1984

Cold-formed angle columns under biaxial bending.

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COLD-FORMED ANGLE COLUMNS UNDER BIAXIAL BENDING

by

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A thesis
presented to the University of Windsor
in partial fulfillment of the
requirements for the degree of
Master of Applied Science
in
Civil Engineering

Windsor , Ontario, 1984

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ABSTRACT

Cross-sectional properties, including cross-sectional area, location of centroid, moments of inertia, torsional and warping constants and location of the shear centre, of both equal and unequal-leg cold-formed angle sections are presented. Two important properties, β_1 and β_2, required for the calculation of theoretical buckling load of eccentrically loaded columns, are also listed.

The computational difficulties encountered in the exact solution of differential equations of equilibrium of columns under biaxial bending are pointed out and theoretical load-deflection relationships using the Galerkin method are presented. These theoretical relationships are then compared with experimental ones, obtained from the tests carried on fifteen 65 X 50 X 4 mm (nine specimens with long leg connected and six with long leg out) and nine 55 X 55 X 4 mm eccentrically loaded cold-formed single angles. The test specimens were connected at ends to the test frame by bolts on one leg and were subjected to eccentric thrust (eccentricity about both principal axes). Three different nominal slenderness ratios (80, 120 and 170) and three different end connections (one-, two- and three- bolt end connections) were used in the test program.

A computer program for pinned-end boundary conditions was developed to predict the load-deflection relationships of cold-formed angles under biaxial bending. This program was also used to estimate the ultimate compressive strengths of such sections connected by one leg. A table giving the ultimate compressive strengths of two commonly used cold-formed angles for various gauge distances is included. Failure is assumed to have occurred when the total stress (compressive or tensile) at any point on the cross-section reaches the value of yield stress or when there is a change of sign for at least one of the deflection components. These predicted loads together with those obtained from ASCE Manual No. 52 and the ECCS Recommendations are compared with experimental failure loads.

The effects of the location of shear centre and magnitude of warping constant on the ultimate compressive strength of cold-formed single angles have been discussed.

ACKNOWLEDGEMENTS

The author wishes to take this opportunity to express his sincerest gratitude to his advisor Dr. Murty K. S. Madugula for his continuous encouragement, guidance, advice and valuable suggestions throughout the preparation of this thesis.

The author is also grateful to Sidbec-Dosco, Montreal, for supplying the cold-formed angles used in this investigation.

The financial support provided by the Natural Sciences and Engineering Research Council of Canada under grant number A1194 is gratefully acknowledged.

Many thanks are also due to :

- Messrs. K. M. Chong and S. P. Khor for carrying out a portion of the experimental work.
- Mr. S. G. Ekhande and other graduate students of the Department of Civil Engineering for their often useful advices.
- Technicians of the Civil Engineering Department for their assistance during the experimental work.
- Computer Centre Staff of the University of Windsor for the help in running the computer program.

LIST OF SYMBOLS

A = area of the cross-section

C_w = warping constant of the cross-section

E = modulus of elasticity

e_x, e_y = x- and y- coordinates of point of application of eccentric thrust with reference to the centroidal principal axes

e_1 = effective eccentricity about the x-x axis

$= e_y - v - (e_x - x_o)\phi$

e_2 = effective eccentricity about the y-y axis

$= e_x - u + (e_y - y_o)\phi$

G = modulus of rigidity = $\frac{E}{2(1+\nu)}$

I_x, I_y = moments of inertia of the cross-section about the major and minor centroidal principal axes (x-x and y-y axes) respectively

$I_{x'}, I_{y'}$ = moments of inertia of the cross-section about the centroidal axes parallel and perpendicular to short leg, x'-x' and y'-y' axes respectively

I_o = polar moment of inertia of the cross-section about the shear centre

J = torsional constant of the cross-section

K_x, K_y = effective length factors for flexural buckling about the x- axis and y- axis respectively

ℓ = length of the member

P = external applied load

P_x = flexural buckling load about x-x axis = $\dfrac{\pi^2 EI_x}{(K_x \cdot \ell)^2}$

P_y = flexural buckling load about y-y axis = $\dfrac{\pi^2 EI_y}{(K_y \cdot \ell)^2}$

$P_{x'y'}$ = product of inertia of the cross-section about x'-x' and y'-y' axes

P_ϕ = torsional buckling load parameter = $(4EC_w\pi^2/\ell^2 + GJ)/r_o^2$

r_y = minimum radius of gyration

r_o^2 = $\beta_1 e_y + \beta_2 e_x + I_o/A$

u, v, ϕ = deflections of the shear centre (u and v) along major and minor centroidal principal axes respectively and rotation of the shear centre (ϕ) about the longitudinal axis (z-z axis)

u_o, v_o, ϕ_o = co-efficients in the expressions for deflections to be determined by the application of the Galerkin method

x, y = x- and y- coordinates of the point at which the stress is being calculated

x', y' = perpendicular distances of the centroid from the outside of the long leg and short leg respectively

x_s, y_s = perpendicular distances of the shear centre from y'-y' and x'-x' axes respectively

x_o, y_o = x- and y- coordinates of the shear centre with reference to the centroidal principal axes

z = location of the cross-section along the length of the member, where the deflection is being calculated

b/t = width-to-thickness ratio

$K\ell/r_y$ = largest effective slenderness ratio according to ASCE Manual No. 52

ℓ/r_y = maximum slenderness ratio

α = angle of inclination of major principal axis with respect to the x'-x' axis

$$\alpha' = \frac{\text{Sinh}\,\lambda - \text{Sin}\,\lambda}{\text{Cosh}\,\lambda - \text{Cos}\,\lambda} = 1.0178$$

$$\beta_1 = \frac{1}{I_x}\left(\int_A y^3 dA + \int_A x^2 y dA\right) - 2y_o$$

$$\beta_2 = \frac{1}{I_y}\left(\int_A x^3 dA + \int_A xy^2 dA\right) - 2x_o$$

λ = 4.73004, an eigenvalue satisfying the characteristic equation $\text{Cos}\lambda\ \text{Cosh}\lambda = 1$

ν = poisson's ratio

σ_{cr} = ultimate maximum stress according to the ASCE Manual No. 52

σ_T = total normal stress at any point

σ_y = yield stress of the material

σ_1 = uniform axial compressive stress = P/A

σ_2 = linearly varying stress due to bending about the major centroidal principal axis (x-x axis)

$$= \frac{Pe_1}{I_x} \cdot y$$

σ_3 = linearly varying stress due to bending about the minor centroidal principal axis (y-y axis)

$$= \frac{Pe_2}{I_y} \cdot x$$

σ_4 = warping normal stress due to non-uniform torsion

$$= E(\bar{\omega}_s - \omega_s)\phi''$$

ω_s = warping function according to Timoshenko and Gere

$\bar{\omega}_s$ = average value of ω_s

λ/λ_y = slenderness ratio according to ECCS Recommendations

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Chapter I

INTRODUCTION

1.1 GENERAL

Structural steel angles are used extensively as leg and diagonal members of latticed towers, as chord and web members of trusses, as bracing members providing lateral support to beams and columns, etc.; they are easy to fabricate and erect because of the basic simplicity of their cross-section. In many of their potential uses, it is often required to use thin bar size angles. However, due to economic reasons, the manufacturers are reluctant to produce thin bar size hot-rolled angles unless the demand is appreciable. In such circumstances, cold-formed angle sections, made from hot-rolled sheet or plate, find their use when they are substituted for hot-rolled angles.

In a majority of the cases, the angles are connected to main members by one leg only, which gives rise to the biaxial bending of the angles. Cold-formed angles being open and thin, have low torsional rigidity and as such, are susceptible to torsional-flexural buckling which greatly reduces their load carrying capacities. For such uses, non-availability of well-formulated and sufficiently established guidelines make proper substitution and independent usage of cold-formed angles difficult, if not altogether impossible. Realising such an information gap, a research project has been undertaken to gain some insight into the behaviour of cold-formed angle columns under biaxial bending.

1.2 OBJECTIVES

The objectives of this investigation are :

1. To compute all the cross-sectional properties of equal and unequal-leg cold-formed angles that a designer may need.
2. To estimate ultimate compressive strengths of equal-leg and unequal-leg cold-formed angle sections under biaxial bending, using the general theory of torsional-flexural buckling.
3. To compare experimental load-deflection relationships with theoretical ones, obtained from the basic differential equations of equilibrium using the Galerkin method.
4. To compare experimental failure loads with the ultimate compressive loads computed from the general theory of torsional-flexural buckling, ASCE Manual No. 52 and ECCS Recommendations.
5. To study the effects of the number of bolts in the end connections on the ultimate strength of the member.

6. To investigate the difference in strengths of unequal-leg angles with long leg connected and with long leg out.
7. To study the effects of the location of the shear centre and the magnitude of the warping constant on the buckling load predicted by the general theory of torsional-flexural buckling.

Chapter II

LITERATURE SURVEY

2.1 GENERAL

The steel angle is a common and almost traditional member in building and industrial construction ---- its popularity can be attributed to its relative lightness, ease of fabrication and connection to the other members of the structure. Its use could be traced since the early days of the construction history. However, in view of its long and widespread use, it is quite surprising to find how little is known of many major aspects of its behaviour as a structural member. In some areas, design information is only available for limited use and often consists of conclusions drawn from the investigation done on other types of cross-section. For some other cases, the design manuals and specifications refer to the use of suitable rational theory or structural performance tests.

The question remains why the problem of single angle columns, under biaxial bending, has not been fully taken care of and the answer is that even though the angle is a very simple section to the common man and the manufacturer, it is not simple for the stress analyst. The principal axes of the cross-section do not coincide with the usual loading directions and any routine loading will cause biaxial bending deflections which are not in the same plane as applied loads. Furthermore, the shear centre does not lie at the centroid due to the lack of symmetry in the cross-section and also is not in the line of action of more commonly applied loads. Thus any loading will force the cross-section to twist, causing torsional-flexural buckling in the angle section.

These angle sections can be broadly classified as hot-rolled or cold-formed depending upon the manufacturing process. However, cold-formed angle sections are relatively new, and as such, very little information on them exists. In the following sections, the available literature is briefly surveyed. For a detailed review of literature, reference may be made to Kennedy and Madugula (1982).

2.2 THEORETICAL SOLUTION OF DIFFERENTIAL EQUATIONS OF EQUILIBRIUM

The differential equations of equilibrium for the general case of biaxial eccentricities were solved by Bleich (1952), Vlasov (1961), Thurlimann (1953), Dabrowski (1961), Prawel and Lee (1964), Culver (1966) and Peköz and Celebi (1969), each using a different approach. Timoshenko and Gere (1961) defined the warping function and also solved the differential equations. They, however, neglected the effect of precritical deflections and therefore, the critical load obtained from their solution is higher than the expected ones, i.e., an upper bound solution results from their approach.

2.3 COLD-FORMED ANGLE SECTIONS

Torsional-flexural buckling of concentrically loaded thin-walled singly-symmetrical columns was first investigated by Chajes and Winter (1965) and Chajes, Fang and Winter (1966). They proposed a simple method of calculating torsional-flexural buckling load of such columns which was also extended to the inelastic range. An interaction equation, accompanied by sets of curves for design use, was suggested. Tests were also done for checking the analytical procedures developed.

In 1969, Peköz and Celebi studied the general behaviour of thin-walled singly-symmetrical open sections under eccentric axial loading in the plane of symmetry. The effect of precritical deflections and other modifications were introduced to the basic theory and the complete range of behaviour of such eccentric members was explored. Simplified design procedures and graphs for some complex parameters were presented together with experimental confirmation of theory. Tests carried out by Carpena, Cauzillo and Nicolini (1976) showed that cold-formed angles had an average buckling strength greater than that of hot-rolled angles.

It is therefore quite clear that the behavioural information available for cold-formed angle sections, connected by one leg, is scarce. That's why the cold-formed design codes and manuals (CSA 1974, Schuster 1975, AISI 1980, Winter 1977) do not give any ultimate strength formula or tables but simply refer the designer to the use of any rational theory or tests for determining structural performance.

All of the required cross-sectional properties of cold-formed sections, to the best of the author's knowledge, are not readily available. Formulas for β_1 and β_2 were developed for equal-leg angles only (Peköz and Celebi 1969). It is also common to use the magnitude of warping constant as zero and location of shear centre at SC_1 or SC_2, as shown in Fig. 4.3.

2.4 HOT-ROLLED ANGLE SECTIONS

Chen and Atsuta (1972) developed exact interaction equations for biaxially loaded doubly symmetrical cross-sections. This was done by curve fitting. In 1974, they extended this method to the case of an unsymmetrical section composed of rectangular elements which meet each other at right angle. This method is applicable to any equal, unequal and built-up angle and is considered powerful and efficient.

The use of angles as laterally unsupported beams was investigated by Leigh and Lay (1970). The solutions are also applicable for restrained beams between the restraint points. The angles were loaded with a uniform moment over the entire span which simulated the most critical lateral buckling situation. The design of the equal-leg and unequal-leg angles --- approximated by the dual rectangle idealization, neglecting the fillet and toe radii --- were found to be governed by stress and deflection criteria rather than by buckling. Safe load tables were also presented. Tests were undertaken by Thomas and Leigh (1970) to compare the theory developed which showed that the angle of twist normally reduces the maximum section stress produced.

Haaijer, Carskaddan and Grubb (1981) studied the feasibility of using finite-element analysis instead of a physical test. Only elastic behaviour was considered, thereby limiting its application to the slender angle columns only.

A total of fifty-seven 90 X 90 X 7 mm angle sections, with eccentricities in the plane of symmetry and also about both the principal axes, were tested by Yokoo, Wakabayashi and Nonaka (1968). Initially twisted specimens were included in test program. It was shown that the buckling behaviour and strength were not affected significantly by the boundary condition for twisting. Considerable torsional deformation was observed for the angles with eccentric loading in the minor principal axis of the cross-section. Also, initial twisting did not affect the buckling behaviour of angles. For the load acting on the major principal axis, bending was predominant resulting in a smaller failure load. On the other hand, in case of load acting on the minor principal axis, failure of the short angles was caused by local buckling or torsional deformation. Long angles were, however, concerned with bending about the major principal plane.

Ishida (1965) carried out tests on a total of 33 concentric and 7 eccentric specimens of 75 X 75 X 6 mm and 65 X 65 X 6 mm semi-killed high strength steel angles, having slenderness ratios from 20 to 100. It was found that the behaviour of rolled high strength steel angles was different from that of rolled mild steel angles due to the presence of higher residual stresses. Correspondingly, the load carrying capacities were lower.

Marsh (1969), after testing 25 X 25 X 2.5 mm and 38 X 38 X 2.5 mm aluminium angles, found that the use of proper effective length factor provided good agreement between the theory he had developed and the experiments carried out.

Eccentrically loaded single angle columns were also investigated theoretically and experimentally by Trahair, Usami and Galambos (1969,1970) and by Usami and Galambos (1971). The single angle compression members were treated as end-restrained columns with biaxially eccentric load and an elastic-plastic behaviour was determined by a numerical analysis which takes the effect of residual stress and initial imperfections of the angle columns into account. Forty-five tests were also done on 51 X 51 X 6.3 and 76 X 51 X 6.3 mm angles to substantiate the proposed numerical procedure. It was observed that the theoretical and experimental results were in good agreement. Load-deflection curves were also presented. AISC Beam-Column interaction equation was found to be a good basis for design recommendations. This numerical procedure also agreed quite well with the tests done on 100 X 100 X 10 mm and 130 X 130 X 9 mm angles by Usami and

Fukumoto (1972), intended to study the behaviour of bracing members of steel bridges. It was also found that the effect of residual stresses was not significant.

Based on the tests of single angle single-bolted connections, Kennedy and Sinclair (1972) developed empirical formulas for ultimate load of bolted connections, for both end- and edge- type failures.

Kennedy and Murty (1972) tested 72 angle struts, with both hinged and fixed end conditions and subjected to axial compressive load. All specimens failed in the inelastic range and a procedure, to estimate the realistic permissible buckling stress for a given angle, was outlined.

A total of 153 equal-leg and unequal-leg angles were tested by the Working Group 08 of Study Committee no. 22 of the International Conference on Large High Voltage Electric Systems(Wood 1975) and it was shown that the critical buckling stress for slenderness ratios between 120 and 250 for crossed diagonals was higher than the Euler critical stress. However, the ratio of the two increased with increasing slenderness ratio.

2.5 DESIGN SPECIFICATIONS

ASCE Manual No. 52 entitled "Guide for the Design of Steel Transmission Tower(1971)" is used extensively for the design of hot-rolled angle members in latticed steel transmission towers. These recommendations, however, have certain limitations. The formula for ultimate compressive stress is based on Euler formula in the elastic range and Column Research Council's basic strength curve in the inelastic range.

AISC Manual of Steel Construction (1980) gives an approximate procedure for the design of single-angle struts, connected by one leg. However, this does not include the effect of torsion caused by the twist of the section and the Manual refers the designer to the appropriate technical publications, if its effect is to be considered.

The European practice for the design of angle members can be found in the Introductory Report of the European Convention for Constructional Steelwork(ECCS 1976). Design procedures were outlined for concentrically and eccentrically loaded angles used in transmission towers or in other structures. It takes into account the effect of the stiffnesses of the other connecting members and reduced torsional rigidity and increase of yield stress at the corners of cold-formed angles.

Chapter III

THEORETICAL FORMULATION

3.1 GENERAL THEORY OF TORSIONAL-FLEXURAL BUCKLING

The basic differential equations of equilibrium of a column, loaded with axial end loads and biaxial eccentricities, are (Timoshenko and Gere, 1961) :

$$EI_y u^{iv} + Pu'' + P(y_o - e_y)\phi'' = 0 \qquad [3.1]$$

$$EI_x v^{iv} + Pv'' - P(x_o - e_x)\phi'' = 0 \qquad [3.2]$$

$$EC_w \phi^{iv} - (GJ - Pe_y\beta_1 - Pe_x\beta_2 - P\frac{I_o}{A})\phi''$$

$$+ P(y_o - e_y)u'' - P(x_o - e_x)v'' = 0 \qquad [3.3]$$

where,

E = modulus of elasticity

I_x, I_y = moments of inertia of the cross-section about the major and minor centroidal principal axes (x-x axis and y-y axis) respectively.

P = external applied load

x_o, y_o = x- and y- coordinates of the shear centre with reference to the centroidal principal axes.

e_x, e_y = x- and y- coordinates of point of application of eccentric thrust with reference to the centroidal principal axes.

C_w = warping constant of the cross-section

G = modulus of rigidity = $\frac{E}{2(1 + \nu)}$

ν = poisson's ratio

I_o = polar moment of inertia of the cross-section about the shear centre.

A = area of the cross-section.

u, v, ϕ = deflection of the shear centre (u and v) along major and minor centroidal principal axes respectively and rotation of the shear centre (ϕ) about the longitudinal axis (z-z axis).

J = torsional constant of the cross-section.

$$\beta_1 = \frac{1}{I_x}\left(\int_A y^3 dA + \int_A x^2 y dA\right) - 2y_o$$

$$\beta_2 = \frac{1}{I_y}\left(\int_A x^3 dA + \int_A xy^2 dA\right) - 2x_o$$

The above differential equations of equilibrium were formulated on the basis of the following assumptions :

1. The column is initially straight.
2. Effects of the residual stresses are negligible.
3. The geometry of the cross-section of the member does not change during buckling. This basic assumption implies that the displacement of the each point in the cross-section can be specified in terms of the displacement components, u, v and ϕ of the shear centre.

4. Shear and axial deformations are insignificant.
5. Deflection components are small (u and v in comparison with the length of the member and ϕ in comparison with $\pi/2$).
6. The material is linearly elastic and the stress-strain curve is elastic-perfectly plastic, having a well-defined yield point.

Exact and approximate solutions of these equations have been given by Vlasov (1961), Culver (1966), Dabrowski (1961), Thurlimann (1953) and Prawel and Lee (1964). However, the exact procedure (Culver 1966), involving twelve unknown integration constants in twelve simultaneous equations, are difficult, if not impossible, to use and therefore, an approximate solution has been resorted to. The Galerkin method, which is sufficiently accurate and simple (Chajes 1974), is used here to obtain a solution of these equations.

The boundary conditions for pinned-end columns can be expressed as follows :

Static:

$$u''_{z = o,\ \ell} = \frac{Pe_x}{EI_y} \qquad [3.4]$$

$$v''_{z = o,\ \ell} = \frac{Pe_y}{EI_x} \qquad [3.5]$$

Geometric:

$$u = v = \phi = \phi' = o \quad \text{at } z = o,\ \ell \qquad [3.6]$$

Since the point of load application is not a point of zero longitudinal displacement, warping is considered to be restrained and the condition $\phi' = 0$ is used (Culver 1966).

Accordingly, the following functions for u,v and ϕ have been selected (Peköz and Celebi 1969) :

$$u = \frac{Pe_x}{2EI_y}(z^2 - \ell z) + u_o \text{ Sin } \frac{\pi z}{\ell} \qquad [3.7]$$

$$v = \frac{Pe_y}{2EI_x}(z^2 - \ell z) + v_o \text{ Sin } \frac{\pi z}{\ell} \qquad [3.8]$$

$$\phi = \phi_o \left[\text{Sin } \frac{\lambda z}{\ell} - \text{Sinh } \frac{\lambda z}{\ell} - \alpha' \left(\text{Cos } \frac{\lambda z}{\ell} - \text{Cosh } \frac{\lambda z}{\ell}\right)\right] \qquad [3.9]$$

where,

ℓ = length of the member.

z = location of the cross-section along the length of the member, where the deflection is being calculated.

u_o, v_o, ϕ_o = co-efficients to be determined by the application of the Galerkin method.

λ = 4.73004, an eigenvalue satisfying the characteristic equation $\text{Cos}\,\lambda\ \text{Cosh}\,\lambda = 1$.

$$\alpha' = \frac{\text{Sinh } \lambda - \text{Sin } \lambda}{\text{Cosh } \lambda - \text{Cos } \lambda} = 1.0178$$

Using Eq. [3.7] to [3.9] and applying Galerkin method to the differential Eq. [3.1] to [3.3], the following algebraic equations are obtained :

$$u_o(EI_y \frac{\pi^4}{\ell^4} \int_o^\ell \text{Sin}^2 \frac{\pi z}{\ell} dz - P \frac{\pi^2}{\ell^2} \int_o^\ell \text{Sin}^2 \frac{\pi z}{\ell} dz)$$

$$+ P(y_o - e_y)\phi_o \frac{\lambda^2}{\ell^2} \int_o^\ell [- \text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell} - \alpha' (- \text{Cos} \frac{\lambda z}{\ell}$$

$$- \text{Cosh} \frac{\lambda z}{\ell})] \text{ Sin} \frac{\pi z}{\ell} dz = - \frac{P^2 e_x}{EI_y} \int \text{Sin} \frac{\pi z}{\ell} dz \qquad [3.10]$$

$$v_o(EI_x \frac{\pi^4}{\ell^4} \int_o^\ell \text{Sin}^2 \frac{\pi z}{\ell} dz - P \frac{\pi^2}{\ell^2} \int_o^\ell \text{Sin}^2 \frac{\pi z}{\ell} dz)$$

$$- P(x_o - e_x)\phi_o \frac{\lambda^2}{\ell^2} \int_o^\ell [- \text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell} - \alpha' (- \text{Cos} \frac{\lambda z}{\ell}$$

$$- \text{Cosh} \frac{\lambda z}{\ell})] \text{ Sin} \frac{\pi z}{\ell} dz = - \frac{P^2 e_y}{EI_x} \int_o^\ell \text{Sin} \frac{\pi z}{\ell} dz \qquad [3.11]$$

$$\phi_o\{EC_w \frac{\lambda^4}{\ell^4} \int_o^\ell [\text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell} - \alpha' (\text{Cos} \frac{\lambda z}{\ell} - \text{Cosh} \frac{\lambda z}{\ell})]^2 dz$$

$$+ (Pe_y\beta_1 + Pe_x\beta_2' + P \frac{I_o}{A} - GJ) \int_o^\ell \frac{\lambda^2}{\ell^2} [- \text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell}$$

$$- \alpha' (- \text{Cos} \frac{\lambda z}{\ell} - \text{Cosh} \frac{\lambda z}{\ell})] [\text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell} - \alpha' (\text{Cos} \frac{\lambda z}{\ell} - \text{Cosh} \frac{\lambda z}{\ell})]dz\}$$

$$- P(y_o - e_y)u_o \frac{\pi^2}{\ell^2} \int_o^\ell [\text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell} - \alpha' (\text{Cos} \frac{\lambda z}{\ell} - \text{Cosh} \frac{\lambda z}{\ell})] \text{ Sin} \frac{\pi z}{\ell} dz$$

$$+ P(x_o - e_x)v_o \frac{\pi^2}{\ell^2} \int_o^\ell [\text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell} - \alpha' (\text{Cos} \frac{\lambda z}{\ell} - \text{Cosh} \frac{\lambda z}{\ell})]$$

$$\text{Sin} \frac{\pi z}{\ell} dz = - P^2 [\frac{e_x(y_o - e_y)}{EI_y} - \frac{e_y(x_o - e_x)}{EI_x}] \int_o^\ell [\text{Sin} \frac{\lambda z}{\ell} - \text{Sinh} \frac{\lambda z}{\ell}$$

$$- \alpha' (\text{Cos} \frac{\lambda z}{\ell} - \text{Cosh} \frac{\lambda z}{\ell})]dz \qquad [3.12]$$

Carrying out the necessary integrations (as shown in Appendix 4) and simplifying, we get

$$\begin{bmatrix} P_y - P & 0 & -P(y_o - e_y)1.4196 \\ 0 & P_x - P & P(x_o - e_x)1.4196 \\ -P(y_o - e_y)0.5507 & P(x_o - e_x)0.5507 & 4.1302EC_w \frac{\pi^2}{\ell^2} + (GJ - Pr_o^2)1.0019 \end{bmatrix} \begin{Bmatrix} u_o \\ v_o \\ \phi_o \end{Bmatrix}$$

$$= \begin{Bmatrix} -\frac{P^2 e_x}{P_y} 1.2732 \\ -\frac{P^2 e_y}{P_x} 1.2732 \\ -P^2 \left[\frac{e_x(y_o - e_y)}{P_y} - \frac{e_y(x_o - e_x)}{P_x} \right] 0.6561 \end{Bmatrix}$$

Substitution of the values of u_o, v_o and ϕ_o in Eq. [3.7] to [3.9] will give the desired deflection components of the shear centre, viz., u, v and ϕ at any particular location z along the longitudinal axis of the column, corresponding to a given applied load P .

Using these deflection components, stress at any point on the cross-section can be computed from (Culver 1966)

$$\sigma_T = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \qquad [3.13]$$

where,

σ_1 = uniform axial compressive stress

= P/A

σ_2 = linearly varying stress due to bending about the major centroidal principal axis (x-x axis)

$$= \frac{P \cdot e_1}{I_x} y$$

$$= \frac{P[e_y - v - (e_x - x_o)\phi]}{I_x} y$$

σ_3 = linearly varying stress due to bending about the minor centroidal principal axis (y-y axis)

$$= \frac{P \cdot e_2}{I_y} x$$

$$= \frac{P[e_x - u + (e_y - y_o)\phi]}{I_y} x$$

σ_4 = warping normal stress due to non-uniform torsion

$= E(\bar{\omega}_s - \omega_s)\phi''$

where,

ω_s = warping function according to Timoshenko and Gere (1961)

$\bar{\omega}_s$ = average value of ω_s

3.2 ASCE MANUAL NO. 52

The American Society of Civil Engineers Manual No. 52, entitled "Guide for Design of Steel Transmission Towers" (ASCE 1971), has been the basis for the design of hot-rolled angle shapes used in latticed steel transmission towers. The recommendations are not to be used when the width-to-thickness ratio, b/t, exceeds 20; the width 'b' is the width measured from the toe of the angle to the root of the fillet. Since there are no fillets in cold-formed angles, the b/t ratio is taken as the flat width-to-thickness ratio for the purpose of calculating the critical stresses of test specimens. The equations given in ASCE Manual No. 52 are in Imperial System of units; in the following, the equations have been converted to SI system.

If the width-to-thickness ratio does not exceed the limiting b/t ratio given by

$$(b/t)_{limit} = 208/\sqrt{\sigma_y} \qquad [3.14]$$

where σ_y is the yield stress of the material in MPa, then the member is sufficiently compact to develop its material yield stress and the ultimate maximum stress, σ_{cr}, is obtained from

$$\sigma_{cr} = [1 - \frac{(K\ell/r_y)^2}{2C_c^2}]\sigma_y \quad \text{if } \frac{K\ell}{r_y} \leq C_c \qquad [3.15]$$

and

$$\sigma_{cr} = \frac{\pi^2 E}{(K\ell/r_y)^2} \quad \text{if } \frac{K\ell}{r_y} > C_c \qquad [3.16]$$

in which

$$C_c = \pi\sqrt{\frac{2E}{\sigma_y}} \qquad [3.17]$$

where,

E = modulus of elasticity of the material

= 205 GPa (assumed in the computation)

and $K\ell/r_y$ = largest effective slenderness ratio.

If the width-to-thickness ratio exceeds limiting b/t ratio, then Eq. [3.15] and [3.17] shall be modified by substituting $\sigma_{y,eff}$ for σ_y where

$$\sigma_{y,eff} = [\,1.8 - \frac{0.8(b/t)}{(b/t)_{limit}}\,]\sigma_y \qquad [3.18]$$

$$\text{if } (b/t)_{limit} \leq (b/t) \leq 1.5(b/t)_{limit}$$

and

$$\sigma_{y,eff} = \frac{57900}{(b/t)^2} \quad \text{if } b/t > 1.5(b/t)_{limit} \qquad [3.19]$$

For members with normal framing eccentricities at both ends and $\ell/r_y < 120$,

$$K\ell/r_y = 60+0.50\,\ell/r_y \qquad [3.20]$$

For members unrestrained against rotation at both ends and $120 < \ell/r_y < 200$,

$$K\ell/r_y = \ell/r_y \qquad [3.21]$$

For members partially restrained against rotation at both ends and $120 < \ell/r_y < 250$,

$$K\ell/r_y = 46.2+0.615\ \ell/r_y \qquad [3.22]$$

Furthermore, single-bolted connection shall not be considered as offering restraint against rotation, whereas a multiple-bolted connection properly detailed to minimise eccentricities shall be considered to offer partial restraint if the connection is made to a sufficiently strong member.

Failure loads computed according to ASCE Manual No. 52 for the specimens included in the experimental investigation are shown in Table 6.7a.

3.3 ECCS RECOMMENDATIONS

The loads in eccentrically loaded cold-formed angle struts are transmitted by bolts or other connectors on only one leg. When these members are the web members of latticed trusses and are relatively flexible, their compressive strength is influenced by the stiffness of the connections to the main members.

For these members, the critical stresses are calculated by entering the curve B3 (for thickness < 20 mm) (Annexure B of European Recommendations for Steel Construction, ECCS 1978) shown in Table 6.4, with :

$$\left(\frac{\lambda}{\lambda_y}\right)_{eff} = 0.60 + 0.57\,\frac{\lambda}{\lambda_y} \qquad \text{if } \frac{\lambda}{\lambda_y} \leq 1.41 \qquad [3.23]$$

$$\left(\frac{\lambda}{\lambda_y}\right)_{eff} = \frac{\lambda}{\lambda_y} \qquad \text{if } \frac{\lambda}{\lambda_y} > 1.41 \qquad [3.24]$$

where

$$\frac{\lambda}{\lambda_y} = \frac{K\ell/r_y}{\pi\sqrt{E/\sigma_y}} \qquad [3.25]$$

where E = 210 GPa for cold-formed angles.

When the chords and the web members do not attain their maximum stress level for the same load conditions

$$\left(\frac{\lambda}{\lambda_y}\right)_{eff} = 0.35 + 0.75\,\frac{\lambda}{\lambda_y} \qquad \text{if } 1.41 \leq \frac{\lambda}{\lambda_y} \leq 3.5 \qquad [3.26]$$

may be used if the chords give good end-restraints to the web members and at least two bolts or rivets in line are present at the end connections of the web members.

Failure loads computed according to ECCS Recommendations for the specimens included in the experimental investigation are shown in Table 6.7a.

Chapter IV

COMPUTER PROGRAM

4.1 GENERAL

Calculation of the deflection components (i.e., u, v and ϕ) from the general theory of torsional-flexural buckling using the Galerkin method is a complicated procedure. This is further aggravated by the need of the actual cross-sectional properties of the specimen. A computer program was developed not only to calculate the actual cross-sectional properties including area, moments of inertia, location of the centroid, warping and torsional constants, minimum radius of gyration, location of the shear centre, β_1 and β_2, but also to estimate the deflection components and calculate the stresses at any point on the cross-section. It then predicts the ultimate compressive strengths based on the failure criteria.

4.2 DESCRIPTION OF THE COMPUTER PROGRAM

The computer program was written in FORTRAN IV language and run on IBM/3031 system. The program can be broadly divided into three parts:

1. Calculation of the actual cross-sectional properties.

2. Estimation of the deflection components using the Galerkin method.
3. Calculation of the total stress, σ_T at any point at any cross-section.

The flow chart of the program is given in Appendix 2 and a complete listing of it is given in Appendix 3. The details of the program are discussed in the following sections.

4.2.1 Main Program

The main program can be broadly divided into the following:

1. Input the data
   - a) Width of the long leg
   - b) Width of the short leg
   - c) Thickness
   - d) Inside bend radius
   - e) Thickness of the connecting gusset plate
   - f) Connected leg
   - g) Gauge distance
   - h) Assumed yield stress
2. Calculate the cross-sectional properties (i.e., A, $I_{x'}$, $I_{y'}$, I_x, I_y, $P_{x'y'}$, I_o, x', y', α, r_y, J, C_w, x_s, y_s, β_1, β_2), for which the theoretical formulation is shown in Appendix 1.
3. Compute P_x, P_y and P_ϕ.

4. Start the iterations at 0.1% of minimum of P_x, P_y and P_ϕ.
5. Find the deflection components (u, v and ϕ) corresponding to this load.
6. Calculate the total stress at each of the 16 points (as shown in Fig. 4.1) for each of the 11 cross-sections (the whole column length being divided into 10 equal segments).
7. Check the failure criteria.
   a) If satisfied, write the input data, calculated cross-sectional properties and estimated ultimate compressive strength.
   b) If not, modify the failure load and repeat the calculations.
8. Exit when all the required information have been obtained.

Output of this program consists of cross-sectional properties and the ultimate compressive strengths for ℓ/r_y ratios ranging from 20 to 240. Ultimate compressive strengths can also be obtained for $C_w = 0$ and two approximate locations of the shear centre, SC_1 and SC_2 (Fig. 4.3).

4.2.2 Subroutines

1. MXCP12, MXCP34, MXCP78: Compute the moment of the shear flow in the curved portion for different conditions.

2. CWACTL: Calculates the magnitude of the actual warping constant.
3. INTGRL: Estimates the value of an integral by adaptive quadrature technique using Simpson's rule (Johnson and Riess 1982).
4. ADSIMP : Estimates the value of an integral in the sub-interval.
5. DEFROT : Calculates the deflection components at any cross-section using the Galerkin method.
6. SIGMA : Computes the stresses at each of the 16 different points of each of the 11 cross-sections and returns the maximum and minimum stresses at any of those 11 X 16 points.
7. SOLVE: Solves the linear equation $A \sin\theta + B \cos\theta + C = 0$ in order to determine the point of maximum or minimum stress, if any, in the curved portion of the angle.

4.3 ADVANTAGES OF THE PROGRAM

This program needs very few input data. For each specimen, only one card is required to feed the program. It generates all the cross-sectional properties and other information required in order to estimate the ultimate compressive strength. It has several built-in error messages so that the user can be warned against some unusual occurrences. This program can be run for any leg size, thickness, inside bend radius, thickness of gusset plate, gauge distance and yield stress. An iterative approach is used for the calculation of ultimate strength and the smallest load increment in the final steps of iteration is taken as 0.001% of the minimum of P_x, P_y and P_ϕ. However, the accuracy of the failure load depends also on the functions used for the deflection components.

With minor modifications, this program is capable of providing load-deflection relationships for cold-formed angles under biaxial bending.

4.4 LIMITATIONS OF THE PROGRAM

This program does not take the effect of material non-linearity and initial out-of-straightness of the columns into account. Also, it is not possible to trace the unloading part of the load-deflection curves.

4.5 FAILURE CRITERIA

It has been assumed that the column has failed if the maximum total stress (tensile or compressive) at any point in the column reaches the yield stress level or at least one of the deflection components changes its sign.

4.6 RESULTS

All the cross-sectional properties are presented in Tables 6.1 and 6.2. Ultimate compressive strengths for two commonly used cold-formed angle sections for ℓ/r_y ratios varying from 20 to 240 for different gauge distances are presented in Table 6.3. Theoretical load-deflection relationships for pinned-end boundary conditions are compared with experimental results and are shown in Fig. 6.1 through 6.7.

Chapter V

EXPERIMENTAL INVESTIGATION

5.1 GENERAL

Tests were carried out on fifteen 65 X 50 X 4 mm and nine 55 X 55 X 4 mm cold-formed angles. Since these thin bar size angle members are principally used as web members in latticed towers, the lower limit of slenderness ratio was set at 80.

Table 6.5 gives the cross-sectional properties while Table 6.6 shows the lengths and slenderness ratios of the cold-formed angles used in this program. These angles were roll-formed from CSA G40.21M 300W grade sheet. The test specimens are cut from fairly straight portion of the factory supplies. However, specific data regarding the magnitude of initial imperfections and residual stress are not available.

5.2 TEST SET-UP

The test set-up shown in Fig. 5.1 consists of four angle members (including the test specimen) connected to the wide flange column of the main test facility. The arrangement resembles one panel of one face of a latticed tower. Load was applied through a ball bearing at joint B (Fig. 5.1). To prevent the transmission of load to member AB, the gusset plate at joint A was loosely connected to the steel wide flange column of the main test facility so that member AB is free to rotate about joint A.

5.2.1 Lateral Support

The test set-up was laterally supported at joints B and C as shown in Fig. 5.3 and 5.4. Rollers were used in the lateral supports to allow vertical but not the lateral movement. However, there could be some out-of-plane rotation of the test specimens.

5.2.2 Deflection Dial Gauges

Four deflection dial gauges with 25 mm travel and a least reading of 0.01 mm were used at midspan of the test specimens. Their arrangement is shown in Fig. 5.5. Two of the dial gauges were used to measure horizontal deflection and twist of the vertical leg, while the other two were used to measure vertical deflection and twist of the horizontal leg.

5.2.3 Supports for Dial Gauges

A specially designed L-shaped extension piece was constructed from metal strips, allowing easy fastening and removal from the specimen by means of a long bolt as shown in Fig. 5.6. This extension piece enabled the measurement of deflection and rotation at midspan. The gauge support bracket, as shown in Fig. 5.7, was made from angle section. This bracket allowed fast and simple attachment of the dial gauges as well as their adjustment on the L-shaped extension piece.

5.3 TESTING PROCEDURE

Test specimens were connected either with one, two or three 16mm diameter ASTM A325M high strength bolts at both ends at a gauge distance of one-half the nominal width of the connected leg. Fig. 5.2 shows the arrangement for load application.

The compressive load was applied by means of a 445 kN mechanical jack. The load was applied in increments of approximately one-twentieth of the predicted ultimate load until failure of the test specimen occurred. The load on the specimen was measured with the help of a previously calibrated load cell. Deflections at each load increment were recorded.

The experimental failure loads are listed in Table 6.7a while the typical load-deflection curves at mid-span for some of the test specimens are presented in Fig. 6.1 through 6.7 and compared with the theoretical results obtained earlier.

Chapter VI.

DISCUSSION OF RESULTS

6.1 CROSS-SECTIONAL PROPERTIES

The angles included in the theoretical investigation have a maximum leg width of 200 mm and correspond to the sizes of hot-rolled angles (CISC 1980). The maximum thickness is limited to 25 mm, it being the upper limit for the applicability of cold-formed steel design specifications (AISI 1980).

The inside bend radius is taken as '2t' for $t \leqslant 4$ mm, and '3t' for $t > 4$ mm, where t is the thickness of the leg of the angle. These bend radii are satisfactory for steels having guaranteed minimum yield stress of 300 MPa (Algoma Steel 1978). For higher strength steels, larger bend radii are required.

6.2 ULTIMATE COMPRESSIVE STRENGTHS

Table 6.3, which shows the ultimate compressive strengths of cold-formed angle sections for ℓ/r_y ratios ranging from 20 to 240 under pinned-end boundary conditions for different gauge distances, is based on an assumed yield stress level of 300 MPa, modulus of elasticity of cold-formed steels of 205 GPa and modulus of rigidity of 77 GPa. In addition, the load is assumed to be acting along the centre-line of the bolt (defined by the gauge distance) on the outer face of the connected leg.

As expected, it can be seen from Table 6.3 that ultimate compressive strength reduces gradually and continuously for increasing ℓ/r_y ratio for a majority of the cases. However, for certain gauge distances, the compressive strength remains more or less constant over a certain range of ℓ/r_y ratio. This can be understood if one bears in mind the coupling of the bending about the x-x axis and y-y axis and twisting about longitudinal z-z axis. This can further be attributed to the nature of the assumed deflection functions.

The effective eccentricities, e_1 and e_2, depend on the variables u, v and ϕ in addition to the constants e_x, e_y, x_o and y_o. For certain combinations of e_x, e_y, u and v, e_2 becomes negative in sign, while e_1 remains positive. Sometimes, the increase of e_2 is much more than that of e_1 (because of the larger increase in the u-component of the deflection) and therefore, the total compressive stress, tends to remain the same. This leads to the almost constant ultimate compressive strength over a certain range of ℓ/r_y ratios for certain gauge distances.

