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Analysis of two-link fixed conveyor systems.

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ANALYSIS OF TWO-LINK FIXED
CONVEYOR SYSTEMS

BY

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ABSTRACT

This thesis considers the steady-state behavior of a fixed conveyor system. The systems design parameters to be considered include the number of men or machines supplying units of production to the conveyor, the number of men removing parts from the conveyor and the length of the conveyor. The conveyor system under study is a queueing system with multiple (M-service) channels supplied by multi-input sources. These sources allow two types of arrivals, (each has different service times), each is governed by a separate independent Poisson distribution. It is assumed that the distribution of service times is exponential.

The steady-state probabilities for n-items in the system and other "measures of effectiveness" are mathematically derived. A simulation analysis is also undertaken to determine the utilization of the service channels and to compare both theoretical and simulation results.

LIST OF APPENDICES

LIST OF NOTATIONS

λ_1 = mean arrival rate of the first type of units

λ_2 = mean arrival rate of the second type of units

μ = mean service rate

ρ_1 = traffic intensity, i.e. λ_1/μ

ρ_2 = traffic intensity, i.e. λ_2/μ

n = service time ratio between two types of units

C = number of service channels

b = length of conveyor

S = number of sources

TABLE OF CONTENTS

Page

CHAPTER 1

INTRODUCTION

Conveyor System

In recent years most modern manufacturing firms have been adhering to the philosophy that a certain amount of work specialization leads to an increased efficiency of production. As a result of this trend it has been becoming more the rule than the exception to find goods being produced on long production lines.

In general, the theory of conveyors is not concerned with the physical construction of the conveyor; for the most part the mechanical design problems have been solved. Conveyor theory is concerned primarily with the determination of minimum cost operating policies for conveyor systems.

The conveyor itself comprises only one portion of a total operating system. Also included as part of a conveyor system are the work stations (or sources) which load the conveyor, the work stations (or service facilities) which unload the conveyor, and the units of production which are being processed, transported, and stored. In determining a system's operating policy, consideration must be given to the rate at which the service facilities are performing operations, the storage capacity of the conveyor, and the production schedule for the manufacturing system of concern.

In recent years, researchers have been devoting more alteration to the area of conveyor theory. This is due to the ever increasing importance of conveyors.

The Fixed Conveyor

The fixed conveyor system is most often used to link together two production centers. This system is exemplified by a group of workers placing units of production onto a gravity feed roller conveyor to be transported to another location where a second group will remove them. For the most part, conveyors of this type are used as means of removing materials from one production center to another and as a location for storing in-process inventories. From Figure 1.1 it can be seen that the general movement of units through this type of system is as follows:

1. As units are completed at the first production center they are loaded onto the conveyor.
2. Units move along the conveyor until they reach the second production center.
3. When a unit reaches the second production center it is moved from the conveyor, operated upon, and passed on to the next system.

As units move through the system certain limiting conditions may be encountered. If all service facilities are occupied

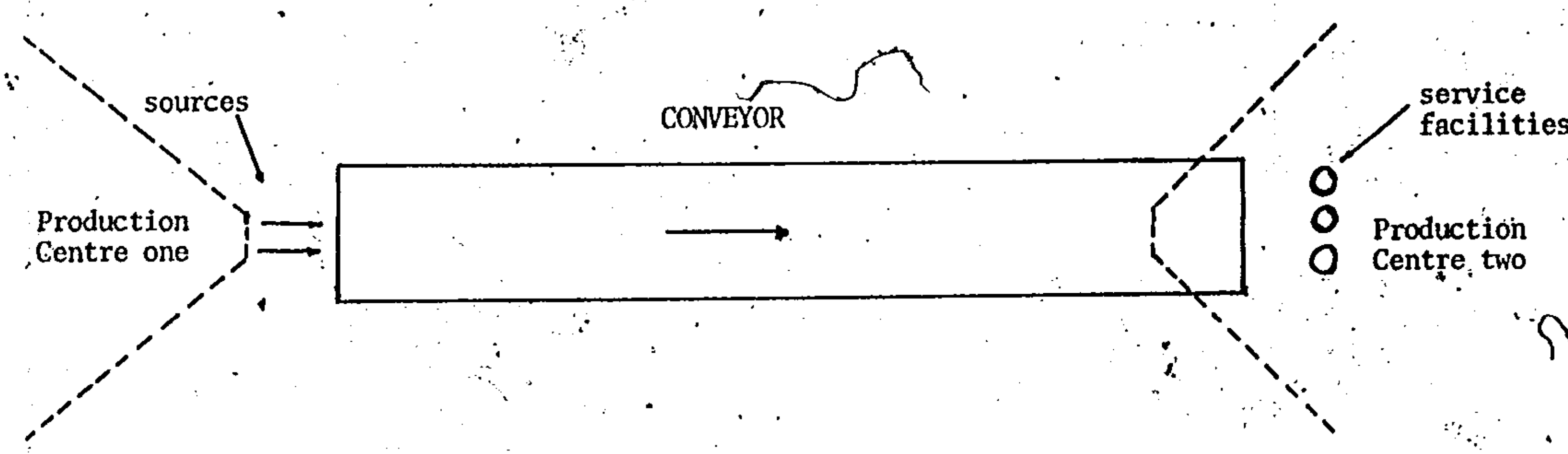


Fig. 1.1. Fixed Conveyor System

when a unit is completed at the source, the unit waits on the conveyor until a facility becomes available. However, if a unit is completed at the source and not only finds all service facilities occupied, but also finds all storage locations on the conveyor filled, it remains at the source.

Queueing Consideration

Queueing theory, also referred to as waiting line analysis, deals with situations which arise when units (customers) arriving at some service mechanism must wait before they can be serviced. In order to study and mathematically analyze a queueing system a prerequisite knowledge of the laws governing the arrival pattern, the service mechanism and the queue discipline must be obtained.

The arrival pattern of units entering a queueing system can be defined by specifying the statistical pattern of arrivals, the average rate of arrivals and the number of units which constitute an arrival (single or bulk arrivals). In some queueing situations it is also necessary to determine whether or not the arrival pattern depends on the status of the system. For example, a long queue may deter units (customers) from entering the system; thus, the status of the system will have a major influence on the pattern of arrivals.

The laws governing a service mechanism for a given queueing system are best described by identifying the statistical distribution of the time to complete a service, by specifying the exact number of arrivals or the statistical distribution which defines the number of arrivals in a given time period, and by giving a description of the physical properties of the mechanism (number of servers and their arrangement).

The queue discipline refers to the order in which units are selected for processing and the manner in which customers (units) behave once they have entered the waiting line. In selecting a unit to be served by the service mechanism, the unit may be chosen according to a first-come-first-serve (FCFS) rule, a random selection rule, or according to some assigned priority scheme. In describing the behavior of units in the queue, one must note if a unit is allowed to leave the system before it has been served (reneging), if a unit is allowed to move freely between waiting lines (jockeying), or if a unit is allowed to refuse to enter the queue when the line length is too long (balking).

Within the literature of queueing theory, a somewhat standardized notation for identifying queueing models is generally used. This notation bears the name of its originators and is referred to as Kendal's notation. Kendal's notations have been introduced in the following symbolic form (18):

$$(a/b/c):(d/e/f)$$

where,

"a" represents the arrival (or interarrival) distribution "b" represents the departure (or service time) distribution.

"c" represents the number of parallel service channels in the system.

"d" represents the service discipline.

"e" represents the maximum number allowed in the system (in service and waiting).

"f" represents the calling source.

The following conventional codes are usually used to replace the symbols "a", "b" and "d",

Symbols "a" and "b"

M = Poisson arrival or departure distributions (or equivalently exponential interarrival or service times distributions).

D = deterministic interarrival or service time.

E_x = erlangian or gamma interarrival or service time distribution.

GI = general independent distribution of arrivals (or interarrival times).

G = general distribution of departures.

Symbol "d"

FCFS = first come, first served.

LCFS = last come, first served.

SIRO = service in random order.

GD = general service discipline.

The symbol "c" is replaced by any positive number representing the number of parallel servers. The symbols "e" and "f" represent finite or infinite numbers in the system and calling source, respectively.

The Conveyor as a Queueing System

Referring back to Fig. 1.1 it can be seen that the system under study possesses many of the properties of a queueing system. Units are processed by the sources and delivered to the conveyor, thus forming a stochastic arrival pattern. Space on the conveyor is provided for the waiting line to form. The service facilities remove parts from the conveyor (queue) and process them in a manner that allows a service time distribution to be defined.

Statement of The Problem

The objectives of this study are: i) to develop a model of the fixed conveyor system; ii) to obtain a methodology for determining the minimum cost combination of sources, servers and conveyor length; iii) comparison of the analytical results with the simulation results.

In the area of fixed conveyors past research has always been concerned mainly with the cases where inputs are identical with respect to the required operations and the service times. So far no efforts have been made in this area to solve the multi-item case, where more than one type of units (multi-item jobs) have been scheduled to be operated upon and transported by the fixed conveyor system.

This study focusses mainly on the fixed conveyor allowing multiple Poisson inputs. The system under consideration has the following properties:

1. Two types of arrivals are allowed, each type is governed by an independent Poisson distribution.

2. M-Channel case is studied where the service rate at each channel has a negative exponential distribution.

3. The service time of a unit from the first type of arrival is n times that of a unit from the second type.

4. The removal of units from the conveyor is based on first come-first serve discipline.

The system's performance is measured by means of certain criterion known as measures of effectiveness, derived by the application of queueing theory. These measures are as follows:

i) steady state probabilities of the system

ii) the probability that system being idle

iii) the expected number of units in the system

iv) the probability that an arrival will have no wait prior to service.

The system design parameters to be considered in this study are:

a) the traffic intensities

b) the number of service channels

c) the number of input sources

d) the length of conveyor

e) the service time ratio between two types of arrivals.

Importance of This Study

The type of the arrival rate used in this study (multiple Poisson input) practically exists as in the case of using conveyors with service stations in the repair and maintenance of large assemblies, each consisting of several identical units. In this study, the steady state solution of multiple jobs, which have been scheduled to be operated on fixed conveyor, will be developed.

The analytical results, cost model and simulation study will be helpful in decision making to determine the optimal operating condition of such conveyor systems.

In this study, the analytical results are compared with the simulation results. The results will be of importance to the designers of such conveyor systems.

CHAPTER 2

REVIEW OF LITERATURE

The paper published by Kwo [10] was the first to view the conveyor as a coupling device between two manufacturing areas. After examining the general conveyor problem, the author gives what he terms 'rules' for satisfactory operation of a conveyor.

The three rules stated are probably the most significant and most often quoted contributions to the theory of conveyor operations. These rules are:

1. The Speed Rule: this rule specifies the range of speeds within which a conveyor must be operated satisfactorily. The general expression is

$$\text{Max } (R_L, R_U) < \frac{V}{S} < \text{Min } \left(\frac{1}{T_L}, \frac{1}{T_U}, \frac{Vc}{S}\right)$$

where: R_L = loading rate

R_U = unloading rate

V = velocity or speed of conveyor

S = spacing between carriers

T_L = average loading time

T_U = average unloading time

Vc = max. technological conveyor speed

2. The Capacity Constraint Rule: this rule states that the conveyor must have enough capacity to accommodate

accumulated items, intentional reserve stock, and stock which must be carried to satisfy the temporary requirements at the loading and unloading points to their physical separation stated in equation form.

$$\frac{MQV}{L} = \frac{MQ}{W} = \frac{QV}{S} > K$$

where: M = total number of carriers on the conveyor

Q = capacity of a carrier

V = the speed of the conveyor

L = the length of the conveyor

W = L/V = revolution time of the conveyor

S = L/M = spacing between carriers

K = constant determined by the specified amount of safety reserve stock to be carried by the conveyor.

3. The Uniformity (Principle) Rule: it states that the conveyor must be loaded as well as unloaded uniformly throughout its entire length. According to this rule, the level of accessibility of parts for the unloading station and spaces for the loading station will be optimized. Helgeson (8) in one of his papers discusses the "how to design" aspects of closed-loop conveyor systems. The operating policy for the system chosen is:

> The conveyor receives parts from loading station at some specified rate R_L. It then delivers these parts to an unloading station, where they are unloaded at some rate R_V. Storage is not allowed at either the input or the output stations. Any part that must be kept in storage is left on the conveyor. To obtain the true storage capacity, consideration must be given

to the fact that the unloading station demands R_V parts per minute and therefore enough spaces must be provided on the conveyor for a possible delay or breakdown of either the loading or unloading station. A third consideration must also be made to allow for a non-uniform distribution of parts on the conveyor.

The author at this point moves on to show how conveyor revolution speeds can be approximated by employing a monograph which he has developed.

Disney (5) presented his paper in the form of a technical note. This was the first published work in which a conveyor system is treated as a multi-channel queueing system with ordered entries. The conveyor chosen for study is of the power and free type.

In a queueing sense, the problem is viewed as a multi-channel problem in which arrivals must enter the first empty channel. He bases his analysis on this model and an extension of the results of work done by Palm who showed that the probability of a loaded hook finding all workers busy and lost to the system is given by Erlang's "Lost Call" formula shown below:

$$P_{lost} = (\rho^r/r!) \Big/ \sum_{i=0}^{r} (\rho)^i/i!$$

where: r = the number of channels

ρ = the load factor

λ = average arrival rate

μ = average service time per service.

P_{lost} = probability that a loaded hook (unit) finds all workers busy and lost to the system.

Using the results of some of his previous work, the author goes on to develop system equations for a two station case. The two storage policies examined are:

1) no storage is allowed at either of the stations, and
2) no storage is allowed at the first station, but N units can be stored at the second station.

The conclusions reached are:

1) For the no storage case (power conveyor) the unbalance created between servers can be partially corrected by increasing μ. However, in this case more parts are lost to the system.
2) Imbalance can be corrected by storage facilities. When balance is achieved, experimentation has shown that only 50% of the design capacity is utilized.
3) Storage facilities reduce the chance of a part recirculating. The major drawback is that as storage allowances are increased the probability of system idleness goes up.

Mayer, as quoted by Cullinane (4), proposed a method for analyzing multiple loading station conveyors. The theory developed is limited to conveyors which carry discrete units of production away from multiple loading points.

In the development of a mathematical model of the system the following assumptions are made.

1) Conveyor speed is constant.
2) All conveyor hooks are empty before arrival at the first work station.

3. There are n stations with equal cycle times.
4. Worker cycle times are equal and independent.
5. Only one conveyor hook was already within a worker's reach.
6. If a worker finds a hook loaded he places the product on the floor.
7. Once production is placed on the floor it is eliminated from subsequent considerations.

Using the concept of expected value and the binomial distribution, a measure of effectiveness is developed. This measure of effectiveness (D) is given by the following expression:

$$D = 1 - \frac{H}{N}\,[1 - q^n]$$

where: N = number of work stations

H = total number of hooks passing per shift

n = total units of production per shift

q = probability no attempt is made to load a given hook.

Within the paper there are many practical examples of how the Measure of Demerit can be applied to operating systems and how they can be used as a criteria for judging the system's effectiveness. Gupta (7) extended Disney's work on the two-channel queueing problem with ordered entry to a general case considering the two-channel case when the maximum number of units allowed in channels one and two are M and N respectively.

Using the generating functions technique, Gupta obtained the queue size distribution in the steady state case.

Pritsker (14) considered a conveyor system which consisted of a conveyor which is not restricted in type (belt, hook, receptacle, etc.), work stations (channels) which remove units from the conveyor and perform some operations on them, and input channels which place units on the conveyor according to some specified distribution.

The approach taken in the analysis was to view the overall conveyor system as a multiple channel queueing system with ordered entries. The major contribution of this study is that through the mathematical development and simulation study it is shown that certain parameters have little or no effect on the steady state performance of the system.

These parameters are shown to be:

1) The distance between service channels.
2) The form of the service distribution if the interarrival distribution is exponential.
3) The distance which units not taken from the conveyor must be fed back.

In the area of fixed conveyors past research has not always been concerned with the analysis of the total system. Most of the early work on fixed conveyors has been qualitative and descriptive in nature. Apple (1) describes gravity feed, roller and belt type conveyors in his book on plant layout. Reed (16) considers some of the mechanical problems that are involved in designing a conveyor system and he proposes methods for determining some of the mechanical design parameters. Carson (3) and Bolz, H.A. (2) concentrated on a description of the physical characteristics of conveyors and a discussion of the types of materials they can carry.

Richman and Elmaghraby (17) reported on the application of queueing theory to a fixed conveyor system. The system considered was a straight line production operation in which the output of one machine was delivered to the next machine by conveyor. The objective of the research was to determine the best conveyor length when in-process inventories were considered. For purposes of analysis the system was viewed as a steady state M/M/1 queueing model with unlimited arrival population. Through the application of conditional probability theory and queueing theory, the length of the conveyor was determined such that it satisfied a requirement that the system would be blocked only some given percentage of the time. Morris (13) applies queueing theory to a roller type gravity feed conveyor. The system studied is a straight line production operation in which, at one stage of manufacture, steel castings are stored in a live bank on a roller conveyor. The objective of the analysis is to determine the length of the conveyor which yields a minimum operating cost. The system is considered to be an M/M/1 queueing model. An objective function is developed and the minimum cost, conveyor length is obtained by direct calculations.

Morgan (12) analyses the steady state behaviour of a two link materials handling system with intermediate storage.

Although it is not specifically a conveyor system, this system is similar to the fixed conveyor in that the first link is a source and the second a service facility. In developing the model, the author uses an example in which a power shovel delivers materials to a storage hopper and trucks then remove this material.

Consideration is first given to the situation which occurs when there is one carrier (source) in the first link. The steady state equations for this sytem are first developed for exponential interarrival times ($E_K/M/C$). Three results are obtained.

1) The mean rate of flow of units through the system per unit time.
2) The proportion of the total time the source is idle, and,
3) The mean number of idle service facilities.

Another study which is similar to those mentioned above was Votaw and Stover (19). They use an example of a chemical plant production process in which reactors correspond to sources, holding tanks to waiting places and stills to service facilities. A unit of production is considered to be one batch of material produced by a reactor. Equations for the state probabilities for the finite source, finite service facility, and limited queue system are given. The results obtained for this system are the average waiting time for a unit, the average length of the waiting line, and the average number of operating service stations and sources.

In a paper devoted primarily to the determination of the upper and lower bounds for the probability that there are N units in the storage area of a multiple source, multiple server system, Weiss (20) described a system very similar to that of the fixed conveyor. The system is assumed to have R independent sources which delivered to K independent service facilities. The arrival of units is taken to be Poisson and the service time exponential.

Jackson (9) has studied queueing systems which described typical production line situations. For the most part his work is purely theoretical. The system studied is a generalized customer-arrival process whose potential customers arrive according to a Poisson distribution, but the probability that a customer actually enters the system depends on the number already in the queue. He has also investigated the case in which the number of servers being employed at a given work center is dependent upon the number of units in the queue. The results of this work is the determination of the conditions necessary for the existence of a unique equilibrium state probability distribution.

Cullinane (4) investigated the transient and steady state behaviour of a fixed conveyor system. The objectives of his research have been (1) to develop a model of the fixed conveyor system for transient operating conditions, and (2) to obtain a methodology for determining the minimum cost combination of sources, servers and conveyor length that can be employed to process short run jobs which have been scheduled to be operated upon and transported by the fixed conveyor system. In the case of multiple job analysis, Cullinane did not allow the interaction (mixing) between the jobs, although it is theoretically possible.

El-sayed et al (15) investigated the multi-item case (interaction between jobs) by applying queueing theory to a closed-loop conveyor system. The system studied is a M/M/2 queueing model with no storage at either of the channels. Steady state probabilities for n-items in the system and their 'measures of effectiveness' are derived.

Having viewed the literature and established the procedure for analyzing the queueing system, the next step is to develop mathematical models for the conveyor system under study.

CHAPTER 3

MULTI-CHANNEL EXPONENTIAL QUEUEING MODELS

In this chapter, the probabilistic queueing models with exponential interarrival times, exponential service times will be developed for the following cases:

1) Single-server, single-source conveyor system with one unit length of conveyor capacity.
2) Two-server, single-source conveyor system with conveyor length as one.
3) Three-server, single-source conveyor system with conveyor length as one.
4) Multi-servers, single-source conveyor system.

The system under study is shown in Fig. 3.1. If "N" denotes the maximum number of units of production, the system can contain at any point in time, "S" specifies the number of sources delivering units to the conveyor, "C" indicates the number of service facilities taking unit from the queue (conveyor) and "b" represents the maximum queue length (conveyor length) allowable, then the following relationship can be stated:

$$N = C + S + b$$

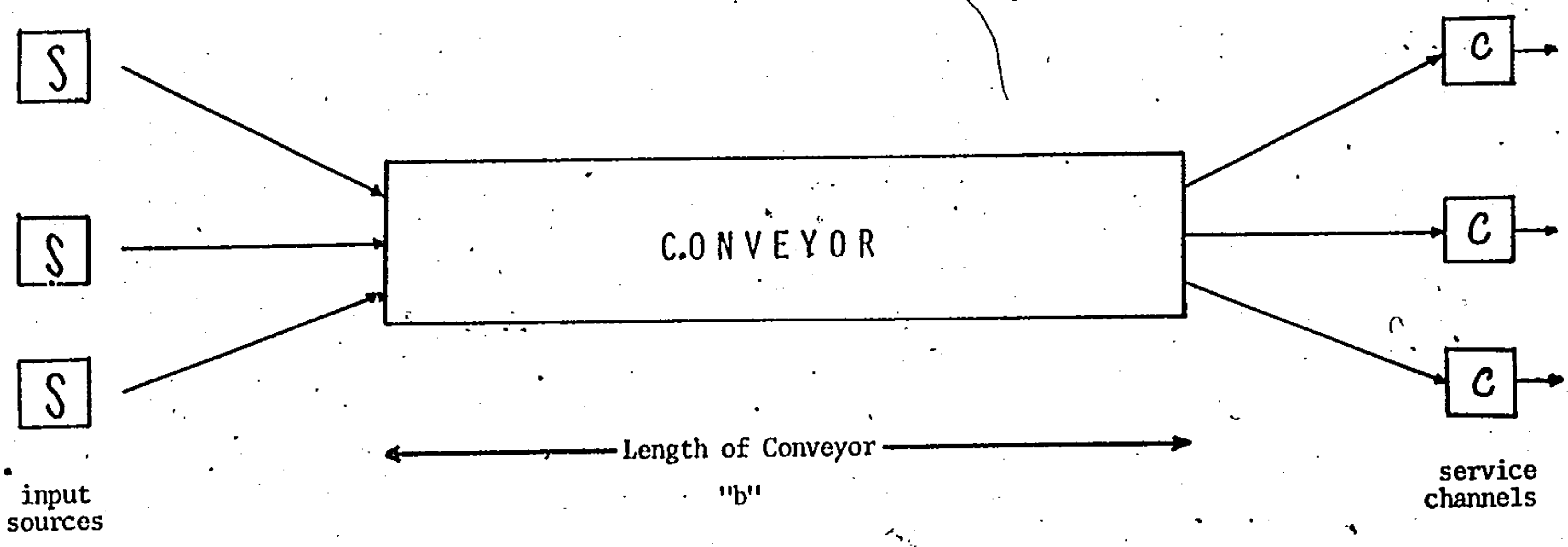


Fig. 3.1. Gravity Feed Roller Conveyor

The following assumptions are made:

1) For each of "S" independent sources, two types of units will be delivered to the conveyor in a Poisson fashion with a mean of λ_1 and λ_2, respectively.

2) The "C" service facilities located along the conveyor are independent of each other and each has a service time which follows an exponential distribution with a mean of $1/\mu$. The service time of the second type of units is $\hat{n}$-times that of the first type.

3) The removal of parts from the conveyor is based on a first come-first serve discipline.

4) The maximum length of the queue is governed by the length of the conveyor.

5) If a source attempts to deliver a unit to the conveyor and finds that there is no space available, the unit remains at the source.

The steady-state probabilities will be derived for all the cases mentioned previously in this chapter. Notations used in deriving these equilibrium probabilities are as follows:

P_{ij} = the probability of having i units from first type and j units from second type in the system

λ_1 = average arrival rate of first type of units

λ_2 = average arrival rate of second type of units

μ = average service rate of service channels

$\hat{n}$ = service time ratio of the two units, i.e.

$$\left(\frac{\text{service rate of second type of units}}{\text{service rate of first type of units}} = \frac{\hat{n}\mu}{\mu}\right)$$

Before the steady-state equations are derived, the basic axioms governing this type of conveyor system are stated:

Basic Axioms

Axiom 1. In an interval of time $\Delta t > 0$, there is a positive probability of arrival (departure) (21).

