calculated as

$$\begin{aligned} C_{r_0 r_0} &= E\left[\left(r_0 - E[r_0]\right)\left(r_0 - E[r_0]\right)^{*}\right] \\ &= E\left[\left(h_0 s_0 + h_1 s_1 + n_0\right)\left(h_0^* s_0^* + h_1^* s_1^* + n_0^*\right)\right] \\ &= E\left[h_0 h_0^* s_0 s_0^* + h_0 s_0 h_1^* s_1^* + h_0 s_0 n_0 + h_1 s_1 h_0^* s_0^* + h_1 h_1^* s_1 s_1^* + h_1 s_1 n_0 + n_0 h_0^* s_0^* + n_0 h_1^* s_1^* + n_0 n_0^*\right] \\ &= \left|h_0\right|^2 + \left|h_1\right|^2 + \sigma_n^2; \end{aligned}$$

$$
\begin{aligned}
\mathrm{C}_{r_0 r_1} &= E\left[\left(r_0 - E\left[r_0\right]\right)\left(r_1 - E\left[r_1\right]\right)^*\right] \\
&= E[(h_0 s_0 + h_1 s_1 + n_0)(-h_0^* s_1 + h_1^* s_0 + n_1^*)] \\
&= E[-h_0 s_0 h_0^* s_1 + h_0 s_0 h_1^* s_0 + h_0 s_0 n_1 - h_1 s_1 h_0^* s_1 + h_1 s_1 h_1^* s_0 + h_1 s_1 n_1 - n_0 h_0^* s_1 + n_0 h_1^* s_0 + n_0 n_1^*] \\
&= 0;
\end{aligned}
$$

$$
\mathrm{C}_{r_1 r_0} = \mathrm{C}_{r_0 r_1} = 0;
$$

$$
\begin{aligned}
\mathrm{C}_{r_1 r_1} &= E\left[\left(r_1 - E\left[r_1\right]\right)\left(r_1 - E\left[r_1\right]\right)^*\right] \\
&= E\left[\left(-h_0 s_1^* + h_1 s_0^* + n_1\right)\left(-h_0 s_1^* + h_1 s_0^* + n_1\right)^*\right] \\
&= \left|h_0\right|^2 + \left|h_1\right|^2 + \sigma_n^2.
\end{aligned}
$$

As a result, the covariance matrix of **r** may be represented as

$$
\mathbf{C}_{\mathbf{rr}} = \begin{bmatrix} \left|h_0\right|^2 + \left|h_1\right|^2 + \sigma_n^2 & 0 \\ 0 & \left|h_0\right|^2 + \left|h_1\right|^2 + \sigma_n^2 \end{bmatrix}. \tag{5.25}
$$

On the other hand, $\mathbf{C}_{\mathbf{sr}}$ is the 2×2 cross-covariance matrix of the complex random vectors **s** and **r** given by

$$
\mathbf{C}_{\mathbf{sr}} = \begin{bmatrix} \mathrm{C}_{s_0 r_0} & \mathrm{C}_{s_0 r_1} \\ \mathrm{C}_{s_1 r_0} & \mathrm{C}_{s_1 r_1} \end{bmatrix}. \tag{5.26}
$$

Elements in $\mathbf{C}_{\mathbf{sr}}$ can be calculated as

$$
\mathrm{C}_{s_0 r_0} = E\left[\left(s_0 - E\left[s_0\right]\right)\left(r_0 - E\left[r_0\right]\right)^*\right] = E\left[h_0^* s_0 s_0^* + h_1^* s_0 s_1^* + s_0 n_0^*\right] = h_0^*
$$

$$C_{s_0 r_1} = E\left[\left(s_0 - E[s_0]\right)\left(r_1 - E[r_1]\right)^*\right] = E\left[-h_0^* s_0 s_1 + h_1^* {s_0}^2 + s_0 n_1^*\right] = h_1^*$$

$$C_{s_1 r_0} = E\left[\left(s_1 - E[s_1]\right)\left(r_0 - E[r_0]\right)^*\right] = E\left[h_0^* s_0^* s_1 + h_1^* s_1 s_1^* + s_1 n_0^*\right] = h_1^*$$

$$C_{s_1 r_1} = E\left[\left(s_1 - E[s_1]\right)\left(r_1 - E[r_1]\right)^*\right] = E\left[-h_0^* {s_1}^2 + h_1^* s_1 s_0 + s_1 n_1^*\right] = -h_0^*$$

Therefore, (5.26) can be rewritten as

$$\mathbf{C_{sr}} = \begin{bmatrix} h_0^* & h_1^* \\ h_1^* & -h_0^* \end{bmatrix}. \tag{5.27}$$

Substituting (5.25), (5.27) into (5.23), the L-MMSE estimator for STBC with perfect synchronization and channel estimation can be derived as

$$\begin{aligned}
\hat{\mathbf{s}}(\mathbf{r}) &= \mathbf{C_{sr}}\mathbf{C_{rr}^{-1}}\mathbf{r} \\
&= \left\{ \begin{bmatrix} h_0^* & h_1^* \\ h_1^* & -h_0^* \end{bmatrix} \begin{bmatrix} |h_0|^2 + |h_1|^2 + \sigma_n^2 & 0 \\ 0 & |h_0|^2 + |h_1|^2 + \sigma_n^2 \end{bmatrix}^{-1} \right\} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} \\
&= \begin{bmatrix} \dfrac{h_0^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} & \dfrac{h_1^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \\ \dfrac{h_1^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} & \dfrac{-h_0^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} \\
&= \begin{bmatrix} \dfrac{\left(|h_0|^2 s_0 + |h_1|^2 s_0^*\right) + \left(h_0^* h_1 s_1 - h_1^* h_0 s_1^*\right) + h_0^* n_0 + h_1^* n_1}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \\ \dfrac{\left(|h_1|^2 s_1 + |h_0|^2 s_1^*\right) + \left(h_1^* h_0 s_0 - h_0^* h_1 s_0^*\right) + h_1^* n_0 - h_0^* n_1}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \end{bmatrix}
\end{aligned} \tag{5.28}$$

This is the closed form of the L-MMSE estimator for ideal STBC system where perfect knowledge of CSI and synchronization are both available. Simulation and numerical results show that for ideal cases, the proposed L-MMSE estimator has the same performance as the conventional detector. Detailed comparisons and analyses will be given in Section 5.4. Next, we derive the L-MMSE estimator for STBC with imperfect channel estimation.

5.3.2 L-MMSE Estimator for Noisy CSI

Now, we study STBC with the appearance of noisy CSI. At this stage, we assume that the synchronization is available both in time and frequency. As shown in Section 5.2, the imperfect estimation of the channel can be represented as:

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{e}, \tag{5.29}$$

where $\hat{\mathbf{h}} = \left[\hat{h}_0, \hat{h}_1\right]^T$, $\mathbf{h} = \left[h_0, h_1\right]^T$, $\mathbf{e} = \left[e_0, e_1\right]^T$. The estimation errors e_0 and e_1 are Gaussian distributed complex random variables with the standard variance of $\sigma_{e_0}^2$ and $\sigma_{e_1}^2$. Without loss of generality, we assume that $\sigma_{e_0}^2 = \sigma_{e_1}^2 = \sigma_e^2$.

