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Aristotle: an ancient mathematical logician

George Boger
Canisius College

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Author: George Boger
Response to this paper by: John Woods

Until recently the difference between Aristotelian, or traditional, logic and Aristotle's own *Introduction* ancient logic had been blurred. Perhaps this is attributable to scholars not having seen a special need to compare the two. However, it was really not possible meaningfully to distinguish the two until modern logicians examined Aristotle's syllogistic through the lens of mathematical logic. As a result, studies of Aristotle's logic since the 1920s have established his genius as a logician of considerable originality and insight. Indeed, we can now recognize many aspects of his logical investigations that are themselves modern, in the sense that modern logicians are making discoveries that Aristotle had already made or anticipated. Not the least of his accomplishments is having treated the process of deduction itself, a fact long overlooked by students of logic. Here we gather five mathematical features of Aristotle's logical investigations in *Prior Analytics*: (1) logic is taken as part of epistemology; (2) syllogistic deduction is treated metalogically; (3) rules of natural deduction are explicitly formulated; (4) the syllogistic system is modeled to demonstrate logical relationships among its rules; and (5) logical syntax is distinguished from semantics. While each of these features is perhaps remarkable in itself, when they are viewed together they reveal the striking philosophical modernity of an ancient logician.

Features of Aristotle's modernity. Aristotle would agree with Alonzo Church and other modern logicians that "(formal) logic is concerned with the analysis of sentences or of propositions and of proof with attention to the *form* in abstraction from the *matter*" (1956: 1). This notion takes the discipline of logic to be a metalogical investigation of underlying logics (1956: 57-58). Part of Aristotle's philosophical genius is having established a formal logic while at the same time making the study of logic a science. He recognized that deductions about a given subject matter are topic specific and pertain to a given universe of discourse,[1]say to geometry or arithmetic or biology, but that such deductions employ a topic neutral deduction system to establish knowledge of logical consequence. In *Prior Analytics* and *Posterior Analytics* and in *Topics* and *Sophistical Refutations*, Aristotle distinguished using a logic to process information on a given subject matter from studying the deduction system of an underlying logic.[2]Prior Analytics successfully established a formal deduction system that could serve as an instrument for demonstrative science, or *apodeiktikê epistêmê*, as this is outlined in *Posterior Analytics*. We are assured of Aristotle's concern with the formal matters of deduction when, at the outset of his treatment of the three figures in *Prior Analytics*, he wrote that "*sullogismos* should be discussed before *apodeixis* (demonstration) because *sullogismos* is more universal: an *apodeixis* is a kind of *sullogismos*, but not every *sullogismos* is an *apodeixis*" (25b28-31). *Prior Analytics* is a study of a system of formal conditions for cogent argumentation.

Thus, when we read in *De Anima* that "the soul ... is like the hand; for the hand is an instrument of instruments, and in the same way the mind is the form of forms" (*De Anima 3.8*, 432a1-2), we can appreciate that Aristotle treated logic as a part of epistemology. In particular, he considered it to be that part used to establish knowledge that a given categorical sentence follows logically or necessarily from other given categorical sentences. Aristotle seconded this notion when in *Posterior Analytics A10* he remarked that "demonstration is not addressed to external argument but to argument in the mind" (76b24-27). A deduction system for Aristotle, then, is an epistemic instrument of the mind by means of which someone is able logically to derive theorems from the first principles of a demonstrative science. The study of such an instrument is focused precisely on the formal conditions of logical consequence. Aristotle and modern logicians share the same notion that establishing knowledge of logical consequence is central to the study of logic.

We might underscore this aspect of Aristotle's investigations by noting that at *Metaphysics 4.3-8* (cf. *11.5*) he treated the most certain principle of all, to wit, the law of non-contradiction, as well as the law of excluded middle. He expressed these laws both as ontological principles and as logical principles. Consequently, given that logic is a part of epistemology concerned with establishing knowledge of logical consequence and, moreover, that a logic has an ontological underpinning that makes it impossible for true sentences to imply a false sentence, we understand that *Prior Analytics* is a metalogical treatise on the syllogistic deduction system and, indeed, on the formal process of deduction itself.

