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Social Responsibility Allocation in Two-echelon Supply Chains: Insights from Wholesale Price Contracts

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Abstract: Corporate social responsibility (CSR) is defined as corporate activities and their impacts on different social groups. In this paper, CSR is considered in a two-echelon supply chain consisting of an upstream supplier and a downstream firm that are bound by a wholesale price contract. CSR performance (the outcome of CSR conduct) of the whole supply chain is gauged by a global variable and the associated cost of achieving this CSR performance is only incurred by the supplier with an expectation of being shared with the downstream firm via the wholesale price contract. As such, the key issue is to determine who should be allocated as the responsibility holder with the right of offering the contract and how this right should be appropriately restricted. Game-theoretical analyses are carried out on six games, resulting from different interaction schemes between the supplier and the firm, to derive their corresponding equilibriums. Comparative institutional analyses are then conducted to determine the optimal social responsibility allocations based on both economic and CSR performance criteria. Main results are furnished in a series of propositions and their implications to the real-world business practice are discussed. The key findings are threefold: Under the current model settings, (1) the optimal allocation scheme is to assign the supplier as the responsibility holder with appropriate restrictions on the corresponding rights to determine the wholesale price; (2) Inherent conflict exists between the economic and CSR performance criteria and, hence, the two maxima cannot be achieved simultaneously; (3) Although integrative channel profit is not attainable, the system-wide profit will be improved by implementing optimal social responsibility allocation schemes.

Keywords: Supply chain management; corporate social responsibility; wholesale price contracts; equilibrium.

1 Introduction

Corporate social responsibility (CSR) is defined as corporate activities and their impacts on

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different social groups, including human rights, environment protection (e.g. recycling used product), pollutant emission control, philanthropy, to name a few (Cater and Jennings 2002). CSR has been receiving considerable attention in the academic community, from the CSR construct in the 1950s (Bowen 1953) to empirical investigations on the relationship between CSR and corporate financial performance (CFP)\(^1\) and, then, to formal modeling of CSR (Baron 2001, 2007, Calveras et al. 2007, Giovanni and Giacinta 2007). In recent years, with the continued trend of globalization, the research on supply chain management has enabled firms to improve their profitability by fostering partnership with other members in their supply chain systems. While firms enjoy improved efficiency, pressures are also accumulating for socially and environmentally responsible supply chain practice (Linton et al. 2007). For instance, many leading brands such as Nike, GAP, Adidas, and McDonalds have been urged to incorporate social responsibility into their supply chains (Amaeshi et al. 2008). In response to this pressure, many supply chain primary firms have introduced codes of conduct to ensure their partners’ business practices to be socially responsible. However, World Bank (2003) reports that implementing codes of conduct is challenged by a plethora of individual CSR codes, the effectiveness of the top-down CSR strategies and insufficient understanding of business benefits.

Note that even if a lobby group (e.g., non-governmental organizations) for social responsibility may only target a particular firm in a supply chain, the pressure can be easily propagated to other members in the system through their business transactions. Therefore, it is necessary to extend the traditional CSR beyond a single firm’s boundary and consider it within a supply chain context (Davis et al. 1997; Mamic 2005). Recent research has started to model social responsibility in supply chain operations. For instance, Savaskan et al. (2004) develop a model for closed-loop supply chains with product remanufacturing and identify an appropriate supply chain structure for original equipment manufacturers. Crutz (2008) introduces a dynamic multi-criteria decision-making framework for modeling and analyzing the equilibrium of supply chain network with environmental responsibility where environmental (social) responsibility is assumed to have no direct impact on market demand and the allocation of environmental (social) responsibility is not explicitly considered. In Hsueh and Chang’s (2008) three-tier (manufacturer, distributor and retailer) supply chain network model, the allocation of CSR for system-wide optimization is captured by additional monetary transfers (via an enforceable agreement), and

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\(^1\) See Orlitzky et al. (2003) for a meta-analysis and Margolis and Walsh (2001) for a survey on empirical studies.
this treatment allows for the assumption of each decentralized manufacturer’s marginal production cost to be the same as that in the centralized network. For empirical studies, Carter et al. (2000) show that environmental purchasing has significant impacts on both income and cost. Carter and Jennings (2002) find a positive relationship between CSR and supplier performance.

Although these studies attempt to incorporate social responsibility into supply chain management, the allocation of social responsibility has not emerged as a main focus, whereas it is a critical issue for supply chain members to collaboratively manage the extended CSR. As OECD (2001) states, “allocating responsibility and determining who is the producer [the responsibility holder] are two of the most important [EPR, Extended Producer Responsibility] policy issues.” On the one hand, the principles of corporate legal personality and separate existence of corporations naturally reject the extension of the responsibility of one member to any others. In this respect, all members in a supply chain are responsible for only their own actions. But on the other hand, the stakeholder theory (Freeman 1984) argues that each supply chain member shares the responsibility for other members’ actions. Now a natural question is how to handle social responsibility in the context of a supply chain: is social responsibility independent for individual firms or shared among different entities? This article follows the second argument and treats social responsibility as shared obligations among supply chain partners. In this case, it is crucial to know how the responsibility is allocated among the firms. Otherwise, unclear allocation is likely to lead to the “tragedy of the commons” and result in lower supply chain efficiency. As an example, Amaeshi et al. (2008) suggest that the more powerful member in a firm-supplier relationship should bear the responsibility to influence the less powerful one(s).

This research aims to address the social responsibility allocation problem in a two-echelon supply chain under wholesale price contracts. The basic settings of the model are outlined as follows: a two-echelon supply chain consists of two members, an upstream supplier (S) and a downstream firm (F). The investment in CSR always incurs by the supplier, which provides a global measurement of the CSR performance for the supply chain and is assumed to be independent of the production quantity\(^2\). This cost is then shared with the firm via a wholesale

---

\(^2\) It is widely observed that the main target in supply chain CSR is at the supplier side. For example, Nike and its subcontractors are often accused of inhumane labor and business practices in its Asian manufacturing facilities (Amaeshi et al. 2008). As the largest specialty apparel retailer, GAP admits to the charge of its substandard working conditions in as many as 3000 of its factories (Merrick 2004). Moreover, CSR activity such as human rights and philanthropy are almost irrelevant to production quantity. Xiao and Yang (2008) and Tsay and Agrawal (2000) make this same assumption in their research as well.
price contract\textsuperscript{3} that is an increasing function of CSR investment by the supplier\textsuperscript{4}. Three power structures are entertained, F as the Stackelberg leader (first mover) and S as the follower (second mover), S as the Stakelberg leader (first mover) and F as the follower (second mover), or F and S are equally powerful and, hence, move simultaneously (Choi 1991). Then, our allocation problem is to determine who should be entrusted with the right of offering the wholesale price contract to enforce social responsibility in the supply chain (hereafter, referred to as the responsibility holder) under each of the three power structures. Depending on whether F or S is the social responsibility holder and which power structure is considered, six scenarios may arise. Game-theoretical analyses are first conducted to obtain the equilibriums for the six cases. The allocation decisions are subsequently assessed based on both economic and CSR performance criteria by employing the methodology of comparative institutional analysis that is widely adopted in institutional economics literature (Coase 1960; Williamson 1985). For the economic performance criterion, the system-wide profit is chosen as a proxy of efficiency to determine the optimal allocation scheme, and this choice is consistent with the concept of strategic CSR (Baron 2001). For the CSR performance criterion, the optimal allocation decision is obtained by maximizing the global CSR performance for the supply chain.

Our model is related to Gurnani et al. (2007), Xiao and Yang (2008) and Tsay and Agrawal (2000) in the following two aspects. First, our CSR-sensitive demand is similar to the quality-sensitive demand in Gurnani et al. (2007) and the service-sensitive demand in Xiao and Yang (2008) and Tsay and Agrawal (2000). Second, for our CSR cost function, Xiao and Yang (2008) and Tsay and Agrawal (2000) assume service cost functions in the same quadratic form that is independent of selling quantity, while Gurnani et al. (2007) introduce a quality cost function with both sales-irrelevant and sales-relevant components. In the aforesaid research, the authors focus on equilibrium variables such as quality/service levels, prices, sales and profits, and it is not a concern how different arrangements of quality/service pricing right affect the supply chain system-wide profit (or efficiency). In this paper, we investigate both equilibrium variables (if CSR were viewed as quality/service level) and the impact of different allocation schemes.

The rest of the article is organized as follows. Sections 2 and 3 present the basic model and the corresponding equilibriums. Section 4 reports our main results, followed by some discussions.

\textsuperscript{3} We use wholesale price contracts because they are commonly observed in practice (Cachon 2003).

\textsuperscript{4} For example, the Starbucks' sustainability conversion and performance price premiums ($0.05 per pound) in its CAFÉ program demand a host of socially responsible practices (Lee et al. 2007).
in Section 5. Finally, some concluding remarks are provided in section 6.

2 The Model

Consider a supply chain with two members, an upstream supplier S and a downstream firm F. The global CSR performance of the supply chain is measured by a variable $y$. To achieve this CSR performance level, certain investment has to be committed. Assume that this cost only incurs by the supplier (but will be shared with the firm F via a wholesale price contract) and takes a quadratic form $c(y) = cy^2/2$, which is independent of the production quantity. In addition to the social cost, a constant unit production cost $c_0$ is also incurred by S. The social responsibility commitment by S is expected to be compensated by F through a wholesale price contract that stipulates F to purchase product from S at a unit social-performance dependent wholesale price $w(y)$:

$$w(y) = w_0 + ky,$$  \hspace{1cm} (1)

where $w_0 > 0$ is a component that is independent of CSR performance, and $k \geq 0$ represents the marginal impact of CSR performance on the wholesale price.

Given a CSR performance level $y$, the larger the $k$ value, the more F is taking on the social responsibility for the supply chain. When $k$ is zero, all social responsibility for the supply chain will be solely assumed by S. On the other hand, when $k$ approaches infinity, all social responsibility will be shifted to F. Therefore, it is reasonable to put a cap $\bar{k}$ on $k$ to make the contract implementable. It is obvious that the wholesale price in (1) serves as a mechanism to share the social responsibility between S and F and $k$ plays a crucial role in achieving an equitable transfer of social cost from S to F. Two key issues in allocating social responsibility between the supply chain members S and F are who should be entrusted with the right of offering the wholesale price contract and what upper limit $\bar{k}$ should be placed on $k$.

F then sells the product in a consumer market characterized by a demand function

$$p = A(y) - \frac{1}{2} bq,$$  \hspace{1cm} (2)

where $p \geq 0$ and $q \geq 0$ are the price and the demand quantity, respectively, $b > 0$ indicates the slope

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5 CSR performance can be measured by investment in CSR activities such as mitigating pollutant emission, improving working conditions, philanthropic donations.

