2013

Optimization of Operational Costs for a Single Supplier - Manufacturer Supply Chain

Hasti Eiliat
University of Windsor

Optimization of Operational Costs for a Single Supplier - Manufacturer Supply Chain

by

Hasti Eiliat

A Thesis

Submitted to the Faculty of Graduate Studies

through Industrial and Manufacturing Systems Engineering

in Partial Fulfillment of the Requirements for

the Degree of Master of Applied Science at the

University of Windsor

Windsor, Ontario, Canada

2013

Optimization of Operational Costs for a Single Supplier – Manufacturer Supply Chain

by

Hasti Eiliat

APPROVED BY:

____
Dr. E. Selvarajah
Odette School of Business

____
Dr. J. Urbanic
Mechanical, Automotive and Materials Engineering

____
Dr. R. Caron, Advisor
Mathematics & Statistics, Cross-appointed in Industrial & Manufacturing Systems Engineering

September 5, 2013

Author's Declaration of Originality

I hereby certify that I am the sole author of this thesis and that no part of this thesis has been published or submitted for publication.

I certify that, to the best of my knowledge, my thesis does not infringe upon anyone's copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledged in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owner(s) to include such material(s) in my thesis.

I declare that this is a true copy of my thesis, including any final revisions, as approved by my thesis committee and the Graduate Studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.

Abstract

This thesis considers a simple supply chain with one supplier and one manufacturer. The supplier provides batches of items to meet the manufacturer's demand targets. The manufacturer tries to choose the right order quantities of items to minimize total operational cost. The cost is made up of inventory and transportation costs which both are functions of order quantity. The inventory costs include holding costs, purchasing costs, ordering costs and financing costs. The transportation costs include the expenses of shipping the items from the supplier to the manufacturer that are calculated with the National Motor Freight Classification (NMFC) standard. The Genetic Algorithm (GA) function in Matlab is used as the solution procedure and the results are compared with the Equal Order Quantity model and the Just-In-Time (JIT) model. The last part of this thesis presents a sensitivity analysis to establish the robustness of the model.

Dedication

This thesis is dedicated to my family, especially...

to my parents for their love;

to my angel sister for her encouragement;

to my brother-in-law for his endless support.

Acknowledgements

I would not be able to pursue my Master's studies and write my thesis without the support and understanding of the following people:

First, I would like to express my sincere gratitude for my supervisor, Professor Richard Caron. I am forever thankful for his understanding and his support in all stages of my studies specifically his help with my proposal preparation and thesis write-up. I believe I learned many life lessons from him that have helped me immensely in my personal life.

Second, I would like to thank Dr. Jill Urbanic and Dr. Esaignani Selvarajah for their time and their helpful feedback on my thesis. I felt very lucky to have them on my thesis committee.

Third, special thanks go to Department of Industrial and Manufacturing Systems Engineering (IMSE) for their financial support through Graduate Assistantship Scholarships and Dr. Patricia Weir and Dr. Mehrdad Saif for hearing me out, believing in me and supporting me to gain back my confidence and progress in my studies.

Lastly, I would love to thank my mother and father for their unconditional and continuous love. They have supported me emotionally and financially to the best of their ability and for that I am always in their debt. Also, I want to thank my sister, Hoda for her true friendship and her open arms and ears. My brother-in-law, Mohammad has also been a great motivator for me. I am thankful for his guidance and encouragement throughout my studies.

Contents

List of Tables

List of Figures

List of Abbreviations

EOQ	Economic Order Quantity
GA	Genetic Algorithm
JIT	Just-In-Time
NMFC	National Motor Freight Classification

List of Notations

Notation	Description
$i \in I$	Items
s_i	Volume of item i (Cubic Foot)
w_i	Weight of item i (Pound)
$\mathcal{S}_i$	Plant warehouse volume capacity for storing item i (Cubic Foot)
$\mathcal{W}_i$	Plant warehouse weight capacity for storing item i (Pound)
α_i	Maximum number of item i that can be stored in the plant warehouse
W_i	Weight per cubic foot of item i
o_i	Fixed ordering cost of item i
h_i	Holding cost of item i per time period
$j \in J$	Time points
n	The last time point
d_i^j	Demand target of item i at time point j
q_i^j	Order quantity of item i at time point j
l_i^j	Inventory level of item i at time point j
$v \in V$	NMFC class types
c_v	Set of items with NMFC class type $v \in V$
$k \in K$	NMFC weight range classes
a_{k-1}	Lower bound of NMFC weight range class k
a_k	Upper bound of NMFC weight range class k
a_τ	Maximum weight that can be shipped at each time point
ϑ_k^v	Transportation cost per 100 pounds of NMFC class type v in weight range k
$e_i \in E_i$	Price break classes for purchasing item i
ϱ_i	Last price break class of item i
$Q_i^{e_i-1}$	Lower bound for price break class e_i of item i
$Q_i^{e_i}$	Upper bound for price break class e_i of item i
$p_i^{e_i}$	Price of item i in price break class e_i
r	Percentage rate of interest
L_i^j	Purchasing cost of q_i^j
F_i^j	Fixed payment at each time point for a loan of L_i^j
$P(q_i^j)$	Total purchasing cost of item i at time point j
$R_v^j(q_i^j : i \in c_v)$	Total transportation cost at time point j with NMFC classification v
$O(q_i^j)$	Total ordering cost of item i at time point j
$H(q_i^j)$	Total holding cost of item i at time point j
$C(q_i^j)$	Total operational cost

CHAPTER 1

Introduction

1.1. Motivation

Manufacturers need to have a good supply chain management system in order to achieve low inventory levels, short lead times and adjustability to meet customer demands at minimal total operational cost [**1**]. This cost is made up of inventory and transportation costs [**2**] that are often minimized separately [**3**]. This minimization strategy may not be able to give an optimal order quantity [**4**] because of the antinomic relationship between inventory cost and transportation cost [**1**]. In this thesis, inventory and transportation costs are minimized together to determine optimal order quantities.

Figure 1.1 shows an example of an antinomic relationship between inventory and transportation costs. It shows that transportation costs increase with frequency and that inventory costs decrease. The manufacturer represented in figure 1.1 should order 7 times in a year to minimize the total cost.

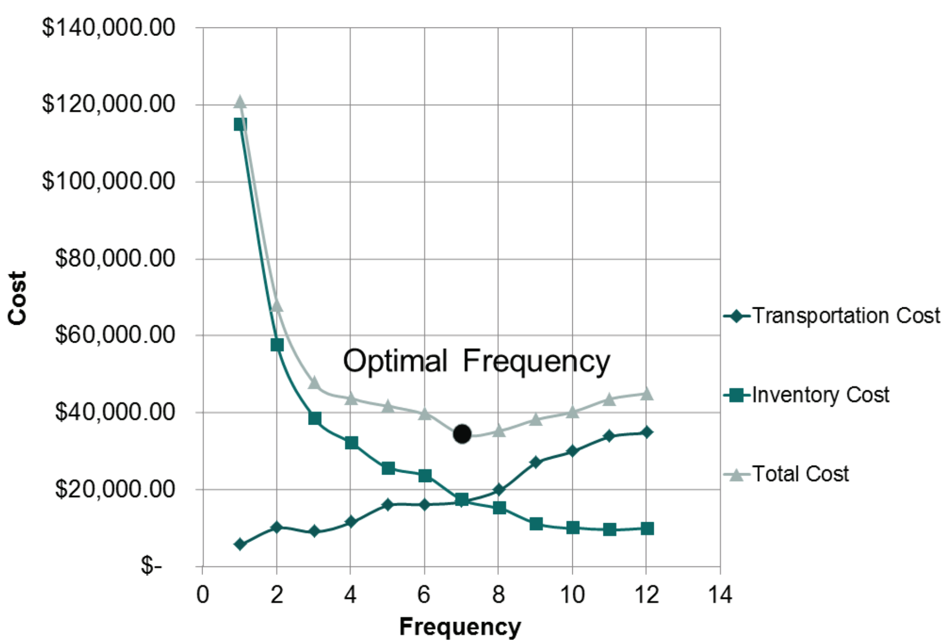

FIGURE 1.1. Relationship Between Inventory, Transportation And Total Costs

1.2. The Need for a New Model

Volatile customer expectations and rapidly changing markets cause short lifecycle items. Manufacturers need a strategy to decrease the risk of short lifecycle items and to increase efficiency. Receiving the items from suppliers at the same time of the demand targets is one of the keys of decreasing the risk for the manufacturers.

A Just-In-Time (JIT) model is one of the ways for achieving this goal, but it may not be the optimal solution. The first reason is, in the JIT model the manufacturers order the items whenever they need to meet the demand targets thus, it covers just pull systems and short planning horizons.

A new model in this thesis covers pull systems, push systems, short planning horizons, and long planning horizons. In pull systems the demand targets for items at each time point are known but may be non-constant during a planning horizon. The manufacturers respond the demand targets and determine order quantities during a planning horizon. In push systems the demand targets and the planning horizon are known and constants. As figure 1.2 from [**5**] shows, push systems have large inventories and waste because of poor communication between buyers (manufacturers) and sellers (suppliers) unlike the pull systems [**5**].

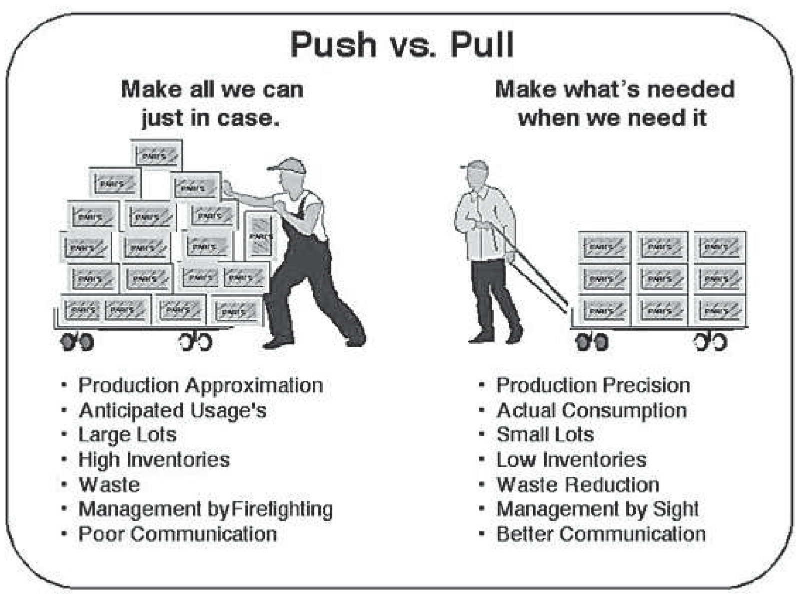

FIGURE 1.2. Push Systems And Pull Systems

The second reason is, by increasing order quantities, the price and shipping cost per item will be decreased, although in a JIT model, the price breaks for purchasing and transportation costs may not happen at all time points.

The last reason is, due to the fact that today most shipping companies and traffic logistics companies use freight classes as standardized freight pricing for their clients such as the FedEx company, the Vimich company, etc. In this thesis the National Motor Freight Classification (NMFC) standard is used as the freight classes, which has not been done before.

1.3. Scope of Research

The scope of this research is to develop and test a mathematical model that will determine the order quantity of each item at each time point to minimize the sum of inventory and transportation costs. Price breaks, NMFC transportation classes and a flexible planning horizon are included in this model. Price breaks are reductions in price for purchasing items. They make the model more accurate and flexible with a non-constant price of items. Transportation costs are calculated by NMFC that makes the model complicated but also reliable. Flexible planning horizon is the planning horizon with non-constant the period between time points. They could be measured in hours, days, weeks, months or years, etc that makes the model complete and also decreases the risk of short lifecycle items.

The Genetic Algorithm (GA) function is used as the solution procedure and the results (costs) of the model are compared to the Equal Order Quantity and JIT costs to define which models determine the optimal order quantity in the same conditions. It also helps to understand the differences between the models and identify the roles of inventory and transportation costs in the total operational cost. Sensitivity is also considered in the research for all parameters to notice the effect of them in the model and again comparing the results of models.

1.4. How this Research is Organized

In chapter 2, a literature review is given to point out how the total cost is minimized, how the order quantities are determined and how inventory and transportation costs are connected.

In chapter 3, the problem, the parameters and the variables are defined. The steps of creating the model and the methodology to solve it with a genetic algorithm in Matlab are also explained.

In Chapter 4, a numerical example is illustrated which includes one supplier and one manufacturer with ten different items and twelve time points. The chapter clearly shows how to solve the model with genetic algorithm in Matlab and gives the results and to compare with JIT and Equal Order Quantity models.

Sensitivity analysis is also performed to recognize the weaknesses and strengths of the model.

In chapter 5 the effectiveness and benefits of the model are discussed and a conclusion is deduced.

CHAPTER 2

Literature Review

2.1. Logistics

Logistics is the planning of the progress of items through a manufacturing process [**6**]. Logistics has two viewpoints: inbound and outbound [**7**]. Inbound logistics focuses on purchasing and shipping the inbound movement of items from suppliers to manufacturers [**8**]. Outbound logistics is the process concerned about the final product and the flows of finished items from the manufacturers to the end clients [**8**]. This thesis covers the inbound logistics.

2.2. Inventory Cost and Transportation Cost

Inventory costs include ordering costs, holding costs and purchasing costs. The ordering cost is a fixed cost [**9**] of tracking trucks from a supplier to a warehouse, labor costs of processing orders, inspection and returning of poor quality products [**10**].

The holding cost is the same as the opportunity cost, which means the potential cost of items that are not being sold while the money could be used elsewhere [**11**]. Holding costs include the cost of handling an item in a warehouse, storage costs of safety stock, rent, labor, utilities, stock deterioration and insurance of items. Manufacturers have special conditions for storing many kinds of items. These conditions might be due to warehouse capacity, special heating or lighting needs and types of insurance, etc.

The purchasing cost relates to the expenses of obtaining the items. The purchasing cost is made up of two components: the expense of buying the items and financing costs. The financing cost is a fixed daily, weekly, monthly or yearly payments for a loan of money for purchasing items with an annual interest rate.

The transportation cost is the cost of shipping the items from a supplier to a manufacturer. This cost can vary from 10% to 40% of the value of the total cost [**12**]. Most manufacturers consider transportation cost as part of the fixed ordering cost [**13**–**21**], although transportation cost depends on the order quantities.

2.2.1. Distribution Strategy

The distribution strategies for truck delivery are:

- Direct: Trucks travel directly from a supplier to a plant [**2**].
- Milk-run: Trucks pick up products at one or several suppliers and deliver them to one or several plants [**2**].
- Cross-dock: Items received at the warehouse are not received into stock, but are prepared for shipment to another location or to retail stores [**10**].

Figure 2.1 shows three different distribution strategies for truck delivery. The strategy between supplier A and plant A and supplier D and plant B is direct distribution. The distribution strategy between suppliers A, B and C to plants C and B is Milk-run. The truck picks up products at supplier A, goes to suppliers B and C respectively, then delivers them to plant C and then plant B. The other distribution strategy is cross-docking to unload materials from suppliers A, B, C and D and loading these materials directly into outbound trucks to deliver them to plants A, B and C.

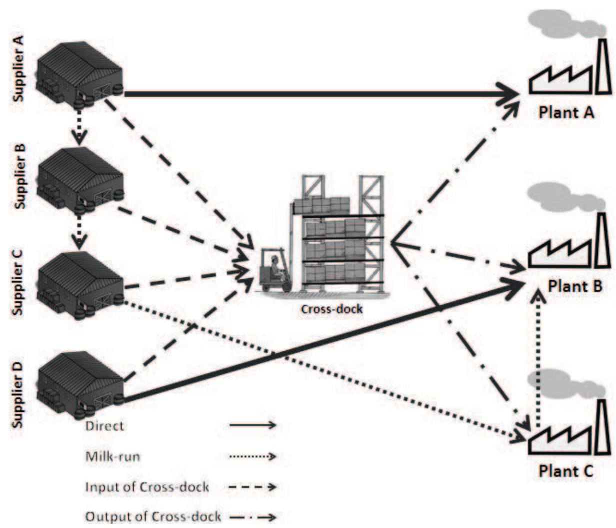

FIGURE 2.1. Comparing Distribution Strategies

Direct delivery has the shortest distance and therefore the shortest delivery time. A delivery through a cross-dock has the longest distance and the longest delivery time. It delivers products from different suppliers to different plants. It leads to high delivery frequency and low plant inventory [**2**]. The differences between the distribution strategies are listed in table 2.1 from Berman's paper [**2**]. This thesis focuses on a direct distribution strategy.

