Inference as growth: Peirce’s ecstatic logic of illation

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ABSTRACT: For Peirce, logic is essentially illative, a relation of inferential growth. It follows that inference and argumentation are essentially ecstatic, an asymmetrical, ampliative movement from antecedent to consequent. It also follows that logic is inherently inductive. While deduction remains an essential and irreplaceable aspect of logic, it should be seen as a more abstract expression of the illative, semiological essence of inference as such.

Keywords: Growth, illation, logic, semiosis, semiological, Peirce, abduction, deduction, induction, inference, evolve.

1. INTRODUCTION

According to Peirce, the fundamental principle of all logic is the relation of illation as expressed by the terms ‘therefore’ or ergo. As I will show, Peirce interprets illation in a radical, semiotic sense as a logic of growth, that is, an inferential movement from some initial state (e.g., a premise) towards a state of increased ‘information’ (e.g., a conclusion). It follows that inference (and argumentation) is essentially ecstatic, an asymmetrical movement from some antecedent state towards some non-reducible consequent.

While Peirce’s illative logic of growth would appear well-suited to inductive and abductive forms of inference (which are ampliative in nature), it is less clear at first blush how it applies to deductive inferences (which are non-ampliative in nature). As I will show, however, Peirce’s account of deductive inferences is also illative in his radical sense where deductive inferences evolve their conclusions in an asymmetrical sense that is consistent with Peirce’s illative logic of growth.

It follows that logic, in Peirce’s illative, ecstatic sense, is better understood as an inductive rather than a deductive science, for the ampliative work of inductive inference better exemplifies, in a richer, fuller sense, the illative, ecstatic essence of inference per se. While deduction still stands as essential and irreplaceable aspect of logic, it remains a purely formal and hence more abstract (and more ‘degenerate’) expression of the illative essence of inference (and argumentation) in its fullest sense.1

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1 I should begin by pointing out that the Peircean account of logic and inference that I will be drawing from and building upon is a hypothesis that is to be tested against experience. What follows is part of my own attempt to test and, if necessary, to modify and revise Peirce’s insights and the implications that might follow from them. I should also note that I am less concerned about whether I have Peirce ‘right’ as it were, and more concerned with trying to develop an account of inference from a broadly Peircean point of view.
2. LOGIC AND INFEREN CE

For Peirce, logic in its broadest sense is “a study of the means of attaining the end of thought.” (CP, 2.198) Viewed with respect to its practical orientation, the end of thought is the reliable and effective attainment of practical ends, a process that is mediated through beliefs (where beliefs stand as habits of mind that function as regulative grounds for attaining ends). Expressed in doxastic terms, logic is a study of the means of attaining reliable and effective beliefs. It is in this spirit that Peirce identifies truthful inquiry as the best means for the “settlement” or “fixation of belief,” for generally speaking, true beliefs are going to have the least chance of being unsettled by experience in the long run (and hence will generally outperform other means of settling or fixing beliefs). Hence Peirce’s characterization of logic as the normative science or “theory of right reasoning,” for as the study of the means of attaining the end of thought, logic must be aimed, generally speaking and in the long run, at determining the best means of attaining that end. (CP, 2.7; 2.29) Thus while logic is certainly concerned with “how we do think” (e.g., psychologically or anthropologically), it is so primarily and essentially as a way of determining “how we ought to think in order to think what is true.” (CP, 2.52) As “a study of the means of attaining the end of thought,” logic in this Peircean sense will therefore be aimed, generally, doxastically, and in the long run, at the advancement of true beliefs, an aim that I will refer to as an alethic orientation.

As a means of advancing true beliefs, logic also stands as a kind of “method of discovering methods,” a capacity for selecting the methods best suited to attaining the end of thought. (CP, 2.108) To this end, the logician should seek “to make out a theory of how knowledge is advanced.” (Peirce 1903b, p. 256) To fully realize this alethic function, logic should thereby be grounded in conditions that: 1) enable the discovery and growth of knowledge, and 2) enable the construction or determination of truthful beliefs. As an alethic mode of being-in-the-world, logic should thereby possess “the vital power of self-correction and of growth” for advancing the end of thought. (Peirce 1898, pp. 44-47; 1913, p. 466) As I hope to show, the alethic orientation of logic developed here is supported by what I will call the semiological, illative structure of inference upon which logic itself is grounded. It is upon the very essence or nature of inference, viewed as a semiological relation of illation, that the possibility of the growth of knowledge and the determination of truthful beliefs ultimately rests. Further, not only is inference in its illative, semiological sense a condition for the possibility of cognitive growth (and the de-

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2 This is one of the central, but often overlooked points of Peirce’s Fixation of Belief. One of the main points of that article is to show that a ‘scientific’ method of securing belief (i.e., one that is aimed at the truth) is the best way of “settling opinions” in the long run precisely because it is the one method that best secures belief against the disruptive force of doubt. Of course it is not the mere ‘settlement’ or ‘fixation’ of belief as such that is the final aim here (e.g., simple consensus), but rather the settlement upon beliefs that stand as reliable and effective guides for attaining ends.

3 Like all the normative sciences, logic is oriented toward the Ideal, not simply the practical. Though arguments in general can certainly be aimed at concrete, practical ends, nevertheless when viewed broadly as an Inductive science, logic is always to be measured against the Ideal toward which it is essentially oriented, and not the practical ends toward which its argument forms may be contingently aimed. (CP, 2.46)

4 For a rigorous presentation of Peirce’s claim about the inferential essence of cognition see “Questions Concerning Certain Faculties Claimed for Man” as well as “Some Consequences of Four Incapacities.”
termination of belief), but I would also suggest that the illative, semiological form of inference is the form of logical growth.

3. SEMIOTIC STRUCTURE OF MIND AND THE LAWS OF INFECTION

From his early Harvard Lectures on logic through the rest of his life’s work, Peirce was captivated by what he called the semiotic structure of thought. One of Peirce’s central insights was that mind or thought was an inferential process whose structure and function was to be found in the structure and function of a sign. In one of his early published papers, Peirce claims that “mind is a sign developing according to the laws of inference.” (Peirce 1868b, p.53) If true, then it follows that logic, as the science or study of thought, must include both the study of signs and the study of the laws of inference.\(^5\)

For Peirce, a sign in its fullest sense (which Peirce often refers to as a Symbol) is a complex, triadic, three-fold structure.