6 Röller (1990) theoretically shows that a quadratic cost function can behave well for analyzing global cost concepts (e.g. diminishing marginal returns) by properly choosing the parameters. In addition, quadratic cost functions are employed in many application studies (see, for example, Perry and Porter 1985, Rath and Zhao 2001, Kwoka 2002). Particularly, in the OM/OR area, Tsay and Agrawal (2000), Gurnani et al. (2007), Xiao and Yang (2008) also make this assumption for their cost function.
of the demand curve, \( A(y) \) characterizes the impact of CSR performance (denoted by \( y \geq 0 \)) on the final consumer market and is assumed to take the following form

\[
A(y) = a_0 + ay,
\]

where \( a_0 > 0 \) captures the base willingness-to-pay from consumers, and \( a > 0 \) stands for the marginal impact of CSR performance on additional willingness-to-pay. This assumption is consistent with Mohr and Webb’s (2005) empirical results from a national sample that CSR has a positive impact on consumer purchase intent.

With these assumptions, the profit function for \( F \) is given as

\[
\Pi^F(q, y, k) = (a_0 + ay - \frac{1}{2}bq)y - (w_0 + ky)q.
\]

Similarly, the profit for \( S \) is

\[
\Pi^S(q, y, k) = (w_0 + ky)q - c_0q - \frac{1}{2}cy^2.
\]

Furthermore, we assume \( w_0 = c_0 \) for the sake of analytical tractability and denote \( A_0 = a_0 - w_0 = a_0 - c_0 > 0 \) for notational simplification, then we have

\[
\Pi^F(q, y, k) = (A_0 + ay - ky)q - \frac{1}{2}bq^2,
\]

\[
\Pi^S(q, y, k) = kyq - \frac{1}{2}cy^2.
\]

The channel profit of the supply chain system is thus derived as

\[
\Pi^T(q, y) = \Pi^F(q, y, k) + \Pi^S(q, y, k) = (A_0 + ay)q - \frac{1}{2}bq^2 - \frac{1}{2}cy^2.
\]

Now \( F \) and \( S \) are treated as two economic agents. Following Choi (1991), the bargaining power in the supply chain is characterized by the Stackelberg leadership model. Three scenarios may arise: (1) Upstream Stackelberg (US) where \( S \) has more bargaining power than \( F \) and, hence, takes the first move; (2) Downstream Stackelberg (DS) where \( F \) has more bargaining power than \( S \) and, hence, moves first; and (3) Vertical Nash (VN) where \( S \) and \( F \) have equal bargaining power and, hence, move simultaneously.

According to the duality of rights and obligations (responsibilities), the responsibility holder

\[\text{The key motivation of assuming } w_0 = c_0 \text{ is to exclude the impact of production cost on the supplier’s CSR decision so that we can isolate the supplier’s CSR behavior and focus on examining how CSR commitments affect supply chain operations, and eventually analyze the impacts of different CSR allocation schemes on the efficiency of the whole supply chain (the system-wide profit).}\]
is assumed to have the right of offering a wholesale price contract \( k \in [0, \bar{k}] \), where \( \bar{k} \in (0, \infty) \)
describes how strong the right corresponds to the social responsibility. Understandably, the
greater \( \bar{k} \) is, the larger the margin of wholesale price contracts from which the responsibility
holder is allowed to choose, corresponding to a stronger right for the responsibility holder. Given
this interpretation, if the social responsibility is allocated to F, it will offer to S a wholesale price
by selecting \( k \in [0, \bar{k}] \) and also order \( q \) units such that its profit \( \Pi_F \) is maximized and S, in this
case, will choose \( y \) to maximize its own profit \( \Pi_S \); on the other hand, if S is allocated as the
social responsibility holder, it will offer to F a contract characterized by \( k \) and determine a CSR
performance level \( y \) to maximize \( \Pi_S \) and F will thus select \( q \) to maximize \( \Pi_F \). Therefore, the
allocation of social responsibility is twofold: who is the responsibility holder to offer \( k \) and what
cap \( \bar{k} \) is placed on \( k \). This allocation decision can thus be depicted by \( (X, \bar{k}) \in \{F, S\} \times (0, \infty) \). As for
the timing of the \( k \) decision, the base model in Sections 3 and 4 assumes that it is made at the
same time as the other decision variable controlled by the responsibility holder. Section 5, on the
other hand, examines the situation that \( k \) is offered by the responsibility holder prior to the other
two decision variables \( q \) and \( y \) are determined by F and S, respectively.

Finally, by combining the choice of a responsibility holder (F or S) and a power structure
(US, DS, or VN), six scenarios arise and are hereafter labeled as S-US, S-DS, S-VN, F-US, F-DS,
and F-VN games, respectively, where the first letter indicates the responsibility holder and the
last two letters identify the power structure. For instance, in the S-US game, the supplier S is the
responsibility holder and the Stackelberg leader and, hence, S is entitled to choose \( k \) as a
responsibility holder and determines its variable \( y \) first as a Stackelberg leader, subsequently, F
as the Stackelberg follower responds with \( q \) to the choices by S. The other five labels can be
interpreted in a similar fashion. Next, the six games are examined and their equilibriums are
obtained.

3 Equilibriums

To make the following analysis meaningful, assume \( a^2 < bc \) to guarantee the system-wide optimal
profit and CSR performance for the supply chain to be greater than zero.

The integrative case

First the integrative case is considered with social responsibility. The first-order conditions
are\(^8\)

\[(A_0 + ay) - bq = 0,\]

\[aq - cy = 0.\]

Solving these two equations simultaneously yields

\[y^*_f = \frac{A_0a}{bc - a^2}; \quad q^*_f = \frac{A_0c}{bc - a^2}.\]

The maximum profit of the supply chain system is

\[\Pi^*_f = \frac{A_0^2c}{2(bc - a^2)}.\]

When social responsibility is not considered in the model with all terms associated with \(y\) being removed, the optimal quantity and system-wide profit can be conveniently obtained as follows

\[q^*_N = \frac{A_0}{b}; \quad \Pi^*_N = \frac{A_0^2}{2b}.\]

**The S-US game**

In the S-US game, the supplier S offers the wholesale price contract \((k)\) and chooses \(y\), then the firm F responds with an order \(q\). By backward induction, from (4), the optimal reaction function for F is

\[q(y, k) = \frac{A_0 + (a - k)y}{b}.\] \hspace{1cm} (7)

Then the supplier’s profit function can be rewritten as

\[\Pi^s_{S-US}(q(y, k), y, k) = \frac{A_k y + (a - k)ky^2}{b} - \frac{1}{2}cy^2.\] \hspace{1cm} (8)

Clearly, this profit function is concave both in \(y\) for a given \(k\) and in \(k\) for a given \(y\), thereby validating Zabel’s (1970) method of first optimizing \(y\) for a given \(k\) and searching over the resulting optimal trajectory to find the optimal \(k\). The first-order condition with respect to \(y\) is

\[\frac{\partial \Pi^s_{S-US}}{\partial y} = \frac{A_k + 2(a - k)ky}{b} - cy = 0 \Rightarrow y(k) = \frac{A_k}{bc - 2k(a - k)}.\] \hspace{1cm} (9)

Substituting (9) into (8) and taking the first-order derivative with respect to \(k\) yield

---

\(^8\) The second order condition is easy to check. For remaining discussions, all second order conditions can be checked in a straightforward manner and, hence, are omitted in the article.
Note that $bc - 2k(a - k) = bc - 2ka + 2k^2 \geq bc - 2ka + k^2 = (a - k)^2 + bc - a^2$. Due to the aforesaid assumption $a^2 < bc$, it is confirmed that the denominator is positive. Therefore, the first-order derivative is positive, or equivalently, $\Pi_{S-US}^k$ increases in $k$, for all $k < bc / a$, and $\Pi_{S-US}^k$ decreases in $k$ for all $k > bc / a$. This indicates that $\Pi_{S-US}^*$ is unimodal in $k$. So, if $k$ is capped before $\Pi_{S-US}^*$ reaches its maximum at $k = bc / a$, i.e., for all $\bar{k} \leq bc / a$, the optimal wholesale price contract will occur at the boundary, $k_{S-US}^* = \bar{k}$. Otherwise, if the cap for $k$ is extended beyond $k = bc / a$, i.e., for all $\bar{k} > bc / a$, $k_{S-US}^* = bc / a$. Plugging the optimal $k_{S-US}^*$ into (9), (7), (4), (5) and (6), we can calculate the equilibrium CSR performance ($y_{S-US}^*$) and product quantity ($q_{S-US}^*$), as well as the equilibrium profits for the firm, the supplier, and the supply chain system. These equilibrium variables are summarized in Proposition 1.

**Proposition 1:** The subgame perfect equilibrium of the S-US game is summarized as

(i) If $\bar{k} \leq bc / a$, the equilibrium variables are

\[
\begin{align*}
    k_{S-US}^* &= \bar{k}; \\
    y_{S-US}^* &= \frac{A_0 \bar{k}}{bc - 2k(a - \bar{k})}; \\
    q_{S-US}^* &= \frac{A_0}{b[bc - 2k(a - \bar{k})]}; \\
    \Pi_{S-US}^s &= \frac{A_0^2 \bar{k}^2}{2b[bc - 2k(a - \bar{k})]^2}; \\
    \Pi_{S-US}^f &= \frac{A_0^2}{2b[bc - 2k(a - \bar{k})]}; \\
    \Pi_{S-US}^{T*} &= \frac{A_0^2}{2b[bc - 2k(a - \bar{k})]} + \frac{bc - \bar{k}^2}{[bc - 2k(a - \bar{k})]^2}. \\
\end{align*}
\]

(ii) If $\bar{k} > bc / a$, the equilibrium variables are

\[
\begin{align*}
    k_{S-US}^* &= \frac{bc}{a}; \\
    y_{S-US}^* &= \frac{A_0 a}{2bc - a^2}; \\
    q_{S-US}^* &= \frac{A_0 c}{2bc - a^2}; \\
    \Pi_{S-US}^s &= \frac{A_0^2 c}{2(2bc - a^2)}; \\
    \Pi_{S-US}^f &= \frac{A_0^2 bc^2}{2(2bc - a^2)^2}; \\
    \Pi_{S-US}^{T*} &= \frac{A_0^2 c(3bc - a^2)}{2(2bc - a^2)^2}. \\
\end{align*}
\]