In *Prior Analytics* Aristotle turned his attention away from object language discourses and toward objectifying the formal deduction apparatus used to establish scientific theorems. Aristotle was especially concerned to determine "how every *sullogismos* is generated" (25b26-31). He accomplished this by exhaustively treating every possible categorical argument pattern3 having a premise-set of two categorical sentence patterns with places for three different terms. Aristotle studied only these patterns because two categorical sentences have the 'the fewest number of terms and premises through which something different than what was initially taken results necessarily'.4 Accordingly, he demonstrated which of these patterns have only valid argument instances and which patterns have only invalid argument instances. The results of his study, particularly in *Prior Analytics A4-7*, serve as elements in his deduction system.

Now, to separate the patterns with only valid instances, that is, the *sullogismoi*, from the patterns with only invalid instances, Aristotle used two metalogical processes: (1) the method of completion and (2) the method of contrasted instances.5 The method of completion (*teleiousthai* or *teleiôsis*) is a deduction process carried out in the metalanguage of *Prior Analytics*.6 This process explicitly employs, as Aristotle's choice of verb suggests, the four *teleioi sullogismoi*, or perfect *sullogismoi*, of the first figure as rules of deduction to establish which second and third figure argument patterns are *sullogismoi*. Aristotle's interest here is to establish which argument patterns have only valid argument instances. Every argument with semantically precise terms fitting one of these patterns is valid. In this way he identified fourteen *sullogismoi* in three figures.7 It is important to recognize that Aristotle treated the *sullogismoi* individually and not axiomatically; his metasystematic treatment of the *sullogismoi* is inductive and not deductive.8 Aristotle identified every argument pattern with only invalid argument instances by the method of contrasted instances.[9] His metasystematic treatment of the non-syllogistic argument patterns is likewise inductive and not axiomatic. The method of contrasted instances is interestingly different from either the modern method of counterargument or the method of counterinterpretation precisely because it invalidates argument patterns and not argument instances. It is noteworthy that Aristotle does not provide even one instance of the method of counterargument in *Prior Analytics A4-7*.[10] We attribute this to his concern with argument patterns and not arguments *per se*.

A *sullogismos*, then, as specifically treated in *Prior Analytics A4-7*, is a relatively uninterpreted object. In fact, it is an elemental argument pattern with only valid argument instances. In *Prior Analytics* Aristotle was principally concerned to treat argument patterns and not arguments in much the same way that a geometer treats triangles and parallelograms apart from a carpenter's concern, for example, with this or that triangle, etc.[11]

It is worth noting in this connection that Aristotle employed, at least implicitly, the semantic principle of form, that two arguments having the same form, or, in Aristotle's case, fitting the same strict syllogistic pattern, are both valid or both invalid. He established a relationship between an argument's pattern and its validity or invalidity. This is especially well stated a number of times in *Prior Analytics* when Aristotle summarized the results of his studies in *A4*, *A5*, and in *A6*. He wrote, for example, of the second figure:

> it is evident both that a *sullogismos* is generated necessarily whenever the terms are related to one another as was stated, and that if there is a *sullogismos*, then it is necessary for the terms to be so related. (*A5*, 28a1-3; cf. *A4*, 26a13-16 & 26b26-28, *A5*, 27a23-25, and *A6*, 29a11-14)

For there to be a *sullogismos* it is necessary and sufficient that terms be formally related as Aristotle stated in a number of rules. Likewise, for there not to be a *sullogismos*, it is necessary and sufficient that terms be formally related in the other ways he systematically examined, likewise stated in rules.[12]

We can now appreciate a third aspect of Aristotle's logical sophistication. Aristotle recognized the epistemic efficacy of the *sullogismoi* – that is, the elemental argument patterns having only valid instances – and he explicitly formulated them as rules of deduction in corresponding sentences. In *Prior Analytics A4-7* Aristotle established a set of deduction rules as part of his natural deduction system.[13] One process of deduction is accomplished by taking pairs of given categorical sentences to generate immediate inferences according to prescribed rules; these inferences are then added to the given sentences and then again taken in pairs, to wit, syllogistically (*sullogistikôs*), until a final conclusion is reached (see esp. *Prior Analytics A25*).[14] Aristotle treated this process in a fashion exactly analogous to chaining immediate inferences by using rules of propositional logic. In *Prior Analytics A4-7* Aristotle treated a *sullogismos* exactly as a topic neutral rule of deduction.

