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Does informal logic have anything to learn from fuzzy logic?

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1. Motivation

Since antiquity, philosophers have been challenged by the apparent vagueness of everyday thought and experience. How, these philosophers have wanted to know, are the things we say and think compatible with logical laws such as Excluded Middle?1 A kindred attraction has been the question of how truth presents itself — relatively and in degrees? in approximations? in resemblances, or in bits and pieces? F.H. Bradley is a celebrated (or as the case may be, excoriated) champion of the degrees view of truth.

> There are, one may say, two main views of error, the absolute and relative. According to the former view there are perfect truths, and on the other side there are sheer errors... . This absolute view I reject... . All truths and all errors in my view may be called relative and the difference in the end between them is a matter of degree.2

The approximation view has been carefully examined in a number of recent publications.3 The resemblance or truthlikeness position has also provoked a sizable literature,4 and the partial truth position a lesser, but no less interesting one.5

Vagueness (or what some commentators take to be vagueness) also constitutes a problem (or part of a problem) for the evaluation of natural language arguments. If we take the propositional calculus as our example, logicians want to be able to say that a fragment of English when reconstructed as an argument of the propositional calculus is valid if its reconstruction is. For this to be true, reconstruction procedures must have the *backwards reflection property* with respect to validity. A shorter way of saying this is that reconstructions must be tight. And for *that* to be true, English on the hoof has to be filtered in all sorts of ways *before* being accepted as inputs to our reconstruction procedures. Filtered out will be such things as ambiguity, tenses, higher-order quantification, adverbs and, presumably, vagueness.6

Vagueness also interests engineers. To get an utterly smooth ride on the new bullet train anywhere in its range of 0 to 300 kilometers per hour, or to get a completely still picture from a bouncing camcorder, or a fluffy, clean and dry wash from an energy-efficient, speedy washing machine, it is necessary that control systems be adept at manipulating vague instructions. Electrical engineers have had good success in this regard by designing control systems which model various structures of what has come to be called *fuzzy set theory*. Fuzzy set theory was not invented by engineers. In 1951 Kaplan and Shott7 proposed to mark degrees of membership in empirical sets by the elements of the unit interval, and to define in like manner the operations of intersection,

union, subset and complementation. Fourteen years later Lotfi Zadeh, an electrical engineer at Berkeley, independently arrived at the same idea and gave it a more elaborate mathematical articulation than the earlier authors.[8] By 1975, Zadeh had hit upon the following thought. The structures that underlie the engineering successes might be adapted in ways that allow us to claim various *philosophical* successes, especially in the theory of human reasoning. In particular, if we had a proper command of fuzziness, we should be able to give principled, theoretical accounts of vagueness, degreed-truth, approximate truth, truthlikeness, partial truth, and inexact reasoning (which is to say, ordinary, everyday reasoning)[9]. Further, instead of having to construct filters to bring natural language arguments up to a condition of input-worthiness for the reconstruction rules of formal logic, we might be able to dispose with filtering altogether by applying a fuzzy logic to natural languages directly.[10]

2. Fuzzy Logic

Fuzzy set theory is a model theoretic adaptation of the nondenumerably many valued logic $\_\aleph_1$ of _ukasiewicz, which is its base logic. We note that $\_\aleph_1$ differs from the countably infinite many valued logic $\_\aleph_0$, in which truth values are the rational numbers in the unit interval [0-1], rather than its real numbers. The set theory arises from the base logic by assigning fuzzy sets to atomic predicates and defining a satisfaction relation on fuzzy sequences. Fuzzy logic, in turn, is an extension of fuzzy set theory got by interpreting truth values as fuzzy subsets of the set of values of the base logic, that is, fuzzy sets of the unit interval. Because the fuzzy power set of the unit interval would encumber the ensuing logic with fearsome complexity, fuzzy logicians restrict the theory to at most countably many truth values. Zadeh calls these *linguistic* truth values, which can be explained as follows. *Truth* (with a capital 'T') is a linguistic variable, with values *true, very true, not very true*, etc. Lower case *true* is primary, and the other values of *True* are defined in terms of *true*. Thus *very true* = $true^2$*, not true* = *true* – *, false p* = *true* ¬*p*, and so on.

