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# City Guarding with Cameras of Bounded Field of View

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# City Guarding with Cameras of Bounded Field of View

By

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City Guarding with Cameras of Bounded Field of View

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DECLARATION OF CO-AUTHORSHIP AND PREVIOUS PUBLICATION

## I. Co-Authorship

I hereby declare that this thesis incorporates material that is the result of joint research, as follows:

Chapter 2 of the thesis contains the results of a collaborative publication with Dr. Ahmad Biniaz as a co-author. Within this chapter, Dr. Biniaz initially suggested the idea of addressing the city guarding problem, and we proposed a solution approach for this problem. We then collaboratively authored the paper and proofread it before its submission for publication.

Chapter 3 of the thesis incorporates unpublished material co-authored with Dr. Ahmad Biniaz. In Chapter 2, we addressed two problems and proposed the idea that these two could be generalized in terms of the shape of the buildings' bases or polygons. Furthermore, we were responsible for writing up the results and proofreading them.

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| Thesis Chapter | Publication title/full citation | Publication Status |
|---|---|---|
| Chapter 2 | City Guarding with Cameras of Bounded Field of View<br>A. Biniaz, M. Hashemi<br>In proceedings of the 35th Canadian Conference on Computational Geometry (CCCG), pp. 71–76, 2023. | Published |

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ABSTRACT

We study two problems related to the City Guarding and the Art Gallery problems.

1. Given a city with $k$ rectangular buildings, we prove that $3k+1$ cameras of $180^\circ$ field of view (*half-sphere guards*) are always sufficient to guard the free space (the ground, walls, roofs, and the sky). This answers a conjecture of Daescu and Malik (CCCG, 2020).

2. Given $k$ orthogonally convex polygons of total $m$ vertices in the plane, we prove that $\frac{m}{2}+k+1$ cameras of $180^\circ$ field of view are always sufficient to guard the free space (avoiding all the polygons). This answers another conjecture of Daescu and Malik (Theoretical Computer Science, 2021).

Both upper bounds are tight in the sense that there are input instances that require these many cameras. Our proofs are constructive and suggest simple polynomial-time algorithms for placing these many cameras.

We then generalize each of the two mentioned problems in some sense with tight bounds.

1. Given a city involving $k$ buildings with any convex-shape base and a total of $m$ top corners, we prove that $m-k+1$ cameras of $180^\circ$ field of view (*half-sphere guards*) are sometimes necessary and always sufficient to guard the ground, walls, roofs, and the sky of the city.

2. Given $k$ simple polygons (convex or non-convex) of total $m$ vertices in the plane which contains $r$ reflex vertices, we prove that $m-k-r+1$ cameras of $180^\circ$ field of view are sometimes necessary and always sufficient to guard the free space.

# DEDICATION

To my friends, countrymen, and women who are enduring challenging living conditions, particularly economic hardships. I salute those who are committed to the cause of freedom, tirelessly persevering in their fight for liberty until the end.

## ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisor, Dr. Biniaz, for his invaluable support and encouragement. I also appreciate Dr. Urbanic and Dr. Mukhopadhyay for their invaluable contributions as members of my thesis committee.

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# *Introduction*

The increasing availability and affordability of modern technologies have led to a growing interest in the use of drones, or unmanned aerial vehicles (UAVs), in metropolitan areas. The technology has advanced to a point where UAVs are now being used for a wide range of applications, from delivery services and emergency response to aerial photography and surveying. Although these uncrewed aerial objects are approvingly advanced in terms of security, they may cause safety issues in the cities, making monitoring the whole city space (ground and sky) crucial. Therefore, this is an issue that needs to be addressed adequately.

On the other hand, fixed cameras are used widely all over the world to monitor the streets and buildings in cities. These cameras usually monitor the ground and walls, but changing the placement and field of view lets us use them to guard the total space of the city. While using cameras is an excellent solution to the guarding problem, the budget is always limited, and the cost of digital devices is relatively high. So, the goal is to minimize the number of cameras required to accomplish the job.

The problem of monitoring the entire space with minimum number of cameras is usually referred to as the *City Guarding* problem in computational geometry.

## 1.1 Basic Terminology and Concepts

Before diving into the topic of City Guarding as a geometric problem, it is important to first understand the main parts involved. This problem is a version of the famous Art Gallery problem, which both fall under the category of visibility problems. The Art Gallery problem is a well-studied problem that was first posed in 1973 by Victor Klee. In this problem, we are given an art gallery in the form of a simple polygon in 2D, and the goal is to place the minimum number of guards/cameras to cover the entire polygon (interior of the gallery) [11]. In other words, each point of the polygon is visible by some guard. A point $p$ is said to be *visible* by a guard $g$ if the line segment $pg$ lies inside the polygon.

In these kinds of problems, there are essential elements like the guards, the area they need to watch or guard, and anything that might block their view. Each of these parts has its own special features based on how we define the problem, which helps us decide which specific version of the problem we are looking at.

Here are some of the key parts and their features:

- Visibility Region: The area which the goal of the problem is to guard. In different versions of visibility problems, this region is mostly categorized as follows:
  - The interior space of a polygon or polyhedron. (e.g., the gallery's interior) See Figure 1.1.1.
  - The exterior space of a polygon or polyhedron. (e.g., the free space around a fortress)

  There are problems where we want to guard both the interior and exterior of a polygon (e.g., Prison Yard Problem).

- Guards: Cameras or persons who are monitoring the region we want to be guarded. Based on the possible places we can put a guard, it can be categorized into the following groups:

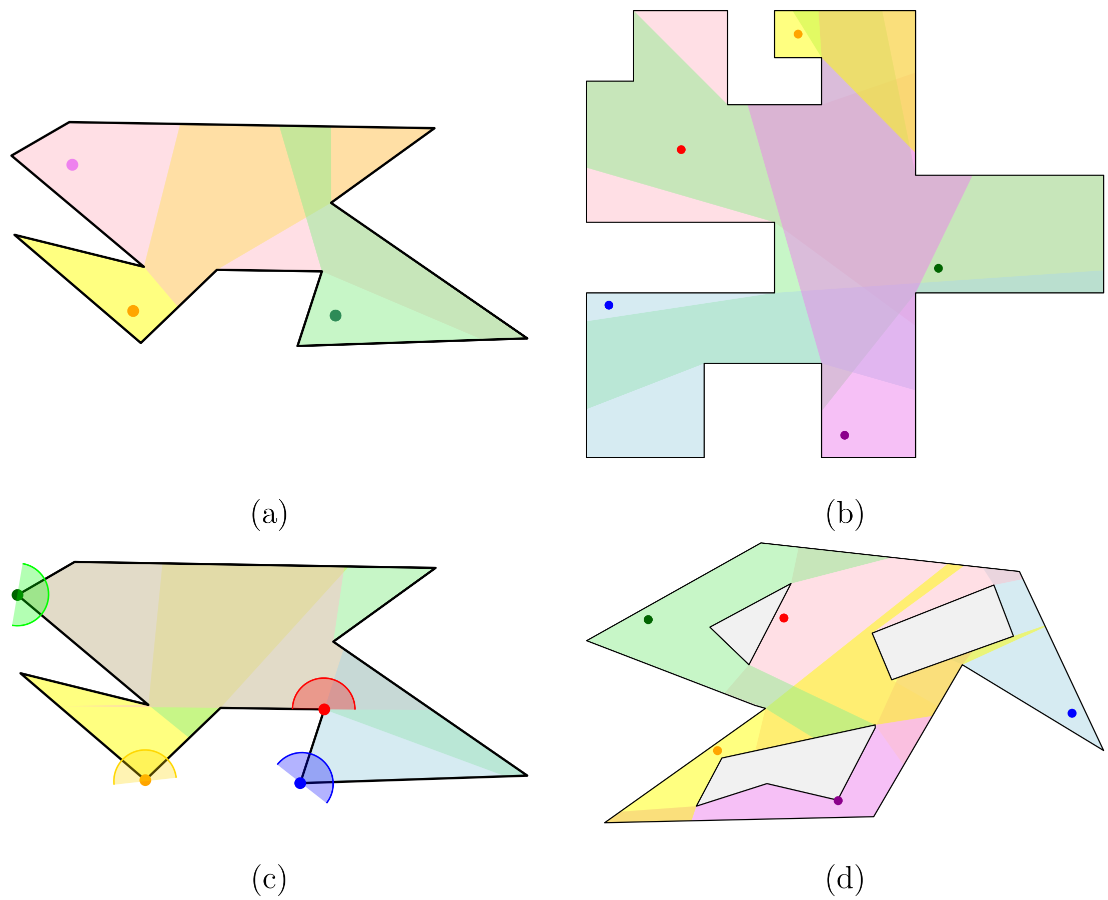


Fig. 1.1.1: Some Art Gallery problem variants: (a) Point guards (360°). (b) Orthogonal polygon & point guards (360°). (c) Vertex guards (180°). (d) Polygon with holes & point guards (360°).