He used the expression ‘*technê sullogistikê*’, or the syllogistic art, in *Sophistical Refutations 11* (172a35) to capture his thinking.[15]

We can now appreciate a fourth mathematical feature of Aristotle’s study of logic. *Prior Analytics* (esp. at *A1-2*, *4-7*, *23*, *45*) is a proof-theoretic treatise in which Aristotle demonstrated certain of the logical relationships among syllogistic deduction rules. Aristotle even modeled his syllogistic logic, albeit in a rudimentary way, to describe and to study the system in order to establish theorems about the system’s properties. Indeed, we see that Aristotle developed an artificial language, although not strictly a formal language, to help model his logic better to reveal its properties. Although it is stretching the point to say that Aristotle approximated inventing an uninterpreted language, we might, nevertheless, recognize a move in this direction.[16]

Again, it is worth noting on a related matter that Aristotle expected scientists to construct what amounts to a logically perfect language for each universe of scientific discourse. This requirement for demonstrative knowledge is forcefully expressed at *Metaphysics 11.5* (cf. *Metaphysics 4.4* and *Prior Analytics B3*) where he exhorted philosophers to eliminate equivocation and ambiguity and to eschew the use of metaphor in scientific discourse. He wrote:

> Those, therefore, who are to communicate with one another by way of argument must have some common understanding … Every word must therefore be intelligible and indicate something definite, not many things, but only one; and if it has more than one signification, it must be made plain in which of these the word is being used. He, therefore, who says that ‘this is and is not’ denies what he affirms, with the consequence that he declares the word to signify what it does not signify; but this is impossible. Consequently, if ‘this is’ signifies something, it is impossible to assert truly its contradictory. (1062a11-19)

In *Prior Analytics* Aristotle’s interests were not immediately focused on the practice of performing object language deductions but on questions that compass the foundations of deductive sciences. Aristotle treated syllogistic entities and their relationships just as modern mathematical logicians have treated the relationships among the operators and deduction rules of propositional logic. And he understood his logical investigations in just this way.

We have already mentioned the metalogical process of completion, whose epistemic import is to establish which argument patterns can serve as rules of deduction. Aristotle’s proof-theoretic theorem concerning completion is that “all the *ateleis sullogismoi* are completed by means of the first figure *sullogismoi* using probative and *reductio* proofs” (*A7*, 29a30-33).[17] Aristotle used metalogical deductions to establish the *sullogismoi*. The complement to completion, the method of contrasted instances, has as its epistemic import to eliminate certain elemental argument patterns as possible rules of deduction.

But one of Aristotle's more remarkable proof-theoretic results is accomplished at *Prior Analytics A7* where he treated the reduction (*anagein* or *anagôgê*) of *sullogismoi*. This matter has caused considerable difficulty for interpreters, a part of which consists in their having confused reduction with analysis (*analuein* or *analusis*) in their zeal to axiomatize Aristotle's syllogistic.[18] However, a careful reading shows that for Aristotle reduction is a proof-theoretic process that establishes certain deduction rules to be redundant or unnecessary in his deduction system: the same deductive results can be obtained by using a select number of rules. At *A7* he treated the logical relationships among all the *sullogismoi* taken as rules. Thus, when treating the reduction of the *sullogismoi*, Aristotle was not concerned to demonstrate the validity or invalidity of a given argument or to show that a given argument pattern is a *sullogismos* (as at *A5-6*). Rather, he was concerned to demonstrate that a conclusion is shown to follow for each of the established *sullogismoi* by using only the two universal *teleioi sullogismoi* as deduction rules. While his process of reducing the *sullogismoi* is metasystematic, it is nevertheless an inductive process that employs a deductive step; Aristotle tests each possible result individually and not axiomatically.[19] Aristotle's theorem concerning reduction is that "all the *sullogismoi* can be reduced to the two universal *sullogismoi* in the first figure" (*A7*, 29b1-2). Expressing this in modern terms, we see that Aristotle demonstrated the logical independence of the two universal *sullogismoi* of the first figure.[20]