It may strike us that Zadeh is unaware of Lindenbaum's proof of the equivalence of $\_\aleph_0$ and $\_\aleph_1$, an equivalence which assures Zadeh's base logic of its desired countability and calls into question his *imposition* of countability constraints on $\_\aleph_1$. On the other hand, in quantificational extensions of infinitely valued propositional logics, a well-defined model theory requires for any set S of degrees of truth that S have a greatest lower bound and a least upper bound. The requirement is met by the reals of the unit interval, but not the rationals. So there is a good semantic reason for sticking with $\_\aleph_1$ as the base logic.

Here is an example of how fuzzy logic (*FL*) works. Object language predicates determine fuzzy sets. Any member of the fuzzy set TALL is a member to some or other degree. Say that Charlie is a member of TALL to degree 0.4. Then it is true to a degree that Charlie is tall if Charlie is tall to degree 0.4. But keep in mind that the metalinguistic "true" is also a vague predicate. Statements are

true or not only in degrees. Now since Charlie is only 0.4 degrees tall, that he is tall in fact is not very true, where not being very true is being true to only a slight degree, say 0.3.

Zadeh defines *not very true* in terms of *true*, and *true*, while "primitive", is not indefinable. Indeed *true* is defined as

È(0.3/0.6,0.5/0.7,0.7/0.8,0.9/0.9,1/1)

that is, as the fuzzy set to which the degree of truth 0.6 belongs to *true* to degree 0.3, degree of truth 0.7 has a 0.5 degree of membership, and so on.

It is not to our purpose to expose *FL* to detailed scrutiny. But it is well worth noting[11] that its truth values are not closed under propositional operations. Its deduction rules are semantic rather than syntactic. Axiomatization issues are "peripheral"[12], and this makes otiose questions of consistency and completeness. This is enough to persuade some people that *FL* is a logic in name only.[13] The harsher-minded of these critics incline to think of *FL* as the "cocaine of science"[14], or as "pornography".[15] Notwithstanding their provenance, these are silly remarks requiring nothing by way of serious reply.
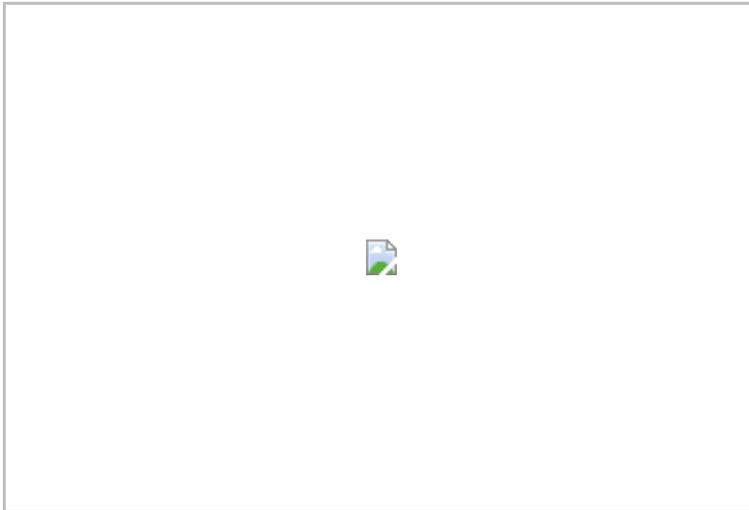
More to the point, the reasoning routines of *FL* are very complex — expensive things to work through in real time. Taking the measure of even simple propositions such as "It is not very true that Charlie is tall" or, in one of Zadeh's own examples, "It is very true that Lisa is young", requires the computation of what Zadeh calls *compatability functions*, themselves compositional functions. The function is graphed in Figure 3 of "Fuzzy Logic and Approximate Reasoning" (page 425); and is reproduced on this page. Of course, there is complexity and complexity. Some critics have charged the fuzzy rules with combinatorial explosiveness. In fact, increases in complexity are not exponential, as Combs has shown.[16] But still there is a lot of complexity in fuzzy models, enough certainly to make them suspect from the point of view of psychological reality.