**The F-US game**

The F-US game is similar to the S-US case except that F rather than S offers the wholesale price contract characterized by $k$. Given $y$ from S, F determines $k$ and $q$ to maximize $\Pi_{F-US}^f(q, y, k) = \Pi^f(q, y, k)$. If S sets $y = 0$, the profit for F will only depend on $q$, and the first-order condition with respect to $q$ immediately implies that, for any $k \in [0, \bar{k}]$, F orders $q = A_0 / b$ with a maximal profit $\Pi_{F-US}^f(q, y, k) = \Pi^f(A_0 / b, 0, k) = A_0^2 / (2b)$. As $y = 0$ in this case, $k$ becomes irrelevant. Therefore, although $k$ may assume any value between 0 and $\bar{k}$, for convenience, we set it at...
\(k(y) = 0\) to break ties. On the other hand, if S commits to \(y > 0\), for any \(q > 0\), \(\pi'_{F-US}(q,y,k)\) strictly decreases in \(k\), then the optimal wholesale price contract is \(k = 0\), and the corresponding optimal order is given as \(q = (A_y + ay)/b\). In this case, \(\pi'_{F-US}(q,y,k) = \Pi'(A_y + ay)/b, y, 0) = (A_y + ay)^2/(2b) > 0\). In addition, \(y > 0\) and \(q = 0\) together imply that \(\pi'_{F-US}(q,y,k) = \Pi'(0,y,k) = 0\) for any \(k \geq 0\). Thus, the optimal reaction to \(y > 0\) can be expressed as \(q = (A_y + ay)/b\) and \(k = 0\). In summary, the optimal reaction from F is

\[
(q(y), k(y)) = \left(\frac{A_y + ay}{b}, 0\right).
\]

(10)

By backward induction, the profit function for S is rewritten as \(\pi'_S(q,y,k) = \Pi^s(q(y),y,k(y))\), where \(q(y)\) and \(k(y)\) are given in (10). If S chooses \(y = 0\), then its profit becomes \(\pi'_{F-US}(y) = \Pi^s(A_y/b, 0) = 0\). But if it selects \(y > 0\), its profit is \(\Pi^s_{F-US}(y) = \Pi^s(A_y + ay)/b, y, 0) = -cy^2/2 < 0\). Thus, the optimal decision for S is \(\dot{y}_{F-US} = 0\). Therefore, the subgame perfect equilibrium of the F-US game can be obtained as shown in Proposition 2.

**Proposition 2**: The subgame perfect equilibrium of the F-US game can be summarized as

\[
k^*_F = 0; \quad y^*_F = 0; \quad q^*_F = \frac{A_y}{b};
\]

\[
\Pi^s_{F-US} = 0; \quad \Pi^s_{F-US} = \frac{A_y^2}{2b}; \quad \Pi^s_{F-US} = \frac{A_y^2}{2b}.
\]

**The S-DS game**

In the S-DS game, F chooses \(q\) first and, then given \(q\), S reacts with \(k\) and \(y\) to maximize

\(\pi'_{S-DS}(q,y,k) = \Pi^s(q,y,k)\). Once again, backward induction is employed to obtain its equilibrium. First, if F orders \(q = 0\), it becomes trivial with \(y = 0\) and \(k = k^*\) by S, where \(k^*\) is a real number arbitrarily picked from \(k \in [0, \bar{k}]\). On the other hand, if \(q > 0\), the optimal reaction \((k,y)\) by S must satisfy the first-order condition with respect to \(y\), i.e. \(kq = cy\). Notice that \(k = 0\) implies \(y = 0\), thereby \(\pi^s_{S-DS}(q,0,0) = 0\). Note further that \(k > 0\) means \(y > 0\), and it follows that

\[
\partial \pi^s_{S-DS}(q,y,k)/\partial k = qk > 0,
\]

indicating that the profit for S strictly increases in \(k\) and, hence, reaches its maximum at \(\bar{k}\). In addition, for all \((k,y) > 0\) with \(kq = cy\), \(\pi^s_{S-DS}(q,y,k) = (kq)^2/(2c) > 0\). Therefore, the optimal reaction \((k,y)\) to \(q > 0\) is \((\bar{k}, \bar{k}q/c)\). To summarize, the reaction function from S is expressed as
Given (11), if the firm chooses \( q = 0 \), its profit is \( \Pi^F_{S-D}(q) = \Pi^F(q,y(q),k(q)) = 0 \). For \( q > 0 \), its profit function can be rewritten as

\[
\Pi^F_{S-D}(q) = \Pi^F(q,y(q),k(q)) = A_y q + \left( \frac{k(a-k)}{c} - \frac{b}{2} \right) q^2.
\]

Due to the assumption \( a^2 - bc < 0 \), \( 2k(a-k) - bc = 2ak - 2k^2 - bc = -(a-k)^2 + a^2 - bc < 0 \), so \( \Pi^F_{S-D}(q) \) is concave. The first-order condition with respect to \( q \) immediately yields

\[
q^*_{S-D} = \frac{Ac}{bc - 2k(a-k)}.
\]

Finally, given that \( q^*_{S-D} > 0 \) as its denominator and numerator are positive, the optimal response \((k(q),y(q))\) from S can be easily obtained from (11). Plugging them into (4), (5) and (6), one can determine the equilibrium profit for S, F, and the supply chain system. All equilibrium variables of the S-DS game can thus be furnished as Proposition 3 below.

**Proposition 3**: The subgame perfect equilibrium of the S-DS game can be summarized as

\[
k^*_{S-D} = \bar{k} ; \quad y^*_{S-D} = \frac{A_y \bar{k}}{bc - 2k(a-k)} ; \quad q^*_{S-D} = \frac{Ac}{bc - 2k(a-k)} ;
\]

\[
\Pi^S_{S-D} = \frac{A_y^2 \bar{k}^2}{2[bc - 2k(a-k)]^2} ; \quad \Pi^F_{S-D} = \frac{A^2 y_c}{2[bc - 2k(a-k)]} ; \quad \Pi^T_{S-D} = \frac{A^2 y_c (bc - 2k a + 3k^2)}{2[bc - 2k(a-k)]^3}.
\]

**The F-DS game**

In the F-DS game, F chooses \( k \) and \( q \) first, followed by S selecting \( y \) to maximize \( \Pi^F_{F,D}(q,y,k) = \Pi^F(q,y,k) \) under the given \( k \) and \( q \). The reaction function for S is thus

\[
y(k,q) = \frac{cq}{c}.
\]

Given (12), the profit function for F is

\[
\Pi^F_{F,D}(q,k) = \Pi^F(q,y(k,q),k) = A_y q + \left( \frac{k(a-k)}{c} - \frac{b}{2} \right) q^2.
\]

This profit function is concave both in \( q \) for a given \( k \) and in \( k \) for a given \( q > 0 \), permitting the application of Zabel’s (1970) method for optimization. Next, we first optimize \( q \) for a given \( k \) and, then find the optimal \( k \). From the first-order condition with respect to \( q \), we have
Substituting (14) into (13) and taking the derivative with respect to \( k \) yield

\[
\frac{\partial \Pi^*_{F,DS}}{\partial k} = \frac{(a - 2k)[q(k)]^2}{c}.
\]

This indicates that \( \Pi^*_{F,DS} \) increases in \( k \) for \( k < a/2 \) and decreases in \( k \) if \( k > a/2 \) and, hence, \( \Pi^*_{F,DS} \) is unimodal. Thus, \( k^*_{F,DS} = \bar{k} \) if \( k < a/2 \), otherwise \( k^*_{F,DS} = a/2 \). By (14), (12), (4), (5) and (6), we can determine the equilibrium variables as shown in Proposition 4.

**Proposition 4**: The subgame perfect equilibrium of the F-DS game can be summarized as

(i) If \( \bar{k} \leq a/2 \), the equilibrium variables are the same as those in the S-DS game;

(ii) If \( \bar{k} > a/2 \), the equilibrium variables are

\[
\begin{align*}
k^*_{F,DS} &= \frac{a}{2}; \quad y^*_{F,DS} = \frac{A_0 a}{2bc - a^2}; \quad q^*_{F,DS} = \frac{2A_0 c}{2bc - a^2}; \\
\Pi^s_{F,DS} &= \frac{A_0^2 a^2 c}{2(2bc - a^2)^2}; \quad \Pi^r_{F,DS} = \frac{A_0^2 c (4bc - a^2)}{2(2bc - a^2)^2}.
\end{align*}
\]

**The S-VN game**

Under the assumption of the S-VN game, S and F determine their variables simultaneously, where S furnishes \( k \) and \( y \) and F provides a quantity \( q \). It is easy to verify that (7) and (11) are the reaction functions for F and S, respectively. If \( q = 0 \), (11) implies that \( y = 0 \) and \( k \) is arbitrary, but (7) indicates that \( q(0,k) = A_0 / b > 0 \). This means that \( q \) cannot be zero in the equilibrium. For \( q > 0 \), (11) implies that \( k = \bar{k} \) and \( y = \bar{k} q / c \). Substituting these two equations into (7) and solving it for \( q \), one can have

\[
q^*_{S-VN} = \frac{A_0 c}{bc - \bar{k}(a - \bar{k})}.
\]

With this result, it is straightforward to derive other variables in the equilibrium as given in the following proposition.

**Proposition 5**: The Nash equilibrium of the S-VN game can be summarized as

\[
\begin{align*}
k^*_{S-VN} &= \bar{k}; \quad y^*_{S-VN} = \frac{A_0 \bar{k}}{bc - \bar{k}(a - \bar{k})}; \quad q^*_{S-VN} = \frac{A_0 c}{bc - \bar{k}(a - \bar{k})}; \\
\Pi^s_{S-VN} &= \frac{A_0^2 \bar{k}^2 c}{2[bc - \bar{k}(a - \bar{k})]}; \quad \Pi^r_{S-VN} = \frac{A_0^2 c (bc - \bar{k}a + 2\bar{k}^2)}{2[bc - \bar{k}(a - \bar{k})]^2}.
\end{align*}
\]
The F-VN game

In the F-VN game, F determines \( k \) and \( q \) at the same time as S gives \( y \). Clearly, (10) and (12) are the reaction functions for F and S, respectively. (10) and (12) are next solved simultaneously. If \( y = 0 \), we have \( q = A_y / b \) from (10), and (12) further implies \( k = 0 \). Thus, \((k, q, y) = (0, A_y / b, 0)\) is a Nash equilibrium for the F-VN game. It is actually the unique Nash equilibrium. As a matter of fact, if \( y > 0 \), (10) implies that \( k = 0 \) and \( q = (A_y + ay) / b > 0 \), contradictory to (12). Therefore, \((k, q, y) = (0, A_y / b, 0)\) is the unique triplet that satisfies both (10) and (12) simultaneously, leading to the following proposition.