A sign, in Peirce’s sense of the term, is an irresolvable, irreducible, trivalent processional structure. A sign is a relation of mediated connection whereby an ‘object,’ i.e., whatever is being conveyed or communicated (what the sign is of), is brought into connection with a recipient or “interpretant,” i.e., whatever is being conveyed or communicated to (what the sign is for), and where the connecting condition (i.e., the principle or ground of the connection or relation as such) is itself constitutive of the relation or connection between the ‘object’ and ‘interpretant’ in question. These three components, ‘object,’ ‘interpretant,’ and connecting condition, principle, or ground (which I will call the connective) are to be taken together as essential, constitutive conditions of a sign qua sign.

Since a sign always stands as a mediated, trivalent relation from some ‘object’ to some recipient or interpretant, then a sign will always have a determinate direction or ‘vector quality’ (to borrow a phrase from Whitehead). This means that the relation between the ‘object’ and ‘interpretant’ in a sign is asymmetrical in the sense that a sign is always oriented towards an interpretant as its culminating reference or end. Thus, like thought and logic in general, a sign has an essential orientation, namely, its reference to some interpretant.\(^6\) Importantly, what is essential to a sign is the orientation toward or

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\(^5\) So important is semiosis to logic and the study of thought that at times Peirce goes so far as to characterize logic itself as “the science of the general laws of signs.” (CP 2.191)

\(^6\) Not surprisingly, the triadic orientation of a sign parallels the asymmetrical, triadic structure of tensed time viewed as a relation of past, present, and future. I accept Peirce’s claim, however, that we should not think of signs as having a temporal structure, but rather should see time as having the structure of a sign. Understood in this sense, the structure of a sign will be taken as fundamental metaphysically and ontologically as the elementary condition against which other similar structures and relations are to be compared and understood. (Peirce 1865, pp. 325-329). I will therefore refer to the position being de-
reference to an interpretant, not the actual presence of an interpretant per se. Hence a sign’s reference to its interpretant will still be satisfied if the interpretant is merely hypothetical, and not actual.

It will be necessary to recognize two different operations, because of the difference between the relation of a symbol to its object and to its interpretant. Illative transformation (the only transformation, relating solely to truth, that a system of symbols can undergo) is the passage from a symbol to an interpretant, generally a partial interpretant. But it is necessary that the interpretant shall be recognized without the actual transformation. Otherwise the symbol is imperfect. There must, therefore, be a sign to signify that an illative transformation would be possible. That is to say, we must not only be able to express “A therefore B,” but “If A then B.” The symbol must, besides, separately indicate its object. The object must be indicated by a sign, and the relation of this to the significant element of the symbol is that both are signs of the same object. This is an equiparant, or commutative relation. It is therefore necessary to have an operation combining two symbols as referring to the same object. This, like the other operation, must have its actual and potential state. (CP, 4.375)

As I hope to show, if we accept Peirce’s claim that “mind is a sign developing according to the laws of inference,” then it follows that thought or mind will have the trivalent structure and essential orientation of a sign. But mind or thought here is also more than a mere sign in a purely formal or structural sense, for mind is also an active or processional semiotic structure that is developing according to the laws of inference. This suggests firstly that mind is not a static state, but an essentially triadic developmental process. It follows secondly that cognition is not primarily a state of knowing in some static, contemplative sense of that term, but is best and most fully illustrated in the process of learning. Third, this also suggests that mind, in its fullest expression as a process of learning, is itself an inferential process, that is, a semiological process of evolving conclusions from premises according to the logic of illation.

4. ILLATION AS THE FUNDAMENTAL LAW OF INference

For Peirce, the fundamental law of thought is to be found, not in the traditional principles of identity, contradiction, or excluded middle, but in the principle of illation as expressed in such commonplaces as ‘follows,’ ‘therefore,’ or ergo. (CP, 2.593; 2.599; 3.175; 3.440; 3.450) Put simply, all inference in its most basic, fundamental sense is illative.

Illation in a Peircean sense of the term is not merely a relation in a purely formal, abstract sense, but is instead a dynamic process or developmental relation between relata. More specifically, illation expresses, one might even say denotes, a connection between relata (e.g., antecedent and consequent) that enables a kind of transition, movement, or ‘flow’ as it were between the relata in question. Further, illation is distinguished from identity in being essentially and constitutively asymmetrical where the connection between relata is one-directional (with symmetrical relations such as those of identity standing as special, complex instances of the more fundamental and asymmetrical rela-

fended here as a semiological account that sees being itself in a fundamental metaphysical sense of that term, as exhibiting or at least including the structure of signs.
It should be apparent that illation is strikingly similar to the asymmetrical, trivalent structure of a sign. Like a sign, the asymmetrical nature of illation has a vector quality or orientation, one that is compatible with, and so can be mapped onto, the trivalent, asymmetrical orientation of the semiological order. It is in this sense that we are to understand the Peircean claim that “mind is a sign developing according to the laws of inference,” for mind or thought is the trivalent, asymmetrical structure of a sign “developing according to” the processional, inferential structure of illation. It is these two conditions, trivalent semiotic structure and the inferential connectivity of illation, that underlie the alethic orientation of logic and mind, an orientation that expresses itself experientially as a temporal orientation toward the future.

Thus the connection between semiosis and illation is indeed quite strong, so strong in fact that one might go so far as to claim that illation is nothing else than a sign viewed in its dynamic, active sense. Peirce himself suggests as much. In fact, he claims that illation, as the basis of inference per se, is also the essence of cognition as well.

To speak summarily and use a symbol of abbreviation, rather than an analytical and iconical idea, we may say that the purpose of signs—which is the purpose of thought—is to bring truth to expression. The law under which a sign must be true is the law of inference; and the signs of a scientific intelligence must, above all other conditions, be such as to lend themselves to inference. Hence, the illative relation is the primary and paramount semiotic relation.

It might be objected that to say that the purpose of thought is to bring the truth to expression is to say that the production of propositions, rather than that of inferences, is the

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7 Peirce argues that illation, as expressed in what he calls the “copula of inclusion,” is more fundamental than the copula of identity, for where the copula of inclusion can account for relations of identity, the copula of identity cannot easily account for relations of inclusion or illative relations (which are essentially asymmetrical). Thus where the principle of identity entails that if A is a necessary and sufficient condition for B, then B would also be a necessary and sufficient condition for A (as a kind of simultaneous or entangled relation of dependence), but within an illative relation (or Peirce’s copula of inclusion) if A is a necessary and sufficient condition for B, it does not follow that B is not also a necessary and sufficient condition for A, hence the essential asymmetry (or perhaps better, non-symmetry) of illative relations.

