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Temperature Anomalies in a Simple Overlapping Generations Model

Chizua Mesigo
chizzymesigo@yahoo.com

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CHIZUA MESIGO

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by

CHIZUA MESIGO

Approved By

Y. Zhang

Department of Economics

M. Arbex, Advisor

Department of Economics

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Abstract

The main aim of the paper is to examine the impact of temperature anomaly in an overlapping generations (OLG) model. The rise in temperature captured by the damage function has a direct effect on production. As temperature rises above the pre-industrial level, output and capital accumulation decline, making representative agent worse off as the lifetime utility declines. The result of the analysis predicts that a temperature anomaly of $2.5°C$ requires a consumption equivalent of 1.04 percent of GDP. The model further shows that the more dependent an economy is on capital, the more significant the losses will be as temperature increases.

Keyword: Temperature anomaly, Overlapping generations model, Climate change.

Dedication

To God Almighty

Acknowledgment

I would like to express my deepest gratitude to my professor and supervisor, Dr. Marcelo Arbex, for his patience, guidance, enthusiastic encouragement, and unwavering support in the course of writing this major paper. He taught me more than I could ever give him credit for here; thank you for creating the platform for me to learn and work with Python.

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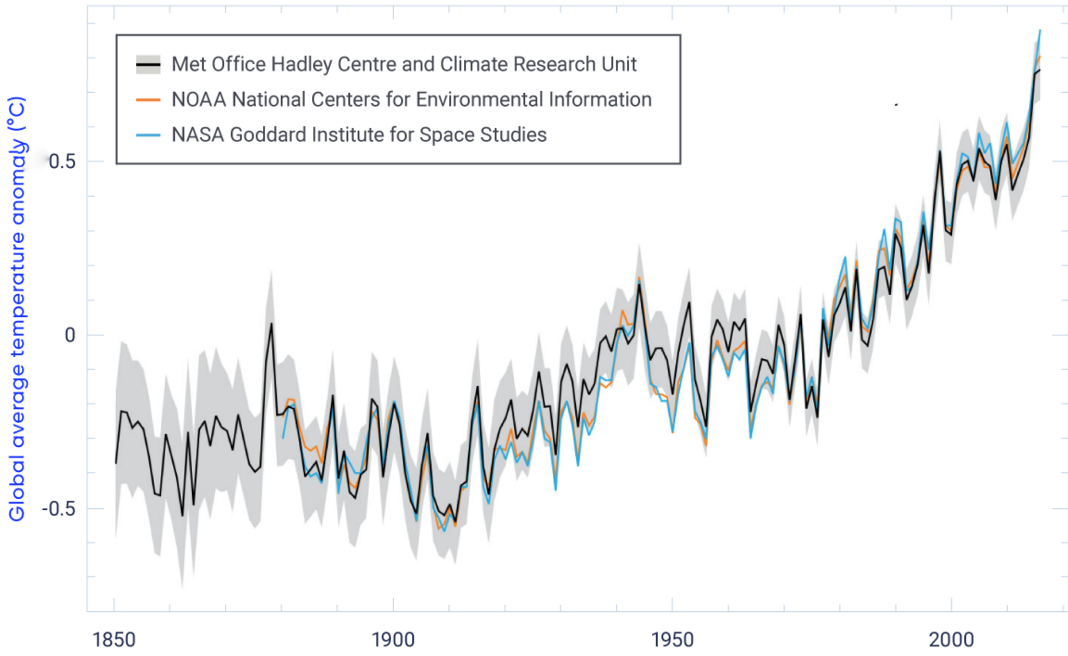
1 Introduction

Climate change is a defining issue of our time. It has emerged as a critical environmental problem. It is already attracting attention at the highest levels, both domestically and internationally. There has been a significant increase in global temperature over the years, with experts projecting more increase in temperature in the future. Human activity (anthropogenic) is considered the leading cause of climate change, and this is majorly from increased emission of greenhouse gases, particularly CO_2. Natural factors external to the climate system, such as changes in volcanic activity, solar output, and the Earth's orbit around the Sun, are also causes of climate change. The Paris Agreement enacted under the United Nations Framework Convention on Climate Change (UNFCCC) aims to strengthen the global response to climate change by seeking to keep global temperature rise this century well below $2°C$ above pre-industrial levels, and also, to pursue efforts to limit the temperature increase even further to $1.4°C$. Furthermore, the Agreement strives to strengthen the ability of countries to deal with the impacts of climate change.

The global mean temperature has risen from $0.85°C$ to $1.06°C$ over the period 1880 to 2012. It is expected to further increase to about $5.8°C$ by the year 2100. Figure 1 below provides a visual representation of temperature anomalies over the years, 1850 - 2016, analyzed independently with three separate sets of data for the reference years.

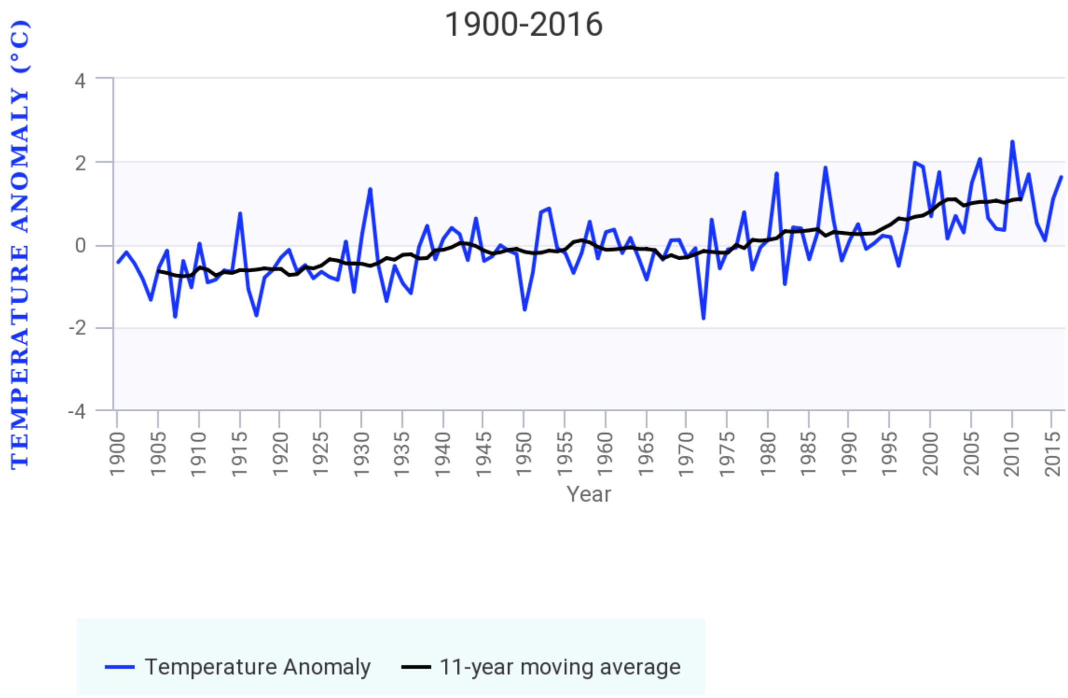
Figure 1: Observed global mean annual surface temperatures anomalies 1850 - 2016



