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On the Relation of Informal to Formal Logic

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ON THE RELATION OF INFORMAL TO FORMAL LOGIC

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Abstract:

The distinction between formal and informal logic is clarified as a prelude to considering their ideal relation. Aristotle's syllogistic describes forms of valid inference, and is in that sense a formal logic. Yet the square of opposition and rules of middle term distribution of positive or negative propositions in an argument's premises and conclusion are standardly received as devices of so-called informal logic and critical reasoning. I propose a more exact criterion for distinguishing between formal and informal logic, and then defend a model for fruitful interaction between informal and formal methods of investigating and critically assessing the logic of arguments.

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1. A Strange Dichotomy

In the history of logic a division between formal and informal methods has emerged. The distinction is not always sharply drawn, and there are theoretical disagreements with practical implications about the logical techniques that should be classified as formal or informal, and about the scope and limits and paradigms of each of these two main categories of logic.

The distinction arises because of unresolved questions about how logical relations are best expressed. At its furthest extremes, the distinction manifests itself as a difference of ideology and methodology between practioners of the most highly formalized symbolic mathematical logics and the purely informal treatment of logical inference in the rhetoric of argumentation, using only the discursive resources of ordinary language.

The polemics associated with disputes between formal and informal logicians are instructive even when they are not especially worthy of imitation. Formal logicians of a radical stripe often dismiss informal logical techniques as insufficiently rigorous, precise, or general in scope, while their equally vehement counterparts in the informal logic camp typically regard algebraic logic and set theoretical semantics as nothing more than an empty formalism lacking both theoretical significance and practical application when not informed by the informal logical content that formal logicians pretend to despise.

I shall not try to document these attitudes, because they are not the sort of views to appear explicitly in the writings of proponents on either side. But I think that anyone who has spent much time working in formal or informal logic, and certainly those who have crossed over from time to time to try their hand at both styles of logic or associated with outspoken members of each side will know well enough from anecdotal experience about the professional polarization to which I refer. I do not insist that the antagonism between formal and informal logicians cannot be friendly and respectful, although I suspect that unfortunately it is often something less.

There is no reason for example why a healthy intellectual rivalry between formal and informal logicians should not be fostered in order to promote advances on both sides in the sort of free market competition of ideas that can help logic to flourish. Such is evidently the case with the tolerance among logicians and mathematicians of differences between classical and intuitionistic and other nonstandard logics, Euclidean and non-Euclidean geometry, and even more prosaic distinctions of method such as that between axiomatic and natural deduction methods in symbolic logic. My impression is that the reality at least of the current situation in logic with its deep-seated bifurcation between formal and informal methods is in this respect far less than ideal. The division is something more like that described by C. P. Snow in his landmark discussion of *The Two Cultures*.1

Indeed, the analogy is quite appropriate and may just be a special instance of the general case, with formal logicians falling on the same side as the exact sciences, and informal logic finding its natural home with the humanities.

Now I find the division between formal and informal logic to be a rather strange dichotomy. Since I consider myself to be both a formal and informal logician, I see the formal and informal logical extremes as two complementary poles along a continuum that insensibly grade off into one another without a definite boundary, and hence as offering no clearcut basis for partisanship. I agree with the informal logicians' claim that symbolic logic by itself is an empty formalism that needs to have a content supplied by informal reasoning. But I also agree with formal logicians when they complain that purely informal logic is often too blunt an instrument to handle highly sophisticated problems involving exact or universal concepts, and that informal reasoning can sometimes overlook logical problems and solutions that formal logic is uniquely able to reveal. I am inclined on pragmatic grounds therefore to accept whatever logical methods are best suited for my analytic purposes in trying to understand many different kinds of logical problems. I think it is only reasonable not to stand on any exclusionary principle in confronting the logic of arguments, but to adopt a style of doing logic that appears to work best in offering the most satisfying results in formulating ideas and testing inferences, in the expression and critical analysis of concepts and arguments, and generally speaking as a guide to good thinking.

I assume that just as there are adversaries who gravitate toward one side of the formal-informal logic distinction in dialectical opposition to the other, so there must also be many others like myself who will prefer a sensible principled compromise between these two extremes, who hope that formal and informal logic can work together in a productive partnership. It is to give substance to this promise of integrating formal and informal logic that I propose first to give a more exact criterion of the distinction between formal and informal logic, and then to defend a model for the fruitful interaction between formal and informal logical methods.

2. Toward a Definition of Formal and Informal Logic

If the attempt to find middle ground between exclusionary formal and informal logic programs is to succeed, then the first task is to clarify the concepts of formal and informal logic. Then it may be possible to describe a way of reconciling formal and informal logical techniques in a more comprehensive though not necessarily syncretic philosophy of logic.

The concepts of formal and informal logic are best approached by beginning with some faulty attempts at definition, and trying to understand exactly how and why they go wrong. We have already observed that Aristotle's logic of syllogisms with its rules for valid inferences and square of opposition is a kind of formal logic, even though it is usually classified as a branch of informal logic.

The same must then be true of Beardsley argument diagramming and Venn, Euler, and Lewis Carroll diagramming. These methods of representing the structure and assessing the deductive validity especially of syllogistic arguments involving Boolean relations as set theoretical inclusions or exclusions of entities in the extensions of predicates intuitively concern the formal logical properties of propositions and arguments. The use of schematic variables, mathematically regimented syntax, and graphic or set theoretical semantic techniques moreover are not necessary to distinguish formal from informal logics. A formal logic in one sense can always be presented in ordinary albeit stinted English. The difference between formulating in symbolic logic an expression such as $(\exists x)(F = \lambda\ y\ [\check{z}\ (y = x)])$ and articulating in nonsymbolic formal terms the equivalent proposition that 'There is an entity such that a certain property is identical to the abstract property of being necessarily identical to the entity'. In the English paraphrase there is no 'formalism' or symbolic notation as usually understood; yet the phrase as much describes formal relations as does the more straightforwardly 'formal' symbolism containing the existential quantifier, lambda operator, modal necessity operator, parentheses and brackets. All such elements are legitimately part of formal logic insofar as they involve logical forms of relations and logical forms of arguments, regardless of whether they are written in a special symbolic notation or in modified English.

It is tempting in light of these conclusions to consider eliminating the distinction between formal and informal logic altogether. This is an attractive option if we interpret all of logic as involving the forms and structures of syntax, semantic relations, and argumentative patterns. Then, while the term 'logical form' designates a special kind of form, the phrase 'formal logic' is technically redundant. It may then be preferable to substitute the terminology of symbolic-nonsymbolic logic for the formal-informal logic distinction. The disadvantage of the approach is that the distinction between formal and informal logic is well-entrenched in logicians' parlance. As a result, we do less violence to established usage if in good linguistic conscience we can maintain the formal-informal logic vocabulary by providing acceptable definitions of 'formal' and 'informal' logic. The problem is that I am not at all sure that there can be a satisfactory definition of the distinction between formal and informal logic that is both intuitively correct and fully in agreement with current terminological usage. The reason, I believe, is that established linguistic practice among logicians is itself not entirely intuitively correct. I shall therefore try to negotiate a commonsense compromise that distinguishes between formal and informal logic in a way that does not exactly conform to the standard convention, but that I think many and perhaps even most logicians on both sides of the distinction may find acceptable.