Fuzzy logic, says Zadeh, "may be viewed as an attempt at accommodation with the *pervasive reality* of fuzziness and vagueness in human cognition. In this sense, fuzzy logic represents a retreat from what may well be an *unrealizable* objective, namely, the construction of a rigorous mathematical foundation for human reasoning and rational behaviour."[17] If this is so, fuzzy logic lays some claim as the logic or a basic part of the logic of actual human thinking. And if *this* is so, the economic costs of real-time computation of compatibility functions must somehow be offset. But how?

3. *FL* Reasoning

If it fulfills the promise of its inventor, *FL* can be said to model actual, everyday, real-time human reasoning. Does it?

There is ample evidence that the world is fuzzy (and fractal, too). At molecular levels, all is a cloud of witness, so to speak. It is also evident that although we know how to conceptualize the world at such levels — after all, chemistry is a mature science — this is not indeed how we experience the world or could experience it. In these matters I find myself siding with Quine. Like Quine I notice that the human subject

> is accorded... certain patterns of irradiation in certain frequencies... and in the fullness of time [he] delivers as output descriptions of the three-dimensional external world and its history.[18]

The story of the journey from such "meager traces" to so "torrential [an] output" is the story of how "from impacts on our sensory surfaces, we in our collective and cumulative creativity down through the generations have projected our systematic theory of the world."[19] The "baffling tangle of relations between"[20] temporarily ordered sets of exteroceptors on given occasions and our command of the world of everyday experience, as well of those swampishly below and galactically beyond, benefits from naturalistic scrutiny — the combined insights of "neurology, psychology, psycholinguistics, genetics and history".[21]

Perceptually and discursively we experience the world in a remarkably hard-edged way. We may take it that our disposition to hard-edged, smooth-curved representation derives in part from how we engage the world at what I am loosely (fuzzily) calling molecular levels. For this to happen, information processing mechanisms must engage the fuzzy. At this level, information is that of which Shannon and Weaver's theory gives a merely comparative quantitative analysis, with an emphasis on how cardinal weighting weighs in.[22] Processing information at this level, I assume to be a necessary condition on noticing at what I'm going to call the macro-level that it is very true that Lisa is young. Information processing at the molecular level is subconscious. Once consciousness enters the picture, representation becomes a dominant factor, and a linear one. But real-time linear treatments of highly complex phenomena are inordinately inefficient. I infer from this that the real-time information

processes are offset by a correspondingly economical displacement of fuzzy phenomena with hard-edged, smooth-curved representations. But again, how?

It may be that the high cost of computing Zadeh's compositional functions is made manageable by high speed calculations using a parallel distributed processing architecture. It may be, that is to say, that something *like FL* operations are at work down below. Certainly PDP programs execute rather perkily when the embedded logic is fuzzy. If so, they are probably subcortical, and certainly subconscious, prelinguistic and non-linear. I say that it "may be" that information processing down below has something of the character of *FL* reasoning. *Something* of that character, perhaps, but not much. To make of *FL* a plausible candidate for subconscious engagement of the fuzziness down below, realm, certain of its defining features must go, including (especially) all those in virtue which *FL* is a model theory laid atop the output of a complex context-free *grammar*. And some will think that, shorn of such structure, too little remains to justify the name of *FL*.

At the macro-level, information graduates from Shannon and Weaver quantities to something that flows in something like the manner of Dretske[23] or (better) Timothy Schroeder[24]. It is now that, the processing of which, takes representation, linearity, and time. (At the graduating juncture, by the way, we see the point of the distinction between relevance *to* and relevance *for*.[25]) The offset, as I say, is the substantial simplification offered by a hard-edged, smooth-curved world. At the molecular level we loose *perspective*, which is one of the things offered by hard-edged and smooth-curved worlds. At this level the inefficiencies of linear representation and information processing are offset by the benefits of effective risk aversion that hard-edged, smooth-curved worlds make possible. (For one thing, it is discernible in such worlds that the tiger is approaching rather than departing.) This suggests an answer the question of two paragraphs ago.