Axiom 2. In a sufficiently small interval of time, at most one arrival (departure) can occur; that is, probability of two or more arrivals (departures) in time Δt is zero.

After writing the steady-state equations, the following three measures of effectiveness will be derived. These measures help to know about the effectiveness of the queueing system.

(i) a measure of the idle time of the system, i.e., the probability of the system being idle, P_{00}

(ii) expected number of units in the system, $E[n]$

(iii) some measure of the waiting time that a unit might be forced to endure, i.e., the probability that a customer (arrival) will have no wait prior to service, $P_r[w=0]$.

The probability equations for different cases can now be written in the following manner.

Case 1. $C = 1;\ b = 1;\ S = 1;\ \hat{n}$ = service time ratio

$$P_{00}(t+\Delta t) = P_{00}(t)[1-S(\lambda_1+\lambda_2)\Delta t] + P_{10}(t)(\mu)\Delta t$$

$$+ P_{01}(t)(\tfrac{1}{\hat{n}}\mu)\Delta t \tag{1}$$

$$P_{10}(t+\Delta t) = P_{10}(t)[1-\{S(\lambda_1+\lambda_2)+\mu\}\Delta t] + P_{00}(t)\cdot S\lambda_1\cdot\Delta t$$

$$+ P_{20}(t)\ \mu\Delta t + P_{11}(t)\ (\tfrac{1/\hat{n}}{2})\mu\Delta t \tag{2}$$

$$P_{01}(t+\Delta t) = P_{01}(t)[1-\{S(\lambda_1+\lambda_2) + \tfrac{1}{\hat{n}}\mu\}\Delta t] + P_{00}(t)\cdot S\lambda_2\ \Delta t$$

$$+ P_{02}(t)\cdot 1\cdot(\tfrac{2/\hat{n}}{2})\mu\Delta t + P_{11}(t)\cdot\tfrac{1}{2}\mu\Delta t \tag{3}$$

$$P_{11}(t+\Delta t) = P_{11}(t)[1-\{S(\lambda_1+\lambda_2) + (\tfrac{1+1/\hat{n}}{2})\mu\}\Delta t] + P_{01}(t)\cdot$$

$$S\lambda_1\Delta t + P_{10}(t)\cdot S\lambda_2\Delta t + P_{21}(t)\cdot\tfrac{2}{3}\ \mu\Delta t$$

$$+ P_{12}(t)\ \tfrac{2/\hat{n}}{3}\ \mu\Delta t \tag{4}$$

$$P_{20}(t+\Delta t) = P_{20}(t)[1-\{S(\lambda_1+\lambda_2) + \mu\}\Delta t] + P_{10}(t)\ S\lambda_1\Delta t$$

$$P_{21}(t)\cdot\tfrac{1/\hat{n}}{3}\ \mu\Delta t + P_{30}(t)\cdot(\mu\Delta t) \tag{5}$$

$$P_{02} = P_{02}(t)[1-\{S(\lambda_1+\lambda_2) + \frac{2/\hat{n}}{\hat{n}}\mu\}\Delta t] + P_{01}(t) \quad S\lambda_2\Delta t$$

$$+ P_{12}(t)\cdot\frac{1}{3}\mu\Delta t + P_{03}(t) \quad [\frac{3/\hat{n}}{\hat{n}}]\mu\Delta t \tag{6}$$

$$P_{12}(t+\Delta t) = P_{12}(t+\Delta t) \quad [1-(\frac{1+2/\hat{n}}{3})(\mu\Delta t)] + P_{02}(t) \quad \lambda_1\Delta t$$

$$+ P_{11}(t) \quad \lambda_2\Delta t \tag{7}$$

$$P_{21}(t+\Delta t) = P_{21}(t+\Delta t) \quad [1-\frac{(2+1/\hat{n})}{3} \mu\Delta t] + P_{20}(t) \quad \lambda_2\Delta t$$

$$+ P_{11}(t) \quad \lambda_1\Delta t \tag{8}$$

$$P_{30}(t+\Delta t) = P_{30}(t)[1-\mu\Delta t] + P_{20}(t) \quad \lambda_1\Delta t \tag{9}$$

$$P_{03}(t+\Delta t) = P_{03}(t)[1-\frac{1}{\hat{n}} \mu\Delta t] + P_{02}(t) \quad \lambda_2\Delta t \tag{10}$$

Rearranging the terms, dividing by Δt, taking the limits as $\Delta t \to 0$, and recognizing that

$$\lim_{\Delta t \to 0} \frac{Pn(t+\Delta t) - Pn(t)}{\Delta t} = Pn'(t)$$

the above equations (1) through (10) can be written as follows.

$$P_{00}'(t) = -S(\lambda_1+\lambda_2)\, P_{00}(t) + \mu\, P_{10}(10) + \frac{\mu}{\hat{n}} P_{01}(t) \tag{1}$$

$$P_{10}''(t) = -[S(\lambda_1+\lambda_2)+\mu]P_{10}(t) + S\lambda_1 P_{00}(t)$$

$$+ \mu\, P_{20}(t) + \frac{\mu}{2\hat{n}} P_{11}(t) \tag{2}$$

$$P_{01}(t) = -[S(\lambda_1+\lambda_2) + \frac{1}{\hat{n}}\mu]P_{01}(t) + S\lambda_2\, P_{00}(t)$$

$$+ \frac{1}{\hat{n}}\mu P_{02}(t) + \tfrac{1}{2}\mu P_{11}(t) \tag{3}$$

$$P_{11}'(t) = -[S(\lambda_1+\lambda_2) + \frac{(1+1/\hat{n})}{2}\mu]P_{11}(t) + S\lambda_1\, P_{01}(t)$$

$$+ S\lambda_2\, P_{10}(t) + \frac{2}{3}\mu\, P_{21}(t) + (2/3\hat{n}\cdot\mu)\, P_{12}(t) \tag{4}$$

$$P_{20}'(t) = -[S(\lambda_1+\lambda_2) + \mu]P_{20}(t) + S\lambda_1\, P_{10}(t) + \frac{1}{3\hat{n}}\mu P_{21}(t)$$

$$+ \mu P_{30}(t) \tag{5}$$

$$P_{02}'(t) = -[S(\lambda_1+\lambda_2) + \frac{\mu}{\hat{n}}]\, P_{02}(t) + S\lambda_2\, P_{01}(t)$$

$$+ \frac{1}{3}\mu P_{12}(t) + \frac{\mu}{\hat{n}} P_{03}(t) \tag{6}$$

$$P_{12}'(t) = -\,[\frac{1+2/\hat{n}}{3}\mu]P_{12}(t) + \lambda_1\, P_{02}(t) + \lambda_2\, P_{11}(t) \tag{7}$$

$$P_{21}'(t) = -\,[\frac{2+1/\hat{n}}{3}\mu]P_{21}(t) + \lambda_2\, P_{20}(t) + \lambda_1 P_{11}(t) \tag{8}$$

$$P_{30}'(t) = -\mu\, P_{30}(t) + \lambda_1 P_{20}(t) \tag{9}$$

$$P_{03}'(t) = -\frac{\mu}{\hbar} P_{03}(t) + \lambda_2 P_{02}(t) \tag{10}$$

The above equations are known as the birth-death equations of the system.

It has been shown that (17) for a system of birth-death equations of the type given above, the limits of $P_n(t)$ exist as t approaches infinity. That is,

$$\lim_{t\to\infty} P_n(t) = P_n$$

and

$$\lim_{dt\to\infty} \frac{dP_n(t)}{dt} = 0$$

where P_n is the steady state probability that the system is in state n at time (t).

If equations (1) through (10) are set equal to zero and divided by μ, the following steady state equations are obtained, where $\lambda_1/\mu = \rho_1$ and $\lambda_2/\mu = \rho_2$.

$$(S\rho_1 + S\rho_2)P_{00} = P_{10} + \frac{1}{\hbar} P_{01} \tag{1}$$

$$(S\rho_1 + S\rho_2 + 1)P_{10} = S\rho_1 P_{00} + P_{20} + \frac{1}{2\hbar} P_{11} \tag{2}$$

$$(S\rho_1 + S\rho_2 + \frac{1}{\hbar})P_{01} = S\rho_2 P_{00} + \frac{1}{\hbar} P_{02} + \tfrac{1}{2} P_{11} \tag{3}$$

$$(S\rho_1 + S\rho_2 + \frac{\hat{n}+1}{2\hat{n}})P_{11} = S\rho_1 P_{01} + S\rho_2 P_{10}$$

$$+ \frac{2}{3} P_{21} + \frac{2}{3\hat{n}} P_{12} \qquad (4)$$

$$(S\rho_1 + S\rho_2)P_{20} = S\rho_1 P_{10} + P_{30} + \frac{1}{3\hat{n}} P_{21} \qquad (5)$$

$$(S\rho_1 + S\rho_2 + \frac{1}{\hat{n}})P_{02} = S\rho_2 P_{01} + \frac{1}{3} P_{12} + \frac{1}{\hat{n}} P_{03} \qquad (6)$$

$$(\frac{\hat{n}+2}{3\hat{n}})P_{12} = \rho_1 P_{02} + \rho_2 P_{11} \qquad (7)$$

$$(\frac{2\hat{n}+1}{3\hat{n}})P_{21} = \rho_2 P_{20} + \rho_1 P_{11} \qquad (8)$$

$$P_{30} = \rho_1 P_{20} \qquad (9)$$

$$P_{03} = \rho_2 P_{02} \qquad (10)$$

Solving the simultaneous equations (1) through (10) the steady-state probabilities can be expressed in the following forms.

$$P_{10} = S\rho_1 P_{00} = \rho_1 P_{00} \text{ (since } S=1) \qquad (11)$$

$$P_{01} = \hat{n}S\rho_2 P_{00} = \hat{n}\rho_2 P_{00} \qquad (12)$$

$$P_{11} = 2\hat{n}S^2\rho_1\rho_2 P_{00} = 2\hat{n}\rho_1\rho_2 P_{00} \qquad (13)$$

$$P_{20} = S^2\rho_1{}^2\ P_{00} = \rho_1{}^2\ P_{00} \tag{14}$$

$$P_{02} = \hat{n}^2 S^2 \rho_2{}^2\ P_{00} = \hat{n}^2\rho_2{}^2\ P_{00} \tag{15}$$

$$P_{12} = 3\hat{n}^2\rho_1\rho_2{}^2\ P_{00} \tag{16}$$

$$P_{21} = 3\hat{n}\rho_1{}^2\rho_2\ P_{00} \tag{17}$$

$$P_{30} = \rho_1{}^3\ P_{00} \tag{18}$$

$$P_{03} = \hat{n}^3\rho_2{}^3\ P_{00} \tag{19}$$

From the boundary condition $\sum_{i=0}^{n} \sum_{j=0}^{n} P_{ij} = 1$, P_{00} can be obtained

$$P_{00} + P_{10} + P_{01} + P_{11} + P_{20} + P_{02} + P_{30} + P_{03} = 1 \tag{19.1}$$

Using eqn. (19.1) with eqns. (11) through (19), we get

$$P_{00}(1 + \rho_1 + \hat{n}\rho_2 + 2\hat{n}\rho_1\rho_2 + \rho_1{}^2 + \hat{n}^2\rho_2{}^2 + 3\hat{n}^2\ \rho_1\rho_2{}^2$$

$$+ 3\hat{n}\rho_1{}^2\ \rho_2 + \rho_1{}^3 + \hat{n}^3\rho_2{}^3) = 1$$

$$P_{00} = \frac{1}{1 + (\rho_1 + \hat{n}\rho_2) + (\rho_1 + \hat{n}\rho_2)^2 + (\rho_1 + \hat{n}\rho_2)^3} \tag{20}$$

Expected number of units in this system is given by

$$E[n] = \sum_{i=0}^{3} \sum_{j=0}^{3} (i+j)\, P_{ij}\ ,\ i+j < N$$

$$= P_{10}+P_{01}+2(P_{11}+P_{20}+P_{02}) + 3(P_{30}+P_{03}+P_{21}+P_{12})$$

$$= [(\rho_1+\hat{n}\rho_2)+2(\rho_1+\hat{n}\rho_2)^2+3(\rho_1+\hat{n}\rho_2)^3]\ P_{00} \qquad (21)$$

The probability that an arrival will not have to wait prior to service is

$$P_r[w=0] = P_{00}$$

In the case of one-channel model, the probability that an arrival will have no wait prior to service is equal to the probability of the system being idle.

i.e. $$Pr[w=0] = \frac{1}{1+(\rho_1+\hat{n}\rho_2) + (\rho_1+\hat{n}\rho_2)^2 + (\rho_1+\hat{n}\rho_2)^3} \qquad (22)$$

<u>Case 2</u>

Steady-state solution for the M/M/2 model:

no. of servers = 2
no. of sources = 1
ratio of service time = $\hat{n}$
length of conveyor = 1

The difference equations for $P_{ij}(t)$ can be written as follows:

$$P_{00}(t+\Delta t) = P_{00}(t)\ [1-(\lambda_1+\lambda_2)\Delta t] + P_{10}(t)\mu\Delta t + P_{01}(t)\ \frac{1}{\hat{n}}\mu\Delta t$$

$1 < i+j < C$

$$P_{10}(t+\Delta t) = P_{10}(t)\,[1-\{(\lambda_1+\lambda_2)+\mu\}\Delta t] + P_{00}(t)\ \lambda_1\Delta t$$

$$+ P_{20}(t)\ 2\mu\Delta t + P_{11}(t)\ (\frac{\mu}{\hat{n}})\ \Delta t$$

$$P_{01}(t+\Delta t) = P_{01}(t)\ [1-\{(\lambda_1+\lambda_2) + \frac{\mu}{\hat{n}}\}\Delta t] + P_{00}(t)\ \lambda_2\Delta t$$

$$+ P_{02}(t)\ \frac{2}{\hat{n}}\ \mu\Delta t + P_{11}(t)\ \mu\Delta t$$

$$P_{11}(t+\Delta t) = P_{11}(t)\ [1-\{(\lambda_1+\lambda_2) + (1+\frac{1}{\hat{n}})\mu\}\Delta t] + P_{01}(t)\ \lambda_1\Delta t$$

$$+ P_{10}(t)\ \lambda_2\Delta t + P_{21}(t)\,\frac{4}{3}\ \mu\Delta t + P_{12}(t)\ \frac{4}{3\hat{n}}\ \mu\Delta t$$

$$P_{20}(t+\Delta t) = P_{20}(t)\,[1-\{(\lambda_1+\lambda_2) + 2\mu\}\Delta t] + P_{10}(t)\ \lambda_1\Delta t$$

$$+ P_{30}(t)\ 2\mu\Delta t + P_{21}(t)\ \frac{2}{3\hat{n}}\ \mu\Delta t$$

$$P_{02}(t+\Delta t) = P_{02}(t)\,[1-\{(\lambda_1+\lambda_2) + \frac{2}{\hat{n}}\mu\}\Delta t] + P_{01}(t)\ \lambda_2\Delta t$$

$$+ P_{12}(t)\ \frac{2}{3}\ \mu\Delta t + P_{03}(t)\ \frac{2}{\hat{n}}\ \mu\Delta t$$

$$P_{21}(t+\Delta t) = P_{21}(t)\,[1-\{(\lambda_1+\lambda_2) + \frac{2(2+1/\hat{n})}{3}\mu\}\Delta t] + P_{11}(t)\cdot\lambda_1\Delta t$$

$$+ P_{20}(t)\cdot\lambda_2\Delta t + P_{31}(t)\cdot\frac{3}{2}\mu\ \Delta t + P_{22}(t)\cdot\frac{1}{\hat{n}}\mu\ \Delta t$$

$$P_{12}(t+\Delta t) = P_{12}(t)[1-\{(\lambda_1+\lambda_2) + \frac{2(1+2/\hat{n})}{3}\mu\}\Delta t]$$

$$+ P_{02}(t)\ \lambda_1\Delta t + P_{11}(t)\ S\lambda_2\Delta t + P_{13}(t)\ \frac{3}{2\hat{n}}\mu\Delta t$$

$$+ P_{22}(t)\ \mu\Delta t$$

$$P_{30}(t+\Delta t) = P_{30}(t)[1-\{(\lambda_1+\lambda_2) + 2\mu\}\Delta t] + P_{20}(t)\ \lambda_1\Delta t$$

$$+ P_{31}(t)\ \frac{1}{2\hat{n}}\mu\Delta t + P_{40}(t)\ 2\mu\Delta t$$

$$P_{03}(t+\Delta t) = P_{03}(t)[1-\{(\lambda_1+\lambda_2) + \frac{2}{\hat{n}}\mu\}\Delta t] + P_{02}(t)\ \lambda_2\Delta t$$

$$+ P_{13}(t)\ \tfrac{1}{2}\mu\Delta t + P_{04}(t)\ \frac{2}{\hat{n}}\mu\Delta t$$

$$P_{22}(t+\Delta t) = P_{22}(t)[1-(1+1/\hat{n})\mu\Delta t] + P_{12}(t)\ \lambda_1\Delta t + P_{21}(t)\ \lambda_2\Delta t$$

$$P_{31}(t+\Delta t) = P_{31}(t)[1- \frac{2(3+1/\hat{n})}{4}\mu\Delta t] + P_{21}(t)\ \lambda_1\Delta t$$

$$+ P_{30}(t)\ \lambda_2\Delta t$$

$$P_{13}(t+\Delta t) = P_{13}(t)[1- \frac{2(1+3/\hat{n})}{4}\mu\Delta t] + P_{03}(t)\ \lambda_1\Delta t$$

$$+ P_{12}(t)\ \lambda_2\Delta t$$

$$P_{40}(t+\Delta t) = P_{40}(t)[1-2\mu\Delta t] + P_{30}(t)\ \lambda_1\Delta t$$

$$P_{04}(t+\Delta t) = P_{04}(t)\ [1 - \frac{2}{\hat{n}}\ \mu\Delta t] + P_{03}(t)\ \lambda_2\Delta t$$

Rearranging the terms, dividing by Δt, taking the limit as $\Delta t \to 0$, and if the resulting equations are set equal to zero, the following steady-state equations are obtained, where $\rho_1 = \lambda_1/\mu$ and $\rho_2 = \lambda_2/\mu$.

$$(\rho_1 + \rho_2)P_{00} = P_{10} + \frac{1}{\hat{n}}\ P_{01}$$

$$(\rho_1 + \rho_2 + 1)P_{10} = \rho_1 P_{00} + 2P_{20} + \frac{1}{\hat{n}}\ P_{11}$$

$$(\rho_1 + \rho_2 + \frac{1}{\hat{n}})P_{01} = \rho_2 P_{00} + \frac{2}{\hat{n}}\ P_{02} + P_{11}$$

$$(\rho_1 + \rho_2 + \frac{\hat{n}+1}{\hat{n}})P_{11} = \rho_1 P_{01} + \rho_2 P_{10} + \frac{4}{3}\ P_{21} + \frac{4}{3\hat{n}}\ P_{12}$$

$$(\rho_1 + \rho_2 + 2)P_{20} = \rho_1 P_{10} + 2P_{30} + \frac{2}{3\hat{n}}\ P_{21}$$

$$(\rho_1 + \rho_2 + \frac{2}{\hat{n}})P_{02} = \rho_2 P_{01} + \frac{2}{3}\ P_{12} + \frac{2}{\hat{n}}\ P_{03}$$

$$(\rho_1 + \rho_2 + \frac{4\hat{n}+2}{3\hat{n}})P_{21} = \rho_1 P_{11} + \rho_2 P_{20} + \frac{3}{2}\ P_{31} + \frac{1}{\hat{n}}\ P_{22}$$

$$(\rho_1 + \rho_2 + \frac{2\hat{n}+4}{3\hat{n}})P_{12} = \rho_1 P_{02} + \rho_2 P_{11} + \frac{3}{2\hat{n}}\ P_{13} + P_{22}$$

$$(\rho_1 + \rho_2 + 2)P_{30} = \rho_1 P_{20} + \frac{1}{2\hat{n}}\ P_{31} + 2P_{40}$$

$$(\rho_1 + \rho_2 + \frac{2}{\hat{n}})P_{03} = \rho_2 P_{02} + \frac{1}{2}\ P_{13} + \frac{2}{\hat{n}}\ P_{04}$$

$$(\frac{\hat{n}+1}{\hat{n}})P_{22} = \rho_1 P_{12} + \rho_2 P_{21}$$

$$(\frac{3\hat{n}+1}{2})P_{31} = \rho_1 P_{21} + \rho_2 P_{30}$$

$$(\frac{\hat{n}+3}{2})P_{13} = \rho_1 P_{03} + \rho_2 P_{12}$$

$$2P_{40} = \rho_1 P_{30}$$

$$\frac{2}{n} P_{04} = \rho_2 P_{03}$$

By solving the above steady state equations, the probabilities in terms of P_{00} are obtained:

$\underline{1 < i+j < c}$

$$P_{10} = \rho_1 P_{00}$$

$$P_{01} = \hat{n}\rho_2 P_{00}$$

$$P_{11} = \hat{n}\rho_1\rho_2 P_{00}$$

$$P_{20} = \frac{1}{2} \rho_1^2 P_{00}$$

$$P_{02} = \frac{\hat{n}^2}{2} \rho_2^2 P_{00}$$

$\underline{c < i+j < b+c}$

$$P_{21} = \frac{3}{4} \hat{n} \rho_1^2 \rho_2 P_{00}$$

$$P_{12} = \frac{3}{4} \hat{n}^2 \rho_1 \rho_2^2 P_{00}$$

$$P_{30} = \frac{1}{4}\,\rho_1^{\,3}\,P_{00}$$

$$P_{03} = \frac{1}{4}\,\hat{n}^3\,\rho_2^{\,3}\,P_{00}$$

<u>$b+c < i+j < b+c+s$</u>

$$P_{22} = \frac{3}{4}\,\hat{n}^2\,\rho_1^{\,2}\rho_2^{\,2}\,P_{00}$$

$$P_{31} = \frac{1}{2}\,\hat{n}\,\rho_1^{\,3}\rho_2\,P_{00}$$

$$P_{13} = \frac{1}{2}\,\hat{n}^3\,\rho_1\rho_2^{\,3}\,P_{00}$$

$$P_{40} = \frac{1}{8}\,\rho_1^{\,4}\,P_{00}$$

$$P_{04} = \frac{1}{8}\,\hat{n}^4\,\rho_2^{\,4}\,P_{00}$$

Again, P_{00} can be obtained from the boundary condition, i.e. $\sum_{i=0}^{n}\sum_{j=0}^{n} P_{ij} = 1$. Probability that the system being an idle is,

$$P_{00} = \frac{1}{1+(\rho_1+\hat{n}\rho_2)+\frac{1}{2}(\rho_1+\hat{n}\rho_2)^2+\frac{1}{4}(\rho_1+\hat{n}\rho_2)^3+\frac{1}{8}(\rho_1+\hat{n}\rho_2)^4}$$

Expected number in the system

$$E[n] = \sum_{i=0}^{n}\sum_{j=0}^{n}(i+j)\,P_{ij}$$

$$= [(P_{01}+P_{10}) + 2(P_{11}+P_{20}+P_{02}) + 3(P_{12}+P_{30}+P_{03}+P_{21})$$

$$+ 4(P_{13}+P_{31}+P_{22}+P_{40}+P_{04})]$$

$$= [(\rho_1+\hat{n}\rho_2) + \frac{2}{2}(\rho_1+\hat{n}\rho_2)^2 + \frac{3}{4}(\rho_1+\hat{n}\rho_2)^3 + \frac{4}{8}(\rho_1+\hat{n}\rho_2)^4]P_{00}$$

The probability that an arrival will have no wait prior to service is

$$Pr\{w=0\} = \sum_{i=0}^{c-1}\sum_{j=0}^{c-1} P_{ij} \qquad i+j < c$$

$$= P_{00} + P_{10} + P_{01}$$

$$= (1 + S\rho_1 + \hat{n}S\rho_2)\, P_{00}$$

$$= [1 + (S\rho_1 + \hat{n}S\rho_2)]\, P_{00}$$

<u>Case 3</u>:

Similar approach is used to solve the model M/M/3 with number of servers equal to one and the length of conveyor is equal to one. The probabilities in terms of P_{00} for M/M/3 are as follows:

<u>$1 \le i+j \le c$</u>

$$P_{10} = \rho_1 P_{00}$$

$$P_{01} = \hat{n}\rho_2 P_{00}$$

$$P_{11} = \hat{n}\rho_1\rho_2\, P_{00}$$

$$P_{20} = \frac{\rho_1^{\,2}}{2!}\, P_{00}$$

$$P_{02} = \frac{\hat{n}^2}{2!}\, \rho_2^{\,2}\, P_{00}$$

$$P_{21} = \frac{1}{2!}\, \hat{n}\rho_1^{\,2}\, \rho_2\, P_{00}$$

$$P_{12} = \frac{1}{2!}\, \hat{n}\rho_1\rho_2^{\,2}\, P_{00}$$

$$P_{30} = \frac{\rho_1^{\,3}}{3!}\, P_{00}$$

$$P_{03} = \frac{\hat{n}^3\rho_2^{\,3}}{3!}\, P_{00}$$

$\underline{c < i+j < b+c}$

$$P_{22} = \frac{4!}{3.3!}\, \hat{n}^2\, \frac{\rho_1^{\,2}}{2!}\, \frac{\rho_2^{\,2}}{2!}\, P_{00}$$

$$P_{31} = \frac{4!}{3.3!}\, \hat{n}\, \frac{\rho_1^{\,3}}{3!}\, \frac{\rho_2}{1!}\, P_{00}$$

$$P_{13} = \frac{4!}{3.3!}\, \frac{\rho_1}{1!}\, \frac{n^3\rho_2^{\,3}}{3!}\, P_{00}$$

$$P_{40} = \frac{4!}{3.3!}\, \frac{\rho_1^{\,4}}{4!}\, P_{00}$$

$$P_{04} = \frac{4!}{3.3!}\, \frac{\hat{n}^4\rho_2^{\,4}}{4!}\, P_{00}$$

$b+c < i+j < b+c+s$

$$P_{23} = \frac{5!}{3^2.3!}\,\frac{\rho_1^{\ 2}}{2!}\,\frac{\hat{n}^3\rho_2^{\ 3}}{3!}\,P_{00}$$

$$P_{32} = \frac{5!}{3^2.3!}\,\frac{\rho_1^{\ 3}}{3!}\,\frac{\hat{n}^2\rho_2^{\ 2}}{2!}\,P_{00}$$

$$P_{41} = \frac{5!}{3^2.3!}\,\frac{\rho_1^{\ 4}}{4!}\,\frac{\hat{n}\rho_2}{1!}\,P_{00}$$

$$P_{14} = \frac{5!}{3^2.3!}\,\frac{\rho_1}{1!}\,\frac{\hat{n}^4\rho_2^{\ 4}}{4!}\,P_{00}$$

$$P_{50} = \frac{5!}{3^2.3!}\,\frac{\rho_1^{\ 5}}{5!}\,P_{00}$$

$$P_{05} = \frac{5!}{3^2.3!}\,\frac{\hat{n}^5\rho_2^{\ 5}}{5!}\,P_{00}$$

Case 4:

Multi-server, Multi-sources Queueing Model M/M/C

$$P_{00}(t+\Delta t) = P_{00}(t)[1-S(\lambda_1+\lambda_2)\Delta t] + P_{10}(t)\ \mu\Delta t$$

$$+ P_{01}(t)\ \frac{\mu}{\hat{n}}\ \Delta t\ , \quad 1 < i+j < c \tag{4.1}$$

$$P_{ij}(t+\Delta t) = P_{ij}(t)[1-\{S(\lambda_1+\lambda_2) + (i+\tfrac{j}{\hat{n}})\ \mu\}\Delta t]$$

$$+ P_{i-1,j}(t)\ S\lambda_1\Delta t + P_{i,j-1}(t)\ S\lambda_2\Delta t$$

$$+ P_{i+1,j}(t)\ (i+1)\mu\Delta t + P_{i,j+1}(t)\ (\tfrac{j+1}{\hat{n}})\ \mu\Delta t$$

$$1 < i+j < c \tag{4.2}$$

$$P_{ij}(t+\Delta t) = P_{ij}(t)[1-\{S(\lambda_1+\lambda_2) + (i+\tfrac{j}{\hat{n}})\mu\}\Delta t]$$

$$+ P_{i-1,j}(t)\ S\lambda_1\Delta t + P_{i,j-1}(t)\ S\lambda_2\Delta t$$

$$+ P_{i+1,j}(t)\ C(\frac{i+1}{i+j+1})\ \mu\Delta t + P_{i,j+1}(t)$$

$$C(\frac{j+\frac{1}{2}}{i+j+1})\ \mu\Delta t\ ,\ i+j = c \qquad (4.3)$$

$$P_{ij}(t+\Delta t) = P_{ij}(t)[1-\{S(\lambda_1+\lambda_2) + C(\frac{i+j/\hat{n}}{i+j})\mu\}\Delta t\ ]$$

$$+ P_{i-1,j}(t)\ S\lambda_1\Delta t + P_{i,j-1}(t)\ S\lambda_2\Delta t$$

$$+ P_{i+1,j}(t)\ C(\frac{i}{i+j})\mu\ \Delta t + P_{i,j+1}(t)$$

$$C(\frac{j/\hat{n}}{i+j})\mu\ \Delta t\ ,\ C < i+j < b+c \qquad (4.4)$$

$$P_{ij}(t+\Delta t) = P_{ij}(t)[1-\{S-[(i+j)-(b+c)]\}(\lambda_1+\lambda_2)\Delta t$$

$$- C(\frac{i+j/2}{i+j})\ \mu\Delta t] + P_{i-1,j}(t)\ [S-\{(i+j)-(b+c)\}+1]$$

$$\lambda_1\Delta t + P_{i,j-1}(t)\ [S-\{(i+j)-(b+c)\}+1]\ \lambda_2\Delta t$$

$$+ P_{i+1,j}(t)\ C(\frac{i}{i+j})\mu\Delta t + P_{i,j+1}(t)\ C(\frac{j/\hat{n}}{i+j})\ \mu\Delta t\ ,$$

$$b+c < i+j < b+c+s \qquad (4.5)$$

$\underline{i+j = b+c+s}$

$$P_{ij}(t+\Delta t) = P_{ij}(t)\ [1-C(\frac{i+j/2}{i+j})\ \mu\Delta t]$$

$$+ P_{i-1,j}(t)\ \lambda_1\Delta t + P_{i,j-1}\ \lambda_2\Delta t\ ,$$

$$i+j = b+c+s \tag{4.6}$$

Expanding terms, dividing by Δt, letting all higher terms of Δt be equal to zero ($\Delta t^2 = 0$), taking the limit as $\Delta t \to 0$, and recognizing that:

$$\lim_{\Delta t \to 0} \frac{P_{ij}(t+\Delta t) - P_{ij}(t)}{\Delta t} = P_{ij}'(t)$$

the above equations can be written as follows

$$P_{00}'(t) = -S(\lambda_1+\lambda_2)P_{00}(t) + \mu P_{10}(t) + \frac{\mu}{\hat{n}}\ P_{01}(t) \tag{4.7}$$

$$P_{ij}'(t) = -[S(\lambda_1+\lambda_2)+(i+\frac{j}{\hat{n}})\mu]P_{ij}(t) + S\lambda_1 P_{i-1,j}(t)$$

$$+ S\lambda_2 P_{i,j-1}(t) + (i+1)\mu\ P_{i+1,j}(t) + (\frac{j+1}{\hat{n}})\mu$$

$$P_{i,j+1}(t)\ ,\quad 1 < i+j < c \tag{4.8}$$

$$P_{ij}'(t) = -[S(\lambda_1+\lambda_2) + (i+\tfrac{j}{\hat{n}})\mu]P_{ij}(t) + S\lambda_1 P_{i-1,j}(t)$$

$$+ S\lambda_2 P_{i,i-1}(t) \pm C(\tfrac{i}{i+j})\mu P_{i+1,j}(t)$$

$$+ C(\tfrac{j/2}{i+j})\mu P_{i,j+1}(t), \quad i+j = C \tag{4.9}$$

$$P_{ij}'(t) = -[S(\lambda_1+\lambda_2) + C(\tfrac{i+j/\hat{n}}{i+j})\mu]P_{ij}(t) + S\lambda_1 P_{i-1,j}(t)$$

$$+ S\lambda_2 P_{i,j-1}(t) + C(\tfrac{i}{i+j})\mu P_{i+1,j}(t) + C(\tfrac{j/\hat{n}}{i+j})\mu$$

$$P_{i,j+1}(t), \quad C < i+j < b+c \tag{4.10}$$

$$P_{ij}'(t) = -[\{S-[(i+j)-(b+c)]\}(\lambda_1+\lambda_2) + C(\tfrac{i+j/\hat{n}}{i+j})\mu] P_{ij}(t)$$

$$+ [S-\{(i+j)-(b+c)\} + 1]\lambda_1 P_{i-1,j}(t)$$

$$+ [S-\{(i+j)-(b+c)\} + 1]\lambda_2 P_{i,j-1}(t)$$

$$+ C(\tfrac{i}{i+j})\mu P_{i+1,j}(t) + C(\tfrac{j/\hat{n}}{i+j})\mu P_{i,j+1},$$

$$b+c < i+j < b+c+s \tag{4.11}$$

$$P_{ij}'(t) = -C(\tfrac{i+j/2}{i+j})\mu P_{ij}(t) + \lambda_1 P_{i-1,j}(t) + \lambda_2 P_{i,j-1}(t),$$

$$i+j = b+c+s \tag{4.12}$$

Equations (4.7) through (4.12) are known as birth-death equations of the system.

If the above equations are set equal to zero, steady-state probabilities P_{10} through P_{ij} can be solved in terms of P_{00}. Then using the relationship

$$\sum_{i=0}^{n} \sum_{j=0}^{n} P_{ij} = 1.0$$

all of the steady-state probabilities can be expressed in the following forms. Letting $\rho_1 = \lambda_1/\mu$ and $\rho_2 = \lambda_2/\mu$,

$$P_{ij} = \frac{(S\rho_1)^i}{i!} \frac{(\hat{n}S\rho_2)^i}{j!} \quad P_{00} \quad , \quad 0 < i+j < C \qquad \text{I}$$

$$P_{ij} = \frac{(i+j)!}{C^{i+j-c}\ C!} \frac{(S\rho_1)^i}{i!} \frac{(\hat{n}S\rho_2)^j}{j!} \quad P_{00} \quad , \quad C \leq i+j < b+c \qquad \text{II}$$

$$P_{ij} = \frac{(i+j)!}{C^{i+j-c}\ C!} \frac{(S\rho_1)^i}{i!} \frac{(\hat{n}S\rho_2)^j}{j!} \frac{1}{S^{2[(i+j)-(b+c)]}} \quad ,$$

$$b+c < i+j < b+c+s \qquad \text{III}$$

Now, measures of effectiveness can be derived for this model utilizing the steady state probabilities given by equations I, II and III in the following manner.

$$P_{00} = \left[\sum_{K=0}^{C} \frac{(\rho_1+\hat{n}\rho_2)^K}{K!} s^K + \sum_{K=C+1}^{b+c} \frac{(\rho_1+\hat{n}\rho_2)^K}{C^{K-1}} s^K \right.$$

$$\left. + \sum_{K=b+c+1}^{b+c+s} \frac{(\rho_1+\hat{n}\rho_2)^K}{C^{K-1}} \frac{s^K}{s^{2(K-b-c)}} \right]^{-1} \qquad \text{IV}$$

Expected number in the system is given by

$$E[n] = \sum_{i=0}^{n} \sum_{j=0}^{n} (i+j)\, P_{ij}$$

$$= (P_{01}+P_{10})+2(P_{11}+P_{20}+P_{02})+\ldots\ldots\ldots P_{nn}$$

$$= \left[\sum_{K=1}^{C} \frac{K}{K!}(\rho_1+\hat{n}\rho_2)^K s^K + \sum_{K=C+1}^{b+c} K(\rho_1+\hat{n}\rho_2)^K s^K/C^{K-1} \right.$$

$$\left. + \sum_{K=b+c+1}^{b+c+s} \frac{K(\rho_1+\hat{n}\rho_2)^K}{C^{K-1}} \frac{s^K}{s^{2(K-b-c)}} \right] P_{00} \qquad \text{V}$$

The probability that an arrival will have no wait is

$$Pr[w=0] = \sum_{i=0}^{C-1} \sum_{j=0}^{C-1} P_{ij}$$

$$= \left[\sum_{K=0}^{C-1} \frac{(\rho_1+\hat{n}\rho_2)^K}{K!} s^K \right] P_{00} \qquad \text{VI}$$

To explain the foregoing developments, the three measures of effectiveness are plotted in figures 1 through 18 against the following independent variables:

i) traffic intensity $\rho_1 = \lambda_1/\mu$

ii) traffic intensity $\rho_2 = \lambda_1/n\mu$

iii) number of service channels - "C"

iv) length of conveyor - "b"

v) number of sources - "s"

vi) service time ratio - "$\hat{n}$"

From fig. 1 and 4, this can be seen that, increase in traffic intensities will result in a decrease in the value of P_{00} (probability that the system is idle). The probability, P_{00}, reaches a very small value (almost zero) when both ρ_1 and ρ_2 are maintained at high values. This is quite apparent that when there are more number of customers in the system, the servers will be more busy and the probability (P_{00}) of the system being an idle will be less. These figures also help us to find the optimum traffic intensities to be allowed so that the system is maintained at the desired value of P_{00}.

From figs. 2 and 5, one can see that the expected number of units in the system also increase with the increase in traffic intensities ρ_1 and ρ_2. At the high values of ρ_1 and ρ_2 (say $\rho_1 = \rho_2 = 1.0$), the expected number of units $E[n]$, reaches the maximum value, then further increase in ρ_1 and ρ_2 will not result in high value of $E[n]$.

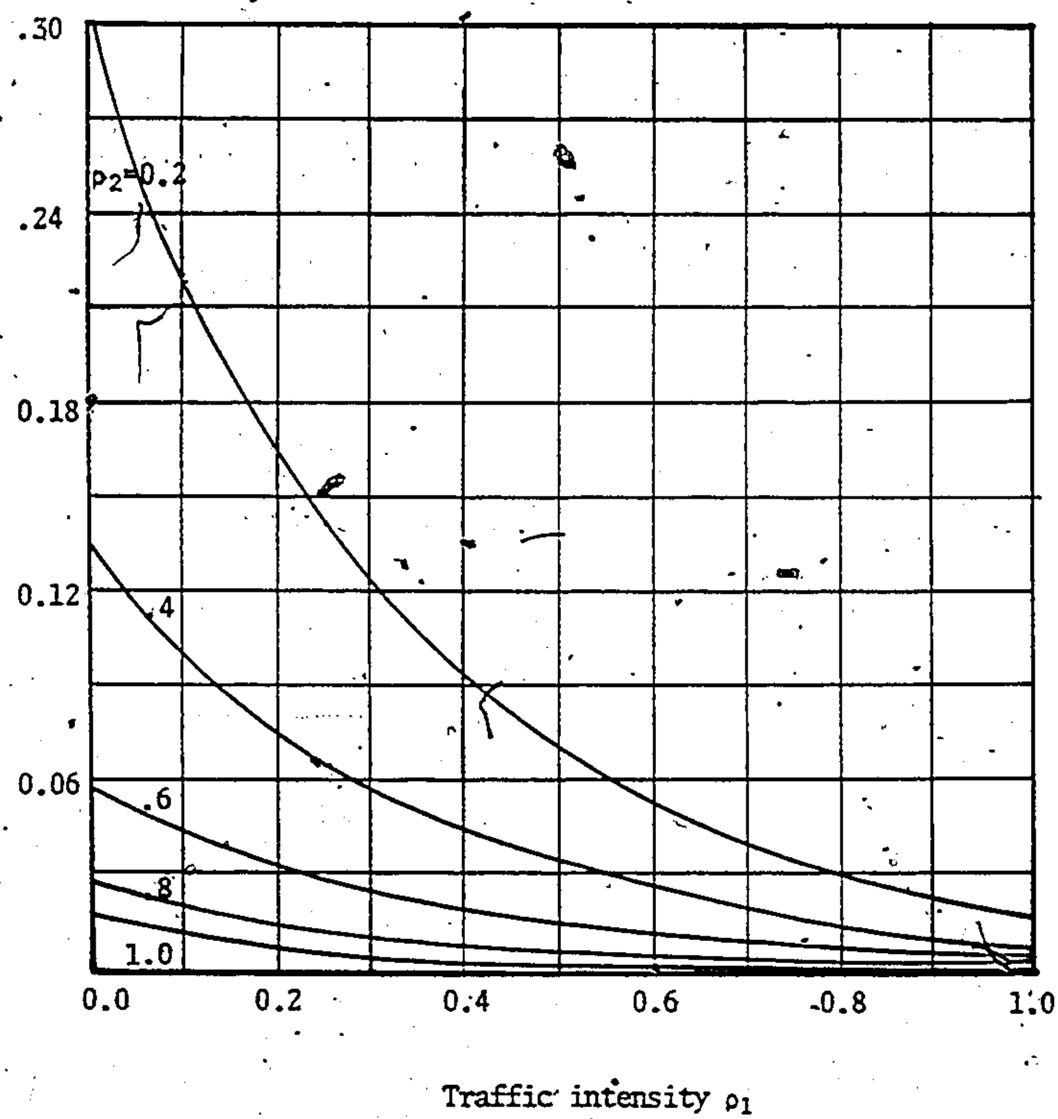


Fig. 1. Probability that the system being idle vs ρ_1.

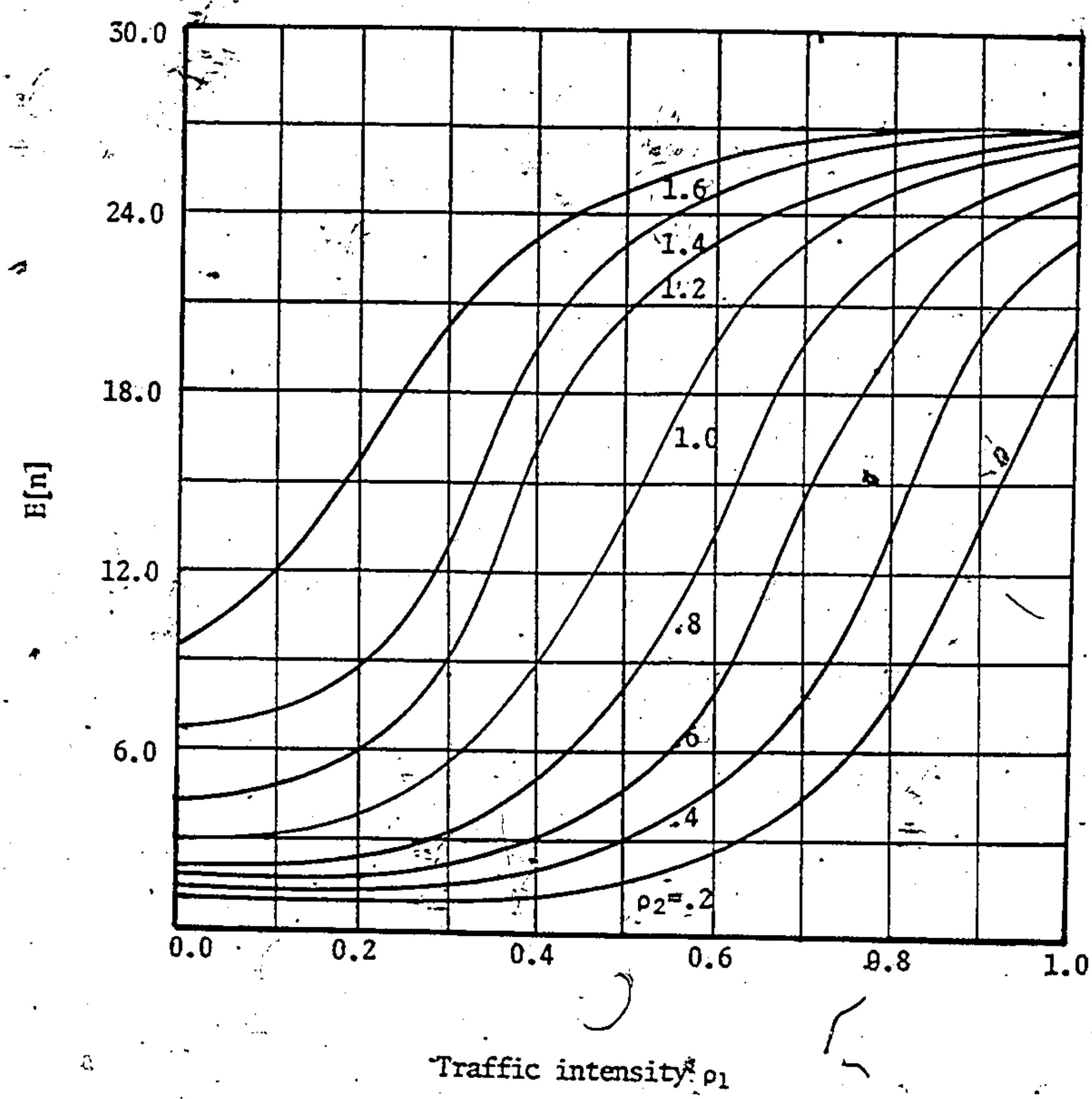


Fig. 2. E[n] versus ρ_1, ρ_2 fixed.

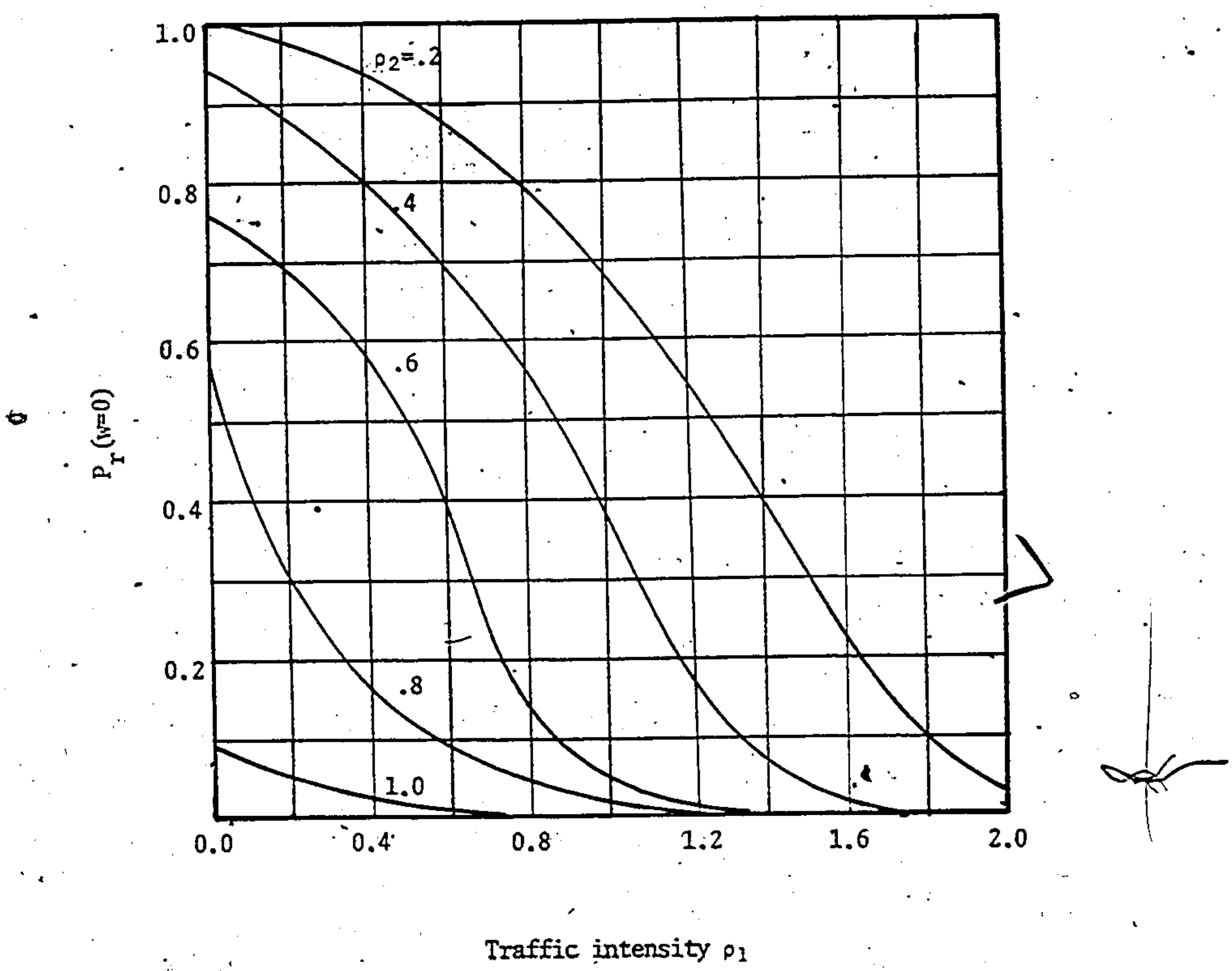


Fig. 3. Probability that an arrival will have no wait vs ρ_1.