**Proposition 6:** The Nash equilibrium of the F-VN game can be summarized as

\[
\begin{align*}
k_{F-VN}^* & = 0; \quad y_{F-VN}^* = 0; \quad q_{F-VN}^* = \frac{A_y}{b}; \\
\prod_{F-VN}^* & = 0; \quad \prod_{F-VN}^* = \frac{A_y^2}{2b}; \quad \prod_{F-VN}^* = \frac{A_y^2}{2b}.
\end{align*}
\]

**Remark:** Propositions 1-6 demonstrate that the power structure has a significant impact on the behavior of the responsibility holder. If a supply chain member is entrusted as a responsibility holder who offers the wholesale price contract characterized by \( k \), it seems to behave in an equitable manner only if it assumes the leadership position. On the one hand, if responsibility holder S is the Stackelberg leader, corresponding to the S-US case, it will always share social responsibility with F at an optimal level of \( k_{S-US}^* = bc / a \) if \( \bar{k} > bc / a \) or \( k_{S-US}^* = \bar{k} \) if \( \bar{k} < bc / a \). Similarly, if responsibility holder F is the Stackelberg leader in the F-DS case, F will offer \( k_{F-DS}^* = a / 2 \) to take its share in achieving the equilibrium CSR performance level. On the other hand, if social responsibility of the supply chain is allocated to S, but it is not the Stackelberg leader in the S-DS or S-VN case, S will always push the \( k \) value to its maximum, i.e., \( k_{S-DS}^* = \bar{k} \) or \( k_{S-VN}^* = \bar{k} \). If there is no restriction on \( \bar{k} \), i.e., \( \bar{k} \to \infty \), S will not pull its weight but transfer all of its social responsibility investment to F via the wholesale price contract. This observation indicates that the right of offering \( k \) for S should come with a restriction on the upper limit of \( k \), which may be imposed by a third party, for instance, a government agency, or through a negotiation between the supply chain partners so that social responsibility is indeed equitably shared. In a similar fashion, one can examine the cases that F is the responsibility holder but not the Stackelberg leader in the F-US or F-VN games. In both cases, F sets \( k^* = 0 \) and refuses to share any CSR investment with S, eventually leading to no CSR performance for the supply chain (\( y_{F-US}^* = 0 \) and
This result shows the other side of the coin: when F is entrusted as the responsibility holder to determine the wholesale price, a lower bound should be placed on $k$ to ensure a reasonable transfer of social responsibility cost from S so that the undesirable case of zero CSR performance is avoided for the supply chain. Once again, this lower limit could be imposed by a third party or negotiated between F and S.

In summary, the equilibrium value of the parameter $k$ characterizes how CSR investment is expected to be shared between S and F, and the CSR investment tends to be shared in an equitable manner if the Stackelberg leader is allocated to decide $k$. Intuitively, in the US and DS cases, the leader’s profit depends on the follower’s response (or threat) and the leader is thus able to take advantage of its leadership position to stimulate (or guide) the follower by choosing a reasonable $k$ to equitably share the CSR investment. On the contrary, if the follower is entrusted with the right of selecting $k$, it knows that its decision on $k$ will be final as the leader has already committed to its actions. As such, the follower does not have any economic incentive to pull its weight. In the VN case, neither stimulation nor threat is possible because S and F have to move simultaneously without any prior knowledge of commitments from their partner. Therefore, each party with the right of determining $k$, in its best economic interests, pushes $k$ towards its boundary ($\bar{k}$ in Proposition 5 and 0 in Proposition 6), thereby forcing its partner to take on as much CSR investment cost as possible.

4 Main results

This section analyzes the equilibriums and derives the optimal allocation of social responsibility according to the methodology of comparative institutional analysis. Implications on business practice are also explored for the resulting social responsibility allocation scheme within a supply chain management context.

4.1 Optimal responsibility allocations based on the economic performance criterion

In Section 3, equilibriums are obtained by examining each supply chain member’s strategic behavior under each of the six aforesaid scenarios. In equilibrium, each member has chosen its optimal strategy to maximize its own profit. Here, we shall employ the comparative institutional analysis approach to investigate the equilibriums and determine the optimal social responsibility allocation scheme that maximizes the total equilibrium profit for the channel under each of the
three power structures.

The US case

From Proposition 1, we have

$$
\Pi^*_s(k) = \begin{cases}
A_1^2 \left( \frac{b}{bc - 2k(a - \bar{k})} \right) + \left[ \frac{bc - \bar{k}(a - \bar{k})}{bc - 2k(a - \bar{k})} \right]^2, & \text{if } \bar{k} \leq \frac{bc}{a} \\
A_1^2 \left( \frac{3bc - a^2}{2(2bc - a^2)} \right), & \text{if } \bar{k} > \frac{bc}{a}
\end{cases}
$$

For all $k \in (0, bc/a)$, we have

$$
\frac{d\Pi^*_s}{dk} = \frac{A_1^2 \left( -2a\bar{k}^3 + 2a^2\bar{k}^2 - bc(a^2 + bc)\bar{k} + ab^2c^2 \right)}{b[bc - 2k(a - \bar{k})]^3} = \frac{A_1^2 F(\bar{k})}{b[bc - 2k(a - \bar{k})]^3},
$$

where $F(\bar{k}) = -2a \bar{k}^3 + 2a^2 \bar{k}^2 - bc(a^2 + bc)\bar{k} + ab^2c^2$.

Then $F'(\bar{k}) = -8a\bar{k}^2 + 6a^2\bar{k} - bc(a^2 + bc) < 0$ due to the assumption $a^2 < bc$, therefore, $F'(\bar{k}) < 0$ for all $\bar{k} \in (0, \infty)$. Note that $F'(a/2) = a^4/2 - bc(a^2 + bc) < 0$ such that $F(\bar{k}) = 0$, $F(\bar{k}) > 0$ for $\bar{k} < \bar{k}^*$ and $F(\bar{k}) < 0$ for $\bar{k} > \bar{k}^*$. Thus (15) implies that $\frac{d\Pi^*_s}{dk} = 0$ at $\bar{k} = \bar{k}^*$, $\frac{d\Pi^*_s}{d\bar{k}} > 0$ for $\bar{k} < \bar{k}^*$ and $\frac{d\Pi^*_s}{d\bar{k}} < 0$ for $\bar{k} > \bar{k}^*$. Therefore, $\Pi^*_s$ is unimodal in $\bar{k}$ and attains its maximum at $\bar{k} = \bar{k}^*$ over $(0, \infty)$.

Proposition 2 indicates that any $\bar{k}$ always leads to the same constant equilibrium channel profit for the F-US game, hence,

$$
\Pi^*_f(k) = \frac{A_1^2}{2b} \left( \frac{a^2}{bc} + 1 \right) = \Pi^*_s(k) < \Pi^*_s(\bar{k}^*), \text{ for any } \bar{k} \in (0, \infty).
$$

Therefore, $(S, \bar{k}^*)$ is the unique optimal responsibility allocation in the US case, meaning that S will be allocated as the responsibility holder with the right to choose $k = \bar{k}^* \in [0, \bar{k}^*)$ when S is the upstream Stackelberg leader.

The DS case

From Proposition 3, we have

9 The comparative institutional analysis (Williamson 1985) suggests that transaction cost savings (efficiency enhancement) via actors' rational behavior drive the evolution of institutions. As such, comparative efficiency advantages dominate the choice of institutions. Applying this idea to our study, to determine who should be allocated the right of offering a wholesale price contract, a rational recommendation is the one who is able to achieve higher system-wide profit for the supply chain.
\[
\frac{d\Pi_{S-DS}^*}{d\bar{k}} = A_0^c \left( -6\bar{k}^2 + 6\bar{k} \left( 2a^2 + bc \right) + abc \right) = \frac{A_0^c c G(\bar{k})}{bc - 2\bar{k}(a - \bar{k})},
\]
(16)

where \( G(\bar{k}) = -6\bar{k}^3 + 6\bar{k} \left( 2a^2 + bc \right) + abc \).

As \( G'(\bar{k}) = -18\bar{k}^2 + 12\bar{k} \left( 2a^2 + bc \right) = -2(3\bar{k} - a)^2 - bc < 0 \), \( G'(\bar{k}) < 0 \) for all \( \bar{k} \in (0, \infty) \), implying that \( G(\bar{k}) \) decreases in \( \bar{k} \) for \( \bar{k} \in (0, \infty) \). Note that \( G(\bar{k}) = a(\bar{k}^2 - \bar{k}^2) = a(2bc - a^2) > 0 \) and \( G(0) = -2a^2 < 0 \), there exists a unique \( \bar{k}_c^* \in (\bar{k}, 0) \) such that \( G(\bar{k}_c^*) = 0 \), \( G(\bar{k}) > 0 \) for \( \bar{k} < \bar{k}_c^* \) and \( G(\bar{k}) < 0 \) for \( \bar{k} > \bar{k}_c^* \). It follows from (16) that \( \frac{d\Pi_{S-DS}^*}{d\bar{k}} = 0 \) at \( \bar{k} = \bar{k}_c^* \), \( \frac{d\Pi_{S-DS}^*}{d\bar{k}} > 0 \) for \( \bar{k} < \bar{k}_c^* \) and \( \frac{d\Pi_{S-DS}^*}{d\bar{k}} < 0 \) for \( \bar{k} > \bar{k}_c^* \). Thus \( \Pi_{S-DS}^* \) is unimodal in \( \bar{k} \) and attains its maximum at \( \bar{k} = \bar{k}_c^* \).

From Proposition 4, we have

\[
\Pi_{F-DS}^* (\bar{k}) = \begin{cases} 
\Pi_{S-DS}^* (\bar{k}), & \text{if } \bar{k} \leq \frac{a}{2} \\
\frac{A_0^c c \left( 4bc - a^2 \right)}{2(2bc - a^2)}, & \text{if } \bar{k} > \frac{a}{2} 
\end{cases}
\]
(17)

It is confirmed that \( \Pi_{F-DS}^* (\bar{k}) \) is continuous in \( \bar{k} \). Given that \( \Pi_{S-DS}^* (\bar{k}) \) increases in \( \bar{k} \) when \( \bar{k} < \frac{a}{2} \), (17) indicates that \( \Pi_{F-DS}^* (\bar{k}) \) reaches its maximum at \( \bar{k} = \frac{a}{2} \). Therefore, for any \( \bar{k} \in (0, \infty) \)

\[
\Pi_{F-DS}^* (\bar{k}) \leq \Pi_{F-DS}^* \left( \frac{a}{2} \right) = \frac{A_0^c c \left( 4bc - a^2 \right)}{2(2bc - a^2)} = \Pi_{S-DS}^* \left( \frac{a}{2} \right) < \Pi_{S-DS}^* (\bar{k}_c^*).
\]

Thus \( (\bar{k}, \bar{k}_c^*) \) arises as the unique optimal responsibility allocation in the DS case.

