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Hyperfine structure of the $2^3P$ state of $^3$He with and without an external magnetic field

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Hyperfine structure of the 2 3P state of 3He with and without an external magnetic field

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Abstract
The hyperfine structure of the 2 3P state of 3He with and without an external magnetic field is precisely calculated. All the linear terms, diamagnetic terms and the \( \alpha^2 \) relativistic correction terms are included in the Zeeman Hamiltonian. The values of the fields for 32 crossings and 5 anticrossings of the magnetic sublevels are theoretically predicted for magnetic field strengths up to 10 000 gauss. The results are compared with experimental data and other theoretical works. All related matrix elements are calculated with high accuracy by the use of double basis set Hylleraas-type variational wavefunctions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The Zeeman effect, the study of the behaviour of atoms in the presence of a magnetic field, is a long and well-established branch of spectroscopy. If the theory of Zeeman effect is sufficiently well understood, then it can be used to extrapolate precise measurements for the fine structure or the hyperfine structure to zero-field strength. Experimentally, level-crossing and level-anticrossing techniques have been used to investigate the fine structure and the hyperfine structure of the excited states of atoms, such as Li [1, 2], 4He [3–9] and 3He [10–13]. As a two-electron atomic system, helium has been the object of extensive investigation for many years. 3He, one of the isotopes of helium, has hyperfine structure because of its nonzero nuclear spin (\( I = 1/2 \)). The splitting of the hyperfine structure levels of 3He with and without an applied magnetic field has been studied in many theoretical and experimental works [14–22]. Recently, the double basis set variational technique developed by Drake and co-workers has been used to carry out high precision calculations of nonrelativistic energy levels, fine structure and the hyperfine structure [18, 23, 24]. The motivations of the present work are (a) to report a new calculation of the hyperfine structure in the 2 3P state of 3He with and without an external magnetic field and (b) to predict the precise values of fields of crossings and anticrossings of the magnetic sublevels.
2. Hyperfine structure without external magnetic field

The total Hamiltonian is

\[ H = H_{NR} + H_{fs} + H_{hfs} \]  

(1)

where \( H_{NR} \) is the nonrelativistic Hamiltonian, \( H_{fs} \) represents the fine structure interaction for helium as described by many authors (see Drake [23, 24] for a review) and \( H_{hfs} \) represents the hyperfine structure discussed further below. Recently, numerical energy levels for \(^4\)He and \(^3\)He have been presented by Morton et al. [18]. Instead of directly using the theoretical energies for \(^3\)He, we combined the theoretical isotope shifts for \(^3\)He relative to \(^4\)He [18] with the best experimental ionization energies for \(^4\)He, as recently measured by, for example, Zelevinsky et al. [25]. This gives higher accuracy because, due to cancellations of the mass-independent QED uncertainties, the calculated isotope shifts are considerably more accurate than the total ionization energies. This higher accuracy (better than \( \pm 100 \) kHz in the isotope shift) has been used to deduce nuclear charge radii for light isotopes and halo nuclei [23, 24, 26–29].

A study of the Zeeman effect is based upon a detailed knowledge of the fine and hyperfine structures. Following the notation of Hinds, Prestage and Pichanick [17], the fine structure parameters are listed in Table 1. \( H_{hfs} \), the hyperfine interaction term coming from the magnetic interaction between electrons and the nuclear spin for \(^3\)He, leads to each fine structure energy level being split into several sublevels with total quantum numbers from \( F = |J - I| \) to \( F = J + I \). Since the interaction for the higher order multipoles (such as magnetic octupole) vanishes for \( I = \frac{1}{2} \), we need only consider the magnetic dipole contribution. According to Bethe and Salpeter [30], the magnetic dipole hyperfine interaction for a two-electron atomic system can be written as

\[ H_{hfs} = -2\mu_0 \sum_{i=1}^{2} \left\{ \frac{-8\pi}{3} (s_i \cdot \mu) \delta(r_i) - \frac{1}{r_i} (l_i \cdot \mu) + \frac{1}{r_i^3} \left( (s_i \cdot \mu) - \frac{3}{r_i} (s_i \cdot r_i)(\mu \cdot r_i) \right) \right\} \]  

(2)

where \( \mu = -\mu_0 g_I I \) is the nuclear magnetic moment, \( \mu_0 = 13.996 \times 10^{-24} \) (CODATA 2002 value) is the Bohr magneton, \( g_I = 2.317 \times 10^{-3} \) is the nuclear \( g \) factor for \(^3\)He [17] and \( r_i, s_i, l_i \) the position vector, spin and orbital angular momentum of the \( i \)th electron, respectively.
The most important interactions are within the manifold of fine structure states with the same \( L, S \) and \( F \), but different \( J \). However, there are also significant off-diagonal mixings between singlet and triplet states.