My suggestion is to distinguish between formal and informal logic by applying the arguably less controversial because criteriologically better-defined distinction between specialized symbolic versus nonspecialized nonsymbolic logic. More specifically, I propose that a logical theory or procedure is formal if and only if it adopts a specialized symbolism for representing logical forms that does not occur in ordinary nonspecialized nonsymbolic thought and language. Although I acknowledge that all of logic has to do with logical form, I do not believe that all expressions of logical form must themselves be formal. This distinction captures much of the received concept, since it includes all of symbolic logic and excludes nonsymbolic evaluations of validity or invalidity. As we might expect, formal logic by the proposed distinction will roughly include everything belonging to what has become the *de facto* criterion for formal logic in relations expressed by means of standard and nonstandard notational variations and extensions of the propositional and predicate-quantificational calculus. But the definition also includes schematic and graphic treatments of syllogistic logic that have traditionally been regarded as more properly within the aegis of informal logic and critical reasoning. Informal logic by contrast on the present proposal is limited to the consideration of a proposition's or argument's logical form by discursive reconstruction within natural language, the use of counterexamples to discredit inferences, identification of arguments as committing any of the so-called rhetorical fallacies, and the like.

The relegation of syllogistic logic, square of opposition, and argument diagramming methods to the genus of informal logic can now be seen as a kind of historical accident. Were it not for the emergence of more powerful algebraic methods of formal logic with the discovery of formal logical techniques in Frege's *Begriffsschrift* and C.S. Peirce's proto-quantificational logic, there is little doubt that the logic of syllogisms, Venn and other styles of diagramming, etc., would constitute the whole of formal logic as opposed to purely informal nonspecialized nonsymbolic logical methods, beyond which Kant in the late eighteenth century was able to declare that no significant advances had been made since the time of Aristotle.[2] Why then should such logical techniques be displaced as informal given the development of contemporary algebraic methods of mathematical logic?

I think it is more appropriate to classify syllogisms and the tools of logic that have standardly been turned over to the informal logic and critical reasoning textbooks as less powerful, general, and technically advanced, but every bit as formal as rigorously symbolic mathematical logics. As a consequence, I include Aristotelian syllogistic and all related graphic paraphernalia as part of genuinely formal logic. I am therefore committed to saying that these methods are properly part of formal logic despite their usually being included in what are called informal logic texts as adjuncts to what is called the informal logic curriculum. If this is true, then it may be time for logicians to admit that insofar as they use syllogistic logic and argument diagramming they are doing formal logic under the mistaken rubric of informal logic, and that it is equally time for formal logicians to admit that there are weaker less universal methods of logic that are just as formal as the algebraic methods of formal symbolic logic which they may prefer to use, but which do not for that reason alone have exclusive title to the category of formal logic.

What then is to be said about the English translation of $(\exists x)(F = \lambda\ y\ [\check{z}\ (y = x)])$? The natural language equivalents of any such expression are strictly informal by my criterion. Yet I imagine that ordinary language translations of expressions in symbolic logic are unlikely to find application in practice as a workable replacement for formal logic methods. The reason quite obviously is that the ordinary language equivalents while available in principle are simply too cumbersome, as they would be in the parallel case of trying to develop an informal mathematics for the purpose of formulating and solving quadratic polynomial equations. With regard to another problem of the practical implications of the distinction, I want to explain that I have no definite pedagogical recommendations to make with respect to the division of logical methods according to the traditional distinction between formal and informal logic. Thus, I see nothing wrong with continuing to teach syllogistic logic and Venn diagramming, to mention two conspicuous examples, as part of nonmathematical logic, critical reasoning, or what is popularly called informal logic. It is just that by my characterization of the distinction between formal and informal logic I find it misleading to refer to such techniques as informal.

I do not claim that my definition of formal and informal logic could not be improved. I have already indicated that I regard the proposal as a tradeoff between conflicting jointly unsatisfiable desiderata. But for present purposes I want to investigate the implications of distinguishing between formal and purely informal logic alternatively as the analysis of logical form by means of specialized symbolisms versus the study of logical form by means of ordinary language.

3. Formal and Informal Logic in Partnership

There are many ways in which formal and informal logic interact. There are many situations in which formal and informal logic may need to cooperate in order to critically evaluate arguments.

Formal symbolic logics are always accompanied by and presented within a discursive framework of informal metalanguage introductions and explanations, or can be traced back through a genealogy of formal conventions to an informal context. Without grounding in ordinary language and a relation to informal ideas, even the formalisms most familiar to practicing logicians lack meaning and application. If symbolic logic is not always needed, if it can be an impediment to understanding, and if it cannot function effectively entirely on its own for theoretical purposes in the explication of logical connections and deductive proof of consequences, then formalisms must be justified by a philosophical rationale. Informal logic is also useful and typically essential in working through a preliminary heuristic analysis of a problem before it can be decided whethere and if so what kind of formal logic to apply in modeling a selection of logical relations or solving a logical problem. Sometimes informal methods provide a better, easier, or more understandable conceptual analysis of the logic of a proposition or argument.

Accordingly, I want to propose a pragmatic principle that allows informal and formal logical methods to be used individually or in combination to achieve the best analysis of the logic of arguments as determined by the specific requirements of each situation. The ideal is for logicians to cultivate proficiency in as many formal and informal logical methods as may be available, not excluding efforts to discover or invent new techniques as each task may demand or that may best answer the exigencies of each analytic problem considered independently on its own terms as a challenge for logical investigation in its own right. I shall illustrate the potential gains from the partnership between formal and informal logic that I visualize by two examples. The first example demonstrates the importance of informal reasoning as a preparation for formalization in the informal analysis of logical problems. The preliminary informal consideration of a logical paradox when properly done can sometimes avoid the need for formalization altogether, while in other cases it may demarcate the extent to which a minimal partial formalization may be useful, or provide a set of parameters by which the crucial concepts and relations to be formalized can first be identified and an exact purpose and justification for formalization determined. The problem I have chosen is the Epimenides or liar paradox, a famous logical puzzle with interesting implications for our understanding of the nature of truth, the limits of sentence meaning in a language, and the semantics of reference, for which many different elaborate formal logical solutions have otherwise been proposed.

The Liar Paradox

The liar paradox is the implicit logical inconsistency entailed by a sentence that asserts its own falsehood. There are several different formulations of liar sentences, which for convenience can be represented by a simplified expression of self-denial in the standard example:

(L) This sentence is false.

Assuming a classical bivalent truth value semantics, the liar paradox is supposed to follow by dilemma from the constructibility of L. This is the orthodox informal explanation of the liar paradox dilemma. Suppose L is true. Since L says of itself that it is false, then, if L is true, it is false. Now suppose that L is false. Then, again because of L's self-denial, it is not the case that L is false, from which it follows that L is true. The conclusion seems inescapable that liar sentence L is true if and only if it is false.3 The contradiction is obviously intolerable. There are two main categories of responses to the liar paradox within standard logic. The force of the liar paradox is often understood to exert pressure on the concept of truth and the bivalent truth value semantics in which the paradox arises. Alternatively, the truth-valuability or meaningfulness of the liar sentence or its constructibility in an ideal language is challenged.

Many solutions have been proposed. Historically, these have included introducing nonstandard many-valued or truth-value gap logics to replace the ordinary two-valued structure, and legislating against self-reference, self-(non)-application, indexicals, demonstratives, or truth value predications within a language. The former strategy has been shown by Brian Skyrms in "Return of the Liar: Three-Valued Logic and the Concept of Truth" to be ineffective by itself against strengthened liar paradoxes that can be constructed for any enlargement of truth-value semantics. The idea is to define a new truth function that collapses $n \leq 2$ truth values into two, by mapping onto falsehood whatever truth value negation receives in the n-valued system.4 The latter strategy is at the heart of Alfred Tarski's influential theory of truth, in which formal languages are stratified into object- and metalanguages, and truth value predications are forbidden within any given language. Truth value predications can then only be made about a lower-level in a higher-level metalanguage in an indefinitely ascending hierarchy in which it is possible to make meaningful pronouncements only about the semantic properties of lower-level languages.5 Saul A. Kripke's "Outline of a Theory of Truth" combines both truth-value gaps and a modified Tarskian stratification of object-and metalanguages, in which there are transfinitely ramified occurrences of metalanguages within each level of the original Tarskian hierarchy. Kripke diagnoses the logical ailment of the liar sentence as a failure of 'grounding', since the sentence seems indefinitely to flip-flop between truth and falsehood without reaching any settled semantic value.6