  - Point Guard: to be placed at any point inside or outside the visibility region (e.g., polygon). See Figures 1.1.1(a), (b), and (d).
  - Vertex Guard: to be placed only at the vertices of the polygon or polyhedron. See Figure 1.1.1(c).
  - Edge Guard: to be placed only on the edges of a polygon or polyhedron. There is also another definition for edge guard where the whole edge represents a guard (like having guards on every point on the edge).
  - Mobile Guard: They are permitted to traverse enclosed line segments (edge or diagonal) within a polygon or polyhedron.

- Obstacles: All objects that obstruct the view of guards. In terms of the Art Gallery problem, the obstacles are some simple polygons, which are called

"Holes," meaning that there is an area inside the main polygon that does not belong to the polygon. Essentially, we have two types of problems in this regard. They either contain holes or do not. See figure 1.1.1(d).

- Field of View and Direction of the Guards: Generally, guards can see 360°; however, in some variants, we can restrict them to only observe a smaller angle like 180°. See figure 1.1.1(c). The field of view can be only horizontal (in 2D problems) or both horizontal and vertical (in higher dimensional problems). If we have any restriction on the field of view of the guards, meaning that it is smaller than 360°, we need to determine each guard is looking toward which direction.

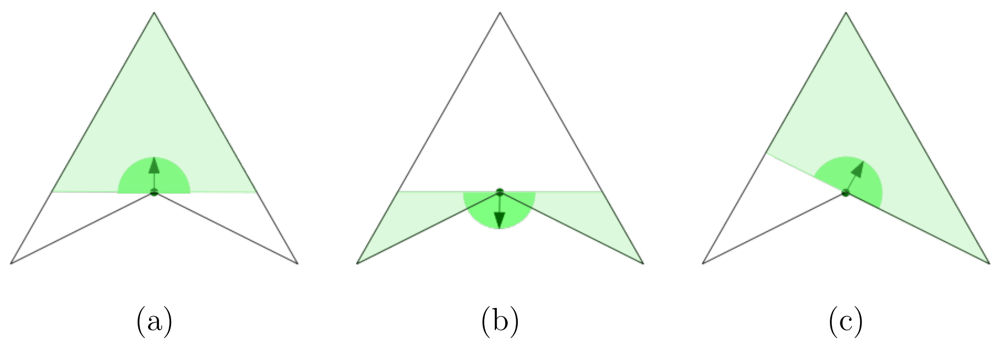


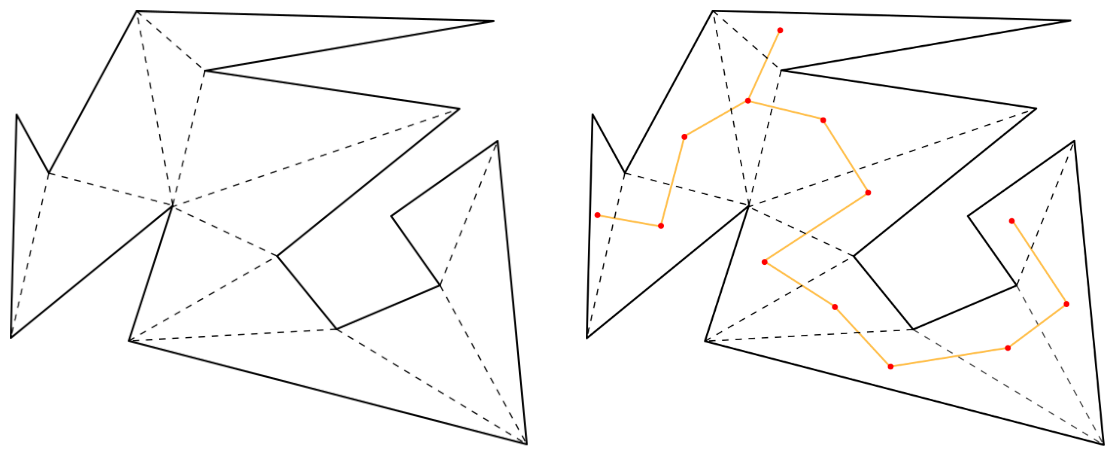

Fig. 1.1.2: Guards' Alignments: (a) Inward-facing. (b) Outward-facing. (c) Edge-aligned.

- Guards' Alignment: If we have vertex guards and the field of view is restricted, we may want to restrict the alignment of the guards as well. There are three possible alignments:
  - Inward aligned (Inward-facing): The guard is placed at vertex $v$, watching through the interior of polygon $P$, ensuring that the interior remains disjoint from the two sides of $P$ incident to $v$. See Figure 1.1.2(a).
  - Outward aligned (Outward-facing): The guard is placed at vertex $v$, with a view into the interior and exterior of polygon $P$, containing the two sides of $P$ incident to $v$. See Figure 1.1.2(b).
  - Edge aligned: The guard is placed at vertex $v$, looking toward the interior

of polygon $P$, aligned with one of the two sides of $P$ incident to $v$. See Figure 1.1.2(c).

In general, we may use any of these alignments, meaning that we do not restrict the alignment of the guards.

Now that we know about the Art Gallery problem and some essential properties involved in this problem and its variants, it is worth mentioning famous approaches or methods that help researchers investigate this problem.



(a) (b)

Fig. 1.1.3: (a) Polygon triangulation. (b) Dual graph of the triangulation.

*Triangulation* is one of the most crucial techniques that is pivotal in addressing the Art Gallery problem, as various bounds are established by leveraging the insights gained from polygon triangulation.

A polygon $P$ is considered triangulated when it is divided into a collection of non-overlapping interior triangles. These triangles consist of edges from the original polygon $P$ or internal diagonals connecting two distinct vertices of $P$. See Figure 1.1.3(a). There are significant works that used triangulation in their proofs for the Art Gallery problem, like [8, 5, 11].

The triangulation of a polygon exhibits intriguing characteristics, and one notable aspect is the ability to create a dual graph based on this triangulation. In this graph, triangles serve as the vertices and an edge exists between two vertices if their corresponding triangles share either a diagonal or a side. A noteworthy attribute of the dual graph derived from polygon triangulation is its tree-like structure. See Figure 1.1.3(b).

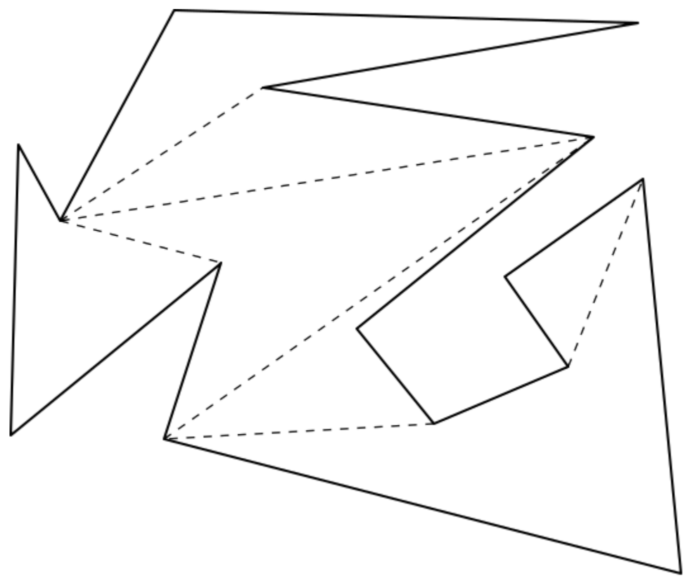


Fig. 1.1.4: Psuedo-triangulation of a polygon

Other related concepts include *Pseudo-triangulation*, where the polygon is subdivided into pseudo-triangles. A pseudo-triangle is defined as a polygon with precisely three convex vertices, referred to as corners. See Figure 1.1.4. Obviously, a triangle is a pseudo-triangle as well. Pseudo-triangulation was first introduced and utilized by Pocchiola and Vetger [14]. Other remarkable works like the work by Speckmann and Tóth employed Pseudo-triangulation [16].

*Convex partitioning* of the polygon is another helpful technique in solving the Art Gallery problem variants, which was first introduced by Chazelle [4]. The procedure of this technique is straightforward. We only need to extend a diagonal from each reflex vertex in any order to make two convex vertices out of them. This method entails dividing the polygon into convex regions, characterized by the unique property that every point within these regions is visible from any point inside or on the boundary of the respective region. Through convex partitioning, the resulting regions are such that each requires only one guard for monitoring. See Figure 1.1.5.

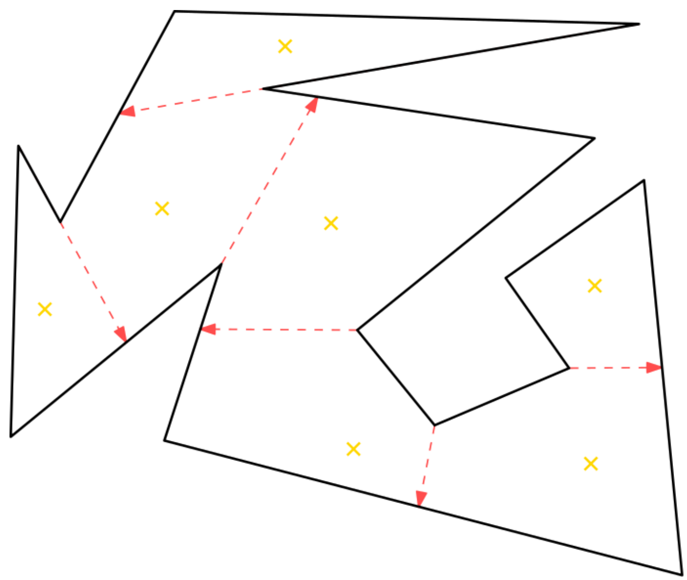


Fig. 1.1.5: Convex partitioning of a polygon

## 1.2 Background

To the best of our knowledge, the problems related to guarding cities were first introduced by Bao et. al [2]. They introduced three different variants of the problem where the goal is to guard (1) only the roofs of the buildings, (2) the walls of the buildings and the ground, and (3) the roofs, walls, and the ground. This latter version is called "City Guarding".

