6-13-2023

Multicriteria Consensus Models to Support Intelligent Group Decision-Making

Hossein Hassani
University of Windsor

MULTICRITERIA CONSENSUS MODELS TO SUPPORT INTELLIGENT GROUP DECISION-MAKING

by

Hossein Hassani

A Dissertation
Submitted to the Faculty of Graduate Studies
through the Department of Electrical and Computer Engineering
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy
at the University of Windsor

Windsor, Ontario, Canada

2023

Multicriteria Consensus Models to Support Intelligent Group Decision-Making

by

Hossein Hassani

APPROVED BY:

____
R. Samavi, External Examiner
Toronto Metropolitan University

____
A. Rahimi
Department of Mechanical, Automotive, and Materials Engineering

____
J. Wu
Department of Electrical and Computer Engineering

____
M. Ahmadi
Department of Electrical and Computer Engineering

____
R. Razavi-Far, Co-Advisor
Department of Electrical and Computer Engineering

____
M. Saif, Co-Advisor
Department of Electrical and Computer Engineering

May 11, 2023

Declaration of Co-Authorship and Previous Publication

I. CO-AUTHORSHIP

I hereby declare that this thesis incorporates material that is result of joint research, as follows:

- Chapter 1 incorporates material co-authored with Dr. Roozbeh Razavi-Far, Dr. Mehrdad Saif, Dr. Jafar Zarei, and Dr. Ferede Blaabjerg. In all cases the key ideas, primary contributions, experimental designs, data analysis, interpretation, and writing were performed by myself; Dr. Roozbeh Razavi-Far and Dr. Mehrdad Saif contributed to formal analysis, writing-review and editing, supervision, and funding; Dr. Jafar Zarei and Dr. Ferede Blaabjerg contributed to writing-review and editing.
- Chapter 2 incorporates material co-authored with Dr. Roozbeh Razavi-Far, Dr. Mehrdad Saif, Dr. Francisco Chiclana, Dr. Ondrej Krejcar, and Dr. Enrique Herrera-Viedma. In all cases the key ideas, primary contributions, experimental designs, data analysis, interpretation, and writing were performed by myself; Dr. Roozbeh Razavi-Far contributed to conceptualization, formal analysis, investigation, resources, writ- ing-review and editing, supervision, project administration, funding; Dr. Mehrdad Saif contributed to formal analysis, writing-review and editing, supervision, and funding; Dr. Francisco Chiclana, Dr. Ondrej Krejcar contributed to formal analysis, investigation, writing-review and editing; Dr. Enrique Herrera-Viedma contributed to formal analysis, writing-review and editing, and funding.
- Chapters 3, 4, 5, and 6 incorporates material co-authored with Dr. Roozbeh Razavi-Far, Dr. Mehrdad Saif, and Dr. Enrique Herrera-Viedma. In all

cases the key ideas, primary contributions, experimental designs, data analysis, interpretation, and writing were performed by myself; Dr. Roozbeh Razavi-Far contributed to conceptualization, formal analysis, investigation, resources, writ- ing-review and editing, supervision, project administration, funding; Dr. Mehrdad Saif contributed to formal analysis, writing-review and editing, supervision, and funding; Dr. Enrique Herrera-Viedma contributed to formal analysis, writing-review and editing, and funding.

I am aware of the University of Windsor Senate Policy on Authorship and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained written permission from each of the co-authors to include the co-authored material in my thesis.

I certify that, with the above qualification, this thesis and the research to which it refers is the product of my own work.

II. PREVIOUS PUBLICATION

This thesis partly includes the original papers that have been previously published in peer reviewed journals as provided in the following table.

Thesis Chapter	Publication Title/Full Citation	Publication Status
Chapters 5, 6	Hassani, Hossein, Roozbeh Razavi-Far, Mehrdad Saif, and Enrique Herrera-Viedma. "Blockchain-Enabled Trust Building for Managing Consensus in Linguistic Opinion Dynamics." IEEE Transactions on Fuzzy Systems (2023).	Published

Chapters 3, 6	Hassani, Hossein, Roozbeh Razavi-Far, Mehrdad Saif, and Enrique Herrera-Viedma. "Reinforcement Learning-Based Feedback and Weight-Adjustment Mechanisms for Consensus Reaching in Group Decision Making." IEEE Transactions on Systems, Man, and Cybernetics: Systems (2022).	Published
Chapter 2	Hassani, Hossein, Roozbeh Razavi-Far, Mehrdad Saif, Francisco Chiclana, Ondrej Krejcar, and Enrique Herrera-Viedma. "Classical dynamic consensus and opinion dynamics models: A survey of recent trends and methodologies." Information Fusion 88 (2022): 22-40.	Published
Chapters 4, 6	Hassani, Hossein, Roozbeh Razavi-Far, Mehrdad Saif, and Enrique Herrera-Viedma. "Consensus-Based Decision Support Model and Fusion Architecture for Dynamic Decision Making." Information Sciences 597 (2022): 86-104.	Published

Chapter 1	Hassani, Hossein, Roozbeh Razavi-Far, Mehrdad Saif, Jafar Zarei, and Frede Blaabjerg. "Intelligent Decision Support and Fusion Models for Fault Detection and Location in Power Grids," in IEEE Transactions on Emerging Topics in Computational Intelligence 6, no. 3 (2022): 530-543.	Published
Chapter 1	Hassani, Hossein, Roozbeh Razavi-Far, and Mehrdad Saif. "Fault location in smart grids through multicriteria analysis of group decision support systems." IEEE Transactions on Industrial Informatics 16, no. 12 (2020): 7318-7327.	Published

I certify that I have obtained a written permission from the copyright owner(s) to include the above published material(s) in my thesis. I certify that the above material describes work completed during my registration as graduate student at the University of Windsor.

I declare that, to the best of my knowledge, my thesis does not infringe upon anyone's copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledged in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owner(s) to include such material(s) in my thesis.

I declare that this is a true copy of my thesis, including any final revisions, as approved by my thesis committee and the Graduate Studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.

Abstract

The development of intelligent systems is progressing rapidly, thanks to advances in information technology that enable collective, automated, and effective decision-making based on information collected from diverse sources. Group decision-making (GDM) is a key part of intelligent decision-making (IDM), which has received considerable attention in recent years. IDM through GDM refers to a decision-making problem where a group of intelligent decision-makers (DMs) evaluate a set of alternatives with respect to specific attributes. Intelligent communication among DMs aims to give orders to the available alternatives. However, GDM models developed for IDM must incorporate consensus support models to effectively integrate input from each DM into the final decision.

Many efforts have been made to design consensus models to support IDM, depending on the decision problem or environment. Despite promising results, significant gaps remain in research on the design of such support models. One major drawback of existing consensus models is their dependence on the type of decision environment, making them less generalizable. Moreover, these models are often static and cannot respond to dynamic changes in the decision environment. Another limitation is that consensus models for large-scale decision environments lack an efficient communication regime to enable DM interactions.

To address these challenges, this dissertation proposes developing consensus models to support IDM through GDM. To address the generalization issue of existing consensus models, reinforcement learning (RL) is proposed. RL agents can be built on the Markov decision process to enable IDM, potentially removing the generalization issue of consensus support models. Contrary to most consensus models, which assume static decision environments, this dissertation proposes a computationally efficient dynamic consensus model to support dynamic IDM. Finally, to facilitate secure

and efficient interactions among intelligent DMs in large-scale problems, Blockchain technology is proposed to speed up the consensus process. The proposed communication regime also includes trust-building mechanisms that employ Blockchain protocols to remove enduring and limitative assumptions on opinion similarity among agents.

Acknowledgements

I would like to reflect on the people who have supported and helped me throughout this period. I would like to express my sincere gratitude to my supervisors, Dr. Mehrdad Saif and Dr. Roozbeh Razavi-Far, for their immense knowledge and for their support throughout my Ph.D. study. In addition, I would like to thank my thesis committee members, Dr. Majid Ahmadi and Dr. Jonathan Wu from the Electrical and Computer Engineering Department, Dr. Afshin Rahimi from Mechanical, Automotive & Materials Engineering Department, and Dr. Reza Samavi from the Toronto Metropolitan Univesity for taking the time out of their busy schedule to participate in my seminars, reviewing my thesis and providing insightful comments to improve this work.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

AAV Average of Absolute Value

ACD Average Consensus Degree

AHD Average Harmony Degree

ALT Alternative to be modified

APS Elements to be modified

BC Bounded Confidence

BMPD Basic Matching Pursuit Decomposition

CCV Canonical Characteristic Value

CD Consensus Degree

CEN Consensus Evolution Network

CODA Continuous Opinion and Discrete Action

CoG Centre of Gravity

CPS Cyber-Physical System

CRP Consensus Reaching Process

DDPG Deep Deterministic Policy Gradient

DG DeGroot Model

DHLPR Double Hierarchy Linguistic Preference Relation

DLTF Distributed Linguistic Trust Function

DM Decision Maker

DMCDM Dynamic Multi-Criteria Decision Making

DW Weisbuch and others

EMD Empirical Mode Decomposition

EV Expected Value

FLE Flexible Linguistic Expression

FPR Fuzzy Preference Relation

GDM Group Decision Making

GG Generator Ground

GO Generator Outage

GNZPWA Generalized Normal Z^+-Value Power Weighted Average

GNZWA Generalized Normal Z^+-Value Weighted Average

HD Harmony Degree

HFLTS Hesitant Fuzzy Linguistic Term Set

HK Hegselmann and Krause

IDM Intelligent Decision Making

IFPR Intuitionistic Fuzzy Preference Relation

ILFPR Intuitionistic Linguistic Fuzzy Preference Relation

IT2FS Interval Type-2 Fuzzy Set

LL Load Loss

LMD Local Mean Decomposition

LOD Linguistic Opinion Dynamics

LODM Linguistic Opinion Dynamic Model

LSF Linguistic Scale Function

LSGDM Large-Scale Group Decision Making

LTS Linguistic Term Set

MAGDM Multiple Attribute Group Decision Making

MDP Markov Decision Process

MeOM Mean of Maxima

MF Membership Function

MST Minimum Spanning Tree

NIS Negative Ideal Solution

NZ Normal Z^+-Value

OD Opinion Dynamics

ODM Opinion Dynamics Model

OMPD Orthogonal Matching Pursuit Decomposition

OWA Ordered Weighted Average

PFLPR Pythagorean Fuzzy Linguistic Preference Relation

PFLV Pythagorean Fuzzy Linguistic Variable

PIS Positive Ideal Solution

PMU	Phasor Measuring Unit
q-ROFS	q-Rung Orthopair Fuzzy Set
RL	Reinforcement Learning
SAC	Soft Actor Critic
SCB	Scenario with Blockchain protocol
SCHB	Scenario with Heterogenous Blockchain protocol
SCOHB	Scenario with Optimal Heterogenous Blockchain protocol
SG	Smart Grid
VMD	Variational Mode Decomposition
WAMS	Wide Area Management System
WMPD	Weak Matching Pursuit Decomposition
WPD	Wavelet Packet Decomposition

Chapter 1

Introduction

The design of intelligent systems is witnessing an expeditious development due to the emergence of information technologies and IoT applications for the sake of intelligent, automatic and collective decision-making by accessing diverse sources of information. Among the developed technologies, cyber-physical systems (CPSs) have been widely spread over the past decades due to their functionalities and efficiency for the deep intertwining of physical and software components, enabling them for the transfer of large amount of data over an interconnected network. In this respect, one can refer to smart grids (SGs) as one of the emerging CPSs that, in contrast to the conventional power systems, benefit from the features of CPSs for a higher reliability and can be simply modified in case of expansion planning to cover power demands [5]. However, the geographical dispersion property of SGs make them suffer from physical vulnerabilities such as outages of generation units, transmission lines and loads, and short-circuit events [6]. Other than physical vulnerabilities, SGs suffer from the security and safety of facilities that enable the networking of embedded components, which can be threatened by an adversary's actions.

Many efforts have been devoted to the design of diagnostic frameworks for SGs to deal with physical and cyber threats, where they can generally be divided into model-based and data driven-based techniques [7]. Model-based techniques rely on an explicit input-output model of the system for the sake of diagnosing faults and attacks. These models, however, could become complex especially in dealing with the complex dynamics of SGs [8]. On the contrary, data driven techniques do not rely on the system model and are not concerned with the complexity of the constructed model

[9, 10]. They rely on data measurements collected from the system under different operational states, which has been enabled by means of wide area measuring system (WAMS) [11]. A successful WAMS implementation for SGs usually requires proper communications between the generation nodes and consuming units [12]. This could be achieved by means of intelligent measuring devices such as phasor measuring units (PMUs), however, it is then a must to deal with the emerging big dimensionality [13, 14]. Other than coping with the curse of dimensionality, the performance of WAMS is highly dependent to signal processing phase for extracting informative features from the collected data measurements.

Generally speaking, signal processing for feature extraction could be performed on three different domains, including time domain, frequency domain, and time-frequency domain. The time-domain features refer to the case, in which statistical measures such as mean, mode, median of the original data measurements are collected to construct the feature set [15]. The frequency-domain features are statistical characteristics of the frequency spectrum of the original data measurements [16]. In order to benefit from the characteristics of both time-domain and frequency-domain features, various methods have been developed to analyze a signal in a time-frequency domain [17]. WPD [18], LMD [19], EMD [20], BMPD [21], and its variants such as OMPD [22] and WMPD [23], and VMD [24] are some well-known signal processing tools in the time-frequency domain to decompose an arbitrary signal into its principal modes. The extracted modes can then be used to reconstruct the original signal and also to construct a set of informative features.

To cope with the curse of dimensionality, data reduction techniques have been widely employed in the design of intelligent diagnostic frameworks. These techniques can generally be divided into dimensionality reduction and feature selection methods in order to reduce the dimension of data so as to improve the efficiency and consistency of the diagnostic models [7, 25, 26, 27]. Dimensionality reduction techniques aim to reduce the dimension of data by employing appropriate transformations on the feature space and can be divided into linear or non-linear categories depending on the type of the employed transformation [28]. Feature selection refers to the process of selecting an informative and adequately-relevant subset of features from the original set of features. These techniques can be categorized into either filters, wrappers, or embedded techniques depending on the employed evaluation metrics [29]. Filters are the earliest techniques used for the feature selection and they just utilize the intrinsic characteristics of data measurements in order to rank the set of features. Therefore, these models are known to be fast and computationally efficient [30]. In contrast to the wrapper methods [31], filters are classifier independent with a better generalization capability, however, they ignore the dependency among features. Wrappers benefit from the interaction with a classification model and take the classification accuracy as the feature selection criterion. These methods are more accurate compared with the filters, however, they are classifier dependent and computationally expensive. Embedded techniques take the advantages of the filters and wrappers by embedding the feature selection into the learning algorithm of the classification models [32].

Other than the signal processing and data reduction that play vital roles in improving the performance of diagnostic systems, the decision-making module could also be of paramount importance in the design of diagnostic frameworks [33, 34, 35]. This module should be scalable and able to efficiently deal with the extracted features through the data processing phase. Furthermore, this module should enable the decision-making process by taking into account multiple criteria designated to the decision task for making efficient decisions. In this regard, collective decision-making, or group decision-making (GDM) could be beneficial for the sake of simultaneously concerning multiple criteria to make the final decision w.r.t. a given set of features. GDM refers to a group of agents that collectively evaluate a decision task w.r.t. a given set of criteria in order to select the best available choice for the given decision problem. However, in order to benefit the most from the opinion of each individual agent in addition to efficiently augment the impact of the given criteria on the decision task, GDM is required to be supported by a consensus model. Therefore, this dissertation is devoted to the design of consensus models to support multiple criteria GDM.

1.1 Background

GDM is a core part of intelligent decision-making (IDM) and has gained much attention in recent years [36]. GDM refers to a decision problem, in which a group of experts is designated to assess a set of alternatives according to a set of attributes through a communication regime to provide rankings for the available set of alternatives [1]. However, decision-makers (DMs) have different backgrounds and levels of knowledge, which result in potential conflicts in the expressed opinions, and, therefore, there is a need to design mechanisms for consensus achievement in the group [37]. Such a mechanism is called the consensus reaching process (CRP) in GDM problems [38].

Ideally, the hope is to reach a total agreement, i.e., a unanimous decision, even though, this is neither practical nor necessary in many real-life decision problems [39]. Instead, the goal could be making decisions that are agreed on by most of the involved DMs, so-called consensual decisions. This has consequently paved the way for a softer consensus methodology that could quantify the level of consensus from absence to the total agreement [40]. To this end, the CRP could be considered as a convergent and multi-stage procedure, where the opinions of DMs are initially assessed, and in case that the level of consensus among them is lower than a given threshold, DMs are encouraged to negotiate in order to bring their opinions closer to a collective one for the sake of consensus reaching. This negotiation process, however, is required to be equipped with an efficient feedback (or adjustment or recommendation) mechanism in order to guide DMs toward the collective opinion of the group.

To guide DMs within a *static* decision framework, the feedback mechanism recommends each individual to modify his/her opinion to some extent so as to get closer to the collective opinion of the group. This mechanism, however, needs to be efficient in terms of the speed and Harmony degree (HD) of DMs. The former is an efficiency measure to be considered in the design of feedback mechanisms, which means how many discussion rounds it would take for the group to reach to the desired level of consensus. The higher the speed, the better the feedback mechanism. The latter, however, is a measure of the deviation between the original opinion of a DM and the modified opinion suggested by the feedback mechanism. This deviation is called *cost* in the GDM taxonomy. Therefore, the lower the cost, the higher the HD, and the better the feedback mechanism [41]. Even though realizing these two efficiency measures is of utmost importance in construction of feedback mechanisms, however, it becomes more challenging for decision making under dynamic decision environments [42].

The counterpart of static decision environments is *dynamic* environments that typically refer to the changes in alternatives, attributes, DMs, and their importance weights. In such a framework, the set of alternatives could be subject to changes during the consensus assessment due to the availability of new alternatives and/or the feasibility of a previous set of alternatives. DMs' weights are the keys in deriving the collective opinion of the group and these weights change from one discussion round to another. As for the dynamic set of attributes, new attributes can be introduced to the problem during the CRP so as to speed up the process and/or to evaluate the decision problem from new viewpoints. And finally, the set of DMs could also be subject to changes due to the fact that some DMs may leave the negotiation and/or new individuals might be invited to participate to the decision problem [43].

In the static and dynamic GDM, which we refer to as *classical dynamic consensus models*, time does not play a role in modeling the dynamism of the feedback mechanism. However, in *opinion dynamics models*, not only time does play an important role in evolution of DMs' opinions, but also the opinion of an individual is modeled to be affected by others through a weighted summation-based aggregation scheme. In such models, the design of a fusion strategy for the feedback mechanism to satisfy the constraint on the evolution of opinions while having time involved in the dynamism is of paramount importance, yet very challenging [44].

Following the given background, we discuss the research problems that are going to be addressed in this dissertation in the following section.

1.2 Problems

We define three main research problems for each model of the decision environment including classical dynamic consensus and opinion dynamics models.

1.2.1 Static GDM: CRP Speed and HD Trade-Off

As mentioned earlier in Section 1.1, the speed of CRP and keeping the HD of DMs at a high level are two important efficiency measures to be considered in the design of feedback mechanisms for static decision-making. Feedback mechanisms usually benefit from two rules called *identification* and *direction*. The former is the phase, in which the inconsistent DMs are identified, who need to modify their initial opinions to get closer to the collective opinion of the group. The latter refers to adjusting the opinions of inconsistent DMs by augmenting a portion of the collective opinion into their original opinions. To speed up the CRP through the feedback mechanism, the portion of the collective opinion to be added to the original opinion needs to be increased. However, this means that there would a higher deviation between the original opinion and the modified opinion of an inconsistent DM, leading to lower HD for an individual. Following this description, considering the trade-off between the speed of CRP and the level of HD is a must in construction of feedback mechanisms for static decision environments. A considerable number of research studies have been devoted to the design of efficient feedback mechanisms by considering the aforementioned trade-off, however, the developed models are dependent to the representation structures used for opinion expressions. This means that the developed models are opinion-dependent and are not generalizable to other decision environments with different types of opinion representation structures.

1.2.2 Dynamic GDM: Dynamic Alternatives

As described in Section 1.1, the CRP in dynamic GDM could be more challenging compared to the static counterpart due to the dynamical changes in environment. The dynamic environment in such models refers to the dynamic changes of the set of alternatives, attributes, DMs, and the importance weights of DMs. In this dissertation, we deal with the situation, in which the set of alternatives is dynamically changing from one discussion round to another. This could be a realistic situation in decision-making due to the fact that alternatives to be assessed could vary in each discussion round based upon the availability of new alternatives and/or the feasibility of the previous set of alternatives [43]. The CRP under such a dynamic environment, however, becomes complicated and computationally inefficient. Therefore, the efficient management of DMs and their interactions within the CRP could be of great importance for reducing the complexity of dynamic decision-making. Furthermore, consensus assessment and recommendation generation for DMs to reach a desired level of consensus should also be designed efficiently to reduce their associated complexity and to speed up the CRP. In this regard, management of the attitude and interest of DMs, proper and dynamic adjustment of the consensus threshold and DMs' importance weights could play an important role in decreasing the associated complexity to dynamic decision-making.

1.2.3 ODMs: Trust Building Through Opinion Similarity

The study of opinion dynamics (OD) aims at understanding how opinions evolve over time among a group of interacting agents. The arithmetic mean of agents' opinions in a previous time-step is used to determine how agents' opinions change over time. It has been decades since opinion dynamics models (ODMs) were developed utilizing time as the main element of dynamism. The developed models can generally be categorized into discrete-in-opinion and continuous-in-opinion models. One can refer to the Voter [45] and Sznajd [46] models as well-known discrete ODMs, where models such as the DG model [47], and BC models including the HK model [48] and DW model [49, 50] are of well-known continuous ODMs. The classic models have been extensively studied, with a variety of variants being proposed in recent years to improve their fusion process. Even though the developed ODMs show encouraging results, however, there still exist several challenges that need to be addressed more efficiently as listed below:

1. It is a new research direction in ODMs to express opinions using linguistic representation structures, and the preliminary results are encouraging. There is, however, room for improvement in the development of linguistic ODMs (LODMs) based on a more generalized opinion representation structure such as Z-numbers.

2. Even though notable efforts have been made to the design of minimum cost consensus models for ODMs, however, the willingness of agents to either accept or refuse the suggested modifications is missing. In ODMs, the willingness is usually addressed through BC models, yet they conduct bias in evolution of agents' opinions.

3. The agents' willingness is typically characterized by BC notion in ODMs. Such models rely on the opinion similarity to build trust among agents, meaning that only agents with similar opinions trust each other and an agent's opinion

is formed by means of the trusted peers. Such models can influence agents' interactions in a biased manner, and agents' opinions might be influenced by within group factors (e.g., peer pressure and group pressure).

1.3 Solutions

The three main research problems throughout this dissertation were discussed in Section 1.2. Following that, here we elaborate more on solutions to the aforementioned problems.

1.3.1 RL-Based Optimization of the CRP and HD Trade-Off

As mentioned in Section 1.2.1, the issue of the trade-off between the CRP speed and HD of DMs arise from the amount of the modifications that inconsistent DMs need to employ to modify their initial opinions. High level of modifications increases the speed of CRP, but lowers the HD of DMs. In order to address this issue, we resort to an reinforcement learning (RL)-based optimization strategy in order to automatically adjust the level of modifications so as to speed up the CRP, while keeping the HD of DMs as high as possible.

One way to realize RL is thorough Markov decision processes (MDPs). Therefore, in order to enable the application of RL in GDM, it is required to firstly convert the decision-making problem into an MDP. To this end, it is proposed to resort to the consensus degree (CD) of DMs as the states of the underlying MDP, where it will be proved that such modelling of states satisfies the Markov property, meaning that CD of DMs at a specific time-step, e.g., $t+1$, does only depend on values taken from the previous time-step, i.e., t, and the rest of the history of states, i.e., $t = 0, 1, \ldots, t-1$, are not required to determine the value of the current state. Following this formulation, RL could be realized for addressing the trade-off problem by adjustment of the amount of modifications to be sent to inconsistent DMs. Other than this, it will be shown that the adjustment of the DMs' importance weights could also contribute to the speed of CRP. Therefore, an RL-based framework will also be developed for the adjustment of DMs' weights to further speeding up the CRP. The reason we resort to the RL is that such a framework is of great generalization capabilities, meaning that once an RL agent is trained to adjust the level of modifications and weights of DMs in a specific decision-making environment, the same agent could also be employed in other decision-making environments without re-training.

1.3.2 Computationally Efficient Dynamic GDM

To address the associated challenges with dynamic GDM mentioned in Section 1.2.2, three major contributions are made to the dynamic decision-making in this dissertation. First, it is proposed to divide the initial set of DMs into several groups depending on the number of available attributes. Each group of DMs are supposed to focus on only one attribute for the sake of opinion expressions. This idea, on the one hand, takes into account the expertise and interest of DMs. On the other hand, it helps with reducing the computational complexity of dynamic decision-making due to the fact that DMs' opinions would be of a lower dimension as they only provide their opinions w.r.t. a single attribute. Following this, the consensus assessment level will also be reduced to a two-level mechanism instead of the typical three-level consensus assessment. Second, due to the fact that alternatives are changing from one discussion round to another, there is a need to look at the history of alternatives to see which one would be the best one, even though it has been removed from the set of alternatives. In order to enable this property, it is proposed to equip the dynamic decision-making framework with a *memory* cell so as to put aside the best alternative at the current time-step to be passed to the next time-step to be evaluated w.r.t. the new set of available alternatives. Third, in order to further reduce the computational complexity of the model, we propose a technique based on the consensus evolution networks (CENs) in order to adjust the consensus threshold in a meaningful way that could help with speeding up the CRP.

1.3.3 Blockchain-Enabled Trust Building

Following the challenges mentioned in Section 1.2.3, we aim to propose a general framework that guides the agents towards a collective opinion considering their willingness in accepting or refusing the suggested modifications. Therefore, we are concerned with the design of an LODM, where agents express their initial opinions in terms of Z-numbers. In order to remove withing group factors (e.g., peer and group pressure) that might impact the agents' opinions, and to model agents' interactions without concerning the opinion similarity, it is proposed to construct a safe and efficient communication regime using Blockchain technology. Within this regime, an agent's identity and opinion are not disclosed for other peers, yet we propose to build trust among agents by just enabling them to see how many of their peers have accepted the suggested modifications by the moderator. To this end, to address the associated challenges of ODMs, we propose the following solutions:

1. Unlike typical ODMs with numerical opinions, a framework is proposed to deal with linguistic opinions in terms of Z-numbers.

2. The willingness of agents toward accepting or refusing the suggested modifications are considered in the constructed framework.

3. Unlike BC models that rely on the similarity of the agents' opinions in order to build trust between them, we propose the construction of a Blockchain-based communication regime for trust building without concerning the opinion similarity, removing bias in agents' interactions.

1.4 Thesis Contributions

GDM problems can be divided into three broad categories including static GDM, dynamic GDM, and ODMs. This dissertation is devoted to make contributions to each of these broad categories by proposing novel decision-making strategies to not only address the associated challenges to each category, but also to contribute to the efficiency of the CRP and computational complexity.

This dissertation contributes to the **static GDM** models by resorting to RL in order to remove the long-standing limitation of such models in terms of their generalization capabilities. The developed static models suffer from the fact that they are dependent to the representation structure of opinions, meaning that a developed model for a specific GDM problem with a specific type of opinion representation structure is not generalizable to another model with a different opinion representation structure. The proposed framework based on the RL removes this barrier by modeling the decision-making problem as an MDP.

Dynamic GDM models are known for suffering from computational complexity due to the dynamical changes of the decision variables. This dissertation, as discussed in Section 1.3.2, proposes a novel and general framework to deal with the complexity of dynamic multiple-attribute GDM (MAGDM) problems through dividing DMs into several groups w.r.t. the available attributes, and by dynamically adjusting the value of consensus threshold so as to speed up the CRP withing groups. The proposed model could deal with the dynamical changes of the set of alternatives.

Within the framework of the third category of models, i.e., **ODMs**, unlike BC models that consider the willingness of the agents by resorting to their opinion similarity that conducts bias in agents' interactions for trust building, this dissertation removes the limitation on opinion similarity for trust building. Other than constructing a novel LODM that benefits from advantages of Z-numbers for opinion expression, the proposed framework makes use of the Blockchain technology to provide a secure and efficient communication regime for agents' interactions, yet building trust among them without concerning the opinion similarity of agents.

Finally, in terms of **applications**, this dissertation makes use of the proposed dynamic GDM model to formulate the *fault location* problem in distributed power systems as a decision-making problem. In this regard, the developed framework is used to locate open-circuit faults such as load-loss (LL), generator-outage (GO), and generator-ground (GG) faults for the practical verification of the proposed dynamic GDM model.

1.5 Outline

The structure of the subsequent chapters of this dissertation is as given below:

Chapter 2 is devoted to the review of state-of-the-art GDM and ODMs. In this chapter, the major elements of GDM and ODMs are firstly introduced and the associated decision-making problems are formulated. This is then followed by a comprehensive review on the recent advancements in both category of models and the leading-edge techniques and technologies are surveyed. Following this review, the research gaps of both models that are either not addressed or are not efficiently addressed are found and are defined as the research problems to be addressed in this dissertation.

Chapter 3 relates to the first research problem of this dissertation, i.e., the *static* GDM. For this research problem, i.e., the trade-off between the speed of the CRP and HD of DMs, this problem is firstly discussed in detail and the associated challenges are highlighted. Following this, the formulation of the problem through MDPs is given and the Markov property is validated for the decision-making problem to verify the applicability of RL in the corresponding decision problem. Algorithms for training the RL agents are precisely discussed and the effectiveness and generalizability of the proposed technique are verified through the presented simulation studies.

Chapter 4 discusses the second research problem, i.e., the *dynamic* GDM with dynamic set of alternatives. In this regard, the general framework of the proposed method is illustrated, which is followed by the problem formulation and the practical verification of the proposed framework.

Chapter 5 is mainly focused on addressing the third research problem, i.e., *ODMs*, where the general limitation of BC models for trust building is firstly demonstrated and discussed. Then, the proposed LODM based on Z-numbers and Blockchain-enabled trust building mechanism is formulated, which is then followed by illustration of the feasibility of the proposed framework and the attained results w.r.t. different Blockchain protocols.

Chapter 6 presents the attained results by each developed model in Chapters 3, 4, and 5. In this chapter, the detailed discussion on the attained results along with the illustrations and tables associated with different experiments for the static, dynamic, and OMDs are presented.

Chapter 7 concludes the dissertation. It firstly begins with a brief review on the general problems addressed in this dissertation along with a short discussion on the proposed solutions. The advantages and disadvantages of the proposed frameworks are discussed and the future research directions have been spotlighted.

Chapter 2

Literature Review

This chapter is devoted to a very comprehensive review on GDM and ODMs. In this regard, we firstly give a brief introduction to both kinds of models, and, then, we specifically focus on the recent developed consensus models under the framework of GDM and ODM. Following this review, we will identify research gaps that are required to be further studied, for which we propose solutions in accordance with what discussed in Chapter 1.

2.1 Background: GDM and ODMs

In GDM, it is usually assumed that a set of DMs $\mathcal{D} = \{d_1, \ldots, d_n\}$, with n being the total number of DMs, aim at giving orders to a set of alternatives $\mathcal{X} = \{x_1, \ldots, x_q\}$ w.r.t. a set of attributes $\mathcal{A} = \{a_1, \ldots, a_m\}$ based on the opinions of the group. The framework for the consensus-based solution to this decision problem through the soft methodology is depicted in Fig. 2.1. Depending on the decision problem and the available set of alternatives and attributes, the initial opinions of DMs are passed into the consensus process block. In case that the consensus level among DMs satisfies a given threshold, the CRP ends up and the selection process gets started. Otherwise, a feedback mechanism gets activated and inconsistent DMs will be provided by recommendations on how to change their opinions for the sake of consensus reaching. As it can be observed, *time* does not play a role in this classical dynamic consensus model. However, in ODMs, time does play an important role in the modelling of dynamism.

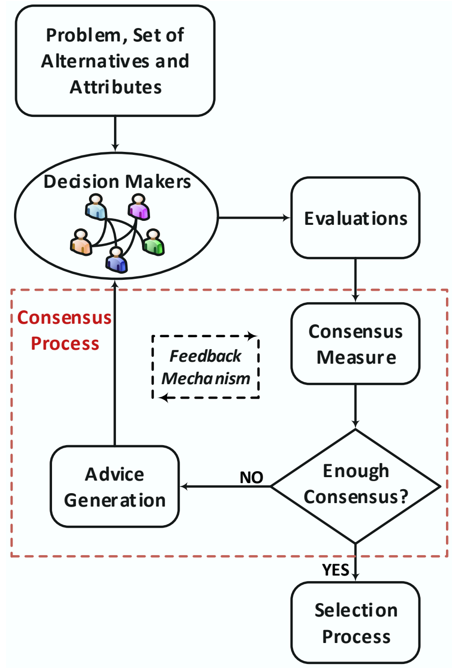

Figure 2.1 – The general framework of the consensus reaching process.

In ODMs, the DMs are usually referred to as *agents*, however, in order to unify this term for both classical dynamic consensus and ODMs, we use the terminology "DM" for both models. In ODMs, it is assumed that each DM d_i $(i = 1, \ldots, n)$ expresses an opinion of the form $\sigma_i(t)$ at time t $(t = 0, 1, 2, \ldots)$. It is also assumed that the ith DM gives a weight to the jth DM as w_{ij} satisfying $w_{ij} \geq 0$ and $\sum_{i=1}^{n} w_{ij} = 1$. Then, the opinion evolution of the ith DM is modelled as follows:

$$\sigma_i(t+1) = \sum_{j=1}^{n} w_{ij}\sigma_j(t) = w_{i1}\sigma_1(t) + \ldots + w_{in}\sigma_n(t), \tag{2.1}$$

or equivalently,

$$\Sigma(t+1) = \mathcal{W} \times \Sigma(t), \tag{2.2}$$

where $\Sigma \in \mathbb{R}^n$ and $\mathcal{W} \in \mathbb{R}^{n \times n}$. This fusion process can lead to a consensus among DMs in case that $\lim_{t\to\infty} \sigma_i(t) = \mathcal{C}$, where $i = 1, \ldots, n$, and $\mathcal{C}$ is a constant and it is called the *consensus opinion*.

Definition 1 ([48]). *DMs $d_1, \ldots, d_n$ will form a consensus if for any initial set of opinions $\Sigma(0) \in \mathbb{R}^n$, there exists a constant value $\mathcal{C} \in \mathbb{R}$ for which $\lim_{t\to\infty} \sigma_i(t) = \mathcal{C}$, with $i = 1, \ldots, n$.*

Following the Definition 1, it is worth mentioning that in case the fusion process ends up with two or more than two different consensus opinions, a polarization or fragmentation will happen, respectively.

2.2 Opinion Representation Structures

In construction of GDM and ODMs, the opinion representation structure plays a vital role. With opinion representation structure, we mean how DMs express their opinions, where it could be in terms of *numerical* or *linguistic* opinions. To this end, we firstly review some of the mostly-used structures for opinion expression in GDM and ODMs.

2.2.1 Preference ordering

This representation format can be used to provide orders for a set of alternatives from the best to the worst. In particular, for a set of alternatives $\mathcal{X} = \{x_1, \ldots, x_q\}$, a DM can express opinions in terms of preference ordering as $\mathcal{O}^k = \{o^k(1), \ldots, o^k(q)\}$, with $o^k(.)$ being a permutation of $\{1, 2, \ldots, q\}$ from the viewpoint of the kth DM. For instance, suppose that four alternatives $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ are put into discussion and the first DM d_1 provides evaluations in terms of preference ordering as $\{3, 2, 4, 1\}$. This means that from the viewpoint of d_1, the best alternative is x_4 and x_3 is the worst.

2.2.2 Multiplicative preference relations

The multiplicative representation format leads to numerical preference relations that interpret the ratio of the preference degree of an alternative over other alternatives in a given scale. Specifically, for the DM d_k, the multiplicative preference relation over the set of alternatives $\mathcal{X} \in \mathbb{R}^q$ could be of the form of a matrix as $\mathcal{P} = [p_{ij}]_{q\times q}$, being p_{ij} belonged exactly to a designated scale to indicate the preference intensity of alternative x_i over the alternative x_j. One of the most-widely used scales is the Saaty 1-9 scale. In this regard, a preference value of $p_{ij} = 1$ denotes no difference between alternatives x_i and x_j from the viewpoint of a DM, while $p_{ij} = 9$ indicates that x_i is absolutely preferred to x_j.

2.2.3 Fuzzy preference relations

Fuzzy preference relations could be referred to as the most widely-used representation structure. It is a numerical representation and could be defined as a fuzzy set on the product set $\mathcal{X} \times \mathcal{X}$. It is often characterized by means of a membership function (MF) $\mu_P : \mathcal{X} \times \mathcal{X} \to [0, 1]$. When the cardinality of the set of feasible solutions $\mathcal{X}$ is small, a fuzzy preference relation can be expressed via a matrix $P = [p_{ij}]_{q\times q}$, where $p_{ij} = \mu_P(x_i, x_j)$ with $i, j \in \{1, \ldots, q\}$, and it indicates the preference intensity of alternative x_i over the alternative x_j. For instance, $p_{ij} = 0.5$ shows the indifference evaluation between alternatives x_i and x_j, or $p_{ij} = 1$ denotes that x_i is absolutely preferred to x_j. In this representation, it is required to set $p_{ii} = 0.5$, with $i = 1, \ldots, q$. In case $p_{ij} + p_{ji} = 1$ ($\forall i, j \in \{1, \ldots, n\}$), it is said that the evaluation matrix P is additive reciprocal and the fuzzy preference relation is often called additive preference relation.

2.2.4 Linguistic preferences

The linguistic assessment of DMs can be enabled by resorting to linguistic term sets (LTSs) and computing with word (CWW) methodologies. A balanced LTS $\mathcal{S} = \{s_i | i = 0, 1, \ldots, 2r\}$, is a completely ordered and finite set with odd cardinality, where r is a nonnegative integer value. In this LTS, s_i represents a linguistic variable, where for two arbitrary linguistic values s_i and s_j, the following criteria hold: 1) it is an ordered set, i.e., $s_i \leq s_j$ iff $i \leq j$, and, 2) there is a negation operator for which $neg(s_i) = s_j$ if $i + j = 2r$. An example of an LTS can be $\mathcal{S} = \{s_0 =$ 'very poor'$, s_1 =$ 'poor'$, s_2 =$ 'slightly poor'$, s_3 =$ 'fair'$, s_4 =$ 'slightly good'$, s_5 =$ 'good'$, s_6 =$ 'very good'$\}$.

The semantics of linguistic terms in an LTS can be extracted by means of type-1 and interval type-2 fuzzy sets, however, to employ LTSs in GDM problems, CWW tools are required to be developed. This has been initiated in [51] by introducing the concept of 2-tuple linguistic modeling.

Definition 2. *Given an LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$, suppose that $\beta \in [0, 2r]$ is resulted by means of a symbolic aggregation operation on $\mathcal{S}$. Then, the equivalent information to β can be expressed in terms of a 2-tuple as follows:*

$$\Delta(\beta) = (s_i, \alpha), \ with \begin{cases} s_i, & i = round(\beta), \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{2.3}$$

where $\Delta : [0, 2r] \to \mathcal{S} \times [-0.5, 0.5)$ and 'round' is used to denote the round operation.

In this regard, the following definition represents how to extract the numerical information from a 2-tuple linguistic assessment.

Definition 3. *Given an LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$ and a 2-tuple (s_i, α), the numerical* *value $\beta \in [0, 2r]$ of this 2-tuple can be evoked by means of the function Δ^{-1} as follows:*

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta, \tag{2.4}$$

where $\Delta^{-1} : \mathcal{S} \times [-0.5, 0.5) \to [0, 2r]$.

Following the above definitions, it is straightforward that a linguistic term $s_i \in \mathcal{S}$ can be represented by means of a 2-tuple as $s_i = \Delta(s_i, 0)$. To this end, in what follows, we review some of the commonly-used linguistic representation structures including fuzzy Z-numbers, hesitant fuzzy linguistic preferences, intuitionistic linguistic preference relations, and interval linguistic preference relations.

Z-numbers contain two different components to describe an uncertain variable and have been extensively used in different applications including decision analysis.

Definition 4 ([52])**.** *A Z-number, denoted by $\mathcal{Z} = (A, B)$, contains two components, where the first component, i.e., A, is a constraint on the values that a real-valued uncertain variable can take. The second component, i.e., B, denotes the certainty of the first component.*

As it can be observed from Definition 4, Z-numbers rely on two LTSs to describe an assessment on a given variable. As an example, the first component of a Z-number can be taken from the LTS $\mathcal{S} = \{s_0 = \text{'very poor'}, \ldots, s_6 = \text{'very good'}\}$ as before, and, the certainty about the first component can be chosen from another LTS defined by $\mathcal{S}' = \{s_0 = \text{'very uncertain'}, s_1 = \text{'uncertain'}, s_2 = \text{'slightly uncertain'}, s_3 = \text{'neutral'}, s_4 = \text{'slightly certain'}, s_5 = \text{'certain'}, s_6 = \text{'very certain'}\}$. To this end, a Z-number can be represented by an ordered pair of fuzzy numbers as $\mathcal{Z} = (s_i, s'_i)$ with $s_i \in \mathcal{S}$ and $s'_i \in \mathcal{S}'$, such as $\mathcal{Z} = (s_6, s'_6) = (\text{'very good'}, \text{'very certain'})$.

Hesitant fuzzy linguistic term sets (HFLTSs) are also useful tools for DMs to express their opinions by making use of several LTSs simultaneously. This is to overcome the limitations of granularity of DMs' knowledge that might not be concurrent with the granularity of a given single LTS.

Definition 5 ([53]). *For a given LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$, a HFLTS, denoted by $\mathcal{H}$, is an ordered and finite subset of consecutive linguistic terms of $\mathcal{S}$.*

By resorting to the above definition, it is evident that different HFLTSs extracted from a given LTS may contain different number of linguistic elements. In this regard, several schemes have been developed to normalize HFLTSs. For instance, two normalization principles, called α-normalization and β-normalization, are proposed in [54] that rely on the risk preferences of DMs to remove some elements from the given HFLTSs (α-normalization) or add elements (β-normalization) to maintain the same number of elements in each HFLTS. To this end, the definition of hesitant fuzzy linguistic preference relations (HFLPRs) can be given as follows.

Definition 6 ([54]). *Assume that $M_{\mathcal{S}}$ is a set of HFLTSs constructed based on the LTS $\mathcal{S}$. An HFLPR can then be represented by a matrix $\mathcal{P} = (p_{ij})_{n\times n}$, where $p_{ij} \in M_{\mathcal{S}}$ and the negation operator holds for p_{ij}, i.e., $neg(p_{ij}) = p_{ji}$.*

As an example, let $\mathcal{S} = \{s_0 = \text{`very poor'}, \ldots, s_6 = \text{`very good'}\}$ be an LTS as before. An HFLPR can then be constructed as follows:

$$\mathcal{P} = \begin{bmatrix} \{s_3\} & \{s_2, s_6\} & \{s_1, s_3, s_4\} \\ \{s_3, s_5\} & \{s_3\} & \{s_4, s_5, s_6\} \\ \{s_1, s_2, s_3, s_4\} & \{s_3, s_5\} & \{s_3\} \end{bmatrix}.$$

Another linguistic representation structure that we review is the intuitionistic linguistic fuzzy preference relations (ILFPRs). The above-mentioned representation structures are mainly used to express the preferred assessments of DMs through either numerical or linguistic preference relations. ILFPRs, however, enable DMs to provide not only their preferred assessments, but also their non-preferred assessments. This representation structure is built upon the intuitionistic fuzzy sets (IFSs) that were firstly introduced by Atanassov [55]. The IFSs are then used by Szmidt and Kacprzyk [56] to propose IFPRs that are constructed based upon numerical values. The operations on IFSs are then extended to linguistic intuitionistic fuzzy sets by Yager [57], which then led to the introduction of the linguistic intuitionistic fuzzy variables [58] to qualitatively represent the preferred and non-preferred assessments of DMs.

Definition 7 ([59])**.** *An intuitionistic linguistic set $\tilde{\mathcal{A}}$ on the set of alternatives $\mathcal{X}$ can be defined as $\tilde{\mathcal{A}} = \{\langle x_i| < s_{\theta(x_i)}, (u(x_i), v(x_i)) >\rangle\}$, where $s_{\theta(x_i)} \in \mathcal{S}$ is a linguistic term, $u(x_i)$ and $v(x_i)$ are used to denote the preferred and non-preferred degrees of the alternative $x_i \in \mathcal{X}$ to the designated linguistic variable $s_{\theta(x_i)}$, with $u(x_i), v(x_i) \in [0, 1]$ and $u(x_i) + v(x_i) = 1, \forall x_i \in \mathcal{X}$.*

With the characteristics of the intuitionistic linguistic sets given in Definition 7, an intuitionistic linguistic variable can be represented by $\tilde{a} = (s_{\theta(a)}, < u(a), v(a) >)$. Then, an ILFPR can be defined as follows.

Definition 8. *An ILFPR on a set of given alternatives $\mathcal{X}$ can be represented by a matrix of the form $\mathcal{P} = (p_{ij})_{n\times n}$, where $p_{ij} = (< s_{\theta_{ij}}, (u_{ij}, v_{ij}) >)$ for $i, j = 1, \ldots, n$, $s_{\theta_{ij}} \in \mathcal{S}$, u_{ij} and v_{ij} being the preferred and non-preferred degrees of alternative x_i over x_j w.r.t. the designated linguistic term θ_{ij}.*

As an example, having the LTS $\mathcal{S} = \{s_0, \ldots, s_6\}$ as before, an element of an ILFPR $\mathcal{P}$ can be represented by $p_{12} =< s_2, (1, 0) >$.