What is common to what I shall call the orthodox response to the liar sentence is the assumption that the mere constructibility of the liar sentence is sufficient to entail the liar paradox, that the liar sentence is true if and only if it is false. The inferability as opposed to the mere constructibility of the liar sentence would be quite another thing. But no one has attempted to prove that the liar sentence can be deduced from an otherwise logically sound set of sentences, such as a scientific or philosophical theory, within a language. The attitude is typically that, unless appropriate precautions are taken, the liar sentence can be formulated, so that we must either prevent its construction or ameliorate its disastrous effects. The liar sentence exists or can be uttered, so we had better do something about it. It is precisely this otherwise reasonable traditional assumption that I want to question.7

If the mere constructibility of the liar sentence in a language entails the liar paradox, then the following dilemma inference must hold for some sentence S:

$$\text{(D)}\quad \frac{L \vee {\sim}L}{S \;\&\; {\sim}S}$$

Within the standard semantic framework in which the liar paradox and its solutions are considered, the inference presumably involves a legitimate relation of a sentence with another self-contradictory sentence. If the liar sentence L is constructible, then a sentence of the form S & ~S, such as 'L is true and it is not the case that L is true' (that is, where S = L), is thought paradoxically to be logically forthcoming in the dilemma by excluded middle from the disjunctive tautology L ∨ ~L, given the self-truth-denying content of L.

Another way to express the dilemma is in this way:

L

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True False

$\downarrow$ $\downarrow$

False True

$\downarrow$ $\downarrow$

True if and only if False

True and False

We now need to ask whether the dilemma inference D is itself logically valid or invalid. There is a dilemma that proves a semantic metaparadox about the liar paradox from the same assumption of a classical semantic background that D is either valid or invalid. Let us consider the possibilities in opposite order. Suppose that D is deductively invalid. Then the liar paradox is not entailed by the mere constructibility or truth or falsehood of the liar sentence L. For in that case the liar sentence whether true or false by hypothesis does not logically imply an explicit syntactical contradiction of the form S & ~S for any sentence S. If there is to be a liar paradox engendered by the constructibility of the liar sentence L, then dilemma inference D must on the contrary be deductively valid. But if D is valid, then, since the conclusion of the inference S & ~S is standardly false, and, indeed, logically necessarily false, it follows that the liar sentence dilemma based on the disjunction L ∨ ~L itself must be logically necessarily false. This is a rather extraordinary, but, I think, unavoidable conclusion, which to my knowledge has nowhere been acknowledged in the extensive philosophical literature on the liar. Now, if the liar sentence dilemma or excluded middle involving the liar sentence cannot be true in order to entail the liar paradox, then there simply is no liar paradox even on the assumption that the liar sentence L is true, since the liar sentence entails the liar paradox by dilemma inference D only if the liar sentence is both true and false, or such that it is true if and only if it is false. We might not be satisfied to have the liar sentence turn out to be necessarily false in this way, but our discomfort cannot be understood as a result of the entailment of a logical paradox from the mere constructibility of the liar sentence.

We can represent the liar metaparadox by a similar dilemma structure as above for the liar paradox:

D

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Valid Invalid

↓ ↓

L∨ ~L False ↓

↓ ↓

No Liar Paradox

We are not yet out of the woods. The liar semantic metaparadox I have described is not offered as in any sense a 'solution' to the liar paradox. We cannot conclude on the basis of the metaparadox that the liar sentence is simply logically necessarily false. For without special provision, that seems to just put us back on the standardly semantically problematic false → true, true → false loop. If a sentence expressing its own falsehood is false or even and especially if it is logically necessarily false, then, apparently, it is true. This is no progress over the usual presentation of the liar paradox. The point I have tried to establish is not that the liar sentence is false, but rather that the liar paradox is not standardly entailed by the mere constructibility of the liar sentence. Yet if the liar paradox is not entailed by the mere constructibility as opposed to the inferability of the liar paradox, then the mere constructibility of the liar paradox poses no threat to classical bivalent truth value or validity semantics, nor to the naive folk theory of truth. Hence, the mere constructibility of the liar sentence in a language provides no adequate philosophical motivation for the elaboration of a special formal semantic policy or revision of the naive theory, such as that represented by a semantics of truth value gaps or Tarskian hierarchy of object- and metalanguage truth value predications, or Kripkean hybrid of these proposals.

There is a semantic confusion that remains to be untangled. We must recall the over-arching assumption in which the necessary falsehood of the liar sentence is considered. It has not been suggested that the liar sentence L is false *simpliciter*, but only that the liar is standardly logically necessarily false *if* or *on the assumption that* dilemma inference D is standardly logically valid. If L is true if and only if it is false if or on the assumption that dilemma inference D is standardly logically valid, then there is only one possible conclusion, which is that dilemma inference D is standardly logically *in*valid. We can, and as a matter of fact, we must proceed in this way. But if we do, then we are back in the first liar metaparadox horn, according to which, where dilemma inference D is logically invalid, the liar paradox once again is not entailed by the mere constructibility of the liar sentence. If dilemma inference D is logically valid, then liar sentence L is logically necessarily false, from which it seems to follow that L, which ostensibly says of itself that it is false, is also logically necessarily true. This conclusion again holds not in any absolute sense, but only conditionally on the assumption that dilemma inference D is logically valid. What follows therefore is rather that dilemma inference D is not logically valid, and that the liar paradox is not entailed by the mere constructibility of the liar sentence.

The semantic metaparadox about the liar paradox shows that whether the liar dilemma is logically valid or invalid, the liar paradox is not logically entailed by the mere constructibility of the liar sentence. The dissolution of the liar paradox via the liar semantic metaparadox shows that the liar paradox cannot be intelligibly formulated as entailed by the mere constructibility of the liar sentence in languages powerful enough to express truth-self-denials. The liar metaparadox suggests that something is wrong with the usual description of how the liar sentence is supposed to entail the liar paradox by way of the liar paradox dilemma-but *what*? What hidden fallacy invalidates the liar paradox argument?

An interesting question remains unanswered if we grasp the invalidity horn of the liar metaparadox dilemma. This is of course the problem of what makes the liar paradox dilemma in D deductively invalid. From an intuitive point of view, it seems inadequate to say merely that the inference is invalid because the liar sentence dilemma L ∨ ~L as a tautology of propositional logic is necessarily true, while the conclusion S & ~S as a contradiction or inconsistency of propositional logic is necessarily false. This naturally makes the paradox dilemma deductively invalid. But a deeper difficulty challenges our understanding of the content of liar sentence L, and requires of any dissolution of the liar paradox by semantic metaparadox a reasonable explanation of why in particular it is deductively invalid in the second liar paradox dilemma horn to make use of either or both of the following inferences (or to validly detach the consequents from their respective material conditional counterparts):

(i) L is true ├— L is false (L is true ⊃ L is false) (L ⊃ ~L)
(ii) L is false ├— L is true (L is false ⊃ L is true) (~L ⊃ L)

Why, we must ask, do these inferences not go through? Given the apparent content of sentence L as denying its own truth, why does the falsehood of sentence L not validly deductively follow from its truth, or its truth from its falsehood, as the colloquial description of the liar paradox proposes?

