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# Abduction as the Mother of All Argumentation

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**ABSTRACT:** Abduction\* is the genus with deduction and induction as species. *Modus tollens* is backward reasoning as an unknown proposition is inferred from a known proposition. *Reductio ad absurdum* is abductive because the conclusion is inferred by deriving a contradiction from an assumption. Inductive reasoning from effect to cause is also backward reasoning. But abduction\* consists of forward reasoning as well. The generic structure of abductive\* argumentation is universal among all cultures, occupations and disciplines.

**KEYWORDS:** abduction, argument to the best explanation, backward reasoning, deduction, hypotheses, induction, inference, parallel postulate, *reductio ad absurdum*, transcendental deduction.

## 1. INTRODUCTION

If we are to give the names of Deduction, Induction, and Abduction to the three grand classes of inference, then Deduction must include every attempt at mathematical demonstration, [...] Induction must mean the operation that induces an assent, [...] Abduction must cover all the operations by which theories and conceptions are engendered (Peirce 1957, p. 237).

Though Peirce demarcates abduction from deduction and induction, “all the operations” is a cue to consider “abduction\*”<sup>1</sup> as all inferential reasoning of which deduction and induction are species that have caught philosophers’ attention. For Peirce the complement subset “abduction” is essential to scientific reasoning.

The inference from agreeing perceptions to physical objects is *abductive* in Peirce’s sense, that is it makes an inference from effects to causes, from observations to their best explanations” (Niiniluoto 1999, p. 39).

If abduction is reasoning backward from effects to causes then most inductive arguments are abductive. However, many empiricists claim that inductive arguments lead to generalizations that may not be causal laws. Hence, some inductive arguments are not backward reasoning. Niiniluoto demarcates abduction from deduction:

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<sup>1</sup> I will use “abduction\*” to designate the genus abduction that includes all forms of inferential reasoning of which deduction and induction are species; and I will use “abduction” to refer to the subset of abduction\* that excludes deduction and induction.

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An abductive argument is not deductive, and it cannot guarantee certainty to its conclusion. It is an ampliative inference, which at best gives some credibility (or epistemic probability) to the conclusion. (Ibid., p. 39)

Abduction is considered backward reasoning: “Systems of abductive, backward reasoning can be exploited to turn up Cartesian proofs.” (Clark 1982, p. 8); “The diagnostic procedure for causal AND/OR/NOT graphs is an abduction procedure, abduction is backward reasoning [...]” (Oleskiak 2004, p. 31); “The most frequently discussed pattern of selective abduction is backward reasoning through given causal laws” (Schurz 2008, p. 4);

Abductive logical programming is a computational framework that extends normal logic programming with abduction. [...] used to generate by means of backward reasoning, [...] (www.absoluteastronomy.com 2009).

Isn't “ampliative” forward rather than backward reasoning, literally, as the conclusion is not contained in the premises, and metaphorically, since according to Peirce science would remain backward without it?

Magnani (2001) claims:

In theoretical analysis, reasoning goes backward from theorems to axioms—from effects to causes—from which they deductively follow (p. 2). [...] In my epistemological model, forward reasoning [...] is consistent with selective abduction while backward reasoning [...] is consistent with the deduction–induction cycle, because both deal with an inference from hypotheses to data (pp. 78-9).

Magnani concludes that deduction and induction are backward reasoning and abduction is forward reasoning:

It is interesting that *conventional curricula* [...] and *problem-based learning curricula* [...] lead students, when they generate explanations, to develop respectively selective abductions (forward reasoning) or to perform the whole abduction-deduction-induction cycle using relevant biomedical information (backward reasoning) (Ibid., p. 91).

Whereas backward reasoning may be associated with abduction when integrated with deduction and induction in a cycle, forward reasoning is associated solely with abduction.

Moreover I have tried to show that the idea of abductive reasoning might be a flexible epistemological interface between other related notions (induction and deduction, best explanation, perception, forward and backward reasoning, defeasibility, discovery, and so on) (Ibid., p. 94).

This is why I include forward as well as backward reasoning in abduction\*.

I demonstrate that the deductive argument forms of *modus tollens* and *reductio ad absurdum* are backward reasoning.

My final motivation comes from another pioneer of abduction in Norwood Russell Hanson:

The logic of Proof (i.e. deductive logic) has claimed philosophers' attention more than the logic of Discovery [...] Logicians of science have described how one might set out reasons in support of

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an hypothesis once it is proposed [...] There are two exceptions: Aristotle and Peirce. When they discussed what Peirce called “retroduction,” both recognized that the proposal of an hypothesis is often a reasonable affair (1958, p. 1073).