With the definitions

\[
W_{J',J}(1) = (-1)^{J+s+f}(2J+1)(2J'+1)^{1/2} \left\{ \begin{array}{ccc} F & I & J' \\ 1 & J & 1 \end{array} \right\} \left\{ \begin{array}{ccc} I & 1 & 1 \\ -I & 0 & 1 \end{array} \right\}
\]

(3)

and

\[
X_{S,S'} = -[(2S+1)(2S'+1)^{1/2} \left\{ \begin{array}{ccc} S' & 1/2 & \frac{1}{2} \\ S & 1/2 & 1 \end{array} \right\}
\]

(4)

then the hyperfine interaction matrix element in the coupled representation \( |LSJIF\rangle \) is expressed as

\[
\langle L'S'J'IF| H_{\text{hfs}} |LSJIF\rangle = W_{J',J}(1) I \left[ C_{S,S} \sqrt{\hat{g}}(-1)^{L+S+F} X_{S,S'} \left\{ \begin{array}{ccc} S' & J' & L \\ S & J & 1 \end{array} \right\} \right.
\]

\[
- D_S \delta_S S'(-1)^{J+s+M} \left\{ \begin{array}{ccc} L & J' & S \\ J & L & 1 \end{array} \right\} \left\{ \begin{array}{ccc} L & 1 & L \\ -M & 0 & M \end{array} \right\} 
\]

\[
+ E_{S,S'} \frac{12}{\sqrt{5}} (-1)^{S'-L+M} X_{S,S'} \left\{ \begin{array}{ccc} S' & L & 2 \\ S & J' & J \end{array} \right\} \left\{ \begin{array}{ccc} L & 2 & L \\ -M & 0 & M \end{array} \right\} \right].
\]

(5)

The three hyperfine structure parameters \([17, 19]\), extended to include off-diagonal matrix elements, are defined by

\[
C_{S,S} = -\frac{8\pi}{3} g_I \mu_0^2 \{ 2S+1 \} L M |\hat{g}(\hat{r}_1) + (-1)^{S'-S} \hat{g}(\hat{r}_2) | 2S+1 L M \}
\]

(6)

\[
D_S = -2g_I \mu_0^2 \{ 2S+1 \} L M |l_{1,z}r_1^{-3} + l_{2,z}r_2^{-3} | 2S+1 L M \}
\]

(7)

\[
E_{S,S'} = -\frac{5}{2} \delta_{S} \mu_0^2 \{ 2S+1 \} L M |(-1)^{S'-S} C_2^{(0)}(\hat{r}_1) r_1^{-3} + C_2^{(0)}(\hat{r}_2) r_2^{-3} | 2S+1 L M \}
\]

(8)

evaluated with \( M = L \) throughout equations (5)–(8). Here, \( C_2^{(0)}(\hat{r}) = \sqrt{4\pi/5} Y_2^{(0)}(\hat{r}) \), the conversion factor from atomic units to MHz is \( g_I \mu_0 = 202.998 \, 180(61) \) MHz [18].

High precision values for all the hyperfine structure parameters in equations (6)–(8) can be calculated by using the double basis set wavefunctions in Hylleraas co-ordinates with and without the mass polarization correction. The details are described in full by Drake [23, 24]. Thus, the linear mass polarization coefficient \( \delta_{\text{MP}} \) can be deduced and applied to any helium isotope. The final expressions, including the linear mass polarization \( \delta_{\text{MP}} \), reduced mass correction and higher order relativistic, QED, and finite nuclear size corrections, are [31]

\[
C_{S',S} = C_{S',S}^{(0)} \left[ 1 + (\delta_{\text{MP}} - 3) \mu/M + \alpha/2\pi + \delta_{\text{ho}} \right]
\]