Source: World Meteorological Organization (2017)

The effects of climate change are not uniform but differentiated by regions. Some adverse physical effects of temperature anomalies include flooding, droughts, wildfire, heat-waves. Some positive effects come with it, for instance, improved agricultural practices in regions located in high latitudes. However, the adverse effects outweigh the positive effects of climate change.

Figure 2: Temperature Anomalies Trend in Canada



Source: Vincent et al. (2015)

The rate of temperature warming is not uniform, as it varies across regions. Canada has seen a rising mean temperature over the years, varying across regions. The observed and projected mean temperature increase is about twice the corresponding increases in the global mean temperature. Experts expect that a persistent rise in temperature over the years will be accompanied by extended growing season, fewer heating degree days, and longer cooler degree days (Bush and Lemmen, 2019).

Between 1948 and 2016, the increase in temperature has been from 1.1°C to 2.3°C, with an estimated mean annual temperature of 1.7°C. Northern Canada has experienced more temperature warming than any other region in the country; with a temperature increase ranging from 1.7°C to 3.0°C from 1948 to 2016, and a mean annual temperature of 2.3°C. The average annual temperature in the province of Ontario, a province in Southern Canada, has increased by up to 1.4°C, with scientists predicting that by 2050, the average annual temperature in Ontario will increase by 2.5°C to 3.7°C from the 1961-1990 baseline average.

Adaptation and mitigation are both viable strategies for combating climate change. However, their approach to climate change is different. According to Bosello et al. (2009), mitigation is always needed to avoid irreversibly and potentially unmanageable consequences. In contrast, adaptation is necessary to address unavoidable climate change damages. While measures are being put in place to abate the causes of climate change, for instance, reduction of greenhouse gas emissions, adopting renewable energy sources such as solar and hydro; there is a need to lessen the impact of climate change on the agents to prevent substantive welfare loss.

This paper seeks to study the effect of rising temperatures on the agent's welfare and the economy. An overlapping generation (OLG) model is employed, where the agent works and saves when young, and then survives on his savings when old.

A key finding of this study is the negative impact of rising mean temperature on output and capital accumulation. The reduction in the level of production comes from the direct effect of temperature anomalies through the damage function. The lifetime utility of the representative agent declines as the mean temperature rises. This decline in utility comes from the indirect effect of temperature anomalies on wage rate and savings, causing the agent to consume less during his lifetime. Given the temperature increase of 2.5°C, the agent consumes 5.01 percent less over his lifetime. The welfare loss is 1.65 percent; for this level of welfare loss, a consumption equivalent of 1.04 percent of the GDP is needed to keep the agent at the same utility as before the rise in temperature.

Compare two economies facing the same climate change situation, but differentiated in their dependence on capital. The result shows that for the same range of temperature increase, the impact of rising temperature is exacerbated in the economy with a greater dependence on capital. The more reliant on capital an economy is, the more significant the effects of temperature anomalies on its macroeconomic variables will be. Furthermore, the welfare loss of young and old agents is more substantive.

Studies have shown that there is a negative relationship between climate change and economic output, and this relationship exists irrespective of the level of development obtainable in these nations. Wade and Jennings (2016) highlights that persistent temperature anomalies had the potential to weaken economic growth through its negative impact on capital stock and labor supply. Adding that developing economies are more likely to receive an enormous hit. The study by Fankhauser and Tol (2005) shows that given a constant savings rate, climate change effect on output leads to a lower output level, leading to a proportional decrease in investment (new capital stock). As a result, future production and consumption per capita will further decline. Given an endogenous savings rate, the effect will cause the agent to change their savings rate to accommodate the impact of a future rise in temperature.

Using an overlapping generations model, Moretto and Tamborini (1997) analyzed the effects of climate change on the economy. The result highlights that increasing temperature lowers productivity over time, adding that future generations may experience a permanent loss of utility as their total endowment declines. Arbex and Batu (2020) developed a DSGE model with a direct impact of climate change on the agent's utility and production through the utility and production damage function. The outcome from their analysis shows that the direct effect of the damage function on output causes a decrease in production, which in the long run, translates to lower consumption. For instance, a $2°C$ permanent increase in temperature lowers long-run GDP by as much as 1.4 percent and that the consumption equivalent is around 3 percent of GDP. Their analysis further shows that direct temperature damages to the agent's preferences aggravate the effects of temperature anomalies on the economy and welfare.

Although our model does not allow for adaptation or mitigation, these are important issues considered in the literature and focus of several debates about climate change. There has been a widespread agreement on the need to have an integrated portfolio of policies involving mitigation and adaptation strategies. However, some studies have advocated for either of the two approaches as being more efficient. For instance, Schumacher (2019) advocated for a policy focused on mitigation. The argument rests on the observation that mitigation is a public good and, as such, has a far-reaching impact on the economy and the country at large than the private good, adaptation. The study by Urwin and Jordan (2008) explained that the optimal policy mix depends on the nature of the economies in view. The need for either mitigation and adaptation varies across countries, and as such, what is obtainable in one country might not be the case in another.