2.2.2. National Motor Freight Classification (NMFC)

The National Motor Freight Traffic Association (NMFTA) defines freight classes and includes the NMFC or National Motor Freight Classification [**22**]. Freight

Distribution Strategy	Delivery Time	Delivery Frequency	Inventory
Direct	Short	Low	High
Milk-run	Medium	Medium	Medium
Cross-dock	Long	High	Low

TABLE 2.1. Comparing Distribution Strategies

classes are based on weight per cubic foot, ease of handling, value and liability from things like theft, damage, breakability and spoilage [**22**]. Table 2.2 from "www.nmfta.org" describes the NMFC classes and is meant for general guidance in picking the correct freight class [**22**].

The table is based on an evaluation of the four transportation characteristics: density, stowability, handling and liability [**22**]. The first class in the table is Class 50. It has a high density per cubic foot, over 50 lbs/ft^3. Items included in Class 50 are heavy manufactured items and must be durable and not be subject to breakage. Although, the items from Class 500, the last class in the table, are the lightest manufactured items like ping pong balls and also for the most expensive items like bags of gold dust [**22**].

Class Name	Cost	Notes, Examples	Weight Range Per Cubic Foot
Class 50 – Clean Freight	Lowest Cost	Fits on standard shrink-wrapped 4X4 pallet, very durable	over 50 lbs
Class 55		Bricks, cement, mortar, hardwood flooring	35-50 pounds
Class 60		Car accessories & car parts	30-35 pounds
Class 65		Car accessories & car parts, bottled beverages, books in boxes	22.5-30 pounds
Class 70		Car accessories & car parts, food items, automobile engines	15 to 22.5 pounds
Class 77.5		Tires, bathroom fixtures	13.5 to 15 pounds
Class 85		Crated machinery, cast iron stoves	12-13.5 pounds
Class 92.5		Computers, monitors, refridgerators	10.5-12 pounds
Class 100		boat covers, car covers, canvase, wine cases, caskets	9-10.5 pounds
Class 110		cabinets, framed artwork, tablesaw	8-9 pounds
Class 125		Small Household appliances	7-8 pounds
Class 150		Auto sheetmetal parts, bookcases,	6-7 pounds
Class 175		Clothing, couches stuffed furniture	5-6 pounds
Class 200		Auto sheetmetal parts, aircraft parts, aluminum table, packaged matresses,	4-5 pounds
Class 250		Bamboo furniture, mattress and boxspring, plasma tv	3-4 pounds
Class 300		wood cabinets, tables, chairs setup, model boats	2-3 pounds
Class 400		Deer antlers	1-2 pounds
Class 500 – Low Density or High Value	Highest Cost	Bags of gold dust, ping pong balls	Less than 1 lbs

TABLE 2.2. National Motor Freight Classification (NMFC)

Each of these classes has a specific transportation cost which is the price for shipping 100 pounds. By increasing the weight of the items shipped from the supplier to the manufacturer, the transportation price per 100 pounds decreases. The maximum weight that can be shipped depends on the freight company. For example, for FedEx, the limitation is 200,000 pounds.

2.3. Aggregation of Inventory and Transportation Costs

Harris [**23**] was the first researcher who minimized inventory and transportation costs with the EOQ model. He studied the problem under demand certainty and continuous delivery. Federgruen and Zipkin [**24**] claimed that they were the first researchers who optimized inventory and transportation costs. They assumed demands were random and proved that minimizing inventory-transportation costs can benefit manufacturers.

Burns et al [**25**] focused on one product and uncertain demand targets. The paper developed a method for minimizing the cost of distributing freight by truck from a supplier to many customers. Speranza and Ukovich [**26**] determined the frequencies at which several products have to be shipped on a common link to minimize the sum of transportation and inventory costs. In that paper, vehicles may or may not be carried out as complete items and the items may or may not share the same truck. They showed that allowing products to be shipped in different frequencies makes truck shipping at high frequencies become filled up completely.

In recent years, aggregation of inventory and transportation costs was also widely studied by many researchers. Berman and Wang [**2**] extended a model to include the strategy of milk-run distribution especially for expensive products with small physical sizes. Berman and Wang included all distribution strategies in his model and compared them. A heuristic and a branch-and-bound algorithm were developed based on the Lagrangian relaxation of the nonlinear program. Ertogral et al [**21**] analyzed the vendor-buyer lot-sizing problem under equal-size shipment policy. All-unit-discount transportation cost structures with or without over-declaration have been considered. The model suggested that production and inventory decisions are affected when transportation is considered in the model.

Baboli et al [**27**] used an algorithm for the determination of the economic order quantity in a two-level supply chain with transportation costs and compared decentralized with centralized decision. The paper considers a two-level supply chain consisting of one warehouse and one retailer. The demand rate by the retailer is known and shortages are not allowed.

Wang et al [**28**] studied logistics scheduling to minimize inventory and transportation costs where a manufacturer receives raw materials from a supplier, makes products in a factory and delivers the finished products to a customer. The supplier, factory, and customer were assumed in three parts to minimize the total cost. Li et al [**1**] optimized transportation and inventory costs based on time in the supply chain logistics system. They considered that the resources available and the total time of inventory and transportation are limited. Hong et al [**29**] integrated inventory and transportation decision for ubiquitous supply chain management. They assumed that the demand for a product could be a linear, a convex or a concave function of the price. Chen et al [**3**] designed an integrated inventory and transportation system to minimize the total costs of inventory and transportation. The results were also compared with the traditional approach, which is based on the EOQ model.

Bertazzi and Speranza [**30**] considered the problem of shipping several products from a common origin to a common destination. The problem is to determine a shipping strategy to minimize the summation of inventory and transportation costs. They assumed continuous frequency and with a set of given frequencies as particular cases with discrete shipping times. Bertazzi and Cherubini [**31**] minimized the inventory-transportation cost with stochastic demand. The transportation was performed either directly or through an intermediate depot. The problem was to determine the quantity of each product to deliver at each time period from the supplier to the depot, from the depot to each retailer and from the supplier to each retailer.

From the literature mentioned above, the decisions in optimizing inventory and transportation costs are [**3**]:

(1) Defining the demand as certain or uncertain.
(2) Defining the delivery as continuous or discrete.
(3) Defining the capacity as finite or infinite.
(4) Defining the allowance of shortage.
(5) Defining the distribution strategy.
(6) Defining the transportation variables.
(7) Defining the number of manufacturers, suppliers and retailers.

These steps help explain the problem clearly and formulate the model. Most researchers minimize inventory and transportation cost by optimizing the order quantity [**3**]. In this research, demand is certain at each time point and the model is between one supplier and one manufacturer. Shortages are not allowed and freight classes are used for the transportation cost. Total cost is optimized by finding the value of the order quantity at each time point for each item.

CHAPTER 3

Model Development

3.1. Problem Definition

This thesis considers a simple supply chain with one supplier and one manufacturer as explained in chapter 1. Although, in modern supply chain a manufacturer receives items from several suppliers, the presented model in this thesis includes just one supplier. The reason is each supplier provides independent items from the other suppliers.

The manufacturer needs to order the items to meet their demand targets at a minimal total cost. Each item has different known demand targets at different time points. It also has fixed cost for each order placed and fixed holding cost. For purchasing and shipping costs, the price is dependent on the number of items that would be ordered. The model determines the number of each item to be purchased, at each time point, to meet the demand targets at these time points, in order to minimize the total cost.

3.1.1. Assumptions

The assumptions that are used in the thesis model are:

(1) Items are always available for shipment.
(2) Transportation costs are determined by the NMFC freight classes.
(3) Each item has constant holding and ordering costs.
(4) The demand targets are known and non-constant.
(5) The lead time is fixed shipping time.
(6) The purchase and transportation costs are vary with order quantity.
(7) The period between time points of planning horizon could be measured in hours, days, months, etc.
(8) Items, which are in a same transportation classification, share shipping.

3.2. Parameters and Variables

We have a planning horizon with n time points, where the period between time points could be measured in hours, days, weeks, months or years, depending on the application. The set of all time points is $J = \{0, 1, 2, 3, \ldots, n\}$.

At time point $j \in J$, which is the beginning of time period j, item $i \in I$ has demand target d_i^j and inventory level l_i^j where I is the index set of all items to be delivered by the supplier to the manufacturer and the initial inventory level l_i^0 is known. Each item i has volume s_i and weight w_i, the unit of volume is a cubic foot and the unit of weight is a pound. The plant warehouse has a limited stock capacity for each item $i \in I$. Let $\mathcal{S}_i$ be the maximum volume and $\mathcal{W}_i$ be the maximum weight of plant warehouse for item i. The volume limitation of plant warehouse for item i is

$$l_i^j s_i \leqslant \mathcal{S}_i;\ \forall i, j \tag{3.1}$$

and the weight limitation of plant warehouse for item i is

$$l_i^j w_i \leqslant \mathcal{W}_i;\ \forall i, j. \tag{3.2}$$

Thus, the maximum capacity for storing item i in the plant warehouse is

$$\alpha_i = \min[\lfloor \frac{\mathcal{S}_i}{s_i} \rfloor, \lfloor \frac{\mathcal{W}_i}{w_i} \rfloor];\ \forall i, \tag{3.3}$$

where $\left\lfloor \frac{\mathcal{S}_i}{s_i} \right\rfloor$ is the maximum number of units of item i that can be stored with volume limitation of the warehouse and $\left\lfloor \frac{\mathcal{W}_i}{w_i} \right\rfloor$ is the maximum number of units of item i that can be stored with weight limitation of the warehouse.

The delivery or order quantity of item i at time point j is determined by nonnegative integer variable q_i^j. We set that $q_i^0 = 0,\ \forall i \in I$. The inventory level of item $i \in I$ at the beginning of time period $j \in J$ is

$$l_i^j = l_i^{j-1} + q_i^j - d_i^{j-1},\ \forall i \in I,\ \forall j \in J \setminus \{0\} \tag{3.4}$$

The order quantity and demand target of item i must be set so that

$$l_i^0 + \sum_{j \in J} q_i^j \geqslant \sum_{j \in J} d_i^j,\ \forall i \in I. \tag{3.5}$$

This inequality means that summation of the initial inventory level and the order quantities should be greater than or equal to all demand targets during the planning horizon. At each time point j, the inventory level of item i should be greater than or equal to the demand target at the same time point. Thus,

$$l_i^j \geqslant d_i^j\ \ \forall i \in I,\ \forall j \in J \setminus \{0\}. \tag{3.6}$$

From equations (3.6) and (3.3), the range of inventory level of item i at time point j is

$$d_i^j \leqslant l_i^j \leqslant \alpha_i;\ \forall i \in I,\ \forall j \in J \setminus \{0\} \tag{3.7}$$

and we assume that at time point 0 we have $l_i^0 \leqslant \alpha_i;\ \forall i \in I$. From equation (3.4), we can rewrite equation (3.7) as

$$d_i^j + d_i^{j-1} - l_i^{j-1} \leqslant q_i^j \leqslant \alpha_i + d_i^{j-1} - l_i^{j-1};\ \forall i, j \tag{3.8}$$

and if we rewrite l_i^{j-1} to $l_i^{j-2} + q_i^{j-1} - d_i^{j-2}$, we will have

$$d_i^j + d_i^{j-1} + d_i^{j-2} - l_i^{j-2} \leqslant q_i^j + q_i^{j-1} \leqslant \alpha_i + d_i^{j-1} + d_i^{j-2} - l_i^{j-2};\ \forall i, j. \tag{3.9}$$

Continuing this backtracking yields

$$-l_i^0 + \sum_{\mathcal{J}=0}^{j} d_i^{\mathcal{J}} \leqslant \sum_{\mathcal{J}=0}^{j} q_i^{\mathcal{J}} \leqslant \alpha_i - l_i^0 + \sum_{\mathcal{J}=0}^{j-1} d_i^{\mathcal{J}};\ \forall i, j. \tag{3.10}$$

Equation 3.10 shows the manufacturer should order $\sum_{\mathcal{J}=0}^{j} q_i^{\mathcal{J}}$ units of item i from time point 0 to time point j ,which is between $-l_i^0 + \sum_{\mathcal{J}=0}^{j} d_i^{\mathcal{J}}$ and $\alpha_i - l_i^0 + \sum_{\mathcal{J}=0}^{j-1} d_i^{\mathcal{J}}$.

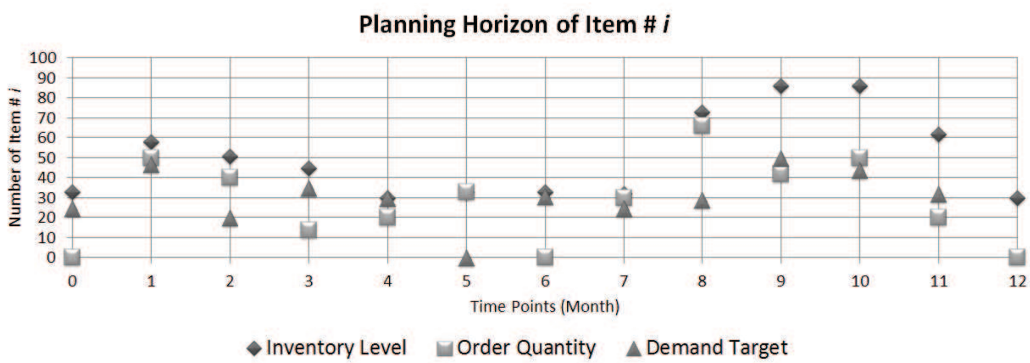

FIGURE 3.1. Planning Horizon of Item i

Time Points(j)	Demand Target(d_i^j)	Order Quantity(q_i^j)	Inventory Level(l_i^j)
0	25	0	33
1	47	50	58
2	20	40	51
3	35	14	45
4	30	20	30
5	0	33	33
6	31	0	33
7	25	30	32
8	29	66	73
9	50	42	86
10	44	50	86
11	32	20	62
12	0	0	30

TABLE 3.1. Demand Targets, Order Quantities And Inventory Levels

Figure 3.1 shows an example of the planing horizon of an item $i \in I$. Table 3.1 gives the values of d_i^j, q_i^j and l_i^j, which are shown as triangles, squares and lozenges in figure 3.1, respectively. The period between time points for this example is one month, the set of time points is $J = \{0, 1, 2, \ldots, 12\}$, the initial inventory level l_i^0 is 33 and all the other inventory levels are calculated from equation 3.4.

As we see in figure 3.1 in each month, the inventory level is greater than the demand target. For example at time point 6 the demand target of item i, d_i^6 is 31 and the inventory level l_i^6 is 33. At the end of the planning horizon, the inventory level is 30 which might be the initial inventory level of the next planning horizon.

In the model, each item i is grouped into one of the 18 classes of the National Motor Freight Classification (NMFC). This classification is a standard freight classification from a low of class 50 to a high of class 500, see table 2.2.

Let the set of NMFC classification be $V = \{1, 2, 3, \ldots, 18\}$. Each NMFC class type $v \in V$ depends on pounds per cubic foot, $W_i = w_i/s_i$, $\forall i \in I$. Table 3.2 shows the relationship between NMFC classification and the pounds per cubic foot, W_i.

v	Class Type	W_i	
		Min	Max
1	Class 50	50.0	∞
2	Class 55	35.0	50.0
3	Class 60	30.0	35.0
4	Class 65	22.5	30.0
5	Class 70	15.0	22.5
6	Class 77.5	13.5	15.0
7	Class 85	12.0	13.5
8	Class 92.5	10.5	12.0
9	Class 100	9.0	10.5
10	Class 110	8.0	9.0
11	Class 125	7.0	8.0
12	Class 150	6.0	7.0
13	Class 170	5.0	6.0
14	Class 200	4.0	5.0
15	Class 250	3.0	4.0
16	Class 300	2.0	3.0
17	Class 400	1.0	2.0
18	Class 500	0	1.0

TABLE 3.2. Relationship Between v, W_i And NMFC Class Types

Each NMFC class type has a set of ranges for different weights, $K = \{1, 2, 3, \ldots, \tau\}$ and c_v is the set of all items with NMFC class type v. The NMFC weight range is k when a total order quantities of items of c_v, is between the lower bound, a_{k-1}

and the upper bound of weight range k, a_k, see equation 3.11.

$$a_{k-1} \leqslant \sum_{i \in c_v} q_i^j w_i < a_k, \ \forall j, v. \tag{3.11}$$

We assume that $a_0 = 0$, the maximum number of units of items $\forall i \in c_v$ that can be shipped is $\sum_{i \in c_v} q_i^j \leqslant \lfloor \frac{a_\tau}{w_i} \rfloor$, $\forall i$ and ϑ_k^v is transportation cost per 100 pounds for NMFC class type $v \in V$ in weight range $k \in K$.

