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Impact of Quality Grading and Uncertainty on Recovery Behaviour in a Remanufacturing Environment

Mohannad Radhi
University of Windsor

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By

Mohannad Radhi

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Mohannad Radhi

APPROVED BY:

____
Dr. Ben Chaouch
Department of Management Science

____
Dr. Zbigniew Pasek
Department of Industrial and Manufacturing System Engineering

____
Dr. Guoqing Zhang, Advisor
Department of Industrial and Manufacturing System Engineering

____
Dr. Waguih ElMaraghy, Chair of Defense
Department of Industrial and Manufacturing System Engineering

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ABSTRACT

This research considers a remanufacturing enterprise that constitutes a stage in a closed loop supply chain. Each returned item is precisely tested and assigned a quality grade between zero and a hundred. Consequently, acceptance to the facility, acquisition price and remanufacturing cost are all quality dependants. The research implements the newsvendor modeling techniques to model the system when a single remanufacturing facility satisfies a single market's demand or when multiple remanufacturing facilities satisfy multiple markets' demand. Thus, non-linear programming or mixed integer non-linear programming models are proposed to maximize the total profit by selecting facilities to operate, optimal minimum quality to accept into each operating facility and market's demand to satisfy from each operating facility. Returns' quality is considered to be stochastic, while markets' demand could be either stochastic or deterministic. The impact of changing returns, quality and demand uncertainties, and transportation cost on remanufacturing systems are studied.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

1.1 Evolution of Closed Loop Supply Chain (CLSC)

Products take-back is one of the activities that many enterprises are obliged or encouraged to do in today's industrial environment. There are many motives that encourage the process of take-back such as: profitability, ethical responsibility, environmental legislation and social benefits (Seitz, 2007). Profitability is the major pillar among all and the degree of profitability is also very important in this context. Also, the ethical responsibility is one of the major motives and it is revived by notions such as 'desirable carrying capacity', 'corporate social responsibility', 'corporate citizenship' and 'environmental issues and ethics' (Seitz, 2007).

Since certain products are to be returned or taken back to certain collection points, a type of management has emerged. This management is called reverse logistics, because it manages the flow of products from the end customers to the collection points. Indeed, compared to the well known direction of product flow from producers to customers, this flow of product is in reverse. If the traditional forward product flow is integrated, coordinated or harmonised with the reverse product flow, the enterprise will create what is called a Closed Loop Supply Chain (CLSC). According to (Akcali & Cetinkaya, 2011), "the purpose of the Forward Supply Chain (FSC) is to provide value to the end consumer in terms of products, whereas the purpose of the Reverse Supply Chain (RSC) is to recover economic and environmental value from used products in a cost effective manner. As the Closed Loop Supply Chain is characterised by recovered material, component, and product flows between the FSC and RSC, the CLSC's objective, in turn, is to supply the recovered value to the end consumer in a cost effective manner".

As a consequence of this take-back, product recovery processes were born in the form of reuse, recycling, cannibalization, repair, and remanufacturing. The following descriptions briefly explain each process:

1. Reuse is the process of directly reusing a product without carrying any repair activity apart from simple cleaning and minor maintenance. Examples of such an industry include reused pallets, bottles and containers.
2. Recycling is the process of recovering material in such a way that is destructive to the shape or structure of the products.
3. Cannibalization is the process of reusing in good condition parts and components that are recovered from used equipments to perform customer services such as maintenance work.
4. Repair or reconditioning is accomplished by restoring failed products to working condition, but not to as good as new condition. With such a process customers expect deterioration in the quality compared to new products.
5. Remanufacturing or refurbishing is the process of transforming or restoring the condition of the used product to its original condition or as good as new without performing structure destructive processes. It includes processes such as: collection, disassembly, inspection, testing, grading, cleaning, identification of parts, parts recovery and product re-assembly.

Indeed, some of today's markets do not only receive their products from manufacturing facilities around the globe, but also from remanufacturing facilities that are run by financially alert and morally responsible management. Remanufacturing has increased drastically in the last two decades due to the increase in producer as well as consumers awareness regarding remanufacturing outcomes. Remanufacturing is an approach used by many companies from different industries such as Dell, Hewlett-Packard (HP), IBM, Kodak, Xerox, General Motors (GM), Jasper Engines and Transmissions and Goodyear. An example of the different remanufactured products include: photocopiers, cellular telephones, single-use cameras, car's engines and transmissions and retreaded tires. Intuitively, the reader might ask whether the remanufactured parts/products will have the exact selling value as the new product or not? In another word, does remanufacturing occur under perfect substitution environment or not? The answer to this question depends on whether the customer will be able to distinguish between the new product and the remanufactured product or not. If not, in terms of performance, shape, or even mentally, then both products manufactured and remanufactured will have the same monetary value and thus they are perfectly substitutable. Mental distinction between products can occur due to customer behaviour or customer misconception towards remanufactured products. Moreover, the following table, according to (Akcali & Cetinkaya, 2011), will categorise remanufacturable products and associate each category with its market properties:

Product Categories	Example	Demand Volume	Usage Period	Product Life Cycle (PLC)	Product Structure and Remanufacturing Operation	Perfect Substitution
Refillable Containers	Single use camera and Printer cartridges	High	Short	Long	Both Simple	Perfectly substituted
Durable products	Ships and automotive parts	Low	Long	Long	Both Complex	Can be either ways
Technology products	Cellular phones	High	Short	Short	Structure is complex Operation is simple	No Perfect Substitution

Table 1: Remanufacturable Products Categorization

The system considered in this research is related to the durable products, e.g. tires in the Canadian markets, or any system that is similar in nature.

1.2 Model Motivation

1.2.1 Quality Uncertainty Recognition

This research is inspired by a very deep relationship between quality and many remanufacturing attributes (e.g. returns' acquisition price, remanufacturing cost, remanufacturing lead time and pre-remanufacturing holding cost). Such a relationship could be found in many durable products that have a long usage period and low demand volume. For the purpose of comprehension, let us assume the presence of a facility that remanufactures a certain type of returns. The returns will be delivered to the remanufacturing facility after being used by the customers. Those returns will be thoroughly inspected by the remanufacturer and graded. The grading of returns could be performed using the weighted average method. For example, each return that comes to the facility will be evaluated against its model, age, condition, number of repairs needed and the associated material cost, skills and equipment needed to perform the remanufacturing process and/or amount of work needed to remanufacture the return. Once the return is graded, it will be given a quality score ranging from 0 to 100. While zero is the lowest quality and is considered to be near scrap, one hundred is the best quality and will need the lowest remanufacturing or repair efforts. As a result, quality is not considered to be deterministic, but rather uncertain just like other uncertainties that are greatly addressed in many production planning and inventory control literatures. This uncertainty is represented by an appropriate quality distribution. Indeed, literatures concerning remanufacturing systems and closed loop supply chains have not adequately addressed quality uncertainty as this study will. This fact is stated by (Akcali & Cetinkaya, 2011) when they say "although the uncertainties associated with the timing and quantity of demand and return have been taken into account by developing stochastic Inventory and Production Planning I&PP models, the quality uncertainty associated with returns is rarely taken into account in an explicit fashion". Generally speaking, uncertainties in remanufacturing include: timing of used products return, location where each used product is returned, fraction of products that are returned, i.e. quantity of returns, and quality of returns.

1.2.2 Costs – Quality Relationships

Since the quality of returns vary, then the price associated with a particular return as well as its remanufacturing cost will depend on the return's quality. Indeed, in terms of quality, the return's acquisition price could be described by an increasing linear function, while remanufacturing cost could be described by a decreasing linear function. Indeed, relating acquisition price to return's quality was not considered in previous literatures. Also, the summation of those two costs will introduce a decreasing linear function in terms of quality. As a result, better quality returns are always more appreciated as they are associated with lower total spending and thus more profit. The above relations and a visualization of the quality distribution are presented in Fingure-1 below. The shaded area under the quality distribution curve represents the portion of returns that are accepted to the remanufacturing facility. Returns that fall out of the shaded area, are considered to be low quality returns and the facility is better off not to remanufacture them.

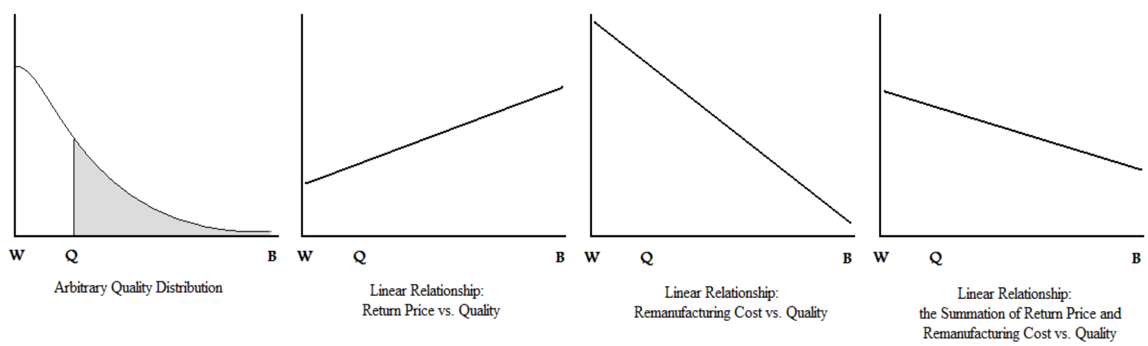


Figure 1: Quality Distribution and the Linear Relationships Between Return Price, Remanufacturing Cost and Quality

According to (Akcali & Cetinkaya, 2011), "the processing costs and revenues can be quality, volume, and/or lead-time dependent. For example, the cost associated with remanufacturing an automotive engine that is driven in a cold climate might be higher as the engine is subject to more wear and tear (due to extreme temperature changes). Similarly, the remanufacturing cost for a two-year-old transmission may be much lower than the remanufacturing cost for a five-year-old transmission as more components have to be replaced for the latter. For instance, as we noted earlier, the recoverable value residing in remanufacturable technology products deteriorates over time".