It may be worthwhile at this juncture to take notice of some of the standard and nonstandard reactions to the content of the liar sentence in allegedly giving rise to the liar paradox independently of concern with the liar semantic metaparadox. Some logicians have concluded that both the liar sentence L and its negation ~L are meaningless, and hence neither true nor false. This maneuver forestalls the valid inferences or true material conditionals in (i) and (ii) standardly needed to derive the liar paradox from the constructability of the liar sentence. Wittgenstein in the *Tractatus Logico-Philosophicus* dismisses the liar sentence as meaningless because of the picture theory of meaning, on the grounds that no meaningful sentence can 'get outside itself' or include within itself a picture of itself with the same logical form or mathematical multiplicity of one-one corresponding elements under analysis, as any construction *per impossibile* must do in order to picture an existent or nonexistent fact about itself.8 Tarski in "The Concept of Truth in Formalized Languages" as we have mentioned argues that the liar paradox motivates an indefinitely ascending hierarchy of object-languages and metalanguages to implement the formation principle by which no meaningful sentence in a formalized language can express the truth or falsehood of other sentences belonging to the same language.9 More recently, paraconsistent logics that in different ways tolerate syntactical contradictions allow the liar sentence and its negation to be regarded as both true and false. This strategy accommodates the intuitive sense by which the liar paradox is supposed to follow from the content of the liar sentence in the orthodox informal characterization. Graham Priest, for example, in *Beyond the Limits of Thought* (without reference to the liar in this immediate context), maintains that: "...if α is both true and false, so is $\neg\alpha$, and so is $\alpha \wedge \neg\alpha$. Hence, a contradiction can be true (if false as well)."10

I do not strongly feel the need to refute any of these resolutions, each of which in my opinion has some merits and some disadvantages. But I shall briefly remark some of the difficulties I find in each. The sustained criticism Wittgenstein offers of the picture theory of meaning in the first third of his *Philosophical Investigations* constitutes good grounds for preferring an alternative semantics.11 I pass without further comment on the most trenchant criticisms frequently raised against Tarski's theory of truth. These are that by refusing to permit any truth predications to be made of sentences within the same language Tarski throws out the baby with the bathwater, and that the indefinite hierarchy of object-and metalanguages for truth value predications never achieves a final characterization of the concept of truth. Tarski's metalinguistic restrictions on truth predications not only disallow diagonal constructions like the liar sentence, to the relief of those who are concerned about its constructibility entailing semantic paradox, but also all logically perfectly harmless truth value pronouncements, such as "This sentence is true". The indefinitely ascending stratification of metalanguages in which the truth or falsehood of sentences in lower tiers of the hierarchy can be made never reaches an endpoint at which the theorist can say that truth has finally been defined. The Tarskian semantic hierarchy furthermore is self-consciously *ad hoc*, supported by no independent justification other than its apparent usefulness in solving the liar paradox.12 Kripke's solution mentioned earlier, involving truth value gaps in truth predications infinitely iterated within each tier of a Tarskian metalanguage structure seems unnecessarily complicated. The paraconsistent analysis of the liar, despite the appeal of its frank avowal of the liar sentence as dialethically both true and false, strikes me as bizarre. If the liar sentence is both true and false, then the liar dilemma inference to the liar paradox would appear to be both valid and invalid.13

As an alternative to these received resolutions of the liar, I now want to extend the liar metaparadox I have already explained to provide semantic considerations that eliminate the paradox altogether in a new and distinctively deflationary way. The liar sentence L in the account I favor is itself a disguised contradiction. It therefore standardly validly supports the derivation of any other contradiction, as we see in the first horn of the liar paradox dilemma, necessitating the trivial valid deduction or default logical truth of the inference or counterpart material conditional in (i) that takes us from L to S & ~S (and in particular from L to ~L). This is an interesting result only in revealing the contradiction concealed within L, in somewhat the manner of the Moore pragmatic paradox sentence, "It is raining, but I don't believe it".14 Yet the liar paradox can be seen to fail in the second dilemma horn. For, standardly, if the liar sentence L is judged to be a disguised contradiction that validly implies any other contradiction, then the negation of the liar sentence ~L must be a disguised tautology. I shall first try to justify these interpretations of L and ~L, paying special attention to the intuitive rationale for regarding ~L as tautologous. Then I shall explain how this interpretation of the negation of the liar sentence in ~L blocks the standard valid inference from ~L to S & ~S (and in particular from ~L to L) in the second liar paradox dilemma horn. By the same analogy I conclude that the negation of the liar sentence is logically no more problematic and in many ways comparable to the pragmatic tautologous redundancy of the negation of the Moore sentence, "Either it is not raining, or I believe that it is".

The negation of the liar sentence, "It is false that this sentence is false", is standardly logically equivalent to the affirmation, "This sentence is true". It is clear that the content of the liar sentence in denying its own truth is self-contradictory, for what the sentence says about itself implies that it is true if and only if it is false.15 This should entail that the negation of the contradictory liar sentence is a tautology. But in what further intuitive sense is the negation of the liar sentence, "This sentence is true", tautologous? Standardly, the negation of the liar like any other sentence is either true or false. But the negation of the liar sentence on pain of contradiction cannot be false, because then it is logically equivalent to the liar sentence itself, "It is false that this sentence is true" or "This sentence is false", which by the above standardly implies an outright logical contradiction. Hence, the negation of the liar sentence must be true. This is to say that the negation of the liar sentence is a disguised tautology in the same sense and ultimately for the same reason that the liar sentence is a disguised contradiction. That there can be such disguised contradictions and tautologies in natural language should not astonish anyone. To interpret the negation of the liar sentence in "It is false that this sentence is false" or "This sentence is true" as a tautology moreover engenders no further paradox, beyond surprising the uninitiated with the conclusion that a sentence that asserts its own truth in a kind of limiting case is necessarily trivially vacuously true.16

Although L as a disguised contradiction standardly validly implies S & ~S, ~L as a disguised tautology standardly implies only other tautologies, and therefore no contradictions. The liar paradox is blocked in this way at the second paradox dilemma horn. There is thus a confusion in the orthodox informal characterization of how the liar paradox is supposed to follow from the constructibility of the liar sentence. The semantic analysis of the content of the liar and its negation which has been sketched reveals the fallacy in the inference. The first part of the dilemma relying on inference or material conditional (i) above goes through well enough ((i) is valid or true). If the liar sentence is true, then it is false. If, on the contrary, the liar sentence is false, then the liar sentence itself is not true; rather, what follows logically is the literal negation of the liar sentence, which simply states, 'This sentence is true'. *This* sentence, the *negation of the liar*, is a tautology, and therefore (necessarily) true. But the assumption in the second liar paradox dilemma horn that the liar sentence is false in applying the inference or material conditional in (ii) does not make the liar sentence itself true ((ii) is invalid or false). There is manifestly nothing paradoxical to be validly derived from the necessary truth of the disguised tautology in the negation of the liar sentence, 'It is false that this sentence is false' or 'This sentence is true'. The liar dilemma is avoided by grasping the second horn, which produces only the potentially unexpected result that a sentence asserting its own truth is a tautology, and does not imply any logical antinomy by itself or in conjunction with the contradiction or necessary falsehood of the liar sentence.17

I conclude that all the fuss made about the liar sentence and liar paradox is logically gratuitous. The liar sentence is not validly deducible in any otherwise sound application of language. If we want to avoid what we may perceive as a contradiction in the liar, despite the metaparadox conclusion that no such contradiction is validly entailed by the mere constructibility of the liar, then we should take pains not to utter it. Should philosophical semantics adopt a special policy to prevent us from lying? I do not think so, no more than semantics should legislate against the constructibility of just plain false sentences, or even more explicitly less controversially outright contradictory sentences such as S & ~S, or, say, the Moore sentence. There is a family of what might be called semantically degenerate constructions in languages, which include at one end of the spectrum explicit contradictions like S & ~S, with the liar and the Moore sentence somewhere toward the other extreme. If we are interested in clarity of thought and language, and if we are interested in truth, then we should avoid using these sentences. Similarly, if we are interested in making significant assertions, then we should avoid using disguised tautologies, like the negation of the liar sentence or pragmatically the negation of the Moore sentence. But the mere fact that a given language is rich enough to permit the construction of these oddities does not seem to me to be a good enough reason to prohibit them or to impose any special semantic restrictions on their formulation or interpretation.

