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Improvement to an existing multi-level capacitated lot sizing problem considering setup carryover, backlogging, and emission control

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Abstract

This paper presents a multi-level, multi-item, multi-period capacitated lot-sizing problem. The lot-sizing problem studies can obtain production quantities, setup decisions and inventory levels in each period fulfilling the demand requirements with limited capacity resources, considering the Bill of Material (BOM) structure while simultaneously minimizing the production, inventory, and machine setup costs. The paper proposes an exact solution to Chowdhury et al. (2018)'s[1] developed model, which considers the backlogging cost, setup carryover & greenhouse gas emission control to its model complexity. The problem contemplates the Dantzig-Wolfe (D.W.) decomposition to decompose the multi-level capacitated problem into a single-item uncapacitated lot-sizing sub-problem. To avoid the infeasibilities of the weighted problem (WP), an artificial variable is introduced, and the Big-M method is employed in the D.W. decomposition to produce an always feasible master problem. In addition, Wagner & Whitin's[2] forward recursion algorithm is also incorporated in the solution approach for both end and component items to provide the minimum cost production plan. Introducing artificial variables in the D.W. decomposition method is a novel approach to solving the MLCLSP model. A better performance was achieved regarding reduced computational time (reduced by 50%) and optimality gap (reduced by 97.3%) in comparison to Chowdhury et al. (2018)'s[1] developed model.

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Keywords: Capacitated Lot Sizing; Multi-level capacitated lot-sizing; Emission control; Dantzig-Wolfe decomposition; Artificial Variable.

1. Introduction

1.1. Research background:

Nowadays, most industries have multi-level, multi-item product structures, where simultaneously, all the items need to be planned in a predetermined time horizon. For such a production schedule, multiple complexities can arise from the capacity of a resource (Such as man, machine, and material), material/inventory shortages, setup decisions (such as line setup, machine breakdown, and maintenance), legal/environmental costs (Such as excess stack air emission, sewage water discharge, noise pollution control). Lot sizing decision needs to consider the earlier mentioned complexity while fulfilling the demand of an internal and external item at minimum cost. A simplified classical lot sizing problem was explored by many researchers considering single-item capacitated lot sizing items. Later, different complexities were considered by adding variable setup time & cost, overtime cost, backlogging cost, and emission control.

The multi-level lot-sizing problem (MLCLSP) is an extension of the capacitated lot-sizing problem, which deals with the issue of determining time-phased production quantities. This problem was introduced by Billington et al. [3], which is considered the theoretical basis for material requirements planning by Sahling et al. [4]. During any manufacturing operations, one of the critical decisions is making setup decisions (Such as pre-heat machine, tool setup & change, machine parameter change, and material change). Setup decisions can be of two types 1) carryover an existing

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setup and 2) Opt-in a new setup. Setup carryover means maintaining the same setup between consecutive periods. Haase et al. [5] identified that solutions change considerably if setup carryover is considered.

Many countries are now applying various laws and regulations for manufacturing industries to measure and reduce the overall carbon footprint to tackle the effect of climate change. The government provides incentives to companies for lowering stack air emissions and enforces penalties if industries are unable to fulfil them. In manufacturing industries, increased production causes an increase in greenhouse gas GHG) emissions (Such as CO2, CO, SOx, and NOx). Warehouse storage, setup, and other processes also play a role in increasing gas emissions. An optimum production lot or batch size with the right amount of inventory storage and minimum setup is required fulfilling the emission cap. As a result, Benjaafar et al. [6] introduced an emission control constraint for lot-sizing problems where different practical scenarios were considered. For example, a firm either can control their Carbon emission (ETS system developed by E.U., followed by 11,000 facilities all over Europe) or pay a tax/penalty for carbon emissions (U.S., a cap-and-trade system of U.S. climate bill).

This paper adapts an existing Mixed Integer Linear Programming (MILP) model for a Multi-level Capacitated Lot-Sizing Problem. The proposed model considers setup carryover, product backlogging, and emission control constraints. The Dantzig-Wolfe decomposition method has been used by adding artificial variables and Big M to the model's objective function. We followed a similar route as Chowdhury et al. [1] by solving an Integer linear programming for the setup decision problem. The Wagner-Whitin model is employed to calculate demand.

1.2. Contribution:

This paper aims to provide optimal solutions for classical multi-level, multi-item capacitated lot-sizing problems considering setup carryover, Backlogging, and emission control using a computationally efficient decomposition technique. Decomposing the problem is required for two reasons; dimensionality and complexity. As will be seen in the literature review section, optimal solutions to the problem under study are not existing in the literature.

Chowdhury et al. [7] developed a solution algorithm to solve a SIULSP (Single item uncapacitated lot sizing problem). The algorithm provides an optimum production schedule plan that meets the demand and generates a quicker solution. Later, as an extension, Chowdhury et al.[1] generated a heuristic approach to solve a Classical MLCLSP model that deals with Setup carryover, Backlogging, and emission control constraints. The author used a novel capacity allocation heuristic to overcome the infeasibility caused by the complicated constraints of the model. The Dantzig Wolfe using capacity allocation heuristic (DW-CA) uses the data of demand gained from the Wagner-Whitin method and then allocates that demand over periods without violating the capacity constraint. We aim to create a solution method to reach optimality, quickly avoiding all heuristics. The Dantzig Wolfe using Big M (DW-M) has been implemented to remove recurring infeasibilities of the Weighted Problem (WP) level to reach the optimal solution.

The study contributes toward achieving a substantially lower optimality gap and faster processing of the solution method to reduce computational time.

2. Literature review

Lot-sizing problem is a very well-known problem. Many studies regarding lot-sizing have been considered by researchers depending on the different problem structures (production level, item quantity, resource level, supply chain level), problem complexity (Setup time, Overtime, Setup carryover, Backlogging, Inventory constraint), solution approach (Exact method, Heuristic method, Meta-heuristic method).