2.3 Consensus Models to Support Classical GDM

Consensus models are equipped with a feedback mechanism, which is typically referred to as recommendation mechanism, and it aims to help inconsistent DMs with modifying their opinions to be guided toward the collective opinion of the group through either a couple of discussion rounds or in one step. The former scheme is usually employed by means of identification and direction rules, while the latter scheme can be realized in the context of optimization models. Despite of this general categorization, we aim to review the most-recent advances in the design of feedback mechanisms developed for novel preference structures and operators by considering the behaviour of DMs, the size of the group, and the employed optimization schemes.

2.3.1 Developed preference structures

The design of a feedback mechanism is highly dependent to the preference structure and requires the development of tools for the sake of consensus reaching. For instance, linguistic preference relations with self-confidence (LPRs-SC) is proposed in [60], where a two-step feedback mechanism is suggested in [61] to not only modify the opinions of DMs, but also to modify their corresponding level of self-confidence. This has been enabled by proposing an aggregation operator and a self-confidence score function to meaningfully adjust the weights of DMs. The authors in [62] introduce the new concept of Pythagorean fuzzy linguistic preference relations (PFLPRs) along with the Pythagorean fuzzy linguistic values (PFLVs) that account for the linguistic membership and non-membership degrees and are driven from the Pythagorean fuzzy sets theory proposed by Yager et al. in 2013 [63]. Based upon the definition of consistency, individual CD, and group CD for PFLPRs, a multi-step feedback mechanism is proposed to adjust the individual CD of the worst DM at each iteration. The interesting feature of the proposed mechanism is that the consistency level of evaluations is retained even after the employed adjustments. In [64], authors proposed a novel preference structure, called flexible linguistic expressions (FLEs), where DMs are allowed to express their opinions by utilizing different subsets of a given LTS along with the distribution information over the expressed subsets. This structure could be referred to as an extension to the linguistic distribution (LD) structure, where not only the LDs, but also incomplete LDs, possibility distribution for HFLTSs and proportional HFLTSs can be extracted from this representation. To deal with uncertainties, an aggregation operator with accuracy and minimum preference loss is proposed for FLEs to construct the collective evaluation and the feedback mechanism benefits from consensus rules with minimum preference loss to adapt inconsistent

Table 2.1 – Recently-developed preference representation structures for GDM.

Reference	Representation Structure
[74, 75]	Pythagorean linguistic preference relations
[76]	Flexible Linguistic Expressions
[77]	Double hierarchy linguistic preference relations
[78]	Comparative linguistic expressions
[79, 80]	Z-numbers and their extensions
[81]	Nonlinear preference relations
[82]	Self-confident linguistic preference relations
[83]	q-rung orthopair fuzzy preference relations
[84]	Complex intuitionistic fuzzy preference relations
[85, 86]	Probabilistic linguistic preference relations
[87]	Heterogeneous preference relations

opinions. More recently, a preference structure is proposed in [65] based upon augmenting the concepts of self-confidence degree and double hierarchy linguistic preference relation (DHLPR). The consensus model is then proposed to be built based on the individual and collective priority vectors, where a feedback mechanism based on the identification and direction rules is proposed to adjust inconsistent DHLPRs. Other representation structures based on the extended versions of Z-numbers such as Z^E-numbers [66] and Z probabilistic LTSs [67], Atanassov's interval valued intuitionistic fuzzy sets and trapezium clouds [68], nonlinear preferences [69], unbalanced probabilistic LTSs [70], incomplete q-rung and interval valued q-rung orthopair FPRs [71, 72], and complex LTSs [73] have also been developed for the sake of decision making. Table 2.1 summarizes the developed preference representation structures in the recent literature works.

2.3.2 Developed operators

New preference structures typically require the introduction of novel operational tools for the sake of consensus reaching in GDM problems. In this section, we review some recent efforts toward the development of useful operators to enable consensus reaching through feedback mechanisms under different preference structures.

Interval type-2 fuzzy sets (IT2FSs) have attracted the attention of researchers due to their efficiency in modelling uncertainties. The authors in [88] proposed the conversion of classical linguistic terms into triangular IT2FSs and by developing weighted mean and weighted semi-absolute deviation operators for IT2FSs, they constructed a consensus model for portfolio allocation. The feedback mechanism in this work considers the acceptable tolerance level of DMs in adjusting their preferences and maximum return and minimum risk models are then suggested for preference adjustment. An improved version of the Euclidean and Hamming distance measures for ILFPRs are proposed in [89], and, accordingly, a feedback mechanism is built upon adjusting the preference elements based on their closeness to a collective one. Various operational laws for probabilistic linguistic q-rung orthopair fuzzy sets (PLq-ROFS) have been recently proposed in [90], where the authors proposed to extract the semantics of PLq-ROFS by means of novel linguistic scale functions (LSFs). The comparison between PLq-ROFSs is enabled by introducing new score and accuracy functions, where the aggregation of PLq-ROFSs is performed by means of PLq-ROF weighted averaging and PLq-ROF ordered weighted averaging. The feedback mechanism adjusts the DMs' preferences by basic operations on PLq-ROFSs and by involving the correlation measures of each DM. Later in [91], authors proposed the integration of neutrality aggregation into the q-ROFSs to construct a power aggregation operator for the sake of GDM. For dual hesitant q-ROFSs [92] and dual probabilistic linguistic environments [93], required operational laws are developed based on the Dombi and Bonferroni mean operators for aggregating preferences and ordering alternatives in the selection process. Furthermore, some attempts have been recently devoted to the design of operators for Z-numbers based on the Archimedean t-norms and t-conorms [94], distance operators for HFLTSs [95] and pair-wise preference relations [96]. Table 2.2 summarizes the developed operators in the recent literature works.

Table 2.2 – Developed operators in the recent literature works.

Reference	Developed Operators
[93, 92]	Dombi operators and Bonferroni mean operators
[94]	Archimedean t-norms and t-conorms
[91]	Power neutrality aggregation operator
[96]	Distance operator for evidential preferences
[95]	Distance operator for hesitant information
[89]	Intuitionistic multiplicative distance measures
[90]	q-rung orthopair fuzzy weighted averaging operator
[88]	Information measures for IT2FSs

2.3.3 Behavioural mechanisms

We generally refer to feedback mechanisms that reflect the DMs' interests, trust relations, attitude, and cooperative or non-cooperative behaviour in the consensus process as the *behavioural* mechanisms. In what follows, we review the most-recent advances in this type of feedback mechanisms.

Due to differences in the nature of decision environments or knowledge and experience of DMs, it is a common practice to take into account the interest of DMs in selecting an attribute or a set of attributes to evaluate a predefined set of alternatives [97]. Following this and for a diverse set of DMs, the construction of a heterogeneous decision environment is beneficial due to providing an opportunity for DMs to express their opinions in terms of their preferred preference structures. Developed techniques for heterogeneous decision environments are usually relying on proposing and performing proper transformations to augment different structures into a homogeneous structure while ensuring the consistency among preference relations [98]. In this regard, the most-recent techniques have focused on the unbalanced LTSs to address the nonlinearities in DM's cognition [99], case-based reasoning for emergency decision-making [100], criteria interactions [101], and to deal with dynamic contexts [102].

Another consideration in behaviour modelling for consensus reaching is the trust relationships between a set of anonymous DMs, which is usually realized through a social network-based mechanism. We categorize these techniques under the large-scale decision-making models, which will be given in the next section. However, it is worth mentioning that in contrast to conventional trust or distrust models, recently-developed techniques treat the trust among DMs as a matter of degree and novel trust functions and trust scores have been proposed to model relationship among DMs [103, 104]. The attitude of DMs could also be considered in behavioural mechanisms to reflect the attitude of DMs toward consensus reaching. To quantify the attitude of DMs in a continuous ranging scale to reflect the pessimistic attitudes to indifferent attitudes in construction of the trust relationships, an attitudinal trust degree is proposed in [105], which makes use of an ordered weighted average (OWA) operator guided by a unit-monotonic function. Considering the risk attitude of DMs in alternative ranking through an evidential reasoning methodology [106] and construction of linguistic quantifiers based upon the attitude of DMs [107] are of recent trends in attitude-based feedback mechanisms.

As the last category of behavioural mechanisms, we review some recent advances on managing the non-cooperative behaviour of DMs toward consensus reaching by means of a feedback mechanism. The non-cooperative behaviour refers to the case, in which the inconsistent DMs are reluctant to modify their opinions according to the provided recommendations through the feedback mechanism. In particular cases, even some DMs intentionally take opposite actions to the recommended adjustments. Therefore, identifying and managing the non-cooperative DMs are of paramount importance for consensus reaching due to their negative impacts in terms of adjustment cost and consensus time. Weight punishment and exit-delegation are two commonly used approaches to manage non-cooperative DMs. The former aims to penalize non-cooperative DMs by reducing their designated weights so as to make them have less impact on the decision made by the group. In the latter, the non-cooperative DMs are removed from the group. In [108], the degree of conflict of DMs are used to identify non-cooperative DMs, where a weight penalty based on the triangular fuzzy numbers are considered for internal DMs, while external non-cooperative DMs are removed from the group. For uncertain decision-making during the COVID-19 outbreak, a co-operation degree is devised in [109] to assign DMs into multiple clusters, where clusters with low co-operation degree are penalized with a low weight. By resorting to the number of adjustments of each DM, a co-operation index is introduced in [110] and it is proposed to take different actions for semi-cooperative and fully non-cooperative DMs in terms of weight penalties. An anti-biased statistical mechanism based upon a Biasedness index is proposed in [111] to manage non-cooperative DMs through extreme, moderate, and soft weight punishment schemes.

2.3.4 Large-Scale GDM

Large-scale GDM (LSGDM) is usually referred to a decision problem that involves at least twenty DMs [112]. Other than the size of the involved DMs, LSGDM approaches need to deal with heterogeneous information due to the diversity of DMs in terms of their background and level of expertise. Furthermore, the management of non-cooperative DMs who interact through a designated social network platform could also be referred to as other challenges that LSGDM are facing with. In this regard, the most-recent works in LSGDM have focused on managing non-cooperative DMs by considering their trust relationships in an interactive social network framework.

Management of the non-cooperative DMs is an inevitable part of LSGDM for the sake of dimension reduction. This is usually performed by means of assigning DMs into multiple clusters based upon some constructed similarity indexes, where DMs with a lower value of the designated similarity index compared with other members of a cluster can be excluded. Therefore, there is a trend of works on attempting toward the design of efficient clustering-based mechanisms to deal with non-cooperative DMs w.r.t. preference representation structures. By resorting to the CEN of a large group of DMs, authors in [113] made use of the Louvain two-phase clustering algorithm [114] to extract communities. For heterogeneous representation structures, an extended version of the k-means clustering algorithm could be employed based on the Euclidean distance between the normalized preference relations and the cluster centres. In this regard, cooperative and non-cooperative indexes of DMs and clusters can be constructed based on the enlargement of the deviation between the original and modified preferences [115] to manage non-cooperative DMs. The same structure could be implemented based on the weight punishment mechanism for k-means [116] or grey clustering [117] algorithms.

Another trend of LSGDM works follow the trust-based feedback mechanisms, which are realized through social networks. This study is important owing to the fact that trust relationships not only have impacts on the clustering process for dimension reduction in LSGDM, but also can influence the CRP. Trust relationships are usually modelled via directed and weighted trust graphs, where the nodes are assumed to be DMs, edges of the graph denotes the trust relationships, and the designated weights show the trust score from one DM to another. This modelling of trust relationships, which is enabled by means of social network analysis, has a significant impact on reducing the complexity of aggregation process by identifying the leadership behaviour of DMs. This could also help with managing the non-cooperative DMs. The idea is to divide DMs by means of clustering algorithms such as the one proposed in [118], where a leader will be assigned to each cluster. In the feedback mechanism, followers (ordinary members of a cluster) are suggested to follow the behaviour of the leader of the cluster so as to adjust their opinions, while non-cooperative members will be assigned a lower weight in the consensus process [119]. Opinion similarity could also be augmented with trust relationships in construction of clustering algorithms for LSGDM in order to involve the level of difference among opinions of DMs [120]. Other than building consensus based upon the opinion of trusted peers for a DM, recent studies show that the opinions of distrusted peers could also help with consensus reaching [121]. This could be employed for social networks with high or medium density because for low density social networks, the collective intelligence level will be diminished when the scope of distrust increases [122]. Besides the techniques developed for managing non-cooperative DMs, a novel scheme is presented in [123] in order to prevent individual manipulation behaviour by an attitudinal weight-adjustment mechanism and to prevent group manipulation behaviour through a minimum adjustment cost framework under social network GDM.

A worthwhile research field in social network-based GDM is the trust propagation in trust networks. A recent review on trust propagation in social networks is provided in [124]. As it was mentioned earlier, trust relationships can be modelled via directed and weighted graphs, where DMs are connected via either a direct or an indirect path. In case of indirect paths, there is a need to estimate the value of trust among DMs, which can be done by means of trust propagation techniques. The most-recent literature works in this field of research are devoted to multi-path trust propagation [125], linguistic trust propagation [126], and DMs' weight adjustment through trust propagation [127]. Managing the minority opinions [128], optimization schemes for consensus reaching [129, 130], minimizing the information loss [131], and dealing with incomplete preferences [132] are some interesting and open problems in social network-based GDM.

2.3.5 Minimum adjustment cost

As it was mentioned earlier, the feedback mechanism can be realized through either identification and direction rules or minimum adjustment cost mechanisms. The former relies on an iterative approach to modify the opinion of inconsistent DMs through multiple discussion rounds. This can in turn have some disadvantages such as deviation of modified opinions from original ones in a great context, imposing high computational cost, and delaying the CRP. In this regard, in the last decade, we have witnessed the emergence of the minimum adjustment cost notion, where the aim is to adjust the opinions of inconsistent DMs (DMs with lower CD than the given consensus threshold) in one step through optimization problems that are subject to different constraints. The reader is referred to a detailed review on these techniques given in [133]. In what follows, we review the most-recent feedback mechanisms constructed based upon the minimum adjustment cost notion.

The basic minimum cost consensus model that is realized by means of an aggregation operator could be constructed as follows:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{n} c_i|\sigma_i - \sigma_i'| \\
\text{s.t.} \quad & |\sigma_i' - \sigma_c'| \leq \epsilon, \quad i = 1, \ldots, n \\
& \sigma_c' = \sum_{i=1}^{n} w_i \sigma_i',
\end{aligned} \tag{2.5}$$

where c_i denotes the unit adjustment cost of DM d_i, σ_i and σ_i' show the initial and modified opinions of DM d_i, respectively, n is the total number of DMs, and σ_c' is the collective opinion. The unit adjustment cost is usually assumed to be constant, however, in realistic decision-making this value is uncertain and the uncertainty has been realized by means of interval values or distribution uncertainty. To this end, an estimation mechanism is proposed in [134] to estimate c_i by augmenting three different constraints for giving higher costs to DMs who change their preferences frequently, to model its uncertainty by means of an ellipsoidal set, and to force the sum of total adjustments costs to be lower than the compensation cost of the moderator. The minimum adjustment feedback mechanism developed in [135] is subject to a maximum compromise limit, i.e., the adjusted preferences are required to be within a pre-defined compromise interval which is rarely studied in social network GDM. Finally, a two-stage feedback mechanism is proposed in [136], where in the first stage, the aim is to determine reference points and to adjust the individual positional ordering of DMs, which are then fed into the second stage for the recommendation generation. Table 2.3 summarizes the developed feedback mechanisms in the recent literature works.

Table 2.3 – Developed feedback mechanisms in the recent literature works.

Feedback Mechanism	Description	Reference
Behavioral Mechanisms	Mechanisms based on the trust relationships among DMs	[97, 103, 104]
	Developed mechanisms by considering the attitude of DMs	[107, 106]
	Management of the non-cooperative behaviour of DMs	[108, 109, 110]
	Management of the biased DMs	[111]
Large-Scale GDM Models	Trust-based mechanisms	[127, 129, 120]
	Trust propagation under social network	[125, 122, 126]
	Leadership and non-cooperative behaviours	[119, 116, 117, 115, 113]
Minimum Adjustment Cost	Behavioural mechanisms	[123]
	Developed mechanisms under social network analysis	[135, 134, 126]

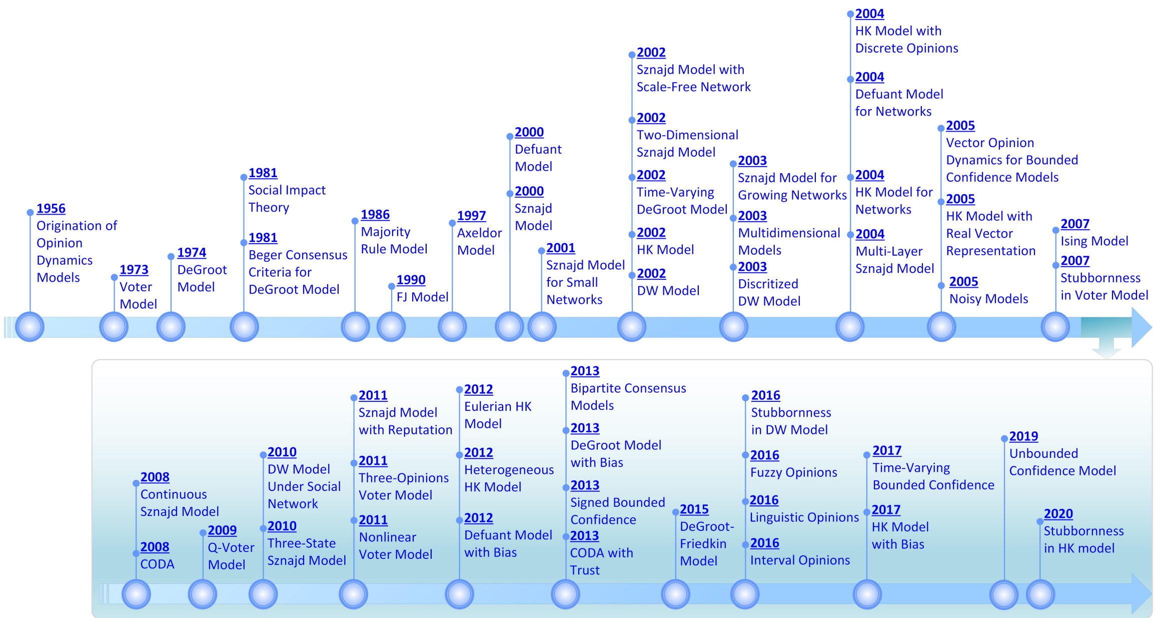

Figure 2.2 – Timeline of some of most important milestones in ODMs.

2.4 Opinion Dynamics for Consensus Reaching

ODMs can be categorized into the time-modelling category of dynamic consensus approaches. This means that time is involved in opinion evolution of DMs and is an important parameter to model dynamism in the consensus process. The timeline of some of the most important milestones in ODMs is represented in Fig. 2.2. In this regard, we have arguably categorized these models into multiple categories by considering the DMs' behaviour, developed models based on the social network analysis, minimum adjustment cost or optimization models, and linguistic models. We then provide a detailed description of the new insights that have been brought by means of RL algorithms in classical dynamic consensus and ODMs.

2.4.1 Decision makers' behaviour

To consider the willingness of DMs in accepting the provided recommendations through feedback mechanisms, BC models provide the opportunity for DMs to only consider

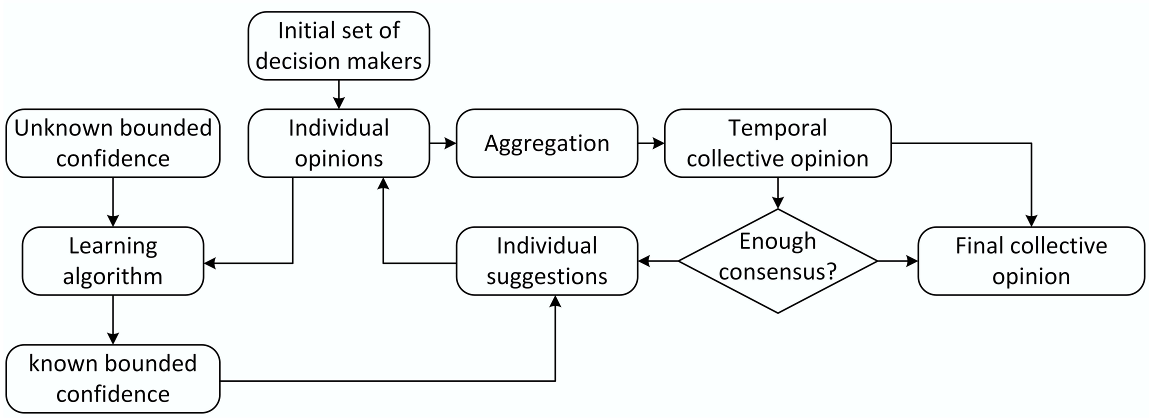

Figure 2.3 – The general framework of recommendation mechanisms with unknown bounded confidence [3].

preferences that do not exceed their designated confidence levels. The BC level could be either known or unknown, where the unknown levels are required to be estimated. Fig. 2.3 depicts a general framework of BC models with known or unknown confidence levels. The general idea is that for $P_k = (p_{ij}^k)_{n\times n}$ being the original opinion of DM d_k, and $P_f = (p_{ij}^f)_{n\times n}$ being the recommended advice generated through the feedback mechanism, then, the DM d_k will accept this recommendation if $D_{kf} \leq \epsilon_k$, where D_{kf} is some distance function and $\epsilon \in [0,1]$ is the confidence bound. One way to deal with unknown bound of confidence is to estimate it via an interval $[\underline{b}^k, \overline{b}^k]$ and by setting a BC threshold τ. The estimation would be assumed accurate in case that $\overline{b}^k - \underline{b}^k \geq \tau$ [3]. Then, based upon D_{kc}, i.e., the distance between opinion of d_k and the collective opinion, feedback rules can be generated. For instance, when $\overline{b}^k - \underline{b}^k \geq \tau$ and $D_{kc} > \underline{b}^k$, the generated advice will be $P_f = P_k + \underline{b}^k / D_{kc} \times (P_c - P_k)$.

Self-persistence behaviour refers to the DMs' adherence to their opinions, which should be considered in the weight-adjustment phase of ODMs. One way to realize this behaviour is through a trust network, where the self-persistence degree of DM

d_i, i.e., α_i, can form the diagonal elements of the weight matrix $\mathcal{W}$ as follows:

$$w_{ii} = \begin{cases} \alpha_i, & deg_i^- > 0, \\ 1, & deg_i^- = 0, \end{cases} \tag{2.6}$$

where deg_i^- denotes the sum of the incoming edges to node d_i in the constructed trust network of DMs. Other non-diagonal elements could also be shaped based on α through an influence index,

$$\digamma_i = \frac{\alpha_i + \varkappa + \varpi}{3}, \tag{2.7}$$

where $\varkappa = deg_i^+/(n-1)$, $\varpi = deg_i^-/3$, deg_i^+ denotes the sum of outgoing edges, and n is the total number of DMs. The self-persistence guided weight assignment could then be as follows:

$$w_{ij} = \frac{\digamma_j}{\sum_k \digamma_k}(1-\alpha_i)a_{ij}, \;\; i \neq j, \tag{2.8}$$

where $\sum_k \digamma_k$ denotes the sum of influence of one-step neighbours of d_i and a_{ij} denotes the adjacency elements. This mechanism is extended the case that considers the influence of two-step neighbours in [137].

The cognitive dissonance of DMs could also shape their communications and updating rule of the ODMs [138]. One case of the cognitive dissonance is the situation, in which a DM aims to eliminate the uncomfortable feelings, meaning that when $D_{ij}(t)$ (the distance between opinions of d_i and a trusted peer d_j at time-step t) is larger than some confidence threshold ϵ, i.e., $D_{ij}(t) > \epsilon$, DM d_i feels uncomfortable and breaks the connection with DM d_j. Another case refers to a realistic situation that DMs aim to build more connections so they feel the support of more DMs. Let $\mathcal{I}(d_i, \Sigma(t)) = \{d_j | D_{ij} \leq \epsilon, a_{ij} = 1\}$ be the confidence set of d_i. Then, in case DMs d_i and d_j have a common trusted peer, shown by d_k, where $d_k \in \mathcal{I}(d_i, \Sigma(t)) \cap \mathcal{I}(d_j, \Sigma(t))$, and $D_{ij}(t) \leq \epsilon$, DMs d_i and d_j can make a connection. Once the connections and eliminations are done at time-step t, the weight-adjustment can be simply fulfilled as follows:

$$w_{ij} = \begin{cases} \frac{1}{|\mathcal{I}(d_i, \Sigma(t))|}, & d_j \in \mathcal{I}(d_i, \Sigma(t)), \\ 0, & \text{otherwise.} \end{cases} \tag{2.9}$$

The concept of leadership behaviour has also been used to guide feedback mechanisms in ODMs [139, 140]. The leader is usually referred to DMs with high influence in the trust network, where different approaches are proposed for identifying the set of leaders. One common way is to divide the complex network of DMs into multiple sub-networks [141], construct the accessibility matrix [142], and perform iterative searches in each sub-network to identify DMs with more influential connections [143]. Other than leadership in a group of DMs, the pressure imposed by the group could also be categorized in the behavioural category of ODMs [144]. This is proposed to model the situation, in which a DM feels pressured to give away an opinion which is similar to the collective opinion of the group. A scheme based on the BC model is proposed in [145] that accounts for the group pressure, where the updating rule of opinions is formed as follows:

$$\sigma_i(t+1) = \frac{(1-\rho_i)\sum_{j \in \mathcal{I}(d_i, \Sigma(t))} \sigma_j(t)}{|\mathcal{I}(d_i, \Sigma(t))|} + \rho_i \sigma_c(t), \tag{2.10}$$

where $\sigma_c(t)$ is the weighted average of DMs' opinions and ρ_i is used to show the group pressure. Other than the group pressure, a DM may also suffer from the peer pressure [146]. Other behavioural actions such as stubbornness [147] and prejudice [148] could also affect the ODMs. To model these all behavioural actions, a stress function of the following form is proposed in [149]:

$$\Theta_i(\sigma_i(t), \sigma_i(t-1), t) = \zeta_i \left(\sigma_i(t) - \sigma_i^+(t)\right)^2 + \upsilon(t) \sum_{j=1}^{n} |a_{ij}| \left(\sigma_i(t) - \text{sign}(a_{ij})\sigma_j(t-1)\right)^2, \tag{2.11}$$

where ζ_i is used to model the prejudice of the DM d_i, $\sigma_i^+(t)$ shows the constant prejudice of the DM d_i, and $\upsilon(t)$ denotes the peer pressure. Following this structure, the aim is to minimize the stress function so as to find the update rule of DMs. It is resulted that the following update rule will minimize the stress function given in (2.11):

$$\sigma_i(t) = \frac{\zeta_i \sigma_i^+ + \upsilon(t) \sum_{j=1}^{n} a_{ij}\sigma_j(t-1)}{\zeta_i + \upsilon(t)\gamma_i}, \tag{2.12}$$

where $\Gamma = diag[\gamma_1, \ldots, \gamma_n] = L + A$, $\gamma_i = \sum_{j=1}^{n} a_{ij}$, and L and A are the Laplacian and signed adjacency matrix of the DMs' signed network. By resorting to the graph theory, the willingness of DMs [150], the problem of unilateral DMs [151] and antagonistic and indifference DMs [152] have also been recently addressed under the opinion dynamics framework.

2.4.2 Social networks

Most of the recent research works fall into this category of methods for ODMs. One of the most-recent advances rely on the continuous opinion and discrete action (CODA) model [153], which can be categorized as a continuous ODM. Developing ODMs with the simultaneous evolution of opinions and actions under social network analysis is an interesting research topic. A model is recently developed in [154] under the assumption that DMs' opinions are private [155] and cannot be obtained by others unless they are directly connected in the social network. The actions, however, are public and DMs are aware of others' actions. The relationship between actions and opinions is modelled as follows:

$$A_i(t) = \begin{cases} 0, & \sigma_i(t) \in [0, h_i) \\ 1, & \sigma_i(t) \in [h_i, 1], \end{cases} \tag{2.13}$$

where h_i is a threshold for action selection of DM d_i. Then, based upon the relationships among DMs, the update rule given in (2.14) has been constructed, where $\mu \in (0, 0.5]$ is a convergence parameter.

$$\sigma_i(t+1) = \begin{cases} \sigma_i(t), & a_{ij} = 1 \wedge |\sigma_i(t) - \sigma_j(t)| > \epsilon, \\ \sigma_i(t) + \mu\left(\sigma_j(t) - \sigma_i(t)\right), & a_{ij} = 1 \wedge |\sigma_i(t) - \sigma_j(t)| \leq \epsilon, \\ \sigma_i(t), & a_{ij} = 0 \wedge |\sigma_i(t) - A_j(t)| > \epsilon, \\ \sigma_i(t) + \mu\left(A_j(t) - \sigma_i(t)\right), & a_{ij} = 0 \wedge |\sigma_i(t) - A_j(t)| \leq \epsilon. \end{cases} \tag{2.14}$$

Recently, a novel model under the structure of a social graph is proposed in [156], where the DMs' interactions do not rely on the proximity of their opinions, but on the influence of their opinions on one topic to other topics. The continuous opinion evolution of DMs is modelled as follows:

$$\sigma_i(t + dt) = \sigma_i(t) + \mathcal{C}\Delta_{\sigma_i}(t), \tag{2.15}$$

where $\mathcal{C}$ is used to denote the influence of opinions and Δ_{σ_i} is as follows:

$$\begin{aligned} \Delta_{\sigma_i} = & \frac{1 - \beta(P_i)}{n-1} \sum_{j \neq i} \zeta(P_i, P_j)[\sigma_j(t) - \sigma_i(t)]dt \\ & + \beta(P_i)[\mathbf{u}(P_i) - \sigma_i(t)]dt + \gamma \mathbf{w}_i(t), \end{aligned} \tag{2.16}$$

where $\beta(P_i) \in (0, 1]$ is the insensitivity of DM d_i that holds the P_i personality [157], $\zeta(P_i, P_j)$ is used to model intensity of interactions among DMs, $\mathbf{u}(P_i)$ accounts for the prejudice of a DM, and $\mathbf{w}_i(t)$ denotes the endogenous process of opinion evolution for each DM [158]. Other models are also developed for different types of interactions in opinion evolution of DMs. For a set of homogeneous DMs, the effect of interaction intensity is investigated in [159] for biased (opinion-dependent) and unbiased (opinion-independent) intensity, where the results are then extended to heterogeneous DMs in [160]. Furthermore, by considering the dependency of DMs' interactions to their current and past opinions, a memory-based connectivity mechanism for ODMs under social network is proposed in [161]. In addition, for social networks with switching topology, an ODM is proposed in [162], where under an arbitrary switching signal, the system bipartite (polarization) consensus or consensus is guaranteed. The evolution of the network over time is studied in [163] by resorting to constructing a rule-base using a distance matrix, which contains the proximity of opinions of paired DMs. The network could also evolve w.r.t. temporal activity patterns such as contact strength of DMs and daily patterns, where the impact of these temporal activities on the speed of consensus is investigated in [164].

Another interesting research trend in social network-based ODMs is to handle uncertainties in DMs' opinions [165]. One way to consider uncertainties is to introduce novel preference structures for DMs to express their opinions. Recently, the concept of interval-valued opinions by considering the uncertainty tolerance of DMs is proposed in [166]. It is proposed to model opinions by numerical intervals $\sigma_i(t) = [\underline{\sigma}_i(t), \overline{\sigma}_i(t)] \subseteq [0, 1]$, with $\underline{\sigma}_i(t) \leq \overline{\sigma}_i(t)$. Then, for the DMs with uncertainty tolerances, the opinion evolution follows the following updating rule:

$$\underline{\sigma}_i(t+1) = T_i\underline{\sigma}_i(t) + \sum_{j \neq i} w_{ij}\underline{\sigma}_j(t), \tag{2.17}$$

$$\overline{\sigma}_i(t+1) = T_i\overline{\sigma}_i(t) + \sum_{j \neq i} w_{ij}\overline{\sigma}_j(t), \tag{2.18}$$

where T_i is the trust of DM d_i. As for DMs without uncertainty tolerances, the update rules are the same as above, however, the terms $\underline{\sigma}_j(t)$ and $\overline{\sigma}_j(t)$ in the summations are replaced with $f_{ij}(t)$, which is an accurate estimation of opinion d_j from d_i. Linguistic models have also been proposed to deal with associated uncertainties, which will be reviewed in the next section.

2.4.3 Linguistic models

As it was mentioned earlier in Section 2.2.4, uncertainty in opinions can also be modelled through linguistic models. This is a new concept in ODMs and some efforts have been devoted to the design of linguistic models based on the 2-tuple and fuzzy linguistic preference structures for opinions evolution [167]. In what follows, the most-recent literature works on linguistic models are reviewed.

In [168], authors propose a personalized individual semantic LODM under the BC framework. Following Definition 3 and the idea of numerical scale models for LTSs [169], the numerical scale of an LTS $\mathcal{S} = \{s_0, \ldots, s_{2r}\}$ for (s_i, α) is defined as follows:

$$NS(s_i, \alpha) = \begin{cases} NS(s_i) + \alpha \left(NS(s_{i+1}) - NS(s_i) \right), & \alpha \geq 0, \\ NS(s_i) + \alpha \left(NS(s_i) - NS(s_{i+1}) \right), & \alpha < 0. \end{cases} \tag{2.19}$$

Then, the process of a linguistic model with personalized individual semantic consists of three steps; (1) semantics translation, in which a linguistic term is translated into a semantic in the interval $[0, 1]$; (2) numerical computation, which takes semantics as input and outputs a numerical value in interval $[0, 1]$; (3) semantic retranslation, in which the output of numerical computation step will be retranslated into a 2-tuple. In this regard, the proposed model can be constructed by following three main steps discussed below.

The first step for DMs is to estimate the semantics of other peers as given below:

$$e_{ij}(t) = \kappa NS_j(\sigma_j(t)) + (1 - \kappa) NS_i(\sigma_j(t)), \tag{2.20}$$

where $e_{ij}(t)$ with $i, j = 1, \ldots, n$ and $i \neq j$ denotes the estimated semantic of DM d_j by DM d_i based on their familiarity modeled by κ. Then, the confidence set of DM d_i can be constructed as follows:

$$\mathcal{I}(d_i, \sigma_i(t)) = \{d_j | \; \|NS_i(\sigma_i(t) - e_{ij}(t))\| \leq \epsilon\}, \tag{2.21}$$

and, then, the weights of DMs can be adjusted in the same way as discussed in Eq. (2.9). The update rule of semantics is proposed to be as follows:

$$NS_i(\sigma_i(t+1)) = w_{i1}(t)e_{i1}(t) + \ldots + w_{in}(t)e_{in}(t). \tag{2.22}$$

Finally, the evolution of opinions can be modeled as given below:

$$\sigma_i(t+1) = NS_i^{-1}(NS_i(\sigma_i(t+1))), \tag{2.23}$$

where NS_i^{-1} is given in Definition 2 in [168]. This scheme has enabled the emergence of other ODMs under multi-granular [170] and probabilistic linguistic models [171].

2.4.4 Reinforcement learning-based models

The essence of RL is learning by interacting with an environment by taking actions. An RL agent takes an action a_t in its environment and based upon the consequences of its actions, which is the reward r_t it receives from the environment, it can learn how to alter its behaviour toward collecting more rewards. For each state transition s_{t+1} in the environment, the agent receives a feedback through a scalar reward r_{t+1}. The agent aims at learning a policy that maximizes the expected return (also known as discounted reward). In brief, in case that the environment satisfies the Markov property, that is the current state is only dependent to the previous state, RL can be realized through an MDP. The consensus process in GDM models and the fusion process in ODMs (despite of the memory-based mechanisms discussed earlier [161]), can be treated as MDPs and the solutions can be achieved by means of RL algorithms. A very limited number of literature opinion dynamics and GDM models have addressed the application of RL, which we aim to review them in this section.

For consensus boost and recommendations to guide DMs in ODMs, a framework based on the RL is presented in [172]. The authors proposed a state space to contain opinions as $S = \{s_i | s_i \in [0, 1], i = 1, \dots, n\}$, and, each agent can take an action from the constructed action space $A = \{a_i | a_i \in [-1, 1], i = 1, \dots, n\}$. Then, a reward signal is constructed as follows:

$$r_t = w_1 r_{ac}(t) + (1 - w_1) r_{cd}(t), \tag{2.24}$$

where r_{ac} and r_{cd} account for the adjustment cost and consensus boost, respectively, and w_1 is used to model the trade-off between them. The adjustment cost is modelled as in the following:

$$r_{ac}(t) = -\sum_{i=1}^{n} |a_i(t)|, \tag{2.25}$$

where it is the negative sum of actions taken by agents. For the consensus boost part, it is required to find the state transition rule, which is realized by means of HK model. In this regard, for those agents who do not adopt adjustment actions, the following transition rule is adopted:

$$s'_i(t + 1) = \frac{1}{|\mathcal{I}(s'_i(t))|} \sum_j s'_j(t), \tag{2.26}$$

where $\mathcal{I}(s'_i) = \{s'_j(t) | \ s'_j(t) - s'_i(t)| \leq \epsilon\}$, with ϵ being the BC threshold. Then, for other agents, the transition law is as follows:

$$s_i(t + 1) = \frac{1}{|\mathcal{I}(s_i(t) + a_i(t))|} \sum_j s_j(t) + a_j(t), \tag{2.27}$$

where $\mathcal{I}(s_i(t) + a_i(t)) = \{s_j(t) + a_j(t) | \, (s_j(t) + a_j(t)) - (s_i(t) + a_i(t))| \leq \epsilon\}$. Finally, $r_{cd}(t)$ is constructed as follows:

$$r_{cd}(t) = n[\text{cd}(t) - \text{cd}'(t)], \tag{2.28}$$

with $\text{cd}(t) = 1 - \frac{\sum_{i=1}^{n} |s_i(t+1) - \frac{\sum_{i=1}^{n} s_i(t+1)}{n}|}{n}$ and $\text{cd}'(t) = 1 - \frac{\sum_{i=1}^{n} |s'_i(t+1) - \frac{\sum_{i=1}^{n} s'_i(t+1)}{n}|}{n}$. Once the set of actions, rewards, and transition laws are constructed, any RL algorithm (depending on the nature of actions and states) can be employed in the learning process of the agent, where an actor-critic learning algorithm is used in [172] for the sake of learning. By considering the effect of stubborn, controlled, and uncontrolled agents, an RL-based mechanism is proposed in [173] for opinion shaping in ODMs by moderating the behaviour of influential DMs. The opinion evolution is modelled via a value iteration mechanism, where the policy evaluation is then converted into a shortest path problem. A model based on the Q-learning algorithm for RL agents in presented in [174], where agents' opinions are assumed to be binary, i.e., $\sigma_i(t) \in \{-1, +1\}$, and at each time instant, an agent is randomly selected and expresses its opinion to a randomly selected neighbour. By considering an internal evaluation function $\mathcal{Q}$ based on the social response of other peers, an update rule of the following form is constructed:

$$\mathcal{Q}_i(\sigma_i(t+1)) = (1-\alpha)\mathcal{Q}_i(\sigma_i(t)) + \alpha r_i(t), \tag{2.29}$$

where $r_i(t) = \sigma_i(t)\sigma_j(t)$ is the reward signal. This is treated as Q-values required in training of an agent based on the Q-learning algorithm. For the same opinion dynamics structure, a game theoretic-based mechanism is employed in [175] to model agents' interactions, where the Q-learning algorithm is used for each agent to learn the optimal policy, which is gaining more rewards in their interactions with other peers. In case that a neighbour of an agent has the same opinion, the agent will receive a reward of 1, otherwise, -1. Agents opinions are also supposed to be binary and to be selected from $\{-1, +1\}$. This framework is extended in [176] to the case, in which agents can take more than two actions. Another game theoretic-based ODM is proposed in [177], where agents' communication is random, however, each agent who decides to express its expression is penalized with a cost, and it will be penalized more in case the neighbouring agent decides not to reply to its opinions or express disagreeing opinions. Without considering the exploration and exploitation [178] in taking actions, a framework based on RL is developed in [179], where agents are assumed to express their opinions randomly from a continuous set of actions to communicate in a social network toward maximizing the number of their followers in mainstream media. RL has also been used for conventional GDM models for DMs' weight adjustment in context-aware heterogeneous decision environments [180, 181]. Table 2.4 summarizes the developed ODMs.

2.5 Challenges and Research Gaps

Many research works have been recently devoted to the design of CRP for GDM, which have been reviewed in the present work. Based on the reviewed papers, we have found some challenges that need to be addressed in future works that aim to design feedback mechanisms for the sake of consensus reaching.

1. The first issue is regarding the recently-developed representation structures for opinion expression. As it was mentioned in Section 2.3.1, new representation structures such as Z^E-numbers are recently developed, where, on the one hand, the development of operational tools such as aggregation and similarity-checking measures, could be an important research attempt toward evoking the information of such representation structures as much as possible. On the other hand, these newly-developed representation structures pave the way for the design of novel and efficient CRPs. For instance, the problem of minimum adjustment

Table 2.4 – Developed ODMs in the recent literature works.

Category	Model	Characteristics	Reference
DMs' behaviour	Bounded confidence	Willingness of DMs, known and unknown confidence bound	[3]
		Cognitive dissonance behaviours	[138]
		Opinion natural reversals dynamics	[139]
		Leadership (opinion leaders and opinion followers)	[140, 143]
		Group and peer pressure	[144, 145]
		Antagonistic and indifference behaviours between individuals	[152]
	DeGroot	Self-persistence of DMs	[137]
		Leadership with minimum number of interactions	[141]
		Peer pressure and stubbornness of DMs	[146, 149]
		Willingness and self-confidence of DMs	[150]
Social networks	Bounded confidence	Opinion and action evolution, modified expressed private opinions	[154, 155]
		Stochastic interactions	[158]
		Dynamic interactions among DMs	[163]
		Fuzzy inference approach to describe bounded confidence	[182]
	Stochastic models	Repulsive interactions between DM's opinions	[156]
		Modulation of the interaction intensity	[160]
		Centralized tuning of the strength of interactions between DMs	[159]
	Hybrid model	Interactions depend on current and past opinions	[161]
	DeGroot	Competition between DMs and switching topology	[162]
		Failure mode and effect analysis	[165]
		Numerical interval opinions and uncertainty tolerances	[166]
	Deffuant	Temporal networks with ordering of interactions	[164]
Optimization models	Bounded confidence	Willingness of DMs	[183, 184]
		Self-trust and fuzzy trust sets	[185]
		Network rewiring for maximizing influence on overall opinion	[186]
	DeGroot	Network partitioning algorithm	[187]
	Hybrid model	Network partitioning algorithm	[188]
		Combining pairwise and group interactions for DMs	[189]
	Interconnected dynamics	Distributed optimization problems over an unbalanced digraph	[190]
Linguistic models	Bounded confidence	Two-tuples linguistic model with numerical scale	[167]
		Personalized individual semantics model	[168]
		Multi-granular unbalanced linguistic term sets	[170]
		Opinion similarity, DMs' credibility and bounded rationality	[191]
RL-based models	bounded confidence	Consensus boost and recommendation mechanism	[172]
	Stochastic	Stubbornness of DMs	[173]
	Binary opinions	Internal evaluation function based on the social responses	[174]
	Game-theoretic model	Reward shaping through interactions with peers	[175, 176, 177]
	Gossiper-Media model	Maximizing the number of followers in mainstream media	[179]
	Fuzzy consensus model	Context-aware heterogeneous decision environment	[180, 181]

cost, social network-based analysis of GDM, LODMs, and managing the behaviour of DMs could all be addressed for these new representation structures.

2. RL has been recently deployed in many control and learning applications. Throughout our review on CRPs for GDM, we witnessed the lack of application of this powerful tool in literature. The CRP is a dynamic mechanism by its nature, because it is modelling the evolution of the consensus among DMs. What makes the application of RL in GDM possible is the fact that regardless of other involved parameters such as the weight of DMs or attributes, the consensus among DMs at each discussion round depends only on the consensus of the previous discussion round. This conducts and satisfies the Markov property in MDP, and, therefore, RL is applicable in modelling the CRP in conventional GDM models. RL can be implemented for the adjustment of the weights of DMs, attributes, alternatives, and even in adjustment of the feedback parameter for consensus reaching through feedback mechanisms. In this regard, the environment would be discrete and depending on the purpose of the RL agent, its actions could be either discrete or continuous. The same is true in the design of feedback mechanisms based on the ODMs, where an RL agent can be assigned to the fusion process for managing the evolution of opinions. The old problem of the trade-off between the consensus speed and HD of DMs (which states that DMs aim to keep their original opinions as much as possible) can be realized by means of RL by modelling the consensus process through game-theoretic mechanisms.

3. Even though some advancements have been made to the LODMs, however, the results are required to be extended to other linguistic representation structures as well. This is of paramount importance due to the fact that different DMs might need to express their opinions using different preference structures due to their level of knowledge or background. Following this, the design of novel

heterogeneous GDM models under opinion dynamics would be another challenge and future trend toward paving the way of the application of the developed LODMs.

4. A common assumption in the reviewed works is that agents with similar opinions which are less than a given threshold, i.e., the bound of confidence of agents, are able to communicate in order to modify their opinions. In this mechanism, other neighbouring agents who do not fall into the confidence bound of agents are ignored. However, it is quite possible in the real life situations where agents might have friends with quite different opinions. Taking the opinions of these long-range neighbours who are out of the confidence bound could also help with consensus reaching. This idea is missing in the most-recent research works.

Chapter 3

Reinforcement Learning-Based Consensus Management for Static Group Decision-Making

In accordance with the associated problems with static GDM described in Chapter 1, the number of discussion rounds (also referred to as the CRP speed) and HD of DMs are two crucial efficiency measures to be considered in the development of CRP for static models. Adjusting the level of opinion modification and importance weights of the DMs in the feedback mechanism has a considerable impact on these efficiency measures. The level of opinion modification is typically determined by means of the *feedback parameter* in the direction rule of the feedback mechanism that directly affects the trade-off between the CRP speed and HD of DMs. The importance weights of DMs, however, are involved in the construction of the collective opinion that could consequently impact the speed of CRP. Therefore, the feedback parameter and importance weights of DMs are required to be carefully designed to account for the aforementioned efficiency measures.

