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**BATCHING FOR MULTI-ITEM SINGLE-MACHINE
AND MULTI-ITEM MULTI-MACHINE**

By

Hamda Halleb

A Thesis

**Submitted to the Faculty of Graduate Studies and Research
Through the Department of Industrial Engineering
in Partial Fulfillment of the Requirement
for the Degree of Master of Applied Science at the
University of Windsor**

Windsor, Ontario, Canada

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ABSTRACT

In this thesis, a heuristic procedure to find the average waiting time for multi-item single machine is presented. The heuristic is based on the assumption that the ratio of run time to setup time is constant across all items. The heuristic is tested against two nonlinear optimizers (MINOS and Simulating Annealing) and for all the tests the average waiting times for the Heuristic are very close to the values given by both Optimizers and this shows that the Heuristic is a good one. The second part of this research is a cost model for the multi-item multi-machine where the problem is converted to multi-item single machine by equally allocating the workload across all the machines. A procedure to allocate items to machines is developed and a solution method to find the number of machines in order to minimize the sum of machinery cost, maintenance cost and delay cost.

DEDICATION

To the two people without whom I would not be here.

(my dad, Mahmoud & my Mother, Halima)

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CHAPTER 1

INTRODUCTION

Queueing delays constitute a major component of manufacturing lead times. Lead times are the primary determinants of manufacturing performances since they affect WIP inventory levels, safety stock and schedule performance. In many manufacturing facilities, it is possible to control the queueing behavior by batching. Karmarkar (1983) presented a single facility multi-item model relating batching to queueing delays as an M/G/1 model. The objective is to find a set of batch sizes in order to minimize the average batch waiting time. In this model, a setup is required before the process of a new batch. Only one setup is required for each batch independent of the batch size.

Research Focus

In Chapter 2, a complete review for the relevant literature is presented.

In Chapter 3, a heuristic procedure is presented to solve the multi-item single facility problem. The heuristic is based on the assumption that the ratio of batch run time to setup time is constant across all items. The above assumption is based on the fact that lot sizes should be higher for items with larger setup times and smaller for items with smaller setup times¹. The solution procedure developed here reduces the size of the

¹ Yang (1990)

problem from being multi-dimensional to single dimension, it also gives good results and this is proven in Chapter 4. A complete analysis of the effects of adding a new item to the product mix or changing the demand requirement of any of the items is included.

In Chapter 4, The Heuristic developed in chapter 3 is tested against two nonlinear optimizers, (MINOS and Simulating Annealing).

In Chapter 5, the case of multi-product multi-machine is considered. The objective is to minimize the sum of queueing delay cost, machinery cost and maintenance cost.

CHAPTER 2

LITERATURE REVIEW

In any manufacturing environment, the total lead time taken to manufacture a product is an important issue. Large lead times impose costs due to work in process (WIP) inventory, increased uncertainty about requirements in terms of quantity as well as quality and also higher safety stocks to cover up for the uncertainties. Because of the increased competition in the marketplace, management is more and more concerned about lead times. When lead times are reduced, work in process inventories are minimized, due dates are met and the firm survives the competition.

The definition of lead time depends on the production system utilized. There are basically two types of production systems: 1) open jobs, where job orders arrive from customers and, 2) closed job shops where production is done for stock or assembly. In the open job shop, manufacturing lead time is divided into two components: backlog time and shop flow time. Backlog is defined as the time period between receiving a customer's order and the time the order is released to the shop floor. Shop flow time is the time that elapses between releasing the order to the floor and the job completion. In the closed job shop, manufacturing lead time is the job flow time and it consists of the following major components: 1) setup time before a production run, 2) batch processing

time, 3) move or transportation time between work centers, 4) waiting time in the queue at each facility before processing. The importance of minimizing the total lead time is one of the major problems that faces the management. The problem that has to be answered is where improvement can be done. By looking at the components of the lead times, one can see that in order to minimize the total lead time, the waiting time is the major element where improvements are more likely to take place and this is due to the simple reason that usually setup times, processing times, and move times are predetermined characteristics of the shop and depend on the machinery and transportation mechanism used in the shop. Improvements can also be done on speeding up the setup times, processing times and move times between stations but that can only be done through more investments. As it can be seen, the question of reducing queue waiting is still possible without any extra investments.

The recent literature emphasizes the importance of waiting time in the queue on the total lead time. These queues can be very substantial and products can spend more than 90% of their time in the shop waiting in the queues (Karmarkar et al., 1985). The queueing behavior in complex job shop is affected by several decision variables which include:

- 1) Lot size choices for each product in the product mix
- 2) Release times of batches to the shop floor
- 3) Sequencing at the machines

- 4) Capacity limitations at the work centers
- 5) Product mix and heterogeneity of the products

The above decision variables are interrelated and the effect of changing one variable will affect the rest of the variables.

2.1 Early Lotsizing Models

A large proportion of manufacturing in the U.S. occurs in closed job shops which produce for stock or assembly. The characteristic of these shops is that they produce a wide variety of products where it is necessary to have a setup time when a switch in production from one product to a different product occurs. It is usually not feasible to setup for one product and run a production until its total demand requirement is met and then switch to another product. It is also not feasible to produce one unit and switch to produce another unit of a different product. Because of the presence of setup times, the production of items is accomplished through batching. In early literature, the most common representation for studying this type of production system is the use of the well known Camp's economic batch size rule. The Camp rule treats each product independently and it only considers batch ordering costs and inventory carrying costs. Early extensions for the basic model include the rules of Wagner and Whitin (1958) and Silver and Meal (1973), dealing with dynamic deterministic demand. There now is a

sizeable literature that extended the basic model to consider capacity limitations (Baker et al., 1978; Florian and Klein, 1971), multiple items (Karmarkar and Schrage, 1985; Manne, 1958; Dogramaci et al., 1981), and multi stage (Afentakis et al., 1984; Maclaren, 1976; Zagwill, 1969). Decision rules have been developed to account for the effect of demand uncertainties on economic batch size (Gardner and Dannenbring, 1979), and for the effect of an aggregate restriction on inventory (Hadley and Whitin, 1963). The effect of multi-item multi-level production situation on the economic batch sizes has been studied, especially in the context of the use of Material Requirements Planning as a multi-level activity technique (Carlson et al., 1979; Blackburn and Millen, 1982; Schwartz, 1981). Most of the above models fail to capture the effects of the lotsizing choices on manufacturing lead times.

2.2 Effect of Lotsizing Choices on Manufacturing Lead Times

Most of the early research attempts to make a tradeoff between the production losses from making too many small batches and the opportunity costs of tying up capital inventory as larger batch sizes are produced. These costs are treated as fixed setup costs and variable holding costs respectively. This common representation fails to completely capture the nature of the batching problem. In particular, there is often no real setup cost in the sense of cash flow being affected. Thus, the idea of a fixed setup cost,

independent of the solution, can be misleading because it is often a consequence of the solution. Rummel (1989) showed that cost models based on lead times are much more accurate at modelling actual cash flows than models based on the conventional setup and holding costs paradigm. As a result, it can be seen that setup time, not the setup cost, is a crucial variable. In recent literature, the effects of lotsizing decisions on batch flow times in manufacturing systems have become the focus of a lot of researchers. The literature includes a variety of manufacturing environments, starting with the simplest case of one product and one machine to the most complicated case of multi-product multi-machine with multi-stages and different routing possibilities.

2.3 Single Facility Models

In many production environments, the management has to answer two important questions: how much and when to produce. The objective may vary depending on the situation. The objectives include: minimize the flow time, minimize the total production cost, minimize the total job lateness, etc. The simplest manufacturing environment is the case of a single facility which processes different products. It seems that the single facility problem is easy to tackle, but it is actually very challenging and it gives a lot of insight for the more complicated situations. The lotsizing problem for the single facility case has been studied by a lot of researchers. The batching decisions on the

manufacturing facility characteristics (in terms of setup time, processing time, demand requirements, etc.) and also on the constraints imposed on the problem. Different approaches have been applied to the problem of multi-item single facility models. The decision is to find the quantity of each product to processed on the facility and to accommodate a feasible schedule for all the products. One approach is the use of the cyclic schedule, where the schedule is designed so that the entire system is periodic. The decision calls for the cyclic length and the batch sizes for each product. Within the cyclic schedule, there are two major variants. One approach is to choose a basic period and to stipulate that the cycle time for each item is an integer multiple of that basic period. In this approach, all lots of each item are of the same size. The approach was adopted by Hsu (1983), Schweitzer and Silver (1983), Lee and Denardo (1985) and Axsater (1987). The second approach is the use of a cycle time T , where some items may be produced several times during a cycle and in different production runs of an item, the batches may differ in size. The use of the cycle time approach was introduced by Maxwell (1964) and studied also by Delporte and Thomas (1977), Gunter (1986), Dobson (1987), Gallego (1988) and Roundy (1989). Zipkin (1991) showed how to compute the optimal lot sizes and cycle length, given the sequence of items in a cycle. His problem requires solving a parametric quadratic program. In all the above approaches, the decision is to choose the batch sizes for each item and the solution for the optimal sequencing is largely dependent on the batch size choices.

Researchers have also addressed the question of multi-item inventory problem with single facility with capacity limitations. A capacity constraint determines the maximum inventory levels that are allowed over time. Such a constraint may be dictated by limitations on volume, weight or the number of units allowed in the warehouse at any particular time. There are a number of approaches in the literature that analyze the multi-item inventory with capacity constraint. One approach is the use of independent cycle times for each of the various items. Planning the production schedule is designed to assure that total available capacity is not exceeded at any point in time. The planning problem is formulated as a cost minimization problem with a single capacity constraint, and Lagrangian multipliers are usually used to solve the corresponding optimization problem. The independent cycle approach was adopted by Hadley and Whitin (1963), Johnson and Montgomery (1974), Phillips et al. (1976). Another approach is based on a joint cycle for all the items where the orders of the individual items are phased within the cycle. Under this approach, the same fixed order cycle is used for all items. Phasing is used to avoid situations peak inventories are reached simultaneously. The problem is to decide on the joint cycle and the phasing within the cycle in order to minimize the total cost while satisfying capacity constraints. This approach was used by Krone (1964), Homer (1966), Goyal (1974).

The recent research for the multi-item single facility includes the following:
Rosenblatt and Rothblum (1990) studied the multi-item inventory system under a single

resource capacity constraint by treating the capacity as decision variable instead of a hard constraint. They provided two solution procedures for deriving an optimal policy within the class of policies that considers a fixed cycle for all the items. Each of the solution procedures is designed to compute the optimal values for the capacity level, the joint cycle length of all items and the interval of phasing orders. Hwang et al. (1993) presented a multiproduct capacitated economic production quantity model of a system in which setup reduction and quantity improvement can be achieved with one time investment. In their model, lot sizes are determined in such a way that processing times and setup times must not exceed a given time capacity of the system and that is accomplished by using the common cycle approach. Olhager and Rapp (1991) presented a model where they considered the effects of reduction in setup time and setup cost on the performance of the production system in terms of inventory turnover rate. They also discussed the impact of the setup time on the lot sizes and on the queueing time. Woodruff et al. (1992) formulated a general sequencing problem that includes two classes of jobs with setup times, setup costs, holding costs and deadlines. The formulation shows that producing in batches, there is an opportunity for increasing the production capacity.