The Multi-Level capacitated lot sizing problem (MLCLSP) is considered by many researchers for its complexity and practical use. The production system can be described as either single-level or multi-level based on the presence of a predecessor/successor relationship. The multi-level production system has one or more end items (Finished goods) and component items with a parent-child structure. Sahling et al. [8] solve an MLCLSP problem using a fixed and optimized heuristic with several end products. Buschkühl et al. [10] provided a meaningful summary of all solution approaches used to solve all variants of MLCLSP & CLSP. The solution approach consisted of mathematical programming heuristics, decomposition heuristics, meta-heuristics, and problem-specific heuristics. Maes et al. [11] presented a new heuristic model to solve the MLCLSP since these problems fall under the class of NP-hard problems. Lagrangean Relaxation (L.R.) was used by Tempelmeier & Derstroff [4] to solve MLCLSP by breaking down the problem into single-level uncapacitated lot sizing problems. Berreta et al. [14] continued the research, comprised a non-zero lead time to the MLCLSP problem, and found a solution by introducing a hybrid Simulated Annealing-based tabu search Method. Population-based heuristic was initially used by Pitakaso et al . [15]. In this paper, multi-item and multi-level lot sizing are where each machine can produce multiple items.

Backlogging is considered in many MLCLSP problems to overcome overtime costs Kimms,[16]. Wu et al. [17] introduced two MIP formulations capable of generating tight lower bounds when solving the MLCLSP-B problem. Toledo et al. [18] proposed a new hybrid method combining a multi-population genetic algorithm with F.O. heuristic and mathematical programming for the MLCLSP-B problem. Zhao et al. [19] consider a combination of accurate MIP & Variable Neighbourhood Decomposition Search (VNDS). Seeanner et al. [20] hybridized the VNDS and the MIP-based FO approach as another method for solving the MLCLSP. Duda & Stawowy [21] considered a VNS approach to solving the MLCLSP with backlogging. A similar problem was solved by Toledo et al. [22], where the author used the relax and fix to build the initial solution and then improved by using a fix and optimize heuristic.

Setup carryover is often considered a part of the lot-sizing problem that requires minimization, and a proper setup carryover assignment can reduce setup time. Wu et al. [23] propose a MIP formulation considering backlogging and setup

carryover. A progressive time-oriented heuristic framework was proposed in this paper. Later, Chen [24] proposed a new F.O. approach to solving two dynamic MLCLSP problems, one considering setup carryover and another without setup carryover. Almeder et al. [15] Propose two methods for MLCLSP with a setup carryover problem. The first method considers batch production, which assumes the whole amount of predecessor must be available to start final product production. The second method is lot-streaming production (non-batching), where the earlier assumption is relaxed. In 2009, Tempelmeier & Buschkühl [10] addressed a lagrangean heuristic solution approach for a linked lot-size dynamic capacitated lot sizing problem. Sahling et al. [9] advanced the research on the dynamic capacitated lot sizing problems. The author used an iterative F.O. approach to solve the problem.
Identical setup costs and times for all items were first addressed by Schimdt et al.[26] while solving the single-level CLSP with setup carryover. Later, Gopal [27] considered item-dependent setup times and costs. Haase [5] proposed the CLSP problem with linked lot sizes of adjacent periods, where the scheduling procedure is backwards-oriented. Sox & Gao [28] considered the CLSP problem in terms of the big-time bucket without limiting the number of products produced in each period and generated an efficient lagrangean decomposition heuristic. Briskorn [29] later adjusted the dynamic programming approach of Sox & Gao [28] so that the subproblem could be solved optimally. Tabu search was employed by Karimi et al. [30] to solve a CLSP with setup carryover and backlogging. Oztürk & Ornek [31] formulate an MLCLSP problem with setup carryover and backlogging in a job shop environment and add two more constraints to penalize and obtain a feasible production plan. In case of a lot sizing problem, setup carryover decision can reduce cost significantly by reducing setup time and resources.

In recent times, all production facilities have been working on reducing their carbon footprint. This capping on carbon emissions can also be linked with lot-sizing decisions to reduce carbon emissions. Benjaafar et al. [6] introduced emission control as lot sizing problem constraints. Later, Retel Helmrich et al. [32] proves that lot-sizing with emission constraints is NP-hard. Masmoudi et al. [33] considered an MLCLSP problem with new objective energy optimization Iijima [34]. Ghosh [35] identified the two most adopted carbon policies are (i) Carbon tax/cost policy and (ii) Carbon cap-and-trade policy and solved the MINLP problem for both policies. Ahmadini et al. [36], In their paper, solved a multi-objective inventory model with back-ordered quantity incorporating green investment to save the environment. Reddy et al. [37] considered Mixed-integer linear programming to minimize total cost, comprising the carbon emission cost due to transportation and production in the facilities using "Improved bender's decomposition." The paper reviews a carbon emission constraint for holding, production and setup processes while solving the lot sizing problem.

From the literature review, it is observable that researchers gain advantages by decomposing MLCLSP to SIULSP. The Dantzig-Wolfe decomposition heuristic is another example of the decomposition technique used by researchers. This technique helps to obtain a better lower bound Duarte et al. [38]; Jans et al. [39]. Manne[40] was the first researcher to use the D.W. decomposition technique for CLSP. The goal is to find a convex combination of a given single-item production plan, which keeps the capacity constraints of the original master problem and leads to minimal cost. Later, Jans et al. [39] propositioned a new D.W. reformulation and a Branch-and-Price (B&P) algorithm. Pimentel et al., [41] compare item and period DW decompositions of a multi-item CLSP. Hamadi & Schoenauer [42] propose D.W. decomposition for the multi-item, multiperiod CLSP with setup times in a meta-heuristic framework. According to Duarte & de Carvalho [38], a discrete Lot-Sizing and Scheduling Problem (DSLP) with setup costs and inventory holding is decomposed using D.W. They develop a B&P and C.G. procedure to solve the problem optimality. The CLSP with setup time was studied by De Araujo et al. [42], and a period of D.W. decomposition was proposed. A hybrid scheme combining L.R. and C.G. is developed to find promising lower bounds. The hybrid algorithms developed by Fiorotto et al. [43] apply L.R. and D.W. decomposition together to obtain lower bounds on CLSPs with multiple items, setup time, and unrelated parallel machines. Zhang et al. [44] studied a capacitated lot sizing problem that considers a scheduling problem based on the machine. The setup time is considered sequence dependent on the option of setup carryover. They used reformulated D.W. decomposition and the branching and selection method to obtain the solution to the problem.