(9)

\[
D_S = D_S^{(0)} \left[ 1 + (\delta_{\text{MP}} - 3) \mu/M + \delta_{\text{ho}} \right]
\]

(10)

\[
E_{S,S'} = E_{S,S'}^{(0)} \left[ 1 + (\delta_{\text{MP}} - 3) \mu/M + \alpha/2\pi + \delta_{\text{ho}} \right]
\]

(11)

where the ratio of reduced mass to nuclear mass (from CODATA 2002 adjustment values) is \( \mu/M = 1.819 \, 212 \, 065(4) \times 10^{-4} \). In order to achieve higher accuracy for the hyperfine
structure, it is necessary to estimate $\delta C_{ho}$ in equation (6) for the dominate Fermi contact term $C_{11}$. As suggested by Hambro [19], Hinds [17] and the detailed comparison in table 5 of Riis et al [31], we assume $\delta C_{ho} = 0.000 507(4)$ for the 2 P states with the uncertainty of 0.000 004 being the difference of $\delta C_{ho}$ between He(1s2s) and He$^+$(1s). The calculated values for all hyperfine structure parameters are listed in table 1.

Finally, once the fine structure parameters and the hyperfine structure parameters are available, the hyperfine energy structure can be obtained by diagonalizing the complete 11 × 11 matrix of the Hamiltonian operator in the coupled representation $|LSJIF⟩$. Since the Hamiltonian in equation (1) is rotationally invariant, the associated magnetic quantum number $M_F$ can be dropped. Table 2 tabulates our hyperfine splittings in 2 3P states of 3He and compares them with the experimental data and other theoretical works. The comparison shows that our results are in the good agreement with experiment, but with higher accuracy.

### 3. Hyperfine structure with external magnetic field

The total Hamiltonian of the 3He atom in an applied magnetic field is

$$H = H_{NR} + H_{fs} + H_{hfs} + H_{Zeeman}$$  \hspace{1cm} (12)

where $H_{NR}$, $H_{fs}$ and $H_{hfs}$ have the same significance as before, and the Zeeman term $H_{Zeeman}$ represents the interaction between the atom with external magnetic field.

In order to obtain Zeeman sublevels with higher accuracy, we follow the formalism of Yan and Drake [32], in which the Zeeman interaction includes the linear terms, diamagnetic terms and $\alpha^2$ relativistic corrected terms. The general form of the Zeeman interaction between the atom and magnetic field was derived from the Breit interaction by Perl and Hughes [33], later by Lewis et al [34] and by Lewis and Hughes [35]. In [32], Yan and Drake published the detailed description of the evaluation of the various terms using their double basis set variational technique for the states of 2 3P, 2 1P, 2 3S, 3 3P of 4He. All the matrix elements corrected to order $\alpha^2$ in their work are considered to be very precise and were cited by many authors such as Zelevinsky et al [25] and Courtade et al [20].

Using standard angular momentum theory, the matrix element of $H_{Zeeman}$ in the coupled representation $|LSJIFM_F⟩$ can be written in the form

$$\langle LSJ'IF'M'_F | H_{Zeeman} | LSJIFM_F⟩ = H_Z(1) + H_Z(2) + H_Z(3) + H_Z(4)$$  \hspace{1cm} (13)

where

Table 2. The theoretical and experimental hyperfine splitting without external magnetic field. The quoted errors of the calculated quantities in the present work reflect the contribution from $\delta C_{ho}$. Units are MHz.

<table>
<thead>
<tr>
<th>Hyperfine splitting $(J, F) - (J', F')$</th>
<th>Present work</th>
<th>Experimental [15]</th>
<th>Other theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1/2) - (2, 3/2)</td>
<td>27 424.837(12)</td>
<td>27 425</td>
<td>27 424.8$^b$</td>
</tr>
<tr>
<td>(2, 3/2) - (1, 1/2)</td>
<td>668.033(9)</td>
<td>668$^a$</td>
<td>668.0$^b$</td>
</tr>
<tr>
<td>(1, 1/2) - (1, 3/2)</td>
<td>4512.191(12)</td>
<td>4512</td>
<td>4512.2$^b$</td>
</tr>
<tr>
<td>(1, 3/2) - (2, 5/2)</td>
<td>1780.880(1)</td>
<td>1781</td>
<td>1780.9$^b$</td>
</tr>
</tbody>
</table>