From such a statement we can notice that the cost-quality relationships have not been addressed properly in the previous literatures.

1.2.3 Perfect Substitution

This paper discusses a remanufacturing system in which perfect substitution does not exist. Therefore, remanufactured products have less monetary values and can not substitute the newly manufactured products as discussed above. Literatures concerning closed loop supply chain and remanufacturing systems, in the last two decades, have heavily considered the case of perfect substitution and not the case without perfect substitution, although the second one is much more applicable in today's market if durable products are considered. "In the automotive replacement parts industry, however, we observe more complex interactions. Specifically, the remanufactured and manufactured parts are perfectly substitutable if the vehicle is under warranty, as the OEM decides whether to use a manufactured or a remanufactured part for replacement. If the product is beyond warranty, however, then the customer's willingness to substitute a remanufactured part for a manufactured one (or vice versa) depends critically on the age of the vehicle and the price difference between the remanufactured and manufactured parts" (Akcali & Cetinkaya, 2011).

Indeed, we have intended to make this work beneficial to as many remanufacturing systems as possible. Therefore, perfect substitution is not allowed in this paper. This work will, also, be applicable beyond the warranty period which is three years in most cases. Thus, excluding the warranty periods, automotive parts fall in the category where perfect substitution is not applicable most of their useful life.

1.2.4 Modeling for a Single Item

According to (Akcali & Cetinkaya, 2011), durable products have to be modeled in a multi-items setting. Indeed, this is true if the company does not take back products in a selective way. For example, if the enterprise chooses to accept all grades of returns for a certain model, then good and bad returns will be accepted to the remanufacturing facility. Most probably, the returns in good condition will be remanufactured and any broken parts will be either taken from the new stocks or taken from the faulty returns if such parts are in sufficient condition. Therefore, the bad returns are used only for cannibalization purposes. On the other hand, if the enterprise is selective in terms of quality, then only the returns with better quality than the optimal minimum quality will be accepted. In another word, the enterprise takes back only the higher quality portion of the return. As a result, all returns will be remanufactured to meet demand and no cannibalization is permitted in normal circumstances. Indeed, the enterprise might remanufacture more than one type of returns. In this case, multi item model could be used.

1.2.5 Networking in Remanufacturing Systems

The case where the enterprise runs multiple remanufacturing facilities to satisfy multiple markets' demand when quality is uncertain was not considered in previous literatures although it is very common in today's market. Each facility might experience different parameters such as return and average quality. At the same time, each market might behave differently by having different average demand as an example. Such a system should be thoroughly tested and this work will have the precedence in doing so.

1.3 Industry Selection

As could be noticed from the previous section, quality grading should be carried over prior to purchasing or dismantling returns. Finding such quality grades in many industries is impossible due to the lack of knowhow, absence of technological enablers or complexity of the return's structure. Thus, choosing a proper industry is a vital step before we could implement our study. One of the industries that has a great potential to benefit from this study is the tires remanufacturing or retreading industry.

Retreading is the process of replacing the tire's tread for a scrapped tire and produce as good as new retreaded tire. Indeed, there are many other options to properly scrap tires such as; tires export, tire-derived fuel (TDF) applications, civil engineering applications, agriculture applications and ground rubber production. A general supply chain chart for the tire industry is represented in the figure below.