2.4 Application of Queueing Theory in Manufacturing

As stated previously, the lot sizing choices for a lot of production systems influence the shop performance. In a lot of situations, the major measure of performance is the job lead time which is mostly influenced by the time batches spend in the queues waiting to be processed. It is conventionally assumed that at the operational level, the performance of manufacturing shops with queues is controlled by sequencing and dispatching at machines but in fact the major determinant of queueing behavior is the lot size policy employed. In lot of cases, waiting time can be determined by using queueing theory results. The initial application of queueing theory in manufacturing was used by Jackson (1963) where he modeled a general job shop as a network of queues to study the behavior of job waiting time under given shop conditions. The work of Jackson was followed by Gordon and Newell (1967), Buzen (1973), Solberg (1977), Kapadia and Hsi (1978), Stecke and Solberg (1981), and Suri (1981). All of the above used the closed queueing network where arrival and departure rates to the system are assumed to be equal. The recently developed decomposition technique for approximate analysis of open queueing networks, served as starting point for modelling manufacturing models as open networks. This approach was first used by Kuehn (1979) and recent work include that of Shanthikumar and Buzacott (1981), Whitt (1982), Zipkin (1983). These models decompose a network of queues into single node models. Each node is described by a

single stream of arrivals and the characteristics of the departing stream is deduced from this model. The arrival stream at each node is the superposition of departure streams from the other nodes in the network that feed it. Local properties of the network, such as queue length at a machine, are determined from the single node models.

Most of the above models do not consider the effect of batching decisions on the queueing behavior of the system or the determination of batch sizes with the exception of work of Zipkin (1986). Models that consider the effect of batching policies on queueing delays have been developed by Karmarkar et al. (1983), Bertrand(1985), Karmarkar (1987a), Karmarkar et al. (1985), Kekre (1987), Williams (1984). For most of the part these models do not analyze the relationship between batch sizes and system parameters. The effect of batching in a deterministic context schedule has been explored in papers by Karmarkar (1987b), Dobson et al. (1987, 1989), Santos and Magazine (1985), Naddef and Santos (1988). In most of the literature, the most common queueing model used to study the behavior of queueing delay for manufacturing systems is the M/G/1 model.

Karmarkar et al. (1983) modeled the multi-item single machine as an M/G/1 queueing model. Their objective is to find the batch sizes for each item that minimize the average waiting time per batch. They presented upper and lower bounds on the average waiting time and based on that they developed three procedures to solve the problem. The first procedure is based on the assumption that all setup times are equal,

the second is developed under the assumption that the utilization level is very high and the third is based on choosing setup times in such a way that for each item the setup time is linear to its contribution to the total utilization. The drawback about the above procedures, is that the demand and the processing rates are not explicitly presented. Yang (1990) presented a searching Algorithm based on a Quasi-Newton optimization technique with sequential search procedure to solve for batch sizes for the model presented by Karmarkar et al. (1983). He also presented a model for lot size reduction where the objective is to reduce the lot sizes with a little sacrifice in shop congestion and average job flow time. Finally, based on the lot size choice model, he developed a total cost minimization model where the major cost components considered are: queueing delay cost, processing time cost and finished inventory carrying cost. Kekre (1987) modified the model presented by Karmarkar et al. (1983) where he studied the effect of look ahead policy for processing. This type of policy saves some setup time when more than one batch of the same product are processed simultaneously. The drawback of this policy is that it cause the processing of a particular product for an extended period of time while other products are waiting in the queue.

2.5 Optimization Techniques

In this research, a Heuristic procedure to find the batch sizes for the M/G/1

model is developed. By studying the form of the waiting time, the objective function can be regarded as a nonlinear optimization with one linear constraint and upper and lower bounds on the variables. This type of function can be solved using MINOS (a Modular In-core Nonlinear Optimization System) which is a computer program designated to solve linear or nonlinear functions subject to linear constraints. Another optimization technique that is used in this research is Simulating Annealing, introduced by Kirkpatrick et al. (1983) and Cerny (1985). Simulating annealing is a powerful general purpose algorithm, based on simulation method of Metropolis et al. (1953), for solving optimization problems. It is useful in finding globally good solutions for a large variety of problems and has several appealing features: it is easy to code, adaptable and finds very high quality solutions.

CHAPTER 3

MULTI-ITEM SINGLE-MACHINE MODEL

3.1 Karmarkar's Model

Karmarkar considered the case of a multi-item single machine problem where the objective is to minimize the mean waiting time of a batch in the queue. The manufacturing facility is modelled as a single server queue with Markovian arrivals and general processing time (Karmarkar et al., 1983).

The notation below is followed throughout this chapter.

i : Index set for items; $i = 1, 2, \dots, I$

D_i : Demand (requirement) for item i (units/ time)

P_i : Processing rate for item i (units/time)

Q_i : Batch size for item i (units)

τ_i : Setup time for item i (independent of the batch size)

X_i : Processing time for a batch of item i

λ_i : Expected arrival rate of batches of item i (batches/time)

λ : Expected total arrival rate of batches of all items

The processing time of a batch Q_i of item i is given by

$X_i = \tau_i + (Q_i / P_i)$ and the number of such batches is $\lambda_i = (D_i / Q_i)$.

The mean service time is given by:

$$\bar{X} = E(X) = \frac{\sum_i (D_i / Q_i) (\tau_i + (Q_i / P_i))}{\sum_i (D_i / Q_i)} \quad (1)$$

The mean waiting time in the queue of a batch is given by the Pollaczek-Khinchin formula as

$$W = \frac{\lambda E(X^2)}{2(1-\rho)} \quad (2)$$

In the above equation $\rho = \lambda E(X)$, representing the traffic intensity and $\lambda = \sum_i \lambda_i$. The average time spent in the system by a batch of item i is $(X_i + W)$. By substituting for λ , ρ , and $E(X^2)$, the mean waiting time in the queue is expressed as:

$$W = \frac{\sum_i (D_i / Q_i) (\tau_i + Q_i / P_i)^2}{2(1 - \sum_i [(D_i / P_i) + (D_i \tau_i / Q_i)])} \quad (3)$$

The problem is to find the batch sizes for each item in order to minimize the average waiting time in the queue. Therefore we can state the problem as follows:

Min W

Subject to

$\rho < 1$ (Stability condition)

Where ρ is given by the following:

$$\rho = \sum_i [(D_i/P_i) + (D_i\tau/Q_i)] \quad (4)$$

3.2 Complexity of the Problem

The problem is "hard" to solve because it is impossible to separate an individual Q_i within the objective function. All Q_i 's appear both in the numerator and the denominator. Therefore, there is no closed form solution to the problem. So, in order to solve this problem, it is necessary to rely on some heuristic methods.

3.3 A Heuristic Solution Procedure

It is suggested that the ratio of batch run time to setup time should be constant across all items. The analysis that will follow is therefore based on the following

assumptions:

1. The ratio of batch run time to setup time is constant across items

$$\frac{\tau_i + Q_i/P_i}{\tau_i} = C \quad (\text{Constant}) \quad (5)$$

2. The batch sizes (Q_i 's) are continuous variables.

Based on the first assumption, we have the following relationships:

The batch sizes have the following expression

$$\forall i, \quad Q_i = (C-1)\tau_i P_i \quad (6)$$

Substituting for all Q_i 's in equation 3, the mean waiting time in the queue is now

$$W = \frac{C^2 \sum_i \frac{D_i \tau_i}{P_i}}{2[C(1 - \sum_i \frac{D_i}{P_i}) - 1]} \quad (7)$$

The stability condition, equation 4, requires that ρ should be less than 1, therefore we have the following condition on the constant C to satisfy the stability condition

$$C > \frac{1}{1 - \sum_i \frac{D_i}{P_i}} \quad (8)$$

It is also important that none of the batch sizes exceeds its demand ($\forall i, Q_i \leq D_i$).

Therefore, we have a second condition on the constant C which is the following:

$$C \leq \min_i \left(\frac{D_i}{\tau_i P_i} \right) + 1 \quad (9)$$

The problem can now be stated as follows:

Min W

Subject to

$$\frac{1}{1 - \sum_i (D_i/P_i)} \leq C \leq \min_i \left(\frac{D_i}{\tau_i P_i} \right) + 1 \quad (10)$$

The above problem is nonlinear with one variable and two constraints. Before going further with the analysis, let α and β represent the followings:

$$\alpha = \sum_i \frac{D_i \tau_i}{P_i} \quad \wedge \quad \beta = \sum_i \frac{D_i}{P_i} \quad (11)$$

Then the average waiting time in the queue, equation 7, can be simplified to the following form

$$W = \frac{C^2 \alpha}{2[C(1-\beta)-1]} \quad (12)$$

Therefore the derivative of W with respect to C is the following

$$\frac{dW}{dC} = \frac{C\alpha[C(1-\beta)-2]}{2[C(1-\beta)-1]^2} \quad (13)$$

The derivative is zero if $C = 0$ or $C = 2/(1-\beta)$. Since the domain requires that C is strictly positive, therefore $C = 2/(1-\beta)$. It is easy to check that the second derivative of W with respect to C at $C = 2/(1-\beta)$ is α which is strictly greater than zero and therefore the mean waiting time is minimized when $C = 2/(1-\beta)$.

Note:

$$\text{If } \frac{2}{1-\beta} > \min_i \left(\frac{D_i}{\tau_i P_i} \right) + 1, \text{ Then } C = \min_i \left(\frac{D_i}{\tau_i P_i} \right) + 1 \quad (14)$$

The above remark is due to the constraint on the upper limit on C (the second condition on the constant C , equation 9).

3.4 Analysis

When $C = 2/(1-\beta)$, we have the following results:

1. Batch sizes: Substitute $C = 2/(1-\beta)$ into equation 6 and we get the following

$$\forall i, Q_i = \frac{1+\beta}{1-\beta} \tau_i P_i \quad (15)$$

2. The average waiting time per batch: Substitute $C = 2/(1-\beta)$ into equation 12 and we get the following

$$W = \frac{2\alpha}{(1-\beta)^2} \quad (16)$$

However, when C takes the value given by equation 14, let C^* be the following:

$$C^* = \min_i \left(\frac{D_i}{\tau_i P_i} \right) + 1 \quad (17)$$

and the batch sizes and the average waiting time are as follows:

1. Batch sizes: Substitute the value for C^* into equation 6 and we get the following

$$\forall i, Q_i = (C^* - 1) \tau_i P_i \quad (18)$$

2. Average waiting time per batch: Substitute the value for C^* into equation 12 and we get the following

$$W = \frac{C^{*2} \alpha}{2[C^*(1-\beta) - 1]} \quad (19)$$

3.4.1 Effect of increasing the utilization level

The utilization level can be increased by increasing the demand requirements for one or more units. The increase of the utilization level effects the batch sizes as well as the average waiting time and this can be illustrated as follows:

If the demand rate for item j increases from D_j to D_j' , then the batch sizes for each item in the product mix increase and the mean waiting time in the queue increases also.