Our final observation about Aristotle's logical acumen concerns his having distinguished logical syntax from semantics in a way familiar to mathematical logicians. While it is doubtful that Aristotle had a modern theory of language, and surely did not work with a string-theoretic formal language, it is nevertheless true that he recognized different logical patterns to underlie sentences involving, for example, ambiguity and equivocation. This, of course, applies *a fortiori* also to arguments. One way sufficient for determining whether a logician distinguishes logical syntax from semantics is to ascertain whether a logician works with a notion of interpretation or re-interpretation. We believe that Aristotle worked with neither notion. However, another equally sufficient way is to determine whether a logician works with a notion of substitution, a process by which one changes the language, or the content words in a given argument, while leaving their meanings and the logical form fixed. Aristotle's pervasive use of schematic letters – not variables[21] – to mark places for terms, his naming terms by their schematic positions, and his practice of substitution indicate his having distinguished semantics and syntax. Moreover, throughout his discussion Aristotle systematically treated patterns of sentences, patterns of two premises with places for three different terms and their corresponding argument patterns schematically. And he did this according to a strict syllogistic syntax without reference to particular arguments.[22] He even explicitly named four logical constants in just the same manner as modern logicians (*A4*, 26b30-33): "belongs to every", "belongs to none", "belongs to some", "does not belong to some". We can turn to *Sophistical Refutations* to corroborate our point about Aristotle distinguishing semantics and syntax.

In *Sophistical Refutations* Aristotle used the word '*sullogismos*' to denote an argument that fits an argument pattern having only valid instances, and he used the expression '*phainomenos sullogismos*', or apparent *sullogismoi*, to denote an argument that appears to fit such a pattern but which really fits another, non-syllogistic pattern, for example, one with four terms as in a case of equivocation. In such cases Aristotle recognized that a given word or expression may have two different meanings and thus fall into two different semantic domains or denote two different terms. Thus, while a two-premise categorical argument with an equivocal term has a given grammatical pattern that makes it appear to be a *sullogismos*, it really has an underlying logical pattern different from a *sullogismos*. And these logical patterns were precisely Aristotle's concern in *Prior Analytics*, and they may even have been presupposed for his study in *Sophistical Refutations*. These patterns are strictly formal and independent of a given object language. Moreover, these patterns are independent of the particular use to which a given object language argument might be put, whether as a hypothetical argument, or as a dialectical, a didactic, a demonstrative, or an eristic argument.23

Modern mathematical logicians, we know, make a clear distinction between logical syntax and semantics, but they believe that Aristotle was not sophisticated enough to make this distinction and that, as a result, he did not define "logical consequence" or "following necessarily". However, if we turn to *Metaphysics 5.5* we recognize a sophistication precisely in his defining "necessary" just as he used the concept in *Prior Analytics*: "that which is necessary is that having no other relationship" (1015a34). At *Metaphysics 4.5* he wrote much the same: "for it is not possible for what is necessary to be one way and another, and so if something is of necessity, it cannot be so and not so" (1010b28-30). Interestingly, he also made an explicit reference to demonstration at *Metaphysics 5.5* in connection with the passage cited above:

> demonstration is of necessary things, because, if there is a demonstration proper, it is not possible for there to be any other relations; the reason for this is the premises, for if there is a *sullogismos* it is [logically] impossible for there to be another relationship among them. (1015b7-9)

Thus, a *sullogismos* is such that no other result is logically possible.24 We have seen above how Aristotle understood the relationship of terms in premises fitting given argument patterns.

In this connection, we also recognize Aristotle to have very clearly distinguished truth from validity. This is especially evident at *Prior Analytics B2-4* where he systematically treated the various possibilities of valid arguments with combinations of true and false sentences as premises and conclusions in the three figures. Aristotle was keenly aware of the difference between (1) establishing knowledge of the truth or falsity of a given sentence (whether by induction or by deduction) and (2) establishing knowledge of the validity or invalidity of a given argument. And, likewise he keenly grasped the difference between the necessity in each case. In addition, he also distinguished validity from deducibility. In this respect it is interesting to consider his metasystematic claim at *Prior Analytics A30*:

> For if nothing that truly belongs to the subjects has been left out of our collection of facts, then concerning every fact, if a demonstration for it exists, we will be able to find that demonstration and demonstrate it, while if it does not naturally have a demonstration, we will be able to make that evident. (46a24-27)