A considerable number of efforts have been devoted to address the aforementioned efficiency measures in static GDM models. However, the developed models suffer from two critical shortcomings listed below:

1. The majority of devoted efforts to speeding up the CRP fall under a general category of GDM models, called *minimum adjustment cost* or *minimum cost consensus*. These methods convert the GDM problem into an optimization problem and try to achieve consensus within the group of DMs in just one

discussion round. Even though such a framework deals with speeding up the CRP, however, the HD of DMs is ignored [192, 115, 193, 194, 195]. It will be presented in the upcoming sections that such models do not necessarily lead to the best HD of DMs, especially when a higher level of consensus is required.

2. To address the challenge discussed in the first item, some efforts have been recently dedicated to address the trade-off between the CRP speed and HD of DMs under the minimum adjustment cost or minimum cost consensus frameworks [196, 197, 198, 199]. Even though such models account for the trade-off problem, however, they are dependent to the type of opinion representation structure and a developed model cannot be generalized to deal with other types of decision environments.

Therefore, the research gap in static GDM problems is the development of an efficient consensus management model for the CRP that not only addresses the trade-off problem, but also it is generalizable and not dependent to the opinion representation structure. In this chapter, this research gap is addressed by proposing novel and efficient RL-based adjustment mechanisms. To employ these adjustment mechanisms, it is proposed to extract the dynamics of state transition from consensus models based on the distributed linguistic trust functions (DLTFs) and Z-numbers in order to convert the decision environment into an MDP. Two independent RL agents are then trained by the deep deterministic policy gradient (DDPG) algorithm to adjust the feedback parameter and importance weights of DMs. The first agent is trained toward reducing the number of discussion rounds, while ensuring the highest possible level of HD among DMs. The second agent merely speeds up the CRP by adjusting the importance weights of the DMs.

3.1 Developed consensus models

This section presents the prerequisites and development procedure of two consensus models based on the DLTFs and Z-numbers.

3.1.1 Preliminaries

Following the model proposed in [200], suppose that $\mathcal{S} = \{s_\alpha | \alpha = 0, \ldots, 2\kappa\}$ is a fully ordered LTS. The concept of distribution assessment for an LTS with linguistic proportions assigned to each element has been proposed in [201] as given below:

Definition 9. *$p = \{(s_k, \beta_k) | k = 0, \ldots, 2\kappa\}$ is a distribution assessment of $\mathcal{S}$, in which $\beta_k \geq 0$ with $\sum_{k=0}^{2\kappa} \beta_k = 1$, is the symbolic proportion of $s_k \in \mathcal{S}$.*

Definition 10. *Having $\mathcal{S}$ and p as before, the expectation of p can be defined as follows:*

$$\mathcal{E}(p) = \sum_{k=0}^{2\kappa} \beta_k s_k. \tag{3.1}$$

Following the concept of distributed linguistic trust given in [202], the DLTFs can be defined as follows:

Definition 11. *For an LTS $\mathcal{S}$, the DLTFs can be defined as $\mathcal{R} = \{(s_\alpha, \phi_\alpha) | \alpha = 0, \ldots, 2\kappa\}$, where ϕ_α is the proportion of s_α with $\phi_\alpha \geq 0$ and $\sum_{\alpha=0}^{2\kappa} \phi_\alpha = 1$.*

Definition 12. *For a set of DLTFs $\{\mathcal{R}^1, \ldots, \mathcal{R}^n\}$ with a designated set of weights $\{W^1, \ldots, W^n\}$, where $\mathcal{R}^i = \{(s_\alpha, \phi_\alpha^i) | \alpha = 0, \ldots, 2\kappa\}$ and $W^i \in [0, 1]$ and $\sum_{i=1}^n W^i = 1$, a distributed trust weighted average operator can be defined as follows:*

$$DTWA(\mathcal{R}^1, \ldots, \mathcal{R}^n) = \{(s_\alpha, \bar{\phi}_\alpha) | \alpha = 0, \ldots, 2\kappa\}, \tag{3.2}$$

with $\bar{\phi}_\alpha = \sum_{i=1}^n W^i \phi_\alpha^i$.

Definition 13. *Let $\mathcal{R}$ be as before, the expectation degree of this DLTF can be defined as follows:*

$$\mathcal{E}(\mathcal{R}) = \sum_{\alpha=0}^{2\kappa} \mathcal{S}_{\alpha} \times \phi_{\alpha} = \mathcal{S}_{\sum_{\alpha=0}^{2\kappa} \alpha\phi_{\alpha}}. \tag{3.3}$$

By modeling the trust degree between DMs e_i and e_j via a linguistic trust function t_{ij} and their connections via an adjacency matrix $\mathcal{J}$, a sociomatrix of the form $T_L = (t_{ij})_{n\times n}$ can then be constructed, with n being the total number of DMs.

Definition 14. *For a sociomatrix $T_L = (t_{ij})_{n\times n}$ associated with a directed graph $\mathcal{G} = (\mathcal{D}, E, \nu)$ with $\mathcal{D} = \{d_1, \ldots, d_n\}$ being the set of nodes, $E = \{e_1, \ldots, e_q\}$ being the set of edges, and distributed linguistic trust weights $\nu = \{\nu_1^E, \ldots, \nu_q^E\}$ associated to each edge, the relative in-degree centrality can be formulated as follows:*

$$C^E(d_k) = \frac{1}{n-1} \sum_{i=1}^{n} t_{ik}. \tag{3.4}$$

Definition 15. *Let $\mathcal{G}$ and $\mathcal{D}$ be as before, and $\{C^E(d_1), \ldots, C^E(d_n)\}$ be the set of in-degree centrality values. The importance weight of a DM d_i (a node of $\mathcal{G}$) can then be computed as follows:*

$$W^i = \frac{\mathcal{E}(\mathcal{G}^E(d_i))}{\sum_{i=1}^{n} \mathcal{E}(\mathcal{G}^E(d_i))}. \tag{3.5}$$

Definition 16. *The distance between two DLTFs $\mathcal{R}^i$ and $\mathcal{R}^j$ is measured as given below:*

$$dist_r(\mathcal{R}^i, \mathcal{R}^j) = \frac{1}{2\kappa+1} \sum_{\alpha=0}^{2\kappa} \left|\phi_{\alpha}^i - \phi_{\alpha}^j\right|. \tag{3.6}$$

For a fully-ordered LTS $\mathcal{S}$, the definition of LSFs is as given below [203]:

Definition 17. *Let s_{α} be a linguistic term in $\mathcal{S}$. A linguistic scale function can then* *be defined as a mapping $\mathcal{F} : s_\alpha \to \theta_\alpha$ with $0 \leq \theta_\alpha \leq 1$.*

The following LSFs are used in this chapter:

$$\begin{aligned} \mathcal{F}_1(s_\alpha) = &\ \ \theta_\alpha = \tfrac{\alpha}{2\kappa}, \quad (\alpha = 0, \ldots, 2\kappa), \\ \mathcal{F}_2(s_\alpha) = &\begin{cases} \frac{a^\kappa - a^{(\kappa-i)}}{2a^{(\kappa-2)}}, & (i = 0, \ldots, \kappa), \\ \frac{a^\kappa + a^{(i-\kappa)}}{2a^\kappa - 2}, & (i = \kappa+1, \ldots, 2\kappa), \end{cases} \end{aligned} \tag{3.7}$$

where $a \in [1.36, 1.4]$ [204].

Definition 18. *For two LTSs $\mathcal{S}$ and $\mathcal{S}'$, a Z-number can be represented by $z_i = (s_i, s_i')$, with $s_i \in \mathcal{S}$ and $s_i' \in \mathcal{S}'$. Following the procedure given in [205], the corresponding numerical characteristics of z can be extracted by means of LSFs in terms of NZs as $NZ_i = ((\mu_i, \sigma_i), (ex_i, en_i, h_i))$.*

Definition 19. *The distance between two NZs can be calculated as follows:*

$$\begin{aligned} dist_z(NZ_i, NZ_j) = \Big|\frac{\mu_i \sigma_j^2}{\sigma_i^2 + \sigma_j^2} - \frac{\mu_j \sigma_i^2}{\sigma_i^2 + \sigma_j^2}\Big| + \Big|\frac{en_j^2 + h_j^2}{en_i^2 + h_i^2 + en_j^2 + h_j^2} \times \frac{\mu_i ex_i}{\sigma_i} - \\ \frac{en_i^2 + h_i^2}{en_i^2 + h_i^2 + en_j^2 + h_j^2} \times \frac{\mu_j ex_j}{\sigma_j}\Big|. \end{aligned} \tag{3.8}$$

Definition 20. *For a set of NZs $NZ_i (i = 1, \ldots, n)$, with an associated set of weights $\{w_1, \ldots, w_n\}$, where $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, the aggregated NZ value can be* *obtained based on a GNZPWA as follows:*

$$GNZPWA(NZ_1, \ldots, NZ_n) = \Big(\big((\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda})^{1/\lambda},$$
$$(\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda})^{1/\lambda - 1} \times \sqrt{\sum_{i=1}^{n} \gamma_i \mu_i^{2\lambda-2} \sigma_i^2} \big), \big((\frac{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda} ex_i^{\lambda}}{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda}})^{1/\lambda},$$
$$(\frac{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda} ex_i^{\lambda}}{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda}})^{1/\lambda - 1} \times \sqrt{\frac{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda} ex_i^{(2\lambda-2)} en_i^2}{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda}}},$$
$$(\frac{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda} ex_i^{\lambda}}{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda}})^{1/\lambda - 1} \times \sqrt{\frac{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda} ex_i^{(2\lambda-2)} h_i^2}{\sum_{i=1}^{n} \gamma_i \mu_i^{\lambda}}} \big) \Big), \tag{3.9}$$

where,

$$\gamma_i = \frac{w_i(1 + \mathcal{P}(NZ_i))}{\sum_{i=1}^{n} w_i(1 + \mathcal{P}(NZ_i))}, \tag{3.10}$$

with $\mathcal{P}(NZ_i) = \sum_{j=1, j\neq i}^{n} SD(NZ_i, NZ_j)$ *and* $SD(NZ_i, NZ_j) = 1 - \frac{dist_z(NZ_i, NZ_j)}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} dist_z(NZ_i, NZ_j)}$ *being the support degree of* NZ_i *from* NZ_j*, and* λ *models the thinking mode of DMs.*

3.1.2 Consensus model with DLTFs

Suppose that $\mathcal{D} = \{d_1, \ldots, d_n\}$ is the set of n DMs who are aiming at evaluating and providing ranking for a set of q alternatives $\mathcal{X} = \{x_1, \ldots, x_q\}$ w.r.t. a set of m attributes $\mathcal{A} = \{a_1, \ldots, a_m\}$. Having a pre-defined *consensus threshold* γ and *feedback parameter* δ, the goal is to design a consensus model based on DLTFs to support this GDM problem.

Given the DMs' connections in terms of an adjacency matrix $\mathcal{J}$, the sociomatrix T_L for a group of DMs with the LTS $\mathcal{S}_\alpha$ can be constructed. By extracting the relative in-degree centrality values from T_L as discussed in Definition 14, the importance weights of DMs $\{W^1, \ldots, W^n\}$ can then be calculated following Definition 15, which can be used to construct the collective opinion of the group as follows.

Definition 21. *For a set of DLTFs represented in a matrix format* $\mathcal{R}^d = [r_{ij}]_{(q\times(2\kappa+1))\times m}$ *with* $d = 1, \ldots, n$*, where* q *denotes the number of available alternatives,* $2\kappa + 1$ *is the number of linguistic terms in the LTS* $\mathcal{S}_\alpha$*, and* m *is the number of available attributes, the collective evaluation matrix* $\bar{\mathcal{R}} = [\bar{r}_{ij}]_{(q\times(2\kappa+1))\times m}$ *can be constructed as follows:*

$$\bar{r}_{ij} = \sum_{d=1}^{n} W^d \times r_{ij}^d, \tag{3.11}$$

where W^d *is the importance weight of the* d*th DM as defined in Eq. (3.5), and* r_{ij}^d *is the* (i, j)*th element in the decision matrix of the* d*th DM.*

For the (i, j)th element, the *element-level* consensus can be computed as follows:

$$\mathcal{CE}_{ij}^d = 1 - \text{list}_r(r_{ij}^d, \bar{r}_{ij}^d). \tag{3.12}$$

Next, the *alternative-level* consensus for the dth DM w.r.t. the ith alternative can be calculated:

$$\mathcal{CA}_i^d = \frac{1}{m} \sum_{j=1}^{m} \mathcal{CE}_{ij}^d. \tag{3.13}$$

Finally, the *relation-level* consensus for the dth DM can be constructed as follows:

$$\mathcal{CI}^d = \frac{1}{q} \sum_{i=1}^{q} \mathcal{CA}_i^d. \tag{3.14}$$

When the consensus assessment is completed, the recommendation mechanism gets activated to generate advice for inconsistent DMs. In this regard, given a consensus threshold γ, the feedback mechanism is built based upon the following identification and direction rules:

1. Construct the set of inconsistent DMs. This set is represented by EXPCH, where EXPCH $= \{d | \mathcal{CI}^d < \gamma\}$.

2. Find the set of alternatives that are required to be modified. This set is represented by ALT and is defined as ALT $= (d, i)|d \in \text{EXPCH} \wedge \mathcal{CA}_i^d < \gamma\}$.

3. Find the set of elements that need to be modified. This set is represented by APS and is defined as APS $= \{(d, i, j)|(d, i) \in \text{ALT} \wedge \mathcal{CE}_{ij}^d < \gamma\}$.

Inconsistent DMs are then recommended to modify their corresponding evaluations in the APS as follows:

$$rr_{ij}^d = (1 - \delta).r_{ij}^d + \delta.\bar{r}_{ij}, \tag{3.15}$$

where rr_{ij}^d is the (i, j)th modified evaluation of the dth DM, $\bar{r}_{ij}$ is the (i, j)th element in the collective evaluation, and $\delta \in [0, 1]$ is the feedback parameter. The CRP will be continued until the consensus degree of each DM satisfies $\mathcal{CI}^d \geq \gamma$.

3.1.3 Consensus model with Z-numbers

Following the consensus model discussed in the previous section, we extend the results to the case, in which DMs' evaluations are expressed in terms of Z-numbers.

Initial evaluations of DMs are in the form of matrices $\mathcal{R}^d = (r_{ij}^d)_{(q\times 2m)}$, where each element is a Z-number as discussed in Definition 18. Following the steps summarized in Algorithm 3 in [1], initial evaluations can then be translated into NZs to construct the translated evaluations in terms of matrices $\tilde{\mathcal{R}}^d = (\tilde{r}_{ij}^d)_{q\times 5m}$. The same procedure discussed in the previous section can be implemented to construct the importance weights of DMs $\{W^1, \dots, W^n\}$. Then, the collective evaluation can be constructed as given below.

Definition 22. *For a set of DMs with translated evaluations $\tilde{\mathcal{R}}_{ij}^d = NZ_{ij}^d$ with $d = 1, \dots, n$, and the constructed set of weights $\{W^1, \dots, W^n\}$, the (i, j)th element of the* *collective evaluation* $\bar{r}^c_{ij}$ *can be obtained as follows:*

$$\bar{r}^c_{ij} = GNZPWA(NZ^1_{ij}, \ldots, NZ^n_{ij}). \tag{3.16}$$

Having the collective evaluation constructed, the consensus assessment can get started by computing the consensus on three different levels. Compared with the three-level consensus given in the previous section, Eq. (3.12) will be modified as follows for NZs:

$$\mathcal{CE}^d_{ij} = 1 - \mathrm{list}_z(\tilde{r}^d_{ij}, \bar{r}^c_{ij}). \tag{3.17}$$

Then, the same procedure can be followed by inconsistent DMs to modify their evaluations.

In this section, two consensus models are proposed based on DLTFs and Z-numbers. These two models are constructed to check for the *generalizability* of the trained RL agents in dealing with different decision environments. The construction of these agents will be presented next and it will be discussed that as long as the observation of an agent does not change from one decision environment to another, the same agents can be employed in different decision environments.

3.2 RL-Based Consensus Reaching Process

Following the given discussion in Section 3.1, the ultimate goal is to present a generalizable mechanism to set the feedback parameter and DMs' importance weights that accounts for the trade-off between the CRP speed and HD of DMs, regardless of the representation structure of evaluations. Toward this end, two consensus models were presented in the previous section with distinct representation structures of evaluations. It is intended to demonstrate that once an RL agent is trained on the decision environment based on the DLTFs, the same agent could also be employed in the decision environment based on the Z-numbers.

3.2.1 Problem Description

As discussed in Section 3.1, minimum adjustment cost models fail to address the trade-off problem. This is due to the fact that such models aim to solve the decision problem through an optimization model given below:

$$\min \sum_{(d,i,j)\in APS} \delta |r^d_{ij} - \overline{r}_{ij}|,$$
$$s.t. \begin{cases} \mathcal{CI}^d \geq \gamma, \\ \overline{\mathcal{R}} = DTWA(\mathcal{R}^1, \ldots, \mathcal{R}^n). \end{cases} \tag{3.18}$$

By solving the above optimization problem, the number of discussion rounds for the consensus achievement will be one, i.e., the CRP speed is optimal. However, experiments show that such a model does not necessarily lead to the best HD for DMs, especially for the cases with $\gamma > 0.9$. By defining the HD of a DM as follows:

$$\mathrm{HD}^d = 1 - \frac{1}{q \times m} \sum_{i=1}^{q} \sum_{j=1}^{m} \mathrm{dist}_r(r^d_{ij}, rr^d_{ij}), \tag{3.19}$$

the results for the minimum adjustment cost model (referred to by 'MinAdj') are compared with those of the proposed consensus model in Section 3.1.2 with $\delta = 0.33$, and the average HD (AHD) of DMs for 50 simulation runs are collected in Table 3.1. The attained results denote that the 'MinAdj' technique sacrifices the HD for a higher speed, while it is possible to achieve a higher AHD with a lower δ through the proposed consensus model. Therefore, the trade-off problem could not be addressed well through the 'MinAdj' model and it is required to develop a generalizable model that accounts for this problem.

Table 3.1 – The attained AHD values by means of the 'MinAdj' and the proposed consensus model in Section 3.1.2.

Method	γ	0.91	0.92	0.93	0.94	0.95
'MinAdj'	δ	0.4995	0.5572	0.6354	0.6704	0.6753
	AHD	0.9287	0.9228	0.9071	0.9052	0.9127
Proposed	δ	0.33	0.33	0.33	0.33	0.33
	AHD	0.9295	0.9281	0.9117	0.9104	0.9156

3.2.2 Problem Formulation

In order to address the trade-off problem by means of RL and to train RL agents, it is required to firstly convert the decision problem into an MDP and extract the state transition rule for the constructed MDP from the decision environment. The extracted transition rule must follow the Markov property, which states all that is needed to determine the next state is information contained in the current state. Following this concept, it is proposed to consider the DMs' consensus degree, i.e., $\mathcal{CI}^d$, as the state of the MDP.

Having the consensus index $\mathcal{CI}^d$ calculated at discussion round t, shown by $\mathcal{CI}^d(t)$, it needs to be verified that the next state, i.e., $\mathcal{CI}^d(t+1)$, does only depend on $\mathcal{CI}^d(t)$. Taking the Markov property into consideration, define $\mathcal{CI}^d$ with $d = 1, \ldots, n$ as the state of the environment, for which the state transition should be formulated. Suppose that the total number of given evaluations by DMs is $p = h + \varrho$, where h denotes the modified evaluations according to Eq. (3.15), and ϱ stands for the unchanged evaluations at discussion round t. Then, the set of modified evaluations is $\{rr^h_{ij}(t) | rr^h_{ij}(t) = (1-\delta)r^h_{ij}(t) + \delta\bar{r}^h_{ij}(t)\}$ and the set of unchanged evaluations can also be constructed as $\{rr^{\varrho}_{ij}(t) | rr^{\varrho}_{ij}(t) = r^{\varrho}_{ij}(t)\}$. In this regard, the collective evaluation at the next discussion round is as follows:

$$\bar{r}_{ij}(t+1) = W^h rr^h_{ij}(t) + \sum_{k=1, i\neq h}^{z} W^k rr^k_{ij}(t) = W^h rr^h_{ij}(t) + \bar{r}_{ij}(t) - W^h r^h_{ij}(t). \quad (3.20)$$

Then, following Eq. (3.12), it could be concluded that:

$$
\begin{aligned}
\mathcal{CE}^h_{ij}(t+1) &= 1 - \left| rr^h_{ij}(t) - \bar{r}_{ij}(t+1) \right| \\
&= 1 - \left| (1-\delta) r^h_{ij}(t) + \delta \bar{r}_{ij}(t) - \left(W^h rr^h_{ij}(t) + \bar{r}_{ij}(t) - W^h r^h_{ij}(t) \right) \right| \\
&= 1 - \Big| (1-\delta) r^h_{ij}(t) + \delta \bar{r}_{ij}(t) - \\
&\qquad\qquad \Big(W^h \left(\delta r^h_{ij}(t) + (1-\delta) r^h_{ij}(t) \right) + \bar{r}_{ij}(t) - W^h r^h_{ij}(t) \Big) \Big| \\
&= 1 + \left(\delta \left(1 - W^h \right) - 1 \right) \left| r^h_{ij}(t) - \bar{r}^h_{ij}(t) \right| \\
&= \mathcal{CE}^h_{ij}(t) + \delta \left(1 - W^h \right) \left| r^h_{ij}(t) - \bar{r}^h_{ij}(t) \right|. \qquad (3.21)
\end{aligned}
$$

For the ϱ elements that do not belong to APS, $r^{\varrho}_{ij}(t+1) = r^{\varrho}_{ij}(t)$, meaning that these evaluations remain unchanged from discussion round (t) to the discussion round $(t+1)$. Therefore, the $\mathcal{CE}^{\varrho}$ on these evaluations remain unchanged, i.e., $\mathcal{CE}^{\varrho}(t+1) = \mathcal{CE}^{\varrho}(t)$. In a matrix format, $\mathcal{CE}^d(t+1) = \mathcal{CE}^{\varrho}(t+1) + \mathcal{CE}^h(t+1)$, denoting that the overall $\mathcal{CE}$ for the dth DM can be resulted by the summation of $\mathcal{CE}$ over unchanged evaluations ϱ and the modified elements h. By referring to Eq. (3.20), it can be concluded that:

$$
\begin{aligned}
\mathcal{CE}^d_{ij}(t+1) &= \mathcal{CE}^{\varrho}_{ij}(t+1) + \mathcal{CE}^h_{ij}(t) + \delta(1-W^h)|r^h_{ij}(t) - \bar{r}^h_{ij}(t)| \\
&= \mathcal{CE}^z_{ij}(t) + \mathcal{CE}^h_{ij}(t) + \delta(1-W^h)|r^h_{ij}(t) - \bar{r}^h_{ij}(t)| \\
&= \mathcal{CE}^d_{ij}(t) + \delta(1-W^h)|r^h_{ij}(t) - \bar{r}^h_{ij}(t)|.
\end{aligned}
$$

Then, the following state transition rule could be resulted, which satisfies the Markov property:

$$
\mathcal{CI}^d(t+1) = \frac{1}{q \times m} \sum_{i=1}^{q} \sum_{j=1}^{m} \left[\mathcal{CE}^d_{ij}(t) + \delta \left(1 - W^h \right) \times \left| r^h_{ij}(t) - \bar{r}^h_{ij}(t) \right| \right]. \qquad (3.22)
$$

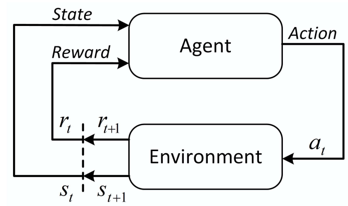

Figure 3.1 – Interactions of an RL agent with its environment.

3.2.3 Training of Agents

In this section, a brief introduction to the DDPG algorithm is given and it is discussed how this algorithm is used to train agents. Briefly, RL refers to learning by interacting with an environment [206]. This interaction is shown in Fig. 3.1. An agent takes action a_t within state s_t and receives a reward r_{t+1} and next state s_{t+1} that offers the agent guidance on how to alter its behaviour toward collecting more rewards. The agent's ultimate goal is to learn an optimal policy that maximizes the expected reward.

One way to realize RL is through MDPs. In this study, the agent interacts with the CRP as its environment, and the state of this environment is the consensus at evaluation matrix level $\mathcal{CI}$. As discussed in Section 3.2.2, the CRP can be formulated as an MDP and it was proved that the state transition satisfies the Markov property. As a result, the CRP is an MDP, and RL can be used to address the trade-off problem outlined above.

By considering the aforementioned trade-off problem, an RL agent, called the δ-Agent, is trained to adjust the feedback parameter in the CRP. Furthermore, another agent, called the W-Agent, is trained to dynamically adjust the importance weights of DMs. For both agents, the observations are the evolution of $\mathcal{CI}$ (3.22). The action of δ-Agent is adjustment of the feedback parameter, while it is the adjustment of DMs' weights for the W-Agent. Both agents take the necessary actions irrespective of the representation structure of DMs' evaluations. The interactions between the δ-Agent and its environment are illustrated in Fig. 3.2. It shows that the agent observes

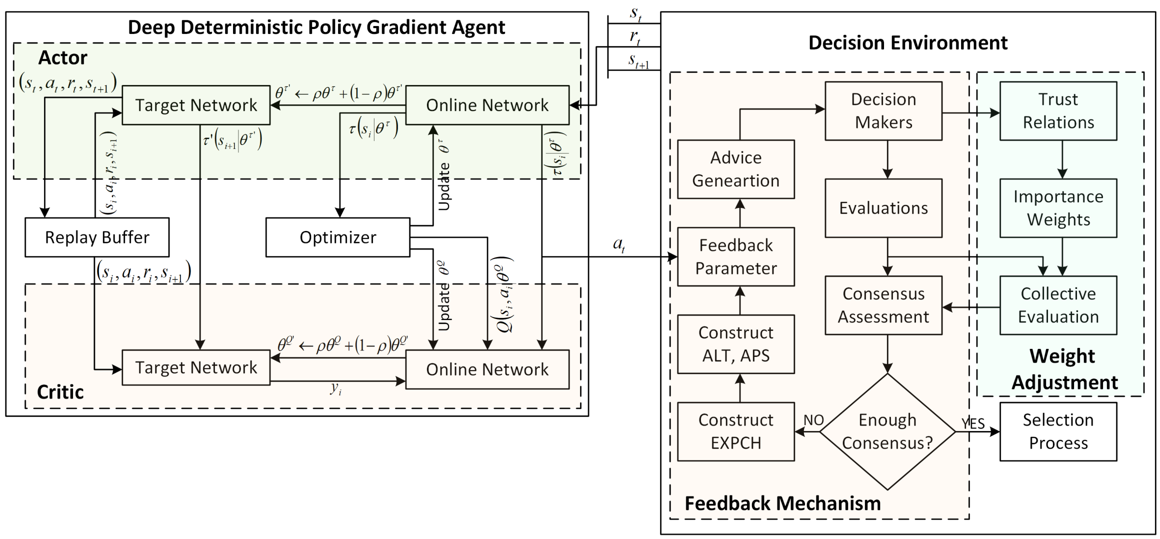

Figure 3.2 – Interactions between the proposed deep deterministic policy gradient agent and the decision environment [4].

the state of the system s_t, takes an action a_t, and receives the next state s_{t+1} and a reward r_t. The δ-Agent sets a value for the feedback parameter at the beginning of each training episode and receives a reward at the end of the corresponding episode. The value of this reward is as follows:

$$r = \begin{cases} \frac{\sum_{d=1}^{n} HD^d}{n\times\delta}, & \hbar \leq 6, \\ \frac{\sum_{d=1}^{n} HD^d}{n\times\delta\times\hbar}, & \text{otherwise}, \end{cases} \tag{3.23}$$

where $\hbar$ is the total number of steps in each training episode. This reward encourages the agent toward maximizing the HD with lower values of δ and discussion rounds. It is worth mentioning that the δ-Agent is trained toward automatic adjustment of the feedback parameter by considering the trade-off between the consensus speed and HD of DMs. It supports the direction rules in the feedback mechanism given in (3.15), where it automatically adjusts the level of modifications for the collected evaluations in APS.

As for the W-Agent, it is aimed at adjusting the importance weights of DMs in each step of each training episode in a way that it reduces the number of discussion rounds. This can be achieved by maximizing the level of consensus among DMs through the following reward function:

$$r = \sum_{i=1}^{n} W^i \sum_{j=1, j\neq i}^{n} W^j c_{ij}, \tag{3.24}$$

$$c_{ij} = 1 - \frac{1}{q \times m} \sum_{k=1}^{q} \sum_{l=1}^{m} \frac{|r^i_{kl} - r^j_{kl}|}{\max\{|r^i_{kl} - r^j_{kl}|\}}. \tag{3.25}$$

As it can be observed from Eq. (3.24), the reward signal is a weighted summation of the consensus among all pairs of DMs. This means that W-Agent attempts toward adjusting W to increase the level of consensus among DMs that will consequently help with speeding up the CRP. It should be noted that a larger number of alternatives and/or attributes results in higher-dimensional evaluations, and, therefore, construction of the reward requires more computational effort. However, the reward value is directly affected by the amount of deviations in the DMs' evaluations. The higher the deviation, the lower the reward. Having the reward signal constructed, one can collect training data in order for each agent to learn the optimal policy.

Training of agents is done by employing the DDPG algorithm. This is due to the fact that environment states (consensus of DMs) and agents' actions (the value of δ and W) take continuous values. To manage high dimensional state and action spaces, the DDPG algorithm is adopted in training of agents. DDPG agents benefit from two networks called 'actor' and 'critic.' The actor network, shown by $\tau(s|\theta^\tau)$, takes the environment states as input and exploits an action. The critic network, shown by $Q(s, a|\theta^Q)$, makes use of the environment states and the generated action by the actor network to estimate the expected reward. To train the δ-Agent through the DDPG algorithm, i.e., to adjust the parameters of the critic θ^Q and actor θ^τ networks, it is firstly required to construct a replay buffer $\mathcal{M}$ with tuples of the form (s_1, r, s_F), where s_1 is the initial state of the environment at the beginning of a training episode, r is the collected reward by the agent, and s_F is the final state of the environment. By randomly sampling a minibatch of size M from the replay buffer, the critic network can be updated by minimizing the following loss function:

$$L(\theta^Q) = \frac{1}{M}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2, \tag{3.26}$$

where M is the size of the minibatch, and $y_i = r_i + \gamma_l Q'(s_{i+1}, \tau'(s_{i+1}|\theta^{\tau'})|\theta^Q)^2$, with γ_l being the discount factor. It is worth noting that y_i is called the target because we want the value of agent to be close to this target value. However, in order to improve the stability of optimization, this target is modeled via target critic Q' and actor τ' networks. Target networks are periodically updated by the agent during the learning process. In the same vein, parameters of the actor network are updated by maximizing the following policy objective function:

$$\bigtriangledown_{\theta^\tau} J \approx \frac{1}{M}\sum_i \bigtriangledown_a Q(s, a|\theta^Q)|_{s=s_i, a=\tau(s_i)} \bigtriangledown_\theta^\tau \tau(s|\theta^\tau)|_{s_i}. \tag{3.27}$$

Training of δ-Agent is summarized in Algorithm 1. In the initialization phase, the replay buffer, actor, critic, and target networks are initialized. In particular, the initialization of the replay buffer includes defining its capacity (100K), observation dimension ($n \times 1$), and action dimension, that is 1×1 for the δ-Agent, and $n \times 1$ for the W-Agent. As for the actor, critic, and target networks, the weights are randomly initialized with the values taken from $[0, 1]$ interval. Then, for N training episodes, it is required to reset the environment to observe the initial state (Line 5). The initial state is then used by the actor network to exploit an action (Line 6) by considering the exploration noise $\mathcal{N}$. The taken action is indeed the value assigned to the feedback parameter δ. The CRP is then executed and the reward is collected from environment (Lines 7 to 10). The tuple (s_1, r, s_F) will then be stored in the replay buffer and by using a minibatch of these collected tuples, parameters of the actor, critic, and target networks can be updated (Lines 13 to 14). The same procedure can be followed to train W-Agent as shown in Algorithm 2. In each episode, the agent takes an action to adjust W in each step and it will be continued until the group of DMs reaches to the desired level of consensus. This stopping criterion is shown by 'IsDone.'

Algorithm 1: Training of the δ-Agent.

Result: δ-Agent
Inputs: critic learning rate (ρ_c), actor learning rate (ρ_a), discount factor (γ_l), total number of episodes (N), exploration noise ($\mathcal{N}$);

Initialize the replay buffer $\mathcal{M}$;
Initialize the critic network $Q(s, a|\theta^Q)$ with θ^Q;
Initialize the actor network $\tau(s|\theta^\tau)$ with θ^τ;
Initialize the target networks Q' and τ' with $\theta^{Q'} \leftarrow \theta^Q$ and $\theta^{\tau'} \leftarrow \theta^\tau$;
for *episode=1 to N* **do**
    Reset the decision environment and compute the initial observation s_I. Select action $\delta = \tau(s_I|\theta^\tau) + \mathcal{N}$;
    Execute the CRP with δ;
    $\hbar \leftarrow$ the number of discussion rounds;
    $s_F \leftarrow$ the state at discussion round $\hbar$;
    Compute r based on the Eq. (3.23).;
    Store the tuple (s_1, r, s_F) in the replay buffer $\mathcal{M}$;
    Sample a minibatch M from $\mathcal{M}$;
    Update the critic and actor using (3.26) and (3.27);
    Update target critic $\theta^{Q'} \leftarrow \rho_c\theta^Q + (1-\rho_c)\theta^{Q'}$ and the target actor $\theta^{\tau'} \leftarrow \rho_a\theta^\tau + (1-\rho_a)\theta^{\tau'}$;
end

3.3 Illustration of the Proposed Consensus Models

This section illustrates the implementation procedure of the proposed consensus models in Section 3.1.2 and Section 3.1.3. The results attained by means of the trained δ-Agent and W-Agent are discussed in detail in Chapter 6 in terms of different experiments.

Algorithm 2: Training of the W-Agent.

Result: W-Agent

Inputs: critic learning rate (ρ_c), actor learning rate (ρ_a), discount factor (γ_l), total number of episodes (N), exploration noise ($\mathcal{N}$);

Initialize the replay buffer $\mathcal{M}$;
Initialize the critic network $Q(s, a|\theta^Q)$ with θ^Q;
Initialize the actor network $\tau(s|\theta^\tau)$ with θ^τ;
Initialize the target networks Q' and τ' with $\theta^{Q'} \leftarrow \theta^Q$ and $\theta^{\tau'} \leftarrow \theta^\tau$;
for *episode=1 to N* **do**
    Reset the decision environment and compute the initial observation s_I;
    Set $t = 1$;
    while $IsDone \neq 1$ **do**
        Select action $W^t = \tau(s_t|\theta^\tau) + \mathcal{N}$ according to the current policy;
        Execute W^t, compute s_{t+1} and r_t (3.24);
        Store the tuple (s_t, W^t, r_t, s_{t+1}) in the replay buffer $\mathcal{M}$;
        Sample a minibatch M from $\mathcal{M}$;
        Update the critic and actor using (3.26) and (3.27);
        Update target critic $\theta^{Q'} \leftarrow \rho_c\theta^Q + (1-\rho_c)\theta^{Q'}$ and the target actor $\theta^{\tau'} \leftarrow \rho_a\theta^\tau + (1-\rho_a)\theta^{\tau'}$;
        $t \leftarrow t + 1$;
    end
end

3.3.1 Consensus Model with DLTFs

Suppose that four DMs $\{d_1, \ldots, d_4\}$ are asked to evaluate three alternatives $\{x_1, x_2, x_3\}$ w.r.t. three designated attributes $\{c_1, c_2, c_3\}$. Trust connections among DMs are modeled via an adjacency matrix as given below:

$$\mathcal{J} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}. \tag{3.28}$$

Consider an LTS $\mathcal{S} = \{s_1, s_2, s_3\}$, with s_1 denoting 'low,' s_2 denoting 'middle,' and s_3 denoting 'high.' Based on the constructed adjacency matrix in (3.28), suppose that the level of trust among DMs can be modeled via the following sociomatrix:

$$T_L = \begin{bmatrix} - & - & - & \begin{Bmatrix} (s_1, 0.6) \\ (s_2, 0.2) \\ (s_3, 0.2) \end{Bmatrix} \\ \begin{Bmatrix} (s_1, 0.31) \\ (s_2, 0.5) \\ (s_3, 0.19) \end{Bmatrix} & - & - & \begin{Bmatrix} (s_1, 0.16) \\ (s_2, 0.53) \\ (s_3, 0.31) \end{Bmatrix} \\ \begin{Bmatrix} (s_1, 0.18) \\ (s_2, 0.43) \\ (s_3, 0.39) \end{Bmatrix} & \begin{Bmatrix} (s_1, 0.18) \\ (s_2, 0.73) \\ (s_3, 0.09) \end{Bmatrix} & - & - \\ \begin{Bmatrix} (s_1, 0.75) \\ (s_2, 0.17) \\ (s_3, 0.08) \end{Bmatrix} & \begin{Bmatrix} (s_1, 0.32) \\ (s_2, 0.37) \\ (s_3, 0.31) \end{Bmatrix} & \begin{Bmatrix} (s_1, 0.05) \\ (s_2, 0.39) \\ (s_3, 0.56) \end{Bmatrix} & - \end{bmatrix}.$$

Then, the trust centrality degrees can be calculated according to Definition 14, where the attained results are given below:

$$\mathcal{C}^E(d_1) = \{(s_1, 0.4121), (s_2, 0.3671), (s_3, 0.2207)\},$$
$$\mathcal{C}^E(d_2) = \{(s_1, 0.2488), (s_2, 0.2297), (s_3, 0.5215)\},$$
$$\mathcal{C}^E(d_3) = \{(s_1, 0.0556), (s_2, 0.3889), (s_3, 0.5556)\},$$
$$\mathcal{C}^E(d_4) = \{(s_1, 0.3789), (s_2, 0.3632), (s_3, 0.2579)\}.$$

Having the centrality degrees calculated, one can compute the importance weights of DMs according to Definition 15 as $W^1 = 0.2138$, $W^2 = 0.2686$, $W^3 = 0.2955$, and $W^4 = 0.2221$.

Now, assume that the initial evaluations of DMs in terms of DLTFs are as given in Table 3.2. Then, the collective evaluation can be constructed according to Definition 21 as given in Table 3.3. Following the definition of the three-level consensus index, the attained results for each DM are collected in Table 3.4. Having $\gamma = 0.85$, it can be observed that DMs d_2 and d_4 are inconsistent and the following set of elements is required to be modified:

$$\text{APS} = \{(2, 1, 1), (2, 1, 2), (4, 2, 1), (4, 2, 2), (4, 2, 3), (4, 3, 3)\}.$$

Selecting $\delta = 0.3$ improves the consensus of each DM in the first discussion round as $\mathcal{CI}^1 = 0.8742$, $\mathcal{CI}^2 = 0.8559$, $\mathcal{CI}^3 = 0.8977$, and $\mathcal{CI}^4 = 0.8527$. Therefore, at the end of the first round of discussion, the consensus index of each DM is larger than the given threshold γ, and, therefore, the CRP terminates.

3.3.2 Consensus Model with Z-numbers

For the same example given in Section 3.3.1, the step-by-step implementation of the consensus model with Z-numbers is discussed in this section. Given the fact that DMs' interactions and level of trust remain unchanged compared with the previous

Table 3.2 – The initial evaluations of the DMs in the consensus model with DLTFs.

$\mathcal{X}$	d_1			d_2		
	c_1	c_2	c_3	c_1	c_2	c_3
x_1	$\begin{bmatrix}(s_1,0.6)\\(s_2,0.1)\\(s_3,0.3)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.4)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.4)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.5)\\(s_2,0.5)\\(s_3,0.0)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.2)\\(s_2,0.8)\\(s_3,0.0)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.6)\\(s_2,0.2)\\(s_3,0.2)\end{bmatrix}$
x_2	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.5)\\(s_3,0.1)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.6)\\(s_3,0.0)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.0)\\(s_3,0.6)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.4)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.0)\\(s_2,0.4)\\(s_3,0.6)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.4)\\(s_3,0.2)\end{bmatrix}$
x_3	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.2)\\(s_3,0.5)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.6)\\(s_2,0.1)\\(s_3,0.3)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.6)\\(s_2,0.2)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.5)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.2)\\(s_2,0.3)\\(s_3,0.5)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.5)\\(s_3,0.1)\end{bmatrix}$

	d_3			d_4		
	c_1	c_2	c_3	c_1	c_2	c_3
x_1	$\begin{bmatrix}(s_1,0.8)\\(s_2,0.0)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.5)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.4)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.1)\\(s_3,0.5)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.3)\\(s_3,0.3)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.5)\\(s_2,0.5)\\(s_3,0.0)\end{bmatrix}$
x_2	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.3)\\(s_3,0.3)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.0)\\(s_2,0.9)\\(s_3,0.1)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.6)\\(s_2,0.2)\\(s_3,0.2)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.8)\\(s_2,0.2)\\(s_3,0.0)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.6)\\(s_2,0.0)\\(s_3,0.4)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.8)\\(s_2,0.2)\\(s_3,0.0)\end{bmatrix}$
x_3	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.1)\\(s_3,0.5)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.6)\\(s_3,0.1)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3)\\(s_2,0.4)\\(s_3,0.3)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.2)\\(s_2,0.5)\\(s_3,0.3)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4)\\(s_2,0.5)\\(s_3,0.1)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.0)\\(s_2,0.6)\\(s_3,0.4)\end{bmatrix}$

Table 3.3 – The collective evaluation for the consensus model with DLTFs.

$\mathcal{X}$	c_1	c_2	c_3
x_1	$\begin{bmatrix}(s_1,0.5844)\\(s_2,0.1919)\\(s_3,0.2237)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.2808)\\(s_2,0.5170)\\(s_3,0.2022)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.4652)\\(s_2,0.3486)\\(s_3,0.1862)\end{bmatrix}$
x_2	$\begin{bmatrix}(s_1,0.5046)\\(s_2,0.3453)\\(s_3,0.1501)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.2314)\\(s_2,0.4836)\\(s_3,0.2850)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.5040)\\(s_2,0.2312)\\(s_3,0.2648)\end{bmatrix}$
x_3	$\begin{bmatrix}(s_1,0.3206)\\(s_2,0.2876)\\(s_3,0.3918)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3627)\\(s_2,0.3821)\\(s_3,0.2553)\end{bmatrix}$	$\begin{bmatrix}(s_1,0.3139)\\(s_2,0.4412)\\(s_3,0.2449)\end{bmatrix}$

Table 3.4 – The three-level consensus indexes w.r.t. the initial DMs' evaluations.

	$\mathcal{CE}$			$\mathcal{CA}$	$\mathcal{CI}$
	0.9309	0.9273	0.9603	0.9395	
d_1	0.8793	0.8100	0.7492	0.8125	0.8668
	0.8934	0.8286	0.8219	0.8480	
	0.8087	0.7806	0.8746	0.8213	
d_2	0.9253	0.7860	0.8650	0.8588	0.8453
	0.8565	0.8105	0.9007	0.8559	
	0.8721	0.9542	0.9236	0.9166	
d_3	0.9374	0.7427	0.9478	0.8760	0.8919
	0.8376	0.8524	0.9594	0.8831	
	0.8437	0.8474	0.8759	0.8557	
d_4	0.7708	0.6776	0.8175	0.7553	0.8169
	0.8702	0.8582	0.7907	0.8397	

Table 3.5 – The initial evaluations of the DMs in the consensus model with Z-Numbers.

$\mathcal{X}$	d_1			d_2		
	c_1	c_2	c_3	c_1	c_2	c_3
x_1	(s_5, s'_3)	(s_7, s'_4)	(s_4, s'_4)	(s_3, s'_3)	(s_3, s'_4)	(s_4, s'_3)
x_2	(s_4, s'_5)	(s_5, s'_4)	(s_4, s'_5)	(s_5, s'_3)	(s_3, s'_6)	(s_7, s'_7)
x_3	(s_6, s'_7)	(s_6, s'_5)	(s_7, s'_3)	(s_5, s'_7)	(s_5, s'_5)	(s_4, s'_5)
	d_3			d_4		
	c_1	c_2	c_3	c_1	c_2	c_3
x_1	(s_5, s'_3)	(s_7, s'_3)	(s_5, s'_7)	(s_5, s'_4)	(s_5, s'_5)	(s_7, s'_6)
x_2	(s_4, s'_6)	(s_7, s'_7)	(s_7, s'_3)	(s_6, s'_6)	(s_4, s'_5)	(s_5, s'_3)
x_3	(s_4, s'_7)	(s_5, s'_5)	(s_6, s'_7)	(s_5, s'_6)	(s_5, s'_7)	(s_6, s'_4)

Table 3.6 – Translated evaluations of each DM.