From (5.29), we can derive that

$$\mathbf{h} = \hat{\mathbf{h}} - \mathbf{e}.$$

The mean square error based on the measurements of received signals and imperfect channel estimation can be given by:

$$MSE(\hat{\mathbf{s}}) = E\left[(\mathbf{s}-\hat{\mathbf{s}})^H (\mathbf{s}-\hat{\mathbf{s}}) | \hat{\mathbf{h}}\right] = E\left[\left|\mathbf{s}-\hat{\mathbf{s}}\right|^2 | \hat{\mathbf{h}}\right]$$

The L-MMSE estimator given in (5.23) can be rewritten as:

$$\hat{\mathbf{s}}(\mathbf{r}) = \mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}} \mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}^{-1} \mathbf{r} \tag{5.30}$$

Elements in $\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}$ can be calculated as

$$\begin{aligned}
C_{r_0 r_0} &= E\left[(r_0 - E[r_0])(r_0 - E[r_0])^* | \hat{\mathbf{h}}\right] \\
&= E\left[\left(\left(\hat{h}_0 - e_0\right)s_0 + \left(\hat{h}_1 - e_1\right)s_1 + n_0\right)\left(\left(\hat{h}_0 - e_0\right)^* s_0^* + \left(\hat{h}_1 - e_1\right)^* s_1^* + n_0^*\right)\right] \\
&= E\left[\left(\hat{h}_0 - e_0\right)\left(\hat{h}_0 - e_0\right)^*\right] + E\left[\left(\hat{h}_1 - e_1\right)\left(\hat{h}_1 - e_1\right)^*\right] + \sigma_n^2 \\
&= \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2;
\end{aligned}$$

$$\begin{aligned}
C_{r_0 r_1} &= E\left[(r_0 - E[r_0])(r_1 - E[r_1])^* | \hat{\mathbf{h}}\right] \\
&= E\left[\left(\left(\hat{h}_0 - e_0\right)s_0 + \left(\hat{h}_1 - e_1\right)s_1 + n_0\right)\left(-\left(\hat{h}_0 - e_0\right)^* s_1 + \left(\hat{h}_1 - e_1\right)^* s_0 + n_1^*\right)\right] \\
&= 0;
\end{aligned}$$

$$C_{r_1 r_0} = C_{r_0 r_1} = 0;$$

$$\begin{aligned}
C_{r_1 r_1} &= E\left[(r_1 - E[r_1])(r_1 - E[r_1])^* | \hat{\mathbf{h}}\right] \\
&= E\left[\left(-\left(\hat{h}_0 - e_0\right)s_1^* + \left(\hat{h}_1 - e_1\right)s_0^* + n_1\right)\left(-\left(\hat{h}_0 - e_0\right)s_1^* + \left(\hat{h}_1 - e_1\right)s_0^* + n_1\right)^*\right] \\
&= \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2.
\end{aligned}$$

Elements in $\mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}}$ can be calculated as

$$C_{s_0 r_0} = E\left[\left(s_0 - E[s_0]\right)\left(r_0 - E[r_0]\right)^* \mid \hat{\mathbf{h}}\right] = \hat{h}_0^*$$

$$C_{s_0 r_1} = E\left[\left(s_0 - E[s_0]\right)\left(r_1 - E[r_1]\right)^* \mid \hat{\mathbf{h}}\right] = \hat{h}_1^*$$

$$C_{s_1 r_0} = E\left[\left(s_1 - E[s_1]\right)\left(r_0 - E[r_0]\right)^* \mid \hat{\mathbf{h}}\right] = \hat{h}_1^*$$

$$C_{s_1 r_1} = E\left[\left(s_1 - E[s_1]\right)\left(r_1 - E[r_1]\right)^* \mid \hat{\mathbf{h}}\right] = -\hat{h}_0^*$$

The coefficient matrixes $\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}$ and $\mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}}$ can therefore be given by

$$\mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}} = \begin{bmatrix} \hat{h}_0^* & \hat{h}_1^* \\ \hat{h}_1^* & -\hat{h}_0^* \end{bmatrix}, \tag{5.31}$$

$$\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}} = \begin{bmatrix} \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2 & 0 \\ 0 & \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2 \end{bmatrix}. \tag{5.32}$$

Substituting (5.31), (5.32) into (5.30), the L-MMSE estimator for STBC with imperfect channel estimation can be derived as

$$\begin{aligned} \hat{\mathbf{s}}(\mathbf{r}) &= \mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}} \mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}^{-1} \mathbf{r} \\ &= \begin{bmatrix} \dfrac{\hat{h}_0^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} & \dfrac{\hat{h}_1^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} \\ \dfrac{\hat{h}_1^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} & \dfrac{-\hat{h}_0^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} \end{aligned} \tag{5.33}$$

This is the closed form of the L-MMSE estimator for STBC with imperfect channel estimation. Please be noted that at this stage, we assume the synchronization is available.

As shown in (5.33), the cross-terms cannot be cancelled due to the imperfect channel estimation. Therefore, estimation errors will be introduced into the ML-detector and degrade system performance in terms of error rate. Here, modify the decision rule by utilizing the error information derived from the channel estimator.

For simplification, we define the cross-correlation matrix:

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{bmatrix}, \tag{5.34}$$

where ρ_{ij} is the cross-correlation coefficient of r_i and h_j given by

$$\rho_{00} = \frac{C_{r_0 h_0}}{\sigma_{r_0}\sigma_{h_0}} = \frac{s_0 \sigma_h^2}{\sigma_r \sigma_{\hat{h}}}$$

$$\rho_{01} = \frac{C_{r_0 h_1}}{\sigma_{r_0}\sigma_{h_1}} = \frac{s_1 \sigma_h^2}{\sigma_r \sigma_{\hat{h}}}$$

$$\rho_{10} = \frac{C_{r_1 h_0}}{\sigma_{r_1}\sigma_{h_0}} = -\left(\frac{s_1 \sigma_h^2}{\sigma_r \sigma_{\hat{h}}}\right)^*$$

$$\rho_{11} = \frac{C_{r_1 h_1}}{\sigma_{r_1}\sigma_{h_1}} = \left(\frac{s_0 \sigma_h^2}{\sigma_r \sigma_{\hat{h}}}\right)^*$$

We further define

$$
\begin{aligned}
|\rho|^2 &= |\rho_{00}|^2 + |\rho_{01}|^2 \\
&= |\rho_{10}|^2 + |\rho_{11}|^2 \\
&= \left(|s_0|^2 + |s_1|^2\right)\frac{\sigma_h^4}{\sigma_r^2\sigma_{\hat{h}}^2} .
\end{aligned} \tag{5.35}
$$

The pdf of the received signal conditioned on the channel estimation and transmitted signals can be expressed as [29]:

$$
\begin{aligned}
p_{r|s,\hat{h}} = &\frac{1}{2\pi\sigma_r^2\left(1-|\rho|^2\right)}\exp\left[\frac{-1}{2\sigma_r^2\left(1-|\rho|^2\right)}\left|r_0 - \left(\rho_{00}\hat{h}_0 + \rho_{01}\hat{h}_1\right)\frac{\sigma_r}{\sigma_h}\right|^2\right] \\
&\times\frac{1}{2\pi\sigma_r^2\left(1-|\rho|^2\right)}\exp\left[\frac{-1}{2\sigma_r^2\left(1-|\rho|^2\right)}\left|r_1 - \left(\rho_{00}^*\hat{h}_1 - \rho_{01}^*\hat{h}_2\right)\frac{\sigma_r}{\sigma_h}\right|^2\right].
\end{aligned} \tag{5.36}
$$