Proof:

Batch sizes:

Initially the utilization level is β . Now D_j is replaced by D_j' , where $D_j' > D_j$, the new utilization level is β'

$$\beta' = \sum_{i \neq j} \frac{D_i}{P_i} + \frac{D_j}{P_j} = \sum_i \frac{D_i}{P_i} - \frac{D_j}{P_j} + \frac{D_j'}{P_j} \quad (20)$$

$$\beta' = \beta + \frac{D_j' - D_j}{P_j} \quad (21)$$

since $D_j' > D_j$, then $\beta' > \beta$ and hence

$$\forall i, \frac{1+\beta'}{1-\beta'} \tau_i P_i > \frac{1+\beta}{1-\beta} \tau_i P_i \quad (22)$$

■ Therefore the batch size for each item increases

If the constant C takes the value of C^* given by equation 17, then the batch sizes do not change.

Average waiting time

The original average waiting time per batch is given by equation 16 when $C = 2/(1-\beta)$ and by equation 19 when $C=C^*$. With the change in the demand rate for item j , the new value of α denoted by α' is as follow

$$\alpha' = \sum_{i \neq j} \frac{\tau_i D_i}{P_i} + \frac{\tau_j D'_j}{P_j} = \sum_i \frac{\tau_i D_i}{P_i} + \frac{\tau_j D'_j}{P_j} - \frac{\tau_j D_j}{P_j} \quad (23)$$

$$\alpha' = \alpha + \tau_j \frac{D'_j - D_j}{P_j} \quad (24)$$

clearly $\alpha' > \alpha$ and therefore

$$\frac{2\alpha'}{(1-\beta')^2} > \frac{2\alpha'}{(1-\beta)^2} > \frac{2\alpha}{(1-\beta)^2} \quad (25)$$

If the average waiting time is represented by equation 19, it is clear to notice that the average waiting time increases since the numerator decreases.

- Therefore the average waiting time increases when the demand requirement for some item increases

3.4.2 Effect of adding a new product to the product mix

Originally, we have items $i=1,2,\dots,I$. Suppose we add a new product, item $I+1$, with demand requirement D_{I+1} , setup time τ_{I+1} and processing rate P_{I+1} . The effect of

adding this product to the product mix is the same as increasing the demand requirement for some item j . The batch sizes for items $i = 1, 2, \dots, I$ increase and the average waiting time increases. The proof of this section is the same as the previous section with the exception that α' is replaced by α_{I+1} and β' is replaced β_{I+1} , where α_{I+1} and β_{I+1} are given by:

$$\alpha_{I+1} = \sum_{i=1}^{I+1} \frac{\tau_i D_i}{P_i} \quad \wedge \quad \beta_{I+1} = \sum_{i=1}^{I+1} \frac{D_i}{P_i} \quad (26)$$

it is clear that $\alpha_{I+1} > \alpha$ and $\beta_{I+1} > \beta$, therefore the results are consistent.

3.5 Yang's Search Algorithm

Yang, (1990) presented a procedure to find the optimal batch sizes and the average waiting time for the multi-item single machine. The procedure is based on the fact that the partial derivatives are zero at the optimal point. Setting the first-order partial derivatives for equation 3 equal to zero, we get the following:

$$\forall i, i=1, 2, \dots, I \quad \frac{(Q_i/P_i)^2 - \tau_i^2}{2\tau_i} = W \quad (27)$$

From equation 27, there must be a set of batch sizes represented by:

$$\forall i, i=1, \dots, I \quad Q_i^* = (2\tau_i W + \tau_i^2)^{1/2} P_i \quad (28)$$

To find an initial solution, the optimality condition of equation 28 can be approximated as:

$$\forall i, i=1,2,\dots,I \quad Q_i^* = (2\tau_i W^*)^{1/2} P_i \quad (29)$$

Then, from equation 29, for item i and item j , the "optimal" lot size ratio can be expressed as:

$$\frac{Q_i^*}{Q_j^*} = \frac{(2\tau_i W^*)^{1/2} P_i}{(2\tau_j W^*)^{1/2} P_j} = \frac{\tau_i^{1/2} P_i}{\tau_j^{1/2} P_j} \quad (30)$$

Now, let:

$$\begin{aligned} b_i &= \tau_i^{1/2} P_i \\ \sigma &= \max_i \{b_i, i=1,2,\dots,I\} \\ a_i &= b_i / \sigma \end{aligned}$$

Q_b = the lot size of the item with the largest b_i . Then all lot sizes at optimal point can be expressed in terms of Q_b only as follows:

$$Q_i^* = a_i Q_b \quad (i=1,2,\dots,I) \quad (31)$$

Substituting equation 31 into W , the objective function becomes a simple function with one unknown Q_b :

$$W(Q_b) = \frac{A Q_b^2 + B Q_b + C}{E Q_b - M} \quad (32)$$

where

$$A = \sum D_i a_i / P_i^2$$

$$B = \sum D_i \tau_i / P_i$$

$$C = \sum D_i \tau_i^2 / a_i$$

$$E = 2 (1 - \sum D_i / P_i)$$

$$M = 2 \sum D_i \tau_i / a_i$$

Equation 32 can be easily solved with the resulting value of Q_b^* :

$$Q_b^* = M/E + [(B.M + C.E)/(E.A) + (M/E)^2]^{1/2} \quad (33)$$

An initial solution for the problem is attained as:

$$Q_i^* = (2\tau_i W(Q_b^*) + \tau_i^2)^{1/2} P_i \quad \forall i, i=1,2,...,I \quad (34)$$

The Searching Algorithm developed by Yang is based on the Feasible Direction Method (as given by Bazaraa and Shetty ,1979), which is a Quasi-Newton optimization technique with a sequential search procedure used to solve nonlinear multi-variable optimization problems.

Searching Procedure:

Step 0: Obtain an initial solution for the batch sizes from given parameters by equations 32, 33, and 34. Find the value of W

Step 1: For $i=1,2,...,I$

$$FOD_i = [(Q_i / P_i)^2 - \tau_i^2] / (2 \tau_i)$$

$$e_i = (FOD_i - W) / W$$

if $e_i \leq 0.001$ go to next i

else if $FOD_i > W$, then $Q_i = Q_i - 1$

else $Q_i = Q_i + 1$

end if (end if)

updating W with updated Q_i 's ($i=1,2,\dots,I$)

next i

Step 2: Repeat step 1 until there is no improvement

A Fortran program which implements the above Searching Algorithm is presented in APPENDIX I.

3.6 Illustrative Examples

To illustrate the results for the Heuristic procedure, some numerical examples are included. The first example, part a, shows the variation of average waiting time with respect to the ratio of run time to setup time. In part b, we study the effect of increasing the demand rate for one of the items in the product mix on the batch sizes. In the third example, we show the effect of adding a new item to the product mix on the batch sizes.

In both examples, the value for the average waiting time is also included using the Yang's Search algorithm.

Example 1:

Part a

Variation of average waiting time with respect to ratio of run time to setup time

We consider the case when we have 6 items in the product mix. The demand requirements, the processing rates and the setup times for all the items are presented in Table 1. From the data provided, we can calculate the values for β (the utilization level) and α given by equation 11.

$$\beta = (100/800) + (120/900) + (100/700) + (150/800) + (150/1000) + (50/500) = 0.8387$$

$$\begin{aligned}\alpha &= 0.002(100/800) + 0.001(120/900) + 0.002(100/700) + 0.004(150/800) \\ &\quad + 0.0025(150/1000) + 0.001(50/500) = 0.001894\end{aligned}$$

The average waiting time obtained using the Heuristic is given by equation 16.

$$\text{The average waiting time per batch is } W = 2 * 0.001894 / (1 - 0.8387)^2 = 0.1456$$

The value of the average waiting time using Yang's Searching procedure is 0.1388

The variation of average waiting time with respect to ratio of run time to setup time is summarized in Table 2 and Figure 1. Equation 12 is used to evaluate the average waiting time.

Table 1

Demand requirements, processing rates and setup times for six items

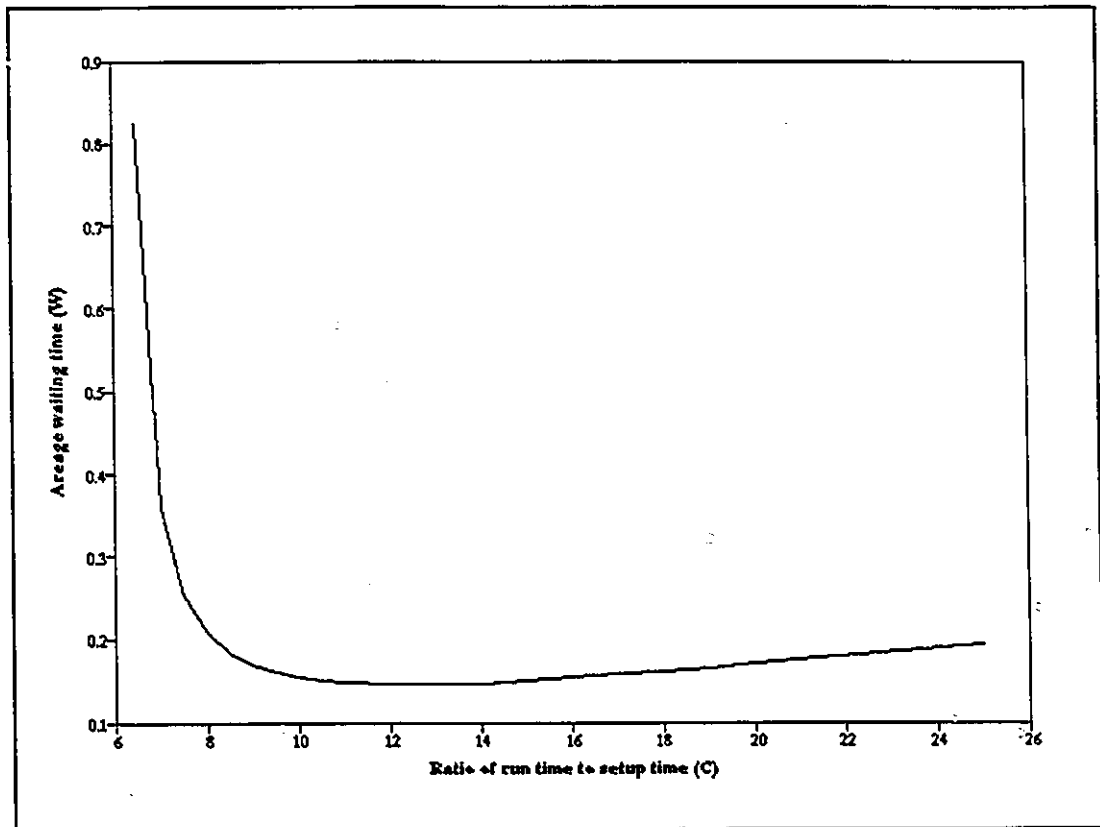
Item #	Demand (units/year)	Processing rate (units/year)	Setup time (year)
1	100	800	0.002
2	120	900	0.001
3	100	700	0.002
4	150	800	0.004
5	150	1000	0.0025
6	50	500	0.001

Table 2

Variation of average waiting time with respect to the ratio of run time to setup time

Ratio (C)	Waiting time (W)
6.5	0.8247
7	0.3592
7.5	0.2538
8	0.2086
8.5	0.1843
9	0.1697
9.5	0.1605
10	0.1544
10.5	0.1504
11	0.1479
12	0.1457
12.5	0.1455
13	0.1458
14	0.1475
15	0.1501
16	0.1533

Figure 1: Average waiting time vs. ratio of run time to setup time



Part b

Effect of increasing the utilization level on the average waiting time and batch sizes

If the demand requirement for item 6 is 100 units instead of 50 units, the new values for β and α denoted by β' and α' can be calculated using equations 21 and 24 respectively

$$\beta' = 0.8387 + (100-50)/500 = 0.9387$$

$$\alpha' = 0.001894 + 0.001(100-50)/500 = 0.001994$$

therefore, the new average waiting time is $2 * 0.001994 / (1-0.9387)^2 = 1.0613$

The average waiting time using Yang's Search Algorithm is 0.9983

The summary for the batch sizes obtained using the Heuristic are presented in Table 3, Table 4 and Figure 2.