Bosello et al. (2010) concludes that it will be welfare improving to have a mix of adaptation and mitigation policies, adding that the burden of adaptation will be higher on developing economies than on developed economies and, hence, a call for these developed economies to lend a hand in easing the burden on developing economies. Fankhauser and Jotzo (2009) highlights the complementary nature of mitigation and adaptation, demonstrating that there no mitigation policy that does not require a substantive level of adaptation. Furthermore, an adaptation strategy can reverse the worst impacts of climate change. In terms of timing, Bosello et al. (2009) concludes that mitigation strategy is immediate, because of its delayed effects, which are driven by environmental inertia. At the same time, adaptation can be postponed until damages from climate change are more significant. The decision on the optimal mix to adopt also depends on the cost associated with it. Shalizi and Lecocq (2009) highlights that mitigation might be the cheapest long-term solution to issues from climate change, and also essential to avoid high-temperature anomalies that may trigger catastrophic consequences. Ingham et al. (2005) demonstrates that mitigation and adaptation, in terms of economic cost, are substitutes, implying that if the cost to abatement falls, the economy's optimal response would be to adopt mitigation strategies.

The remainder of the paper is structured as follows. Section 2 describes the theoretical model. Section 3 presents numerical results. Section 4 concludes.

2 Theoretical Model

I present a simple overlapping generations (OLG) model drawn from Jung (2019) with permission[1], McCandless (2008) and Champ and Freeman (2001), with specific changes, including extension to two economies and the introduction of temperature anomalies.

[1]Material from Jung (2019) was used in the Econ 8040 course, Macroeconomic Theory II

2.1 Preference

The model includes two economies represented as economy A and economy B. The agents in these economies live for two periods: young and old, with a population of young and old agents represented as N_y and N_o, respectively. They consume and save during their productive period (young), and when old, they consume from their savings. These agents value consumption when young (c_y) and when old (c_o), and thus, derive utility from consuming over their lifetime. The functional form of the lifetime utility for these agents is

$$U^i(c^i_y, c^i_o) = ln(c^i_{y,t}) + \beta ln(c^i_{o,t+1}) \qquad i = A, B \tag{1}$$

with β being the subjective discount factor.

2.2 Technology and the Firm

Firms produce final output Y_t, using inputs capital K_t and labor L_t. A damage function is introduced to the production function. This damage function captures the direct effect of a given temperature anomaly T_t, at time t, on production.

A Cobb-Douglas production function is assumed for the final output in both economies, as follows

$$Y^i_t = D^Y(T^i_t)F(K^i_t, L^i_t) = D^Y(T^i_t)A^i K^{i\alpha}_t L^{i1-\alpha}_t \qquad i = A, B \tag{2}$$

The damage function is in line with the Integrated Assessment Model, first presented by Nordhaus (1991). The climate damage function quantifies the risk the economy faces as a result of temperature anomalies.

The function $D^Y(T_t^i)$ captures these economic damages from climate change as a fraction of the final output. The temperature anomaly, represented numerically by T_t^i, is the deviation of the temperature from the average global warming (pre-industrial level). The global pre-industrial level is set at $2°C$. Damages from climate change are multiplicative, and a convex function of temperature (Nordhaus, 2007; Weitzman, 2012) with the level of damages dependent on the degree of convexity (Bretschger and Pattakou, 2019). I employed in the model, Nordhaus's damage function of global temperature, with damages as a quadratic function of the temperature level. The functional form is specified as

$$D^Y(T_t^i) = \frac{1}{1 + \theta_Y (T_t^i)^2} \tag{3}$$

where T_t^i is the deviation of the temperature above the pre-industrial level.

Firms rent physical capital from households and hire workers at prices q_t (the return to physical capital) and w_t (the real wage). The profit maximization function for the firms is represented thus,

$$\underset{K_t^i, L_t^i}{max} D^Y(T_t^i) F(K_t^i, L_t^i) - w_t^i L_t^i - q_t^i K_t^i \tag{4}$$

The maximization problem is solved to yield the following equations

$$w_t^i = D^Y(T_t^i) F_L \tag{5}$$

$$q_t^i = D^Y(T_t^i) F_K \tag{6}$$

$$\tag{7}$$

The additional definitions required are specified below:

$$r_t^i = q_t^i - \delta^i \tag{8}$$

$$R_t^i = 1 + (1 - \tau_K^i)(q_t^i - \delta^i) \tag{9}$$

With δ being the rate of depreciation, r the interest rate and R the after-tax gross interest rate.

These first-order conditions are similar to the standard, except for the inclusion of damages to production due to temperature anomalies captured by the damage function, $D^Y(T_t^i)$. Not only is the economy's level of production impacted by these temperature anomalies, but also, the factor prices and the interest rates.

2.3 Government

In both economies, the government collects taxes on capital τ_K^i and labor τ_L^i. The tax on labor is payable during the productive period of the agent, while the tax on capital is payable in the second period of the agent's life. Both taxes constitute the revenue of the government. The expenditure of the government includes government consumption, G, and transfers T_y^i and T_o^i to the young and old, respectively. The taxes and transfers to households are exogenous in the model. The equilibrium condition for the government is specified as

$$G_t^i + T_{y,t}^i + T_{o,t}^i = \tau_{L,t}^i \times w_t^i L_t^i + \tau_{K,t}^i \times r_t^i K_t^i \tag{10}$$

2.4 Household's Problem

The household's preferences in both economies are identical, and so is their maximization problem. In this regard, the superscript i will be dropped in this section.

$$\max_{c_{y,t},c_{o,t+1}} \quad ln(c_{y,t}) + \beta ln(c_{o,t+1}) \tag{11}$$

subject to

$$c_{y,t} + s_t = (1 - \tau_{L,t})w_t + t_{y,t} \tag{12}$$

$$c_{o,t+1} = R_{t+1}s_t + t_{o,t+1} \tag{13}$$

where $R_{t+1}s_t = 1 + (1 - \tau_{K,t})r_t$ is the after tax gross interest rate.
The functional form for the utility function is given as $u(c_y) = \ln(c_y)$ and $u(c_o) = \ln(c_o)$. The substitution method is employed here: I substituted consumption out of the utilities using the budget constraint, to have one choice variable, savings, s.

$$\max_{s_t} \; ln((1 - \tau_L)w_t + t_{y,t} - s_t) + \beta ln(R_{t+1}s_t + t_{o,t+1}) \tag{14}$$

The first order condition with respect to s_t yields,

$$\frac{1}{(1 - \tau_{L,t})w_t + t_{y,t} - s_t} = \frac{\beta R_{t+1}}{R_{t+1}s_t + t_{o,t+1}} \tag{15}$$

This gives the expression for the optimal savings, s_t^* for the household.

$$s_t^* = \frac{\beta R_{t+1}((1 - \tau_{L,t})w_t + t_{y,t}) - t_{o,t+1}}{(1 + \beta)R_{t+1}} \tag{16}$$

The household's intertemporal choices are affected by the impact of temperature anomalies on the production level of the economy. To see this, recall that from the firm's maximization equilibrium conditions, wage and after-tax gross interest rates are impacted by the damage function. To this end, the household's optimal saving is as well affected by the damage function, translating to the agent's utility being indirectly affected by these temperature anomalies.