The pdf shown in (5.36) may be represented as a simplified form of

$$
p_{r|s,\hat{h}} = p_{r_0|s,\hat{h}} \times p_{r_1|s,\hat{h}} . \tag{5.37}
$$

Therefore, the conditional distributions of r_0 and r_1 can be expressed as independent Gaussian distributed random variables with conditional expectations and variances of

$$
E\left[r_0\right] = \left(\rho_{00}\hat{h}_0 + \rho_{01}\hat{h}_1\right)\frac{\sigma_r}{\sigma_h} ,
$$

$$
E\left[r_1\right] = \left(\rho_{00}^*\hat{h}_1 - \rho_{01}^*\hat{h}_2\right)\frac{\sigma_r}{\sigma_h} ,
$$

$$
E\left[r_0^2\right] = E\left[r_1^2\right] = 2\pi\sigma_r^2\left(1-|\rho|^2\right).
$$

For modulations with equal energy constellations (the PSK signals for example), the maximum likelihood detector at the receiver is to choose a pair of symbols in the constellation to minimize the metric

$$d^2\left(r_0, E[r_0]\right) + d^2\left(r_1, E[r_1]\right), \tag{5.38}$$

Substitute $E[r_0]$ and $E[r_1]$ in (5.38) with the expectations derived above, the metric can be rewritten as

$$\left| r_0 - \left(\rho_{00}\hat{h}_0 + \rho_{01}\hat{h}_1\right)\frac{\sigma_r}{\sigma_h}\right|^2 + \left| r_1 - \left(\rho_{00}^*\hat{h}_1 - \rho_{01}^*\hat{h}_2\right)\frac{\sigma_r}{\sigma_h}\right|^2 \tag{5.39}$$

After some expanding and manipulation, (5.39) can be further written as

$$\begin{aligned} &\left| \left(r_0\hat{h}_0^* + r_1^*\hat{h}_1\right)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_0 \right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1 \right)\left|s_0\right|^2 \\ &+ \left| \left(r_0\hat{h}_1^* - r_1^*\hat{h}_0\right)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_1 \right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1 \right)\left|s_1\right|^2 \end{aligned} \tag{5.40}$$

Obviously, the above expression consists of two parts:

$$\left| \left(r_0\hat{h}_0^* + r_1^*\hat{h}_1\right)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_0 \right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1 \right)\left|s_0\right|^2$$

$$\left| \left(r_0\hat{h}_1^* - r_1^*\hat{h}_0\right)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_1 \right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1 \right)\left|s_1\right|^2$$

Therefore, the modified decision rule for STBC with imperfect channel estimation can be given by the decision metric

$$\left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_0 - s_i\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_i\right|^2 \le \left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_0 - s_k\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_k\right|^2, \quad (5.41)$$

for detecting s_0 and the decision metric

$$\left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_1 - s_i\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_i\right|^2 \le \left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_1 - s_k\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_k\right|^2, \quad (5.42)$$

for detecting s_1. Compared with the decision rules for perfect knowledge of CSI, which are given by the decision metric

$$\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 - 1\right)\left|s_i\right|^2 + d^2\left(\hat{s}_0, s_i\right) \le \left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 - 1\right)\left|s_k\right|^2 + d^2\left(\hat{s}_0, s_k\right), \quad \forall i \ne k$$

for detecting s_0 and the decision metric

$$\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 - 1\right)\left|s_i\right|^2 + d^2\left(\hat{s}_1, s_i\right) \le \left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 - 1\right)\left|s_k\right|^2 + d^2\left(\hat{s}_1, s_k\right), \quad \forall i \ne k$$

for detecting s_1. The modified decision rule is optimum since it includes the estimation error information and makes the maximum likelihood decisions based on the pdf(s) conditioned on the estimation of the channel fading factors [29]. Next, we investigate the case where synchronization is unavailable in both time and frequency.

5.3.3 L-MMSE Estimator for Imperfect Synchronization

In this subsection, we derive the L-MMSE estimator for STBC with imperfect synchronization. To simplify the deduction process, we consider the synchronization problem alone and assume that the receiver has perfect knowledge of CSI. As shown in Section 5.2, the simplified system models for the received STBC symbol sequences under imperfect synchronization within two symbol time slots can be represented as

$$r_0^{'}(i) = h_0 s_0(i) + h_1 s_1(i) + I_{00} + I_{01} + n_0(i); \tag{5.43}$$

$$r_1^{'}(i) = -h_0 s_1^{*}(i) + h_1 s_0^{*}(i) + I_{10} + I_{11} + n_1(i), \tag{5.44}$$

or in the matrix form:

$$\mathbf{r}^{'} = \mathbf{r} + \mathbf{I}_0 + \mathbf{I}_1$$

where I_{mn} represents the ISI experienced by $r_m^{'}(i)$ from the symbol transmitted over channel n. Here, we assume that the ISI is incurred only because of the imperfect synchronization between two paths, and the ISI from the same symbol over multipath will not be considered. We also assume that the relative delay between two transmitters is smaller than one symbol period T.

It is obviously that if we apply $r_0^{'}(i)$ and $r_1^{'}(i)$ directly to the L-MMSE estimator derived in Subsection 5.3.1, the system will fail due to the ISI. Some mechanism therefore needs to be involved in the system to mitigate the impact of this interference. Among the many ISI combating techniques is the parallel interference cancellation (PIC) [5], [34],

[37] and [38], which provides a near-optimum reception for wireless signals undergoing ISI with very low computational complexity. In this part, we propose a simple PIC least-squares (LS) detector. The PIC-LS detector is to utilize the received coarse synchronized symbol sequences to estimate the ones for ideal cases, and then to send the estimations to the detection device. In other words, we manipulate with the imperfectly synchronized signals and make them as close to the ones without time errors as possible.

With perfect synchronization, the received STBC coded symbols within arbitrary 2 symbol time slots are given by

$$r_0(i) = h_0 s_0(i) + h_1 s_1(i) + n_0(i);$$

$$r_1(i) = -h_0 s_1^*(i) + h_1 s_0^*(i) + n_1(i).$$

From (5.43) and (5.44), the optimum estimations of $r_0(i)$ and $r_1(i)$ based on $r_0^{'}(i)$ and $r_1^{'}(i)$ can be given as

$$\hat{r}_0(i) = r_0^{'}(i) - ISI_{00} - ISI_{01}; \tag{5.45}$$

$$\hat{r}_1(i) = r_1^{'}(i) - ISI_{10} - ISI_{11}. \tag{5.46}$$

In practice, the actual ISI_{mn} consist of interference components ranging from $s_n(-\infty)$ to $s_n(+\infty)$ and have no closed forms. Therefore, simplified models need to be built to represent the ISI approximately. The requirements for the new models are to be as close to the actual ISI as possible, as well as to have simple closed forms.

Considering a scenario shown in the Figure 5.2, without perfect synchronization, the ISI from neighbor symbols will be introduced at the receiver due to the relative delay τ between r_0 and r_1.