3.3. Objective Function

The price of each item decreases when the number of items increases. Let $E_i = \{1, 2, 3, \ldots, \varrho_i\}$ be the set of price breaks of item i, $Q_i^{e_i - 1}$ and $Q_i^{e_i}$ be the lower bound and upper bound for price break e_i of item i, respectively and ϱ_i be the last price break of item i. If q_i^j is between $Q_i^{e_i-1}$ and $Q_i^{e_i}$, the price of one item i will be $p_i^{e_i}$. Let L_i^j be the purchasing cost of order quantity of item i at time point j. Thus, we have

$$\begin{aligned} &\text{if } Q_i^{e_i-1} \leqslant q_i^j < Q_i^{e_i}, \text{ then} \\ &L_i^j = p_i^{e_i} q_i^j; \ \forall i, j. \end{aligned} \tag{3.12}$$

The manufacturer for purchasing the items needs a loan and the most typical loan payment type is the payment in which each time point has the same value over the planning horizon [**32**]. Let r be the interest rate per time period. For example, an annual interest rate is 0.06 and the period between time points is one month, a monthly interest rate will be $r = \dfrac{0.06}{12} = 0.005$.

The fixed payment, F_i^j, at each time point from time point j to time point n for a loan of L_i^j with the interest rate r is

$$F_i^j = L_i^j \frac{r(1+r)^{n-j+1}}{(1+r)^{n-j+1} - 1}; \ \forall i, j. \tag{3.13}$$

The manufacturer gets the loan, L_i^j at the beginning of time point j and starts paying the fixed payments at the end of time point j. Let $P(q_i^j)$ be the total purchasing cost of item i at time point j, which is the total amount repaid including interest of item i from time point j to the end of the planning horizon [**33**]. Thus, we have.

$$P(q_i^j) = (n - j + 1) * F_i^j; \ \forall i, j. \tag{3.14}$$

To see how equation 3.14 is used consider following example. Consider $j = n$, thus, $P(q_i^n)$ is calculated from

$$
\begin{aligned}
\text{if} \quad Q_i^{e_i-1} \leqslant \; q_i^n \; & < Q_i^{e_i}, \text{ then} \\
P(q_i^n) \; & = \; (n-n+1)F_i^n. \\
& = \; L_i^n \frac{r(1+r)^{n-n+1}}{(1+r)^{n-n+1}-1}. \\
& = \; p_i^{e_i} q_i^n \frac{r(1+r)}{(1+r)-1}.
\end{aligned} \tag{3.15}
$$

The equation 3.15 means that if the manufacturer orders q_i^n the price per item i is $p_i^{e_i}$ and the loan that the manufacturer needs is $L_i^n = p_i^{e_i} * q_i^n$. Thus, the total amount repaid including interest from beginning time point n to the end of time point n is $p_i^{e_i} q_i^n \dfrac{r(1+r)}{(1+r)-1}$ which is equal to the fixed payment, F_i^n.

Let $R_v^j(q_i^j : i \in c_v)$ be the transportation cost for shipping the items, which are included in c_v. As mentioned in section 3.2, each NMFC classification v has a specific cost for each different weight range $k \in K$. From equation 3.11, we have

$$
\begin{aligned}
& \text{if } a_{k-1} \leqslant \sum_{i \in c_v} q_i^j w_i < a_k, \text{ then} \\
& R_v^j(q_i^j : i \in c_v) = 0.01 \min[\vartheta_k^v \sum_{i \in c_v} q_i^j w_i, \vartheta_{k+1}^v a_k]; \; \forall k, v, j.
\end{aligned} \tag{3.16}
$$

From equation 3.16, if the weight range is k, the manufacturer for calculating the transportation cost, $R_v^j(q_i^j : i \in c_v)$ will compare the transportation cost of the current weight range, $\vartheta_k^v \sum_{i \in c_v} q_i^j w_i$ and the next weight range, $\vartheta_{k+1}^v a_k$ to choose the minimal transportation cost.

A fixed ordering cost o_i incurred each time item i is ordered. The ordering cost for item i at time point j is

$$O(q_i^j) = o_i(\min[q_i^j, 1]), \; \forall i, \; j. \tag{3.17}$$

Item i has a unit holding cost h_i per time period. The total holding cost for storing order quantities of item i between time points j and $j+1$ is

$$H(q_i^j) = h_i l_i^j; \; \forall i, j. \tag{3.18}$$

The total holding cost for $\forall i \in I$ during whole planning horizon is

$$\begin{aligned}
\sum_{i\in I}\sum_{j\in J} H(q_i^j) &= \sum_{i\in I}(h_i l_i^0 + \sum_{j\in J} h_i l_i^j). \\
&= \sum_{i\in I}(h_i l_i^0 + h_i l_i^1 + h_i l_i^2 + \ldots + h_i l_i^n). \\
&= \sum_{i\in I}(h_i l_i^0 + h_i(l_i^0 + q_1^j - d_i^0) + h_i(l_i^1 + q_2^j - d_i^1) + \ldots + h_i(l_i^{n-1} + q_n^j - d_i^{n-1})). \\
&\vdots \\
&= \sum_{i\in I} h_i[(n+1)l_i^0 + \sum_{j\in J}((n-j+1)q_i^j - (n-j)d_i^j) - nd_i^0]. \\
&= \sum_{i\in I}\sum_{j\in J} h_i[(n+1)l_i^0 - (n-j)d_i^j - nd_i^0 + (n-j+1)q_i^j].
\end{aligned} \tag{3.19}$$

3.4. Model

The total cost is the summation of purchasing, ordering, transportation and holding costs. Let $C(q_i^j)$ be the total cost. From equations [(3.15),(3.16),(3.17) and (3.19)], we have

$$C(q_i^j) = \sum_{j\in J}\sum_{i\in I}\left(P(q_i^j) + O(q_i^j) + H(q_i^j)\right) + \sum_{j\in J}\sum_{v\in V} R_v^j(q_i^j : i \in c_v),$$

where

- $P(q_i^j) = (n-j+1)\dfrac{r(1+r)^{n-j+1}}{(1+r)^{n-j+1}-1} p_i^{e_i} q_i^j$ if $Q_i^{e_i-1} \leqslant q_i^j < Q_i^{e_i}$; $\forall i, j.$
- $O(q_i^j) = o_i(\min[q_i^j, 1]);\ \forall i, j.$
- $H(q_i^j) = h_i[(n+1)l_i^0 - (n-j)d_i^j - nd_i^0 + (n-j+1)q_i^j];\ \forall i, j.$
- $R_v^j(q_i^j : i \in c_v) = 0.01\min[\vartheta_k^v \sum_{i\in c_v} q_i^j w_i, \vartheta_{k+1}^v a_k]$ if $a_{k-1} \leqslant \sum_{i\in c_v} q_i^j w_i < a_k; \forall k, v, j.$

(3.20)

For finding the optimal q_i^j, we need to minimize the total cost, $C(q_i^j)$. Thus, the model is to

$$
\begin{aligned}
&\text{minimize} \quad C(q_i^j) \\
&\text{subject to} \\
&\qquad \textstyle\sum_{\mathcal{J}=0}^{j} q_i^{\mathcal{J}} \geqslant -l_i^0 + \sum_{\mathcal{J}=0}^{j} d_i^{\mathcal{J}};\ \forall i, j, \\
&\qquad \textstyle\sum_{\mathcal{J}=0}^{j} q_i^{\mathcal{J}} \leqslant \min[\lfloor \frac{a_{\mathcal{T}}}{w_i} \rfloor, \alpha_i - l_i^0 + \sum_{\mathcal{J}=0}^{j-1} d_i^{\mathcal{J}}];\ \forall i, j \text{ and} \\
&\qquad q_i^j \in \mathbb{N}^0;\ \forall i, j.
\end{aligned}
\tag{3.21}
$$

3.5. Solution of the Model

The Genetic Algorithm (GA) module in Matlab's global optimization toolbox is used to solve equation 3.21. The Genetic Algorithm (GA) is a stochastic search method for solving both constrained and unconstrained optimization problems that is based on a natural selection process that mimics biological evolution [**34**]. It explores the solution space by using concepts taken from natural genetics [**35**] and evolution theory [**36**].

GA starts with an initial set of solutions which is known as a population. The individuals of the population are called chromosomes which are evaluated according to a predefined fitness function, in our case the total cost. Each chromosome includes several genes. The gene represents an order quantity of item i at time point j. For example, if we have 10 items and 12 time points, we will have 120 genes (order quantities) in one chromosome, figure 3.2.
The chromosomes evolve through successive iterations called generations [**35**]. A new generation is created by changing chromosomes in the existing population through crossover and mutation [**37**], figure 3.3.

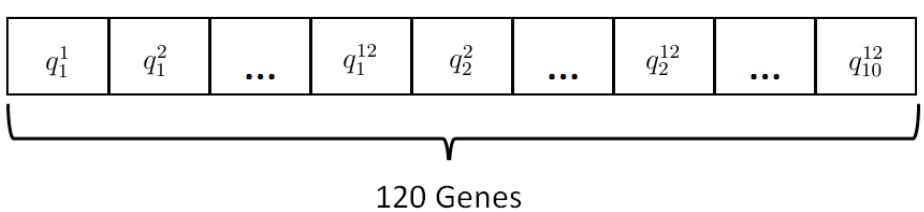

FIGURE 3.2. A Chromosome With 120 Genes

To solve instances of our model we use the syntax given in [**34**] which the initial population, generation, crossover and mutation are random.

$[x, fval, exitflag, output] = ga(fitnessfcn, nvars, A, b, [\,], [\,], LB, UB, [\,], IntCon, options).$

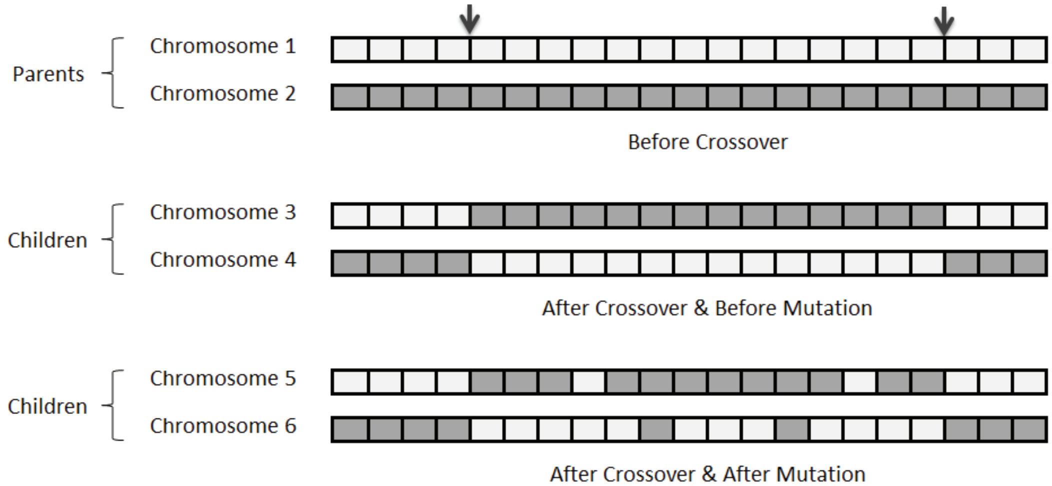

FIGURE 3.3. Crossover And Mutation

We first deal with the input parameters. The fitness function, our total cost, is $fitnessfcn$. The number of integer variables is $nvars$.
A is a matrix for linear inequality constraints and b is a vector for linear inequality constraints of the form $Ax \leqslant b$. In our case

$$-\sum_{\mathcal{J}=1}^{j} q_i^{\mathcal{J}} \leqslant -\sum_{\mathcal{J}=0}^{j} d_i^{\mathcal{J}} + l_i^0;\ \forall i, j.$$

$$\sum_{\mathcal{J}=1}^{j} q_i^{j} \leqslant \min[\lfloor \frac{a_\tau}{w_i} \rfloor, \alpha_i - l_i^0 + \sum_{\mathcal{J}=0}^{j-1} d_i^{\mathcal{J}}];\ \forall i, j.$$

The symbol "[ ]" represents a placeholder for a matrix of linear or nonlinear equality constraints, a vector of linear or nonlinear equality constraints and function of nonlinear constraints, respectively of which there are non in my model.
LB and UB are the vectors of lower and upper bounds, respectively. The last input is *Options* which its structure is

$options = gaoptimset('Generations', value1, 'PopulationSize', value2, 'EliteCount', value3)$ [**34**].

$Populationsize$, $value1$ specifies how many chromosomes are in each $Generation$, $value2$. With a large *Populationsize*, the genetic algorithm searches the solution space more thoroughly to increase the chance of having a global minimum rather than a local minimum [**34**]. $EliteCount$ is the number of chromosomes that are survive and go on to the next generation [**34**]. For an integer problem the minimum $EliteCount$ is
$value3 = 0.05 * min(max(10 * nvars, 40), 100)$.

The output parameters of GA are, x which is the best point that GA located during its generations and $fval$ is a fitness function evaluated at x. The next parameter is $exitflag$, when there are integer constraints, GA uses the penalty fitness value instead of the fitness value for stopping criteria. The last output parameter is *output* that gives information about algorithm performance.

CHAPTER 4

Numerical Example

4.1. Parameters of the Numerical Example

This chapter illustrates the model with a numerical example with 10 items and 12 time points, where the period between time points is one month.
The demand targets of the example d_i^j, are given in table 4.1. The table shows items 1 and 9 have constant demand targets every month in a year, items 2, 3 and 7 have fixed demand targets in constant time periods, items 4, 5 and 8 have specific patterns for their demand targets and items 6 and 10 do not have any specific patterns for their demand targets.

		$j \in J$											
		0	1	2	3	4	5	6	7	8	9	10	11
$i \in I$	1	43	43	43	43	43	43	43	43	43	43	43	43
	2	145	0	145	0	145	0	145	0	145	0	145	0
	3	117	0	0	0	117	0	0	117	0	0	117	0
	4	17	20	17	20	17	20	17	20	17	20	17	20
	5	322	0	334	0	284	0	290	0	275	0	287	0
	6	38	19	57	42	52	51	37	34	34	41	52	39
	7	0	0	0	0	0	364	0	0	0	0	0	364
	8	500	300	83	500	300	83	500	300	83	500	300	83
	9	101	101	101	101	101	101	101	101	101	101	101	101
	10	126	124	128	0	105	119	122	128	0	121	122	120

Table 4.1. Demand Targets Of The Example

i	l_i^0	s_i	w_i	$\mathcal{S}_i$	$\mathcal{W}_i$	α_i	W_i	v
1	45	5.120	163.840	409.60000	13107.2000	80	32.00	3
2	140	10.000	228.800	10000.00000	254196.8000	1000	22.88	4
3	100	18.290	457.330	6402.67000	182933.3300	350	25.00	4
4	17	5.530	66.910	552.96000	6021.7300	90	12.10	7
5	300	8.230	275.660	9874.28571	358354.2857	1200	33.50	3
6	40	9.600	119.040	2400.00000	28569.6000	240	12.40	7
7	0	4.096	116.163	4014.08000	104546.3040	900	28.36	4
8	100	4.320	150.552	17280.00000	752760.0000	4000	34.85	3
9	0	5.830	76.399	1283.04000	19099.8000	220	13.10	7
10	128	9.410	316.240	17882.35290	632470.5882	1900	33.60	3

Table 4.2. Parameters Of The Example

Table 4.2 shows all other parameters for the example. The first column of the table is the item index and the next column includes the initial inventory levels

l_i^0. The third and fourth columns are item volumes, s_i and item weights, w_i, respectively. The maximum volumes of the plant warehouse for item i, $\mathcal{S}_i$ are in column five and the maximum weights, $\mathcal{W}_i$ are in column six.