D.W. decomposition is used in various other problem-solving methods. Iijima[34] studied practical airline crew scheduling problems and reformulated them as a set partitioning problem by Dantzig-Wolfe decomposition. Leao et al. [45] Investigated the one-dimensional cutting-stock problem integrated with the lot-sizing problem and Dantzig–Wolfe decompositions while developing column generation techniques to obtain upper and lower bounds for the integrated problem in the context of paper industries. In research from Chowdhury et al. [1], it can be seen that multi-level capacitated lot sizing problems with setup carryover and backlogging cost can be divided into single-item uncapacitated lot sizing problem (SIULSP) by using Dantzig-Wolfe (D.W.) decomposition method. The author has used the Capacity allocation heuristic to obtain a feasible solution for the capacitated lot sizing problem. And compared the result with earlier studies and found improvement in optimality gap & computation time. Shahvari et al. considered a bi-criteria objective to simultaneously minimize the total weighted completion time and, at the same time, minimize the total weighted tardiness of jobs by decomposing the mathematical model with the help of the Dantzig-Wolfe-Decomposition technique and, Branch-and-Price algorithm. Oskorbin & Khvalynskiy [46] consider the applicability of the Dantzig-Wolfe method for Large-Scale Nonlinear Programming with a composite (block) structure of the function and constraints. Jaumard et al. [47] revisit the latter linear program model proposed for the AMD problem and introduce a new one with a polynomial number of variables. Dupin et al. [48] solved a variant for the vehicle routing problem with time windows, site dependencies, multiple depots, and outsourcing goals.

Karateke et al. [49] applied D.W. decomposition to find the exact solution of the network flow model. Karateke et al. [49] applied the D.W. decomposition technique to solve Multi floor facility layout problem. Borgwardt & Patterson [44] is used for the minimum-cost mass transport problem.

Gupta et al. [50]presented the optimum time coordination of overcurrent relays solved in a distribution network. Toloo et al. [51] considered using the Big M method to solve Data envelopment analysis (DEA) with an application on Bank. Putcha [52] introduces a new and uniform method of linear programming. Soleimani-damaneh [53] introduces a modified Big M method that reduces the number of iterations for this method when it is used to identify the presence of infeasibility in linear systems. Arsham [54] used a three-phase simplex-type solution algorithm to solve general linear programs instead of using artificial variables and constraints.

An alternate method was explained by Conejo (2006) [55], where the author showed an example of solving a problem with complicated constraints using Dantzig Wolfe with the M method and artificial variable. As the M method and artificial variable remove any infeasibility from the problem and calculate the minimum production cost planning schedule, there is an advantage to solving the multi-level capacitated lot sizing problem using this technique which hasn't been exploited in the literature.

To summarize the overall study, The complexity of the developed model MLCLSP with Setup carryover, Backlogging and Emission control is novel in the literature and solved with the heuristic method. And there is an opportunity in the literature for solving such a complex model with an exact method using the Big M method and artificial variables.

Many researchers applied different meta-heuristic and evolutionary algorithms (Genetic Algorithm, Memetic Algorithm, Ant colony-based algorithm, Bee's fix and optimize algorithm) to solve the Lot sizing problems quickly and efficiently. Some researchers applied unique algorithms not typically used in general lot sizing studies. Such as, In the research of Berretta & Rodrigues [14], a memetic algorithm (Similar to a genetic algorithm) was used for a multi-stage capacitated lot sizing problem. Another example of a population-based heuristic was used by Boonmee & Sethanan [56], Where the author introduced a particle swarm optimization (PSO) variant in the poultry industry. Furlan & Santos [57] developed a heuristic bees-and-fix-and-optimize (BFO) to solve multi-level lot sizing problems that start with an initial solution and then search for a random solution with a bee's algorithm from a subproblem and use fix and optimize the method to get the solution. Behnamian et al. [58] used a model of a multi-level, multi-item, multi-period capacitated lot sizing problem with uncertainty in levels. They used the Markov chain concept on it. Witt [59] proposed a silver meal heuristic for the MLCLSP, where the heuristic starts calculating solutions for an uncapacitated problem and performs backwards and forward moves of the lot sizes. Wei et al. [60] studied MLCLSP with a focus on replaceability.

Table 1 chronologically presents relevant studies conducted based on the complexity of the problem, the proposed solution approach, and the properties of the lot-sizing problem.

Table 1: Synthesis Matrix (Classical lot-sizing problem)

	Problem Solved								
Reference	Single-	Multi-level	Setup Time	SCB	Multi-Item	Backlogging	E.C.	Overtime	Solution approach
Manne (1958)	√	-	-	-	-	-	-	√	DW
Billington, McClain, and Thomas (1986)	-	√	√	-	-	-	-	-	LR and B&B
Tempelmeier and Derstroff (1996)	-	√	√	-	-	-	-	-	LDH
Sox & Gao (1999)	√	-	√	√	-	-	-	-	LDH
Jans and Degraeve (2004)	√	-	√	-	√	-	-	-	DW,CG , B&B
Tempelmeier and Buschkühl (2009)	-	√	√	√	√	-	-	-	LDH
Pimentel et. al. (2010)	√	-	√	-	√	-	-	-	DW , B&P
Caserta & Voß (2012)	√	-	√	-	-	-	-	-	D.W., CG
Wu et al. (2013)	-	√	√	-	-	√	-	-	R&F
Toledo et al. (2013)	-	√	√	-	-	√	-	-	GA , F&O
Gören & Tunalı (2015)	√	-	√	√	-	-	-	-	G.A., F&O
Fiorotto et al. (2015)	√	-	√	-	√	-	-	-	L.R., DW
Araujo et al. (2015)	√	-	√	-	√	-	-	-	L.R., D.W.,CG
Chowdhury, Baki, Azab (2016)	-	√	√	√	√	√	√	-	C.A., D.W.,CG
Witt A. (2019)	-	√	√	-	-	-	-	-	SMH
Wei M., Qi M., Wu T., Zhang C. (2019)	-	√	√	-	-	-	-	-	R&F, BS
Duda J., Stawowy A. (2019)	-	√	√	-	-	√	-	-	VNS
Claudio Toledo· (2017)	-	√	√	-	-	√	-	-	R&F, F&O
Furlan M.M., Santos M.O.(2017)	-	√	√	-	-	-	-	-	BFO
Zhang C. et al. (2021)	√	-	√	√	-	-	-	-	D.W., B&S

Abbreviation: Solution Approach: B&B = Branch and Bound, L.R. = Lagrangian Relaxation, LDH = Lagrangian Decomposition Heuristic, DW= Dantzig–Wolfe, C.G. = Column Generation, B&P = Branch and Price, GA = Genetic Algorithm, R.F. = Relax and Fix, F.O. = Fix and Optimize, CA = Capacity Allocation heuristic, Bees & Fix & Optimize =BFO, Silver Meal Heuristic=SMH, Branch & Price = B&P, Branch & Selection= B&S.