**The VN case**

From Proposition 5, for \( \bar{k} \in (\bar{k}, \infty) \), we have

\[
\frac{d\Pi_{S-VN}^*}{d\bar{k}} = A_0^c \left( -4\bar{k}^3 + 3a\bar{k}^2 - a^2 \bar{k} + abc \right) = \frac{A_0^c c H(\bar{k})}{2bc - \bar{k}(a - \bar{k})},
\]
(18)

where \( H(\bar{k}) = -4\bar{k}^3 + 3a\bar{k}^2 - a^2 \bar{k} + abc \).

Then \( H'(\bar{k}) = -12\bar{k}^2 + 6a\bar{k} - a^2 = -3\bar{k}^2 - a^2 < 0 \), indicating that \( H(\bar{k}) \) decreases in \( \bar{k} \). As \( H(\bar{k}) = a(\bar{k}^2 - \bar{k}^2) = a(2bc - a^2) < 0 \) (due to \( a^2 < bc \) ), there exists a unique \( \bar{k}_c^* \in (\bar{k}, 2\sqrt{bc}) \) such that \( H(\bar{k}_c^*) = 0 \), \( H(\bar{k}) > 0 \) for \( \bar{k} < \bar{k}_c^* \) and \( H(\bar{k}) < 0 \) for \( \bar{k} > \bar{k}_c^* \). Given that the denominator of (18) is positive, it follows that \( \frac{d\Pi_{S-VN}^*}{d\bar{k}} = 0 \) at \( \bar{k} = \bar{k}_c^* \), \( \frac{d\Pi_{S-VN}^*}{d\bar{k}} > 0 \) for \( \bar{k} < \bar{k}_c^* \) and \( \frac{d\Pi_{S-VN}^*}{d\bar{k}} < 0 \) for \( \bar{k} > \bar{k}_c^* \). Therefore, \( \Pi_{S-VN}^* \) is unimodal in \( \bar{k} \) and achieves its maximum at \( \bar{k} = \bar{k}_c^* \) over \( (0, \infty) \).

Furthermore, Proposition 6 indicates that a constant equilibrium channel profit is always attained for any \( \bar{k} \) when \( F \) is responsible for determining \( k \in [0, \bar{k}] \) and, hence,
\[ \Pi^*_{F_{-VNF}}(k) = \frac{A_1 k^2}{2b} < \frac{A_2 (bc + a^2)}{2b^2 c} = \Pi^*_{S_{-SNF}}(a) \leq \Pi^*_{S_{-SNF}}(k_{**}) . \]

Therefore \((S,k_{**})\) is the unique optimal responsibility allocation in the VN case.

These results can now be summarized as Proposition 7.

**Proposition 7**: According to the economic performance criterion, \((S,k_{*})\), \((S,k_{**})\) and \((S,k_{***})\) are the unique optimal responsibility allocation for the US, DS and VN cases, respectively.

Proposition 7 furnishes the optimal allocation schemes as well as the corresponding \(k\) values at optimality under the three power structures. Next, Corollaries 1-3 further establish that it remains optimal to entrust \(S\) with the right of offering the wholesale contract over a certain range of \(k\) values, even if they are not set at their corresponding optimality.

**Corollary 1**: If \(bc \leq (3 + \sqrt{5})a^2 / 2\), then \(\Pi^*_{S_{-US}}(k) \geq \Pi^*_{F_{-US}}(k)\) for \(k \in (0, \infty)\); otherwise, if \(bc > (3 + \sqrt{5})a^2 / 2\), there exists a unique \(k^*_0 > k^*_e\) such that \(\Pi^*_{S_{-US}}(k) \geq \Pi^*_{F_{-US}}(k)\) for \(k \in (0, k^*_e]\) and \(\Pi^*_{S_{-US}}(k) < \Pi^*_{F_{-US}}(k)\) for \(k \in [k^*_e, \infty)\).

**Proof**: Note that for all \(k \in (0, \infty)\), \(\Pi^*_{F_{-US}}(k) = A_0 / (2b)\) and \(\lim_{k \to \infty} \Pi^*_{F_{-US}}(k) = A_0 / (2b)\). Earlier arguments indicate that \(\Pi^*_{S_{-US}}(k)\) increases in \(k\) for \(k \in (0, k^*_e]\) and decreases in \(k\) for \(k \in [k^*_e, \infty)\). One can verify that

\[ \Pi^*_{F_{-US}}(k) = \frac{A_0 c(3bc - a^2)}{2(2bc - a^2)} \geq \Pi^*_{S_{-US}}(k) \]

Therefore, if \(bc \leq (3 + \sqrt{5})a^2 / 2\), then \(\Pi^*_{S_{-US}}(k) \geq \Pi^*_{F_{-US}}(k)\) for all \(k \in (0, \infty)\). On the other hand, if \(bc > (3 + \sqrt{5})a^2 / 2\), there exists a unique \(k^*_0 \in (k^*_e, bc / a)\) such that \(\Pi^*_{S_{-US}}(k) > \Pi^*_{F_{-US}}(k)\) for all \(k \in (0, k^*_0]\) and \(\Pi^*_{S_{-US}}(k) < \Pi^*_{F_{-US}}(k)\) for all \(k \in [k^*_0, \infty)\). Corollary 1 is thus proved.

**Corollary 2**: There exists an unique \(k^*_0 > k^*_e\) such that \(\Pi^*_{S_{-OS}}(k) \geq \Pi^*_{F_{-OS}}(k)\) for \(k \in (0, k^*_0]\) and \(\Pi^*_{S_{-OS}}(k) < \Pi^*_{F_{-OS}}(k)\) for \(k \in [k^*_0, \infty)\).

**Proof**: Since \(\Pi^*_{F_{-OS}}(k)\) reaches its maximum at \(k^*_e < (a / 2, a)\), then (17) implies that \(\Pi^*_{F_{-OS}}(k) > \Pi^*_{S_{-OS}}(a / 2)\) for all \(k > a / 2\). Further, due to \(\lim_{k \to \infty} \Pi^*_{F_{-OS}}(k) = 0\) and the unimodality of \(\Pi^*_{S_{-OS}}(k)\), it follows that there is a unique \(k^*_0 > k^*_e\) such that \(\Pi^*_{S_{-OS}}(k) \geq \Pi^*_{F_{-OS}}(k)\) for all \(k \in (0, k^*_0]\) and \(\Pi^*_{S_{-OS}}(k) < \Pi^*_{F_{-OS}}(k)\) for all \(k \in [k^*_0, \infty)\). Corollary 2 is then proved.

**Corollary 3**: There exists an unique \(k^*_0 > k^*_e\) such that \(\Pi^*_{S_{-SNF}}(k) \geq \Pi^*_{F_{-SNF}}(k)\) for \(k \in (0, k^*_0]\) and
Proof: it is trivial to verify that $\lim_{k \to -\infty} \Pi^*_S(k) = 0$ and $\lim_{k \to 0} \Pi^*_F(k) = \frac{A^*_b}{2b} = \Pi^*_I(k)$ for all $k \in (0, \infty)$. Then the unique maximum of $\Pi^*_S(k)$ at $k^* = \frac{a}{2\sqrt{bc}}$ implies that part (iii) holds. Corollary 3 is thus proved.

Remark: Under the basic model setting that CSR performance-related cost incurs only by the supplier, to maximize the channel profit of the supply chain, Proposition 7 indicates that the right to price CSR performance via a wholesale price contract should be allocated to the supplier regardless of the power structure. The corresponding optimal $k$ values are derived therein for the three power structures, US, DS, and VN, respectively. When $k$ is set at a value other than its optimality, Corollaries 1-3 further reveal a range of values within which it remains optimal to allocate the right to S for each of the three power structures. Except for the US case with $bc \leq (3 + \sqrt{5})a^2 / 2$ where it is always better, in terms of system-wide profit, to allocate the right to S, Corollaries 1-3 highlight the importance of placing appropriate caps on $k$ ($k^*_b, k^*_e, k^*_w$): within these limits, the system-wide profit will be higher if the right is allocated to S; once these thresholds are exceeded, it would be better to entrust the right to F. Intuitively, if S’s right of pricing CSR performance into a wholesale price contract is not appropriately restricted, it tends to abuse the right by shifting too much cost to F, thereby hurting the overall channel profitability. These results demonstrate that the responsibility holder allocation depends on how to restrict the right by placing a cap on $k$ rather than the power structure within a supply chain. In contrary to the suggestion of Amaeshi et al. (2008) that the more powerful member in a supply chain should be held responsible, Proposition 7 tends to partially support the argument based on the principles of corporate legal personality and separate existence of a corporation that each member is responsible for only its own activity if the right corresponding to the responsibility is appropriately restricted. Note further that given $c_0$ (then $w_0$ is fixed), the wholesale price $w(y)$ is determined by $k$ and $y$. Therefore, Proposition 7 indicates that, with an appropriate restriction on the right to price CSR performance, the system-wide optimal economic performance can be achieved by allocating the right to the supplier who incurs the investment in social responsibility.

It is reasonable to question who controls the allocation right of the contract and how the optimal allocation scheme is implemented. Note that this research assumes that information is complete and symmetric for both parties, and the decision-makers are rational. When the channel
profit is chosen as the economic criterion for the supply chain, the comparative institutional analysis suggests that the profit maximization drives S and F to reach the optimal allocation scheme given in Proposition 7. As for the implementation issue, for the US and VN cases, it is confirmed that \( \Pi_{S-US}^* (k_e^*) \geq \Pi_{F-US}^* \) and \( \Pi_{S-US}^* (k_e^*) \geq \Pi_{F-US}^* \), and \( \Pi_{S-VN}^* (k_e^{**}) \geq \Pi_{F-VN}^* \) and \( \Pi_{S-VN}^* (k_e^{**}) \geq \Pi_{F-VN}^* \), indicating that the optimal allocation scheme not only increases the system-wide profit, but also enhances each party’s individual profitability. Therefore, the implementation of the optimal solution is not an issue as it is in the economic interest of each participant in these two cases. On the other hand, in the DS case, we have \( \Pi_{S-DS}^* (k_e^{**}) > \Pi_{F-DS}^* \) and \( \Pi_{S-DS}^* (k_e^{**}) < \Pi_{F-DS}^* \), indicating that F’s profit actually goes down by implementing the optimal allocation scheme although the system-wide profit increases. In this case, due to the complete and symmetric information assumption, an appropriate lump-sum transfer payment from S to F exists such that the optimal allocation becomes a win-win solution for both parties. As a matter of fact, let \( \Delta \) be the transfer payment, as long as \( \Delta > \Pi_{F-DS}^* - \Pi_{S-DS}^* (k_e^{**}) \) and \( \left[ \Pi_{S-DS}^* (k_e^{**}) - \Pi_{F-DS}^* \right] - \Delta > 0 \), the optimal allocation of S being the responsibility holder makes both S and F better off. Due to the fact that \( \Pi_{S-DS}^* (k_e^{**}) + \Pi_{F-DS}^* (k_e^{**}) > \Pi_{S-DS}^* + \Pi_{F-DS}^* \) as per the argument leading to Proposition 7, the existence of such a \( \Delta \) is guaranteed. Furthermore, the assumption of complete and symmetric information allows for establishing this transfer payment as an enforceable clause of the wholesale contract, which is consistent with the implicit assumption of enforceability based on transfer payments in Hsueh and Chang (2008) as well.