In equilibrium, total household savings equal the total capital stock for the economy. Aggregate capital stock for each economy is therefore: $K = S = N_y s^*$

$$K_t = N_{y,t} \times s_t^* = N_{y,t} \frac{\beta R_{t+1}((1-\tau_{L,t})w_t + t_{y,t}) - t_{o,t+1}}{(1+\beta)R_{t+1}} \tag{17}$$

Substituting the expression for the wage and the after-tax gross interest rate, the capital stock in the economy can be expressed as

$$K_t = N_{y,t} \frac{\beta(1+(1-\tau_{K,t})(q_t-\delta))((1-\tau_{L,t})D^Y(T_t)F_L + t_{y,t}) - t_{o,t+1}}{(1+\beta)(1+(1-\tau_{K,t})(q_t-\delta))} \tag{18}$$

To simplify the system of equations, the labor supply in the economy, L, is normalized to 1. This gives the expression for output, factor prices, and interest rate as a function of the capital stock. Given the values for the model parameters,$N_y, N_o, \beta, \delta, A$, capital share, α, and the government parameters; τ_K, τ_L, t_y, t_o, T_y, T_o, a solution will be derived for the capital stock from equation (15) and then back out other variables which are a function of the capital stock.

2.5 Aggregation

For each economy $i = A, B$, the following aggregation holds:

Aggregate consumption:

$$C^i = N_y^i \times c_y^i + N_o^i \times c_o^i \tag{19}$$

Aggregate government consumption:

$$G^i = N_y^i(\tau_L^i \times wL) + N_o^i(\tau_K^i \times rK) - N_y^i \times t_y^i - N_o^i \times t_o^i \tag{20}$$

Aggregate Resource Constraint: This is also the market clearing condition. It is important to ascertain the accuracy of the solution.

$$C^i + N_y^i s^{i*} + G^i = Y^i + (1 - \delta^i)K^i \tag{21}$$

2.6 Aggregate Economy

An aggregate economy is introduced in the model, which consists of the weighted average of the economies A and B key macroeconomic variables. This is done to further examine the effects of temperature anomalies on an aggregate level. The following equations represent the expression of the variables in this economy.

$$K_t = \phi_t K_t^A + (1-\phi_t) K_t^B \tag{22}$$

$$Y_t = \phi_t Y_t^A + (1-\phi_t) Y_t^B \tag{23}$$

$$C_{y,t} = \phi_t c_{y,t}^A + (1-\phi_t) c_{y,t}^B \tag{24}$$

$$C_{o,t} = \phi_t c_{y,t}^A + (1-\phi_t) c_{o,t}^B \tag{25}$$

ϕ_t is defined as the weight on the macroeconomic variables for economy A, on the share of economy A in the aggregate economy.

3 Numerical Results

To illustrate the impact of temperature anomalies on the economy and welfare, numerical exercises are employed. These exercises are analyzed in this section. Going forward, the superscript i is dropped.

3.1 Parameters

The parameters employed in the analysis of the model will be explained and specified in this section. Functional forms of production($D^Y(T_t)F(K_t, L_t)$), damage function ($D^Y(T_t)$) and utility U($c_{y,t}, c_{o,t+1}$) functions have been specified in the previous section.

The share of capital in production set to 0.30 (Golosov et al., 2014). Capital is assumed to depreciate annually at the rate of 10 percent, that is $\delta = 0.10$. Labor (L) is normalized to 1 to make the numerical exercise simpler. For the output damage function, I employed the Nordhaus's damage function of global temperature. In the analysis, the benchmark temperature anomaly is set to zero, indicating a normalization of temperature. The damage function parameter is set to $\theta_Y = 0.0028388$ (Arbex and Batu, 2020).

Table 1 presents a summary of the parameters employed in the analysis.

Table 1: Model Parameters

Parameter	Description	Source	Value
Preferences			
β	Discount factor	1	0.985
ϕ_t	Weight on economy i's macroeconomic variables	5	0.5
N_y^i	Number of young agents in the economy	5	1
N_0^i	Number of old agents in the economy	5	1
Production inputs			
α	Capital share of output	2	0.3
δ	Capital depreciation rate	3	0.10
Government			
τ_L	Tax on labor	5	0.2
τ_K	Tax on capital	5	0.15
t_y	Transfer to the young household	5	0.0
t_o	Transfer to the young household	5	0.0
Damage function			
θ_Y	Damage function coefficient	4	0.0028

Note: The parameter values were sourced as follows: 1: Nordhaus (1991); 2. Golosov et al. (2014); 3. Prescott (1986); 4. Arbex and Batu (2020); 5. Jung (2019).

3.2 One Agent Economy

The focus in this section is a single economy with the number of agents normalized to one. The analysis was done based on two scenarios; an economy with no increase in the mean temperature ($T_t = 0$). The next scenario involves the economy experiences anomalies in temperature ($T_t \neq 0$), with mean temperature increase ranging from 0.5°C to 2.5°C. Table 2 displays the result of the analysis, which demonstrates that rising mean temperature has a negative economic impact.