	$\mathcal{X}$	c_1	c_2	c_3
d_1	x_1	[(0.7,0.2),(0.4,0.2,0.04)]	[(1.0,0.3),(0.5,0.2,0.05)]	[(0.5,0.2),(0.5,0.2,0.05)]
	x_2	[(0.5,0.2),(0.6,0.2,0.04)]	[(0.7,0.2),(0.5,0.2,0.05)]	[(0.5,0.2),(0.6,0.2,0.04)]
	x_3	[(0.8,0.3),(1.0,0.3,0.03)]	[(0.8,0.3),(0.6,0.2,0.04)]	[(1.0,0.3),(0.4,0.2,0.04)]
d_2	x_1	[(0.4,0.2),(0.4,0.2,0.04)]	[(0.5,0.3),(0.5,0.2,0.05)]	[(0.5,0.2),(0.5,0.2,0.04)]
	x_2	[(0.6,0.2),(0.5,0.2,0.04)]	[(0.4,0.2),(0.7,0.3,0.03)]	[(0.9,0.3),(0.9,0.3,0.02)]
	x_3	[(0.7,0.2),(1.0,0.3,0.01)]	[(0.7,0.2),(0.6,0.2,0.04)]	[(0.5,0.2),(0.6,0.2,0.04)]
d_3	x_1	[(0.7,0.2),(0.4,0.2,0.04)]	[(1.0,0.3),(0.4,0.2,0.04)]	[(0.7,0.2),(1.0,0.3,0.01)]
	x_2	[(0.5,0.2),(0.8,0.3,0.02)]	[(1.0,0.3),(1.0,0.3,0.01)]	[(1.0,0.3),(0.4,0.2,0.04)]
	x_3	[(0.5,0.2),(1.0,0.3,0.01)]	[(0.7,0.2),(0.6,0.2,0.04)]	[(0.8,0.3),(1.0,0.3,0.01)]
d_4	x_1	[(0.7,0.2),(0.5,0.2,0.05)]	[(0.7,0.2),(0.6,0.2,0.04)]	[(0.9,0.3),(0.7,0.3,0.02)]
	x_2	[(0.8,0.3),(0.8,0.3,0.02)]	[(0.5,0.2),(0.6,0.2,0.04)]	[(0.7,0.2),(0.4,0.2,0.04)]
	x_3	[(0.7,0.2),(0.8,0.3,0.02)]	[(0.7,0.2),(0.9,0.3,0.03)]	[(0.9,0.3),(0.5,0.2,0.05)]

example, therefore, the same importance weights W^d can be resulted.

Suppose that DMs can choose the fuzzy restriction on their evaluations from the LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_6\}$; s_0, very poor; s_1, poor; s_2, slightly poor; s_3, fair; s_4, slightly good; s_5, good; and s_6, very good. They can also choose the certainty of their evaluations from $\mathcal{S}' = \{s'_0, s'_2, \ldots, s'_6\}$; s'_0, strongly uncertain; s'_1, uncertain; s'_2, somewhat uncertain; s'_3, neutral; s'_4, somewhat certain; s'_5, certain; and s'_6, strongly certain. Based on these LTSs, the initial evaluations of DMs for ranking alternatives w.r.t. the given attributes can be expressed using Z-numbers as illustrated in Table 3.5. Then, these evaluations are required to be translated into NZs. The translated evaluations are collected in Table 3.6. By selecting $\gamma = 0.85$ and $\delta = 0.3$, it takes five discussion rounds ($t = 1, \ldots, 5$) in order for all DMs to satisfy the designated consensus threshold, where the evolution of $\mathcal{CI}^d$ w.r.t. each discussion round is represented in Table 3.7. As it can be observed, the proposed consensus model effectively improves the consensus among DMs.

Table 3.7 – The evolution of the $\mathcal{CI}^d$ w.r.t. each discussion round t.

$\mathcal{CI}$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$\mathcal{CI}^1$	0.9367	0.9497	0.9589	0.9619	0.9643
$\mathcal{CI}^2$	0.6580	0.7485	0.8005	0.8364	0.8568
$\mathcal{CI}^3$	0.7995	0.8431	0.8650	0.8658	0.8659
$\mathcal{CI}^4$	0.6765	0.7483	0.8013	0.8363	0.8553

Chapter 4

Consensus and Fusion Models to Support Dynamic Group Decision Making

In contrast to the static decision environment, a dynamic environment in CRP refers to the changes in the set of alternatives, attributes, DMs, and their importance weights. In this regard, the set of alternatives could be subject to changes during the consensus assessment due to the availability of new alternatives and/or the feasibility of the previous set of alternatives [43]. The dynamic set of attributes models the situations where new attributes are introduced to the problem during the CRP in order to speed up the process and/or to evaluate the decision problem from new viewpoints [207, 208]. The set of DMs can also be subject to changes due to the fact that some DMs may leave the negotiation and/or new individuals might be invited to participate to the decision problem [209, 210]. DMs' importance weights, which are of paramount importance in constructing the collective evaluation of the group, can also be subject to changes from one discussion round to another in a dynamic decision environment. Some efforts have been made to address this issue through the transition probabilities of DMs' evaluations [211], prior knowledge on the DMs' personal characteristics [212], and eigenvector-based weight assigning approach [213].

Decision-making with consensus reaching under dynamic environments, however, becomes more complex compared with static environments due to changes in the decision parameters including the set of alternatives. Efficient management of DMs and their interactions within the CRP can help with reducing the complexity of dynamic decision-making. Furthermore, consensus assessment and recommendation generation for DMs to reach a desired level of consensus should also be efficient to reduce their associated complexity and to speed up the CRP. In this regard, management of the attitude and interest of DMs, proper and dynamic adjustment of the consensus threshold and DMs' importance weights could play an important role in coping with the complexity of decision-making in dynamic environments.

In order to deal with the complexity of dynamic decision-making, this chapter is devoted to the introduction of a novel and computationally efficient framework based on the Z-numbers for the sake of dynamic decision-making. In this regard, DMs are initially divided into several groups, where DMs with the same interest form a group. This is to reduce the dimension of initial evaluations of DMs and to convert the multiple attribute decision problem into multiple single attribute problems to ease the consensus assessment. At each time-step, a set of available alternatives, which is subject to changes from one time-step to another, is put into discussion for each group. To construct the collective evaluation of each group based on the provided evaluations, trust relationships of DMs are integrated into their attitudes through an attitudinal quantifier, where an OWA operator is then employed to construct the collective opinion for the sake of consensus assessment. In this regard, the consensus threshold value is dynamically adjusted through a computationally efficient mechanism that relies on extracting the minimum spanning tree (MST) of the constructed CEN in each group. The collective evaluations of groups are then aggregated through an optimal fusion model to be fed into the selection process. The selected alternative in the selection process will then be carried over to the next time-step by a designated memory. This process will be continued until all alternatives are evaluated at least once. To this end, this chapter makes the following contributions to the dynamic MAGDM models:

1. A computationally efficient framework is proposed for dynamic decision-making based on Z-numbers.

2. A mechanism is proposed to integrate trust relationships among DMs with their

attitudes to adjust DMs' importance weights.

3. The consensus threshold value for each group is determined by means of an efficient and dynamic mechanism.

4. A novel fusion model is finally proposed to optimally aggregate the collective evaluations of all groups to be fed into the selection process.

4.1 Preliminaries

Supposed that $\mathcal{S}$ is an LTS as defined in Section 3.1.1. In this regard, the definition of LSFs can be given as follows [214].

Definition 23. *Suppose that s_i is a linguistic term in $\mathcal{S}$, then, an LSF can be defined as a mapping $\mathcal{F} : s_i \rightarrow \theta_i\,(i = 0, 1, 2, \ldots, 2p)$, where $0 \leq \theta_0 \leq \theta_1 \leq \ldots \leq \theta_{2p} \leq 1$.*

The following LSFs are used in this chapter:

$$\mathcal{F}_1(s_i) = \theta_i = \frac{i}{2p}, (i = 0, 1, \ldots, 2p) \tag{4.1}$$

$$\mathcal{F}_2(s_i) = \theta_i = \begin{cases} \frac{a^p - a^{(p-i)}}{2a^{(p-2)}} & (i = 0, 1, \ldots, p) \\ \frac{a^p + a^{(i-p)} - 2}{2a^p - 2} & (i = p+1, p+2, \ldots, 2p) \end{cases} \tag{4.2}$$

$$\mathcal{F}_3(s_i) = \theta_i = \begin{cases} \frac{p^\kappa - (p-i)^\kappa}{2p^\kappa} & (i = 0, 1, \ldots, p) \\ \frac{p^\varsigma + (i-p)^\varsigma}{2p^\varsigma} & (i = p+1, p+2, \ldots, 2p) \end{cases} \tag{4.3}$$

where p is the number of linguistic terms, a belongs to the interval $[1.36, 1.4]$, and $\kappa, \varsigma \in [0, 1]$ denote the curvature of the subjective functions for gain and loss, respectively [215].

Definition 24. *Following the Definition 20, in case $SD\,(NZ_i, NZ_j) = 0$, the GNZPWA operator can then degenerate a GNZWA operator, that is $\mathcal{B}\,(NZ_1, \ldots, NZ_n)$.*

Definition 25. *[216] By assuming that the set of DMs $\mathcal{D}$ are vertices of a weighted undirected graph, this graph can be represented by $\mathcal{G}(\mathcal{D}, \zeta, \mathcal{W})$ with q vertices, a finite set of edges $\zeta = \{\zeta_{hl}\}\,(h, l = 1, 2, \ldots, q,\ h \neq l)$, where an edge ζ_{hl} denotes the connection between the hth and lth DMs, and a set of weights $\mathcal{W} = \{w_{hl}\}\,(h, l = 1, 2, \ldots, q)$ with $h \neq l$.*

Definition 26. *[217] In a simple undirected graph $\mathcal{G}(\mathcal{D}, \zeta)$, the neighborhood N_k for a vertex e_k is defined as $N_k = \{d_p : \zeta_{kp} \in \zeta\}$, where the local clustering coefficient for G can then be defined as:*

$$\mathcal{LCC}_k = \frac{2|\{\zeta_{pa} : e_p, e_a \in N_k, \zeta_{pa} \in \zeta\}|}{N(e_k)\,[N(e_k) - 1]} \tag{4.4}$$

where e_p and e_a are neighbors of e_k in N_k, $N(e_k)$ stands for the number of neighbors of e_k, the $|.|$ operator denotes the cardinality of the enclosed set, and $\frac{N(e_k)[N(e_k)-1]}{2}$ shows the total number of edges in $\mathcal{G}$. Therefore, the overall clustering coefficient can be computed as the average of all local clustering coefficients as given below:

$$\mathcal{CC} = \frac{1}{q}\sum_{i=1}^{q} \mathcal{LCC}_i. \tag{4.5}$$

Definition 27. *[218]: For a weighted undirected graph $\mathcal{G}$, the MST $\mathcal{M}(\mathcal{D}', \zeta', \mathcal{W}')$ is a subset of the edges of a connected, weighted undirected graph that connects all the vertices, without any cycles and with the minimum total edge weight possible.*

4.2 The General Framework

Traditional MAGDM problems can be referred to as providing rankings for a set of alternatives $\mathcal{X} = \{x_1, x_2, \ldots, x_q\}$ by a set of DMs $\mathcal{D} = \{d_1, d_2, \ldots, d_n\}$ w.r.t. multiple attributes $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$. Such a framework does not benefit the most from the DMs' expertise and does not take their interest into consideration. Furthermore, the

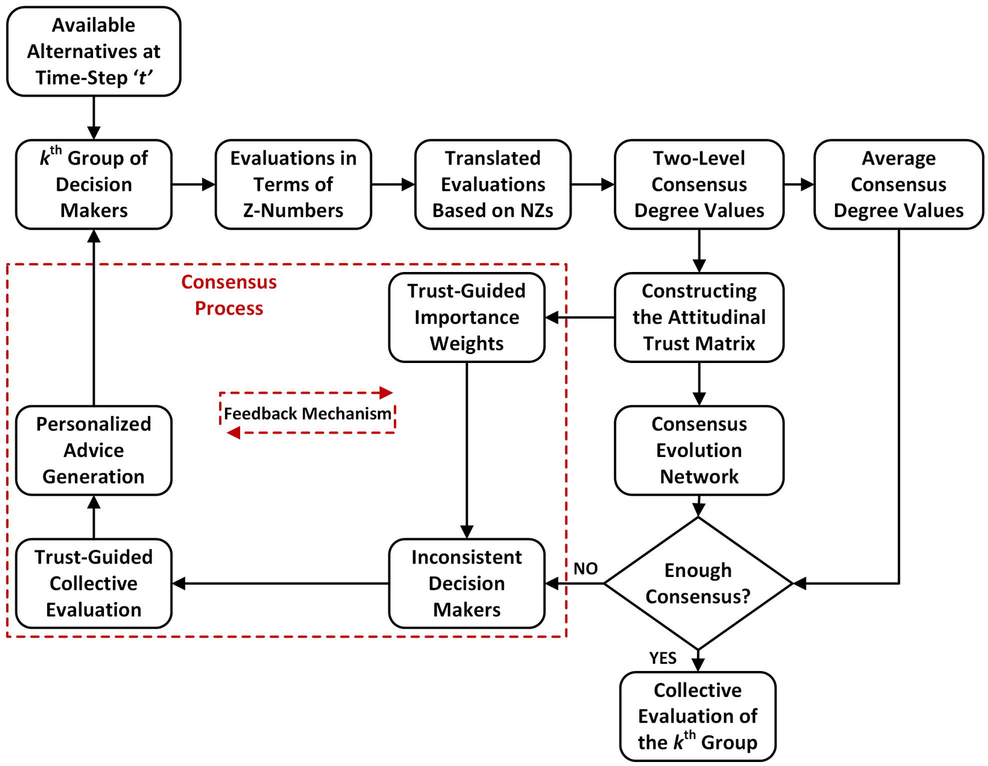

Figure 4.1 – The general framework of the proposed method.

practical decision environments are mostly dynamic and statistical GDM frameworks are not suitable for such environments. This chapter puts forward an MAGDM framework to deal with these challenges.

It is proposed to initially divide the DMs into m groups w.r.t. m available attributes. This is to take the interest of DMs into account so that each DM evaluates alternatives w.r.t. only one attribute. Such a mechanism provides an opportunity for DMs to choose the most relevant attribute to their expertise. This idea leads to breaking up the multiple-attribute decision problem into multiple single-attribute decision problems that consequently helps with reducing the computational burden of the CRP. The general structure of the CRP and the construction of the collective evaluation in each group of DMs are illustrated in Fig. 4.1.

Specifically, the designated alternatives at time-step $t = 1$ will be put into discussion first. This set of alternatives is given to each group of DMs, where the initial evaluations w.r.t. each attribute are provided in terms of Z-numbers, which are then translated into NZs. In each group, the CRP is fulfilled, and, the inconsistent DMs with a CD lower than a specified threshold are advised to modify their evaluations through a trust-based recommendation model. The collective evaluation of each group is then constructed through the GNZWA operator $\mathcal{B}$, and, then, the collective evaluations of all groups are fed into the fusion model to allocate appropriate weights to each attribute $\{w^{(1,1)}, w^{(1,2)}, \ldots, w^{(1,m)}\}$. The proposed fusion model then makes use of the GNZPWA operator $\mathcal{H}$ to construct ratio systems. By extracting the PIS and the NIS from the constructed ratio systems, the alternatives are ranked based on a closeness coefficient and the best one is carried over to the next time-step. In the next time-step, i.e., $t = 2$, a new subset of alternatives plus the alternative in the memory (the selected alternative in the previous time-step) are selected to be evaluated. This process will be continued until all the initial alternatives are evaluated at least once. The selected alternative at the last time-step is the best one and is considered as the solution to the decision problem.

4.3 MAGDM with Dynamic Alternatives

Let $\mathcal{D}$, $\mathcal{A}$, and $\mathcal{X}$ be the initial set of DMs, attributes, and alternatives, respectively. DMs are initially divided into m groups based on their selected attributes. Let $\mathcal{D}^{(k)} = \{e_1^{(k)}, e_2^{(k)}, \ldots, e_{n(k)}^{(k)}\}$ $(k = 1, 2, \ldots, m)$ be the kth group of DMs focused on the kth attribute with $n(k)$ being the number of DMs in the kth group, and $\mathcal{X}^{(t)} = \{x_1^t, x_2^t, \ldots, x_{q(t)}^t\}$ be the set of alternatives at time-step t, where $q(t)$ shows the number of alternatives at time-step t.

The initial evaluations of the kth group at time-step t in terms of Z-numbers is $\mathcal{R}^{(k,t)} = \{\mathcal{R}_1^{(k,t)}, \mathcal{R}_2^{(k,t)}, \ldots, \mathcal{R}_{n(k)}^{(k,t)}\}$, where each evaluation relation can be represented by a matrix $\mathcal{R}_p^{(k,t)} = [r_{p,i}]_{q(t)\times 1}^{(k,t)} = [(s_{p,i}, s'_{p,i})]_{q(t)\times 1}^{(k,t)}$, in which $(s_{p,i}, s'_{p,i})$ denotes a Z-number given by the pth DM with $s_{p,i} \in \mathcal{S}$ and $\mathcal{S}$ being an LTS of fuzzy *restrictions*

Table 4.1 – Evaluations provided by all groups of DMs.

	Group 1				Group 2			
x	e_1^1	e_2^1	e_3^1	e_4^1	e_1^2	e_2^2	e_3^2	e_4^2
x_1	(s_3, s'_6)	(s_3, s'_6)	(s_4, s'_4)	(s_4, s'_5)	(s_5, s'_5)	(s_5, s'_6)	(s_4, s'_4)	(s_4, s'_6)
x_2	(s_3, s'_5)	(s_5, s'_3)	(s_3, s'_3)	(s_4, s'_5)	(s_6, s'_4)	(s_4, s'_4)	(s_5, s'_4)	(s_5, s'_5)
x_3	(s_5, s'_3)	(s_4, s'_5)	(s_5, s'_4)	(s_4, s'_4)	(s_4, s'_3)	(s_5, s'_4)	(s_4, s'_3)	(s_5, s'_3)
x_4	(s_4, s'_5)	(s_5, s'_4)	(s_4, s'_5)	(s_4, s'_4)	(s_3, s'_6)	(s_3, s'_5)	(s_3, s'_5)	(s_4, s'_5)
	Group 3				Group 4			
x	e_1^3	e_2^3	e_3^3	e_4^3	e_1^4	e_2^4	e_3^4	e_4^4
x_1	(s_5, s'_3)	(s_4, s'_3)	(s_4, s'_6)	(s_4, s'_5)	(s_6, s'_6)	(s_5, s'_4)	(s_5, s'_2)	(s_5, s'_3)
x_2	(s_3, s'_5)	(s_5, s'_4)	(s_3, s'_3)	(s_3, s'_4)	(s_4, s'_5)	(s_2, s'_3)	(s_4, s'_4)	(s_5, s'_2)
x_3	(s_4, s'_6)	(s_3, s'_4)	(s_4, s'_5)	(s_5, s'_3)	(s_4, s'_5)	(s_5, s'_6)	(s_5, s'_5)	(s_5, s'_4)
x_4	(s_6, s'_3)	(s_4, s'_5)	(s_4, s'_5)	(s_4, s'_6)	(s_5, s'_6)	(s_6, s'_5)	(s_5, s'_6)	(s_6, s'_2)

on the evaluations, $s'_{p,i} \in \mathcal{S}'$ with $\mathcal{S}'$ being an LTS of *certainty degree* of the given evaluation, $p = 1, 2, \ldots, n(k)$, $i = 1, 2, \ldots, q(t)$, and $k = 1, 2, \ldots, m$.

Example 1. *Suppose that an initial group of sixteen DMs $\mathcal{D} = \{e_1, e_2, \ldots, e_{16}\}$ are supposed to review the air pollution potential of four areas $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ w.r.t. four attributes $\mathcal{A} = \{a_1, a_2, a_3, a_4\}$. The moderator divides DMs into four groups based on their interests and suppose that there exist four DMs in each group. DMs in each group can choose the fuzzy restriction on their evaluations from $\mathcal{S} = \{s_0, s_1, \ldots, s_6\}$; s_0, very poor; s_1, poor; s_2, slightly poor; s_3, fair; s_4, slightly good; s_5, good; and s_6, very good. They can also choose the certainty of their evaluations from $\mathcal{S}' = \{s'_0, s'_2, \ldots, s'_6\}$; s'_0, strongly uncertain; s'_1, uncertain; s'_2, somewhat uncertain; s'_3, neutral; s'_4, somewhat certain; s'_5, certain; and s'_6, strongly certain. The initial evaluations provided by all groups of DMs are collected in Table 4.1.*

The initial evaluations in terms of Z-numbers are then used to extract a NZ from each element. This transformation is used to provide a quantitative representation for a Z-number. The set of evaluations $\mathcal{R}^{(k,t)}$ are translated into $\widetilde{\mathcal{R}}^{(k,t)} = \{\widetilde{\mathcal{R}}_1^{(k,t)}, \widetilde{\mathcal{R}}_2^{(k,t)}, \ldots, \widetilde{\mathcal{R}}_{n(k)}^{(k,t)}\}$, where $\widetilde{\mathcal{R}}_p^{(k,t)}$ $(p = 1, 2, \ldots, n\,(k))$ denotes the translated evaluations of the pth DM in the kth group. Each evaluation term in $\mathcal{R}_p^{(k,t)}$ is then decom-

Table 4.2 – Translated evaluations of all DMs in the first group.

DM	x	Translated Evaluation
e_1^1	x_1	$[(0.6016, 0.2152), (0.7822, 0.2553, 0.0297)]$
	x_2	$[(0.6100, 0.2161), (0.7673, 0.2522, 0.0306)]$
	x_3	$[(0.6325, 0.2186), (0.7212, 0.2414, 0.0335)]$
	x_4	$[(0.6325, 0.2186), (0.7441, 0.2491, 0.0309)]$
e_2^1	x_1	$[(0.6006, 0.2236), (0.7058, 0.2491, 0.0307)]$
	x_2	$[(0.6540, 0.2357), (0.6582, 0.2373, 0.0351)]$
	x_3	$[(0.6090, 0.2252), (0.7008, 0.2479, 0.0312)]$
	x_4	$[(0.6315, 0.2276), (0.7036, 0.2486, 0.0309)]$
e_3^1	x_1	$[(0.7642, 0.2562), (0.6185, 0.2173, 0.0399)]$
	x_2	$[(0.7360, 0.2468), (0.6674, 0.2293, 0.0362)]$
	x_3	$[(0.7585, 0.2543), (0.6468, 0.2218, 0.0382)]$
	x_4	$[(0.7360, 0.2468), (0.6477, 0.2221, 0.0381)]$
e_4^1	x_1	$[(0.6982, 0.2337), (0.7079, 0.2434, 0.0319)]$
	x_2	$[(0.7233, 0.2425), (0.6796, 0.2336, 0.0349)]$
	x_3	$[(0.7008, 0.2347), (0.7023, 0.2415, 0.0325)]$
	x_4	$[(0.7008, 0.2347), (0.6815, 0.2343, 0.0347)]$

posed into a five-element NZ. Therefore, the translated evaluations can be expressed by a matrix of the form $\widetilde{\mathcal{R}}_p^{(k,t)} = [\widetilde{r}_{p,i}^{(k,t)}]_{q(t)\times 5}^{(k,t)} = [((\mu_{p,i}, \sigma_{p,i}), (\vartheta_{p,i}, \varpi_{p,i}, \varrho_{p,i}))]_{q(t)\times 5}^{(k,t)}$.

Example 2. *(Example 1 continuation) Let $\mathcal{S}$ and $\mathcal{S}'$ be as before and $\mathcal{U} = [0, 1]$. The constructed NZs for all DMs in the first group are given in Table 4.2.*

Once the initial evaluations are translated, the CRP in each group gets started. In what follows, a two-level CD has been suggested to be used for the sake of consensus assessment.

4.3.1 Consensus Degree based on NZs

A two-level CD is proposed as each group of DMs evaluate a given set of alternatives w.r.t. only one attribute. Therefore, the proposed scheme aims to break a multiple-attribute decision problem into multiple single-attribute decision problems to reduce the consensus assessment in each group into just two levels compared with the typical three-level assessments. This consequently helps with making the proposed approach computationally efficient.

Let $\widetilde{\mathcal{R}}_h^{(k,t)}$ and $\widetilde{\mathcal{R}}_l^{(k,t)}$ be two decision matrices in terms of NZs provided by DMs $e_h^{(k)}$ and $e_l^{(k)}$ in the kth group, respectively, on a set of alternatives $\mathcal{X}^{(t)} = \{x_1^{(t)}, x_2^{(t)}, \ldots, x_{q(t)}^{(t)}\}$.

Level 1. *CD on alternatives*: For DMs $e_h^{(k)}$ and $e_l^{(k)}$ in the kth group with $k = 1, 2, \ldots, m$, their CD on an alternative $x_i^{(k,t)}\,(i = 1, 2, \ldots, q\,(t))$ is defined as follows:

$$\mathcal{CE}_{h,l}^{(k,i)} \left(\widetilde{\mathcal{R}}_h^{(k,t)}, \widetilde{\mathcal{R}}_l^{(k,t)} \right) = 1 - \mathrm{dist}_z \left(\widetilde{r}_{h,i}^{(k,t)}, \widetilde{r}_{l,i}^{(k,t)} \right), \tag{4.6}$$

where $\mathrm{list}_z(\widetilde{r}_{h,i}^{(k,t)}, \widetilde{r}_{l,i}^{(k,t)})$ denotes the distance between two NZs as defined in Definition 19. By setting $\mathcal{U} = [0, 1]$, it can be concluded that $\mathcal{CE}_{h,l}^{(k,i)}(\widetilde{\mathcal{R}}_h^{(k,t)}, \widetilde{\mathcal{R}}_l^{(k,t)}) \in [0, 1]$, $\forall i, h, l$.

Level 2. *CD on decision matrix*: For DMs $e_h^{(k)}$ and $e_l^{(k)}$, their CD on decision matrices $\widetilde{\mathcal{R}}_h^{(k,t)}$ and $\widetilde{\mathcal{R}}_l^{(k,t)}$ is defined as follows:

$$\mathcal{CD}_{h,l}^{(k,t)} \left(\widetilde{\mathcal{R}}_h^{(k,t)}, \widetilde{\mathcal{R}}_l^{(k,t)} \right) = \frac{1}{q\,(t)} \sum_{i=1}^{q(t)} \mathcal{CE}_{h,l}^{(k,i)} \left(\widetilde{\mathcal{R}}_h^{(k,t)}, \widetilde{\mathcal{R}}_l^{(k,t)} \right).$$

Same as what stated for the $\mathcal{CE}_i$, the effective domain of $\mathcal{U} = [0, 1]$ leads to $\mathcal{CD}_{h,l}^{(k)}(\widetilde{R}_h^{(k,t)}, \widetilde{R}_l^{(k,t)}) \in [0, 1]$. For a DM in the kth group $e_h^{(k)}$, the following average CD (ACD) can then be defined so as to check his CD with the rest of the peers in a group:

$$\mathcal{ACD}_h^{(k,t)} = \frac{1}{n\,(k) - 1} \sum_{l=1, l \neq h}^{n(k)} \mathcal{CD}_{h,l}^{(k,t)} \left(\widetilde{\mathcal{R}}_h^{(k,t)}, \widetilde{\mathcal{R}}_l^{(k,t)} \right).$$

As for the given illustrative example, $\mathcal{CD}$ on the decision matrix and $\mathcal{ACD}$ values are collected in Table 4.3 and Table 4.4, respectively.

Table 4.3 – $\mathcal{CD}$ on decision matrix for all groups of DMs.

	Group 1					Group 2			
	e_1^1	e_2^1	e_3^1	e_4^1		e_1^2	e_2^2	e_3^2	e_4^2
e_1^1	-	0.9413	0.9642	0.9503	e_1^2	-	0.9267	0.9285	0.9140
e_2^1	$*$	-	0.9319	0.9640	e_2^2	$*$	-	0.9047	0.9181
e_3^1	$*$	$*$	-	0.9634	e_3^2	$*$	$*$	-	0.9135
e_4^1	$*$	$*$	$*$	-	e_4^2	$*$	$*$	$*$	-
	Group 3					Group 4			
	e_1^3	e_2^3	e_3^3	e_4^3		e_1^4	e_2^4	e_3^4	e_4^4
e_1^3	-	0.9444	0.9529	0.9504	e_1^4	-	0.9394	0.9389	0.9209
e_2^3	$*$	-	0.9532	0.9560	e_2^4	$*$	-	0.9184	0.9454
e_3^3	$*$	$*$	-	0.9525	e_3^4	$*$	$*$	-	0.9123
e_4^3	$*$	$*$	$*$	-	e_4^4	$*$	$*$	$*$	-

Table 4.4 – The $\mathcal{ACD}$ of each DM in each group.

	$k=1$	$k=2$	$k=3$	$k=4$
$\mathcal{ACD}_1^{(k)}$	0.8262	0.8688	0.7986	0.8019
$\mathcal{ACD}_2^{(k)}$	0.7607	0.8722	0.8515	0.7762
$\mathcal{ACD}_3^{(k)}$	0.7912	0.8651	0.8831	0.7534
$\mathcal{ACD}_4^{(k)}$	0.8097	0.8217	0.8525	0.7596

4.3.2 Consensus Threshold

The consensus threshold for the kth group during the rth discussion round at time-step t is denoted by $\gamma_r^{(k,t)}$, and it is proposed to be determined based on the $\mathcal{CD}$. If the $\mathcal{CD}$ among DMs $e_h^{(k)}$ and $e_l^{(k)}$ in the rth discussion round at time-step t, i.e., $\mathcal{CD}_{h,l}^{(k,r)}$, is larger than $\gamma_r^{(k,t)}$, it can be said that there exist a consensus relationship between these two DMs. This idea is used to define CENs based on weighted undirected graphs.

Definition 28. *In a group of DMs $\mathcal{D}^{(k)} = \{e_1^{(k)}, e_2^{(k)}, \ldots, e_{n(k)}^{(k)}\}$ $(k = 1, 2, \ldots, m)$, the CEN is a weighted undirected graph $\mathcal{G}(\mathcal{D}, \zeta, \mathcal{C})$ with $n(k)$ vertices, consensus relations $\zeta = \{\zeta_{hl}^{(k)}, h, l = 1, 2, \ldots, n(k), h \neq l\}$, and consensus relation values $\mathcal{C} = \{c_{hl}^{(k)} = \mathcal{CD}_{hl}^{(k)} | h, l = 1, 2, \ldots, n(k), h \neq l, \mathcal{CD}_{hl}^{(k)} \geq \gamma^{(k)}\}$. If $c_{hl}^{(k)} \geq \gamma^{(k)}$, there is an edge in $\mathcal{G}$ connecting DMs $e_h^{(k)}$ and $e_l^{(k)}$ with the weight of $c_{hl}^{(k)}$, otherwise, there is no edge between these two DMs.*

Based on the above definition, $\mathcal{G}_r^{(k,t)}(\mathcal{D}^{(k)}, \zeta_r^{(k,t)}, \mathcal{C}_r^{(k,t)})$ denotes the CEN of the kth group in the rth discussion round at time-step t.

Definition 29. *For the kth group in the rth discussion round at time-step t, the complete CEN can be denoted via $\mathcal{G}_{c,r}^{(k,t)}$ that consists of $n(k)$ DMs as vertices, consensus relations $\zeta_{c,r}^{(k,t)}$ and consensus relation values $\mathcal{C}_{c,r}^{(k,t)}$ with $\mathcal{CD}_{r,hl}^{(k,t)} \geq \gamma_{c,r}^{(k,t)}$, where $\gamma_{c,r}^{(k,t)} = \min\{\mathcal{CD}_{r,hl}^{(k,t)}\}$.*

A complete CEN is a completely connected network and according to the Definition 26, the overall $\mathcal{CC}$ for a complete CEN is equal to one. Any $\gamma_r^{(k,t)} > \gamma_{c,r}^{(k,t)}$ will result in an incomplete CEN, where for an incomplete CEN, the overall $\mathcal{CC}$ is smaller than one. Furthermore, there exists $\gamma_{e,r}^{(k,t)}$, for which there is no connection among all vertices, leading to an empty CEN, where the overall $\mathcal{CC}$ is zero. The proposed method to find the consensus threshold is based on the fact that a lower $\mathcal{CC}$ in a CEN conducts less stable relationships among DMs. There exist a sensitive threshold value for which a CEN becomes more vulnerable and unstable. This threshold value

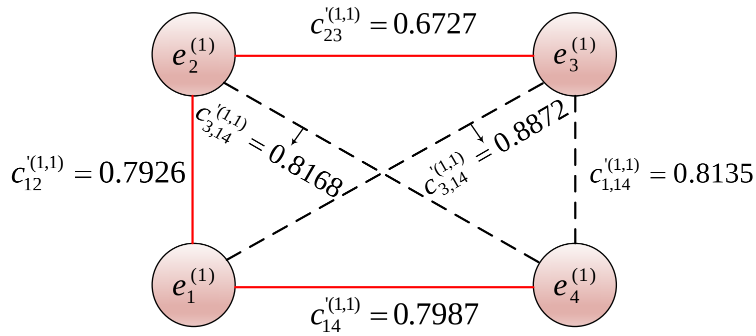

Figure 4.2 – The complete CEN and its corresponding MST for the first group of DMs.

results in a sensitive CEN and can be selected as a proper consensus threshold. The following procedure based on the CENs and MSTs is proposed to extract the sensitive CEN so as to find a proper consensus threshold.

Let $\mathcal{G}_{c,r}^{(k,t)}(\mathcal{D}^{(k)}, \zeta_r^{(k,t)}, \mathcal{C}_r^{(k,t)})$, k, r and t be as before. We then define the MST of $\mathcal{G}_{c,r}^{(k,t)}$ as $\mathcal{M}_r^{(k,t)}(\mathcal{D}'^{(k)}, \zeta_r'^{(k,t)}, \mathcal{C}_r'^{(k,t)})$. We further define a set of edges from $\zeta_r^{(k,t)}$ for which, $|\zeta_r^{(k,t)}| = \epsilon_1$, and they are not included in $\zeta_r'^{(k,t)}$, where $|\zeta_r'^{(k,t)}| = \epsilon_2$, and denote it by $N\zeta_r^{(k,t)} = \{N\zeta_{p,hl}^{(k,t)}, N\zeta_{p+1,hl}^{(k,t)}, \ldots, N\zeta_{\epsilon_1-\epsilon_2,hl}^{(k,t)}\}$, where its elements are set in a descent order w.r.t. their corresponding weights, i.e., $c_{p,hl}'^{(k,t)} \leq c_{p+1,hl}'^{(k,t)}$. Following the fact that the overall $\mathcal{CC}$ for an MST is zero, in order to determine the sensitive CEN and its corresponding consensus threshold, we suggest the following simple rule. Determine the MST of $\mathcal{G}_{c,r}^{(k,t)}$ and compute the overall $\mathcal{CC}$, which must be zero. Then, add the first edge in $N\zeta_r^{(k,t)}$, i.e., $N\zeta_{p,hl}^{(k,t)}$, to the constructed MST and compute the overall $\mathcal{CC}$. If the overall $\mathcal{CC}$ is not zero, the maximum weight of edges in $\zeta_r'^{(k,t)}$ is the consensus threshold; otherwise, add the first two edges in $N\zeta_r^{(k,t)}$ to the constructed MST and compute the overall $\mathcal{CC}$. If it is not zero, the consensus threshold is $c_{p,hl}'^{(k,t)}$; otherwise, repeat the previous step. This simple rule should be repeated until the first sub-graph with a nonzero overall $\mathcal{CC}$ is resulted. Algorithm 3 summarizes the required steps in forming the sensitive CEN to find the consensus threshold.

Example 3. *(Example 1 continuation) In order to set the consensus threshold for the*

Algorithm 3: Consensus Threshold

Input : $\mathcal{G}_{c,r}^{(k,t)}\left(\mathcal{D}^{(k)}, \zeta_r^{(k,t)}, \mathcal{C}_r^{(k,t)}\right)$: complete CEN of the kth group.

ϵ_1: The cardinality of $\zeta_r^{(k,t)}$, $|\zeta_r^{(k,t)}| = \epsilon_1$.

ϵ_2: The cardinality of $\zeta_r^{\prime(k,t)}$, $|\zeta_r^{\prime(k,t)}| = \epsilon_2$.

Output: $\gamma_r^{(k,t)}$: Consensus threshold of the kth group at round r.

1 **while** $\mathcal{CC} \neq 0$ **do**

2   Extract the MST $\mathcal{M}_r^{(k,t)}(\mathcal{D}^{\prime(k)}, \zeta_r^{\prime(k,t)}, \mathcal{C}_r^{\prime(k,t)})$ from the complete CEN $\mathcal{G}_{c,r}^{(k,t)}(\mathcal{D}^{(k)}, \zeta_r^{(k,t)}, \mathcal{C}_r^{(k,t)})$.

3   Construct $N\zeta_r^{(k,t)} = \{N\zeta_{p,hl}^{(k,t)}, N\zeta_{p+1,hl}^{(k,t)}, \ldots, N\zeta_{\epsilon_1-\epsilon_2,hl}^{(k,t)}\}$.

4   **for** $i = 1 : \epsilon_1 - \epsilon_2$ **do**

5   Remove the i-first element of $N\zeta_r^{(k,t)}$ and add it (them) to the set of edges of the MST $\zeta_r^{\prime(k,t)}$.

6   Compute the overall $\mathcal{CC}$ of $T_r^{(k,t)}(\mathcal{D}^{\prime(k)}, \zeta_r^{\prime(k,t)}, \mathcal{C}_r^{\prime(k,t)})$.

7   **if** $i = 1 \wedge \mathcal{CC} \neq 0$ **then**

8             $\gamma_r^{(k,t)} = \max\{c_{hl}^{\prime(k,t)} | \zeta_{hl}^{(k,t)} \in \zeta_r^{\prime(k,t)}\}$ and terminate the algorithm.

9   **else if** $i \neq 0 \wedge \mathcal{CC} \neq 0$ **then**

10             $\gamma_r^{(k,t)} = c_{i-1,hl}^{\prime(k,t)}$ and terminate the algorithm.

11   **else**

12   Return to step 6.

13   **end**

14   **end**

15 **end**

first group of DMs, their CDs are used to construct a CEN. As there are four DMs in the first group, the constructed CEN consists of four vertices and the corresponding complete CEN contains six edges. The complete CEN has been shown in Fig. 4.2, where its MST has been highlighted by red lines. As the overall $\mathcal{CC}$ *for the MST is zero, therefore, the next edge that will be added to the MST is* $\zeta_{34}^{(1,1)}$*, where its corresponding weight is 0.8135. By computing the overall* $\mathcal{CC}$*, it will be zero. Therefore, the next edge to be added to the MST is* $\zeta_{24}^{(1,1)}$*, where its corresponding weight is 0.8168. In this case, the overall* $\mathcal{CC}$ *will be obtained as 0.83 that is not zero. Therefore, the consensus threshold for the first group of DMs is* $\gamma_1^{(1,1)}$*=0.8135. By doing the same procedure, one can conclude that* $\gamma_1^{(2,1)} = 0.8344$*,* $\gamma_1^{(3,1)} = 0.8326$*, and* $\gamma_1^{(4,1)} = 0.7969$*.*

4.3.3 Trust Model for Recommendation Generation

In order to generate the adjustment rules for inconsistent DMs in each group, three main steps are required to be fulfilled.

1. Determine inconsistent DMs in each group. Let k, r, and t be as before. Then, $\mathcal{ACD}_n^{(k,t)}$ denotes the ACD of the nth DM in the kth group at time-step t and the set of inconsistent DMs can be constructed as follows:

$$\mathrm{EXPCH}_r^{(k,t)} = \{e_n^{(k)} | \mathcal{ACD}_n^{(k,t)} < \gamma_r^{(k,t)} \in [0,1]\}.$$

Example 4. *(Example 1 continuation) By comparing the attained results collected in Table 4.4 with the consensus thresholds obtained in Example 3, the following sets of inconsistent DMs can be constructed:* $EXPCH_1^{(1,1)} = \{e_2^{(1)}, e_3^{(1)}, e_4^{(1)}\}$*,* $EXPCH_1^{(2,1)} = \{e_4^{(2)}\}$*,* $EXPCH_1^{(3,1)} = \{e_1^{(3)}\}$*, and* $EXPCH_1^{(4,1)} = \{e_2^{(4)}, e_3^{(4)}, e_4^{(4)}\}$*.*

Definition 30. *For a pair of DMs* $e_h^{(k)}$ *and* $e_l^{(k)}$ *in the same group, their trust rela-*

tionship $\mathcal{TR}$ is based on the measured $\mathcal{CD}$ and can be defined as:

$$\mathcal{TR}_{r,hl}^{(k,t)} = \mathcal{CD}_{r,hl}^{(k)}(\widetilde{\mathcal{R}}_h^{(k,t)}, \widetilde{\mathcal{R}}_l^{(k,t)}), \tag{4.7}$$

and, then, the trust matrix of the group $\mathcal{TRM}$ can be defined as:

$$\mathcal{TRM}_r^{(k,t)} = \left[\mathcal{TR}_{r,hl}^{(k,t)}\right]_{q(k)\times q(k)}. \tag{4.8}$$

2. Construct the trust model $\mathcal{TRM}_r^{(k,t)}$.

Suppose that $e_n^{(k)}$ is an inconsistent DM in EXPCH$_r^{(k,t)}$. Let the trust relationship of this DM with other DMs in the group be as $\{\mathcal{TR}_{r,hl}^{(k,t)} | l = 1, 2, \dots, n(k), l \neq h\}$. Suppose that $\sigma(i) : \{1, 2, \dots, n(k) - 1\} \to \{l = 1, 2, \dots, n(k) | l \neq h\}$ is a permutation that verifies $\mathcal{TR}_{r,h\sigma(i)}^{(k,t)} \geq \mathcal{TR}_{r,h\sigma(i+1)}^{(k,t)}$. Then, the trust degrees based on the DMs' attitude can be calculated through an OWA operator, which is guided by a basic unit-monotonic function $\mathcal{Q}$ as follows:

$$\beta_{\sigma(i)}^{(k,t)} = \mathcal{Q}(\frac{i}{n(k)-1}) - \mathcal{Q}(\frac{i-1}{n(k)-1}),\ i = 1, 2, \dots, n(k) - 1.$$

For the inconsistent DM $e_n^{(k)}$, compute the trust weights $\beta_{\sigma(i)}^{(k,t)}$. Then, the collective evaluation matrix needs to be constructed to generate recommendations for the inconsistent DM, where a regularly-increasing monotone quantifier of the form $\mathcal{Q}(p) = p^{\alpha}$ can be used, with $\alpha \in [0, 1]$ being the attitudinal parameter to conduct the attitude of a DM.

Suppose that $\widetilde{\mathcal{R}}_n^{(k,t)}$ is the evaluation of the inconsistent DM $e_n^{(k)}$, and $\widetilde{\mathcal{R}}_l^{(k,t)}$ with $l = 1, 2, \dots, n(k) - 1$, and $l \neq h$, are the evaluations of other DMs in the group. Then, a collective decision matrix $\overline{\mathcal{R}}_{r,n}^{(k,t)} = (\overline{r}_{ij}^{(k,t)})$ can be constructed based on the attitudinal-based trust degrees where:

$$\overline{r}_{ij}^{(k,t)} = \mathcal{B}\left(\beta_{\sigma(1)}^{(k,t)}\widetilde{r}_{ij,\sigma(1)}^{(k,t)}, \ldots, \beta_{\sigma_{r,1}^{(k,t)}}^{(k,t)}\widetilde{r}_{ij,\sigma(q(k)-1)}^{(k,t)}\right),$$

with $\mathcal{B}$ being the GNZWA operator.

Example 5. *(Example 1 continuation) Suppose that we aim at constructing the collective decision matrix for the inconsistent DM in $EXPCH_1^{(2,1)} = \{e_4^{(2)}\}$. There are four DMs in the second group ($n(2) = 4$), and, therefore, there exist three ($(n(2)-1)$) attitudinal-based trust degrees for this DM. It can be concluded that $\sigma(1) = 2$, $\sigma(2) = 3$, and $\sigma(3) = 1$. By choosing $\alpha = 4/6$, the attitudinal trust degrees for the inconsistent DM are $\beta_2^{(2,1)} = 0.4807$, $\beta_3^{(2,1)} = 0.2824$, and $\beta_1^{(2,1)} = 0.2369$. Then, a collective evaluation can be constructed based on the GNZWA operator as follows:*

$$\bar{\mathcal{R}}_{1,4}^{(2,1)} = \begin{bmatrix} 0.7468 & 0.2505 & 0.9036 & 0.2774 & 0.0231 \\ 0.7532 & 0.2526 & 0.6600 & 0.2266 & 0.0369 \\ 0.7863 & 0.2633 & 0.5600 & 0.2022 & 0.0439 \\ 0.5395 & 0.2082 & 0.7790 & 0.2663 & 0.0223 \end{bmatrix}.$$

3. The inconsistent DM $e_n^{(k)}$ is then suggested to modify his evaluations $\widetilde{r}_{ij}^{(k,t)}$ with $i = 1, 2, \ldots, q(k)$ and $j = 1, 2, \ldots, 5$ as in the following:

$$\begin{aligned} r\widetilde{r}_{ij}^{(k,t)} = \Big(&\Big([1-\delta]\widetilde{\mu}_{ij}^{(k,t)} + \delta\overline{\mu}_{ij}^{(k,t)}, \sqrt{[1-\delta](\widetilde{\sigma}_{ij}^{(k,t)})^2 + \delta(\overline{\sigma}_{ij}^{(k,t)})^2}\Big), \\ &\Big(\frac{[1-\delta]\widetilde{\mu}_{ij}^{(k,t)}\widetilde{\vartheta}_{ij}^{(k,t)} + \delta\overline{\vartheta}_{ij}^{(k,t)}}{[1-\delta]\widetilde{\mu}_{ij}^{(k,t)} + \delta\overline{\mu}_{ij}^{(k,t)}}, \sqrt{\frac{[1-\delta]\widetilde{\mu}_{ij}^{(k,t)}(\widetilde{\varpi}_{ij}^{(k,t)})^2 + \delta\overline{\mu}_{ij}^{(k,t)}(\overline{\varpi}_{ij}^{(k,t)})^2}{[1-\delta]\widetilde{\mu}_{ij}^{(k,t)} + \delta\overline{\mu}_{ij}^{(k,t)}}}, \\ &\sqrt{\frac{[1-\delta]\widetilde{\mu}_{ij}^{(k,t)}(\widetilde{\varrho}_{ij}^{(k,t)})^2 + \delta\overline{\mu}_{ij}^{(k,t)}(\overline{\varrho}_{ij}^{(k,t)})^2}{[1-\delta]\widetilde{\mu}_{ij}^{(k,t)} + \delta\overline{\mu}_{ij}^{(k,t)}}}\Big)\Big), \end{aligned} \tag{4.9}$$

where $\delta \in [0,1]$ is the feedback parameter.