According to Bao et al. [2] the City Guarding problem can be interpreted as a 2.5-dimensional version of the well-studied Art Gallery problem. As we stated, in the standard Art Gallery problem, we are given a simple polygon and the goal is to place the minimum number of guards/cameras to cover the entire polygon [11]. The Art Gallery problem and its variations have been well-studied in recent years [19]. The variations usually enforce constraints on the shape of the polygon, the existence of holes, the shape of holes, the orientation of holes, locations of guards, guards' field and range of vision, to name a few. We will mention important related ones in the next section and elaborate on some of them. The City Guarding problem has the same flavor as the Art Gallery problem with rectangular holes.

The City Guarding also has the same flavor as a free-space illuminating problem, studied by Blanco et al. [3], in which the input consists of pairwise disjoint rectangles in the plane and the goal is to place minimum number of lights at the corners of the rectangles to light up the free space (the entire plane minus the rectangles).

In the City Guarding problem, we should take into account many factors, such as the city's layout, buildings' orientation, and the cameras' field of view. These factors usually led to different variations of the City Guarding problem.

In the next section, we will elaborate on the works on City Guarding, each of which has studied a version considering some factors and constraints.

What we have studied (in Chapter 2) is a version of the City Guarding problem that is introduced by Daescu and Malik [6]: *Given $k$ pairwise disjoint rectangular-base buildings with arbitrary orientation, find a minimum number of cameras that guard the city such that (i) each camera is a half-sphere with* $180°$ *field of view and infinite range, and (ii) each camera is placed at a corner on top of the roof of a building in a direction orthogonal to a wall.*

We have also studied another problem by Daescu and Malik [7]: *Given $k$ disjoint arbitrary oriented orthogonally convex polygons with a total of $m$ vertices, find a minimum number of guards required to guard the free space and the boundaries of the polygons while the field of vision of each guard is limited to* $180°$.

## 1.3 Related Works

In this section, our initial topic of discussion will be the Art Gallery problem and its related versions to the City Guarding problem to give the reader a sense of how guarding problems have been extensively studied over the past few decades and how these two problems are closely related.

### 1.3.1 Art Gallery Problem

As previously stated, Victor Klee posed the Art Gallery problem to Chvátal in 1973 [11]. In 1975, Chvátal answered Klee's question by demonstrating that $\lfloor \frac{n}{3} \rfloor$ vertex guards are sufficient with $n$ representing the number of vertices [5]. The approach of

Chvátal was a bit sophisticated. So, later on in 1978, Fisk proved the same bound of $\lfloor \frac{n}{3} \rfloor$ vertex guards by applying a 3-coloring scheme to the triangulation of the polygon and positioning the guards on the vertices with the fewest occurrences of color [8]. This gave a simpler proof.

In 1980, Chazelle established his naive convex partitioning, asserting that any polygon can be divided into a maximum of $r + 1$ convex segments [4]. Subsequently, in 1982, O'Rourke utilized this finding to demonstrate that, for a simple $n$-gon with $r > 1$ reflex vertices, $r$ guards are sometimes necessary and always sufficient for guarding the polygon's interior [11].

We previously highlighted the close connection between City Guarding and the Art Gallery problem, specifically, the variant involving polygons with holes, as depicted in Figure 1.1.1(d). Some variants of City Guarding can be reduced to this version of the Art Gallery.

Regarding the polygon with holes version, O'Rourke established a sufficiency bound of $\lfloor \frac{n+2h}{3} \rfloor$ vertex guards using the dual graph of the polygon with holes triangulation in 1983 [11], and in 1987, Shermer conjectured that the sufficiency bound is $\lfloor \frac{n+h}{3} \rfloor$ vertex guards and proved it for $h = 1$ [15]. This conjecture is still open after more than 37 years. Although the conjecture is not proven yet, in 1990, for *point guards* version of the problem, Hoffmann et al. demonstrated a sufficient bound of $\lfloor \frac{n+h}{3} \rfloor$ for *point guards* [9], where $n$ is the sum of vertices of the polygon and holes, and $h$ is the number of holes.

In 2000, Tóth showed that for the $\pi$ field-of-view version, $\lfloor \frac{n}{3} \rfloor$ point guards with $n$ as the number of vertices is sufficient [18]. Tóth also established some lower bounds for the versions with a range of vision alpha that is less than $\pi$ in 2002 [17]. Although the bound of $\lfloor \frac{n}{3} \rfloor$ was the same bound of Chvátal and Fisk regardless of the restriction on the field of view, Tóth had used *point guards* for guarding the polygon, and the bound for vertex guards version could have been different. Therefore, in 2005, Speckmann and Tóth proved a bound for vertex guards, saying that any simple polygon with $n$ vertices, $k$ of which are convex, can be monitored by at most $\lfloor \frac{2n-k}{3} \rfloor$ *edge-aligned* vertex $\pi$-guards [16].

To achieve this, they employed a specific type of pseudo-triangulation known as *pointed pseudo-triangulation*, which minimizes the number of pseudo-triangles within the polygon to $k-2$. They then demonstrated that each pseudo-triangle can be guarded using $\lfloor \frac{2\ell-3}{3} \rfloor$ guards where $\ell$ is the number of vertices in the pseudo-triangle. Since the dual graph of pseudo-triangulation is a tree, Spekmann and Tóth utilized a directed approach to identify three potential guard sets. Each of these sets was capable of overseeing all the pseudo-triangles, and they showed that the union of these sets encompassed all the vertices of each pseudo-triangle. Through detailed calculations, they determined that the total number of vertices guarded across all three sets amounted to $2n-k$. Consequently, by selecting one of the three sets with the minimum cardinality, they achieved a guard placement of $\lfloor \frac{2n-k}{3} \rfloor$.

The Art Gallery problem has been well-studied in the past, and we just explained some important and related versions here. There is a rich literature on the Art Gallery problem for which we refer the reader to [1, 3, 4, 5, 9, 11, 12, 16, 18, 17].

### 1.3.2 City Guarding

To the best of our knowledge there are two works giving bounds for the City Guarding problem. The first one is the work by Bao et al. [2], and the latter is a work by Daescu and Malik [6].

Bao et al. [2] studied the City Guarding problem for a city with a rectangular border containing $k$ rectangular-base buildings that are orthogonal (to the city boundary) with arbitrary positive widths, lengths, and heights. In their study, the cameras are assumed to have 360° field of view and be positioned only at the top corners of the buildings or the four corners of the city border. They showed that $\lfloor \frac{2(k-1)}{3} \rfloor + 1$ guards are always sufficient and sometimes necessary to guard the roofs. They also showed that $k + \lfloor \frac{k}{4} \rfloor + 1$ guards are sufficient to guard walls and ground. For the city guarding (roofs, walls, the ground), they showed the sufficiency of $k + \lfloor \frac{k}{2} \rfloor + 1$ guards.

For Roof Guarding, they made the assumption that the building heights can be arranged in a way that avoids cycles for roof visibility. This allowed them to simplify the problem by transforming it into a graph. In this graph, each roof is denoted by a node, and if a potential guard position on the roof $a$ can monitor the roof $b$ completely, a directed edge from the roof $a$ to roof $b$ is added. Due to the ordered heights of the buildings, the resulting graph takes the form of a directed acyclic graph (DAG).

In Ground and Wall Guarding, they simply reduced the problem to the Art Gallery problem for orthogonal polygons with holes. it can be done by projecting each building vertically to the ground, which gives $k$ rectangular holes inside a rectangle (the city border) as the polygon. According to Hoffmann et al. results for this reduced problem, $\lfloor \frac{n+h}{4} \rfloor$ vertex guards can always monitor an orthogonal polygon with $h$ holes, where $n$ is the total number of vertices of polygon and holes [10]. Bao et al. applied this bound to the Ground and Wall Guarding problem where $n = 4 + 4k$ and $k$ is the number of buildings (which are the $h$ holes, respectively). So, by calculation, $1 + k + \lfloor \frac{k}{4} \rfloor$ is achievable.