The maximum numbers of item i that can be stored in the plant warehouse, α_i, are in column seven. Columns eight and nine include weight per cubic foot, W_i and the set of NMFC class types, $v = \{3, 4, 5\}$.

Table 4.3 shows the price costs, $p_i^{e_i}$; $\forall i \in I$, $\forall e_i \in E_i$. For example, the table shows that, for item 5, the price cost is $p_5^{2_5} = \$9.00$ per unit if the number of units purchased is between $Q_5^{1_5} = 51$ and $Q_5^{2_5} = 151$, inclusive. Thus, purchasing cost with $r = 0.005$ would be

$$\text{If } Q_5^{1_5} \leqslant q_5^j < Q_5^{2_5}, \text{ then}$$
$$P(q_5^j) = (13 - j)\frac{0.005(1.005)^{13-j}}{(1.005)^{13-j} - 1} p_5^{2_5} q_5^j; \ \forall j. \tag{4.22}$$

Item $i \in I$	Price Cost $e_i \in E_i$: 1	2	3	4	5	6	7	8	9
1	1-21	21-51	51-121	121-351	351-∞				
	\$ 12.50	\$ 11.25	\$ 10.50	\$ 10.25	\$ 10.06				
2	1-51	51-101	101-136	136-501	501-∞				
	\$ 18.75	\$ 17.75	\$ 16.25	\$ 15.75	\$ 15.00				
3	1-∞								
	\$ 19.25								
4	1-101	101-∞							
	\$ 8.75	\$ 8.00							
5	1-51	51-151	151-251	251-401	401-801	801-∞			
	\$ 9.50	\$ 9.00	\$ 8.75	\$ 8.13	\$ 7.50	\$ 7.16			
6	1-16	16-22	22-61	61-251	251-∞				
	\$ 13.75	\$ 13.50	\$ 13.25	\$ 12.50	\$ 12.25				
7	1-101	101-201	201-301	301-401	401-501	501-601	601-701	701-801	801-∞
	\$ 10.00	\$ 9.50	\$ 9.25	\$ 9.00	\$ 8.75	\$ 8.50	\$ 8.00	\$ 7.75	\$ 7.50
8	1-501	501-2001	2001-∞						
	\$ 6.00	\$ 5.25	\$ 5.00						
9	1-101	101-201	201-∞						
	\$ 7.75	\$ 7.50	\$ 7.25						
10	1-121	121-501	501-1201	1201-∞					
	\$ 11.00	\$ 10.75	\$ 10.50	\$ 10.00					

TABLE 4.3. Relationship Between Price Costs Per Unit And $[Q_i^{e_i-1} - Q_i^{e_i})$ Range

Table 4.4 shows the transportation costs, ϑ_k^v. To see how table 4.4 is used, consider $j = 4$ and $v = 3$. From the last column of table 4.2 we determine that

$c_3 = \{1, 5, 8, 10\}$. Table 4.2 also provides the value $w_i,\ i \in c_3$. If $\sum_{i \in c_3} q_i^4 w_i$ is 1800 lb, we have

$$\sum_{i \in c_3} q_i^4 w_i = 163.84\ q_1^4 + 275.66\ q_5^4 + 150.552\ q_8^4 + 316.24\ q_{10}^4 = 1800$$

and from equation 3.16, the total transportation cost is

If $1000 \leqslant \sum_{i \in c_3} q_i^4 w_i < 2000$, then

$$R_3^4(q_i^4 : i \in c_3) = 0.01 \min[2.35 * 1800, 2.07 * 2000] = 41.40.$$

A lower rate of $2.07 * 2000 = 41.40$ is applied and the manufacturer is billed for $k = 4$. If $\sum_{i \in c_3} q_i^4 w_i$ had been 1500 lb then the total transportation cost would have been as follows

If $1000 \leqslant \sum_{i \in c_3} q_i^4 w_i < 2000$, then

$$R_3^4(q_i^4 : i \in c_3) = 0.01 \min[2.35 * 1500, 2.07 * 2000] = 35.25.$$

A lower rate of $2.35 * 1500 = 35.25$ is applied and the manufacturer is billed for $k = 3$.

		$k \in K$						
		1	2	3	4	5	6	7
		$a_{k-1} - a_k$						
		0-500	500-1000	1000-2000	2000-5000	5000-10000	10000-20000	20000-200000
$v \in V$	3	\$2.90	\$2.57	\$2.35	\$2.07	\$1.57	\$1.38	\$0.78
	4	\$3.25	\$2.67	\$2.55	\$2.46	\$1.88	\$1.64	\$0.99
	7	\$4.00	\$3.70	\$3.20	\$2.50	\$2.16	\$2.03	\$1.09

TABLE 4.4. Transportation Costs Per 100 Pounds Of The Example

Table 4.5 and 4.6 show fixed ordering costs, o_i and holding cost per month, h_i, respectively.

i	1	2	3	4	5	6	7	8	9	10
o_i	\$69.13	\$170.37	\$110.36	\$19.35	\$201.05	\$76.63	\$82.83	\$216.39	\$104.17	\$156.41

TABLE 4.5. Ordering Costs Of The Example

i	1	2	3	4	5	6	7	8	9	10
h_i	\$5.17	\$7.53	\$4.59	\$1.65	\$4.31	\$5.96	\$4.37	\$2.48	\$3.98	\$4.94

TABLE 4.6. Holding Costs Of The Example

4.2. Results of the Numerical Example

Table 4.7 shows an optimal set of order quantities, q_i^j, $\forall i, j$ for the example. The results are given after 200 runs and 13 hours with generation 3000 and population size 5000 with GA in Matlab. At each run Matlab gives a different total cost with a different set of order quantities, then compares them to give an optimal answer. Appendix B shows the Matlab code. Figure 4.1 shows all results of 200 runs of Matlab, the optimal answer is given at 83rd run which has the minimal total cost, \$140,185.12.

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$	Total Cost
1	41	43	43	80	6	43	43	43	80	43	6	0	471	471	
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730	
3	17	350	0	1	0	0	0	0	0	0	0	0	368	368	
4	20	90	0	37	0	37	0	21	0	0	0	0	205	205	
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491	
6	17	240	0	97	0	51	48	0	0	0	0	0	453	453	
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728	
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432	
9	202	194	26	202	0	101	101	202	0	184	0	0	1212	1212	
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087	\$ 140,185.12

Table 4.7. Optimal Order Quantity q_i^j Of The Example With My Model

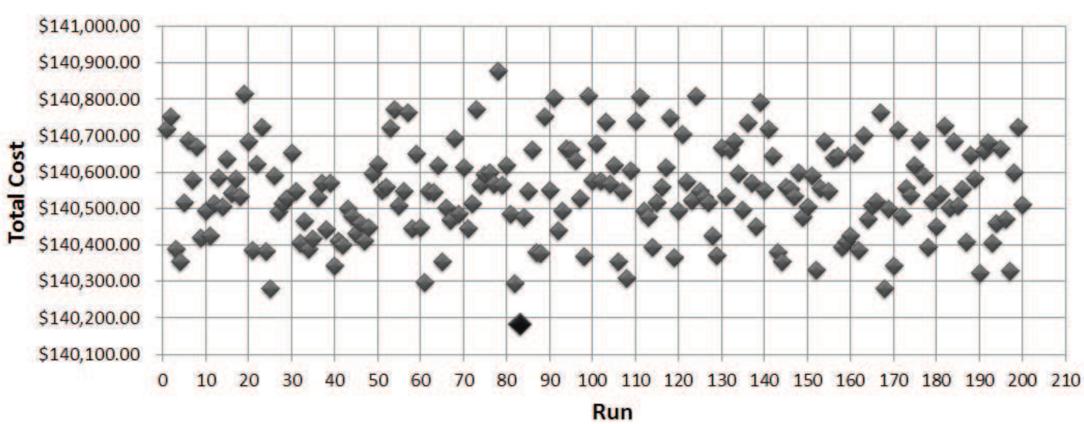

Figure 4.1. Comparing Matlab Runs

An equality of $\sum_{j=1}^{12} q_i^j$ and $\sum_{j=0}^{12} d_i^j - l_i^0$ columns of table 4.7 shows that all demand targets are met during the planning horizon. The table shows, each item has a specific ordering pattern, for example a pattern for ordering item 7 is ordering its annual demand targets in second month and for item 1 is ordering non-constant units every month.

Tables 4.8 and 4.9 shows inventory levels and holding costs which are calculated from equations 3.4 and 3.18, respectively. The largest holding cost is for item 8, $5,353.83 because its annual demand target is ordered in first two months. Although it has the largest holding cost, the price per unit of item 8 is its lowest price, $20 and the frequency is 2 instead of 12 times in a year.

$i \in I$	$j \in J$												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	45	43	43	43	80	43	43	43	43	80	80	43	0
2	140	0	725	580	580	435	435	290	290	145	145	0	0
3	100	0	350	350	234	234	234	117	117	117	0	0	0
4	17	20	90	73	90	73	90	73	74	57	37	20	0
5	300	0	1200	866	1135	851	851	561	561	287	287	0	0
6	40	19	240	184	240	189	189	200	166	132	91	39	0
7	0	0	728	728	728	728	364	364	364	364	364	364	0
8	100	300	2732	2649	2149	1849	1766	1266	966	883	383	83	0
9	0	101	194	119	220	119	119	119	220	119	202	101	0
10	128	124	965	837	837	732	613	491	363	363	242	120	0

TABLE 4.8. Optimal Inventory Level, l_i^j Of The Example With My Model

$i \in I$	$j \in J$													Holding Cost
	0	1	2	3	4	5	6	7	8	9	10	11	12	
1	33.24	31.76	31.76	31.76	59.09	31.76	31.76	31.76	31.76	59.09	59.09	31.76	0.00	$ 464.62
2	150.67	0.00	780.25	624.20	624.20	468.15	468.15	312.10	312.10	156.05	156.05	0.00	0.00	$ 4,051.89
3	65.52	0.00	229.31	229.31	153.31	153.31	153.31	76.65	76.65	76.65	0.00	0.00	0.00	$ 1,214.02
4	4.02	4.72	21.26	17.25	21.26	17.25	21.26	17.25	17.48	13.47	8.74	4.72	0.00	$ 168.68
5	184.48	0.00	737.93	532.54	697.96	523.32	523.32	344.98	344.98	176.49	176.49	0.00	0.00	$ 4,242.49
6	33.95	16.13	203.70	156.17	203.70	160.42	160.42	169.75	140.89	112.04	77.24	33.10	0.00	$ 1,467.51
7	0.00	0.00	454.49	454.49	454.49	454.49	227.25	227.25	227.25	227.25	227.25	227.25	0.00	$ 3,181.44
8	35.39	106.18	966.99	937.61	760.64	654.45	625.07	448.10	341.91	312.54	135.56	29.38	0.00	$ 5,353.83
9	0.00	57.37	110.20	67.59	124.96	67.59	67.59	67.59	124.96	67.59	114.74	57.37	0.00	$ 927.57
10	90.34	87.51	681.04	590.71	590.71	516.60	432.62	346.52	256.19	256.19	170.79	84.69	0.00	$ 4,103.90
														$ 25,175.96

TABLE 4.9. Optimal Holding Cost $H(q_i^j)$ Of The Example With My Model

Tables 4.10, 4.11 and 4.12 show the optimal ordering costs, purchasing costs and transportation costs for all items in a year which are calculated from 3.17, 3.15 and 3.16, respectively.

The minimal ordering cost is for item 7, it is ordered just once in second month and item 1 has highest ordering cost with frequency 11. The largest purchasing cost is for item 8 since it has the highest order quantities in year, 3432 units. Transportation costs are calculated for three NMFC class types, $v = \{3, 4 \text{ and } 7\}$ out of 18 NMFC class types. From the last column of table 4.2 items 1, 5, 8 and 10 are included in NMFC class type $v = 3$, items 2, 3 and 7 are in $v = 4$ and items 4, 6 and 9 are in $v = 7$. The total transportation cost is \$17,137.06 and the highest transportation cost is for $v = 3$.

$i \in I$	$j \in J$												Ordering Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
1	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	0.00	$ 1,901.08
2	425.93	425.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 851.86
3	275.90	275.90	0.00	275.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 827.70
4	48.38	48.38	0.00	48.38	0.00	48.38	0.00	48.38	0.00	0.00	0.00	0.00	$ 241.89
5	502.27	502.27	0.00	502.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 1,506.82
6	191.14	191.14	0.00	191.14	0.00	191.14	191.14	0.00	0.00	0.00	0.00	0.00	$ 955.70
7	0.00	207.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 207.08
8	540.98	540.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 1,081.96
9	260.43	260.43	260.43	260.43	0.00	260.43	260.43	260.43	0.00	260.43	0.00	0.00	$ 2,083.41
10	390.90	390.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 781.79
													$ 10,439.30

TABLE 4.10. Optimal Ordering Cost $O(q_i^j)$ Of The Example With My Model

$i \in I$	$j \in J$												Purchasing Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
1	476.38	498.38	497.15	861.14	76.70	493.47	492.25	491.03	850.53	488.60	75.56	0.00	$ 5,301.19
2	96.82	11203.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 11,300.79
3	337.98	6941.31	0.00	19.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 7,299.02
4	180.74	811.32	0.00	331.90	0.00	330.26	0.00	186.52	0.00	0.00	0.00	0.00	$ 1,840.73
5	215.85	8854.99	0.00	2242.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 11,312.86
6	237.03	3090.75	0.00	1243.01	0.00	689.33	647.18	0.00	0.00	0.00	0.00	0.00	$ 5,907.30
7	0.00	5812.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 5,812.67
8	3795.53	14073.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 17,868.74
9	1512.53	1449.05	207.08	1501.36	0.00	746.97	745.12	1486.54	0.00	1347.36	0.00	0.00	$ 8,996.00
10	1354.51	10439.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 11,793.52
													$ 87,432.81

TABLE 4.11. Optimal Purchasing Cost $P(q_i^j)$ Of The Example With My Model

$v \in V$	$j \in J$												Transportation Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
3	1224.73	8237.67	110.61	681.78	23.45	110.61	110.61	110.61	156.27	110.61	23.45	0.00	$ 10,900.40
4	164.33	4064.08	0.00	13.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	$ 4,241.75
7	217.47	537.28	50.03	320.28	0.00	217.47	217.47	217.47	0.00	217.47	0.00	0.00	$ 1,994.92
													$ 17,137.06

TABLE 4.12. Optimal Transportation Cost $R_v^j(q_i^j : i \in c_v)$ Of The Example With My Model

Table 4.13 gives a summary of all costs of the example and figure 4.2 summarized the percentage of these costs. The biggest slice of figure 4.2 is for purchasing cost, 62% and the smallest one is for ordering cost, 8%. It means that purchasing cost plays the most important role in the total cost. The holding cost with 18% and transportation cost with 12% are the other roles. In our example the inventory cost with 88% is greater than the transportation cost with 12%.