The same trend (i.e., greater increase in e_2 over e_1) is also confirmed by the numerical results presented by Culver (1966). For identical boundary conditions in his investigation, it was found that the rate of increase of σ_3 is much more than that of σ_2 when ℓ/r_y ratio was increased from 60 to 140, due to the rapid increase in the u-component of the deflection together with the rotation, ϕ.

This behaviour occurs only when the point of load application is close to the point of intersection of the y-y axis and the centre-line of the connected leg.

It is to be noted that the failure criterion adopted here, viz. failure at first yield, leads to a lower bound solution. However, if the residual stresses are considered, the critical point, i.e., the point at which the resultant total stress reaches the yield stress, may be different from the one determined from theoretical formulation. In some cases, this may lead to actual failure loads being less than those predicted by the present theory.

Angles with large width-to-thickness ratio tend to fail by local buckling of the legs, much before their overall failure. Such local instability can be taken into account by appropriately modifying the yield stress of the material, as is done in ASCE Manual No. 52.

6.3 LOAD-DEFLECTION RELATIONSHIPS

6.3.1 Experimental Results

The experimental results, consisting of horizontal and vertical deflections and twists of the legs, are resolved into three deflection components, u, v and ϕ with respect to shear centre and are shown in Fig. 6.1 through 6.7. Results shown here are those at mid-span only.

6.3.2 Exact Solution of Differential Equations

Culver's exact solution of differential equations, involving twelve unknown constants of integration in twelve simultaneous equations, includes trigonometric and hyperbolic functions. The hyperbolic functions are difficult to evaluate for large values of arguments making the use of the exact solution computationally impossible. When the author tried to use this method for the test specimens under investigation, for some combinations of load, length and flexural rigidities, algorithmic singularity was encountered in the solution of those twelve equations.

6.3.3 Solution of Differential Equations applying the Galerkin method

A close observation of Fig. 6.1 through 6.7 will reveal that, for a given load, two of three experimental deflection components (usually the u-component and rotation, ϕ) are less than the theoretically computed values which establishes the fact that the experimental set-up offers a boundary condition which is different from pinned-ends, probably due to the rigidity of the connection and the influence of the other members of the test frame. In general, the experimental v-components of deflection are more than those of the theoretical deflections. Besides the usual assumptions involved in the theoretical formulation, some of the parameters possibly influencing the experimental results are discussed below.

Eccentricity: Depending on the gauge distance, the shape of the load-deflection curve changes. It can be shown theoretically that the load-deflection curve is quite sensitive to the gauge distance.

Effect of the test setup: In all these comparisons, the length of the member is assumed to be the distance from gauge to gauge of the connecting members. However, the test specimen actually includes a gusset plate at both ends. Since the properties of the gusset plate are quite different from those of the test specimen, ignoring the presence of the gusset plate and assuming the test specimen to be a member having uniform cross-sectional properties from gauge to gauge of the connecting members, results in discrepancies between theoretical and experimental load-deflection curves.

The most important feature of this comparison is the confirmation of the theoretical load-deflection curves by experimental results. So far, to the best of author's knowledge, no such information exists for cold-formed angles. In spite of some discrepancies, the trends of the respective curves are all alike. The shapes of all these load-deflection curves are very much similar to the corresponding curves of hot-rolled angle sections reported by Usami (1970).

6.3.4 Comparison of Exact and Galerkin Solutions

Table 6.9 shows a comparison of results obtained from exact and Galerkin solutions for 65 X 50 X 4 mm (long leg out) angle with a gauge distance of 25 mm and slenderness ratio of 20. This is one of the few cases for which the "exact" method worked, except for the smallest load. Loads shown in the table are those which the program calculated at each step of iteration, hence the decimal fractions. For the range of loading shown, per cent error varies from 0.0 to -2.0 for u- and v- components showing that the Galerkin method is sufficiently accurate. However, the error in the evaluation of ϕ is surprisingly high. But, the values of ϕ being very small, it is quite unlikely that such an error will have any significant effect on the eccentricities, e_1 and e_2. Therefore, it can be assumed that the calculated stresses would hardly differ from the exact stresses. Differences, if any, would be acceptable for all practical purposes.

6.4 COMPARISON OF EXPERIMENTAL FAILURE LOADS WITH THE PREDICTED LOADS

6.4.1 General

From a study of Table 6.5, it can be seen that the actual cross-sectional areas were 3-4% less than the nominal areas while the actual moments of inertia were about 9-10% less than those specified.

The experimental and theoretical loads are summarized in Table 6.7a. The computed failure loads based on actual dimensions are 5-8% less than those based on nominal dimensions. In the following sections, the predicted failure loads according to ASCE Manual No. 52, ECCS Recommendations and the general theory of torsional-flexural buckling are compared with the experimental failure loads.

6.4.2 ASCE Manual No. 52

6.4.2.1 Slenderness ratios less than 120

The calculated failure load is based only on eccentricity of loads; the number of bolts does not enter the computations. However, experimental failure loads for two- and three- bolted end connections are significantly higher than those for single-bolted end connections. An examination of Table 6.7a reveals that for two- and three- bolted end connections, the computed loads are in reasonable agreement with the experimental failure loads.

For single-bolted connections, the computed loads are upto 30% higher than the experimental failure loads. The discrepancy between the computed and experimental failure loads is higher for specimens having smaller slenderness ratios. This point should be kept in mind if ASCE Manual is applied for the design of cold-formed angles.

6.4.2.2 Slenderness ratios exceeding 120

For all the nine specimens tested, the experimental failure loads are 10-50% higher than the computed loads; therefore, the ASCE Manual can be safely used in the design of slender members. For such members, the Manual distinguishes between single- and multiple- bolted connections, higher values being allowed for multiple-bolted connections. However, the Manual does not make any distinction between two-bolted and three-bolted connections, whereas the experimental failure loads of specimens with three bolts are in all cases higher than the failure loads of specimens with two bolts.

6.4.3 <u>ECCS Recommendations</u>

For all slenderness ratios, the computed failure loads are either less or very close to the experimental failure loads. They are more conservative than ASCE Manual No. 52 and can be applied with confidence to the design of cold-formed angles connected by one leg.

6.4.4 <u>General Theory of Torsional-Flexural Buckling</u>

The failure loads according to the general theory of torsional-flexural buckling are given in Table 6.7b for all single-bolted columns included in the test program. Single-bolted connections are assumed to provide insignificant restraint to bending and therefore, pinned-end connections are assumed in the analysis. This is also in conformity with ASCE Manual No. 52 (1971). Since the line of action of the eccentric thrust does not pass through the point of zero longitudinal displacement (defined by the warping function), warping is restrained (Culver 1966).

A study of Tables 6.7a and 6.7b shows that the theoretical failure loads vary from 55-90% of the experimental failure loads. The discrepancy is more for slender columns compared to others. One possible reason for this discrepancy is the fact that slender columns are more sensitive to the end connections than stocky or intermediate columns. In the theoretical computations, the ends are conservatively assumed as pinned while for the test columns, the rigidity of the steel frame may have provided partial restraint.

Also, one end of the test specimens was allowed to move in the vertical plane while the other end, being connected to a rigid wide flange column, could be considered as partially fixed. This differential movement of the joints probably has produced additional curvature, and therefore bending moments, in the test specimens which has not been considered in the theoretical formulation.

6.5 LONG LEG CONNECTED VERSUS LONG LEG OUT

Neither the ASCE Manual No. 52 nor ECCS Recommendations makes any distinction in the failure loads of the angles with long leg connected or long leg out. However, tests carried on 65 X 50 X 4 mm cold-formed angles show that the failure loads are 8-30% higher if the long leg is out. Failure loads based on the general theory of torsional-flexural buckling also lead to the same conclusion.

6.6 EFFECT OF THE OTHER VARIABLES

To study the effect of the location of the shear centre and the magnitude of warping constant, failure loads are computed according to the general theory of torsional-flexural buckling for the following cases :

1. Exact location of the shear centre
   a) warping constant = 0
   b) warping constant calculated according to Timoshenko and Gere (1961)
2. Shear centre assumed at point 'SC_1' (Fig. 4.3)
   a) warping constant = 0
   b) warping constant calculated according to Timoshenko and Gere (1961)
3. Shear centre assumed at point 'SC_2' (Fig. 4.3)
   a) warping constant = 0
   b) warping constant calculated according to Timoshenko and Gere (1961)

The results for 65 X 50 X 4 mm cold-formed angle with long leg connected are summarized in Table 6.8.

A study of Table 6.8 shows that the effect of all these

Chapter VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 GENERAL

Though cold-formed angles are currently available only in thin bar-size sections, the properties of heavier sections are also included here for use in future, when cold-formed angles might become competitive with hot-rolled angles.

Cold-formed angles connected by one leg generally have decreasing strengths with increasing ℓ/r_y ratios, except when the point of load application is close to the point of intersection of the minor principal axis (y-y axis) with the centre-line of the connected leg. In such cases, ultimate compressive strength may remain more or less constant over a certain range of ℓ/r_y ratios. This behaviour could be attributed to the nature of assumed deflection functions.

7.2 CONCLUSIONS

From the theoretical and limited experimental study and for the test condition specified herein, the following conclusions can be drawn about the behaviour of thin bar-size cold-formed angles :

1. For certain combinations of load, length and bending rigidities of the member, computational problems will

arise in the determination of the constants of integration if Culver's "exact" procedure is followed for the solution of differential equations of equilibrium for columns under biaxial bending.

2. The application of the Galerkin method to predict theoretical load-deflection relationships is a simple, fast and sufficiently accurate alternative to the exact solution.
3. Failure loads computed according to ECCS Recommendations are, in general, conservative.
4. For stocky and intermediate specimens with single-bolted end connection, ASCE Manual No. 52 predicts failure loads which are higher than the experimental failure loads. For multiple-bolted connection, the predicted failure loads are either less than or very close to the experimental failure loads.
5. Loads computed according to general theory of torsional-flexural buckling (assuming pinned-end connection and warping restrained) are approximately 55-90% of experimental failure loads; the discrepancy is more for slender columns.
6. For all slenderness ratios, the strength of member increases when the number of bolts in the end connection is increased from one to three.
7. Unequal leg cold-formed angles are capable to carry higher loads when the long leg is out.

8. The usual assumptions regarding the location of the shear centre (SC_1 and SC_2, as shown in Fig. 4.3) and the magnitude of warping constant ($C_w = 0$) are sufficiently accurate to predict failure loads according to the general theory of torsional-flexural buckling.

7.3 RECOMMENDATIONS FOR FUTURE RESEARCH

Based on the experience gained during the investigation, the following recommendations are being made :

1. Further research may be directed to include the effect of residual stresses in the calculation of ultimate strengths.
2. Initial out-of-straightness could also be investigated in order to estimate its effect on the ultimate compressive strength.
3. Finally, a simple and useful interaction type formula may be obtained for cold-formed angle sections under biaxial bending.

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VITA AUCTORIS

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Table 6.1 Cross-Sectional Properties (full section) of Cold-Formed Angles - Equal Angles

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x'		Axis y'-y'		Product of Inertia	Axis x-x		Axis y-y		J	Location of Shear Center		I_0	C_w	β_1*	β_2*
						$I_{x'}$	y'	$I_{y'}$	x'		I_x	α	I_y	r_y		x_s	y_s				
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
200 x 200	25	75	66.2	0.649	8440	31.3	66.0	31.3	66.0	-23.1	54.4	45.0	8.24	31.3	1760	41.3	41.3	90.9	1520	0.00	292
	20	60	54.9	0.539	7000	26.7	62.5	26.7	62.5	-18.8	45.5	45.0	7.90	33.6	933	44.5	44.5	81.0	721	0.00	290
	16	48	45.2	0.443	5760	22.4	59.8	22.4	59.8	-15.2	37.7	45.0	7.19	35.3	491	46.6	46.6	69.8	308	0.00	288
	13	39	37.5	0.368	4780	18.9	57.8	18.9	57.8	-12.5	31.4	45.0	6.39	36.6	269	47.9	47.9	59.7	131	0.00	287
	10	30	29.4	0.289	3750	15.0	55.9	15.0	55.9	- 9.69	24.7	45.0	5.33	37.7	125	48.9	48.9	48.0	41.9	0.00	286
150 x 150	20	60	39.2	0.385	5000	10.3	50.5	10.3	50.5	- 7.72	18.1	45.0	2.61	22.8	667	30.0	30.0	29.5	326	0.00	220
	16	48	32.7	0.320	4160	8.86	47.6	8.86	47.6	- 6.32	15.2	45.0	2.55	24.7	355	32.8	32.8	26.6	161	0.00	218
	13	39	27.3	0.268	3480	7.57	45.5	7.57	45.5	- 5.21	12.8	45.0	2.36	26.1	196	34.5	34.5	23.4	75.1	0.00	217
	10	30	21.6	0.212	2750	6.10	43.5	6.10	43.5	- 4.05	10.2	45.0	2.05	27.3	91.7	35.8	35.8	19.3	25.9	0.00	216
125 x 125	16	48	26.4	0.259	3360	4.85	41.6	4.85	41.6	- 3.59	8.44	45.0	1.26	19.4	287	25.5	25.5	14.0	97.2	0.00	183
	13	39	22.2	0.218	2830	4.20	39.4	4.20	39.4	- 2.98	7.17	45.0	1.22	20.8	159	27.5	27.5	12.6	49.5	0.00	181
	10	30	17.7	0.173	2250	3.42	37.4	3.42	37.4	- 2.33	5.75	45.0	1.10	22.1	75.0	29.2	29.2	10.7	18.4	0.00	180
	8	24	14.4	0.142	1840	2.84	36.1	2.84	36.1	- 1.88	4.72	45.0	0.965	22.9	39.3	30.0	30.0	8.99	7.31	0.00	180
100 x 100	16	48	20.1	0.197	2560	2.27	35.7	2.27	35.7	- 1.78	4.05	45.0	0.495	13.9	218	17.9	17.9	6.12	44.1	0.00	149
	13	39	17.1	0.168	2180	2.01	33.4	2.01	33.4	- 1.49	3.50	45.0	0.515	15.4	123	20.3	20.3	5.78	26.7	0.00	146
	10	30	13.7	0.135	1750	1.67	31.2	1.67	31.2	- 1.18	2.84	45.0	0.494	16.8	58.3	22.3	22.3	5.06	11.3	0.00	145
	8	24	11.3	0.111	1440	1.40	29.9	1.40	29.9	- 0.953	2.35	45.0	0.450	17.7	30.7	23.3	23.3	4.37	4.81	0.00	144

Table 6.1 cont'd

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x' $I_{x'}$	Axis x'-x' y'	Axis y'-y' $I_{y'}$	Axis y'-y' x'	Product of Inertia	Axis x-x I_x	Axis x-x α	Axis y-y I_y	Axis y-y r_y	J	Location of Shear Center x_s	Location of Shear Center y_s	I_0	C_w	β_1*	β_2*
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
	6	18	8.71	0.085	1110	1.10	28.6	1.10	28.6	-0.723	1.82	45.0	0.379	18.5	13.3	24.1	24.1	3.49	1.45	0.00	144
90 x 90	13	39	15.0	0.148	1920	1.41	31.0	1.41	31.0	-1.07	2.48	45.0	0.335	13.2	108	17.2	17.2	3.93	18.8	0.00	133
	10	30	12.2	0.119	1550	1.18	28.8	1.18	28.8	-0.850	2.03	45.0	0.333	14.7	51.7	19.4	19.4	3.52	8.66	0.00	131
	8	24	10.0	0.099	1280	1.00	27.4	1.00	27.4	-0.691	1.69	45.0	0.310	15.6	27.3	20.6	20.6	3.08	3.86	0.00	130
	6	18	7.77	0.076	990	0.791	26.1	0.791	26.1	-0.525	1.32	45.0	0.266	16.4	11.9	21.5	21.5	2.50	1.21	0.00	129
75 x 75	13	39	12.0	0.118	1530	0.748	27.6	0.748	27.6	-0.597	1.35	45.0	0.151	9.94	86.0	12.3	12.3	1.94	9.12	0.00	112
	10	30	9.81	0.096	1250	0.646	25.2	0.646	25.2	-0.483	1.13	45.0	0.163	11.4	41.7	15.0	15.0	1.85	5.10	0.00	110
	8	24	8.16	0.080	1040	0.554	23.8	0.554	23.8	-0.395	0.949	45.0	0.159	12.4	22.2	16.4	16.4	1.66	2.52	0.00	109
	6	18	6.36	0.062	810	0.444	22.4	0.444	22.4	-0.301	0.745	45.0	0.142	13.3	9.72	17.5	17.5	1.38	0.857	0.00	108
	5	15	5.40	0.053	687	0.381	21.8	0.381	21.8	-0.253	0.635	45.0	0.128	13.7	5.73	17.9	17.9	1.20	0.405	0.00	108
65 x 65	10	30	8.24	0.081	1050	0.397	22.9	0.397	22.9	-0.307	0.704	45.0	0.090	9.24	35.0	12.0	12.0	1.09	3.07	0.00	96.4
	8	24	6.91	0.068	880	0.346	21.4	0.346	21.4	-0.254	0.599	45.0	0.092	10.2	18.8	13.5	13.5	1.01	1.71	0.00	94.9
	6	18	5.42	0.053	690	0.280	20.0	0.280	20.0	-0.195	0.475	45.0	0.086	11.1	8.28	14.8	14.8	0.859	0.633	0.00	94.0
	5	15	4.61	0.045	587	0.242	19.3	0.242	19.3	-0.164	0.406	45.0	0.079	11.6	4.90	15.3	15.3	0.757	0.310	0.00	93.7

Table 6.1 con'td

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x'		Axis y'-y'		Product of Inertia	Axis x-x		Axis y-y		J	Location of Shear Center		I_0	C_w	β_1*	β_2*
						$I_{x'}$	y'	$I_{y'}$	x'		I_x	α	I_y	r_y		x_s	y_s				
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
55 x 55	10	30	6.67	0.065	850	0.221	20.6	0.221	20.6	-0.179	0.400	45.0	0.042	7.06	20.3	8.52	8.52	0.558	1.52	0.00	83.1
	8	24	5.65	0.055	720	0.197	19.0	0.197	19.0	-0.150	0.347	45.0	0.047	8.05	15.4	10.5	10.5	0.549	0.994	0.00	81.1
	6	18	4.47	0.044	570	0.163	17.5	0.163	17.5	-0.117	0.279	45.0	0.046	9.01	6.84	11.9	11.9	0.487	0.424	0.00	79.9
	5	15	3.83	0.038	487	0.142	16.8	0.142	16.8	-0.098	0.240	45.0	0.044	9.46	4.06	12.5	12.5	0.436	0.219	0.00	79.5
	4	8	3.19	0.031	407	0.121	15.9	0.121	15.9	-0.078	0.199	45.0	0.043	10.2	2.17	13.3	13.3	0.384	0.030	0.00	77.9
	3	6	2.44	0.024	311	0.094	15.3	0.094	15.3	-0.059	0.153	45.0	0.034	10.5	0.934	13.5	13.5	0.301	0.008	0.00	77.9
45 x 45	8	24	4.39	0.043	560	0.098	16.7	0.098	16.7	-0.079	0.177	45.0	0.019	5.06	11.9	7.16	7.16	0.251	0.443	0.00	67.7
	6	18	3.53	0.035	450	0.084	15.1	0.084	15.1	-0.063	0.146	45.0	0.021	6.85	5.40	9.01	9.01	0.239	0.238	0.00	66.0
	5	15	3.04	0.030	387	0.074	14.4	0.074	14.4	-0.053	0.127	45.0	0.021	7.33	3.23	9.71	9.71	0.220	0.135	0.00	65.4
	4	8	2.57	0.025	327	0.064	13.4	0.064	13.4	-0.042	0.106	45.0	0.022	8.17	1.74	10.7	10.7	0.202	0.021	0.00	63.7
	3	6	1.97	0.019	251	0.050	12.8	0.050	12.8	-0.032	0.082	45.0	0.018	8.45	0.754	10.9	10.9	0.160	0.006	0.00	63.7

*For the definition of β_1 and β_2, refer to Timoshenko and Gere, 1961.

Table 6.2 Cross-Sectional Properties (full section) of Cold-Formed Angles - Unequal Angles

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x′-x′ $I_{x'}$	Axis x′-x′ y'	Axis y′-y′ $I_{y'}$	Axis y′-y′ x'	Product of Inertia	Axis x-x I_x	Axis x-x α	Axis y-y I_y	Axis y-y r_y	J	Location of Shear Center x_s	Location of Shear Center y_s	I_0	C_w	β_1*	β_2*
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
200 x 150	25	75	56.4	0.553	7190	27.0	75.4	13.6	47.1	-14.5	36.3	32.6	4.35	24.6	1500	22.5	39.0	54.8	1680	65.7	215
	20	60	47.1	0.462	6000	23.5	71.2	11.7	43.7	-11.9	30.9	31.9	4.31	26.8	800	25.9	46.3	51.9	690	69.2	231
	16	48	38.9	0.382	4960	19.9	68.1	9.93	41.2	- 9.70	25.8	31.4	4.02	28.5	423	28.1	50.7	46.5	281	73.2	237
	13	39	32.4	0.318	4130	16.9	65.9	8.41	39.4	- 7.97	21.7	31.0	3.62	29.6	232	29.9	53.5	40.8	103	76.2	240
200 x 100	20	60	39.2	0.385	5000	18.9	83.5	3.59	27.5	- 5.95	21.0	18.9	1.55	17.6	667	18.8	47.8	35.6	464	123	-12.2
	16	48	32.7	0.320	4160	16.5	79.7	3.06	25.1	- 4.89	18.1	18.1	1.46	18.8	355	17.6	55.4	33.5	214	128	85.0
	13	39	27.3	0.268	3480	14.2	77.0	2.61	23.3	- 4.04	15.4	17.5	1.34	19.6	196	17.1	59.9	30.2	96.2	132	127
	10	30	21.6	0.212	2750	11.5	74.4	2.12	21.7	- 3.16	12.4	17.0	1.15	20.5	91.7	16.9	63.3	25.4	37.1	136	152
150 x 100	16	48	26.4	0.259	3360	7.30	57.0	2.76	29.1	- 3.28	9.02	27.7	1.03	17.6	287	15.1	33.5	14.5	132	65.8	139
	13	39	22.2	0.218	2830	6.34	54.5	2.38	27.2	- 2.73	7.73	27.0	0.991	18.7	159	17.4	38.4	13.7	45.7	69.1	156
	10	30	17.7	0.173	2250	5.19	52.1	1.94	25.4	- 2.13	6.25	26.3	0.885	19.8	75.0	19.2	41.9	11.9	18.6	72.8	165
	8	24	14.4	0.142	1840	4.32	50.6	1.61	24.3	- 1.72	5.16	25.9	0.776	20.5	39.3	19.4	43.3	10.1	7.39	75.1	166

Table 6.2 cont'd

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x'		Axis y'-y'		Product of Inertia	Axis x-x		Axis y-y		J	Location of Shear Center		I_0	C_w	β_1*	β_2*
						$I_{x'}$	y'	$I_{y'}$	x'		I_x	α	I_y	r_y		x_s	y_s				
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
125 x 90	16	48	22.0	0.216	2800	4.08	48.3	1.87	28.4	- 2.11	5.36	31.2	0.599	14.6	239	12.8	23.6	7.91	100	45.5	120
	13	39	18.6	0.183	2370	3.60	45.7	1.63	26.3	- 1.76	4.64	30.4	0.600	15.9	134	14.6	28.3	7.62	47.1	47.4	136
	10	30	14.9	0.146	1900	2.99	43.3	1.35	24.4	- 1.39	3.78	29.7	0.550	17.1	63.3	16.6	32.2	6.81	14.4	50.5	143
	8	24	12.2	0.120	1560	2.50	41.8	1.13	23.2	- 1.12	3.13	29.3	0.500	17.9	33.3	17.9	34.2	5.95	5.23	52.7	146
125 x 75	13	39	17.1	0.168	2180	3.27	49.2	0.960	21.3	- 1.29	3.85	24.1	0.381	13.2	123	11.5	28.8	6.29	22.7	64.2	85.1
	10	30	13.7	0.135	1750	2.75	46.6	0.796	19.5	- 1.02	3.18	23.2	0.358	14.3	58.3	13.6	34.1	5.89	11.6	67.8	114
	8	24	11.3	0.111	1440	2.32	45.0	0.669	18.3	- 0.830	2.66	22.6	0.323	15.0	30.7	13.8	36.3	5.15	5.39	70.6	122
	6	18	8.71	0.085	1110	1.82	43.4	0.527	17.2	- 0.631	2.08	22.1	0.271	15.6	13.3	13.8	37.8	4.15	1.59	73.0	126
100 x 90	13	39	16.1	0.158	2050	1.91	35.1	1.48	29.5	- 1.26	2.97	40.2	0.413	14.2	115	16.4	19.8	4.71	25.6	13.5	136
	10	30	13.0	0.127	1650	1.60	32.8	1.24	27.4	- 0.998	2.43	39.9	0.402	15.6	55.0	18.2	22.5	4.20	11.5	14.6	135
	8	24	10.7	0.105	1360	1.34	31.4	1.04	26.1	- 0.810	2.02	39.7	0.370	16.5	29.0	19.3	23.9	3.66	5.14	15.4	135
	6	18	8.24	0.081	1050	1.06	30.1	0.821	24.8	- 0.615	1.57	39.5	0.314	17.3	12.6	20.1	25.0	2.96	1.63	16.2	135

Table 6.2 cont'd

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x'		Axis y'-y'		Product of Inertia	Axis x-x		Axis y-y		J	Location of Shear Center		I_0	C_w	β_1*	β_2*
						$I_{x'}$	y'	$I_{y'}$	x'		I_x	α	I_y	r_y		x_s	y_s				
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
100 x 75	13	39	14.5	0.143	1850	1.73	38.1	0.872	23.9	-0.936	2.33	32.7	0.270	12.1	104	11.0	18.7	3.44	29.3	32.7	105
	10	30	11.8	0.115	1500	1.47	35.6	0.733	21.9	-0.745	1.93	31.9	0.269	13.4	50.0	12.9	23.1	3.24	10.8	34.6	116
	8	24	9.73	0.095	1240	1.25	34.1	0.621	20.6	-0.606	1.62	31.4	0.251	14.2	26.5	14.1	25.4	2.90	4.40	36.6	119
	6	18	7.54	0.074	960	0.988	32.6	0.492	19.4	-0.461	1.26	30.9	0.216	15.0	11.5	15.3	27.2	2.41	1.10	38.6	121
90 x 75	13	39	13.5	0.133	1720	1.27	33.8	0.828	25.2	-0.798	1.88	37.2	0.222	11.4	97.0	11.5	15.5	2.72	20.8	19.7	110
	10	30	11.0	0.108	1400	1.09	31.4	0.702	23.1	-0.638	1.56	36.6	0.228	12.8	46.7	13.6	19.4	2.57	8.08	20.8	114
	8	24	9.10	0.089	1160	0.927	29.9	0.597	21.7	-0.520	1.31	36.2	0.216	13.7	24.7	14.7	21.4	2.30	4.21	22.1	115
	6	18	7.06	0.069	900	0.738	28.4	0.474	20.5	-0.396	1.02	35.8	0.189	14.5	10.8	15.7	22.9	1.90	1.34	23.5	116
	5	15	5.99	0.059	762	0.632	27.7	0.407	19.9	-0.332	0.870	35.6	0.169	14.9	6.35	16.1	23.5	1.66	0.579	24.1	116
90 x 65	10	30	10.2	0.100	1300	1.01	33.4	0.464	19.5	-0.504	1.31	30.7	0.164	11.2	43.3	10.1	19.4	2.09	8.82	33.3	95.6
	8	24	8.48	0.083	1080	0.869	31.8	0.396	18.2	-0.413	1.11	30.1	0.157	12.1	23.0	11.5	22.2	1.94	3.27	35.3	102
	6	18	6.59	0.065	840	0.696	30.2	0.316	16.9	-0.315	0.874	29.5	0.138	12.8	10.1	12.7	24.4	1.65	0.858	37.5	105
	5	15	5.59	0.055	712	0.598	29.5	0.272	16.3	-0.265	0.746	29.2	0.124	13.2	5.94	13.2	25.1	1.44	0.422	38.5	106

Table 6.2 cont'd

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x' $I_{x'}$	Axis x'-x' y'	Axis y'-y' $I_{y'}$	Axis y'-y' x'	Product of Inertia	Axis x-x I_x	Axis x-x α	Axis y-y I_y	Axis y-y r_y	J	Location of Shear Center x_s	Location of Shear Center y_s	I_0	C_w	β_1*	β_2*
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
80 x 60	10	30	9.03	0.089	1150	0.692	30.1	0.349	18.8	-0.371	0.929	32.6	0.111	9.84	38.3	8.98	15.6	1.40	6.86	26.3	86.0
	8	24	7.53	0.074	960	0.601	28.5	0.300	17.5	-0.305	0.791	31.9	0.110	10.7	20.5	10.4	18.5	1.33	2.83	27.7	92.5
	6	18	5.89	0.058	750	0.485	26.9	0.242	16.2	-0.234	0.627	31.2	0.100	11.5	9.00	11.5	20.7	1.14	0.855	29.7	95.4
	5	15	5.00	0.049	637	0.419	26.2	0.208	15.6	-0.196	0.536	30.9	0.091	11.9	5.31	12.1	21.6	1.02	0.348	30.7	96.3
75 x 50	8	24	6.59	0.065	840	0.456	28.5	0.172	14.6	-0.205	0.564	27.7	0.065	8.77	17.9	7.57	16.8	0.909	2.06	32.9	69.7
	6	18	5.18	0.051	660	0.374	26.8	0.140	13.3	-0.158	0.454	26.8	0.060	9.55	7.92	9.05	19.9	0.826	0.509	35.2	79.9
	5	15	4.42	0.043	562	0.325	26.0	0.121	12.7	-0.133	0.390	26.3	0.055	9.92	4.69	9.60	21.0	0.744	0.291	36.4	82.3
65 x 50	8	24	5.96	0.058	760	0.303	24.1	0.162	15.7	-0.167	0.413	33.5	0.052	8.25	16.2	7.77	12.9	0.634	1.90	19.9	73.5
	6	18	4.71	0.046	600	0.250	22.5	0.133	14.3	-0.129	0.333	32.7	0.050	9.13	7.20	9.06	15.5	0.576	0.637	21.4	77.9
	5	15	4.02	0.039	512	0.218	21.8	0.116	13.7	-0.109	0.287	32.4	0.047	9.54	4.27	9.61	16.5	0.520	0.301	22.3	78.8
	4	8	3.35	0.033	427	0.185	20.7	0.098	12.9	-0.086	0.238	31.5	0.045	10.2	2.28	10.5	17.9	0.466	0.031	24.0	78.8
55 x 35	6	18	3.53	0.035	450	0.130	21.4	0.044	10.2	-0.056	0.158	26.2	0.017	6.08	5.40	5.16	12.1	0.251	0.282	25.9	43.7
	5	15	3.04	0.030	387	0.116	20.5	0.039	9.57	-0.048	0.139	25.5	0.016	6.46	3.23	5.81	13.8	0.241	0.122	26.9	52.0
	4	8	2.57	0.025	327	0.101	19.3	0.033	8.73	-0.038	0.118	24.0	0.017	7.13	1.74	6.54	15.9	0.232	0.022	29.3	57.2
	3	6	1.97	0.019	251	0.080	18.6	0.026	8.24	-0.029	0.092	23.7	0.014	7.37	0.754	6.62	16.4	0.184	0.006	30.2	58.0

Table 6.2 cont'd.

Size Long leg x Short leg	Thickness	Inside Bend Radius	Mass	Dead Load	Area	Axis x'-x'		Axis y'-y'		Product of Inertia	Axis x-x		Axis y-y		J	Location of Shear Center		I_0	C_w	β_1*	β_2*
						$I_{x'}$	y'	$I_{y'}$	x'		I_x	α	I_y	x_y		x_s	y_s				
mm x mm	mm	mm	kg/m	kN/m	mm^2	$10^6 mm^4$	mm	$10^6 mm^4$	mm	$10^6 mm^4$	$10^6 mm^4$	Deg.	$10^6 mm^4$	mm	$10^3 mm^4$	mm	mm	$10^6 mm^4$	$10^6 mm^6$	mm	mm
45 x 30	6	18	2.83	0.028	360	0.067	18.2	0.026	9.55	-0.032	0.084	28.7	0.008	4.78	4.32	4.07	7.92	0.120	0.176	19.3	29.3
	5	15	2.45	0.024	312	0.061	17.3	0.023	8.86	-0.028	0.075	27.8	0.008	5.18	2.60	4.43	9.72	0.119	0.113	19.6	40.3
	4	8	2.09	0.021	267	0.054	16.0	0.020	7.96	-0.022	0.065	26.0	0.009	5.93	1.42	5.68	12.4	0.124	0.015	21.7	48.3
	3	6	1.62	0.016	206	0.043	15.3	0.016	7.45	-0.017	0.051	25.6	0.008	6.18	0.619	5.78	13.0	0.101	0.005	22.6	49.0

*For the definition of β_1 and β_2, refer to Timoshenko and Gere, 1961.

Table 6.3 Ultimate Compressive Strength of Cold-Formed Angle Columns Under Biaxial Bending

Slenderness Ratio	ULTIMATE COMPRESSIVE STRENGTH (kN)											
	55 x 55 x 3 x 6						55 x 35 x 4 x 8 [Long Leg Connected]				55 x 35 x 4 x 8 [Long Leg Out]	
	g=15	g=20	g=25	g=30	g=35	g=40	g=20	g=25	g=30	g=35	g=15	g=20
20	32.5	40.4	51.0	35.7	27.1	21.7	35.1	44.1	51.4	37.6	42.6	27.8
30	31.9	39.4	50.4	35.6	26.9	21.5	34.3	42.8	51.3	37.5	42.4	27.5
40	30.9	38.0	48.3	35.6	26.6	21.3	33.2	41.2	51.1	37.5	41.9	27.2
50	29.8	36.4	45.9	35.5	26.3	20.9	31.9	39.2	49.7	37.5	41.1	26.9
60	28.5	34.6	43.1	35.5	25.9	20.5	30.5	37.0	46.2	37.5	40.2	26.4
70	27.1	32.6	40.2	35.5	25.4	20.1	28.9	34.7	42.3	37.4	39.0	25.8
80	25.6	30.5	37.2	35.5	24.8	19.5	27.2	32.3	37.9	37.3	37.5	25.2
90	24.1	28.5	34.3	35.5	24.1	18.9	25.5	29.6	34.1	37.2	35.8	24.4
100	22.6	26.4	31.4	35.4	21.0	18.3	23.9	26.9	30.7	35.8	33.7	23.2
110	21.1	24.4	28.7	33.0	17.0	17.6	21.9	24.5	27.8	32.1	31.5	19.2
120	19.6	22.6	26.2	29.9	14.1	16.8	20.1	22.4	25.2	28.9	29.2	16.1
130	18.3	20.8	23.9	27.1	12.0	16.0	18.5	20.4	22.9	26.0	26.9	13.7
140	17.0	19.2	21.8	24.5	10.3	15.2	17.1	18.7	20.8	23.5	24.6	11.8
150	15.8	17.7	19.9	22.2	8.82	14.4	15.8	17.2	19.0	21.3	22.5	10.3
160	14.7	16.3	18.1	20.2	7.71	13.6	14.6	15.9	17.4	19.4	20.5	9.07
170	13.7	15.1	16.6	18.4	6.81	12.8	13.5	14.6	16.0	17.7	18.7	8.03
180	12.7	13.9	15.2	16.7	6.05	12.1	12.6	13.5	14.7	16.2	17.1	7.16
190	11.9	12.9	14.0	15.3	5.43	11.4	11.7	12.5	13.6	14.8	15.7	6.43
200	11.1	11.9	12.9	14.0	4.90	10.7	10.9	11.6	12.5	13.7	14.4	5.80
210	10.3	11.1	11.9	12.9	4.44	10.0	10.2	10.8	11.6	12.6	13.3	5.26
220	9.65	10.3	11.0	11.8	4.05	9.43	9.51	10.1	10.8	11.6	12.2	4.80
230	9.03	9.61	10.2	10.9	3.70	8.86	8.91	9.44	10.1	10.8	11.3	4.39
240	8.47	8.96	9.50	10.1	3.40	8.34	8.36	8.84	9.38	10.0	10.5	4.03

Note: 1. The size of angle is expressed as: Width of Long Leg (mm) x Width of Short Leg (mm) x Thickness (mm) x Inside Bend Radius (mm)

2. g = gauge distance (mm)

Table 6.4 Dimensional Buckling Curve, B3 (Limiting Stress in MPa)

λ	0	1	2	3	4	5	6	7	8	9	λ
0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	0
10	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	249.3	10
20	248.4	247.4	246.5	245.5	244.6	243.6	242.6	241.6	240.5	239.5	20
30	238.4	237.3	236.2	235.1	234.0	232.8	231.7	230.6	229.5	228.5	30
40	227.4	226.3	225.2	224.1	223.0	221.9	220.7	219.5	218.2	217.0	40
50	215.7	214.4	213.1	211.7	210.4	209.0	207.6	206.2	204.7	203.3	50
60	201.5	200.4	198.9	197.4	195.8	194.3	192.8	191.2	189.6	188.0	60
70	186.4	184.8	183.1	181.5	179.8	178.1	176.4	174.7	173.0	171.2	70
80	169.4	167.6	165.7	163.9	162.1	160.3	158.5	156.7	155.0	153.2	80
90	151.5	149.8	148.0	146.3	144.6	142.9	141.2	139.6	137.9	136.3	90
100	134.7	133.1	131.5	129.9	128.3	126.7	125.2	123.7	122.1	120.6	100
110	119.1	117.6	116.2	114.7	113.3	111.8	110.4	109.1	107.1	106.4	110
120	105.0	103.8	102.5	101.2	100.0	98.7	97.5	96.3	95.2	94.0	120
130	92.8	91.7	90.6	89.5	88.4	87.3	86.3	85.2	84.2	83.2	130
140	82.2	81.3	80.3	79.4	78.5	77.6	76.7	75.8	74.9	74.0	140
150	73.2	72.3	71.5	70.5	69.6	69.0	68.2	67.4	66.7	65.8	150
160	65.1	64.5	63.8	63.1	62.5	61.8	61.2	60.5	59.9	59.3	160
170	58.7	58.0	57.4	56.8	56.2	55.6	55.1	54.5	53.9	53.4	170
180	52.8	52.3	51.8	51.3	50.8	50.3	49.9	49.4	48.9	48.5	180
190	48.0	47.5	47.1	46.6	46.2	45.8	45.4	45.0	44.6	44.2	190
200	43.8	43.3	42.9	42.5	42.2	41.8	41.5	41.1	40.7	40.3	200
210	40.0	39.6	39.2	38.9	38.5	38.2	37.9	37.6	37.2	36.9	210
220	36.6	36.3	36.0	35.7	35.4	35.1	34.9	34.6	34.3	34.0	220
230	33.7	33.5	33.2	33.0	32.8	32.6	32.3	32.0	31.7	31.5	230
240	31.2	31.0	30.8	30.6	30.4	30.1	29.9	29.7	29.5	29.3	240
250	29.0										250

Table 6.5 Cross-Sectional Properties of Test Specimens

Property	Nominal Size of Angle (mm) 65 x 50 x 4	55 x 55 x 4
Actual Width of Long Leg (mm)	63.30	53.20
Actual Width of Short Leg (mm)	48.80	53.20
Thickness (mm)	4.00	3.96
Inside Bend Radius (mm)	8.00	8.00
Nominal Area (mm^2)	427	407
Actual Area, A (mm^2)	415	389
Maximum Moment of Inertia, I_x (mm^4)	219600	177400
Nominal Minimum Moment of Inertia (mm^4)	44790	42670
Minimum Moment of Inertia, I_y (mm^4)	40990	37620
Torsional Constant, J (mm^4)	2210	2030
Polar Moment of Inertia about the Shear Centre, I_o (mm^4)	428960	342430
x-Coordinate of Shear Centre, x_o (mm)	-17.82	-18.10
y-Coordinate of Shear Centre, y_o (mm)	-9.37	0.00
β_1 (mm)	23.19	0.00
β_2 (mm)	76.84	75.35
Warping Constant, C_w (mm^6)	29890	27600

Table 6.6 Lengths and Slenderness Ratios of the Test Specimens

Angle Size (mm)	Specimen No.	No. of bolts at each end-connection	Length (mm)	Maximum Slenderness Ratio	
				Nominal	Actual
65 x 50 x 4 Long Leg Connected	ES65-1-1 1-2 1-3	1 2 3	838	82	84
	2-1 2-2 2-3	1 2 3	1218	119	122
	3-1 3-2 3-3	1 2 3	1752	171	176
65 x 50 x 4 Long Leg Out	ES65-4-1 4-2 4-3	1 2 3	838	82	84
	5-1 5-2 5-3	1 2 3	1752	171	176
55 x 55 x 4	ES55-1-1 1-2 1-3	1 2 3	838	82	85
	2-1 2-2 2-3	1 2 3	1218	119	123
	3-1 3-2 3-3	1 2 3	1752	171	178