In addressing the City Guarding variant, they applied an approach based on O'Rourke's method of partitioning orthogonal polygons into *L-shaped polygons* [13, 11]. They projected the buildings to the ground like the previous variant to achieve rectangular holes. Then, they used a vertical cut to connect the top right vertex of each hole to the enclosing rectangle to obtain an orthogonal polygon $P$ without holes. After this, Bao et al. conducted partitioning using the adapted L-shaped partition algorithm as outlined in [13, 11]. For each histogram, they sequentially numbered all reflex vertices from right to left, starting from 1. Subsequently, they partitioned each histogram by introducing vertical cuts at every even-numbered indexed reflex vertex. The outcomes of these steps resulted in a collection of $k + \lfloor \frac{k}{2} \rfloor + 1$ L-shaped orthogonal polygons, for which they then allocated guards to the possible positions on the right.

Recently, Daescu and Malik [6] studied the City Guarding problem for cameras with 180° field of view. They explored two versions: (a) axis-aligned buildings and (b) arbitrary-orientated buildings. They proved that $2k+\lfloor \frac{k}{4} \rfloor+4$ cameras are sufficient to guard axis-aligned buildings. For arbitrarily oriented buildings, they gave an example that requires $3k+1$ cameras for any $k \geq 1$. They conjectured that $3k+1$ cameras are also sufficient. See Figure 1.4.1(a) for an example of arbitrary-oriented rectangular buildings.

The versions of the problem they have explored differ from the version studied by Bao et al., primarily due to a crucial assumption regarding guard placement. While Bao et al. positioned guards on the top corners of buildings and the four corners of the city, Daescu and Malik constrained guard placement exclusively to the top corners of buildings. This exclusion of city border's corners differentiates the new version from the former version notably.

Additionally, considering the cameras' 180° field of view, it is essential to specify the alignment of the guards. In this version, each guard is positioned in a way that the visible and obscured regions are divided by a vertical plane parallel to one of the building's walls where the camera is installed. If we imagine buildings to be projected into the plane, we can say the guards are *edge-aligned*.

It is worth mentioning that the field of view is restricted both horizontally and vertically, and each guard is in the form of a half-sphere.

For problem version (a), in which the structure of the city is similar to Bao et al., Daescu and Malik proved a tight bound of $k$ for the Roof Guarding variant. Sufficiency of the bound is obvious as we have $k$ buildings. They proved the necessity of the bound by giving an example of ordered buildings in terms of heights from high to low in a way that each building blocks the vision of the previous building' guards over the next buildings. You can see this example and possible guard placements for each of two consecutive buildings in Figure 1.3.1. As version (a) can be categorized as a special case of version (b), this tight bound can be applied to both of the versions.

Daescu and Malik established a theorem that enables a more focused approach to guard by specifically targeting the roofs, walls, and ground of buildings. According to this theorem, if guards are positioned to cover the roofs, walls, and ground of the city, then every point within the city's aerial space is monitored.

Working on version (a) by using the aforementioned theorem, they concentrated on guarding the walls and the ground in a way that the guards placement maintains guarding roofs as well. They employed the strategy of constructing staircases and

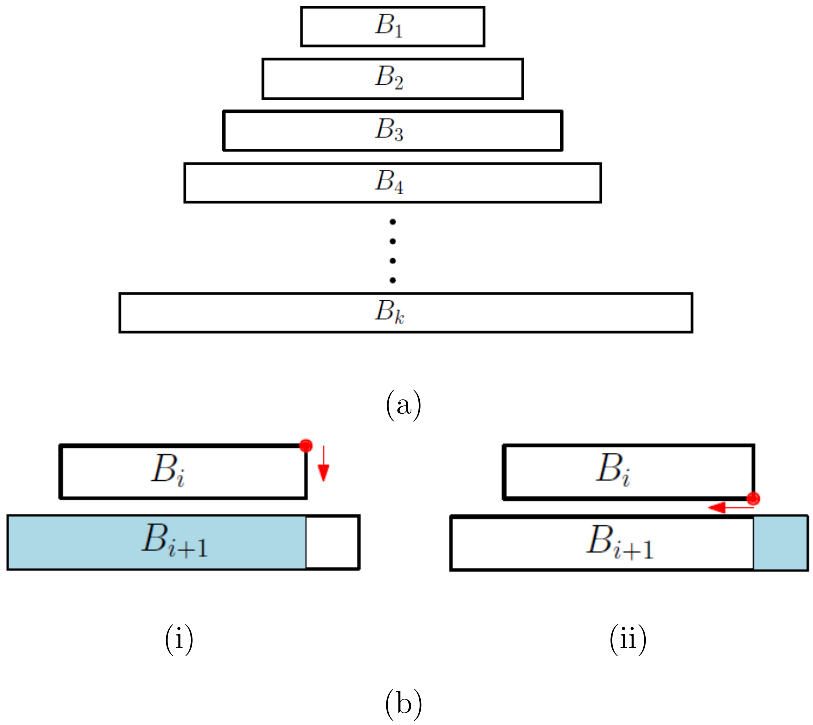



Fig. 1.3.1: (a) Configuration of buildings. (b) Potential guard positions, trying to see both of the buildings; borrowed from [6].

extending walls to establish the sufficiency bound of $2k + \lfloor \frac{k}{4} \rfloor + 4$. Various possible staircases are illustrated in Figure 1.3.2 sourced from [6]. Identifying the staircase with the minimum number of involved buildings, denoted as $\delta$ to be equal to $\lfloor \frac{k}{4} \rfloor + 3$, they placed $\delta + 1 = \lfloor \frac{k}{4} \rfloor + 4$ cameras to guard the staircase. Subsequently, by extending the walls of all the buildings in the opposite direction of the staircase, they demonstrated that $2k$ cameras suffice to guard the remaining regions. The guards are positioned, ensuring that each building has a guard that is looking at both its roof and the designated region it is assigned to monitor. This resulted in a total bound of $2k + \lfloor \frac{k}{4} \rfloor + 4$. Importantly, this bound remains consistent for both the Walls and Ground Guarding variant and the City Guarding variant.

In version (b), where buildings have arbitrary orientations, Daescu and Malik provided an example to establish the necessity bound of $3k+1$ cameras. The illustrated example can be seen in Figure 1.3.3. In this scenario, $B_i$ is situated within the span of $B_j$ for all $j < i$. None of the edges of $B_i$ are visible, either partially or completely,

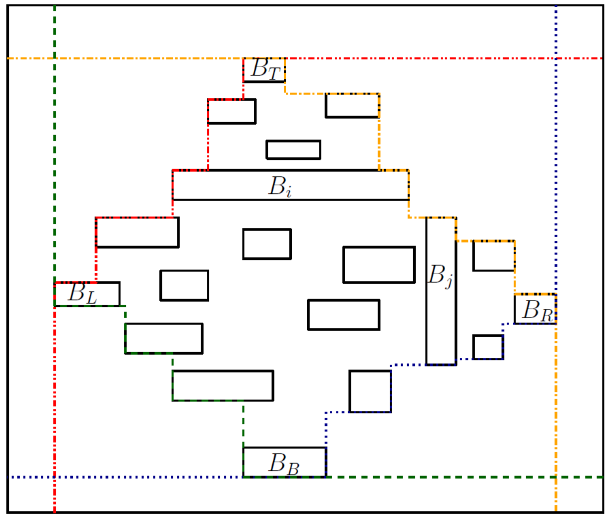



Fig. 1.3.2: Various staircases, which are different in orientation, are shown with red dash-dot, orange dash-dot, blue dotted, and green dashed; borrowed from [6].

from any vertex of $B_j$, where $j < i-1$, and from any vertex of $B_m$, where $m > i+1$. For each potential position of a vertex guard on $B_i$, the guard can observe at most one edge of $B_{i+1}$. Essentially, there is no guard position on $B_i$ from which both an edge of $B_{i-1}$ and an edge of $B_i$ are visible. Given this setup, they demonstrated that each area between two successive buildings, encompassing a wall of $B_i$ and two walls of $B_{i+1}$, can be guarded by at least two guards leading to $2(k-1)$ cameras. Furthermore, each left wall of a building necessitates at least one guard (since guards in the intermediate regions cannot monitor them), requiring an additional $k$ cameras. The top and right walls of $B_1$ and the lower wall of $B_k$ remain uncovered, each requiring a camera, adding a total of 3 more cameras. Consequently, they reached the necessity bound of $2(k-1)+k+3 = 3k+1$.