Now it is to be observed that the illative relation is not simply convertible; that is to say, that “from A necessarily follows B” does not necessarily imply that “from B necessarily follows A.” Among the vagaries of some German logicians of some of the inexact schools, the convertibility of illation (like almost every other imaginable absurdity) has been maintained; but all the other inexact schools deny it, and exact logic condemns it, at once. Consequently, the copula of inclusion, which is by the ergo freed from the accident of asserting the truth of its antecedent, is equally inconvertible. For though “men include only mortals,” it does not follow that “mortals include only men,” but, on the contrary, what follows is “mortals include all men.” Consequently, again, the fundamental relative copula is inconvertible. That is, because “Tom loves (if anybody) only a servant (or servants) of Dick,” it does not follow that “Dick is served (if at all) only by somebody loved by Tom,” but, on the contrary, what follows is “Dick is master of every person (there may be) whose is loved by Tom.” We thus see, clearly, first, that, as the fundamental relative copula, we must take that particular mode of junction; secondly, that that mode is at bottom the mode of junction of the ergo, and so joins a relative of antecedental character to a relative of consequential character, and thirdly, that the copula is inconvertible, so that the two kinds of constituents are of opposite characters. There are, no doubt, convertible modes of junction of relatives, as in “lover of servant”; but it will be shown below that these are complex and indirect in their constitution. (CP, 3.474)
primary object. But the production of propositions is of the general nature of inference, so that inference is the essential function of the cognitive mind. (CP, 2.444, Note)

Cognition in a Peircean sense is an inferential, semiotic movement or connection between some initial condition (e.g., a case or fact) and its semiological, illative result (e.g., an explanation or conclusion). Cognition is the sense of an illative, semiological connection (of a certain type) between relata as expressed in the language of ‘therefore’ or ergo. Thus, if inference is indeed “the essential function of the cognitive mind” as Peirce suggests, and inference is an illative transition or semiotic movement from one condition to another (through the essential, asymmetric connectivity that underlies all such relations), then as already noted, cognition in its purest sense is not to be thought of as a static mental state (e.g., knowing as a kind of mental relation to eternal or fixed truths), but is instead an inferential, semiotic process of epistemic growth. Thus, cognition in its purest sense (i.e., cognition abstracted from other non-epistemic orientations), is learning.8

In the first place every kind of consciousness enters into cognition. Feelings, in the sense in which alone they can be admitted as a great branch of mental phenomena form the warp and whoof of cognition and even in the objectionable sense of pleasure and pain, they are constituents of cognition. The will, in the form of attention, constantly enters, and the sense of reality or objectivity, which is what we have found ought to take the place of will, in the division of consciousness, is even more essential yet, if possible. But that element of cognition which is neither feeling nor the polar sense, is the consciousness of a process, and this in the form of the sense of learning, of acquiring, of mental growth is eminently characteristic of cognition… This is the consciousness that binds our life together. It is the consciousness of synthesis. (Peirce 1887-88, p. 260)

Cognition in the sense being developed here is what Peirce calls the “genuine synthetic consciousness,” the mental process of discerning the real or factual nature of a connection between relata. Thus, cognition is experienced as “the sense of the process of learning,” a sense that “is the prominent ingredient and quintessence of reason.” This claim is further supported by the alethic orientation of logic, for insofar as logic is the “study of the means for attaining the end of thought,” and since a reference to an end “essentially involves growth,” then the means of attaining the end of thought must involve a process of growth toward that end.9 (Peirce 1908, p. 439-440)

8 I would add that the more traditional view of cognition as a kind of contemplative or ‘knowing’ state is not cognition in its fullest sense, but merely cognition viewed in its aesthetic or practical aspects: 1) as something aesthetically pleasing or ‘beautiful,’ or 2) as a means of achieving some given practical end. On the view being developed here, while the aesthetic and practical considerations are real aspects or ingredients of cognition, they are not to be confused with cognition per se. Cognition is the process of learning, and its aesthetic or practical aspects, while important for understanding cognition in its broadest sense, are importantly distinct from the inferential, illative learning process that constitutes cognition as such. It follows from this, of course, that when we cease to learn, then cognition in its fullest, elementary sense also ceases or diminishes as a prominent aspect of experience, leaving one in a state of mere aesthetic enjoyment accompanied by actions of an increasingly habitual character, for when learning ceases, aesthetic enjoyment and habitual response are pretty much all that remain.

9 The semiological, illative notion of inference as growth outlined here is not to be restricted to humans, but is also an essential feature of animal life as well. Put simply, animals also make inferences about their world, but of a less thoroughly self-conscious sense. (see Peirce 1913, pp. 467-470)
5. INFERENCE AS GROWTH: ECSTASIS

If I am right, then the semiological account of inference, cognition, and mind outlined here has a number of important implications. In this paper I will draw attention to one of the more important of these, namely, that all inference, and hence all argumentation as such, must necessarily involve some form of growth. Put simply, conclusions must be seen as growing or evolving from their premises: premises evolve conclusions.

To say that a premise evolves its conclusion means: 1) The premise stands in an asymmetrical relation to its conclusion, 2) The asymmetrical relation between premise and conclusion is a relation of determination, 3) As determined, a conclusion is not reducible to its premises, and 4) The relation between premise and conclusion thereby represents a growth in information from premise to conclusion. Space prohibits me from going into all these points in any detail, so I will provide an introductory gloss of each. Since we have already seen that the very structure of sign and illation involve an essential asymmetry, we can accept this as given.