4. Classical Logic

Let us recur to the claim that fuzzy logic

> may be viewed as an attempt to accommodation with the pervasive reality of fuzziness and vagueness in human cognition. In this sense, fuzzy logic represents a retreat from what my well be an unrealizable objective, namely, the construction of a rigorous mathematical foundation for human reckoning... .[26]

The passage suggests that Zadeh is one of very many people who think that classical logic fails for natural languages. It would matter if this skepticism were justified. If classical logic did fail for natural languages, there would be an important vacancy in the theory of practical argument, and we could at least consider fuzzy logic as a candidate for the job. All the more so if, as Zadeh appears to believe, it is the "pervasive reality of fuzziness and vagueness" that puts paid to classical theories. A negative answer to our question would also

matter. If the "pervasive unity of fuzziness and vagueness" didn't topple classical logic, then the following argument would seem to be all but unstoppable.

1. The pervasive reality of fuzziness and vagueness leaves classical logic in tact.

2. Therefore, classical logic is just as good as theory of human reasoning with fuzzy vocabularies as it is a theory of reasoning in vocabularies that are entirely precise.

It is easy to show that the fuzziness and vagueness of natural language cause classical logic no embarrassment.

What would it be for *QT*, classical first order logic, to fail in English? It would fail if there were valid forms in *QT* or logical truths of *QT* which fail for certain interpretations of their atomic constituents. *QT*-formalization is such that properties such as validity and logical truth have the *backwards reflection property*. Let Σ be an English construction. If its logical form is valid in *QT* , it too is valid. If its logical form in *QT* is a logical truth, it too is a logical truth.[27] Our question, then, is whether any *QT* principle fails on a fuzzy interpretation of its atomic constituents. In other words, do fuzzy instantiations cancel the backwards reflection mechanism for validity and logical truth? The answer is No. No logical law of *QT* is overturned by adding a vague predicate to its classical language. Under such lexical supplementation, no logical particle would be semantically altered, no new semantic categories would be added, and no new kinds of grammatical structure. The logic of the resulting language would be unchanged. It would still be a classical logic. It would still be a logic that honoured the principle that a logical truth is a sentence whose truth value owes nothing to meaning, save the meaning of its logical particles.

Against this it could be insisted that Bivalence fails in such a language, and since Bivalence is a classical law, classical logic fails. It does not. Bivalence is proclaimed for the formal language of *QT*, a language whose atoms are uninterpreted. A *QT* formalization of a language containing vague, predicates does not have the backwards reflection property with regard to bivalence, but this no more disturbs classical logic than does the non-backwards reflection of invalidity. We can admit that "The present king of France is bald" is (twice-over) non-bivalent. This means in particular that negation does not *in fact* turn a designated truth value into an anti-designated truth value. Even so, the following argument is valid.

1. The present kind of France is bald or the Banach-Tarski

theorem is defective.

1. ¬(The present king of France is bald).

2. Therefore, the Banach-Tarski theorem is defective.

For what does a judgement of validity come to? It judges that *if* the premisses are true, so too, necessarily, in the conclusion. And this is true.

We are a long way from saying that no case can be made for fuzzy logic, anymore than we would say that no case can be made for supervaluational logic[28] or free logic. Even if we suppose wholly satisfactory motivations for such logics, it cannot be that they make any case for the collapse of classical logic. Of course, there are nonclassical logics which exhibit the logical forms of English constructions and which *QT* cannot capture. This makes it true to say that *QT* is not the logic of English. This is so, but before we make too much of it, we should remark that the same is true of the propositional calculus *PC* in relation to *QT* itself. *QT* isn't the logic of English, but it comes closer to being so than *PC*. It is one thing for a logic not to be the logic of English. It is another thing entirely whether not being the logic of English is being a logic that English falsifies. *QT* is not the logic of English; and English abounds in vague predicates. But vague predicates do nothing to invalidate *QT's* valid argument forms or the logical forms of its logical truths. So I say again that classical logic is as good a theory of natural language arguments in which vague predicates are deployed as it is a theory of such arguments in which no vague predicates are deployed.