In a companion paper, Daescu and Malik [7] studied another problem of the same flavor; guard free space formed by orthogonally convex polygons. Given $k$ pairwise disjoint orthogonally convex polygons with total $m$ vertices, the goal is to place cameras of 180° field of view to guard the free space and the boundaries of the polygons (cameras should be placed at corners of polygons and orthogonal to its sides). An *orthogonal polygon* is a polygon whose edges are orthogonal to each other (not necessarily orthogonal to the $xy$-axis). An orthogonal polygon is *orthogonally*

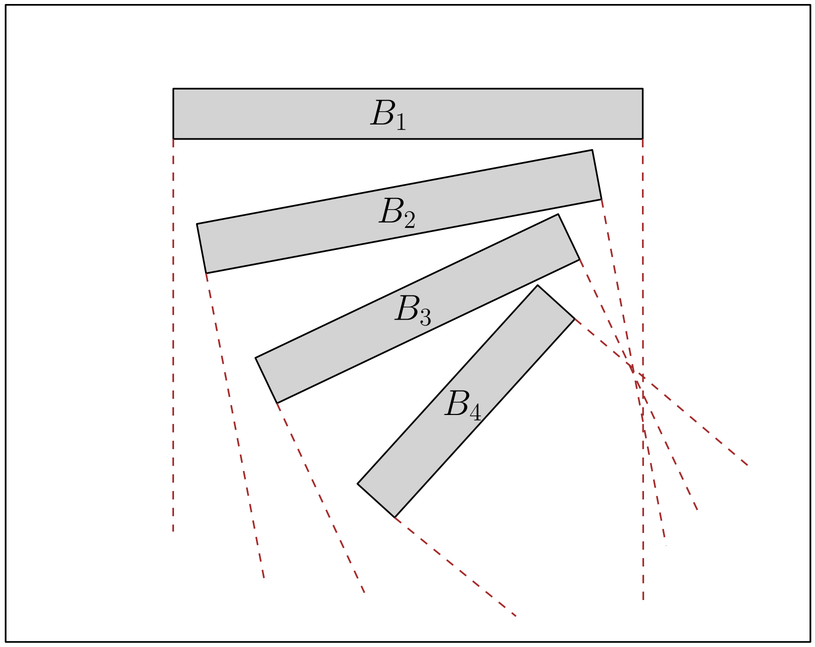



Fig. 1.3.3: A city with $k = 4$ buildings that needs $3k + 1$ guards; borrowed from [6].

*convex* if its intersection with any line orthogonal to its edges is either empty or a single line segment; see for example, polygon $C$ in Figure 1.4.1(b). Daescu and Malik showed that for axis-aligned polygons $\frac{m}{2} + \lfloor \frac{k}{4} \rfloor + 4$ cameras are always sufficient, and for arbitrary-oriented polygons $\frac{m}{2} + k + 1$ cameras are sometimes necessary for any $k \geq 1$ and any valid $m$. They conjectured that $\frac{m}{2} + k + 1$ cameras are also sufficient. See Figure 1.4.1(b) for an example of arbitrary-oriented orthogonally convex polygons.

The primary distinction of Guarding Orthogonally Convex Polygons, setting it apart from the City Guarding problem (Walls and Ground variant), is that, unlike dealing with $k$ specific shapes like rectangles, each with precisely four vertices, orthogonally convex polygons vary in the number of vertices, and these vertices can be either reflex or convex. Hence, this latter problem is more generalized than City Guarding in the context of guarding free space. Correspondingly, as we mentioned, in tackling this problem, Daescu and Malik generalized their approach of using staircases and extensions on the orthogonally convex polygons utilizing these polygons' special characteristic, that is, the count of convex vertices for each polygon equals to $\frac{m_i}{2} + 2$, where $m_i$ represents its number of vertices. Likewise, the number of reflex vertices is $\frac{m_i}{2} - 2$. Therefore, the total number of convex vertices and reflex vertices can be calculated as follows:

$$c = m - r = \sum_{i=1}^{k} m_i - r_i = \sum_{i=1}^{k} m_i - (\frac{m_i}{2} - 2) \sum_{i=1}^{k} \frac{m_i}{2} + 2 = \frac{m}{2} + 2k$$

$$r = m - c = m - (\frac{m}{2} + 2k) = \frac{m}{2} - 2k$$

where $c$ is the total number of convex vertices, and $r$ is the total number of reflex vertices.

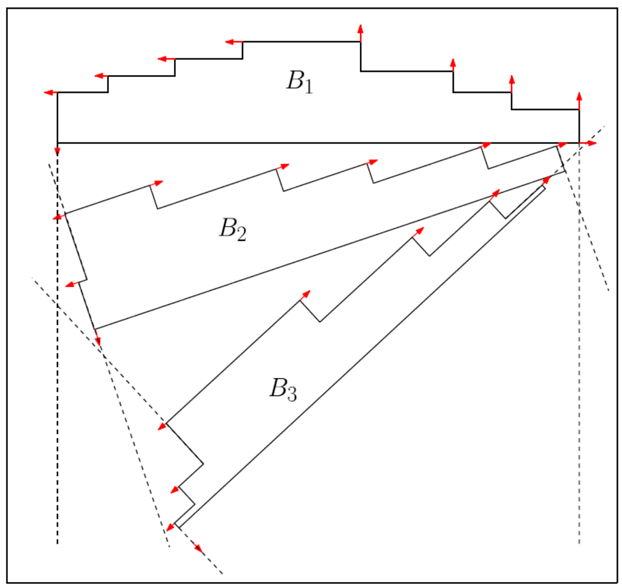
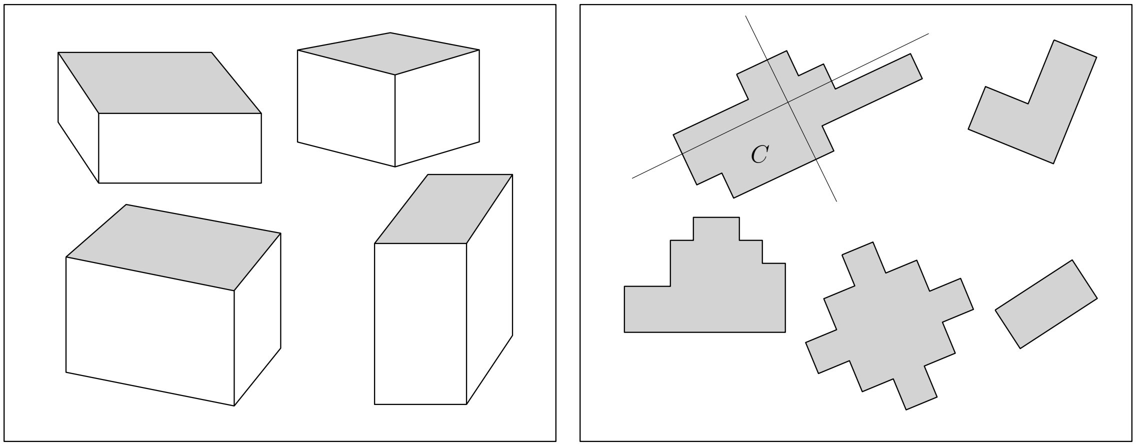



Fig. 1.3.4: A configuration of $k = 3$ polygons with $m = 44$ that needs $\frac{m}{2} + k + 1 = 26$ guards; sourced from [7]

Moreover, the proposed example of polygons' configuration and the given argument for delivering the necessity bound for the arbitrary version of this problem is similar to the one for City Guarding, but in this case, they employ orthogonally convex polygons instead of rectangles. The example is shown in Figure 1.3.4.




(a) (b)

Fig. 1.4.1: (a) A city with rectangular buildings. (b) Orthogonally convex polygons.

## 1.4 Our Contributions

In Chapter 2, We prove both conjectures of Daescu and Malik [6, 7] that $3k+1$ cameras are sufficient to guard arbitrary-oriented rectangular buildings, and $\frac{m}{2}+k+1$ cameras are sufficient to guard arbitrary-oriented orthogonally convex polygons. Our proofs are constructive and suggest polynomial-time algorithms for finding these many guards. The two proofs share some similarities in the sense that both partition the free space into convex regions and then provide an upper bound for the number of these regions. We explain our proof for rectangular buildings first as it is easier to explain. Then, we give a short description of how to generalize it for monotone orthogonal polygons.

In Chapter 3, we generalize each of the two mentioned problems with tight bounds. We prove that $m-k+1$ cameras of $180°$ field of view (*half-sphere*) are sometimes necessary and always sufficient to guard the city containing $k$ buildings with any convex-shape base and a total of $m$ top corners, and $m-k-r+1$ such cameras are sometimes necessary and always sufficient to guard $k$ polygons (convex or non-convex) of total $m$ vertices in the plane which contains $r$ reflex vertices.

# References

[1] Mikkel Abrahamsen, Anna Adamaszek, and Tillmann Miltzow. "The art gallery problem is $\exists\mathbb{R}$-complete". In: *Proceedings of the 50th Annual ACM SIGACT Symposium on Theory of Computing*. 2018, 65–73.

[2] Lichen Bao, Sergey Bereg, Ovidiu Daescu, Simeon Ntafos, and Junqiang Zhou. "On some city guarding problems". In: *International Computing and Combinatorics Conference*. 2008, 600–610.

[3] Gregoria Blanco, Hazel Everett, Jesus Garcia Lopez, and Godfried Toussaint. "Illuminating the free space between quadrilaterals with point light sources". In: *Proceedings of Computer Graphics International, World Scientific*. 1994.

[4] Bernard Chazelle. "Computational geometry and convexity". In: 1980.

[5] Vasek Chvátal. "A combinatorial theorem in plane geometry". In: *Journal of Combinatorial Theory* Series B.18 (1975), 39–41.

[6] Ovidiu Daescu and Hemant Malik. "City guarding with limited field of view". In: *Proceedings of 32nd Canadian Conference on Computational Geometry (CCCG)*. 2020, 300–311.

[7] Ovidiu Daescu and Hemant Malik. "New bounds on guarding problems for orthogonal polygons in the plane using vertex guards with halfplane vision". In: *Theoretical Computer Science* 882 (2021), pp. 63–76.