Table 3

Optimal batch sizes when demand for item 6 is 50 units

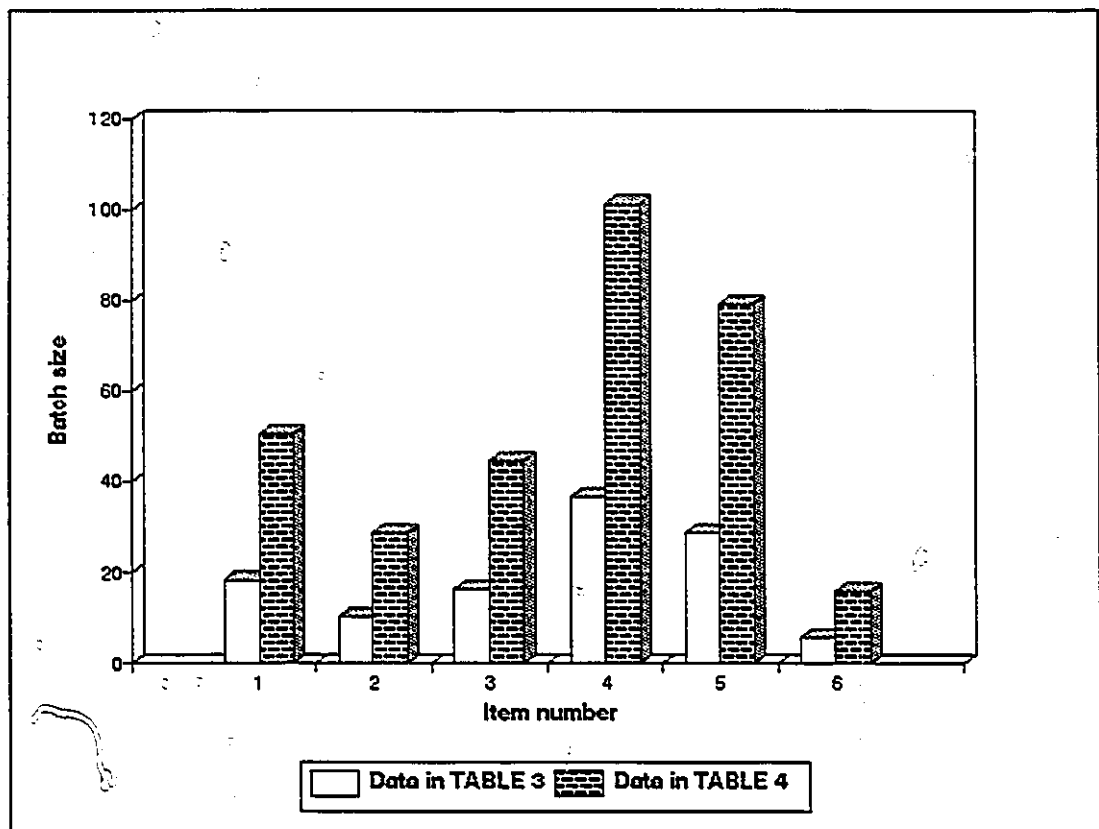
Item #	1	2	3	4	5	6
Batch size	18.23	10.26	15.96	36.47	28.49	5.69

Table 4

Optimal batch sizes when demand for item 6 is increased to 100 units

item #	1	2	3	4	5	6
Batch size	50.59	28.46	44.27	101.18	79.05	15.81

Figure 2: Effect of changing the demand for item 6 from 50 to 100 on the batch sizes



Example 2:

Effect of adding a new item to the product mix

Now, we add a new product to the data of example 1 (item 7) with demand rate, processing time and setup time parameters included in Table 5.

The new values for β and α denoted by β_7 and α_7 can be calculated using equation 26.

$$\beta_7 = 0.8387 + (60/1000) = 0.8987$$

$$\alpha_7 = 0.001894 + 0.001(60/1000) = 0.001954$$

therefore, the new average waiting time is $2 * 0.001954 / (1 - 0.8987)^2 = 0.3808$

The average waiting time using Yang's Search Algorithm is 0.3600

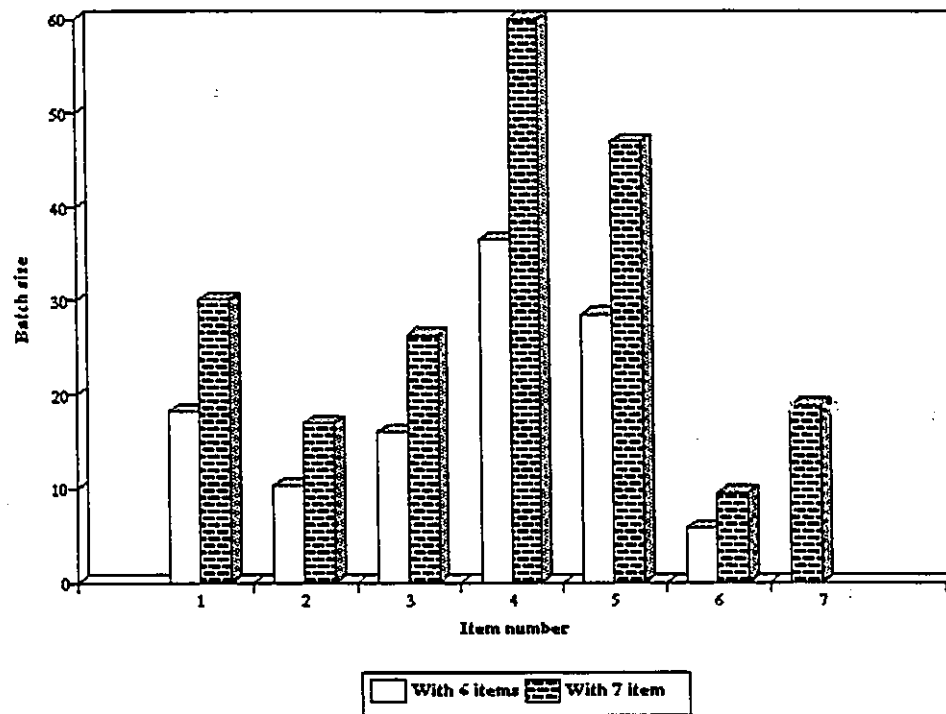
The summary for the batch sizes obtained using the Heuristic are presented in Table 5 and the comparison between having 6 and 7 items in the product mix is presented in Figure 3.

Table 5

Effect of adding a new product to the product mix

Item#	Demand (units/year)	Processing Rate (units/year)	Setup Time (year)	Batch Size (units)
1	100	800	0.002	29.98
2	120	900	0.001	16.86
3	100	700	0.002	26.23
4	150	800	0.004	59.97
5	150	1000	0.0025	46.85
6	50	500	0.001	9.37
7	60	1000	0.001	18.74

Figure 3: Effect of adding a new item to the product mix



CHAPTER 4

COMPARING THE HEURISTIC WITH TWO OPTIMIZERS

4.1 MINOS Optimizer

4.1.1 Overview

MINOS stands for " a Modular In-core Nonlinear Optimization System ". It is a computer software designed to optimize a certain linear or non linear objective function $f(x)$ by finding a point x which makes $f(x)$ as close to $\pm \infty$ as possible.

Since it is not normally meaningful to go quite that far, MINOS allows the user to restrict the variables x to some feasible region specified by a set of linear constraints $(Ax \leq \text{or} \geq b)$ and a set of upper and lower bounds $(l \leq x \leq u)$.

In algebraic form MINOS is designed to solve problems of the form

$$\text{optimize} \quad f(x) + c^T x$$

$$\text{Subject to} \quad Ax \leq \text{or} \geq b, \quad l \leq x \leq u$$

The MINOS optimizer requires two subroutines: CALCFG and CALCON. Subroutine CALCFG is required for the computation of the objective function and its Gradients. Subroutine CALCON is only required in the presence of nonlinear

constraints. If all constraints are linear, then CALCON is optional.

4.1.2 Minimizing the average waiting time using MINOS

Before using MINOS to find the "optimal" average waiting time per batch for the original problem, a change of variables is introduced. Let $x_i = D_i/Q_i$, the problem can now be stated as:

Minimize

$$W = \frac{\sum_i x_i (\tau_i + \frac{D_i}{P_i x_i})^2}{2(1 - \sum_i \frac{D_i}{P_i} + \tau_i x_i)} \quad (35)$$

Subject to

$$\sum_i \frac{D_i}{P_i} + \tau_i x_i < 1 \quad (36)$$

$$1 \leq x_i \leq D_i \quad \forall i \quad i=1,2,\dots,I$$

The first constraint is the stability constraint, and the rest of the constraints correspond to the fact that each batch size is least one and at most D_i . In other words for each item, we have the following:

$$1 \leq Q_i \leq D_i \text{ which implies that } 1 \leq D_i/Q_i \leq D_i.$$

As it can be seen, the above problem is nonlinear with a single linear constraint (stability constraint) plus upper and lower bounds on the variable.

The files supplied to the optimizer MINOS are the followings:

- * SPECS file: The major components of this file are: problem type (Minimization), the number of nonlinear variables and bounds on the variables.
- * MPS file: This file includes the following: the data specifying the linear constraint and the upper bounds on each variable .
- * Subroutine CALCFG: It computes the objective function and its gradients.
- * Basis file: It contains an initial feasible solution.
- * A data file which contains the input data corresponding to demand, processing rate and setup time for each item.

A sample example for the above files is included in **APPENDIX II.**

4.2 Simulating Annealing Optimizer

4.2.1 Overview

The development of the standard simulated annealing method was motivated by the behavior of mechanical systems with very large number of degrees of freedom. According to the general principles of physics, any such system will, given the necessary freedom, tend toward the state of minimum energy. Therefore a mathematical model of the behavior of such a system will contain a method for minimizing a certain function, namely, the total energy of the system.