In a Peircean sense of the term, to determine is to make something otherwise than it would have been. (Peirce 1865, pp. 217-219) Put simply the semiotic process of bringing an ‘object’ in relation to an interpretant is a determination of that object whereby the process of semiological inference makes the ‘object’ as received or interpreted otherwise than it would have been (the same applies to the interpretant viewed first as a potential state and after as a determinate state). As already noted, this has two important implications. First it entails that the interpretant in a semiological relation cannot be reduced to the object interpreted (as a relation of identity). The interpretant will contain conditions that are new or novel with respect to the object interpreted. Thus, a conclusion is always a further determination of the premise in some important sense. As a consequence, the conclusion of an inference represents a state of increased information with respect to the premise. Put simply, the conclusion always tells us something more than the premise.10

Thus, on our semiological account of inference, any and every inference must be said to evolve its conclusion in the sense that a conclusion will always contain more information than was contained in the premise. Put simply, inference evolves. I will add that the semiological, illative form of inference outlined here is itself the condition for the possibility of growth in a logical sense of the term. Put simply, the illative, semiological nature of inference is the condition, ground, or ‘principle’ if you will of all logical or mental growth; semiological illation is the logic of growth.11 It is in this general sense that an inference is seen to be inherently ecstatic, for with its essential, semiological orientation towards or reference to an interpretant, it is always reaching beyond its antecedent or present state.12 As I hope to show in future work (and as Peirce himself glimpsed

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10 In his early lectures in logic Peirce actually calls the growth of Information associated with the semiotic structure of thought was “the great and fundamental secret of the science of logic.” (Peirce 1866, p. 465)

11 Following Peirce’s claim that metaphysics should build upon logic, then any attempt to develop a metaphysical account of growth should take the illative, semiological account of inference outlined here (as the logical form of growth) as its starting point. Part of my future work will be to develop an account of growth in general based on the illative, semiological account of logical growth outlined here.

12 My reference to the ecstatic nature of semiological illation is directed at Heidegger’s critique of logic. I am trying to show that what Heidegger calls the ecstatic character of genuine thinking (as something contrary to what he characterizes as the totalizing, self-enclosing nature of ‘logic’) is itself a feature of logic properly understood. I am claiming, in effect, that Heidegger’s critique of logic fails to think logic
time and time again), the implications of this are potentially profound. In the present paper, however, we will limit the rest of our discussion to showing how this semiological account of inference applies to the kinds of demonstrative inferences commonly referred to as Deductions. For the purposes of this paper, I will accept the Peircean claim that all forms of argumentation are variations of three basic argument forms: Abduction, Induction, and Deduction.

6. THE SEMIOLOGICAL, ILLATIVE STRUCTURE OF DEDUCTION

As stated earlier, illation is an asymmetrical connection between relata. It is now time to say a little more about the nature or character of the illative connection or semiological connective as such. I will simply note here that for illation to be comprehensive enough to cover all argument forms, it is sufficient that the illative connection between relata be equivalent in character or argumentative force to the weakest of Peirce’s argument forms, namely, abduction.

Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis.

Deduction proves that something must be; Induction shows that something actually is operative; Abduction merely suggests that something may be.

Its only justification is that from its suggestion deduction can draw a prediction which can be tested by induction, and that, if we are ever to learn anything or to understand phenomena at all, it must be by abduction that this is to be brought about.

No reason whatsoever can be given for it, as far as I can discover; and it needs no reason, since it merely offers suggestions. (CP, 5.171)

As an inference or argument suggesting what “may be,” abduction is most often expressed in the general form of a hypothetical. In fact, in his earlier work Peirce actually referred to the abductive form of argument as ‘Hypothesis’ or ‘Presumption.’ Thus within an abduction the connection between the premise and conclusion (i.e., the relata of illation) is a hypothetical connection, one that may or may not be true of the world as experienced. As Peirce notes, such a hypothetical connection needs no justification, for it simply states what may be the case as part of an inferential standpoint.

Presumption is the only kind of reasoning which supplies new ideas, the only kind which is, in this sense, synthetic. Induction is justified as a method which must in the long run lead up to the truth, and that, by the gradual modification of the actual conclusion. There is no such warrant for presumption. The hypothesis which it problematically concludes is frequently utterly wrong itself, and even the method need not ever lead to the truth; for it may be that features of the phenomena which it aims to explain have no rational explanation at all. Its only justification is that its method is the only way in which there can be any hope of attaining a rational explanation. (CP, 2.777)

As a general rule, abductive inferences or hypotheses have such a loose ‘logical’ connection that they flow or happen quite readily and easily, with little or no effort, much like a

through to its own fundamentally ecstatic, illative, semiological ground. (For a more complete discussion of Heidegger’s critique of logic, see his Metaphysical Foundations of Logic, Trans. M Heim, Bloomington: Indiana Univ. Press. 1992)
simple association of ideas whose only justification is that it happens to ‘feel right’ as it were. (CP, 1.608) The only thing that can be determined with any assurance through an abduction is whether the conditions in question are “logically compossible or not.” (CP, 4.86) Since abduction covers all that is “logically compossible,” and an inference must also convey what is logically compossible (i.e., what may be illatively or inferentially connected), then the connection between relata found within an inference in its illative, semiological sense, should be comparable to the kind of connection found or expressed within abduction. Hence, all inference in its illative, semiological sense will involve some logically compossible connection of what may be in an abductive or hypothetical sense of the term. In fact, so fundamental is this illative, abductive power of making connections and forming hypotheses that Peirce himself identifies it with the traditional ‘light of nature’ and ‘light of reason.’ (CP, 2.23-2.25)

At this point we will address two important and related questions. First, how can we account for the possibility of deduction from the abductive character of inference in the illative, semiological sense defended here, that is, how can the certainty or necessity associated with deductive inference emerge from the abductive or hypothetical character of the illative, semiological form of inference upon which it is grounded? A second, related question is how can a deduction be said to evolve its conclusion, that is, what sense are we to make of the claim that “deduction merely evolves the necessary consequences of a pure hypothesis”? (CP, 5.171)

7. THE ‘INNER’ WORLD AND THE ‘OUTER’ WORLD

To address these questions requires that we build upon a distinction between what Peirce calls the ‘inner’ world and the ‘outer’ world. (Peirce 1895, p. 24-25) Put simply, the ‘inner’ world is the world of pure hypothesis, a world of pure mental construction governed solely by the ‘laws’ of thought, i.e., those rules of inference or habits of mind that Peirce refers to as leading principles (the more contingent of which are called material principles and the less contingent and more deeply inherited are called logical principles). In contrast, the ‘outer’ world is the world of concrete, natural fact, the world of experience as governed by the laws of nature. ‘Inner world’ inferences will involve the simple application of the laws of thought to one’s hypothetically constructed conditions, while ‘outer world’ inferences, if they are to be ‘true,’ will require that the laws of mind and its hypothetical conditions conform to the facts of experience in its broadest sense (including the contingent natural laws that happen to govern ‘outer’ occurrences). (Peirce 1903a, p. 194) Thus, put simply, ‘inner world’ inferences in the sense intended here will tend toward the a priori, while ‘outer world’ inferences will tend toward the a posteriori. (Peirce 1865, p. 245)