Table 6.7a Experimental and Theoretical Failure Loads

Specimen Number	Experimental Failure Load in kN	Computed Failure Load (kN) based on			
		Nominal Dimensions		Actual Dimensions	
		ASCE	ECCS	ASCE	ECCS
ES65-1-1	59.5	76.7	59.8	74.8	57.4
1-2	71.0	76.7	59.8	74.8	57.4
1-3	75.0	76.7	59.8	74.8	57.4
2-1	45.7	60.4	45.4	56.4	42.6
2-2	56.8	60.4	45.8	57.2	43.3
2-3	59.2	60.4	45.8	57.2	43.3
3-1	37.0	29.5	24.8	27.1	22.9
3-2	37.0	37.7	28.7	35.2	26.8
3-3	40.0	37.7	28.7	35.2	26.8
ES65-4-1	57.0	76.7	59.8	74.8	57.4
4-2	81.0	76.7	59.8	74.8	57.4
4-3	81.0	76.7	59.8	74.8	57.4
5-1	40.0	29.5	24.8	27.1	22.9
5-2	44.0	37.7	28.7	35.2	26.8
5-3	51.5	37.7	28.7	35.2	26.8
ES55-1-1	58.0	75.9	57.0	71.2	53.4
1-2	70.0	75.9	57.0	71.2	53.4
1-3	75.0	75.9	57.0	71.2	53.4
2-1	54.5	57.6	43.3	52.0	39.3
2-2	58.6	57.6	43.6	53.0	40.2
2-3	63.8	57.6	43.6	53.0	40.2
3-1	40.5	28.1	23.6	24.8	21.0
3-2	40.0	35.9	27.3	32.5	24.6
3-3	43.8	35.9	27.3	32.5	24.6

Table 6.7b Predicted Failure Loads for Test Specimens

Specimen Number	Computed Failure Loads (kN) based on General Theory of Torsional-Flexural Buckling
ES65-1-1	49.8
2-1	34.2
3-1	21.2
4-1	50.4
5-1	27.5
ES55-1-1	52.5
2-1	37.6
3-1	22.6

Table 6.8 Effect of Variables on the Ultimate Compressive Strength of 65 x 50 x 4 mm Cold-Formed Angle (Long Leg Connected)

Slenderness Ratio	Failure Loads (kN) based on					
	Exact Shear Centre		Shear Centre at SC_1		Shear Centre at SC_2	
	$C_w = 0$	Actual C_w	$C_w = 0$	Actual C_w	$C_w = 0$	Actual C_w
20	71.8	70.1	71.8	70.3	71.8	70.0
30	71.0	70.0	71.2	70.2	70.8	69.8
40	67.6	68.5	67.9	68.8	67.4	68.4
50	63.6	64.4	64.1	64.7	63.4	64.1
60	59.3	59.8	59.8	60.3	59.0	59.6
70	54.8	55.2	55.3	55.7	54.6	55.0
80	50.4	50.7	50.8	51.1	50.2	50.4
90	46.1	46.0	46.5	46.6	45.9	45.7
100	41.5	41.4	42.0	41.9	41.3	41.2
110	37.5	37.4	37.8	37.8	37.3	37.2
120	33.9	33.9	34.2	34.1	33.8	33.7
130	30.8	30.7	31.0	31.0	30.6	30.6
140	28.0	28.0	28.2	28.1	27.9	27.9
150	25.5	25.5	25.7	25.7	25.5	25.4
160	23.4	23.4	23.5	23.5	23.3	23.3
170	21.4	21.4	21.5	21.5	21.4	21.3
180	19.7	19.7	19.8	19.8	19.6	19.6
190	18.1	18.1	18.2	18.2	18.1	18.1
200	16.8	16.8	16.8	16.8	16.7	16.7
210	15.5	15.5	15.6	15.6	15.5	15.5
220	14.4	14.4	14.4	14.4	14.4	14.4
230	13.4	13.4	13.4	13.4	13.4	13.4
240	12.5	12.5	12.5	12.5	12.5	12.5

Table 6.9 Comparison of Exact and Galerkin Solutions

Size of the angle = 65 x 50 x 4 mm (Long Leg Out)

Gauge Distance = 25 mm

Length of the member = 189.6 mm (ℓ/r_y = 20)

Load kN	u-component			v-component			rotation, $\phi \times 10^3$		
	Exact mm	Galerkin mm	% error	Exact mm	Galerkin mm	% error	Exact rad	Galerkin rad	% error
1.3276	-	0.0014	-	-	0.0025	-	-	0.0002	-
14.6036	0.0150	0.0150	0.0	0.0273	0.0273	0.0	-0.1314	0.0202	-115.4
27.8796	0.0287	0.0288	0.0	0.0522	0.0522	0.0	-0.0753	0.0742	-198.5
41.1556	0.0427	0.0427	0.0	0.0771	0.0771	0.0	0.0163	0.1630	900.0
54.4316	0.0568	0.0567	-0.2	0.1020	0.1020	0.0	0.1452	0.2875	98.0
67.7076	0.0711	0.0708	-0.4	0.1270	0.1270	0.0	0.3131	0.4485	43.2
69.0352	0.0726	0.0723	-0.4	0.1295	0.1295	0.0	0.3321	0.4666	40.5
70.3629	0.0740	0.0737	-0.4	0.1320	0.1320	0.0	0.3515	0.4852	38.0
80.9837	0.0856	0.0851	-0.6	0.1521	0.1521	0.0	0.5215	0.6470	24.1
134.0877	0.1460	0.1432	-2.0	0.2528	0.2525	-0.1	1.7959	1.8376	2.3

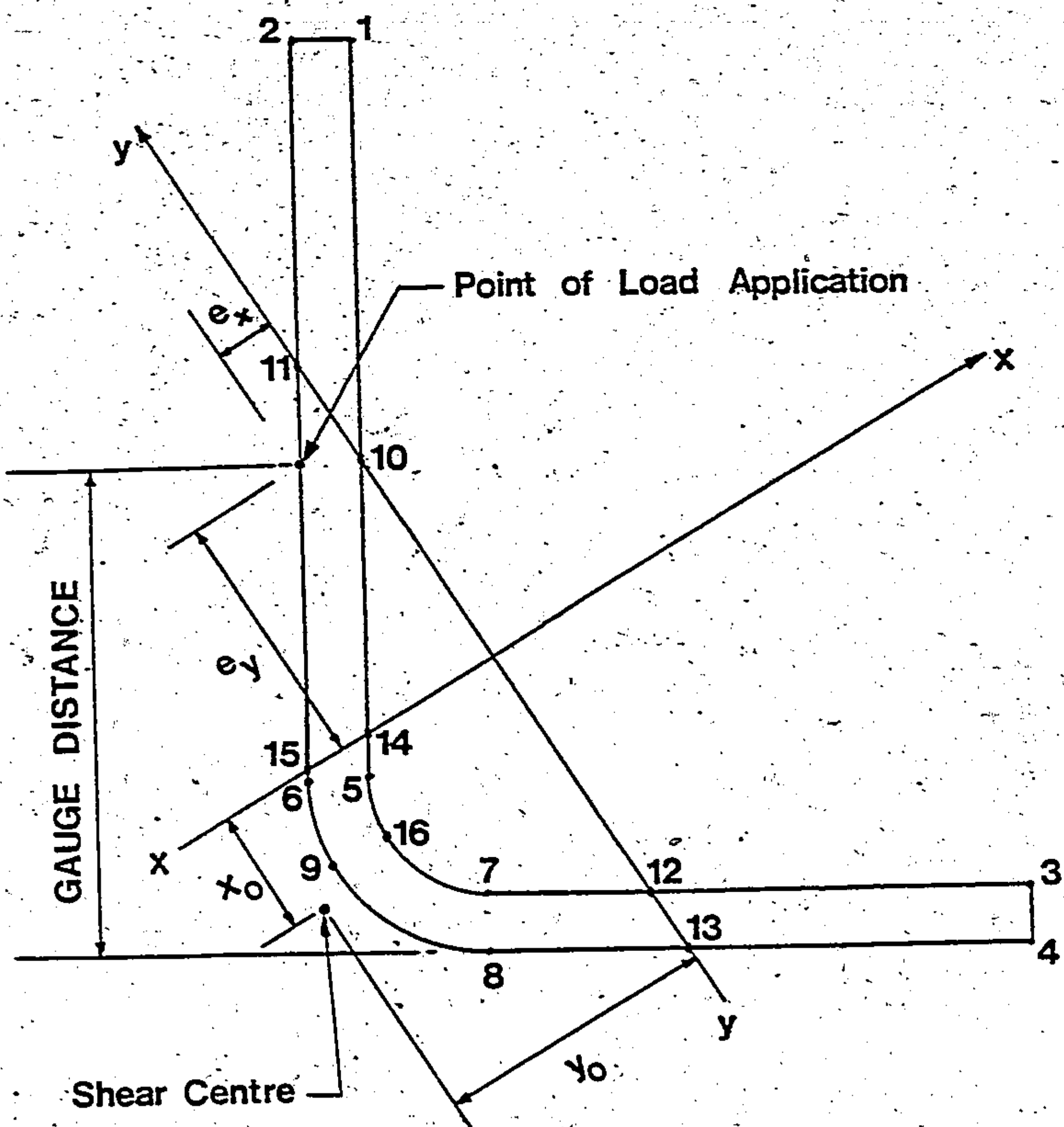

Figure 4.1 Location of critical points and the load on the cross-section.

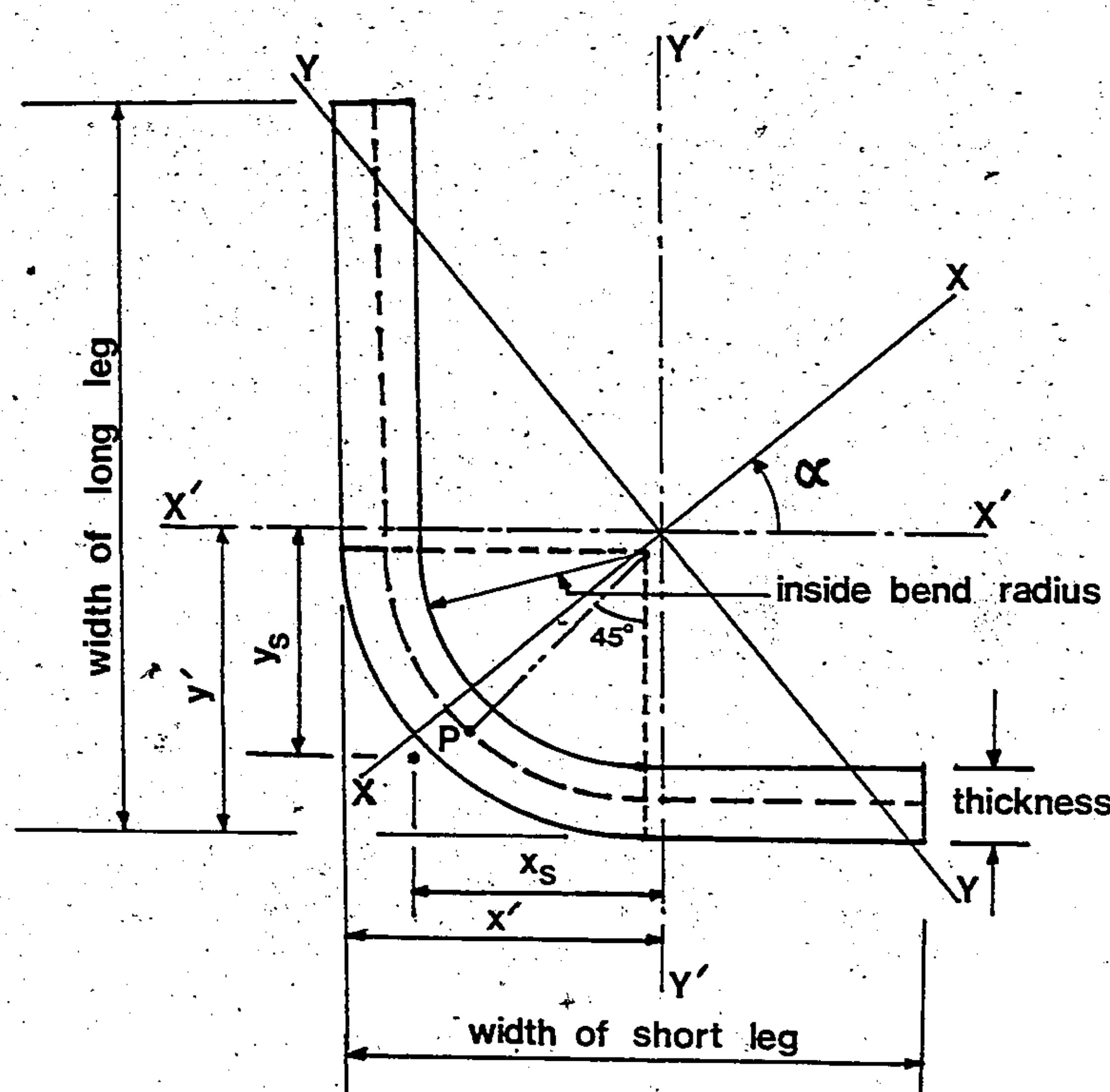

Figure 4.2 Cross-section of cold-formed angle.

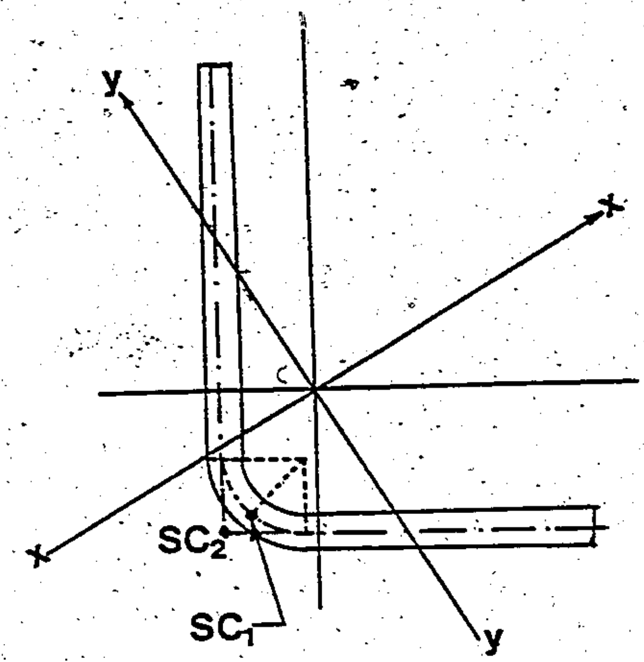

Figure 4.3 Assumed locations of shear centre.

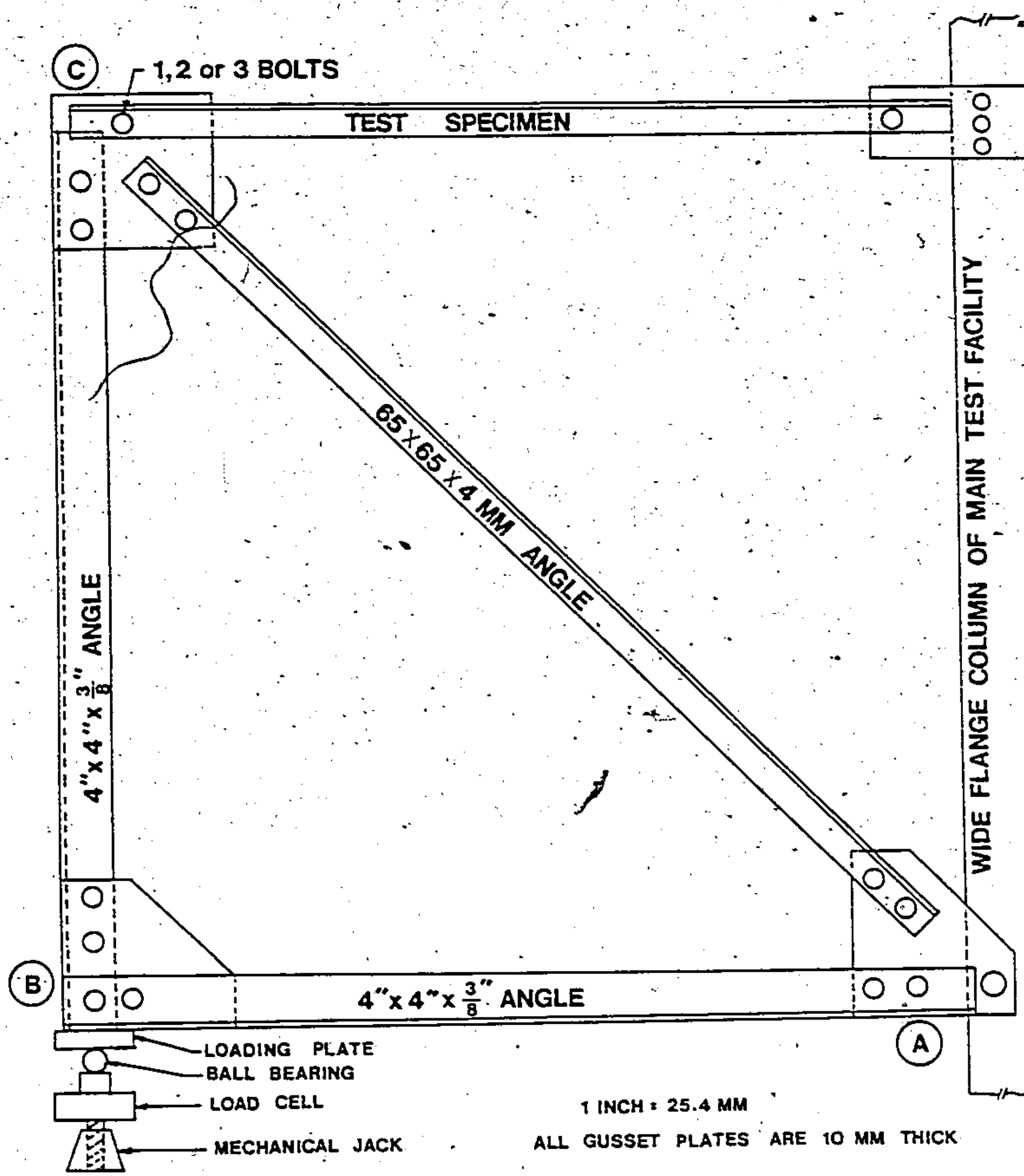

Figure 5.1 Schematic diagram of test set-up.

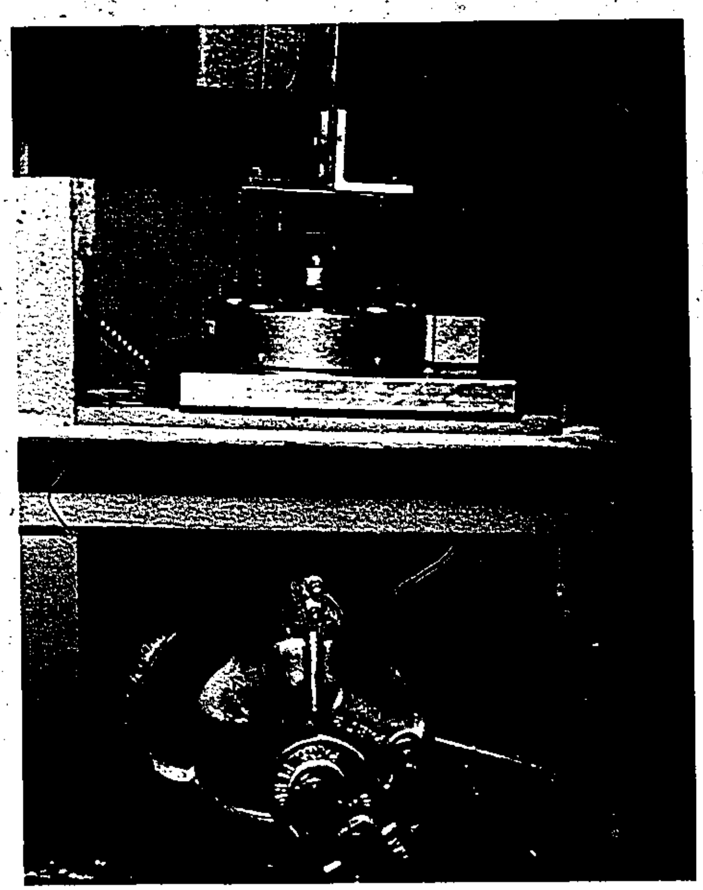

Figure 5.2 Arrangement for load application at joint B.

Figure 5.3 Lateral support at joint B.

Figure 5.4 Lateral support at joint C.

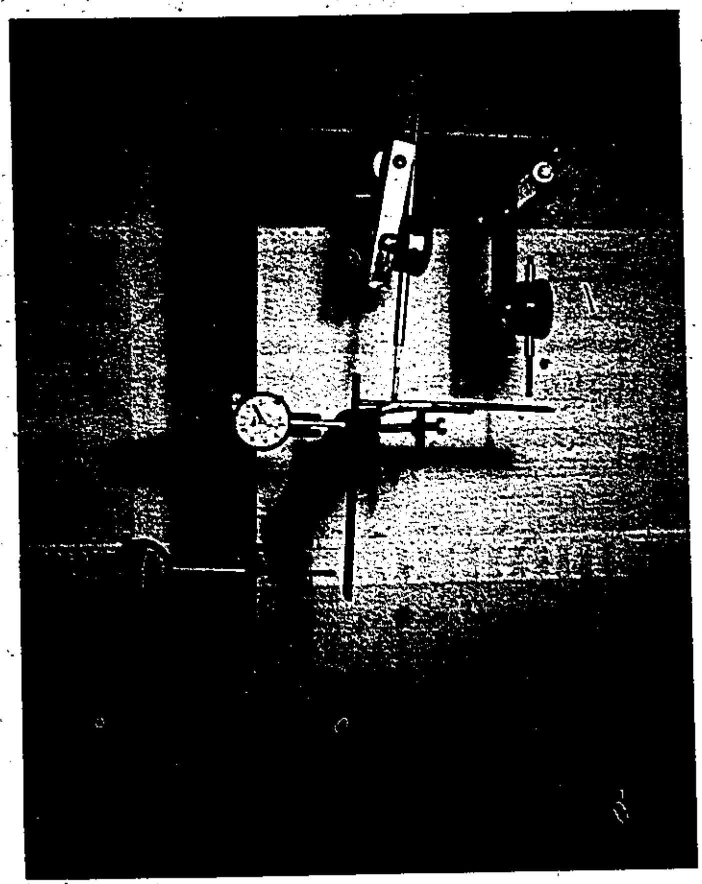

Figure 5.5 Arrangement of dial gauges at midspan of the test specimen.

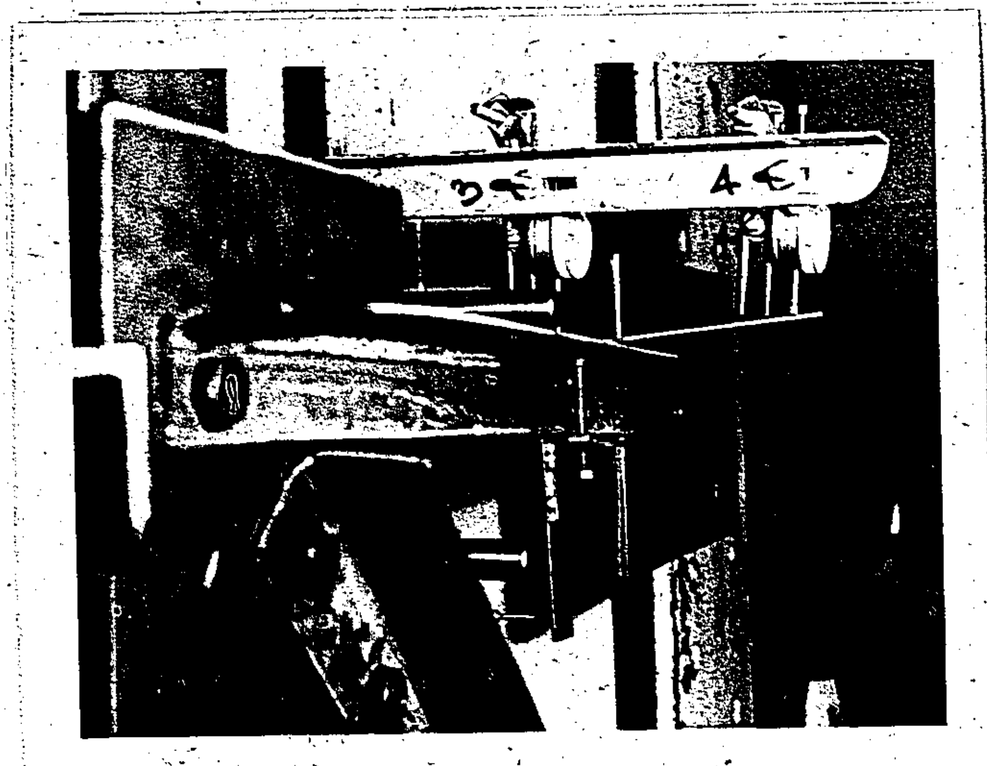

Figure 5.6 L-shaped extension piece at midspan.

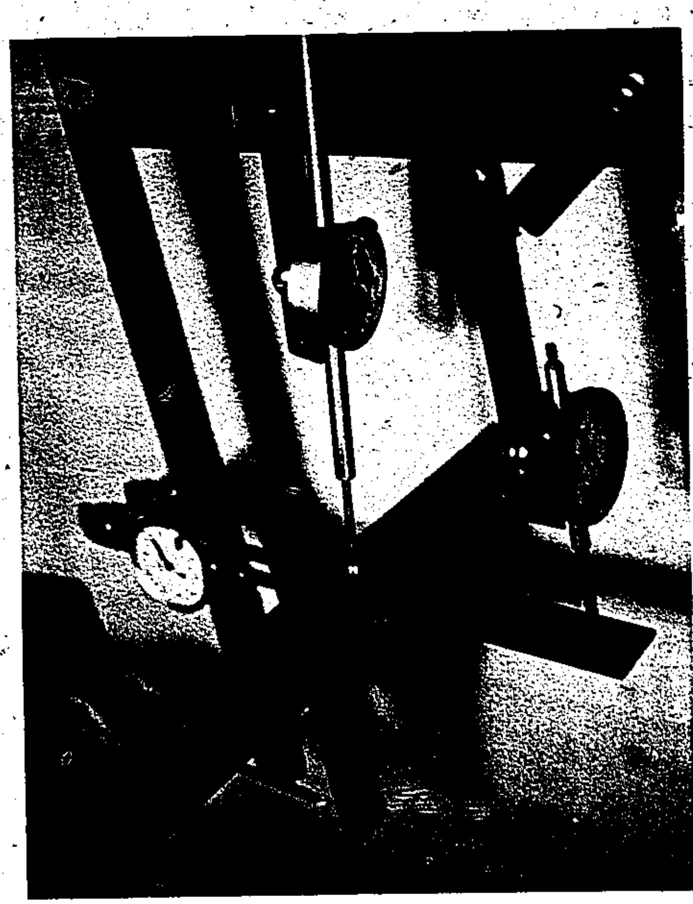

Figure 5.7 Gauge support bracket.

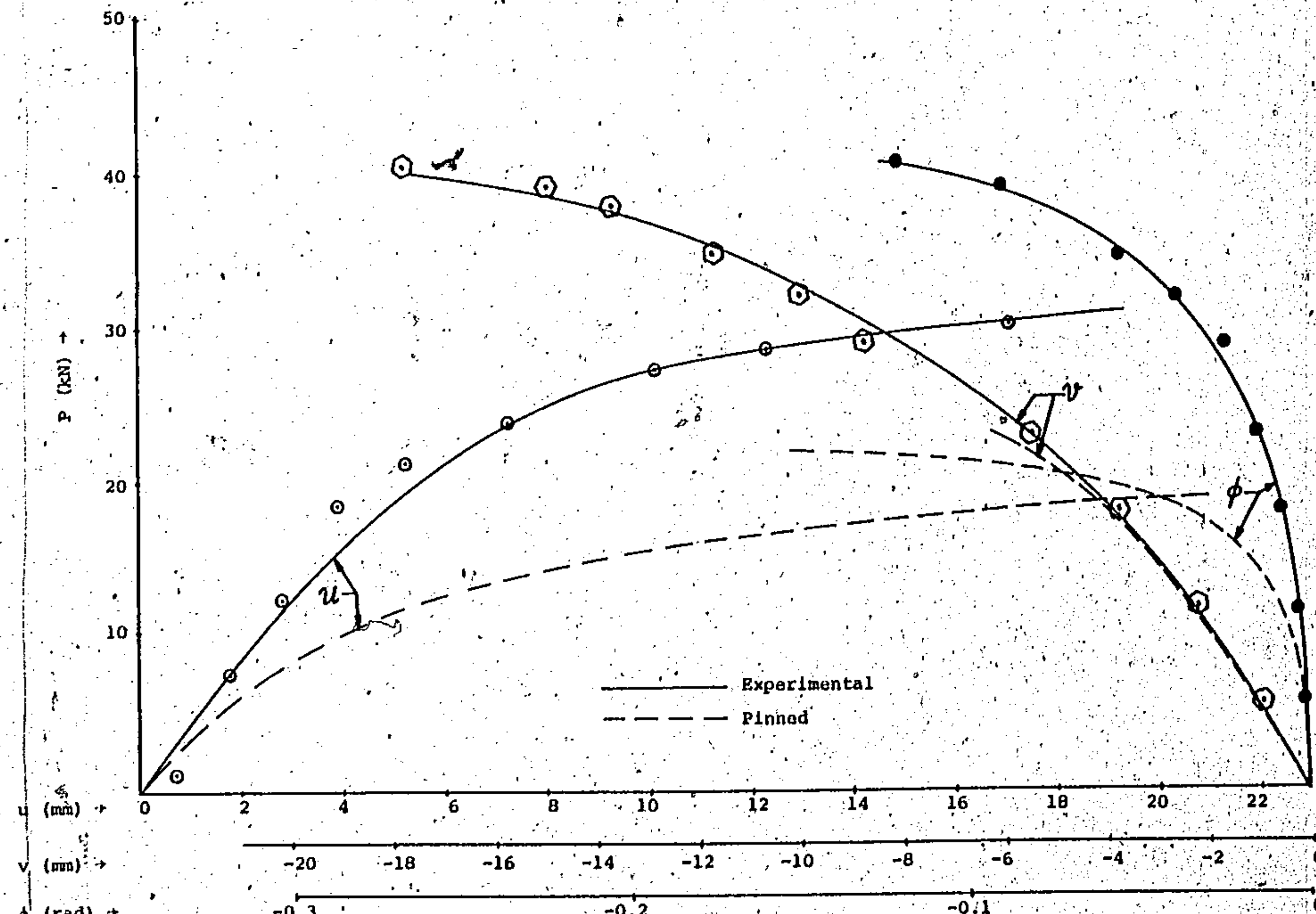

Figure 6.1 Load-deflection and load-rotation curves at midspan for Specimen ES55-3-1

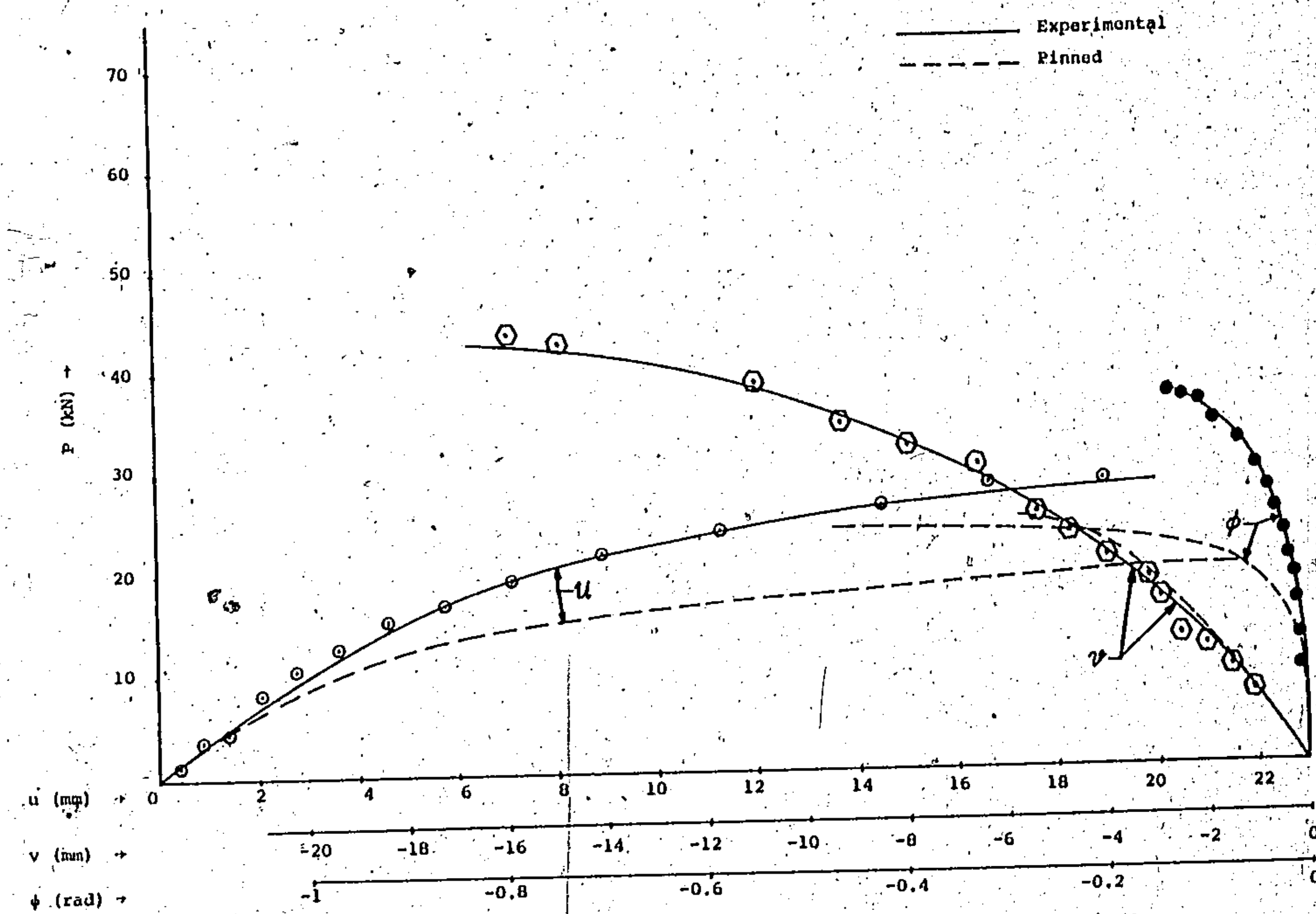

Figure 6.2 Load-deflection and load-rotation curves at midspan for Specimen ES65-3-2

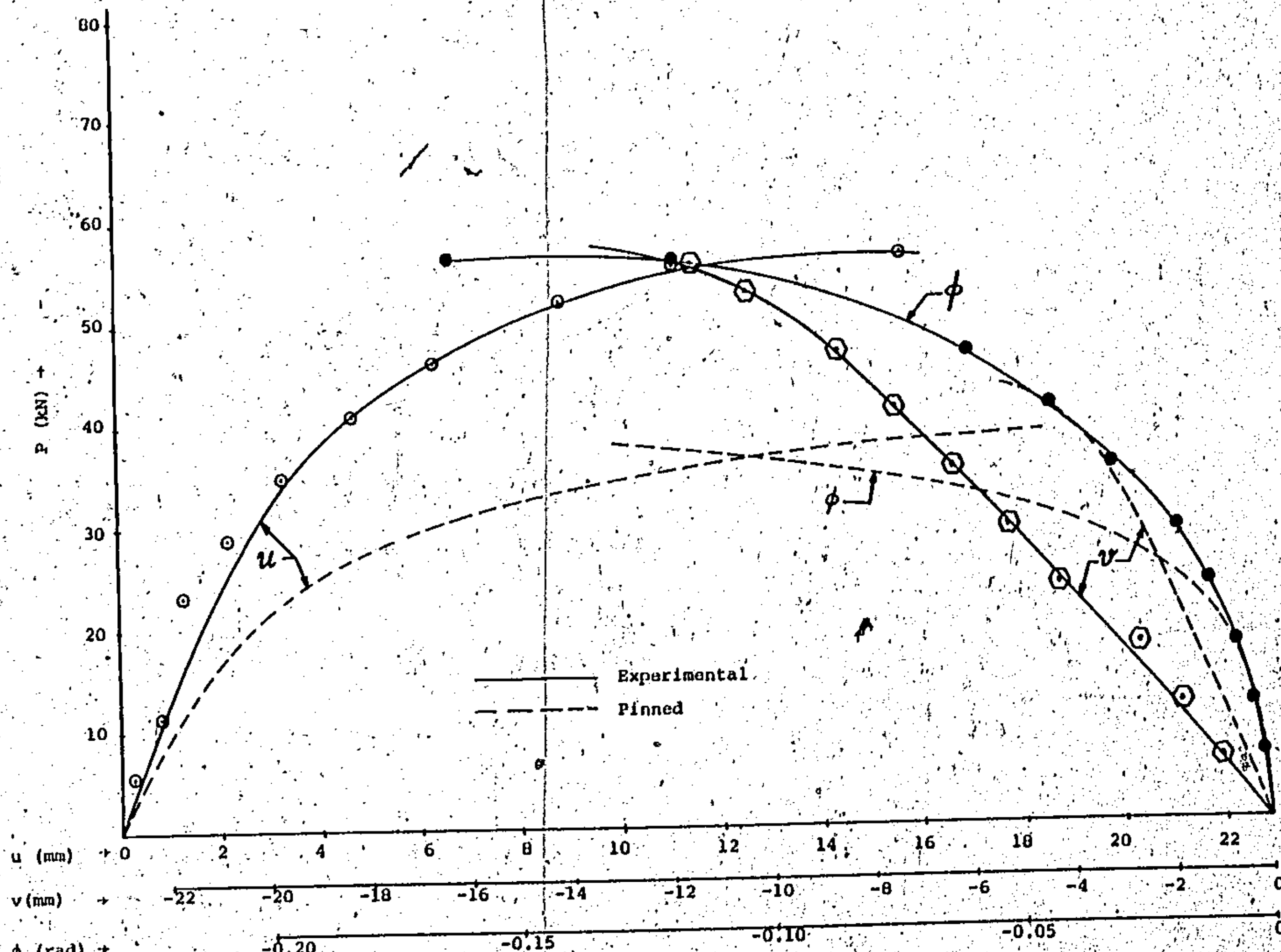

Figure 6.3 Load-deflection and load-rotation curves at midspan for Specimen ES65-2-2

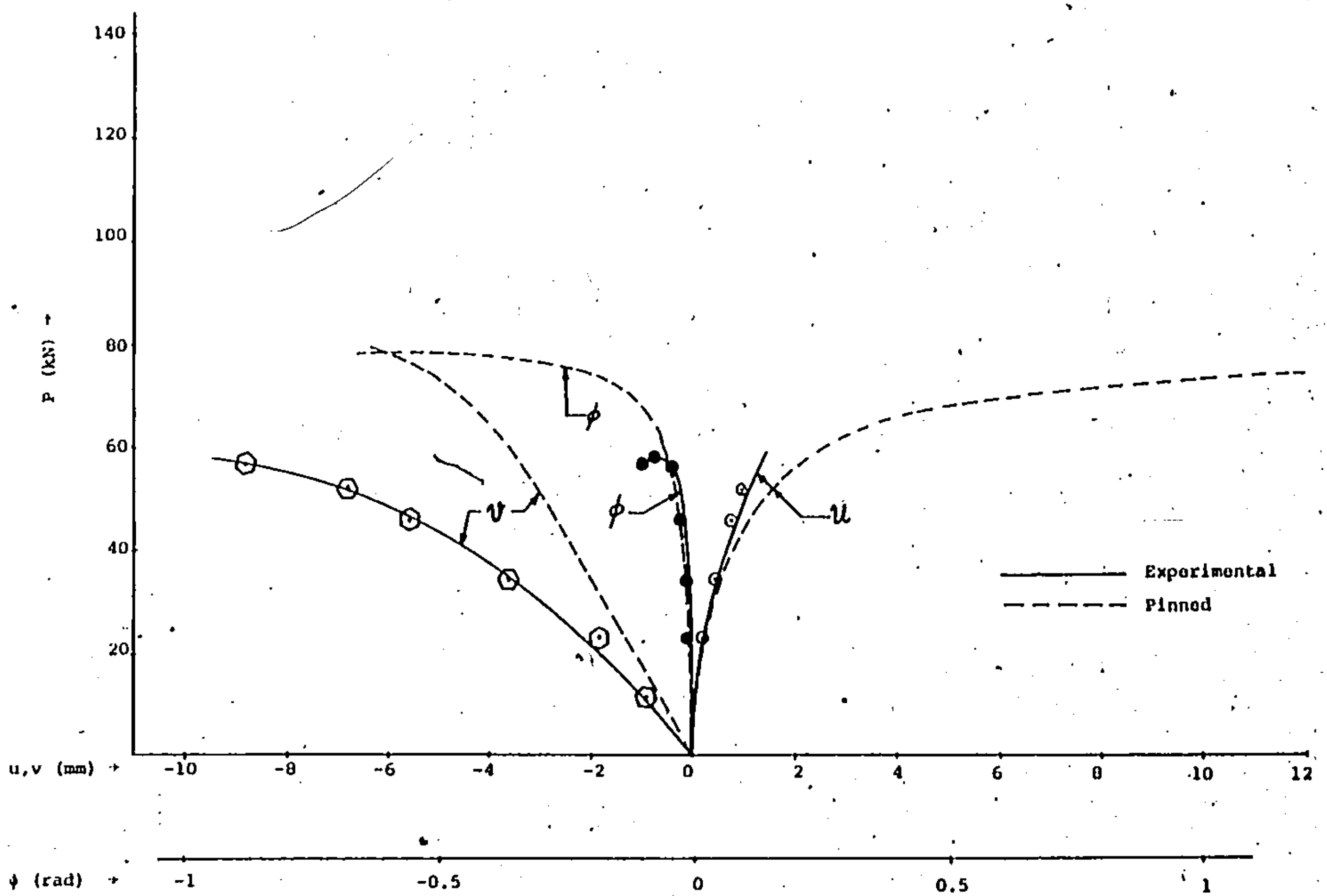

Figure 6.4 Load-deflection and load-rotation curves at midspan for Specimen ES55-1-1