Hanson goes against mainstream philosophy of science which focuses on the context of justification rather than on the context of discovery. Abduction\* covers both contexts.

### 2. HISTORIC ROOTS OF ABDUCTION

Western philosophy began in Asia with the quest for ultimate constituents as Aristotle states: Most of the first philosophers thought that principles in the form of matter were the only principles of all things; [...] Thales [...] says it is water” (Kirk, Raven and Schofield 1983, #85, p. 89). As a scientist Thales observed that the earth floats on water but to derive “everything is water” from this is neither inductive nor deductive. Rather, “[...] the reasons given for Thales’ choice of water are professedly conjectural” (Ibid., p. 90).

Conjecturing hypotheses is based on complex reasons that are not *ad hoc*. For Aristotle Thales’ reasons were “mainly physiological” (Ibid., p. 91). But they had a wider historical and cultural grounding besides being founded on all the scientific knowledge available at the time:

Thales would no doubt be encouraged and gratified to have the apparently native Homeric precedents. Thus Thales’ view that the earth floats on water seems to have been most probably based upon direct contact with near eastern mythological cosmology. We have already seen that he had associations both with Babylonia and with Egypt. The idea that the earth actually floats upon water was more clearly and more widely held in the latter of these countries; and the conjecture may be hazarded that Thales was indebted to Egypt for this element of his world-picture (Ibid., p. 93).

A comprehensive philology has to invert the Eurocentrism that conventionally pervades in recording history. The standard history takes us to the Egyptian civilization vertically as the antecedent of the Ancient Greek civilization so that Thales would be directly influenced by the Homeric tradition of Ancient Greece that preceded him, which in turn would be influenced by the earlier Egyptians. The authors of *The Presocratic Philosophers* give this credibility since it is Darwinian in terms of tracing origins, but they simultaneously state that the horizontal influences of the Greeks, Egyptians and the near east were factors. Guthrie (1962) supports these horizontal influences:

Some point to the undoubted fact that he lived in a country familiar with both Babylonian and Egyptian ideas, and, according to an unchallenged tradition, had himself visited Egypt. In both these civilizations water played a preponderant part which was reflected in their mythology” (p. 58).

Hence, Thales’ reasoning for this conjecture is abductive.

Chrysippus (3<sup>rd</sup> Century BCE) formulates *modus tollens*: “If the first, then the second; but not the second; therefore, not the first” (Spade 2002, p. 39). Earlier Plato and Aristotle made abundant use of *modus tollens*. One of Thales’ students probably constructed the following objection:

If everything is water, then everything is wet.  
Some things are not wet.  
Therefore, Not everything is water.

The following form is generated after converting “some things are not wet” to “not everything is wet”:

$$\begin{aligned} p &\supset q \\ \sim q & \\ \therefore \sim p & \end{aligned}$$

In *modus tollens* like in all deductively valid arguments the conclusion follows from the premises, hence it comes after the premises.

The *Rg Veda*<sup>2</sup> provides another early formulation:

If everything is A, then A is nothing in particular.  
A is not nothing in particular.  
Therefore, not everything is A. (McEvelley 2003, p. 30)

This generalized form can be used against positing any element as *urstoff* of the world.

These historic arguments begin with the intention to demonstrate  $\sim p$ . *Modus tollens* is commonly used in modern scientific reasoning. A hypothesis can be stated as a conditional statement and a counterexample that negates the consequent hence leads to the denial of the antecedent:

If x is within the atmosphere of Earth with no counteracting forces in play, then x will fall to the ground when dropped.  
x does not fall to the ground when dropped.  
Therefore, it is not the case that x is within the atmosphere of Earth with no counteracting forces in play.

From the observation of x not falling to the ground we conclude that either we are not within the Earth’s atmosphere which would surely be observed at the moment or that there are counteracting forces, which may not at the moment be observed. Hence, *modus tollens* is backward reasoning from something observed to something unobserved.

*Modus tollens* is also used to reject hypotheses:

If Newton’s principle of reaction is correct, then the principle of relativity is incorrect.  
The principle of relativity is correct (not incorrect).  
Therefore, Newton’s principle of reaction is incorrect.<sup>3</sup>

This is not forward reasoning because some empirical evidence leads us to reject Newton’s third law. Rather, the principle of relativity, confirmed or verified by experiment, leads to the rejection of the principle of reaction which is inconsistent with it.

*Modus tollens* generates *reductio ad absurdum* arguments:

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<sup>2</sup> Although there is a controversy about when the *Rg Veda* was written, it is usually dated as being earlier than 1000 BCE. See for example: <http://en.wikipedia.org/wiki/Rigveda>.