Table 2: Result for One Agent Economy

Variables	Benchmark	Mean Temperature Increase				
	0°C	0.5°C	1.0°C	1.5°C	2.0°C	2.5°C
Y^*	0.5776	0.5771	0.5755	0.5728	0.5692	0.5645
K^*	0.1605	0.1604	0.1599	0.1592	0.1582	0.1569
q^*	1.0796	1.0796	1.0796	1.0796	1.0796	1.0796
r^*	0.9796	0.9796	0.9796	0.9796	0.9796	0.9796
R^*	1.8326	1.8326	1.8326	1.8326	1.8326	1.8326
w^*	0.4043	0.4040	0.4028	0.4010	0.3984	0.3951
s^*	0.1605	0.1604	0.1599	0.1592	0.1582	0.1569
c_y^*	0.1630	0.1628	0.1624	0.1616	0.1606	0.1593
c_o^*	0.2942	0.2939	0.2931	0.2917	0.2899	0.2875
C^*	0.4571	0.4567	0.4554	0.4533	0.4504	0.4467
G^*	0.1045	0.1044	0.1041	0.1036	0.1029	0.1021
U^*	-3.0195	-3.0213	-3.0269	-3.0360	-3.0488	-3.0652

We can see a steady decline in the macroeconomic variables as temperature increases over the range of 0.5°C to 2.5°C. Table 3 displays how these variables respond to temperature anomalies as a percentage of the benchmark, which is the normalization of temperature ($T_t = 0$).

Table 3: Responses of Economy's Variables to Anomalies in Temperature

	Benchmark	Mean Temperature Increase				
Losses(% of Benchmark)	0°C	0.5°C	1.0°C	1.5°C	2.0°C	2.5°C
Output	0.00	0.09	0.40	0.83	1.45	2.32
Capital stock	0.00	0.06	0.37	0.81	1.43	2.29
wage	0.00	0.07	0.37	0.82	1.46	2.28
Consumption young	0.00	0.12	0.43	0.86	1.47	2.28
Consumption old	0.00	0.10	0.41	0.85	1.46	2.27
Utility	0.00	0.07	0.26	0.55	0.97	1.50
Consumption equivalent(% of GDP)	0.00	0.05	0.17	0.38	0.67	1.04

A rise in the mean temperature from 0.5°C to 2.5°C lowers output from 0.09 to 2.32 percent. This decrease in output comes from the direct effect of temperature anomalies through the damage function. As total output in the economy decreases, so will capital accumulation and factor prices decline, though not at the same rate. For the same range of temperature increase, the capital stock decreases from 0.06 to 2.29 percent.

The agent's income declines as the mean temperature rise in this economy. The reduced income comes from the decrease in the real wage paid for the agent's productive service. With a temperature increase range of 0.5°C to 2.5°C, the wage rate decreases from 0.07 to 2.47 percent. The reduction in income impact the intertemporal allocation of resources as the agent seeks to maximize his lifetime utility. The agent values consumption in both periods of life, and desires to keep consuming irrespective of the reduced resources. The total savings of the agent when young constitute the total capital stock in the economy, and therefore, both decrease at the same rate as temperature rises. As consumption level in both periods decreases, it lowers the agent's lifetime utility. The decline in utility comes through the effect of temperature anomalies on wage rate and savings.

As temperature increases to $2.5°C$, the income of the young agent decreases by 2.28 percent, which in turn lowers consumption when young and savings in the first period by 2.28 and 2.29 percent, respectively. The consumption level of the agent, when old, also declines by 2.27 percent. With this, the lifetime utility decreases by 1.50 percent. To compensate for the loss in welfare, and still keep the agent at the same utility level as before the temperature increase, a consumption equivalent welfare of 1.04 percent of GDP is needed. The consumption equivalent welfare attempts to keep the agent at the same utility level as before the temperature anomaly, hence, to eliminate welfare loss. To put this in perspective, consider the economy of Canada with a 2019 GDP of \$1.7 trillion, the model predicts that for a temperature anomaly of $2.5°C$, the consumption equivalent welfare in Canada would be \$18 billion or would be $\$46,000$ per person, per year.

3.3 Two Agents Economies

For this exercise, assumptions are made regarding the climate change situation of the two economies. Normalizing the mean temperature in economy A to zero, $(T_t^A = 0)$ and a mean temperature increase range of $0.5°C$ to $2.5°C$ in economy B.

The outcome for economy A is the same for the one agent economy with no temperature change, $(0°C)$. Furthermore, for economy B, the result from the analysis is the same as the one agent economy with a mean temperature range of 0.5°C to 2.5°C. These results are presented in Table 2 from the previous subsection. As mean temperature rises above pre-industrial level, output and capital accumulation decline, though not at the same rate. Also, lowering the marginal product of inputs, with the marginal product of physical capital decreasing by a minimal amount, unlike the marginal product of labor. The agent's intertemporal choices are affected by the impact of temperature anomalies on production, resulting in a reduction in the consumption level of the agents, which ultimately lowers their lifetime utility.

Comparing the economy that is suffering from temperature anomaly with an economy that is not affected, we can see that its macroeconomic variables are lower, and the welfare loss is more significant. Consumption equivalent welfare, therefore, is needed to keep the lifetime utility of these agents at the same level as before the temperature increase.

Even though all parameters can be changed to study this impact of temperature anomalies, however, I am going to concentrate on the following examples. I considered a scenario in which economy B is more reliant on capital with a capital share of 0.5 ($\alpha = 0.5$), still keeping the capital share of economy A constant at 0.3 ($\alpha = 0.3$).

Table 4: Responses of Economy's B Variables to Anomalies in Temperature

	Benchmark	Mean Temperature Increase				
Losses(% of Benchmark)	0°C	0.5°C	1.0°C	1.5°C	2.0°C	2.5°C
Output	0.00	0.15	0.50	0.83	1.16	3.17
Capital stock	0.00	0.25	0.51	1.27	2.03	3.30
wage	0.00	0.10	0.50	1.11	2.02	3.13
Consumption young	0.00	0.25	0.50	1.25	2.00	3.25
Consumption old	0.00	0.08	0.50	1.16	2.08	3.16
Utility	0.00	0.05	0.19	0.44	0.77	1.21
Consumption equivalent(% of GDP)	0.00	0.03	0.08	0.19	0.33	0.51

The result in Table 4 ($\alpha = 0.5$), when compared to table 3 ($\alpha = 0.3$), shows that the more an economy depends on capital ($\alpha = 0.5$), the more sensitive the macroeconomic variables will be to temperature anomalies. We can see this in the increasing significance of the losses recorded as temperature increases. For instance, a 2.5°C rise in temperature lowers output by 3.17 percent in the more capital dependent economy as compared to a decline of 2.32 percent in output in an economy with a lower dependence on capital. For the same temperature anomaly, the loss in consumption when young and old in economy B ($\alpha = 0.5$) is 0.97 and 0.89 percent higher than what is obtainable in economy A ($\alpha = 0.3$).