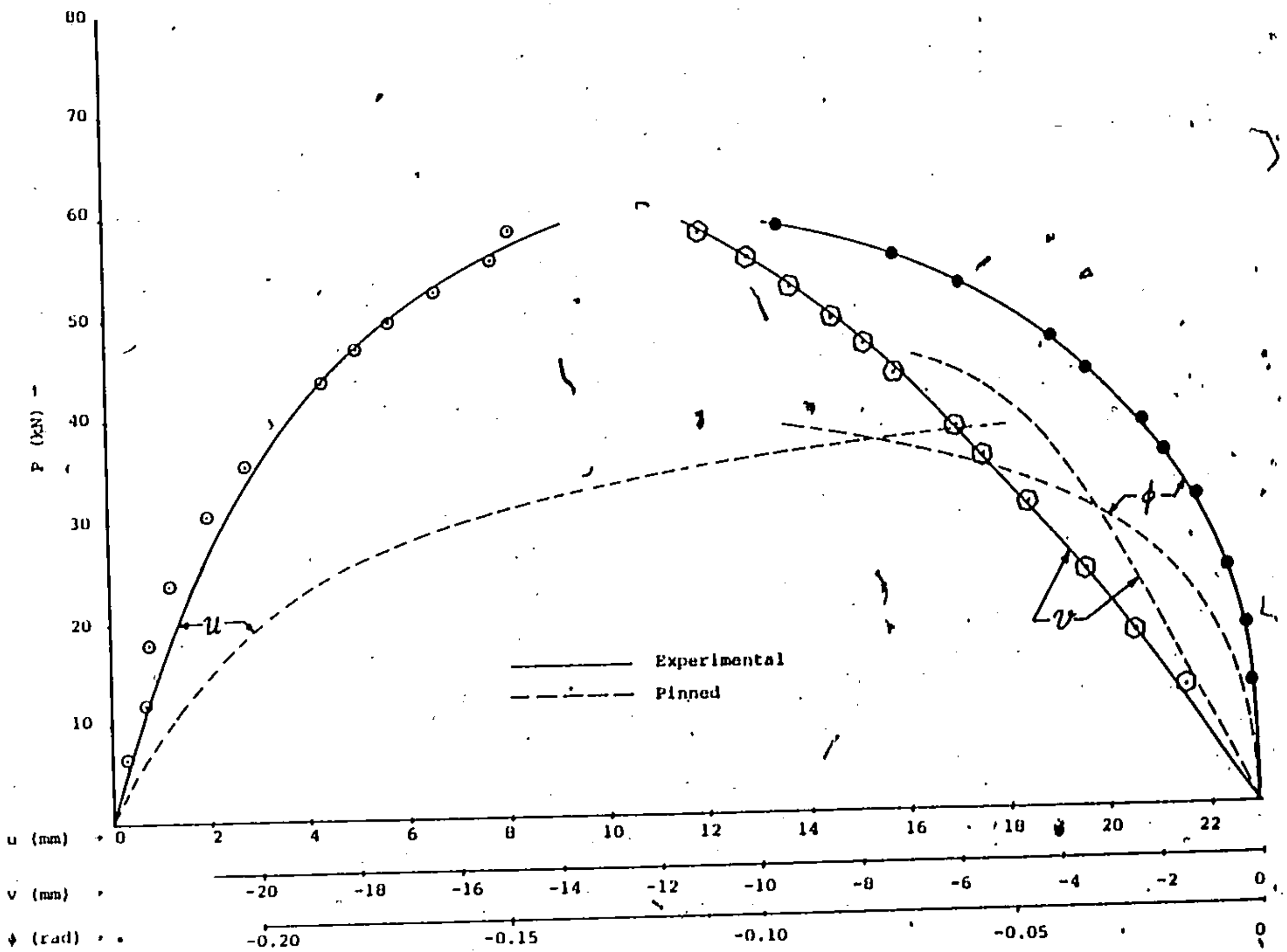

Figure 6.5 Load-deflection and load-rotation curves at midspan for Specimen ES55-2-2

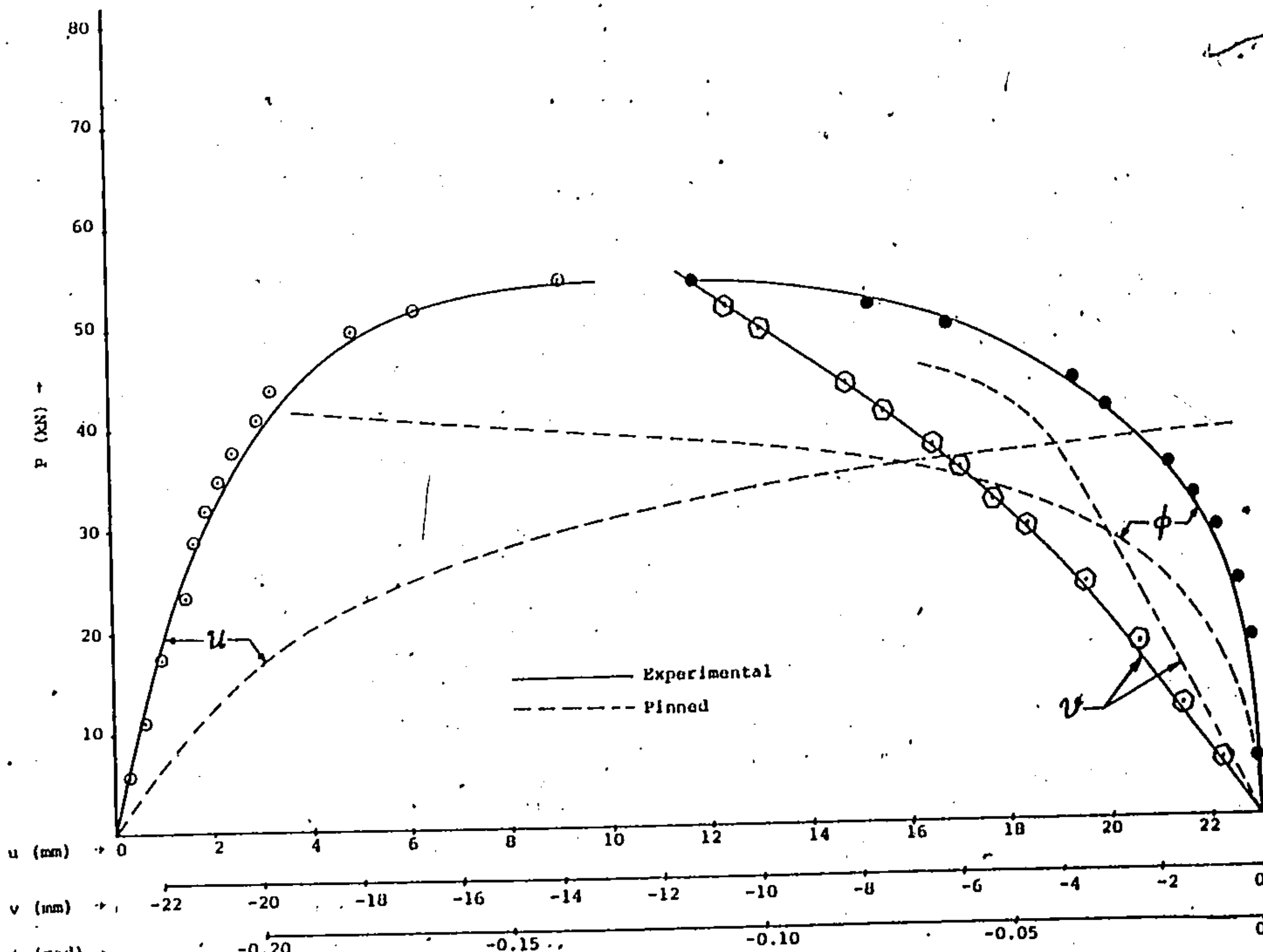

Figure 6.6 Load-deflection and load-rotation curves at midspan for Specimen ES55-2-1

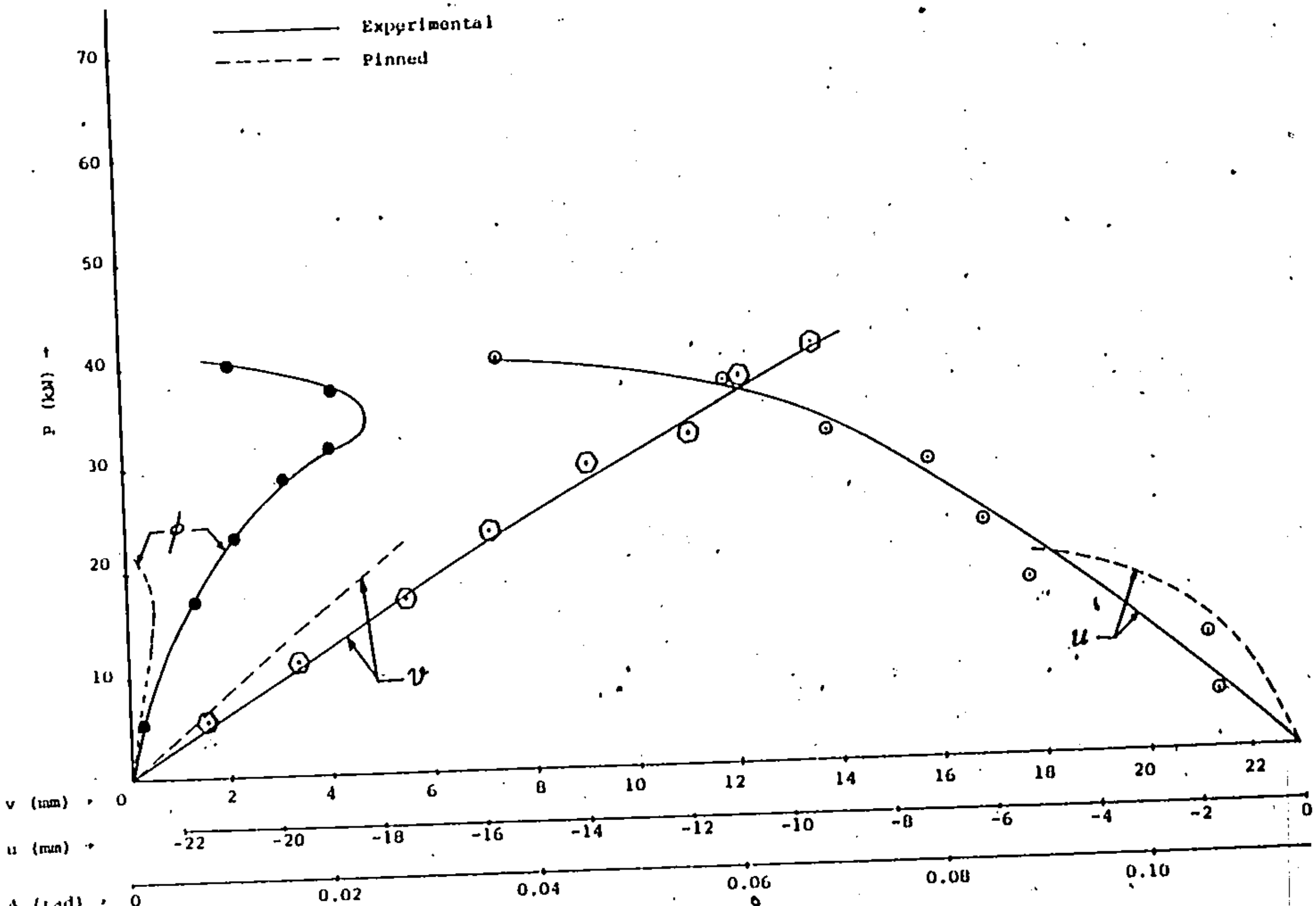

Figure 6.7 Load-deflection and load-rotation curves at midspan for Specimen ES65-5-1

APPENDIX 1

THEORETICAL FORMULATION FOR THE CROSS-SECTIONAL PROPERTIES OF COLD-FORMED ANGLES

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The formulas for the calculation of the cross-sectional properties are as given below:

1. Area (Ref. Fig. 1)

[1] $$A = (c_1 + c_2)\, t + \frac{\pi}{4} [(r + t)^2 - r^2]$$

2. Location of Centroid (Ref. Fig. 1)

[2] $$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - A_4 x_4}{A_1 + A_2 + A_3 - A_4}$$

[3] and $$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 - A_4 y_4}{A_1 + A_2 + A_3 - A_4}$$

where $A_1 = c_1 t$

$A_2 = c_2 t$

$A_3 = \frac{\pi}{4}(r+t)^2$

$A_4 = \frac{\pi}{4} r^2$

$x_1 = y - c_1/2$

$x_2 = t/2$

$x_3 = (r+t) - \frac{4(r+t)}{3\pi}$

$x_4 = (r+t) - \frac{4r}{3\pi}$

$y_1 = t/2$

$y_2 = x - c_2/2$

$y_3 = (r+t) - \frac{4(r+t)}{3\pi}$

$y_4 = (r+t) - \frac{4r}{3\pi}$

3. <u>Moments of inertia</u> (Ref. Fig. 1)

Final expressions for I_{xx}, I_{yy} and P_{xy} are as follows:

[4] $$I_{xx} = \frac{c_1 t^3}{12} + c_1 t(\bar{y}-t/2)^2 + \frac{c_2^3 t}{12} + c_2 t(x-\bar{y}-c_2/2)^2$$
$$+ \{ \frac{1}{16}\pi(r+t)^4 - \frac{\pi(r+t)^2}{4} [\frac{4(r+t)}{3\pi}]^2 + \frac{\pi(r+t)^2}{4} [\bar{y}-(r+t)+ \frac{4(r+t)}{3\pi}]^2\}$$
$$- \{ \frac{1}{16}\pi r^4 - \frac{\pi r^2}{4} (\frac{4r}{3\pi})^2 + \frac{\pi r^2}{4} [\bar{y} - (r+t) + \frac{4r}{3\pi}]^2\}$$

[5] $$I_{yy} = \frac{c_1^3 t}{12} + c_1 t(y - \bar{x} - c_1/2)^2 + \frac{c_2 t^3}{12} + c_2 t(\bar{x} - t/2)^2$$
$$+ \{ \frac{1}{16}\pi(r+t)^4 - \frac{\pi(r+t)^2}{4} [\frac{4(r+t)}{3\pi}]^2 + \frac{\pi(r+t)^2}{4} [\bar{x}-(r+t)+ \frac{4(r+t)}{3\pi}]^2\}$$
$$- \{ \frac{1}{16}\pi r^4 - \frac{\pi r^2}{4} (\frac{4r}{3\pi})^2 + \frac{\pi r^2}{4} [\bar{x} - (r+t) + \frac{4r}{3\pi}]^2\}$$

[6] and $P_{xy} = c_1 t[-(\bar{y} - t/2)](y - \bar{x} - c_1/2) + c_2 t[-(\bar{x} - t/2)](x - \bar{y} - c_2/2)$

$$+ \frac{\pi(r+t)^2}{4}\{[d_2 + \frac{4(r+t)}{3\pi}][d_1 + \frac{4(r+t)}{3\pi}]\}$$

$$- \frac{\pi r^2}{4}[(d_2 + \frac{4r}{3\pi})(d_1 + \frac{4r}{3\pi})] + \frac{(r+t)^4}{8} - \frac{r^4}{8} - \frac{\pi(r+t)^2}{4}[\frac{4(r+t)}{3\pi}]^2$$

$$+ \frac{\pi r^2}{4}(\frac{4r}{3\pi})^2$$

Minimum and maximum moments of inertia are computed by using the following expressions:

[7] $I_{min} = \frac{I_{xx}+I_{yy}}{2} - \sqrt{(\frac{I_{xx}-I_{yy}}{2})^2 + P_{xy}^2}$

[8] and $I_{max} = I_{xx} + I_{yy} - I_{min}$

4. Inclination of major principal axis to horizontal

The angle α between the major principal axis (u-u axis) and the horizontal (x-x axis) is calculated as follows:

[9] $\tan 2\alpha = \frac{2P_{xy}}{I_{xx}-I_{yy}}$

5. Minimum radius of gyration.

[10] $r_{min} = \sqrt{\frac{I_{min}}{Area}}$

6. Torsional constant (Ref. Fig. 1)

St. Venant's torsional constant has been computed by using the following expression:

[11] $J = \frac{1}{3} bt^3$ where b = centre-line length of the cross-section

$= c_1 + c_2 + \pi(r+t/2)/2$

and t = thickness of the section.

7. Location of shear centre (Ref. Fig. 2 to 13)

(a) v-coordinate (Bending about v-v axis) (Ref. Figs. 2 to 10)

Depending on the size of the angle, thickness and inside bend radius, there are several possible cases as shown in Table I

Case I - Figure 2

[12] $m_1 = (\bar{y} - t/2)t \cos \alpha\ \ell_1^3/(3I_{min})$

[13] $m_2 = (\bar{y} - t/2)t \cos \alpha\ \ell_2(\ell_1^2 - \ell_2^2/3)/(2I_{min})$

[14] $m_3 = (\bar{x} - t/2)t \sin \alpha\ \ell_3^3/(3I_{min})$

[15] $m_4 = (\bar{x} - t/2)t \sin \alpha\ \ell_4(\ell_3^2 - \ell_4^2/3)/(2I_{min})$

[16] $$m_5 = \int_0^{\pi/4} \{r_o^2 q_{c1} + r_o d q_{c1} \sin(\pi/2 - \phi + \theta) - r_o^3 td \cos(\alpha + \phi - \pi/2)\theta / I_{min} - \frac{2r_o^4 t}{I_{min}} \sin \theta/2 \sin(\alpha + \theta/2)$$

$$- \frac{r_o^2 d^2 t \sin(\pi/2 - \phi + \theta)}{I_{min}} \cos(\alpha + \phi - \pi/2)\theta$$

$$- \frac{2r_o^3 td}{I_{min}} [\sin \theta/2 \sin(\alpha + \theta/2) \sin(\pi/2 - \phi + \theta)]\} d\theta$$

[17] and $m_6 = \int_0^{\pi/4} \{r_o^2 q_{c2} + r_o d q_{c2} \cos(\pi/2 - \phi - \theta) - \frac{r_o^3 dt \cos(\pi/2 - \alpha - \phi)}{I_{min}} \theta$

$- \frac{2r_o^4 t}{I_{min}} \sin \theta/2 \sin(\pi/2 - \alpha + \theta/2) - \frac{r_o^2 d^2 t}{I_{min}} \cos(\pi/2 - \alpha - \phi)\theta \cos(\pi/2-\phi-\theta)$

$- \frac{2r_o^3 dt}{I_{min}} [\cos(\pi/2 - \phi - \theta)\sin \theta/2 \sin(\pi/2 - \alpha + \theta/2)]\}d\theta$

[18] $\therefore v_s = m_1 + m_2 + m_5 - m_3 - m_4 - m_6$

<u>Case II - Figure 3</u>

m_1, m_2, m_3, m_4 = same as eqn. nos. [12], [13], [14] and [15], respectively.

[19] $m_5 = \int_0^{\pi/4} \{r_o^2 q_{c1} - r_o d q_{c1} \sin(\pi/2 - \phi + \theta) + \frac{r_o^3 td \cos(\alpha + \phi - \pi/2)}{I_{min}} \theta$

$- \frac{2r_o^4 t}{I_{min}} \sin \theta/2 \sin(\alpha + \theta/2) - \frac{r_o^2 d^2 t \sin(\pi/2 - \phi + \theta)}{I_{min}} \cos(\alpha + \phi - \pi/2)\theta$

$+ \frac{2r_o^3 td}{I_{min}} [\sin \theta/2 \sin(\alpha + \theta/2)\sin(\pi/2 - \phi + \theta)]\}d\theta$

[20] and $m_6 = \int_0^{\pi/4} \{r_o^2 q_{c2} - r_o d q_{c2} \cos(\pi/2 - \phi - \theta) + \frac{r_o^3 dt \cos(\pi/2 - \alpha - \phi)}{I_{min}} \theta$

$- \frac{2r_o^4 t}{I_{min}} \sin \theta/2 \sin(\pi/2 - \alpha + \theta/2) - \frac{r_o^2 d^2 t}{I_{min}} \cos(\pi/2 - \alpha - \phi)\theta \cos(\pi/2-\phi-\theta$

$+ \frac{2r_o^3 dt}{I_{min}} [\cos(\pi/2 - \phi - \theta)\sin \theta/2 \sin(\pi/2 - \alpha + \theta/2)]\}d\theta$

[21] $\therefore\ v_s = m_1 + m_2 + m_5 - m_3 - m_4 - m_6$

<u>**Case III - Figure 4**</u>

[22] $m_1 = (\bar{y} - t/2)t \cos\alpha\ \ell_1^2(\ell_1'/2 - \ell_1/6)/I_{min}$

[23] $m_3 = (\bar{x} - t/2)t \sin\alpha\ \ell_3^2(\ell_3'/2 - \ell_3/6)/I_{min}$

[24] $m_5 = \int_0^{\beta}$ (Integrand same as eqn. no. [19])

[25] $$m_6 = \int_0^{\pi/4-\beta} \{r_o^2 q_{c3} - r_o d q_{c3} \sin(\pi/2 - \phi + \theta + \beta) + \frac{r_o^3 td \cos(\alpha + \phi - \pi/2)}{I_{min}}\theta$$

$$- \frac{2r_o^4 t}{I_{min}} \sin\theta/2 \sin(\alpha + \beta + \theta/2) - \frac{r_o^2 d^2 t \sin(\pi/2 - \phi + \theta + \beta)}{I_{min}}\theta \cos(\alpha+\phi-\pi/2)$$

$$+ \frac{2r_o^3 td}{I_{min}} [\sin\theta/2 \sin(\alpha + \beta + \theta/2)\sin(\pi/2 - \phi + \theta + \beta)]\}d\theta$$

[26] $m_7 = \int_0^{\beta}$ (Integrand same as eqn. no. [20])

[27] and $$m_8 = \int_0^{\pi/4-\beta} \{r_o^2 q_{c4} - r_o d q_{c4} \cos(\pi/2 - \phi - \theta - \beta) + \frac{r_o^3 td \cos(\pi/2 - \alpha - \phi)}{I_{min}}\theta$$

$$- \frac{2r_o^4 t}{I_{min}} \sin\theta/2 \sin(\pi/2 - \alpha + \beta + \theta/2) - \frac{r_o^2 d^2 t}{I_{min}} \cos(\pi/2 - \alpha - \phi)\theta\cos(\pi/2 - \phi - \theta - \beta)$$

$$+ \frac{2r_o^3 td}{I_{min}} [\sin\theta/2 \sin(\pi/2 - \alpha + \beta + \theta/2)\cos(\pi/2 - \phi - \theta - \beta)]\}d\theta$$

[28] $\therefore\ v_s = m_1 + m_5 + m_6 - m_3 - m_7 - m_8$

Case IV – Figure 5

m_1, m_3 = same as eqn. nos. [22] and [23], respectively.

[29] $m_5 = \int_0^{\beta}$ (Integrand same as eqn. no. [19])

[30] $m_6 = \int_0^{\delta}$ (Integrand same as eqn. no. [25])

[31] $m_7 = \int_0^{\nu}$ (Integrand same as eqn. no. [20])

[32] $m_8 = \int_0^{\pi/2-\beta-\nu-\delta}$ (Integrand same as eqn. no. [27])

[33] $\therefore v_s = m_1 + m_5 + m_6 - m_3 - m_7 - m_8$

Case Va – Figure 6

m_1, m_2, m_3, m_4 = same as eqn. nos. [12], [13], [14], and [15], respectively.

[34] $m_5 = \int_0^{\delta}$ (Integrand same as eqn. no. [19])

[35] and $m_6 = \int_0^{\pi/2-\delta}$ (Integrand same as eqn. no. [20])

[36] $\therefore v_s = m_1 + m_2 + m_5 - m_3 - m_4 - m_6$

Case Vb – Figure 6

m_1, m_2, m_3 = same as eqn. nos. [12], [13], and [14], respectively.

[37] $m_4 = (\bar{x} - t/2)t \sin\alpha(\ell_4 - x')[\ell_3^2 - (\ell_4 - x')^2/3]/(2I_{min})$

[38] $m_{4a} = (\bar{x} - t/2)q_{c5}x'(1 + \frac{x'^2}{3c'} + \frac{x'b'}{2c'})$

where $b' = -2\ell_4$

and $c' = 2q_{c5}I_{min}/(t \sin\alpha)$

[39] $m_5 = \int_0^{\pi/2}$ (Integrand same as eqn. no. [19])

[40] $\therefore \; v_s = m_1 + m_2 + m_{4a} + m_5 - m_3 - m_4$

Case VIa - Figure 7

m_1, m_2, m_3, m_4 = same as eqn. nos. [12], [13], [14] and [15], respectively.

[41] $$m_5 = \int_0^{\delta} \{r_o^2 q_{c1} + \frac{r_o^3 td\cos(\pi/2 + \alpha + \phi)}{I_{min}}\theta - \frac{2r_o^4 t}{I_{min}}\sin\theta/2\,\sin(\alpha + \theta/2)$$

$$- r_o dq_{c1}\cos(\pi + \phi - \theta) - \frac{r_o^2 d^2 t}{I_{min}}\cos(\pi/2 + \alpha + \phi)\theta\,\cos(\pi + \phi - \theta)$$

$$+ \frac{2r_o^3 td}{I_{min}}[\sin\theta/2\,\sin(\alpha + \theta/2)\cos(\pi + \phi - \theta)]\}d\theta$$

[42] and $$m_6 = \int_0^{\pi/2-\delta} \{r_o^2 q_{c2} + \frac{r_o^3 dt\cos(\pi/2 + \alpha + \phi)}{I_{min}}\theta - \frac{2r_o^4 t}{I_{min}}\sin\theta/2\,\sin(\pi/2-\alpha+\theta/2)$$

$$- r_o dq_{c2}\sin(-\theta-\phi) - \frac{r_o^2 d^2 t\cos(\pi/2 + \alpha + \phi)}{I_{min}}\theta\,\sin(-\theta-\phi)$$

$$+ \frac{2r_o^3 td}{I_{min}}[\sin\theta/2\,\sin(\pi/2 - \alpha + \theta/2)\sin(-\theta-\phi)]\}d\theta$$

[43] $\therefore \; v_s = m_1 + m_2 + m_5 - m_3 - m_4 - m_6$

Case VIb - Figure 7

$m_1, m_2, m_3, m_4, m_{4a}$ = same as eqn. nos. [12], [13], [14], [37] and [38], respectively.

[44] $m_5 = \int_0^{\pi/2}$ (Integrand same as eqn. no. [41])

[45] $\therefore \; v_s = m_1 + m_2 + m_{4a} + m_5 - m_3 - m_4$

Case VIIa - Figure 8

m_1, m_3, m_4 = same as eqn. nos. [22], [14] and [15], respectively.

[46] $$m_5 = \int_0^{\beta} \{r_o^2 q_{c1} + r_o d q_{c1} \sin(\pi/2 + \phi - \theta) - \frac{r_o^3 td \sin(\alpha + \phi)}{I_{min}} \theta$$

$$- \frac{2r_o^4 t}{I_{min}} \sin \theta/2 \sin(\alpha + \theta/2) - \frac{r_o^2 d^2 t \sin(\alpha + \phi)}{I_{min}} \theta \sin(\pi/2 + \phi - \theta)$$

$$- \frac{2r_o^3 td}{I_{min}} [\sin \theta/2 \sin(\alpha + \theta/2) \sin(\pi/2 + \phi - \theta)]\} d\theta$$

[47] $$m_6 = \int_0^{\delta} \{r_o^2 q_{c3} + \frac{r_o^3 dt \cos(\pi/2 + \alpha + \phi)}{I_{min}} \theta - \frac{2r_o^4 t}{I_{min}} \sin \theta/2 \sin(\alpha + \beta + \theta/2)$$

$$- r_o d q_{c3} \cos(\pi + \phi - \beta - \theta) - \frac{r_o^2 d^2 t \cos(\pi/2 + \alpha + \phi) \cos(\pi + \phi - \beta - \theta)}{I_{min}} \theta$$

$$+ \frac{2r_o^3 td}{I_{min}} [\sin \theta/2 \sin(\alpha + \theta/2 + \beta) \cos(\pi + \phi - \beta - \theta)]\} d\theta$$

[48] and $m_7 = \int_0^{\pi/2-\beta-\delta}$ (Integrand same as eqn. no. [42])

[49] $\therefore \; v_s = m_1 + m_5 + m_6 - m_3 - m_4 - m_7$

Case VIIb - Figure 8

m_1, m_3, m_4, m_{4a} = same as eqn. nos. [22], [14], [37] and [38], respectively.

[50] $m_5 = \int_0^{\beta}$ (Integrand same as eqn. no. [46])

[51] and $m_6 = \int_0^{\pi/2-\beta}$ (Integrand same as eqn. no. [47])

[52] $\therefore \; v_s = m_1 + m_{4a} + m_5 + m_6 - m_3 - m_4$

Case VIIIa - Figure 9

m_1, m_3, m_4 = same as eqn. nos. [22], [14] and [15], respectively.

[53] $m_5 = \int_0^{\beta}$ (Integrand same as eqn. no. [19])

[54] $m_6 = \int_0^{\delta}$ (Integrand same as eqn. no. [25])

[55] and $m_7 = \int_0^{\pi/2-\beta-\delta}$ (Integrand same as eqn. no. [20])

[56] $\therefore \; v_s = m_1 + m_5 + m_6 - m_3 - m_4 - m_7$

Case VIIIb - Figure 9

m_1, m_3, m_4, m_{4a} = same as eqn. nos. [22], [14], [37] and [38], respectively.

[57] $m_5 = \int_0^{\beta}$ (Integrand same as eqn. no. [19])

[58] and $m_6 = \int_0^{\pi/2-\beta}$ (Integrand same as eqn. no. [25])

[59] $\therefore \; v_s = m_1 + m_{4a} + m_5 + m_6 - m_3 - m_4$

Case IXa - Figure 10

m_1, m_2, m_3, m_4 = same as eqn. nos. [12], [13], [14] and [15], respectively.

[60] $m_5 = \int_0^{\delta}$ (Integrand same as eqn. no. [16])

[61] and $m_6 = \int_0^{\pi/2-\delta}$ (Integrand same as eqn. no. [17])

[62] $\therefore \; v_s = m_1 + m_2 + m_5 - m_3 - m_4 - m_6$

Case IXb - Figure 10

m_1, m_2, m_3, m_4, m_{4a} = same as eqn. nos. [12], [13], [14], [37] and [38], respectively.

[63] and $m_5 = \int_0^{\pi/2}$ (Integrand same as eqn. no. [16])

[64] $\therefore\ v_s = m_1 + m_2 + m_{4a} + m_5 - m_3 - m_4$

(b) u-coordinate (Bending about u-u axis) (Ref. Fig. 11 to 13)

Depending on the location where the u-u axis intersects the centre-line of the cross-section, one of the following cases will be applicable.

Case I - Figure 11 [u-u axis intersects the centre-line of the cross-section just at the beginning of curve]

[65] $$m_1 = \frac{(\bar{y} - t/2)t\, c_1^2}{2I_{max}} \left[\frac{\bar{y} - t/2}{\cos^2\alpha} + (\ell_1 - \frac{c_1}{3})\sin\alpha \right]$$

[66] $$m_2 = (\bar{x} - t/2)t \cos\alpha\; c_2^3/(3I_{max})$$

[67] $$\text{and } m_3 = \int_0^{\pi/2} \{ r_o^2 q_{c6} + r_o d q_{c6} \sin(\theta + \phi) + \frac{r_o^3 td \cos(\alpha + \phi)}{I_{max}}\theta$$

$$+ \frac{r_o^2 d^2 t \sin(\theta + \phi)\theta \cos(\alpha + \phi)}{I_{max}} + \frac{2r_o^4 t}{I_{max}} \sin\theta/2 \sin(\pi/2 - \alpha - \theta/2)$$

$$+ \frac{2r_o^3 td}{I_{max}} [\sin\theta/2 \sin(\theta + \phi)\sin(\pi/2 - \alpha - \theta/2)] \} d\theta$$

[68] $$\therefore\; u_s = m_1 + m_2 + m_3$$

Case II - Figure 12 [u-u axis intersects the centre-line of the cross-section in the curved portion)

m_1 = same as eqn. no. [65].

[69] $$m_2 = (\bar{x} - t/2)t \cos\alpha\; c_2^2(\ell_5 - c_2/3)/(2I_{max})$$

[70] $$m_3 = \int_0^{\beta} \text{(Integrand same as eqn. no. [67])}$$

[71] $$\text{and } m_4 = \int_0^{\pi/2-\beta} \{ r_o^2 q_{c7} - \frac{r_o^3 td \cos(\alpha + \phi)}{I_{max}}\theta + \frac{2r_o^4 t}{I_{max}} \sin\theta/2 \sin(\alpha - \theta/2)$$

$$+ r_o q_{c7} d \sin(\theta + \phi) - \frac{r_o^2 d^2 t \cos(\alpha + \phi)}{I_{max}}\theta \sin(\theta + \phi)$$

$$+ \frac{2r_o^3 td}{I_{max}} [\sin(\theta + \phi)\sin\theta/2 \sin(\alpha - \theta/2)] \} d\theta$$

[72] $\therefore \; u_s = m_1 + m_2 + m_3 + m_4$.

However, if $d_1 < 0$, the eqn. nos. [70] and [71] shall be replaced by eqn. nos. [73] and [74], respectively.

[73] $$m_3 = \int_0^{\beta} \{r_o^2 q_{c6} - r_o d q_{c6} \sin(\theta + \phi) - \frac{r_o^3 td \cos(\alpha + \phi)}{I_{max}} \theta$$

$$+ \frac{r_o^2 d^2 t \sin(\theta + \phi)\theta \cos(\alpha + \phi)}{I_{max}} + \frac{2r_o^4 t}{I_{max}} [\sin \theta/2 \sin(\pi/2 - \alpha - \theta/2)]$$

$$- \frac{2r_o^3 td}{I_{max}} [\sin \theta/2 \sin(\theta + \phi)\sin(\pi/2 - \alpha - \theta/2)]\}d\theta$$

[74] and $$m_4 = \int_0^{\pi/2-\beta} \{r_o^2 q_{c7} + \frac{r_o^3 td \cos(\alpha + \phi)}{I_{max}} \theta - r_o q_{c7} d \sin(\theta + \phi)$$

$$- \frac{r_o^2 d^2 t \cos(\alpha + \phi)}{I_{max}} \theta \sin(\theta + \phi) + \frac{2r_o^4 t}{I_{max}} [\sin \theta/2 \sin(\alpha - \theta/2)]$$

$$- \frac{2r_o^3 td}{I_{max}} [\sin (\theta + \phi)\sin \theta/2 \sin(\alpha - \theta/2)]\}d\theta$$

<u>Case III - Figure 13</u> [u-u axis intersects the centre-line of the cross-section in the straight portion of the long leg]

m_1 = same as eqn. no. [65]

[75] $m_2 = -(\bar{x} - t/2)t \cos \alpha \; \ell_6^3/(3I_{max})$

[76] $m_3 = (\bar{x} - t/2)\ell_5[t \cos \alpha \; \ell_5^2/(3I_{max}) + q_{c8}]$

[77] and $m_4 = \int_0^{\pi/2}$ (Integrand same as eqn. no. [67])

[78] $\therefore \; u_s = m_1 + m_2 + m_3 + m_4$

8. Polar moment of inertia about the shear centre

[79] $I_{ps} = I_{max} + I_{min} + \text{Area}(u_s^2 + v_s^2)$

where u_s and v_s are the computed u- and v-coordinates of the shear centre.

9. Warping constant (Ref. Fig. 14)

The following are the basic expressions used in the calculation of the magnitude of warping constant:

[80] $$w_{s_1} = -\int_0^s r_1 ds \qquad 0 \leq s \leq c_1$$

[81] $$w_{s_2} = -r_1 c_1 - \int_0^\theta r_2(r + t/2)d\theta \qquad 0 \leq \theta \leq \pi/2$$

[82] $$w_{s_3} = -r_1 c_1 - (r + t/2)^2\pi/2 - r'(r + t/2)[\cos(\nu + \pi/2) - \cos\nu] - \int_0^s r_3 ds$$

$$0 \leq s \leq c_2$$

[83] $$w_{\bar{s}} = \frac{1}{[c_1+c_2+\pi(r+t/2)/2]}\left[\int_0^{c_1} w_{s_1} ds + \int_0^{\pi/2} w_{s_2}(r + t/2)d\theta + \int_0^{c_2} w_{s_3} ds\right]$$

$$= \frac{1}{[c_1+c_2+\pi(r+t/2)/2]}\{-r_1c_1[c_1 + \pi(r + t/2)]/2 + c_2(c_3 - r_3c_2/2)$$

$$- \frac{\pi^2(r+t/2)^3}{8} - r'(r+t/2)^2[\sin(\nu + \pi/2) - \sin\nu - \pi/2\cos\nu]\}$$

where $c_3 = -r_1c_1 - (r + t/2)^2\pi/2 - r'(r + t/2)[\cos(\nu + \pi/2) - \cos\nu]$

[84] and finally warping constant,

$$C_w = \int (w_{\bar{s}} - w_s)^2 t ds$$

$$= \int_0^{c_1} (w_{\bar{s}} - w_{s_1})^2 t ds + \int_0^{c_2} (w_{\bar{s}} - w_{s_3})^2 t ds + \int_0^{\pi/2} (w_{\bar{s}} - w_{s_2})^2 t(r+t/2)d\theta$$

which becomes

$$t[w_s^{-2} + r_1^2 c_1^2/3 + w_s^- r_1 c_1]c_1 + t[w_s^{-2} + c_3^2 + r_3^2 c_2^2/3$$

$$- c_3 r_3 c_2 - 2w_s^- c_3 + w_s^- r_3 c_2]c_2 + \{[w_s^{-2} + r_1^2 c_1^2 + r'^2 (r + t/2)^2$$

$$\cos^2 \nu - 2r_1 c_1 r'(r + t/2)\cos \nu + 2w_s^- r_1 c_1 - 2w_s^- r'(r + t/2)\cos \nu]$$

$$+ (r + t/2)^4 \pi^3/24 + r'^2 (r + t/2)^2 (\pi/4 - .5 \sin 2\nu) - [-r_1 c_1 (r + t/2)^2$$

$$+ r'(r + t/2)^3 \cos \nu - w_s^- (r + t/2)^2]\pi^2/4 - [-2r_1 c_1 r'(r + t/2) +$$

$$2r'^2 (r + t/2)^2 \cos \nu - 2w_s^- r'(r + t/2)][\sin(\nu + \pi/2) - \sin \nu] +$$

$$2(r + t/2)^3 r'[.5 \pi \sin(\nu + \pi/2) + \cos(\nu + \pi/2) - \cos \nu]\}t(r + t/2)$$

10. β_1 and β_2 (Ref. Fig. 15 and 16)

(a) For long leg only (Ref. Fig. 15)

[85] $x_{\ell\ell} = [x'_\beta - p_\ell]\cos(\pi/2 - \alpha) - q_1 - y_1 \cos(\pi/2 - \alpha)$

[86] $y_{\ell\ell} = [x'_\beta - p_\ell]\sin(\pi/2 - \alpha) - y_1 \sin(\pi/2 - \alpha)$

[87] and $dA = t dp_\ell$

where p_ℓ varies from 0 to $(x'_\beta - r_o)$

(b) For short leg only (Ref. Fig. 15)

[88] $x_{s\ell} = [y'_\beta - p_s]\cos \alpha - y_1 \cos(\pi/2 - \alpha) - q_1$

[89] $y_{s\ell} = -\{(y'_\beta - p_s)\sin \alpha + y_1 \sin(\pi/2 - \alpha)\}$

[90] and $dA = t dp_s$

where p_s varies from 0 to $(y'_\beta - r_o)$

(c) For curved portion only (Ref. Fig. 16)

[91] $x_{cp} = -[\bar{x}' - r_o + r_o \sin\theta]$

[92] $y_{cp} = -[\bar{y}' - r_o + r_o \cos\theta]$

[93] $x_c = y_{cp}\cos\alpha + x_{cp}\sin\alpha$

[94] $y_c = x_{cp}\cos\alpha - y_{cp}\sin\alpha$

[95] and $dA = r_o\, t\, d\theta$

where θ varies from 0 to $\pi/2$

Finally, β_1 and β_2 are given by the following expressions:

[96] $$\beta_1 = \Big(\int_0^{x'_\beta - r_o}(y^3_{\ell\ell} + x^2_{\ell\ell}\, y_{\ell\ell})\, t dp_\ell + \int_0^{y'_\beta - r_o}(y^3_{s\ell} + x^2_{s\ell}\, y_{s\ell})\, t dp_s$$

$$+ \int_0^{\pi/2}(y^3_c + x^2_c\, y_c) r_o\, t\, d\theta\Big)/I_{max} - 2v_s$$

[97] and $$\beta_2 = \Big(\int_0^{x'_\beta - r_o}(x^3_{\ell\ell} + y^2_{\ell\ell}\, x_{\ell\ell})\, t dp_\ell + \int_0^{y'_\beta - r_o}(x^3_{s\ell} + y^2_{s\ell}\, x_{s\ell})\, t dp_s$$

$$+ \int_0^{\pi/2}(x^3_c + y^2_c\, x_c) r_o\, t\, d\theta\Big)/I_{min} - 2u_s$$

The integrals are evaluated by adaptive quadrature technique using Simpson's rule [Ref.: Johnson, L.W. and Riess, R.D. 1982. Numerical Analysis. Addison-Wesley Publishing Company, Don Mills, Ontario. pp. 313-317].