<sup>3</sup> This labeling of the principles is from (Darrigol 1995, p. 1).

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- (1) Assume not- $p$
- (2) Provide argumentation that derives any contradiction ( $q \bullet \sim q$ ) from this assumption.
- (3) Maintain  $p$  on this basis (Rescher, 2006).

This is schematized as:

$\sim p$	As the assumption of the <i>reductio</i>
$\sim p \supset (q \bullet \sim q)$	Derived from premises and conditional proof
$\sim(q \bullet \sim q)$	The law of non contradiction
$\sim\sim p$	<i>Modus tollens</i> on the previous two lines
$p$	Double negation on previous line

Zeno employed *reductio* arguments abundantly. Rescher states:

The use of such *reductio* argumentation was common in Greek mathematics and was also used by philosophers in antiquity and beyond. Aristotle employed it in the *Prior Analytics* to demonstrate the so-called imperfect syllogisms when it had already been used in dialectical contexts by Plato [...] Kant's entire discussion of the antinomies in his *Critique of Pure Reason* was based on *reductio* argumentation (Ibid.).

The proof of Euclid's proposition 4 uses *reductio*:

[...] hence the base  $BC$  will coincide with the base  $EF$ .

[For if,  $B$  coincides with  $E$  and  $C$  with  $F$ , the base  $BC$  does not coincide with the base  $EF$ , two straight lines will enclose a space: which is impossible. Therefore the base  $BC$  will coincide with the base  $EF$ ] and will be equal to it. [C.N. 4] (Heath 1956, p. 248)

Euclid employed *reductio* in the proof of the first proposition that was not a construction. *Reductio* arguments are paradigms of backward reasoning as the conclusion is in mind before beginning the *reductio* that is generated on the assumption of the negation of the conclusion. Hence, intuitionist logicians are weary of the viability of *reductios* (Ibid.), based on the assumption that natural deduction provides forward reasoning that is ampliative: "The nine rules of inference listed above represent ways of inferring something new from previous steps in a deduction" (Klement, 2006).

### 3. SACCHERI'S *REDUCTIO* PROOF OF EUCLID'S FIFTH POSTULATE

Saccheri provided a proof of Euclid's fifth postulate (Halsted, 1986). In the process of showing the contradiction that follows from each of the alternatives to the parallel postulate, Saccheri invented non-Euclidean geometry but did not recognize "the legitimacy of his creation" (Ibid., p. viii).

In Book I Saccheri uses a *reductio* proof to establish the parallel postulate. He begins by forming the Saccheri quadrilateral in which the base angles are right angles. Now he needs to show that the summit angles will also be right angles. First, the two summit angles are proven to be equal, so we need only show that one of the summit angles is a right angle. Saccheri proposes the exhaustive disjunction that this angle is either (a) right angle, or (b) obtuse angle, or (c) acute angle. He then uses *reductio* arguments to demonstrate why the assumptions of (b) and (c) lead to contradictions so that each is false. Hence, by disjunctive syllogism (a) is true. Saccheri did not use

Euclid’s fifth postulate, nor definition 23 of “parallel” nor any of Euclid’s propositions that have used either the fifth postulate or definition 23 in their proofs. Hence, Saccheri successfully proved Euclid’s fifth postulate deductively without begging the question.

However, “In the 33rd theorem is found Saccheri’s ‘flaw,’ where he breaks away from his rigorous logic and carefully crafted ‘perfect’ structure and, as it were in disbelief, remarks ‘[...] but this is contrary to our intuitive knowledge of a straight line’” (Fairfield 2005). The proof of proposition XXXIII contains a *reductio* transformed into *modus tollens*:

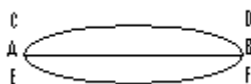
- ( $\alpha$ ) If two straight lines AB and CD fall on the same straight line then they enclose a space.
- ( $\beta$ ) Neither AB nor CD can enclose a space.

Therefore, ( $\gamma$ ) AB and CD do not fall on the same line (Ibid., pp. 173-7).