For example, atoms of a molten metal when cooled to a freezing temperature will tend to assume relative positions in a lattice in such a way as to minimize the potential energy of their mutual forces. Because of the large number of atoms and the possible arrangements, the final state will most likely correspond to a local energy minimum and not a global one. The solidified metal may be reheated and cooled slowly with the hope that it will then migrate to a lower energy state. In metallurgy, that process is called annealing; therefore, the method that mathematically models it is called simulating annealing.

Simulating annealing is a convenient way to find a global extremum of a function that has many local extrema and may not be smooth (the method does not require calculation of derivatives). The method is based on random walk that samples the objective function in the space of the independent variables. The execution of the

method is summarized in the following steps:

1. Let $\Phi(x)$ be the function to be minimized with x restricted to Ω , a subset of R^n . Let Φ_m be the value of Φ at the global optimum. Let x_0 be an arbitrary starting point (either specified or randomly chosen from Ω).
2. Set $\Phi_0 = \Phi(x_0)$. If $|\Phi_0 - \Phi_m| < \epsilon$, stop.
3. Random direction. Generate n independent standard normal variates Y_1, Y_2, \dots, Y_n and compute components of U :

$$U_i = Y_i / (Y_1^2 + Y_2^2 + \dots + Y_n^2)^{1/2}, i=1, 2, \dots, n.$$
4. Set $x^* = x_0 + (\Delta r)U$. Where (Δr) is the step size.
5. If $x^* \notin \Omega$, return to step 3. Otherwise, set $\Phi_1 = \Phi(x^*)$ and $\Delta\Phi = \Phi_1 - \Phi_0$.
6. If $\Phi_1 \leq \Phi_0$, set $x_0 = x^*$ and $\Phi_0 = \Phi_1$. If $|\Phi_0 - \Phi_m| < \epsilon$, stop. Otherwise, go to step 3.
7. If $\Phi_1 > \Phi_0$, set $p = \exp(-\beta \Delta\Phi)$.
 - (a) Generate a uniform 0-1 variate V .
 - (b) If $V \geq p$, go to step 3.
 - (c) If $V < p$, set $x_0 = x^*$, $\Phi_0 = \Phi_1$, and go to step 3.

4.2.2 Minimizing the average waiting time using Simulating Annealing

A Fortran computer program that uses the steps presented in the previous section have been developed with some slight modifications. The program uses two stopping criteria: 1) reach the optimal objective value for the objective function or 2) stop after a certain number of iterations. Since the optimal objective value is not known in advance, an approximated value (an objective value much smaller than the one given by the Heuristic) is inputted to the program. The program also keeps record for the minimum objective value obtained up to the last iteration.

A flow chart for the program is presented below. In order to make it easy for the reader to understand the program, most of the notation used in the chart is the same as the one used in the program. The program uses the following variable names:

Fnot: objective value at the current iteration

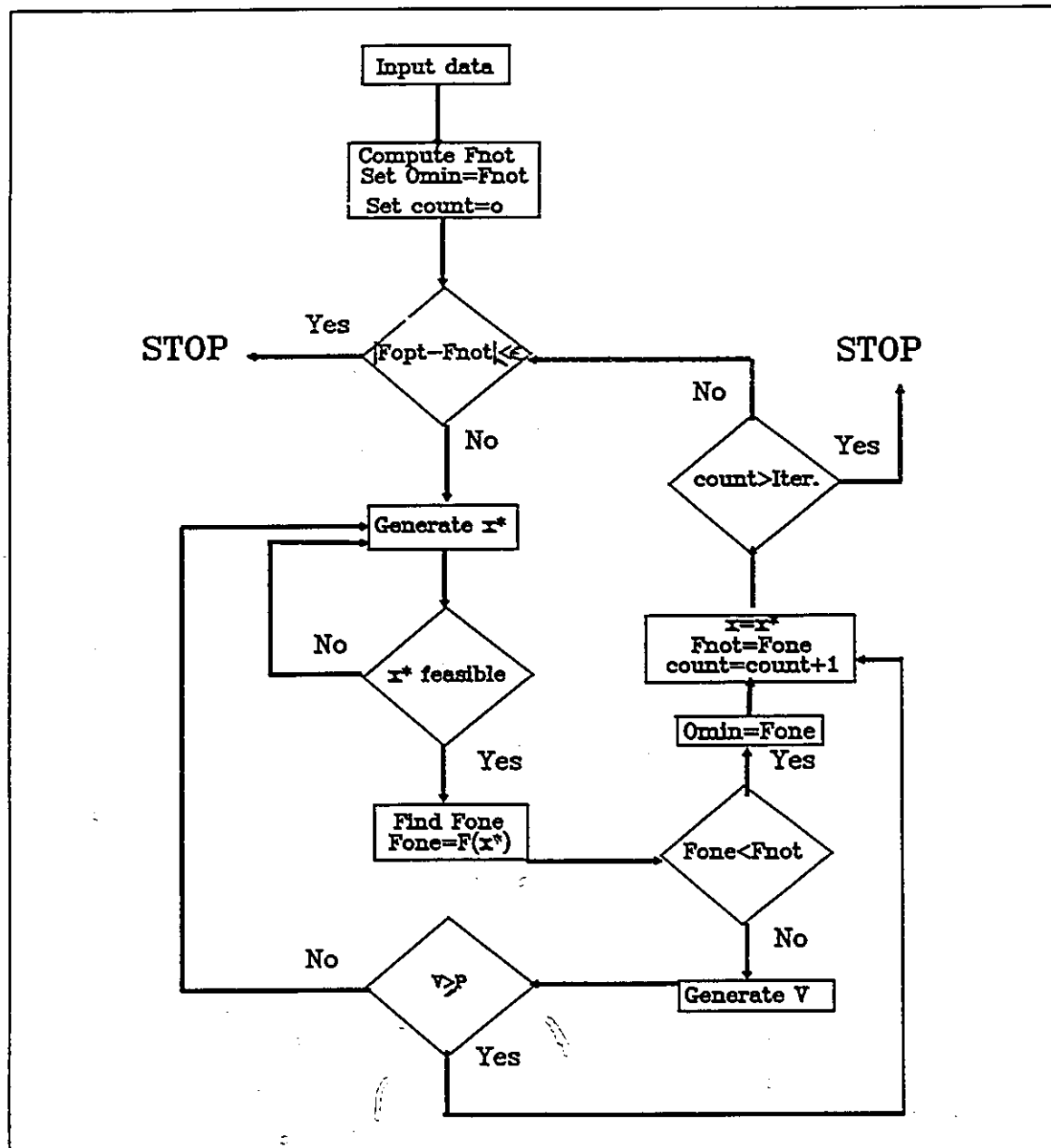
Fopt: optimal objective value

Omin: the minimum objective value obtained up to the current iteration

PRB: (denoted by P in the chart) represent the probability of jumping out of the current local minimum.

The remaining of the variable names can be read from the program. The complete program is include in **APPENDIX III**.

Figure 4: Flow chart for the Simulating Annealing Algorithm



4.3 Comparison between the Heuristic, MINOS and Simulating Annealing

In order to compare the Heuristic with MINOS and Simulating Annealing, three arbitrary problems are chosen. In each problem four different tests are included. The problems are chosen as follows:

- 1) The number of items in the product mix is 20
- 2) Since the batch sizes in the Heuristic are linear to the product of processing rate and setup time, the problems are chosen in increasing mean of the product of processing time and setup time.
- 3) In each test, the standard deviation of the product of processing rate and setup time is modified by changing the processing rates for each item.

The data used for each test are provided in **APPENDIX IV**.

A complete summary for all the tests is presented in **Table 6**.

From all the tests, it can be noticed that the objective values given by all three methods are very close and relative errors are small. Therefore, the Heuristic proposed here can be regarded as a good approximation in finding the batch sizes that minimize the average waiting time. However, if the standard deviation is larger, it is expected that the Heuristic will be less accurate than the two other optimizers.

Table 6

Results summary for the average batch waiting time from all the tests

Problem#	Mean ($(\sum \tau_i, P_i)/n$)	Standard Deviation	Heuristic	MINOS	Simulating Annealing	Test#
I	9.8640	1.719934	0.044088	0.044132	0.044102	1
		2.24100	0.049580	0.049570	0.049567	2
		3.00533	0.070550	0.071640	0.070571	3
		2.90554	0.099191	0.100201	0.097521	4
II	12.0080	2.33367	0.032373	0.032432	0.032396	1
		2.83979	0.035166	0.035151	0.035147	2
		3.68277	0.046214	0.046209	0.046202	3
		3.85627	0.065467	0.065414	0.065481	4
III	14.1455	2.99265	0.027381	0.027370	0.027358	1
		3.58364	0.029217	0.029230	0.029181	2
		4.55312	0.036434	0.036429	0.036414	3
		5.71150	0.059791	0.059712	0.059743	4

CHAPTER 5

A COST MINIMIZATION MODEL FOR MULTI-ITEM MULTI-MACHINE

In this chapter, the focus is to develop a model that minimizes the sum of the following costs: Investment cost, maintenance cost and the delay cost. A procedure to find the "optimal" solution is included.

5.1 Model Environment and Assumptions

The problem of multi-item multi-machine has not received a great attention in the literature due to the difficulty involved in solving it. In order to tackle this problem, a procedure that modifies the problem from a multi-item multi-machine to a multi-item single machine is proposed through the remainder of this chapter. The problem is tackled from the view point of Management. In any job shop, the average waiting time can be minimized by having parallel machines and therefore, the Management has to decide on the number of machines that to be purchased in order to minimize the cost associated with the job shop. The relevant costs that are considered here are: Machinery cost, maintenance cost and the delay cost. The more parallel machines we have in the shop, the less time products wait to be processed and hence the average waiting time decreases. However, when we have more machines, it results in additional machinery and maintenance costs.

Therefore, we have to find the number of identical machines to be purchased such that the relevant cost is minimized.

Assumptions:

1. We assume that all the machines are identical.
2. If more than one machine share the processing of the products, then each machine is allocated enough work so that all the machines have the same processing work load. Some items may be processed on two machines in order to have equal processing work load. If an item is processed on two machines, then it has a batch size associated with each machine. That is, the demand for a particular item may be processed on different machines, but each batch size is only processed on the machine that was allocated to it and there is no splitting of batches.
3. The job shop costs are the following:

Investment (or machinery) cost

The investment cost represents the initial cost for purchasing the machines. Suppose that cost of one machine is C_1 and the total budget available is B , then the maximum number of identical machines (K) that can purchased is restricted by the constraint:

$$C_1 K \leq B \quad (37)$$

Assume that the planning horizon is T periods (years) and the value of any machine at the end of the planning horizon is 0, then the investment cost per year is :

$$IC(K) = C_i K (A/P, i, T) \quad (38)$$

where i represents the interest rate and $(A/P, i, T)$ is given by the following:

$$(A/P, i, T) = \frac{i(1+i)^T}{(1+i)^T - 1} \quad (39)$$

Maintenance cost

Assume that each machine requires some maintenance in order to keep functioning. Suppose that the maintenance cost can be represented by a linear function. Therefore the maintenance cost for each machine can be represented by:

$M(t) = A + B t$ and if there are K identical machines the cost is $K M(t)$. Therefore the total maintenance cost per year is :

$$MC(K) = (A/P, i, T) \sum_{t=1}^{t=T} K(A+Bt)(P/F, i, t) \quad (40)$$

where $(P/F, i, t)$ is given by the following:

$$\frac{1}{(1+i)^t} \quad (41)$$

Delay cost

The delay cost is associated with the time jobs wait in the queue before being served. Let C_w represents the cost per unit time for each item waiting in the queue. The total delay cost is denoted by DC and the expression for this cost is presented in the next section.