Transportation Cost	Inventory Cost			Total Cost
	Purchasing	Ordering	Holding	
$ 17,137.06	$ 87,432.81	$ 10,439.30	$ 25,175.96	$ 140,185.12
	$ 123,048.06			

TABLE 4.13. Comparing Costs Of The Example With My Model

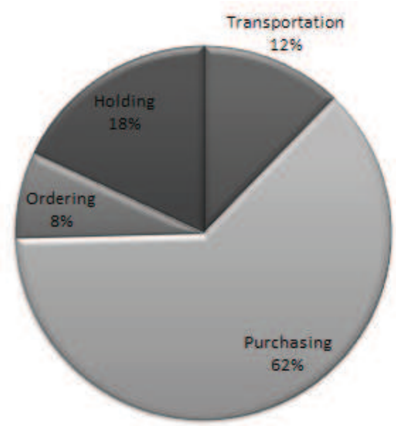

FIGURE 4.2. Dividing Total Cost Of My Model

4.3. Comparative Studies

In this section, the optimal answer of the model of this thesis which is named "My Model" is compared to the results of the Just-In-Time and the Equal Order Quantity models to show which model produces the minimal total cost. Tables 4.14 and 4.15 show sets of order quantities and inventory levels of JIT model, respectively. To see how table 4.15 is calculated consider following example of item 6 at time point 3. From equation 3.4 and table 4.1, we have

$$\begin{aligned} l_6^3 &= l_6^2 - d_6^2 + q_3^6, \\ &= l_6^0 - d_6^0 + q_1^6 - d_6^1 + q_2^6 - d_6^2 + q_3^6 = d_6^3. \end{aligned} \tag{4.23}$$

Tables 4.16, 4.17 and 4.18 show holding costs, ordering costs and purchasing

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$	Total Cost
1	41	43	43	43	43	43	43	43	43	43	43	0	471	471	
2	5	145	0	145	0	145	0	145	0	145	0	0	730	730	
3	17	0	117	0	0	117	0	0	117	0	0	0	368	368	
4	20	17	20	17	20	17	20	17	20	17	20	0	205	205	
5	22	334	0	284	0	290	0	274	0	287	0	0	1491	1491	
6	17	56	41	51	51	37	34	34	41	52	39	0	453	453	
7	0	0	0	0	364	0	0	0	0	0	364	0	728	728	
8	700	83	500	300	83	500	300	83	500	300	83	0	3432	3432	
9	202	101	101	101	101	101	101	101	101	101	101	0	1212	1212	
10	122	128	0	105	119	122	128	0	121	122	120	0	1087	1087	$ 140,899.91

Table 4.14. Order Quantity q_i^j Of The Example With The JIT Model

$i \in I$	$j \in J$: 0	1	2	3	4	5	6	7	8	9	10	11	12
1	45	43	43	43	43	43	43	43	43	43	43	43	0
2	140	0	145	0	145	0	145	0	145	0	145	0	0
3	100	0	0	117	0	0	117	0	0	117	0	0	0
4	17	20	17	20	17	20	17	20	17	20	17	20	0
5	300	0	334	0	284	0	290	0	274	0	287	0	0
6	40	19	56	41	51	51	37	34	34	41	52	39	0
7	0	0	0	0	0	364	0	0	0	0	0	364	0
8	100	300	83	500	300	83	500	300	83	500	300	83	0
9	0	101	101	101	101	101	101	101	101	101	101	101	0
10	128	124	128	0	105	119	122	128	0	121	122	120	0

Table 4.15. Inventory Level l_i^j Of The Example With The JIT Model

costs of the JIT model, respectively. The total holding cost with the JIT model is less than the holding cost with My Model because in the JIT model at each time point, the items are ordered as same as their demand targets. The ordering cost with the JIT model is more than the ordering cost with My Model, the reason is the frequencies in the JIT model is more than My Model.

$i \in I$	$j \in J$													Holding Cost
	0	1	2	3	4	5	6	7	8	9	10	11	12	
1	33.24	31.76	31.76	31.76	31.76	31.76	31.76	31.76	31.76	31.76	31.76	31.76	0.00	$ 382.63
2	150.67	0.00	156.05	0.00	156.05	0.00	156.05	0.00	156.05	0.00	156.05	0.00	0.00	$ 930.91
3	65.52	0.00	0.00	76.65	0.00	0.00	76.65	0.00	0.00	76.65	0.00	0.00	0.00	$ 295.48
4	4.02	4.72	4.02	4.72	4.02	4.72	4.02	4.72	4.02	4.72	4.02	4.72	0.00	$ 52.45
5	184.48	0.00	205.39	0.00	174.64	0.00	178.33	0.00	168.49	0.00	176.49	0.00	0.00	$ 1,087.83
6	33.95	16.13	47.53	34.80	43.29	43.29	31.40	28.86	28.86	34.80	44.14	33.10	0.00	$ 420.14
7	0.00	0.00	0.00	0.00	0.00	227.25	0.00	0.00	0.00	0.00	0.00	227.25	0.00	$ 454.49
8	35.39	106.18	29.38	176.97	106.18	29.38	176.97	106.18	29.38	176.97	106.18	29.38	0.00	$ 1,108.57
9	0.00	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	0.00	$ 631.07
10	90.34	87.51	90.34	0.00	74.10	83.98	86.10	90.34	0.00	85.40	86.10	84.69	0.00	$ 858.89
														$ 6,222.46

TABLE 4.16. Holding Cost $H(q_i^j)$ Of The Example With The JIT Model

$i \in I$	$j \in J$												Ordering Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
1	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	0.00	$ 1,901.08
2	425.93	425.93	0.00	425.93	0.00	425.93	0.00	425.93	0.00	425.93	0.00	0.00	$ 2,555.59
3	275.90	0.00	275.90	0.00	0.00	275.90	0.00	0.00	275.90	0.00	0.00	0.00	$ 1,103.60
4	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	0.00	$ 532.15
5	502.27	502.27	0.00	502.27	0.00	502.27	0.00	502.27	0.00	502.27	0.00	0.00	$ 3,013.64
6	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	0.00	$ 2,102.53
7	0.00	0.00	0.00	0.00	207.08	0.00	0.00	0.00	0.00	0.00	207.08	0.00	$ 414.17
8	540.98	540.98	540.98	540.98	540.98	540.98	540.98	540.98	540.98	540.98	540.98	0.00	$ 5,950.80
9	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	0.00	$ 2,864.68
10	390.90	390.90	0.00	390.90	390.90	390.90	390.90	0.00	390.90	390.90	390.90	0.00	$ 3,518.07
													$ 23,956.31

TABLE 4.17. Ordering Cost $O(q_i^j)$ Of The Example With The JIT Model

$i \in I$	$j \in J$												Purchasing Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
1	476.38	498.38	497.15	495.92	494.70	493.47	492.25	491.03	489.81	488.60	487.38	0.00	$ 5,405.08
2	96.82	2352.83	0.00	2341.22	0.00	2329.65	0.00	2318.12	0.00	2306.63	0.00	0.00	$ 11,745.28
3	337.98	0.00	2314.65	0.00	0.00	2297.52	0.00	0.00	2280.47	0.00	0.00	0.00	$ 7,230.63
4	180.74	153.25	179.85	152.49	178.96	151.74	178.08	150.99	177.19	150.24	176.31	0.00	$ 1,829.84
5	215.85	2797.56	0.00	2367.03	0.00	2405.09	0.00	2261.15	0.00	2356.68	0.00	0.00	$ 12,403.36
6	237.03	764.45	558.30	692.76	691.04	500.10	458.42	457.28	550.06	695.90	520.63	0.00	$ 6,125.96
7	0.00	0.00	0.00	0.00	3350.14	0.00	0.00	0.00	0.00	0.00	3300.59	0.00	$ 6,650.73
8	3795.53	513.06	3083.12	1845.30	509.27	3060.30	1831.63	505.49	3037.59	1818.03	501.74	0.00	$ 20,501.07
9	1512.53	780.41	778.49	776.56	774.64	772.73	770.81	768.90	766.99	765.09	763.19	0.00	$ 9,230.34
10	1354.51	1417.62	0.00	1184.07	1338.62	1337.86	1400.18	0.00	1317.05	1324.64	1329.91	0.00	$ 12,004.46
													$ 93,126.74

TABLE 4.18. Purchasing Cost $P(q_i^j)$ Of The Example With The JIT Model

Table 4.19 shows a summary of transportation, purchasing, ordering and holding costs of the JIT model. In figure 4.3 the main slice is for purchasing cost as same as My Model but the smallest slice is for holding cost with 4%.

Transportation Cost	Inventory Cost			Total Cost
	Purchasing	Ordering	Holding	
$ 17,594.39	$ 93,126.74	$ 23,956.31	$ 6,222.46	$ 140,899.91
	$ 123,305.51			

TABLE 4.19. Comparing Costs Of The Example With The JIT Model

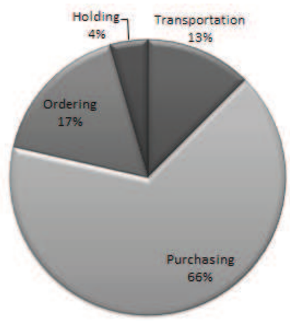

FIGURE 4.3. Dividing The JIT Model Total Cost

In the Equal Order Quantity model, the manufacturer orders the equal amount of items at each time point. To see how the Equal Order Quantity model works, consider the annual order quantity of item 8 is 3432. The manufacturer has 12 ways to order item 8 in a year, table 4.20 shows all these possible ways. Each way has a specific total cost, after comparing the total costs, the optimal solution is ordering 858 units of item 8 every three months.

Possible ways to order item 8	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
1	286	286	286	286	286	286	286	286	286	286	286	286	3432	3432
2	572	0	572	0	572	0	572	0	572	0	572	0	3432	3432
3	858	0	0	858	0	0	858	0	0	858	0	0	3432	3432
4	1144	0	0	0	1144	0	0	0	1144	0	0	0	3432	3432
5	1430	0	0	0	0	1430	0	0	0	0	572	0	3432	3432
6	1716	0	0	0	0	0	1716	0	0	0	0	0	3432	3432
7	2002	0	0	0	0	0	0	1430	0	0	0	0	3432	3432
8	2288	0	0	0	0	0	0	0	1144	0	0	0	3432	3432
9	2574	0	0	0	0	0	0	0	0	858	0	0	3432	3432
10	2860	0	0	0	0	0	0	0	0	0	572	0	3432	3432
11	3146	0	0	0	0	0	0	0	0	0	0	286	3432	3432
12	3432	0	0	0	0	0	0	0	0	0	0	0	3432	3432

TABLE 4.20. All Possible Ways Of Ordering Item 8 With The Equal Order Quantity Model

Table 4.21 shows the optimal order quantities with the Equal Order Quantity model for all items. Table 4.22 shows the inventory level of all items at each time points with the Equal Order Quantity model.

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$	Total Cost
1	39	39	39	39	39	39	39	39	39	40	40	40	471	471	
2	121	0	121	0	122	0	122	0	122	0	122	0	730	730	
3	92	0	0	92	0	0	92	0	0	92	0	0	368	368	
4	17	17	17	17	17	17	17	17	17	17	17	18	205	205	
5	248	0	248	0	248	0	249	0	249	0	249	0	1491	1492	
6	37	37	37	38	38	38	38	38	38	38	38	38	453	454	
7	121	0	121	0	122	0	121	0	121	0	122	0	728	728	
8	858	0	0	858	0	0	858	0	0	858	0	0	3432	3432	
9	202	101	101	101	101	101	101	101	101	101	101	0	1212	1212	
10	90	90	90	90	91	91	91	91	90	91	91	91	1087	1087	
															$ 140,736.67

TABLE 4.21. Order Quantity q_i^j Of The Example With The Equal Order Quantity Model

$i \in I$	$j \in J$: 0	1	2	3	4	5	6	7	8	9	10	11	12
1	45	41	37	33	29	25	21	17	13	9	6	3	0
2	140	116	116	92	92	69	69	46	46	23	23	0	0
3	100	75	75	75	50	50	50	25	25	25	0	0	0
4	17	17	14	14	11	11	8	8	5	5	2	2	0
5	300	226	226	140	140	104	104	63	63	38	38	0	0
6	40	39	57	38	35	22	9	10	14	18	15	1	0
7	0	121	121	242	242	364	0	121	121	242	242	364	0
8	100	458	158	75	433	133	50	408	108	25	383	83	0
9	0	101	101	101	101	101	101	101	101	101	101	101	0
10	128	92	58	20	110	96	68	37	0	90	60	29	0

TABLE 4.22. Inventory Level l_i^j Of The Example With The Equal Order Quantity Model

Tables 4.23, 4.24, 4.25 and 4.26 show holding costs, ordering costs, purchasing costs and summary of all costs of the Equal Order Quantity model. The total holding cost with this model is smallest than two other models and the total ordering cost and purchasing cost are bigger than My Model. Figure 4.4 shows the percentages of the all costs. As the figure shows the highest rate is for purchasing cost as same as two other models and the lowest one is for holding cost as same as the JIT model.

$i \in I$	$j \in J$													Holding Cost
	0	1	2	3	4	5	6	7	8	9	10	11	12	
1	33.24	30.29	27.33	24.38	21.42	18.47	15.51	12.56	9.60	6.65	4.43	2.22	0.00	$ 206.09
2	150.67	124.84	124.84	99.01	99.01	74.26	74.26	49.51	49.51	24.75	24.75	0.00	0.00	$ 895.40
3	65.52	49.14	49.14	49.14	32.76	32.76	32.76	16.38	16.38	16.38	0.00	0.00	0.00	$ 360.34
4	4.02	4.02	3.31	3.31	2.60	2.60	1.89	1.89	1.18	1.18	0.47	0.47	0.00	$ 26.93
5	184.48	138.98	138.98	86.09	86.09	63.95	63.95	38.74	38.74	23.37	23.37	0.00	0.00	$ 886.75
6	33.95	33.10	48.38	32.25	29.71	18.67	7.64	8.49	11.88	15.28	12.73	0.85	0.00	$ 252.93
7	0.00	75.54	75.54	151.08	151.08	227.25	0.00	75.54	75.54	151.08	151.08	227.25	0.00	$ 1,360.98
8	35.39	162.11	55.92	26.55	153.26	47.08	17.70	144.41	38.23	8.85	135.56	29.38	0.00	$ 854.43
9	0.00	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	57.37	0.00	$ 631.07
10	90.34	64.93	40.93	14.11	77.63	67.75	47.99	26.11	0.00	63.52	42.34	20.47	0.00	$ 556.13
														$ 6,031.04

TABLE 4.23. Holding Cost $H(q_i^j)$ Of The Example With The Equal Order Quantity Model

$i \in I$	$j \in J$												Ordering Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
1	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	172.83	$ 2,073.91
2	425.93	0.00	425.93	0.00	425.93	0.00	425.93	0.00	425.93	0.00	425.93	0.00	$ 2,555.59
3	275.90	0.00	0.00	275.90	0.00	0.00	275.90	0.00	0.00	275.90	0.00	0.00	$ 1,103.60
4	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38	$ 580.53
5	502.27	0.00	502.27	0.00	502.27	0.00	502.27	0.00	502.27	0.00	502.27	0.00	$ 3,013.64
6	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	191.14	$ 2,293.67
7	207.08	0.00	207.08	0.00	207.08	0.00	207.08	0.00	207.08	0.00	207.08	0.00	$ 1,242.51
8	540.98	0.00	0.00	540.98	0.00	0.00	540.98	0.00	0.00	540.98	0.00	0.00	$ 2,163.93
9	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	260.43	0.00	$ 2,864.68
10	390.90	390.90	390.90	390.90	390.90	390.90	390.90	390.90	390.90	390.90	390.90	390.90	$ 4,690.76
													$ 22,582.81

TABLE 4.24. Ordering Cost $O(q_i^j)$ Of The Example With The Equal Order Quantity Model

$i \in I$	$j \in J$												Purchasing Cost
	1	2	3	4	5	6	7	8	9	10	11	12	
1	453.14	452.02	450.91	449.79	448.68	447.57	446.46	445.35	444.25	454.51	453.38	452.25	$ 5,398.30
2	2030.74	0.00	2020.73	0.00	2027.37	0.00	2017.34	0.00	2007.34	0.00	1997.38	0.00	$ 12,100.89
3	1829.08	0.00	0.00	1815.57	0.00	0.00	1802.12	0.00	0.00	1788.74	0.00	0.00	$ 7,235.51
4	153.63	153.25	152.87	152.49	152.12	151.74	151.36	150.99	150.61	150.24	149.87	158.29	$ 1,827.46
5	2241.17	0.00	2230.12	0.00	2219.11	0.00	2217.04	0.00	2206.05	0.00	2195.10	0.00	$ 13,308.59
6	506.33	505.08	503.83	516.17	514.89	513.62	512.35	511.08	509.81	508.54	507.28	506.02	$ 6,115.00
7	1187.20	0.00	1181.35	0.00	1185.23	0.00	1169.70	0.00	1163.90	0.00	1167.70	0.00	$ 7,055.08
8	4652.23	0.00	0.00	4617.86	0.00	0.00	4583.66	0.00	0.00	4549.62	0.00	0.00	$ 18,403.37
9	1512.53	780.41	778.49	776.56	774.64	772.73	770.81	768.90	766.99	765.09	763.19	0.00	$ 9,230.34
10	1022.47	1019.95	1017.43	1014.91	1023.65	1021.12	1018.59	1016.06	1002.41	1011.03	1008.51	1006.01	$ 12,182.14
													$ 92,856.70

TABLE 4.25. Purchasing Cost $P(q_i^j)$ Of The Example With The Equal Order Quantity Model

Transportation Cost	Inventory Cost			Total Cost
	Purchasing	Ordering	Holding	
$ 19,266.12	$ 92,856.70	$ 22,582.81	$ 6,031.04	$ 140,736.67
	$ 121,470.56			

TABLE 4.26. Comparing Costs Of The Example With The Equal Order Quantity Model

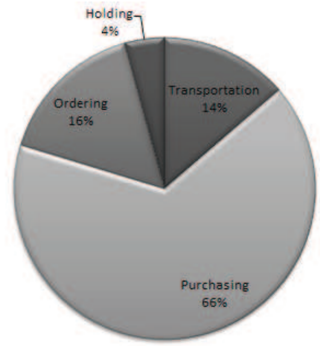

FIGURE 4.4. Dividing The Equal Order Quantity Model Total Cost

Table 4.27 shows the summary of all costs of the three models. The total cost of My Model is much less than the total cost of the JIT and the Equal Order Quantity models. It means that My Model gives an optimal set of order quantities of the example. The holding cost in My Model is larger that the other models and its ordering, purchasing and transportation costs are less. The JIT model has the highest total cost, ordering cost and purchasing cost, and the Equal Order Quantity model has the highest transportation cost. Figure 4.5 shows the relationship between the purchasing and transportation costs in all three models. These two costs are the most important roles in supply chain. As figure 4.5 shows, the saving of these two costs in My Model is more than $7,550. As mentined before, this numerical example is a fictional case study and potential savings in real supply chain could be much more than the illustrated results.