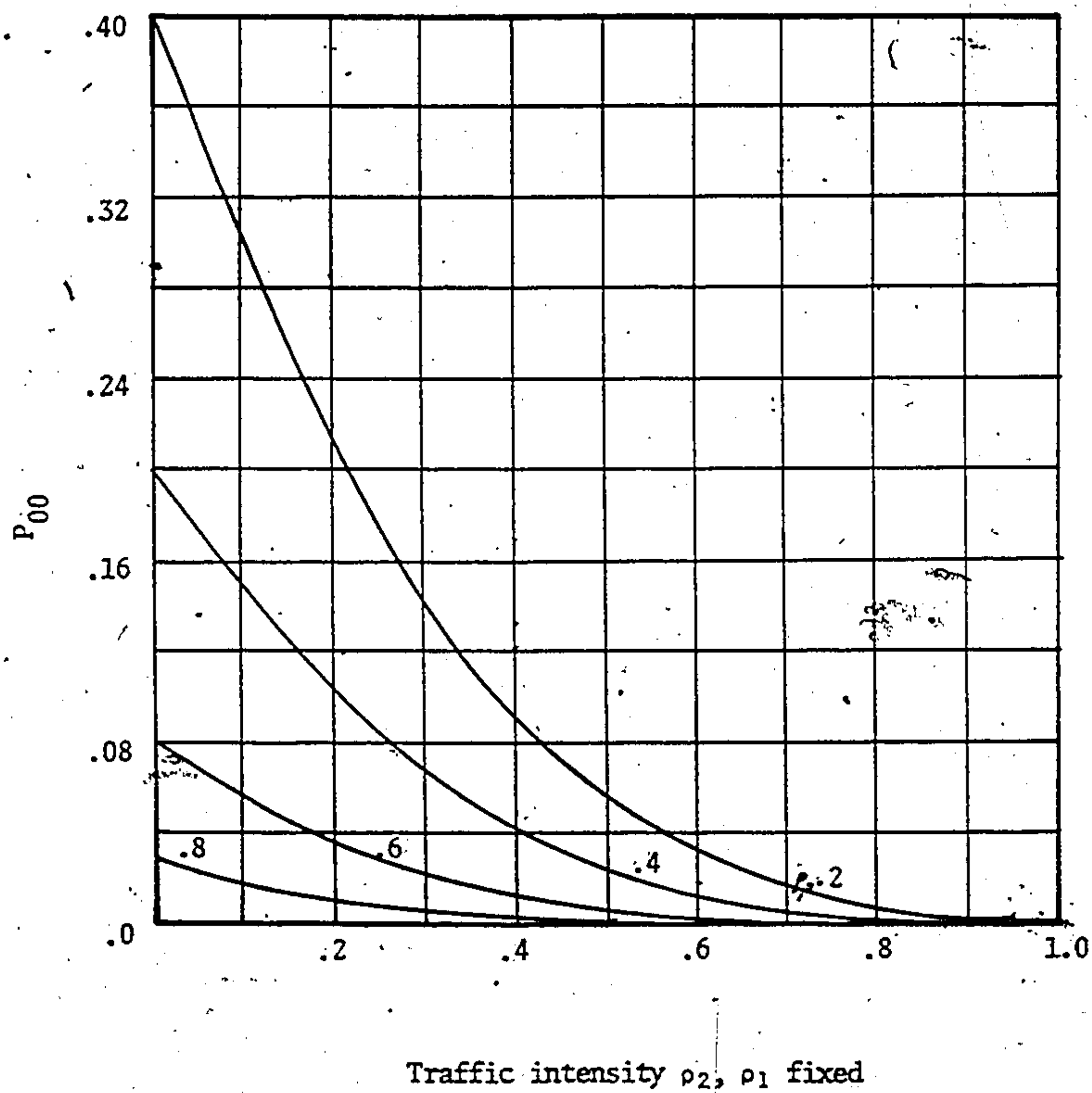


Fig. 4. Probability that the system being idle vs ρ_2.

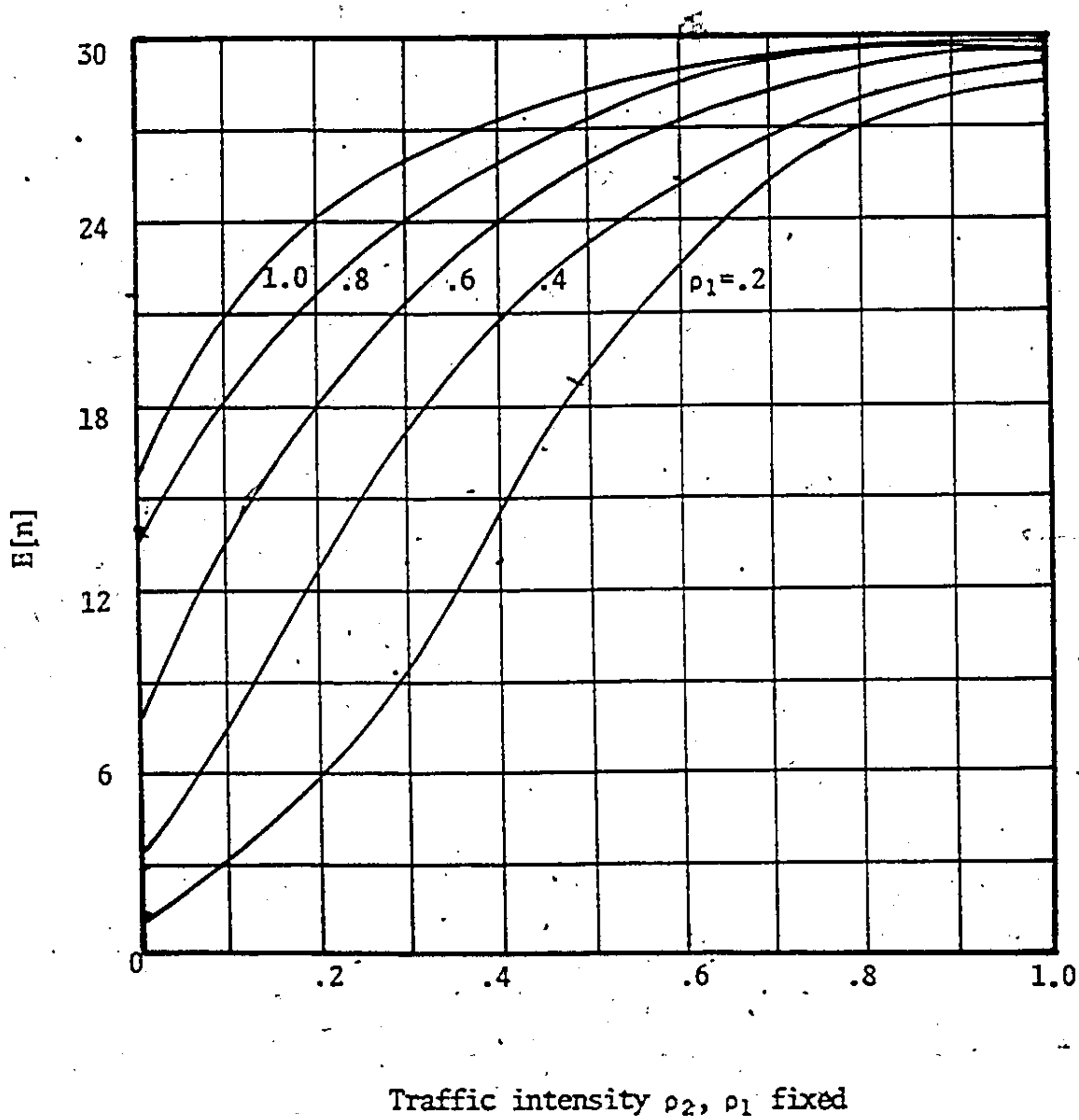


Fig. 5. E[n] versus traffic intensity.

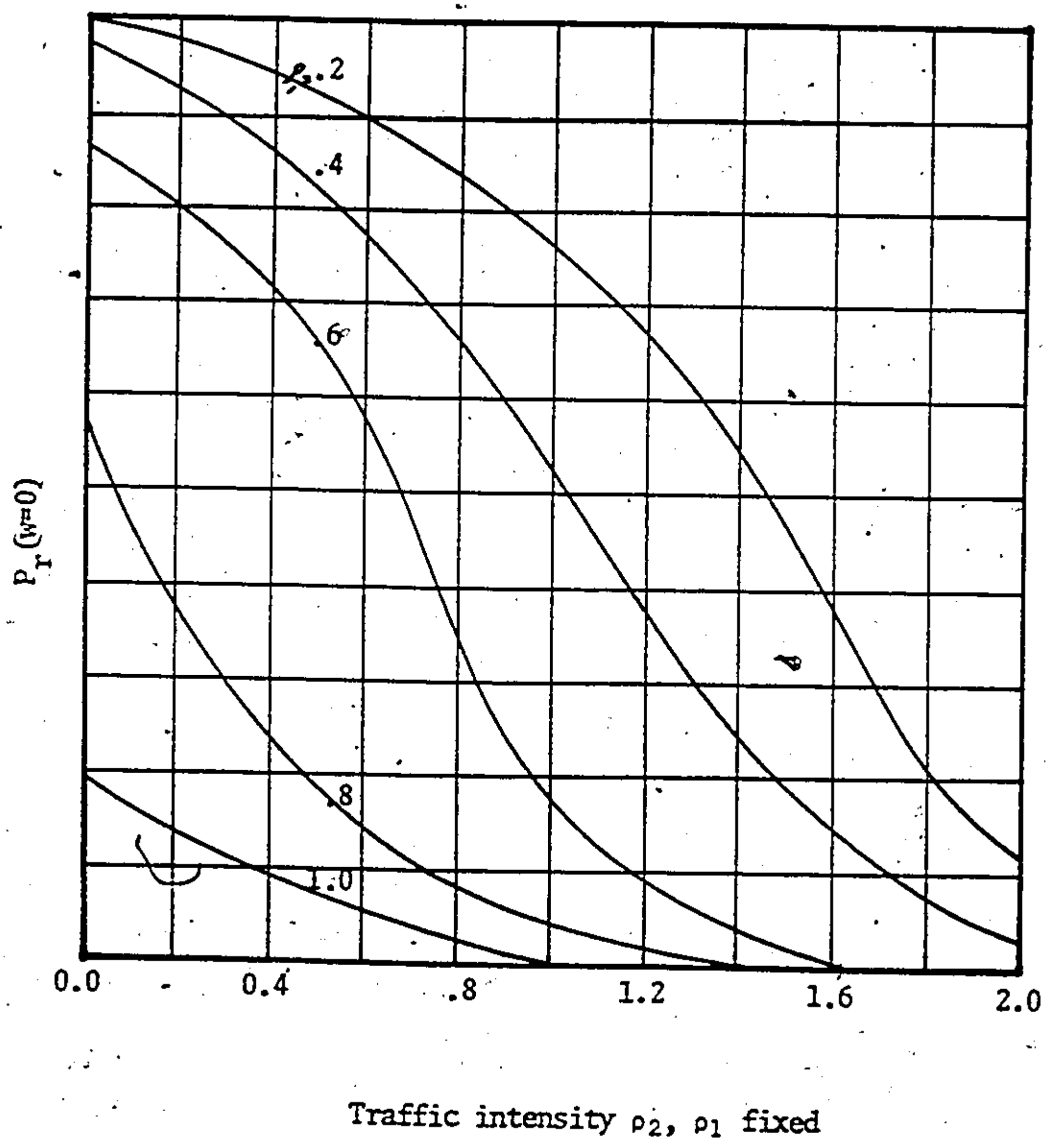


Fig. 6. Probability, $P_r(w=0)$, versus traffic intensity.

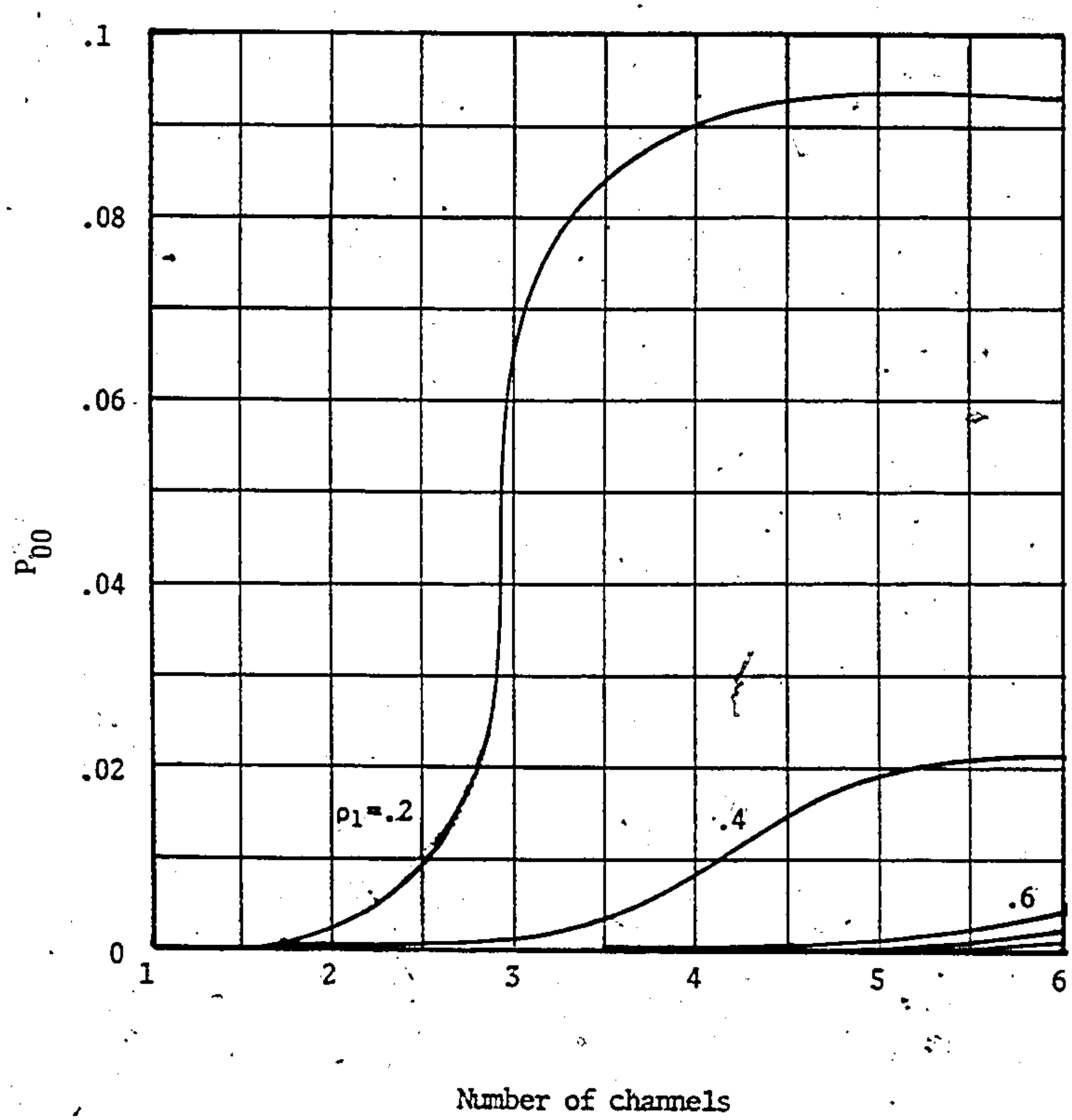


Fig. 7. Probability that the system being idle vs number of service channels.

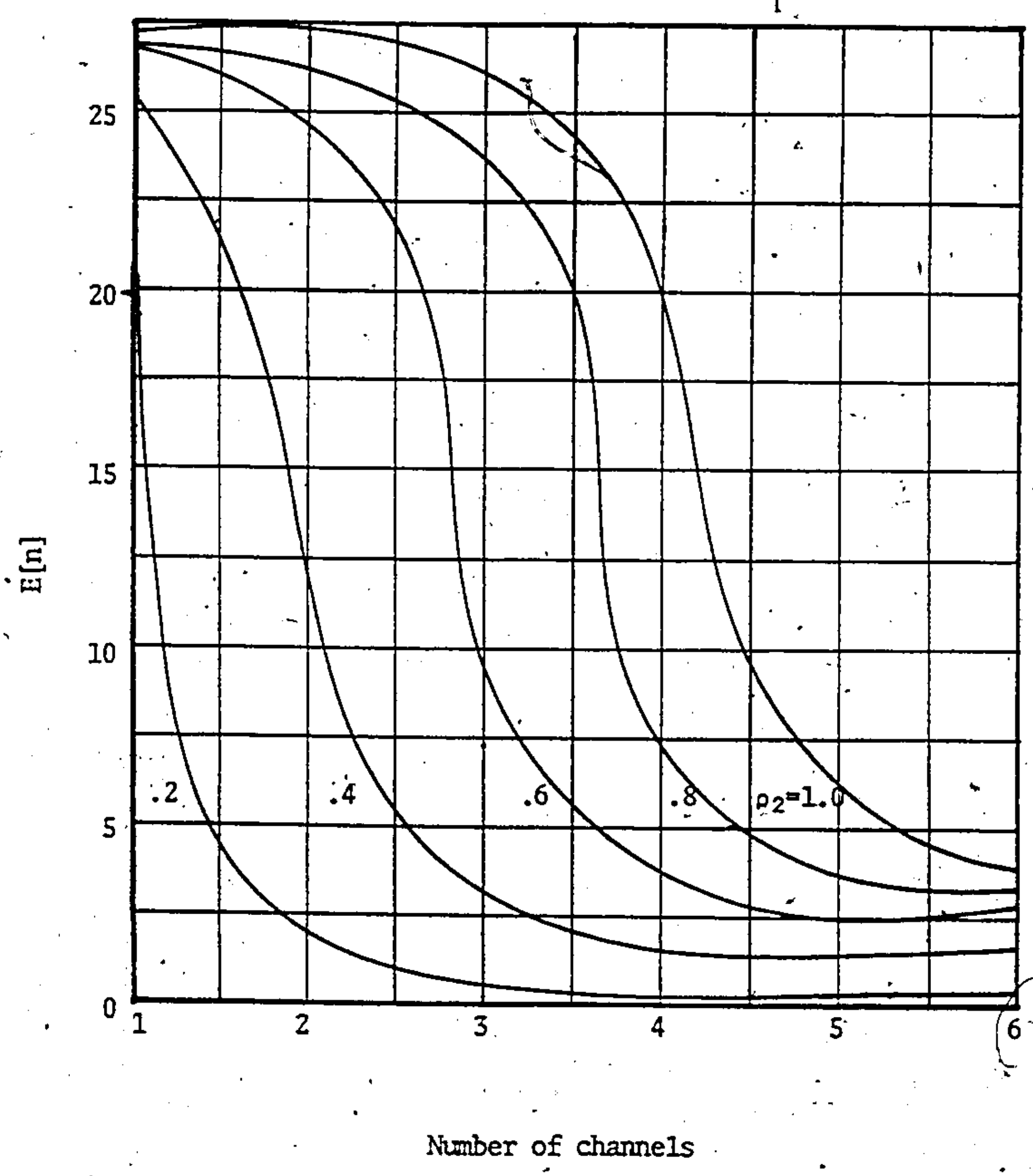


Fig. 8. Expected number in the system versus number of channels.

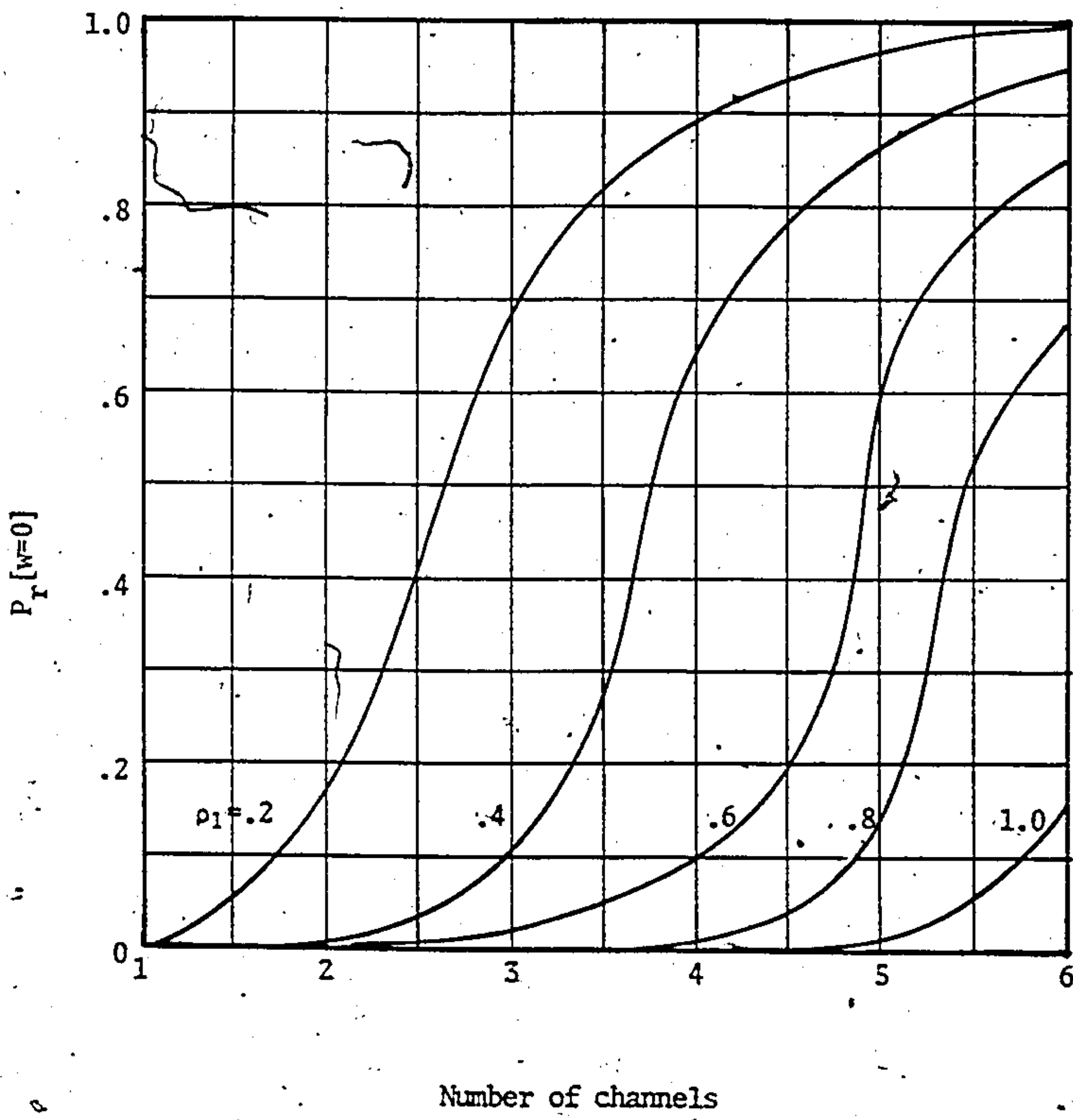


Fig. 9. Probability of no wait versus number of service channels.