On the Peircean account of learning being developed here, we gain knowledge of the ‘outer world’ through experience. As we experience various phenomena (e.g., facts, events, occurrences, and so on) we discern that the ‘outer’ world does not always conform to the expectations, associations, or laws of our ‘inner’ world, and this may prompt us to try and make our inferences better conform to ‘outer’ experience (as prompted by logic’s alethic orientation). When surprised by some experience, a hypothesis usually suggests itself as a possible ground for ‘explaining’ or accounting for the phenomena in question. The formation of such a hypothesis typically happens quite easily and effortlessly in keeping with the illative, semiological account of inference outlined
above. Approached philosophically or scientifically, we next deduce what might follow from accepting such a hypothesis, and then proceed to test our hypothesis (and its deduced implications or ‘predictions’) against experience through induction (which is a sort of hermeneutic process of experimentation that engenders informational feedback from the ‘outer’ environment). (Peirce 1868b, pp. 28-55)

I want to suggest that because the illative, inferential process outlined here implies only the minimal kind of connection between relata associated with abductive arguments, that this makes it possible for us to modify our inferences so that our hypotheses can become better adjusted to the experiential constraints of the ‘outer’ world. Put simply, the weak hypothetical, evolutionary connection characteristic of the illative, semiological form of inference means that the form of inference per se is marked by an essential adaptability to various kinds of inferential inputs, constraints, or other variables. Thus, the character of an inference will in large part be a function of the kinds of conditions upon and within which it is operating. It is this essential adaptability of the form of illative, semiological inference that allows our knowledge of the ‘outer’ world to grow or evolve as experience demands, thereby enabling thought (and logic) to attain its proper end.

Taking reasoning and argumentation to be examples of more ‘deliberate’ or controlled inferential processes, then a posterior inferences (i.e., those that are oriented towards experience or the ‘outer’ world) will tend to produce arguments that are abductive or inductive in character, while a prior inferences (i.e., those that are oriented towards the purely hypothetical constructions of the ‘inner’ world) will tend to produce arguments with the more demonstrative character generally associated with deduction.13 While I cannot say whether Peirce would agree with my conclusion about the essential adaptability of the illative, semiological form of inference, I can safely say that my position is at least consistent with his account of the ‘worldly’ difference between demonstrative and non-demonstrative reasoning.

But reasoning based upon the laws of the inner world is not thus uncertain. It is called demonstrative reasoning, or demonstration. For instance, if you add up a column of five hundred figures, you get the sum total by mathematical reasoning. It is said to be absolutely certain that your result will be correct. This is an exaggeration. We have seen that it depends upon observation; and observation is always subject to error. But experimentation is so handy upon the creations of our own imagination. The trials can so quickly and at so little cost be repeated; and doing it very frequently we get to be so extremely expert at it, that the probability of error is reduced to the point at which those people who only make dual distinctions, and who class questions into those which we positively know the answers and those of which we guess the answers prefer to class our knowledge of such inferences as positive certainty. In truth, positive certainty is unattainable by man. (Peirce 1895, pp. 25-26)

Put simply, when inner world inferences are brought under the formative rule or guidance of the evolved laws of thought (i.e., its leading ‘logical’ principles), then such inferences will tend to have a demonstrative or deductive character (while inferences that must conform to the general facts and laws of nature will tend to be more abductive and inductive).

13 By ‘argumentation’ I mean the inferential process of following or being carried along by argument as it develops or evolves, not the effective use of argumentation as a means to some other end. The latter would also be part of the general account of logic being developed here, but would fall under what Peirce calls ‘speculative rhetoric.’
in character). For Peirce, this process is most vividly illustrated in the work of mathematics, where one begins by positing certain conditions, e.g., axioms, operators, etc., and then proceeds to determine, following established laws of thought or leading principles, what conditions might follow from this hypothetical starting point. Hence Peirce’s somewhat flippant claim that “Mathematical reasoning holds. Why should it not? It relates only to the creations of mind…. ” (CP, 2.192)

It clearly belongs to logic to evolve the consequences of its own forms. Hence, the whole of the theory of numbers belongs to logic; or rather, it would do so, were it not, as pure mathematics, prelogical, that is, even more abstract than logic. (CP, 4.90)

And again,

The third elementary way of reasoning is deduction, of which the warrant is that the facts presented in the premisses could not under any imaginable circumstances be true without involving the truth of the conclusion, which is therefore accepted with necessary modality. But though it be necessary in its modality, it does not by any means follow that the conclusion is certainly true. When we are reasoning about purely hypothetical states of things, as in mathematics, and can make it one of our hypotheses that what is true shall depend only on a certain kind of condition—so that, for example, what is true of equations written in black ink would certainly be equally true if they were written in red—we can be certain of our conclusions, provided no blunders have been committed. This is “demonstrative reasoning.” Fallacies in pure mathematics have gone undetected for many centuries. It is to ideal states of things alone—or real states of things as ideally conceived, always more or less departing from the reality—that deduction applies. The process is as follows, at least in many cases:

We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination. If visual, it will either be geometrical, that is, such that familiar spatial relations stand for relations asserted in the premisses, or it will be algebraical, where the relations are expressed by objects which are imagined to be subject to certain rules, whether conventional or experiential. This diagram, which has been constructed to represent intuitively or semi-intuitively the same relations which are abstractly expressed in the premisses, is then observed, and a hypothesis suggests itself that there is a certain relation between some of its parts—or perhaps this hypothesis had already been suggested. In order to test this, various experiments are made upon the diagram, which is changed in various ways. This is a proceeding extremely similar to induction, from which, however, it differs widely, in that it does not deal with a course of experience, but with whether or not a certain state of things can be imagined. Now, since it is part of the hypothesis that only a very limited kind of condition can affect the result, the necessary experimentation can be very quickly completed; and it is seen that the conclusion is compelled to be true by the conditions of the construction of the diagram. This is called “diagrammatic, or schematic, reasoning.” (CP, 2.778)