List of Symbols

d = distance between the centre of curvature and the centroid

d_1 = $\bar{y} - (r + t)$

d_2 = $\bar{x} - (r + t)$

q_{c1} = shear flow at the beginning of the curve from the short leg (bending about v-v axis)

q_{c2} = shear flow at the beginning of the curve from the long leg (bending about v-v axis)

q_{c3} = shear flow at the point where the v-v axis intersects the short leg, if it is in the curved portion, i.e. at the end of angle β [Ref. Fig. 5] (bending about v-v axis)

q_{c4} = shear flow at the point where the v-v axis intersects the long leg, if it is in the curved portion, i.e., at the end of angle γ [Ref. Fig. 5] (bending about v-v axis)

q_{c5} = shear flow at the end of the curve from the short leg (bending about v-v axis)

q_{c6} = shear flow at the beginning of the curve from the short leg (bending about u-u axis)

q_{c7} = shear flow at the beginning of the curve from the long leg (bending about v-v axis)

q_{c8} = shear flow at the end of the curve from the short leg (bending about u-u axis)

r_o = $r + t/2$

x' = distance of the point of zero shear flow from the end of the curve, if it lies in the straight portion of the long leg

Table I

Case	Figure	Coordinates of centre of curvature		v-v axis intersects		Location of point of zero shear flow	Remarks
		x-coord.	y-coord.	long leg	short leg		
Equal leg angles							
I	2	-	-	straight portion	straight portion	curved portion	
II	3	+	+	"	"	"	
III	4	+	+	curved portion	curved portion	"	
Unequal leg angles							
IV	5	+	+	"	"	"	The angle δ+β from short leg gives the point of zero shear flow
Va	6	+	+	straight portion	straight portion	"	The angle δ from short leg gives the point of zero shear flow
Vb	6	+	+	"	"	straight portion	-
VIa	7	+	-	"	"	curved portion	"
VIb	7	+	-	"	"	straight portion	-
VIIa	8	+	-	"	curved portion	curved portion	The angle δ+β from short leg gives the point of zero shear flow
VIIb	8	+	-	"	"	straight portion	-
VIIIa	9	+	+	"	"	curved portion	"
VIIIb	9	+	+	"	"	straight portion	-
IXa	10	-	-	"	straight portion	curved portion	The angle δ from short leg gives the point of zero shear flow
IXb	10	-	-	"	"	straight portion	-