5. A Heideggerian Connection

Heidegger high-jacked from Husserl the concept of intentionality. For Husserl, intentionality is a defining feature of consciousness. It is that which gives consciousness its aboutness; it is that in virtue of which consciousness is always consciousness *of*.

To see what Heiddegger was up to in Division I of *Sein und Zeit*, we must emphasize that he ditched the equation of wakefulness — even alert, busy-as-a-bee wakefulness — and consciousness. As Heidegger saw it, most of our waking actions are unattended by and unshaped by mental states. This mindlessness of ordinary waking human behaviour Heidegger calls the *intentionality of coping*. Consider a case in which you are watching a short-order cook working at full blast at a New York midday. It is easy to see his behaviour as connectionist and mindless, as behaviour reflecting repetoires of different skills which he draws upon concurrently and distributively, and without a jot of reflection when things are going well. Consciousness enters the Heideggerian picture when coping meets with resistance, where resistance can be seen as any event which prompts the coper to attend.

Here is a view that carries important consequences for the analysis of propositional assent — indeed for conversation generally. On the received view, when I assert, e.g., that the cat is on the mat, I in effect report a current mental state (belief), an object of my present attention. My assertion is sincere if the belief actually exists, and true if the belief is also true. Conversation more generally still is a sequence of exchanges of transparent propositional contents *modulo* the usual speech acts.

If Heidegger is right, the received view is wholly wrong. Conversation is just linguistic coping, as Heidegger sees it. If so, then you and I are less often in a state of belief than many theorists suppose; and when you are telling me about the amenities of, say, Amsterdam, though you tell me the truth, you are not transmitting your beliefs and you are not inducing new beliefs in me, unless perhaps what you tell me is surprising. ("No kidding, the best intellectuals' bar is right in the middle of the red light district!"). When I stop and think — when I put a temporary (and expensive) halt to coping — I find that in what I do in the world I am infrequently the owner of mental states, infrequently the possessor of beliefs. It is a respectable way of being mindless.

Heidegger's conjectures about consciousness and coping turn out to have been rather prescient, although no one would say that the now massive research programme on consciousness and its limitations owes anything directly to the German existentialist. Not everything is known about consciousness, needless to say But what is known is of critical importance for fuzzy theory. Consciousness has a surprisingly narrow bandwidth. It processes information very slowly. The rate of processing from the five senses combined — the sensorsium, as the Mediaevals used to say — is in the neighbourhood of 11 million bits per second. For any of those seconds, something fewer than 40 bits makes its way into consciousness. Consciousness therefore is highly entropic. At any given time there is an extraordinary quantity of information processed by the human system — *in* it to speak — which consciousness cannot gain access to. Consciousness is a thermodynamically costly state for a human system to be in. Equally, the bandwidth of *language* is far, far smaller than the bandwidth of *sensation*. A great deal of what we know — most in fact — we aren't able to tell one another. Our sociolinguistic intercourse is a series of exchanges whose bandwidth is 16 bits per second. More surprising still are Benjamin Libet's experiments[29] which indicate that when a human subject consciously decides to perform an action A — say, phoning his colleague in London — that action is underway a full half-second *before* consciousness enters the picture. Intentional actions start up prior to the generation of conscious intentions to perform them.

Libet was inclined to think that his discoveries harboured an important metaphysical consequence. He thought that they constituted a counterexample to materialism. Roger Penrose is similarly minded. He has argued that a materialistic account of Libet's data entails a substantial rejigging of basic physics.[30] Even so, not everyone has accepted Libet's basic thesis (anti-materialist consequences aside).[31] Dennett, for example, argues that Lipet's thesis is put in doubt by its reliance upon subjects' introspections, concerning which Dennett holds that there are no facts of the matter.[32]

Conscious experience is dominantly linear. Human beings are notoriously inadept at being in multiples of conscious states at once. And time flows. Taken together these facts loosely amount to an operational definition of the linearity of consciousness. Linearity plays a role in the cognitive economy that tight money plays in the other economy. It slows things down and it simplifies

them. Linearity is a suppressor of complexity; and reductions in complexity coincide with reductions in information. The sheer paucity of information possessed by human consciousness at any given time contrasts with environments known to be fuzzy. Fuzziness, unlike probability, is unchanged by arbitrarily large increases in information.