$^a$ Reference [22].
$^b$ Reference [17].
$^c$ Reference [21].
Hyperfine structure of $^3\text{He}$ with magnetic fields

The various magnetic field effect. The last term

\[ H_z = (-1)^{F+F'+1-M_F+J'} \{ \begin{array}{c} L \cr J \cr S \end{array} \} + (-1)^{L+S-J'} \{ \begin{array}{c} J \cr S \cr L \end{array} \} \]

\[ \times \left[ g'_{L} \left( \begin{array}{c} J \cr F \cr I \end{array} \right) \{ \begin{array}{c} L \cr J \cr S \end{array} \} \right] \left( \begin{array}{c} F' \cr 1 \cr F \end{array} \right) \{ \begin{array}{c} J' \cr F' \cr I \end{array} \} \left( \begin{array}{c} M_F \cr 0 \cr M_F \end{array} \right) \delta_{M_F, M_F} \frac{2}{R_{\infty}} \frac{(\mu_0 B)^2}{g_{Q_2}} \]

\[ H_z(2) = (-1)^{2F+I-M_F+J+L+S}(2F + 1)(2J + 1) \left( \begin{array}{c} F \cr 0 \cr F \end{array} \right) \{ \begin{array}{c} J \cr F \cr I \end{array} \} \left( \begin{array}{c} M_F \cr 0 \cr M_F \end{array} \right) \delta_{M_F, M_F} \frac{2}{R_{\infty}} \frac{(\mu_0 B)^2}{g_{Q_2}} \]

\[ H_z(3) = (-1)^{F+F'+1-M_F+J'+L+S+1}(2F' + 1)(2F + 1)(2J' + 1)(2J + 1) \]

\[ \times \left( \begin{array}{c} F' \cr 2 \cr F \end{array} \right) \{ \begin{array}{c} F \cr J \cr S \end{array} \} \left( \begin{array}{c} M_F \cr 0 \cr M_F \end{array} \right) \delta_{M_F, M_F} \frac{2}{R_{\infty}} \frac{(\mu_0 B)^2}{g_{Q_2}} \]

\[ H_z(4) = (-1)^{2F-M_F+1-J+L+S-1}(2F' + 1)(2F + 1)(2J + 1)(I + 1) \]

\[ \times \left( \begin{array}{c} F' \cr 1 \cr F \end{array} \right) \{ \begin{array}{c} I \cr F' \cr J \end{array} \} \left( \begin{array}{c} M_F \cr 0 \cr M_F \end{array} \right) \delta_{M_F, M_F} \frac{2}{R_{\infty}} \frac{(\mu_0 B)^2}{g_{Q_2}} \]

where $B$ is the external magnetic field strength and $R_{\infty} = 3.289 \times 10^9$ (MHz) is the Rydberg constant. The physical origin of these terms is as follows. The linear term $H_z(1)$ consists of the lowest order Zeeman effect ($g_L$ and $g_S$ term), the correction for the motion of the centre of mass ($m/M$ term) and the relativistic corrections ($\alpha^2$ term). $H_z(2)$ and $H_z(3)$ arise from the scalar quantity $\frac{1}{4} (\mu_0 B)^2 \sum_{i=1}^2 \{1 - C_2^2(r_i^2)\}$ due to the quadratic magnetic field effect. The last term $H_z(4)$ represents the linear interaction between the nuclear spin and the external magnetic field. The various $g$ factors are defined as

\[ g'_{L} = \sqrt{\frac{(2L + 1)L(L + 1)}{6}} g_L + \frac{2}{\sqrt{6}} m \frac{F_1}{M} \frac{a}{\sqrt{6}} (F_2 + F_3 - F_4) \]

\[ g'_{S} = \sqrt{\frac{(2S + 1)S(S + 1)}{6}} g_{S'\alpha^2} (-1)^{\frac{2S+1}{2}} \frac{a}{\sqrt{2L + 1}} \left( \begin{array}{c} S \cr 1/2 \cr 1/2 \cr 1 \end{array} \right) \left( \begin{array}{c} F_5 + \frac{Z}{2} F_6 - \frac{1}{2} F_7 \end{array} \right) \]