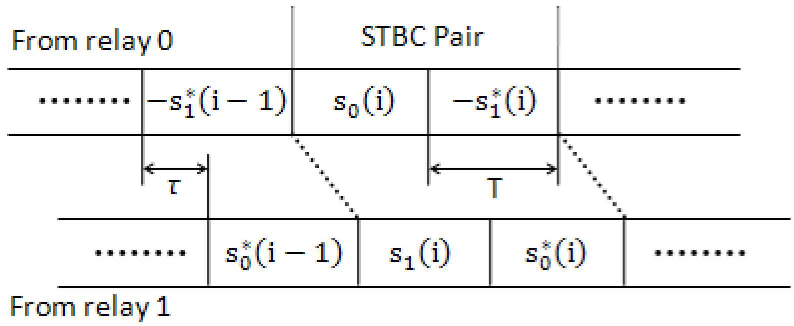


Figure 5.2: STBC Transmission with Imperfect Synchronization.

We assume that the relative delay is no greater than the symbol period. This assumption is easy to meet in practice since rough synchronization is always required. Without loss of generality, we also assume that the receiver is perfectly synchronized with relay 0. Therefore, the simplified models for the received signals over 2 symbol time slots can be given by [37]

$$r_0^{'}(i) = h_0 s_0(i) + h_1 s_1(i) + \beta h_1 s_0^*(i-1) e^{j\varphi} + n_0(i); \tag{5.47}$$

$$r_1^{'}(i) = -h_0 s_1^*(i) + h_1 s_0^*(i) + \beta h_1 s_1(i) e^{j\varphi} + n_1(i), \tag{5.48}$$

where $\beta h_1 s_0^*(i-1) e^{j\varphi}$ and $\beta h_1 s_1(i) e^{j\varphi}$ are the simplified ISI models for $r_0^{'}(i)$ and

$r_1^{'}(i)$, respectively. They are attenuated and phase shifted versions of the previous symbols transmitted over the second channel. Coefficients β and φ represent the impact of time errors on signals amplitude and phase, respectively. Since the imperfect synchronization we considered is only caused by the relative delay, the impact of φ can be ignored. Also, since the defined model only contains the inter-symbol interference from adjacent previous symbols, it achieves low computational complexity by some sacrifice of performance. However, for most practical baseband pulse shaping waveforms such as the raised cosine, the simplified model is already the dominant part of ISI and covers most of the interference energy [37].

Based on the ISI models in (5.47) and (5.48), the near-optimum estimators for $r_0(i)$ and $r_1(i)$ using $r_0^{'}(i)$ and $r_1^{'}(i)$ can be derived as

$$\hat{r}_0\left(r_0^{'}\right) = r_0^{'}(i) - \beta h_1 s_0^{*}(i-1) e^{j\varphi}; \tag{5.49}$$

$$\hat{r}_1\left(r_1^{'}\right) = r_1^{'}(i) - \beta h_1 s_1(i) e^{j\varphi}. \tag{5.50}$$

or in the matrix form:

$$\hat{\mathbf{r}} = \mathbf{r}^{'} - \mathbf{I}$$

The complex coefficient $\beta e^{j\varphi}$ can be estimated by minimizing the following least-squares estimator:

$$\arg\min \left| r_0(i) - \left(r_0^{'}(i) - \beta e^{j\varphi} h_1 s_0^{*}(i-1) \right) \right|^2. \tag{5.51}$$

The estimation process needs to be initialized by pilot symbol sequences. The PIC-LS detection procedure is shown as follows [37].

Step 1: The estimation of $\beta e^{j\varphi}$.

Before the transmission of data symbols, orthogonal pilot matrix are inserted to the head of the data given by

$$\mathbf{P} = \begin{bmatrix} p_0(0) & p_0(1) \\ p_1(0) & p_1(1) \end{bmatrix},$$

where columns represent different transmit antennas and rows are different time slots. After the transmission, the received signals with time errors are given by

$$\begin{aligned} \mathbf{r}_\mathbf{p}^{'} &= \mathbf{h}^T\mathbf{P} + \mathbf{n} + \mathbf{I} \\ &= \mathbf{r}_\mathbf{p} + \mathbf{I} \end{aligned} \tag{5.52}$$

where $\mathbf{r}_p$ is the ideal received pilot symbol matrix. $\mathbf{I}$ represents the inter-symbol interference. Take a sample at time slot (1), the received signal is

$$\begin{aligned} r_p^{'}(1) &= h_0 p_0(1) + h_1 p_1(1) + n(1) + I(1) \\ &= r_p(1) + I(1) \\ &= r_p(1) + \beta e^{j\varphi} h_1 p_1(0) \end{aligned}$$

Therefore, the estimation of the complex coefficient $\beta e^{j\varphi}$ can be derived as

$$\beta e^{j\varphi} = \frac{r_p^{'}(1) - r_p(1)}{h_1 p_1(0)}. \tag{5.53}$$

For imperfect channel estimation,

$$\beta e^{j\varphi} = \frac{r_p^{'}(1) - r_p(1)}{\hat{h}_1 p_1(0)}$$

At this stage, we assume that this coefficient keeps the same for the entire data packet. Therefore, the estimation of $\beta e^{j\varphi}$ does not need to be done repeatedly and the insertion of pilot symbols is only required at the beginning of the detection.

Step 2: The PIC iteration.

After the insertion of the pilot symbols, we transmit the data symbols by applying STBC code. We define the first STBC code matrix as

$$\mathbf{S}(0) = \begin{bmatrix} s_0(0) & -s_1^*(0) \\ s_1(0) & s_0^*(0) \end{bmatrix}.$$

The received signals with ISI at 2 time slots are given by

$$r_0^{'} = h_0 s_0(0) + h_1 s_1(0) + n_0 + I_0; \tag{5.54}$$

$$r_1^{'} = -h_0 s_1^*(0) + h_1 s_0^*(0) + n_1 + I_1. \tag{5.55}$$

Upon examining (5.54), we notice that based on our simplified ISI model, I_0 is the interference from the pilot symbol $p_1(1)$ since the first STBC pair is adjacent to the pilot symbols. Therefore, we can remove I_0 at the initialization stage and derive the optimum estimation for ideal received signal r_0 as

$$\hat{r}_0 = r_0^{'} - \beta e^{j\varphi} h_1 p_1(1). \tag{5.56}$$

We utilize $\hat{r}_0$ and $r_1^{'}$ to start the iteration and input them to the rest part of the receiver [37]. After the detection, we have the initial guess of the transmitted symbols as $s_0^{(0)}(0)$, $s_1^{(0)}(0)$. Our next step is to input $s_1^{(0)}(0)$ back to the PIC detector and calculate the first guess of I_1 as

$$I_1^{(0)} = \beta e^{j\varphi} h_1 s_1^{(0)}(0). \tag{5.57}$$

Therefore, the initial guess of the ideal received signal r_1 can be calculated as

$$\hat{r}_1^{(0)} = r_1^{'} - \beta e^{j\varphi} h_1 s_1^{(0)}(0). \tag{5.58}$$

Then we utilize $\hat{r}_0$ and $\hat{r}_1^{(0)}$ to run the next iteration. After K iterations, the estimated signals $\hat{r} = \left[\hat{r}_0, \hat{r}_1^{(K)}\right]^T$ will be used as the optimum estimation of the ideal received signals [37]. Since the channel fading factors keep the same for the entire data package, the fading factor h_1 in both $\hat{r}_0$ and $\hat{r}_1^{(K)}$ will be cancelled in the component $\beta e^{j\varphi} h_1$, therefore, the impact of imperfect channel estimation does not affect the performance of the PIC detector. This feature makes the PIC detector an ideal pre-processor for joint channel estimation and synchronization problems.