5.2 Model Formulation

This is a multi-item multi-machine case. In order to convert it into a multi-item single machine case, the following assignment is applied:

The total processing work load of all machines is given by $\sum_i (D_i/P_i)$. It represents the total time it takes to process all the jobs (not including the setup times). If the total processing work load is greater than one, it is not feasible to have a single machine to process all the items because the stability condition is not satisfied. If there is more than one machine available to process all the products, all the machines will be allocated the work in such a way that the processing work loads are equal across all the machines. For example, if the total processing work load is 1.4 and we have two available machines, then we allocate the products to the two machines in such a way that each machine has a processing work load of 0.7.

5.2.1 Assigning products to machines

Let the total processing work load of all machines be N , all the products have to be allocated to the available machines in such a way that the processing work load is distributed equally across all machines. The procedure to allocate all the products to the available machines is as follow:

1. The total processing work load of all machines is $\sum_i (D_i/P_i) = N$
2. When K machines share the processing of all the products, the processing work load

for each machine is N/K .

3. Assign jobs to each machine according to the following:

Machine 1:

Find the index n_1 such that the following relation holds

$$\sum_{i=1}^{i=n_1} \frac{D_i}{P_i} \leq \frac{N}{K} < \sum_{i=1}^{i=n_1+1} \frac{D_i}{P_i} \quad (42)$$

Then assign products $1, 2, \dots, n_1$ to be processed on machine 1 and a fraction $F(n_1+1)$ of the demand of item n_1+1 to be processed on machine 1 such

$$\sum_{i=1}^{i=n_1} \frac{D_i}{P_i} + F(n_1+1) \frac{D_{n_1+1}}{P_{n_1+1}} = \frac{N}{K} \quad (43)$$

Machine 2:

First, the remaining fraction $(1-F(n_1+1))$ of the demand for item n_1+1 is assigned to be processed on machine 2; then Find the index n_2 such that

$$(1-F(n_1+1)) \frac{D_{n_1+1}}{P_{n_1+1}} + \sum_{i=n_1+2}^{i=n_2} \frac{D_i}{P_i} \leq \frac{N}{K} < (1-F(n_1+1)) \frac{D_{n_1+1}}{P_{n_1+1}} + \sum_{i=n_1+2}^{i=n_2+1} \frac{D_i}{P_i} \quad (44)$$

then assign products n_1+1, n_1+2, \dots, n_2 and a fraction $F(n_2+1)$ of the demand for item n_2+1 to be processed on machine 2 such that

$$(1-F(n_1+1)) \frac{D_{n_1+1}}{P_{n_1+1}} + \sum_{i=n_1+2}^{i=n_2} \frac{D_i}{P_i} + F(n_2+1) \frac{D_{n_2+1}}{P_{n_2+1}} = \frac{N}{K} \quad (45)$$

The same procedure is followed to allocate the rest of the items to the rest of the

machines. At the end the allocation process, all the products will be allocated to the K machines and the processing work load is equal across all the machines.

5.2.2 Queue delays

Using the procedure of the previous section, each machine can have its own queue and the batching lot sizes are determined separately for each machine. Now each machine can be modeled as M/G/1 model.

Using the results from the previous chapter and from the procedure described earlier, we get the following results:

Machine 1:

Machine 1 has been allocated items $1, 2, \dots, n_1$ and a fraction $F(n_1+1)$ of item n_1+1 .

Find the for the above items so that average waiting time per batch is minimized.

Let β_1 represents the processing work load for machine 1, then β_1 is given by:

$$\beta_1 = \frac{N}{K} = \sum_{i=1}^{i=n_1} \frac{D_i}{P_i} + F(n_1+1) \frac{D_{n_1+1}}{P_{n_1+1}} \quad (46)$$

and let α_1 be the following

$$\alpha_1 = \sum_{i=1}^{n_1} \frac{\tau_i D_i}{P_i} + F(n_1+1) \frac{\tau_{n_1+1} D_{n_1+1}}{P_{n_1+1}} \quad (47)$$

Using the results of the previous chapter, the average waiting time per batch in the queue of machine 1 is given by

$$W_1 = \frac{C_1^{*2} \alpha_1}{2[C_1^*(1-\beta_1)-1]} \quad (48)$$

where C_1^* is given by the following:

$$C_1^* = \min\left(\frac{2}{1-\beta_1}, \min_{i=1, \dots, n_1} \left(\frac{D_i}{\tau_i P_i}\right) + 1, \left(\frac{F(n_1+1)D_{n_1+1}}{\tau_{n_1+1} P_{n_1+1}}\right) + 1\right) \quad (49)$$

The batch sizes for items $i=1, 2, \dots, n_1$ and a fraction $F(n_1+1)$ of item n_1+1 are given by the following:

$$\forall i=1, 2, \dots, n_1 \quad Q_i = (C_1^* - 1) \tau_i P_i \quad \wedge \quad Q_{n_1+1} = (C_1^* - 1) \tau_{n_1+1} P_{n_1+1} \quad (50)$$

The average delay for each unit of all the items processed on machine 1 is equal to: (average waiting time for each batch) times (the total number of batches) divided by (the total number of all the units of all the items). Therefore the total average delay for all the units of all the items is equal to: (the average delay for each unit of all the items) times (the total number of all the units of all the items). Hence the delay cost due to machine 1 is

$$DC_1 = C_w W_1 \left(\sum_{i=1}^{n_1} \frac{D_i}{Q_i} + F(n_1+1) \frac{D_{n_1+1}}{Q_{n_1+1}} \right) \quad (51)$$

Machine 2:

Machine 2 has been allocated a fraction $(1-F(n_1+1))$ of item n_1+1 , items n_1+2 ,

n_1+3, \dots, n_2 and a fraction $F(n_2+1)$ of item n_2+1 . The batch size for each item is to be found so that the average waiting time per batch for machine 2 is minimized. Let β_2 represents the processing work load for machine 2, then

$$\beta_2 = \beta_1 = \frac{N}{K} = (1-F(n_1+1)) \frac{D_{n_1+1}}{P_{n_1+1}} + \sum_{i=n_1+2}^{n_2} \frac{D_i}{P_i} + F(n_2+1) \frac{D_{n_2+1}}{P_{n_2+1}} \quad (52)$$

and let α_2 be the following

$$\alpha_2 = (1-F(n_1+1)) \frac{\tau_{n_1+1} D_{n_1+1}}{P_{n_1+1}} + \sum_{i=n_1+2}^{n_2} \frac{\tau_i D_i}{P_i} + F(n_2+1) \frac{\tau_{n_2+1} D_{n_2+1}}{P_{n_2+1}} \quad (53)$$

The average waiting time for a batch in the queue for machine 2 is given by

$$W_2 = \frac{C_2^{*2} \alpha_2}{2[C_2^*(1-\beta_2)-1]} \quad (54)$$

where C_2^* is given by the following

$$C_2^* = \min\left(\frac{2}{1-\beta_2}, \min_{i=n_1+2, \dots, n_2} \left(\frac{D_i}{\tau_i P_i} + 1, \left(\frac{(1-F(n_1+1))D_{n_1+1}}{\tau_{n_1+1} P_{n_1+1}} + 1, \left(\frac{F(n_2+1)D_{n_2+1}}{\tau_{n_2+1} P_{n_2+1}} + 1\right)\right)\right)\right) \quad (55)$$

Using the same analysis used for machine 1, the delay cost due to machine 2 is

$$DC_2 = C_w W_2 \left[(1-F(n_1+1)) \frac{D_{n_1+1}}{\tau_{n_1+1} P_{n_1+1}} + \sum_{i=n_1+2}^{n_2} \frac{D_i}{\tau_i P_i} + F(n_2+1) \frac{D_{n_2+1}}{\tau_{n_2+1} P_{n_2+1}} \right] \quad (56)$$

The same procedure is used to find the delay costs for the rest of the machines. After finding the delay costs for each machine, the form for the model's total cost is as follows

$$TC(K) = IC(K) + MC(K) + \sum_{j=1}^{j=K} DC_j \quad (57)$$

The problem is to find the number of machines (K) for which the total cost is minimized. The number of machines that can be purchased is controlled by the budget available for investment and the procedure to find the optimal number of machines is discussed in the next section.

5.3 Frocedure to Find the Optimal Number of Machines

The problem is to find the number of machines that minimizes the total relevant cost. If the total processing work load is greater than one, it is not possible to have a single machine to process all the products. As the number of machines increases, the machinery cost increases and the maintenance cost increases also; however, the delay cost decreases. A procedure to find the "optimal" number of machines that minimizes the total cost is the following:

1. Set $K = 1$, $TC(K-1) = \infty$ and go to 2
2. If $N > K$, then set $K = K + 1$ and return to step 1; otherwise go to step 3.
3. If $C_1 K \leq B$, then assign all the items to the K available machines as described in the previous section and find the total cost $TC(K)$ and go to step 4. Otherwise go to step 5.
4. If $TC(K) < TC(K-1)$, then increase K by 1 and go to step 3; otherwise go to step 5
5. Stop. The optimal number of machines is K-1

Numerical example:

Suppose, we have 20 different items where the demands, processing rates and setup times presented in Table 7, and suppose, we also have the followings:

$T=10$ years (the planning horizons)

$i=10\%$ (the interest rate)

$B=\$35,000$ (budget)

$C_1=\$8,000$ (cost for one machine)

$C_w =\$240/\text{year}$ for each unit waiting in the queue

$M(t)=200+70 t$ (maintenance cost)

The question is to find the number of machines to minimize the total cost

Applying the procedure presented above

$N=1.576032$ (the total utilization)

1. $K=1, TC(0)=\infty$

2. $N>1$ go back to step 1

1. $K=2, TC(1)=\infty$

2. $N<2$

3. $8,000(2)<B=35,000$

Assign all the items to the two available machines by balancing the work load

by going through all the steps, we get the following results

$\beta_1 = \beta_2 = 0.78816$

Assign items 1 through 11 to machine 1. Assign 0.639485 of item 12 to machine 1 also

Assign $(1-0.639485)$ of item 12 to machine 2 and also assign items 13 through 20 to

machine 2.

$$W_1=0.06795, W_2=0.054753$$

The total delay cost is \$1,839.17

Machinery cost is \$2,603.2

Maintenance cost is 1,061.26

the total cost is \$5503.42

4. $TC(2) < TC(1)$ increase K to 3 and go to step 3

$$3. (8,000)(3) < 35,000$$

Assign all the items to the three available machines by balancing the work load.

by going through all the steps, we get the following results

$$\beta_1 = \beta_2 = \beta_3 = 0.525344$$

Assign items 1 through 6 and a fraction 0.6476055 of item 7 to machine 1

Assign a fraction $(1-0.6476055)$ of item 7 and items 8 through 12 and a fraction 0.797456 of item 13 to machine 2

Assign a fraction $(1-0.797456)$ of item 13 and items 14 through 20 to machine 3

The machinery cost is \$3,904.8

The maintenance cost is \$1,591.9

The delay cost is \$302.3

The total cost is \$5,799

4. $TC(2) > TC(3)$ go to 5

5. STOP

the number of machines that are needed to minimize the cost is 2

Table 7

Data for 20 items to be processed on K machines

D_i (units/year)	P_i (units/year)	τ_i (year)
350	4000	0.001
420	5000	0.0012
360	4600	0.0014
400	6200	0.0013
420	6400	0.0011
340	4400	0.0018
400	3800	0.002
300	3300	0.0022
280	4000	0.0016
350	3800	0.0018
280	3500	0.0014
240	2800	0.0024
340	3900	0.0014
350	5300	0.0015
280	4800	0.0017
260	2700	0.0018
250	4900	0.001
320	4200	0.0014
190	2400	0.0016
250	3100	0.0014

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In this thesis a heuristic procedure to minimize the average waiting time for the multi-item single machine based on the $M/G/1$ formulation proposed by Karmarkar is proposed. The heuristic is based on the assumption that the ratio of run time to setup time is constant across all items. The effects of increasing the demand rate for the items or adding a new product to the product mix on the batch sizes as well as the average waiting time is discussed. It is shown that increasing the utilization level will increase the average waiting time per unit time and also will increase the batch sizes for each item.