### 4.2 Optimal responsibility allocations according to the CSR performance criterion

In the US case, Propositions 1 and 2 clearly indicate that

\[
y_{S-US}^*(k) > 0 = y_{F-US}^*(k), \quad \text{for } k \in (0, \infty).
\]

Then \( y_{S-US}^* \) increases for all \( k \in (0, \sqrt{bc/2}] \) and decreases for all \( k \in (\sqrt{bc/2}, bc/a] \). Thus \( y_{S-US}^* \) is unimodal in \( k \) and attains its maximum at \( k^*_s = \sqrt{bc/2} \) over \( (0, bc/a) \). Note that for \( k \in (bc/a, \infty) \),

\[
y_{S-US}^*(k) = y_{S-US}^*(bc/a) < y_{S-US}^*(k),
\]

so \( y_{S-US}^* \) reaches its global maximum at \( k^*_s \). Moreover, (19) implies that \( (S, k^*_s) \) is the unique optimal responsibility allocation in the US case according to the CSR
performance criterion.

In the DS case, part (i) of Proposition 4 indicates that \( y_{S,DS}(\kappa) = y_{F,DS}(\kappa) \) for all \( \kappa \leq a/2 \). For \( \kappa > a/2 \), we have

\[
\frac{dy_{S,DS}}{d\kappa} = \frac{A_b \left( bc - 2\kappa^2 \right)}{\left[ bc - 2\kappa (a - \kappa) \right]}.
\]

Then \( y_{S,DS} \) is unimodal in \( \kappa \) and attains its maximum at \( \kappa^*_{DS} = \kappa^*_F = \sqrt{bc/2} \). Note that for \( \kappa > a/2 \),

\[
y_{F,DS}(\kappa) = y_{F,DS}(a/2) = y_{F,DS}(a/2) < y_{S,DS}(\kappa^*_F) \cdot
\]

Thus \( (S,\kappa^*_F) \) is the unique optimal responsibility allocation according to the CSR performance criterion in the DS case.

In the VN case, from Proposition 5 and 6, it follows that for all \( \kappa \in (0, \infty) \),

\[
y_{S,VN}(\kappa) = \frac{A_b \kappa}{bc - \kappa (a - \kappa)} > 0 = y_{F,VN}(\kappa),
\]

and

\[
\frac{dy_{S,VN}}{d\kappa} = \frac{A_b \left( bc - \kappa^2 \right)}{\left[ bc - \kappa (a - \kappa) \right]}.
\]

Then \( y_{S,VN} \) is unimodal in \( \kappa \) and reaches its maximum at \( \kappa^*_{V\!N} = \sqrt{bc} \). Note that for all \( \kappa \in (0, \infty) \),

\[
y_{F,VN}(\kappa) = 0 < y_{S,VN}(\kappa) \leq y_{S,VN}(\kappa^*_V) \cdot
\]

Thus \( (S,\kappa^*_V) \) is the unique optimal responsibility allocation according to the CSR performance criterion in the VN structure.

**Proposition 8**: According to the CSR performance criterion, \( (S,\kappa^*_F) \), \( (S,\kappa^*_V) \) and \( (S,\kappa^*_U) \) are the unique optimal responsibility allocations in the US, DS, and VN cases, respectively.

**Corollary 4**: (i) For all \( \kappa \in (0, \infty) \), \( y_{S,US}(\kappa) > y_{F,US}(\kappa) \) and \( y_{S,VN}(\kappa) > y_{F,VN}(\kappa) \). (ii) There is a unique \( \kappa^*_F > \kappa^*_V > \kappa^*_U \) such that \( y_{S,US}(\kappa) \geq y_{F,US}(\kappa) \) for all \( \kappa \in (0, \kappa^*_F) \) and \( y_{S,VN}(\kappa) < y_{F,VN}(\kappa) \) for all \( \kappa \in (\kappa^*_V, \infty) \).

**Proof**: Part (i) is straightforward. For part (ii), when \( \kappa > a/2 \), since \( \lim_{\kappa \to \infty} y_{S,US}(\kappa) = 0 \) and

\[
y_{F,US}(\kappa) = y_{F,US}(a/2) = y_{S,US}(a/2) < y_{S,US}(\kappa^*_F),
\]

then the unimodality of \( y_{S,US} \) at \( \kappa^*_F \) implies that there is a unique \( \kappa^*_F > \kappa^*_V \) such that \( y_{S,US}(\kappa) \geq y_{F,US}(\kappa) \) for all \( \kappa \in (a/2, \kappa^*_F) \) and \( y_{S,US}(\kappa) < y_{F,US}(\kappa) \) for all \( \kappa \in (\kappa^*_V, \infty) \). Note further that \( y_{S,US}(\kappa) = y_{F,US}(\kappa) \) for all \( \kappa \leq a/2 \). Then part (ii) is proved. This completes the proof of Corollary 4.

**Remark**: When the objective is to maximize the channel CSR performance, the current model demonstrates that the optimal social responsibility allocation is to designate S as the responsibility holder and entrust it with the (optimally restricted) right to price CSR performance
in a wholesale price contract under each of the three power structures. Corollary 4 further reveals that, even if $\bar{k}$ is not set at its optimality, a higher CSR performance is always achieved by assigning S as the responsibility holder in the US and VN cases where S is stronger (US) or equally powerful (VN). But for the DS structure where the downstream F is more powerful, to make the weaker player S to be the responsibility holder, an appropriate restriction on the right ($y_k^*$) has to be imposed; otherwise, the more powerful F will arise as a better choice. Therefore, Corollary 4 is by and large compatible with the suggestion of Amaeshi et al. (2008) that the more powerful player should bear social responsibility.

4.3 Conflict between the economic and the CSR performance criteria

Due to the uniqueness of $y_k^*$, $y_k^{**}$, $y_k^{***}$, $e_k^*$, $e_k^{**}$ and $e_k^{***}$, we show the conflict between the social and economic performance criteria by asserting $e_k^* \neq e_k^{**}$, $e_k^{**} \neq e_k^{***}$ and $e_k^{***} \neq e_k^{****}$.

For the US case, as $\bar{k}_s = \sqrt{bc}/2 < bc/a$, substituting $\bar{k}_s = \sqrt{bc}/2$ into the expression of $\frac{d \Pi_{S-US}^*}{dk}$ in (15) yields

$$\left. \frac{d \Pi_{S-US}^*}{dk} \right|_{\bar{k}_s = \sqrt{bc}/2} = -\frac{A_5^2 \sqrt{2bc}}{8b(\sqrt{2bc-a})} < 0.$$ 

Given that $\frac{d \Pi_{S-US}^*}{\bar{k}}$ reaches its maximum at $\bar{k}_s^*$, $e_k^* \neq e_k^{**}$. Furthermore, the unimodality of $\Pi_{S-US}^*$ with respect to $\bar{k}$ implies that $e_k^* > e_k^{**}$.

For the DS and the VN cases, we can similarly ascertain that

$$\left. \frac{d \Pi_{S-DS}^*}{dk} \right|_{\bar{k}_s = \sqrt{bc}/2} = -\frac{A_5^2}{2b(\sqrt{2bc-a})} < 0,$$

and

$$\left. \frac{d \Pi_{S-VN}^{**}}{dk} \right|_{\bar{k}_s = \sqrt{bc}/2} = -\frac{A_5^2 \left[4bc(\sqrt{bc} - a) + a^2 \sqrt{bc} \right]}{2(2bc-a\sqrt{bc})} < 0.$$ 

As $\frac{d \Pi_{S-DS}^{**}}{\bar{k}}$ and $\frac{d \Pi_{S-VN}^{**}}{\bar{k}}$ are both zero, we have $e_k^{**} > e_k^{***}$ and $e_k^{***} > e_k^{****}$.

These results are now summarized in Proposition 9.

**Proposition 9**: Assume that $\bar{k}_s^*$, $\bar{k}_s^{**}$, and $\bar{k}_s^{***}$ are the optimal $\bar{k}$ values corresponding to the three power structures as given in Proposition 7 and $\bar{k}_s^* = \bar{k}_s^{***} = \sqrt{bc}/2$, and $\bar{k}_s^{**}$ are the optimal $\bar{k}$ values corresponding to the three power structures as given in Proposition 8, then $e_k^* > e_k^{**}$, $e_k^{**} > e_k^{***}$, and $e_k^{***} > e_k^{****}$. 

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Remark: Propositions 7 and 8 indicate that the optimal economic and CSR performances could be attained by allocating S as the social responsibility holder with appropriate restrictions on $k$ when each criterion is independently considered as a single objective. Proposition 9 further points out that these two criteria are inherently in conflict with each other and it is impossible to achieve both optimality simultaneously under any of the three power structures. In other words, if the economic performance is to be maximized, the channel CSR performance measured by $y$ will not achieve its maximum, and vice versa. Proposition 9 highlights the tradeoff between the economic and CSR performance criteria. This finding sheds significant insights for supply chain managers (the primary member, in particular) who are under increasing pressure for socially responsible business practices: it might well be the case of finding a right trade-off between social and economic performances. Recent research indicates that supply chain managers have started to address consumer confidence and trust about whether goods and services are provided without compromising ethical and environmental standards (New 2003).

4.4 Comparisons of economic and social responsibility performance

This subsection compares the channel optimal profits, sales quantities, and CSR performance for the decentralized system under the three power structures with those of the integrative case with and without social responsibility considerations. The results are summarized in Proposition 10.

Proposition 10: Let $q_i^*$ and $q_N^*$ be the optimal sales quantities for the integrative case with and without considering social responsibility, and $(q_{S-US}^*(\bar{k}_v^*), q_{S-DS}^*(\bar{k}_v^*), q_{S-VNS}^*(\bar{k}_v^*))$ and $(y_{S-US}^*(\bar{k}_v^*), y_{S-DS}^*(\bar{k}_v^*), y_{S-VNS}^*(\bar{k}_v^*))$ be the optimal quantity and the CSR performance vectors as per the optimal social responsibility allocation schemes for the three power structures as given in Proposition 7. The corresponding profits below are distinguished by their subscripts in a similar fashion. Then

(i) $q_i^* > q_{S-US}^*(\bar{k}_v^*) > q_N^*$, $q_i^* > q_{S-DS}^*(\bar{k}_v^*) > q_N^*$, and $q_i^* > q_{S-VNS}^*(\bar{k}_v^*) > q_N^*$;
(ii) $\Pi_i^* > \Pi_{S-US}^*(\bar{k}_v^*) > \Pi_N^*$, $\Pi_i^* > \Pi_{S-DS}^*(\bar{k}_v^*) > \Pi_N^*$, and $\Pi_i^* > \Pi_{S-VNS}^*(\bar{k}_v^*) > \Pi_N^*$ and
(iii) If $bc < 2a^2$, then $y_{S-US}^*(\bar{k}_v^*) < y_i^*$, $y_{S-DS}^*(\bar{k}_v^*) < y_i^*$ and $y_{S-VNS}^*(\bar{k}_v^*) < y_i^*$.