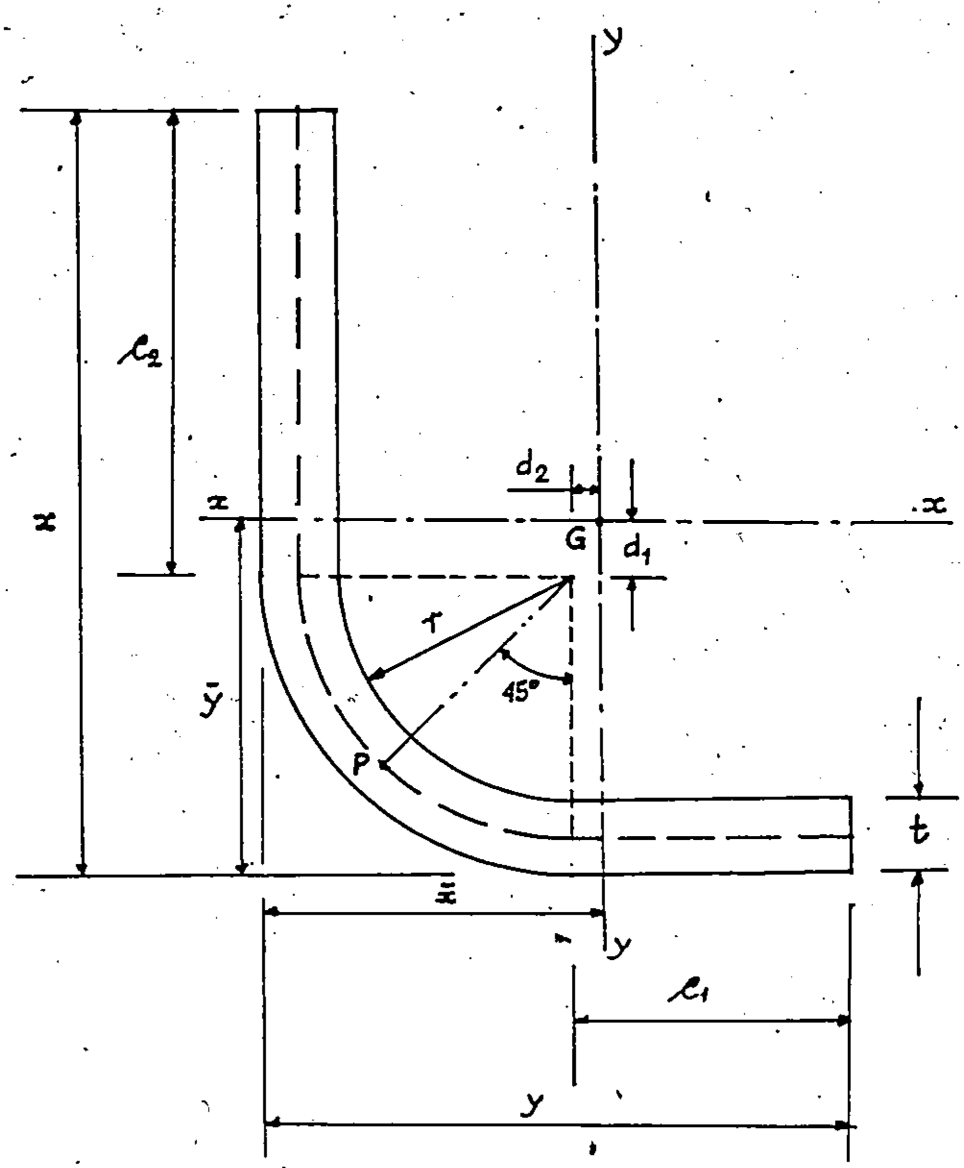


Figure 1

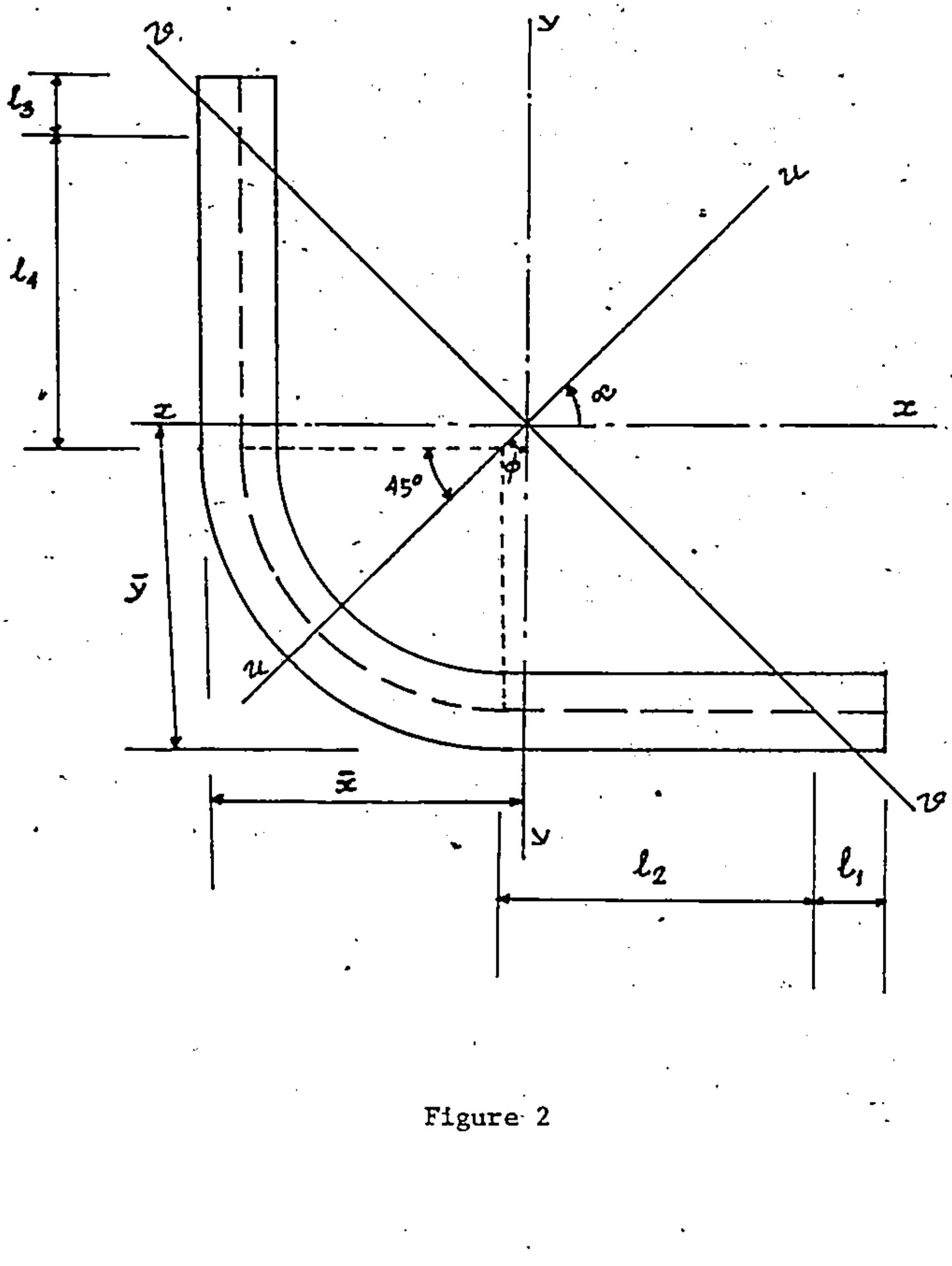


Figure 2

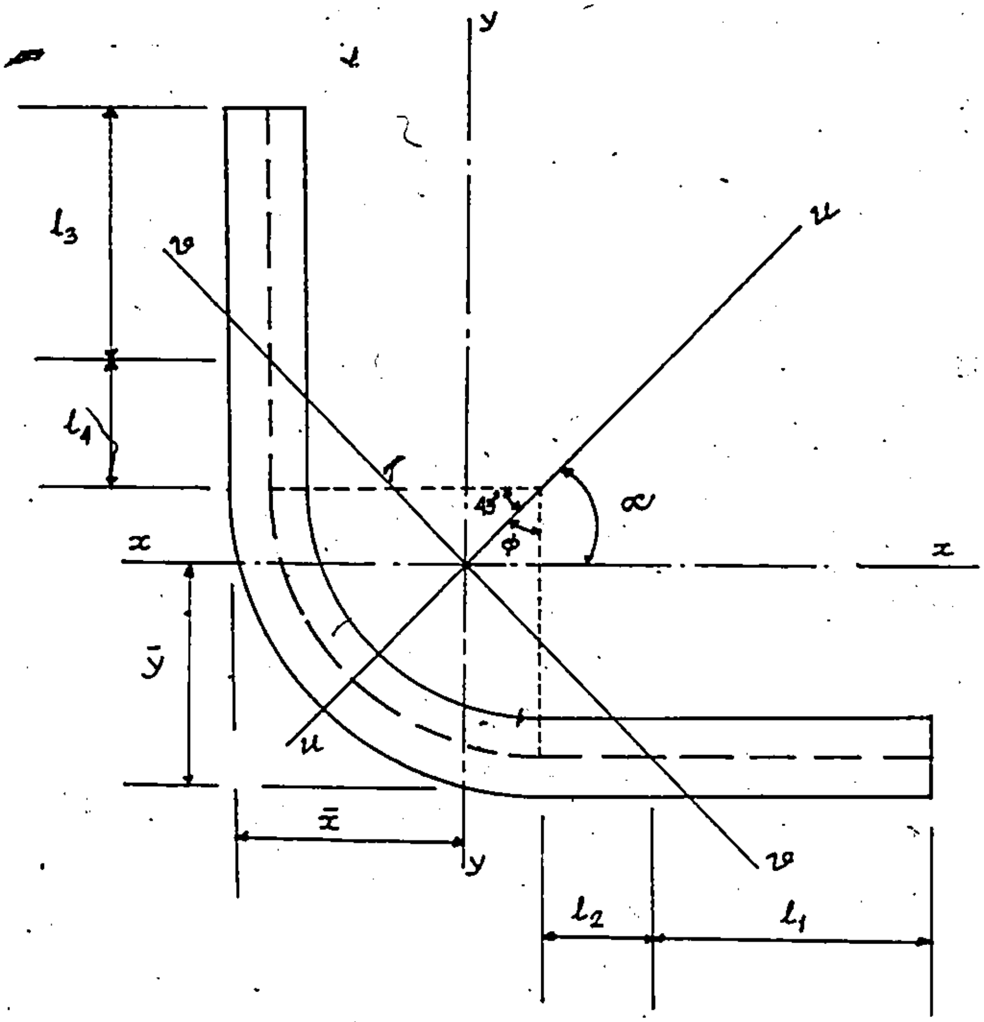


Figure 3

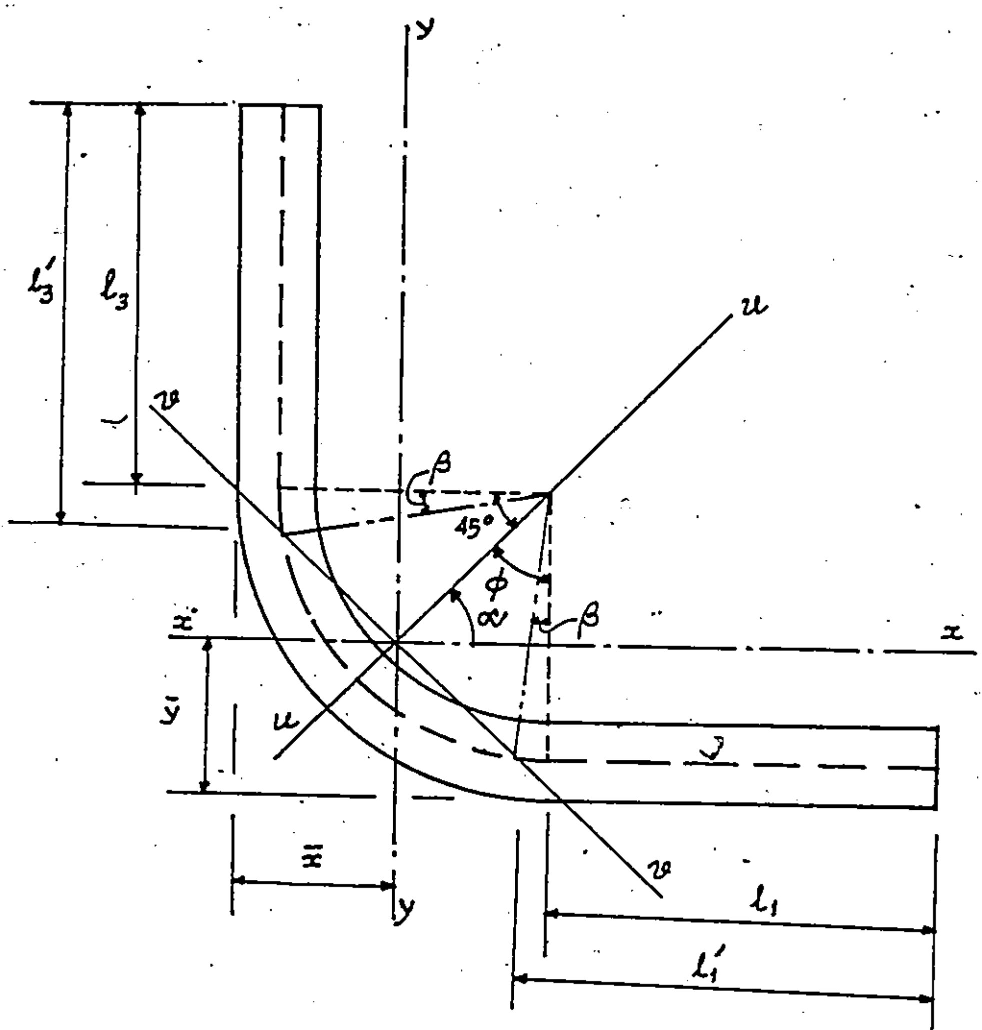


Figure 4

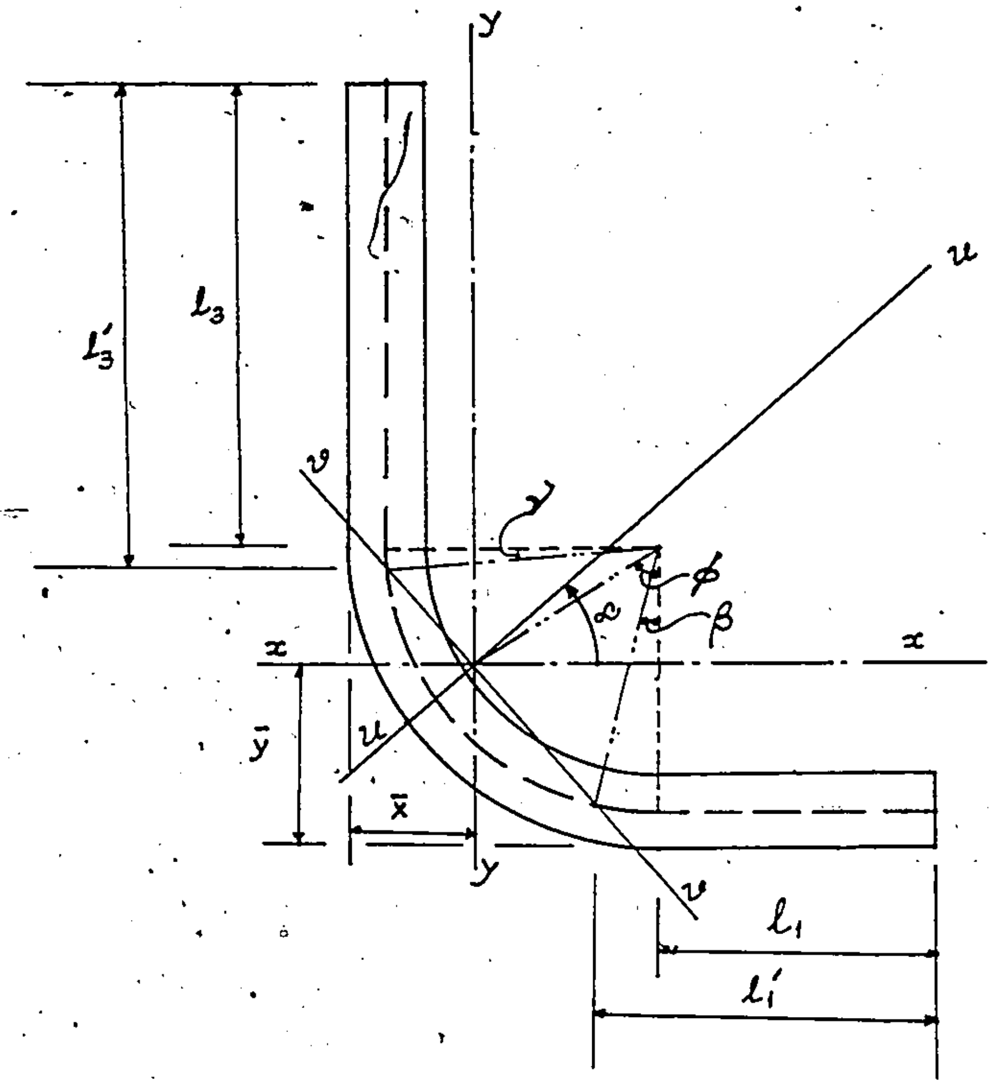


Figure 5

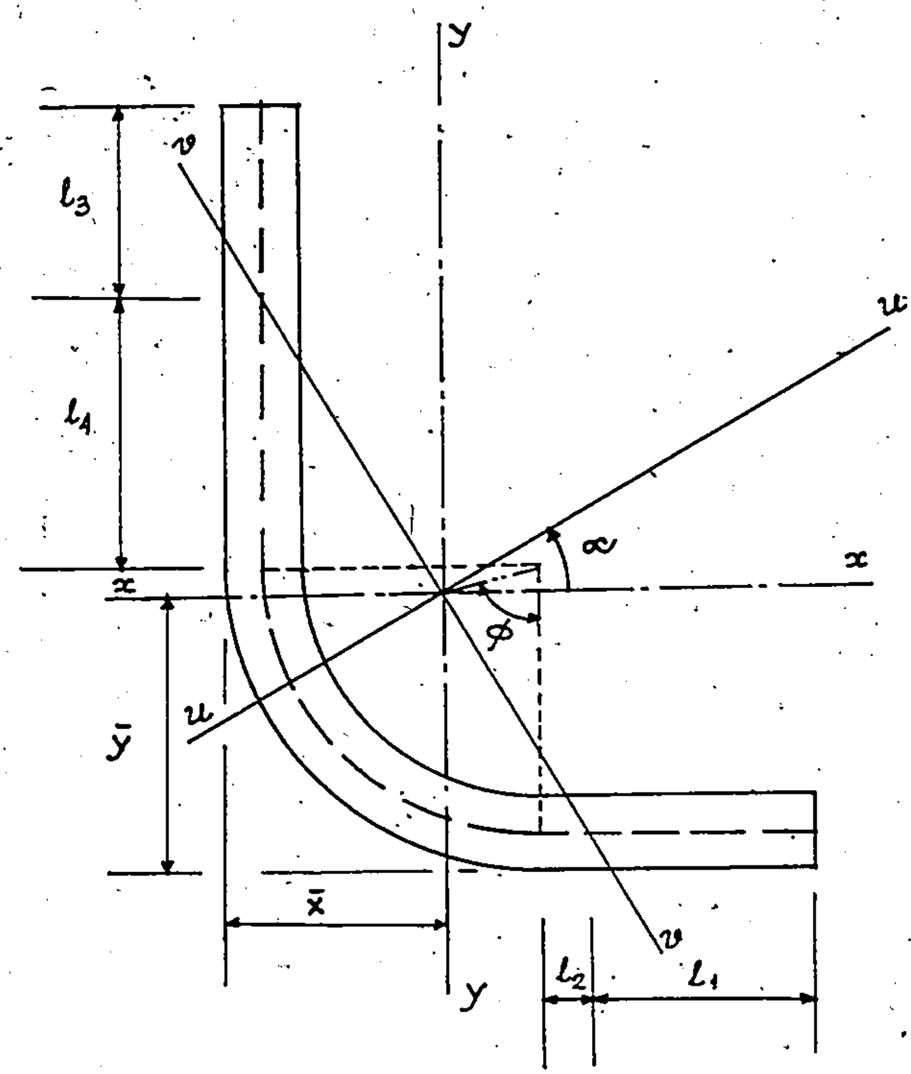


Figure 6

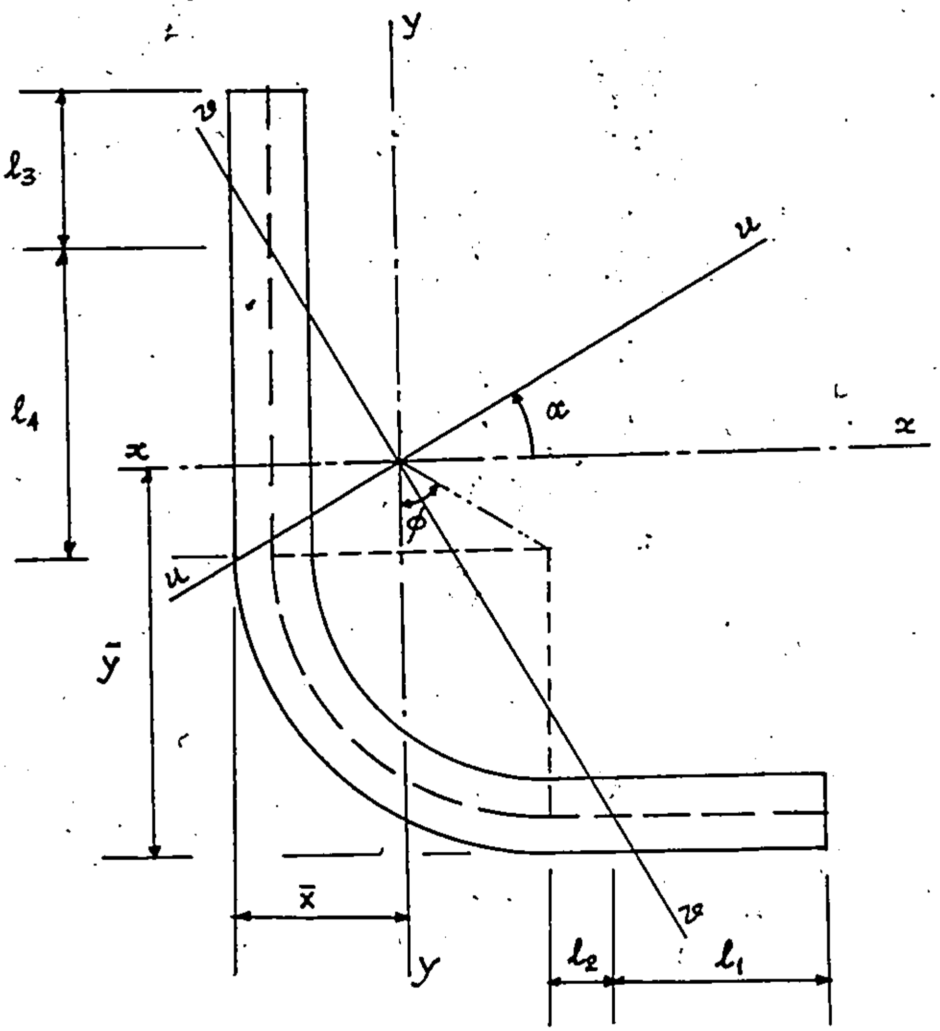


Figure 7

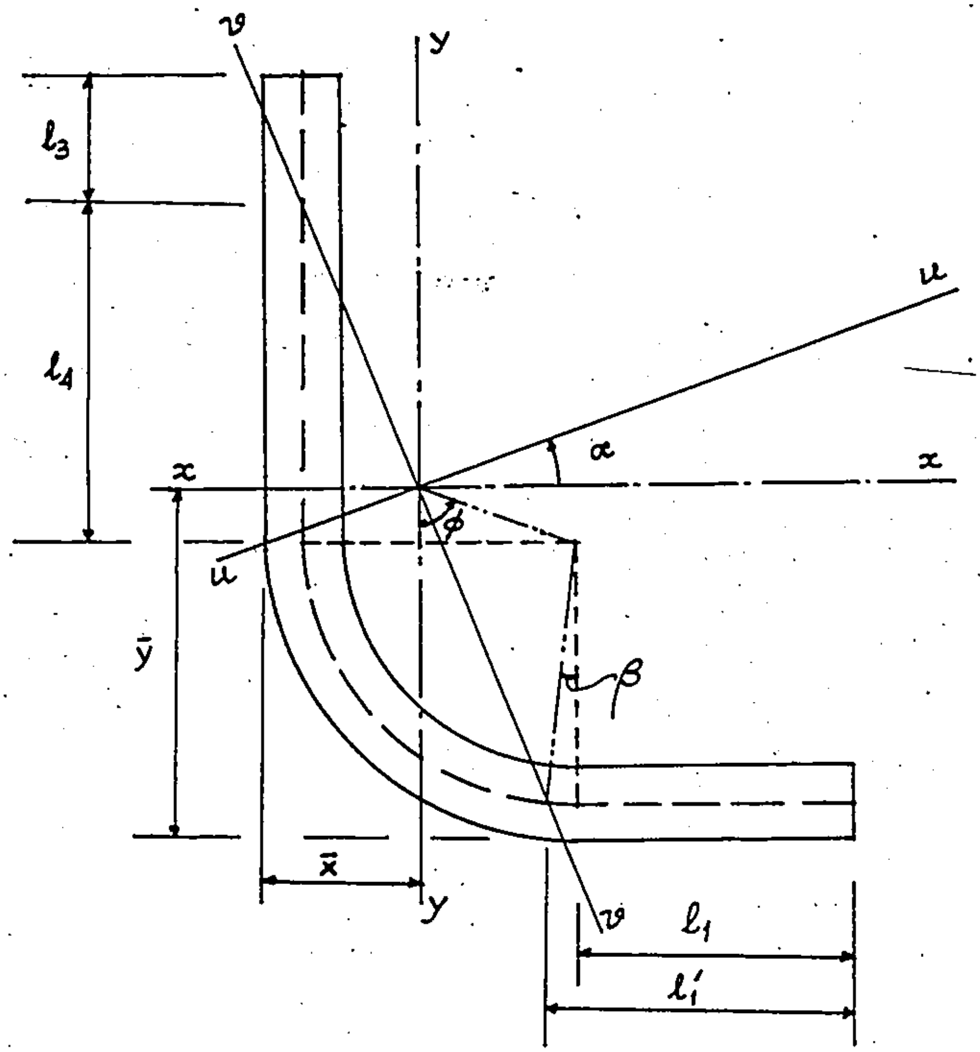


Figure 8

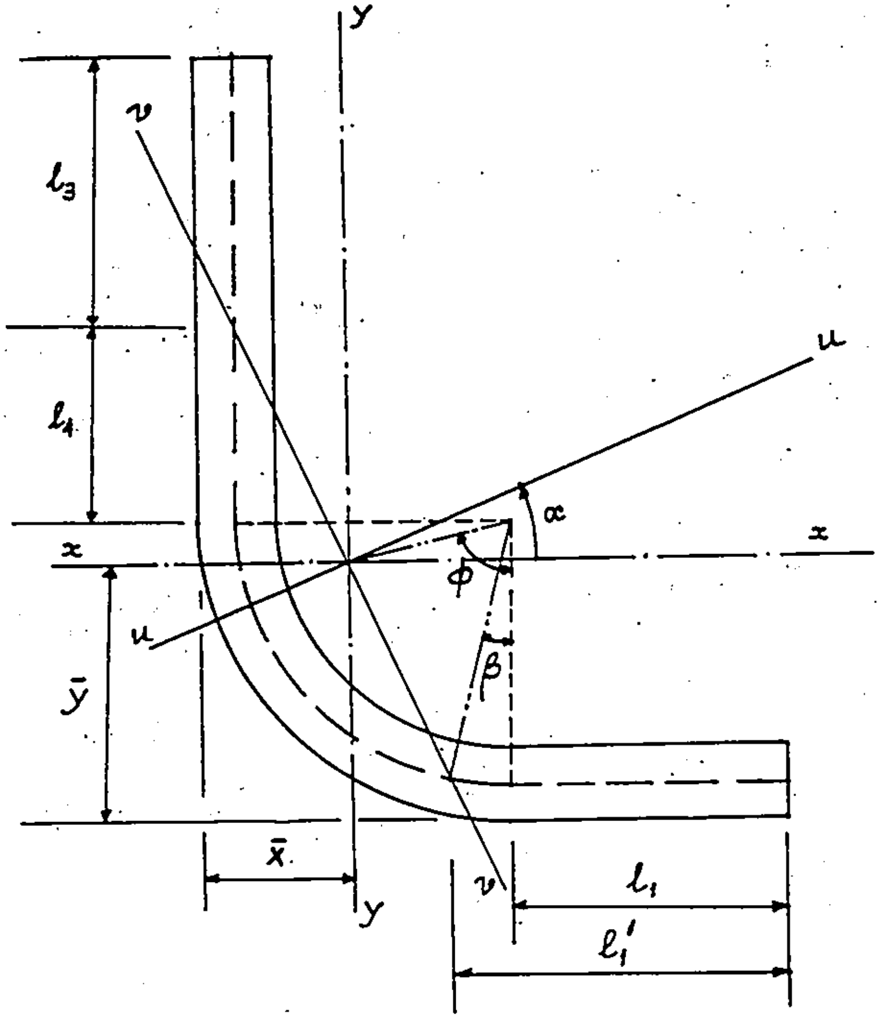


Figure 9

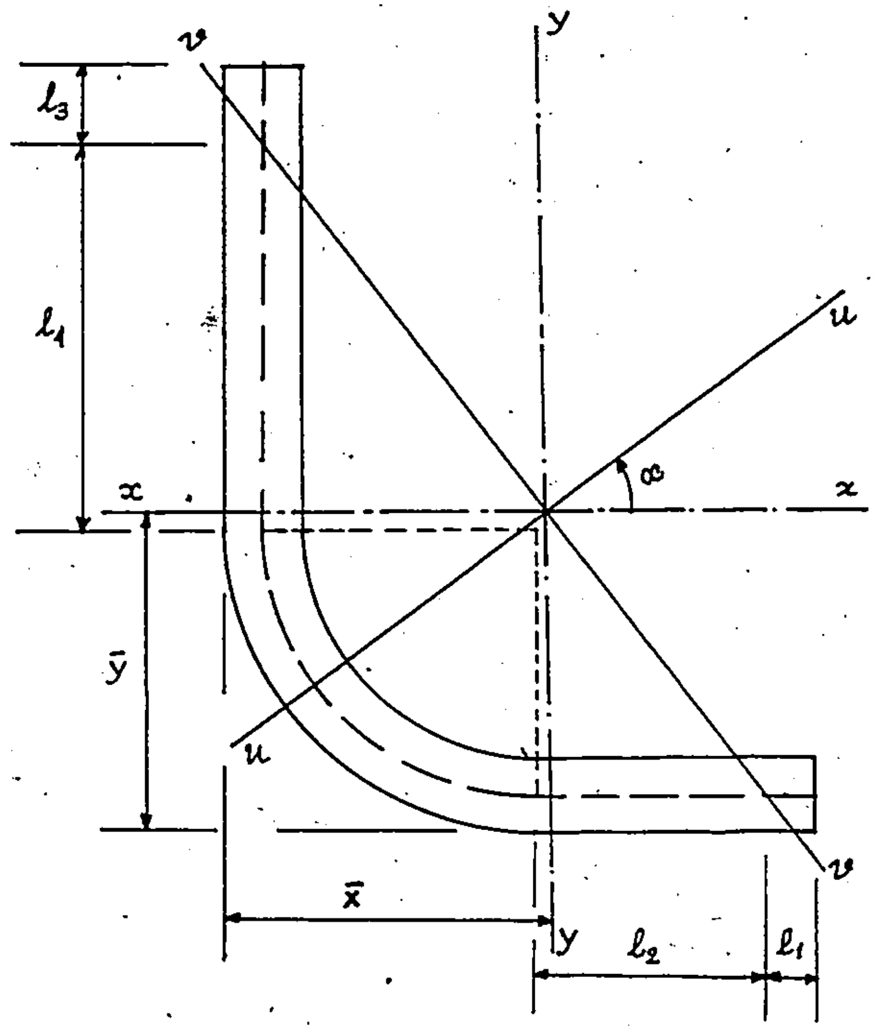


Figure 10

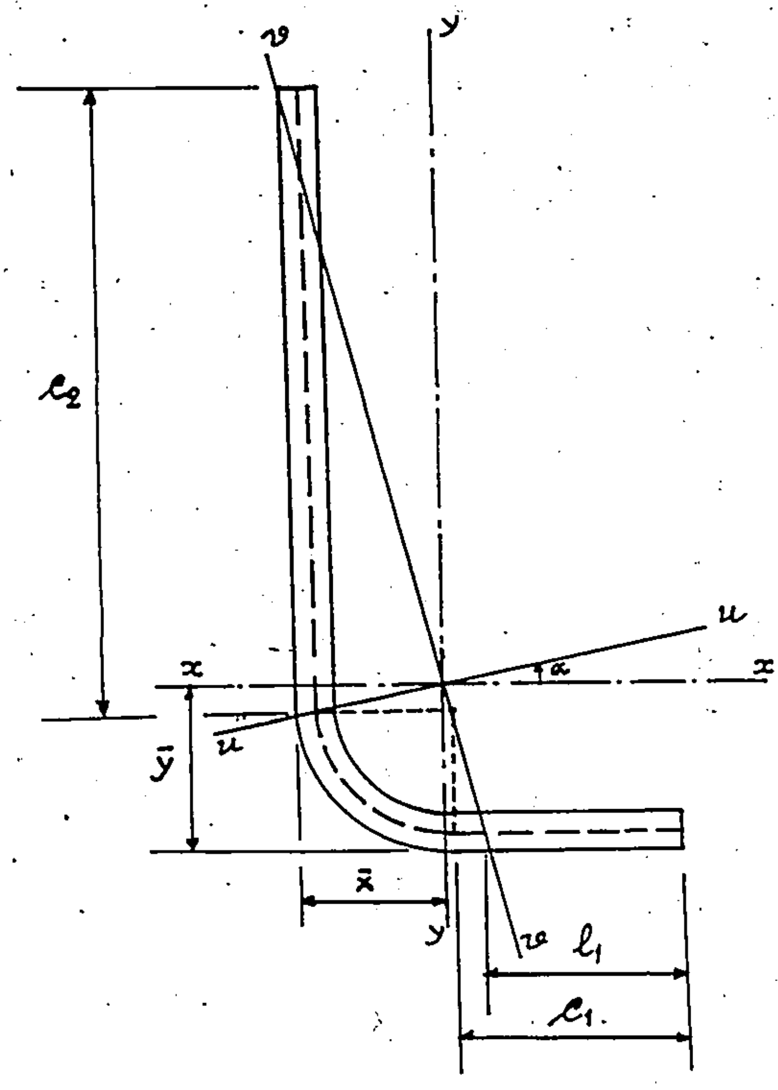


Figure 11

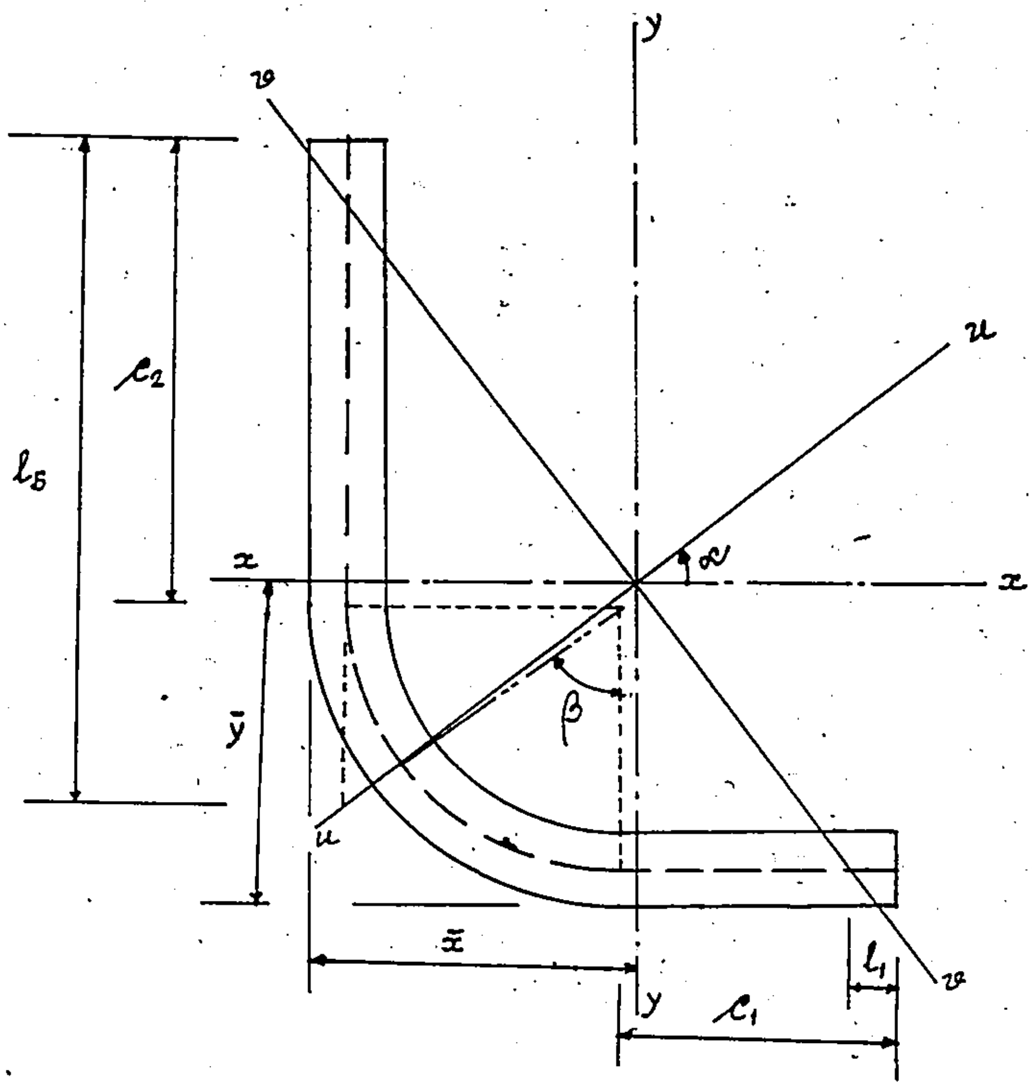


Figure 12

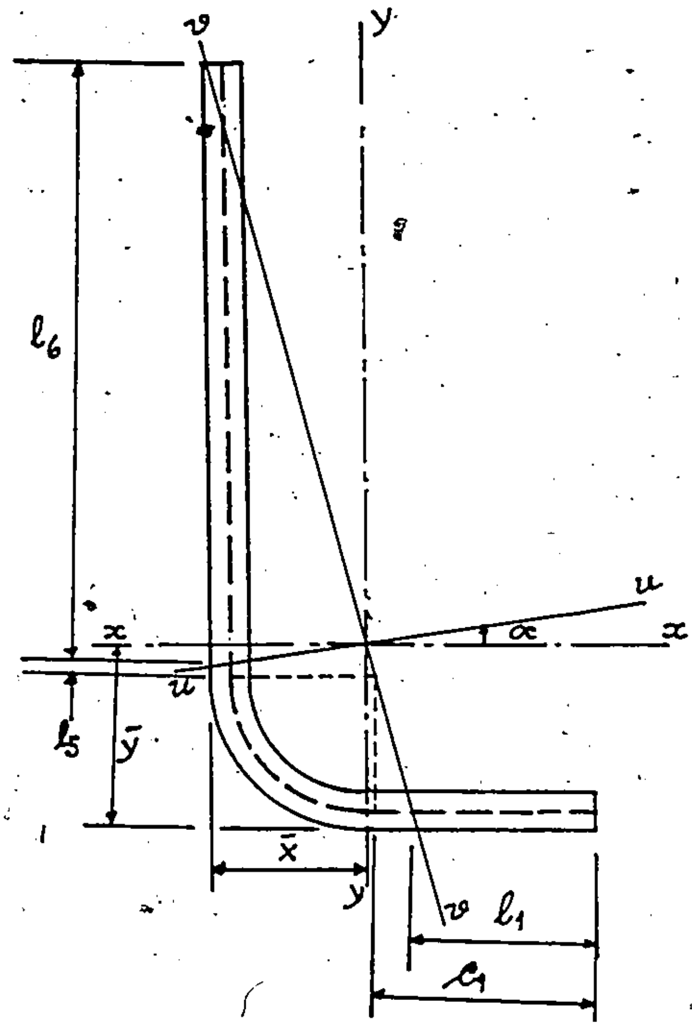


Figure 13

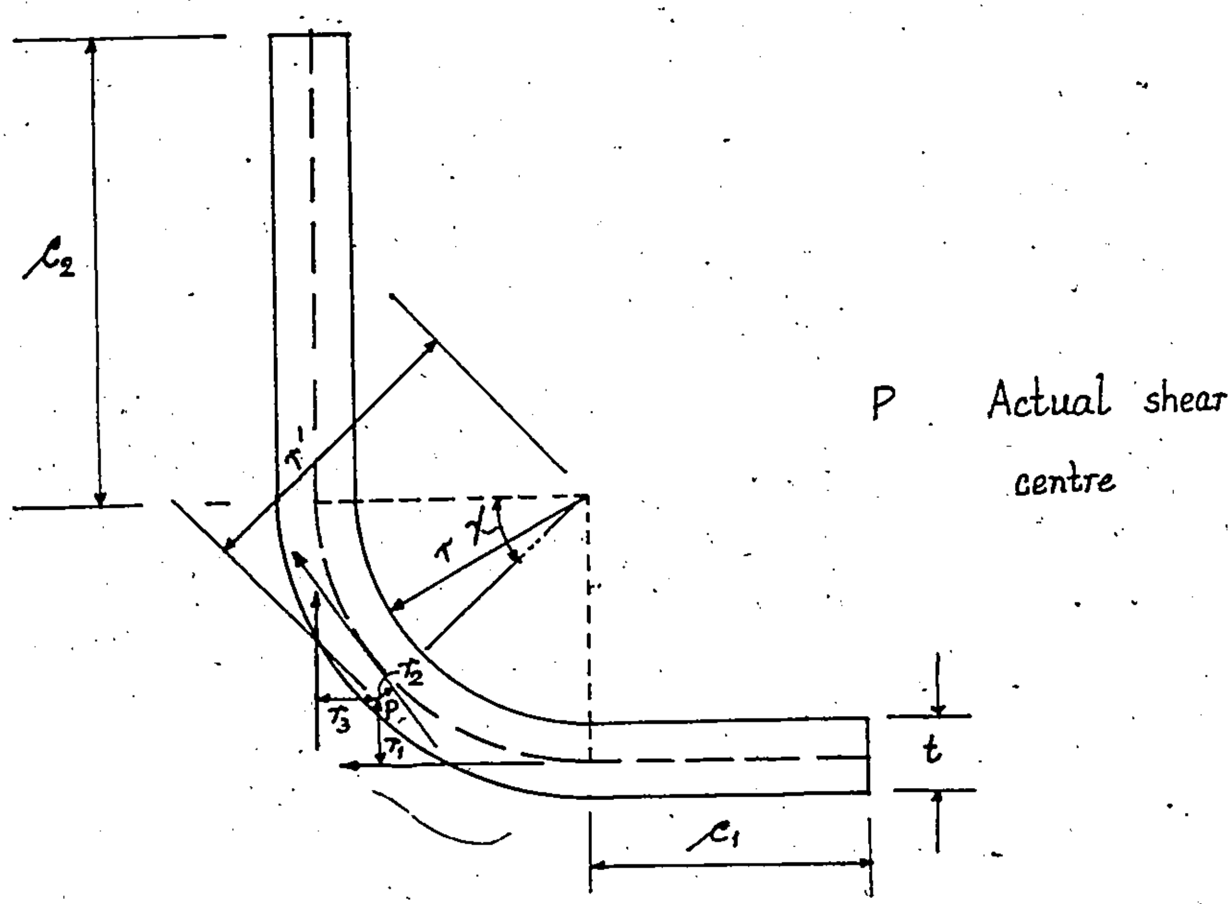


Figure 14

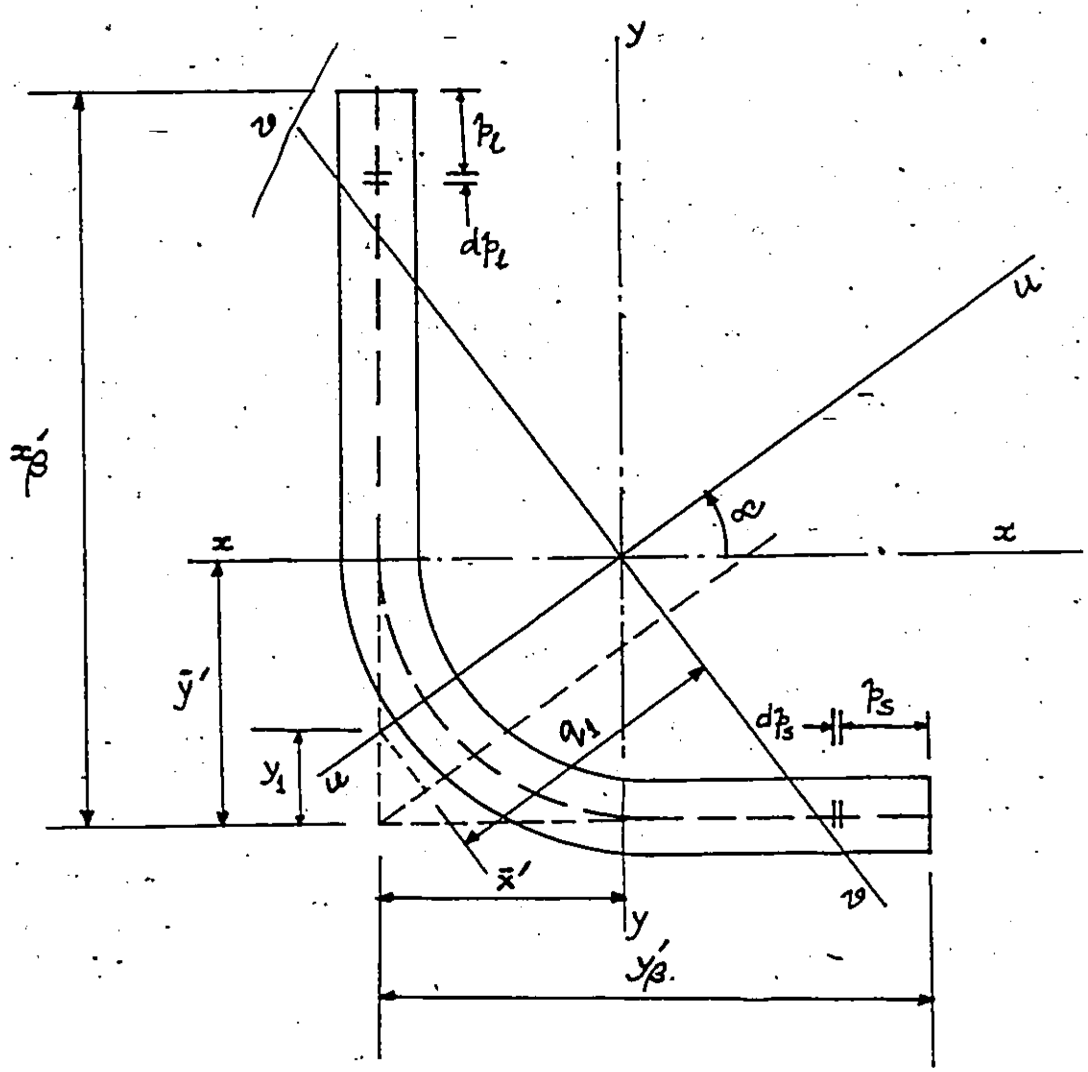


Figure 15

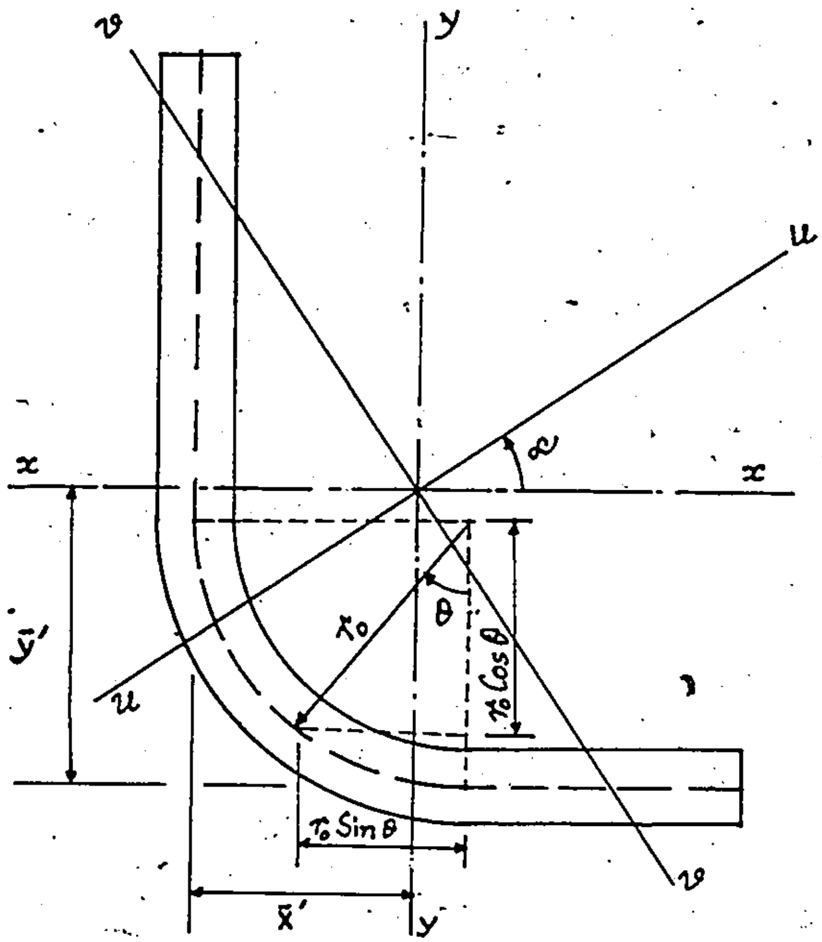


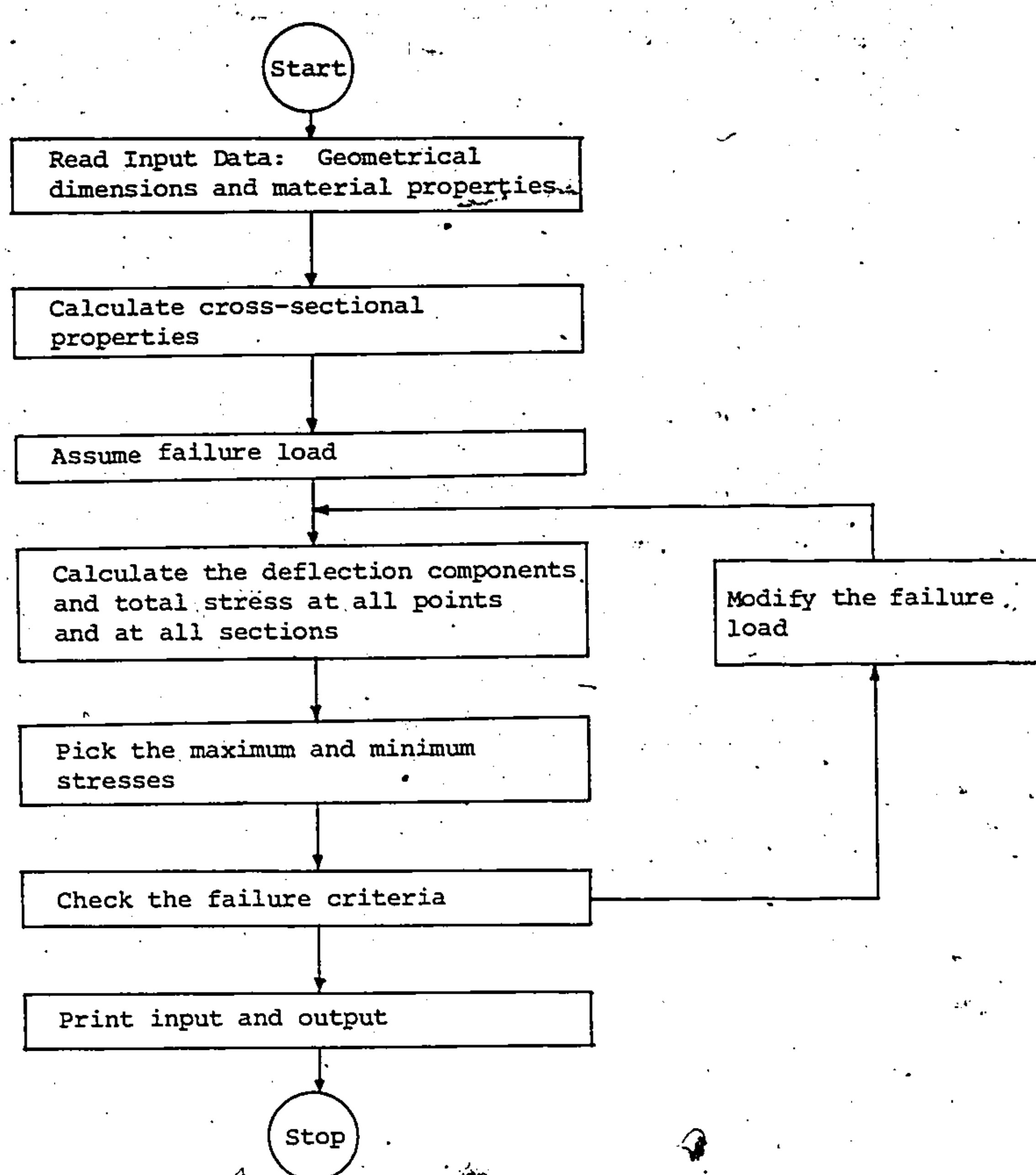
Figure 16

APPENDIX 2

FLOW CHART OF COMPUTER PROGRAM

APPENDIX 2

FLOW CHART OF COMPUTER PROGRAM



APPENDIX 3

SOURCE LISTING OF COMPUTER PROGRAM

```
C   ------------------------------------------------------------
C   -                     RAY, SUJIT KUMAR                     -
C   ------------------------------------------------------------
C   -                                                          -
C   -                  SENSITIVITY  ANALYSIS                   -
C   ------------------------------------------------------------
C ***************************************************************
C *  THIS PROGRAM CALCULATES THE THEORETICAL CRITICAL LOAD      *
C *  OF A COLD-FORMED ANGLE COLUMN, UNDER TORSIONAL-            *
C *  FLEXURAL BUCKLING, FOR DIFFERENT VALUES OF WARPING         *
C *  CONSTANT ,SHEAR CENTER LOCATION & SLENDERNESS RATIO. THE   *
C *  ASSUMPTIONS MADE ARE MENTIONED IN THE RESPECTIVE PARTS     *
C *  OF THE PROGRAM. HOWEVER, THE BASIC ASSUMPTION OF THIS      *
C *  ANALYSIS IS THE CONSIDERATION OF THE CURVED PORTION        *
C *  OF THE ANGLE SECTION AS LINE ELEMENT.                      *
C ***************************************************************
C *                        INPUT  DATA                          *
C *                                                             *
C *     X=LONG LEG OF THE ANGLE SECTION                         *
C *     Y=SHORT LEG OF THE ANGLE SECTION                        *
C *     T=THICKNESS, ASSUMED CONSTANT THROUGHOUT                *
C *     R=INTERNAL RADIUS OF CURVED PORTION                     *
C *     GPLATE=THICKNESS OF THE CONNECTED GUSSET PLATE          *
C *     CONLEG=SIZE OF THE CONNECTED LEG, X OR Y                *
C *     GAGDIS=LOCATION OF THE BOLT HOLE                        *
C *     SIGMAY=YIELD STRESS OF THE MATERIAL                     *
C *       ( NOTE : ALL INPUTS SHALL BE IN N AND MM ONLY)        *
C ***************************************************************
C *                   VARIABLE IDENTIFICATION                   *
C *                                                             *
C *     C=SECTIONAL AREA, MM**2                                 *
C *     Y11=DISTANCE OF CENTROID ALONG X-X AXIS, MM             *
C *     X11=DISTANCE OF CENTROID ALONG Y-Y AXIS, MM             *
C *     IXX=MOMENT OF INERTIA ABOUT X-X AXIS, MM**4             *
C *            (X-X AXIS BEING THE AXIS PARALLEL TO             *
C *              SHORT LEG)                                     *
C *     IYY=MOMENT OF INERTIA ABOUT Y-Y AXIS, MM**4             *
C *            (Y-Y AXIS BEING THE AXIS PARALLEL TO             *
C *               LONG LEG)                                     *
C *     PXY=PRODUCT OF INERTIA, MM**4                           *
C *     IMIN=MOMENT OF INERTIA ABOUT V-V AXIS, MM**4            *
C *            (V-V AXIS BEING THE AXIS MAKING AN ANGLE         *
C *               ALPHA WITH Y-Y AXIS)                          *
C *     IMAX=MOMENT OF INERTIA ABOUT U-U AXIS, MM**4            *
C *            (U-U AXIS BEING THE AXIS MAKING AN ANGLE         *
C *               ALPHA WITH X-X AXIS)                          *
C *     APHD=THE ANGLE ALPHA BETWEEN THE X-X AXIS & U-U         *
```

```
C    *         AXIS,  MEASURED COUNTER-CLOCKWISE FROM X-X AXIS        *
C    *                                                                 *
C    *    UZRACT,VZRACT=U- & V-  COORDINATES  OF  THE  ACTUAL          *
C    *                      LOCATION  OF  THE  SHEAR  CENTER,  MM      *
C    *                                                                 *
C    *    UZRAS1,VZRAS1,UZRAS2,VZRAS2=U- & V-  COORDINATES  OF  THE    *
C    *                             ASSUMED  LOCATION  OF  THE          *
C    *                               SHEAR  CENTER, MM                 *
C    *                                                                 *
C    *    RMIN=MINIMUM  RADIUS  OF  GYRATION, MM                       *
C    *                                                                 *
C    *    J3=TORSIONAL  CONSTANT , MM**4                               *
C    *                                                                 *
C    *    EU,EV=U- & V- ECCENTRICITES  OF  THE  POINT  OF  THE         *
C    *               LOAD  APPLICATION, MM                             *
C    *                                                                 *
C    *    BETA1,BETA2=PARAMETERS  REQD.  FOR  THE  CALCULATION         *
C    *                 OF  THE  CRITICAL  LOAD, MM                     *
C    *                                                                 *
C    *    CWACT=ACTUAL  MAGNITUDE  OF  THE  WARPING  CONSTANT,         *
C    *                                                 MM**6           *
C    *                                                                 *
C    *    PXE=EULER  LOAD  OF  THE  MEMBER ( BENDING  ABOUT U-U        *
C    *                                         AXIS ), N               *
C    *                                                                 *
C    *    PYE=EULER  LOAD  OF  THE  MEMBER ( BENDING  ABOUT V-V        *
C    *                                         AXIS ), N               *
C    *                                                                 *
C    *    POE=CRITICAL  LOAD  OF  THE  MEMBER ( UNDER PURE             *
C    *                                        TORSION ), N.            *
C    *                                                                 *
C    *    CRILOD=CRITICAL  LOAD  UNDER  TORSIONAL-FLEXURAL             *
C    *                       BUCKLING, N                               *
C    *                                                                 *
C    ******************************************************************
      IMPLICIT REAL*8 (A-Z)
      INTEGER I,J,K,L,M,N,MM,IK,IJ,IL,KLM,MMP
      DIMENSION UP(16),VP(16),U(11),V(11),BETA(11),PHIDER(11),
     *          UCH(11),VCH(11),BETACH(11)
      READ(5,55555) N
      DO 10  I=1,N
      READ(5,33333) X,Y,T,R,GPLATE,CONLEG,GAGDIS,SIGMAY
55555 FORMAT(I3)
33333 FORMAT(8F6.2)
      X1=R+T
      C1=Y-X1
      C2=X-X1
      C3=T*.5
      C4=C1*C3*T
      C5=C2*C3*(X+X1)
      C6=C1*C3*(Y+X1)
      C7=C2*C3*T
      C8=(R+C3)*1.570796327
C    ******************************************************************
C    *  CALCULATION  OF  THE  SECTIONAL  AREA                         *
C    ******************************************************************
      C=(C1+C2+C8)*T
C    ******************************************************************
C    *  CALCULATION  OF  THE  XBAR & YBAR                             *
C    ******************************************************************
      CXY=(X1-C8*.40528473)*C8*T
      CY=C6+C7+CXY
      CX=C4+C5+CXY
      Y11=CY/C.
      X11=CX/C
C    ******************************************************************
C    *  CALCULATION  OF  IXX, IYY & PXY                               *
C    ******************************************************************
      CYY=C1*T*((Y+X1)*.5-Y11)**2+T*C2*(Y11-C3)**2
      CXX=C2*T*((X+X1)*.5-X11)**2+T*C1*(X11-C3)**2
      P=Y11-X1+C8*.40528473
      Q=X11-X1+C8*.40528473
      C9=.14877839*(R+C3)**3*T
      C10=T/12.
      C11=T*T*C10
      IS1=C1**3*C10+C2*C11+C9+C8*T*P**2
      IYY=IS1+CYY
      IS2=C2**3*C10+C1*C11+C9+C8*T*Q**2
      IXX=IS2+CXX
      C12=R+C3
```

```
      D1=X11-X1
      D2=Y11-X1
      PXY=-C1*T*(X11-C3)*((Y+X1)*.5-Y11)-C2*T*(Y11-C3)
     *    *((X+X1)*.5-X11)+T*C12*(D1*D2*1.570796327+C12*D1+C12*D2+
     *    C12*C12*.5)
C    ***************************************************************
C    *  CALCULATION  OF  IMIN, IMAX & ALPHA                        *
C    ***************************************************************
      IMIN=(IXX+IYY)*.5-DSQRT(((IXX-IYY)*.5)**2+PXY*PXY)
      IMAX=IXX+IYY-IMIN
      IF(IXX.EQ.IYY)GO TO 191
      QNT=2*PXY/(IXX-IYY)
      ANGLE=DATAN(DABS(QNT))
      APHD=ANGLE*28.64788976
      GO TO 121
  191 APHD=45.00
  121 APHR=APHD*.017453293
      APH1=DSIN(APHR)
      APH2=DCOS(APHR)
      APH3=DTAN(APHR)
C    ***************************************************************
C    *  CALCULATION  OF  ACTUAL  SHEAR  CENTER                     *
C    ***************************************************************
C    *  CALCULATION  OF  Y- COORDINATE  OF  THE  ACTUAL  SHEAR     *
C    *  CENTER                                                     *
C    ***************************************************************
      L1=Y-Y11-(X11-C3)*APH3
      D3=DSQRT(D1*D1+D2*D2)
      PHI=DATAN(D2/D1)
      SPHI=DSIN(PHI)
      CPHI=DCOS(PHI)
      IF(D2.LT.0.0.AND.L1.GT.C1)GO TO 510
C    ***************************************************************
C    *  CALCULATION  OF  THE  MOMENT  OF  THE  STRAIGHT  PORTION  OF *
C    *  THE  SHORT  LEG                                            *
C    *  ALSO,  TT=0.0      AND      L10=0.0                        *
C    ***************************************************************
      L2=Y-L1-X1
      QXC=T*APH2/(2.*IMIN)
      QX1L1=QXC*L1*L1
      CMX=(X11-C3)*T*.5*APH2/IMIN
      MX1=2.*CMX*L1**3/3.
      QX2L2=QXC*(L1+L2)*(L1-L2)
      MX2=CMX*L2*(L1*L1-L2*L2/3.)
      L10=0.0
      TT=0.0
      GO TO 511
  510 CONTINUE
C    ***************************************************************
C    *  CALCULATION  OF  THE  MOMENT  OF  THE  STRAIGHT  PORTION  OF *
C    *  THE  SHORT  LEG                                            *
C    *  ALSO,  L2=0.0                                              *
C    ***************************************************************
      L10=L1
      L1=C1
      QXC=T*APH2/IMIN
      QX2L2=QXC*L1*(L10-.5*L1)
      CMX=(X11-C3)*T*APH2/IMIN
      MX1=CMX*L1*L1*(L10*.5-L1/6.)
      MX2=0.0
      L2=0.0
      Q1=QX2L2
C    ***************************************************************
C    *  CALCULATION  OF  THE  ANGLE  "TT"                          *
C    ***************************************************************
      CC1=1./APH3
      CC2=X1-Y11
      AX=1.+CC1*CC1
      BX=2.*(CC2*CC1*CC1-D1*CC1)
      CX=D1*D1+CC2*CC2*CC1*CC1-2.*D1*CC1*CC2-C12*C12
      DIS=BX*BX-4.*AX*CX
      XI=(-BX+DSQRT(DIS))/(2.*AX)
      XII=(-BX-DSQRT(DIS))/(2.*AX)
      XXX=XI
      TT=DARSIN(DABS(XXX)/C12)
      C13=QX2L2-T*C12*C12*APH2/IMIN
      C14=-T*C12*D3*DSIN(APHR+PHI)/IMIN
      IF(D1.LT.0.0)C14=-C14
      C15=T*C12*C12/IMIN
      QX2L2=C13+C14*TT+C15*DCOS(APHR+TT)
```

```
 511  C13=QX2L2-T*C12*C12*DCOS(APHR+TT)/IMIN
      C14=T*C12*D3*DSIN(APHR+PHI)/IMIN
      IF(D1.LT.0.0)C14=-C14
      C15=T*C12*C12/IMIN
      THT=1.570796327
      IF(D2.LT.0.0.AND.L10.GT.C1) THT=1.570796327-TT
C     ****************************************************************
C     *  FINDING  THE  POINT  OF  ZERO  SHEAR  FLOW                  *
C     ****************************************************************
      AA=C13-C14*THT+C15*DCOS(THT+APHR+TT)
      IF(AA.GE.0.0) GO TO 51
      THT1=0.0
      DTH=.1
      THT2=THT1+DTH
   15 A=C13-C14*THT1+C15*DCOS(APHR+THT1+TT)
      B=C13-C14*THT2+C15*DCOS(APHR+THT2+TT)
      IF(A.GT.0.0.AND.B.LT.0.0) GO TO 20
      THT1=THT2
      THT2=THT2+DTH
      GO TO 15
   20 THT1=THT1+.01
      DTH=DTH*.1
      THT2=THT1+DTH
   25 A=C13-C14*THT1+C15*DCOS(APHR+THT1+TT)
      B=C13-C14*THT2+C15*DCOS(APHR+THT2+TT)
      IF(A.LT.0.0.AND.B.LT.0.0)GO TO 38
      IF(A.GT.0.0.AND.B.LT.0.0) GO TO 30
      THT1=THT2
      THT2=THT2+DTH
      GO TO 25
   38 THT1=THT1-.1
   30 THT1=THT1+.001
      DTH=DTH*.1
      THT2=THT1+DTH
   35 A=C13-C14*THT1+C15*DCOS(APHR+THT1+TT)
      B=C13-C14*THT2+C15*DCOS(APHR+THT2+TT)
      IF(A.LT.0.0.AND.B.LT.0.0)GO TO 42
      IF(A.GT.0.0.AND.B.LT.0.0) GO TO 40
      THT1=THT2
      THT2=THT2+DTH
      GO TO 35
   40 CONTINUE
C     ****************************************************************
C     *  THE  FOLLOWING  THT,  WHICH  IS  THE  ANGLE  MEASURED       *
C     *  CLOCKWISE  FROM  SHORT  LEG,  GIVES  THE  POINT  OF  ZERO   *
C     *  SHEAR  FLOW.                                                *
C     ****************************************************************
      THT=.5*(THT1+THT2)
      ULMT=THT
      LLMT=0.0
      GO TO 45
   42 THT=THT1-.001
      ULMT=THT
      LLMT=0.0
      GO TO 45
   51 IF(D1.GT.0.0.AND.D2.LT.0.0) GO TO 902
      IF(TT.NE.0.0.AND.D1.LT.0.0.AND.D2.LT.0.0) GO TO 559
C     ****************************************************************
C     *  CALCULATION  OF  THE  MOMENT  OF  THE  CURVED  PORTION      *
C     *  ABOUT  CENTROID                                             *
C     ****************************************************************
      ULMT=1.570796327
      LLMT=0.0
      C16=T/IMIN
      INT1=C12*C12*QX2L2*ULMT
      INT2=C12*QX2L2*D3*(-CPHI-SPHI)
      INT3=C12**3*C16*.5*D3*DSIN(APHR+PHI)*ULMT*ULMT
      INT4=C12**4*C16*(ULMT*APH2-DSIN(APHR+ULMT)+APH1)
      INT5=C12*C12*C16*DSIN(APHR+PHI)*D3*D3*(SPHI+
     *     .570796327*CPHI)
      INT6=C12**3*C16*D3*(APH2*(SPHI+CPHI)-DCOS(APHR+PHI)
     *     *ULMT*.5+.5*DSIN(APHR-PHI))
C     ****************************************************************
C     *  MXCP  GIVES  THE  MOMENT  OF  THE  CURVED  PORTION  ABOUT   *
C     *  CENTROID                                                    *
C     ****************************************************************
      IF(D1.LT.0.0)GO TO 528
      MXCP=INT1-INT2-INT3-INT4-INT5-INT6
      GO TO 112
  528 MXCP=INT1+INT2+INT3-INT4-INT5+INT6
```

```
      GO TO 112
559   WRITE(6,19999)
19999 FORMAT(/,'NON-SDTANDARD  ANGLE'/)
  112 L3=X-X11-(Y11-C3)/APH3
      L4=X-L3-X1
      IF(AA.GT.0.0) GO TO 52
C   ***************************************************************
C   *  CALCULATION  OF  THE  MOMENT  OF  THE  STRAIGHT  PORTION  OF *
C   *  THE  LONG  LEG                                               *
C   ***************************************************************
      CQX=T*APH1/(2.*IMIN)
      QX3L3=CQX*L3*L3
      MCX=(Y11-C3)*T*APH1/(2.*IMIN)
      MX3=MCX*(L3**3)*2./3.
      QX4L4=CQX*(L3+L4)*(L3-L4)
      MX4=MCX*L4*(L3*L3-L4*L4/3.)
      IF(AA.EQ.0.0)GO TO 95
      IF(D1.GT.0.0.AND.D2.LT.0.0)GO TO 908
      IF(D1.LT.0.0.AND.L3.GT.C2)GO TO 655
      LLMT=0.0
      ULMT=1.5708-PHT
      IF(D2.LT.0.0.AND.L10.GT.C1)ULMT=ULMT-TT
      TT2=0.
      IF(L4.GT.0.0)GO TO 656
C   ***************************************************************
C   *  CALCULATION  OF  THE  MOMENT  OF  THE  CURVED  PORTION       *
C   *  ABOUT  CENTROID                                              *
C   ***************************************************************
 352  INT1=C12*C12*QX4L4*ULMT
      INT2=C12**3*T*D3*DSIN(APHR+PHI)/IMIN*ULMT*ULMT*.5
      INT3=C12*D3*QX4L4*(DSIN(1.570796-ULMT-PHI)-CPHI)
      INT4=C12*C12*D3*D3*T*DSIN(APHR+PHI)/IMIN*(-ULMT*DSIN(1.570796-ULMT
     *       -PHI)+DSIN(ULMT+PHI)-SPHI)
      INT5=C12**4*T/IMIN*(-DSIN(1.570796-APHR+ULMT)+APH1*ULMT+APH2)
      INT6=2.*C12**3*T*D3/IMIN*(-.5*APH1*(DSIN(1.570796-ULMT-PHI)-CPHI)
     *       +.25*DCOS(APHR+PHI)*ULMT-.125*(DSIN(2.*ULMT+PHI-APHR)-DSIN
     *       (PHI-APHR)))
C   ***************************************************************
C   *  MXCRV  GIVES  THE  MOMENT  OF  THE  CURVED  PORTION  ABOUT   *
C   *  CENTROID                                                     *
C   ***************************************************************
      IF(D1.LT.0.0)GO TO 339
      MXCRV=INT1+INT2-INT3+INT4+INT5+INT6
      GO TO 340
 339  MXCRV=INT1+INT2+INT3-INT4-INT5+INT6
 340  MXCP=MXCP-MXCRV
      IF(ULMT.EQ.TT2)GO TO 656
C   ***************************************************************
C   *  VZRACT  GIVES  THE  V- COORDINATE   OF  THE  ACTUAL  SHEAR   *
C   *  CENTER                                                       *
C   ***************************************************************
 95   VZRACT=-(MX1+MX2+MXCP-MX3-MX4)
      GO TO 14
 655  CONTINUE
C   ***************************************************************
C   *  CALCULATION  OF  THE  MOMENT  OF  THE  STRAIGHT  PORTION  OF *
C   *  THE  LONG  LEG                                               *
C   ***************************************************************
      L11=L3
      L3=C2
      QX4L4=CQX*2.*L3*(L11-L3*.5)
      MX3=MCX*2.*L3*L3*(L11*.5-L3/6.)
      MX4=0.
      L4=0.
C   ***************************************************************
C   *  CALCULATION  OF  THE  ANGLE  "TT2"                           *
C   ***************************************************************
      TT2=DARCOS(DABS(XII)/C12)
      ULMT=TT2
      LLMT=0.
      GO TO 352
 656  CONTINUE
C   ***************************************************************
C   *  CALCULATION  OF  THE  MOMENT  OF  THE  CURVED  PROTION  ABOUT*
C   *  CENTROID  IF THE  CENTROID  LIES  IN  THE  SECOND  QUADRANT  *
C   *  WITH  RESPECT  TO  CENTER  OF  CURVATURE  AND  IF  THE  POINT*
C   *  OF  ZERO  SHEAR  FLOW  IS  IN  THE  CURVED  PORTION          *
C   ***************************************************************
      ULMT=1.5708-PHT-TT-TT2
      LLMT=0.
```

```
      C400=QX3L4-C12*C12*T/IMIN*APH1
      C401=C12*D3*T*DSIN(APHR+PHI)/IMIN
      C402=T*C12*C12/IMIN
      Q3=C400+C401*TT2-C402*DSIN(TT2-APHR)
      INT1=C12*C12*Q3*ULMT
      INT2=C12**3*D3*T*DSIN(APHR+PHI)/IMIN*.5*ULMT*ULMT
      INT3=C12*D3*Q3*(DSIN(1.570796-PHI-ULMT-TT2)-DSIN(1.570796-PHI-
     *     TT2))
      INT4=C12*C12*D3*D3*T/IMIN*DSIN(APHR+PHI)*(-ULMT*DSIN(
     *     1.570796-PHI-ULMT-TT2)+DCOS(1.570796-PHI-ULMT-TT2)
     *     -DCOS(1.570796-PHI-TT2))
      INT5=C12**4*T/IMIN*(DSIN(APHR-TT2)*ULMT-DSIN(-APHR+ULMT+TT2
     *     +1.570796)+DSIN(1.570796-APHR+TT2))
      INT6=2.*C12**3*T*D3/IMIN*(-.125*(DSIN(2.*ULMT+PHI-APHR+2.*TT2)-
     *     DSIN(PHI-APHR+2.*TT2))+.25*DCOS(APHR+PHI)*ULMT-.5*DSIN
     *     (APHR-TT2)*(DSIN(1.570796-ULMT-PHI-TT2)-DSIN(1.570796-PHI-
     *     TT2)))
C     ****************************************************************
C     *  MXCRV  GIVES  THE  MOMENT  OF  THE  CURVED  PORTION  ABOUT  *
C     *  CENTROID                                                    *
C     ****************************************************************
      IF(D1.GT.0.0.AND.D2.GT.0.0) GO TO 658
      MXCRV=INT1+INT2+INT3-INT4-INT5+INT6
      MXCP=MXCP-MXCRV
      GO TO 95
  658 MXCRV=INT1-INT2-INT3-INT4-INT5-INT6
      MXCP=MXCP-MXCRV
      GO TO 95
   52 CONTINUE
C     ****************************************************************
C     *  CALCULATION  OF  THE  POINT  OF  ZERO  SHEAR  FLOW  IF  IT  *
C     *  LIES  BEYOND  THE  CURVED  PORTION,  THAT  IS,  IN  THE     *
C     *  STRAIGHT  PORTION  OF  THE  LONG  LEG                       *
C     ****************************************************************
      AQ=1.
      BQ=-2.*L4
      CQ=2.*AA*IMIN/(T*APH1)
      DISC=BQ*BQ-4.*AQ*CQ
      XPP=(-BQ+DSQRT(DISC))/(2.*AQ)
      XPS=(-BQ-DSQRT(DISC))/(2.*AQ)
      XP=XPS
C     ****************************************************************
C     *  CALCULATION  OF  THE  MOMENT  OF  THE  STRAIGHT  PORTION  OF *
C     *  THE  LONG  LEG                                              *
C     ****************************************************************
      MCX=Y11-C3
      MX=MCX*AA*XP*(1.+(XP*XP/(3.*CQ))+(XP*BQ/(2.*CQ)))
      CQX=T*APH1/(2.*IMIN)
      QX3L3=CQX*L3*L3
      MX3=2.*MCX*(L3**3)*CQX/3.
      QX4L4=CQX*(L3*L3-(L4-XP)*(L4-XP))
      MX4=MCX*CQX*(L4-XP)*(L3*L3-(L4-XP)*(L4-XP)/3.)
C     ****************************************************************
C     *  VZRACT  GIVES  THE  V- COORDINATE  OF  THE  ACTUAL  SHEAR    *
C     *  CENTER                                                      *
C     ****************************************************************
      VZRACT=-(MX1+MX2+MXCP+MX-MX3-MX4)
      GO TO 14
   45 CONTINUE
C     ****************************************************************
C     *  CALCULATION  OF  THE  MOMENT  OF  THE  CURVED  PORTION      *
C     *  ABOUT  CENTROID                                             *
C     ****************************************************************
      IF(D1.GT.0.0.AND.D2.LT.0.0) GO TO 905
      C16=T/IMIN
      INT1=C12*C12*QX2L2*(ULMT-LLMT)
      INT2=C12*QX2L2*D3*(DCOS(1.570796+TT+ULMT-PHI)-DCOS(1.570796+TT+
     *     LLMT-PHI))
      INT3=C12**3*C16*.5*D3*DCOS(APHR+PHI-1.570796)*(ULMT*ULMT-LLMT
     *     *LLMT)
      INT4=C12**4*C16*((ULMT-LLMT)*DCOS(APHR+TT)-DSIN(ULMT+TT+APHR)
     *     +DSIN(APHR+LLMT+TT))
      INT5=C12*C12*C16*D3*D3*DCOS(APHR+PHI-1.570796)*(-ULMT*DCOS(
     *     1.570796
     *     -PHI+ULMT+TT)+LLMT*DCOS(1.570796-PHI+LLMT+TT)+DSIN(1.570796
     *     -PHI+TT+ULMT)-DSIN(1.570796-PHI+TT+LLMT))
      INT6=C12**3*C16*D3*(-DCOS(APHR+TT)*(DCOS(1.570796-PHI+ULMT+TT)-
     *     DCOS
     *     (1.570796-PHI+LLMT+TT))+.25*(DCOS(1.570796+2.*ULMT+2.*TT+APHR
     *     -PHI)-DCOS(1.570796+2.*LLMT+2.*TT+APHR-PHI))-.5*DSIN(1.570796
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```
     *        -APHR-PHI)*(ULMT-LLMT))
C    **************************************************************
C    *  MXCP GIVES THE MOMENT OF THE CURVED PORTION ABOUT          *
C    *  CENTROID                                                    *
C    **************************************************************
      IF(D1.LT.0.0) GO TO 525
      MXCP=-INT6-INT5-INT4+INT1-INT2-INT3
      GO TO 526
 525  MXCP=INT1+INT2+INT3-INT4-INT5+INT6
 526  IF(D2.LT.0.0.AND.L10.GT.C1) GO TO 555
      GO TO 112
 555  LLMT=0.0
      ULMT=TT
C    **************************************************************
C    *  CALCULATION OF THE MOMENT OF THE CURVED PORTION ABOUT*
C    *  CENTROID                                                    *
C    **************************************************************
      INT1=C12*C12*Q1*(ULMT-LLMT)
      INT2=C12*Q1*D3*(DSIN(-ULMT+PHI)-DSIN(-LLMT+PHI))
      INT3=C12**3*C16*.5*D3*DSIN(APHR+PHI)*(ULMT*ULMT-LLMT
     *     *LLMT)
      INT4=C12**4*C16*((ULMT-LLMT)*APH2-DSIN(ULMT+APHR)
     *     +DSIN(APHR+LLMT))
      INT5=C12*C12*C16*D3*D3*DSIN(APHR+PHI)*(-ULMT*DSIN
     *     (-ULMT+PHI)+DCOS(-ULMT+PHI)+LLMT*DSIN(PHI-LLMT)-DCOS(PHI-
     *     LLMT))
      INT6=C12**3*C16*D3*(-APH2*(DSIN(-ULMT+PHI)-DSIN(-LLMT
     *     +PHI))-.5*DSIN(1.570796-APHR-PHI)*(ULMT-LLMT)+.25*(DCOS(2.*
     *     ULMT+1.570796+APHR-PHI)-DCOS(2.*LLMT+APHR-PHI+1.570796)))
C    **************************************************************
C    *  MXCP GIVES THE MOMENT OF THE CURVED PORTION ABOUT          *
C    *  CENTROID                                                    *
C    **************************************************************
      MXCP=INT1+INT2+INT3-INT4-INT5+INT6+MXCP
      GO TO 112
 902  CONTINUE
C    **************************************************************
C    *  CALCULATION OF THE MOMENT OF THE CURVED PORTION ABOUT*
C    *  CENTROID IF THE CENTROID LIES IN THE SECOND QUADRANT *
C    *  WITH RESPECT TO CENTER OF CURVATURE AND IF POINT OF *
C    *  ZERO SHEAR FLOW IS NOT IN THE CURVED PORTION              *
C    **************************************************************
      IF(TT.EQ.0.0)GO TO 904
C    **************************************************************
C    *  HERE, L1.GT.C1                                              *
C    **************************************************************
      ULMT=TT
      LLMT=0.
      CALL MXCP78(C12,Q1,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX)
      MXCP=MXX
      ULMT=1.570796-TT
      LLMT=0.
      CALL MXCP12(C12,QX2L2,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX,IT)
      MXCP=MXCP+MXX
      GO TO 112
 904  CONTINUE
C    **************************************************************
C    *  HERE, L1.LE.C1                                              *
C    **************************************************************
      ULMT=1.570796
      LLMT=0.
      CALL MXCP12(C12,QX2L2,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX,TT)
      MXCP=MXX
      GO TO 112
 905  CONTINUE
C    **************************************************************
C    *  CALCULATION OF THE MOMENT OF THE CURVED PORTION ABOUT*
C    *  CENTROID IF THE CENTROID LIES IN THE SECOND QUADRANT *
C    *  WITH RESPECT TO CENTER OF CURVATURE AND IF THE POINT*
C    *  OF ZERO SHEAR FLOW IS IN THE CURVED PORTION               *
C    **************************************************************
      IF(TT.EQ.0.0)GO TO 906
C    **************************************************************
C    *  HERE, L1.GT.C1                                              *
C    **************************************************************
      ULMT=TT
      LLMT=0.
      CALL MXCP78(C12,Q1,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX)
      MXCP=MXX
      ULMT=THT
```

```
      LLMT=0.
      CALL MXCP12(C12,QX2L2,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX,TT)
      MXCP=MXCP+MXX
      GO TO 112
  906 CONTINUE
C     ************************************************************
C     *  HERE,  L1.LE.C1                                         *
C     ************************************************************
      ULMT=THT
      LLMT=0.
      CALL MXCP12(C12,QX2L2,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX,TT)
      MXCP=MXX
      GO TO 112
  908 CONTINUE
C     ************************************************************
C     *  CALCULATION OF THE MOMENT OF THE CURVED PORTION ABOUT*
C     *  CENTROID IF THE CENTROID LIES IN THE SECOND QUADRANT *
C     *  WITH RESPECT TO CENTER OF CURVATURE AND IF POINT OF *
C     *  ZERO SHEAR FLOW IS IN THE CURVED PORTION               *
C     ************************************************************
      ULMT=1.570796-THT-TT
      LLMT=0.
      CALL MXCP34(C12,QX4L4,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX)
      MXCRV=MXX
      MXCP=MXCP-MXCRV
      GO TO 95
   14 CONTINUE
C     ************************************************************
C     *  CALCULATION OF U-COORDINATE OF THE ACTUAL SHEAR        *
C     *  CENTER                                                 *
C     ************************************************************
C     *  CALCULATION OF THE MOMENT OF THE STRAIGHT PORTION      *
C     *  OF THE SHORT LEG                                       *
C     ************************************************************
      QY1SC=T*C1*((X11-C3)/APH2+(L1-C1*.5)*APH1)/IMAX
      MY1=(X11-C3)*T*C1*C1*.5*((X11-C3)/APH2+(L1-C1/3.)*
     *    APH1)/IMAX
      C18=T/IMAX
      CO1=(Y11-C3)*APH3
      IF(IXX.EQ.IYY)GO TO 31
C     ************************************************************
C     *  FINDING THE POINT AT WHICH U-U AXIS INTERSECTS THE     *
C     *  CENTER-LINE OF THE CROSS-SECTION                        *
C     ************************************************************
      IF(CO1.LE.D1)GO TO 52
      AB=1.570796327-PHI-APHR
      IF(D1.GT.0.0)GO TO 330
      CC=1.570796327+PHI+APHR
      BB=DARSIN((D3*DSIN(CC))/(R+C3))
      AB1=AB-BB
      THTA=1.570796327-AB1-PHI
      GO TO 332
  330 BB=DARSIN((D3*DSIN(AB))/(R+C3))
      CC=3.141592654-AB-BB
      CO2=(R+C3)*DSIN(CC)/DSIN(AB)
C     ************************************************************
C     *  THE FOLLOWING THTA, WHICH IS THE ANGLE MEASURED        *
C     *  COUTER-CLOCKWISE FROM THE LONG LEG, GIVES THE POINT    *
C     *  AT WHICH U-U AXIS INTERSECTS THE CENTER-LINE OF THE    *
C     *  CROSS-SECTION                                           *
C     ************************************************************
      THTA=DARCOS((CO2*APH2-D2)/(R+C3))
  332 LLMT=0.0
      ULMT=1.570796327-THTA
      GO TO 32
   31 LLMT=0.0
      ULMT=.785398163
      THTA=ULMT
   32 CONTINUE
C     ************************************************************
C     *  CALCULATION OF THE MOMENT OF THE CURVED PORTION ABOUT *
C     *  CENTROID                                               *
C     ************************************************************
   63 INT1=C12*C12*QY1SC*(ULMT-LLMT)
      INT2=C12**3*.5*C18*D3*DCOS(APHR+PHI)*(ULMT**2-LLMT**2)
      INT3=C12**4*C18*(-DCOS(ULMT+APHR)+DCOS(LLMT+APHR)
     *     +(LLMT-ULMT)*APH1)
      INT4=C12*D3*QY1SC*(-DCOS(ULMT+PHI)+DCOS(LLMT+PHI))
      INT5=C12*C12*C18*D3*D3*DCOS(PHI+APHR)*(-ULMT*DCOS(
     *     ULMT+PHI)+LLMT*DCOS(LLMT+PHI)+DSIN(ULMT+PHI)-DSIN(LLMT+PHI))
```

```
      INT6=C12**3*2.*C18*D3*(.125*(-DSIN(2.*ULMT+APHR+PHI)
     *     +DSIN(2.*LLMT+APHR+PHI))+.25*(ULMT-LLMT)*DCOS(APHR-PHI)
     *     +.5*APH1*(DCOS(ULMT+PHI)-DCOS(LLMT+PHI)))
C     ****************************************************************
C     * MYC1 GIVES THE MOMENT OF THE CURVED PORTION ABOUT          *
C     * CENTROID                                                   *
C     ****************************************************************
      IF(D1.LT.0.0)GO TO 530
      MYC1=INT1+INT2+INT3+INT4+INT5+INT6
      GO TO 539
  530 MYC1=INT1+INT3+INT5-INT6-INT2-INT4
  539 IF(ULMT.EQ.1.5708)GO TO 602
      GO TO 603
   62 ULMT=1.5708
      LLMT=0.0
      GO TO 63
  602 IF(CO1.NE.D1)GO TO 605
C     ****************************************************************
C     * CALCULATION OF THE MOMENT OF THE STRAIGHT PORTION OF       *
C     * THE LONG LEG IF U-U AXIS INTERSECTS THE CENTER-LINE        *
C     * OF THE CROSS-SECTION JUST AT THE END OF THE CURVED         *
C     * PORTION                                                    *
C     ****************************************************************
      MY2=(Y11-C3)*T*APH2/IMAX*C2**3/3.
C     ****************************************************************
C     * UZRACT GIVES THE U- COORDINATE OF THE ACTUAL SHEAR         *
C     * CENTER                                                     *
C     ****************************************************************
      UZRACT=-(MY1+MY2+MYC1)
      GO TO 100
  605 CONTINUE
C     ****************************************************************
C     * CALCULATION OF THE MOMENT OF THE STRAIGHT PORTION OF       *
C     * THE LONG LEG IF U-U AXIS INTERSECTS THE CENTER-LINE        *
C     * OF THE CROSS-SECTION IN THE STRAIGHT PORTION OF THE        *
C     * LONG LEG                                                   *
C     ****************************************************************
      L5=D1-CO1
      C20=.785398164-APHR
      C21=D3*DCOS(APHR+PHI)+.900316316*C12*DSIN(
     *    C20)
      QAT90=QY1SC+C18*C21*C12*1.570796327
      MY3=(Y11-C3)*L5*(T*APH2*L5*L5/(3.*IMAX)+QAT90)
      L6=X-X1-L5
      MY2=(Y11-C3)*T*APH2*L6**3/(3.*IMAX)
C     ****************************************************************
C     * UZRACT GIVES THE U- COORDINATE OF THE ACTUAL SHEAR         *
C     * CENTER                                                     *
C     ****************************************************************
      UZRACT=-(MY1+MY2+MY3+MYC1)
      GO TO 100
  603 CONTINUE
C     ****************************************************************
C     * CALCULATION OF THE MOMENT OF THE STRAIGHT PORTION OF       *
C     * THE LONG LEG IF U-U AXIS INTERSECTS THE CENTER-LINE        *
C     * OF THE CROSS-SECTION IN THE CURVED PORTION                 *
C     ****************************************************************
      L5=C2+CO1-D1
      QL=T*C2*APH2/IMAX*(L5-C2*.5)
      MY2=(Y11-C3)*T*APH2*.5/IMAX*C2*C2*(L5-C2/3.)
      LLMT=0.0
      ULMT=THTA
C     ****************************************************************
C     * CALCULATION OF THE MOMENT OF THE CURVED PORTION ABOUT*     
C     * CENTROID                                                   *
C     ****************************************************************
      INT1=QL*C12*C12*ULMT
      INT2=C12**3*C18*D3*.5*DCOS(PHI+APHR)*ULMT*ULMT
      INT3=C12**4*C18*(DSIN(ULMT-APHR)-ULMT*APH2+APH1)
      INT4=C12*D3*QL*(-DCOS(ULMT+PHI)+CPHI)
      INT5=C12*C12*C18*D3*D3*(-ULMT*DCOS(ULMT+PHI)+DSIN(ULMT+PHI)-SPHI)
     *     *DCOS(APHR+PHI)
      INT6=C12**3*2.*C18*D3*(.125*(-DCOS(2.*ULMT-APHR+PHI)
     *     +DCOS(APHR-PHI))+.25*DSIN(APHR+PHI)*ULMT+.5*APH2*
     *     (DCOS(ULMT+PHI)-CPHI))
C     ****************************************************************
C     * MYC2 GIVES THE MOMENT OF THE CURVED PORTION ABOUT          *
C     * CENTROID                                                   *
C     ****************************************************************
      IF(D1.LT.0.0)GO TO 540
```

```
      MYC2=INT1-INT2+INT3+INT4-INT5+INT6
      GO TO 542
  540 MYC2=INT1+INT3-INT5-INT6+INT2-INT4
C     ***************************************************************
C     *  UZRACT  GIVES  THE  U-  COORDINATE  OF  THE  ACTUAL  SHEAR  *
C     *  CENTER                                                      *
C     ***************************************************************
  542 UZRACT=-(MY1+MY2+MYC1+MYC2)
  100 CONTINUE
C     ***************************************************************
C     *  CALCULATION  OF  THE  BETA1 & BETA2  (REFERENCE: PAGE 245   *
C     *  OF "THEORY  OF  ELASTIC  STABILITY"  BY  TIMOSHENKO)         *
C     ***************************************************************
C     *  CALCULATION  FOR  LONG  LEG                                 *
C     ***************************************************************
      C400=(Y11-C3)/APH2
      C401=X11-C01-C3
      C402=X-C3
      C403=Y-C3
      MM=1
      CALL  INTGRL(0.,X-X1,SUM11,MM,C400,C401,C402,C403,APH1,APH2,T,
     *            X11,Y11,R,IMIN)
      MM=2
      CALL  INTGRL(0.,X-X1,SUM12,MM,C400,C401,C402,C403,APH1,APH2,T,
     *            X11,Y11,R,IMIN)
C     ***************************************************************
C     *  CALCULATION  FOR  SHORT  LEG                                *
C     ***************************************************************
      MM=3
      CALL  INTGRL(0.,Y-X1,SUM21,MM,C400,C401,C402,C403,APH1,APH2,T,
     *            X11,Y11,R,IMIN)
      MM=4
      CALL  INTGRL(0.,Y-X1,SUM22,MM,C400,C401,C402,C403,APH1,APH2,T,
     *            X11,Y11,R,IMIN)
C     ***************************************************************
C     *  CALCULATION  FOR  CURVED  PORTION                           *
C     ***************************************************************
      MM=5
      ULMT=1.5707963
      CALL  INTGRL(0.,ULMT,SUM31,MM,C400,C401,C402,C403,APH1,
     *            APH2,T,X11,Y11,R,IMIN)
      MM=6
      CALL  INTGRL(0.,ULMT,SUM32,MM,C400,C401,C402,C403,APH1,
     *            APH2,T,X11,Y11,R,IMIN)
   26 SUM1=SUM11+SUM21+SUM31
   27 SUM2=SUM12+SUM22+SUM32
      GAMMA1=1.570796327-PHI-APHR
      GAMMA2=.785398163-APHR
C     ***************************************************************
C     *  UZRAS1,VZRAS1,UZRAS2 & VZRAS2  GIVE  THE  U- & V-           *
C     *  COORDINATES  OF  THE  ASSUMED  SHEAR  CENTERS               *
C     ***************************************************************
      IF(D1.GT.0.0) GO TO 370
      VZRAS1=-(C12*DSIN(GAMMA2)-D3*DSIN(GAMMA1))
      UZRAS1=-(C12*DCOS(GAMMA2)-D3*DCOS(GAMMA1))
      GO TO 374
  370 VZRAS1=-(C12*DSIN(GAMMA2)+D3*DSIN(GAMMA1))
      UZRAS1=-(C12*DCOS(GAMMA2)+D3*DCOS(GAMMA1))
  374 H1=X11-C3-C01
      H2=(Y11-C3)/APH2
      VZRAS2=-H1*APH2
      UZRAS2=-(H2+H1*APH1)
C     ***************************************************************
C     *  XC1,YC1,XC2 & YC2  GIVE  THE  X- & Y-  COORDINATES  OF THE  *
C     *  ASSUMED  SHEAR  CENTERS                                     *
C     ***************************************************************
      XC1=UZRAS1*APH2-VZRAS1*APH1
      YC1=UZRAS1*APH1+VZRAS1*APH2
      XC2=UZRAS2*APH2-VZRAS2*APH1
      YC2=UZRAS2*APH1+VZRAS2*APH2
C     ***************************************************************
C     *  CALCULATION  OF  THE  CO-ORDINATES  OF  THE  CRITICAL       *
C     *  POINTS,  ON  THE  CROSS-SECTION,  FOR  THE  PURPOSE  OF     *
C     *  CHECKING  STRESSES                                          *
C     ***************************************************************
      UP(1)=(X-X11)*APH1-(Y11-T)*APH2
      VP(1)=(X-X11)*APH2+(Y11-T)*APH1
      UP(2)=(X-X11)*APH1-Y11*APH2
      VP(2)=(X-X11)*APH2+Y11*APH1
      UP(3)=-(X11-T)*APH1+(Y-Y11)*APH2
```

```
      VP(3)=-(X11-T)*APH2-(Y-Y11)*APH1
      UP(4)=-X11*APH1+(Y-Y11)*APH2
      VP(4)=-X11*APH2-(Y-Y11)*APH1
      UP(5)=-(Y11-T)*APH2-(X11-X1)*APH1
      VP(5)=-(X11-X1)*APH2+(Y11-T)*APH1
      UP(6)=-Y11*APH2-(X11-X1)*APH1
      VP(6)=-(X11-X1)*APH2+Y11*APH1
      UP(7)=-(Y11-X1)*APH2-(X11-T)*APH1
      VP(7)=-(X11-T)*APH2+(Y11-X1)*APH1
      UP(8)=-(Y11-X1)*APH2-X11*APH1
      VP(8)=-X11*APH2+(Y11-X1)*APH1
      PRJ1=Y11*APH2/APH1
      PRJ2=(Y11-T)*APH2/APH1
      DY1=X11+PRJ1
      DY2=X11+PRJ2
      DLY=DY1-X1
      IF(DLY.LE.0.)GO TO 7010
      UP(11)=-Y11*APH2+PRJ1*APH1
      VP(11)=Y11*APH1+PRJ1*APH2
      GO TO 7011
7010  UP(11)=0.
      VP(11)=0.
7011  DLYM=DY2-X1
      IF(DLYM.LE.0.)GO TO 7012
      UP(10)=-(Y11-T)*APH2+PRJ2*APH1
      VP(10)=(Y11-T)*APH1+PRJ2*APH2
      GO TO 7014
7012  UP(10)=0.
      VP(10)=0.
7014  IF(DABS(UP(10)).LE.1.0D-07.AND.DABS(UP(11)).LE.1.0D-07)GO TO 7015
      WRITE(6,7879)
7879  FORMAT(//,10X,'POINT  MARKED  10  AND  11  ARE  NOT  CORRECTLY',
     *       ' IDENTIFIED',/)
      GO TO 10
7015  UP(10)=0.
      UP(11)=0.
      PRJ3=Y11*APH1/APH2
      PRJ4=(Y11-T)*APH1/APH2
      DY3=X11-PRJ3
      DY4=X11-PRJ4
      DLY=DY3-X1
      IF(DLY.LE.0.)GO TO 7016
      UP(15)=-Y11*APH2-PRJ3*APH1
      VP(15)=-PRJ3*APH2+Y11*APH1
      GO TO 7017
7016  UP(15)=0.
      VP(15)=0.
7017  DLYM=DY4-X1
      IF(DLYM.LE.0.)GO TO 7018
      UP(14)=-(Y11-T)*APH2-PRJ4*APH1
      VP(14)=-PRJ4*APH2+(Y11-T)*APH1
      GO TO 7019
7018  UP(14)=0.
      VP(14)=0.
7019  IF(DABS(VP(14)).LE.1.0D-07.AND.DABS(VP(15)).LE.1.0D-07)GO TO 7020
      WRITE(6,78799)
78799 FORMAT(//,10X,'POINT  MARKED  14  AND  15  ARE NOT  CORRECTLY',
     *       ' IDENTIFIED',/)
      GO TO 10
7020  VP(14)=0.
      VP(15)=0.
      PRJ5=X11*APH1/APH2
      PRJ6=(X11-T)*APH1/APH2
      DX1=Y11+PRJ5
      DX2=Y11+PRJ6
      DLX=DX1-X1
      IF(DLX.LE.0.)GO TO 7021
      UP(13)=PRJ5*APH2-X11*APH1
      VP(13)=-X11*APH2-PRJ5*APH1
      GO TO 7022
7021  UP(13)=0.
      VP(13)=0.
7022  DLXM=DX2-X1
      IF(DLXM.LE.0.)GO TO 7023
      UP(12)=PRJ6*APH2-(X11-T)*APH1
      VP(12)=-(X11-T)*APH2-PRJ6*APH1
      GO TO 7024
7023  UP(12)=0.
      VP(12)=0.
7024  IF(DABS(UP(12)).LE.1.0D-07.AND.DABS(UP(13)).LE.1.0D-07)GO TO 7025