The simulation results show that the optimum iteration number for 2-branch STBC system is 2 or 3, which means the increase in computational complexity is very moderate. On the other hand, since the PIC detection consists of linear calculation only, the proposed receiver preserves linear decoder. The estimations of the ideal received signals are then applied to the L-MMSE estimator designed for ideal cases to estimate the transmitted symbols. The coefficient matrixes for the L-MMSE estimator with perfect synchronization are derived in Subsection 5.2.1 and given by

$$\mathbf{C}_{\mathbf{sr}} = \begin{bmatrix} h_0^* & h_1^* \\ h_1^* & -h_0^* \end{bmatrix},$$

$$\mathbf{C}_{\mathbf{rr}} = \begin{bmatrix} |h_0|^2 + |h_1|^2 + \sigma_n^2 & 0 \\ 0 & |h_0|^2 + |h_1|^2 + \sigma_n^2 \end{bmatrix}.$$

The L-MMSE estimator for STBC with imperfect synchronization can be represented as

$$\begin{aligned} \hat{\mathbf{s}}(\hat{\mathbf{r}}) &= \mathbf{C}_{\mathbf{sr}|\hat{\mathbf{r}}(i)} \mathbf{C}_{\mathbf{rr}|\hat{\mathbf{r}}(i)}^{-1} \hat{\mathbf{r}} \\ &= \left\{ \begin{bmatrix} h_0^* & h_1^* \\ h_1^* & -h_0^* \end{bmatrix} \begin{bmatrix} |h_0|^2 + |h_1|^2 + \sigma_n^2 & 0 \\ 0 & |h_0|^2 + |h_1|^2 + \sigma_n^2 \end{bmatrix}^{-1} \right\} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{h_0^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} & \dfrac{h_1^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \\ \dfrac{h_1^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} & \dfrac{-h_0^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \end{bmatrix} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \end{bmatrix} \end{aligned} \tag{5.59}$$

5.3.4 Proposed Receiver

So far, we have derived the L-MMSE estimator for STBC with imperfect channel estimation and synchronization separately. Since the synchronization problem and the channel estimation problem are independent with each other, the solution for joint problem can be easily derived through the linear combination of the two. When both synchronization and perfect channel estimation are unavailable, we have the following information at the receiver:

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{e};$$

$$\hat{\mathbf{r}} = [\hat{r}_0, \hat{r}_1]^T,$$

where $\hat{\mathbf{h}} = [\hat{h}_0, \hat{h}_1]^T$; $\mathbf{h} = [h_0, h_1]^T$; $\mathbf{e} = [e_0, e_1]^T$. The estimation errors e_0 and e_1 are Gaussian distributed complex random variables with the standard variance of $\sigma_{e_0}^2$ and $\sigma_{e_1}^2$. Without loss of generality, we assume that $\sigma_{e_0}^2 = \sigma_{e_1}^2 = \sigma_e^2$. $\hat{r}_0$ and $\hat{r}_1$ are the estimated ideal received signals given by

$$\hat{r}_0(r_0^{'}) = r_0^{'}(i) - \beta e^{j\varphi}\hat{h}_1 s_0^{(K)*}(i-1),$$

$$\hat{r}_1(r_1^{'}) = \hat{r}_1^{(K)}(r_1^{'}) = r_1^{'}(i) - \beta e^{j\varphi}\hat{h}_1 s_1^{(K-1)}(i).$$

The L-MMSE estimator for STBC based on above information can be written as

$$\hat{\mathbf{s}}(\mathbf{r}) = \mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}},\hat{\mathbf{r}}}\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}},\hat{\mathbf{r}}}^{-1}\hat{\mathbf{r}}. \tag{5.60}$$

Since the ISI caused by imperfect synchronization has been pre-cancelled by the PIC detector, $\hat{\mathbf{r}}$ can be treated as the ideal received signals. Therefore, the covariance matrixes in (5.60) are actually only conditioned on the imperfect channel estimation and can be written as

$$\hat{\mathbf{s}}(\mathbf{r}) = \mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}}\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}^{-1}\hat{\mathbf{r}} \ .$$

where $\mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}}$ and $\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}$ have already been derived in Subsection 5.3.2 given by

$$\mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}} = \begin{bmatrix} \hat{h}_0^* & \hat{h}_1^* \\ \hat{h}_1^* & -\hat{h}_0^* \end{bmatrix}; \tag{5.61}$$

$$\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}} = \begin{bmatrix} \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2 & 0 \\ 0 & \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2 \end{bmatrix}. \tag{5.62}$$

From (5.61) and (5.62), the L-MMSE estimator for STBC with imperfect channel estimation and synchronization can be derived as

$$\begin{aligned}
\hat{\mathbf{s}}(\mathbf{r}) &= \mathbf{C}_{\mathbf{sr}|\hat{\mathbf{h}}}\mathbf{C}_{\mathbf{rr}|\hat{\mathbf{h}}}^{-1}\hat{\mathbf{r}} \\
&= \left\{ \begin{bmatrix} \hat{h}_0^* & \hat{h}_1^* \\ \hat{h}_1^* & -\hat{h}_0^* \end{bmatrix} \begin{bmatrix} \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2 & 0 \\ 0 & \left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2 \end{bmatrix}^{-1} \right\} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \end{bmatrix} \\
&= \begin{bmatrix} \dfrac{\hat{h}_0^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} & \dfrac{\hat{h}_1^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} \\ \dfrac{\hat{h}_1^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} & \dfrac{-\hat{h}_0^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} \end{bmatrix} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \end{bmatrix}
\end{aligned} \tag{5.63}$$

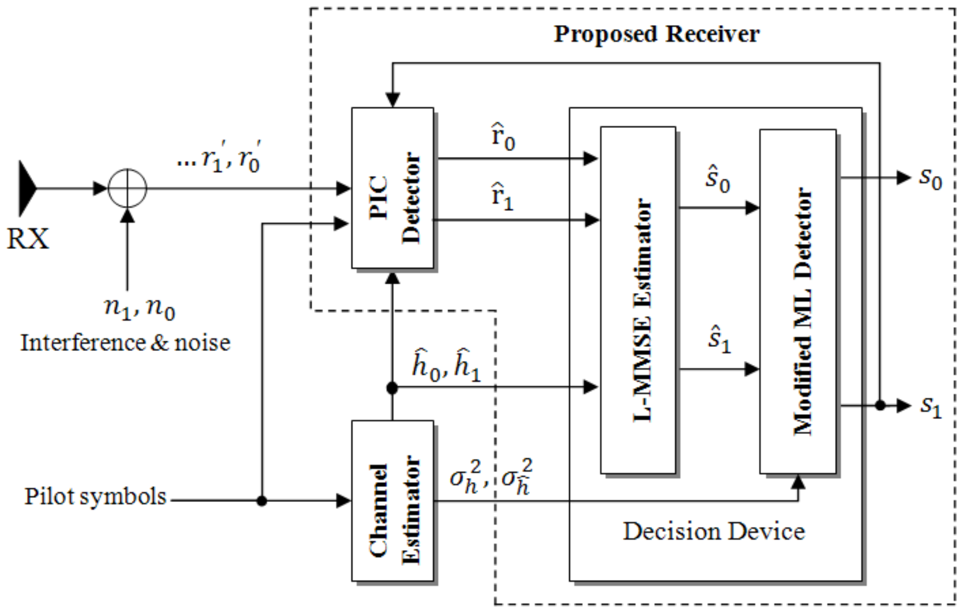


Figure 5.3: Block Diagram of the Proposed Receiver.