3. Methodology

3.1. Introduction to the Dantzig-Wolfe Decomposition technique

The decomposition method allows some problems to be solved in a distributed manner. Dantzig–Wolfe (D.W.) decomposition was initially developed for linear programs with a block-angular structure of the constraint matrix and later generalized to the mixed integer case (Vanderbeck & Savelsbergh, (2006) [61]. For example, research from Chowdhury et al. (2018)[1] has shown that a multi-level item capacitated lot sizing problem can be divided into two or more single-item uncapacitated lot sizing sub-problems.

D.W. decomposition with Complicated Constraint: Any linear programming problem with appropriate structure can be divided into subproblems, but the complicating constraints involving two or multiple variables from different blocks drastically complicate the problem's solution. Complicating constraints prevent a straightforward solution from being considered. [55]. Decomposition procedures are computational techniques that indirectly consider the complicating constraints. The penalty for such a simplification is repetition. Instead of solving the original problem with complicating constraints, two problems are solved iteratively (i.e., repetitively): a simple so-called master problem/weighted problem and a problem similar to the original one but without complicating constraints (relaxed problem/sub-problem). [55]

3.2. D.W. method by using Big M

In some problems, the relaxed/sub-problem is unable to formulate a feasible master problem. In that case, an always feasible master problem can be formulated with artificial variables [55]. The artificial variable helps to attain the minimum amount needed to reach feasibility.

The weighted problem with artificial variables takes the following form:

Minimize $$\sum_{i=1}^{p} zi\,u_i + \left\{(M\left(\sum_{j=1}^{n} v_j + \omega\right)\right\} \qquad (3.1)$$
$$u1, \ldots, up\ ;\ v1, \ldots, vm, w$$

Subject to,

$$\sum_{i=1}^{p} r_j u_i + v_J - \omega = bj \qquad (3.2)$$

$$\sum_{i=1}^{p} u_i = 1 \qquad (3.3)$$

$$u_i \geq 0 \qquad (3.4)$$

Here, v_j & ω is the artificial variable. M is a large number. The difference between v_j & ω is the amount required to attain the minimum feasibility.

3.2. Employing artificial variables in the application of Dantzig-Wolfe Decomposition:

The following steps can be pursued to solve an L.P. problem with the D.W. decomposition technique:

Step 1: Find a basic feasible solution for the relaxed Master problem (Disregarding complicated constraints) by using *p* (any) number of arbitrary master problem solutions.

Step 2: If possible, Create a non-relaxed weighted problem with the basic feasible solution of the relaxed problem. Identify a solution for the non-relaxed weighted problem (Satisfy the complicated constraint & convexity constraint). Also, calculate the dual variable value of the complicated constraint λ and convexity constraint σ.

Step 3: Else, find any one (1) arbitrary objective function and add an artificial variable to the non-relaxed weighted problem to initialize the iteration. Calculate the dual value of the complicated constraint λ and convexity constraint σ.

Let's assume C, A, and E is the coefficient of the Objective function, Complicated constraint & Easy constraint. "x" is the variable, and "b_a" and "b_e' is the right-hand sides of the complicated and easy constraint. The steps are:

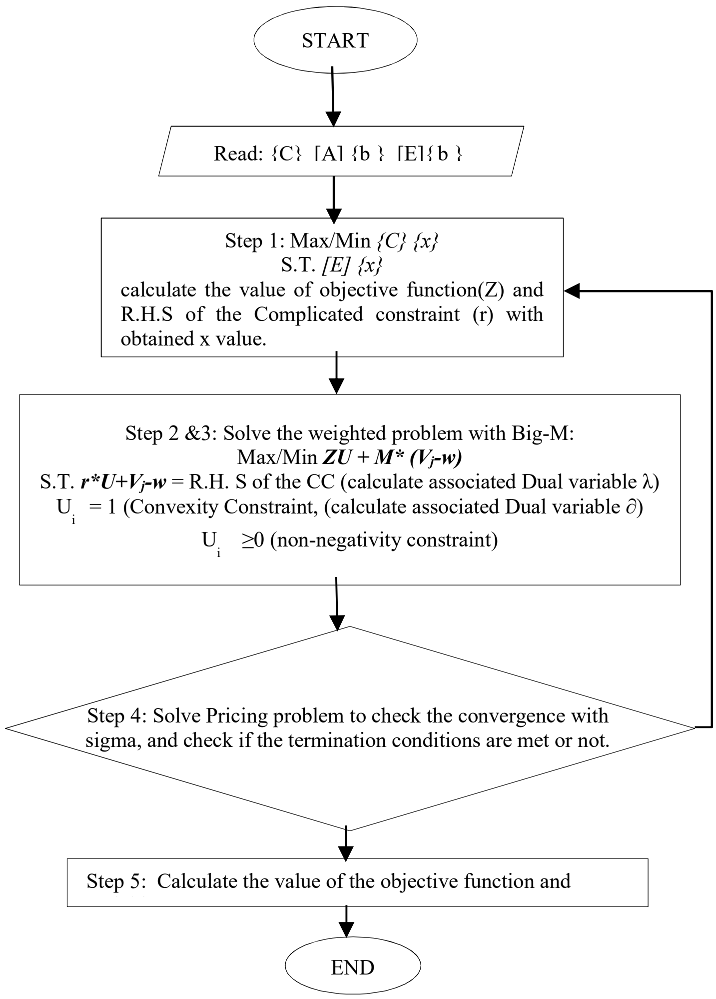

Fig 1 Flowchart of the steps followed to decompose a problem using D.W. and Big-M methods.