Table 4.5 – The evolution of $\mathcal{ACD}$ for each DM in each group.

	time-step $t=1$									
DM	Group 1					Group 2				
	r=1	r=2	r=3	r=4	r=5	r=1	r=2	r=3	r=4	r=5
1st	0.8262	0.8368	0.8918	0.9279	0.9519	0.8688	0.8772	0.8839	0.8912	0.9231
2nd	0.7607	0.8216	0.8798	0.9192	0.9457	0.8722	0.8816	0.8891	0.9028	0.9165
3rd	0.7912	0.8440	0.8955	0.9301	0.9532	0.8651	0.8714	0.8768	0.9035	0.9156
4th	0.8097	0.8592	0.9074	0.9387	0.9592	0.8217	0.8459	0.8655	0.8935	0.9152
	Group 3					Group 4				
1st	0.7986	0.8487	0.8962	0.9288	0.9492	0.8019	0.8458	0.8992	0.9331	0.9331
2nd	0.8515	0.8663	0.9050	0.9321	0.9512	0.7762	0.8479	0.9009	0.9344	0.9344
3rd	0.8831	0.9003	0.9245	0.9419	0.9528	0.7534	0.8273	0.8849	0.9232	0.9232
4th	0.8525	0.8708	0.9077	0.9342	0.9530	0.7596	0.8318	0.8888	0.9262	0.9262

Example 6. *(Example 1 continuation) For the inconsistent DM $e_4^{(2)}$, the modified evaluation is as follows with $\delta = 0.33$:*

$$\mathcal{R}\widetilde{\mathcal{R}}_{1,4}^{(2,1)} = \begin{bmatrix} 0.6931 & 0.2319 & 0.9657 & 0.2899 & 0.0169 \\ 0.8069 & 0.2697 & 0.7424 & 0.2547 & 0.0277 \\ 0.8178 & 0.2731 & 0.5190 & 0.1958 & 0.0459 \\ 0.6247 & 0.2177 & 0.7790 & 0.2663 & 0.0223 \end{bmatrix}.$$

Inconsistent DMs are recommended to modify their evaluations according to the above-mentioned trust-based mechanism. The moderator decides about the number of discussion rounds r. Suppose that the moderator suggests five discussion rounds, i.e., $r = 5$. The evolution of the $\mathcal{ACD}$ for each DM is collected in Table 4.5. Furthermore, the evolution of the consensus threshold for each group of DMs is represented in Table 4.6.

4.3.4 Fusion Model and the Selection Process

Once the CRP finishes by reaching to the maximum number of discussion rounds, the selection process gets started that makes use of a fusion model. This process contains

Table 4.6 – The evolution of the consensus threshold for each group.

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$
$\gamma_r^{(1,1)}$	0.8135	0.8765	0.9179	0.9452	0.9634
$\gamma_r^{(2,1)}$	0.8344	0.8628	0.8813	0.9008	0.9140
$\gamma_r^{(3,1)}$	0.8326	0.8841	0.9189	0.9412	0.9525
$\gamma_r^{(4,1)}$	0.7969	0.8538	0.9080	0.9209	0.9209

two main steps. In the first step, the importance of attributes is quantified by a set of optimal weights that are constructed in the fusion model. In the second step, the ratio systems are constructed based on the GNZPWA operator. A general rule is used for weight-assigning in the fusion model. That is, an attribute with a low support degree from other attributes should be given a low weight. Based on this concept, and by resorting to the maximum-deviation procedure, an optimization problem is proposed so as to determine the weight of each attribute.

1. Suppose that $\overline{\mathcal{R}}_c^{(k,t)} = [\overline{r}_i^{(k,t)}]_{q(k,t)\times 5}$ is the final collective evaluation matrix of the kth group, which has been obtained based on the GNZWA operator, and $w^{(k,t)}$ is the weight assigned to the kth attribute. Then, the solution of the following optimization problem leads to an appropriate weight for each attribute:

$$\max \sum_{k=1}^{m} w^{(k,t)} \sum_{j=1, j\neq k}^{m} w^{(j,t)} \sum_{i=1}^{q(t)} SD\left(\overline{r}_i^{(k,t)}, \overline{r}_i^{(j,t)}\right)$$
$$s.t. \ \sum_{k=1}^{m} w^{(k,t)} = 1; \ \ w^{(k,t)} \geq 0, \tag{4.10}$$

where $SD(.,.)$ is the support degree function defined in Definition 20.

Example 7. *(Example 1 continuation) By solving the optimization problem (4.10), the weight of each attribute at time-step* $t=1$ *can be computed as* $w^{(1,1)} = 0.2334$, $w^{(2,1)} = 0.2126$, $w^{(3,1)} = 0.279$, *and* $w^{(4,1)} = 0.275$.

2. Let $\overline{\mathcal{R}}_f^{(k,t)} = [\overline{r}_i^{(k,t)}]_{q(t)\times 5}$ and $w^{(k,t)}$ be as obtained in the previous step. Then, the ratio systems can be constructed as follows by integrating the collective evaluations in the fusion model:

$$\mathcal{RS}_i^t = \mathcal{H}\left(w^{(1,t)}\overline{r}_i^{(1,t)}, \ldots, w^{(m,t)}\overline{r}_i^{(m,t)}\right), \tag{4.11}$$

where $\mathcal{H}$ is the GNZPWA operator. In order to select the best alternative, the PIS and the NIS can be determined based on the constructed ratio systems as follows:

$$\begin{aligned}
p_+^t &= \Big(\max_i(\overline{\mu}_{\mathcal{RS}_i^t}), \min_i(\overline{\sigma}_{\mathcal{RS}_i^t}), \max_i(\overline{\vartheta}_{\mathcal{RS}_i^t}), \min_i(\overline{\varpi}_{\mathcal{RS}_i^t}), \min_i(\overline{\varrho}_{\mathcal{RS}_i^t})\Big),\\
n_-^t &= \Big(\min_i(\overline{\mu}_{\mathcal{RS}_i^t}), \max_i(\overline{\sigma}_{\mathcal{RS}_i^t})), \min_i(\overline{\vartheta}_{\mathcal{RS}_i^t}), \max_i(\overline{\varpi}_{\mathcal{RS}_i^t}), \max_i(\overline{\varrho}_{\mathcal{RS}_i^t})\Big).
\end{aligned}$$

Finally, the closeness coefficient of $\mathcal{RS}_i^t$ can be calculated as:

$$d_{\mathcal{RS}_i^t}^t = \frac{\mathrm{dist}_z\left(\mathcal{RS}_i^t, n_-^t\right)}{\mathrm{dist}_z\left(\mathcal{RS}_i^t, n_-^t\right) + \mathrm{dist}_z\left(\mathcal{RS}_i^t, p_+^t\right)}. \tag{4.12}$$

The closeness coefficient is then used to rank alternatives; the larger the value of $d_{\mathcal{RS}_i^t}^t$, the better the alternative x_i^t.

Example 8. *(Example 1 continuation) By applying the GNZPWA operator to $\overline{\mathcal{R}}_f^{(k,t)}$ w.r.t. the attribute weights $w^{(k,t)}$, the ratio systems can be constructed as in the following:*

$$\mathcal{RS}^t = \begin{bmatrix} 0.7288 & 0.2465 & 0.7436 & 0.2528 & 0.0314 \\ 0.6355 & 0.2341 & 0.6060 & 0.2206 & 0.0391 \\ 0.7485 & 0.2528 & 0.6852 & 0.2367 & 0.0351 \\ 0.7053 & 0.2421 & 0.7785 & 0.2631 & 0.0260 \end{bmatrix}.$$

Then, following the given formula in (4.12), the closeness coefficients can be computed as $d_{\mathcal{RS}^t}^t = \{0.4845, 0.4326, 0.5220, 0.4410\}$. Then, the ordered set of alternatives is $\mathcal{X}_{\mathcal{O}}^t = \{x_3^t, x_1^t, x_4^t, x_2^t\}$, i.e., $x_3^t > x_1^t > x_4^t > x_2^t$.

4.4 Comparative Analysis and Discussion

In this section, the sensitivity of the proposed MAGDM framework to the design parameters is discussed in detail. Its practical verification is then investigated through introducing the application of MAGDM in locating faults in power distribution systems, where the attained results are presented in Chapter 6.

Parameter λ is used in the GNZPWA operator to model the thinking mode of DMs. A larger value of λ denotes that DMs are more optimistic, while lower values denote that DMs are more pessimistic. The sensitivity of the proposed method to λ is shown in Fig. 4.3. As it can be observed, smaller values speed up the CRP and lead to higher $\mathcal{ACD}$. The graphs shown in Fig. 4.3 represent the average $\mathcal{ACD}$ of 16 DMs used in the illustrative example. The attained results denote that being more pessimistic helps with speeding up the CRP, while optimistic thinking modes lower the speed of the CRP and lead to lower consensus among DMs.

Parameter a in $\mathcal{F}_2(s_i)$ and parameters κ and ς in $\mathcal{F}_3(s_i)$ play an important role in the extraction of NZs from Z-numbers. The evolution of $\mathcal{ACD}$ for each group of DMs w.r.t. to a is represented in Table 4.7. As it can be observed, the larger values have just slightly increased the $\mathcal{ACD}$, denoting that the sensitivity to this parameter is not significant. On the other hand, nine combinations of κ and ς are selected and the evolution of $\mathcal{ACD}$ is represented in Table 4.8. The attained results show that the selection of lower values for both parameters speeds up the CRP.

In extraction of NZs from Z-numbers, two LSFs are required, where the first one $f(s_i)$ extracts information from $s_i \in \mathcal{S}$ to construct the first part of NZs, while the second LSF $g(s_i')$ is applied to $s_i' \in \mathcal{S}'$ to construct the second part of NZs. The combinations of LSFs and the evolution of $\mathcal{ACD}$ w.r.t. each combination is represented in Table 4.9. The attained results denote that the best combination is

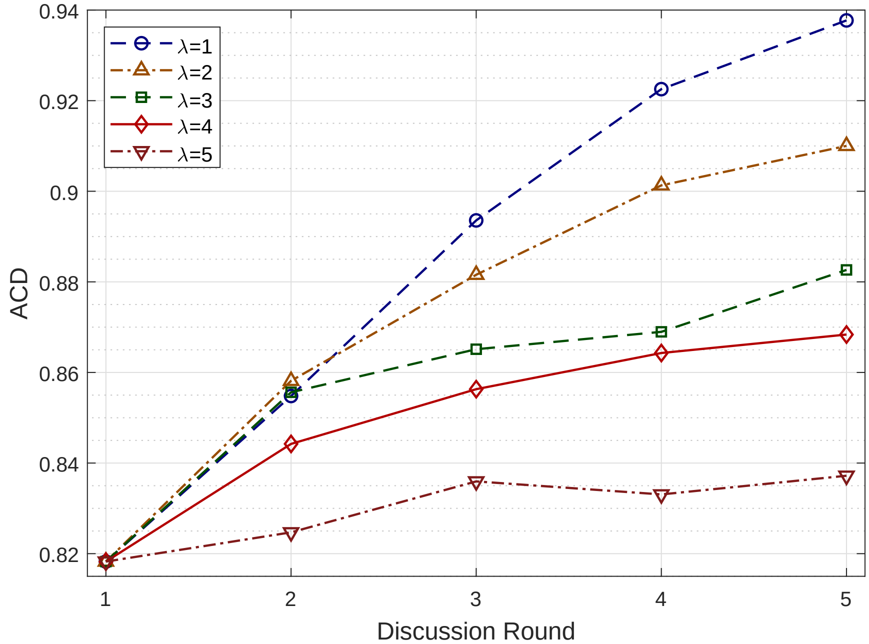

Figure 4.3 – The variation of $\mathcal{ACD}$ w.r.t. the changes of λ.

Table 4.7 – The evolution of $\mathcal{ACD}$ w.r.t. the changes of 'a'.

	Group 1					Group 2				
a	r=1	r=2	r=3	r=4	r=5	r=1	r=2	r=3	r=4	r=5
1.36	0.7968	0.8403	0.8935	0.9289	0.9525	0.8568	0.8689	0.8787	0.8977	0.9175
1.37	0.7969	0.8404	0.8936	0.9290	0.9525	0.8569	0.8690	0.8788	0.8978	0.9176
1.38	0.7971	0.8406	0.8937	0.9290	0.9526	0.8571	0.8692	0.8790	0.8978	0.9177
1.39	0.7972	0.7407	0.8938	0.9291	0.9526	0.8572	0.8693	0.8791	0.8979	0.9178
1.40	0.7974	0.8409	0.8939	0.9292	0.9527	0.8574	0.8695	0.8793	0.8980	0.9179
a	Group 3					Group 4				
1.36	0.8443	0.8697	0.9071	0.9335	0.9511	0.7716	0.8374	0.8929	0.9288	0.9288
1.37	0.8450	0.8703	0.9075	0.9338	0.9512	0.7728	0.8382	0.8935	0.9292	0.9292
1.38	0.8457	0.8709	0.9079	0.9340	0.9514	0.7739	0.8391	0.8940	0.9296	0.9296
1.39	0.8464	0.8715	0.9084	0.9343	0.9516	0.7751	0.8399	0.8946	0.9300	0.9300
1.40	0.8471	0.8716	0.9085	0.9344	0.9517	0.7763	0.8407	0.8951	0.9303	0.9303

Table 4.8 – The evolution of $\mathcal{ACD}$ w.r.t. to κ and ς.

κ	ς	r=1	r=2	r=3	r=4	r=5
0.1	0.5	0.7466	0.7833	0.8183	0.8590	0.8722
0.9	0.5	0.4865	0.6006	0.6944	0.7407	0.7724
0.5	0.1	0.5685	0.6656	0.7312	0.7596	0.7949
0.5	0.9	0.4710	0.5816	0.6709	0.7274	0.7875
0.5	0.5	0.5189	0.6243	0.6882	0.7432	0.7734
0.9	0.9	0.4386	0.5727	0.6557	0.7084	0.7448
0.1	0.1	0.7957	0.8346	0.8566	0.8851	0.8984
0.1	0.9	0.6958	0.7353	0.7820	0.8247	0.8367
0.9	0.1	0.5356	0.6391	0.7101	0.7460	0.7871

Table 4.9 – The evolution of $\mathcal{ACD}$ w.r.t. to linguistic scale function combinations.

$f(s_i)$	$g(s'_i)$	r=1	r=2	r=3	r=4	r=5
$\mathcal{F}_1(s_i)$	$\mathcal{F}_2(s'_i)$	0.8188	0.8552	0.8939	0.9228	0.9380
$\mathcal{F}_1(s_i)$	$\mathcal{F}_3(s'_i)$	0.7957	0.8346	0.8566	0.8851	0.8984
$\mathcal{F}_2(s_i)$	$\mathcal{F}_1(s'_i)$	0.8172	0.8450	0.8688	0.8864	0.9028
$\mathcal{F}_3(s_i)$	$\mathcal{F}_1(s'_i)$	0.8166	0.8445	0.8682	0.8863	0.9025
$\mathcal{F}_2(s_i)$	$\mathcal{F}_3(s'_i)$	0.8009	0.8388	0.8603	0.8832	0.8961
$\mathcal{F}_3(s_i)$	$\mathcal{F}_2(s'_i)$	0.8006	0.8386	0.8602	0.8831	0.8960

$f(s_i) = \mathcal{F}_1(s_i)$ and $g(s'_i) = \mathcal{F}_2(s'_i)$, however, no significant changes occur from one combination to another that denotes the robustness of the proposed scheme in coping with different LSFs.

Next, the effect of recommendation policies w.r.t. α is discussed. $\alpha = 0$ leads to the case, in which the inconsistent DM trusts only one peer with the closest evaluations, while $\alpha = 1$ denotes an indifferent policy, in which the inconsistent DM trusts all peers equally. Other values than zero and one, e.g., $\alpha = \frac{4}{6}$, gives a higher weight to peers with closer evaluations. The attained results in terms of the $\mathcal{ACD}$ for five discussion rounds are collected in Table 4.10 and the average values for all groups are represented in Table 4.11. The attained results in Table 4.11 denote the superiority of the recommended policy in this work, i.e., the policy with $\alpha = \frac{4}{6}$, for the sake of speeding up the CRP that consequently leads to higher $\mathcal{ACD}$ values.

Table 4.10 – Comparison of different recommendation policies.

	Group 1					Group 2				
α	r=1	r=2	r=3	r=4	r=5	r=1	r=2	r=3	r=4	r=5
0	0.7968	0.8114	0.8245	0.8423	0.8582	0.8568	0.8610	0.8650	0.8688	0.8782
$\frac{4}{6}$	0.7968	0.8099	0.8290	0.8461	0.8616	0.8568	0.8603	0.8637	0.8669	0.8669
1	0.7968	0.8095	0.8286	0.8458	0.8613	0.8568	0.8599	0.8629	0.8657	0.8684
α	Group 3					Group 4				
0	0.8471	0.8534	0.8593	0.8720	0.8836	0.7716	0.7946	0.8011	0.8191	0.8333
$\frac{4}{6}$	0.8471	0.8543	0.8610	0.8704	0.8815	0.7716	0.7925	0.8104	0.8302	0.8480
1	0.8471	0.8547	0.8616	0.8710	0.8817	0.7716	0.7914	0.8092	0.8246	0.8382

Table 4.11 – The average values of $\mathcal{ACD}$ for all groups w.r.t. different values of α.

α	r=1	r=2	r=3	r=4	r=5
0	0.8181	0.8301	0.8375	0.8506	0.8633
$\frac{4}{6}$	0.8181	0.8292	0.8410	0.8534	0.8652
1	0.8181	0.8289	0.8406	0.8518	0.8624

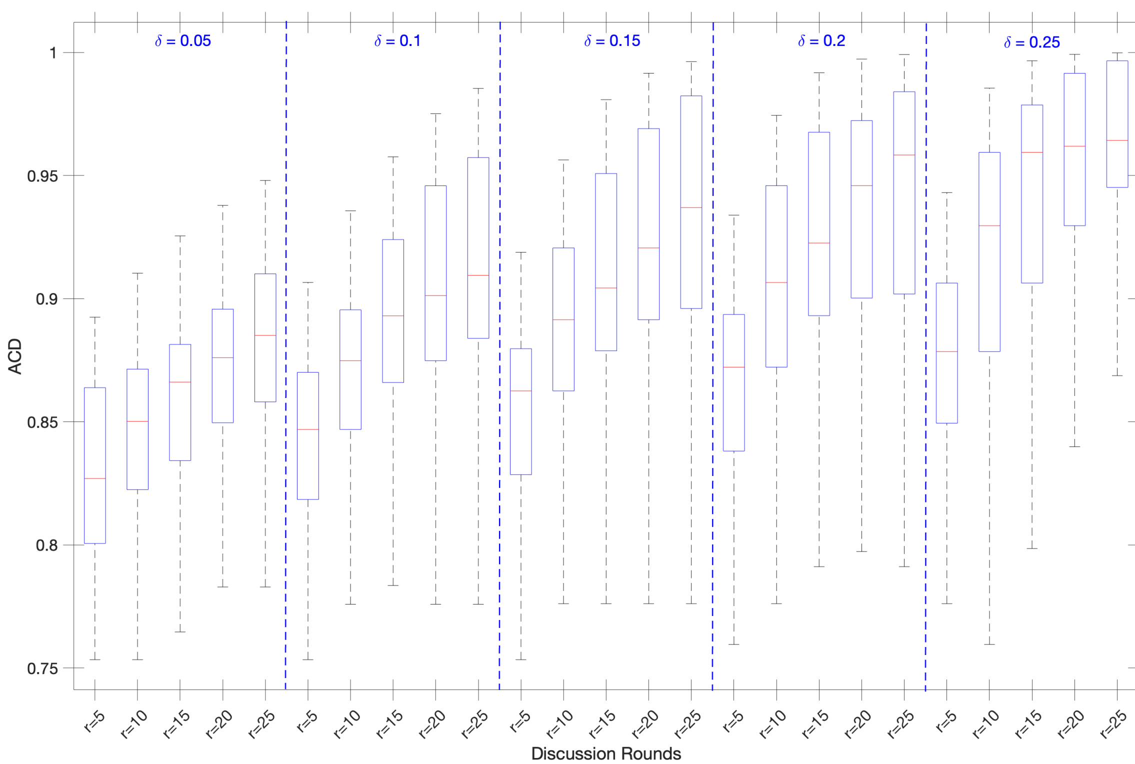

Figure 4.4 – The variation of ACD w.r.t. the changes of feedback parameter δ and the number of discussion rounds r

.

In the next experiment, the effect of the feedback parameter δ and discussion rounds r on the proposed framework is investigated. The general rule is that a larger value of δ increases the $\mathcal{ACD}$, however, imposes a higher cost to a DM to modify the initial evaluations. Furthermore, a successful CRP increases the $\mathcal{ACD}$ when more discussion rounds are assigned. Variation of the $\mathcal{ACD}$ w.r.t. $\delta = \{0.05, 0.1, 0.15, 0.2, 0.25\}$ and $r = \{5, 10, 15\}$ is given in Fig. 4.4. As it can be observed, for a specific δ, the $\mathcal{ACD}$ increases when the number of discussion rounds is increased. Furthermore, regardless of the number of discussion rounds, a larger value of δ increases the $\mathcal{ACD}$. This experiment verifies the validness of the proposed dynamic decision-making framework.

4.5 Complexity Analysis

This section discusses the time complexity of the proposed dynamic GDM model. The proposed model relies on three algorithms discussed before. For the first algorithm, which is used to find the consensus threshold value, suppose that the number of iterations that it takes for having $\mathcal{CC} \neq 0$ is $\aleph_1$, and the number of inconsistent DMs in each group is at most $\aleph_2$. Then, the complexity of this algorithm is $\mathcal{O}\left(\aleph_1 \times (\zeta_1 - \zeta_2) \times n \log n\right)$. The second algorithm is concerned with the recommendation mechanism. The time complexity of this algorithm is $\mathcal{O}\left(\aleph_2 \times q\right)$. The third algorithm deals with the fusion model, where the complexity of this algorithm is $\mathcal{O}\left(m \times q^2\right)$. Therefore, the time complexity of this algorithm will be in total $\mathcal{O}\left(\aleph_1 \times (\zeta_1 - \zeta_2) \times n \log n + \aleph_2 \times q + m \times q^2\right)$. Due to the fact that in dynamic GDM, the set of alternatives and DMs take high values, therefore, the time complexity could be summarized as $\mathcal{O}\left(m \times q^2\right)$, because $\aleph_1$ and $\aleph_2$ could be ignored compared with n and q.

Chapter 5

Blockchain-Enabled Consensus Management in Linguistic Opinion Dynamics

The study of opinion dynamics aims at understanding how opinions evolve over time among a group of interacting agents. The arithmetic mean of agents' opinions in a previous time-step is used to determine how agents' opinions change over time. It has been decades since ODMs were developed utilizing time as the main element of dynamism [219]. The developed models can generally be categorized into discrete-in-opinion and continuous-in-opinion models. One can refer to the Voter [45] and Sznajd [46] models as well-known discrete ODMs, where models such as DeGroot model [47], and BC models including Hegselmann and Krause (HK) model [48] and Weisbuch and others (DW) model [220, 50] are of well-known continuous ODMs. The classic models have been extensively studied, with a variety of variants being proposed in recent years to improve their fusion process. These models can arguably be divided into four categories including models that consider agents' behaviour, social network-based models, models based on the minimum adjustment notion, and linguistic models, where we provided a very comprehensive review on such models in Chapter 2.

Even though the developed ODMs have shown encouraging results, however, there still exist several challenges that need to be addressed more efficiently.

1. It is a new research direction in ODMs to express opinions using linguistic representation structures, and preliminary results are encouraging. There is,

however, room for improvement in the development of LODMs based on a more generalized opinion representation structure such as Z-numbers.

2. Even though notable efforts have been made to the design of minimum cost consensus models for opinion dynamics, however, the willingness of agents to accept or refuse the suggested modifications is missing. In opinion dynamics, the willingness is usually addressed through the BC models, yet they conduct bias in evolution of agents' opinions.

3. The agents' willingness is typically characterized by BC notion in ODMs. Such models rely on the opinion similarity to build trust, meaning that only agents with similar opinions trust each other and an agent's opinion is formed by means of the trusted peers. Such models can influence agents' interactions in a biased manner, and agents' opinions might be influenced by within group factors (e.g., peer pressure or group pressure).

Following the aforementioned challenges, a general framework that guides agents toward a consensus opinion by considering their willingness is the subject of this chapter. This chapter is concerned with the design of an LODM, where agents express their opinions in terms of Z-numbers. In order to remove within-group factors (e.g., group and peer pressure) that might impact agents' opinions, and to model agents' interactions without concerning the opinion similarity, it is proposed to construct a safe and efficient communication regime using the Blockchain technology. Within this regime, an agent's identity and opinion are not disclosed for other peers, yet it proposes to build trust among agents by just enabling them to see how many of their peers have accepted the suggested modifications by the moderator. To this end, this chapter contributes to the design of an ODM that accounts for the aforementioned challenges as summarized below [221]:

1. Construction of an LODM based on Z-numbers;

2. Considering agents' willingness to accept or refuse the suggested modifications;
3. Constructing a Blockchain-enabled trust-building mechanism to model agents' interactions.

5.1 Preliminaries

This section gives an introduction to the concepts of Z-numbers and their transformation into numerical values.

5.1.1 Z-numbers

Z-numbers consist of two components to describe an uncertain variable to address the reliability issue of information.

Definition 31 ([52]). *A Z-number, denoted by $\mathcal{Z} = (\mathcal{A}, \mathcal{B})$, is made up of an ordered pair of fuzzy numbers, where the first component, $\mathcal{A}$, is a restriction on the values that an uncertain variable $\mathcal{X}$ can take, and the second component, $\mathcal{B}$, is a measure of reliability of the first component.*

Components of Z-numbers are typically expressed in natural language, e.g., $\mathcal{Z} =$ ('very good', 'certain'), indicating that Z-numbers can be realized through LTSs, which are represented by ordinal linguistic scales.

Definition 32 ([222]). *$\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$ is a finite and completely ordered LTS with odd cardinality, where r is a nonnegative integer value. s_i denotes a linguistic variable and for two arbitrary linguistic variables s_i and s_j, the following two conditions hold: (1) $\mathcal{S}$ is an ordered set, therefore, $s_i \leq s_j$ iff $i \leq j$; (2) There is a negation operator for which $neg(s_i) = s_j$ if $i + j = 2r$.*

5.1.2 Transformation of Z-numbers

A fuzzy set $\mathcal{A}$ defined on universe $\mathcal{X}$ can be represented by $\mathcal{A} = \{< x, \mu_{\mathcal{A}}(x) > | x \in \mathcal{X}\}$, where $\mu_{\mathcal{A}} : \mathcal{X} \rightarrow [0, 1]$ is an MF that quantifies the degree for which $x \in \mathcal{X}$ belongs to $\mathcal{A}$. For the fuzzy set $\mathcal{A}$, its fuzzy expectation can be defined as follows.

Definition 33 ([223])**.** *For a fuzzy set $\mathcal{A} = \{< x, \mu_{\mathcal{A}}(x) > | x \in \mathcal{X}\}$, its fuzzy expectation can be defined by:*

$$\mathcal{E}_{\mathcal{A}}(x) = \int_{\mathcal{X}} x \mu_{\mathcal{A}}(x) dx, \tag{5.1}$$

which is a measure of information strength supporting the fuzzy set $\mathcal{A}$. Given the above definition, transformation of a Z-number into a crisp value can be perfected in four steps.

For a given Z-number $\mathcal{Z} = (\mathcal{A}, \mathcal{B})$, suppose that $\mathcal{A}$ and $\mathcal{B}$ take linguistic terms from an LTS, where the semantics of $\mathcal{A}$ and $\mathcal{B}$ are given in terms of trapezoidal and triangular MFs. Given $\mathcal{A} = \{< x, \mu_{\mathcal{A}}(x) > | x \in [0, 1]\}$ and $\mathcal{B} = \{< x, \mu_{\mathcal{B}}(x) > | x \in [0, 1]\}$, the first step is to convert $\mathcal{B}$ into a crisp value. This crisp value can be calculated by means of the centroid deffuzification as given below:

$$\alpha = \frac{\int_{\mathcal{X}} x \mu_{\mathcal{B}}(x) dx}{\int_{\mathcal{X}} \mu_{\mathcal{B}}(x) dx}. \tag{5.2}$$

Having α calculated, the next step is to transfer the weight of $\mathcal{B}$ to the first part, through which the weighted $\mathcal{A}^{\alpha}$ can be resulted as $\mathcal{A}^{\alpha} = \{< x, \mu_{\mathcal{A}^{\alpha}}(x) > | \mu_{\mathcal{A}^{\alpha}}(x) = \alpha \mu_{\mathcal{A}}(x) | x \in [0, 1]\}$. It is worth mentioning that the fuzzy expected value of $\mathcal{A}^{\alpha}$ through this process becomes α times the expected value of $\mathcal{A}$. This can be shown as given below:

$$\mathcal{E}_{\mathcal{A}^{\alpha}}(x) = \int_{\mathcal{X}} x \mu_{\mathcal{A}^{\alpha}}(x) dx = \alpha \int_{\mathcal{X}} x \mu_{\mathcal{A}}(x) dx = \alpha \mathcal{E}_{\mathcal{A}}(x).$$

The weighted $\mathcal{A}^\alpha$ is therefore an irregular fuzzy number. The next step is then to convert this irregular fuzzy number into a regular one. The regularized fuzzy number can be represented by $\tilde{\mathcal{A}} = \{\langle x, \mu_{\tilde{\mathcal{A}}}(x)\rangle | \mu_{\tilde{\mathcal{A}}}(x) = \mu_{\mathcal{A}}(\frac{x}{\sqrt{\alpha}}), x \in [0,1]\}$, and it can be shown that the regularized fuzzy number is of the same fuzzy expectation value as that of $\mathcal{A}^\alpha$:

$$\mathcal{E}_{\tilde{\mathcal{A}}}(x) = \int_{\sqrt{\alpha}\mathcal{X}} x\mu_{\mathcal{A}}(\frac{x}{\sqrt{\alpha}})dx = \alpha \int_{\mathcal{X}} t\mu_{\mathcal{A}}(t)dt = \alpha\mathcal{E}_{\mathcal{A}}(x),$$

where the last integral is solved by the following change of variable $x = \sqrt{\alpha}t$.

Thus far, the conversion of a Z-number into a regularized fuzzy number has been discussed. The last step of the proposed conversion technique is to make use of canonical characteristic values (CCVs) of linguistic variables in order to assign a crisp value to a fuzzy number. Following the formal definition of CCVs given in [224], the authors defined three functions including expected value (EV), centre of gravity (CoG), and mean of maxima (MeOM), for which the following combination could also be referred to as a CCV function:

$$\phi(s) = \vartheta_1 \mathrm{EV}(s) + \vartheta_2 \mathrm{CoG}(s) + \vartheta_3 \mathrm{MeOM}(s), \tag{5.3}$$

where s is a linguistic term and $\vartheta_i \in [0,1]$ with $\sum_{i=1}^{3} \vartheta_i = 1$. Assuming the semantic of s being modeled by a trapezoidal MF, $\mathcal{F}[a,b,c,d]$, EV, CoG, and MeOM could then be calculated:

$$\mathrm{EV}(\mathcal{F}) = \frac{a+b+c+d}{4}, \tag{5.4}$$

$$\mathrm{CoG}(\mathcal{F}) = \begin{cases} a, & \text{if } a=b=c=d, \\ \frac{c^2+d^2-a^2-b^2+cd-ab}{3(c+d-a-b)}, & \text{otherwise,} \end{cases} \tag{5.5}$$

$$\mathrm{MeOM}(\mathcal{F}) = \frac{a+c}{2}. \tag{5.6}$$

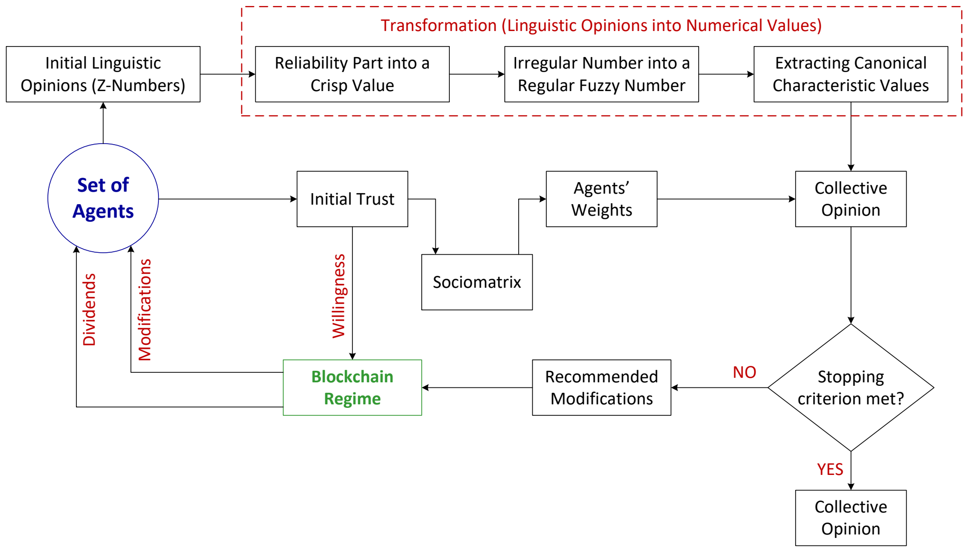

Figure 5.1 – The general framework of the proposed ODM.

Therefore, the combined CCV function given in Eq. (5.3) could result in a crisp value from a Z-number.

5.2 The Proposed Opinion Dynamics Model

This section discusses the major components of the proposed LODM and its implementation procedure. This proposed model is developed to cope with situations, in which agents might not be interested in updating their initial opinions due to either lack of trust to the group, or internal, or external reasons. Therefore, the aim is to build trust among agents through Blockchain protocols to guide the group of agents toward a consensus opinion.

Framework of the proposed ODM is illustrated in Fig. 5.1. The general idea is that agents conveniently express their initial opinions in terms of Z-numbers. Linguistic opinions are then converted into crisp values through the transformation block. This block generates the translated opinions that are fed into the collective opinion block, which is constructed based upon a minimum cost consensus strategy to extract the collective opinion from the initial ones. The moderator sends back to each agent the required change of opinion in the event an stopping criterion (will be discussed shortly) is not met. Agents make up their minds to accept or refuse the suggested modifications based on their trust to the group, internal reasons, or external reasons for refusal. Agents, however, anonymously express their decisions following a Blockchain protocol. This process continues until the group reaches to either the full consensus or the number of discussion rounds exceeds a threshold (stopping criteria).

5.2.1 Initialization

Consider a group of n agents a_i, with $i = 1, 2, \dots, n$. Linguistic opinion of agent a_i is expressed in terms of a Z-number $\mathcal{Z}_i = (s_i, s'_i)$, where $s_i \in \mathcal{S} = \{s_0, \dots, s_{2r}\}$ and $s' \in \mathcal{S}' = \{s'_0, \dots, s'_{2r}\}$. $\mathcal{S}$ and $\mathcal{S}'$ are two LTSs, from which agents initially select the restriction and reliability values. Without loss of generality, we assume that semantics of linguistic terms in $\mathcal{S}$ and $\mathcal{S}'$ are characterized by means of trapezoidal $\mathcal{F}[a_k, b_k, c_k, d_k]$ and triangular $\mathcal{T}[a'_k, b'_k, c'_k]$ fuzzy MFs, respectively, where $k = 0, 1, \dots, 2r$.

Initial opinions, i.e., $\mathcal{Z}_i = (s_i, s'_i)$, $i = 1, \dots, n$, go through the transformation block. Within this block, following the presented results in Section 5.1, the reliability part of initial opinions, i.e., s'_i, is defuzzified by means of centroid calculation for the semantic of s'_i that is characterized by triangular MF $\mathcal{T}_i$. The resulted defuzzified value is shown by α_i. The weight of reliability part, i.e., α_i, is then added to the restriction part to construct an irregular fuzzy number $\mathcal{Z}_i^{\alpha}$, which is then converted into a regularized one $\tilde{\mathcal{Z}}_i$ according to the method presented in Section 5.1.2. The regularized fuzzy number is characterized by means of a trapezoidal MF $\mathcal{F}_i$, where according to Eqs. (5.4)-(5.6), it is finally converted into a numerical opinion. This numerical opinion of agent a_i is denoted by o_i, $i = 1, \dots, n$. Furthermore, it is assumed that the level of initial trust among agents is iT with $iT \in [0\%, 100\%]$. The level of trust iT is a key parameter in our design and it serves two purposes.

First, the initial trust of each agent to the group is assumed to be selected from a normal distribution with the mean of iT and variance of $\sigma\%$, i.e., $T_i(1) \in \mathcal{N}(iT, \sigma)$, where $T_i(1)$ is the initial trust of agent a_i at time-step $t = 1$. Second, we construct a sociomatrix $\mathcal{A} = [a_{i,j}]_{n\times n}$, where $a_{i,j} = 1$ denotes that agents a_i and a_j are initially trusted peers, while $a_{i,j} = 0$ denotes that agents are not initially connected. In the sociomatrix $\mathcal{A}$, there exist $n \times (n-1)/2$ elements that can either take a value of 0 or 1. To construct this matrix, it is supposed that $iT\%$ of elements are randomly set to 1, and the rest of the elements are 0. Following this assumption, the weight of agent a_i is:

$$w_i = \frac{\sum_{j=1, j\neq i}^{n} a_{i,j}}{n-1}. \tag{5.7}$$

It is worth noting that for the case with $iT = 0\%$, meaning that there is no initial connections among agents, the importance weights of agents are equal and are set to $w_i = 1/n$, $i = 1, 2, \dots, n$.

Therefore, in the initialization phase, linguistic opinions of agents $\mathcal{Z}_i$ are converted into numerical opinions o_i and importance weights of agents w_i are determined.

5.2.2 Construction of the Collective Opinion

Given the translated initial opinions o_i, with $i = 1, \dots, n$, it is assumed that moderator could provide agents with a satisfactory opinion o' that relatively meets agents' preferences. To extract o' from o_i, a minimum cost consensus model is proposed. Let $f_i(o') = |o' - o_i|$. Further to this, assume ϖ_i is the unit cost paid by the moderator to convince agent a_i to accept the suggested modification. Following this, it is evident that $\varpi_i f_i(o')$ is the total cost to be paid to the agent a_i and $\sum_{i=1}^{n} \varpi_i f_i(o')$ is the total cost to be paid to all agents. Obviously, the smaller the value of $\sum_{i=1}^{n} \varpi_i f_i(o')$, the higher the degree of consensus among agents. Therefore, the ultimate goal is to build a model to minimize $\sum_{i=1}^{n} \varpi_i f_i(o')$ to construct the collective opinion o'.

In this regard, we construct the following model:

$$
\begin{aligned}
\min \quad & \Delta = \sum_{i=1}^{n} \varpi_i f_i(o') = \sum_{i=1}^{n} \varpi_i |o' - o_i| \\
\text{s.t.} \quad & \begin{cases} o' \in O, \\ o_i \in [0,1], i = 1, \ldots, n, \\ o' = \sum_{i=1}^{n} w_i o_i. \end{cases}
\end{aligned}
\tag{5.8}
$$

The nonlinear programming (5.8) can be transformed into a linear one by introducing $u_i = \left[|o' - o_i| + (o' - o_i)\right]/2$ and $v_i = \left[|o' - o_i| - (o' - o_i)\right]/2$, where $u_i \geq 0$, $v_i \geq 0$, $u_i * v_i = 0$, $|o' - o_i| = u_i + v_i$, and $o' - o_i = u_i - v_i$. The linear programming can then be constructed as follows:

$$
\begin{aligned}
\min \quad & \Delta = \sum_{i=1}^{n} (w_i u_i + w_i v_i) \\
\text{s.t.} \quad & \begin{cases} o' \in O, \\ o_i \in [0,1], i = 1, \ldots, n, \\ o' = \sum_{i=1}^{n} w_i o_i, \\ o' - u_i + v_i = o_i, \\ u_i \geq 0, \\ v_i \geq 0. \end{cases}
\end{aligned}
\tag{5.9}
$$

Other than u_i and v_i, solution to model (5.9) provides a collective opinion o' that leads to the minimum cost consensus opinion. Furthermore, the collective opinion is a weighted summation of agents' opinions.

5.2.3 Blockchain-Enabled Trust Building

The moderator collects agents' opinions $o_i(t)$ and sends back the required modifications to agents, those are $o_i(t)-o'$ with $i = 1, \dots, n$. Agents who accept the suggested modifications update their opinions through the following rules:

$$o_i(t+1) = \begin{cases} o_i(t) + (|o_i(t) - o'|), & o_i(t) - o' < 0, \\ o_i(t) - (|o_i(t) - o'|), & o_i(t) - o' > 0. \end{cases} \tag{5.10}$$

These update rules compel agents to apply the required modifications at once. However, there could be another scheme, in which agents only apply a portion of the suggested modifications and wait for the next discussion rounds:

$$o_i(t+1) = \begin{cases} o_i(t) + \lambda_i(|o_i(t) - o'|), & o_i(t) - o' < 0, \\ o_i(t) - \lambda_i(|o_i(t) - o'|), & o_i(t) - o' > 0, \end{cases} \tag{5.11}$$

where $\lambda_i \in [0, 1]$.

To model agents' interactions, we introduce a Blockchain-based communication regime. Through this regime, agents anonymously express their willingness toward accepting or refusing the suggested modifications. Every agent can then observe how many of the n agents have accepted the modification, however, they are not aware of the identity and opinion of other agents. Due to the fact that agents are not aware of others' opinions, this communication regime must be accompanied by a trust-building mechanism. This is of utmost importance for agents' opinions to be closer to the collective opinion and a consensus opinion sought. In order to accomplish this, we propose a Blockchain-enabled trust building mechanism.

In the Blockchain protocol, we assume there exists a smart contract logged in a ledger and it is shared and monitored by agents and the moderator. This contract will be executed only if $\mu\%$ of n agents accept to apply the modifications. We show this set of agents by $\mathcal{H}^t$, where $|\mathcal{H}^t| \leq n$ and $|.|$ denotes the cardinality of the enclosed set. If $|\mathcal{H}^t|/n \geq \mu\%$, agents in $\mathcal{H}^t$ modify their opinions, however, all the n agents receive the same dividend from moderator, that is:

$$div_i(t) = \left[\sum_{i \in \mathcal{H}} |o_i(t) - o'|\right] /n, \quad i = 1, \ldots, n. \tag{5.12}$$

On the other hand, if $|\mathcal{H}^t|/n < \mu\%$, the contract will not be executed, no modifications will be applied, and no dividend will be received by any of agents. Therefore, within such a protocol, agents become more willing to trust others and to accept the modification so that they can ultimately receive more dividends. However, there is a possibility of betrayal by agents, because they might become tempted to not change their opinions, yet receive their dividends. We formulate this problem as follows.

To formulate this protocol, at time-step t, agents a_i, $i = 1, \ldots, n$, express their willingness $E_i(t)$ to either accept ($E_i(t) = 1$) or refuse ($E_i(t) = 0$) the modification. It is evident that $\mathcal{H}^t = \{a_i | E_i(t) = 1\}$. The number of agents who accept the modification is then $\sum_{i=1}^{n} E_i(t)$. Therefore, the contract will be executed only if:

$$\frac{\sum_{i=1}^{n} E_i(t)}{n} \geq \mu\%. \tag{5.13}$$

The value of $E_i(t)$ for agent a_i, however, depends on three separate items: (1) trust to the group $T_i(t)$; (2) internal reasons for betrayal $\rho_i(t)$; and (3) external reasons for betrayal $\epsilon_i(t)$.

Agents' trusts $T_i(t)$ are initialized by $iT \in [0\%, 100\%]$. Initial trust values can either be increased by $\eta_1\%$ or decreased by $\eta_2\%$, depending on the number of betrayals among agents. The higher the rate of betrayals, the lower the trust of an agent to the group. This can be formulated as follows:

$$T_i(t+1) = \begin{cases} T_i(t) + \eta_2, & \sum_{i=1}^{n} E_i(t) \leq \sum_{i=1}^{n} E_i(t-1), \\ T_i(t) + \eta_1, & \text{otherwise.} \end{cases} \tag{5.14}$$

Following this formulation, we model the first item that impacts $E_i(t)$, by defining a trust function as given below:

$$\psi(T_i(t)) = \begin{cases} 1, & T_i(t) > \gamma_i, \\ 0, & \text{otherwise}, \end{cases} \tag{5.15}$$

where $\gamma_i \in \mathbb{U}(0, 100)$ is the designated trust threshold to the agent a_i, with $\mathbb{U}$ being a randomly-generated uniform number in the $[0\%, 100\%]$ interval. Therefore, betrayal to the group can negatively impact an agent' trust, and delays the consensus reaching process.

To model the second item that impacts $E_i(t)$, i.e., internal reasons of betrayal, it is assumed that ρ_i belongs to a binomial distribution $\mathbb{B}$ with $\mathbb{P}(0) = 10\%$, meaning that the chance of betrayal due to internal reasons is set to 10% for each agent. The internal reasons of betrayal are actually proposed to model the characteristics of individual agents, who might betray the group due to their attitude toward the decision problem, cognitive dissonance, antagonistic and/or indifference behaviour, and stubbornness.

Last but not least, external reasons for betrayal are also modelled by ϵ_i that belongs to a binomial distribution with $\mathbb{P}(0) = 30\%$, meaning that the chance of betrayal due to external reasons is 30% for an agent. One can refer to situations, in which an agent is unable to continue the negotiation, or the case, in which an agent does not update his/her opinion as he/she has already received enough dividends.