The use of largely informal and dispensably quasi-formal logic in this analysis of the liar sentence shows the extent to which complicated formal symbolic notations and semantic relations can be avoided in the treatment of some of the most difficult problems in logic. The paradox disappears when we subject it to a thorough-going critical analysis using only the most modest and informal logical apparatus. To the extent that this is possible with other logical problems, the pragmatic principle I have proposed recommends that informal rather than formal logical methods be employed. That this is not always the most expedient use of logic is indicated by the second example I shall introduce. Here with respect to a notorious argument in philosophical theology it turns out to be more advantageous to use formal symbolic logic to discover a hidden logical fallacy in St. Anselm's ontological proof for the existence of God.

Anselm's Ontological Proof

Anselm's ontological proof for the existence of God has a precise modal structure. By formalizing the argument, it is possible to identify the intensional modal fallacy it contains. The deductive invalidity of Anselm's inference embodied in the fallacy defeats his argument, even if, contrary to Kant's famous objection, existence is included as a 'predicate' or identity-determining constitutive property of particulars.

Norman Malcolm in "Anselm's Ontological Arguments" distinguishes two forms of Anselm's inference.[18] Malcolm acknowledges that "There is no evidence that [Anselm] thought of himself as offering two different proofs."[19] Some commentators have indeed understood what Malcolm refers to as the second ontological proof as Anselm's official or final formulation, interpreting the first version as a preliminary attempt to express or preparatory remarks for the demonstration's later restatement.[20] Anselm presents the so-called second ontological proof in the *Proslogion* III:

> For there can be thought to exist something whose non-existence is inconceivable; and this thing is greater than anything whose non-existence is conceivable. Therefore, if that than which a greater cannot be thought could be thought to exist, then that than which a greater cannot be thought would not be that than which a greater cannot be thought-a contradiction. Hence, something than which a greater cannot be thought exists so truly that it cannot even be thought not to exist. And You are this being, O Lord our God. Therefore, Lord my God, You exist so truly that You cannot even be thought not to exist.[21]

This passage contains what I regard as the heart of Anselm's proof. For present purposes, I want to avoid the controversy of whether the text offers one or two distinct arguments. I shall therefore concentrate on this formulation of Anselm's argument, without trying to decide whether it is essentially the same as or relevantly different than related inferences about God's existence appearing elsewhere in Anselm's writings. I shall also follow Charles Hartshorne's recommendation in *The Logic of Perfection* by referring to the second statement of Anselm's ontological argument as a kind of modal proof, and of the 'irreducibly modal structure' of this form of Anselm's argument.[22] Yet I differ sharply from Hartshorne in interpreting the modality of Anselm's proof as intensional rather than alethic.

Where q abbreviates $(\exists x)Px$, that a perfect being or perfection exists, Hartshorne attributes this form to Anselm's proof:

Hartshorne's Formalization of Anselm's Alethic Modal Proof for the Existence of God

1. $q \rightarrow$ ž q	Anselm's principle
2. ž $q \vee \sim$ ž q	Excluded middle
3. $\sim$ ž q $\rightarrow$ ž $\sim$ ž q	Becker's postulate
4. ž $q \vee$ ž $\sim$ ž q	(3 logical equivalence)
5. ž $\sim$ ž $q \rightarrow$ ž $\sim q$	(1 modal form of *modus tollens*)
6. ž $q \vee$ ž $\sim q$	(4,5 dilemma and detachment)

7. ~ ž ~ q	Perfection not logically impossible
8. ž q	(6,7 disjunctive dilemma)
9. ž $q \rightarrow q$	Modal axiom
10. q	(8,9 detachment)

This is an elegant but defective derivation. Hartshorne presents Becker's Postulate in proposition (3) as though it were a universal modal truth. But the principle holds at most only in modal systems like S_5 with latitudinarian semantic transworld-accessibility relations.[23] There is furthermore a logical difficulty in a key assumption of the proof that renders the entire inference unsound.[24]

Proposition (5), which Hartshorne says follows from (1) as a modal form of *modus tollens*, is clearly false. Hartshorne glosses the assumption by maintaining that: "...the necessary falsity of the consequent [of (1)] implies that of the antecedent...".[25] The principle Hartshorne applies to proposition (1) to obtain (5) is thus: ($\alpha \rightarrow \beta$)→ (ž ~ $\beta \rightarrow$ ž ~ α). This conditional is not generally true, as an obvious counterexample shows. Let α = Snow is red, and β = 2+2 = 5. Then the instantiation of $\alpha \rightarrow \beta$ in Snow is red → 2+2 = 5 is true by default, if the → conditional is interpreted classically, since it is false that snow is red. But it is false that ž ~ $\beta \rightarrow$ ž ~ α in the instantiation ž ~ (2+2 = 5) → ž ~ (Snow is red), because although it is true that ž ~(2+2 = 5), it is false that ž ~ (Snow is red). If Hartshorne tries to avoid the counterexample by interpreting the conditional → nonstandardly in a relevance logic, then the proof is either deprived of excluded middle in proposition (2), or logically disabled in its crucial inference from the disjunction in (2) to the conditional in (3).[26]

The problem is magnified in Hartshorne's application of this 'modal form of *modus tollens*' to proposition q in proof step (5). Consider that if it is necessary that it is not necessary that snow is white, it by no means follows that it is necessary (particularly because *a fortiori* it is not actually the case) that snow is not white. This refutes the general truth of (5) along with the general principle on which it is supposed to depend. What about proposition (5), in Hartshorne's specific interpretation, according to which q means $(\exists x)Px$, that a perfect being or perfection exists? After all, proposition (1) is also not generally true, but at most, Hartshorne believes, when q abbreviates $(\exists x)Px$. Here is a dilemma. By Hartshorne's appeal to excluded middle, either proposition q or its negation is true, $q \vee$ ~q. If ~q, then Anselm's ontological argument is logically unsound on any interpretation. If q, then, given Hartshorne's other assumptions in his formalization, the following inference holds:

Classical Logical Triviality of Hartshorne's Modal Principle (5)

1. q	Assumption
2. $q \rightarrow$ ž q	Anselm's principle (Hartshorne)

3. ž *q* → ◊ ž *q* Modal axiom (α → ◊ α)

4. ◊ ž *q* → ~ ž ~ ž *q* Modal duality (◊ α ↔ ~ ž ~ α)

5. *q* → ~ ž ~ ž *q* (1-4 hypothetical syllogism)

6. ~ ž ~ ž *q* (1,5 detachment)

If *q* is true, as Hartshorne's conclusion (10) states, that a perfect being or that perfection exists, then Hartshorne's proposition (5) is logically trivial. For then the antecedent of (5) is false, making ž ~ ž *q* → ž ~ *q* an empty truism on the classical interpretation of the conditional →. This makes it equally (classically) uninterestingly true both that ž ~ ž *q* → ~ ž ~ *q* and ž ~ ž *q* → ž *q*.But since Hartshorne's formalization relies on proposition (5) in its derivation of (6) from (4) and (3), it follows that the conclusion of Anselm's proof that God exists is true only if Anselm's proof for the conclusion as Hartshorne interprets it is (classically) logically trivial.

If Hartshorne has correctly represented Anselm's modal ontological argument, then the proof is clearly in bad shape. Hartshorne's reconstruction makes no use and takes no notice of Anselm's idea about the conceivability of God as a being than which none greater can be conceived, nor does Hartshorne construe Anselm's proof as the *reductio ad absurdum* Anselm intends when he argues as above: "...if that than which a greater cannot be thought could be thought to exist, then that than which a greater cannot be thought would not be that than which a greater cannot be thought-a contradiction".27 The principle of charity therefore requires an effort to locate an alternative formalization of Anselm's modal argument that avoids Hartshorne's commitment to the manifestly unsound principle in proposition (5).