This statement surely points to Aristotle's concern with the practical power of his syllogistic system as an epistemic instrument for obtaining scientific knowledge. Moreover, this passage suggests a modern concern with the completeness of a deduction system, that is, with whether every logical consequence of a set of sentences is deducible using a given set of deduction rules (cf. *A23*). As we know, making a distinction between syntax and semantics is thought to be necessary for asking about the completeness of a set of deduction rules. The statement from *A30* cited above surely makes evident that Aristotle thought about the foundations of a system of logic, although his completeness proof in this connection has features different from that of a modern logician's proof.[25]

Summary. We have highlighted five aspects of the remarkable modernity of Aristotle's thinking about logic. (1) Aristotle took logic to be a part of epistemology. A logic is used to establish knowledge of logical consequence, and the science of logic takes this as its principal concern. (2) *Prior Analytics* is a metalogical treatise on the syllogistic deduction system. Aristotle exhaustively treated all possible combinations of 'syllogistic' argument patterns to determine which have only valid argument instances. (3) Aristotle recognized the epistemic efficacy of certain elemental argument patterns having only valid instances, and he explicitly formulated them as rules of natural deduction in corresponding sentences. (4) *Prior Analytics* is a proof-theoretic treatise in which Aristotle described a natural deduction system and demonstrated certain of the logical relationships among syllogistic rules. In fact, Aristotle modeled his syllogistic in a rudimentary way for this purpose. One important metasystematic result is to have established the independence of a set of deduction rules. Finally, (5) Aristotle worked with a notion of substitution sufficient for distinguishing logical syntax and semantics. In this connection he also distinguished validity from deducibility sufficiently well to note the completeness of his logic.

Our reading of *Prior Analytics* takes Aristotle to have treated the process of deduction much as modern mathematical logicians do and not to have been confused about some fundamental matters of logic. Least of all was he confused, as commentators such as Günther Patzig (1968: 16-42) believe, about a distinction between "following necessarily" and "being necessary", both in respect of the distinction between a *sullogismos*, or a deduction, and a demonstration (*apodeixis*) and of the distinction between assertoric logic and modal logic. Aristotle clearly distinguished between (1) a given sentence's following necessarily from other sentences and (2) a given sentence denoting a state of affairs to be necessary (or possible). Grasping him to be concerned with the deduction process helps us to avoid such an error. In any case, Aristotle recognized that, while the conclusion of a given argument follows necessarily from its premises, this necessity may not be evident to a participant. He knew that the epistemic process of deduction produces knowledge, or makes evident, that a given sentence follows necessarily from other given sentences. He considered the product of this epistemic process to be an argumentation that includes a deductive chain of reasoning in addition to the given premises and conclusion. He recognized using deduction rules in the epistemic process for establishing validity. Furthermore, Aristotle distinguished (1) the subject matter of a given argument from (2) the use to which a given argument might be put from (3) the varying expertise of a participant. All these matters are distinct from (4) the formal matters underlying any of them. And precisely to examine these formal matters was his project in *Prior Analytics*. In this connection, then, we understand Aristotle to have distinguished two kinds of knowledge that cannot be otherwise: (1) knowledge of what is true or false, which pertains to sentences, and (2) knowledge of what is valid or invalid, which pertains to arguments.

Concluding remarks. Only recently have we been able to recover something of Aristotle's promethean accomplishments relating to logic. Indeed, we are recognizing more and more that part of the history of modern logic is to have re-invented the wheel that Aristotle turned many years ago. It is astonishing that for hundreds of years, perhaps dating to before the *Port Royal Logic*, Aristotelian logic, or traditional logic, has been taught without a single reference to the process of deduction. It has been the practice of R. Whately, W. S. Jevons, H. W. B. Joseph, J. N. Keynes, R. M. Eaton, and many others. It is still the practice in untold numbers of introductory textbooks on categorical logic to test a syllogism according to rules of quality, quantity and distribution and entirely to overlook the deduction process of chaining syllogisms, not to mention the glaring traditionalist error to take a syllogism to be either a valid or invalid argument.