All in all, it can be concluded that:

$$E_i(t) = \psi(T_i(t)) \wedge \rho_i \wedge \epsilon_i, \tag{5.16}$$

meaning that agent a_i accepts the modification ($E_i(t) = 1$) if agent's trust is larger than γ_i, 'and' the agent does not have internal 'and' external reasons for betrayal. We call this the scenario with Blockchain protocol and denote it by 'SCB.'

In a second protocol, we consider the agents' willingness toward adjustment of the threshold for contract execution $\mu\%$. We call this 'SCHB' scenario that stands for the scenario with heterogenous Blockchain protocol. In this scenario, μ is not constant and differs for each agent, i.e., the threshold for contract execution for agent a_i is $\mu_i \in [0\%, 100\%]$.

Finally, our third scenario considers the situation, in which agents' internal reasons for betrayal is updated in each time-step. This scenario is denoted by 'SCOHB' that stands for the optimal heterogeneous Blockchain-enabled trust building. It is meaningful due to the fact that agents' belief could be affected by the number of agents that accept modifications in previous time-steps. The following update rule is then constructed for this scenario:

$$\rho_i(t+1) = \frac{n \times \rho_i(t) + \sum_{i=1}^{n} E_i(t) - \mu_i}{n + \sum_{i=1}^{n} E_i(t) - \mu_i}. \tag{5.17}$$

In summary, the required modifications in response to the model (5.9) are sent to the agents. To either accept or refuse the suggested modifications, agents are given access to a ledger supported by the Blockchain technology. In case the number of agents who accept the required modifications are larger than $\mu\%$, the smart contract logged in the ledger will be executed, by which agents' opinions will be updated and agents will receive their dividends.

5.2.4 Algorithm

In this section, we illustrate the pseudo-codes of the proposed trust-building mechanism w.r.t. 'SCB,' where the other two scenarios, i.e., 'SCHB' and 'SCOHB,' can be conducted by this algorithm with slight changes. Algorithm 4 summarizes the step-by-step implementation of the 'SCB.' Furthermore, the function '*minimum_cost*' in line 2 is indeed the presented model by (5.9), where it takes o_i, w_i, and ϖ_i as inputs to result the collective opinion o' and the minimum cost (denoted by '*min_cost*' in

Algorithm 4) to construct this opinion.

5.3 Illustration of the Proposed Model

This section discusses how the proposed ODM can be implemented. The sensitivity and comparison analysis of the proposed model are presented in detail in Chapter 6.

Consider five agents, i.e., $n = 5$, who initially express their opinions in terms of Z-numbers $Z_i = (s_j, s'_j)$, $j = 1, \ldots, 2r+1$, where the fuzzy restrictions are taken from the LTS $\mathcal{S} = \{s_0, \ldots, s_6\}$ with a reliability value from $\mathcal{S}' = \{s'_0, \ldots, s'_6\}$. Semantics of $\mathcal{S}$ and $\mathcal{S}'$ are given in Table 5.1 in terms of trapezoidal $\mathcal{F}$ and triangular $\mathcal{T}$ fuzzy MFs. Let $\mathcal{Z}_1 = (s_5, s'_1)$, $\mathcal{Z}_2 = (s_4, s'_1)$, $\mathcal{Z}_3 = (s_1, s'_1)$, $\mathcal{Z}_4 = (s_1, s'_6)$, and $\mathcal{Z}_5 = (s_4, s'_5)$ be the initial opinions of agents. These linguistic opinions are firstly converted into numerical values. As an example, the conversion of $\mathcal{Z}_1$ is given here. The second part of $\mathcal{Z}_1$, i.e., s'_1, can be converted into a crisp value through the centroid method:

$$\alpha = \frac{\int x\mu_{s'_1}(x)dx}{\int \mu_{s'_1}(x)dx} = 0.0467.$$

Weight of the reliability part can then be added to the restriction part as given below:

$$\begin{aligned} Z_1^{\alpha} &= \sqrt{0.0467} \times [0.7, 0.75, 0.85, 0.9; 1] \\ &= [0.1512, 0.1620, 0.1836, 0.1944; 1]. \end{aligned}$$

Consider $\vartheta_1 = \vartheta_2 = \vartheta_3 = \frac{1}{3}$. By calculating $\text{EV}_1 = 0.1728$, $\text{COG}_1 = 0.1512$, and $\text{MeOM}_1 = 0.1728$, the CCV of Z_1 is obtained as $\text{CCV}_1 = o_1(1) = 0.1656$.

Similarly, $o_2(1) = 0.1404$, $o_3(1) = 0.0360$, $o_4(1) = 0.1789$, and $o_5(1) = 0.5240$. Given $iT = 70\%$ and $\sigma = 1\%$, the initial translated opinions, $\varpi_i = [2, 1, 3, 3, 3]$, and $w = [0.2660, 0.3256, 0.1642, 0.1340, 0.1102]^T$, $o' = 0.1772$ is the solution to model (5.9) with a minimum cost of 1.5291.

Algorithm 4: Blockchain-Enabled Trust-Building

Inputs:

Number of agents (n), number of time-steps ($\mathcal{T}$),contract execution threshold (μ),unit cost paid to agent a_i (ϖ_i),initial trust (iT), variance of the initial trust (σ),trust increment (η_1), trust decrement (η_2), trust thresholds (γ_i), agents' weights (w_i),internal reasons of betrayal (ρ_i), external reasons of betrayal (ϵ_i),number of agents who do not betray at $t = 0$ (p).

for *i=1:n* **do**
- $\mathcal{Z}_i = (s_i, s'_i) \leftarrow$ initial opinion of agent a_i;
- $o_i(1) \leftarrow \vartheta_1 \mathrm{EV}(\tilde{\mathcal{Z}}_i) + \vartheta_2 \mathrm{CoG}(\tilde{\mathcal{Z}}_i) + \vartheta_3 \mathrm{MeOM}(\tilde{\mathcal{Z}}_i)$;

$[o', \text{min\_cost}] \leftarrow \text{minimum\_cost}(o_i(1), \varpi_i, w_i)$; $t \leftarrow 1$;

while $t < \mathcal{T}$ **do**
- **for** *i=1:n* **do**
  - **if** $\epsilon_i(t) = 1 \wedge \rho_i(t) = 1 \wedge T_i(t) > \gamma_i$ **then**
    - $E_i(t) = 1$;
  - **else**
    - $E_i(t) = 0$;
- **if** $\sum_{k=1}^{n} E_k(t)/n > \mu/100$ **then**
  - **for** *i=1:n* **do**
    - **if** $E_i(t) = 1$ **then**
      - $o_i(t+1) \leftarrow o'$;
    - **else**
      - $o_i(t+1) \leftarrow o_i(t)$;;
    - $div_i(t) \leftarrow \left[\sum_{i \in \mathcal{H}} |o_i(t) - o'|\right]/n$;
- **else**
  - **for** *i=1:n* **do**
    - $o_i(t+1) \leftarrow o_i(t)$;$d_i(t) \leftarrow 0$;
- **if** $t = 1$ **then**
  - **if** $\sum_{i=1}^{n} E_i(t)/n > p/n$ **then**
    - **for** *i=1:n* **do**
      - $T_i(t+1) \leftarrow T_i(t) + \eta_1$;
  - **else**
    - $T_i(t+1) \leftarrow T_i(t) + \eta_2$;
- **else**
  - **if** $\sum_{i=1}^{n} E_i(t) \geq \sum_{i=1}^{n} E_i(t-1)$ **then**
    - **for** *i=1:n* **do**
      - $T_i(t+1) \leftarrow T_i(t) + \eta_1$
  - **else**
    - **for** *i=1:n* **do**
      - $T_i(t+1) \leftarrow T_i(t) + \eta_2$

Table 5.1 – Semantics of linguistic terms in $\mathcal{S}$ and $\mathcal{S}'$.

$\mathcal{S}$			$\mathcal{S}'$		
s_0	'very poor'	$\mathcal{F}[0, 0, 0.1, 0.15]$	s'_0	'strongly uncertain'	$\mathcal{T}[0, 0, 0.15]$
s_1	'poor'	$\mathcal{F}[0.1, 0.15, 0.25, 0.3]$	s'_1	'uncertain'	$\mathcal{T}[0.05, 0.2, 0.35]$
s_2	'slightly poor'	$\mathcal{F}[0.25, 0.3, 0.4, 0.45]$	s'_2	'somewhat uncertain'	$\mathcal{T}[0.2, 0.35, 0.5]$
s_3	'fair'	$\mathcal{F}[0.4, 0.45, 0.55, 0.6]$	s'_3	'neutral'	$\mathcal{T}[0.35, 0.5, 0.65]$
s_4	'slightly good'	$\mathcal{F}[0.55, 0.6, 0.7, 0.75]$	s'_4	'somewhat certain'	$\mathcal{T}[0.5, 0.65, 0.8]$
s_5	'good'	$\mathcal{F}[0.7, 0.75, 0.85, 0.9]$	s'_5	'certain'	$\mathcal{T}[0.65, 0.8, 0.95]$
s_6	'very good'	$\mathcal{F}[0.85, 0.9, 1, 1]$	s'_6	'strongly certain'	$\mathcal{T}[0.85, 1, 1]$

Table 5.2 – The betrayal index for each agent at the first time-step E_i^1.

Agent	ρ	ϵ	$T^1(\%)$	$\gamma(\%)$	E^1
a_1	0	0	69.21	29.99	0
a_2	1	1	69.73	80.71	0
a_3	1	1	69.97	91.37	0
a_4	1	1	69.64	76.51	0
a_5	1	1	70.70	33.42	1

Required modifications will then be suggested to agents, where they can either accept or refuse them due to lack of trust to the group ($T_i^1 < \gamma_i$), or internal (ρ with $\mathbb{P}(0) = 10\%$), or external reasons (ϵ with $\mathbb{P}(0) = 30\%$). With this setup, the betrayal index for each agent at the first time-step (E_i^1) can be resulted as given in Table 5.2. Following this, it is evident that only agent a_5 is willing ($E_5^1 = 1$) to employ the modification. Having $\mu = 50\%$, the contract will not be executed ($\frac{1}{5} = 20\% < \mu$), and, therefore, no initial opinion is modified toward the collective opinion. For 15 time-steps, the evolution of opinions is shown in Fig. 5.2. The results are obtained for the case with $\eta_1 = 5\%$ and $\eta_2 = -2\%$. The attained results denote that it takes 7 time-steps for three agents to accept modifications. The fourth agent accepts the modification at time-step $t = 10$, that is $t = 11$ for the last agent. This is due to the increase in agents' trust to the group, where the evolution of average trust for the group is shown in Fig. 5.3. The group reaches to the consensual opinion at $t = 11$, which is the end of group negotiations. Afterwards, even though the group reaches

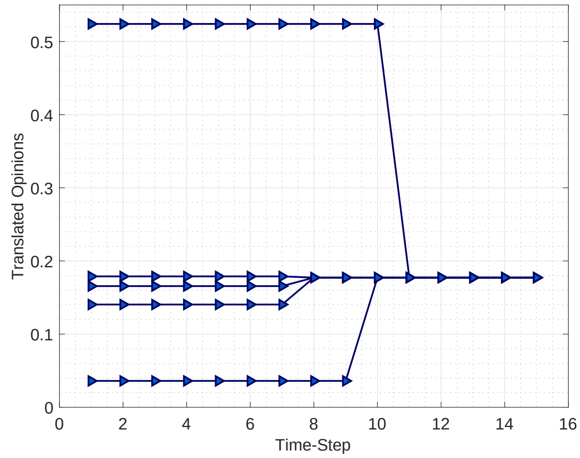

Figure 5.2 – The evolution of opinions for 15 time-steps.

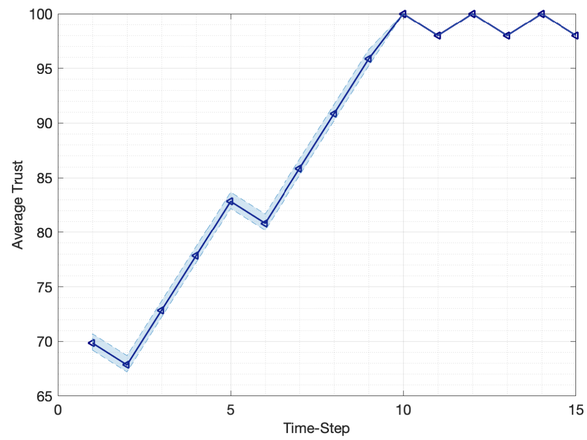

Figure 5.3 – The evolution of average trust (%) in the group. The shaded area shows the trust interval of all agents.

to the maximum level of trust, however, due to the fact that some agents might have internal or external reasons for betrayal, the level of trust fluctuates. In this regard, even though there exist fluctuations in the level of trust, however, the group stick to the consensual opinion and there is no change in the group opinion. In this example, therefore, the moderator pays 1.5291 in total through 11 time-steps to persuade all agents. The agents' dividends are then $div_1 = 0.0232$, $div_2 = 0.0368$, $div_3 = 0.4237$, $div_4 = 0.0050$, and $div_5 = 1.0404$.

Chapter 6

Sensitivity Analysis and Practical Verification of the Proposed Consensus Models

Following the presented results in Chapter 3, Chapter 4, and Chapter 5, this chapter is devoted to a more detailed analysis of the proposed consensus models in the previous chapters. In this regard, we firstly verify the applicability and superiority of the proposed RL-based consensus model for static decision-making through three different experiments. This discussion is then followed by the practical verification of the dynamic model proposed in Chapter 4 for locating faults in distributed power systems. Last but not least, the proposed Blockchain-based trust building mechanism is checked for its sensitivity to the design parameters along with addressing its superiority compared with two state-of-the-art models.

6.1 Sensitivity Analysis of the RL-based Consensus Model

The trained δ-Agent and W-Agent in Chapter3 are employed in three different experiments. It should be noted that both agents are only trained on the developed consensus model based on the DLTFs, and, the same agents are used for the consensus model based on the Z-numbers.

For the following experiments, we set the critic and actor learning rates to $\rho_c =$

0.005, $\rho_a = 0.0001$, the total number of episodes to $N = 15,000$, the size of minibatch to $M = 256$, and $\gamma_l = 0.99$. Five scenarios are considered, where the number of alternatives can be a random integer number from intervals $[2, 5]$, $[6, 10]$, $[11, 15]$, and $[16, 20]$. For any of these cases, we set the number of DMs to $n = 5$ and the number of attributes to $m = 5$. Furthermore, the consensus threshold is set to take values from $\gamma \in [0.85, 0.95]$. Each scenario is repeated for 50 times.

6.1.1 First Experiment

In this experiment, the δ-Agent is trained toward speeding up the CRP by properly adjusting the feedback parameter. For the sake of comparison, the proposed δ-Agent is trained by means of both DDPG and soft actor-critic (SAC) algorithms. For the SAC agent, the number of hidden layers in the critic network is two, that is three for the actor network. The learning rate for both networks is 0.001 and the optimizer is Adam. Sample time is 1, the size of buffer is 100,000, and the discount factor and minibatch size are 0.99 and 256, respectively. Following this experiment, the attained AHD, average value of the feedback parameter $\bar{\delta}$, and the average number of discussion rounds $\bar{t}$ are collected in Table 6.1. The attained results suggest that for the DDPG-based agent, the average number of discussion rounds in all scenarios is 4.2, that is 3.5 for the SAC-based agent. Furthermore, the average value of AHD in all scenarios for the DDPG-based and SAC-based agents are 0.9380 and 0.9321, respectively. The results of this experiment, on the one hand, verify the applicability of the proposed δ-Agent in adjusting the feedback parameter in the CRP. On the other hand, it can be concluded that SAC-based agent sacrifices the HD for a lower number of discussion rounds to speed up the CRP, while the DDPG-based agent leads to higher HDs, and, therefore, higher discussion rounds.

We repeat the same experiment for the proposed consensus model based on Z-numbers, where the attained results are collected in Table 6.2. It is worth noting that the same agents that were trained on the consensus model with DLTFs are

Table 6.1 – The attained results by means of the δ-Agent w.r.t. different number of available alternatives in the consensus model based on the DLTFs.

Method	$\mathcal{X}$	$q \in [2,5]$	$q \in [6,10]$	$q \in [11,15]$	$q \in [16,20]$
DDPG	AHD	0.9398	0.9394	0.9337	0.9392
	$\bar{t}$	4.52	4.14	4.14	4.00
	$\bar{\delta}$	0.1977	0.1990	0.1967	0.1987
SAC	AHD	0.9262	0.9293	0.9252	0.9477
	$\bar{t}$	3.88	3.72	3.54	2.86
	$\bar{\delta}$	0.3136	0.2898	0.3204	0.3021

Table 6.2 – The attained results by means of the δ-Agent w.r.t. different number of available alternatives in the consensus model based on the Z-numbers.

Method	$\mathcal{X}$	$q \in [2,5]$	$q \in [6,10]$	$q \in [11,15]$	$q \in [16,20]$
DDPG	AHD	0.9807	0.9826	0.9819	0.9825
	$\bar{t}$	3.58	3.30	3.24	3.20
	$\bar{\delta}$	0.3243	0.3222	0.3216	0.3222
SAC	AHD	0.9855	0.9825	0.9846	0.9844
	$\bar{t}$	4.14	3.78	3.74	3.32
	$\bar{\delta}$	0.2707	0.3162	0.2837	0.3123

employed for this consensus model, too. This is due to the fact that both agents receive the same observations in terms of dimension and range of values, but from different decision environments. This is to investigate how scalable is the trained δ-Agent. The presented results in Table 6.2 denote high values of AHD, meaning that the original evaluations of DMs do not undergo significant changes during the CRP, while the number of discussion rounds is reduced compared with the previous case. The average discussion rounds for the DDPG-based agent is 3.33, that is 3.745 for the SAC-based agent. In the same vein, the AHD is 0.9819 for the DDPG-based agent, that is 0.9842 for the SAC-based agent.

6.1.2 Second Experiment

In this experiment, we only employ the W-Agent in CRP of models based on DLTFs and Z-numbers to adjust the importance weights of DMs. Two agents are trained

Table 6.3 – The attained results by means of the W-Agent w.r.t. different number of available alternatives in the consensus model based on the DLTFs.

Method	$\mathcal{X}$	$q \in [2,5]$	$q \in [6,10]$	$q \in [11,15]$	$q \in [16,20]$
DDPG	AHD	0.9304	0.9320	0.9365	0.9369
	$\bar{t}$	3.76	3.67	3.42	3.16
SAC	AHD	0.9361	0.9333	0.9362	0.9319
	$\bar{t}$	3.94	3.91	3.65	3.55
'TrRel'	AHD	0.9260	0.9257	0.9332	0.9315
	$\bar{t}$	4.88	4.71	4.39	4.39

utilizing the DDPG and SAC algorithms. In this regard, the attained results for the consensus model based on DLTFs are collected in Table 6.3. It is worth mentioning that for this experiment, the value of feedback parameter is constant and is equal to $\delta = 0.25$. Furthermore, we make a comparison with the case, in which the importance weights of DMs are conventionally set based on the trust relationships between DMs according to Eq. (3.5), shown by 'TrRel' in Table 6.3.

The attained results denote that W-Agent outperforms the conventional trust-based weight-assignment mechanism in terms of AHD and the consensus speed. Turning into details, the average values of AHD and $\bar{t}$ for the DDPG-based agent are 0.9340 and 3.5, respectively, those are 0.9433 and 3.76 for the SAC-based agent. As for the 'TrRel' scenario, the average values of AHD and $\bar{t}$ are 0.9291 and 4.59, respectively. The results of this experiment highlight the efficiency of the proposed RL-based CRP, which can guarantee higher HD values and lower number of discussion rounds compared with conventional techniques.

We then do the same experiment for the consensus model based on Z-numbers to check for the scalability of the proposed W-Agent. In this regard, the attained results are collected in Table 6.4. Results verify the superiority of the proposed W-Agent over the 'TrRel' method. Turning into details, the average values of AHD and number of discussion rounds, for the DDPG-based agent, are 0.9856 and 4.50, respectively. These values are 0.9842 and 5.18 for the SAC-based agent. As for the 'TrRel' method,

Table 6.4 – The attained results by means of the W-Agent w.r.t. different number of available alternatives in the consensus model based on the Z-numbers.

Method	$\mathcal{X}$	$q \in [2, 5]$	$q \in [6, 10]$	$q \in [11, 15]$	$q \in [16, 20]$
DDPG	AHD	0.9854	0.9856	0.9854	0.9859
	$\bar{t}$	4.86	4.52	4.40	4.22
SAC	AHD	0.9834	0.9856	0.9835	0.9842
	$\bar{t}$	5.54	5.20	5.06	4.90
'TrRel'	AHD	0.9786	0.9789	0.9792	0.9797
	$\bar{t}$	6.16	6.08	5.68	5.62

AHD and number of discussion rounds are 0.9791 and 5.89, respectively.

6.1.3 Dual-Agent Experiment

In this experiment, we examine four different combinations of DDPG and SAC agents. For instance, 'DDPG-SAC' is a combination, in which the first agent, i.e., DDPG agent, is employed for adjustment of the feedback parameter, and the second one, i.e., the SAC agent, is used for the weight assignment. In this regard, the previously-trained agents are simultaneously employed in the consensus model based on DLTFs and the attained results are collected in Table 6.5 for each combination. The first combination leads to the highest averaged AHD value, that is 0.9436, while the third combination results in the lowest number of discussion rounds, that is 3.18. The second combination have the lowest averaged value of the feedback parameter, that is 0.1996, and, therefore, leads to the highest number of discussion rounds.

By repeating the same experiment for the consensus model based on Z-numbers, the attained results are collected in Table 6.6. In this case, the highest value of the averaged AHD is obtained for the third combination, that is 0.9831, while the lowest number of discussion rounds is resulted by the first combination, that is 3.25. The fourth combination has the highest number of discussion rounds and the second combination leads to the lowest AHD.

Table 6.5 – The attained results by employing δ-Agent and W-Agent simultaneously in the developed consensus model based on the DLTFs.

Combination	$\mathcal{X}$	$q \in [2,5]$	$q \in [6,10]$	$q \in [11,15]$	$q \in [16,20]$	Avg.
#1 DDPG-DDPG	AHD	0.9535	0.9409	0.9392	0.9410	0.9436
	$\bar{t}$	4.06	3.92	3.82	3.90	3.93
	$\bar{\delta}$	0.2006	0.2015	0.1990	0.2001	0.2003
#2 DDPG-SAC	AHD	0.9372	0.9365	0.9365	0.9420	0.9381
	$\bar{t}$	4.70	4.62	4.18	4.08	4.40
	$\bar{\delta}$	0.2008	0.1999	0.1999	0.1978	0.1996
#3 SAC-DDPG	AHD	0.9372	0.9389	0.9324	0.9345	0.9357
	$\bar{t}$	3.42	3.04	3.00	3.26	3.18
	$\bar{\delta}$	0.2914	0.3158	0.3223	0.2959	0.3064
#4 SAC-SAC	AHD	0.9341	0.9311	0.9402	0.9324	0.9345
	$\bar{t}$	4.21	3.23	3.02	3.10	3.39
	$\bar{\delta}$	0.3091	0.3107	0.3241	0.3316	0.3189

Table 6.6 – The attained results by employing δ-Agent and W-Agent simultaneously in the proposed consensus model based on the Z-numbers.

Combination	$\mathcal{X}$	$q \in [2,5]$	$q \in [6,10]$	$q \in [11,15]$	$q \in [16,20]$	Avg.
#1 DDPG-DDPG	AHD	0.9819	0.9818	0.9834	0.9827	0.9824
	$\bar{t}$	3.38	3.32	3.22	3.06	3.25
	$\bar{\delta}$	0.3221	0.3276	0.3219	0.3221	0.3234
#2 DDPG-SAC	AHD	0.9838	0.9817	0.9818	0.9820	0.9823
	$\bar{t}$	3.70	3.46	3.24	3.22	3.41
	$\bar{\delta}$	0.3165	0.3266	0.3219	0.3252	0.3226
#3 SAC-DDPG	AHD	0.9805	0.9844	0.9838	0.9838	0.9831
	$\bar{t}$	3.68	3.72	3.40	3.34	3.54
	$\bar{\delta}$	0.3203	0.3028	0.3148	0.3252	0.3158
#4 SAC-SAC	AHD	0.9807	0.9819	0.9840	0.9828	0.9824
	$\bar{t}$	4.00	3.90	3.88	4.30	4.02
	$\bar{\delta}$	0.3355	0.3353	0.3131	0.2921	0.3190

Table 6.7 – Comparison between the W-Agent and the case with no RL agent for the weight adjustment in the consensus model based on DLTFs.

Agent		$q \in [2,5]$	$q \in [6,10]$	$q \in [11,15]$	$q \in [16,20]$	Avg.
DDPG	AHD	0.9226	0.9224	0.9230	0.9254	0.9233
	$\bar{t}$	5.1	4.83	4.62	4.54	4.77
SAC	AHD	0.9246	0.9235	0.9248	0.9153	0.9221
	$\bar{t}$	5.27	5.12	5	5.12	5.13
No Agent	AHD	0.9210	0.9191	0.9140	0.9194	0.9184
	$\bar{t}$	5.37	5.29	5.30	5.23	5.30

6.1.4 Comparison with the No-Agent Scenario

To further investigate the efficiency of the trained RL agents compared with the case, in which no RL agent is employed, the second experiment is used for the W-Agent with $\gamma \in [0.91, 0.95]$. In this experiment, the feedback parameter is kept constant ($\delta = 0.25$). For the 'No Agent' scenario, the weights of DMs are adjusted based on the in-degree centrality values and the constructed sociomatrix T_L of DMs. In this regard, the attained results for the consensus model based on DLTFs are collected in Table 6.7. As it can be observed from this table, both W-Agents, i.e., DDPG and SAC agents, outperform the 'No Agent' scenario in terms of the average number of discussion rounds $\bar{t}$ and AHD. Specifically, the average number of discussion rounds for the DDPG and SAC agents are 4.77 and 5.13, respectively, whereas it is 5.30 for the 'No Agent' scenario. Furthermore, the AHD of the DDPG and SAC agents are 0.9233 and 0.9221, respectively, that is 0.9184 for the 'No Agent' scenario. Therefore, the attained results denote that not only the number of discussion rounds can be reduced effectively by means of RL agents, but also they improve the HD of DMs.

6.1.5 Robustness of the trained agents

Finally, we perform a sensitivity analysis to verify the robustness of the trained agents against the changes of 'λ' in the aggregation operator (3.9) and 'a' in the LSF (3.7). In this regard, we firstly employ the proposed selection process in [1] to provide rankings of alternatives. We then resort to Spearman's rank-correlation test [225] to check whether the provided rankings are correlated. This test relies on the following statistic,

$$r_s = 1 - 6\frac{\sum_{i=1}^{q}(\varpi_i)^2}{q(q^2-1)}, \tag{6.1}$$

where q is the number of alternatives, and ϖ_i is the difference between two provided rankings for the alternative x_i. The statistic r_s belongs to the $[-1, +1]$ interval, for which values closer to $+1$ (-1) denote a stronger positive (negative) correlation between two rankings. In this experiment, we set $\lambda = 1$ and $a = 1.36$ as the baseline model to provide rankings of alternatives. Then, by the change of λ over $\lambda = \{2, 3, 4, 5\}$ and $a = \{1.37, 1.38, 1.39, 1.40\}$, different combinations of parameters are created. For each combination, 200 simulation runs are completed over a different number of alternatives $q \in [2, 5]$, attributes $m \in [2, 5]$, and initial evaluations. We then compare the rankings provided by each combination with the baseline model. In this regard, the average absolute value of r_s for all simulation runs w.r.t. each combination are collected in Table 6.8. On the one hand, the attained results indicate that absolute values of r_s are higher than 0.5 in all combinations, denoting a strong correlation between the provided rankings despite the changes in λ and a. On the other hand, the attained results denote that the changes of a do not significantly affect the rankings; however, a larger value of λ slightly reduces the correlation between the rankings, meaning that when DMs become more optimistic (larger λ), there exists less correlation between their provided rankings and the baseline model.

6.1.6 Discussion

These experiments verified that RL-based mechanisms show promising results in speeding up the CRP of static GDM models and opens a new pathway toward the design of CRP.

Table 6.8 – The attained average absolute values of r_s for each combination.

a \ λ	2	3	4	5
1.37	0.7485	0.7930	0.7745	0.7360
1.38	0.8110	0.7415	0.7360	0.7080
1.39	0.8335	0.7545	0.7315	0.7115
1.40	0.8615	0.7845	0.7345	0.6780

Other than the high computational cost of 'MinAdj' techniques, the presented results in Table 3.1 showed that these techniques do not necessarily lead to the best HD. The results of the first experiment denoted that by observing the initial consensus among DMs, an RL agent can decide about the required feedback parameter to speed up the CRP, while DMs can keep their original evaluations in a great context.

The results of the second experiment, on the one hand, indicated the superiority of the proposed RL-based weight-assignment technique over techniques that rely on trust relationships among DMs. On the other hand, and compared with the results of the first experiment, it could be concluded that δ-Agent is more efficient than W-Agent for improving the CRP, meaning that adjustment of the feedback parameter can help more with speeding up the CRP than adjustment of the DMs' weights.

In the third experiment, the first and third combinations led to the best performance in dealing with both decision environments, meaning that these combinations could be better choices to be implemented in the CRP. In these two combinations, the DDPG agent is employed for the weight assignment, showing that the DDPG agent is of better performance in weight-assignment. Furthermore, the attained results verified that the combination of both agents is more effective than implementing them individually for either feedback adjustment or weight-assignment.

6.2 Practical Verification of the Proposed Dynamic Model

This section discusses the practical verification of the proposed dynamic GDM model in Chapter 4. In this regard, the problem of fault location in distributed power systems is formulated into a dynamic decision problem to be addressed by the proposed model.

Power distribution systems are distributed systems and due to their vast geographical spread, locating a faulty component is challenging. This framework is developed to address this challenging issue by integrating the opinions of DMs into diagnostic systems. Its effectiveness and applicability have been verified for locating LL faults in the IEEE 39-bus power distribution system. As the name recalls, this system contains 39 buses, where 19 of them are load buses, i.e., a load has been attached to the corresponding bus.

The initialization is performed as previously mentioned in Chapter 4 by assuming the number of available DMs is sixteen. Four attributes are constructed based on the frequency measurements collected from each bus in the system. In the presence of a fault, therefore, there would be 39 potential alternatives $\{x_1, \ldots, x_{39}\}$, as the system has 39 buses. In what follows, the procedure to locate an LL fault on bus 24 is given.

An LL fault occurs at bus 24, therefore, the solution to the decision problem must be x_{24}. The moderator decides about the alternatives to be put into discussion at each time-step t. Suppose that 10 alternatives are assigned to time-steps $t = 1, 2, 3$, that is nine for the time-step $t = 4$. Therefore, there exist four time-steps and each alternative will be evaluated at least once. At the time-step $t = 1$, the set of $\mathcal{X}^{(1)} = \{x_1, \ldots, x_{10}\}$ is put into discussion in each group. Suppose that the moderator sets five discussion rounds for each group, i.e., $r = 5$, and $\alpha = \frac{4}{6}$, $\delta = 0.22$. The $\mathcal{ACD}$ evolution at time-step $t = 1$ is represented in Table 6.9 and the closeness coefficients for the first set of alternatives d^1 are collected in Table 6.10. The best alternative is $\mathcal{L}^{(1)} = x_3$, where $\mathcal{L}^{(t)}$ denotes the memory. In the time-step $t = 2$, a new set of alternatives along with the alternative in the memory are put into the discussion. Therefore, $\mathcal{X}^{(2)} = \{x_{11}, \ldots, x_{20}, \mathcal{L}^{(1)}\}$. With five discussion rounds, the $\mathcal{ACD}$ evolution is represented in Table 6.9 and the closeness coefficients are collected in Table 6.10. x_{16} has the largest closeness coefficient, and, therefore, $\mathcal{L}^{(2)} = x_{16}$. In the same vein, $\mathcal{L}^{(3)} = x_{24}$ and $\mathcal{L}^{(4)} = x_{24}$. After four time-steps, all alternatives are evaluated at least once, and, $\mathcal{L}^{(4)}$ denotes the solution of the decision problem, that is x_{24} in this example. Therefore, after four time-steps, the location of fault is determined, which is bus 24.

The same setup is used to locate other LL faults, where the attained results are collected in Table 6.11. The proposed framework has only failed in locating the fault on bus 12, where a performance of 94.73% is achieved in making the right decision about the location of faults.

6.2.1 Comparative Analysis of the Proposed Dynamic Model

In this section, a comparison is provided with the literature work [1] that puts forward an MAGDM framework for locating faults in smart grids. This comparison is meaningful as both works are established based on Z-numbers for locating faults. The evolution of $\mathcal{ACD}$ for locating a fault on bus 8 is represented in Table 6.12 for the proposed dynamic model and the model developed in [1]. As it can be observed, the proposed dynamic framework is superior in speeding up the CRP and leads to a higher final $\mathcal{ACD}$ for the same number of discussion rounds, which verifies the superiority of the proposed dynamic model in terms of speeding up the CRP.

In order to further investigate the superiority of the proposed method, a comparative study has been performed with a recent dynamic multi-criteria decision-making (DMCDM) model [2]. DMCDM can be categorized into the multi-period dynamic models, where it develops an extended version of the classical alternative queuing method [226] by means of fuzzy preference relations to provide ranking for alternatives. The opinion of DMs are expressed in terms of intuitionistic fuzzy numbers and

Table 6.9 – The evolution of ACD for each DM in locating a load loss at bus 24.

	time-step $t = 1$									
DM	Group 1					Group 2				
	r=1	r=2	r=3	r=4	r=5	r=1	r=2	r=3	r=4	r=5
1st	0.7753	0.8143	0.8427	0.8427	0.8427	0.8475	0.8561	0.8629	0.8685	0.8798
2nd	0.8201	0.8312	0.8378	0.8378	0.8378	0.8284	0.8484	0.8536	0.8623	0.8746
3rd	0.7867	0.8162	0.8427	0.8427	0.8427	0.7641	0.7975	0.8242	0.8449	0.8679
4th	0.8079	0.8293	0.8366	0.8366	0.8366	0.8119	0.8228	0.8315	0.8378	0.8610
	Group 3					Group 4				
1st	0.8564	0.8640	0.8699	0.9075	0.9154	0.7831	0.8311	0.8654	0.8900	0.9081
2nd	0.8079	0.8276	0.8452	0.8930	0.9089	0.8661	0.8840	0.8968	0.9062	0.9131
3rd	0.8092	0.8356	0.8582	0.9005	0.9166	0.8719	0.8860	0.8960	0.9030	0.9081
4th	0.8397	0.8501	0.8593	0.9011	0.9173	0.8704	0.8864	0.8978	0.9061	0.9122
	time-step $t = 2$									
	Group 1					Group 2				
1st	0.8121	0.8475	0.8948	0.8948	0.8948	0.7947	0.8466	0.8533	0.8583	0.9021
2nd	0.8312	0.8424	0.8914	0.8914	0.8914	0.7744	0.8361	0.8433	0.8634	0.9073
3rd	0.8245	0.8455	0.8942	0.8942	0.8942	0.8206	0.8426	0.8505	0.8578	0.9039
4th	0.8322	0.8463	0.8934	0.8934	0.8934	0.7792	0.8254	0.8471	0.8548	0.9005
	Group 3					Group 4				
1st	0.8621	0.8724	0.8724	0.8724	0.8724	0.8170	0.8288	0.8361	0.8504	0.8618
2nd	0.8861	0.8943	0.8943	0.8943	0.8943	0.7681	0.8011	0.8118	0.8412	0.8642
3rd	0.8440	0.8660	0.8660	0.8660	0.8660	0.7871	0.7955	0.8240	0.8467	0.8655
4th	0.8455	0.8686	0.8686	0.8686	0.8686	0.8147	0.8274	0.8379	0.8541	0.8676

Table 6.10 – The closeness coefficients of the set of alternatives in each time step.

t	Alternatives										$\mathcal{L}^{(t)}$
$\mathcal{X}^{(1)}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	-	$\boldsymbol{x_3}$
d^1	0.06	0.07	0.45	0.42	0.34	0.24	0.24	0.22	0.04	0.24	-	**0.45**
$\mathcal{X}^{(2)}$	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_3	$\boldsymbol{x_{16}}$
d^2	0.03	0.03	0.06	0.05	0.43	0.46	0.35	0.24	0.07	0.07	0.06	**0.46**
$\mathcal{X}^{(3)}$	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{16}	$\boldsymbol{x_{24}}$
d^3	0.17	0.09	0.16	0.53	0.06	0.04	0.04	0.03	0.06	0.05	0.45	**0.53**
$\mathcal{X}^{(4)}$	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	x_{24}	-	$\boldsymbol{x_{24}}$
d^4	0.11	0.05	0.04	0.08	0.06	0.07	0.07	0.08	0.07	0.55	-	**0.55**

Table 6.11 – The real location of faults (x_f), the decisions made by the DMs after four time steps ($\mathcal{L}^4$), and the closeness coefficients (d^4) of the selected alternative at time step $t = 4$.

x_f	$\mathcal{L}^4$	d^4	x_f	$\mathcal{L}^4$	d^4
x_3	x_3	0.57	x_4	x_4	0.56
x_7	x_7	0.58	x_8	x_8	0.55
x_{12}	x_8	0.55	x_{15}	x_{15}	0.57
x_{16}	x_{16}	0.6	x_{18}	x_{18}	0.6
x_{20}	x_{20}	0.58	x_{21}	x_{21}	0.6
x_{23}	x_{23}	0.6	x_{24}	x_{24}	0.6
x_{25}	x_{25}	0.58	x_{26}	x_{26}	0.58
x_{27}	x_{27}	0.6	x_{28}	x_{28}	0.54
x_{29}	x_{29}	0.5	x_{31}	x_{31}	0.46
x_{39}	x_{39}	0.6	-	-	-

Table 6.12 – The evolution of $\mathcal{ACD}$ by the proposed method compared with [1] in locating a load-loss fault at bus 8.

Method	r=1	r=2	r=3	r=4	r=5
Dynamic Model	0.8056	0.8275	0.8389	0.8500	0.8598
[1]	0.7775	0.8049	0.8129	0.8294	0.8326

an entropy-based mechanism is introduced to adjust the importance weights of DMs. A common way to compare decision-making models is to check the correlation between their provided ranking results. In this regard, DMCDM is also employed for the sake of fault location. Four time periods ($t = 1, 2, 3, 4$) are considered, where the best alternative in each period is carried over to the next one. Same as before, the number of alternatives in the first, second, and third periods is 10, whereas it is 9 in the fourth period. Four criteria are considered for each period and the weights of criteria are set to $[0.25, 0.25, 0.25, 0.25]$.

As for comparison, the ranking results of DMCDM are compared with those of the proposed method w.r.t. different quantifiers used for adjustment of the DMs' weights. This is to check for the sensitivity of the provided rankings by both models to the quantifier used for adjustment of the DMs' weights. In this regard, the Spearman's rank-correlation test [225] is employed, which is a common test to verify the relationships between provided rankings of each model w.r.t. different quantifiers. The quantifier used in this work,

$$Q(p) = p^{\alpha}, \tag{6.2}$$

and the one used in DMCDM,

$$Q(p) = (-p^2 + 3p)/2, \tag{6.3}$$

are considered for the adjustment of DMs' weights. For each employed quantifier, the ranking results for all time periods are collected and compared by means of the following statistic of the Spearman's rank-correlation test:

$$r_s(t) = 1 - 6\sum_{i=1}^{q(t)}(\Delta_i)^2/q(t)(q(t)^2 - 1), \tag{6.4}$$

where $q(t)$ denotes the number of alternatives in the tth period and Δ_i indicates the difference of alternative x_i between two different rankings provided by means of the aforementioned quantifiers. $r_s(t) \in [-1, +1]$ and $r_s(t) = -1$ ($r_s(t) = +1$) denotes a completely negative (positive) relationship between two rankings and a closer value of $r_s(t)$ to -1 or $+1$ indicates a stronger relationship between two rankings.

DMCDM is compared with the proposed dynamic model for locating ten different faults and the attained $r_s(t)$ and average of absolute values (AAV) are collected in Table 6.13. As it can be observed from the attained AAVs, the proposed method outperforms DMCDM in seven out of ten experiments, which are highlighted by bold text in Table 6.13. In these cases, the AVVs attained by means of the proposed method are closer to $+1$ compared with those of the DMCDM method, indicating stronger relationships between rankings provided by the proposed method. Therefore, the provided comparative experiment verifies the superiority of the proposed method

Table 6.13 – The attained $r_s(t)$ and average of absolute values (AAVs) by means of DMCDM [2] and the proposed method in this paper for locating 10 different faults.

Fault	Method	$t=1$	$t=2$	$t=3$	$t=4$	AAV
x_3	Ours	0.45	0.60	0.77	0.77	**0.65**
	DMCDM	0.48	0.65	0.15	0.38	0.42
x_4	Ours	0.44	0.95	0.52	0.84	**0.69**
	DMCDM	-0.03	0.93	0.45	0.56	0.49
x_7	Ours	0.98	0.93	0.79	0.79	**0.87**
	DMCDM	0.96	0.85	0.10	0.54	0.61
x_8	Ours	1.00	0.76	0.25	0.58	**0.65**
	DMCDM	0.68	0.86	-0.20	0.36	0.53
x_{12}	Ours	0.43	0.63	0.65	0.58	0.57
	DMCDM	-0.19	0.95	0.46	0.71	0.58
x_{15}	Ours	0.45	-0.78	0.55	0.38	0.54
	DMCDM	0.78	-0.96	0.32	0.37	0.61
x_{16}	Ours	0.98	-0.41	0.62	0.05	**0.52**
	DMCDM	-0.10	0.11	0.87	-0.02	0.28
x_{18}	Ours	-0.38	0.55	0.02	0.13	0.27
	DMCDM	0.89	0.11	0.13	0.44	0.39
x_{20}	Ours	-0.09	-0.97	0.41	0.39	**0.47**
	DMCDM	0.52	-0.63	-0.17	0.39	0.43
x_{21}	Ours	0.94	-0.18	0.80	0.39	**0.58**
	DMCDM	-0.44	-0.76	0.80	0.08	0.52

and indicates that despite of the type of quantifier used for adjustment of DMs' weights, the proposed method outperforms DMCDM in terms of robustness against the changes in the quantifiers.

6.3 Sensitivity Analysis of the Blockchain-Enabled Trust Building Mechanism

Following the presented results in Chapter 5, in this section, a comprehensive sensitivity and comparative study of the proposed Blockchain-enabled trust building mechanism is provided.

Table 6.14 – The attained average trust $\overline{T}$, consensus achievement ratio $\overline{r}$, and average number of required time-steps for consensus achievement $\overline{t}$ w.r.t. the changes in variance of the initial trust $iT = 70\%$.

$\sigma(\%)$	$\overline{T}(\%)$	$\overline{r}(\%)$	$\overline{t}$
1	68.92	68.14	27.04
5	68.18	68.14	26.97
10	68.50	66.48	29.76
15	68.50	67.31	34.79
20	68.25	65.37	36.84

6.3.1 Sensitivity Analysis: First Experiment

The amount of trust increment or decrement following the execution status of contracts, i.e., η_1 and η_2, play an important role in trust improvement and opinion evolution. In what follows, we firstly explore the impact of η_1 and η_2 on trust improvement within the group, which is then followed by a discussion on how these parameters affect the evolution of opinions.

Remark 1. *In the following experiments, the value of σ is set to 1%, however, there is no restriction on this selection. It is a design parameter and can take any value depending on the decision problem. Nevertheless, by resorting to the attained results collected in Table 6.14, it could be concluded that a higher value of σ does not significantly affect the average trust and consensus achievement ratio, however, it slightly increases the required number of time-steps for consensus achievement.*

In the first experiment, we assume that agent a_i, who is persuaded to apply the modification at time-step t, updates the opinion $o_i(t)$ as $o_i(t+1) = o'$, i.e., the agent conducts the total required modification at once. To check for trust building w.r.t. η_1 and η_2 under this experiment, it is assumed that η_1 increases from 1% to 10% with an 0.5% increment step, whereas η_2 decreases from -1% to -10% with a -0.5% decrement step. The number of agents is $n = 30$, ϖ_i are pseudo-random integers drawn from the uniform distribution $[1, 3]$, $\mu = 50\%$ ($\mu \in [30\%, 50\%]$ for the 'SCHB' scenario), $iT = 70\%$, trust thresholds γ_i are randomly selected from the

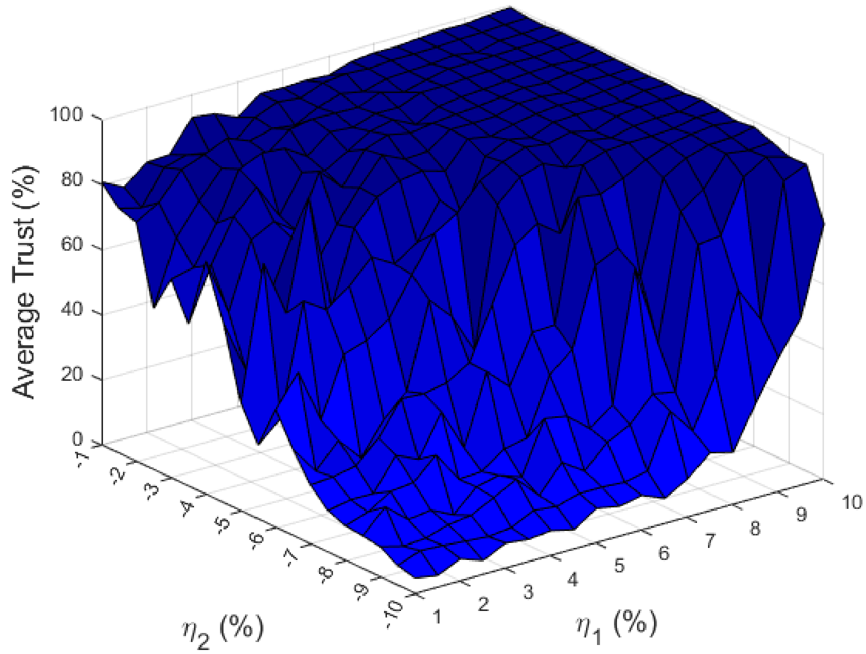

Figure 6.1 – Average trust (%) of all agents in the scenario 'SCB' w.r.t. the changes of η_1 and η_2.

uniform distribution $[0\%, 100\%]$, and the number of time-steps is set to $\mathcal{T} = 200$. The internal ρ and external ϵ reasons of betrayal are modeled by binomial distributions $\mathbb{B}$ with $\rho(0) = 10\%$ and $\epsilon(0) = 30\%$, respectively. Following this setup, the average value of agents' trust for $\mathcal{T}$ trials for 'SCB' and 'SCNB' (the scenario with no Blockchain protocol) are illustrated in Fig. 6.1 and Fig. 6.2, respectively.