\[ g_{\alpha^2} = \alpha^2 (-1)^{\frac{S}{2}} (2S + 1) \frac{\sqrt{5}}{6} \left( \begin{array}{c} S \cr 1 \cr 1 \cr 1 \end{array} \right) \left( \begin{array}{c} \frac{\alpha}{\sqrt{\pi}} \cr 0 \cr -Z \frac{\alpha}{\sqrt{\pi}} + \frac{3}{2} F_6 \end{array} \right) \]

with [36, 37]

\[ g_{Q_1} = F_{10} \]

\[ g_{Q_2} = F_{11} \]

\[ g_{L} = 1 - \frac{m}{M} \]

\[ g_{S} = 2 \left[ 1 + \frac{\alpha}{2\pi} - 0.328478965 \left( \frac{\alpha}{\sqrt{\pi}} \right)^2 + 1.17611 \left( \frac{\alpha}{\sqrt{\pi}} \right)^3 + \cdots \right] \]
where \( m/M \) is the ratio of the electron mass to the nuclear mass for \(^3\text{He}\), \( \alpha^{-1} = 137.035 999 11(46) \) is the fine structure constant, \( F_i \) \((i = 1–11)\) are the matrix elements with respect to the nonrelativistic wavefunction of helium and \( Z = 2 \) is the atomic number of He.

All the matrix elements \( F_i \) in equations (18)–(22) for \(^3\text{He}\) can be directly derived from those of \(^4\text{He}\), as each \( F_i \) for \(^4\text{He}\) are available from the work of Yan and Drake (1994) by means of the expansion

\[
\langle F_i \rangle = \left( \frac{m}{\mu} \right)^n \left[ \langle F_i^\infty \rangle + \delta_{\text{MP}} \left( \frac{\mu}{M} \right) \right]
\]

where \( \langle F_i \rangle \) represents the matrix element for the finite nuclear mass case (including mass polarization via the \( \delta_{\text{MP}} \) term) and \( F_i^\infty \) is the matrix element for infinite nuclear mass. Also, \( n \) is the degree of homogeneity with respect to \( r \) for each operator. For example, \( n = 1 \) for \( F_6 \). The derived values of \( F_i \) for the \(^2\ 3P\) and \( ^2\ 1P\) states of \(^3\text{He}\) are tabulated in table 3. For comparison and further application, all reduced \( g \) values in our work and those of other theoretical works and experiments are summarized in table 4.
Using the tabulated values for the parameters and matrix elements, the energies of Zeeman sublevels of the hyperfine structure of $^3$He are then obtained as a function of the applied magnetic field strength by diagonalizing the complete $24 \times 24$ matrix of the Hamiltonian in the coupled representation $|LSJIFM_F\rangle$, including all basis states of a given $L$, $n$ and spin multiplicity. Figures 1 and 2 illustrate the behaviour of the $2\,^3P$ state as the magnetic field strength is varied from 0 to 10000 gauss ($2\,^1P$ state is ignored here), where each sublevel is labelled by the three quantum numbers $J, F, M_F$.

4. Zeeman sublevel crossings and anticrossings

The calculation of Zeeman sublevels of the hyperfine structure carried out above allows us to determine the sublevel-crossing points and sublevel-anticrossing points. The level crossing is referred to a situation when sublevel energies coincide, while the level anticrossing is referred to a situation when two sublevels repel one another and their wavefunctions interchange their identities [1, 2]. Since there are no terms in equation (12) that mix states with different values of $M_F$, the anticrossings can only happen between the levels with the same $M_F$ where off-diagonal matrix elements are nonzero. Numerical studies of the Zeeman pattern in the region of magnetic field strengths up to 10000 gauss show that the total number of crossings of magnetic sublevels in the $2\,^3P$ state of $^3$He is 32 and the total number of anticrossing of magnetic sublevels is 5. The details are clearly demonstrated in tables 5 and 6, where the available experimental values of the crossing field are also tabulated for comparison.

What is particularly noteworthy in table 5 is that half of the calculated field strengths for the crossings shows significant disagreements with the 18 values that have been measured.
Most of the discrepancies involve the $2\ ^3\!P$ state with $J' = 2$ and $F' = 5/2$. However, there is no particular reason why the calculated crossings for this state should be less accurate than for the others, and the discrepancies are in some cases much larger than what can be accounted for theoretically. It would be interesting to repeat the 1967 measurements reported in [13] since some of these older measurements may be in error.