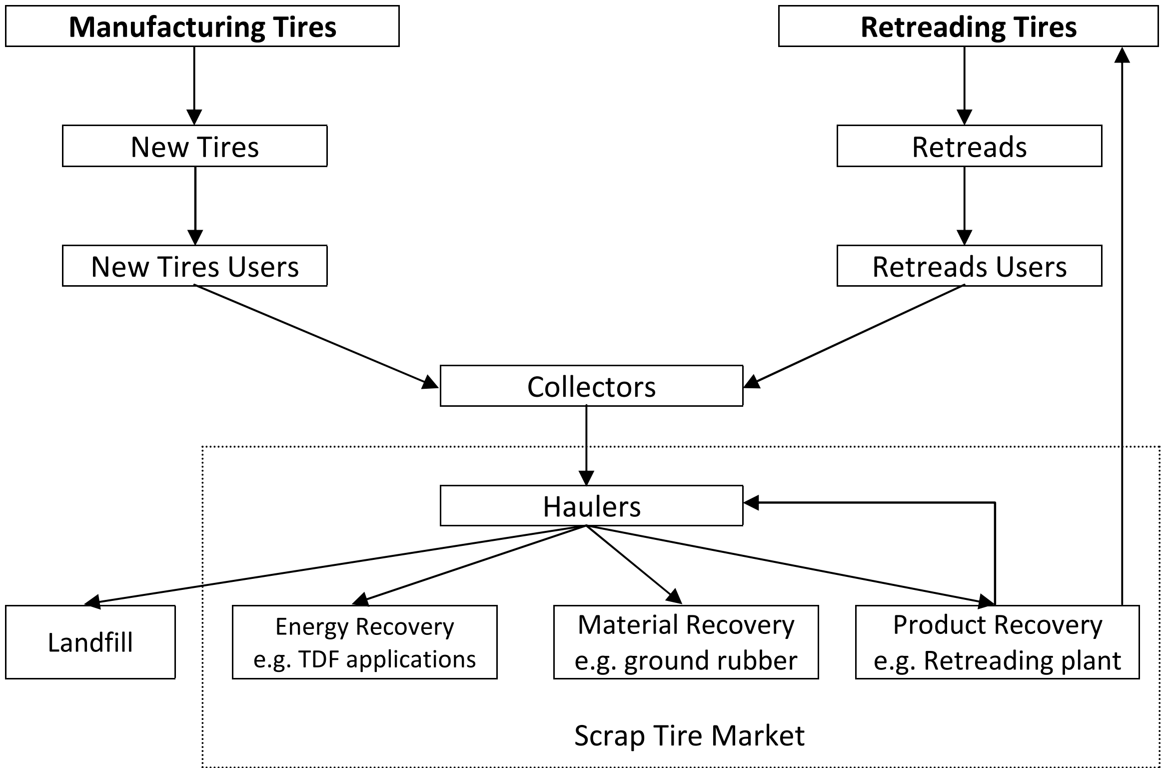


Figure 2: Generic Overview of the Tire Supply Chain

The process of tires retreading or remanufacturing could be the business of two main parties and they are: original equipment manufacturer (OEM) or independent remanufacturer. There are about 1200 plants that perform tire retreading in North America ranging from small plants that produce 20 retreads per day to large plants that produce 100 retreads per day. According to (Boustani, Sahni, Gutowski, & Graves, 2010), "the tire retreading industry is reportedly the largest sector of remanufacturing industry in the United States in terms of the number of remanufacturing (retreading) plants".

In North America, 80 to 100 percent of the aircraft and heavy truck tires get retreaded, while about 30 percent of the light and off the road (OTR) truck tires get retreaded. Due to the incorrect perception about retreads safety, only 2 percent of the passenger car tires are retreaded in North America. All North American retreads generate only 3 billion dollars in revenue. If compared to the 2.23 billion dollars generated only by Goodyear North America in its sales for new tires in the third quarter of the fiscal year 2011, retreading should be better managed. Thus, the results reported in this thesis could be of use to the retreading facilities as well as many other industries that have similar settings.

Since, the retreading process removes only 10 to 20 percent of the tires total construction weight and preserves the core, it is considered to be the ultimate option for managing scrap tires. Retreaded tires could be as much as 70 percent cheaper than newly manufactured tires. Moreover, tires could be retreaded three to five times depending on the remanufacturability of the tires and the condition or quality of the returned tires.

There are many key players in the retreading industry. Some of those players and their associated tasks are briefly discussed below:

1. Collectors or tire retailers collect scrap or used tires after being replaced with new or retreaded tires by customers. An example of such retailers in Canada includes Costco, Canadian Tires and many other tire garages.
2. Haulers are businesses that make a profit by aggregating, sorting and delivering collected tires to the appropriate processors. The sorting is performed to separate retreadable from non-retreadable tires in an effort to sell those retreadable tires to a remanufacturing facility. Indeed, retreadable tires could bring up to one third

of their original price in revenue for the Haulers. Consequently, every retreadable tire is potentially associated with high revenue. This aggregation and sorting processes are assumed to be performed periodically.

3. Processors are the many parties that process scrap or used tires to produce useful and environmentally responsible products. A retreading plant is among those processors that were briefly mentioned above.