Jan Lukasiewicz can be credited with shedding light on the syllogistic by being the first to examine it with the theoretical apparatus of mathematical logic. But Lukasiewicz and his followers really only 'improved' the traditionalist interpretation with a sophistication afforded by mathematical logic. Both lines of interpretation took Aristotle's presentation in *Prior Analytics* to be his own axiomatization of the syllogistic. While traditionalists awkwardly drew lines between sentences in different syllogisms to indicate their logical relationships (their analyses or transformations), axiomaticists such as Lukasiewicz cleverly turned a *sullogismos* into a logically true conditional proposition that could be processed by a propositional logic. In this way the axiomaticists indicated the logical relationships among the syllogisms. Again, the epistemic process of deduction treated in *Prior Analytics* was overlooked. It was not until the early 1970s with the work of John Corcoran and Timothy Smiley that the case for Aristotle's reputation as a logician of consummate intelligence and originality was well argued. They established Aristotle to be concerned with a deduction process just as many modern logicians are. Corcoran and Smiley also used mathematical logic to model Aristotle's logic. However, instead of finding an axiomatization of a logic, they discovered a natural deduction system. But they remained puzzled by reduction, in part, we believe, because they did not think that Aristotle modeled his own system of deduction rules nor that he could envision distinguishing syntax and semantics. Our interpretation builds on the work of Corcoran and Smiley, and now on that of Robin Smith whose recent translation of *Prior Analytics* (1989) has incorporated their findings. We believe, however, that Aristotle did model his own system. In particular, we read him as treating a *sullogismos* as a rule of deduction at *Prior Analytics A4-7*, and that he himself was able proof-theoretically to determine certain properties of his deduction system. He was able to refine the system by eliminating redundant rules, and he affirmed his system's completeness. These are Aristotle's own accomplishments, not merely those of modern logicians who, using mathematical logic, believe themselves to have discovered features of the syllogistic unknown to Aristotle. Indeed, modern logicians might wonder at their 'having spoken' Aristotelian logic their whole lives without any idea of it.

ENDNOTES

1Aristotle regularly employed the notion of "universe of discourse". See, for example, his discussion at *Posterior Analytics A10*

2An underlying logic consists in a language, a semantics, and a deduction system; see J. Corcoran 1974: 87-88. *Prior Analytics*, *Categories*, and *De Interpretatione* together comprise Aristotle's treatment of an underlying logic.

3We use 'pattern' and 'form' (J. Corcoran 1993: xxxi-xxxvii) as, for example, I. Copi (1986: 288-291) respectively uses 'form' and 'specific form' or as W. O. Quine uses "general [logical] schema" and "special case [logical schema]" (1982: 44; cf. 1970: 47-51).

4See *Prior Analytics B2*, 53b18-20 and *Posterior Analytics A3*, 73a7-11 and *B11*, *B24-27*.

5We have followed Ross (1949: 302) in using the expression 'contrasted instances' to name Aristotle's method of invalidation.

6Aristotle also used '*teleiousthai*' (to complete) to denote the syllogistic deduction process carried out in a given object language.

7Aristotle recognized 14 *sullogismoi* in three figures whereas traditional logicians, or logicians referring to traditional logic, consider there to be 24 syllogisms in four figures.

8The following analogy helps to explain Aristotle's procedure. Just as a geologist might use a hammer to break open a given rock to determine whether or not it is a geode, and upon making the determination place the object in one of two piles, so Aristotle uses a metalogical deduction to determine in each case when a given argument pattern belongs in the set of *sullogismoi*.

9It is evident that Aristotle considered the non-syllogistic argument patterns, which consist in two premises with three different terms, to have no valid instances.

10Aristotle's method of invalidating argument patterns consists in substituting two sets of three different terms into two argument patterns to produce two arguments all of whose sentences are true. Each argument has premises fitting the same premise-pair pattern, but the one has a universal affirmative (*a*) sentence as a conclusion and the other a universal privative (*e*) sentence as a conclusion. However, it is a simple matter to adapt his method to the method of counterargument, and thus to produce two arguments each with true premises and a false conclusion, by switching his substitution instances. Consider, in this respect, his invalidating any syllogistic argument having two universal privative sentences in the first figure at *A4*, 26a11-13.

11While we take Aristotle to have explicitly treated argument patterns and not arguments, we do not consider him to have apprehended these 'forms' in a platonistic fashion.