14 Peirce’s leading principles should be read as general rules, guidelines or ‘habits’ of mind that have been formed from experience. Some leading principles are inherited as part of the historical development of thought or mind in general (e.g., customs or evolutionarily developed ‘rules’ or ‘laws’ of thought), while others are constructed or formed individually as part of the more locally situated cognitive process of learning from experience. As I hope to show in a future paper that builds upon some of my own earlier work, the most deeply inherited leading principles are not to be found in philosophy, but in myth. If I am right, then myth evolved philosophy.
Importantly it follows that the difference between deduction, induction, and abduction is not a difference of kind, for while deductions may appear to have a ‘demonstrative’ character, no deduction is ever able to achieve the kinds of demonstrative certainty or necessity that is often associated with this argument form. The demonstrative character of a deduction is the expression of the high degree of confidence that we typically ascribe to such arguments, but such confidence can never be verified and hence warranted in any absolute sense. The best that can be said of such ‘demonstrative’ arguments is that they have not been, and seemingly cannot be, falsified.

The truth is our ideas about the distinction between analytical and synthetical judgments is much modified by the logic of relatives, and by the logic of probable inference. An analytical proposition is a definition or a proposition deducible from definitions; a synthetical proposition is a proposition not analytical. Deduction, or analytical reasoning, is, as I have shown in my “Theory of Probable Reasoning,” a reasoning in which the conclusion follows (necessarily, or probably) from the state of things expressed in the premisses, in contradistinction to scientific or synthetical reasoning, which is a reasoning in which the conclusion follows probably and approximately from the premisses, owing to the conditions under which the latter have been observed, or otherwise ascertained. The two classes of reasoning present, besides, some other contrasts that need not be insisted upon in this place. They also present significant resemblances. Deduction is really a matter of perception and of experimentation, just as induction and hypothetic inference are; only, the perception and experimentation are concerned with imaginary objects instead of with real ones. The operations of perception and of experimentation are subject to error, and therefore it is only in a Pickwickian sense that mathematical reasoning can be said to be perfectly certain. It is so only under the condition that no error creeps into it; yet, after all, it is susceptible of attaining a practical certainty. So, for that matter, is scientific reasoning; but not so readily. Again, mathematics brings to light results as truly occult and unexpected as those of chemistry, only they are results dependent upon the action of reason in the depths of our own consciousness, instead of being dependent, like those of chemistry, upon the action of Cosmical Reason, or Law. Or, stating the matter under another aspect, analytical reasoning depends upon associations of similarity, synthetical reasoning upon associations of contiguity. The logic of relatives, which justifies these assertions, shows accordingly that deductive reasoning is really quite different from what it was supposed by Kant to be; and this explains how it is that he and others have taken various mathematical propositions to be synthetical which in their ideal sense, as propositions of pure mathematics, are in truth only analytical. (CP, 6.595; Appendix A)

And again,

Nevertheless, there is undoubtedly a great distinction between inference resting on merely inward observation, which a moderate amount of attention can put beyond all reasonable doubt, and inferences based upon our attempts to catch the regularities of Nature, essays in which can never hope to attain more than a somewhat close approach to the truth, and whose surmised regularities we can hardly be much surprised to find are only quite exceptionally even at all near the veritable law. This is the great distinction of demonstrative and experimental reasoning. Of these two kinds of reasoning the demonstrative, depending upon inward observation and inward regularities, has to be studied first. (Peirce 1895, p. 26)

Thus, the difference between deduction, induction, and abduction is more a difference of degree than of kind, one that arises from differences in the particular, worldly orientation of the shared illative, semiological form of inference from which they arise and upon which they rest.
There is an immense distinction between the Inward and Outward truth. I know them alike by experimentation only. But the distinction lies in this, that I can glut myself with experiments in the one case, while I find it most troublesome to obtain any that are satisfactory in the other. Over the Inward, I have considerable control, over the Outward very little. It is a question of degree only. Phenomena that inward force puts together appear similar, phenomena that outward force puts together appear contiguous. We can try experiments establishing similarity so easily that it seems as if we could see through and through that; while contiguity strikes us as a marvel. The young chemist precipitates Prussian blue from two nearly colorless fluids a hundred times over without ceasing to marvel at it. Yet he finds no marvel in the fact that any one precipitate when compared in color with the other seems similar every time. It is quite as much a mystery, in truth, and you can no more get at the heart of it than you can get at the heart of an onion. (CP, 4.87)

8. HOW DEDUCTION EVOLVES ITS CONCLUSION

We are now in a better position to address the question of how a deduction can be said to evolve its conclusion. As illative, semiological inferences with a particular ‘inner worldly’ orientation, deductions must evolve their conclusions in at least a minimal sense whereby the conclusions are a determination of the premises from which they evolve. (CP, 1.484) Relative to the premise, deductive conclusions will develop and emerge as something new, something that could not be what it is apart from the premise from which it evolves (and hence involves the premise), but which is nonetheless somehow ‘more’ in some important sense than the premise from which it evolved.

What was peculiar to Kant—it came from his thin study of the syllogistic figure—was his way of putting the distinction, when he says we necessarily think the explicatory proposition although confusedly, whenever we think its subject. This is monstrous! The question whether a given thing is consistent with a hypothesis, is the question of whether they are logically compossible or not. I can easily throw all of the axioms of number, which are neither numerous nor complicated, into the antecedent of a proposition—or into its subject, if that be insisted upon—so that the question of whether every number is the sum of three cubes, is simply a question of whether that is involved in the conception of the subject and nothing more. But to say that because the answer is involved in the conception of the subject, it is confusedly thought in it, is a great error. To be involved, is a phrase to which nobody before Kant ever gave such a psychological meaning. Everything is involved which can be evolved. But how does this evolution of necessary consequences take place? We can answer for ourselves after having worked a while in the logic of relatives. It is not by a simple mental state, or strain of mental vision. It is by manipulating on paper, or in the fancy, formulæ or other diagrams—experimenting on them, experiencing the thing. Such experience alone evolves the reason hidden within us and as utterly hidden as gold ten feet below ground—and this experience only differs from what usually carries that name in that it brings out the reason hidden within and not the reason of Nature, as do the chemist’s or physicist’s experiments. (CP, 4.86)

While we may still hold that deductive inferences add little, if anything, to our knowledge of outer-worldly ‘matters of fact’ (which must be learned a posteriori through experience), such inferences must nevertheless be understood to be more than a simple restatement of the premises. Put simply, deductive inferences must involve the growth of information in some broad, non-experiential sense of that term.