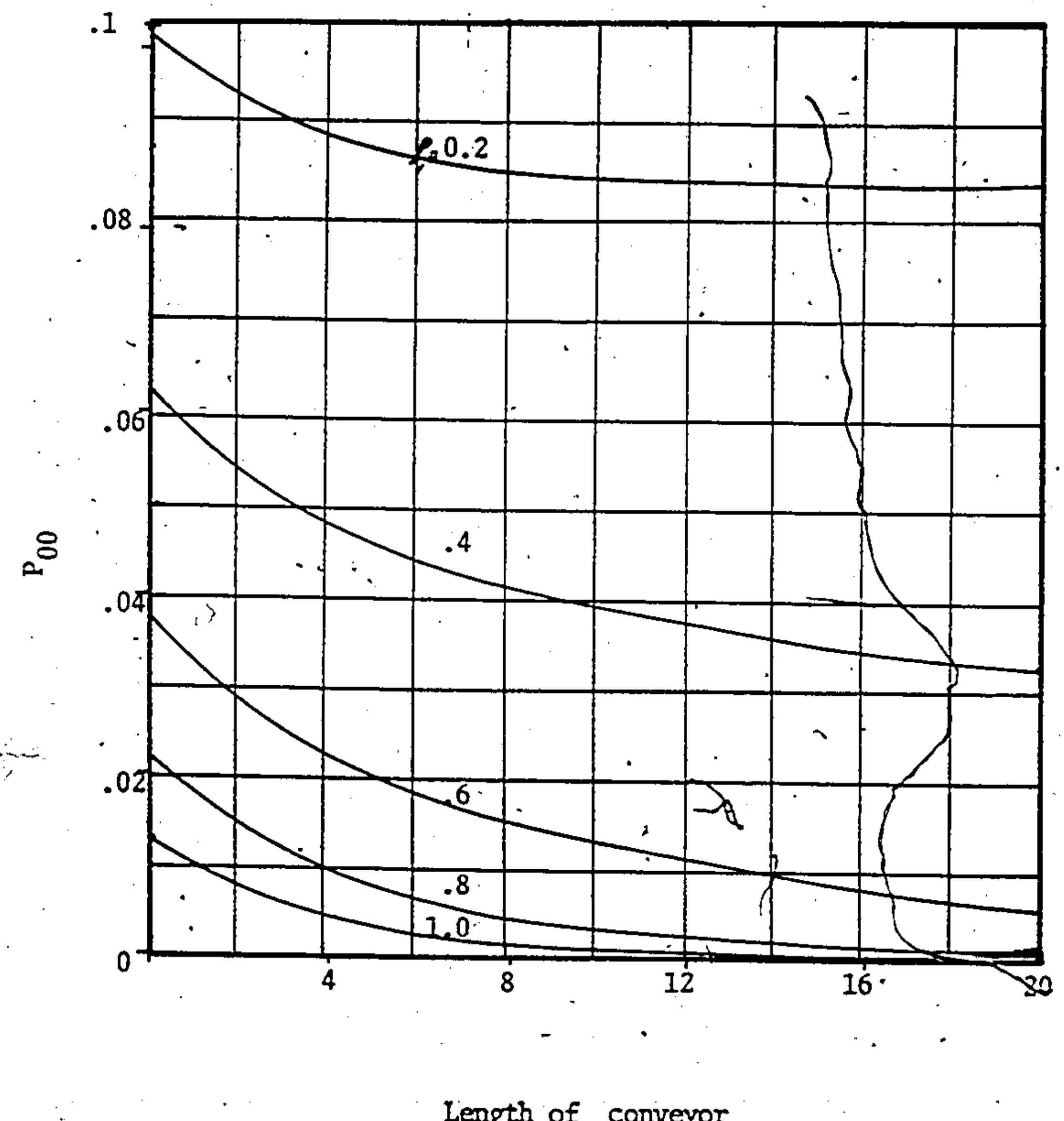


Fig. 10. Probability that the system being idle versus conveyor length.

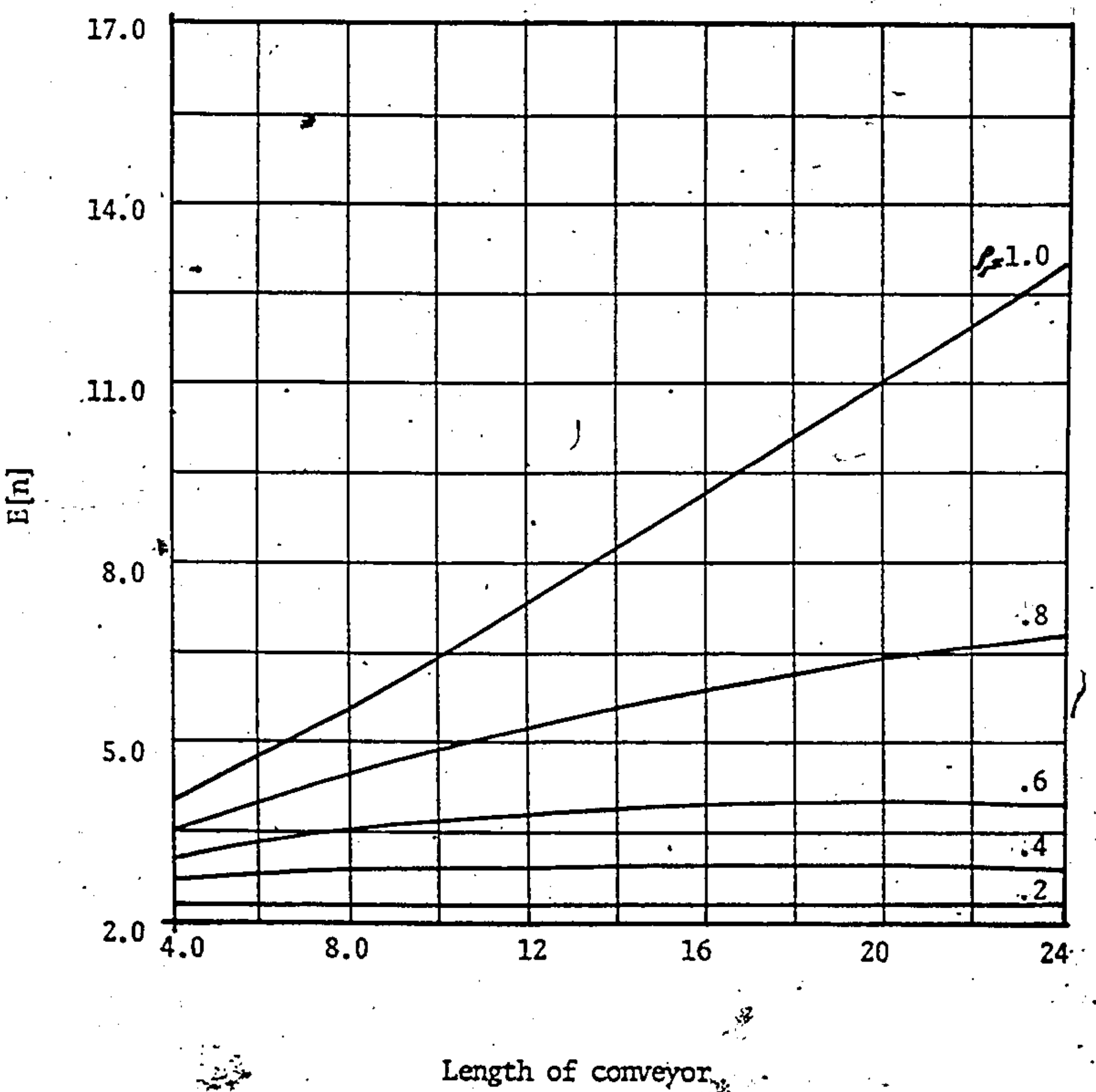


Fig. 11. Expected number in the system versus conveyor length.

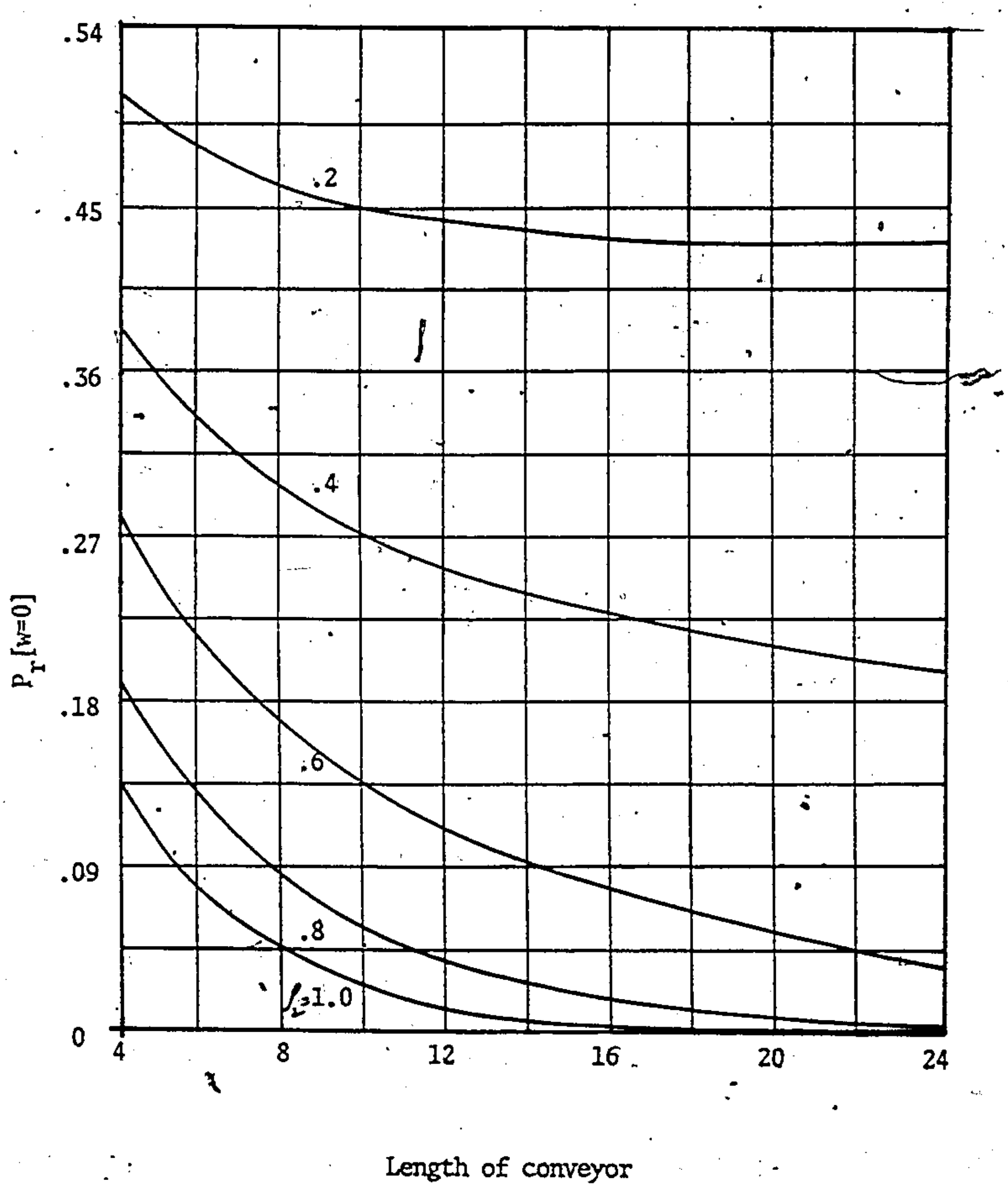


Fig. 12. Probability of no wait versus conveyor length.

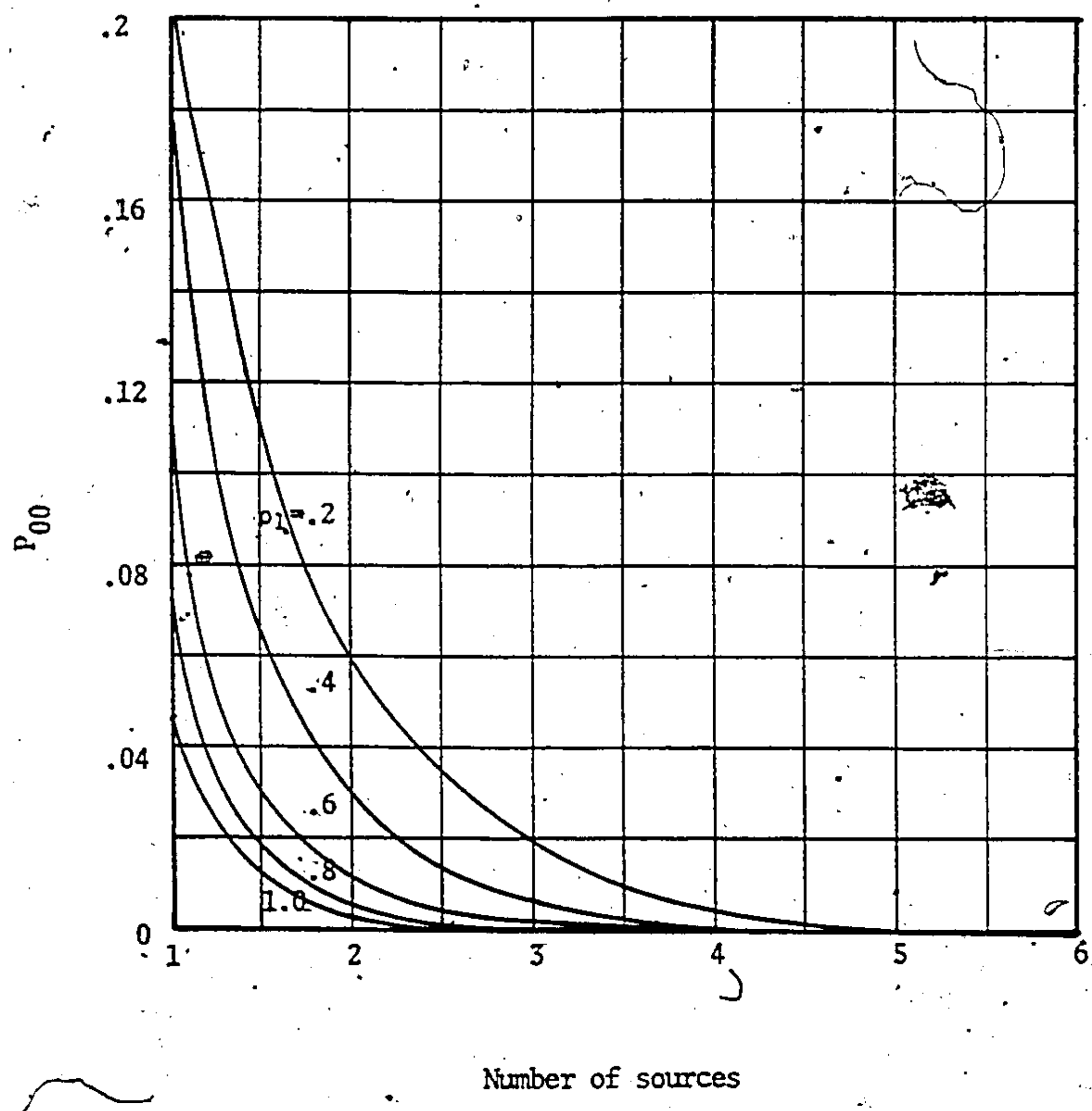


Fig. 13. Probability that the system being idle versus number of sources.

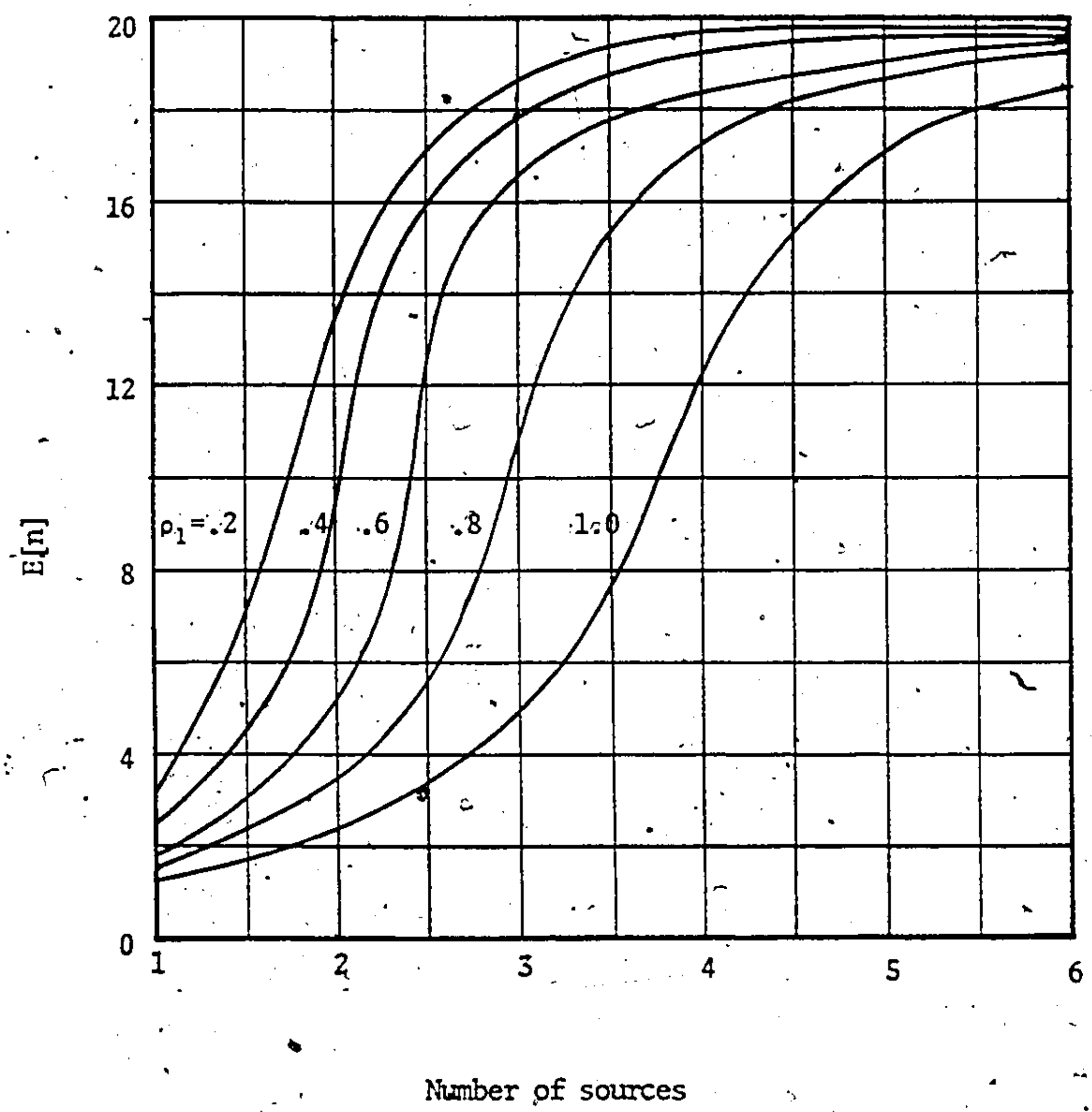


Fig. 14. Expected number of units versus number of sources.

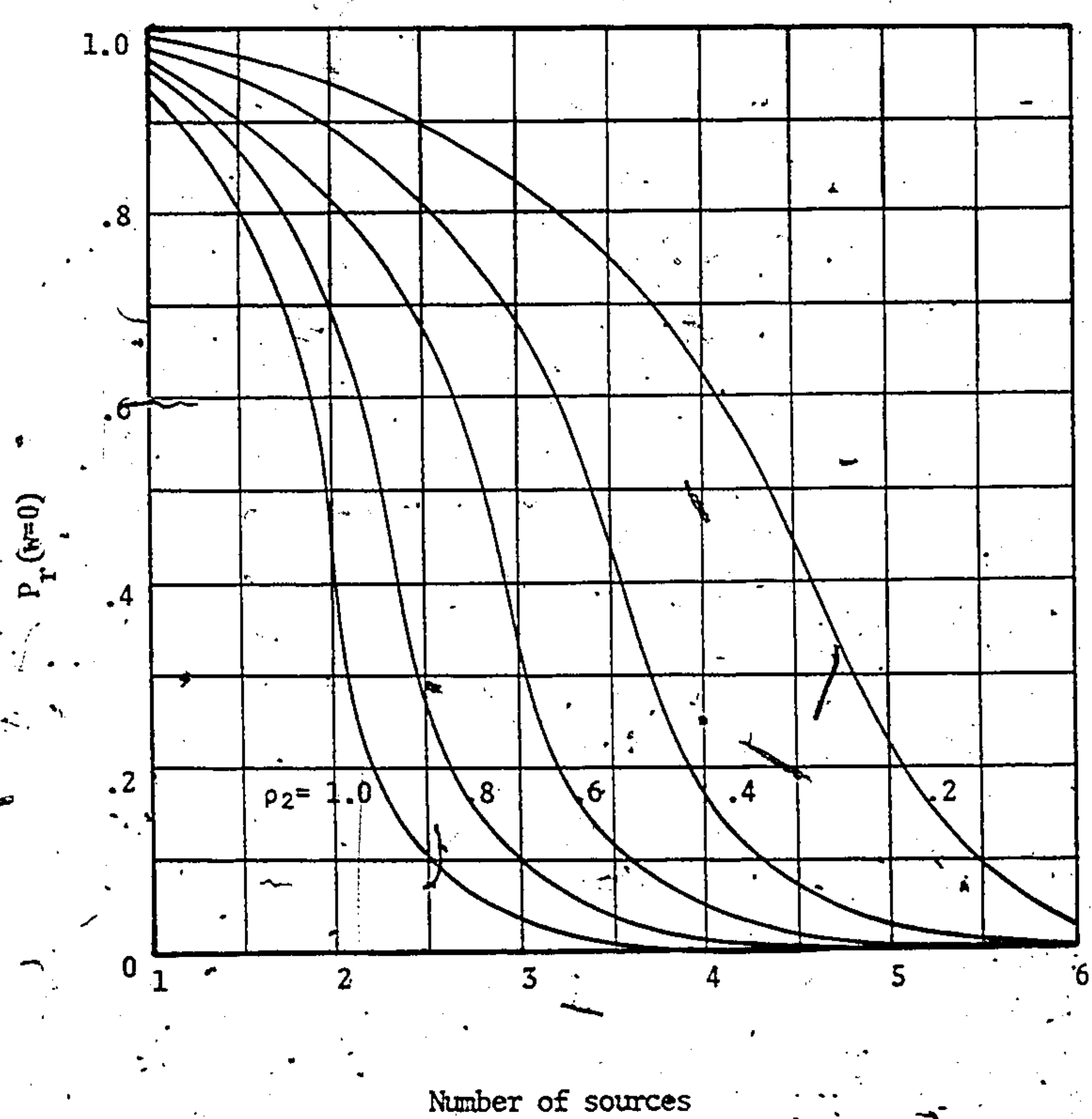


Figure 15. Probability of no wait versus number of sources.