5. Summary

In our work, fine structure parameters used to calculate the hyperfine structure are obtained by combining our best available isotope shifts and very recent high precision experimental transition frequencies of $^4\text{He}$. The hyperfine structure parameters $C_{S,S}, D_S$ and $E_{S,S}$ also reach high accuracy due to the use of the unique double basis set variational wavefunctions. The hyperfine structure of $^3\text{He}$ with and without the external magnetic field is obtained by diagonalizing the related complete matrix of the Hamiltonian operator. In the region of magnetic field strength up to 10000 gauss studied in this work, we find 32 crossings of the magnetic sublevels. Comparisons show that there exist several substantial discrepancies between our calculations and the 1967 measurements of German [13] (see table 5). On the one hand, new theoretical formulations are needed, such as suggested by Pachucki [39] who considered the relativistic and the second-order contributions to the hyperfine structure of the $2\ ^3\!S_1$ state of $^3\text{He}$. An extension of his work to higher angular momentum states would be helpful in further investigation of the hyperfine structure, and work on this is still in progress. On the other hand, the refined experiments on the hyperfine structure of this isotope will be useful to provide further comparison between theory and measurement. In addition, five anticrossings of the magnetic sublevels are predicted in our numerical calculation. Their
Table 5. Field for crossing of magnetic sublevels. The quoted errors in the calculated quantities in the present work reflect the contribution from $\delta_{\text{tr}}$. Significant differences between theory and experiment are indicated by an asterisk (*). Units are gauss.