The attained results highlight two important features of the proposed trust-building mechanism. First, the proposed Blockchain-enabled trust building mechanism ('SCB' in Fig. 6.1) has led to a significant improvement in trust among agents compared with the case with no Blockchain-based interactions ('SCNB' in Fig. 6.2), especially for $\eta_1 \geq |\eta_2|$ combinations. Second, for $\eta_1 \simeq |\eta_2|$ or $\eta_1 \gg |\eta_2|$ combinations, the average trust is dramatically increased through the 'SCB' scenario, meaning that such combinations of η_1 and η_2 not only can help with minimizing the likelihood of betrayal by agents, but also can increase the chance of consensus achievement.

For other Blockchain-enabled trust building scenarios mentioned in Chapter 5,

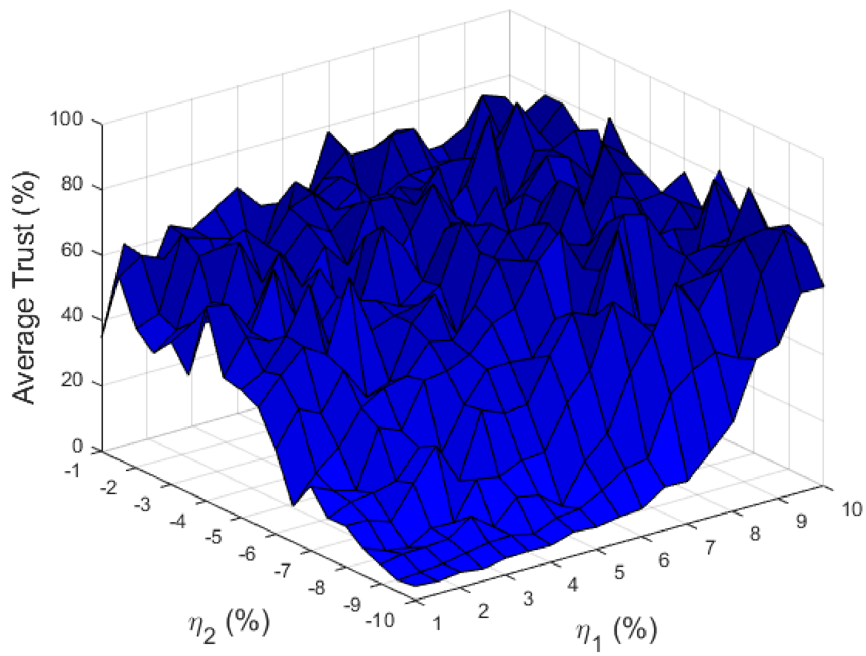

Figure 6.2 – Average trust (%) of all agents in the scenario 'SCNB' w.r.t. the changes of η_1 and η_2.

the same experiment is conducted and these scenarios are compared in terms of the average agents' trust for all combinations of $\eta_1 \in [1\%, 10\%]$ and $\eta_2 \in [-10\%, -1\%]$. The attained results are collected in Table 6.15. For example, for the case with $iT = 70\%$, compared to 'SCNB,' the attained results verify that Blockchain protocols have notably improved the group trust by about 18%. However, all Blockchain protocols have almost yielded the same average trust under this experiment. The difference between these protocols goes back to their impact on consensus achievement and consensus speed, which are discussed next.

Table 6.15 – The attained average trust $\overline{T}$, consensus achievement ratio $\overline{r}$, and average number of required time-steps for consensus achievement $\overline{t}$ for different protocols under the first experiment w.r.t. the changes of $\eta_1 \in [1\%, 10\%]$ and $\eta_2 \in [-10\%, -1\%]$ and the initial trust iT.

$iT(\%)$	'SCNB'			'SCB'			'SCHB'			'SCOHB'		
	$\overline{T}(\%)$	$\overline{r}(\%)$	$\overline{t}$	$\overline{T}(\%)$	$\overline{r}(\%)$	$\overline{t}$	$\overline{T}(\%)$	$\overline{r}(\%)$	$\overline{t}$	$\overline{T}(\%)$	$\overline{r}(\%)$	$\overline{t}$
0	1	0	-	61.34	65.10	42.29	61.37	65.93	44.38	61.13	66.76	47.45
10	5.24	0	-	61.71	65.93	45.27	61.30	65.93	41.19	61.74	67.04	45.07
20	11.81	0	-	62.54	66.76	43.28	62.05	67.59	45.12	61.83	65.90	44.55
30	19.20	0	-	63.17	65.37	42.49	63.51	65.65	41.81	63.31	67.04	41.07
40	27.02	0	-	64.67	67.04	39.98	64	65.65	38.86	64.55	67.04	39.93
50	35.48	0	-	65.63	66.20	35.10	66.20	66.48	32.36	62.93	67.59	36.27
60	44.30	0	-	67.54	68.14	29.78	67.95	68.98	29.60	67.29	68.70	34.26
70	51.62	0	-	68.92	68.14	27.04	68.73	69.53	23.80	69	68.14	25.87
80	59.09	0	-	70.07	71.47	20.98	69.69	68.98	18.85	70.26	72.02	21.38
90	64.73	0.01	73	71.87	77.84	13.10	70.82	76.73	12.31	71.22	77.84	11.11
100	67.31	0.05	77.35	72.28	92.52	6.27	72.01	94.18	6.52	71.60	93.91	6.18
Avg.	35.16	0.03	75.25	66.34	70.41	31.42	66.16	70.51	30.44	65.90	71.10	32.10

For the previous experiment, we compare the aforementioned protocols in terms of the ratio of combinations of η_1 and η_2, for which the full consensus is achieved $\overline{r}$, and, also, in terms of the average number of required time-steps for achieving the full consensus $\overline{t}$. With the full consensus, we mean all the agents' opinions reach the same final state within $\mathcal{T} = 200$ time-steps. To begin with, consider the case with $iT = 70\%$. Our simulations denote that 'SCNB' leads to no consensus among agents for all combinations of η_1 and η_2. However, those combinations for which the consensus is achieved under different Blockchain protocols are illustrated in Fig. 6.3. Following the results presented in Fig. 6.3 and the collected results in Table 6.15, it can be observed that $\overline{r} = 68.14\%$ of all combinations has led to a consensus under the 'SCB' protocol, whereas it is 69.53% for 'SCHB,' and 68.14% for 'SCOHB.' The attained results under this experiment, on the one hand, denote that Blockchain protocols have improved the ratio of consensus achievement with at least 68% compared with 'SCNB'. One the other hand, the 'SCHB' scenario resulted the highest ratio due to the fact that $\mu \in [30\%, 50\%]$ assigns lower thresholds for contract execution for some agents. Furthermore, one can compare the consensus speed of protocols by resorting to $\overline{t}$. In this regard, the required number of time-steps for consensus achievement under the 'SCB' are illustrated in Fig. 6.4. As it can be observed, for the cases with $\eta_1 \geq |\eta_2|$, a lower number of time-steps are required for the full consensus. For other protocols, the value of $\overline{t}$ is collected in Table 6.15 for the case with $iT = 70\%$. The best performance is achieved under the 'SCHB' scenario ($\overline{t} = 23.80$) due to lower contract execution thresholds, which is then followed by the 'SCOHB' ($\overline{t} = 25.87$) and 'SCB' ($\overline{t} = 27.04$), respectively.

Other than η_1 and η_2, the initial trust level iT could also affect the average trust in the group $\overline{T}$, the rate of consensus achievement $\overline{r}$, and the consensus speed $\overline{t}$. We repeat the first experiment for different initial trust levels. Specifically, the initial trust is changed from 0% to 100% and the attained $\overline{T}$, $\overline{r}$, and $\overline{t}$ values are collected in Table 6.15. Firstly, this comprehensive experiment denotes that 'SCNB' leads to

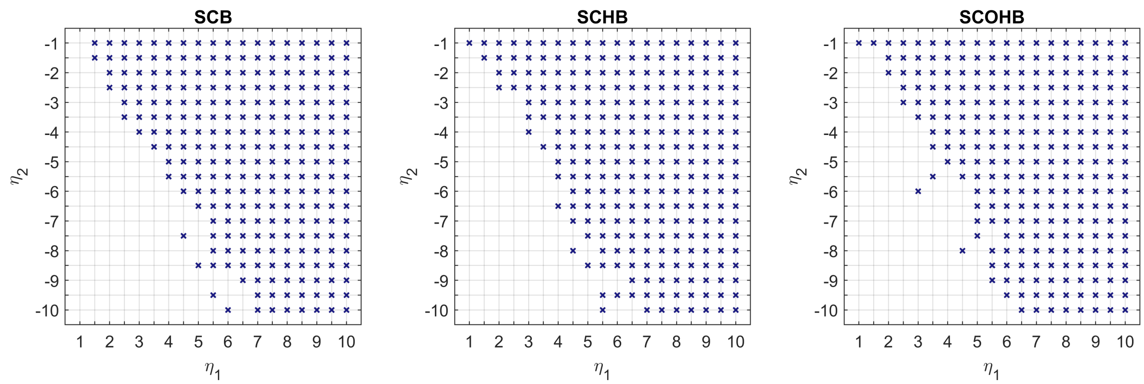

Figure 6.3 – Consensus achievement under different Blockchain protocols with $iT = 70\%$. The combination led to the full consensus is marked by a '×' symbol.

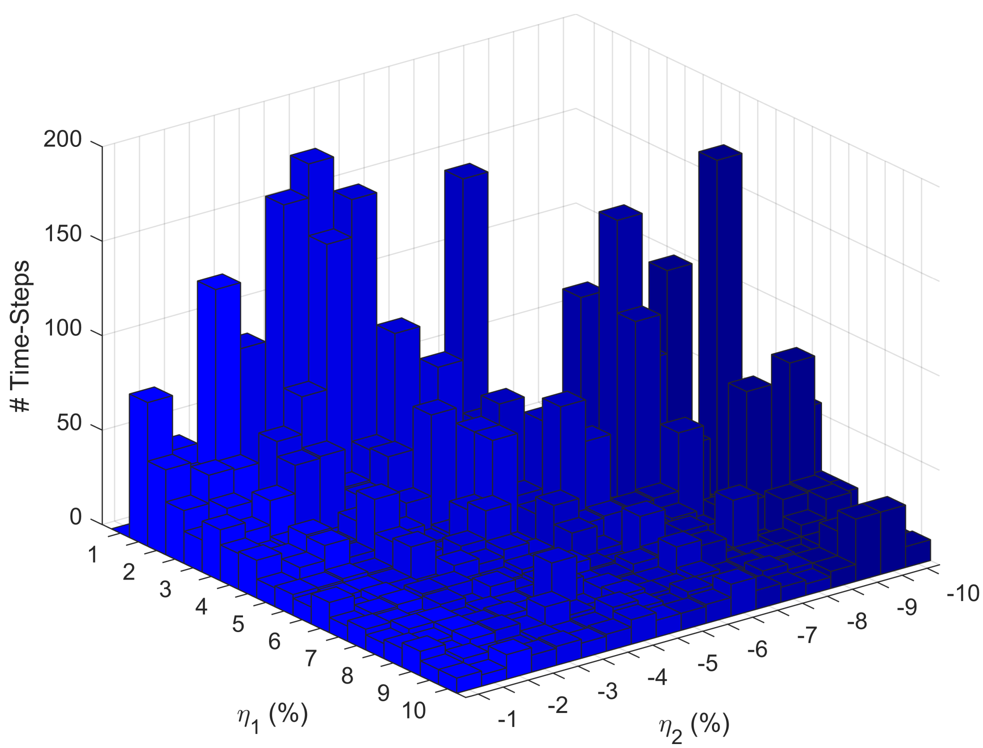

Figure 6.4 – The required number of time-steps for consensus achievement under the 'SCB' scenario w.r.t. the changes of η_1 and η_2 for the case with $iT = 70\%$ within the framework of the first experiment.

no consensus when the initial trust level changes from 0% to 80%, and for the cases with $iT = 90\%$ and $iT = 100\%$, a very low rate of consensus achievement (0.01 and 0.05, respectively) with a very low consensus speed (73 and 77.35, respectively) is achieved in comparison with the Blockchain-based scenarios; Secondly, regardless of the type of scenario, the increase in the initial trust level improves the average trust, the rate of consensus achievement, and the consensus speed. Therefore, the higher the initial trust level, the higher the average trust within the group, the higher the rate of consensus achievement, and the faster the consensus speed. Thirdly, even though all Blockchain protocols lead to almost the same trust with the group in average, however, the 'SCOHB' protocol results in a higher rate of consensus achievement, which is then followed by 'SCHB' and 'SCB,' respectively.

In addition to η_1, η_2, and iT, the internal ρ and external ϵ reasons of betrayal could also impact the rate of consensus achievement. Within the framework of the first experiment, we set the initial trust level to $iT = 50\%$ and assume that $\eta_1 \in [1\%, 10\%]$ and $\eta_2 \in [-10\%, -1\%]$. For this setup, the values of ρ and ϵ are changed from 0.05 to 0.5 with an 0.05 increment step. For each combination of ρ and ϵ, the rate of consensus achievement is illustrated in Fig. 6.5. As expected, the lower the values of ρ and ϵ, the higher the rate of consensus achievement. Therefore, the attained results denote that by managing the internal and external reasons of betrayal, one can improve the rate of consensus achievement by the group of agents.

6.3.2 Sensitivity Analysis: Second Experiment

In the first experiment, it was assumed that agents apply the total amount of required modifications once they accept the recommended modifications. However, in our second experiment, it is assumed that agents do not entirely apply the recommended modifications. Instead, they apply a portion of the modification. i.e., $o_i(t)$ by $o_i(t+1) = o_i(t) - \lambda_i \times \delta_i(t)$, where $\delta_i = o_i(t) - o'$. Under this experiment, the values of $\overline{T}$, $\overline{r}$, and $\overline{t}$ are collected in Table 6.16. Obviously, by applying just a portion of the

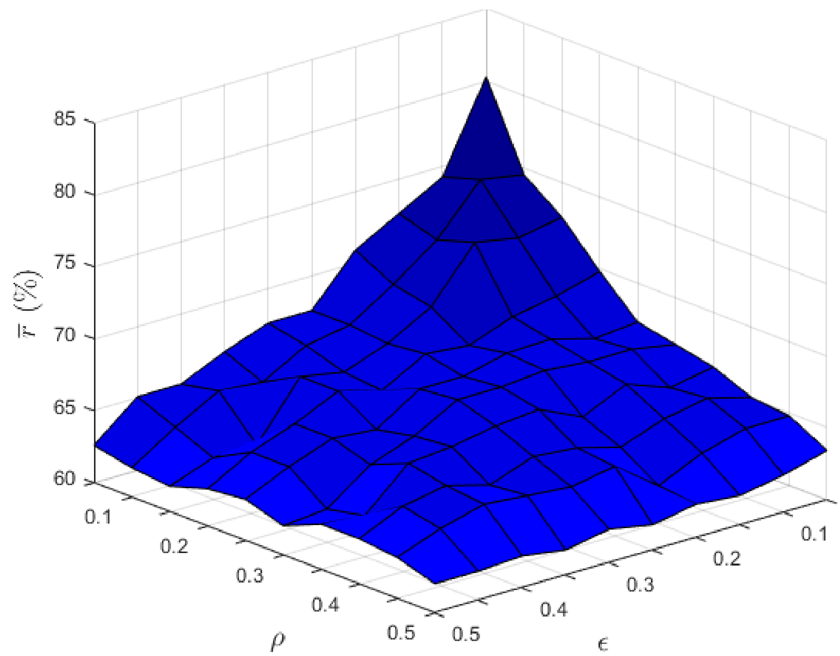

Figure 6.5 – The rate of consensus achievement under the 'SCB' scenario w.r.t. the changes of η_1 and η_2 with $iT = 50\%$ under the framework of the first experiment.

recommended modification, the rate of consensus achievement will be reduced compared with the first experiment. In the same vein, the number of required time-steps for consensus achievement will also be increased (lower consensus speed). This can be concluded by comparing the average results presented in Table 6.15 (first experiment) and Table 6.16 (second experiment). However, the trust-building protocols work well for both experiments, denoting that the proposed protocols are independent of the way that modifications are employed. Furthermore, it is worth noting that for the case with $iT = 90\%$, a better consensus speed ($\bar{t} = 81$) is obtained for the 'SCNB' compared with other Blockchain protocols (those are $\bar{t} = 113.81$, $\bar{t} = 116.25$, and $\bar{t} = 113.04$ for 'SCB,' 'SCHB,' and 'SCOHB,' respectively). However, as the rate of consensus achievement is $\bar{r} = 0.28\%$, it denotes that consensus is achieved in only one combination of 361 possible combinations of η_1 and η_2, which is for the case with $\eta_1 = 2\%$ and $\eta_2 = -7\%$. By checking for the same combination of η_1 and η_2 for the 'SCB,' the consensus speed is 68, which is better than the 'SCNB' scenario.

Therefore, the attained average value does not reflect the better performance of the 'SCNB' compared with 'SCB.'

The proposed trust-building mechanism incorporates variables that take random values ($\rho, \epsilon, \gamma, iT$), and also variables that help with consensus management, i.e., η_1, and η_2. Following the provided comprehensive sensitivity analysis and the results presented in Fig. 6.3, two general properties of the given algorithm could be given as below, despite the values of the aforementioned random variables:

1. The choice of $\eta_1 \geq |\eta_2|$ leads to a consensus opinion, even if the selection of $\eta_1 \simeq \eta_2$ could take more time-steps for the algorithm to converge to the collective opinion.

2. The choice of $\eta_1 < |\eta_2|$ could either result in consensus, polarization, or fragmentation, depending on the values of the random variables.

6.3.3 Comparative Analysis

To demonstrate its novelty, we compare the proposed Blockchain-enabled trust building mechanism with other existing methods.

1. Despite the fact that willingness of agents to accept the advice is typically modeled through the BC framework in ODMs that in turn can conduct bias in agents' interactions [227, 228, 219, 154, 229], these techniques do not simultaneously consider the internal and external factors that might impact agents' judgments and interactions. Further to this, trust building using opinion similarity typically involves a trust propagation mechanism to estimate the level of trust among agents. This mechanism suffers from a high computational complexity that could be as high as $\mathcal{O}(2^V)$ for a total number of V agents. On the contrary, the proposed model in this work removes the opinion similarity constraint for trust building, and, also, it efficiently incorporates the internal and external factors that affect agents' interactions.

2. Even though some efforts have been devoted to the design of LODMs [228, 230], however, the recently developed models are mostly concerned with numerical opinions [231, 232, 233, 234, 235] and less attention is paid to the LODMs. Our proposed model can be categorized under the LODMs, where we propose to make use of Z-numbers in order for the agents to conveniently express their opinions and to model opinion uncertainties more effectively.

Table 6.16 – The attained average trust $\overline{T}$, consensus achievement ratio $\bar{r}$, and average number of required time-steps for consensus achievement $\bar{t}$ for different protocols under the second experiment w.r.t. the changes of $\eta_1 \in [1\%, 10\%]$ and $\eta_2 \in [-10\%, -1\%]$ and the initial trust iT.

$iT(\%)$	'SCNB'			'SCB'			'SCHB'			'SCOHB'		
	$\overline{T}(\%)$	$\bar{r}(\%)$	$\bar{t}$	$\overline{T}(\%)$	$\bar{r}(\%)$	$\bar{t}$	$\overline{T}(\%)$	$\bar{r}(\%)$	$\bar{t}$	$\overline{T}(\%)$	$\bar{r}(\%)$	$\bar{t}$
0	0.13	0	-	61.27	44.04	126.92	61.54	46.26	126.83	61.30	44.04	130.17
10	5.04	0	-	61.28	44.32	133.04	61.47	43.77	129.31	61.51	44.88	127.02
20	12.02	0	-	62.39	44.32	124.54	61.88	43.49	125.96	62.29	42.94	122.31
30	19.29	0	-	63.80	44.04	124.71	63.46	43.49	123.53	63.31	45.98	126.76
40	27.43	0	-	64.57	45.43	121.36	64.41	45.43	123.20	64.39	45.71	124.39
50	30.66	0	-	65.56	45.15	121.53	66.39	44.32	121.23	65.93	45.43	121.62
60	43.52	0	-	67.96	47.37	122.19	67.02	46.54	113.92	67.23	47.92	119.87
70	53.21	0	-	68.70	48.75	115.57	68.88	50.14	118.80	68.43	47.92	114.75
80	59.58	0	-	70.22	47.09	111.02	70.15	49.86	115.44	70.10	49.31	117.56
90	65.32	0.28	81	71.65	50.14	113.81	71.50	49.03	116.25	71.38	50.69	113.04
100	67.69	3.6	86	71.99	49.58	107.36	71.73	50.69	108.22	71.98	49.86	106.62
Avg.	34.93	1.94	83.5	66.31	46.39	120.19	66.22	46.64	120.25	66.19	46.79	120.37

Table 6.17 – The average number of required time-steps to reach consensus using different methods.

n	5	15	25	35	50
Li et al., [231]	11.725	46.27	76.19	100.89	116.28
Zha et al., [227]	8.61	34.28	53.65	80.85	107.78
‘SCB’	7.25	9.31	12.56	13.21	15.87
‘SCHB’	7.49	8.79	12.16	13.01	14.44
‘SCOHB’	6.83	9.38	11.05	12.78	15.63

Furthermore, we provide a numerical comparison with two models [231, 227] in terms of the required number of time-steps for consensus achievement to illustrate the superiority of the proposed Blockchain-enabled trust building mechanisms. For the proposed model in [231], the number of alternatives and attributes are set to 4, $\epsilon \in [0, 0.15]$, $\beta \in [0, 1]$, and the incomplete social trust matrix initially contains $iT = 90\%$ of all possible connections. The proposed model in [227] is also set to deal with four alternatives, the bounded confidence is set to $\epsilon \in [0, 0.5]$, and the consensus threshold is $\mu = 0.85$. Following this setup, the average number of time-steps for different number of agents $n \in \{5, 15, 25, 35, 50\}$ are collected in Table 6.17 and are compared with those of our proposed Blockchain protocols with $iT = 90\%$. On the one hand, the superiority of the proposed protocols over the proposed models in [231, 227] is evident. On the other hand, the proposed protocols appear more robust to the increase in number of agents, whereas the other two techniques fail to converge quickly.

6.4 Summary

In this dissertation, three novel consensus models were introduced to facilitate group decision-making and opinion dynamics. The first model aimed to enhance the speed of the consensus reaching process while maintaining a high Harmony degree among decision-makers. This model is versatile and applicable to various group decision-making environments. However, its limitation lies in addressing dynamic environments where the number of decision-makers fluctuates between discussion rounds. Adjustments in the action of trained reinforcement learning agents are needed to adaptively modify the action space size. The second proposed model addressed dynamic environments where the number of alternatives can dynamically change during the negotiation process. However, the model's limitation arises when the set of attributes also changes dynamically, requiring the dynamic formation of decision-maker groups, which may not be efficient. Lastly, a Blockchain-based trust-building protocol was proposed within the opinion dynamics framework to establish trust among decision-makers without relying solely on opinion similarity. However, the model's limitation lies in the involvement of random variables that impact the evolution of opinions and trust among decision-makers. A more efficient mechanism is required to optimally select these random variables to expedite the consensus reaching process.

Chapter 7

Conclusions and Future Work

Intelligent group decision-making is beneficial for integrating decisions made by diverse intelligent decision makers to come up with the solution to a decision problem that suits all the decision makers. Intelligent group decision-making, however, is required to be equipped with consensus support models to guid the individual intelligent agents toward the collective decision of the group. In this regard, the design of consensus support models that efficiently integrate the opinions of decision makers and speed up the consensus reaching process is of utmost importance for the sake of intelligent group decision-making. Toward this end, the design of consensus support models has gained much attention recently and encouraging results have been reported in state-of-the-art works. However, there still exist several shortcomings that have not been addressed well.

This dissertation is thus devoted to the design of consensus support models for intelligent group decision-making by concerning the shortcomings of the previous state-of-the-art works. These shortcomings can be categorized into three distinct categories such as (i) generalization of static consensus models, (ii) dynamic consensus models to support dynamic group decision-making, and (iii) the efficient modelling of the decision makers interactions in large-scale or opinion dynamics models.

The problem (i) refers to the fact that most of the developed consensus models for static decision-making are specifically designed for a decision environment with a particular type of representation structure for opinions. This is a limitation of such models, because a developed model for a specific decision environment is not extendable to other environments and such models only work well under the assumptions hold for that particular decision environment and they are either not applicable in other decision environments or their performance could be degraded once the decision environment changes. Therefore, there is a need to develop consensus models for static decision environments that are more general and are of good performance in different decision environments.

The second problem (ii) arises in dynamic environment, in which the decision variables are subject to changes from one discussion round to another. The developed models mainly suffer from their computational complexity due to the fact that in dynamic environments, the consensus reaching process needs to be fulfilled at each time-step. Therefore, such models could be computationally expensive and there is a need to develop models that mitigate this issue by efficiently managing the group of decision makers and their interactions.

The third problem (iii) is mostly concerned with large-scale group decision-making and opinion dynamics models. Due to the availability of a large group of decision makers, managing the willingness and interactions of such a group could be challenging for the sake of consensus reaching. Many efforts have been devoted to the design of consensus models to build trust among decision makers so as to speed up the consensus reaching process. However, the developed models try to build trust between decision makers through the level of opinion similarity which conducts bias in their interactions. Therefore, it is of paramount importance to not only remove this limitation, but also to propose more efficient communication regimes so that decision makers can securely interact to collectively decide on the solution to the decision problem.

This dissertation addressed the aforementioned challenges by proposing novel consensus support models for intelligent decision-making. To deal with the first issue (i), it was proposed to resort to reinforcement learning in order to construct a general consensus model that could deal with different decision environments irrespective to the type of representation structure used for the opinion expression. This could be achieved due to the fact that reinforcement learning is a model-free learning algorithm and once an agent is trained for a specific environment, the same agent could deal with other decision environments, too. To enable the application of reinforcement learning, however, it was required to convert the decision environment into a Markov decision process to construct the state transition rules. Following this conversion, it was proposed to train two agents for adjusting the feedback parameter and importance weights of decision makers so as to speed up the consensus reaching process along with keeping the Harmony degree of decision makers at a high level. The attained results not only verified the applicability of the proposed model, but also denoted the trained agents are generalizable and the same agents could deal with diverse static decision environments.

For the second problem (ii), a dynamic framework was proposed that helped with reducing the computational complexity of dynamic decision environments. This was done by proposing to divide decision makers into several groups depending on the number of available attributes, where each group of decision makers was only focusing on a single attribute for opinion expression. This idea not only helps with reducing the computational effort of the consensus reaching process by decreasing the dimensions of the opinions, but also it reduced the required level of consensus assessment into two levels in contrast to the typical three-level consensus assessment procedure. Further to this, designing a meaningful, yet effective consensus threshold for the sake of speeding up the consensus reaching process was another tool that was proposed in order to reduce the computational complexity of such models. The attained results verified the applicability of the proposed framework and its practical verification was also shown by addressing the fault location problem in distributed power systems.

Last but not least, to address the third problem (iii), it was proposed to make use of the Blockchain technology in order to provide a safe and secure communication regime in order for decision makers to interact with each other. The proposed strategy was able to remove the long-standing problem in opinion dynamics models, where the developed models make use of opinion similarity to build trust among decision makers. This traditional modelling of interactions not only conducts bias, but also are computationally expensive as they usually involve a trust propagation mechanism to estimate the level of trust among decision makers. To remove such boundaries, the proposed Blockchain-enabled trust building mechanism does not concern opinion similarity for the sake of trust building, yet it was able to build trust among decision makers and to speed up the consensus reaching process.

7.1 Future Work

The following research directions could be suggested based on the developed consensus models in this dissertation:

1. The proposed reinforcement learning-based consensus model in Chapter 3 deals with the situation, in which the decision environment is static. However, extending this framework to dynamic environments, in which the set of decision makers could be subject to changes from one discussion round to another, could be a challenging, yet interesting research direction due to the fact the agent needs to adjust its actions w.r.t. the number of decision makers.

2. The proposed dynamic framework in Chapter 4 was only concerning the case, in which the dynamism was modelled by the changes in the number of alternatives. However, extending this framework or devising new dynamic frameworks that could deal with the situations that other than the alternatives, the set of decision makers and attributes are subject to changes, could also be an interesting and challenging research direction.

3. The proposed Blockchain-enabled trust building is the first attempt toward the use of this technology in opinion dynamics models. Making use of the proposed

protocols for decision making under different decision conditions by considering the attitude and non-cooperative behaviour of decision makers could be a worthwhile research direction. Furthermore, the proposed protocols control the agents-moderator interactions, however, there could be a potential application of such protocols to manage the within-group interactions of agents, too.

References

[1] Hossein Hassani, Roozbeh Razavi-Far, and Mehrdad Saif. Fault location in smart grids through multicriteria analysis of group decision support systems. *IEEE Transactions on Industrial Informatics*, 16(12):7318–7327, 2020.

[2] Ran Tao, Zeyi Liu, Rui Cai, and Kang Hao Cheong. A dynamic group mcdm model with intuitionistic fuzzy set: Perspective of alternative queuing method. *Information Sciences*, 555:85–103, 2021.

[3] Quanbo Zha, Yucheng Dong, Hengjie Zhang, Francisco Chiclana, and Enrique Herrera-Viedma. A personalized feedback mechanism based on bounded confidence learning to support consensus reaching in group decision making. *IEEE Trans. Syst., Man, Cybernet.: Syst.*, 51(6):3900–3910, 2021.

[4] Yan Liu, Zhiyuan Jiang, Shunqing Zhang, and Shugong Xu. Deep reinforcement learning-based beam tracking for low-latency services in vehicular networks. In *ICC 2020-2020 IEEE International Conference on Communications (ICC)*, pages 1–7. IEEE, 2020.

[5] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Vasile Palade. Generative adversarial network-based scheme for diagnosing faults in cyber-physical power systems. *Sensors*, 21(15):5173, 2021.

[6] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Gérard-André Capolino. Regression models with graph-regularization learning algorithms for accurate fault location in smart grids. *IEEE Systems Journal*, 15(2):2012–2023, 2020.

[7] Hossein Hassani, Ehsan Hallaji, Roozbeh Razavi-Far, and Mehrdad Saif. Unsupervised concrete feature selection based on mutual information for diagnosing

faults and cyber-attacks in power systems. *Engineering Applications of Artificial Intelligence*, 100:104150, 2021.

[8] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Enrico Zio. Deep learning with long short-term memory networks for diagnosing faults in smart grids. *European Journal for Security Research*, 6(2):151–169, 2021.

[9] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Jafar Zarei. Unknown input observers design for real-time mitigation of the false data injection attacks. In *2020 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pages 3612–3617. IEEE, 2020.

[10] Maryam Farajzadeh-Zanjani, Ehsan Hallaji, Roozbeh Razavi-Far, and Mehrdad Saif. Generative-adversarial class-imbalance learning for classifying cyber-attacks and faults-a cyber-physical power system. *IEEE Transactions on Dependable and Secure Computing*, 19(6):4068–4081, 2021.

[11] Hossein Hassani, Maryam Farajzadeh-Zanjani, Roozbeh Razavi-Far, Mehrdad Saif, and Vasile Palade. Design of a cost-effective deep convolutional neural network–based scheme for diagnosing faults in smart grids. In *2019 18th IEEE International Conference On Machine Learning And Applications (ICMLA)*, pages 1420–1425. IEEE, 2019.

[12] Hossein Hassani, Roozbeh Razavi-Far, and Mehrdad Saif. Locating faults in smart grids using neuro–fuzzy networks. In *2019 IEEE International Conference on Systems, Man and Cybernetics (SMC)*, pages 3281–3286. IEEE, 2019.

[13] Hossein Hassani, Roozbeh Razavi-Far, and Mehrdad Saif. Dynamic group decision support models for locating faults in power systems. In *2021 4th IEEE International Conference on Industrial Cyber-Physical Systems (ICPS)*, pages 321–327, 2021.

[14] Hossein Hassani, Roozbeh Razavi-Far, and Mehrdad Saif. A comparative assessment of dimensionality reduction techniques for diagnosing faults in smart grids. In *2020 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pages 3618–3623. IEEE, 2020.

[15] Riku-Pekka Nikula, Konsta Karioja, Mika Pylvänäinen, and Kauko Leiviskä. Automation of low-speed bearing fault diagnosis based on autocorrelation of time domain features. *Mechanical Systems and Signal Processing*, 138:106572, 2020.

[16] Amandeep Sharma, Rajvardhan Jigyasu, Lini Mathew, and Shantanu Chatterji. Bearing fault diagnosis using frequency domain features and artificial neural networks. In *Information and Communication Technology for Intelligent Systems*, pages 539–547. Springer, 2019.

[17] Boualem Boashash and Larbi Boubchir. On the selection of time-frequency features for improving the detection and classification of newborn eeg seizure signals and other abnormalities. In *International Conference on Neural Information Processing*, pages 634–643. Springer, 2012.

[18] Van Huan Pham, Soonyoung Han, Minh Duc Do, and Hae-Jin Choi. A wavelet packet spectral subtraction and convolutional neural network based method for diagnosis of system health. *Journal of Mechanical Science and Technology*, 33(12):5683–5687, 2019.

[19] Jonathan S Smith. The local mean decomposition and its application to eeg perception data. *Journal of the Royal Society Interface*, 2(5):443–454, 2005.

[20] J Ji, J Qu, Y Chai, Y Zhou, and Q Tang. Sensor fault diagnosis using ensemble empirical mode decomposition and extreme learning machine. In *Chinese Intelligent Systems Conference*, pages 199–209. Springer, 2016.

[21] Huaiguang Jiang, Jun J Zhang, Wenzhong Gao, and Ziping Wu. Fault detection, identification, and location in smart grid based on data-driven computational methods. *IEEE Transactions on Smart Grid*, 5(6):2947–2956, 2014.

[22] Yagyensh Chandra Pati, Ramin Rezaiifar, and Perinkulam Sambamurthy Krishnaprasad. Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition. In *Proceedings of 27th Asilomar conference on signals, systems and computers*, pages 40–44. IEEE, 1993.

[23] Simon Foucart. Stability and robustness of weak orthogonal matching pursuits. In *Recent advances in harmonic analysis and applications*, pages 395–405. Springer, 2012.

[24] Konstantin Dragomiretskiy and Dominique Zosso. Variational mode decomposition. *IEEE transactions on signal processing*, 62(3):531–544, 2013.

[25] Ehsan Hallaji, Maryam Farajzadeh-Zanjani, Roozbeh Razavi-Far, Vasile Palade, and Mehrdad Saif. Constrained generative adversarial learning for dimensionality reduction. *IEEE Transactions on Knowledge and Data Engineering*, 35(3):2394–2405, 2023.

[26] Hossein Hassani, Roozbeh Razavi-Far, and Mehrdad Saif. Fault diagnosis in smart grids using a deep long short-term memory-based feature learning architecture. In *Proceedings of the 30th European Safety and Reliability Conference and the 15th Probabilistic Safety Assessment and Management Conference*. IEEE, 2020.

[27] Roozbeh Razavi-Far, Daoming Wan, Mehrdad Saif, and Niloofar Mozafari. To tolerate or to impute missing values in v2x communications data? *IEEE Internet of Things Journal*, 9(13):11442–11452, 2021.

[28] Feng Wang, Jiye Liang, and Chuangyin Dang. Attribute reduction for dynamic data sets. *Applied Soft Computing*, 13(1):676–689, 2013.

[29] Abbas Jalilvand and Naomie Salim. Feature unionization: a novel approach for dimension reduction. *Applied Soft Computing*, 52:1253–1261, 2017.

[30] Manoranjan Dash and Huan Liu. Feature selection for classification. *Intelligent data analysis*, 1(1-4):131–156, 1997.

[31] Majdi Mafarja and Seyedali Mirjalili. Whale optimization approaches for wrapper feature selection. *Applied Soft Computing*, 62:441–453, 2018.

[32] Giorgio Roffo. Ranking to learn and learning to rank: On the role of ranking in pattern recognition applications. *arXiv preprint arXiv:1706.05933*, 2017.

[33] Roozbeh Razavi-Far and Michel Kinnaert. Incremental design of a decision system for residual evaluation: A wind turbine application. *IFAC Proceedings Volumes*, 45(20):343–348, 2012.

[34] Luis J De Miguel and L Felipe Blázquez. Fuzzy logic-based decision-making for fault diagnosis in a dc motor. *Engineering Applications of Artificial Intelligence*, 18(4):423–450, 2005.

[35] Roozbeh Razavi-Far, Hadi Davilu, Vasile Palade, and Caro Lucas. Neuro-fuzzy based fault diagnosis of a steam generator. *IFAC Proceedings Volumes*, 42(8):1180–1185, 2009.

[36] Chonghui Zhang, Weihua Su, Shouzhen Zeng, Tomas Balezentis, and Enrique Herrera-Viedma. A two-stage subgroup decision-making method for processing large-scale information. *Expert Systems with Applications*, 171:114586, 2021.

[37] Enrique Herrera-Viedma, Iván Palomares, Cong-Cong Li, Francisco Javier Cabrerizo, Yucheng Dong, Francisco Chiclana, and Francisco Herrera. Revisiting fuzzy and linguistic decision making: Scenarios and challenges for making

wiser decisions in a better way. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 51(1):191–208, 2021.

[38] Xiangyu Zhong, Xuanhua Xu, and Bin Pan. A non-threshold consensus model based on the minimum cost and maximum consensus-increasing for multi-attribute large group decision-making. *Information Fusion*, 77:90–106, 2022.

[39] I.J. Pérez, F.J. Cabrerizo, S. Alonso, Y.C. Dong, F. Chiclana, and E. Herrera-Viedma. On dynamic consensus processes in group decision making problems. *Information Sciences*, 459:20–35, 2018.

[40] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, Jafar Zarei, and Frede Blaabjerg. Intelligent decision support and fusion models for fault detection and location in power grids. *IEEE Transactions on Emerging Topics in Computational Intelligence*, 6(3):530–543, 2022.

[41] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Enrique Herrera-Viedma. Reinforcement learning-based feedback and weight-adjustment mechanisms for consensus reaching in group decision making. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2022.

[42] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Enrique Herrera-Viedma. Consensus-based decision support model and fusion architecture for dynamic decision making. *Information Sciences*, 597:86–104, 2022.

[43] I.J. Pérez, F.J. Cabrerizo, and E. Herrera-Viedma. Group decision making problems in a linguistic and dynamic context. *Expert Systems with Applications*, 38(3):1675–1688, 2011.

[44] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, Francisco Chiclana, Ondrej Krejcar, and Enrique Herrera-Viedma. Classical dynamic consensus and

opinion dynamics models: A survey of recent trends and methodologies. *Information Fusion*, 88:22–40, 2022.

[45] Richard A Holley and Thomas M Liggett. Ergodic theorems for weakly interacting infinite systems and the voter model. *The annals of probability*, pages 643–663, 1975.

[46] Katarzyna Sznajd-Weron and Jozef Sznajd. Opinion evolution in closed community. *International Journal of Modern Physics C*, 11(06):1157–1165, 2000.

[47] Morris H DeGroot. Reaching a consensus. *Journal of the American Statistical association*, 69(345):118–121, 1974.

[48] Rainer Hegselmann, Ulrich Krause, et al. Opinion dynamics and bounded confidence models, analysis, and simulation. *Journal of artificial societies and social simulation*, 5(3), 2002.

[49] Guillaume Deffuant, David Neau, Frederic Amblard, and Gerard Weisbuch. Mixing beliefs among interacting agents. *Advances in Complex Systems*, (3):11, 2001.

[50] Gérard Weisbuch, Guillaume Deffuant, Frédéric Amblard, and Jean-Pierre Nadal. Meet, discuss, and segregate! *Complexity*, 7(3):55–63, 2002.

[51] F. Herrera and L. Martinez. A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 8(6):746–752, 2000.

[52] Lotfi A Zadeh. A note on z-numbers. *Information Sciences*, 181(14):2923–2932, 2011.

[53] Rosa M. Rodriguez, Luis Martinez, and Francisco Herrera. Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, 20(1):109–119, 2012.

[54] Bin Zhu and Zeshui Xu. Consistency measures for hesitant fuzzy linguistic preference relations. *IEEE Transactions on Fuzzy Systems*, 22(1):35–45, 2014.

[55] Krassimir Atanassov. Intuitionistic fuzzy sets. *International journal bioautomation*, 20:1, 2016.

[56] E Szmidt and J Kacprzyk. Group decision making under intuitionistic fuzzy preference relations. In *IPMU: information processing and management of uncertainty in knowledge-based systems (Paris, 6-10 July 1998)*, pages 172–178, 1998.

[57] Ronald R. Yager. Multicriteria decision making with ordinal/linguistic intuitionistic fuzzy sets for mobile apps. *IEEE Transactions on Fuzzy Systems*, 24(3):590–599, 2016.

[58] Fanyong Meng, Jie Tang, and Hamido Fujita. Linguistic intuitionistic fuzzy preference relations and their application to multi-criteria decision making. *Information Fusion*, 46:77–90, 2019.

[59] JQ Wang and JJ Li. The multi-criteria group decision making method based on multi-granularity intuitionistic two semantics. *Science & Technology Information*, 33(1):8–9, 2009.

[60] Wenqi Liu, Yucheng Dong, Francisco Chiclana, Francisco Javier Cabrerizo, and Enrique Herrera-Viedma. Group decision-making based on heterogeneous preference relations with self-confidence. *Fuzzy Optim. Decis. Mak.*, 16(4):429–447, 2017.

[61] Shennan Zhu, Jing Huang, and Yejun Xu. A consensus model for group decision making with self-confident linguistic preference relations. *Int. J. Intell. Syst.*, 2021.

[62] Jinpei Liu, Mengdi Fang, Feifei Jin, Zhifu Tao, Huayou Chen, and Pengcheng Du. Pythagorean fuzzy linguistic decision support model based on consistency-adjustment strategy and consensus reaching process. *Soft Comput.*, 25(13):8205–8221, 2021.

[63] Ronald R Yager and Ali M Abbasov. Pythagorean membership grades, complex numbers, and decision making. *Int. J. Intell. Syst.*, 28(5):436–452, 2013.

[64] Yuzhu Wu, Yucheng Dong, Jindong Qin, and Witold Pedrycz. Flexible linguistic expressions and consensus reaching with accurate constraints in group decision-making. *IEEE Trans. Cybernet.*, 50(6):2488–2501, 2019.

[65] Xunjie Gou, Zeshui Xu, Xinxin Wang, and Huchang Liao. Managing consensus reaching process with self-confident double hierarchy linguistic preference relations in group decision making. *Fuzzy Optim. Decis. Mak.*, 20(1):51–79, 2021.

[66] Ye Tian, Xiangjun Mi, Yunpeng Ji, and Bingyi Kang. Ze-numbers: A new extended z-numbers and its application on multiple attribute group decision making. *Eng. Appl. Artific. Intell.*, 101:104225, 2021.

[67] Jiahui Chai, Sidong Xian, and Sichong Lu. Z probabilistic linguistic term sets and its application in multi-attribute group decision making. *Fuzzy Optim. Decis. Mak.*, pages 1–38, 2021.

[68] Qianlei Jia, Jiayue Hu, Qizhi He, Weiguo Zhang, and Ehab Safwat. A multicriteria group decision-making method based on aivifss, z-numbers, and trapezium clouds. *Inf. Sci.*, 566:38–56, 2021.

[69] Diego García-Zamora, Álvaro Labella, Rosa M Rodríguez, and Luis Martínez. Nonlinear preferences in group decision-making. extreme values amplifications and extreme values reductions. *Int. J. Intell. Syst.*, 2021.

[70] Fei Teng, Peide Liu, and Xia Liang. Unbalanced probabilistic linguistic decision-making method for multi-attribute group decision-making problems with heterogeneous relationships and incomplete information. *Artific. Intell. Review*, 54(5):3431–3471, 2021.

[71] Zhiming Zhang and Shyi-Ming Chen. Group decision making with incomplete q-rung orthopair fuzzy preference relations. *Inf. Sci.*, 553:376–396, 2021.

[72] Ziyu Yang, Liyuan Zhang, and Tao Li. Group decision making with incomplete interval-valued q-rung orthopair fuzzy preference relations. *Int. J. Intell. Syst.*

[73] Dimple Rani and Harish Garg. Complex intuitionistic fuzzy preference relations and their applications in individual and group decision-making problems. *Int. J. Intell. Syst.*, 36(4):1800–1830, 2021.

[74] Mostafa Hajiaghaei-Keshteli, Zeynep Cenk, Babek Erdebilli, Yavuz Selim Özdemir, and Fatemeh Gholian-Jouybari. Pythagorean fuzzy topsis method for green supplier selection in the food industry. *Expert Systems with Applications*, 224:120036, 2023.

[75] Jishu Jana and Sankar Kumar Roy. Linguistic pythagorean hesitant fuzzy matrix game and its application in multi-criteria decision making. *Applied Intelligence*, 53(1):1–22, 2023.