For a very long time in the Western intellectual tradition, logic has been a linguistic enterprise. This includes the logic of practical reasoning. There exists abundant evidence that a good deal of reasoning is subconscious. To the extent that there is so, fuzzy logic may well be a model of the right type. To the extent that reasoning is a conscious matter, it is not a fit candidate for fuzzy analysis, not anyhow if we are looking for models that are psychologically real. For consciousness cannot abide high levels of information. Fuzziness, on the other hand, like the thin starlet who can wolf down 6000 calories a day, is an informational glutton. Zadeh knows this, surprisingly. In the 1965 paper it is elevated to a law — the Law of Incompatibility:

> As the complexity of a system increases, our ability to make precise yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.[33]

There we have it. When practical reasoning is conscious, fuzzy logic is not the appropriate model. It is not even the appropriate *ideal* model. But what, then, of the impressive success of fuzzy engineering? Impressive a success as it surely is, it is perhaps not an impressively impressive success.[34] Zadeh's engineering work was designed to help with pattern recognition and like issues in computing. It was also directed toward the provision of frameworks within which humans supply computer-controlled systems with vaguely formulated information. And let's not forget fuzzy bullet trains and fuzzy washing-machines. It is rather striking that all these engineering applications are generated within a logico-mathematical framework that is entirely classical. Bearing in mind that a fuzzy set is nothing more than a classical function from a universe to the unit internal, the success of the engineering does nothing to show the classical scaffolding to be defective.

Endnotes

[1]The locus classicus of the vagueness literature is the sorites paradox of the Miletian logician Eubulides, also the true father of the liar paradox and contemporary of Aristotle. In our own day Timothy Williamson's book, Vagueness, London: Routledge 1996, stands out for the excellence of its historical survey and for the originality of its own "logic of clarity".

[2]F.H. Bradley, Essays on Truth and Reality, Oxford: Oxford University Press 1914, 25c.

[3]T.A.F Kuipers, "Naive and Refined Truth Approximation", Synthese 93

(1992), 299-341 and "'The Dual Foundation of Qualitative Truth Approximation", Erkenntnis 47 (1997), 145-179. See also S.D. Zwart, Approach to the Truth, Amsterdam: Institute for Logic, Language and Computation 1998, and Bryson Brown, "Approximate Truth: A Paraconsistent Account", in Revising (or Living with) Inconsistent Theories: On the Role of Inconsistencies in the History and Philosophy of Science, Dordrecht and Boston: Kluwer, to appear.

4See, for example, I. Nimiluoto, Truthlikeness, Dordrecht and Boston: Reidel 1987; G. Oddie, Likeness to Truth, Dordrecht and Boston: Reidel 1986. See also Zwart, op.cit.

5For example, M. Bunge, The Myth of Simplicity, Englewood Cliffs, NJ: Prentice-Hall 1963.

6But see below.

7A. Kaplan and H.F. Schott, "A Calculus for Empirical Classes", Methodos 3 (1951), 165-188.

8L.A. Zadeh, "Fuzzy Sets", Information and Control, 8 (1965), 338-353. Zadeh's philosophical or logical interest in fuzziness was prompted by Max Black's 1963 article in Dialogue, Vol 2, "Reasoning with Loose Concepts" (pp.1-12). 9I won't take the time here to query the plausibility of a single new logic for a fine-grained treatment of things as different as these.

10L.A. Zadeh, "Fuzzy Logic and Approximate Reasoning", Synthese, 30 (1975), 407-428.

11Zadeh op.cit. 1975, 245 and LA. Zadeh and RE. Belman, "Local and Fuzzy Logics", in J.M. Dunn and G. Epstein (eds.), Modern Uses of Multiple-Valued Logic, Dodrecht and Boston: Reidel 1977,105-106; 106-109; 151.