Proof: For part (i), we only prove that $q_i^* > q_{S-US}^*(\bar{k}_v^*) > q_N^*$ as the other two cases can be shown in a similar fashion. From part (i) of Proposition 1, for $\bar{k} \in (0, bc/a]$, we have

$q_i^* > q_{S-US}^*(\bar{k}) \iff W(\bar{k}) = a^2 bc - a(a^2 + bc)\bar{k} + (a^2 + bc)\bar{k}^2 > 0$.

Since $a^2 < bc$, we have
Thus the equation $w(k) = 0$ does not have any root over $(0,bc/a)$, and the convexity of $w(k)$ and $w(0) = a^2bc > 0$ imply that $w(k) > 0$ for all $k \in (0,bc/a)$. For $k \in (bc/(a^2-2), \infty)$, as $q_i^* = (A,c)/(bc-a^2)$ and $q_{S,E}^* = (A,c)/(2bc-a^2)$ and they share the same numerator with the latter having a larger denominator, it is obvious $q_i^* > q_{S,E}^*(k)$. Then $q_i^* > q_{S,E}^*(k)$ for all $k \in (0, \infty)$. Therefore, $q_i^* > q_{S,E}^*(k)$.

Note that $E_x < a < b/a$ as per the proof of Proposition 7, Proposition 1(i) yields

$$q_{S,E}^*(k) = \frac{A_b(bc-k_a^*(a-k_a^*))}{b(bc-2k_a^*(a-k_a^*))} > \frac{A_b}{b} = q^*_a.$$  

For part (ii), for the same reason, we only prove $\Pi_i^* > \Pi_{S,E}^*(k_x^*) > \Pi_{S,K}^*$. From (6), $\Pi_i^*(q,y)$ is strictly concave in $(q,y)$. Then $\Pi_i^* = \Pi_i^*(q_i^*,y) > \Pi_i^*(q,y)$ for any $(q,y) \neq (q_i^*,y_i^*)$. From part (i), $q_i^* > q_{S,E}^*(k_x^*)$, hence $\Pi_i^* > \Pi_{S,E}^*(k_x^*)$. Furthermore, the optimality of $E_x^*$ implies that $\Pi_{S,E}^*(k_x^*) > \Pi_{S,E}^*(k_E^*) = A^*_i/(2b) = \Pi_{S,E}^*$.

For part (iii), we first prove $y_{S,E}^*(k_x^*) < y_i^*$. Note that

$$y_{S,E}^*(k_x^*) < \max_k y_{S,E}^*(\sqrt{bc / 2}) = y_{S,E}^*(\sqrt{bc / 2}) = \frac{A_b}{2(\sqrt{2bc-a})} = \frac{A_b}{2a(\sqrt{2bc-a})}$$

where the first equality is implied in the deduction of Proposition 8, the second and the third inequalities are due to $bc < 2a^2$ and $bc > a^2$, respectively.

$y_{S,E}^*(k_x^*) < y_i^*$ can be proved in a similar fashion. Now we prove $y_{S,E}^*(k_x^*) < y_i^*$. By $bc < 2a^2$, we have $H(a) = -a(2a^2-bc) < 0$ ($H(\cdot)$ is introduced in Eq. (18)). Since $H(a/2) > 0$, $H(k_x^*) = 0$, and $H(\cdot)$ is a decreasing function as per the earlier discussions, it is ascertained that $k_x^* \in (a/2, a)$.

Further, the deduction of Proposition 8 implies that $y_{S,E}^*(k)$ increases in $k$ in $(0, \sqrt{bc})$. Since $k_x^* < a$, we have

$$y_{S,E}^*(k_x^*) < y_{S,E}^*(a) = \frac{A_b}{bc} < \frac{A_b}{bc-a^2} = y_i^*$$

Proposition 10 is thus proved.

Remark: Proposition 10 clearly demonstrates that, with the presence of CSR, the integrative system-wide optimal profit and sales quantity are not attainable via a decentralized system regardless of how CSR is allocated between the two members (S and F) due to double-marginalization. Nevertheless, it does point out that the channel profit and sales can be improved
by implementing the optimal social responsibility allocation schemes in the decentralized system compared to the integrative case without considering social responsibility. An intuitive interpretation is that the sales are improved because the market demand curve is shifted upwards by socially responsible activities (Propositions 1, 3 and 5 show that equilibrium $y$ is strictly greater than 0, while in the case without CSR, $y$ is always equal to 0), leading to a higher system-wide profit. This enhanced profitability, as discussed at the end of Section 4.1, provides a basis for both parties to improve their individual profitability either automatically or via an appropriate credible transfer payment. Proposition 10 thus helps to explain the recent trend in the business world: more and more companies (often primary firms of global supply chains) commit resources to socially and environmentally responsible activities such as establishing and implementing certain codes of conduct as a means to eventually improving their economic performance\(^\text{10}\). And the prediction of efficiency improvement justifies the empirical findings that CSR is positively related to corporate financial performance (Margolis and Walsh 2001; Orlitzky et al. 2003).

In the proof of Proposition 10 (iii), the assumption of $bc < 2a^2$ is introduced together with $bc > a^2$. The following arguments are furnished to justify these two assumptions: (1) For a given market demand characterized by $a$ and $b$, the impact of the CSR investment on the supplier’s cost should be restricted to a reasonable range (i.e. $a^2 / b < c < 2a^2 / b$); (2) the upper bound assumption of $bc < 2a^2$ ensures that the optimal $\bar{k}'$’s under all three power structures (i.e. $\bar{k}_e^*$, $\bar{k}_e^\ast$, and $\bar{k}_e^\ast\ast$) is less than $a$. As such, by implementing the optimal allocation scheme, the firm’s unit profit margin increases in $y$ (as $(a - k)y$ appears in the profit function (4)), leading to the firm’s interests in the supplier’s CSR investments (otherwise the firm always prefers to $y = 0$ because any increase in $y$ will result in a decrease in its unit profit margin).

5 Discussions

In Section 4, when the optimal allocation decision is considered, it is assumed that the responsibility holder simultaneously determines $k$ along with the other variable. This section examines the case that $k$ is first determined by the responsibility holder and then other decision variables are subsequently decided as per each of the six aforesaid games.

\(^{10}\) For example, Cone/Roper Cause Related Trends Report (1999) shows that nearly 50% of larger corporations have programs associated with social issues.
Corresponding to the six games, S-US, F-US, S-DS, F-DS, S-VN, and F-VN, defined in Section 2, we now modify them by assuming that \( k \) is first determined by the responsibility holder, followed by other decision variables. The modified games are denoted as SS-US, FF-US, SS-DS, FF-DS, SS-VN, and FF-VN games, where SS and FF indicate that the supplier and the firm are, respectively, assigned as the responsibility holder and decide \( k \) prior to other decision variables. Then we change the subscripts of the equilibrium and optimal decision variables in Sections 2-4 in a similar fashion to reflect the corresponding modified scenarios. For example, \( \pi^*_{SS-US} \) and \( \pi^*_{FF-US} \) represent the equilibrium supply chain system profits of the SS-US and FF-US games, respectively.

According to Zabel’s (1970) method, it is easy to check that the equilibrium variables for the SS-US and FF-DS cases are identical to those in the S-US and F-DS cases, respectively. Especially, we have \( \pi^*_{SS-US} = \pi^*_{S-US} \) and \( \pi^*_{FF-DS} = \pi^*_{F-DS} \) for all \( \bar{k} \). Now let us turn to the other four scenarios. First, the equilibrium variables are derived as follows.

(I) The FF-US game:

(i) if \( \bar{k} \leq a/2 \), the equilibrium variables are

\[
\begin{align*}
  k^*_{FF-US} &= \bar{k} ; \\
  y^*_{FF-US} &= \frac{A_y \bar{k}}{bc - 2\bar{k}(a - \bar{k})} ; \\
  q^*_{FF-US} &= \frac{A_y [bc - \bar{k}(a - \bar{k})]}{b[bc - 2\bar{k}(a - \bar{k})]} ; \\
  \pi^*_{FF-US} &= \frac{A_y^2 \bar{k}^2}{2b[bc - 2\bar{k}(a - \bar{k})]} ; \\
  \pi^*_{FF-US}' &= \frac{A_y^2 [bc - \bar{k}(a - \bar{k})]^2}{2b[bc - 2\bar{k}(a - \bar{k})]^2} ; \\
  \pi^*_{FF-US}'' &= \frac{A_y^2 \bar{k}^2}{2b[bc - 2\bar{k}(a - \bar{k})]} \left[ \frac{bc - \bar{k}(a - \bar{k})}{[bc - 2\bar{k}(a - \bar{k})]^2} \right] .
\end{align*}
\]

(ii) if \( \bar{k} > a/2 \), the equilibrium variables are

\[
\begin{align*}
  k^*_{FF-US} &= \frac{a}{2} ; \\
  y^*_{FF-US} &= \frac{A_y a}{2bc - a^2} ; \\
  q^*_{FF-US} &= \frac{A_y (4bc - a^2)}{2b(2bc - a^2)} ; \\
  \pi^*_{FF-US} &= \frac{A_y^2 a^2}{4b(2bc - a^2)} ; \\
  \pi^*_{FF-US}' &= \frac{A_y^2 (4bc - a^2)^2}{8b(2bc - a^2)^2} ; \\
  \pi^*_{FF-US}'' &= \frac{A_y^2 (16b^2c^2 - 4bca^2 - a^4)}{8b(2bc - a^2)^2} .
\end{align*}
\]

(II) The SS-DS game:

(i) if \( \bar{k} \leq \sqrt{bc}/2 \), the equilibrium variables are

\[
\begin{align*}
  k^*_{SS-DS} &= \bar{k} ; \\
  y^*_{SS-DS} &= \frac{A_y \bar{k}}{bc - 2\bar{k}(a - \bar{k})} ; \\
  q^*_{SS-DS} &= \frac{A_y c}{bc - 2\bar{k}(a - \bar{k})} ; \\
  \pi^*_{SS-DS} &= \frac{A_y^2 c^2}{2b[bc - 2\bar{k}(a - \bar{k})]^2} ; \\
  \pi^*_{SS-DS}' &= \frac{A_y^2 c}{2b[bc - 2\bar{k}(a - \bar{k})]} ; \\
  \pi^*_{SS-DS}'' &= \frac{A_y^2 c (bc - 2\bar{k}a + 3\bar{k}^2)}{2b[bc - 2\bar{k}(a - \bar{k})]^2} .
\end{align*}
\]

(ii) if \( \bar{k} > \sqrt{bc}/2 \), the equilibrium variables are
By examining the equilibrium variables for the six modified games, we can establish Proposition 11 as follows.