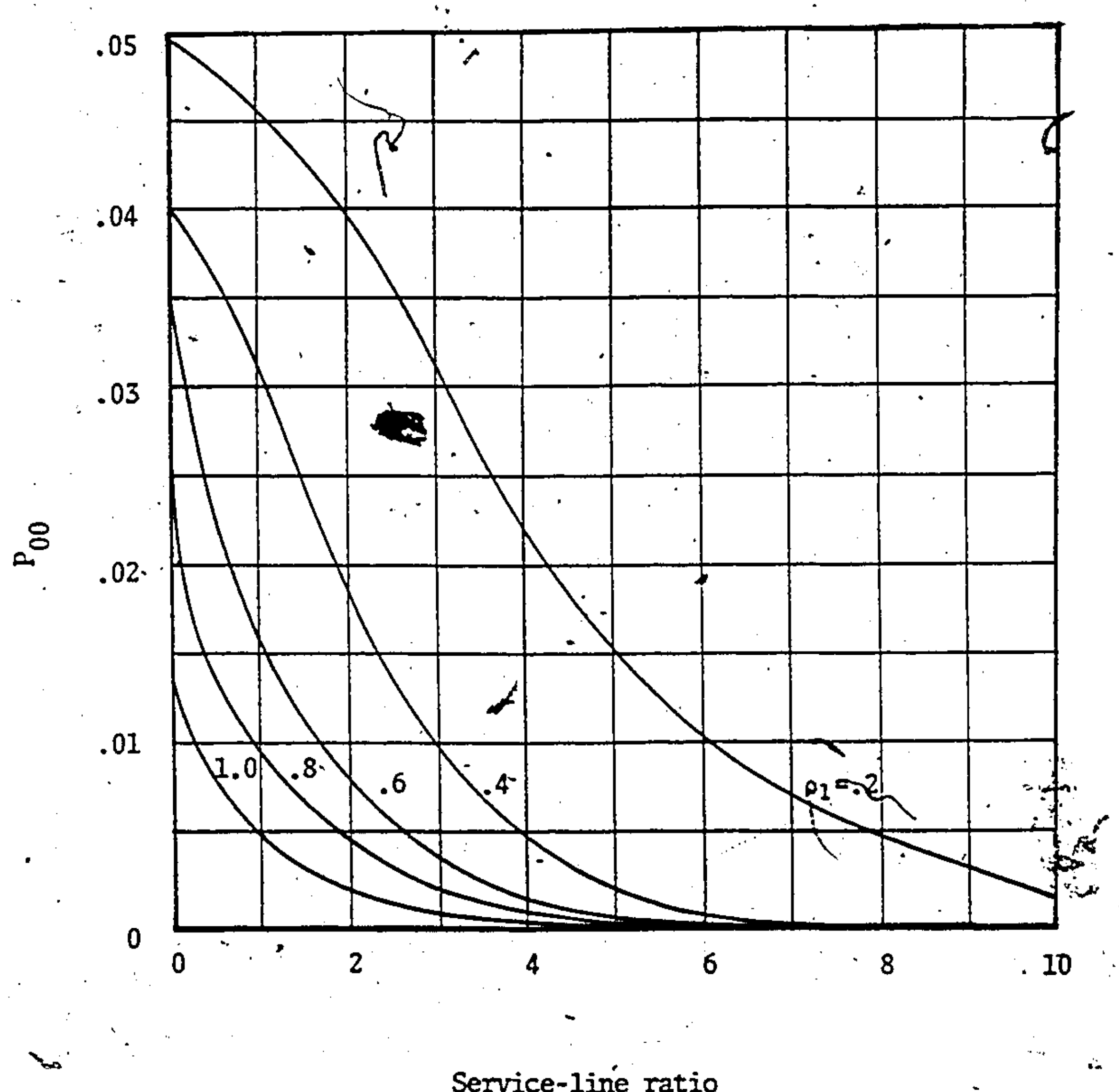


Fig. 16. Probability that the system being idle versus service time ratio.

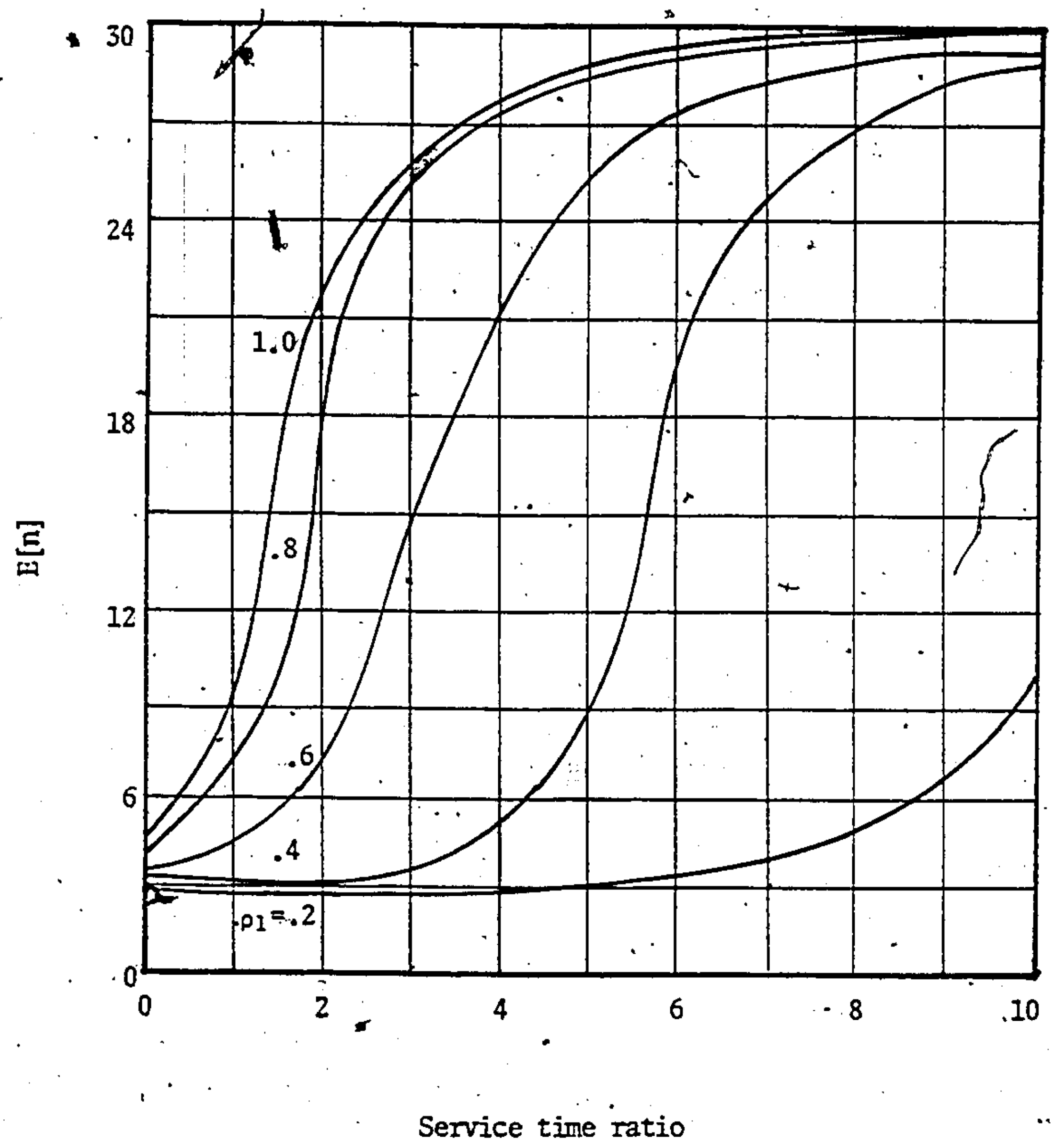


Fig. 17. Expected number of units versus service time ratio

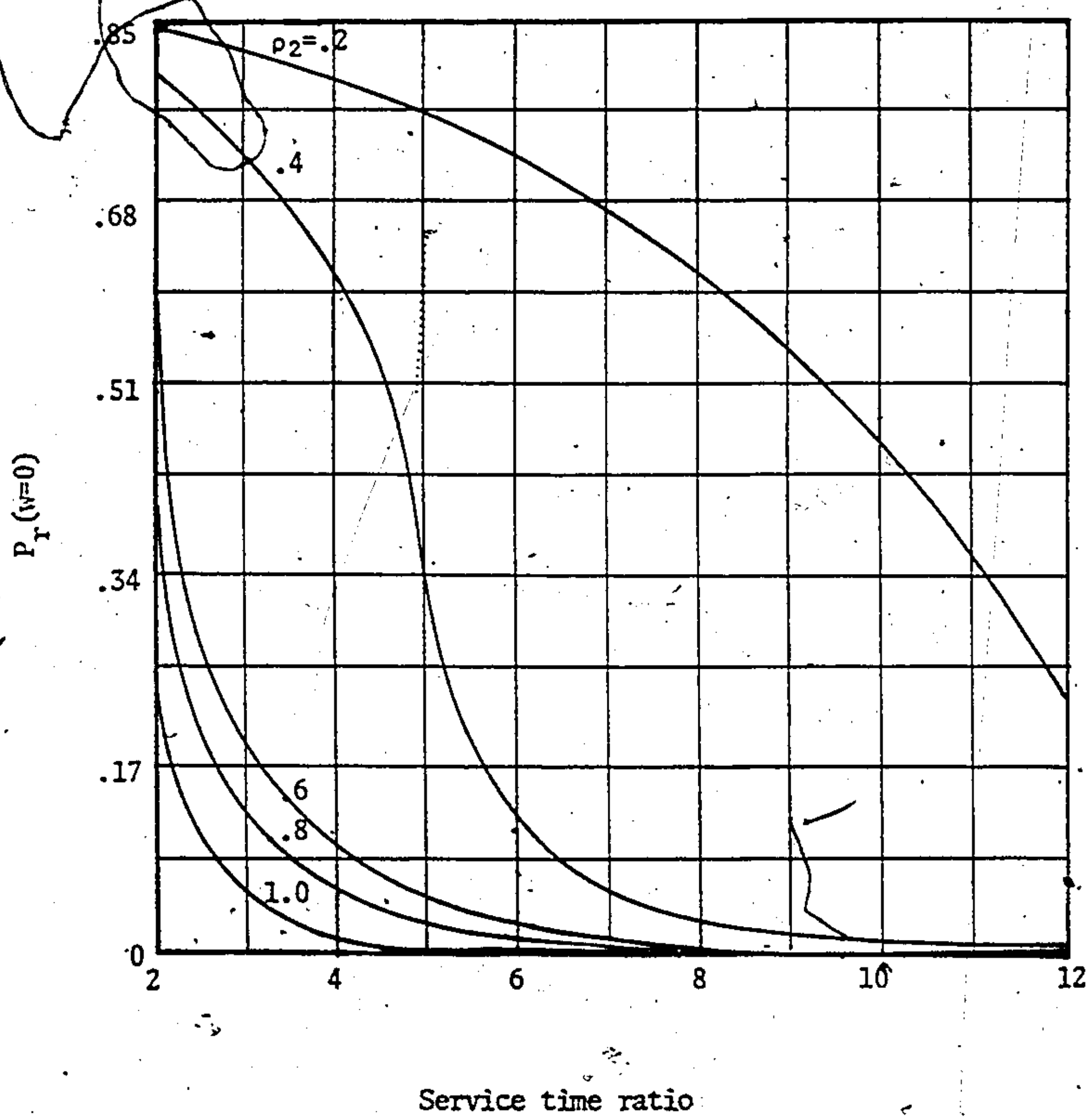


Fig. 18. Probability of no wait versus service time ratio.

The probability that an arrival will have no wait ($Pr[w=0]$) decreases with increasing ρ_1 and ρ_2, as shown in figs. 3 and 6. To maintain the desired value of $Pr[w=0]$, say 0.9, number of service channels should also be increased with the increase in ρ_1 and ρ_2.

Figs. 7 through 9 show the effect of number of service channels on the measures of performance. An increase in the number of channels, having ρ_1, ρ_2 and other variables constant, will increase the probability, P_{00}, till it attains a constant value. This is shown in fig. 7.

The value of $E[n]$ decreases as the number of channels are increased as shown in fig. 8.

Fig. 9 shows that an increase in number of channels, having other variables, some constant value, will reduce the waiting time of customers.

Similarly, the effects of length of conveyors, number of sources and service time ratio are shown in figs. 10 through 18.

This would be worthwhile to clarify here that the results and graphs are presented to help in designing any new conveyor system according to their technological constraints. In this study, instead of emphasizing any specific example, consideration has been given to solve any problems arising in designing such systems. Every industrial situation will have different technological constraints placed on the total number of servers, sources and unit lengths of conveyor that can be utilized to process a given set of jobs. So, the model developed for multi-item, multi-channel fixed conveyor system and graphs presented in this chapter will be helpful to the designers.

CHAPTER 4

COST MODEL DEVELOPMENT AND THE DETERMINATION OF THE MINIMUM COST COMBINATION OF SOURCES, SERVERS AND CONVEYOR LENGTH

The most important aspect in the development of a methodology for analyzing a production system is the transformation of theoretical concepts into workable decision making tools. In this chapter, the previously developed ideas and procedures will be converted into a form that will enhance the quality of managerial decision making.

The work to follow will be initially concerned with the development of economic models for the fixed conveyor system. Once having obtained the economic model for the system, a procedure for determining the minimum cost combination of sources, servers and conveyor length will be given.

Economic Model

In the development of an economic model some basic assumptions will be made. First, it will be assumed that since the conveyor system is part of a larger industrial system, the cost of providing a service and the cost of waiting for service will both be internal to the same organization. A second assumption will be that since the processing of units through the fixed conveyor system is so far removed from the actual profit making mechanism of the organization, it will be far more meaningful to develop the economic model in terms of costs rather than profits.

With the above assumptions in mind, it is possible to define the cost which must be included in the cost models for the fixed conveyor.

The costs are given by the following terms:

C_1 = The cost of having the system idle for one time unit.

C_2 = The cost per unit time per unit in the system.

C_3 = The cost of providing each unit length of conveyor during the processing period.

C_4 = The cost of having a source available for delivering units to the conveyor for one unit of time.

C_5 = The cost of having a server available for servicing units of a given job for one unit of time.

Using the following measures of effectiveness, (1) P_{00} (probability of the system being an idle), and (2) $P_r[w=0]$ (probability that an arrival will not have to wait prior to service, and (3) $E[n]$ (expected number in the system), the element of total cost equation can be defined. Now let,

T_t = the total time to process a job,

N_b = the total number of unit spaces of conveyor length made available during T_t.

The total cost of processing a job can be expressed as follows:

$$TC = C_1 P_{00} T_t + C_2 E[n] T_t + C_3 N_b + C_4 T_t C + C_5 T_t S$$

where TC = Total cost of processing a given job.

$C_1 P_{00} T_t$ = The cost of having the system idle during the complete operation.

$C_2 E[n] T_t$ = The cost of having expected units in the system during the complete operation.

$C_3 N_b$ = The cost of providing total length of conveyor during the processing period.

$C_4 T_t C$ = The cost of having "C" servers available for servicing units during the processing period.

$C_5 T_t S$ = The cost of having "S" sources available for delivering units during the processing period.

Determining the Minimum Cost Combination

Having formulated the total cost equation for the operation of a fixed conveyor system, it now becomes possible to develop a procedure for determining the minimum cost combination of system's components. In general, two possible classes of problems will be encountered.

1) The first class considers the situation in which all of the systems' components are viewed as being variable. That is, for a class of one problem, a minimum cost combination of sources, servers, and conveyor length will need to be determined.

2) The second class of problem involves a situation in which the conveyor is fixed in length. Problems

of the second class are the onces most frequently encountered, since in an actual operating system the distance between work stations usually remains constant. When determining a minimum cost combination of systems components for a problem of the second class, only the combination of sources and servers need to be considered.

In order to fully define the cost minimization problem for both classes, it must first be realized that for any industrial situation there will be technological constraints placed on the total number of servers, sources and unit lengths of conveyor that can be utilized to process a given set of jobs. The technological constraints result from physical space limitations at the work centers or labor availability. In addition to the constraints which result from the physical restrictions of the system, there will usually be a constraint on the maximum number of time units a given set of jobs can consume.

The following procedure can be employed to determine the minimum cost combination of sources (S), servers (C) and conveyor length (b) which satisfies the constraints given below:

$$1 < S < 4$$

$$1 < C < 4$$

$$10 < b < 25$$

$$T_t < 200$$

1. Set 'b' equal to any fixed value, say b_0.
2. Test all combinations of C and S with $b = b_0$ and select optimum C(C*) and optimum S(S*).
3. Holding C and S constant at C* and S*, search over the feasible range of conveyor length, b, until the total minimum cost is found.

Example Problem:

Consider the following situation:

The jobs that are listed in Table A are to be scheduled through a fixed conveyor system. The problem is to determine the minimum cost combination of sources, servers, and conveyor length which will satisfy the requirements that, "25 units of production will be completed for each job in a maximum time span of 200 time units".

TABLE A

The total no. of jobs being processed = 4

Job Number	Number of Units	Traffic Intensity λ/μ ρ_1	ρ_2
1	25	0.2	0.2
2	25	0.2	1.0
3	25	0.4	1.0
4	25	1.0	0.4

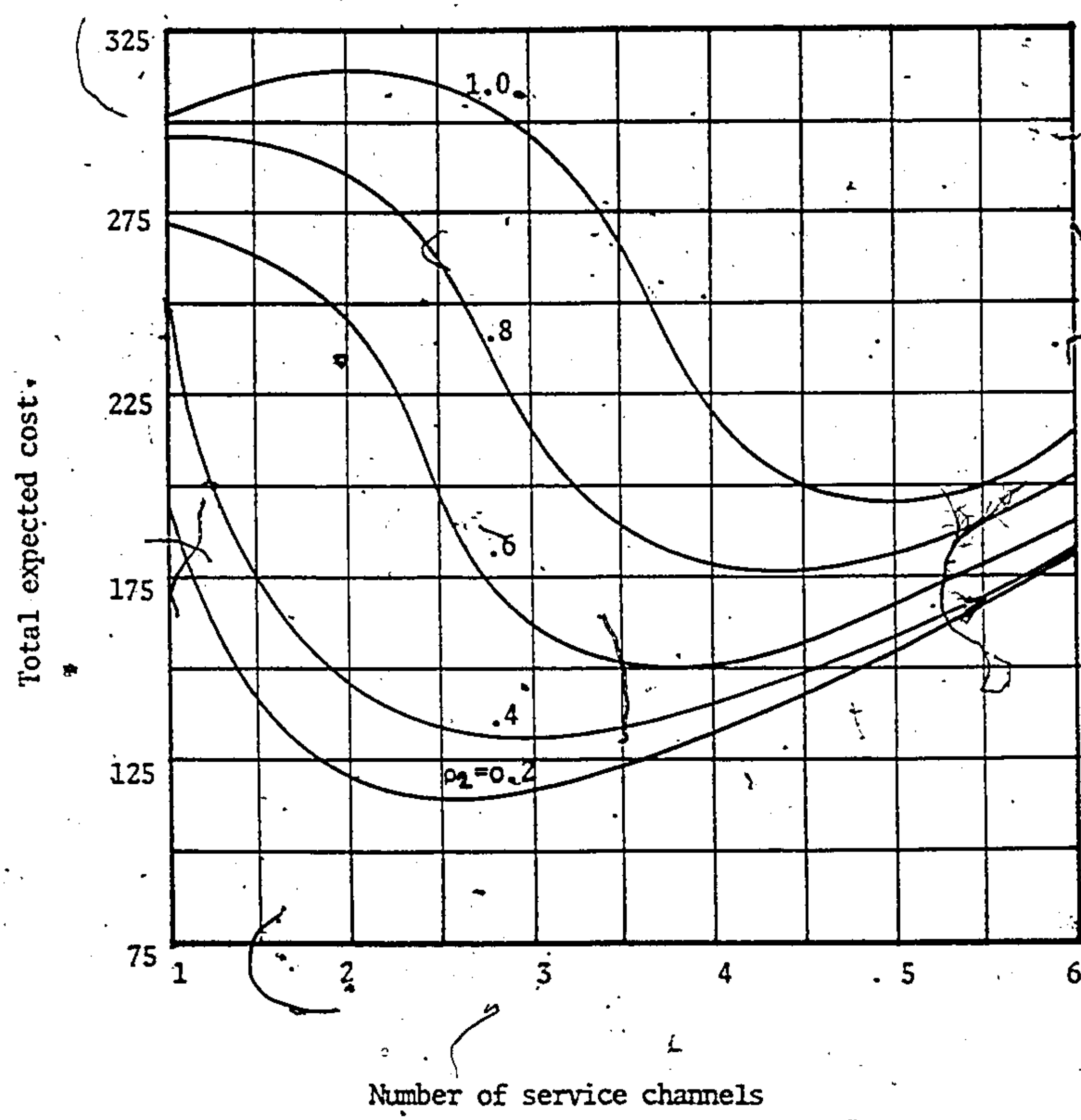


Fig. 19. Total expected cost versus number of service channels.

The physical requirements of the system and the cost associated with operating the system (Thomas, Paul, Cullinane) are such that the problem can be formulated as follows.

Minimize Total Cost,

such that $1 < S < 4$

$1 < C < 4$

$10 < b < 25$

$T < 200$

$C_1 = 1.0 \qquad C_2 = 0.05$

$C_3 = 0.07 \qquad C_4 = 0.1$

$C_5 = 0.1$

S, C and b are integers.

Computer program has been written to perform the necessary calculations. The program for determining the minimum cost combination of sources, servers and conveyors length is given in Appendix C. The results of the analysis of the above problem are given in Tables 1 thru 4.

Fig. 19 shows the total cost combination of sources and servers.

TABLE 1

RESULTS OBTAINED FROM
COST ANALYSIS FOR JOB ONE

$\rho_1=0.2$ $\rho_2=0.2$

Combination Number	Conveyor Length (b)	Sources (S)	Servers (C)	Total Cost
1	25	1	1	combination infeasible
2	25	1	2	combination infeasible
3	25	1	3	combination infeasible
4	25	1	4	combination infeasible
5	25	2	1	combination infeasible
6	25	2	2	combination infeasible
7	25	2	3	combination infeasible
8	25	2	4	145.98
9	25	3	1	249.21
10	25	3	2	144.22
11	25	3	3	130.99*
12	25	3	4	145.09
13	25	4	1	267.71
14	25	4	2	258.81
15	25	4	3	149.32
16	25	4	4	153.62

*minimum cost

TABLE 2

RESULTS OBTAINED FROM
COST ANALYSIS FOR JOB TWO

$\rho_1=0.2$ $\rho_2=1.0$

Combination Number	Conveyor Length (b)	Sources (S)	Servers (C)	Total Cost
1	25	1	1	227.99
2	25	1	2	197.39
3	25	1	3	99.99*
4	25	1	4	109.96
5	25	2	1	248.28
6	25	2	2	263.20
7	25	2	3	274.19
8	25	2	4	246.62
9	25	3	1	264.08
10	25	3	2	279.85
11	25	3	3	297.84
12	25	3	4	314.37
13	25	4	1	278.03
14	25	4	2	294.83
15	25	4	3	314.54
16	25	4	4	334.19

*minimum cost

TABLE 3

RESULTS OBTAINED FROM
COST ANALYSIS FOR JOB THREE

$\rho_1=0.4$ $\rho_2=1.0$

Combination Number	Conveyor Length (b)	Sources (S)	Servers (C)	Total Cost
1	25	1	1	228.89
2	25	1	2	220.51
3	25	1	3	104.37*
4	25	1	4	108.62
5	25	2	1	248.95
6	25	2	2	264.51
7	25	2	3	278.09
8	25	2	4	274.66
9	25	3	1	265.00
10	25	3	2	280.66
11	25	3	3	299.00
12	25	3	4	316.71
13	25	4	1	279.04
14	25	4	2	295.40
15	25	4	3	315.20
16	25	4	4	335.25

*minimum cost

TABLE 4

RESULTS OBTAINED FROM
COST ANALYSIS FOR JOB FOUR

$\rho_1=1.0$ $\rho_2=0.4$

Combination Number	Conveyor Length (b)	Sources (S)	Servers (C)	Total Cost
1	25	1	1	224.87
2	25	1	2	114.94
3	25	1	3	100.99*
4	25	1	4	115.09
5	25	2	1	246.56
6	25	2	2	259.19
7	25	2	3	253.07
8	25	2	4	147.25
9	25	3	1	262.02
10	25	3	2	277.85
11	25	3	3	294.21
12	25	3	4	304.08
13	25	4	1	276.00
14	25	4	2	293.52
15	25	4	3	312.71
16	25	4	4	330.82

*minimum cost

CHAPTER 5

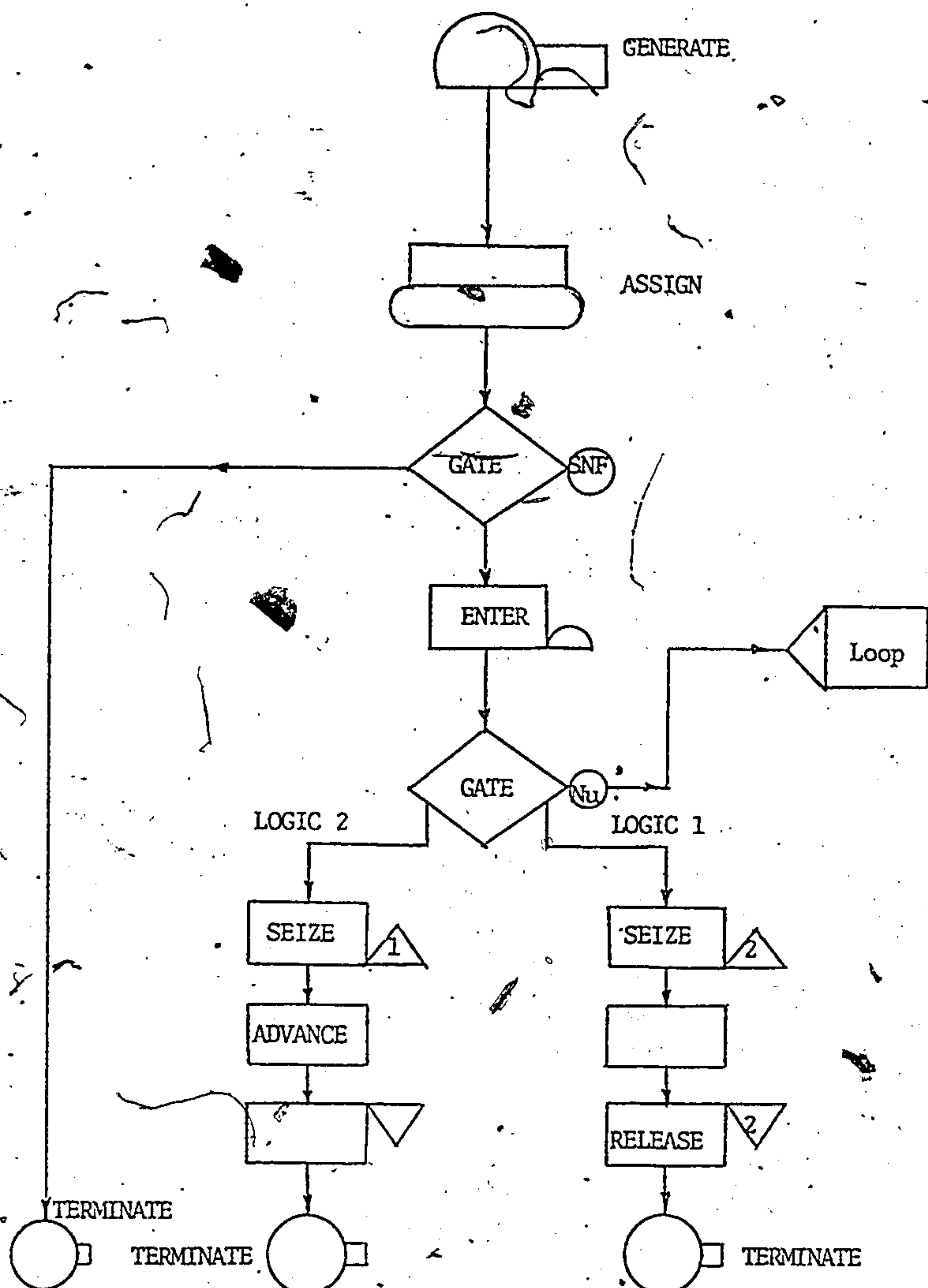
SIMULATION OF THE SYSTEM

As the title implies, this chapter will deal with the evaluation of the conveyor system under study. Simulation has long been considered an important approach to study problems and also to compare the analytical results with the simulation results. Simulation is nothing more than the technique of performing sampling experiments on a computer model.