Every retreaded tire should go through each of the following processes in order to be legally remanufactured in Canada:

1. First Stage Inspection: this inspection is conducted visually by the haulers to sort tires into retreadable and non-retreadable tires.
2. Second Inspection Stage: the inspections conducted by the retreading plant in this stage are more rigorous inspection methods as they are performed using high technology equipments. For example, Shearography is a machine that possesses two digital laser cameras and able to detect any air trapped or any separation in the tire casing. If any is found, the tire will be rejected from the retreading plant and directed toward the waste stream. Also, the nail hole detection equipment is used in this stage to perform a non-destructive test. This test induces electricity into the tire's casing to detect all nail holes or flaws in the tire's casing. There is no limit to the number of flaws that could be repaired during the retreading process as long as they do not overlap. In order to apply this study, each tire should be quality graded based on number, location and type of flaws exist in the casing as well as type of repair needed. Indeed, a nail-hole costs less than a section repair,

because it requires cheaper repair material and less repair time. Thus, the existence of one section repair will drop the quality grade much more than one nail-hole.

3. Buffing Stage: is the stage where the tires' tread is peeled off using sharp blades.
4. Patch and Repair Stage: is where all tires get repaired and patched from any damages in the tire casing. Compared to all other stages, this stage is labour intensive, because every tire is repaired based on its needs and no two tires are identical. As a result, automation can not be implemented in this stage. Also, the more repairs are performed, the more expenses the retreading plant will experience. Consequently, the plant should identify the minimum quality grade that is acceptable for retreading.
5. Tread Application Stage: tread can be applied using one of two processes that exist in the market. The first option is the cold retreading process where the tread is already pre-moulded and cured before the tread application. The process is completed by adhering the pre-moulded tread to the buffed tire. The second option is the hot retreading process where rubber is applied on the buffed tire and then they all placed in a mould. The mould including the tire is then heated until the tread is eventually formed.
6. Final Inspection and Shipping: in this stage tires will be re-inspected using the same NDT test as before and prepared for shipping.

As a result of what have been addressed in this section, the retreading plants could boost their financial performance by adapting this study or similar studies that encourage quality grading for the returns. Also, tires can be easily quality graded before dismantling or before starting the remanufacturing process. Thus, this industry is a great candidate that can benefit from our study.

1.4 Research Objective

My research objective is to propose newsvendor like models to study the system's performance and return management when quality is uncertain and the enterprise runs either a single remanufacturing facility to satisfy a single market's demand or multiple remanufacturing facilities to satisfy multiple markets' demand. In the single facility and single market case, the research will propose non-linear programming models and differential-type formulas to maximize total profit by choosing the optimal minimum quality that should be accepted into the remanufacturing facility. In the case of multiple facilities and multiple markets case, the research will develop mixed integer non-linear programming models to maximize the total profit by selecting facilities to operate, optimal minimum quality to accept into each operating facility and market's demand to satisfy from each operating facility.

CHAPTER 2: LITERATURE REVIEW

During our literature review, we have noticed that most of the papers concerning CLSC and remanufacturing systems did not consider quality uncertainty. On the other hand, few papers have considered quality uncertainty under CLSC and remanufacturing systems. Among those that considered quality uncertainty, very few papers assigned quality distribution to quality. The rest have considered the yield of the returned lot. All of those issues and more will be detailed in two sections below.

2.1 Literature on Remanufacturing with Quality Uncertainty

This work has enormously benefited from the review paper of (Akcali & Cetinkaya, 2011) as it offers a general overview of the existing models in the remanufacturing industry. This work has reviewed many papers that deal with modeling I&PP systems in remanufacturing environment and, at the same time, evaluated many papers that reviewed the same topic. These authors' state that "none of these reviews provides a comprehensive and comparative examination of the up-to-date literature that allows for a critical assessment of the existing I&PP models". Indeed, the paper was able to discuss many issues related to I&PP and their operational as well as modeling effect, classify the existing literature into different categories based on several system parameters, relate the system parameters to the used solution methodologies and modeling techniques, and suggest several major research paths. In terms of demand and return, the paper was able to recognise that the existing mathematical models have been following one of two paths, either deterministic or stochastic. The models under the deterministic class consider time either as continuous or discrete. On the other hand, the models under the stochastic class were further divided into either continues or periodic review. Each one of these subcategories is even divided further into several minor subcategories with several papers as an example of each. The minor subcategories were classified based on the following:

1. Modelling parameters or decision variables.
2. System structure and characteristic.
3. Time setting and planning horizon.
4. Type of product and number of items.
5. Allowance for perfect substitution.
6. Modeling assumption (e.g. backordering, disposal, lead time, demand and return distribution).
7. Objective function whether to maximize profit or minimize cost.
8. Solution methodologies and modeling techniques.
9. Dependence between demand and return.

One of the most important points that the paper has made, is that the purpose of almost all the reviewed papers was to optimise system performance under an I&PP policy by controlling its parameters. Only handful papers dealt with finding optimal policies. Also, one of the major recommendations in this paper is the need to address quality uncertainty and its effects on closed loop supply chains and remanufacturing systems.