**Proposition 11**: For the modified games where $k$ is determined by the responsibility holder before the other decision variables $q$ and $y$ are furnished by F and S, respectively, it remains true for the optimal responsibility allocation schemes derived in Propositions 7 and 8 under each of the three power structures as well as the comparative statics established in Propositions 9 and 10.

**Proof**: We first verify Proposition 7. For the FF-US game, we have
Clearly, $\Pi^*_{FF-US}(\bar{k}) = \Pi^*_{US}(\bar{k}) = \Pi^*_{SS-US}(\bar{k})$ for all \( \bar{k} \leq a / 2 \leq a \leq bc / a \). Recall that $F(\bar{k}) = -2ak^4 + 2a^2k^2 - bc(a^2 + bc)k + ab^2c^2$ is introduced in analyzing the US case in Section 4.1, it is easy to verify that $F(a / 2) = abc(bc - a^2) / 2 > 0$. Then $\Pi^*_{US}(\bar{k})$ increases at $\bar{k} = a / 2$. By the definition of $\bar{k}^*$ in Section 4.1, we have $\Pi^*_{US}(\bar{k}^*) > \Pi^*_{US}(a / 2) = \Pi^*_{FF-US}(a / 2) \geq \Pi^*_{US}(\bar{k})$, where the last inequality is derived due to the fact that the supply chain system profit function (20) increases over $(0, a / 2)$ and remains constant for $\bar{k} \in [a / 2, \infty)$. Thus, $(S, \bar{k}^*)$ remains the optimal responsibility allocation for the US case according to the economic performance criterion, even if $k$ is first decided by the firm. In a similar way, we can also confirm that for all $\bar{k}$,

\[
\Pi^*_{S-DSS}(\bar{k}^*) = \Pi^*_{S-DS}(\bar{k}^*) \geq \Pi^*_{S-DS}(\bar{k}),
\]

(21)

(22)

(21) and (22) imply that $(S, \bar{k}^*)$ and $(S, \bar{k}^{***})$ are the optimal responsibility allocations for the DS and VN cases, respectively.

Now we prove that Proposition 8 remains true. We can easily determine that for all $\bar{k}$,

\[
y^*_{US}(\bar{k}^*) > y^*_{US}(a / 2) = y^*_{FF-US}(a / 2) \geq y^*_{US}(\bar{k}),
\]

(23)

\[
y^*_{S-DS}(\bar{k}^{**}) = y^*_{SS-DS}(\bar{k}^{**}) \geq y^*_{SS-DS}(\bar{k}),
\]

(24)

\[
y^*_{S-VN}(\bar{k}^{***}) = y^*_{SS-VN}(\bar{k}^{***}) \geq y^*_{SS-VN}(\bar{k}) \text{ and } y^*_{S-VN}(\bar{k}^{***}) > y^*_{FF-VN}(a / 2) \geq y^*_{FF-VN}(\bar{k}).
\]

(25)

Then (23), (24) and (25) imply that $(S, \bar{k}^*)$, $(S, \bar{k}^{**})$ and $(S, \bar{k}^{***})$ are the optimal responsibility allocations for the US, DS and VN cases, respectively.

Finally, since the assumption that $k$ is first decided by the corresponding responsibility holder does not have any impact on the optimal responsibility allocations, Propositions 9 and 10 follow immediately. Proposition 11 is thus proved.

**Remark:** Two points are worth mentioning here. First, as the responsibility holder’s $k$ decision induces the subsequent US, DS or VN game, it has to take into account the subsequent equilibrium variables due to backward induction. This consideration helps to avoid the extreme case of not sharing the CSR investment at all. Second, Proposition 11 shows that Propositions 7-
are robust to the change of the sequence of determining $k$ as long as $\bar{k}$ is appropriately specified.

To illustrate the shapes of and relationships among the channel profit functions, a numerical example has been developed for the US case as shown in Table 1, and the resulting graph is depicted in Fig. 1 (As a matter of fact, the relative relationships among the curves in Fig. 1 can be theoretically confirmed). Fig. 1 clearly points out the optimal allocation at $\bar{k}_e^*$ (Proposition 11). If $\bar{k}$ is not set at its optimality $\bar{k}_e^*$ and $k$ is offered by the responsibility holder ahead of the other two decision variables, $q$ and $y$, Fig. 1 also furnishes the ranges of $\bar{k}$ values within which the profit is indifferent ($0 < \bar{k} \leq a / 2$), the channel profit is higher if S is the responsibility holder ($a / 2 < \bar{k} < \bar{k}_e^*$), or it is better to entrust F as the responsibility holder ($\bar{k} > \bar{k}_e^*$). Fig. 1 also schematically confirms Proposition 7 and Corollary 1 when $\bar{k}$ is not set at its optimality and $k$ is determined with the responsibility holder’s other decision variable simultaneously. For the DS and VN cases, similar numerical experiments and graphical representations can be obtained and are omitted here for the sake of space.

Table 1. Channel profit for the S-US (SS-US), F-US, FF-US cases

<table>
<thead>
<tr>
<th>$\bar{k}$</th>
<th>$bc &gt; (3 + \sqrt{5})a / 2$</th>
<th>$bc \leq (3 + \sqrt{5})a / 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Pi_{S-US}^* \left( \Pi_{SS-US}^* \right)$</td>
<td>$\Pi_{F-US}$</td>
</tr>
<tr>
<td>0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>0.3</td>
<td>0.309299</td>
<td>0.25</td>
</tr>
<tr>
<td>0.6</td>
<td>0.336389</td>
<td>0.25</td>
</tr>
<tr>
<td>0.9</td>
<td>0.322674</td>
<td>0.25</td>
</tr>
<tr>
<td>1.2</td>
<td>0.297115</td>
<td>0.25</td>
</tr>
<tr>
<td>1.5</td>
<td>0.275543</td>
<td>0.25</td>
</tr>
<tr>
<td>1.8</td>
<td>0.259758</td>
<td>0.25</td>
</tr>
<tr>
<td>2.1</td>
<td>0.248293</td>
<td>0.25</td>
</tr>
<tr>
<td>2.4</td>
<td>0.239755</td>
<td>0.25</td>
</tr>
<tr>
<td>2.7</td>
<td>0.237387</td>
<td>0.25</td>
</tr>
<tr>
<td>3.0</td>
<td>0.237387</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Parameter values are set as follows: $w_0 = e_0 = 1, a_0 = 2, A_0 = a_0 - w_0 = 1$;
For the \( bc > (3 + \sqrt{5})a / 2 \) case, \( a = 0.8, b = 2, c = 1 \); For the \( bc \leq (3 + \sqrt{5})a / 2 \) case, \( a = 1, b = 1.5, c = 1 \).

**Fig. 1 The Comparison of equilibrium profits (the US case)**

6 Concluding Remarks

Social responsibility allocation is considered in a two-echelon supply chain, consisting of a downstream firm F and an upstream supplier S bound by a wholesale price contract. The CSR performance of the supply chain is assumed to be a global variable \( y \) and the related cost is incurred only by S and is expected to be shared with F via the wholesale price contract that is characterized by a parameter \( k \). With the duality of responsibility and rights, the allocation is conceived as a two-dimensional vector. The first dimension assigns a supply chain member as the responsibility holder and entrusts it with the right to price the CSR performance in the contract. The second dimension specifies an upper bound \( \bar{k} \) for the key parameter \( k \) in the wholesale price contract, which effectively places a restriction on the right of the responsibility holder. The power structure of the supply chain is captured as the Stackelberg leader-follower relationship. Different combinations of responsibility holder assignment and power structures lead to six distinct games and their corresponding equilibriums are derived accordingly in Propositions 1 through 6. By analyzing the equilibriums as per the methodology of comparative institutional analysis, the following key results are obtained:

1. Under each of the three power structures, the optimal social responsibility allocation scheme is always to assign the supplier as the responsibility holder with appropriate restrictions on \( k \) based on both the economic and the CSR performance criteria (Propositions 7 and 8). When the economic performance drifts away from its maximum, such restrictions are mandatory...
for the supplier to be the responsible holder under all the three power structures with a minor exception (Corollaries 1-3). Otherwise, if the cap on $k$ is set at a level that exceeds a threshold, $F$ will turn out to be a better responsibility holder for the channel profit. In the model setting in this research, the investment for ensuring the global social performance $y$ incurs by $S$ only. Therefore, the optimal allocation scheme of entrusting $S$ with the right of pricing $y$ tends to support arguments based on the principles of corporate legal personality and separate existence of a corporation. On the other hand, if the maximal CSR performance is not attained, to make the supplier a better responsibility holder, an appropriate restriction on the right is only required for the DS case when the supplier is a relatively weaker player (Corollary 4). From this perspective, this result is compatible with the suggestion of Amaeshi et al. (2008) that more powerful members should be held accountable.

(2) Under all the three power structures, it is impossible to achieve optimal economic and CSR performance simultaneously. Inherent conflict exists between these two criteria when social responsibility allocation decisions are made (Proposition 9). This result highlights the need for finding an appropriate tradeoff between these two criteria for supply chain managers who are faced with increasing social responsibility pressures in practice, as observed by New (2003).

(3) Under all the three power structures, the integrative channel profit is not attainable due to double-marginalization, but the system-wide profit will be improved by implementing optimal social responsibility allocation schemes compared to the case without considering social responsibility at all (Proposition 10). This result helps us understand the recent trend of investing in social responsibility in the business world, and justifies the empirical findings that CSR is positively related to corporate financial performance (Margolis and Walsh 2001; Orlitzky et al. 2003).

Finally, Proposition 11 shows that Propositions 7-10 are robust relative to the sequence change of determining $k$ (i.e., $k$ is first offered by the responsibility holder) as long as $k$ is appropriately specified.

The current model assumes that information on both the cost parameter of social responsibility investment and the parameter of the market impact of CSR performance is symmetric for the supplier and the firm. In reality, the supplier may possess more information on the cost parameter while the firm is likely to understand the market impact better. This information asymmetry raises a new question: How do moral hazard and/or adverse selection
influence social responsibility allocation? Another potential extension of this research is to consider other well-known contract structures such as the buy back contract, the revenue sharing contract, the quantity flexibility contract, to name a few. Still another consideration is to explore the situation that both the supplier and firm incur their individual CSR costs. These open questions warrant further investigations.

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