[8] Steve Fisk. "A short proof of Chvátal's Watchman Theorem". In: *Journal of Combinatorial Theory, Series B* 24.3 (1978), p. 374.

[9] Frank Hoffmann. "On the rectilinear art gallery problem". In: *International Colloquium on Automata, Languages, and Programming*. Springer, 1990, pp. 717–728.

[10] Frank Hoffmann, Klaus Kriegel, and Csaba D. Tóth. "Vertex Guards in Reclinear Polygons with Holes". In: *Kyoto International Conference on Computational Geometry and Graph Theory*. 2007.

[11] Joseph O'Rourke. *Art Gallery Theorems and Algorithms*. Oxford University Press, Inc., 1987.

[12] Joseph O'Rourke. "Galleries need fewer mobile guards: A variation on Chvátal's theorem". In: *Geometriae Dedicata* 14 (1983), pp. 273–283.

[13] Joseph O'Rourke. "An alternate proof of the rectilinear art gallery theorem". In: *Journal of Geometry* 21.1 (1983), pp. 118–130.

[14] Michel Pocchiola and Gert Vegter. "The Visibility Complex". In: *International Journal of Computational Geometry; Applications* 06.03 (1996), pp. 279–308.

[15] Thomas Shermer. *Triangulation graphs that require extra guards.* Tech. rep. 3D-13. NYIT Computer Graphics Technical Report, 1984.

[16] Bettina Speckmann and Csaba D. Tóth. "Allocating Vertex $\pi$-Guards in Simple Polygons via Pseudo-Triangulations". In: *Discrete Computational Geometry* 33.2 (2004), pp. 345–364.

[17] Csaba D. Tóth. "Art galleries with guards of uniform range of vision". In: *Computational Geometry* 21.3 (2002), pp. 185–192.

[18] Csaba D. Tóth. "Art gallery problem with guards whose range of vision is 180". In: *Computational Geometry* 17.3 (2000), pp. 121–134.

[19] Jorge Urrutia. "Art gallery and illumination problems". In: *Handbook of computational geometry*. 2000, 973–1027.

# CHAPTER 2

# *City Guarding with Cameras of Bounded Field of View*

Ahmad Biniaz, Mohammad Hashemi

In Proceedings of the 35th Canadian Conference on Computational Geometry

In this Chapter, we first provide our proofs for the City Guarding problem, and then we will similarly present the proofs on Guarding Orthogonal Polygons.

## 2.1 City Guarding

In this section, we present our algorithm for the City Guarding problem. The following lemma, borrowed from [3], implies that to guard the entire space, it suffices to guard roofs, walls, and the ground. Therefore, in the algorithm, we focus on guarding roofs, walls, and the ground.

**Lemma 1** (Daescu and Malik [3])**.** *If in a city the roofs, walls, and the ground are guarded by a set of cameras, then every point in the aerial space of the city is visible by a camera.*

Recall that the city consists of $k$ arbitrary-oriented buildings with rectangular basis, and that the cameras have $180°$ field of view (*half-sphere*) and should be placed at corners on top of the roofs orthogonal to a wall. (We clarify that a camera could be placed in such a way that it sees the roof of the building, as in Figure 2.1.1.)

As we explained in the previous Chapter, Daescu and Malik [3] gave an example that requires $3k+1$ cameras. This example is given in Figure 1.3.3. They conjectured that the bound $3k + 1$ is tight.

We show how to guard the city with at most $3k+1$ cameras, and thus proving the conjecture of [3]. We project the buildings onto the plane to obtain rectangles (in dimension 2). Then, we guard the rectangles (representing roofs), their sides (representing walls), and the space between them (representing the ground). By Lemma 1, this would give a guarding of the city in dimension 3.

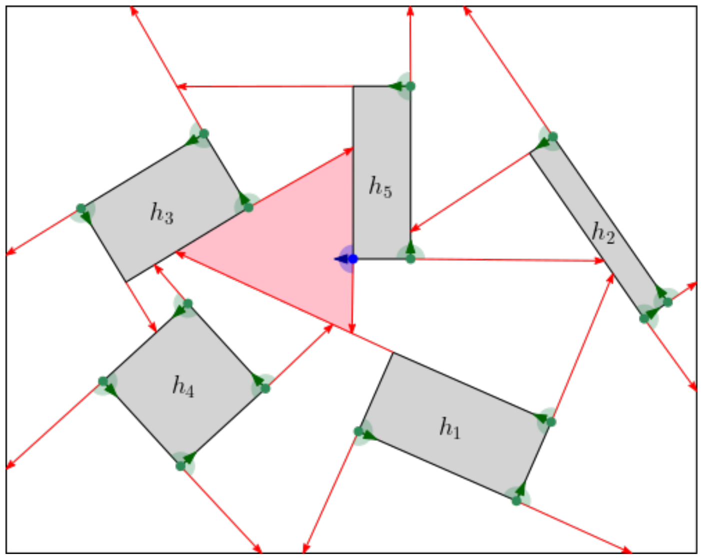



Fig. 2.1.1: A city with $k = 5$ buildings. The sides are extended in order $h_1, h_2, h_3, h_4, h_5$. The pink area is a bad region. The green marks are corner guards and the blue mark is an boundary guard.

We start by projecting the buildings vertically into the plane; this is a typical first step for problems of this type, see e.g. [1, 3]. Thus we obtain $k$ pairwise disjoint rectangles in the plane. We may assume without loss of generality that the $k$ rectangles lie in a bigger rectangle called $P$. One can think of $P$ as a polygon and of rectangles as holes. Thus after this projection, each building becomes a hole in $P$ and each wall becomes a side of some hole. One can think of this as an instance of the Art Gallery problem consisting of a polygon with rectangular holes.

Our next step is to guard $P$ by cameras with 180° field of view. This would give (after lifting the rectangles back to their original height) a guarding of walls and the ground. As we will see later, our placement of cameras would guard the roofs as well.

Let $h_1, h_2, \ldots, h_k$ denote the rectangular holes ordered arbitrarily. For each $h_i$ in this order, we extend the sides of $h_i$ in counterclockwise direction and stop as soon as reaching another hole, an extension of a previous side, or the boundary of $P$; see Figure 2.1.1. Each extension is essentially a *directed line segment* whose initial point is a hole corner. These extensions partition $P$ into some regions that we denoted $R_1, R_2, \ldots$; notice that we exclude the holes.

**Lemma 2.** *Each region $R_i$ is convex.*

*Proof.* The region $R_i$ is an intersection of a set of quadrants (which are convex). Each quadrant is defined by extensions of two adjacent sides of the same hole. Since the intersection of any set of convex objects is known to be convex, the region $R_i$ is convex. □

Therefore, by extending the sides of the holes, the free space of the polygon is divided into some convex regions. Chazelle, by a similar approach of extending diagonals, proved a convex partitioning of $r + 1$ regions where $r$ is the number of reflex vertices [2].

**Lemma 3.** *The number of regions $R_1, R_2, \ldots$ is $3k + 1$.*

*Proof.* We define a plane graph $G = (V, E)$ as follows. The vertex set $V$ consists of the corners of the holes and the intersection points of the extended sides. We refer to them by *corner* and *intersection* vertices, respectively. The edges in $E$ are formed by the sides of the holes, the extensions of sides, and the boundary of $P$.

We claim that each vertex of $G$ has degree 3, and thus $G$ is 3-regular. Each corner vertex is incident to two sides of a hole and an extension, thus has degree 3. Each intersection vertex is incident to an extension and two segments obtained from the intersected segment, and thus has degree 3. Degenerate cases are rather easy to handle, for example if two extensions hit a segment at the same point $p$, then we treat $p$ as two vertices of degree 3 instead of one vertex of degree 4.

The number of corner vertices is $4k$. Each extension (of a side of a hole) defines an intersection vertex. Thus the number of intersection vertices is the same as the total number of sides of holes, which is $4k$. Therefore $|V| = 8k$. Since the sum of the vertex degrees in any graph is twice the number of edges ($\sum_{v \in V} deg(v) = 2|E|$) and $G$ is 3-regular ($\sum_{v \in V} deg(v) = 3|V|$), we have the following equality,

$$2|E| = 3|V|.$$

Therefore,

$$|E| = \frac{3|V|}{2} = \frac{3 \cdot 8k}{2} = 12k.$$

Let $F$ be the set of faces of $G$, which includes the holes, the outerface (exterior of $P$), and the regions $R_1, R_2, \dots$. Using Euler's formula for connected planar graphs, we have

$$|F| = |E| - |V| + 2 = 12k - 8k + 2 = 4k + 2.$$

Excluding the outerface and the $k$ holes, the number of regions $R_1, R_2, \dots$ is $3k + 1$. □

**Lemma 4.** *Each region $R_i$ contains a corner of a hole on its boundary.*

*Proof.* Recall the extensions of $h_1, \dots, h_k$ in this order. Observe that the boundary of $R_i$ contains (parts of) some extensions. Consider the last extension that was added to the boundary of $R_i$, or say, closes the region $R_i$. The entire directed line segment that defines this extension is part of the boundary of $R_i$. The initial point of this directed line segment is a corner of a hole. □

By Lemma 4 each region $R_i$ has a hole corner on its boundary. If the boundary of $R_i$ has a 90° angle at some corner, then we call it a *good region*, and otherwise a *bad region*; see Figure 2.1.1.