The heuristic is tested against two nonlinear optimizers and the computational experiments show that the heuristic procedure gives very close results to the ones given by both optimizers.

Based on the heuristic procedure and the allocation of items in such a way that the processing work load is equal across all available machines, a cost minimization model for the multi-item multi-machine problem is formulated. Each machine is formulated as an $M/G/1$ queueing system and is assigned a specific number of items from the product mix. The relevant costs that are considered in this research include: the

machinery investment cost, machinery maintenance cost and delay cost. A procedure to find the number of machines that are required for the production system is proposed.

6.2 Recommendations

For further research the following ideas could be explored:

1. Develop a model for the multi-item multi-machine problem where the machines available have different characteristics. For example, one can have a mix of multipurpose machines and the problem is to find the proper mix of machines to minimize a certain manufacturing cost or optimize a certain shop performance measure
2. Study the effect of machine breakdowns on the optimal batch sizes for the multi-item multi-machine case.
3. Incorporate the effect of defective parts which can be reworked for the case of multi-item single . The defectives can either be reworked with the rest of the parts or may require special processing.
4. Apply the results from this research for the multi-item multi-machine with multi stages

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APPENDIX I

Implementation of Yang's Search Algorithm

```

IMPLICIT REAL *8(A-H,O-Z)
REAL*8 P(99),TOW(99),D(99)
REAL*8 FOD(99),E(99),A(99),B(99),Q(99)
CALL INDATA(N,D,P,TOW)
BMAX=0.0
DO 10 I=1,N
  B(I)=(SQRT(TOW(I)))*P(I)
10  CONTINUE
  DO 11 I=1,N
    BIG=B(I)
    IF(BIG .GE. BMAX)THEN
      BMAX=BIG
    ENDIF
11  CONTINUE
  DO 12 I=1,N
    A(I)=B(I)/BMAX
12  CONTINUE
    T1=0.0
    T2=0.0
    T3=0.0
    T4=0.0
    T5=0.0
    DO 13 I=1,N
      T1=T1+D(I)*A(I)/(P(I)**2)
      T2=T2+2*D(I)*TOW(I)/P(I)
      T3=T3+D(I)*(TOW(I)**2)/A(I)
      T4=T4+(D(I)/P(I))
      T5=T5+2*(D(I)*TOW(I)/A(I))
13  CONTINUE
      T4=2*(1-T4)
      QB=(T5/T4)+(SQRT(((T2*T5+T3*T4)/(T4*T1))+((T5/T4)**2)))
      FQB=(T1*(QB**2)+T2*QB+T3)/(T4*QB-T5)
      DO 15 I=1,N
        Q(I)=SQRT((2*TOW(I)*FQB)+(TOW(I)**2))*P(I)
15  CONTINUE
      ITER=0
21  NCOUNT=0
      CALL OBJECT(N,D,P,TOW,Q,F)
      ITER=ITER+1
      DO 20 I=1,N

```

```

35  CALL OBJECT(N,D,P,TOW,Q,F)
    FOD(I)=(((Q(I)/P(I))**2)-(TOW(I)**2))/(2*TOW(I))
    E(I)=(FOD(I)-F)/F
    IF(E(I) .LT. 0.001) GO TO 20
    IF(FOD(I) .GT. F) THEN
        NCOUNT=NCOUNT+1
        Q(I)=Q(I)-1.0
    ELSE
        Q(I)=Q(I)+1.0
        NCOUNT=NCOUNT+1
    END IF
    GO TO 35
20  CONTINUE
    IF (NCOUNT .GT. 0 .AND. ITER .LT. 500 ) GO TO 21
    WRITE(6,*)'OPTIMAL OBJECTIVE VALUE ---- : ',F
    WRITE(6,*)'OPTIMAL BATCH SIZES -----:'
    DO 50 I=1,N
        WRITE(6,3)I, Q(I)
50  CONTINUE
3   FORMAT (2X,'Q(',I2,')=' ,F5.2)
    STOP
    END
    SUBROUTINE INDATA(N,D,P,TOW)
    IMPLICIT REAL*8(A-H,O-Z)
    REAL*8 P(99),D(99),TOW(99)
    READ (5,*) N
    DO 70 I=1,N
        READ (5,*) D(I),P(I),TOW(I)
70  CONTINUE
    RETURN
    END
    SUBROUTINE OBJECT(N,D,P,TOW,Q,F)
    IMPLICIT REAL*8(A-H,O-Z)
    REAL*8 Q(99),D(99),P(99),TOW(99)
    ANUMER=0.0
    DENOM=0.0
    DO 80 I=1,N
        ANUMER=ANUMER+(D(I)/Q(I))*((TOW(I)+(Q(I)/P(I))**2)
        DENOM=DENOM+(D(I)/P(I))+TOW(I)*D(I)/Q(I)
80  CONTINUE
    ADENOM=2.0*(1.0-DENOM)
    F=ANUMER/ADENOM
    RETURN
    END

```

APPENDIX II

Solving the Problem using MINOS

CCC -----SPECS file-----

```
BEGIN
  MINIMIZE
  NONLINEAR VARIABLES      20
  PROBLEM NUMBER           1
  JACOBIAN                 SPARSE
  RADIUS OF CONVERGENCE    1.00E-4
  LINESEARCH TOLERANCE     0.99999
  RHS                      RHS
  BOUNDS                   20
  LOWER BOUND              1.0
  ROWS                     5
  COLUMNS                 100
  ELEMENTS                 500
  COMPLETION               YES
  LIST LIMIT               100
  CRASH OPTION              1
  VERIFY GRADIENTS         NO
  ITERATIONS LIMIT         2000
  MAJOR ITERATIONS         50
  MINOR ITERATIONS         50
  SOLUTION                  YES
  CALL FUNCTION ROUTINES WHEN OPTIMAL
END
```

CCC -----This is the MPS file-----

CCC It contains the only linear constraint and the upper bound on the variables

```
NAME      HAMDA
ROWS
L ROW1
COLUMNS
  X1  ROW1  0.0013
  X2  ROW1  0.0015
  X3  ROW1  0.0018
  X4  ROW1  0.0020
  X5  ROW1  0.0014
  X6  ROW1  0.0028
  X7  ROW1  0.0023
```

X8	ROW1	0.0026	
X9	ROW1	0.0019	
X10	ROW1	0.0021	
X11	ROW1	0.0018	
X12	ROW1	0.0025	
X13	ROW1	0.0014	
X14	ROW1	0.0018	
X15	ROW1	0.0021	
X16	ROW1	0.0023	
X17	ROW1	0.0016	
X18	ROW1	0.0021	
X19	ROW1	0.0017	
X20	ROW1	0.0024	
RHS	RHS	ROW1	0.31444

CCC ----Upper bounds on the variables----

BOUNDS

UP BOUND	X1	250.0
UP BOUND	X2	360.0
UP BOUND	X3	300.0
UP BOUND	X4	400.0
UP BOUND	X5	360.0
UP BOUND	X6	240.0
UP BOUND	X7	270.0
UP BOUND	X8	200.0
UP BOUND	X9	170.0
UP BOUND	X10	280.0
UP BOUND	X11	180.0
UP BOUND	X12	180.0
UP BOUND	X13	240.0
UP BOUND	X14	270.0
UP BOUND	X15	220.0
UP BOUND	X16	220.0
UP BOUND	X17	210.0
UP BOUND	X18	280.0
UP BOUND	X19	160.0
UP BOUND	X20	230.0

CCC-- This is a basis file which is optional. it provides an initial feasible point as input to MINOS--

```

FX INITIAL  X1      3.3
FX INITIAL  X2      4.5
FX INITIAL  X3      3.7
FX INITIAL  X4      3.5
FX INITIAL  X5      4.40
FX INITIAL  X6      2.0
FX INITIAL  X7      3.1
FX INITIAL  X8      1.9
FX INITIAL  X9      2.4
FX INITIAL  X10     3.14
FX INITIAL  X11     3.5
FX INITIAL  X12     2.6
FX INITIAL  X13     5.2
FX INITIAL  X14     3.6
FX INITIAL  X15     2.94
FX INITIAL  X16     3.15
FX INITIAL  X17     3.2
FX INITIAL  X18     2.8
FX INITIAL  X19     4.3
FX INITIAL  X20     2.83
ENDATA

```

CCC --- Subroutine CALCFG that evaluates the objective function and its gradients---

```

SUBROUTINE CALCFG( MODE,N,X,F,G,NSTATE,NPROB )
  IMPLICIT REAL*8(A-H,O-Z)
  REAL*8 X(N),G(N),D(100),TOW(100),P(100)

  ANUMER = 0.0
  DENOM  = 0.0
  FUNC0  = 0.0
  DO 5 I = 1,N
5    READ(7,*) D(I),P(I),TOW(I)
      DO 10 I = 1,N
        TERM1 = TOW(I) + (D(I)/( X(I)*P(I) ))
        FUNC0 = FUNC0 + X(I)*(TERM1**2)
        ANUMER = ANUMER + X(I)*(TERM1**2)
        DENOM  = DENOM  + (D(I)/P(I))+TOW(I)*X(I)
10    CONTINUE
  ADENOM = 2.0*(1.0-DENOM)

```

```

CCC Evaluate the objective function value
  F =ANUMER/ADENOM
  DO 20 I = 1,N
    FUNC1 =TOW(I)*(TOW(I) +(D(I)/( X(I)*P(I) )))
CCC Evaluate the objective gradients
  G(I) =(FUNC1*ADENOM + 2.0*TOW(I)*FUNC0 )/( ADENOM**2 )
20  CONTINUE
  REWIND(7)
  RETURN
  END