One of the closest work to this research is the paper written by (Ferguson, Guide Jr., Koca, & Souza, 2009). The paper claims that by considering return sorting and grading, the enterprise will be able to increase the profit by 4 percent. Thus, return has been classified into three quality grades: scrap for materials, harvest for parts, or fit-for-remanufacturing. Furthermore, the last grade was divided even further to include: good, better and best grades. Using a general representation, each return is given a grade between 0 and 1, where 0 is total scrap and 1 is best quality possible. The paper assumed that return could be represented by a probability distribution in terms of their quality. Moreover, the paper was able to recognise a decreasing function for the remanufacturing cost vs. quality relationship, but the acquisition price vs. quality relationship was not addressed in this paper. Also, the pre-remanufacturing holding cost vs. quality relationship is represented by a decreasing function. Such a relationship is not considered in our model, because all accepted return will be directly remanufactured to increase the responsiveness of the system and to cope with market's conditions. Also, if the return is given a probability distribution, it is not expected that all products under one category will cost the same during the remanufacturing process. This is due to the fact that, under one category there are still many differences between returns in terms of quality. This is why our model is much more related to the quality due to the fact that it cares more about the quality of the return rather than the quantity in each quality category. The model optimises the system performance when both demand and return are considered to be deterministic. Demand certainty comes from the fact that modeling is applied to a system where lease trading prevails. Thus, managers can almost precisely forecast returns. For both cases, the objective of the firm is "to maximize its total expected profit across a finite discrete-time horizon, by deciding optimal returns to remanufacture and how many to salvage, and in turn how many returns to hold for future periods".

Many papers have addressed quality uncertainty in terms of yield of remanufacturable parts. In another word, the percentage of parts, out of one lot purchased, that can be remanufactured to supply demand. Such a direction was followed by (Bakal & Akcali, 2006) when their paper discussed the automotive remanufacturing industry in the U.S. The paper has taken a bigger picture of the automotive remanufacturing industry since they included in their study not only remanufacturers, but also dismantlers and brokers as well as the possible integration between them. Ford was given as an example to illustrate the unsuccessful business model in the remanufacturing industry. According to the paper, such a lack of success was due to the wrong decisions in terms of selling and acquisition prices. Therefore, the objective of the paper's model was to maximise the firm's profit by choosing the optimal selling and acquisition prices. In reality, a firm can set the selling price of the remanufactured parts that belong to an old generation of cars (e.g. 10 – 15 years old), because there is no more production from OEM. This assumption is not applicable for the newer generation of cars because the OEM still produces and thus the market will control the selling price. Intuitively, return quantity and quality are price dependants, while only demand quantity is price dependant.

Quality in terms of yield is argued to be uncertain and have a random behaviour. Thus, the higher the acquisition price, the more, in terms of quantity, and the better, in terms of quality or yield, the returns will be. Indeed, we can notice here that the system will carry on products that are not remanufacturable and salvage them to recycling third parties later on. Thus, the business model is overwhelming the performance of the system by allowing unwanted returns to be accepted into the system. In contrast, our business model forces the firm to qualify the product before accepting it to the remanufacturing process. Also, we can notice here a vital difference between the two papers. A higher price in our model provides better return quality that requires lower remanufacturing cost. In contrast, this paper assign a fixed remanufacturing cost to each return no matter what quality it has. Moreover, return and demand quantities are assumed to be deterministic. The modeling of the system was performed with deterministic as well as random yields. For the random case we have two scenarios. First, the yield will be realized first and then the selling price could be set. Second, the selling price should be determined before realizing the yield. A final note regarding this paper is that the model deals with durable products and thus perfect substitution is not considered.

(Zikopoulosa & Tagarasa, 2008) have presented a study in order to encourage the remanufacturing firms to design or develop a sorting or quality evaluation mechanism for the returns. This mechanism could be a small electronic or any other device that is not necessarily very accurate, but will give an idea about the remanufacturability of the return. Thus, the paper compares between the profitability of a system that has no sorting before disassembly and another with sorting before disassembly. Returns in the studied system could be either remanufacturable or non-remanufacturable. In addition, quality uncertainty was presented as the yield or the proportion of return that is remanufacturable. In contrast to disassembling returns before sorting, sorting using such a mechanism is not very accurate and thus there are two types of errors: the error of accepting non-remanufacturable returns and the error of rejecting remanufacturable returns. Moreover, the model has applied the newsvendor concepts as there would be under and over stocking costs for any shortage or excess in the return and remanufactured inventories. Also, demand was presented as uncertain variable and the model was set to suit a single period setting. Finally, the paper optimises the system's performance to increase the total profit by optimising the remanufacturing as well as procurement quantities.