I now want to apply some of the symbolizations Priest recommends in formalizing the argument from Anselm's discussion of God's inconceivability to Anselm's ontological proof for God's existence. Like Priest, I use an indifferently definite or indefinite description operator δ, and I adopt an informal ontically neutral interpretation of the quantifiers, expressing existence by means of a predicate, E! (E-shriek). I also follow Priest in symbolizing Anselm's relation of "being greater than" by the convenient predicate '>'. However, I revise Priest's conceivability operator to extend its scope. I allow the τ-operator to range over a greater than relation to express the conceivability that the relation holds between two objects, instead of merely attaching to an individual object term as a way of expressing the corresponding object's conceivability. Then I can logically represent Anselm's definition of God as that than which none greater is conceivable by the expression τ(*y* > *x*) in *g* = δ *x* ¬ ∃*y* τ (*y* > *x*).28

By contrast with Hartshorne, I will not emphasize the proof's alethic modality, in the proposition that if a perfect being exists then it necessarily exists. Instead, I wish to call attention to the proof's hitherto neglected intensional modality implied by Anselm's reliance on the concept of God as a being than which none greater is conceivable. This formalization more accurately reflects Anselm's thinking in the ontological proof, which I take to be an improvement over Hartshorne's. I shall nevertheless argue that the intensional modality implied by wide-scope conceivability in Anselm's definition of God renders the ontological argument deductively invalid. For as such it requires a violation *salva non veritate* of the extensionality of Priest's definite or indefinite description operator. Then, even if Kant's objection that existence is not a 'predicate' or identity-determining constitutive property is overturned, the logical structure of Anselm's argument on the most charitable reconstruction nevertheless fails by virtue of instantiating an intensional modal fallacy.

The ontological proof is formalized by the following inference, in which two new principles are adduced. The first thesis maintains the extensionality of δ, stating that anything identical to a definitely or indefinitely described object has whatever properties are attributed to the object by the description. The second is a conceivable greatness thesis, which states that there is always an existent or nonexistent object which is conceivably (if not also actually) greater in Anselm's sense than any nonexistent object. Anselm's argument can now be symbolized in this way:

Anselm's Intensional Modal Proof for the Existence of God

1. $g = \delta\, x \neg \exists y\, \tau\, (y > x)$	Definition g
2. $\forall x\, (\, \delta\, y \ldots y \ldots = x \rightarrow \ldots x \ldots)$	Extensionality δ
3. $\forall x\, (\neg\, \mathrm{E!}\, x \rightarrow \exists y\, \tau\, (y > x))$	Conceivable greatness
4. $\neg\, \mathrm{E!}g$	Hypothesis for *reductio*
5. $\neg\, \mathrm{E!}g \rightarrow \exists y\, \tau\, (y > g)$	(3 instantiation)
6. $\exists y\, \tau\, (y > g)$	(4,5 detachment)
7. $\neg\, \exists y\, \tau\, (y > g)$	(1,2 instantiation)
8. $\mathrm{E!}g$	(4,6,7 *reductio ad absurdum*)

As in Hartshorne's formalization, Anselm is interpreted as drawing inferences from propositons containing modal contexts. But here, as opposed to Hartshorne's rendition, the modal contexts in question are intensional rather than alethic, expressing the intentionality of wide-scope conceivability in the interpolated conceivable greatness principle.

The proof has several advantages over previous attempts to symbolize Anselm's ontological argument. The argument as reconstructed is extremely compact, reflecting about the same level of complexity as Anselm's original prose statement. Anselm's definition of God as that than which none greater is conceivable is explicit in proposition (1). The *punctum saliens* of Anselm's proof, that if God does not exist, then there is after all something conceivably greater than God, is featured prominently in proposition (5), derived via instantiation from the more general conceivable greatness thesis in (3). Finally, unlike Hartshorne's version, the proposed formalization explicitly represents Anselm's proof as a *reductio*. To define God as that than which none greater is conceivable, and to suppose that God does not exist, is to be embroiled in outright logical contradiction, if, as Anselm seems to assume, we can always conceive of something greater than anything that does not actually exist.

The proposed method of formalizing Anselm's argument makes it easy to discover the proof's logical weakness. The problem arises in proposition (7). The conceivability context $\tau \ulcorner \_\_\_\_ \urcorner$ is modal, because it represents an intensional mode of whatever sentence or proposition is inserted. The context's intensionality is seen in the fact that coreferential singular denoting terms and logically equivalent propositions cannot be freely intersubstituted in the context *salve veritate*. Thus, whereas virtually any short and sweet tautology is conceivable, not every infinitely long or monstrously complicated tautology logically equivalent to it is also thereby conceivable.

The intensionality of conceivability invalidates the inference from (1) and (2) to (7), by requiring the substitution of *g* in definition (1) for the definitely or indefinitely description-bound variable *x* by the extensionality of δ in principle (2). The standard Kantian objection to Anselm's proof can also be pinpointed in this symbolization to the conceivable greatness thesis in proposition (3). Kant refutes the ontological argument in the section on 'The Ideal of Pure Reason' in the *Critique of Pure Reason* A599/B627-A600/B628. Kant's claim that existence has no part in the identity-determining constitutive properties of 100 real or unreal gold Thalers challenges the principle in (3) that we can always conceive of something greater than any nonexistent object. Anselm's ontological argument cannot succeed if relative greatness is judged only by a comparison of the constitutive properties that make two or more objects the particular objects they are, to the exclusion of all extraconstitutive properties that categorize such objects' ontic status as existent or nonexistent.

Kant is certainly right to draw this conditional conclusion. The important question is whether Kant is entitled to claim that existence is not a 'predicate' or identity-determining constitutive property. There are intriguing proposals for avoiding Kant's 100 gold Thalers criticism in the philosophical literature, and for reconciling ourselves to accepting existence as a 'predicate' in the one special logically unique case of God.29 Without entering into the merits of these replies to Kant, it is worth remarking that even if Kant's objection to the assumption in (3) is forestalled, the intensional modal structure of Anselm's argument remains deductively invalid.30 The intensional fallacy arises in Anselm's proof because of its attempt to apply the description context extensionality principle in assumption (2) at inference step (7) to the definition of God in assumption (1). Since Kant's conceptual-metaphysical or grammatical refutation of the ontological argument is controversial, the deductive logical invalidity entailed by the argument's committing the intensional fallacy may now appear to be the more fundamental and decisive objection to Anselm's ontological proof. Anselm might avoid venially transgressing Kant's injunction against treating existence as a 'predicate'. But the intensional fallacy in Anselm's modal ontological argument for the existence of God is its more deadly cardinal sin. The logical error in Anselm's proof has been entirely overlooked in the history of purely informal discussions of the inference, and it is only by formalizing the logical structure of Anselm's argument that it becomes possible to identity its deductive invalidity.

4. Conclusion

The examples I have considered are not intended as the only or even as the most useful applications of the pragmatic principle I have endorsed for a partnership between formal and informal logic. They are meant to be illustrative only in an area where many other solutions or attitudes are possible and might even be preferable. The general point is partly to voice a plea for tolerance in logical investigations, to recognize and take full advantage of the continuum of methods available in logic, any of which might be useful and none of which should be overlooked because of ideological or methodological prejudice.31

Notes

1. C.P. Snow, *The Two Cultures: And A Second Look* (New York: New American Library, 1964).

2. Immanuel Kant, *Critique of Pure Reason* [1787], translated by Norman Kemp Smith (New York: St. Martin's Press, 1965), Bviii: "That logic has already, from the earliest times, proceeded upon this sure path is evidenced by the fact that since Aristotle it has not required to retrace a single step, unless, indeed, we care to count as improvements the removal of certain needless subtleties or the clearer exposition of its recognised teaching, features which concern the elegance rather than the certainty of the science. It is remarkable also that to the present day this logic has not been able to advdance a single step, and is thus to all appearance a closed and complete body of doctrine."