```

```
      WRITE(6,78781)
78781 FORMAT(//,10X,'POINT  MARKED  12  AND  13  ARE  NOT  CORRECTLY',
     *       ' IDENTIFIED',/)
      GO TO 10
 7025 UP(12)=0.
      UP(13)=0.
      D11=X-DY1
      D10=X-DY2
      D14=X-DY4
      D15=X-DY3
      D12=Y-DX2
      D13=Y-DX1
      VZERO=VZRACT
      UZERO=UZRACT
C   ***********************************************************************
C   *  CALCULATION  OF  THE  TORSIONAL  CONSTANT, MINIMUM  RADIUS        *
C   *  OF  GYRATION  &  POLAR  MOMENT  OF  INERTIA.                      *
C   *                                                                     *
C   *  ASSUMPTION  MADE:   1)  TORSIONAL  CONSTANT,J3=B*T**3/3           *
C   *                           WHERE,T=THICKNESS   &                     *
C   *                                 B=CENTER-LINE  LENGTH  OF           *
C   *                                   THE  CROSS-SECTION                *
C   ***********************************************************************
      J3=C*T*T/3.
      RMIN=DSQRT(IMIN/C)
      IF(IXX.EQ.IYY)GO TO 28
      IABSC=IMIN+IMAX+C*(UZERO*UZERO+VZERO*VZERO)
      GO TO 29
   28 IABSC=IMIN+IMAX+C*UZERO*UZERO
   29 CONTINUE
C   ***********************************************************************
C   *  CALCULATION  OF  THE  COORDINATES  OF  THE  CENTER  OF           *
C   *  THE  CURVED  PORTION                                               *
C   ***********************************************************************
      IF(IXX.EQ.IYY)GO TO 310
      AB=1.570796327-PHI-APHR
      IF(D1.GT.0.0) GO TO 360
      GO TO 361
  360 UCPR=-D3*DCOS(AB)
      VCPR=-D3*DSIN(AB)
      GO TO 303
  361 UCPR=D3*DCOS(AB)
      VCPR=D3*DSIN(AB)
      GO TO 303
  310 IF(D1.LT.0.0.AND.D2.LT.0.0) GO TO 320
      UCPR=-D3
      VCPR=0.0
      GO TO 303
  320 UCPR=D3
      VCPR=0.0
  303 CONTINUE
C   ***********************************************************************
C   *  CALCULATION  OF  THE  DISTANCE  BETWEEN  THE  SHEAR  CENTER      *
C   *  AND  THE  CENTER  OF  THE  CURVED  PORTION                        *
C   ***********************************************************************
      RPR=DSQRT((VZERO-VCPR)**2+(UZERO-UCPR)**2)
      IF(IXX.EQ.IYY)GO TO 315
      UPPR=UCPR-C12*APH2
      VPPR=C12*APH1+VCPR
      RRR=DSQRT((UPPR-UZERO)**2+(VPPR-VZERO)**2)
      LMT=DARCOS((RPR*RPR+C12*C12-RRR*RRR)/(2.*C12*RPR))
C   ***********************************************************************
C   *  XC & YC  GIVE  THE  X- & Y-  COORDINATES  OF  THE  ACTUAL        *
C   *  SHEAR  CENTER                                                      *
C   ***********************************************************************
      YC=-(D1+RPR*DSIN(LMT))
      XC=-(D2+RPR*DCOS(LMT))
      GO TO 333
  315 CONTINUE
C   ***********************************************************************
C   *  XC & YC  GIVE  THE  X- & Y-  COORDINATES  OF  THE  ACTUAL        *
C   *  SHEAR  CENTER                                                      *
C   ***********************************************************************
      XC=-(D2+RPR*.70710678)
      YC=-(D1+RPR*.70710678)
      LMT=.785398163
  333 WRITE(6,200)
      WRITE(6,300)X,Y,T,R,C,Y11,X11,CONLEG,GAGDIS,GPLATE,SIGMAY,IXX,IYY,
     *       IMAX,IMIN,PXY,RMIN,APHD,J3,IABSC
```

```
200  FORMAT(////,10X,19('*'),'SECTIONAL   PROPERTIES',19('*'),//,10X,12
    *     ('*'),'( CURVED PORTION AS A LINE ELEMENT )',12('*'),/)
308  FORMAT(////,15X,'SIZE  OF  THE  ANGLE',10X,'=',1X,F6.2,
    *     '*',F6.2,'*',F6.2,//,15X,'INTERNAL  RADIUS',14X,
    *     '=',7X,F5.2,//,15X,'AREA  OF  THE  CROSS-SECTION  =',
    *     F12.2,' MM**2',//,15X,'DIST. OF CENTROID ALONG X-AXIS=',
    *     F12.2,' MM',//,15X,'DIST. OF CENTROID ALONG Y-AXIS=',
    *     F12.2,' MM',
    *     //,15X,'CONNECTED  LEG',16X,'=',F12.2,' MM',//,15X,
    *     'GAUGE  DISTANCE',15X,'=',F12.2,' MM',//,15X,
    *     'THICKNESS  OF  GUSSET  PLATE  =',F12.2,' MM',//,15X,
    *     'ASSUMED MATERIAL YIELD STRESS =',F12.2,' N/MM**2',
    *     //,15X,'M.  I.  ABOUT  X-X  AXIS',6X,'=',F10.0,3X,'MM**4'
    *     ,//,15X,'M.  I.  ABOUT  Y-Y  AXIS',6X,'=',F10.0,3X,
    *     'MM**4',//,15X,'MAX.  M. I. ABOUT  U-U  AXIS  =',F10.0,
    *     3X,'MM**4',//,15X,'MIN.  M. I. ABOUT  V-V  AXIS  =',F10.0
    *     ,'    MM**4',//,15X,'PRODUCT  OF  INERTIA',10X,'=',F10.0,
    *     '    MM**4',//,15X,'MINIMUM  RADIUS  OF  GYRATION =',
    *     F12.2,' MM',//,15X,'ALPHA  MEASURED  FROM  X AXIS =',
    *     F12.2,' DEGREES',//,15X,'TORSIONAL  CONSTANT',11X,'=',
    *     F12.2,' MM**4',//,15X,'POLAR M.I. ABOUT SHEAR CENTR  =',
    *     F10.0,'     MM**4',/)
     WRITE(6,3088)UZRACT,VZRACT,UZRAS1,VZRAS1,UZRAS2,VZRAS2,XC,YC,
    *     XC1,YC1,XC2,YC2
3088 FORMAT(/,15X,'IN  U- & V- SYSTEM  OF  AXES :',
    *     //,15X,'LOC OF ACTUAL  SHEAR  CENTR   = ',F7.2,',',F7.2,
    *     //,15X,'LOC OF ASUMED  SHEAR  CENTR  1= ',F7.2,',',F7.2,
    *     //,15X,'LOC OF ASUMED  SHEAR  CENTR  2= ',F7.2,',',F7.2,
    *     //,15X,'IN X- & Y- SYSTEM  OF  AXES :',//,15X,
    *     'LOC OF ACTUAL  SHEAR  CENTR   = ',F7.2,',',F7.2,//
    *     ,15X,'LOC OF ASUMED  SHEAR  CENTR  1= ',F7.2,',',F7.2,//
    *     ,15X,'LOC OF ASUMED  SHEAR  CENTR  2= ',F7.2,',',F7.2,//
    *     ,10X,60('*'))
     CALL CWACTL(RPH,C12,X11,C3,YC,XC,Y11,LMT,C1,C2,C8,T,CWACT,
    *     AVOMGS,ZSC1,ZSC2)
     IF(CONLEG.EQ.X)GO TO 701
C    **********************************************************
C    *  EU  &  EV  GIVE  THE  CO-ORDINATES  OF  THE  POINT  OF  THE  *
C    *  LOAD  APPLICATION                                         *
C    **********************************************************
     EU=(GAGDIS-Y11)*APH2-(X11+.5*GPLATE)*APH1
     EV=-(X11+.5*GPLATE)*APH2-(GAGDIS-Y11)*APH1
     GO TO 702
701  EU=-(Y11+.5*GPLATE)*APH2+(GAGDIS-X11)*APH1
     EV=(GAGDIS-X11)*APH2+(Y11+.5*GPLATE)*APH1
C    **********************************************************
C    *  ASSUMPTIONS  MADE :                                       *
C    *                        1) MODULUS OF SHEAR RIGIDITY=77000 N/MM**2*
C    *                        2) MODULUS OF ELASTICITY=205000 N/MM**2  *
C    *                                                            *
C    *   THE  MEMBER  IS  ASSUMED  TO  BE  INITIALLY  STRAIGHT  AND  *
C    *  STRESS  FREE.                                             *
C    *   THE  MEMBER  IS  ASSUMED  TO  FAIL  OR  REACH  CRITICAL     *
C    *  LOAD  IF  THE  MAXIMUM  STRESS  AT  ANY  POINT  OF  THE      *
C    *  CROSS-SECTION  REACHES  THE  YIELD  STRESS  OF  THE  MATERIAL*
C    *  OR  THE  DEFLECTION  COMPONENTS  CHANGE  THEIR  SIGN        *
C    **********************************************************
702  G=.77D 05
     ELAS=.205D 06
     IL=1
790  BETA1=SUM1/IMAX-2.*VZERO
     BETA2=SUM2/IMIN-2.*UZERO
     WRITE(6,400)UZERO,VZERO,BETA1,BETA2
400  FORMAT(///,10X,60('*'),//,10X,17('*'),'TABLE  OF  CRITICAL  LOADS'
    *     ,17('*'),//,10X,60('*'),//,21X,'SHEAR. CENTER  CONSIDERED'
    *     ,'=',F6.2,',',F6.2,//,21X,'BETA1=',F9.2,9X,'BETA2=',
    *     F9.2)
     IABSC=IMIN+IMAX+C*(UZERO*UZERO+VZERO*VZERO)
     CCC=(Y11-C3)-DABS(XC)
     CCD=-((X11-C3)-DABS(YC))
77   FORMAT(//,25X,I3,22X,F10.1)
     C70=UZERO-EU
     C71=VZERO-EV
     C72=IABSC/C+BETA1*EV+BETA2*EU
     IJ=1
     CW=CWACT
786  SR=20.
     WRITE(6,7979)CW
7979 FORMAT(//,28X,'CW=',D15.6,' MM**6',//,10X,60('-'),//,18X,
    *     'SLENDERNESS  RATIO',12X,'CRITICAL  LOAD',/,53X,
```

```
     *'(N)',/,10X,80('-'))
770   CLENTH=SR*RMIN
      C73=9.869604404/(CLENTH*CLENTH)
      PYE=ELAS*IMIN*C73
      PXE=ELAS*IMAX*C73
      POE=(4.130221794*ELAS*CW*C73+1.001870809*G*J3)/C72
C     ******************************************************************
C     * FOR FIRST ITERATION, CRITICAL LOAD IS ASSUMED TO BE            *
C     * 0.1% OF THE MINIMUM OF PXE, PYE & POE                          *
C     ******************************************************************
      IF(PYE.GT.POE)GO TO 705
      CRILOD=PYE
      GO TO 706
705   IF(POE.LT.0.)GO TO 7899
      CRILOD=POE
      GO TO 706
7899  CRILOD=PYE
706   INCR=CRILOD/10.
      CRILOD=.001*CRILOD
      MNP=1
762   CALL DEFROT(PYE,PXE,POE,CRILOD,C70,C71,C72,EU,EV,CLENTH,IMIN,
     *      IMAX,ELAS,U,V,BETA,PHIDER)
      CALL SIGMA(CRILOD,EU,EV,UP,VP,C,IMIN,IMAX,MXSTR,SIGMAY,
     *      MNSTR,U,V,BETA,UZERO,VZERO,ELAS,PHIDER,AVONGS,
     *      WSC1,WSC2,LMT,C12,RPR,C1,X1,APH1,APH2,D1,D2,CW,CCC,C2,
     *      R,D10,D11,D12,D13,D14,D15,CCD)
      IF(MNP.EQ.1)GO TO 7585
C     ******************************************************************
C     * CHECKING THE SIGNS OF THE DEFLECTION COMPONENTS                *
C     ******************************************************************
      DO 9898 KLM=2,10
      EEE=DABS(U(KLM))-DABS(UCH(KLM))
      IF(EEE.LT.0.)GO TO 761
      FFF=DABS(V(KLM))-DABS(VCH(KLM))
      IF(FFF.LT.0.)GO TO 761
      GGG=DABS(BETA(KLM))-DABS(BETACH(KLM))
      IF(GGG.LT.0.)GO TO 761
9898  CONTINUE
C     ******************************************************************
C     * CHECKING THE MAXIMUM STRESS                                    *
C     ******************************************************************
7585  IF(DABS(MXSTR).LE.DABS(MNSTR))MXSTR=DABS(MNSTR)
      IF(MXSTR.GT.SIGMAY)GO TO 761
      CRILOD=CRILOD+INCR
C     ******************************************************************
C     * STORING THE CURRENT DEFLECTION COMPONENTS FOR USE IN           *
C     * NEXT STEP OF ITERATION                                         *
C     ******************************************************************
      DO 7789 KLM=1,11
      UCH(KLM)=U(KLM)
      VCH(KLM)=V(KLM)
      BETACH(KLM)=BETA(KLM)
7789  CONTINUE
      MNP=MNP+1
      GO TO 762
761   CRILOD=CRILOD-INCR
      INCR=INCR/10.
      MNP=1
764   CALL DEFROT(PYE,PXE,POE,CRILOD,C70,C71,C72,EU,EV,CLENTH,IMIN,
     *      IMAX,ELAS,U,V,BETA,PHIDER)
      CALL SIGMA(CRILOD,EU,EV,UP,VP,C,IMIN,IMAX,MXSTR,SIGMAY,
     *      MNSTR,U,V,BETA,UZERO,VZERO,ELAS,PHIDER,AVONGS,
     *      WSC1,WSC2,LMT,C12,RPR,C1,X1,APH1,APH2,D1,D2,CW,CCC,C2,
     *      R,D10,D11,D12,D13,D14,D15,CCD)
      IF(MNP.EQ.1)GO TO 7586
C     ******************************************************************
C     * CHECKING THE SIGNS OF THE DEFLECTION COMPONENTS                *
C     ******************************************************************
      DO 9897 KLM=2,10
      EEE=DABS(U(KLM))-DABS(UCH(KLM))
      IF(EEE.LT.0.)GO TO 763
      FFF=DABS(V(KLM))-DABS(VCH(KLM))
      IF(FFF.LT.0.)GO TO 763
      GGG=DABS(BETA(KLM))-DABS(BETACH(KLM))
      IF(GGG.LT.0.)GO TO 763
9897  CONTINUE
C     ******************************************************************
C     * CHECKING THE MAXIMUM STRESS                                    *
C     ******************************************************************
7586  IF(DABS(MXSTR).LE.DABS(MNSTR))MXSTR=DABS(MNSTR)
```

```
      IF(MXSTR.GT.SIGMAY)GO TO 763
      CRILOD=CRILOD+INCR
C   ************************************************************
C   *  STORING  THE  CURRENT  DEFLECTION  COMPONENTS  FOR  USE  IN  *
C   *  NEXT  STEP  OF  ITERATION
C   ************************************************************
      DO 7788 KLM=1,11
      UCH(KLM)=U(KLM)
      VCH(KLM)=V(KLM)
      BETACH(KLM)=BETA(KLM)
 7788 CONTINUE
      MNP=MNP+1
      GO TO 764
 763  CRILOD=CRILOD-INCR
      INCR=INCR/10.
      MNP=1
 765  CALL DEFROT(PYE,PXE,POE,CRILOD,C70,C71,C72,EU,EV,CLENTH,IMIN,
     *     IMAX,ELAS,U,V,BETA,PHIDER)
      CALL SIGMA(CRILOD,EU,EV,UP,VP,C,IMIN,IMAX,MXSTR,SIGMAY,
     *     MNSTR,U,V,BETA,UZERO,VZERO,ELAS,PHIDER,AVOMGS,
     *     WSC1,WSC2,LMT,C12,RPR,C1,X1,APH1,APH2,D1,D2,C4,CCC,C2,
     *     R,D10,D11,D12,D13,D14,D15,CCD)
      IF(MNP.EQ.1)GO TO 7587
C   ************************************************************
C   *  CHECKING  THE  SIGNS  OF  THE  DEFLECTION  COMPONENTS        *
C   ************************************************************
      DO 9896 KLM=2,10
      EEE=DABS(U(KLM))-DABS(UCH(KLM))
      IF(EEE.LT.0.)GO TO 766
      FFF=DABS(V(KLM))-DABS(VCH(KLM))
      IF(FFF.LT.0.)GO TO 766
      GGG=DABS(BETA(KLM))-DABS(BETACH(KLM))
      IF(GGG.LT.0.)GO TO 766
 9896 CONTINUE
C   ************************************************************
C   *  CHECKING  THE  MAXIMUM  STRESS                               *
C   ************************************************************
 7587 IF(DABS(MXSTR).LE.DABS(MNSTR))MXSTR=DABS(MNSTR)
      IF(MXSTR.GT.SIGMAY)GO TO 766
      CRILOD=CRILOD+INCR
C   ************************************************************
C   *  STORING  THE  CURRENT  DEFLECTION  COMPONENTS  FOR  USE  IN  *
C   *  NEXT  STEP  OF  ITERATION
C   ************************************************************
      DO 7787 KLM=1,11
      UCH(KLM)=U(KLM)
      VCH(KLM)=V(KLM)
      BETACH(KLM)=BETA(KLM)
 7787 CONTINUE
      MNP=MNP+1
      GO TO 765
 766  CRILOD=CRILOD-INCR
      INCR=INCR/10.
      MNP=1
 768  CALL DEFROT(PYE,PXE,POE,CRILOD,C70,C71,C72,EU,EV,CLENTH,IMIN,
     *     IMAX,ELAS,U,V,BETA,PHIDER)
      CALL SIGMA(CRILOD,EU,EV,UP,VP,C,IMIN,IMAX,MXSTR,SIGMAY,
     *     MNSTR,U,V,BETA,UZERO,VZERO,ELAS,PHIDER,AVOMGS,
     *     WSC1,WSC2,LMT,C12,RPR,C1,X1,APH1,APH2,D1,D2,C4,CCC,C2,
     *     R,D10,D11,D12,D13,D14,D15,CCD)
      IF(MNP.EQ.1)GO TO 7588
C   ************************************************************
C   *  CHECKING  THE  SIGNS  OF  THE  DEFLECTION  COMPONENTS        *
C   ************************************************************
      DO 9895 KLM=2,10
      EEE=DABS(U(KLM))-DABS(UCH(KLM))
      IF(EEE.LT.0.)GO TO 769
      FFF=DABS(V(KLM))-DABS(VCH(KLM))
      IF(FFF.LT.0.)GO TO 769
      GGG=DABS(BETA(KLM))-DABS(BETACH(KLM))
      IF(GGG.LT.0.)GO TO 769
 9895 CONTINUE
C   ************************************************************
C   *  CHECKING  THE  MAXIMUM  STRESS                               *
C   ************************************************************
 7588 IF(DABS(MXSTR).LE.DABS(MNSTR))MXSTR=DABS(MNSTR)
      IF(MXSTR.GT.SIGMAY)GO TO 769
      CRILOD=CRILOD+INCR
C   ************************************************************
C   *  STORING  THE  CURRENT  DEFLECTION  COMPONENTS  FOR  USE  IN  *
```

```
C    *  NEXT  STEP  OF  ITERATION                                              *
C    ***************************************************************************
      DO 7786 KLM=1,11
      UCH(KLM)=U(KLM)
      VCH(KLM)=V(KLM)
      BETACH(KLM)=BETA(KLM)
 7786 CONTINUE
      MNP=MNP+1
      GO TO 768
  769 CRILOD=CRILOD-INCR
      INCR=INCR/10.
      MNP=1
 7707 CALL DEFROT(PYE,PXE,POE,CRILOD,C70,C71,C72,EU,EV,CLENTH,IMIN,
     *           IMAX,ELAS,U,V,BETA,PHIDER)
      CALL SIGMA(CRILOD,EU,EV,UP,VP,C,IMIN,IMAX,MXSTR,SIGMAY,
     *           MNSTR,U,V,BETA,UZERO,VZERO,ELAS,PHIDER,AVOMGS,
     *           WSC1,WSC2,LMT,C12,APR,C1,X1,APH1,APH2,D1,D2,CW,CCC,C2,
     *           R,D10,D11,D12,D13,D14,D15,CCD)
      IF(MNP.EQ.1)GO TO 7575
C    ***************************************************************************
C    *  CHECKING  THE  SIGNS  OF  THE  DEFLECTION  COMPONENTS                 *
C    ***************************************************************************
      DO 9801 KLM=2,10
      EEE=DABS(U(KLM))-DABS(UCH(KLM))
      IF(EEE.LT.0.)GO TO 7698
      FFF=DABS(V(KLM))-DABS(VCH(KLM))
      IF(FFF.LT.0.)GO TO 7698
      GGG=DABS(BETA(KLM))-DABS(BETACH(KLM))
      IF(GGG.LT.0.)GO TO 7698
 9801 CONTINUE
C    ***************************************************************************
C    *  CHECKING  THE  MAXIMUM  STRESS                                         *
C    ***************************************************************************
 7575 IF(DABS(MXSTR).LE.DABS(MNSTR))MXSTR=DABS(MNSTR)
      IF(MXSTR.GT.SIGMAY)GO TO 7698
      CRILOD=CRILOD+INCR
C    ***************************************************************************
C    *  STORING  THE  CURRENT  DEFLECTION  COMPONENTS  FOR  USE  IN          *
C    *  NEXT  STEP  OF  ITERATION.                                             *
C    ***************************************************************************
      DO 7701 KLM=1,11
      UCH(KLM)=U(KLM)
      VCH(KLM)=V(KLM)
      BETACH(KLM)=BETA(KLM)
 7701 CONTINUE
      MNP=MNP+1
      GO TO 7707
C    ***************************************************************************
C    *  CRILOD  GIVES  THE  CRITICAL  LOAD  FOR  THE  SECTION  AND           *
C    *  SLENDERNESS  RATIO                                                     *
C    ***************************************************************************
 7698 IK=SR
      WRITE(6,77)IK,CRILOD
      IF(SR.GE.240.)GO TO 782
C    ***************************************************************************
C    *  CHANGE  TO  NEXT  HIGHER  SLENDERNESS  RATIO                          *
C    ***************************************************************************
      SR=SR+10.
      GO TO 770
  782 IF(IJ.EQ.2)GO TO 7852
C    ***************************************************************************
C    *  USING  THE  MAGNITUDE  OF  WARPING  CONSTANT = 0.0                    *
C    ***************************************************************************
      CW=0.0
      IJ=2
      WRITE(6,882)
  882 FORMAT(//,10X,60('-'),/,10X,60('*'))
      GO TO 786
 7852 IF(IL-2)787,788,1010
C    ***************************************************************************
C    *  CHANGE TO  ANOTHER  LOCATION  OF THE  SHEAR-CENTER                   *
C    ***************************************************************************
  787 UZERO=UZRAS1
      VZERO=VZRAS1
      IL=2
      WRITE(6,884)
  884 FORMAT(//,10X,60('-'),/,10X,60('*'),/,10X,60('*'))
      GO TO 790
C    ***************************************************************************
C    *  CHANGE  TO  ANOTHER  LOCATION  OF  THE  SHEAR-CENTER                 *
```

```
C   ************************************************************
  788 UZERO=UZRAS2
      VZERO=VZRAS2
      IL=3
      WRITE(6,884)
      GO TO 790
 1010 WRITE(6,885)
  885 FORMAT(//,10X,60('-'),/,10X,60('*'),/,10X,20('END'))
   10 CONTINUE
      STOP
      END
C   ************************************************************
C   *  THIS  SUBROUTINE  COMPUTES  THE  MOMENT  OF  THE  CURVED  *
C   *  PORTION (NEEDED  FOR  THE  CALCULATION  OF V-COORDINATE   *
C   *  OF  ACTUAL  SHEAR  CENTER)  IF  THE  CENTROID  IS  IN  THE *
C   *  SECOND  QUADRANT  WITH  RESPECT  TO  CENTER  OF  CURVATURE *
C   *  AND IS VALID  BETWEEN  THE  LIMITS  0 AND THT FROM  THE   *
C   *  SHORT  LEG                                                *
C   ************************************************************
      SUBROUTINE MXCP12(R,QX,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX,TT)
      IMPLICIT REAL*8 (A-Z)
      INT1=R*R*QX*(ULMT-LLMT)
      INT2=R*D3*QX*(DSIN(3.141592-TT+PHI-ULMT)-DSIN(3.141592-TT+PHI-
     *     LLMT))
      INT3=R**3*D3*T/IMIN*.5*DCOS(1.570796+APHR+PHI)*(ULMT*ULMT-LLMT*
     *     LLMT)
      INT4=R**4*T/IMIN*(DCOS(APHR+TT)*(ULMT-LLMT)-DSIN(APHR+TT+ULMT)+
     *     DSIN(APHR+TT+LLMT))
      INT5=R*R*D3*D3*T/IMIN*DCOS(1.570796+APHR+PHI)*(-ULMT*DSIN(3.141592
     *     -TT+PHI-ULMT)+LLMT*DSIN(3.141592+PHI-TT-LLMT)+DCOS(3.141592+
     *     PHI-TT-ULMT)-DCOS(3.141592+PHI-TT-LLMT))
      INT6=2.*R**3*D3*T/IMIN*(-DCOS(APHR+TT)*.5*(DSIN(3.141592-TT+PHI-
     *     ULMT)-DSIN(3.141592+PHI-TT-LLMT))-.25*DCOS(3.141592+PHI+APHR)
     *     *(ULMT-LLMT)-.125*(DSIN(2.*ULMT+2.*TT-PHI-3.141592+APHR)-DSIN
     *     (APHR-3.141592+2.*TT-PHI+2.*LLMT)))
      MXX=INT1+INT2+INT3-INT4-INT5+INT6
      RETURN
      END
C   ************************************************************
C   *  THIS  SUBROUTINE  COMPUTES  THE  MOMENT  OF  THE  CURVED  *
C   *  PORTION (NEEDED  FOR  THE  CALCULATION  OF V-COORDINATE   *
C   *  OF  ACTUAL  SHEAR  CENTER)  IF  THE  CENTROID  IS  IN  THE *
C   *  SECOND  QUADRANT  WITH  RESPECT  TO  CENTER  OF  CURVATURE *
C   *  AND IS VALID  BETWEEN  THE  LIMITS  0 AND 90-THT-TT (IF   *
C   *  TT.NE.0.0)  OR  0 AND 90-THT (IF  TT.EQ.0.0)  FROM  THE   *
C   *  LONG LEG                                                  *
C   ************************************************************
      SUBROUTINE MXCP34(R,QX,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX)
      IMPLICIT REAL*8 (A-Z)
      INT1=R*R*QX*(ULMT-LLMT)
      INT2=R**3*T*D3/IMIN*.5*DCOS(1.570796+APHR+PHI)*(ULMT*ULMT-LLMT
     *     *LLMT)
      INT3=R*D3*QX*(DCOS(-ULMT-PHI)-DCOS(-LLMT-PHI))
      INT4=R**4*T/IMIN*(DCOS(APHR-1.570796)*(ULMT-LLMT)-DSIN(1.570796
     *     -APHR+ULMT)+DSIN(1.570796-APHR+LLMT))
      INT5=R*R*D3*D3*T/IMIN*DCOS(1.570796+APHR+PHI)*(ULMT*DCOS(-ULMT-PHI
     *     )-LLMT*DCOS(-LLMT-PHI)+DSIN(-ULMT-PHI)-DSIN(-LLMT-PHI))
      INT6=2.*R**3*D3*T/IMIN*(.5*DCOS(APHR-1.570796)*(DCOS(-ULMT-PHI)
     *     -DCOS(-LLMT-PHI))-.25*DSIN(1.570796-APHR-PHI)*(ULMT-LLMT)
     *     -.125*(DCOS(2.*ULMT+PHI-APHR+1.570796)-DCOS(2.*LLMT+PHI-
     *     APHR+1.570796)))
      MXX=INT1+INT2-INT3-INT4-INT5+INT6
      RETURN
      END
C   ************************************************************
C   *  THIS  SUBROUTINE  COMPUTES  THE  MOMENT  OF  THE  CURVED  *
C   *  PORTION (NEEDED  FOR  THE  CALCULATION  OF V-COORDINATE   *
C   *  OF  ACTUAL  SHEAR  CENTER)  IF  THE  CENTROID  IS  IN  THE *
C   *  SECOND  QUADRANT  WITH  RESPECT  TO  CENTER  OF  CURVATURE *
C   *  AND IS VALID  BETWEEN  THE  LIMITS  0 AND TT  FROM  THE   *
C   *  SHORT  LEG                                                *
C   ************************************************************
      SUBROUTINE MXCP78(R,QX,D3,T,APHR,PHI,IMIN,ULMT,LLMT,MXX)
      IMPLICIT REAL*8 (A-Z)
      INT1=R*R*QX*(ULMT-LLMT)
      INT2=R*D3*QX*(DCOS(1.570796+PHI-ULMT)-DCOS(1.570796+PHI-LLMT))
      INT3=R**3*D3*T/IMIN*.5*DSIN(APHR+PHI)*(ULMT*ULMT-LLMT*LLMT)
      INT4=R**4*T/IMIN*(DCOS(APHR)*(ULMT-LLMT)-DSIN(APHR+ULMT)+DSIN
     *     (APHR+LLMT))
      INT5=R*R*D3*D3*T/IMIN*DSIN(APHR+PHI)*(ULMT*DCOS(1.570796+PHI-
```

```
      ULMT)-LLMT*DCOS(1.570796+PHI-LLMT)+DSIN(1.570796+PHI
     *     -ULMT)-DSIN(1.570796+PHI-LLMT))
      INT6=2.*R**3*D3*T/IMIN*(.5*DCOS(APHR)*(DCOS(1.570796+PHI-ULMT)
     *     -DCOS(1.570796+PHI-LLMT))-.25*DSIN(1.570796+APHR+PHI)
     *     *(ULMT-LLMT)-.125*(DCOS(-APHR+1.570796+PHI-2.*ULMT)-DCOS
     *     (-APHR+PHI+1.570796-2.*LLMT)))
      MXX=INT1+INT2-INT3-INT4-INT5-INT6
      RETURN
      END
C     ****************************************************************
C     *  THIS  SUBROUTINE  CALCULATES  THE  MAGNITUDE  OF  ACTUAL    *
C     *  WARPING  CONSTANT   AND  RETURNS  THE  VALUE  IN  THE       *
C     *  VARIABLE  CWACT                                              *
C     ****************************************************************
      SUBROUTINE CWACTL(RPR,C12,X11,C3,YC,XC,Y11,LMT,C1,C2,C8,T,CWACT,
     *                  AVOMGS,C300,C301)
      IMPLICIT REAL*8 (A-Z)
      C300=-((X11-C3)-DABS(YC))
      C300A=C12*C12
      C300B=RPR*C12
      C300C=DCOS(1.570796+LMT)
      C300D=DSIN(1.570796+LMT)
      C300E=DCOS(LMT)
      C300F=DSIN(LMT)
      C301=C300*C1-(C300A*1.570796+C300B*(C300C-C300E))
      C302=(Y11-C3)-DABS(XC)
      AVOMGS=(C300*C1*.5*(C1+3.141592*C12)+C2*(C301-C302*.5*C2)-C12*C300
     *       A*1.233701-C12*C300B*(C300D-C300E*1.570796-C300F))/(C1+C2+C
     *       8)
      C303=C300*C1
      C304=AVOMGS*AVOMGS
      C305=2.*AVOMGS
      PART1=(C304+C303*C303/3.-C305*C303*.5)*C1*T
      PART2=(C304+C301*C301+C302*C302*C2*C2/3.-C301*C302*C2-C305
     *       *(C301-C302*C2*.5))*C2*T
      PART3=((C304+C303*C303+C300B*C300B*C300E*C300E+2.*C303*C300B*C300E
     *       -C305*(C303+C300B*C300E))*1.570796+C300A*C300A*1.291928+C300
     *       B*C300B*(.785398163-.5*DSIN(2.*LMT))-C300A*(C303+C300B*C300E
     *       -AVOMGS)*2.467402-C300B*(2.*C303+2.*C300B*C300E-C305)*(C300D
     *       -C300F)+2.*C300A*C300B*(1.570796*C300D+C300C-C300E))*T*C12
      CWACT=PART1+PART2+PART3
      RETURN
      END
C     ****************************************************************
C     *  THIS  SUBROUTINE  ESTIMATES  THE  VALUE  OF  AN  INTEGRAL    *
C     *  BY  ADAPTIVE  QUADRATURE  TECHNIQUE  USING  SIMPSON'S    -   *
C     *  RULE.                                                        *
C     ****************************************************************
C     *                                                               *
C     *   DESCRIPTION OF PARAMETERS:                                  *
C     *                                                               *
C     *   C       = LOWER LIMIT OF INTEGRATION                        *
C     *   D       = UPPER LIMIT OF INTEGRATION                        *
C     *   SIMP    = ESTIMATED VALUE OF THE INTEGRAL                   *
C     *   K       = NUMBER OF DIVISIONS INTO WHICH (C,D) IS PARTITIONED*
C     *   HMIN    = MINIMUM SUB-INTERVAL; THE ALGORITHM IS SAID TO HAVE*
C     *             FAILED IF THE ERROR TEST CANNOT BE SATISFIED WITH- *
C     *             OUT USING  A SUBDIVISION SMALLER THAN 'HMIN'.      *
C     *   IERROR  = A FLAG DENOTING EITHER A SUCCESS OR FAILURE OF    *
C     *             THE ALGORITHM                                     *
C     *   SP      = ESTIMATE USING  ONE PANEL SIMPSON'S RULE          *
C     *   S2P     = ESTIMATE USING  TWO-PANEL COMPOSITE SIMPSON'S RULE *
C     *                                                               *
C     ****************************************************************
      SUBROUTINE INTGRL(C,D,SIMP,MM,C400,C401,C402,C403,APH1,APH2,T,
     *                  X11,Y11,R,IMIN)
      IMPLICIT REAL*8 (A-Z)
      INTEGER I,IERROR,K,MM,II
      SIMP=0.0
      EST=0.0
      IERROR=4
      K=50
      AINC=(D-C)/K
      A=C
      B=C+AINC
      HMIN=(B-A)/2.0E 19
      II=2
      IF(IMIN.GE.1.0E 06)II=4
      IF(IMIN.LE.1.0E 06.AND.IMIN.GE.1.0E 05)II=3
      TOL=10**(II)/K
```

```
      DO 500 I=1,K
      CALL ADSIMP(A,B,HMIN,TOL,EST,IERROR,MM,C400,C401,C402,C403,
     *            APH1,APH2,T,X11,Y11,R)
      SIMP=SIMP+EST
      IF(IERROR.EQ.1) GO TO 400
      A=A+AINC
      B=B+AINC
500   CONTINUE
      GO TO 600
400   CONTINUE
      WRITE(6,11)B
11    FORMAT(//,3X,'ALGORITHM COULD NOT INTEGRATE  F(X) BEYOND ',D16.9)
600   CONTINUE
      RETURN
      END
C     ******************************************************************
C     *  THIS  SUBROUTINE  ESTIMATES  THE  VALUE  OF  AN  INTEGRAL  IN *
C     *  THE  SUB-INTERVAL  (A,B)                                      *
C     ******************************************************************
      SUBROUTINE ADSIMP(A,B,HMIN,TOL,EST,IERROR,MM,C400,C401,C402,C403,
     *                  APH1,APH2,T,X11,Y11,R)
      IMPLICIT REAL*8 (A-Z)
      INTEGER I,IERROR,MM
      DIMENSION HH(20),V(20),FH(20),FS(20),FT(20),FU(20),FV(20)
      FUNCT1(P,FC1,FC2,FC3,FC4,FC5,T)=(((FC1-P)*FC2-FC3-FC4*FC2)**2
     *   *((FC1-P)*FC5-FC4*FC5)+((FC1-P)*FC5-FC4*FC5)**3)*T
      FUNCT2(P,FC1,FC2,FC3,FC4,FC5,T)=(((FC1-P)*FC2-FC3-FC4*FC2)**3
     *   +((FC1-P)*FC2-FC3-FC4*FC2)*((FC1-P)*FC5-FC4*FC5)**2)*T
      FUNCT3(P,FC1,FC2,FC3,FC4,FC5,T)=(-((FC1-P)*FC2+FC4*FC5)**3-((
     *   FC1-P)*FC2+FC4*FC5)*((FC1-P)*FC5-FC3-FC4*FC2)**2)*T
      FUNCT4(P,FC1,FC2,FC3,FC4,FC5,T)=(((FC1-P)*FC5-FC3-FC4*FC2)**3
     *   +((FC1-P)*FC5-FC3-FC4*FC2)*((FC1-P)*FC2+FC4*FC5)**2)*T
      FUNCT5(P,X11,Y11,R,T,APH1,APH2)=((-(Y11-R-T+(R+T*.5)*DSIN(P))*
     *   APH2+(X11-R-T+(R+T*.5)*DCOS(P))*APH1)**3+(-(Y11-R-T+(R+.5*T
     *   )*DSIN(P))*APH2+(X11-R-T+(R+T*.5)*DCOS(P))*APH1)*(-(X11-R-T
     *   +(R+T*.5)*DCOS(P))*APH2-(Y11-R-T+(R+T*.5)*DSIN(P))*APH1
     *   )**2)*T*(R+T*.5)
      FUNCT6(P,X11,Y11,R,T,APH1,APH2)=((-(X11-R-T+(R+T*.5)*DCOS(P))*
     *   APH2-(Y11-R-T+(R+T*.5)*DSIN(P))*APH1)**3+(-(X11-R-T+(R+
     *   .5*T)*DCOS(P))*APH2-(Y11-R-T+(R+T*.5)*DSIN(P))*APH1)*
     *   (-(Y11-R-T+(R+T*.5)*DSIN(P))*APH2+(X11-R-T+(R+.5*T)*
     *   DCOS(P))*APH1)**2)*T*(R+.5*T)
      EST=0.0
      I=1
      HH(1)=A
      V(1)=B
      GO TO (11,12,13,14,15,16),MM
11    FH(1)=FUNCT1(A,C402,APH1,C400,C401,APH2,T)
      FS(1)=FUNCT1(A+(B-A)/4.,C402,APH1,C400,C401,APH2,T)
      FT(1)=FUNCT1(A+(B-A)/2.,C402,APH1,C400,C401,APH2,T)
      FV(1)=FUNCT1(B,C402,APH1,C400,C401,APH2,T)
      FU(1)=FUNCT1(A+3.0*(B-A)/4.,C402,APH1,C400,C401,APH2,T)
      GO TO 50
12    FH(1)=FUNCT2(A,C402,APH1,C400,C401,APH2,T)
      FS(1)=FUNCT2(A+(B-A)/4.,C402,APH1,C400,C401,APH2,T)
      FT(1)=FUNCT2(A+(B-A)/2.,C402,APH1,C400,C401,APH2,T)
      FV(1)=FUNCT2(B,C402,APH1,C400,C401,APH2,T)
      FU(1)=FUNCT2(A+3.*(B-A)/4.,C402,APH1,C400,C401,APH2,T)
      GO TO 50
13    FH(1)=FUNCT3(A,C403,APH1,C400,C401,APH2,T)
      FS(1)=FUNCT3(A+(B-A)/4.,C403,APH1,C400,C401,APH2,T)
      FT(1)=FUNCT3(A+(B-A)/2.,C403,APH1,C400,C401,APH2,T)
      FV(1)=FUNCT3(B,C403,APH1,C400,C401,APH2,T)
      FU(1)=FUNCT3(A+3.*(B-A)/4.,C403,APH1,C400,C401,APH2,T)
      GO TO 50
14    FH(1)=FUNCT4(A,C403,APH1,C400,C401,APH2,T)
      FS(1)=FUNCT4(A+(B-A)/4.,C403,APH1,C400,C401,APH2,T)
      FT(1)=FUNCT4(A+(B-A)/2.,C403,APH1,C400,C401,APH2,T)
      FV(1)=FUNCT4(B,C403,APH1,C400,C401,APH2,T)
      FU(1)=FUNCT4(A+3.*(B-A)/4.,C403,APH1,C400,C401,APH2,T)
      GO TO 50
15    FH(1)=FUNCT5(A,X11,Y11,R,T,APH1,APH2)
      FS(1)=FUNCT5(A+(B-A)/4.,X11,Y11,R,T,APH1,APH2)
      FT(1)=FUNCT5(A+(B-A)/2.,X11,Y11,R,T,APH1,APH2)
      FV(1)=FUNCT5(B,X11,Y11,R,T,APH1,APH2)
      FU(1)=FUNCT5(A+3.*(B-A)/4.,X11,Y11,R,T,APH1,APH2)
      GO TO 50
16    FH(1)=FUNCT6(A,X11,Y11,R,T,APH1,APH2)
      FS(1)=FUNCT6(A+(B-A)/4.,X11,Y11,R,T,APH1,APH2)
      FT(1)=FUNCT6(A+(B-A)/2.,X11,Y11,R,T,APH1,APH2)
```

```
      FV(1)=FUNCT6(B,X11,Y11,R,T,APH1,APH2)
      FU(1)=FUNCT6(A+3.*(B-A)/4.,X11,Y11,R,T,APH1,APH2)
50    SF=((V(I)-RR(I))/6.0)*(FR(I)+4.0*FT(I)+FV(I))
      S2F=((V(I)-RR(I))/12.)*(FR(I)+FV(I)+2.0*FT(I)+4.0*FS(I)+4.0*FU(I))
      ERR=2.0*TOL*(V(I)-RR(I))/(B-A)
      IF(DABS(S2F-SF).LE.ERR) GO TO 100
      RR(I+1)=RR(I)
      V(I+1)=(RR(I)+V(I))/2.0
      FR(I+1)=FR(I)
      FT(I+1)=FS(I)
      FV(I+1)=FT(I)
      GO TO (51,52,53,54,55,56),MM
51    FS(I+1)=FUNCT1(RR(I+1)+((V(I+1)-RR(I+1))/4.),C402,APH1,C400,C401,
     *                APH2,T)
      FU(I+1)=FUNCT1(RR(I+1)+3.*((V(I+1)-RR(I+1))/4.),C402,APH1,C400,
     *                C401,APH2,T)
      GO TO 60
52    FS(I+1)=FUNCT2(RR(I+1)+((V(I+1)-RR(I+1))/4.),C402,APH1,C400,C401,
     *                APH2,T)
      FU(I+1)=FUNCT2(RR(I+1)+3.*((V(I+1)-RR(I+1))/4.),C402,APH1,C400,
     *                C401,APH2,T)
      GO TO 60
53    FS(I+1)=FUNCT3(RR(I+1)+((V(I+1)-RR(I+1))/4.),C403,APH1,C400,C401,
     *                APH2,T)
      FU(I+1)=FUNCT3(RR(I+1)+3.*((V(I+1)-RR(I+1))/4.),C403,APH1,C400,
     *                C401,APH2,T)
      GO TO 60
54    FS(I+1)=FUNCT4(RR(I+1)+((V(I+1)-RR(I+1))/4.),C403,APH1,C400,C401,
     *                APH2,T)
      FU(I+1)=FUNCT4(RR(I+1)+3.*((V(I+1)-RR(I+1))/4.),C403,APH1,C400,
     *                C401,APH2,T)
      GO TO 60
55    FS(I+1)=FUNCT5(RR(I+1)+((V(I+1)-RR(I+1))/4.),X11,Y11,R,T,APH1,
     *                APH2)
      FU(I+1)=FUNCT5(RR(I+1)+3.*((V(I+1)-RR(I+1))/4.),X11,Y11,R,T,APH1,
     *                APH2)
      GO TO 60
56    FS(I+1)=FUNCT6(RR(I+1)+((V(I+1)-RR(I+1))/4.),X11,Y11,R,T,APH1
     *                ,APH2)
      FU(I+1)=FUNCT6(RR(I+1)+3.*((V(I+1)-RR(I+1))/4.),X11,Y11,R,T,APH1,
     *                APH2)
60    I=I+1
      IF((V(I)-RR(I)).GE.HMIN) GO TO 200
      B=RR(I)
      IERROR=1
      GO TO 300
200   CONTINUE
      GO TO 50
100   CONTINUE
      EST=EST+S2F
      IF(V(I).EQ.B) IERROR=0
      IF(V(I).EQ.B) GO TO 800
      RR(I-1)=V(I)
      FR(I-1)=FV(I)
      FT(I-1)=FU(I-1)
      GO TO (71,72,73,74,75,76),MM
71    FS(I-1)=FUNCT1(RR(I-1)+((V(I-1)-RR(I-1))/4.),C402,APH1,C400,C401
     *                ,APH2,T)
      FU(I-1)=FUNCT1(RR(I-1)+3.*(V(I-1)-RR(I-1))/4.,C402,APH1,C400,C401
     *                ,APH2,T)
      GO TO 80
72    FS(I-1)=FUNCT2(RR(I-1)+((V(I-1)-RR(I-1))/4.),C402,APH1,C400,C401
     *                ,APH2,T)
      FU(I-1)=FUNCT2(RR(I-1)+3.*(V(I-1)-RR(I-1))/4.,C402,APH1,C400,C401
     *                ,APH2,T)
      GO TO 80
73    FS(I-1)=FUNCT3(RR(I-1)+((V(I-1)-RR(I-1))/4.),C403,APH1,C400,C401
     *                ,APH2,T)
      FU(I-1)=FUNCT3(RR(I-1)+3.*(V(I-1)-RR(I-1))/4.,C403,APH1,C400,C401
     *                ,APH2,T)
      GO TO 80
74    FS(I-1)=FUNCT4(RR(I-1)+((V(I-1)-RR(I-1))/4.),C403,APH1,C400,C401,
     *                APH2,T)
      FU(I-1)=FUNCT4(RR(I-1)+3.*(V(I-1)-RR(I-1))/4.,C403,APH1,C400,C401
     *                ,APH2,T)
      GO TO 80
75    FS(I-1)=FUNCT5(RR(I-1)+((V(I-1)-RR(I-1))/4.),X11,Y11,R,T,APH1,
     *                APH2)
      FU(I-1)=FUNCT5(RR(I-1)+3.*(V(I-1)-RR(I-1))/4.,X11,Y11,R,T,APH1,
     *                APH2)
```

```
      GO TO 80
76    PS(I-1)=FUNCT6(RR(I-1)+((V(I-1)-RR(I-1))/4.),X11,Y11,R,T,APH1,
     *                APH2)
      PU(I-1)=FUNCT6(RR(I-1)+3.*(V(I-1)-RR(I-1))/4.,X11,Y11,R,T,APH1,
     *                APH2)
80    I=I-1
      GO TO 50
300   CONTINUE
800   CONTINUE
      RETURN
      END
C     ****************************************************************
C     *  THIS  SUBROUTINE  CALCULATES  THE  DEFLECTIONS & ROTATION   *
C     *  AT  ANY  CROSS-SECTION  FOR  ANY  PARTICULAR  VALUE  OF  LOAD*
C     *  USING  GALERKIN  METHOD.                                    *
C     ****************************************************************
      SUBROUTINE DEFROT(PYE,PXE,PCE,P,C13,C14,C15,EU,EV,CLENTH,
     *                  VVI,UUI,E,U,V,PHI,PHIDER)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3,3),B(3),WKAREA(80),U(11),V(11),PHI(11),
     *          PHIDER(11)
      A(1,1)=PYE-P
      A(1,2)=0.
      A(1,3)=-P*C14*1.419594418
      A(2,1)=0.
      A(2,2)=PXE-P
      A(2,3)=P*C13*1.419594418
      A(3,1)=-P*C14*.5507
      A(3,2)=P*C13*.5507
      A(3,3)=C15*(POE-1.001870809*P)
      B(1)=-P*P/PYE*EU*1.273239545
      B(2)=-P*P/PXE*EV*1.273239545
      B(3)=-P*P*.6506108964*(EU*C14/PYE-EV*C13/PXE)
      IA=3
      M=1
      N=3
      IDGT=9
      CALL LEQT2P(A,M,N,IA,B,IDGT,WKAREA,IER)
      C1=B(1)
      C2=B(2)
      C3=B(3)
      D0=4.73004/CLENTH
      D1=3.1415927/CLENTH
      D2=P*EU/(2.*E*VVI)
      D3=P*EV/(2.*E*UUI)
      Z=0.
      DEL=CLENTH/10.
      DO 25 I=1,11
      U(I)=D2*Z*(Z-CLENTH)+C1*DSIN(D1*Z)
      V(I)=D3*Z*(Z-CLENTH)+C2*DSIN(D1*Z)
      PHI(I)=C3*(DSIN(D0*Z)-DSINH(D0*Z)-1.017809411*(DCOS(D0*Z)-DCOSH
     *       (D0*Z)))
      PHIDER(I)=D0*D0*(-DSIN(D0*Z)-DSINH(D0*Z)+1.017809411*(DCOS(D0*Z)
     *          +DCOSH(D0*Z)))*C3
      Z=Z+DEL
25    CONTINUE
      U(11)=0.
      V(11)=0.
      PHI(11)=0.
      RETURN
      END
C     ****************************************************************
C     *  THIS  SUBROUTINE  CALCULATES  THE  STRESSES AT  THE  PRE-   *
C     *  DEFINED 16  DIFFERENT  POINTS  OF  EACH  OF  THE  11        *
C     *  DIFFERENT  CROSS-SECTIONS  AND  RETURNS  THE  MAXIMUM  AND  *
C     *  MINIMUM  STRESSES,  AT  ANY  OF  THOSE  11  CROSS-SECTIONS  *
C     *  AND  11 X 16  POINTS  THRUOUT  THE  LENGTH  OF  THE  COLUMN *
C     ****************************************************************
      SUBROUTINE SIGMA(PLOAD,EU,EV,UP,VP,C,IMIN,IMAX,MXSTR,SIGMAY,
     *                 MNSTR,U,V,BETA,UZERO,VZERO,Z,PHIDER,AVOMGS,
     *                 WSC1,WSC2,LAT,CO2,APR,C1,X1,APH1,APH2,D1,D2,CW,
     *                 CCC,C2,R,D10,D11,D12,D13,D14,D15,CCD)
      IMPLICIT REAL*8 (A-Z)
      INTEGER I,II,JX,JN,KK,JFK
      DIMENSION STRESS(16),VP(16),UP(16),WS(16),U(11),V(11),
     *          BETA(11),PHIDER(11),MAXSTR(11),MINSTR(11)
      C10=PLOAD/C
      DO 50 II=1,11
      C11=(EV-V(II)-(EU-UZERO)*BETA(II))*PLOAD/IMAX
      C12=(EU-U(II)+(EV-VZERO)*BETA(II))*PLOAD/IMIN
```

```
      C13=R*PHIDER(II)
      C14=1.5707963
      IF(CW.EQ.0.)GO TO 182
      A=-C11*X1*APH1+C12*X1*APH2+C13*RPR*CO2*DSIN(LMT)
      B=-C11*X1*APH2-C12*X1*APH1+C13*RPR*CO2*DCOS(LMT)
      CONS=-C13*CO2*CO2
      M=DMAX1(A,B,CONS)
      A=A/M
      B=B/M
      CONS=CONS/M
      CALL SOLVE(A,B,CONS,THTA)
      GO TO 75
182   A=-C11*X1*APH1+C12*X1*APH2
      B=-C11*X1*APH2-C12*X1*APH1
      THTA=DATAN(-B/A)
75    XP=-(D2+X1*DCOS(THTA))
      YP=-(D1+X1*DSIN(THTA))
      UP(9)=YP*APH1+XP*APH2
      VP(9)=YP*APH2-XP*APH1
      XP=-(D2+R*DCOS(THTA))
      YP=-(D1+R*DSIN(THTA))
      UP(16)=YP*APH1+XP*APH2
      VP(16)=YP*APH2-XP*APH1
      IF(CW.EQ.0.)GO TO 185
      DO 5 I=1,4
      STRESS(I)=C10+C11*VP(I)+C12*UP(I)+C13*AVOMGS
5     CONTINUE
      DO 25 I=7,8
      STRESS(I)=C10+C11*VP(I)+C12*UP(I)+C13*(AVOMGS-WSC1*C1)
25    CONTINUE
      DO 92 I=5,6
      STRESS(I)=C10+C11*VP(I)+C12*UP(I)+C13*(AVOMGS-WSC2)
92    CONTINUE
      WS(9)=AVOMGS-(WSC1*C1-CO2*CO2*(C14-THTA)-RPR*CO2*
     *     (DCOS(LMT+(C14-THTA))-DCOS(LMT)))
      STRESS(9)=C10+C11*VP(9)+C12*UP(9)+C13*WS(9)
      WS(16)=WS(9)
      STRESS(16)=C10+C11*VP(16)+C12*UP(16)+C13*WS(16)
      WS(10)=AVOMGS-(WSC2-CCC*(C2-D10))
      WS(11)=AVOMGS-(WSC2-CCC*(C2-D11))
      WS(14)=AVOMGS-(WSC2-CCC*(C2-D14))
      WS(15)=AVOMGS-(WSC2-CCC*(C2-D15))
      WS(12)=AVOMGS-(CCD*D12)
      WS(13)=AVOMGS-(CCD*D13)
      DO 1212 I=10,15
      STRESS(I)=C10+C11*VP(I)+C12*UP(I)+C13*WS(I)
1212  CONTINUE
      GO TO 187
185   DO 99 I=1,16
      STRESS(I)=C10+C11*VP(I)+C12*UP(I)
99    CONTINUE
187   DO 11 I=1,16
      IF(UP(I).EQ.0..AND.VP(I).EQ.0.)STRESS(I)=0.
11    CONTINUE
C   ***************************************************************
C   *  SORTING  OF  THE  MAXIMUM  AND  MINIMUM  STRESSES  AT  A   *
C   *  CROSS-SECTION                                              *
C   ***************************************************************
      MAXSTR(II)=STRESS(1)
      MINSTR(II)=STRESS(1)
      DO 10 I=2,16
      IF(MAXSTR(II).GT.STRESS(I))GO TO 12
      MAXSTR(II)=STRESS(I)
12    IF(MINSTR(II).LE.STRESS(I))GO TO 10
      MINSTR(II)=STRESS(I)
10    CONTINUE
50    CONTINUE
C   ***************************************************************
C   *  SORTING  OF  THE  MAXIMUM  AND  MINIMUM  STRESSES  IN  THE *
C   *  COLUMN                                                     *
C   ***************************************************************
      MXSTR=MAXSTR(1)
      MNSTR=MINSTR(1)
      DO 90 II=2,11
      IF(MXSTR.GT.MAXSTR(II))GO TO 95
      MXSTR=MAXSTR(II)
95    IF(MNSTR.LE.MINSTR(II))GO TO 90
      MNSTR=MINSTR(II)
90    CONTINUE
      RETURN
```

```
      END
C     ******************************************************************
C     * THIS  SUBROUTINE  CALCULATES  THE ANGLE  THETA, THAT IS, THE *
C     * POINT  OF  MAXIMUM  STRESS  IN  THE  CURVED  PORTION, IF ANY *
C     ******************************************************************
      SUBROUTINE SOLVE(A,B,C,THTA)
      IMPLICIT REAL*8  (A-Z)
      INTEGER I
      I=1
      THTA=0.
      INCR=.1
5     X=A*DSIN(THTA)+B*DCOS(THTA)+C
      Y=A*DSIN(THTA+INCR)+B*DCOS(THTA+INCR)+C
      IF((X.GT.0..AND.Y.LT.0.).OR.(X.LT.0..AND.Y.GT.0.))GO TO 10
      THTA=THTA+INCR
      IF(THTA.GT.1.5708)GO TO 100
      GO TO 5
10    GO TO (15,25,35,40),I
15    INCR=INCR/10.
      I=2
      GO TO 5
25    INCR=INCR/10.
      I=3
      GO TO 5
35    INCR=INCR/10.
      I=4
      GO TO 5
40    THTA=THTA+.5*INCR
      GO TO 101
100   THTA=0.
101   RETURN
      END
```