Another study that refers to the quality uncertainty as the yield is the paper presented by (Mukhopadhyaya & Ma, 2009). This study is similar to the studies explained above in term of system characteristics. For example, the model is designed to suit random demand, perfect substitution, single product, and single period. Yield is studied in two different settings: deterministic and stochastic. Also, optimality occurs by finding the optimal returns to take back, items to acquire or to order, and items to produce. The only major novel contribution in this paper is its consideration for delivery lead time in the case of random yield. Delivery lead time is considered in the context of whether the system will be able to place the order for new material to produce new items after obtaining the yield or not. If such a lead time is long, then acquisition process will occur before the yield is revealed. In contrast, if the lead time is short, then acquisition process will occur after the yield is revealed, thus system's performance will increase.

Most of the available literatures classify returns into remanufacturable or non-remanufacturable returns when quality uncertainty is considered. One of the few studies that classified return into more than two classes is the study presented by (Teunter & Flapper, 2011). Therefore, they assumed that returns can be divided into k different quality types and each type has a certain probability. Thus, quality is presented by a multinomial distribution. Also, their model is configured to suit single period setting, certain and uncertain demand, newsboy-type setting, sorting is performed after acquisition, and acquisition price is either fixed or quantity dependant. One of the common points between this paper and our work is the fact that remanufacturing cost is quality dependant. Unfortunately, due to the sequence of events in their system, quality dependency did not consider acquisition price. Optimality was found by choosing the proper acquisition quantity that minimises the total expected cost.

Most of the previous studies are tailored to suit the automotive industry, but the study (Robotis, Bhattacharya, & Van Wassenhove, 2005) is tailored to suit a special case in the cell phone industry or some other cases related to products with short PLC. The study presents a reseller who has two suppliers with two different quality trends high and low as well as two groups of customers with two different quality trends high and low. Cell phones are acquired in bulk and have certain quality distributions related to both suppliers' trends. Acquisition cost, in this setting, is supplier dependant. Also, demand from the secondary market has certain distributions and minimum acceptable qualities for both market trends. As long as the quality of the cell phone is above the market's acceptable quality, it is a potential sell and has a fixed price that does not depend on the quality of the cell phone. If the quality of the cell phone is below the acceptable quality, it is either gets disposed of or remanufactured. Thus, the paper objective's is to compare between the performances of the two system orientations, with and without remanufacturing activities. Optimality is sought in the optimal acquired quantity, in the optimal remanufactured quantity for the later system, and in the quality level that a product should have in order to be remanufactured in the later system. We have noticed that this study is different from our study since it does not require all products to be remanufactured to as good as new condition. Also, acquisition price is not quality dependant as the case of our study. Also, this study allows for downward substitution rather than perfect substitution. Downward substitution is the process of supplying lower quality demand with higher quality items to capture the market and to avoid inventory accumulation.

Simulation has also been used in various studies to verify the importance of quality classification. Such a study is presented by (Behret & Korugan, 2009). They have tested a hybrid system with a perfect substitution option. Both demand and return are stochastic and follow Poisson distributions. In their study, there are many quality related parameters that are either different or similar to the quality related parameters presented in our study. Such parameters include, yield or recovery rate, remanufacturing processing cost, remanufacturing effort, operational disposal cost, and remanufacturing overflow disposal cost. Simulation was run for a benchmark system with no quality classification and for an alternative system with quality classification. In the alternative system, returns are classified into three qualities: good, average and bad. As a result of the study, quality classification is vital in the following cases:

1. When the quality difference between returned products is high.
2. When the return rate is high and the demand rate is low.
3. When the proportions of different qualities in the returned products are close to each other.
4. When the difference between the remanufacturing costs of different qualities is high.
5. When the difference between the acquisition costs of different qualities is high.
6. When the difference between the operation disposal costs of different qualities is high.
7. When the difference between the overflow disposal costs of different qualities is high.
8. When the difference between the recovery rates of different qualities is high.

The problem settings defined in the study (ARAS, BOYACI, & VERTER, 2004) is very similar to that defined in the paper presented by (Behret & Korugan, 2009). They have presented very similar cases in which quality classification is a better approach. We noticed few differences between the two studies such as number of quality classifications for returns. Also, remanufacturing lead time is considered to be a quality dependant parameter in the later study. Moreover, this paper is one of the few papers ,encountered in our literature review, that calculated holding cost based on quality level using operating cost of remanufacturing and opportunity cost of capital. Finally, this study used the Continuous Markov Chain optimization technique to solve for the minimum long-run average system cost.