**Camera Placement**: Take any region $R_i$. If $R_i$ is a bad region then let $c$ be an arbitrary corner on the boundary of $R_i$. We place a camera at $c$ facing towards the interior of $R_i$ and perpendicular to the boundary segment of $R_i$ containing $c$. We call this camera a *boundary guard*—it lies on the boundary of $R_i$. If $R_i$ is a good region then let $c$ be the lowest (i.e. with the smallest $y$-coordinate) corner at which the boundary of $R_i$ has angle 90°. We place a camera at $c$ facing towards the interior of

$R_i$ and perpendicular to the clockwise boundary segment at $c$ (which is the extension at $c$). We call this camera a *corner guard*—it lies on a corner of $R_i$.

Since $R_i$ is convex (by Lemma 2) the camera that is placed on the boundary of $R_i$ covers the entire interior of $R_i$. Since we place exactly one camera for each region $R_i$, (i) all regions $R_1, R_2, \ldots$ are guarded, and (ii) the number of cameras is equal to the number of regions $R_i$ which is $3k+1$ by Lemma 3. Therefore we have guarded the polygon $P$ by $3k+1$ guards. As discussed earlier, this gives a guarding of walls and the ground in the city.

We claim that our camera placement, also guards the roofs. Observe that for each hole $h$ it holds that one of its corners is the lowest corner of angle 90° on the boundary of some good region $R_i$. Notice that such a lowest corner of $R_i$ is uniquely defined by $h$. The camera that is placed at that corner (perpendicular to the extended side), guards the roof of $h$. The following theorem summarizes our result of this section.

**Theorem 5.** *Given $k$ arbitrary-oriented rectangular-base buildings, we can guard the entire space (the ground, walls, roofs, and the sky) with at most $3k+1$ cameras of 180° field of view that are placed at top corners of buildings orthogonal to a wall. The bound $3k+1$ is the best achievable.*

## 2.2 Guarding Orthogonally Convex Polygons

In this section we present our algorithm for guarding the free space formed by orthogonally convex polygons. Recall that the scene consists of $k$ arbitrary-oriented orthogonally convex polygons, and that the cameras have 180° field of view and should be placed on corners of polygons orthogonal to a side. We may assume without loss of generality that the $k$ polygons lie in a rectangular polygon called $P$. The free space, that we need to guard, is the interior of $P$ minus the $k$ given polygons.

As outlined in the preceding chapter, Daescu and Malik [4] presented an instance requiring $\frac{m}{2}+k+1$ cameras. The details of this example are provided in Figure 1.3.4. They conjectured that the bound $\frac{m}{2}+k+1$ is tight and here we prove this conjecture.

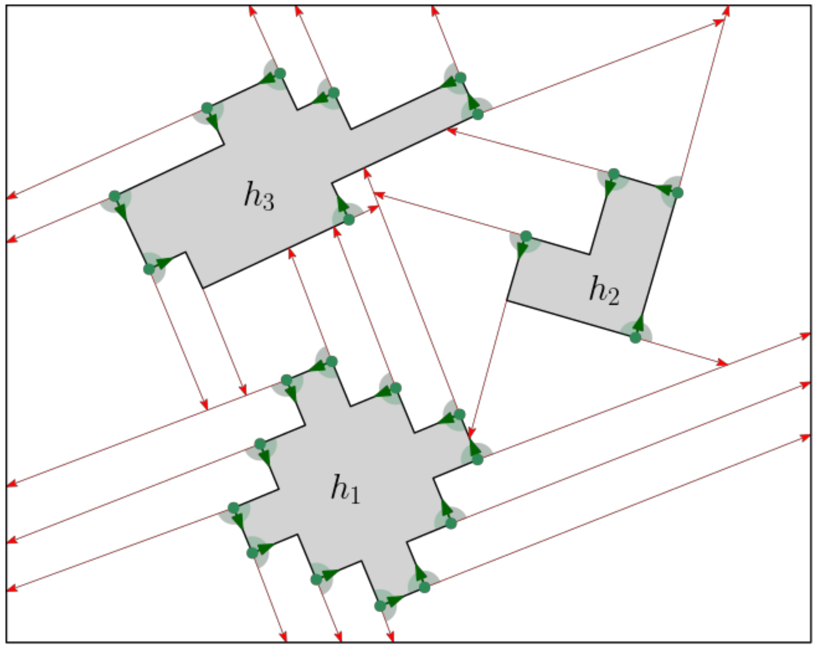



Fig. 2.2.1: Three orthogonally convex polygons in the plane. The green marks are corner guards.

Similar to our algorithm for the City Guarding in previous section we extend the sides of the polygons to partition the free space into convex region and then use one camera for each region. Let $h_1, h_2, \ldots, h_k$ denote the polygons in an arbitrary order. For each $h_i$ in this order, we extend the sides of $h_i$ in counterclockwise direction and stop as soon as reaching another polygon, an extension of a previous side, or the boundary of $P$. We only extend the sides whose extensions do not intersect the interior of $h_i$; see Figure 2.2.1. Thus we extend one side for every convex corner of a polygon. These extensions partition the free space into some regions that we denoted $R_1, R_2, \ldots$.

By an argument similar to that of Lemma 2 we can show that each $R_i$ is convex.

By an argument similar to that of Lemma 3 we can show that the number of regions $R_i$ is $\frac{m}{2} + k + 1$. We define a 3-regular plane graph $G = (V, E)$ as before. Among all corners, we only introduce vertices for convex ones. By a simple counting argument that we showed in the preceding Chapter, the total number of convex corners is $c = \frac{m}{2} + 2k$; see also [4]. Thus the number of vertices of $G$ is $2c$, one vertex for each convex corner and one vertex for its extension. Thus $|V| = 2c = m + 4k$.

Since the graph is 3-regular, the total degree is $3|V| = 3m + 12k$, which is equal to $2|E|$. Hence $|E| = \frac{3m}{2} + 6k$. Thus, for the number of faces we get

$$|F| = \left(\frac{3m}{2} + 6k\right) - (m + 4k) + 2 = \frac{m}{2} + 2k + 2.$$

Excluding the outerface and the $k$ holes, the number of regions $R_i$ is $\frac{m}{2}+k+1$. Similar to Lemma 4 we can show that each $R_i$ has a corner on its boundary. We classify the regions by *good* and *bad* and then place cameras on the corners (one camera for each $R_i$) similar to our placement in the previous section. This would guard the free space with $\frac{m}{2}+k+1$ cameras. The following theorem summarizes our result in this section.

**Theorem 6.** *Given $k$ pairwise disjoint arbitrary-oriented orthogonally convex polygons of total $m$ vertices in the plane, we can guard the entire free space with at most $\frac{m}{2}+k+1$ cameras of $180°$ field of view that are placed at the corners of the polygons orthogonal to a side. The bound $\frac{m}{2}+k+1$ is the best achievable.*

**Remark.** It is easily seen that the algorithm of this subsection can be generalized to guard cities with buildings that have orthogonally convex bases. In fact, the City Guarding in the previous subsection is a special case of this problem where $m = 4k$.

# References


[1] Lichen Bao, Sergey Bereg, Ovidiu Daescu, Simeon Ntafos, and Junqiang Zhou. "On some city guarding problems". In: *International Computing and Combinatorics Conference*. 2008, 600–610.

[2] Bernard Chazelle. "Computational geometry and convexity". In: 1980.

[3] Ovidiu Daescu and Hemant Malik. "City guarding with limited field of view". In: *Proceedings of 32nd Canadian Conference on Computational Geometry (CCCG)*. 2020, 300–311.

[4] Ovidiu Daescu and Hemant Malik. “New bounds on guarding problems for orthogonal polygons in the plane using vertex guards with halfplane vision”. In: *Theoretical Computer Science* 882 (2021), pp. 63–76.

# CHAPTER 3

# *Guarding Free-space of Polygons using Vertex Half Guards*

Ahmad Biniaz, Mohammad Hashemi

In this Chapter, we first generalize the City Guarding problem, and then we will similarly generalize Guarding Orthogonally Convex Polygons to Guarding the Free-space of Simple Polygons.

## 3.1 City Guarding with Convex-shape Base Buildings

We have observed that our established bounds in the City Guarding problem can be extended when the city border and the ground bases of buildings are any convex polygon, not limited to rectangles.

The achieved result obtained for the City Guarding problem in Chapter 2 was expressed in terms of $k$, representing the number of buildings. To generalize the problem, we may face buildings with varying numbers of walls (at least 3), resembling the scenario encountered in Guarding Orthogonally Convex Polygons. This implies the need to introduce the total number of top corners of the buildings as a variable in the problem.