A simulation model can be considered as consisting of two basic phases: (1) data generation and (2) bookkeeping. Data generation involves the production of representative interarrival times and service times where needed throughout the queueing system. Generally, this involves producing representative observations from prespecified probability distributions.

The bookkeeping phase of a simulation model deals with updating the system when new events (arrivals and departures) occur, monitoring and recording the system states as they change, and keeping track of quantities of interest such as waiting time and idle time.

GPSS (General Purpose Simulation System) is used for this study as it is one of the most capable languages for modeling quite complex queueing systems.



G.P.S.S. Flow Chart

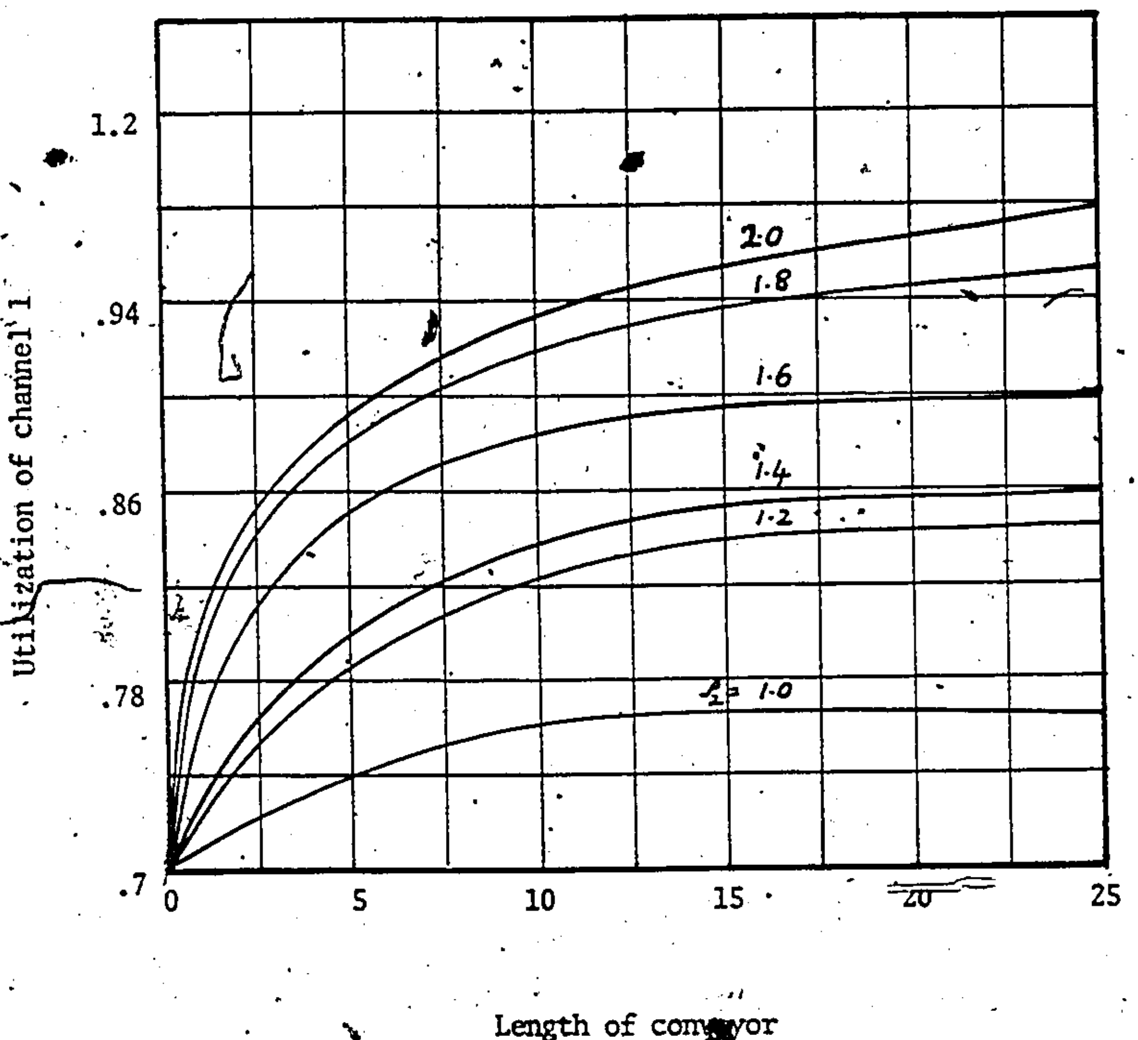


Fig. 20. Utilization of channel 1 versus length of conveyor.

The following measures of performance are obtained for two and three channel conveyor:

1) utilization of each individual channel
2) utilization of conveyor
3) probability that an arrival will have no wait prior to service

In order to attain the first two measures of performance, the following cases are studied.

i) length of conveyor traffic intensity (ρ_1) are kept constant while the values of traffic intensity (ρ_2) are increased
ii) length of conveyor and traffic intensity (ρ_2) as kept constant while the values of traffic intensity (ρ_1) are increased
iii) effect of length of conveyor for different values of ρ_1 and ρ_2 is also studied

This can be seen from the fig. 20 that as traffic intensity and length of conveyor increase the utilization of channels also increases. And also, from fig. 23 it can be seen that as length of conveyor is increased, keeping ρ_1 and ρ_2 constant, the percentage utilization of conveyor decreases.

The third measure of performance (i.e., the probability that an arrival will have no wait prior to service) is obtained by finding the total number of entries to the channels and the number of lost arrivals. The probability that an arrival will have no wait is given by the following expression:

$$Pr[w=0] = \frac{\text{total no. of transactions generated - no. of lost arrivals}}{\text{total no. of transactions generated}}$$

The simulation results of Pr[w=0] and Pr[w=0] obtained by analytical methods are plotted in fig.24 for the various values of ρ_1 and ρ_2. This comparison shows that there is no significant difference between the results obtained from simulation and the analytical model developed.

TABLE 1

Length of Conveyor = 5
Traffic Intensity ρ_1 = 1.0

UTILIZATION OF				TRAFFIC INTENSITY ρ_2
CHANNEL 1	CHANNEL 2	CHANNEL 3	CONVEYOR	
0.742	0.639	0.529	0.11	1.0
0.792	0.704	0.609	0.165	1.2
0.812	0.760	0.669	0.218	1.4
0.854	0.803	0.73	0.272	1.6
0.883	0.837	0.702	0.323	1.8
0.898	0.863	0.823	0.385	2.0

TABLE 2

Length of Conveyor = 10
Traffic Intensity ρ_1 = 1.0

UTILIZATION OF				TRAFFIC INTENSITY ρ_2
CHANNEL 1	CHANNEL 2	CHANNEL 3	CONVEYOR	
0.761	0.675	0.571	0.094	1.0
0.801	0.721	0.681	0.127	1.2
0.841	0.778	0.708	0.187	1.4
0.864	0.828	0.768	0.227	1.6
0.918	0.878	0.837	0.349	1.8
0.921	0.892	0.861	0.389	2.0

TABLE 3

Length of Conveyor = 15
Traffic Intensity ρ_1 = 1.0

UTILIZATION OF				TRAFFIC INTENSITY ρ_2
CHANNEL 1	CHANNEL 2	CHANNEL 3	CONVEYOR	
0.761	0.675	0.567	0.062	1.0
0.813	0.754	0.668	0.102	1.2
0.846	0.789	0.729	0.155	1.4
0.890	0.843	0.799	0.208	1.6
0.920	0.899	0.854	0.321	1.8
0.948	0.928	0.908	0.430	2.0

TABLE 4

Length of Conveyor = 20
Traffic Intensity ρ_1 = 1.0

UTILIZATION OF				TRAFFIC INTENSITY ρ_2
CHANNEL 1	CHANNEL 2	CHANNEL 3	CONVEYOR	
0.761	0.675	0.567	0.046	1.0
0.828	0.767	0.677	0.091	1.2
0.850	0.799	0.732	0.122	1.4
0.886	0.846	0.799	0.184	1.6
0.996	0.933	0.910	0.386	1.8
0.967	0.963	0.949	0.506	2.0

TABLE 5

Length of Conveyor = 25
Traffic Intensity ρ_1 = 1.0

UTILIZATION OF				TRAFFIC INTENSITY ρ_2
CHANNEL 1	CHANNEL 2	CHANNEL 3	CONVEYOR	
0.790	0.730	0.620	0.032	1.0
0.812	0.740	0.645	0.055	1.2
0.851	0.800	0.725	0.118	1.4
0.913	0.879	0.837	0.218	1.6
0.946	0.927	0.900	0.328	1.8
0.968	0.954	0.933	0.450	2.0

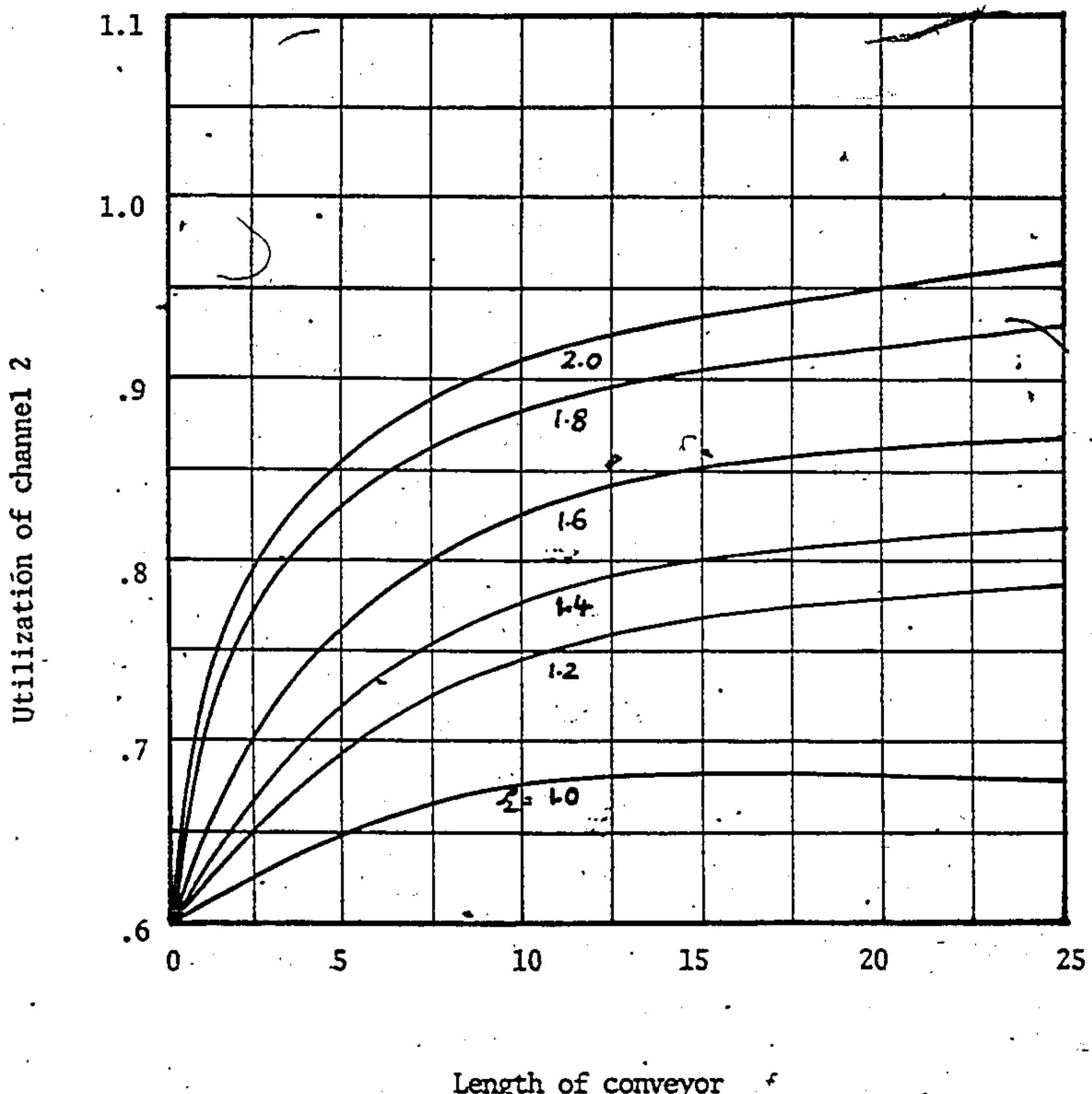


Fig. 21. Utilization of channel 2 versus length of conveyor.

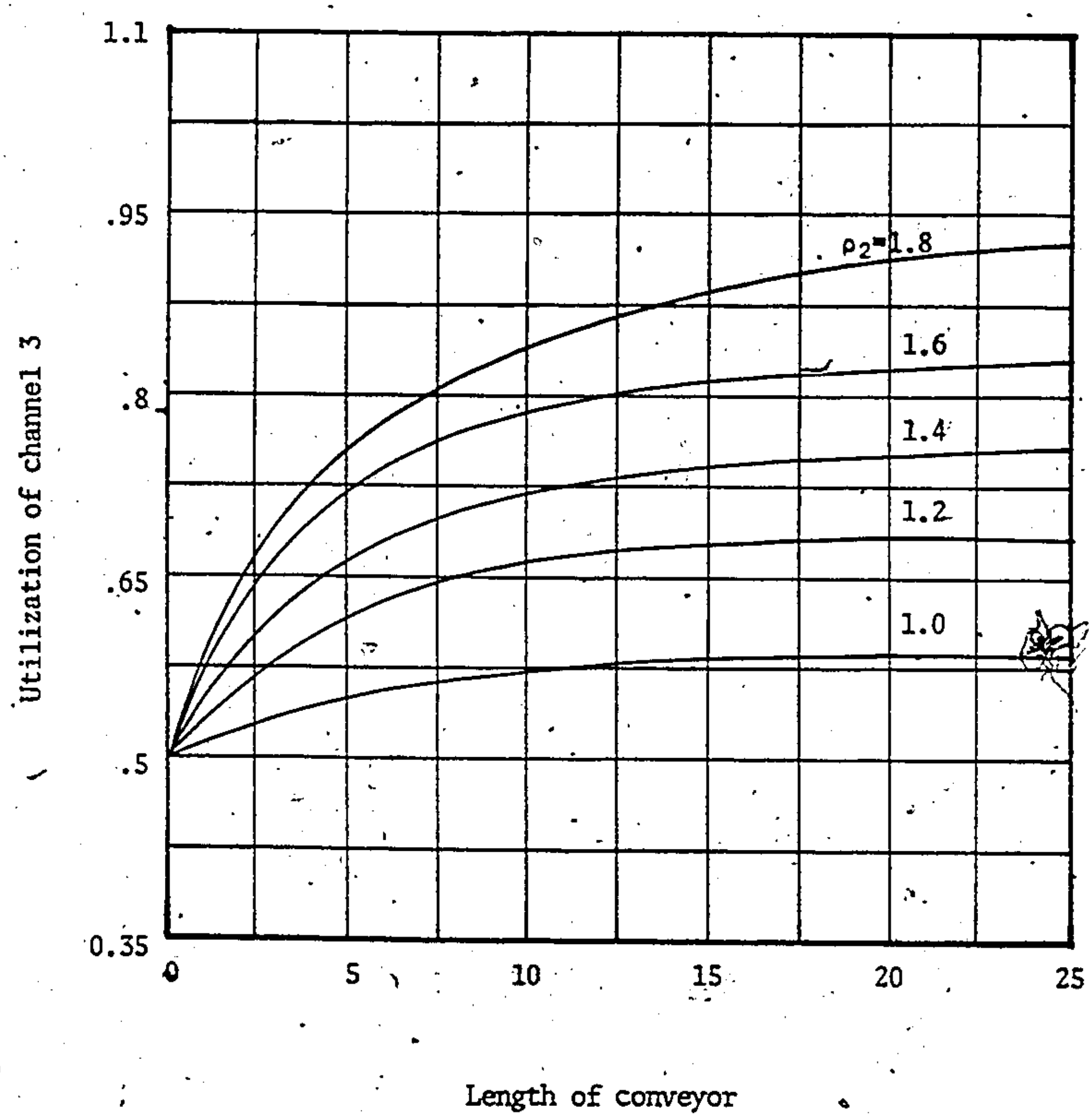


Fig. 22. Utilization of channel 3 versus length of conveyor.

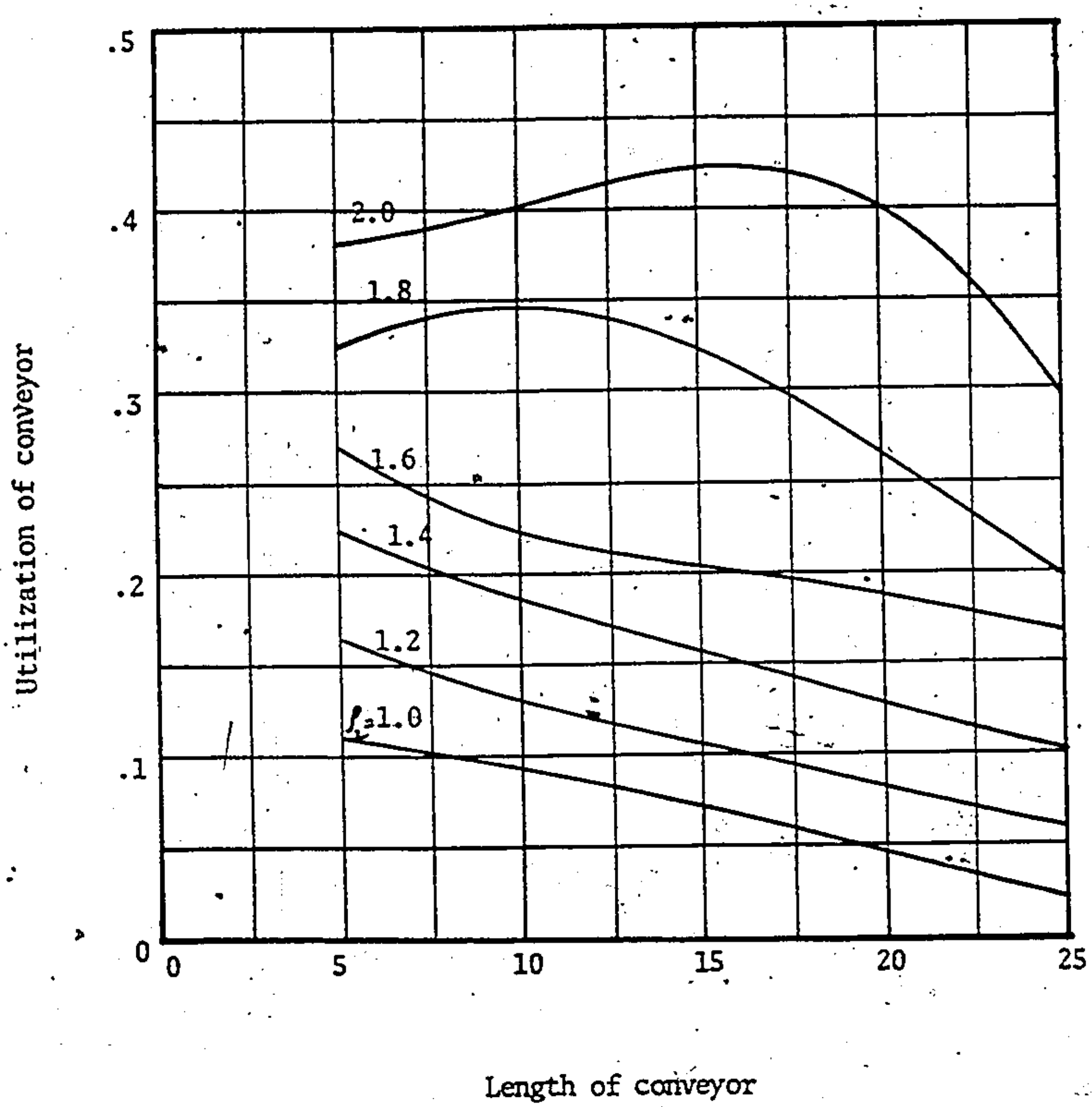
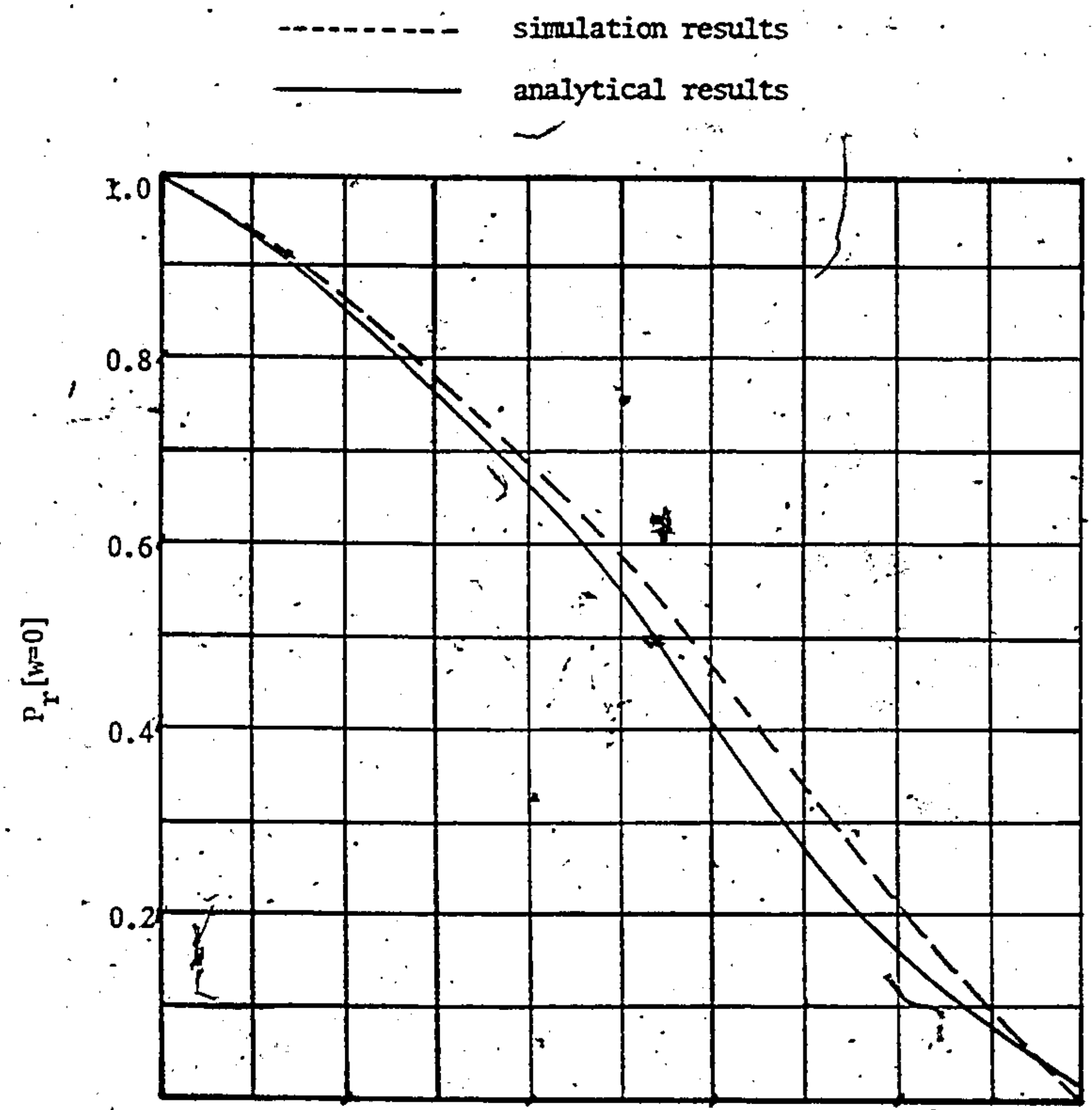


Fig. 23. Utilization of conveyor versus length of conveyor.