3. An excellent introduction is Robert L. Martin, editor, *The Paradox of the Liar* (New Haven: Yale University Press, 1970). See also Jon Barwise and John Etchemendy, *The Liar: An Essay on Truth and Circularity* (Oxford: Oxford University Press, 1987), and Martin, editor, *Recent Essays on Truth and the Liar Paradox* (Oxford: The Clarendon Press, 1984). The most complete historical discussion of the Epimenides paradox (Paul, Epistle to Titus 1 verses 12-13) is Alexander Rünstow, *Der Lüngner: Theorie, Geschichte und Auflösung* (New York: Garland Publishing, Inc., 1987).

4. See Brian Skyrms, "Return of the Liar: Three-Valued Logic and the Concept of Truth", *American Philosophical Quarterly*; 7, 1970, pp. 153-161. If we try to forestall the liar paradox dilemma by introducing a third truth value and assigning it to the liar sentence, then we can define a new truth table and reformulate the liar paradox in this way. For nonstandard trivalent negation with a third undetermined (U) truth value, we might have:

p	*~p*
T	F
F	T
U	U

Now we define:

p	*@p*
T	F
F	T
U	F

The strengthened liar paradox then presumably holds when we construct a strengthened liar sentence L* that says '@(This sentence is true)' (instead of this equivalent of the standard formulation, '~(This sentence is true)'), and then subject the strengthened liar sentence L* to a modified dilemma that takes as its basis the disjunction L* ∨ @L* instead of L ∨ ~L. The method is generalizable to any expanded many-valued truth value matrix involving any number of truth values.

5. Alfred Tarski, "The Concept of Truth in Formalized Languages", *Logic, Semantics, Metamathematics:*

Papers from 1923 to 1938, translated by J.H. Woodger (Oxford: The Clarendon Press, 1956), pp. 152-278.

6. Saul A. Kripke, "Outline of a Theory of Truth", *The Journal of Philosophy*, 72, 1975, pp. 690-716.

7. A typical statement of the orthodox account by which the constructibility of the liar sentence alone entails the liar paradox appears in Tarski, "The Concept of Truth in Formalized Languages", p. 158: "The source of this contradiction is easily revealed: in order to construct the assertion (β) ['*c is not a true sentence*'*is a true sentence if and only if c is not a true sentence*] we have substituted for the symbol '*p*' in the scheme (2) [*x is a true sentence if and only if p*] an expression which itself contains the term 'true sentence'... . Nevertheless no rational ground can be given why such substitutions should be forbidden in principle."

8. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, edited by C.K. Ogden (London: Routledge & Kegan Paul, 1922), 2.174; 3.332-3.333 (concerning Russell's theory of types). In 3.332 Wittgenstein writes: "No proposition can say anything about itself because the propositional sign cannot be contained in itself...", and in 4.442: "A proposition cannot possibly assert of itself that it is true."

9. Tarski, "The Concept of Truth in Formalized Languages", pp. 157-165 and *passim*.

10. Graham Priest, *Beyond the Limits of Thought* (Cambridge: Cambridge University Press, 1995), pp. 156-161. For indications of dialethic intutions about the proper response to the liar paradox, see also Priest and Richard Routley "First Historical Introduction: A Preliminary History of Paraconsistent and Dialethic Approaches", in Priest, Routley and Jean Norman, editors, *Paraconsistent Logic: Essays on the Inconsistent* (Munich and Vienna: Philosophia Verlag, 1989), pp. 12-13, 22-23, 36-44, 48.

11. Wittgenstein, *Philosophical Investigations*, third edition, edited by G.E.M. Anscombe (New York: Macmillan Publishing Co., Inc., 1958), especially §§ 1-64.

12 See Priest, *Beyond the Limits of Thought*, pp. 167-171; also Priest, *In Contradiction: A Study of the Transconsistent* (Dordrecht and Boston: Martinus Nijhoff Publishers, 1987), pp. 23-28.

13. The dialethic logician need not blink at such a contradiction. Yet even Priest, in his chapter on 'Entailment' in *In Contradiction*, pp. 104-105, writes: "...a necessary condition for entailment is truth-preservation from antecedent to consequent... Hence we may say that an entailment is false if it is possible for the antecedent to be true and the consequent false." Where a liar sentence is dialethically both true and false, it is not only possible but actually the case that the assumption (the liar sentence itself) of the liar dilemma inference is true (and false) while the conclusion (any contradiction) is both true and false. If by Priest's definition the liar paradox dilemma inference is deductively invalid (full stop), then the paradox is blocked. But if the inference works by virtue of matching the truth (not the falsehood) of the assumption with the truth (not the falsehood) of the conclusion, or by matching the (necessary) falsehood of the assumption and the truth or (and) falsehood of the conclusion, then the paradox goes through by deductively valid dilemma, though the true (and false) contradiction it entails may be semantically harmless. I can only conclude that I do not sufficiently understand the concept of deductive validity or entailment in a dialethic context to be able to assess its role and therefore to be able to endorse this type of

paraconsistency solution to the liar paradox.

14. The name 'Moore's Paradox' was given by Wittgenstein in response to a series of problems G.E. Moore mentioned in a paper delivered to the Moral Science Club at Cambridge University in 1944. Wittgenstein reportedly was more impressed by the paradox than Moore, who dismissed it as a psychological rather than logical absurdity. See Ray Monk, *Wittgenstein: The Duty of Genius* (New York: Penguin Books, 1991), pp. 544-547.

15. The problem is not just that in the first liar paradox dilemma horn we obtain the conditional $L \supset \sim L$ ($\equiv \sim L$). If we assume (that) L (is true) and from its content validly deduce or detach its negation $\sim L$ via inference or counterpart material conditional (i), then we derive the outright syntactical inconsistency $L \,\&\, \sim L$.

16. Other examples abound: 'John has been a married bachelor for the last fifteen years'; 'Russell was the greatest philosopher of the 20th Century, and so was Wittgenstein'; 'First order logic with arithmetic is both syntactically consistent and deductively complete'.

17. This method of resolving the liar paradox is not intended to provide a panacea for logical and semantic paradoxes generally. Compare Russell's paradox in set theory, which validly deduces an explicit contradiction from two propositions that from a naive set theoretical viewpoint are apparently true by stipulation. The inference has this form:

(R)

$R = \{x \mid x \in x\}$
$R' = \{x \mid x \notin x\}$

$R' \in R \equiv R' \notin R$

The dissolution of the liar paradox by contrast is blocked because the concept of validity is itself standardly defined in terms of truth. The necessary falsehood of the liar paradox conclusion reflects back on the content of the liar sentence as the premise of the attempt to deduce the liar paradox from the mere constructibility of the liar. This is not typically a feature of logical or semantic paradox, as the Russell paradox shows. But it is a requirement of the liar paradox that in particular renders the effort to deduce the liar paradox from the liar sentence logically invalid in the second liar paradox dilemma horn, and hence logically and semantically innocuous.

18. Norman Malcolm, "Anselm's Ontological Arguments", *The Philosophical Review*, 69, pp. 41-62.

19. Ibid., p. 45. Malcolm explicitly states that Anselm does not separate the inferences, but maintains that the ontological argument can be better understood and defended if the two versions are distinguished.

20. See Gregory Schufreider, *An Introduction to Anselm's Argument* (Philadelphia: Temple University Press, 1978), pp. 40-45; Robert Brecher, "Hartshorne's Modal Argument for the Existence of God", *Ratio*, 17, pp. 140-146; Ermanno Bencivenga, *Logic and Other Nonsense: The Case of Anselm and his God* (Princeton:

Princeton University Press, 1993), pp. 113-123.