As discussed in Subsection 5.3.2, we need to modify the decision rule in the ML-detector based on the channel estimation error [29]. We apply the decision metric

$$\left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_0 - s_i\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_i\right|^2 \leq \left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_0 - s_k\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_k\right|^2, \quad \forall i \neq k$$

for detecting s_0 and the decision metric

$$\left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_1 - s_i\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_i\right|^2 \leq \left|\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\hat{s}_1 - s_k\right|^2 + \left(\frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2\right) - 1\right)\left|s_k\right|^2, \quad \forall i \neq k$$

for detecting s_1.

The PIC detector for the ideal received signal sequence, the L-MMSE estimator for the transmitted symbol sequence and the modified decision rule together constitute the proposed receiver for STBC with imperfect channel estimation and synchronization. Figure 5.3 shows a simplified block diagram of the proposed receiver.

5.4 Comparison with Alamouti's Receiver

In this section, we compare the proposed receiver with the conventional receiver proposed in [1] and investigate their relationship. The Alamouti's conventional STBC receiver consists of two major parts: the linear combiner and the ML-detector. Since the comparison has been made in previous section for the decision rules, we now concentrate on comparing the proposed L-MMSE estimator and conventional linear combiner. The conventional linear combination rule can be represented in matrix form as

$$\hat{\mathbf{s}}(\mathbf{r}) = \begin{bmatrix} h_0^* & h_1 \\ h_1^* & -h_0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix}. \tag{5.64}$$

The proposed L-MMSE estimator is given by

$$\hat{\mathbf{s}}(\mathbf{r}) = \begin{bmatrix} \dfrac{\hat{h}_0^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} & \dfrac{\hat{h}_1^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} \\ \dfrac{\hat{h}_1^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} & \dfrac{-\hat{h}_0^*}{\left|\hat{h}_0\right|^2 + \left|\hat{h}_1\right|^2 + 2\sigma_e^2 + \sigma_n^2} \end{bmatrix} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \end{bmatrix}$$

Obviously, with perfect channel estimation and synchronization, $\hat{h}_m = h_m$, $\hat{r}_m = r_m$,

$m = 0,1$ and $\sigma_e^2 = 0$. The L-MMSE estimator can be rewritten as

$$\hat{\mathbf{s}}(\mathbf{r}) = \begin{bmatrix} \dfrac{h_0^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} & \dfrac{h_1^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \\ \dfrac{h_1^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} & \dfrac{-h_0^*}{|h_0|^2 + |h_1|^2 + \sigma_n^2} \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}$$

For BPSK modulation, the receiver needs to make the decision that whether +1 or -1 has been transmitted. The denominators of all elements in above matrix have no influence on decision making since they are always positive. Therefore, the L-MMSE estimator can be further simplified as

$$\hat{\mathbf{s}}(\mathbf{r}) = \begin{bmatrix} h_0^* & h_1^* \\ h_1^* & -h_0^* \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}$$

After some complex conjugate manipulation, the conventional linear combiner can be derived as

$$\hat{\mathbf{s}}(\mathbf{r}) = \begin{bmatrix} h_0^* & h_1 \\ h_1^* & -h_0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix}$$

From above deductions, conclusions can be made that Alamouti's linear combiner is the optimum L-MMSE estimator for ideal cases. The proposed L-MMSE estimator is the general solution for STBC coded signals where both the channel estimation error and timing misalignment error have been introduced and utilized. The performance analysis and simulations will be given in detail later in Chapter 6.

Chapter 6

Simulation Results

In this chapter, we present simulation results to illustrate the performance of the proposed receiver. We consider the conventional Alamouti's STBC scheme with two transmit and one receive antennas. The channel model used in our simulations is the Rayleigh fading channel model originally presented in [14]. We assume that the channel is quasi-static and flat fading and the additive noise is composed of $\mathbb{CN}\left(0,\sigma_n^2\right)$ distributed samples. For fare comparisons, same transmitted energy per frame for both data symbols and pilot symbols is guaranteed for different decoding schemes. The STBC system is simulated using BPSK modulation. The modulated source data symbols are encoded by Alamouti's STBC coding matrix. For imperfect synchronization, we only study the time error caused by the relative delay between two transmitters.

The BER performance of the STBC receiver using the derived L-MMSE estimator under perfect conditions is plotted in Figure 6.1. The performance of Alamouti's conventional linear combiner is also simulated for comparison. Since the channel estimation in this stage is perfect, we use the Alamouti's hard decision rule for detection. As it is expected, the L-MMSE estimator is shown to be the same as the conventional combiner for ideal cases, since Alamouti's linear combiner is one of the simplified forms of the general L-MMSE estimator as we discussed in Section 5.4.

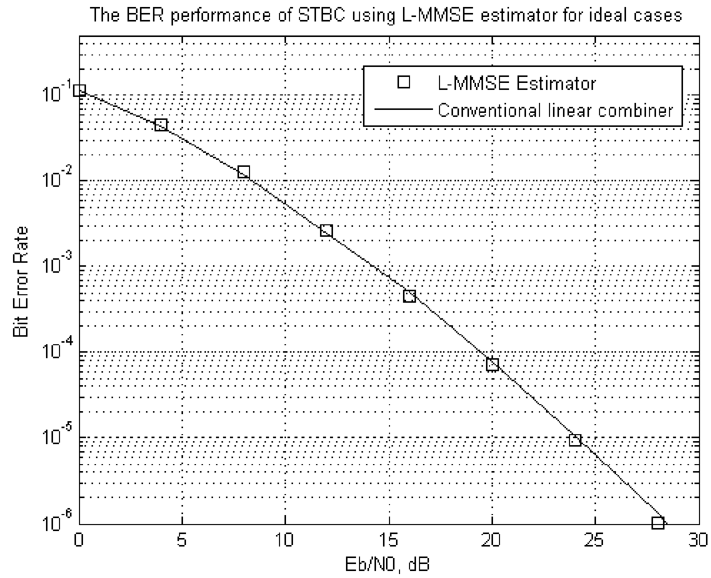


Figure 6.1: The BER Performance of Alamouti’s STBC with Perfect Channel Estimation and Synchronization Using L-MMSE Estimator and Conventional Linear Combiner.