Step 4: Once the new solution for the non-relaxed weighted problem is found, check whether the new solution of the non-relaxed problem is optimum or not by checking convergence with a dual variable value. If the termination is not met, add one more solution to the relaxed problem to create a new non-relaxed weighted problem and move to step 2.

Step 5: If the termination condition is met, use the weighted variable to calculate the final objective function solution of the master problem and the value of the variables.

4. Proposed model

4.1. D.W. Decomposition method for MLCLSP with Setup Carryover, Backlogging, and Emission control (MLCLSP with SCBE):

Chowdhury et al. [1] derived a multi-level multi-period lot-sizing problem model while considering the cost of setup times, setup carryover, Backlogging, and emission control. The production plan has a finite planning horizon. All items are produced with a single resource (Machine) with a predetermined capacity. A setup cost can be incurred through setup time or cost of setup. The objective is identifying the optimum production quantity required to meet demand with a given capacity restriction. The model considers the Bill of material (BOM) structure to calculate the internal demand. The assumption of the model is set to be identical to Chowdhury et al. [1] model to compare and obtain an exact solution. The assumptions are given below:

- There is a fixed number of machines (resources) with period-specific capacities.
- The planning timeline has a limit which is divided into T periods (Shift/Day/Week).
- The external demands are arranged in a general product/process structure which contains assemblies and sub-assemblies. And resource (machine) needs to be eligible to produce certain assemblies and sub-assemblies, and the eligibility information is known. Also, the item is assigned to a single machine.
- Production cost can vary with time, and setup cost is fixed over time.
- The setup is sequence-independent.
- Full demand is known at the beginning of each production period.
- No inventory shortage is allowed for the component item. The inventory shortage is allowed only for end items.
- Multiple items can be processed in a single resource.
- Inventory at the beginning and end is zero.

The model used by Chowdhury et al. [1] is given below; in the model, T is the number of periods available, n is the number of items, and m is the total number of the machine. The decision variables are as follows:

I_{jt} Inventory level of item $j \in \{1, n\}$ at the end of period $t \in \{1, T\}$

X_{jt} Production quantity of item $j \in \{1, n\}$ in period $t \in \{1, T\}$

$$Y_{jt} = \begin{cases} 1 & \text{if there is a setup for item } j \in \{1,n\} \text{ on machine } i \in \{1,m\} \text{ in period } t \in \{1,T\} \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_{jt} = \begin{cases} 1 & \text{if the setup state of } \textit{machine } i \mid j \in \varphi(i) \text{ at the end of period } t \in \{1,T\} \text{ and at the beginning of period(need) } (\boldsymbol{t+1}) \text{ is item } j \in \{1,n\} \\ 0 & \text{Otherwise} \end{cases}$$

b_{jt} Quantity back ordered for item $j \in \{1, n\}$ in period $t \in \{1, T\}$

E_t Emission due to production, inventory, and setup in period $t \in \{1, T\}$

The parameters used are as follows:

a_{jk} Quantity of item (parts) $j \in \{1, n\}$ required to produce one unit of item $k \in \{1, n\}$

D_{jt} External demand of item $j \in \{1, n\}$ in period $t \in \{1, T\}$

h_j Holding cost of item $j \in \{1, n\}$

c_j Setup cost for item $j \in \{1, n\}$

s_j Setup time for item $j \in \{1, n\}$

M A large number

I_{j0} Initial inventory level of item $j \in \{1, n\}$

R_{it} Available capacity of machine $i \in \{1, m\}$ in period $t \in \{1, T\}$ (in time units)

$\Gamma(j)$ Set of immediate successors of item $j \in \{1, n\}$

P_{jt} Production cost per unit of finished item $j \in \{1, n\}$ at period $t \in \{1, T\}$

$\varphi(i)$ Set of items that can be assigned to machine $i \in \{1, m\}$

ω Set of end items (items with external demand only)

$\mu(j)$ Set of immediate predecessors of item $j \in \{1, n\}$

$\rho(j)$ Set of machines eligible to process item $j \in \{1, n\}$

p_j Processing time required to produce one unit of item $j \in \{1, n\}$

β_j Backlogging cost for one unit of item $j \in \{1, n\}$ per period.

$\hat{s}_j$ Carbon emission related to the setup of item $j \in \{1, n\}$

$\hat{p}_j$ Carbon emission related to per unit production of item $j \in \{1, n\}$

$\hat{h}_j$ Carbon emission related to per unit holding inventory of item $j \in \{1, n\}$

C_{cap} Total allowable carbon emission cap

Model MLCLSP_SCBE:

The objective function minimizes the entire cost of production, holding, setup & backlogging cost.

$$Min \sum_{j=1}^{n} \sum_{t=1}^{T} (P_{jt} X_{jt} + h_j I_{jt} + c_j Y_{jt} + \beta_j b_{jt}) \tag{4.1}$$

Subject to,

$$I_{j0} = b_{j0} = \alpha_{j0} = 0 \quad \forall j \in \{1, n\} \tag{4.2}$$

$$I_{jt} = I_{j(t-1)} + X_{jt} + b_{jt} - b_{j(t-1)} - D_{jt} \forall j \in \{1, n\},\ t \in \{1, T\} \ | j \in \omega \tag{4.3}$$

$$I_{jt} = I_{j(t-1)} + X_{jt} - \sum_{k \in \Gamma(j)} a_{jk} X_{kt} \quad \forall j \in \{1, n\}, t \in \{1, T\} \ | j \notin \omega \tag{4.4}$$

$$X_{jt} \leq \mathrm{M}(Y_{jt} + \alpha_{j(t-1)}) \ \forall\, i \in \{1, m\}, j \in \varphi(i), t \in \{1, T\} \tag{4.5}$$

$$\sum_{j \in \varphi(i)} p_j X_{jt} + \sum_{j \in \varphi(i)} s_j Y_{jt} \leq R_{it} \ \forall\, i \in \{1, m\},\ t \in \{1, T\} \tag{4.6}$$

$$\alpha_{jt} \leq Y_{jt} + \alpha_{j(t-1)} \quad \forall\, i \in \{1, m\}, j \in \varphi(i), t \in \{1, T\} \tag{4.7}$$

$$\sum_{j \in \varphi(i)} \alpha_{jt} \leq 1 \ \ \forall\, i \in \{1, m\},\ t \in \{1, T\} \tag{4.8}$$

$$E_t = \sum_{j=1}^{n} (\hat{p}_j X_{jt} + \hat{h}_j I_{jt} + \hat{s}_j Y_{jt}) \ \ \forall\, t \in \{1, T\} \tag{4.9}$$

$$\sum_{t=1}^{T} E_t \leq C_{cap} \tag{4.10}$$

$$I_{jt} \geq 0,\ X_{jt} \geq 0,\ b_{jt} \geq 0, \forall\, j \in \{1, n\}, t \in \{1, T\} \tag{4.11}$$

$$b_{jT} = 0 \quad \forall\, j \in \{1, n\} \tag{4.12}$$

$$Y_{jt},\ \alpha_{jt} \in \{0,1\} \qquad \forall\, i \in \{1, m\}, j \in \varphi(i), t \in \{1, T\} \tag{4.13}$$