The generalized theorem we have derived is as follows:

**Theorem 7.** *Given $k$ arbitrary-oriented convex-shape-base buildings with a total of $m$ corners inside a convex-shape city border, we can guard the entire space* (*the ground,* *walls, roofs, and the sky) with at most $m - k + 1$ cameras of $180°$ field of view (half-sphere guards) that are placed at top corners of buildings orthogonal to a wall. The bound $m - k + 1$ is tight.*

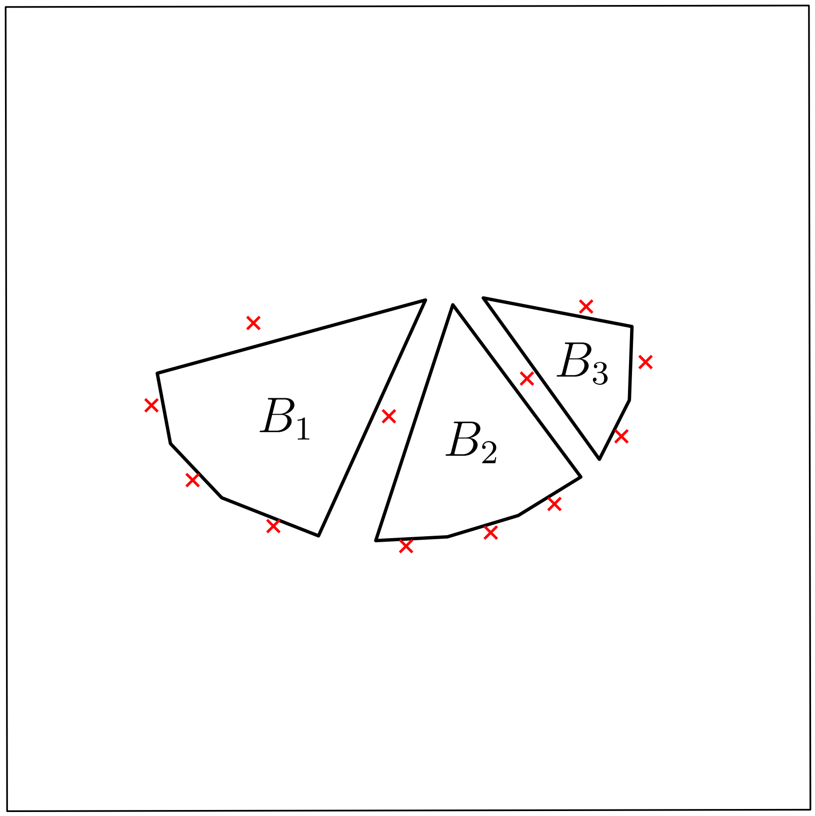



Fig. 3.1.1: A city with k = 3 buildings and a total of $m$ buildings' corners that needs $m - k + 1$ guards. $m - k + 1$ points are shown with a red cross, each of which can be guarded by one distinct guard.

*Proof.* The sufficiency bound can be derived as a corollary of Theorem 5. Since each side is extended, resulting in the creation of a new vertex with each extension, we maintain a 3-regular graph. Therefore, the bound can be attained by substituting $2m$ as the number of vertices in the formula for the sum of the degrees and the Euler Formula.

$$|E| = \frac{3|V|}{2} = \frac{3 \cdot 2m}{2} = 3m.$$

$$|F| = |E| - |V| + 2 = 3m - 2m + 2 = m + 2.$$

Subsequently, by excluding both the outerface and the $k$ holes, we obtain $m + 2 -$

$k-1 = m-k+1$ convex regions. Likewise, each of these regions can be monitored by either a corner or a boundary guard, ensuring that each roof is covered by at least one of the designated corner guards.

To establish the necessity bound, we propose a city structure illustrated in Figure 3.1.1. This city structure involves $k$ buildings with any arbitrary size which have $m$ corners in total. Given this arrangement, we have demonstrated $m-k+1$ points near the walls of the buildings with a red cross. Considering the restrictions on the potential cameras (they can be placed on the top corners orthogonal to a wall with $180°$ horizontal and vertical field of view), it is clear that no potential camera can monitor two of these points. Therefore, we need at least one camera to guard these points, and we can calculate the number of required cameras to be $m-k+1$, which gives us the necessary bound.

Consequently, the bound of $m-k+1$ is tight.

□

## 3.2 Guarding Free-space of Simple Polygons

In monitoring the open spaces within polygons, as opposed to City Guarding, there is no requirement to cover the interior of the polygons. This flexibility enables us to extend this problem to contain any simple polygon.

The generalization can be suggested as a corollary as follows:

**Theorem 8.** *Given $k$ pairwise disjoint arbitrary-oriented simple polygons of total $m$ vertices in the plane surrounded by a rectangular border, we can guard the entire free space with at most $m-r-k+1$ cameras of $180°$ field of view that are placed at the corners of the polygons orthogonal to a side. This achieved bound is tight.*

*Proof.* The sufficiency bound can be derived as a corollary of Theorem 6. Likewise, only sides ending to a convex vertex are extended, resulting in the creation of $c = m-r$ new vertices, where $c$ is the total number of convex vertices, and $r$ is the total number of reflex vertices. By a similar argument, this gives us a 3-regular planar graph (we ignore reflex vertices and consider each segment of the boundary of polygons or the rectangular border as a single edge). Therefore, the bound can be attained by substituting $2c$ as the number of vertices in the formula for the sum of the degrees and the Euler Formula:

$$|E| = \frac{3|V|}{2} = \frac{3 \cdot 2c}{2} = 3c.$$

$$|F| = |E| - |V| + 2 = 3c - 2c + 2 = c + 2.$$

Subsequently, by excluding both the outerface and the $k$ holes, we obtain $c+2-k-1 = c - k + 1 = m - r - k + 1$ convex regions. Each of these regions can be monitored by either a corner or a boundary guard, ensuring that each roof is covered by at least one of the designated corner guards.

To establish the necessity bound, we can simply add some reflex corners to the example in Figure 3.1.1. In fact, we can show that reflex corners do not increase the need for cameras. in that example, $c = m$ and $c - k + 1$ were necessary, and after adding reflex corners, we have $c = m - r$. So, subbing it in the $c - k + 1$ gives us the bound of $m - r - k + 1$. $\square$

# CHAPTER 4

# *Conclusion*

In this thesis, we addressed two problems related to city guarding and the art gallery. First, we established that for a city with $k$ rectangular buildings, it always takes $3k + 1$ cameras with a $180°$ field of view to sufficiently guard the entire free space, including ground, walls, roofs, and the sky, proving a previously proposed conjecture. Similarly, for $k$ orthogonally convex polygons with a total of $m$ vertices in the plane, we demonstrated that it requires at least $\frac{m}{2} + k + 1$ cameras with a $180°$ field of view to guard the free space.

Furthermore, we generalized the previously mentioned problems to more complex scenarios. For a city composed of $k$ buildings with arbitrary convex-shaped bases and a total of $m$ top corners, we established that to guard the entire city, including its ground, walls, roofs, and sky, it is sometimes necessary and always sufficient to use $m - k + 1$ cameras with a 180-degree field of view. Similarly, for a set of $k$ polygons, whether convex or non-convex, with a combined total of $m$ vertices in the plane, which includes $r$ reflex vertices, it is sometimes necessary and always sufficient to employ $m - k - r + 1$ cameras with a 180-degree field of view to guard the free space. These results provide precise bounds for camera placement in these generalized scenarios.

## 4.1 Discussion

We investigated a specific version of city guarding, considering particular constraints and assumptions, and we also extended our analysis to more generalized scenarios.

However, it is worth mentioning that this problem has many applications, and to address them, various assumptions in the problem can be generalized. Below, we explain some ideas for discussion, each presenting potential future research:

One of the assumptions in the studied versions of city guarding is that the buildings are disjoint and have some arbitrary width, length, and height. However, one can suggest a scenario where buildings are stacked on top of each other like the tower structures commonly found in metropolitan cities. It can be an amazing case to study.

Another limitation was that the shape of the buildings' bases were polygonal. An interesting idea can be to consider buildings with a curved base or explore the concept of circular structures. The idea of this scenario comes from the modern architectural designs of circular buildings.

In urban environments, specific high-importance areas, including buildings, streets, and open spaces, require enhanced monitoring, especially during peak hours or periods of high traffic. The need for effective coverage in these regions increases the number of required cameras. For this purpose, there exists a guarding problem known as the $k$-guarding problem, which has been explored in previous studies such as [1, 2]. The objective of this problem is to place a minimum of $k$ guards to ensure the security of the designated area. We can study $k$-guarding in the context of city guarding.

In the studied versions of city guarding, each guard can see an infinite range if there is no obstacle within its vision. However, it is clear that this assumption is not real, as even the most advanced cameras experience a loss of resolution when monitoring over long distances. Therefore, one awesome idea is to study a version of the problem where the range of vision is given, meaning each guard can only observe within a defined radius.

While we have explored several interesting versions of the problem, particularly those involving constraints on the definition, there are still many more unexplored variations to study.

# References

[1] Patrice Belleville, Prosenjit Bose, Jurek Czyzowicz, Jorge Urrutia, and Joseph Zaks. "K-Guarding Polygons on the Plane". In: *Canadian Conference on Computational Geometry*. 1994.

[2] Daniel Busto, William S. Evans, and David G. Kirkpatrick. "On k-Guarding Polygons." In: (2013).

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