<table>
<thead>
<tr>
<th>Crossing $(J, F, M_J) \rightarrow (J', F', M'_{J'})$</th>
<th>Present work</th>
<th>Experiment [13]</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2, 3/2, −3/2)−(1, 1/2, 1/2)</td>
<td>160.8422(20)</td>
<td>160.831(2)</td>
<td>0.011(3)*</td>
</tr>
<tr>
<td>2 (1, 3/2, −3/2)−(2, 5/2, 5/2)</td>
<td>249.26(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 (1, 3/2, −3/2)−(2, 5/2, 3/2)</td>
<td>328.3754(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 (1, 3/2, −1/2)−(2, 5/2, 5/2)</td>
<td>343.102(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 (1, 3/2, −3/2)−(2, 5/2, 1/2)</td>
<td>480.9610(7)</td>
<td>480.963(2)</td>
<td>−0.002(2)</td>
</tr>
<tr>
<td>6 (1, 3/2, −1/2)−(2, 5/2, 3/2)</td>
<td>518.2855(4)</td>
<td>518.285(2)</td>
<td>0.000(2)</td>
</tr>
<tr>
<td>7 (1, 3/2, 1/2)−(2, 5/2, 5/2)</td>
<td>544.7925(5)</td>
<td>544.793(2)</td>
<td>−0.000(2)</td>
</tr>
<tr>
<td>8 (2, 3/2, −1/2)−(1, 1/2, 1/2)</td>
<td>647.7145(4)</td>
<td>647.852(647)</td>
<td>−0.14(65)</td>
</tr>
<tr>
<td>9 (1, 3/2, −3/2)−(2, 5/2, −1/2)</td>
<td>900.728(3)</td>
<td>900.765(5)</td>
<td>−0.037(6)*</td>
</tr>
<tr>
<td>10 (1, 1/2, −1/2)−(1, 3/2, 3/2)</td>
<td>925.3176(29)</td>
<td>925.323(9)</td>
<td>0.005(10)</td>
</tr>
<tr>
<td>11 (1, 3/2, 3/2)−(2, 3/2, −3/2)</td>
<td>947.4447(38)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 (2, 5/2, 5/2)−(1, 1/2, −1/2)</td>
<td>998.9944(26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 (2, 5/2, 3/2)−(2, 3/2, −3/2)</td>
<td>1013.5076(50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 (1, 3/2, −1/2)−(2, 5/2, 1/2)</td>
<td>1111.987(4)</td>
<td>1112.042(4)</td>
<td>−0.055(6)*</td>
</tr>
<tr>
<td>15 (1, 3/2, 3/2)−(2, 5/2, 5/2)</td>
<td>1233.584(3)</td>
<td>1233.566(16)</td>
<td>0.018(16)</td>
</tr>
<tr>
<td>16 (1, 1/2, −1/2)−(2, 5/2, 3/2)</td>
<td>1434.0633(4)</td>
<td>1434.081(4)</td>
<td>−0.018(4)*</td>
</tr>
<tr>
<td>17 (2, 5/2, 3/2)−(2, 3/2, −3/2)</td>
<td>1438.823(9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 (1, 3/2, 1/2)−(1, 1/2, −1/2)</td>
<td>1520.682(9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 (2, 3/2, −3/2)−(1, 3/2, 1/2)</td>
<td>1523.7742(88)</td>
<td>1523.780(15)</td>
<td>−0.006(16)</td>
</tr>
<tr>
<td>20 (2, 3/2, −3/2)−(1, 1/2, −1/2)</td>
<td>1595.704(28)</td>
<td>1595.695(800)</td>
<td>0.009(800)</td>
</tr>
<tr>
<td>21 (2, 5/2, 5/2)−(2, 3/2, −3/2)</td>
<td>1644.203(7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 (1, 1/2, 1/2)−(2, 5/2, 5/2)</td>
<td>1763.790(5)</td>
<td>1763.803(3)</td>
<td>−0.013(6)*</td>
</tr>
<tr>
<td>23 (1, 3/2, 3/2)−(2, 3/2, −1/2)</td>
<td>1907.107(6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 (1, 1/2, 1/2)−(1, 3/2, 3/2)</td>
<td>2205.307(11)</td>
<td>2205.220(44)</td>
<td>0.087(45)*</td>
</tr>
<tr>
<td>25 (2, 5/2, 1/2)−(2, 3/2, −3/2)</td>
<td>2859.490(8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26 (1, 1/2, −1/2)−(2, 5/2, 1/2)</td>
<td>3001.283(5)</td>
<td>3001.414(30)</td>
<td>−0.131(30)*</td>
</tr>
<tr>
<td>27 (2, 5/2, 3/2)−(2, 3/2, −1/2)</td>
<td>3841.227(9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28 (2, 3/2, 1/2)−(2, 5/2, 5/2)</td>
<td>4137.720(11)</td>
<td>4137.743(21)</td>
<td>−0.023(24)</td>
</tr>
<tr>
<td>29 (2, 3/2, 3/2)−(0, 1/2, 1/2)</td>
<td>7436.796(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 (0, 1/2, −1/2)−(2, 3/2, 3/2)</td>
<td>7903.917(4)</td>
<td>7903.978(8)</td>
<td>−0.061(9)*</td>
</tr>
<tr>
<td>31 (0, 1/2, 1/2)−(2, 5/2, 5/2)</td>
<td>8747.236(3)</td>
<td>8747.303(17)</td>
<td>−0.067(17)*</td>
</tr>
<tr>
<td>32 (2, 5/2, 5/2)−(0, 1/2, −1/2)</td>
<td>9262.1496(44)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Field for anticrossing of magnetic sublevels. The quoted errors in the field strengths reflect the contribution from $\delta_{\text{tr}}$. Units are gauss.

<table>
<thead>
<tr>
<th>Anticrossing $(J, F, M_J) \rightarrow (J', F', M'_{J'})$</th>
<th>Magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (1, 3/2, −3/2)−(2, 3/2, 3/2)</td>
<td>1205.7454(94)</td>
</tr>
<tr>
<td>2 (2, 3/2, 1/2)−(1, 1/2, 1/2)</td>
<td>2726.787(11)</td>
</tr>
<tr>
<td>3 (2, 5/2, −3/2)−(1, 3/2, −3/2)</td>
<td>4060.108(14)</td>
</tr>
<tr>
<td>4 (1, 1/2, −1/2)−(1, 3/2, 1/2)</td>
<td>6962.806(29)</td>
</tr>
<tr>
<td>5 (2, 5/2, 1/2)−(1, 3/2, 1/2)</td>
<td>9230.7779(61)</td>
</tr>
</tbody>
</table>

validity may be tested by measurements of the field of anticrossing. Unfortunately, such data are unavailable in the literature for the $2 \, ^3\text{P}$ state, except for $n\Delta \, (n = 3–8)$ states of $^3\text{He}$ [11, 12].
Acknowledgment

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References

[33] Perl W and Hughes V 1953 Phys. Rev. 91 842