	Transportation	Purchasing	Ordering	Holding	**Total**
JIT	$ 17,594.39	$ 93,126.74	$ 23,956.31	$ 6,222.46	**$ 140,899.91**
EOQ	$ 19,266.12	$ 92,856.70	$ 22,582.81	$ 6,031.04	**$ 140,736.67**
My Model	$ 17,137.06	$ 87,432.81	$ 10,439.30	$ 25,175.96	**$ 140,185.12**

TABLE 4.27. Comparing Total Costs Of Three Models

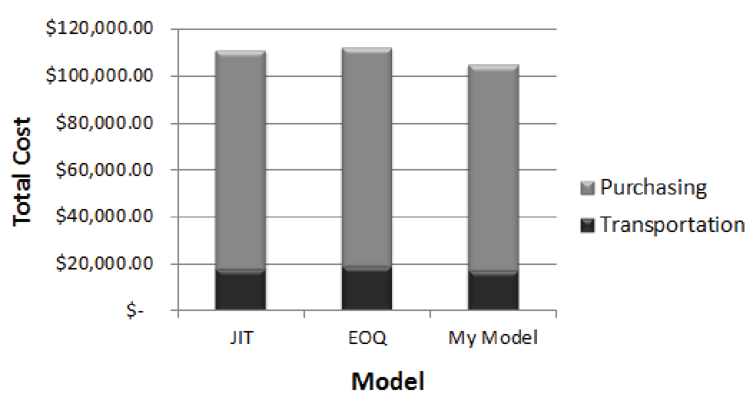

FIGURE 4.5. Comparing Purchasing And Transportation Costs Of Three Model

Table 4.28 and figure 4.6 show relationship between holding costs of three models. The holding cost of My Model is more than the holding costs of the other models. For all items, the Equal Order Quantity has the smallest holding costs except for items 3 and 7 which have the lowest holding costs with the JIT model. Item 9 has the same holding cost with the JIT and the Equal Order Quantity models because it has the same set of order quantities. Item 8 in figure 4.6 has the greatest holding cost with My Model and item 4 has the smallest holding cost with the Equal Order Quantity model.

	$\sum_{j\in J} H(q_i^j)$		
$i \in I$	JIT	EOQ	My Model
1	$ 382.63	$ 206.09	$ 464.62
2	$ 930.91	$ 895.40	$ 4,051.89
3	$ 295.48	$ 360.34	$ 1,214.02
4	$ 52.45	$ 26.93	$ 168.68
5	$1,087.83	$ 886.75	$ 4,242.49
6	$ 420.14	$ 252.93	$ 1,467.51
7	$ 454.49	$1,360.98	$ 3,181.44
8	$1,108.57	$ 854.43	$ 5,353.83
9	$ 631.07	$ 631.07	$ 927.57
10	$ 858.89	$ 556.13	$ 4,103.90
Total	**$6,222.46**	**$6,031.04**	**$25,175.96**

Table 4.28. Comparing Holding Costs Of Three Models

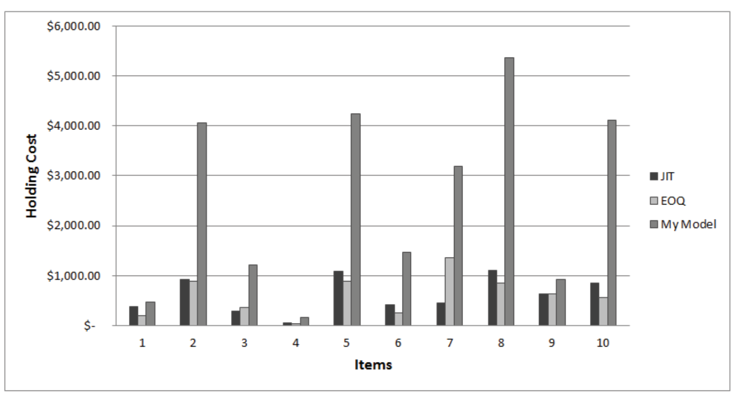

Figure 4.6. Comparing Holding Costs Of Three Models

Table 4.29 and figure 4.7 show ordering cost of all items with all models. My Model has the less ordering cost for all items and the Equal Order Quantity has the biggest ordering cost for all items except item 8 which is ordered 11 times with the JIT model and just 4 times in a year with the Equal Order Quantity model. Items 2, 3, 5 and 9 have the same ordering costs with the JIT and Equal Order Quantity models. The highest ordering cost in figure 4.7 is for item 8 with the JIT model and the lowest one is for item 7 with My Model which is ordered just once in a year.

	$\sum_{j\in J} O(q_i^j)$		
$i \in I$	JIT	EOQ	My Model
1	$ 1,901.08	$ 2,073.91	$ 1,901.08
2	$ 2,555.59	$ 2,555.59	$ 851.86
3	$ 1,103.60	$ 1,103.60	$ 827.70
4	$ 532.15	$ 580.53	$ 241.89
5	$ 3,013.64	$ 3,013.64	$ 1,506.82
6	$ 2,102.53	$ 2,293.67	$ 955.70
7	$ 414.17	$ 1,242.51	$ 207.08
8	$ 5,950.80	$ 2,163.93	$ 1,081.96
9	$ 2,864.68	$ 2,864.68	$ 2,083.41
10	$ 3,518.07	$ 4,690.76	$ 781.79
Total	**$23,956.31**	**$22,582.81**	**$10,439.30**

TABLE 4.29. Comparing Ordering Costs Of Three Models

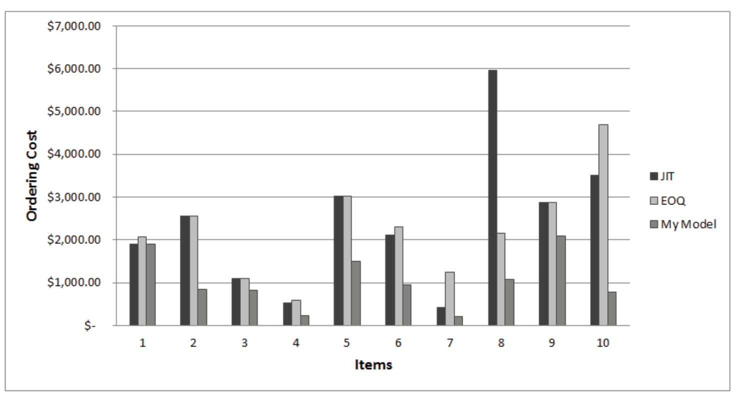

FIGURE 4.7. Comparing Ordering Costs Of Three Models

Table 4.30 and figure 4.8 show the purchasing cost. Items 3 and 4 have the least purchasing costs with the Equal Order Quantity model but for the rest of items My Model has the smallest purchasing costs. Table 4.31 shows annual transportation cost of all NMFC class types with all models. Total transportation cost of My Model is less than two other models. For $v = 3$ and $v = 7$ the transportation costs in My Model is the least and for $v = 4$ the JIT model is the least transportation cost.

	$\sum_{j\in J} P(q_i^j)$		
$i \in I$	JIT	EOQ	My Model
1	$ 5,405.08	$ 5,398.30	$ 5,352.56
2	$11,745.28	$ 12,100.89	$11,300.79
3	$ 7,230.63	$ 7,235.51	$ 7,299.02
4	$ 1,829.84	$ 1,827.46	$ 1,840.73
5	$12,403.36	$ 13,308.59	$11,312.86
6	$ 6,125.96	$ 6,115.00	$ 5,907.30
7	$ 6,650.73	$ 7,055.08	$ 5,812.67
8	$20,501.07	$ 18,403.37	$17,868.74
9	$ 9,230.34	$ 9,230.34	$ 8,996.00
10	$12,004.46	$ 12,182.14	$11,793.52
Total	**$93,126.74**	**$ 92,856.70**	**$87,484.17**

TABLE 4.30. Comparing Purchasing Costs Of Three Models

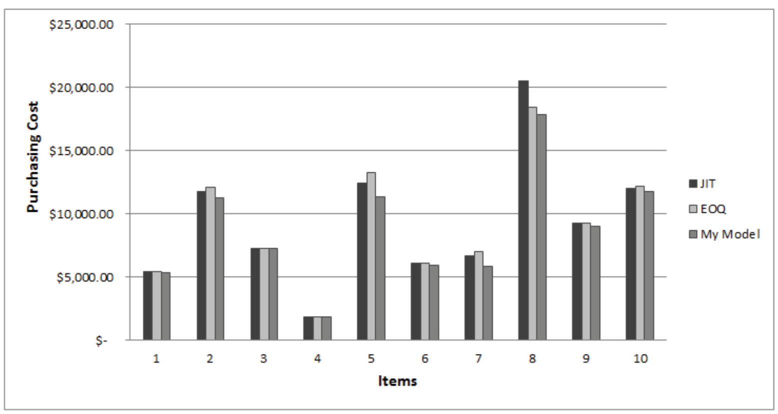

FIGURE 4.8. Comparing Purchasing Costs Of Three Models

	$\sum_{j\in J} R(q_i^j)$		
$v \in V$	JIT	EOQ	My Model
3	$ 342,844.39	$ 334,179.77	$ 327,011.91
4	$ 115,353.82	$ 135,230.83	$ 127,252.36
7	$ 69,633.64	$ 108,572.94	$ 59,847.51
Total	**$527,831.85**	**$577,983.54**	**$514,111.79**

TABLE 4.31. Comparing Transportation Costs Of Three Models

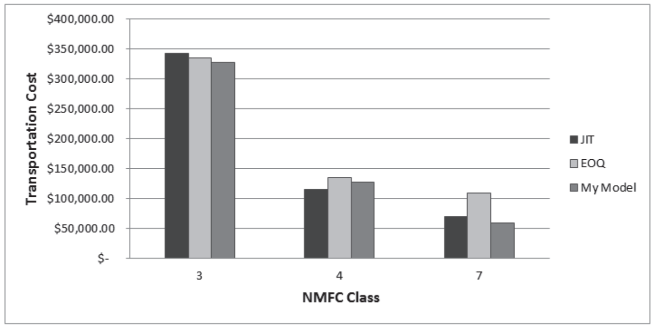

Figure 4.9. Comparing Transportation Costs Of Three Models

4.4. Sensitivity Analysis

Sensitivity analysis is used to notice the effect of the parameters in the model. One variable is changed and the others will be constant. The parameters that will be analyzed are holding cost, h_i, ordering cost, o_i, interest rate, r and transportation cost ϑ_k^v.

4.4.1. Holding Cost of item i per time unit, h_i

Investigating h_i helps note the effect of the holding cost, $H(q_i^j)$, in the total cost. Five possible situations for h_i, are decreasing 80% and 40% and increasing 80% and 40% and without changing. The set of optimal order quantities will be changed by changing h_i. Tables A.1, A.2, A.3 and A.4 in Appendix A illustrate these sets of optimal order quantities. The tables show that by increasing h_i, order quantities of items 2, 3, 5, 7, 8 and 10 do not change. The reason is the total holding costs of these items are bigger than the others as table 4.9 shows. Thus, changing 40% or 80% of the holding cost does not effect on way of ordering these items. For items 1, 4, 6 and 9 changing the holding cost, h_i, makes significant differences in the way of ordering. By increasing the holding cost the number of items for storing are decreased and frequencies are increased. For example, item 9 is ordered 8 times in a year when h_i decreases 80% and it is ordered 10 times when h_i increases 80%. This makes an increasing the ordering cost and a decreasing the holding cost.

h_i	Transportation	Purchasing	Ordering	Holding	**Total**
Decrease 80%	$ 16,975.84	$ 87,265.45	$ 10,487.67	$ 5,073.30	**$ 119,802.27**
Decrease 40%	$ 17,045.07	$ 87,262.25	$ 10,015.20	$ 15,191.04	**$ 129,513.57**
0	$ 17,137.06	$ 87,432.81	$ 10,439.30	$ 25,175.96	**$ 140,185.12**
Increase 40%	$ 17,314.57	$ 87,507.20	$ 11,178.76	$ 34,995.17	**$ 150,995.69**
Increase 80%	$ 17,469.00	$ 87,580.17	$ 11,296.42	$ 44,878.23	**$ 161,223.82**

Table 4.32. Comparing All Costs With Different h_i

Table 4.32 compares the costs of each set of order quantities. It shows by increasing h_i, the transportation, the purchasing and the ordering costs are increasing to balance the total cost and keep the holding costs constant.

4.4.2. Ordering Cost of item i per time unit, o_i

Studying the ordering cost, o_i assists to analyze the role of $O(q_i^j)$ in the total cost. Each of these changes has different sets of optimal order quantities that tables A.5, A.6, A.7, A.8 in Appendix A show them. When the ordering cost is increased, My Model tries to minimize a frequency. For example item 6 is ordered 7 times in a year, when o_i decreases 40% and it is ordered 5 times in a year when o_i increases 80%. As tables 4.9 and 4.10 show for item 6 the ordering cost is less than the holding cost. Thus, My Model balance these costs when the ordering cost is changed, it means that by increasing the ordering cost, the holding cost decreases. For items 2, 3, 5, 7, 8 and 10 the order quantities are not changed, the reason is the holding costs for these items are much more than the ordering costs.

Comparing the costs of each sets of order quantities is shown in table 4.33. When the ordering cost increases; the transportation, the purchasing and the holding costs decrease. By decreasing the ordering cost; purchasing and transportation do not follow a specific pattern. When the ordering cost decreases 40% the transportation and purchasing costs are increasing but when the ordering cost decreases 80%; the purchasing cost decreases and the transportation cost increases, the reason is to balance the total cost.

o_i	Transportation	Purchasing	Ordering	Holding	**Total**
Decrease 80%	$ 17,158.38	$ 87,352.62	$ 2,149.62	$ 25,113.55	**$ 131,774.17**
Decrease 40%	$ 17,231.17	$ 87,468.08	$ 6,707.25	$ 25,091.31	**$ 136,497.82**
0	$ 17,137.06	$ 87,432.81	$ 10,439.30	$ 25,175.96	**$ 140,185.12**
Increase 40%	$ 17,109.40	$ 87,429.37	$ 15,018.07	$ 25,134.11	**$ 144,690.95**
Increase 80%	$ 17,221.66	$ 87,373.66	$ 19,520.74	$ 25,115.32	**$ 149,231.38**

Table 4.33. Comparing All Costs With Different o_i

4.4.3. Interest Rate, r

Changing r changes the purchasing cost, $P(q_i^j)$ and the total cost. The purchasing cost includes the expenses for buying the items and paying a loan. loan payments are monthly payments for a loan with interest rate $r = \frac{\text{(Annual Interest Rate)}}{12}$.