One way of trying to characterize the growth in information characteristic of deduction is to see deductive arguments as a semiological, illative movement from a condition of relative indetermination or vagueness to a condition of increased determination or explication. (CP, 6.191) On this reading, deductive explication would be a process of
making explicit, not in the sense of merely bringing the content of its premise more clearly into view (as if that content was already somehow ‘present’ or ‘pre-given’ in the premise as a kind of tacit, latent, or ‘undeveloped’ relation of identity), but rather as a further differentiation or determination of the premise.\(^\text{15}\) It is better to see the conclusion as a further differentiation of the more undifferentiated premise, where the condition for the possibility of such differentiation lies, not merely in the premise itself (viewed as the ‘object’ of the semiological process or order), but also in the determining nature of the transitional, illative connection (or semiological connective) whereby the premise and conclusion are combined (or conjoined). Thus a deductive inference is still a kind of real discovery as it were, but one that relates more to the ‘inner’ rather the ‘outer’ world. The main difference between the ampliative processes of induction and abduction, and the more explicative process associated with deduction is that where induction and abduction must be tested against the evidence of experience, deductive conclusions follow the less constraining path of imaginative experiments constructed primarily in accordance with the evolved laws of thought. There is the evolution of results in each case, i.e., a genuine sense of discovery generally associated with the growth of an idea, but in the one case the evolution seems to follow necessarily (according to one’s leading ‘logical’ principles) while in the other it follows contingently (according to the leading ‘material’ principles that one has learned from experience). In the end, all three forms of argument are evolutionary in the sense outlined here, with deduction or explication being the real evolution of results whose degree of confidence has the general character of certainty or necessity as follows from the application of one’s hypothetical conditions to the leading ‘logical’ principles that we accept as the evolved laws of mind.

9. LOGIC AS AN INDUCTIVE SCIENCE

The semiological, illative nature of inference as the form of growth combined with the account of logic as the “study of the means for attaining the end of thought,” suggests that the argumentative model or form that best expresses the nature and essence of logic is not deduction (or even abduction), but induction. True belief, as the proper end of thought, is best attained not through the a priori, explicative methods of deduction, nor the loosely bound, ‘free associations’ of abduction, but the hermeneutic, informational feedback processes associated with induction (which incorporates the results of abduction and deduction within its own experimental methods of hermeneutic, informational feedback). Thus, logic in this alethic sense is an inductive science.

Among inductive sciences are commonly placed optics and other branches of physics. Now this is totally wrong; for induction only discovers natural classification—only from neat goats sheep and deer finds that all ruminants are cloven-hoofed—that is erects ruminants into a natural kind or one which possesses universal characters over and above those which serve to define it. Now classification is no part of physics. Optics and such sciences discover causes—remote efficient—and this can only be done by hypothesis. On the other hand, sciences are

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\(^\text{15}\) Expressed in Kantian terms, deductive explication is more akin to synthetic a priori judgment than an analytical judgment, for while the relation of premise to conclusion (or subject to predicate) may indeed be necessary (in a sense that would have to be spelled out more clearly), nevertheless on the semiological, illative account of inference outlined here it would be misleading to claim that the conclusion (or predicate) was already contained in the premise (or subject).
often excluded from the inductive class without any reason—Logic is one of these and therefore purely inductive; and yet it has usually been placed among the deductive sciences merely on account of its great exactitude. (Peirce 1866, p. 87)

As an inductive science, logic is also a “positive” science, that is, “a science of fact” which, with its alethic orientation toward the truth, is importantly distinct from mathematics or formal (mathematical) logic, for the latter are not in-themselves positive sciences, but are merely “the science of the consequences of hypotheses.” (CP, 1.247)

10. POSTSCRIPT

I will conclude by noting that the illative, semiological account of inference being developed here can likely be translated into more familiar and commonly used models. Thus, the semiological structure of object-sign-interpretant can be recast, for example, in functional terms as a functional X-Y relation where the set of ‘objects’ X is being mapped onto the set of ‘interpretants’ Y. Similarly, one may recast the same semiological structure in terms of mathematical ‘machine’ language where the object side of the relation represents the ‘input,’ the sign is the ‘processor,’ and the interpretant is the ‘output.’ Analogously, the process of illation can be reformulated in terms of the modern philosophy of information, where the idea of an asymmetrical connection between relata is recast in terms of the “General Definition of Information” as “data + meaning” (Floridi 2010, p. 20-21), where ‘data’ and ‘meaning’ are taken as relata while the operator ‘+’ represents the connection between them. In fact, I would argue that it would not be much of a stretch to reinterpret Peirce’s signs as a purely logical version of Floridi’s “interconnected informational organisms” or “inforgs”, and Peirce’s general semiotic, illative environment as comparable to what Floridi calls the “infosphere.” (Floridi, p. 9) Such reformulations of the notions being developed here would likely be fine, and perhaps even helpful, as long as we keep a number of important things in sight: 1) The models used must preserve the basic asymmetry of semiological inference, 2) The models must preserve the growth of information that occurs in any semiological inference, and 3) The models must preserve the irreducible trivalency of semiological relationality, and all that this entails. Of course, since the conditions outlined here are themselves hypothetical and would need to be tested against the evidence of experience, then it is possible that we may have to refine and revise our account if needed, but that is to look far beyond the present project.
REFERENCES


Commentary on “INFERENACE AS GROWTH: PEIRCE’S ECSTATIC LOGIC OF ILLATION” by Philip Rose

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1. ECSTASY

Philip Rose starts off with a bang: Peirce’s is an ecstatic logic. It says so right there in the paper’s title. Mainstream logicians will be shocked to hear it, and Peirce scholars reduced to tears. But, not to worry—the ecstasy that logic is neither powder up the nose nor the mystical embrace of transcendence. It is something a lot closer to home, sweet home. Consider a case. Harry has a current set K of beliefs and is now the recipient of some new information H. As it happens, this new information is inconsistent with Harry’s prior beliefs. We might suppose that this is a case in which Harry is faced with a belief-update task. He must form the consistent set K(H).¹ When this sort of consistency maintenance is done right, certain conditions are met. But, by and large, the successful updater won’t be able to specify them. He will proceed without knowing what he is doing. He will proceed, that is to say, ecstatically.