[76] Sha Fan, Haiming Liang, Yucheng Dong, and Witold Pedrycz. A personalized individual semantics-based multi-attribute group decision making approach with flexible linguistic expression. *Expert Systems with Applications*, 192:116392, 2022.

[77] Ruichen Zhang, Zeshui Xu, and Xunjie Gou. Electre ii method based on the cosine similarity to evaluate the performance of financial logistics enterprises

under double hierarchy hesitant fuzzy linguistic environment. *Fuzzy Optimization and Decision Making*, 22(1):23–49, 2023.

[78] Wen He, Rosa M Rodríguez, Bapi Dutta, and Luis Martínez. A type-1 owa operator for extended comparative linguistic expressions with symbolic translation. *Fuzzy Sets and Systems*, 446:167–192, 2022.

[79] Arijit Mondal, Sankar Kumar Roy, and Jianming Zhan. A reliability-based consensus model and regret theory-based selection process for linguistic hesitant-z multi-attribute group decision making. *Expert Systems with Applications*, page 120431, 2023.

[80] Jiahui Chai, Yi Su, and Sichong Lu. Linguistic z-number preference relation for group decision making and its application in digital transformation assessment of smes. *Expert Systems with Applications*, 213:118749, 2023.

[81] Mohuya Byabartta Kar, Raghunathan Krishankumar, Dragan Pamucar, and Samarjit Kar. A decision framework with nonlinear preferences and unknown weight information for cloud vendor selection. *Expert Systems with Applications*, 213:118982, 2023.

[82] Yejun Xu, Shennan Zhu, Xia Liu, Jing Huang, and Enrique Herrera-Viedma. Additive consistency exploration of linguistic preference relations with self-confidence. *Artificial Intelligence Review*, 56(1):257–285, 2023.

[83] Tao Li, Liyuan Zhang, and Zhenglong Zhang. Incomplete linguistic q-rung orthopair fuzzy preference relations and their application to multi-criteria decision making. *Complex & Intelligent Systems*, pages 1–19, 2023.

[84] Harish Garg and Dimple Rani. New prioritized aggregation operators with priority degrees among priority orders for complex intuitionistic fuzzy information.

Journal of Ambient Intelligence and Humanized Computing, 14(3):1373–1399, 2023.

[85] Peide Liu, Ran Dang, Peng Wang, and Xiaoming Wu. Unit consensus cost-based approach for group decision-making with incomplete probabilistic linguistic preference relations. *Information Sciences*, 2023.

[86] Xingli Wu and Huchang Liao. Managing uncertain preferences of consumers in product ranking by probabilistic linguistic preference relations. *Knowledge-Based Systems*, 262:110240, 2023.

[87] Hengjie Zhang, Xiaomin Wang, Weijun Xu, and Yucheng Dong. From numerical to heterogeneous linguistic best–worst method: Impacts of personalized individual semantics on consistency and consensus. *Engineering Applications of Artificial Intelligence*, 117:105495, 2023.

[88] Qun Wu, Xinwang Liu, Jindong Qin, and Ligang Zhou. Multi-criteria group decision-making for portfolio allocation with consensus reaching process under interval type-2 fuzzy environment. *Inf. Sci.*, 2021.

[89] Cheng Zhang, Huchang Liao, Li Luo, and Zeshui Xu. Distance-based consensus reaching process for group decision making with intuitionistic multiplicative preference relations. *Appl. Soft Comput.*, 88:106045, 2020.

[90] Donghai Liu and An Huang. Consensus reaching process for fuzzy behavioral topsis method with probabilistic linguistic q-rung orthopair fuzzy set based on correlation measure. *Int. J. Intell. Syst.*, 35(3):494–528, 2020.

[91] Salih Berkan Aydemir and Sevcan Yilmaz Gunduz. A novel approach to multi-attribute group decision making based on power neutrality aggregation operator for q-rung orthopair fuzzy sets. *Int. J. Intell. Syst.*, 36(3):1454–1481, 2021.

[92] Arun Sarkar and Animesh Biswas. Dual hesitant q-rung orthopair fuzzy dombi t-conorm and t-norm based bonferroni mean operators for solving multicriteria group decision making problems. *Int. J. Intell. Syst.*, 36(7):3293–3338, 2021.

[93] Abhijit Saha, Tapan Senapati, and Ronald R Yager. Hybridizations of generalized dombi operators and bonferroni mean operators under dual probabilistic linguistic environment for group decision-making. *Int. J. Intell. Syst.*

[94] Hong-gang Peng, Xiao-kang Wang, Hong-Yu Zhang, and Jian-qiang Wang. Group decision-making based on the aggregation of z-numbers with archimedean t-norms and t-conorms. *Inf. Sci.*, 569:264–286, 2021.

[95] Qianshan Zhan, Chao Fu, and Min Xue. Distance-based large-scale group decision-making method with group influence. *Int. J. Fuzzy Syst.*, 23(2):535–554, 2021.

[96] Zhang Yiru, Bouadi Tassadit, Wang Yewan, and Martin Arnaud. A distance for evidential preferences with application to group decision making. *Inf. Sci.*, 568:113–132, 2021.

[97] Su-min Yu, Zhi-jiao Du, Jian-qiang Wang, Han-yang Luo, and Xu-dong Lin. Trust and behavior analysis-based fusion method for heterogeneous multiple attribute group decision-making. *Computers Indust. Eng.*, 152:106992, 2021.

[98] Fanyong Meng, Shyi-Ming Chen, and Ruiping Yuan. Group decision making with heterogeneous intuitionistic fuzzy preference relations. *Inf. Sci.*, 523:197–219, 2020.

[99] Fei Teng, Peide Liu, and Xia Liang. Unbalanced probabilistic linguistic decision-making method for multi-attribute group decision-making problems with heterogeneous relationships and incomplete information. *Artific. Intell. Review*, 54(5):3431–3471, 2021.

[100] Kai Zhang, Jing Zheng, and Ying-Ming Wang. Heterogeneous multi-attribute case retrieval method based on group decision making using incomplete weight information. *J. Intell. Fuzzy Syst.*, (Preprint):1–13, 2021.

[101] Yingying Liang, Jindong Qin, Luis Martínez, and Jun Liu. A heterogeneous qualiflex method with criteria interaction for multi-criteria group decision making. *Inf. Sci.*, 512:1481–1502, 2020.

[102] Juan Antonio Morente-Molinera, X Wu, Ali Morfeq, Rami Al-Hmouz, and Enrique Herrera-Viedma. A novel multi-criteria group decision-making method for heterogeneous and dynamic contexts using multi-granular fuzzy linguistic modelling and consensus measures. *Inf. Fusion*, 53:240–250, 2020.

[103] Atefeh Taghavi, Esfandiar Eslami, Enrique Herrera-Viedma, and Raquel Urena. Trust based group decision making in environments with extreme uncertainty. *Knowl.-Based Syst.*, 191:105168, 2020.

[104] Xiao-guo Chen, Gao-feng Yu, Jian Wu, and Yue Yang. A minimum trust discount coefficient model for incomplete information in group decision making with intuitionistic fuzzy soft set. *Int. J. Fuzzy Syst.*, 22(6):2025–2040, 2020.

[105] Jian Wu, Xue Li, Francisco Chiclana, and Ronald Yager. An attitudinal trust recommendation mechanism to balance consensus and harmony in group decision making. *IEEE Trans. Fuzzy Syst.*, 27(11):2163–2175, 2019.

[106] Benhong Peng, Chaoyu Zheng, Xuan Zhao, Guo Wei, and Anxia Wan. Pythagorean fuzzy multiattribute group decision making based on risk attitude and evidential reasoning methodology. *Int. J. Intell. Syst.*

[107] Jingjing Hao and Francisco Chiclana. Attitude quantifier based possibility distribution generation method for hesitant fuzzy linguistic group decision making. *Inf. Sci.*, 518:341–360, 2020.

[108] Huchang Liao, Lisi Kuang, Yuxi Liu, and Ming Tang. Non-cooperative behavior management in group decision making by a conflict resolution process and its implementation for pharmaceutical supplier selection. *Inf. Sci.*, 567:131–145, 2021.

[109] Xiaofang Li, Huchang Liao, and Zhi Wen. A consensus model to manage the non-cooperative behaviors of individuals in uncertain group decision making problems during the covid-19 outbreak. *Appl. Soft Comput.*, 99:106879, 2021.

[110] Ming Tang, Huchang Liao, Xiaomei Mi, Benjamin Lev, and Witold Pedrycz. A hierarchical consensus reaching process for group decision making with noncooperative behaviors. *Eur. J. Oper. Res.*, 293(2):632–642, 2021.

[111] Meysam Rabiee, Babak Aslani, and Jafar Rezaei. A decision support system for detecting and handling biased decision-makers in multi criteria group decision-making problems. *Expert Syst. Appl.*, 171:114597, 2021.

[112] Ming Tang and Huchang Liao. From conventional group decision making to large-scale group decision making: What are the challenges and how to meet them in big data era? a state-of-the-art survey. *Omega*, 100:102141, 2021.

[113] Tong Wu, Xinwang Liu, Jindong Qin, and Francisco Herrera. Balance dynamic clustering analysis and consensus reaching process with consensus evolution networks in large-scale group decision making. *IEEE Trans. Fuzzy Syst.*, 29(2):357–371, 2019.

[114] Vincent D Blondel, Jean-Loup Guillaume, Renaud Lambiotte, and Etienne Lefebvre. Fast unfolding of communities in large networks. *J. Stat. Mech. Theory Exp.*, 2008(10):P10008, 2008.

[115] Zhang-Peng Tian, Ru-Xin Nie, Jian-Qiang Wang, and Ru-Yin Long. Adaptive consensus-based model for heterogeneous large-scale group decision-making:

Detecting and managing noncooperative behaviors. *IEEE Trans. Fuzzy Syst.*, 29(8):2209–2223, 2021.

[116] Xiangrui Chao, Gang Kou, Yi Peng, and Enrique Herrera Viedma. Large-scale group decision-making with non-cooperative behaviors and heterogeneous preferences: an application in financial inclusion. *Eur. J. Oper. Res.*, 288(1):271–293, 2021.

[117] Prasenjit Mandal, Sovan Samanta, and Madhumangal Pal. Large-scale group decision-making based on pythagorean linguistic preference relations using experts clustering and consensus measure with non-cooperative behavior analysis of clusters. *Complex Intell. Syst.*, pages 1–15, 2021.

[118] Tong Wu, Kun Zhang, Xinwang Liu, and Changyan Cao. A two-stage social trust network partition model for large-scale group decision-making problems. *Knowl.-Based Syst.*, 163:632–643, 2019.

[119] Chenxi Zhang, Meng Zhao, Lichao Zhao, and Qinfei Yuan. A consensus model for large-scale group decision-making based on the trust relationship considering leadership behaviors and non-cooperative behaviors. *Group Dec. Neg.*, 30(3):553–586, 2021.

[120] Zhi-jiao Du, Han-yang Luo, Xu-dong Lin, and Su-min Yu. A trust-similarity analysis-based clustering method for large-scale group decision-making under a social network. *Inf. Fusion*, 63:13–29, 2020.

[121] Giovanni F Massari, Ilaria Giannoccaro, and Giuseppe Carbone. Are distrust relationships beneficial for group performance? the influence of the scope of distrust on the emergence of collective intelligence. *Int. J. Prod. Econ.*, 208:343–355, 2019.

[122] Feng Pei, Yu-Wei He, An Yan, Mi Zhou, Yu-Wang Chen, and Jian Wu. A consensus model for intuitionistic fuzzy group decision-making problems based on the construction and propagation of trust/distrust relationships in social networks. *Int. J. Fuzzy Syst.*, 22(8):2664–2679, 2020.

[123] Jian Wu, Mingshuo Cao, Francisco Chiclana, Yucheng Dong, and Enrique Herrera-Viedma. An Optimal Feedback Model to Prevent Manipulation Behavior in Consensus Under Social Network Group Decision Making. *IEEE Trans. Fuzzy Syst.*, 29(7):1750–1763, JUL 2021.

[124] Raquel Urena, Gang Kou, Yucheng Dong, Francisco Chiclana, and Enrique Herrera-Viedma. A review on trust propagation and opinion dynamics in social networks and group decision making frameworks. *Inf. Sci.*, 478:461–475, 2019.

[125] Yujia Liu, Changyong Liang, Francisco Chiclana, and Jian Wu. A knowledge coverage-based trust propagation for recommendation mechanism in social network group decision making. *Appl. Soft Comput.*, 101:107005, 2021.

[126] Jian Wu, Zhiwei Zhao, Qi Sun, and Hamido Fujita. A maximum self-esteem degree based feedback mechanism for group consensus reaching with the distributed linguistic trust propagation in social network. *Inf. Fusion*, 67:80–93, 2021.

[127] Feifei Jin, Meng Cao, Jinpei Liu, Luis Martínez, and Huayou Chen. Consistency and trust relationship-driven social network group decision-making method with probabilistic linguistic information. *Appl. Soft Comput.*, 103:107170, 2021.

[128] Ruxue Ren, Ming Tang, and Huchang Liao. Managing minority opinions in micro-grid planning by a social network analysis-based large scale group decision making method with hesitant fuzzy linguistic information. *Knowl-Based Syst.*, 189:105060, 2020.

[129] Xiaoyang Zhou, Feipeng Ji, Liqin Wang, Yanfang Ma, and Hamido Fujita. Particle swarm optimization for trust relationship based social network group decision making under a probabilistic linguistic environment. *Knowl.-Based Syst.*, 200:105999, 2020.

[130] Dong Cheng, Faxin Cheng, Zhili Zhou, and Yong Wu. Reaching a minimum adjustment consensus in social network group decision-making. *Inf. Fusion*, 59:30–43, 2020.

[131] Jing Xiao, Xiuli Wang, and Hengjie Zhang. Managing classification-based consensus in social network group decision making: An optimization-based approach with minimum information loss. *Inf. Fusion*, 63:74–87, 2020.

[132] Junfeng Chu, Yingming Wang, Xinwang Liu, and Yicong Liu. Social network community analysis based large-scale group decision making approach with incomplete fuzzy preference relations. *Inf. Fusion*, 60:98–120, 2020.

[133] Hengjie Zhang, Sihai Zhao, Gang Kou, Cong-Cong Li, Yucheng Dong, and Francisco Herrera. An overview on feedback mechanisms with minimum adjustment or cost in consensus reaching in group decision making: Research paradigms and challenges. *Inf. Fusion*, 60:65–79, 2020.

[134] Yanling Lu, Yejun Xu, Enrique Herrera-Viedma, and Yefan Han. Consensus of large-scale group decision making in social network: the minimum cost model based on robust optimization. *Inf. Sci.*, 547:910–930, 2021.

[135] Yuxiang Yuan, Dong Cheng, and Zhili Zhou. A minimum adjustment consensus framework with compromise limits for social network group decision making under incomplete information. *Inf. Sci.*, 549:249–268, 2021.

[136] Jie Long, Haiming Liang, Lei Gao, Zhaoxia Guo, and Yucheng Dong. Consensus reaching with two-stage minimum adjustments in multi-attribute group decision

making: A method based on preference-approval structure and prospect theory. *Comput. Ind. Eng.*, 158:107349, 2021.

[137] Qinyue Zhou, Zhibin Wu, Abdulrahman H. Altalhi, and Francisco Herrera. A two-step communication opinion dynamics model with self-persistence and influence index for social networks based on the DeGroot model. *Inf. Sci.*, 519:363–381, 2020.

[138] Ke Li, Haiming Liang, Gang Kou, and Yucheng Dong. Opinion dynamics model based on the cognitive dissonance: An agent-based simulation. *Inf. Fusion*, 56:1–14, 2020.

[139] Renbin Xiao, Tongyang Yu, and Jundong Hou. Modeling and simulation of opinion natural reversal dynamics with opinion leader based on hk bounded confidence model. *Complexity*, 2020, 2020.

[140] Yi Lu, Yiyi Zhao, Jiangbo Zhang, Jiangping Hu, and Xiaoming Hu. Fuzzy Hegselmann-Krause opinion dynamics with opinion leaders. In *2019 Chinese Control Conf. (CCC)*, pages 6019–6024. IEEE, 2019.

[141] Yucheng Dong, Zhaogang Ding, Luis Martínez, and Francisco Herrera. Managing consensus based on leadership in opinion dynamics. *Inf. Sci.*, 397:187–205, 2017.

[142] Stephen Warshall. A theorem on boolean matrices. *J. ACM (JACM)*, 9(1):11–12, 1962.

[143] Zhen Zhang, Yuan Gao, and Zhuolin Li. Consensus reaching for social network group decision making by considering leadership and bounded confidence. *Knowl.-Based Syst.*, 204:106240, 2020.

[144] Chun Cheng, Yaoxian Song, and Changbin Yu. Group pressure leads to consensus of Hegselmann-Krause opinion dynamics. In *2019 Chinese Control Conf. (CCC)*, pages 7945–7949. IEEE, 2019.

[145] Chun Cheng and Changbin Yu. Opinion dynamics with bounded confidence and group pressure. *Phys. A: Stat. Mech. Appl.*, 532:121900, 2019.

[146] Justin Semonsen, Christopher Griffin, Anna Squicciarini, and Sarah Rajtmajer. Opinion dynamics in the presence of increasing agreement pressure. *IEEE Trans. Cybernet.*, 49(4):1270–1278, 2018.

[147] Koresh Khateri, Mahdi Pourgholi, Mohsen Montazeri, and Lorenzo Sabattini. Effect of stubborn agents on bounded confidence opinion dynamic systems: Unanimity in presence of stubborn agents. In *2019 27th Iranian Conf. Elec. Eng. (ICEE)*, pages 875–880. IEEE, 2019.

[148] David Bindel, Jon Kleinberg, and Sigal Oren. How bad is forming your own opinion? volume 92, pages 248–265. Elsevier, 2015.

[149] Guang He, Wenbing Zhang, Jing Liu, and Haoyue Ruan. Opinion dynamics with the increasing peer pressure and prejudice on the signed graph. *Nonlinear Dyn.*, pages 1–13, 2020.

[150] Yanhong Li, Gang Kou, Guangxu Li, and Haomin Wang. Multi-attribute group decision making with opinion dynamics based on social trust network. *Inf. Fusion*, 75:102–115, 2021.

[151] David Angeli and Sabato Manfredi. Criteria for asymptotic clustering of opinion dynamics towards bimodal consensus. *Automatica*, 103:230–238, 2019.

[152] Guang He, Jing Liu, Huimin Hu, and Jian-An Fang. Discrete-time signed bounded confidence model for opinion dynamics. *Neurocomputing*, 2019.

[153] André CR Martins. Continuous opinions and discrete actions in opinion dynamics problems. *Int. J. Mod. Phys. C*, 19(04):617–624, 2008.

[154] Min Zhan, Gang Kou, Yucheng Dong, Francisco Chiclana, and Enrique Herrera-Viedma. Bounded confidence evolution of opinions and actions in social networks. *IEEE Trans. Cybernet.*, pages 1–12, 2021.

[155] Jian Hou, Wenshan Li, and Mingyue Jiang. Opinion dynamics in modified expressed and private model with bounded confidence. *Phys. A: Stat. Mech. Appl.*, 574:125968, 2021.

[156] Alessandro Nordio, Alberto Tarable, Carla-Fabiana Chiasserini, and Emilio Leonardi. Opinion dynamics on correlated subjects in social networks. *IEEE Trans. Netw. Sci. Eng.*, 7(3):1901–1912, 2019.

[157] Andrea Baronchelli. The emergence of consensus: a primer. *Royal Soc. Open Sci.*, 5(2):172189, 2018.

[158] Francois Baccelli, Avhishek Chatterjee, and Sriram Vishwanath. Pairwise stochastic bounded confidence opinion dynamics: Heavy tails and stability. *IEEE Trans. Automat. Contr.*, 62(11):5678–5693, 2017.

[159] Paolo Bolzern, Patrizio Colaneri, and Giuseppe De Nicolao. Opinion influence and evolution in social networks: A markovian agents model. *Automatica*, 100:219–230, 2019.

[160] Paolo Bolzern, Patrizio Colaneri, and Giuseppe De Nicolao. Opinion dynamics in social networks: The effect of centralized interaction tuning on emerging behaviors. *IEEE Trans. Comput. Soc. Syst.*, 7(2):362–372, 2020.

[161] Simone Mariano, IC Morărescu, Romain Postoyan, and Luca Zaccarian. A hybrid model of opinion dynamics with memory-based connectivity. *IEEE Control Syst. Lett.*, 4(3):644–649, 2020.

[162] Guang He, Haoyue Ruan, Yanlei Wu, and Jing Liu. Opinion dynamics with competitive relationship and switching topologies. *IEEE Access*, 9:3016–3025, 2020.

[163] Qingxing Dong, Xin Zhou, and Luis Martinez. A hybrid group decision making framework for achieving agreed solutions based on stable opinions. *Inf. Sci.*, 490:227–243, 2019.

[164] Mingwu Li and Harry Dankowicz. Impact of temporal network structures on the speed of consensus formation in opinion dynamics. *Phys. A: Stat. Mech. Appl.*, 523:1355–1370, 2019.

[165] Hengjie Zhang, Yucheng Dong, Jing Xiao, Francisco Chiclana, and Enrique Herrera-Viedma. Consensus and opinion evolution-based failure mode and effect analysis approach for reliability management in social network and uncertainty contexts. *Reliab. Eng. Syst. Saf.*, 208:107425, 2021.

[166] Yucheng Dong, Min Zhan, Zhaogang Ding, Haiming Liang, and Francisco Herrera. Numerical interval opinion dynamics in social networks: Stable state and consensus. *IEEE Trans. Fuzzy Syst.*, 29(3):584–598, 2019.

[167] Yucheng Dong, Xia Chen, Haiming Liang, and Cong-Cong Li. Dynamics of linguistic opinion formation in bounded confidence model. *Inf. Fusion*, 32:52–61, 2016.

[168] Haiming Liang, Cong-Cong Li, Yucheng Dong, and Francisco Herrera. Linguistic opinions dynamics based on personalized individual semantics. *IEEE Trans. Fuzzy Syst.*, pages 1–1, 2020.

[169] Yucheng Dong, Yinfeng Xu, and Shui Yu. Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model. *IEEE Trans. Fuzzy Syst.*, 17(6):1366–1378, 2009.

[170] Zhen Zhang, Zhuolin Li, and Yuan Gao. Consensus reaching for group decision making with multi-granular unbalanced linguistic information: A bounded confidence and minimum adjustment-based approach. *Inf. Fusion*, 74:96–110, 2021.

[171] Yixin Zhang, Zeshui Xu, Zhinan Hao, and Huchang Liao. Dynamic assessment of internet public opinions based on the probabilistic linguistic bayesian network and prospect theory. *Appl. Soft Comput.*, 106:107359, 2021.

[172] Mingwei Wang, Fangshun Li, and Decui Liang. Opinion dynamics and consensus achievement strategy based on reinforcement learning. In *2020 IEEE Int. Symp. Signal Process. Inf. Tech. (ISSPIT)*, pages 1–6. IEEE, 2020.

[173] Vivek Borkar and Alexandre Reiffers-Masson. Opinion shaping in social networks using reinforcement learning. *arXiv preprint arXiv:1910.08802*, 2019.

[174] Sven Banisch and Eckehard Olbrich. Opinion polarization by learning from social feedback. *J. Math. Sociol.*, 43(2):76–103, 2019.

[175] Raphaël Truffet. Dynamical networks for modeling opinion dynamics in social networks. 2017.

[176] Chao Yu, Guozhen Tan, Hongtao Lv, Zhen Wang, Jun Meng, Jianye Hao, and Fenghui Ren. Modelling adaptive learning behaviours for consensus formation in human societies. *Sci. Rep.*, 6(1):1–13, 2016.

[177] Felix Gaisbauer, Eckehard Olbrich, and Sven Banisch. Dynamics of opinion expression. *Phys. Rev. E.*, 102(4):042303, 2020.

[178] Cédric Colas, Olivier Sigaud, and Pierre-Yves Oudeyer. Gep-pg: Decoupling exploration and exploitation in deep reinforcement learning algorithms. In *Int. Conf. Mach. Learn.*, pages 1039–1048. PMLR, 2018.

[179] Chengwei Zhang, Xiaohong Li, Jianye Hao, Sandip Sen, Wanli Xue, and Zhiyong Feng. The dynamics of opinion evolution in gossiper-media model with WoLS-CALA learning. In *Proc. 17th Int. Conf. Auton. Agents Multi-Agent Syst.*, pages 2159–2161, 2018.

[180] Carmen De Maio, Giuseppe Fenza, Vincenzo Loia, Francesco Orciuoli, and Enrique Herrera-Viedma. A framework for context-aware heterogeneous group decision making in business processes. *Knowl.-Based Syst.*, 102:39–50, 2016.

[181] Carmen De Maio, Giuseppe Fenza, Vincenzo Loia, Francesco Orciuoli, and Enrique Herrera-Viedma. A context-aware fuzzy linguistic consensus model supporting innovation processes. In *2016 IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, pages 1685–1692. IEEE, 2016.

[182] Yiyi Zhao, Min Xu, Yucheng Dong, and Yi Peng. Fuzzy inference based Hegselmann-Krause opinion dynamics for group decision-making under ambiguity. *Inf. Process. Manag.*, 58(5):102671, 2021.

[183] Quanbo Zha, Haiming Liang, Gang Kou, Yucheng Dong, and Shui Yu. A feedback mechanism with bounded confidence-based optimization approach for consensus reaching in multiple attribute large-scale group decision-making. *IEEE Trans. Comput. Soc. Syst.*, 6(5):994–1006, 2019.

[184] Quanbo Zha, Haiming Liang, et al. An optimization based consensus model in multiple attribute group decision making with individual bounded confidences. In *2019 IEEE Int. Conf. Syst., Man Cybernet. (SMC)*, pages 4025–4030. IEEE, 2019.

[185] Ying Ji, Ping Li, Zhong Wu, and Deqiang Qu. Reaching consensus based on the opinion dynamics in social networks. *Arab. J. Sci. Eng.*, 46(2):1677–1690, 2021.

[186] Markus Brede. How does active participation affect consensus: Adaptive network model of opinion dynamics and influence maximizing rewiring. *Complexity*, 2019, 2019.

[187] Xia Chen, Zhaogang Ding, Yucheng Dong, and Haiming Liang. Managing consensus with minimum adjustments in group decision making with opinions evolution. *IEEE Trans. Syst., Man, Cybernet.: Syst.*, 2019.

[188] Yi Liu, Haiming Liang, Lei Gao, and Zhaoxia Guo. Optimizing consensus reaching in the hybrid opinion dynamics in a social network. *Inf. Fusion*, 72:89–99, 2021.

[189] Xiaoxuan Liu, Changwei Huang, Haihong Li, Qionglin Dai, and Junzhong Yang. The combination of pairwise and group interactions promotes consensus in opinion dynamics. *Complexity*, 2021, 2021.

[190] Xinli Shi, Jinde Cao, Guanghui Wen, and Matjaž Perc. Finite-time consensus of opinion dynamics and its applications to distributed optimization over digraph. *IEEE Trans. Cybernet.*, 49(10):3767–3779, 2018.

[191] Yupeng Li, Meng Liu, Jin Cao, Xiaolin Wang, and Na Zhang. Multi-attribute group decision-making considering opinion dynamics. *Expert Syst. Appl.*, 184:115479, 2021.

[192] Benhong Peng, Chaoyu Zheng, Xuan Zhao, Guo Wei, and Anxia Wan. Pythagorean fuzzy multiattribute group decision making based on risk attitude and evidential reasoning methodology. *International Journal of Intelligent Systems*, 36(11):6180–6212, 2021.

[193] Giovanni F. Massari, Ilaria Giannoccaro, and Giuseppe Carbone. Are distrust relationships beneficial for group performance? the influence of the scope of

distrust on the emergence of collective intelligence. *International Journal of Production Economics*, 208:343–355, 2019.

[194] Feng Pei, Yu-Wei He, An Yan, Mi Zhou, Yu-Wang Chen, and Jian Wu. A consensus model for intuitionistic fuzzy group decision-making problems based on the construction and propagation of trust/distrust relationships in social networks. *International Journal of Fuzzy Systems*, 22(8):2664–2679, 2020.

[195] Xiaofang Li, Huchang Liao, and Zhi Wen. A consensus model to manage the non-cooperative behaviors of individuals in uncertain group decision making problems during the covid-19 outbreak. *Applied Soft Computing*, 99:106879, 2021.

[196] Yuxiang Yuan, Dong Cheng, Zhili Zhou, and Faxin Cheng. A minimum adjustment cost consensus framework considering harmony degrees and trust propagation for social network group decision making. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, pages 1–13, 2022.

[197] Mingshuo Cao, Jian Wu, Francisco Chiclana, and Enrique Herrera-Viedma. A bidirectional feedback mechanism for balancing group consensus and individual harmony in group decision making. *Information Fusion*, 76:133–144, 2021.

[198] Yanling Lu, Yejun Xu, Enrique Herrera-Viedma, and Yefan Han. Consensus of large-scale group decision making in social network: the minimum cost model based on robust optimization. *Information Sciences*, 547:910–930, 2021.

[199] Mingshuo Cao, Jian Wu, Francisco Chiclana, Raquel Ureña, and Enrique Herrera-Viedma. A personalized consensus feedback mechanism based on maximum harmony degree. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 51(10):6134–6146, 2021.

[200] Zeshui Xu. Deviation measures of linguistic preference relations in group decision making. *Omega*, 33(3):249–254, 2005.

[201] Guiqing Zhang, Yucheng Dong, and Yinfeng Xu. Consistency and consensus measures for linguistic preference relations based on distribution assessments. *Information Fusion*, 17:46–55, 2014. Special Issue: Information fusion in consensus and decision making.

[202] Jian Wu, Lifang Dai, Francisco Chiclana, Hamido Fujita, and Enrique Herrera-Viedma. A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust. *Information Fusion*, 41:232–242, 2018.

[203] Yucheng Dong, Yinfeng Xu, and Shui Yu. Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model. *IEEE Transactions on Fuzzy Systems*, 17(6):1366–1378, 2009.

[204] Guang-Yu Bao, Xiang-Lei Lian, Ming He, and Ling-Ling Wang. Improved two-tuple linguistic representation model based on new linguistic evaluation scale. *Control and Decision*, 25(5):780–784, 2010.

[205] Hong-Gang Peng and Jian-Qiang Wang. A multicriteria group decision-making method based on the normal cloud model with zadeh's z -numbers. *IEEE Transactions on Fuzzy Systems*, 26(6):3246–3260, 2018.

[206] Hossein Hassani, Roozbeh Razavi-Far, and Mehrdad Saif. Real-time out-of-step prediction control to prevent emerging blackouts in power systems: A reinforcement learning approach. *Applied Energy*, 314:118861, 2022.

[207] Yucheng Dong, Hengjie Zhang, and Enrique Herrera-Viedma. Consensus reaching model in the complex and dynamic magdm problem. *Knowledge-Based Systems*, 106:206–219, 2016.

[208] Rodolfo Lourenzutti and Renato A. Krohling. A generalized topsis method for group decision making with heterogeneous information in a dynamic environment. *Information Sciences*, 330:1–18, 2016. SI Visual Info Communication.

[209] Miguel A Ballester and José Luis García-Lapresta. A recursive group decision-making procedure for choosing qualified individuals. *International Journal of Intelligent Systems*, 24(8):889–901, 2009.

[210] S. Alonso, I.J. Pérez, F.J. Cabrerizo, and E. Herrera-Viedma. A linguistic consensus model for web 2.0 communities. *Applied Soft Computing*, 13(1):149–157, 2013.

[211] Zhenzhen Ma, Jianjun Zhu, and Ye Chen. A probabilistic linguistic group decision-making method from a reliability perspective based on evidential reasoning. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 50(7):2421–2435, 2020.

[212] Thomas L Saaty. *Fundamentals of decision making and priority theory with the analytic hierarchy process*. RWS publications, 1994.

[213] R. Ramanathan and L.S. Ganesh. Group preference aggregation methods employed in ahp: An evaluation and an intrinsic process for deriving members' weightages. *European Journal of Operational Research*, 79(2):249–265, 1994.

[214] Jian qiang Wang, Jia ting Wu, Jing Wang, Hong yu Zhang, and Xiao hong Chen. Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems. *Information Sciences*, 288:55–72, 2014.

[215] Amos Tversky and Daniel Kahneman. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5(4):297–323, 1992.

[216] Joan M Aldous and Robin J Wilson. *Graphs and applications: an introductory approach*. Springer Science & Business Media, 2003.

[217] Duncan J Watts and Steven H Strogatz. Collective dynamics of 'small-world'networks. *nature*, 393(6684):440–442, 1998.

[218] David Cheriton and Robert Endre Tarjan. Finding minimum spanning trees. *SIAM Journal on Computing*, 5(4):724–742, 1976.

[219] Quanbo Zha, Yucheng Dong, Francisco Chiclana, and Enrique Herrera-Viedma. Consensus reaching in multiple attribute group decision making: A multi-stage optimization feedback mechanism with individual bounded confidences. *IEEE Transactions on Fuzzy Systems*, 30(8):3333–3346, 2022.

[220] Guillaume Deffuant, David Neau, Frederic Amblard, and Gérard Weisbuch. Mixing beliefs among interacting agents. *Advances in Complex Systems*, 3(01n04):87–98, 2000.

[221] Hossein Hassani, Roozbeh Razavi-Far, Mehrdad Saif, and Enrique Herrera-Viedma. Blockchain-enabled trust building for managing consensus in linguistic opinion dynamics. *IEEE Transactions on Fuzzy Systems*, 2023, doi:10.1109/TFUZZ.2023.3235411.

[222] Lotfi Asker Zadeh. The concept of a linguistic variable and its application to approximate reasoning—i. *Information sciences*, 8(3):199–249, 1975.

[223] Bingyi Kang, Daijun Wei, Ya Li, and Yong Deng. A method of converting z-number to classical fuzzy number. *Journal of Information &computational Science*, 9(3):703–709, 2012.

[224] Jin-Hsien Wang and Jongyun Hao. An approach to computing with words based on canonical characteristic values of linguistic labels. *IEEE Transactions on Fuzzy Systems*, 15(4):593–604, 2007.

[225] Celik Parkan and Ming-Lu Wu. Decision-making and performance measurement models with applications to robot selection. *Computers & Industrial Engineering*, 36(3):503–523, 1999.

[226] Norberto Navarrete, Masao Fukushima, and Hisashi Mine. A new ranking method based on relative position estimate and its extensions. *IEEE Transactions on Systems, Man, and Cybernetics*, 9(11):681–689, 1979.

[227] Quanbo Zha, Yucheng Dong, Hengjie Zhang, Francisco Chiclana, and Enrique Herrera-Viedma. A personalized feedback mechanism based on bounded confidence learning to support consensus reaching in group decision making. *IEEE Trans. Syst., Man, Cybern.: Syst.*, 51(6):3900–3910, 2019.

[228] Haiming Liang, Cong-Cong Li, Yucheng Dong, and Francisco Herrera. Linguistic opinions dynamics based on personalized individual semantics. *IEEE Trans. Fuzzy Syst.*, 29(9):2453–2466, 2020.

[229] Quanbo Zha, Haiming Liang, Gang Kou, Yucheng Dong, and Shui Yu. A feedback mechanism with bounded confidence-based optimization approach for consensus reaching in multiple attribute large-scale group decision-making. *IEEE Trans. Comput. Soc. Syst.*, 6(5):994–1006, 2019.

[230] Jesús Giráldez-Cru, Manuel Chica, and Oscar Cordón. A framework of opinion dynamics using fuzzy linguistic 2-tuples. *Knowl.-Based Syst.*, 233:107559, 2021.

[231] Yanhong Li, Gang Kou, Guangxu Li, and Haomin Wang. Multi-attribute group decision making with opinion dynamics based on social trust network. *Inf. Fusion*, 75:102–115, 2021.

[232] Zhibin Wu, Qinyue Zhou, Yucheng Dong, Jiuping Xu, Abdulrahman H. Altalhi, and Francisco Herrera. Mixed opinion dynamics based on DeGroot model and

Hegselmann-Krause model in social networks. *IEEE Trans. Syst., Man, Cybern.: Syst.*, 2022, 10.1109/TSMC.2022.3178230.

[233] Lei Shi, Yuhua Cheng, Jinliang Shao, Xiaofan Wang, and Hanmin Sheng. Leader-follower opinion dynamics of signed social networks with asynchronous trust/distrust level evolution. *IEEE Trans. Netw. Sci. Eng.*, 9(2):495–509, 2022.

[234] Yao Zou and Ziyang Meng. Targeted bipartite consensus of opinion dynamics in social networks with credibility intervals. *IEEE Trans. Cybern.*, 52(1):372–383, 2022.

[235] Zijie Zhao, Lei Shi, Tong Li, Jinliang Shao, and Yuhua Cheng. Opinion dynamics of social networks with intermittent-influence leaders. *IEEE Trans. Comput. Soc.*, 2022, 10.1109/TCSS.2022.3145044.

Appendix A

Copyright Permissions

RightsLink

Home | Help | Live Chat | Sign in | Create Account

Requesting permission to reuse content from an IEEE publication

Blockchain-Enabled Trust Building for Managing Consensus in Linguistic Opinion Dynamics

Author: Hossein Hassani

Publication: IEEE Transactions on Fuzzy Systems

Publisher: IEEE

Date: Dec 31, 1969

Thesis / Dissertation Reuse

The IEEE does not require individuals working on a thesis to obtain a formal reuse license, however, you may print out this statement to be used as a permission grant:

Requirements to be followed when using any portion (e.g., figure, graph, table, or textual material) of an IEEE copyrighted paper in a thesis:

1) In the case of textual material (e.g., using short quotes or referring to the work within these papers) users must give full credit to the original source (author, paper, publication) followed by the IEEE copyright line © 2011 IEEE.
2) In the case of illustrations or tabular material, we require that the copyright line © [Year of original publication] IEEE appear prominently with each reprinted figure and/or table.
3) If a substantial portion of the original paper is to be used, and if you are not the senior author, also obtain the senior author's approval.

Requirements to be followed when using an entire IEEE copyrighted paper in a thesis:

1) The following IEEE copyright/ credit notice should be placed prominently in the references: © [year of original publication] IEEE. Reprinted, with permission, from [author names, paper title, IEEE publication title, and month/year of publication]
2) Only the accepted version of an IEEE copyrighted paper can be used when posting the paper or your thesis on-line.
3) In placing the thesis on the author's university website, please display the following message in a prominent place on the website: In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of [university/educational entity's name goes here]'s products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to http://www.ieee.org/publications_standards/publications/rights/rights_link.html to learn how to obtain a License from RightsLink.

If applicable, University Microfilms and/or ProQuest Library, or the Archives of Canada may supply single copies of the dissertation.

BACK CLOSE WINDOW

RightsLink

Home | Help | Live Chat | Sign in | Create Account

Classical dynamic consensus and opinion dynamics models: A survey of recent trends and methodologies

Author:
Hossein Hassani,Roozbeh Razavi-Far,Mehrdad Saif,Francisco Chiclana,Ondrej Krejcar,Enrique Herrera-Viedma

Publication: Information Fusion

Publisher: Elsevier

Date: December 2022

Journal Author Rights

Please note that, as the author of this Elsevier article, you retain the right to include it in a thesis or dissertation, provided it is not published commercially. Permission is not required, but please ensure that you reference the journal as the original source. For more information on this and on your other retained rights, please visit: https://www.elsevier.com/about/our-business/policies/copyright#Author-rights

BACK CLOSE WINDOW

Home | Help | Live Chat | Sign in | Create Account

RightsLink

Requesting permission to reuse content from an IEEE publication

Reinforcement Learning-Based Feedback and Weight-Adjustment Mechanisms for Consensus Reaching in Group Decision Making

Author: Hossein Hassani

Publication: IEEE Transactions on Systems, Man, and Cybernetics: Systems

Publisher: IEEE

Date: 2022

Thesis / Dissertation Reuse

The IEEE does not require individuals working on a thesis to obtain a formal reuse license, however, you may print out this statement to be used as a permission grant:

Requirements to be followed when using any portion (e.g., figure, graph, table, or textual material) of an IEEE copyrighted paper in a thesis:

1) In the case of textual material (e.g., using short quotes or referring to the work within these papers) users must give full credit to the original source (author, paper, publication) followed by the IEEE copyright line © 2011 IEEE.
2) In the case of illustrations or tabular material, we require that the copyright line © [Year of original publication] IEEE appear prominently with each reprinted figure and/or table.
3) If a substantial portion of the original paper is to be used, and if you are not the senior author, also obtain the senior author's approval.

Requirements to be followed when using an entire IEEE copyrighted paper in a thesis:

1) The following IEEE copyright/ credit notice should be placed prominently in the references: © [year of original publication] IEEE. Reprinted, with permission, from [author names, paper title, IEEE publication title, and month/year of publication]
2) Only the accepted version of an IEEE copyrighted paper can be used when posting the paper or your thesis on-line.
3) In placing the thesis on the author's university website, please display the following message in a prominent place on the website: In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of [university/educational entity's name goes here]'s products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to http://www.ieee.org/publications_standards/publications/rights/rights_link.html to learn how to obtain a License from RightsLink.

If applicable, University Microfilms and/or ProQuest Library, or the Archives of Canada may supply single copies of the dissertation.

BACK CLOSE WINDOW

Home | Help | Live Chat | Sign in | Create Account

RightsLink

Consensus-Based Decision Support Model and Fusion Architecture for Dynamic Decision Making

Author: Hossein Hassani,Roozbeh Razavi-Far,Mehrdad Saif,Enrique Herrera-Viedma

Publication: Information Sciences

Publisher: Elsevier

Date: June 2022

Journal Author Rights

Please note that, as the author of this Elsevier article, you retain the right to include it in a thesis or dissertation, provided it is not published commercially. Permission is not required, but please ensure that you reference the journal as the original source. For more information on this and on your other retained rights, please visit: https://www.elsevier.com/about/our-business/policies/copyright#Author-rights

BACK CLOSE WINDOW

RightsLink

Home | Help | Live Chat | Sign in | Create Account

Requesting permission to reuse content from an IEEE publication

Intelligent Decision Support and Fusion Models for Fault Detection and Location in Power Grids

Author: Hossein Hassani

Publication: IEEE Transactions on Emerging Topics in Computational Intelligence

Publisher: IEEE

Date: June 2022

Thesis / Dissertation Reuse

The IEEE does not require individuals working on a thesis to obtain a formal reuse license, however, you may print out this statement to be used as a permission grant:

Requirements to be followed when using any portion (e.g., figure, graph, table, or textual material) of an IEEE copyrighted paper in a thesis:

1) In the case of textual material (e.g., using short quotes or referring to the work within these papers) users must give full credit to the original source (author, paper, publication) followed by the IEEE copyright line © 2011 IEEE.
2) In the case of illustrations or tabular material, we require that the copyright line © [Year of original publication] IEEE appear prominently with each reprinted figure and/or table.
3) If a substantial portion of the original paper is to be used, and if you are not the senior author, also obtain the senior author's approval.

Requirements to be followed when using an entire IEEE copyrighted paper in a thesis:

1) The following IEEE copyright/ credit notice should be placed prominently in the references: © [year of original publication] IEEE. Reprinted, with permission, from [author names, paper title, IEEE publication title, and month/year of publication]
2) Only the accepted version of an IEEE copyrighted paper can be used when posting the paper or your thesis on-line.
3) In placing the thesis on the author's university website, please display the following message in a prominent place on the website: In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of [university/educational entity's name goes here]'s products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to http://www.ieee.org/publications_standards/publications/rights/rights_link.html to learn how to obtain a License from RightsLink.

If applicable, University Microfilms and/or ProQuest Library, or the Archives of Canada may supply single copies of the dissertation.

BACK CLOSE WINDOW

Requesting permission to reuse content from an IEEE publication

Fault Location in Smart Grids Through Multicriteria Analysis of Group Decision Support Systems

Author: Hossein Hassani

Publication: IEEE Transactions on Industrial Informatics

Publisher: IEEE

Date: December 2020

Thesis / Dissertation Reuse

The IEEE does not require individuals working on a thesis to obtain a formal reuse license, however, you may print out this statement to be used as a permission grant:

Requirements to be followed when using any portion (e.g., figure, graph, table, or textual material) of an IEEE copyrighted paper in a thesis:

1) In the case of textual material (e.g., using short quotes or referring to the work within these papers) users must give full credit to the original source (author, paper, publication) followed by the IEEE copyright line © 2011 IEEE.
2) In the case of illustrations or tabular material, we require that the copyright line © [Year of original publication] IEEE appear prominently with each reprinted figure and/or table.
3) If a substantial portion of the original paper is to be used, and if you are not the senior author, also obtain the senior author's approval.

Requirements to be followed when using an entire IEEE copyrighted paper in a thesis:

1) The following IEEE copyright/ credit notice should be placed prominently in the references: © [year of original publication] IEEE. Reprinted, with permission, from [author names, paper title, IEEE publication title, and month/year of publication]
2) Only the accepted version of an IEEE copyrighted paper can be used when posting the paper or your thesis on-line.
3) In placing the thesis on the author's university website, please display the following message in a prominent place on the website: In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of [university/educational entity's name goes here]'s products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to http://www.ieee.org/publications_standards/publications/rights/rights_link.html to learn how to obtain a License from RightsLink.

If applicable, University Microfilms and/or ProQuest Library, or the Archives of Canada may supply single copies of the dissertation.

BACK CLOSE WINDOW

Vita Auctoris

Hossein Hassani obtained his B.Sc. and M.Sc. degrees in Electrical Engineering from Shiraz University of Technology, Shiraz, Iran, in 2014 and 2016, respectively. Currently, he is pursuing his Ph.D. degree at the Department of Electrical and Computer Engineering, University of Windsor, Windsor, ON, Canada, with an expected graduation date in Summer 2023. His primary research interests lie in the fields of machine learning, computational intelligence, and their applications in fault diagnosis and cybersecurity of cyber-physical systems.