Additional exercise, for instance, a capital share of 0.7 ($\alpha = 0.7$), shows that as temperature increases, it exacerbates the decline in output. As the capital share of the economy increases in magnitude, the more significant losses will be experienced in the face of rising temperatures. In other words, as the degree of the capital intensity of an economy increases, the greater the negative impact of temperature anomalies will be.

A key reason for this including the aggregate economy is to highlight the effect of climate change at the aggregate level. If both economies are experiencing the same temperature change, there is not much difference in their macroeconomic variables. Therefore, the aggregate economy's variables are unaffected, irrespective of the weight assigned. However, in the situation where one economy's mean temperature is relatively constant and the other economy's mean temperature is increasing over a range, the value of the aggregate economy will then depend on the weight, ϕ_t, assigned. It also will depend on the capital share (α).

The following exercise illustrates how the aggregate economy's macroeconomic variables are largely dependent on the weight (ϕ_t) assigned. In the next exercise, I normalize the temperature of Economy A, and allowing the mean temperature for economy B to fluctuate over a range of $0.5°C$ to $2.5°C$.

Table 5: Response of the Aggregate Variables to climate change, $\phi_t = 0.5$

	Economy A	Economy B				
Variables	Benchmark	Mean Temperature Increase				
Losses(% of Benchmark)	$0°C$	$0.5°C$	$1.0°C$	$1.5°C$	$2.0°C$	$2.5°C$
Y_t^*	0.00	0.03	0.19	0.42	0.73	1.23
K_t^*	0.00	0.03	0.19	0.37	0.69	1.25
$C_{y,t}^*$	0.00	0.06	0.18	0.43	0.74	1.13
$C_{o,t}^*$	0.00	0.05	0.18	0.42	0.73	1.14

Assuming an equal share of the economies macroeconomic variables ($\phi_t = 0.5$), the result from Table 5 shows that a persistent increase in economy B's temperature aggravates the loss of the aggregate macroeconomic variables. This loss is due to the decline in the economy's B variables as a result of temperature anomalies.

The next sets of exercises further illustrates the crucial role played by the weight (ϕ_t) assigned, presented in Tables 6 and 7. Adjusting the climate change situation in economy A to reflect a temperature anomaly of $1°C$ while keeping the temperature range the same in economy B.

Table 6: Response of the Aggregate Variables to climate change $\phi_t = 0.3$

	Economy A	Economy B				
Variables	Benchmark	Mean Temperature Increase				
Losses(% of Benchmark)	$0°C$	$0.5°C$	$1.0°C$	$1.5°C$	$2.0°C$	$2.5°C$
Y_t^*	0.00	-0.78	-0.58	0.00	0.19	0.80
K_t^*	0.00	-0.53	-0.31	0.00	0.44	1.01
$C_{y,t}^*$	0.00	-0.52	-0.35	0.00	0.43	1.00
$C_{o,t}^*$	0.00	-0.53	-0.34	0.00	0.43	1.01

Even though the macroeconomic variables of economy A are relatively stable, the fluctuations in the macroeconomic variables of the aggregate economy is largely due to the the decline in economy B's variables, irrespective of the weight assigned.

Comparing an assigned weight of 0.3 (Table 6) to a weight of 0.8 (Table 7), the gains and losses of the aggregate variables are more significant with a smaller weight ($\phi_t = 0.3$) as temperature increases.

Table 7: Response of the Aggregate Variables to climate change $\phi_t = 0.8$

	Economy A	Economy B				
Variables	Benchmark	Mean Temperature Increase				
Losses(% of Benchmark)	1.5°C	0.5°C	1.0°C	1.5°C	2.0°C	2.5°C
Y_t^*	0.00	-0.15	-0.09	0.00	0.13	0.29
K_t^*	0.00	-0.15	-0.09	0.00	0.13	0.29
$C_{y,t}^*$	0.00	-0.15	-0.10	0.00	0.12	0.28
$C_{o,t}^*$	0.00	-0.15	-0.10	0.00	0.12	0.29

To conclude this section, it has been seen that agents are made worse off as temperature rises above pre-industrial level. This has been demonstrated in the above exercises. A government policy that cares about the welfare of these individuals would seek to eliminate any substantive loss in welfare. The consumption equivalent welfare measures how much the government should give the agent so that he has the same utility as if no temperature change, hence no substantive loss. This welfare, has been shown to increase as the mean temperature increases. To put it in perspective, consider the economy of Canada with a 2019 GDP of $1.7 trillion, the model predicts that for a temperature anomaly of 2.5°C, the consumption equivalent welfare in Canada would be $18 billion. The consumption equivalent welfare could be in the form of lump-sum transfers to agents.

4 Conclusions

This paper examined the impact of temperature anomalies on welfare and the economy, using a simple overlapping generation model. The introduction of a damage function to the production function captured the direct effect of temperature anomalies on the output level in the economy.

The outcome from the analysis shows that rising temperature above pre-industrial level impacts negatively on the welfare of the agent and the economy as a whole. As mean temperature increases, output level, and capital accumulation decline, which lowers the marginal product of inputs. Also, persistent rise in temperature lowers the lifetime utility of the agent, leading to a substantive loss in welfare. Furthermore, the more reliant an economy is on capital, the greater the impact of temperature anomalies on the economy.

Even though I did not address mitigation and adaptation strategies, however, I have a simple framework that can address these strategies. Future research can incorporate these strategies into the model in studying the impact of temperature anomalies in the economy.

Bibliography

ARBEX, M. AND M. BATU (2020): "Weather, Climate and the Economy: Welfare Implications of Temperature Shocks," *Resource and Energy Economics*.

BOSELLO, F., C. CARRARO, AND E. DE CIAN (2009): "An analysis of adaptation as a response to climate change," *University Ca'Foscari of Venice, Dept. of Economics Research Paper Series*.

——— (2010): "Climate policy and the optimal balance between mitigation, adaptation and unavoided damage," *Climate Change Economics*, 1, 71–92.