Saccheri was careful not to beg the question. But he jumped to the conclusion. Both ( $\alpha$ ) and ( $\beta$ ) are unwarranted. They are not derived from any of the first four postulates, definitions 1 through 22 and the common notions. What else is allowed in a proof? Any logically true propositions are allowed. ( $\alpha$ ) or ( $\beta$ ) are obviously not logically true. Hence, two tacit premises are needed for the proof of the fifth postulate to go through. Lambert took the flaw to be that of applying “properties at infinity that are only true in finite ranges” (McCleary 1994, p. 38).<sup>4</sup>

Saccheri therefore stands refuted. However, the extensive proofs of five lemmas, four corollaries and two scolia to proposition XXXIII based on the Euclidean definitions of point, straight line, and so on; do recursively and rigorously establish the two tacit premises (Ibid., pp. 173-207). Hence, proposition XXXIII, “*The hypothesis of acute angle is absolutely false; because repugnant to the nature of the straight line*” (Ibid., p. 173), is soundly established as Saccheri states: “I shall take the utmost care not to pass over any objection, however pedantic it might seem, since it appears to me that this is appropriate to a highly rigorous proof” (Halsted, p. 251)<sup>5</sup>. Furthermore in Book II Saccheri gave an independent proof of ( $\alpha$ ) and ( $\beta$ ). Neither Euclid nor Saccheri nor any geometer in between had the notion of a curvilinear line though they had the notions of curves and arcs.

After Gauss, Bolyai, Lobachevski and Riemann we have the following picture:



The three lines, one straight AB and two curvilinear CD and EF all lie on the same line though not on the same straight line. If “line” is disambiguated as “straight line” or “curvilinear line” then Saccheri is wrong. The notion of curvilinear lines was

<sup>4</sup> This may not be fair to Saccheri as the 19<sup>th</sup> century of infinity that Cantor probably used is being applied backward to Saccheri who probably did not accept actual infinities as probably Euclid would not have earlier and Kant would not have later.

<sup>5</sup> This is provided by Halsted as an alternative translation to the one provided in the text of theorem XXXIII on page 173 as: “But since I am here to go into the very first principles, I shall diligently take care, that I omit nothing objected almost too scrupulously, which indeed I recognize to the most exact demonstration.

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inconceivable to Saccheri (Ibid., p. ix). However, if “line” is disambiguated only as “straight line” then Saccheri is not wrong. The fifth postulate is then true and proven to be true. Euclidean and non-Euclidean geometries are reconciled with the general meaning of “line” as either “straight line” (Euclidean) or “curvilinear line” (non-Euclidean). This ambiguous notion of “line” lies in *analysis situs* (topology), or “general geometry”:

C'est pourquoi ils ont imaginé ce qu'ils appellent <<la géométrie générale>>, qui comprend comme cas particuliers les trois systèmes d'Euclide, de Lobatchevsky et de Riemann, et qui n'en comprend pas d'autres. Et cette épithète de << générale>> signifie évidemment, dans leur esprit, qu'aucune autre géométrie n'est concevable (Poincaré 1913, p. 162).

Poincaré's conjecture that these three geometries are the only geometries possible is an abductive inference based on the assumption that “line” can only be disambiguated as “straight” or “concavely curvilinear” or “convexly curvilinear.”

With the disambiguation of “line” as “straight or curvilinear” or even as “curvilinear” but not as “straight”; Saccheri's ( $\beta$ ) is false as neither do AB and CD together, nor AB and EF together, nor CD and EF together, nor AB, CD and DE all together enclose a space. Hence, the only fallacy in Saccheri's argument is an inadvertent but happy fallacy of equivocation. The discovery of this equivocation is purely abductive and backward reasoning literally as well as historically. At his time Saccheri was not equivocating but looking at his argument today he is equivocating. Furthermore, the notion of curvilinear lines is not a deductive consequence of the possible falsity of Euclid's fifth postulate, but is a conjecture of a possibility and the actual construction of a curvilinear line. It is purely abductive and it pronounces the victory of constructivism and intuitionism over logicism and formalism.

### 4. KANT'S TRANSCENDENTAL ARGUMENT AS ABDUCTIVE

Kant's transcendental deduction is abductive.<sup>6</sup> I restrict myself to the transcendental argument for space as an intuition in the transcendental aesthetics (Kant 1855, pp. 37-41). The argument is captured by “Space then is a necessary representation a priori, which serves for the foundation of all external intuitions” (Ibid., p. 38). This is called a “deduction” because it has the form of *modus ponens*:

There are perceptions<sup>7</sup>;  
if there are perceptions, then there is an *a priori* intuition of space;  
Therefore, there is an *a priori* intuition of space.

The conditional “if...then” is not material but transcendental as at least in the fourth row of the truth table where both ‘there are perceptions’ and ‘there is an *a priori* intuition of space’ are false, the conditional is also false.

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<sup>6</sup> Lawson considers Kant's transcendental deduction as a species of abduction (Guala, p. 8). Wahid claims: “Transcendental arguments have been described as disclosing the necessary conditions of the possibility of phenomena [...] ‘no different from Peircian abduction’ [...]” (2002, p. 273). Beth (2007) states: “Peirce's abduction has a form similar to transcendental argumentation, in that it infers to a possible explanatory principle, [...]” (p. 16). Overton (2002) claims: “A form of abduction was brought to prominence by Kant” (p. 48).