12Here I follow Haack, op.cit., 234. 13It is Susan Haack's contention that since FL doesn't have a truth-preserving consequence relation that it is not a logic. Period. (It is not a view that would fly very high wherever preservationist logics flourish; in Lethbridge, for example). (Susan Haack, Deviant Logic, Fuzzy Logic, Chicago: University of Chicago Press 1996, 231.)

14William Kahan, quoted in B. Kosko, Fuzzy Thinking: The New Science of Fuzzy Logic; New York: Hyperion 1993, 6.

15Dana Scott in a talk entitled "Deviant Logic: Fact or Fiction?", quoted in Haack op.cit., 230.

16William E. Combs, "The Combs Method for Rapid Inference", reprinted in Earl Cox, The Fuzzy Systems Handbook, 2nd edition, San Diego: AP Professional 1998, 659-680.

17Zadeh, op.cit. 1975, 426. Emphases added.

18 W.V. Quine, Ontological Relativity and Other Essays, New York: Columbia University Press 1969, 83.

19 W.V. Quine, Pursuit of Truth, 2nd edition, Cambridge, MA: Harvard University Press 1992, 1.

20 Idem.

21 Idem.

22 Claude Shannon, "The Mathematical Theory of Communication," Bell System Technical Journal, reprinted with the same title, and an introductory essay by Warren Weaver, by the University of Illinois Press, 1949.

23 Fred I. Dretske, Knowledge and the Flow of Information, Cambridge, MA: MIT Press 1981.

24 Timothy Schroeder, Foundations of Mental Representation, unpublished PhD. Thesis, Stanford University 1998.

25 John Woods, Agenda Relevance: An Essay on Formal Pragmatics, to appear.

26 Zadeh op. cit. 1975, 426.

27 It is well-known that invalidity and logical falsehood do not have the backwards reflection property.

28 Let me say in passing that vagueness seems to me better handled by supervalutaions over metric spaces.

29 B. Libet, W.W. Alberts, E.W. Wright, Jr., L.D. Delatyre, G. Levin, and B. Feinstein, "Production of Threshold Levels of Conscious Sensation by Electrical Stimulation of Human Somatosensory Cortex", Journal of Neurophysicology 27 (1964), 546-578. Benjamiin Libet, Curtis A. Gleason, Elwood W. Wright, and Dennis K. Pearl., "Time of Conscious Intention to Act in Relation to Onset of Cerebral Activity (Readiness - potential)", Brain,106 (1983), 623-642. Benjamin Libet, "Unconscious Cerebral Initiative and the role of Conscious Will in Voluntary Action", The Behavioural and Brain Sciences, 8 (1985), 529-566.

30 Roger Penfold, The Emperor's New Mind, Oxford: Oxford University Press 1989. Paperback edition, Vintage 1990; 569-572 of the 1990 edition.

31 See, for example, Patricia Churchland, "On the Alleged Backwards Referral of Experiences and its Relevance to the Mind-Body Problem", Philosophy of Science, 48(1981), 165-181. There is a reply by Libet, "The Experimental Evidence for Subjective Referral of a Sensory Experience Backwards in Time: Reply to p.5. Churchland", Philosophy of Science, 48(1981), 182-197, and a rejoinder by Churchland, "The Timing of Sensation: A Reply to Libet",

Philosophy of Science 48(1981), 492-497. See also S. Harnad, "Editorial Commentary" Behavioral and Brain Sciences, 12(1989), 183.

[32]Daniel C. Dennett, Consciousness Explained, Boston: Little, Brown 1991, 153-166.

[33]As formulated in Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", IEEE Transactions on Systems, Man, and Cyberneticsl, SMC-3(1), January 1973, 28-44; 28.

[34]Echoing George Boolos', remark about Quine's claim that there is a "remarkable concurrence of diverse definitions of logical truth." The concurrence said Boolos is not a "remarkably remarkable one." See W.V. Quine, Philosophy of Logic, 2nd ed., Cambridge, MA: Harvard University Press, 1986, 91, and George S. Boolos, "Second Order Logic", Journal of Philosophy, 72 (1975), 509-527; 525.