Traffic intensity ρ_2

Probability versus ρ_2

Number of channels = 2; length of conveyor = 25

number of sources = 2; ρ_1 = 0.2

Fig. 24. Comparison between the simulation results and the analytical results.

Probability versus traffic intensity.

Utilization of service channels against length of conveyors are plotted in Figs. 20 through 22. Utilization of conveyor against length of conveyor is also plotted in Fig. 23.

These figures show that as length of conveyor increases utilization of service channels also increases. But, when it reaches to a maximum point, further increase in length of conveyor will not affect the utilization. This is also shown that as traffic intensity increases the utilization of service channels and conveyor will also increase.

The probability that an arrival will be serviced as soon as it enters (no wait, Pr[w=0]) the system is plotted in fig. 24. These values of Pr[w=0] obtained from simulation are compared with the analytical results in fig. 24, and there is no significant difference found between these results.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The objectives of this thesis have been:

1) to develop a model of the fixed conveyor system where multi-item jobs have been scheduled to be processed,

2) to obtain a methodology for determining the minimum cost combination of sources, servers, and conveyor length,

3) simulation of the system by GPSS.

In satisfying the first objective, the fixed conveyor system was modeled as a multiple source, multiple server, infinite population queueing process. Using the concepts of birth-death processes, the system's state equations were developed. Relationships for determining the system's steady state probabilities were given and numerical procedure for determining the system's steady state probabilities was introduced.

Once the method for determining the system's state probabilities as a function of time was tested the following measures of effectiveness were introduced.

1) The probability of the system being an idle.

2) The expected number of units in the system.

3) The probability that an arrival will not have to wait Pr w=0 .

After obtaining mathematical descriptions of the system, the results are plotted to show the system's behavior.

In an effort to satisfy the second objective, a cost model for this conveyor system was introduced by using the measures of performance and the mathematical relationship which described the system's behavior. Having formulated the cost model, a methodology for determining the minimum components was developed.

Finally, simulation analysis was undertaken to give some insight into the response of the fixed conveyor. The results from simulation and analytical methods are compared and no significant difference was found.

CONCLUSIONS

The following conclusions can be drawn from this study:

1) This study shows that the steady state solution of multiple jobs which have been scheduled to be operated on fixed conveyor is feasible through the application of queueing theory.

2) The results obtained from analytical method, cost analysis and simulation studies are helpful in finding the optimum system's components (sources, servers, etc.).

3) An increase in the number of sources (servers or length of conveyor) does not always result in a high performance.

4) The cost analysis shows that the system having less numbers of service facilities and servers may not be necessarily the minimum cost combination. An increase in the numbers of service facilities and servers may give better results.

5) Analytical results also indicate that an increase in service facilities (sources, length of conveyor, etc.) to a certain value gives the satisfactory system's performance, as shown in fig. 24

6) Simulation study shows that utilization of the channels and the conveyor increase with an increase in the length of conveyor. When utilization reaches the constant value, (i.e. further increase in conveyor length will not increase the utilization) an increase in traffic intensity will increase the utilizations of the channel and the conveyor.

In summary, a model has been developed for aiding the decision maker when he is attempting to determine the optimum number of sources, servers and length of the conveyor that should be used when multi-item jobs are being processed and transported by a fixed conveyor system.

SUGGESTIONS FOR FUTURE RESEARCH

Several possible extensions of this research have been recognized and are listed below.

1) The current model can be extended to include the case in which the delivery rates of the sources and the service rates of the service facilities are not all the same. If the model could be extended to reflect non-homogeneous conditions a more accurate duplication of an actual operating system would be achieved.

2) The current model can be extended to include the case in which more than two types of arrival are allowed.

3) Extend the current model to take into consideration the transient behavior of the fixed conveyor system.

4) Consider an extension of the current model to include the case in which some units of production must be reworked or scrapped.

5) Perform an extensive analysis of the costs which make up the total cost of operating the conveyor system; that is, develop a procedure for determining the value of each cost factor.

6) Conduct an investigation into the robustness of the assumptions made in the present study; that is, a simulation of the fixed conveyor system could be performed for many arrival distributions and many service time distributions.

APPENDIX A

The following pages contain a computer program which has been developed to analyse a fixed conveyor system.

The program language used is system 360 Fortran IV. Definitions of the variables have been given as part of the documentation and appear on the program listing.

This program evaluates the measures of performance using equations IV through VI given on page 43. The variables used in this program are as follows:

a) Number of service channels

b) Number of input sources

c) Length of conveyor

d) Traffic intensities

e) Service time ratio.

```
$JOB    WATFIV  XXXXXXXXXX                          M.ALI
C
C
C       THIS PROGRAM HAS BEEN DEVELOPED TO EVALUATE THE MEASURES OF
C       PERFORMANCE
C
C
C       DEFINITIONS OF THE VARIABLES USED IN THIS PROGRAM
C
C       RO1=TRAFFIC INTENSITY
C
C       RO2=TRAFFIC INTENSITY
C
C       IS=NUMBER OF INPUT SOURCES
C
C       IAN=SERVIC TIME RATIO
C
C       N=LENGTH OF THE CONVEYOR
C
C       M=NUMBER OF SERVICE CHANNELS
C
C       POO=PROBABILITY OF THE SYSTEM BEING AN IDLE
C
C       PRW=PROBABILITY THAT AN ARRIVAL WILL HAVE NO WAIT
C
C       EN=THE EXPECTED NUMBER OF UNITS IN THE SYSTEM
C
C
C
        DIMENSION A(15).B(15).P00(15).FAC(15).X(15).Y(15)
        FAC(1)=1.
        DO 10 II=2.14
   10   FAC(II)=FAC(II-1)*II
        IS=3
        DO 40 IK=4.20.4
        F=IK
        RO1=F/20
        DO 50 IL=4.20.4
        P=IL
        RO2=P/20.
        WRITE(6.200)RO1.RO2
  200   FORMAT(6X.' TRAFFIC INTENSITY RO1='.F10.5.6X.'TRAFFIC INTENSITYRO2
       1='.F10.5)
        AA=1.
        IS=4
        M=3
        DO 90 N=10.26.2
        SUM=AA
        C=IS*(RO1+AN*RO2)
        DO 14 I=1.M
   14   SUM=SUM+C**I/FAC(I)
        M1=M+1
        M2=M+N
        DO 24 J=M1.M2
   24   SUM=SUM+(C**(J/2.)/M**((J-1)/2.))*(C**(J/2.)/M**((J-1)/2.))
        N1=M+N+1
        N2=M+N+IS
        DO 11 K=N1.N2
   11   SUM=SUM+(C**(K/3.)/M**((K-1)/3.))*(C**(K/3.)/M**((K-1)/3.))*
       1(C**(K/3.)/M**((K-1)/3.))/(IS**(2.*K-2.*M-2.*N))
        POO=1./SUM
        BB=1.
        SUM2=BB
        IF(M.EQ.1)GO TO 60
        IF(M.GT.1)GO TO 70
   60   PRW=POO
        GO TO 80
   70   M3=M-1
        DO 34 L=1.M3
   34   SUM2=SUM2+C**L/FAC(L)
        PRW=POO*SUM2
   80   SUM3=0.
        DO 35 I=1.M
   35   SUM3=SUM3+(I)*(C**I)/FAC(I)
        DO 45 J=M1.M2
   45   SUM3=SUM3+(J)*(C**(J/2.)/M**((J-1)/2.))*(C**(J/2.)/M**((J-1)/2.))
        DO 55 K=N1.N2
   55   SUM3=SUM3+(K)*(C**(K/3.)/M**((K-1)/3.))*(C**(K/3.)/M**((K-1)/3.))*
       1(C**(K/3.)/M**((K-1)/3.))/(IS**(2.*K-2.*M-2.*N))
        EN=POO*SUM3
        WRITE(6.30)M.N.POO.EN.PRW.IS.TC
```

```
      30   FORMAT(2X,'NO. OF CHANNELS=',I2,2X,' LENGTH OF CONVEYOR=',I2,2X,'
          1P00=',F10.9,3X,'EN=',F10.7,3X,'PRW=',F10.9,2X,'IS=',I2,2X,'TC=',F
          110.5)
      90   CONTINUE
      50   CONTINUE
      40   CONTINUE
           STOP
           END

   $ENTRY
```

APPENDIX B

The program presented in this Appendix has been written to perform a multiple Poisson input analysis of a fixed conveyor system and to plot the analytical results. The program language used is system 360 Fortran IV. A listing of the program is given on the following pages. The variable names used within the program have been defined as part of the program documentation and appear on the program listing.

Using equations IV through VI (on page 43), the measures of performance are plotted against the following design parameters:

1. Number of service channels
2. Number of input sources
3. Length of conveyor
4. Traffic intensities
5. Service time ratio.

```
$JOB    WATFIV  XXXXXXXXXX                                    M.ALI
C
C
C       THIS PROGRAM HAS BEEN WRITTEN TO PLOT THE MEASURES OF PERFORMANCE
C       AGAINST THE DESIGN PARAMETERS.
C
C
C       DEFINITIONS OF THE VARIABLES USED IN THIS PROGRAM
C
C       RO1=TRAFFIC INTENSITY
C
C       RO2=TRAFFIC INTENSITY
C
C       IS=NUMBER OF INPUT SOURCES
C
C       IAN=SERVIC TIME RATIO
C
C       N=LENGTH OF THE CONVEYOR
C
C       M=NUMBER OF SERVICE CHANNELS
C
C       POO=PROBABILITY OF THE SYSTEM BEING AN IDLE
C
C       PRW=PROBABILITY THAT AN ARRIVAL WILL HAVE NO WAIT
C
C       EN=THE EXPECTED NUMBER OF UNITS IN THE SYSTEM
C
C
      DIMENSION FAC(20),X(18),Y(18)
      AMOVE=-1.
      CALL PLOTID('MUSTAFA ALI','G149001331')
      CALL XLIMIT(180.)
      RO2=0.5
      IS=2
      IAN=2
      FAC(1)=1.
      DO 10 II=2,14
   10 FAC(II)=FAC(II-1)*II
      DO 25 M=2,6
      READ(2,13)FX,FY,DX,DY
   13 FORMAT(2(F3.1,E7.0))
      DO 50 IL=4,20,4
      P=IL
      RO1=IL/20.
      WRITE(6,200)RO1,RO2
  200 FORMAT(6X,' TRAFFIC INTENSIY RO1=',F10.5,6X,'TRAFFIC INTENSITY RO2
     1=',F10.5)
      DO 90 N=4,24,4
      AA=1.
      SUM=AA
      C=IS*(RO1+IAN*RO2)
      DO 14 I=1,M
   14 SUM=SUM+C**I/FAC(I)
      M1=M+1
      M2=M+N
      DO 24 J=M1,M2
   24 SUM=SUM+(C**(J/2.)/M**((J-1)/2.))*(C**(J/2.)/M**((J-1)/2.))
      N1=M+N+1
      N2=M+N+IS
      DO 11 K=N1,N2
   11 SUM=SUM+(C**(K/3.)/M**((K-1)/3.))*(C**(K/3.)/M**((K-1)/3.))*
     1(C**(K/3.)/M**((K-1)/3.))/(IS**(2.*K-2.*M-2.*N))
      POO=1./SUM
      BB=1.
      SUM2=BB
      IF(M.EQ.1)GO TO 60
      IF(M.GT.1)GO TO 70
   60 PRW=POO
      GO TO 80
   70 M3=M-1
      DO 34 L=1,M3
   34 SUM2=SUM2+C**L/FAC(L)
      PRW=POO*SUM2
   80 SUM3=0.
      DO 35 I=1,M
   35 SUM3=SUM3+(I)*(C**I)/FAC(I)
      DO 45 J=M1,M2
   45 SUM3=SUM3+(J)*(C**(J/2.)/M**((J-1)/2.))*(C**(J/2.)/M**((J-1)/2.))
      DO 55 K=N1,N2
   55 SUM3=SUM3+(K)*(C**(K/3.)/M**((K-1)/3.))*(C**(K/3.)/M**((K-1)/3.))*
```

```
     1(C**(K/3.)/M**((K-1)/3.))/(IS**(2.*K-2.*M-2.*N))
      EN=POO*SUM3
      WRITE(6,30)M,N,POO,EN,PRW,IS,IAN
 30   FORMAT(2X,'NO. OF CHANNELS=',I2,2X,'LENGTH OF CONVEYOR=',I2,2X,'
     1POO=',F10.9,2X,'EN=',F10.7,2X,'PRW=',F10.9,2X,'IS=',I2,2X,'IAN=',I
     12)
      Y(N/4)=PRW
      X(N/4)=N
 90   CONTINUE
      CALL CALCO2(X,Y,8,AMOVE,5.,5.,FX,DX,FY,DY,Q,1,2)
      AMOVE=0.
 50   CONTINUE
      AMOVE=7.
 25   CONTINUE
      CALL PLTEND(15.)
      STOP
      END

$ENTRY
```

APPENDIX C

The program presented in this Appendix has been developed to determine the minimum cost combination of sources, servers,and length of the conveyor.

The program language used is System 360 Fortran IV. Definitions of the variables have been given as part of the documentation and appear on the program listing.

```
$JOB    WATFIV  XXXXXXXXXX                                 M.ALI
C
C
C       THIS PROGRAM HAS BEEN DEVELOPED TO DETERMINE THE MINIMUM COST
C       COMBINATION OF SOURCES, SERVERS, AND CONVEYOR LENGTH.
C
C
C
C       DEFINITIONS OF THE VARIABLES USED IN THIS PROGRAM
C
C       RO1=TRAFFIC INTENSITY
C
C       RO2=TRAFFIC INTENSITY
C
C       IS=NUMBER OF INPUT SOURCES
C
C       IAN=SERVIC TIME RATIO
C
C       N=LENGTH OF THE CONVEYOR
C
C       M=NUMBER OF SERVICE CHANNELS
C
C       P00=PROBABILITY OF THE SYSTEM BEING AN IDLE
C
C       PRW=PROBABILITY THAT AN ARRIVAL WILL HAVE NO WAIT
C
C       EN=THE EXPECTED NUMBER OF UNITS IN THE SYSTEM
C
C       C1=THE COST OF HAVING THE SYSTEM IDLE FOR ONE TIME UNIT
C
C       C2=THE COST PER UNIT TIME PER UNIT IN THE SYSTEM
C
C       C3=THE COST OF PROVIDING EACH UNIT LENGTH OF CONVEYOR DURING THE
C       PROCESSING PERIOD
C
C       C4=THE COST OF HAVING A SOURCE AVAILABLE FOR DELIVERING UNITS
C       TO THE CONVEYOR FOR ONE UNIT OF TIME.
C
C       C5=THE COST OF HAVING A SERVER AVILABLE FOR SERVICING UNITS
C       OF A GIVEN JOB FOR ONE UNIT OF TIME.
C
C       TT=THE TOTAL TIME TO PROCESS A JOB
C
C       N=THE TOTAL NUMBER OF UNITS SPACES OF  CONVEYOR LENGTH MADE
C       AVILABLE DURING TT.
C
C       TC=THE TOTAL COST OF PROCESSING A GIVEN JOB
C
C
        DIMENSION A(15),B(15),P00(15),FAC(15),X(15),Y(15)
        FAC(1)=1.
        DO 10 II=2,14
   10   FAC(II)=FAC(II-1)*II
        C1=0.5
        C2=0.05
        C3=0.07
        C4=0.1
        C5=0.1
        TT=150.
        IS=3
        DO 40 IK=4,20,4
        F=IK
        RO1=F/20
        DO 50 IL=4,20,4
        P=IL
        RO2=P/20.
        WRITE(6,200)RO1,RO2
  200   FORMAT(6X,' TRAFFIC INTENSITY RO1=',F10.5,6X,'TRAFFIC INTENSITYRO2
       1=',F10.5)
        AA=1.
        IS=4
        M=3
        DO 90 N=10,26,2
        SUM=AA
        C=IS*(RO1+AN*RO2)
        DO 14 I=1,M
   14   SUM=SUM+C**I/FAC(I)
        M1=M+1
        M2=M+N
        DO 24 J=M1,M2
   24   SUM=SUM+(C**(J/2.)/M**((J-1)/2.))*(C**(J/2.)/M**((J-1)/2.))
```

```
      N1=M+N+1
      N2=M+N+IS
      DO 11 K=N1,N2
 11   SUM=SUM+(C**(K/3.)/M**((K-1)/3.))*(C**(K/3.)/M**((K-1)/3.))*
     1(C**(K/3.)/M**((K-1)/3.))/(IS**(2.*K-2.*M-2.*N))
      POO=1./SUM
      BB=1.
      SUM2=BB
      IF(M.EQ.1)GO TO 60
      IF(M.GT.1)GO TO 70
 60   PRW=POO
      GO TO 80
 70   M3=M-1
      DO 34 L=1,M3
 34   SUM2=SUM2+C**L/FAC(L)
      PRW=POO*SUM2
 80   SUM3=0.
      DO 35 I=1,M
 35   SUM3=SUM3+(I)*(C**I)/FAC(I)
      DO 45 J=M1,M2
 45   SUM3=SUM3+(J)*(C**(J/2.)/M**((J-1)/2.))*(C**(J/2.)/M**((J-1)/2.))
      DO 55 K=N1,N2
 55   SUM3=SUM3+(K)*(C**(K/3.)/M**((K-1)/3.))*(C**(K/3.)/M**((K-1)/3.))*
     1(C**(K/3.)/M**((K-1)/3.))/(IS**(2.*K-2.*M-2.*N))
      EN=POO*SUM3
      TC=C1*TT*POO+C2*EN*TT+C3*N+C4*M*TT+C5*IS*TT
      WRITE(6,30)M,N,POO,EN,PRW,IS,TC
 30   FORMAT(2X,'NO. OF CHANNELS=',I2,2X,' LENGTH OF CONVEYCR=',I2,2X,'
     1POO=',F10.9,3X,'EN=',F10.7,3X,'PRW=',F10.9,2X,'IS=',I2,2X,'TC=',F
     110.5)
 90   CONTINUE
 50   CONTINUE
 40   CONTINUE
      STOP
      END

$ENTRY
```

APPENDIX D

The program presented in this Appendix has been developed to evaluate the following measures of performance:

a) utilization of each individual channel

b) utilization of the conveyor

c) probability that an arrival will have no wait prior to service.

The program language used is G.P.S.S. (General Purpose Simulation System). Definitions of the variables have been given as part of the documentation and appear on the program listing.

```
BLOCK
NUMBER  *LOC    OPERATION   A,B,C,D,E,F,G          COMMENTS
                SIMULATE
        *       SIMULATION OF A TWO-LINK FIXED CONVEYOR SYSTEM
        *       ARRIVAL RATE LAMDA 1 IS 1/100
        *       ARRIVAL RATE LAMDA2 IS 1/150
        *       CHAN1=SERVICE FACILITY '1'
        *       CHAN2=SERVICE FACILITY '2'
        *       STORAGE=LENGTH OF THE CONVEYOR
        *       FUNCTION DEFINITIONS
         2      FUNCTION     RN1,C24
        0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.3/.75,1.38
        .8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
        .97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
         LENT   STORAGE      25
         1      VARIABLE     100+P7*200
  1             GENERATE     100,FN2       LAMDA 1 IS 1/100
  2             TRANSFER     ,ABC
  3             GENERATE     150,FN2       LAMDA 2 IS 1/150
  4             ASSIGN       7,K1          REPLACE ENTERING VALUE OF PARAMETER 7
  5      ABC    GATE SNF     LENT,TERM     CHECK WHETHER CONVEYOR HAS SPACE OR NOT
  6             ENTER        LENT          UNITS ARRIVE AT THE CONVEYOR
  7             LOGIC S      2
  8             LOGIC R      1
  9      ABOV   GATE LR      1
 10             SAVEVALUE    4,24
 11             SAVEVALUE    5,34
 12             SAVEVALUE    6,44
 13             ASSIGN       1,4
 14             ASSIGN       4,3
 15             ASSIGN       5,1
 16      TEST   TEST E       X*5,1,X*1
 17             ASSIGN       5+,1
 18             INDEX        5,3
 19             GATE LS      2,ANY
 20      ANY    LOOP         4,TEST
 21             LOGIC S      1
 22             LOGIC R      2
 23             TRANSFER     ,ABOV
 24             SEIZE        CHAN1         SEIZE THE SERVICE CHANNEL 1
 25             LEAVE        LENT          LEAVE THE CONVEYOR
 26             SAVEVALUE    1,1
 27             ADVANCE      V1,FN2        UNITS ARE BEING SERVICED
 28             SAVEVALUE    1,0
 29             ASSIGN       6,X1
 30             LOGIC R      1
 31             RELEASE      CHAN1         RELEASE THE CHANNEL 1
 32             TABULATE     LUCK
 33             TERMINATE    1
         LUCK   TABLE        M1,20,20,200
 34             SEIZE        CHAN2         SEIZE THE CHANNEL 2
 35             LEAVE        LENT          LEAVE THE CONVEYOR
 36             SAVEVALUE    2,1
 37             ADVANCE      V1,FN2        UNITS ARE BEING SERVICED
 38             SAVEVALUE    2,0
 39             ASSIGN       6,X2
 40             LOGIC R      1
```

```
41          RELEASE     CHAN2         RELEASE CHANNEL 2
42          TABULATE    TBLE
43          TERMINATE   1
     TBLE   TABLE       M1,20,20,200
44          SEIZE       CHAN3         SEIZE THE CHANNEL 3
45          LEAVE       LENT          LEAVE THE CONVEYOR
46          SAVEVALUE   3,1
47          ADVANCE     V1,FN2        UNITS ARE BEING SERVICED
48          SAVEVALUE   3,0
49          ASSIGN      6,X3
50          LOGIC R     1
51          RELEASE     CHAN3         RELEASE THE CHANNEL 3
52          TABULATE    HOPE
53          TERMINATE   1
     HOPE   TABLE       M1,20,20,200
54          TERMINATE   1
55   TERM   TERMINATE   1
            START       200,NP
            RESET
            START       10000
            CLEAR
     1      VARIABLE    100+P7*80     SERVICE TIME 1/MU IS 180
            START       10000
            CLEAR
     1      VARIABLE    100+P7*110    SERVICE TIME 1/MU IS 210
            START       10000
            END
```

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