Constraint (4.2) states that the initial inventory, backlogging, and setup condition is zero. Constraints (4.3) and (4.4) describe the inventory balance for items that are required to fulfil external and internal demands, respectively. Constraint (4.5) ensures correct setup state decision or setup carryover decision. Constraint (4.6) is the capacity constraint. This constraint is the complicated constraint in the model. Constraint (4.7) provides a trade-off between setup carryover decisions to make sure that a setup can be carried over from the period t to period $(t+1)$ only if either item j is setup in period t, or the setup state previously has been passed over from period $(t-1)$ to period t. Constraint (4.8) restricts a setup from being carried over more than one period, and the constraint is also a complicated constraint. Constraint (4.9) calculates the amount of carbon emission due to production, inventory, and setup for each period. Constraint (4.10) provides a limit on carbon emission, and as it is related to production, inventory, and setup cost, it is a complicated constraint. Constraints (4.11) declare the nonnegativity of the variables, Constraint (4.12) ensures the ending backlogging quantity is zero, and Constraints (4.13) provide information for the binary constraints.

4.2. Relaxed problem for Component & End item from the Original Master Problem:

The relaxed (also known as subproblems) are generated into two decomposable problems for Single item uncapacitated lot sizing problem. The relaxed problem does not contain any capacity restriction as the complicated constraints are not considered in the sub-problem/relaxed problem. The first Sub-problem is for the final/end item (known external demand, no successors), and the second equation is generated for the component item. Relaxed problems are solved competently using a dynamic programming algorithm proposed by Chowdhury et al. (2018). The Dynamic Programming algorithm will generate an optimal solution for all relaxed problems because each uncapacitated single-item relaxed problem has a W.W. cost structure. Both equations for the end and component item have been given below:

Relaxed problem for end item:

$$Min\ \sum_{i=1}^{m}\sum_{t=1}^{T}\left[(P_{jt}-w_{it}p_j-\gamma\hat{p}_j)X_{jt}+(c_j-w_{it}s_j-\gamma\hat{s}_j)Y_{jt}-\alpha_{jt}y_{it}\right]+\sum_{t=1}^{T}\left[(h_j-\gamma\hat{h}_j)I_{jt}+\beta_j b_{jt}\right]-v_j\quad \forall j \tag{4.14}$$

Subject to,

$$I_{j0}=b_{j0}=\alpha_{j0}=0 \tag{4.15}$$

$$I_{jt}=I_{j(t-1)}+X_{jt}+b_{jt}-b_{j(t-1)}-D_{jt}\ \ \forall t$$

$$I_{kt}=I_{k(t-1)}+X_{kt}-\sum_{k'\in\Gamma(k)}\boldsymbol{a_{kk'}}X_{k't}\quad \forall t \tag{4.16}$$

$$X_{jt}\le M(Y_{jt}+\alpha_{j(t-1)})\ \forall\, i\in\rho(j),t \tag{4.17}$$

$$Y_{jt}+\alpha_{j(t-1)}\ \le 1\ \forall\, i\in\rho(j),t \tag{4.18}$$

$$I_{jt},X_{jt},b_{jt}\ge 0\qquad \forall\, i\in\rho(j),t\ge 1 \tag{4.19}$$

$$Y_{jt},\alpha_{jt}\in\{0,1\}\qquad \forall\, i\in\rho(j),t\ge 1 \tag{4.20}$$

After all end items are scheduled, the next item, $k|k\in\mu(j)$, is scheduled. The decomposed subproblems for all $k|k\in\mu(j)$ are as follows:

Relaxed problem for the component item:

$$Min\ \sum_{i=1}^{m}\sum_{t=1}^{T}[(P_{kt}-w_{it}p_k-\gamma\hat{p}_k)X_{kt}+(c_k-w_{it}s_k-\gamma\hat{s}_k)Y_{kt}-\alpha_{kt}y_{it}]+\sum_{t=1}^{T}\left[(h_k-\gamma\hat{h}_k)I_{kt}\right]-v_k\quad \forall k\in\mu(j) \tag{4.21}$$

Subject to,

$$I_{kt}=I_{k(t-1)}+X_{kt}-\sum_{k'\in\Gamma(k)}\boldsymbol{a_{kk'}}X_{k't}\quad \forall t \tag{4.28}$$

and (4.15), (4.17)– (4.21) for $j=k$.

4.3. Weighted problem:

A weighted problem is formed with the original master problem multiplied by the weighted variable $(\boldsymbol{\lambda_{ju}})$**,** because if the problem has one or more basic feasible solutions of the relaxed/Sub-problem, those solutions can be used to produce a feasible solution to the non-relaxed (restricted) master problem as a linear convex combination of those basic feasible solutions. If the master problem has constraints that can lead to an infeasible solution, a big value, M, and an artificial variable are added to the master problem to remove the infeasibility.