BRETSCHGER, L. AND A. PATTAKOU (2019): "As bad as it gets: how climate damage functions affect growth and the social cost of carbon," *Environmental and resource economics*, 72, 5–26.

BUSH, E. AND D. S. LEMMEN (2019): *Canada's changing climate report*, Government of Canada= Gouvernement du Canada.

CHAMP, B. AND S. FREEMAN (2001): *Modeling Monetary Economies*, Cambridge University Press, 2 ed.

FANKHAUSER, S. AND F. JOTZO (2009): *Perspective Paper on Adaptation as a Response to Climate Change*, Copenhagen Consensus Center.

FANKHAUSER, S. AND R. S. TOL (2005): "On climate change and economic growth," *Resource and Energy Economics*, 27, 1–17.

GOLOSOV, M., J. HASSLER, P. KRUSELL, AND A. TSYVINSKI (2014): "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82, 41–88.

Ingham, A., J. Ma, and A. M. Ulph (2005): "Can adaptation and mitigation be complements," *Tyndall Centre for Climate Change Research, Report*, 79.

Jung, J. (2019): "Computational Economics," Tech. rep., Towson University.

McCandless, G. (2008): "The ABCs of RBCs," *Cambridge, Massachusetts, London: Harvard*.

Moretto, M. and R. Tamborini (1997): "Climate change and event uncertainty in a dynamic model with overlapping generations," Tech. rep., Nota di Lavoro.

Nordhaus, W. (2007): "Accompanying Notes and Documentation on Development of DICE-2007 Model: Notes on DICE-2007. delta. v8 as of September 21, 2007," *Miscellaneous publication, Yale University, New Haven, NE, USA*.

Nordhaus, W. D. (1991): "To slow or not to slow: the economics of the greenhouse effect," *The economic journal*, 101, 920–937.

Prescott, E. C. (1986): "Theory ahead of business-cycle measurement," in *Carnegie-Rochester conference series on public policy*, Elsevier, vol. 25, 11–44.

Schumacher, I. (2019): "Climate Policy Must Favour Mitigation Over Adaptation," *Environmental and Resource Economics*, 74, 1519–1531.

Shalizi, Z. and F. Lecocq (2009): *Climate change and the economics of targeted mitigation in sectors with long-lived capital stock*, The World Bank.

Urwin, K. and A. Jordan (2008): "Does public policy support or undermine climate change adaptation? Exploring policy interplay across different scales of governance," *Global environmental change*, 18, 180–191.

Vincent, L., X. Zhang, R. Brown, Y. Feng, E. Mekis, E. Milewska, H. Wan, and X. Wang (2015): "Observed trends in Canada's climate and influence of low-frequency variability modes," *Journal of Climate*, 28, 4545–4560.

Wade, K. and M. Jennings (2016): "The impact of climate change on the global economy," *Schroders Talking Point*.

Weitzman, M. L. (2012): "GHG targets as insurance against catastrophic climate damages," *Journal of Public Economic Theory*, 14, 221–244.

Windsor Environmental Master Plan, . (2012): *City of Windsor, Climate change adaptation plan*, Windsor Environmental Master Plan.

World Meteorological Organization, W. (2017): *WMO statement on the state of the global climate in 2016*, World Meteorological Organization (WMO).

Appendix

Table 8: Result for Two Agent Economy

	Economy A	Economy B				
Variables	Benchmark	Mean Temperature Increase				
	$0°C$	$0.5°C$	$1.0°C$	$1.5°C$	$2.0°C$	$2.5°C$
K^*	0.1605	0.1604	0.1599	0.1592	0.1582	0.1569
Y^*	0.5776	0.5771	0.5755	0.5728	0.5692	0.5645
q^*	1.0796	1.0796	1.0796	1.0796	1.0796	1.0796
r^*	0.9796	0.9796	0.9796	0.9796	0.9796	0.9796
R^*	1.8326	1.8326	1.8326	1.8326	1.8326	1.8326
w^*	0.4043	0.4040	0.4028	0.4010	0.3984	0.3951
s^*	0.1605	0.1604	0.1599	0.1592	0.1582	0.1569
c_y^*	0.1630	0.1628	0.1624	0.1616	0.1606	0.1593
c_o^*	0.2942	0.2939	0.2931	0.2917	0.2899	0.2875
C^*	0.4571	0.4567	0.4554	0.4533	0.4504	0.4467
G^*	0.1045	0.1044	0.1041	0.1036	0.1029	0.1021
U^*	-3.0195	-3.0213	-3.0269	-3.0360	-3.0488	-3.0652

```
#-*- coding: utf-8 -*-
"""
Created on Fri Apr 24 17:28:54 2020

@author: Chizua
"""
#This code is based on material prepared by Dr. Juergen Jung (Towson U
#with permission, the introduction of temperature to the code was done


import numpy as np
import matplotlib.pyplot as plt
import math as m
from scipy import stats as st
from scipy.optimize import fsolve
import time
```

```
plt.close('all')

N_y         = 2.0
N_o         = 2.0
N_yA        = 1.0
N_oA        = 1.0
N_yB        = 1.0
N_oB        = 1.0
alpha       = 0.3
phi       = 0.5
A           = 1
beta        = 0.9850
delta       = 0.1
tau_L       = 0.2
tau_K       = 0.15
t_y         = 0.0
```

```python
t_o         = 0.0
L           = 1

#the damage function
def D(T):
    return 1/(1 + theta_y*(T**2))


for TB in np.arange(0,2.5,0.5):
    TA = 0
    theta_y = 0.0028388


def func(KA):
    s_A  = -KA + N_yA\
    *((beta*(1+(1-tau_K)*(alpha*D(TA)*A*KA**(alpha-1) - delta))* \
    ((1-tau_L)*((1-alpha)*D(TA)*A*KA**alpha) + t_y) - t_o) \
    /((1+beta)*(1. + (1-tau_K)*(alpha*D(TA)*A*KA**(alpha-1) - delta)))
```

```
    return s_A


def func1(KB, TB):
    s_B = -KB + N_yB\
    *((beta*(1+(1-tau_K)*(alpha*D(TB)*A*KB**(alpha-1) - delta))* \
    ((1-tau_L)*((1-alpha)*D(TB)*A*KB**alpha) + t_y) - t_o) \
    /((1+beta)*(1. + (1-tau_K)*(alpha*D(TB)*A*KB**(alpha-1) - delta))))

    return s_B


def f(K):
    s = phi*func(KA) + (1-phi)*func1(KB, TB)

    return s
```