21. Anselm, *Anselm of Canterbury*, translated by Jasper Hopkins and Herbert Richardson, (Toronto and New York: Edwin Mellen Press, 1974) (4 vols.), I, p. 94. Anselm, *Opera omnia, ad fidem codicum recensuit Franciscus Salesius Schmitt*. (Edinburgh: Nelson & Sons, 1945-1951) (6 vols.), I, pp. 102-103, writes: "*Nam potest cogitari esse aliquid, quod non possit cogitari non esse; quod maius est quam quod non esse cogitari potest. Quare si id quo maius nequit cogitari, potest cogitari non esse: id ipsum quo maius cogitari nequit, non est id quo maius cogitari nequit; quod convenire non potest. Sic ergo vere est aliquid quo maius cogitari non potest, ut nec cogitari possit non esse. Et hoc es tu, domine deus noster. Sic ergo vere es, domine deus meus, ut nec cogitari possis non esse.*"

22. Charles Hartshorne, *The Logic of Perfection and Other Essays in Neoclassical Metaphysics* (LaSalle: Open Court Publishing Co., 1962), pp. 49-57. I have converted Hartshorne's necessity operator 'M' to what is nowadays the more conventional symbol 'ž'. See Hartshorne, *Anselm's Discovery: A Re-Examination of the Ontological Proof for God's Existence* (LaSalle: Open Court Publishing Co., 1965). Alvin Plantinga, *The Nature of Necessity* (Oxford: The Clarendon Press, 1974), offers another approach to the formal modal structure of Anselm's ontological argument.

23. The principle Hartshorne identifies as Becker's Postulate, ~ ž *q* → ž ~ ž *q*, is logically equivalent to the characteristic axiom of modal S_5, more usually formulated as ◊α → ž ◊α. See R. L. Purtill, "Hartshorne's Modal Proof", *The Journal of Philosophy*, 63, 1966, p. 398.

24. Hartshorne's proposition (1) is criticized by John Hick in Hick and Arthur C. McGill, editors, *The Many-Faced Argument: Recent Studies on the Ontological Argument for the Existence of God* (New York: The Macmillan Company, 1967), pp. 349-352. See also Brecher, "Hartshorne's Modal Argument for the Existence of God", *Ratio*, 17, 1976, pp. 140-146; Plantinga, "A Valid Ontological Argument?", *The Philosophical Review*, 70, 1961, pp. 93-101. Hartshorne, "Necessity", *Review of Metaphysics*, 21, 1967, pp. 290-309, replies to Purtill on the proper modal logical concept of necessity as it pertains to Anselm's efforts to prove the existence of God. Another kind of informal criticism is offered by J. O. Nelson, "Modal Logic and the Ontological Proof of God's Existence", *Review of Metaphysics*, 17, 1963, pp. 235-242.

25. Hartshorne, *The Logic of Perfection and Other Essays in Neoclassical Metaphysics*, p. 51.

26. On the unavailability of excluded middle and disjunctive syllogism in relevance logic, see Stephen Read, *Relevant Logic: A Philosophical Examination of Inference* (Oxford: Basil Blackwell, 1988), p. 60.

27. See Desmond Paul Henry, *The Logic of Saint Anselm* (Oxford: The Clarendon Press, 1967), p. 143.

28. The difference is not trivial, but concerns whether Anselm is to be understood as saying merely that God is that than which there is nothing conceivable that is greater, or that than which it is not conceivable that there be anything greater. Compare Graham Priest's definition in *Beyond the Limits of Thought* (Cambridge: Cambridge University Press, 1995), p. 62, n. 2: "Let τx be 'x is conceived'. Then God (g) may be defined as $\delta x \neg\exists y\, (\tau y \wedge y > x)$. (Quantifiers, note, are not existentially loaded.) Let $\varphi(x)$ be the second-order condition $Ex \wedge \forall P\, (P \neq E$

$\rightarrow (Px \leftrightarrow Pg))$, where E is existence. Then the claim is $\forall x(\neg Eg \wedge \varphi(x) \rightarrow x > g)$.' See Priest, "Indefinite Descriptions", *Logique et Analyse*, 22, 1979, pp. 5-21. I make a similar plea for the ontic neutrality of the quantifier in noncircular reconstructions of Anselm's proof in Dale Jacquette, "Meinongian Logic and Anselm's Ontological Proof for the Existence of God", *The Philosophical Forum*, 25, 1994, pp. 231-240.

29. See Jerome Shaffer, "Existence, Predication and the Ontological Argument", *Mind*, 71, 1962, pp. 307-325; S. Morris Engel, "Kant's 'Refutation' of the Ontological Argument", *Philosophy and Phenomenological Research*, 24, 1963, pp. 20-35; Plantinga, "Kant's Objection to the Ontological Argument", *The Journal of Philosophy*, 63, 1966, pp. 537-546.

30. An inference in this same style of logical notation that is subject to Kant's objection can also be formalized. Instead of relying on the extensionality of δ, the method appeals to what Priest calls the Characterization Principle. The argument takes the following form:

1. $\varphi\ (\delta x(\varphi x))$	Characterization
2. $g = \delta x \neg \exists y \tau (y > x)$	Definition *g*
3. $\forall x(\neg \mathrm{E}! \rightarrow \exists y \tau (y > x))$	Conceivable greatness
4. $\neg \mathrm{E}!g$	Hypothesis for *reductio*
5. $\neg \mathrm{E}!g \rightarrow \exists y \tau (y > g)$	(3 instantiation)
6. $\exists y \tau (y > g)$	(4,5 detachment)
7. $\lambda\ \mathrm{z}[z = \neg \exists y \tau (y > z)]\delta x \neg \exists y \tau (y > x)$	(1 instantiation)
8. $\neg \exists y \tau (y > \delta x \neg \exists y \tau (y > x))$	(7 λ-equivalence)
9. $\neg \exists y \tau (y > g)$	(2,8 Substitution of identicals)
10. $\mathrm{E}!g$	(4,6,9 *reductio ad absurdum*)

Kant's 100 gold Thalers criticism balks at the attempt in step (3) to give an instantiation of the Characterization Principle. The problem is that nonexistence like existence for Kant is not a 'predicate', which is to say that nonexistence is not a constitutive identity-determining property. There is moreover a sense in which this alternative method of formalizing Anselm's proof falls back into the same problem as symbolizations involving the extensionality of descriptor δ. By itself, $\neg \exists y \tau (y > x)$ with its unbound variable is not well-formed, and does not designate a property or even a relational property term. To obtain a substitution instance of Characterization as in (8) above, it is necessary to resort as in (7) to instantiation via λ-conversion. Yet there is a logical equivalence between λ-abstractions and descriptions, as expressed in this untyped statement intuitively equating a formal abstraction with the or a definitely or indefinitely described property satisfying certain conditions: $(\forall\ x)(\lambda y[...y...]x \equiv \delta z(\forall\ y)(zy \equiv (...y...)x))$. Applying λ-equivalence to proposition (7) to deduce step (8) thereby again presupposes the extensionality of δ. I was led to consider this version of Anselm's proof in responding to questions raised by Priest in personal correspondence.

31. A version of one main part of this essay was presented at the Conference on Truth, sponsored by the Inter-University Centre of Dubrovnik, Croatia, held in Bled, Slovenia, June 3-8, 1996, under the title, "Truth as a Regulative Concept of Philosophical Semantics". I am grateful to John Biro and Brian McLaughlin for useful comments and criticisms. The material on Anselm is extracted from an unpublished manuscript on "Conceivability, Intensionality, and the Logic of Anselm's Modal Argument for the Existence of God". My research for this project was generously supported by the Commission for Cultural, Educational and Scientific Exchange Between Italy and the United States of America, during my tenure as J. William Fulbright Distinguished Lecture Chair in Contemporary Philosophy of Language at the University of Venice, Italy.

View Commentary by H.V. Hansen
View Index of Papers and Commentaries
Return to Main Menu