APPENDIX 4

DEFINITE INTEGRALS

APPENDIX 4

DEFINITE INTEGRALS

$$\int_0^{\ell} \operatorname{Sin} \frac{\pi z}{\ell}\, dz = 0.6366\ \ell$$

$$\int_0^{\ell} \operatorname{Sin}^2 \frac{\pi z}{\ell}\, dz = 0.5000\ \ell$$

$$\int_0^{\ell} [\operatorname{Sin} \frac{\lambda z}{\ell} - \operatorname{Sinh} \frac{\lambda z}{\ell} - \alpha' (\operatorname{Cos} \frac{\lambda z}{\ell} - \operatorname{Cosh} \frac{\lambda z}{\ell})]dz = 0.8457\ \ell$$

$$\int_0^{\ell} [\operatorname{Sin} \frac{\lambda z}{\ell} - \operatorname{Sinh} \frac{\lambda z}{\ell} - \alpha' (\operatorname{Cos} \frac{\lambda z}{\ell} - \operatorname{Cosh} \frac{\lambda z}{\ell})]^2\, dz = 1.0359\ \ell$$

$$\int_0^{\ell} \operatorname{Sin} \frac{\pi z}{\ell} [\operatorname{Sin} \frac{\lambda z}{\ell} - \operatorname{Sinh} \frac{\lambda z}{\ell} - \alpha' (\operatorname{Cos} \frac{\lambda z}{\ell} - \operatorname{Cosh} \frac{\lambda z}{\ell})]dz = 0.7098\ \ell$$

$$\int_0^{\ell} \operatorname{Sin} \frac{\pi z}{\ell} [- \operatorname{Sin} \frac{\lambda z}{\ell} - \operatorname{Sinh} \frac{\lambda z}{\ell} + \alpha' (\operatorname{Cos} \frac{\lambda z}{\ell} + \operatorname{Cosh} \frac{\lambda z}{\ell})]dz = -0.3131\ \ell$$

$$\int_0^{\ell} [\operatorname{Sin} \frac{\lambda z}{\ell} - \operatorname{Sinh} \frac{\lambda z}{\ell} - \alpha' (\operatorname{Cos} \frac{\lambda z}{\ell} - \operatorname{Cosh} \frac{\lambda z}{\ell})][- \operatorname{Sin} \frac{\lambda z}{\ell} - \operatorname{Sinh} \frac{\lambda z}{\ell}$$

$$+ \alpha' (\operatorname{Cos} \frac{\lambda z}{\ell} + \operatorname{Cosh} \frac{\lambda z}{\ell})]dz = -0.5696\ \ell$$

APPENDIX 5

SAMPLE CALCULATION FOR COMPUTATION OF ACTUAL DIMENSIONS OF TEST SPECIMENS

APPENDIX 5

SAMPLE CALCULATION FOR COMPUTATION OF ACTUAL DIMENSIONS OF TEST SPECIMENS

Nominal dimensions of the specimen: 55 x 55 x 4 mm

Mass = 3.713 kg

Length = 1.208 m

Mass density = 7850 kg/m^3

$$\text{Volume} = \frac{3.713}{7850} = 4.73 \times 10^{-4}\ m^3$$

$$\text{Cross-sectional area} = \frac{\text{Volume}}{\text{Length}} = \frac{4.73 \times 10^{-4}}{1.208} = 3.9155 \times 10^{-4}\ m^2$$

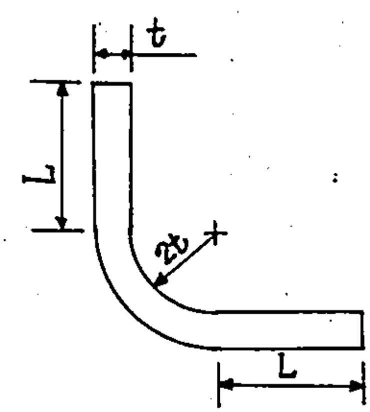
$$= 3.9155 \times 10^{2}\ mm^2$$

	1	2	3	4	5	6
Thickness (t) (in.)	0.156	0.160	0.158	0.155	0.155	0.155

Average thickness = 0.156 in = 3.96 mm

Centre line length of the cross-section of angle

$$= \frac{3.9155 \times 10^2}{3.96} = 98.13\ mm$$



From the figure,

$$L + L + \frac{\pi}{2} \times 2.5t = 98.13\ mm$$

or L = 41.23 mm

∴ Actual dimensions are

(41.23 + 12) x (41.23 + 12) x 3.96

i.e., 53.2 x 53.2 x 3.96 mm.

APPENDIX 6

TYPICAL CALCULATIONS FOR THE DETERMINATION OF FAILURE LOADS

APPENDIX 6

TYPICAL CALCULATIONS FOR THE DETERMINATION OF FAILURE LOADS

A6.1 According to ASCE Manual No. 52 (based on nominal dimensions)

(a) Single-bolted specimen

Consider ES65-3-1

$$\left(\frac{b}{t}\right)_{actual} = \frac{65-12}{4} = 13.25; \; E = 205 \text{ GPa}; \; \sigma_y = 300 \text{ MPa}$$

$$\left(\frac{b}{t}\right)_{limit} = \frac{208}{\sqrt{\sigma_y}} = 12.01 \approx 12 < 13.25$$

$$\therefore \left(\frac{b}{t}\right)_{limit} < \left(\frac{b}{t}\right)_{actual}$$

$$\sigma_{y,eff} = [1.8 - \frac{0.8 \times 13.25}{12}] \times 300 = 275 \text{ MPa}$$

$$C_c = \pi \sqrt{\frac{2E}{\sigma_{y,eff}}} = 121.3$$

$$\frac{K\ell}{r_y} = 171$$

$$\frac{K\ell}{r_y} > C_c$$

$$\sigma_{cr} = \frac{\pi^2 \times 205\,000}{171^2} = 69.19 \text{ MPa}$$

Failure Load = 69.19 x 426.83 = 29530 N = 29.5 kN

(b) Multiple-bolted specimen

Consider ES65-3-3

$$\frac{K\ell}{r_y} = 46.2 + 0.615 \times 171 = 151.37$$

$$\frac{K\ell}{r_y} > C_c$$

$$\sigma_{cr} = \frac{\pi^2 \times 205\,000}{151.37^2} = 88.30 \text{ MPa}$$

Failure Load = 88.30 x 426.83 = 37690 N = 37.7 kN

A6.2 According to ECCS Recommendations (based on nominal dimensions)

(a) Single-bolted specimen

Consider ES65-3-1

$E = 210$ GPa; $\sigma_y = 300$ MPa

$$\lambda = \frac{K\ell}{r_y} = 171$$

$$\lambda_y = \pi \sqrt{\frac{E}{\sigma_y}} = \pi \sqrt{\frac{210\,000}{300}} = 83.12$$

$$\frac{\lambda}{\lambda_y} = \frac{171}{83.12} = 2.057 > 1.41$$

$$\left(\frac{\lambda}{\lambda_y} \right)_{eff} = \frac{\lambda}{\lambda_y}$$

$\lambda_{eff} = \lambda = 171$

$\sigma_{cr} = 58.0$ MPa

Failure Load = 58.0 x 426.83 = 24756 N = 24.8 kN

(b) Multiple-bolted specimen

Consider ES65-3-3

$$\frac{\lambda}{\lambda_y} = 2.057$$

$$1.41 \leq \frac{\lambda}{\lambda_y} \leq 3.5$$

$$\left(\frac{\lambda}{\lambda_y}\right)_{eff} = 0.35 + 0.75 \times 2.057 = 1.893$$

$$\lambda_{eff} = 1.893 \times 83.12 = 157.35$$

$\sigma_{cr} = 67.2$ MPa

Failure Load = 67.2 x 426.83 = 28683 N = 28.7 kN