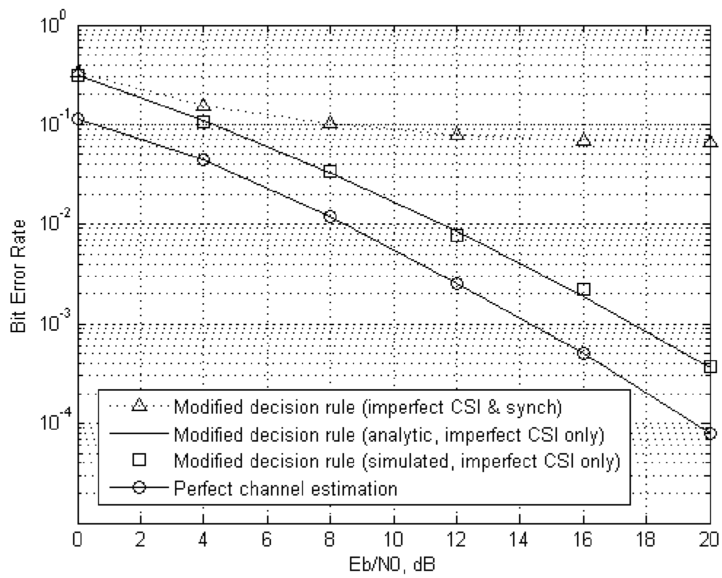


Figure 6.2: The BER Performance of Alamouti’s STBC with Imperfect Channel Estimation Using Tarokh’s Decision Rule.

Figure 6.2 shows the BER performance of Tarokh's decision rule under imperfect channel estimation. The perfect CSI curve is also given as a lower bond of achievable performance. The channel estimation error variance $\sigma_e^2 = 0.2$. It is observed that when synchronization is perfect, improvement of BER performance can be achieved by applying Tarokh's decision rule to the receiver. It is also shown in Figure 6.2 that when synchronization is imperfect, the system applying Tarokh's decision rule only will fail.

In Figure 6.3, we plot the BER performance versus data SNR for STBC under imperfect synchronization using PIC detector. For comparison, the corresponding results of the conventional detector are also shown. Two pilot symbols are used for each transmit antenna to initialize the detection. The simulation is done for time error $\beta = -10, 0, 5$ (dB) and we assume that the relative delay keeps the same for the entire data packet. The PIC iteration number $K = 2$.

As to the optimum number of iteration, the BER performance of the PIC detector for $K = 1, 2$, and 3 iterations are plotted in Figure 6.4. It can be seen that the PIC technique is a very effective way to mitigate the impact of imperfect synchronization and 2 or 3 iterations deliver almost all the gain.

The PIC scheme relies on the detection results for previous symbols, therefore can be viewed as a detection feedback technique. Naturally, any feedback error will have a negative effect on the detection for the current symbol. To examine this critical issue, the PIC detector is carried out with error propagation (for example, with the natural propagation of any feedback errors) and its bit error rate (BER) performance is shown in Figure 6.5. Obviously, the impact of error propagation is very minor.

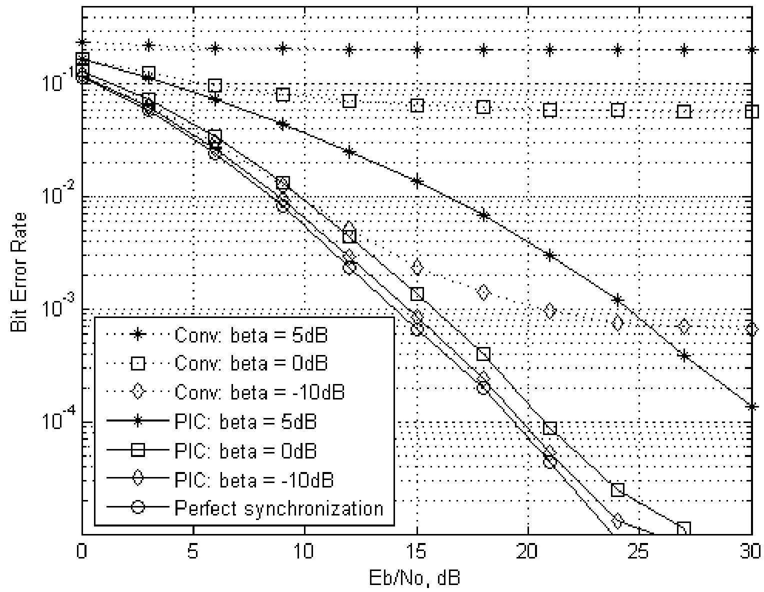


Figure 6.3: The BER Performance of STBC under Imperfect Synchronization with and without PIC Detector; No Feedback Error Propagation; Iteration Number *K*=2.

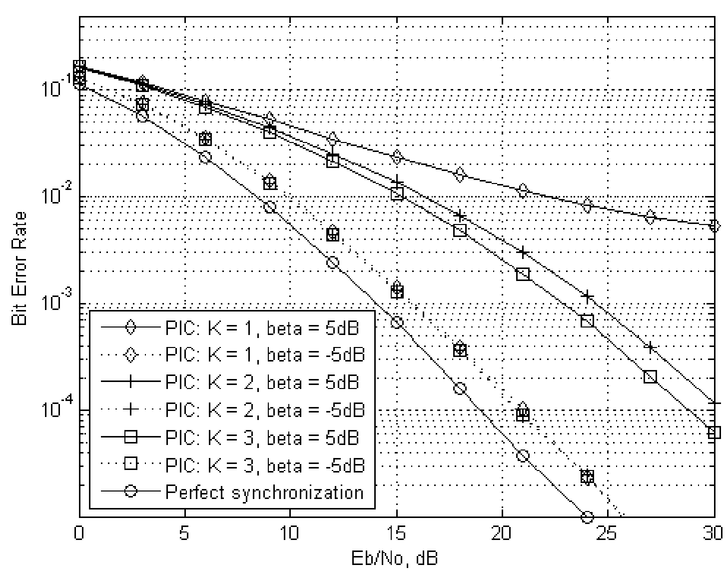


Figure 6.4: The BER Performance of PIC Detection for Different Number of Iterations.

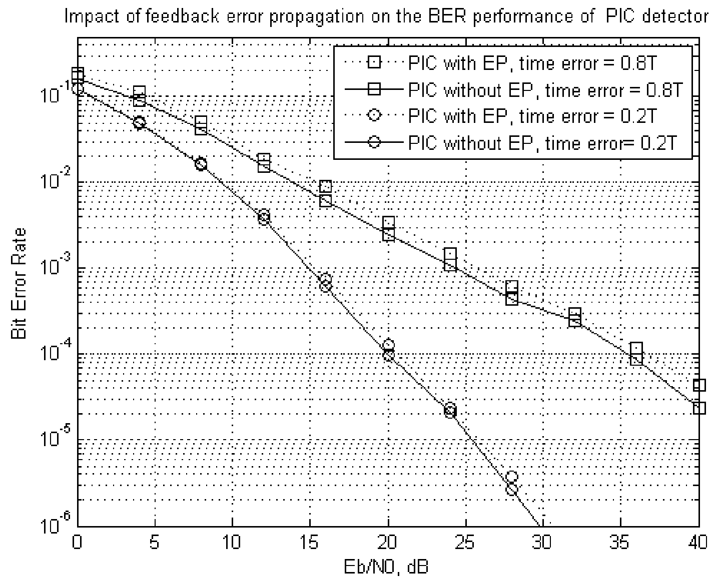


Figure 6.5: The Impact of Error Propagation (EP) on the BER Performance of PIC Detector. Time Error $\tau = 0.2T, 0.8T$.