The annual interest rate of the example is 6% and is compared with 1%, 4% and 11%. Each of these changes of r has different set of optimal order quantities that the tables A.9, A.10 and A.11 in Appendix A illustrate them. Order quantities of item 2, 3, 5, 7, 8 and 10 are fixed with different r. Sets of order quantities of the other items are changed to balance the total cost. The ordering and holding costs are increased and transportation is decreased when r increases. For example item 4 is ordered 7 times with annual interest rate, 11% but is ordered 5 times when the annual interest rate is 1%. It makes the ordering and holding costs increasing and the transportation cost decreasing.

Annual Interest Rate	r	Transportation	Purchasing	Ordering	Holding	Total
Decrease 5%	0.0033	$ 17,208.84	$ 85,460.07	$ 10,439.30	$ 25,121.30	**$ 138,229.51**
Decrease 2%	0.00083	$ 17,055.89	$ 86,616.82	$ 10,227.25	$ 25,218.42	**$ 139,118.38**
0	0.0050	$ 17,137.06	$ 87,432.81	$ 10,439.30	$ 25,175.96	**$ 140,185.12**
Increase 5%	0.00917	$ 16,999.73	$ 89,408.85	$ 10,536.05	$ 25,231.68	**$ 142,176.31**

TABLE 4.34. Comparing All Costs With Different r

4.4.4. Transportation Cost per 100 pounds for NMFC class type v in weight range k, ϑ_k^v

Changing ϑ_k^v effects on the transportation cost, $R_v^j(q_i^j : i \in c_v)$ and the total cost. Each of these changes has different set of optimal order quantities that the tables A.12, A.13, A.14 and A.15 in Appendix A show them. Items 2, 3, 5, 7, 8 and 10 are fixed with different ϑ_k^v. The order quantities of the other items are changed to balance the total cost. By increasing the transportation cost, the purchasing and ordering costs are decreased and the holding cost is increased, table 4.35 shows the details. The reason is when transportation is increased My Model tries to keep the total cost constant, thus, minimizing frequencies helps to decrease the transportation cost. By decreasing frequencies and the transportation cost, the holding cost is increasing so whenever the holding cost is less than the summation of the ordering, transportation and purchasing costs, My Model prefers to increase the holding cost instead of the other costs.

ϑ_k^v	Transportation	Purchasing	Ordering	Holding	Total
Decrease 80%	$ 3,518.52	$ 87,578.92	$ 11,556.85	$ 24,876.37	**$ 127,530.66**
Decrease 40%	$ 10,431.37	$ 87,501.48	$ 11,248.04	$ 25,011.66	**$ 134,192.56**
0	$ 17,137.06	$ 87,432.81	$ 10,439.30	$ 25,175.96	**$ 140,185.12**
Increase 40%	$ 23,861.76	$ 87,439.68	$ 10,727.19	$ 25,174.28	**$ 147,202.90**
Increase 80%	$ 30,543.98	$ 87,396.38	$ 10,727.19	$ 25,258.23	**$ 153,925.78**

TABLE 4.35. Comparing All Costs With Different ϑ_k^v

4.4.5. SUMMARY OF SENSITIVITY ANALYSIS

Table 4.36 summarizes the results of the efficiency of all parameters. The table shows by increasing h_i, the transportation, purchasing and ordering costs are increased and the holding cost is decreased. It means that the items are ordered more frequencies when h_i increases. Thus, the ordering cost is increased because the frequencies are increased.

The next row of the table is increasing o_i, the changes of the transportation cost is unknown, sometimes it is increased or decreased by changing o_i. The items orders less frequencies when the ordering cost is more than the holding cost and it makes the transportation cost decreases. By decreasing frequencies the holding cost increases and the purchasing cost decreases. In the example, items have the different relationship between the holding and ordering costs that make the results complicated. Finally, by increasing o_i, the purchasing and holding costs are decreased and the ordering and total costs are increased.

The third row explains when interest rate is increasing the model tries to order less frequencies thus the holding cost is increasing and the transportation cost is decreasing. The final row is changing ϑ_k^v, that exactly opposite of h_i. The reason is that when the transportation cost is increasing the items are ordered less frequencies than before. That makes ordering, purchasing and the transportation costs decreasing and the holding cost increasing.

By Increasing	Transportation Cost	Purchasing Cost	Ordering Cost	Holding Cost	Total Cost
h_i	Increase	Increase	Increase	Decrease	Increase
o_i	Unknown	Decrease	Increase	Decrease	Increase
r	Decrease	Decrease	Increase	Increase	Increase
ϑ_k^v	Decrease	Decrease	Decrease	Increase	Increase

TABLE 4.36. Summary Of Sensitivity Analysis

CHAPTER 5

Conclusions

5.1. Significance

Optimization of inventory and transportation costs helps manufacturers store and transport the right order quantities at the right time. The presented model in this thesis has given an optimal order quantities of the items which are ordered from a supplier to a manufacturer. The model covers the NMFC transportation classification, price breaks and financing costs with non-constant demand targets, which has not been done before. Moreover, a flexible planning horizon helps the manufacturer uses the model with different time periods such as hours, days or months, etc.

5.2. Contributions

This thesis proposed a mixed integer, non-linear programming model to determine the order quantity at minimal total cost which is made up of inventory and transportation costs.

The new things in the model are price breaks of purchasing and transportation costs and the NMFC transportation classification. They make the model more accurate and completed. Price breaks help the manufacturers spend less on purchasing and transportation costs with increasing order quantities. The NMFC transportation classification also helps the manufacturer the class types of their items and the transportation price ranges. The model covers flexible planning horizons for pull and push systems to decrease the risk of short lifecycle items and increase efficiency.

The Genetic Algorithm function in Matlab is used as the solution procedure and the results of the model are compared to the results of the Equal Order Quantity and the JIT models to define My Model determine the optimal order quantity. Sensitivity is also considered for all parameters to notice the effect of them in the model, it also shows that the holding cost is the most important role in the total cost.

5.3. Future Work

The following research topics can be pursued to bridge current gaps in literature
.

(1) Using other distribution strategies for truck delivery like Milk-run or Cross-dock, etc. In this study the direct strategy is used as a distribution strategy.
(2) Considering a two-level supply chain consisting of one supplier, one manufacturer and one retailer. This thesis considers a simple supply chain without any retailers.

References

[1] Y. Li, Q. Su, and T. Li, "The optimization and integration of the transportation and inventory cost based on time in supply chain logistics system," *2010 International Conference on Logistics Engineering and Intelligent Transportation Systems, LEITS2010 - Proceedings*, pp. 195–198, 2010.

[2] O. Berman and Q. Wang, "Inbound logistic planning: Minimizing transportation and inventory cost," *Transportation Science*, vol. 40, no. 3, pp. 287–299, 2006.

[3] Z. N. Chen, C. K. M. Lee, W. H. Ip, and G. T. S. Ho, "Design and evaluation of an integrated inventory and transportation system," *Transportation Planning and Technology*, vol. 35, no. 4, pp. 491–507, 2012.

[4] J. Carter and B. Ferrin, "Transportation costs and inventory management: Why transportation costs matter," *Production and Inventory Management Journal*, vol. 37, no. 3, pp. 58–61, 1996.

[5] TridenT, "Trident job," URL(http://tridentjob.blogspot.ca), [Online; accessed 14-July-2013].

[6] R. Clavette, "Vimich," URL(http://www.vimich.com/what-is-logistics), [Online; accessed 17-February-2013].

[7] M. Ben-Daya, R. As'Ad, and M. Seliaman, "An integrated production inventory model with raw material replenishment considerations in a three layer supply chain," *International Journal of Production Economics*, vol. 143, no. 1, pp. 53–61, 2013.

[8] G. Svensson, "Sub-contractor and customer sourcing and the occurrence of disturbances in firms' inbound and outbound logistics flows," *Supply Chain Management*, vol. 8, no. 1, pp. 41–56, 2003.

[9] A. S. Onawumi, A. E. Oluleye, and K. A. Adebiyi, "An economic lot size model with shortages and inflation," *International Business Management*, vol. 3, no. 4, pp. 75–79, 2009.

[10] M. Murray, "Logistics," URL(http://logistics.about.com/supplychainglossary/CrossDock.htm), [Online; accessed 17-February-2013].

[11] Investopedia, "Investopedia," URL(http://www.investopedia.com/terms/h/holding-costs.asp), [Online; accessed 15-July-2013].

[12] "Lean manufacture," URL(http://www.leanmanufacture.net/operations/inventorycosts.aspx),[Online; accessed 1-March-2013].

[13] S. Goyal, "Determination of optimum production quantity for a two-stage production system," *Operational Research Quarterly*, vol. 28, no. 4, pp. 865–870, 1977.

[14] A. Banerjee, "A joint economic-lot-size model for purchaser and vendor," *Decision Sciences*, vol. 17, no. 3, pp. 292–311, 1986.

[15] S. Goyal, "A joint economic-lot-size model for purchaser and vendor: A comment," *Decision Sciences*, vol. 19, no. 1, pp. 236–241, 1988.

[16] S. Goyal, "A one-vendor multi-buyer integrated inventory model: A comment," *European Journal of Operational Research*, vol. 82, no. 1, pp. 209–210, 1995.

[17] R. Hill, "The single-vendor single-buyer integrated production-inventory model with a generalised policy," *European Journal of Operational Research*, vol. 97, no. 3, pp. 493–499, 1997.

[18] R. Hill, "Optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory problem," *International Journal of Production Research*, vol. 37, no. 11, pp. 2463–2475, 1999.

[19] S. Goyal and F. Nebebe, "Determination of economic production-shipment policy for a single-vendor-single-buyer system," *European Journal of Operational Research*, vol. 121, no. 1, pp. 175–178, 2000.

[20] S. Goyal and Y. Gupta, "Integrated inventory models: The buyer-vendor coordination," *European Journal of Operational Research*, vol. 41, no. 3, pp. 261–269, 1989.

[21] K. Ertogral, M. Darwish, and M. Ben-Daya, "Production and shipment lot sizing in a vendor-buyer supply chain with transportation cost," *European Journal of Operational Research*, vol. 176, no. 3, pp. 1592–1606, 2007.

[22] P. Levine, "Nmfta," URL(http://www.nmfta.org), [Online; accessed 5-May-2013].

[23] F. Harris, "How many parts to make at once," *Factory, the Magazine of Management*, vol. 10, no. 1, pp. 135–136, 1913.

[24] A. Federgruen and P. Zipkin, "Combined vehicle routing and inventory allocation problem.," *Operations research*, vol. 32, no. 5, pp. 1019–1037, 1984.

[25] L. D. Burns, R. W. Hall, D. E. Blumenfeld, and C. F. Daganzo, "Distribution strategies that minimize transportation and inventory costs.," *Operations research*, vol. 33, no. 3, pp. 469–490, 1985.

[26] M. G. Speranza and W. Ukovich, "Minimizing transportation and inventory costs for several products on a single link," *Operations research*, vol. 42, no. 5, pp. 879–894, 1994.

[27] A. Baboli, M. P. Neghab, and R. Haji, "An algorithm for the determination of the economic order quantity in a two-level supply chain with transportation costs: Comparison of decentralized with centralized decision," *Journal of Systems Science and Systems Engineering*, vol. 17, no. 3, pp. 353–366, 2008.

[28] X. Wang and T. C. E. Cheng, "Logistics scheduling to minimize inventory and transport costs," *International Journal of Production Economics*, vol. 121, no. 1, pp. 266–273, 2009.

[29] K. Hong, S. S. Yeo, H. J. Kim, E. P. Chew, and C. Lee, "Integrated inventory and transportation decision for ubiquitous supply chain management," *Journal of Intelligent Manufacturing*, vol. 23, no. 4, pp. 977–988, 2012.

[30] L. Bertazzi and M. G. Speranza, "Continuous and discrete shipping strategies for the single link problem," *Transportation Science*, vol. 36, no. 3, pp. 314–325, 2002.

[31] L. Bertazzi and S. S. M. Cherubini, "An inventory-transportation system with stochastic demand," *Computational Management Science*, vol. 10, no. 1, pp. 1–20, 2013.

[32] J. Guttentag, "The math behind your home loan," 2007. URL(http://www.washingtonpost.com/wp-dyn/content/article/2007/10/05/AR2007100501165.html), [Online; accessed 20-September-2013].

[33] J. Guttentag, "The mortgage professor," 2009. URL(http://www.mtgprofessor.com/formulas.htm), [Online; accessed 20-September-2013].

[34] "Mathworks." URL(http://www.mathworks.com/discovery/genetic-algorithm.html), [Online; accessed 28-June-2013].

[35] M. A. El Baz, "A genetic algorithm for facility layout problems of different manufacturing environments," *Computers and Industrial Engineering*, vol. 47, no. 2-3, pp. 233–246, 2004.

[36] M. J. Li, D. S. Chen, S. Y. Cheng, F. Wang, Y. Li, Y. Zhou, and J. L. Lang, "Optimizing emission inventory for chemical transport models by using genetic algorithm," *Atmospheric Environment*, vol. 44, no. 32, pp. 3926–3934, 2010.

[37] M. Obitko, "Obitko," URL(http://www.obitko.com/tutorials/genetic-algorithms/ga-basic-description.php), [Online; accessed 28-June-2013].

APPENDIX A

Sensitivity Analysis

	$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
$i \in I$	1	2	3	4	5	6	7	8	9	10	11	12		
1	78	43	6	80	6	80	6	80	6	80	6	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
4	90	0	37	0	20	17	37	4	0	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	221	75	0	41	99	0	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
9	202	200	121	0	101	101	202	0	202	83	0	0	1212	1212
10	123	964	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.1. Order Quantities When h_i Is Decreased 80%

	$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
$i \in I$	1	2	3	4	5	6	7	8	9	10	11	12		
1	78	6	43	80	6	80	6	80	6	43	43	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
4	20	90	0	37	17	20	17	0	4	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	221	19	97	0	99	0	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
9	223	199	0	202	0	202	0	202	0	184	0	0	1212	1212
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.2. Order Quantities When h_i Is Decreased 40%

	$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
$i \in I$	1	2	3	4	5	6	7	8	9	10	11	12		
1	41	43	43	43	80	6	43	80	6	43	43	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	17	350	0	1	0	0	0	0	0	0	0	0	368	368
4	21	69	37	20	0	17	37	4	0	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	221	19	56	41	51	48	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
9	202	119	101	101	101	101	202	0	101	184	0	0	1212	1212
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.3. Order Quantities When h_i Is Increased 40%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	41	43	43	80	6	43	43	43	43	43	43	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
	4	20	90	17	0	20	17	20	17	4	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	240	0	56	41	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	119	101	101	101	101	101	202	0	101	83	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

Table A.4. Order Quantities When h_i Is Increased 80%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	41	43	43	80	6	80	6	80	6	80	6	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
	4	20	90	0	37	0	18	36	4	0	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	240	0	97	0	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	119	101	101	101	103	200	0	202	0	83	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

Table A.5. Order Quantities When o_i Is Decreased 80%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	78	43	6	43	80	6	43	43	43	80	6	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	1	0	0	0	0	0	0	0	0	368	368
	4	20	70	20	17	37	0	37	0	4	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	221	19	56	41	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	200	20	101	101	101	202	0	101	184	0	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

Table A.6. Order Quantities When o_i Is Decreased 40%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	41	80	6	43	80	6	43	43	80	6	43	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
	4	21	69	20	37	17	0	37	4	0	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	221	19	56	92	0	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	119	101	101	202	0	202	0	202	0	83	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

Table A.7. Order Quantities When o_i Is Increased 40%

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
1	41	80	6	80	6	43	43	43	43	80	6	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
4	20	70	20	17	37	20	0	17	4	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	240	0	97	0	51	48	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	701	2731	0	0	0	0	0	0	0	0	0	0	3432	3432
9	202	200	20	101	101	101	202	0	101	184	0	0	1212	1212
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.8. Order Quantities When o_i Is Increased 80%