In my The New Oxford Dictionary of English, the third and final listing has “ecstasy” mean standing outside oneself. The word derives from the Old French (exastie) via late Latin, from the Greek ekstasis, based on ek (for “out”) + histanai (for “to place”).

Consider another case. In the proof theories of the mainstream logics of deduction, a proof is a finite sequence of sentences each line of which is an axiom or arises from prior lines by finite application of the proof rules. Of course, this is a technical sense of “proof”, made so in part by the fact that for a sequence to be a proof it is not necessary that it display the rules which validate its lines. It is precisely the opposite with the proofs that we demand from our students. Suppose that the sequence \( p_1, \ldots, p_n, q \) is a proof in the first, technical, sense. It notoriously fails to be a proof in the second, problem-set, sense, because it fails to accompany each succeeding line with the rule that justifies it. Now when it comes to natural language reasoning, it is simply a fact of everyday life that people are more adept at making arguments than they are at justifying their constituent lines. If we allow that an argument can be good in the absence of those side-comments, then good argument-making doesn’t require a knowledge of its justifications. Good argument-making is ecstatic.

¹ I simplify. For two hundred years mathematicians did good work on infinitesimals, notwithstanding that the calculus was inconsistent and they knew it.
2. ILLATION

Peirce is too original a thinker for traditionalist pigeon-holing across the board. There are exceptions, of course, none more important than his insistence that the theoretical core of logic is illation. A more up-to-date way of saying the same thing is that the heart and soul of logic is its consequence relation or, out of deference to logic’s sprawling and rivalrous pluralism, that the heart and soul of a logic is its consequence relation.

This was an insight coterminal with the very founding of systematic logic. Aristotle insisted on a distinction between consequence and syllogistic consequence, and with it a distinction between consequence-having and consequence-drawing. It is known that Aristotle would not permit the use of immediate inferences in argumentative contexts. He thought (rightly) that when construed as an argument, an immediate inference is question-begging. So syllogisms are required, none of whose premisses immediately implies their respective conclusions. This is the first acknowledgement of the difference (and the importance of it) between consequence-having, which is what immediate inferences instantiate, and consequence-drawing, which is what syllogisms instantiate.

Let S be a set of statements. To know whether p is a consequence of S it suffices to know whether S entails p. To know whether p is a consequence of S which, given the context at hand, should be drawn, it is necessary but not sufficient to know that S entails p. What these further conditions are is a matter of contention. Even so, when the right consequences of S are drawn, the conditions in virtue of which this is so are not in the general case known to the drawer. More ecstasy still.

Peirce is one of a good many logicians who smudge the distinction between consequence-having and consequence-drawing. At CP, 4.375 the confusion is apparent. Speaking of what Peirce calls “illative transformations”, the logician requires “a sign to be able to express [not only] ‘A therefore B’ but [also] ‘If A then B’.” But ‘A therefore B’ gives consequence-drawing and ‘If A then B’ gives consequence-having. So I think we may say that Peirce’s smudge was inadvertent: Illation is consequence-drawing.

3. LOGIC AS INDUCTIVE

At a certain level of abstraction we can say that consequence-havings serve as occasions for consequence-drawings. They do so in something like the following way. Suppose that you recognize that p is a consequence of S. Then, in the circumstances I am presuming for this kind of case, you will have a consequence-drawing problem with regard to ⟨S, p⟩ or, for short, an ⟨S, p⟩-problem. At this level of abstraction, we may suppose it intelligible to say that your drawing of p from S—or your not drawing of it—represent alternative solutions of your ⟨S, p⟩-problem. Either way, these solutions would have required fulfillment of conditions necessary and sufficient to achieve these ends successfully. Call these the K-conditions for the ⟨S, p⟩-problem.

Solving an ⟨S, p⟩-problem involves a search for these K-conditions. Since, whatever they turn out to be case by case, the K-conditions are not known to you, there is ample room for the following two conjectures:
COMMENTARY

(1) One is that solutions of \((S, p)\)-problem are achieved by real-life reasoners \textit{tacitly}.

(2) The other is that the reasoning that underlies these K-condition searches is not in the general case \textit{deductive}, even when the solution provides for the drawing of a deductive consequence of \(S\).

So, then, since logic is centrally the business of consequence-drawing then the logic of deduction has an ineliminably inductive character. (Point to Peirce! Point to Rose!)

4. ILLATIVE GROWTH

Philip Rose says that Peirce understands the logic of illation to be the logic of growth. This includes deductive inference which is illiative in Peirce’s “radical sense [in which] deductive inferences evolve their conclusions in an asymmetrical sense that is consistent with Peirce’s illiative logic of growth.” Another way of saying the same thing is that even deductive consequence-drawing is ampliative.

There are two ways in which this term might be interpreted, one of which is true and the other of which is a bit trickier. The true claim is that deductive reasoning has an unshirkably inductive character, and “ampliative” is just another word for “inductive”. The trickier claim is itself subject to two interpretations. The first is the intuitively plausible claim that when, under the conditions that make it right, you draw a deductive consequence of something, it is sometimes the case that you are bringing to light something that you hadn’t realized before. Now for this to be true in any interesting sense, it cannot be true in any interesting sense that, for beings like us, belief is closed under consequence. On this interpretation, the ampliative claim is that deductive inference can beget new beliefs. And that, I think, is certainly true.

The second interpretation is that, sometimes at least, the conclusion of a deductively valid argument is more informative than its premisses combined. If we here allow “informative” to mean what it means in the mathematical theory of information quantities—or what is sometimes called Shannon information—the thesis is certainly not true.

We should take care with this. On the face of it, the second interpretation of the ampliative thesis contradicts the first. But this is not so. When a system possesses Shannon information, the information is not, as we might say, informative for it. Shannon information doesn’t inform. On the other hand, new belief does inform, at least when true. New belief informs you of something you hadn’t been previously aware of. So, in the relevant sense, the sense of being newly informed, there are cases galore in which even deductive consequence-drawing is ampliative. Point to Peirce! Point to Rose!

\[\text{2} \quad \text{This same inductive character is also present, perhaps more recognizably so, in premiss searches for deductive proofs of fixed propositions. In some cases, this kind of theorem-proving is governed by algorithms, but this is very much the exception not the rule.}\]