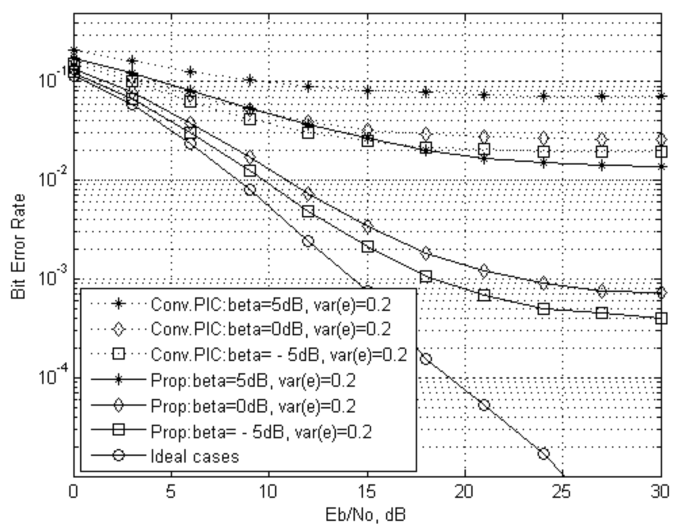


Figure 6.6: The BER Performance of Proposed Receiver Compared with Conventional PIC Detector. Time Error $\beta = -5, 0, 5(\text{dB})$, $\sigma_e^2 = 0.2$.

We compare the performance of the proposed receiver with the conventional PIC detector in Figure 6.6. It can be seen that the proposed receiver outperforms conventional PIC detector significantly when perfect channel estimation is not available. Composite simulation results are shown in Figure 6.7. The proposed receiver is examined under imperfect channel estimation and synchronization. We fix the channel estimation error variance $\sigma_e^2 = 0.2$ and simulate the performance of the system for time error $\beta = -10, 0, 5(\mathrm{dB})$. The PIC detection is carried out without feedback error and the number of iteration is 2.

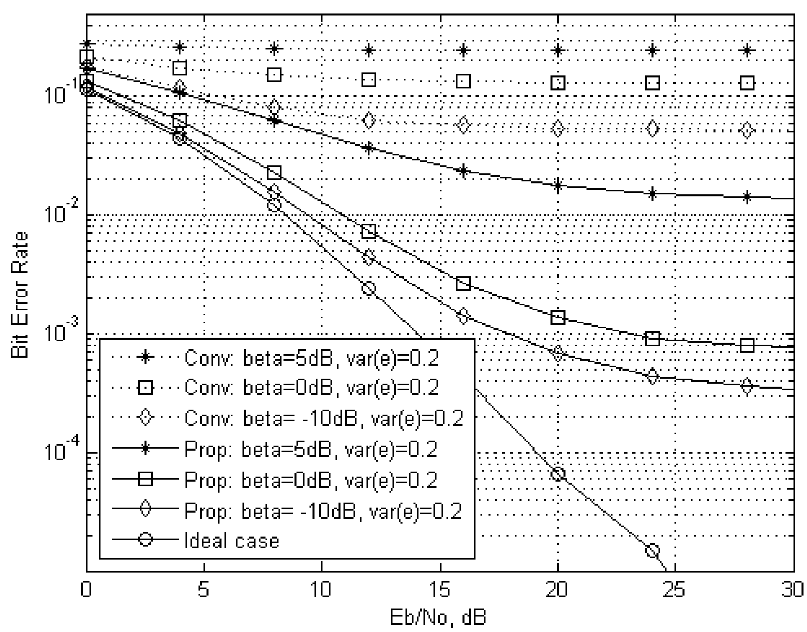


Figure 6.7: The BER Performance of the Proposed STBC Receiver with Imperfect Channel Estimation and Synchronization. Channel Estimation Error Variance $\sigma_e^2 = 0.2$, $\beta = -10, 0, 5(\mathrm{dB})$. PIC Iteration Number $K = 2$. No Error Propagation.

Chapter 7

Conclusions and Future Works

In this thesis, we studied the performance of space-time coded systems in cooperative transmission environment when perfect channel state information (CSI) and synchronization are both unavailable. We proposed a simple receiver for CSI error mitigation and time error cancellation in order to make STBC transmission reliable. This receiver is designed for Alamouti's 2-branch STBC system [1]. Although two transmit and one receive antennas are used throughout the analysis, this receiver can be easily expanded to more general cases. In the proposed receiver, the problem of noisy CSI and imperfect synchronization are tackled separately by a modified ML-detector and a PIC-LS detector [37].

Simulation and numerical results show that, in quasi-orthogonal static fading channel, our proposed STBC receiver is the same as the conventional one under perfect conditions, and would significantly outperform the one for ideal cases when the system is undergoing imperfect channel estimation and synchronization. On the other hand, the proposed STBC receiver requires only one extra PIC detector to estimate the ideal received signals. Therefore, it keeps a relative low decoding complexity and easy to be implemented to practical systems. In addition, the proposed receiver is easy to extend to higher order orthogonal STBC systems which implement more transmit and receive antennas. However, when the PIC detector is not initialized properly, the performance of the proposed receiver may degrade due to the error propagation. Moreover, the proposed receiver cannot deal with time errors greater than one symbol period, which increases the possibility of performance degradation in high speed transmissions.

In our study of imperfect channel estimation problem, a simplified Tarokh's decision rule has been derived based on the pdf of the received signal [29]. This pdf is conditioned on the imperfect channel estimations and transmitted symbol sequences. For the imperfect synchronization problem, a PIC-LS detector is derived based on the following assumptions [37], [38]: *(i)* the imperfect synchronization is only caused by the relative delay between two transmitters and this delay is smaller than one symbol period *T*, *(ii)* the first transmitters is synchronization with the receiver, and *(iii)* the relative delay between two transmitters keeps the same for the entire data packet. For systems with fast moving transmitters, where *(iii)* is hard to achieve, a price of extra pilot symbols needs to be paid for the estimation of the time error coefficients. Simulation and numerical results have shown that the PIC-LS estimator can effectively cancel the ISI caused by imperfect synchronization without being affected by imperfect channel estimation. This attracting feature makes the PIC-LS detector and Tarokh's decision rule orthogonal with each other, and thus, makes their combination easy and reliable. Furthermore, a general linear combiner for Alamouti's 2-branch STBC system is derived from the L-MMSE estimator. We use this estimator in our proposal to replace the conventional linear combiner in order to adapt the system to higher order modulations such as QPSK or QAM.

In summary, the proposed STBC receiver consists of three parts: the PIC-LS detector for time error cancellation, the L-MMSE estimator for transmitted symbol sequence estimation and the modified ML-detector applying simplified Tarokh's decision rule for channel estimation error mitigation. The objective of our study is not to replace

Alamouti's STBC scheme, but to overcome the disadvantage inherent in the conventional STBC system and to enhance its performance in practical environments. Our proposed receiver is aimed at providing a simple alternative solution for STBC with imperfect channel estimation and synchronization. However, for the case of severe misalignment in timing and frequency, our proposed receiver suffers significant performance lost due to the limitation of the system model used in the PIC detector. For the future works, more general system models can be considered, meanwhile, we feel that it may be possible to introduce the time error information into the decision rule to help the decoder to make better decisions.

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