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
1	41	43	43	43	43	80	6	43	43	43	43	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	17	350	0	1	0	0	0	0	0	0	0	0	368	368
4	20	90	0	37	0	37	0	21	0	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	240	0	97	0	51	48	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
9	202	194	26	101	101	202	0	202	0	184	0	0	1212	1212
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.9. Order Quantities When Annual Interest Rate Is 1%

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
1	41	43	43	43	80	6	43	43	43	43	43	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	17	350	0	0	1	0	0	0	0	0	0	0	368	368
4	90	0	20	37	0	37	0	17	4	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	221	75	41	0	99	0	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
9	202	119	101	202	0	202	0	202	0	184	0	0	1212	1212
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.10. Order Quantities When Annual Interest Rate Is 4%

$i \in I$	$j \in J$: 1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - l_i^0$
1	41	43	43	80	6	80	6	80	6	80	6	0	471	471
2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
3	18	349	0	1	0	0	0	0	0	0	0	0	368	368
4	20	70	37	20	0	37	17	4	0	0	0	0	205	205
5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
6	17	240	0	97	0	51	48	0	0	0	0	0	453	453
7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
9	202	119	201	102	0	101	202	0	202	0	83	0	1212	1212
10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.11. Order Quantities When Annual Interest Rate Is 11%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	42	42	43	43	43	43	43	80	6	43	43	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	1	0	0	0	0	0	0	0	0	368	368
	4	20	70	20	37	17	0	20	17	4	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	240	0	56	41	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	119	101	101	101	101	101	101	101	101	83	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.12. Order Qquantities When ϑ_k^v Is Decreased 80%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	41	43	43	43	43	43	43	80	6	80	6	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	1	0	0	0	0	0	0	0	0	368	368
	4	90	20	0	17	20	37	0	17	4	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	240	0	56	41	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	197	23	101	101	101	101	101	101	184	0	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.13. Order Qquantities When ϑ_k^v Is Decreased 40%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	41	43	43	80	6	43	43	80	6	80	6	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	17	350	0	1	0	0	0	0	0	0	0	0	368	368
	4	20	70	20	37	0	37	17	4	0	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	221	75	41	0	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	119	101	202	0	101	202	0	202	0	83	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.14. Order Qquantities When ϑ_k^v Is Increased 40%

		$j \in J$												$\sum_{j=1}^{12} q_i^j$	$\sum_{j=0}^{12} d_i^j - I_i^0$
		1	2	3	4	5	6	7	8	9	10	11	12		
$i \in I$	1	78	43	9	77	6	80	6	43	43	80	6	0	471	471
	2	5	725	0	0	0	0	0	0	0	0	0	0	730	730
	3	42	325	0	1	0	0	0	0	0	0	0	0	368	368
	4	20	70	37	0	20	37	17	0	4	0	0	0	205	205
	5	22	1200	0	269	0	0	0	0	0	0	0	0	1491	1491
	6	17	221	75	0	41	51	48	0	0	0	0	0	453	453
	7	0	728	0	0	0	0	0	0	0	0	0	0	728	728
	8	700	2732	0	0	0	0	0	0	0	0	0	0	3432	3432
	9	202	119	202	0	101	101	202	0	202	83	0	0	1212	1212
	10	122	965	0	0	0	0	0	0	0	0	0	0	1087	1087

TABLE A.15. Order Qquantities When ϑ_k^v Is Increased 80%

APPENDIX B

Matlab Code

```
function F = TotalCost(q)
%Resizing matrix form (1,120) to (10,12).
for i=1:10
    for j=1:12
        q3(i,j)=q((i-1)*12+j);
    end
end
q= q3;
%Changing variables from q to q2.
q2(:,1)=q(:,1);
 for j=2:12
     q2(:,j)=q(:,j)-q(:,j-1);
 end
q=q2;
%Demand Target
d= [43  43  43  43  43  43  43  43  43  43  43  43  0;
      145 0   145 0   145 0   145 0   145 0   145 0   0;
      117 0   0   117 0   0   117 0   0   117 0   0   0;
      17  20  17  20  17  20  17  20  17  20  17  20  0;
      322 0   334 0   284 0   290 0   274 0   287 0   0;
      38  19  56  41  51  51  37  34  34  41  52  39  0;
      0   0   0   0   0   364 0   0   0   0   0   364 0;
      500 300 83  500 300 83  500 300 83  500 300 83  0;
      101 101 101 101 101 101 101 101 101 101 101 101 0;
      126 124 128 0   105 119 122 128 0   121 122 120 0];
%Inventory Level at time point zero
ellzero = [45;
            140;
            100;
            17;
            300;
            40;
            0;
            100;
            0;
            128];
```

```
%Inventory Level
for i = 1:10
      L(i,1)=ellzero(i)+q(i,1)-d(i,1);
  for j=1:12
        if j>1
           L(i,j)= L(i,j-1)+q(i,j)-d(i,j);
        end
  end
end
%w: item weight w(i)
      w= [163.84
      228.8
      457.3333
      66.90816
      275.6571
      119.04
      116.1626
      150.552
      76.3992
      316.2353];

%h: holding cost h(i)
h= [5.17;
7.53;
4.59;
1.65;
4.31;
5.96;
4.37;
2.48;
3.98;
4.94];
```

```
 %o: ordering cost, o(i)
o= [69.13;
170.37;
110.36;
19.35;
201.05;
76.63;
82.83;
216.39;
104.17;
156.41];



% Price Break

for j = 1:12
   if (q(1,j)>= 0 ) && (q(1,j)< 21 );
       p(1,j) = 12.50;
   elseif (q(1,j)>= 21 ) && (q(1,j)< 51 );
       p(1,j) = 11.25;
   elseif (q(1,j)>= 51 ) && (q(1,j)< 121 );
       p(1,j) = 10.50;
   elseif (q(1,j)>= 121 ) && (q(1,j)< 351);
       p(1,j) = 10.25;
   elseif (q(1,j)>= 351 );
       p(1,j) = 10.06;
   end
end

for j = 1:12
    if (q(2,j)>= 0 ) && (q(2,j)< 51 );
        p(2,j) = 18.75;
    elseif (q(2,j)>= 51 ) && (q(2,j)< 101 );
        p(2,j) = 17.75;
    elseif (q(2,j)>= 101 ) && (q(2,j)< 136 );
```

```
        p(2,j) = 16.25;
    elseif (q(2,j)>= 136 ) && (q(2,j)< 501);
        p(2,j) = 15.75;
    elseif (q(2,j)>= 501 );
        p(2,j) = 15.00;
    end
end


for j = 1:12
   if (q(3,j)>= 0 );
       p(3,j) = 19.25;
   end
end

for j = 1:12
    if (q(4,j)>= 0 ) && (q(4,j)< 101 );
        p(4,j) = 8.75;
    elseif (q(4,j)>= 101 ) ;
        p(4,j) = 8.00;
    end
end

for j = 1:12
    if (q(5,j)>= 0 ) && (q(5,j)< 51 );
        p(5,j) = 9.50;
    elseif (q(5,j)>= 51 ) && (q(5,j)< 151 );
        p(5,j) = 9.00;
    elseif (q(5,j)>= 151 ) && (q(5,j)< 251 );
        p(5,j) = 8.75;
    elseif (q(5,j)>= 251 ) && (q(5,j)< 401 );
        p(5,j) = 8.13;
    elseif (q(5,j)>= 401 )&& (q(5,j)< 801);
        p(5,j) = 7.50;
    elseif (q(5,j)>= 801 );
```

```
        p(5,j) = 7.16;
    end

end
for j = 1:12
    if (q(6,j)>= 0 ) && (q(6,j)< 16 );
        p(6,j) = 13.75;
    elseif (q(6,j)>= 16 ) && (q(6,j)< 22 );
        p(6,j) = 13.50;
    elseif (q(6,j)>= 22 ) && (q(6,j)< 61 );
        p(6,j) = 13.25;
    elseif (q(6,j)>= 61 ) && (q(6,j)< 251 );
        p(6,j) = 12.50;
    elseif (q(6,j)>= 251 ) ;
        p(6,j) = 12.25;
    end
end

for j = 1:12
   if (q(7,j)>= 0 ) && (q(7,j)< 101 );
       p(7,j) = 10;
   elseif (q(7,j)>= 101 ) && (q(7,j)< 201 );
       p(7,j) = 9.50;
   elseif (q(7,j)>= 201 ) && (q(7,j)< 301 );
       p(7,j) = 9.25;
   elseif (q(7,j)>= 301 ) && (q(7,j)< 401 );
       p(7,j) = 9;
   elseif (q(7,j)>= 401 ) && (q(7,j)< 501 );
       p(7,j) = 8.75;
   elseif (q(7,j)>= 501 ) && (q(7,j)< 601 );
       p(7,j) = 8.50;
   elseif (q(7,j)>= 601 ) && (q(7,j)< 701 );
       p(7,j) = 8.00;
   elseif (q(7,j)>= 701 ) && (q(7,j)< 801 );
       p(7,j) = 7.75;
```

```
    elseif (q(7,j)>= 801 );
        p(7,j) = 7.50;
    end
end
for j = 1:12
    if (q(8,j)>= 0 ) && (q(8,j)< 501 );
        p(8,j) = 6.00;
    elseif (q(8,j)>= 501 ) && (q(8,j)< 2001 );
        p(8,j) = 5.25;
    elseif (q(8,j)>= 2001 ) ;
        p(8,j) = 5.00;
    end
end

for j = 1:12
    if (q(9,j)>= 0 ) && (q(9,j)< 101 );
        p(9,j) = 7.75;
    elseif (q(9,j)>= 101 ) && (q(8,j)< 201 );
        p(9,j) = 7.50;
    elseif (q(9,j)>= 201 )
        p(9,j) = 7.25;
    end
end

for j = 1:12
    if (q(10,j)>= 0 ) && (q(10,j)< 121 );
        p(10,j) = 11.00;
    elseif (q(10,j)>= 121 ) && (q(10,j)< 501 );
        p(10,j) = 10.75;
    elseif (q(10,j)>= 501 ) && (q(10,j)< 1201 );
        p(10,j) = 10.50;
    elseif (q(10,j)>= 1201 );
        p(10,j) = 10.00;
    end
end
```

```
% Ordering, Holding and Purchasing costs
n= [12];
Annualinterest = [0.06];
r = Annualinterest/n;
for i = 1:10
    for j = 1:12
P(i,j) = (((n-j+1)*(r*((1+r)^(n-
j+1)))*(p(i,j)*q(i,j))))/(((1+r)^(n-j+1))-1);
O(i,j)= o(i)*(min(q(i,j),1));
H(i,j)= h(i)*L(i,j);
Helzero(i)=ellzero(i)*h(i);
    end
end

% Transportation Cost
trRange=[0 501 1001 2001 5001 10001 20001 2000000];

%case 60 , 65 , 85
trPrice = [2.90 2.57 2.35 2.07 1.57 1.38 0.78 0.78;
3.25 2.67 2.55 2.46 1.88 1.64 0.99 0.99;
4.00 3.70 3.20 2.50 2.16 2.03 1.09 1.09];
s = zeros(3,12);
R = zeros(3,12);

for j= 1:12
    s(1,j) = (w(1)* q(1,j)) + (w(5) * q(5,j)) + (w(8) *
q(8,j)) + (w(10)* q(10,j));
    for k= 1:7
        if s(1,j) >= trRange(1,k) && s(1,j) <
trRange(1,k+1)
            R(1,j) = 0.01 * (min ( (trRange(1,k+1)
*trPrice(1,k+1)), (s(1,j)*trPrice(1,k))));
                end
    end
end
```

```
for j= 1:12
    s(2,j) = (w(2)* q(2,j)) + (w(3) * q(3,j)) + (w(7) *
q(7,j));
    for k= 1:7
        if s(2,j) >= trRange(1,k) && s(2,j) <
trRange(1,k+1)
                 R(2,j) = 0.01 * min (trRange(1,k+1)
*trPrice(2,k+1), s(2,j)*trPrice(2,k));

        end
    end
end

for j= 1:12
    s(3,j) = w(4)* q(4,j) + w(6) * q(6,j) + w(9) * q(9,j);
    for k=2:8
        if s(3,j) >= trRange(1,k) && s(3,j) <
trRange(1,k+1)
                R(3,j) =  0.01 * min ( trRange(1,k+1)
*trPrice(3,k+1), s(3,j)*trPrice(3,k));

        end
    end
end

% function F: totalcost(q)
  Kol= P + O + H ;
  TT =  sum(Kol,2);
  Inventory = sum(TT)+sum(Helzero);
  TrTotalC = sum(R,2);
  Transportation = sum(TrTotalC);
         F = Inventory + Transportation ;
  end
```

```
    %Lower bound
   Lower = [41   84  127 170 213 256 299 342 385 428 471
471 5   730 730 730 730 730 730 730 730 730 730 730 17
367 367 367 368 368 368 368 368 368 368 368 20  90  110
127 147 164 184 201 205 205 205 205 22  1222    1222
1491    1491    1491    1491    1491    1491    1491
1491    1491 17 238 257 313 354 405 453 453 453 453 453
453 0   728 728 728 728 728 728 728 728 728 728 728 700
3432    3432    3432    3432    3432    3432    3432
3432    3432    3432    3432 202    321 422 523 624 725
826 927 1028    1129    1212    1212 122    1087    1087
1087    1087    1087    1087    1087    1087    1087
1087    1087];


  %upper bound
  Upper = [78   121 164 207 250 293 336 379 422 465 471
471 730 730 730 730 730 730 730 730 730 730 730 730 367
367 367 368 368 368 368 368 368 368 368 368 90  110 127
147 164 184 201 205 205 205 205 205 1222    1222    1491
1491    1491    1491    1491    1491    1491    1491
1491    1491 238    257 313 354 405 453 453 453 453 453
453 453 728 728 728 728 728 728 728 728 728 728 728 728
3432    3432    3432    3432    3432    3432    3432
3432    3432    3432    3432    3432 321    422 523 624
725 826 927 1028    1129    1212    1212    1212 1087
1087    1087    1087    1087    1087    1087    1087
1087    1087    1087    1087];

            Q(:,1)=Lower;
            Q(:,2)=Upper;
            Best_Fitness=Inf;

for Run=1:100
clc;
```

```
Run
options = gaoptimset('Generations',3000
,'PopulationSize',5000, 'EliteCount',5);

    [x,fval,exitflag,output,population,scores] =
ga(@TotalCost,120,[],[],[],[],Q(:,1),Q(:,2),[],[1:120],opt
ions);

    for i=1:10
        for j=1:12

            Results(i,j)= x((i-1)*12+j);
        end
    end

    T(Run)=fval;
    if Best_Fitness>fval
        Best_Fitness=fval;
                Best_Q=Results;
    end
end
   Best_Q2(:,1)=Best_Q(:,1);
for j=2:12
     Best_Q2(:,j)=Best_Q(:,j)-Best_Q(:,j-1);
 end
```

Vita Auctoris

NAME	Hasti Eiliat
PLACE OF BIRTH	Tehran, Iran
YEAR OF BIRTH	1988
EDUCATION	University of Windsor, Windsor, Ontario, Canada M.A.Sc, Industrial and Manufacturing Systems Engineering Summer 2011 - Summer 2013 GPA: 12.5 out of 13 Thesis: Optimization of Inventory and Transportation Costs
	Azad University of Tehran, Tehran, Iran Bachelor of Science, Industrial Engineering, 2006 - 2010 GPA: A- Thesis: Systematic Layout Planning in a Plastic Factory
	Bahar College, Tehran, Iran Pre-university Degree in Mathematics, 2005 - 2006 GPA: A+
SCHOLARSHIP	Granted International Scholarship, University of Windsor $6000, 2 years, Summer 2011- Winter 2013
Employment Experience	Teaching Assistant, University of Windsor:
	Technical Communications in Winter 2013
	Engineering Economics in Fall 2012 and 2011
	Systems Analysis and Design in Fall 2012
	Treatment of Experiment Data in Winter 2012
	Co-op Student, Shadi Plastic Factory, Tehran, Iran, 2010