12Aristotle provided nine sentences expressing the 14 *sullogismos* rules and fifteen sentences covering every possible non-syllogistic pattern.

13This system consists in four kinds of categorical sentence, two pairs of contradictories and one pair of contraries, three conversion rules, four *sullogismos* rules (reduced to two at *A7*), and direct and indirect proofs.

14The syllogistic process as Aristotle construed it is roughly analogous to adding a series of single digit numbers: two are taken and added, the result is then paired in turn with a third number and added, and so on until a sum is calculated. We might surmise in this connection that Aristotle likely incorporated into his conception of an extended deduction process the possibility of one or more indirect deductions as 'nested steps'.

15Cf. *Sophistical Refutations 9* and *11*, and *Prior Analytics A1*. Also consider Aristotle's view at *A30* (46a10-12/15) and at *Metaphysics* 1005b5-8 and 996b26 concerning 'the principles [*archai*] of the *sullogismoi* and of demonstrations'.

16It is doubtful that Aristotle developed this artificial language to model natural language and more likely that he aimed to standardize scientific discourse or to model his logic.

17'*Teleios*' and '*atelês*' are epistemic terms referring to the evidency of a sentence following logically from two other sentences. A *teleios sullogismos* is completed through itself (*di'hautou*; *A7*, 29b7); in such a case the necessity of the conclusion following necessarily from the premises is immediately evident. In the case of an *atelês sullogismos*, evidency of necessity is not immediate since something other is needed; here a deduction is required to establish knowledge of logical consequence. On this matter, see also *A1* at

24b18-26, *A4* at 26b28-33, *A5* at 27a1-3, 27a15-18, and 28a4-7, and *A6* at 28a15-17 and 29a14-16.

18J. Lukasiewicz (1958), for example, believes that he helps to illuminate Aristotle's own axiomatization of the syllogistic, and J. W. Miller (1938: 14, 25, 28) believes that he completed an undertaking that Aristotle himself had begun. Traditionalists have tried to 'deduce' all the syllogisms from the *dictum de omni et nullo*, while the axiomaticists have tried variously to 'deduce' them as theorems from others taken as axioms.

19Reduction here is not *per se* a deduction process, but a process that employs deduction. Aristotle at *A7* performed deductions (or referred to those at *A5-6*) but with the objective to eliminate redundant rules in order to simplify his deduction system. Aristotle demonstrated that each of the second and third figure *sullogismoi*, as well as the two particular *sullogismoi* of the first figure, can be completed by using only the two universal *sullogismoi* of the first figure as deduction rules. Cf. above note 9 on his procedure.

20At *A4-6* he established the preeminence of the *teleioi sullogismoi* (first figure) among the *sullogismoi* and implicitly established that the *ateleis sullogismoi* (second and third figures) are redundant rules in his deduction system.

21We believe that Aristotle took each letter to be a schematic letter, and not a logical variable, in a way similar to W. O. Quine's meaning of "a dummy to mark a position" (1970: 12; cf. 1982: 33, 145-146, 160-162).

22However, Aristotle did provide many instances of categorical arguments with two premises, particularly in respect of his method of contrasted instances for invalidating argument patterns.

23This collection might perhaps be extended to include the modal syllogistic treated at *Prior Analytics A3, 8-22*.

24This holds notwithstanding that a weakened *a* or *e* sentence (i. e. an *i* or *o* sentence) is a different sentence. This is a trivial truth for Aristotle; see *Prior Analytics B1*, 53a3-14, esp. 53a12. "To be otherwise" refers to contrariety and contradiction.

25The intuitive aspect of Aristotle's completeness proof at *A23* (cf. *A7* on reduction) is roughly captured by the modern notion of mathematical induction. For Aristotle, the elemental syllogistic argument patterns capture every possible valid argument having a premise-set of two sentences. These elemental argument patterns constitute (along with the conversion rules) the elements of (syllogistic) deductive reasoning. Every extended syllogistic discourse (sic. a deduction) is reducible to a chain of *sullogismoi*, that is, to a chain of immediate inferences generated syllogistically (*sullogistikôs*). Every valid categorical argument having more than two premises can be completed, that is, its conclusion can be deduced syllogistically by generating a chain of immediate inferences, to wit, a chain of *sullogismoi*.

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