```

CCC Sample data file required as input to CALCFG subroutine. The data corresponds to the input for problem# III test# 2 The input is read as follows:
Demand, Processing rate, Setup time

```

250,10288,0.0013
360,9643,0.0015
300,8085.8,0.0018
400,10307.3,0.0020
360,10610.4,0.0014
240,7605.2,0.0028
270,6815,0.0023
200,7044,0.0026
170,6844.5,0.0019
280,7673,0.0021
180,5114.1,0.0018
180,5034.2,0.0025
240,5889.6,0.0014
270,7514.1,0.0018
220,6426.4,0.0021
220,5484.9,0.0023
210,7415.9,0.0016
280,8426.4,0.0021
160,3967.9,0.0017
230,6110.6,0.0024

```

APPENDIX III

Implementation of the Simulating Annealing Algorithm

C... NONLINEAR OPTIMIZATION USING THE SIMULATING ANNEALING ALGORITHM

c.. Variables Used

c.. 1. N = Problem dimension

```
IMPLICIT REAL*8(A-H,O-Z)0
REAL*8 P(99),D(99),TOW(99),UB(99),LB(99)
REAL*8 X(99),DELr,XSTAR(99),Xmin(99),S(99)
```

```
EXTERNAL RNNOR,RNSET,RNUN,RNUNF
```

```
ISEED = 124567
```

C.. READ the Data

```
Call INDATA(N,X,DELr,BETA,Fopt,D,P,TOW,UB,LB)
```

C.. EPSILON IS TAKEN AS 10^{-6}

```
ICOUNTER = 0
EPSILON = 10**(-6)
```

```
CALL OBJECT(N,D,P,TOW,X,F)
```

```
Omin = F
```

```
DO 1 I = 1,N
```

```
Xmin(I) = X(I)
```

```
1 CONTINUE
```

```
Fnot = F
```

```
DELF = Fnot - Fopt
```

```
IF(ABS(DELF).LE.EPSILON) THEN
```

```
GO TO 100
```

```
END IF
```

C.. GENERATE A RANDOM DIRECTION

```
20 IF(ICOUNTER.GT.2000) GO TO 100
```

```
CALL Finddir(N,S,ISEED)
```

```

DO 10 I = 1,N

XSTAR(I) = X(I) + DELr*S(I)

10  CONTINUE

CALL FSBLT(N,XSTAR,LB,UB,INDEX,D,P,TOW)

IF(INDEX.EQ.0) GO TO 20

CALL OBJECT(N,D,P,TOW,XSTAR,F)

Fone = F

DELF1 = Fone - Fnot

IF(DELF1.LE.0) THEN
  Fnot = Fone

DO 11 I = 1,N
X(I) = XSTAR(I)
11  CONTINUE

CALL COMPARE(Omin, F, Xmin, X, N)

ICOUNTER = ICOUNTER + 1

DELF = Fnot - Fopt

IF(ABS(DELF).LE.EPSILON) THEN

GO TO 100
END IF

GO TO 20

END IF

IG = 0.0
TEMP = BETA * (Fnot**(IG))*DELF1
PRB = EXP(-TEMP)
CALL Rangen(RPRB,ISEED)

```



```

IF(RPRB.LT.PRB) THEN

DO 12 I = 1,N
X(I) = XSTAR(I)
12  CONTINUE

ICOUNTER = ICOUNTER + 1

Fnot = Fone

END IF

GO TO 20

100  WRITE(6,*) 'OPTIMAL Objective : ',Omin
      Do 2 I = 1,N
      WRITE(6,3) I,Xmin(I)
2    CONTINUE
3    FORMAT(2X,'X(',I2,') = ',F5.2)
      STOP
      END

SUBroutine INDATA(N,X,DELr,BETA,Fopt,D,P,TOW,UB,LB)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 X(99),P(99),D(99),TOW(99),LB(99),UB(99),DELr,Fopt

READ(5,*) N,DELr,BETA,Fopt
READ(5,*) (X(I),I=1,N)

DO 10 I = 1,N
READ(5,*) D(I),P(I),TOW(I),LB(I),UB(I)
10  CONTINUE

RETURN
END

```

SUBROUTINE Finddir(NR,R,ISEED)

IMPLICIT REAL*8(A-H,O-Z)

REAL*8 R(99)

INTEGER ISEED

EXTERNAL RNNOR,RNSET,RNUNF,RNGET

```
30  RN = Rnurf()
    IF(RN.EQ.0.0) GO TO 30
    ISEED = ISEED*(RN +1.0)
    IF(ISEED.GE.2147483646) ISEED = ISEED/3
    Call Rnset(ISEED)
    Call Rnnor(NR,R)
    SUM = 0
    DO 10 I=1,NR
    SUM = SUM + R(I)**2
10  Continue
    DO 20 I=1,NR
    R(I) = R(I)/Sqrt(SUM)
20  Continue
    Return
END
```

SUBROUTINE FSBLT(N,X,LB,UB,INDEX,D,P,TOW)

IMPLICIT REAL*8(A-H,O-Z)

REAL*8 X(N),LB(N),UB(N),D(N),P(N),TOW(N)

C.. DEFINE CONSTRAINTS HERE :-

SUM1 = 0

SUM2 = 0

Do 10 I = 1,N

SUM1 = SUM1 + TOW(I)*X(I)

SUM2 = SUM2 + D(I)/P(I)

10 Continue

C.. CHECK FOR FEASIBILITY :-

NCON = N

DO 20 I = 1,NCON

C.. CHECKING FOR UPPER LIMIT ON VARIABLES :-

```
IF(UB(I).LT.X(I)) THEN  
  INDEX = 0  
  GO TO 30  
END IF
```

C.. CHECKING FOR LOWER LIMIT ON VARIABLES :-

```
IF(X(I).LT.LB(I)) THEN  
  INDEX = 0  
  GO TO 30  
END IF
```

20 CONTINUE

```
CONST = SUM1 + SUM2  
IF (CONST.GE.1) THEN  
  INDEX = 0  
  GO TO 30  
END IF
```

INDEX = 1

30 RETURN
END

SUBROUTINE rangen(RNUMBR,ISEED)

```
IMPLICIT REAL*8(A-H,O-Z)  
REAL*8 R(1)  
EXTERNAL RNUN,RNSET  
REAL RNUMBR  
CALL RNSET(ISEED)  
NR = 1  
CALL RNUN(NR,R)  
RNUMBR = R(1)  
RETURN  
END
```

SUBROUTINE OBJECT(N,D,P,TOW,X,F)

IMPLICIT REAL*8(A-H,O-Z)

REAL*8 X(N),D(N),TOW(N),P(N)

ANUMER = 0.0

DENOM = 0.0

FUNC0 = 0.0

DO 10 I = 1,N

TERM1 = TOW(I) +(D(I)/(X(I)*P(I)))

FUNC0 = FUNC0 +X(I)*(TERM1**2)

ANUMER = ANUMER + X(I)*(TERM1**2)

DENOM = DENOM + (D(I)/P(I))+TOW(I)*X(I)

10 CONTINUE

ADENOM = 2.0*(1.0-DENOM)

F =ANUMER/ADENOM

RETURN

SUBROUTINE COMPARE(Omin, Obj, Xmin, X, N)

IMPLICIT REAL*8(A-H,O-Z)

IF(Obj.LT.Omin) THEN

DO 10 I = 1,N

Xmin(I) = X(I)

10 CONTINUE

END IF

RETURN

END

APPENDIX IV

Data used for testing the Heuristic

Problem # I		Processing rates			
Demand	Setup time	Test# 1	Test# 2	Test# 3	Test# 4
250	0.001	8800	9786.4	10772.8	5840.8
360	0.0012	7600	8422	9244	5134
300	0.0014	6500	7204.571	7909.143	4386.286
400	0.0013	8600	9358.769	10117.54	6323.692
360	0.0011	9500	10396.73	11293.45	6809.818
240	0.0018	6000	6548	7096	4356
270	0.002	5800	6293.2	6786.4	4320.4
200	0.0022	5600	6048.364	6496.727	4254.909
170	0.0016	5800	6416.5	7033	3950.5
280	0.0018	6400	6948	7496	4756
180	0.0014	5400	4695.429	3990.857	7513.714
180	0.0024	4800	4389	3978	6033
240	0.0014	6400	5695.429	4990.857	8513.714
270	0.0015	7400	6742.4	6084.8	9372.8
220	0.0017	6500	5919.765	5339.529	8240.706
220	0.0018	5400	4852	4304	7044
210	0.001	7800	6813.6	5827.2	10759.2
280	0.0014	8600	7895.429	7190.857	10713.71
160	0.0016	3800	3183.5	2567	5649.5
230	0.0014	5200	4495.429	3790.857	7313.714

Problem# II		Production rates			
Demand	Setup time	Test# 1	Test# 2	Test# 3	Test# 4
250	0.0012	9000	10000	11000	6000
360	0.0014	8000	8857.143	9714.286	5428.571
300	0.0016	7000	7750	8500	4750
400	0.0015	9000	9800	10600	6600
360	0.0013	9500	10423.08	11346.15	6730.769
240	0.0021	7000	7571.429	8142.857	5285.714
270	0.0022	6000	6545.455	7090.909	4363.636
200	0.0024	6000	6500	7000	4500
170	0.0018	5800	6466.667	7133.333	3800
280	0.002	6600	7200	7800	4800
180	0.0016	5600	4850	4100	7850
180	0.0024	5000	4500	4000	6500
240	0.0012	6600	5600	4600	9600
270	0.0017	8000	7294.118	6588.235	10117.65
220	0.0019	7000	6368.421	5736.842	8894.737
220	0.002	5800	5200	4600	7600
210	0.0013	8000	7076.923	6153.846	10769.23
280	0.0018	8800	8133.333	7466.667	10800
160	0.0015	4400	3600	2800	6800
230	0.0023	6500	5978.261	5456.522	8065.217

Problem# III		Production rates			
Demand	Setup time	Test# 1	Test# 2	Test# 3	Test# 4
250	0.0013	9200	10288.12	11376.23	12464.35
360	0.0015	8700	9643.033	10586.07	11529.1
300	0.0018	7300	8085.861	8871.722	9657.583
400	0.002	9600	10307.28	11014.55	11721.83
360	0.0014	9600	10610.39	11620.79	12631.18
240	0.0028	7100	7605.196	8110.393	8615.589
270	0.0023	6200	6815.022	7430.043	8045.065
200	0.0026	6500	7044.058	7588.115	8132.173
170	0.0019	6100	6844.5	7589	8333.5
280	0.0021	7000	7673.595	8347.19	9020.786
180	0.0018	5900	5114.139	4328.278	3542.417
180	0.0025	5600	5034.18	4468.36	3902.54
240	0.0014	6900	5889.607	4879.214	3868.821
270	0.0018	8300	7514.139	6728.278	5942.417
220	0.0021	7100	6426.405	5752.81	5079.214
220	0.0023	6100	5484.978	4869.957	4254.935
210	0.0016	8300	7415.906	6531.813	5647.719
280	0.0021	9100	8426.405	7752.81	7079.214
160	0.0017	4800	3967.912	3135.824	2303.735
230	0.0024	6700	6110.604	5521.208	4931.813

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