<sup>7</sup> I mean by “perceptions” here what are for Kant commonly called “external appearances.”



Kant's argument may be disambiguated in four ways: perceptions are not possible without the intuition of space, knowledge through perception is not possible without the *a priori* knowledge of the intuition of space, knowledge through perception is not possible without the intuition of space, and perception is not possible without the *a priori* knowledge of the intuition of space. These may be schematized as:

- [TAS]<sub>m</sub> Perceptions exist<sup>8</sup>;  
     if perceptions exist, then space exists as an intuition;  
     therefore, space exists as an intuition.
- [TAS]<sub>e</sub> Knowledge of perceptions exists;  
     if knowledge of perceptions exists, then *a priori* knowledge of space as an intuition exists;  
     therefore, *a priori* knowledge of space as an intuition exists.
- [TAS]<sub>em</sub> Knowledge of perceptions exist;  
     if knowledge of perceptions exists, then space exists as an intuition;  
     therefore, space exists as an intuition.
- [TAS]<sub>me</sub> Perceptions exist;  
     if perceptions exist, then *a priori* knowledge of space as an intuition exists;  
     therefore, *a priori* knowledge of space as an intuition exists.

Perhaps Kant intends to present all four. [TAS]<sub>m</sub> is a metaphysical argument, [TAS]<sub>e</sub> is an epistemological argument, [TAS]<sub>em</sub> is an epistemo-metaphysical argument as it starts from the knowledge of perceptions and ends with the existence of space as an intuition in the conclusion. [TAS]<sub>me</sub> is a metaphysico-epistemological argument as it starts from the existence of perceptions and ends with the knowledge of space as an *a priori* intuition.

The metaphysical argument is precarious since it is not clear whether by “perception” Kant means the sensation in the mind or the phenomenal object or both. Taken in one direction it leads to idealism. Kant may wish to make the epistemological argument, but it is simply unsound because the conditional is not true. I can have knowledge of perception without having *a priori* knowledge of the intuition of space. The epistemo-metaphysical argument is sound. I do have knowledge of perceptions, and this knowledge implies the existence of space as an intuition since what I perceive exists in space. The metaphysico-epistemological argument is unsound, as perceptions obviously exist whether or not I have *a priori* knowledge of space as an intuition.

The transcendental conditional in the metaphysical argument is not a material conditional but is a causal conditional and that is why from a realist perspective it is false. In the epistemological argument, when interpreted as a causal conditional, it is simply false from any epistemological point of view. As a causal conditional it is true in the epistemo-metaphysical argument and false in the metaphysico-epistemological argument. The standard conception of abduction as backward reasoning is reconciled with Magnani's contention that it is forward reasoning. Abduction\* is the genus of reasoning, either forward or backward, that humans use in supporting claims by providing reasons which can take varying forms from classical deductive and inductive models to analogies to metaphors to allegories to model theoretic reasoning to other complex types of

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<sup>8</sup> Although I am not comfortable with the use of ‘exists’ here as I agree with Leibniz and Russell that existence is not a first order property of objects, I am using it for convenience here.

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modeling. Hence, no matter how we characterize Kant's transcendental deduction it is abductive\*. We may debate about which species of abduction\* it is.

### 5. CONCLUSION

My conjecture is that arguments in the sense of providing reasons for one's claims are common to all cultures historically, vertically as well as horizontally. Emphasis on deductive reasoning, especially of the type that is grounded in presocratic Ionia such as that provided by Thales and perfected later in Egypt by Euclid, perhaps creates an ethnic bias and at the same time decries the universality of deductive reasoning making it culture specific. However, since persons with varied cultural backgrounds use such deductive reasoning the apparent ethnic bias disappears. Induction is perhaps much wider in terms of its presence in all cultures, as it seems like everyone uses induction in their day to day lives and not just the scientists. Even non human animals use induction when they refuse to put their limbs in fire once they realize that fire burns them. Yet, the way induction is presented as the core of the scientific enterprise it is formalized as if it is culture specific to scientific cultures. I claim that all cultures use abduction\* as a form of reasoning which encompasses forward as well as backward reasoning. Whereas the species of deduction and induction may not be common to all cultures, other species of abduction\* are common to all cultures. I am sure that it can be empirically confirmed that all humans across all cultures at present or at any time in history, are capable of all the species of abductive\* reasoning, whether it be deduction, induction, model theoretic, analogical, or abduction.

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