Weighted Problem ((WP):

$$Min\sum_{u\in U_j}\sum_{j=1}^{n}\sum_{t=1}^{T}(P_{jt}X_{jt}^{u}+h_jI_{jt}^{u}+c_jY_{jt}^{u}+\beta_j b_{jt}^{u})\,\boldsymbol{\lambda_{ju}}+\ \mathrm{M}*\{(\mathrm{V1}+\mathrm{V2}+\mathrm{V3})+\mathrm{W}\} \tag{4.29}$$

Subject to:

$$\sum_{u\in U_j}\sum_{j\in\varphi(i)}(p_jX_{jt}^{u}+s_jY_{jt}^{u})\lambda_{ju}+(\mathrm{V1}-\mathrm{W})=R_{it}\!:\boldsymbol{w_{it}}\ \forall\, i,\,t \tag{4.30}$$

$$\sum_{u\in U_j}\sum_{t=1}^{T}\sum_{j=1}^{n}\big(\hat{p}_jX_{jt}^{u}+\hat{h}_jI_{jt}^{u}+\hat{s}_jY_{jt}^{u}\big)\lambda_{ju}+V2-W=\ C_{cap}\ \therefore\boldsymbol{\gamma} \tag{4.31}$$

$$\sum_{u\in U_j}\sum_{j\in\varphi(i)}\alpha_{jt}^{u}\lambda_{ju}+V3-W=1\!:\boldsymbol{y_{it}}\ \forall\, t\ge 0,i \tag{4.32}$$

$$\sum_{u\in U_j}\lambda_{ju}=1\!:\boldsymbol{v_j}\ \forall\, j\ \text{(Convexity Constraint)} \tag{4.33}$$

$$\lambda_{juz}\ge 0\ \forall\, j,u\in U_j\ \text{(Non-negativity Constraint)} \tag{4.34}$$

4.4. Framework of the solution procedure:

The following steps are applied to solve the model:

- Step 1: Fix the dual variables w_{it}, y_{it}, γ, and v_j a value of zero for the end items $j|j\in\omega$.Where, w_{it}, γ , y_{it} are the dual variable of the complicated constraint & , v_j It is the dual variable of the convexity constraint.
- Step 2: Solve the single-item uncapacitated lot sizing problem using dynamic programming per Chowdhury et al. (2018) 's algorithm. And calculate the variable value of X_{jt} and setup decision Y_{jt} for item j in period t.
- Step 3: Derive demand for the component items $k|k\in\lambda$ by using the W.W. algorithm. And repeat steps 1 & 2 for the component item.
- Step 3: Derive demand for the component items $k|k\in\lambda$ by using the W.W. algorithm. And repeat steps 1 & 2 for the component item.
- Step 4: Solve the ILP for maximizing setup cost savings procedure from Chowdhury et al. [1] and obtain the value of the setup carryover decision variable $\alpha_{jt}\forall j,t$.

  Step 5: Use the Y_{jt} values from step 2 and α_{jt} From step 4, As parameters and solve the model's Original Master Problem to obtain an optimal value for X_{jt}, I_{jt} and B_{jt}.

- Step 6: Solve the reformulated *WP* with Big M and artificial variables, and print the dual variable value.
- Step 7: Use the dual variable value for convergence checking using the relationship between reduced cost & dual variable. If a non-negative reduced cost is found, the iteration shall stop; otherwise, use the updated dual variable and continue from Step 2.

5. Computational Study

5.1. Dataset used:

The proposed exact method result has been compared with Chowdhury et al. [1]'s computational result. Tempelmeier and Derstroff [12] first introduced the five sets of problem instances. The dataset used here to compare both methods is the class B dataset.

Table 5.1
Computational study analysis (Average Optimality Gap)

		Utilization rate											
TBO Profile	CV	90%		70%		50%		90/70/50		50/70/90		Mean	
		DW-M	DW-CA	DW-M	DW-CA	DW-M	DW-CA	DW-M	DW-CA	DW-M	DW-CA	DW-M	DW-CA
1	0.1	0.00	0.01	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.02
	0.4	0.00	0.62	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.06	0.00	0.15
	0.7	0.01	0.01	0.00	0.06	0.00	0.07	0.00	0.06	0.00	0.06	0.00	0.05
	mean	0.00	0.21	0.00	0.04	0.00	0.02	0.00	0.04	0.00	0.06	0.00	0.08
2	0.1	0.00	0.01	0.00	0.06	0.00	0.06	0.00	0.06	0.00	0.04	0.00	0.05
	0.4	0.00	0.00	0.00	0.00	0.00	0.01	0.00	*	0.00	0.06	0.00	0.02
	0.7	0.00	0.01	0.00	0.06	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.02
	mean	0.00	0.00	0.00	0.04	0.00	0.02	0.00	0.04	0.00	0.04	0.00	0.03
4	0.1	0.00	0.01	0.00	0.06	0.00	0.06	0.00	*	0.00	0.00	0.00	0.03
	0.4	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.03
	0.7	0.00	0.00	0.00	0.06	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.02
	mean	0.00	0.02	0.00	0.04	0.00	0.02	0.00	0.03	0.00	0.00	0.00	0.03
1/2/4	0.1	0.00	0.01	0.00	0.06	0.00	*	0.00	0.00	0.00	0.01	0.00	0.02
	0.4	0.00	0.00	0.00	0.06	0.00	0.01	0.00	*	0.00	0.00	0.00	0.02
	0.7	0.00	0.00	0.00	0.06	0.00	0.06	0.00	0.06	0.00	0.00	0.00	0.04
	mean	0.00	0.00	0.00	0.06	0.00	0.03	0.00	0.03	0.00	0.00	0.00	0.03
4/2/1	0.1	0.00	0.01	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.02
	0.4	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.06	0.00	0.04
	0.7	0.00	0.01	0.00	0.06	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.03
	mean	0.00	0.03	0.00	0.04	0.00	0.00	0.00	0.04	0.00	0.04	0.00	0.03
												0.001	**0.037**

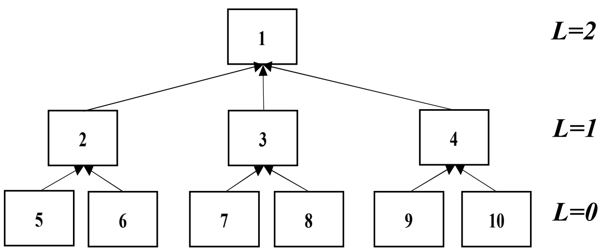

Fig. 2 General Product Structure

Class B dataset contains a capacity of three (3) machines with ten periods to produce ten different items.