Plot the function to see whether it has a root-point

```
KAmin = 0.0001

KAmax = 0.3


KAv = np.linspace(KAmin, KAmax, 200)  #gridpoints between

                                        #Kmin and Kmax


fKA_v = np.zeros(len(KAv),float) #output vectors prefilled with zeros

for i,KA in enumerate(KAv):

    fKA_v[i] = func(KA)


fig, ax = plt.subplots()

ax.plot(KAv, fKA_v)

ax.plot(KAv, np.zeros(len(KAv)), 'r') #plot horizontal line at

                                        #zero with red

ax.set_title('$K^A$')
```

```
KBmin = 0.0001

KBmax = 0.3


KBv = np.linspace(KBmin, KBmax, 200)   #gridpoints between

                                        #Kmin and Kmax


fKB_v = np.zeros(len(KBv),float) #output vectors prefilled with zeros


fig, ax = plt.subplots()

for TB in np.arange(0,4,1.5):

    print('TB = ', TB)

    for i,KB in enumerate(KBv):

        fKB_v[i] = func1(KBv[i],TB)


    ax.plot(KBv, fKB_v)
```

```
ax.set_title('$K^B$')

plt.legend(['TB = 0.0','TB = 1.5','TB= 3.0'], loc = 'best')


plt.show()


print(" ")

print("------------------------ Fsolve ------------------------")

solutionK =  fsolve(f, 2,)

solutionKA = fsolve(func, 2)


TBv = np.arange(0,1,0.5)

for TB in TBv:


   solutionKB = fsolve(func1, 2, (TB) )

   print(TB, solutionKB)
```

to define the expressions for the variables of interest

```
Kstar  = (phi*solutionKA[0])+((1-phi)*solutionKB[0])

KAstar = solutionKA[0]

KBstar = solutionKB[0]


YAstar = D(TA)*A*KAstar**alpha*L**(1-alpha)

qAstar = alpha*D(TA)*A*KAstar**(alpha-1)

rAstar = qAstar - delta

RAstar = 1. + (1-tau_K)*(qAstar - delta)

wAstar = (1.-alpha)*D(TA)*A*KAstar**alpha


YBstar = D(TB)*A*KBstar**alpha*L**(1-alpha)

qBstar = alpha*D(TB)*A*KBstar**(alpha-1)

rBstar = qBstar - delta

RBstar = 1. + (1-tau_K)*(qBstar - delta)
```

```
wBstar = (1.-alpha)*D(TB)*A*KBstar**alpha



Ystar = (phi*YAstar) + ((1-phi)*YBstar)


sstar = Kstar/N_y

sAstar = KAstar/N_yA

sBstar = KBstar/N_yB


cyAstar = (1.-tau_L)*wAstar + t_y - sAstar

coAstar = RAstar*sAstar + t_o


cyBstar = (1.-tau_L)*wBstar + t_y - sBstar

coBstar = RBstar*sBstar + t_o


GAstar = N_yA*tau_L*wAstar + N_oA*tau_K*rAstar*sAstar\
```

```
/ + N_yA*t_y + N_yA*t_o

CAstar = N_yA*cyAstar + N_oA*coAstar

ARC_A = YAstar - delta*KAstar - CAstar - GAstar


GBstar = N_yB*tau_L*wBstar + N_oB*tau_K*rBstar*sBstar\

/ +  N_yB*t_y + N_yB*t_o

CBstar = N_yB*cyBstar + N_oB*coBstar

ARC_B = YBstar - delta*KBstar - CBstar - GBstar


Cstar = phi*CAstar + (1-phi)*CBstar

Gstar = phi*GAstar + (1-phi)*GBstar

ARC = Ystar - delta*Kstar - Cstar - Gstar




print("--------------------------------------")

print(" Root finding ")
```

```
print("----------------------------------------")
print("KA* = {:6.4f}".format(KAstar))
print("KB* = {:6.4f}".format(KBstar))
print("K*  = {:6.4f}".format(Kstar))
print("----------------------------------------")
print("YA* = {:6.4f}".format(YAstar))
print("YB* = {:6.4f}".format(YBstar))
print("Y*  = {:6.4f}".format(Ystar))
print("----------------------------------------")
print("qA* = {:6.4f}".format(qAstar))
print("qB* = {:6.4f}".format(qBstar))
print("----------------------------------------")
print("rA* = {:6.4f}".format(rAstar))
print("rB* = {:6.4f}".format(rBstar))
print("----------------------------------------")
print("RA* = {:6.4f}".format(RAstar))
```

```
print("RB* = {:6.4f}".format(RBstar))
print("-----------------------------------------")
print("wA* = {:6.4f}".format(wAstar))
print("wB* = {:6.4f}".format(wBstar))
print("-----------------------------------------")
print("sA* = {:6.4f}".format(sAstar))
print("sB* = {:6.4f}".format(sBstar))
print("s*  = {:6.4f}".format(sstar))
print("-----------------------------------------")
print("cyA* = {:6.4f}".format(cyAstar))
print("cyB* = {:6.4f}".format(cyBstar))
print("-----------------------------------------")
print("coA* = {:6.4f}".format(coAstar))
print("coB* = {:6.4f}".format(coBstar))
print("-----------------------------------------")
print("CA* = {:6.4f}".format(CAstar))
```

```
print("CB* = {:6.4f}".format(CBstar))
print("C*  = {:6.4f}".format(Cstar))
print("----------------------------------------")
print("GA* = {:6.4f}".format(GAstar))
print("GB* = {:6.4f}".format(GBstar))
print("G*  = {:6.4f}".format(Gstar))
print("----------------------------------------")
print("ARC_A = {:6.4f}".format(ARC_A))
print("ARC_B = {:6.4f}".format(ARC_B))
print("ARC   = {:6.4f}".format(ARC))
```

Vita Auctoris

Name:	Chizua Mesigo
Place of Birth:	Nsukka, Nigeria
Year of Birth:	1991
Education:	University of Nigeria Secondary School, Nsukka 2002 - 2008
	University of Nigeria, Nsukka 2008 - 2012
	University of Windsor, Windor, Ontario 2018 - 2020