- General product structure is considered. General product structures, which represent multiple assemblies, are the most complex since there is no limit on the number of predecessors or successors. (See Fig 2). The structure has three different levels of production.
- Three demand structures with varying coefficients of variance (CV = 0.1, 0.4, 0.7) with a predetermined mean demand are considered for generating demand.
- The capacity utilization profile is determined as 90%,70%,50%,90%/70%/50%, and 50%/70%/90%. Available capacity per period is computed by dividing the mean demand by the target capacity utilization. 90%/70%/50% means three utilization profiles for three levels.
- Five Setup cost profiles are determined with the following equation: Time between Order (TBO) Profiles are 1, 2, 4, 4/2/1, 1/2/4. The time between orders is the production cycle's average length [1]. The profile with slashes means (Example, 4/2/1) different TBO profiles for different hierarchy levels. $\text{Setup cost} = \frac{1}{2} * (TBO)^2 * holding\ cost * Average\ Demand$

Table 5.2

Computational study (Computational time)

TBO Profile	*Utilization rate*							
	90%							
	DW-M			DW-CA			Mean	
	1500	2000	2500	1500	2000	2500	DW-M	DW-CA
	Computational Time (Sec)							
1	2.0	2.3	1.8	3.1	5.4	3.7	2.0	4.1
2	2.1	1.8	1.8	2.6	4.5	3.0	2.6	3.4
4	2.2	2.3	2.0	3.4	5.1	3.2	2.2	3.9
1/2/4	1.5	1.6	1.6	9.7	6.4	3.2	1.6	6.4
4/2/1	2.7	2.3	2.3	3.4	5.3	3.5	2.4	4.1
Average	2.1	2.1	1.9	4.4	5.3	3.3	2.2	4.4

5.2. Results:

A total of 75 instances were generated for both DW-CA & DW-M algorithms. Among all the cases, four instances were found infeasible for the DW-CA algorithm, highlighted with (*) sign. DW-M algorithm provides a 0% optimality gap for 73 out of 75 instances. (See Table 5.1)

Each model is coded using Fico's Mosel (Xpress IVE version:8.13 – 64-bit) algebraic modelling language. All the test instances are run on a P.C. with an Intel(R) Core (T.M.) i7-3770 CPU with a 3.40GHz processor and 16 G.B. of RAM.

To further investigate the computational study in terms of computational time, another 15 instances are generated. The instances were generated considering the emission control constraint as a variant [1] (1500,2000 and 2500 t/MWh), similar to the consideration of Chowdhury et al.,; [1]. The instances had a fixed 90% utilization rate, with a demand structure with a mean of 100 and 0.1 co-efficient of variance. (See Table 5.2)

The average time for code completion status update for the DW-M algorithm is 2.2s, and for DW-CA is 4.4 seconds. A graphical presentation for all TBO profiles at 1500 t/MWh carbon emission cap limit is presented on the chart (Fig 5.2).

5.3. Discussion & Limitation:

The Multi-level capacitated lot sizing problem (MLCLSP) belongs to the class of NP-hard problems [11]. The problem solved here is an extension of Classical MLCLSP and an NP-hard problem.

The presented solution combines the Dantzig Wolfe decomposition and Big M methods to solve a lot-sizing problem scenario. The Dantzig formulation can be used only with a special angular block structure [55]. As per Bergner et al. (2006)[62], Dantzig Wolfe's reformulation needs tailoring for all applications, and the user must know that the problem has an exploitable structure to solve the model algorithmically. So, the Dantzig-Wolfe Decomposition technique cannot solve any model without such a structure. Also, for a larger problem, one might be unable to solve the decomposed model optimally; hence, we might not wait for the model to converge before we terminate the solution procedure fully. Therefore, the quality of the produced optimized solution could be compromised, i.e. we may end with an optimality gap.

Now, a few drawbacks require attention while solving the problem for the Big M method. The value of the Big M while solving the problem needs to be set carefully to guarantee the optimization process's correctness. A lower value could lead to infeasible solutions even if they exist, and a higher value could lead to numerical instabilities. So, the requirement of re-tuning of the M value while solving the real-world scenario may limit its application [63].

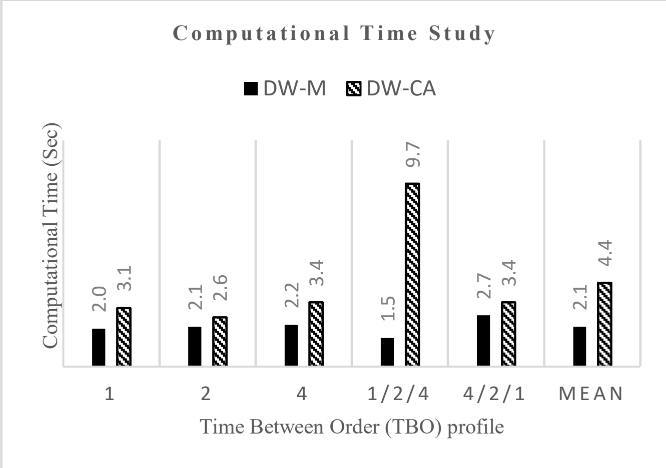

Fig. 3: Computational time study.

6. Conclusion

This chapter summarizes the approach to solving an existing multi-level capacitated lot sizing problem by removing the heuristic model to reach optimality by improvising Big M and artificial variables to the model. A better performance was achieved from the existing heuristic method (DW-CA) compared to the proposed method (DW-M) while solving the MLCLSP with SCBE. The computational study in Table 5.1 shows that, practically DW-M method arrives at a zero-optimality gap. The improvement of the computational time is remarkable as it takes 50% less computational time with the DW-M method compared to the existing heuristic method. (See Table 5.2).

Overall, the performance of the new approach provides a satisfactory result with all feasible solutions and a 0.001% average optimal gap, which allows us to avoid the heuristic model and achieve the goal of the study.

As a part of improvement, in the future, introducing more complexity to the existing model, such as a hybrid manufacturing concept and GHG gas emission reduction from operational vehicle constraints, can be considered. The lot-sizing model can consider the assumption of using hybrid machinery to replicate the present manufacturing world scenario and reduce the time for setup costs. Platform production concept can be adapted in the model to meet mass customization; this advanced concept will allow more savings for classical lot sizing problems. Also, Meta-heuristics can be developed for the model to tackle future complexities. This advanced concept will allow more savings for classical lot sizing problems. Also, Meta-Heuristics are known to produce a quicker result. A solution approach considering meta-heuristics can be developed for the model to tackle future complexities and improve computational time.

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