2.2 Literature on Remanufacturing without Quality Uncertainty

Many papers have addressed uncertainty in demand and return without addressing the uncertainty in quality. An example of such papers is the work presented by (Shi, Zhang, Sha, & Amin, Coordinating production and recycling decisions with stochastic demand and return, 2010). The study presents a perfect substitution environment with manufacturing and recycling options. The study has the precedence in presenting the return quantity as a non-linear function of the acquisition price. The total expected profit has been maximized by optimizing recycling quantity, manufacturing quantity, serviceable inventory stocking level as well as acquisition price. Lagrangian relaxation method, subgradient algorithm and heuristic algorithm all have been used to solve the problem and the results were compared to the results obtained from the GAMS solver. Also, this work has the precedence in considering manufacturing and recycling capacities. Moreover, various relationships have been studied through sensitivity analysis including: profit vs. manufacturing and recycling capacities, stocking level vs. demand uncertainty, production and recycling quantities vs. return uncertainty, and acquisition price vs. return uncertainty. A very similar extension of the previous work is <u>Optimal Production Planning for a Multi-Product Closed Loop System with Uncertain</u>

Demand and Return presented by (Shi, Zhang, & Sha, 2011). One of the few differences between the two papers is that this work is modeled by considering a linear relationship between return quantity and acquisition price. Another difference between the two works is that the return horizon is considered from the beginning of the planning period to the end of remanufacturing period in the later study. Another extension of the earlier work is Optimal Production and Pricing Policy for a Closed Loop System presented by (Shi, Zhang, & Sha, 2011) with similar problem settings. The study optimizes quantities of return and demand by setting the optimal acquisition and selling prices. In contrast to the earlier study, this paper assumes that quantities have linear relationships with their corresponding prices. Similar to the earlier study, many relationships have been studied in this paper such as the effect of uncertainties in demand and return on production and pricing policies.

As per the study (Hsueh, 2010), it has the precedence in presenting dependency between demand and return which takes into account the different phases of the product life cycle. The study assumes that both demand and return follow normal distributions with changing means depending on the specific phase of the PLC. Closed form formulas have been derived to calculate the optimal production quantity, reordering point and safety stock in each phase of the PLC. As per the study, production quantities of new products should continuously increase with time as the mean of demand increases with time in the growth phase. In the later phases (e.g. maturity and decline), production quantities of new products should continuously decrease with time as the return of used products gradually increase in the system until the production facility is saturated with used items that will lead to some disposal activities in the final stages of decline phase.

Deterministic models are those models that assume every variable in the system to be known with certainty. An example of such an approach is the paper written by (Koh, Hwang, Sohn, & Ko, 2002) which analyzes the reusable items industries. This paper models a system where used items perfectly substitute newly purchased items. Intuitively, the system consists of previously known demand to be supplied from serviceable inventory, which has been fed by processed returns or newly purchased items. The returns accumulate in a warehouse to a certain level before they get processed. Also, the model analyzes the system in two different settings. First, one recovery process and one or more orders are allowed (1, n). Second, one order and two or more recovery processes are allowed (n, 1). This modeling classification has been considered as a weakness by the authors themselves, as the optimality might occur with multiple recovery processes and orders. For each setting, two different cases are modeled by the paper depending on the relative demand (d), production (P) and return (r) rates. In the first case, $p>d>r$, while the second case implies that $d \geq p>r$. The objective of the paper was to find EOQ by finding the optimal number of items to be ordered in each order and to find EPQ by finding the optimal quantity of items to start production with. The objective function was to minimise total cost when all fixed and variable cost are considered (e.g. holding, ordering and setup costs). To find the optimal solution three steps are needed. First, we find the optimal inventory level to start production with. Second, we find the optimal number of orders or production processes (n), depending on the setting, by using the research procedure. Third, we find number of items in each order or the EOQ. An extension of this literature is the paper introduced by (Konstantaras, Skouri, & Jaber, 2010) which generally followed the same stream of work. The advancement compared to this work was in the general structure of the business model. The later had the return tested first and then either remanufactured to as good as new or refurbished to meet secondary market.

An extension of the study (Koh, Hwang, Sohn, & Ko, 2002) is the paper prepared by (Chiu, Li, & Wang, 2009) which considered constant ordering lead time and variable normally distributed demand. The paper has two main objectives. The first objective is to find closed form formulas for the economic order quantity (EOQ) and for the economic return quantity (ERQ). The relevant quantities include the optimal number of orders, the optimal quantity in each order and the optimal level of return to start remanufacturing with. The second objective of the paper is to test the significance of ordering lead time L and mean demand quantity μ in the total cost of the system. The importance of considering L and μ was tested using the derived formulas as well as Kruskal-Wallis Test. One of this paper weaknesses is that it examines the case of $p>d>r$ considered in (Koh, Hwang, Sohn, & Ko, 2002), but did not examine the other feasible cases.

CHAPTER 3: MATHEMATICAL MODELS

As depicted in Figure-2, returns received by the remanufacturing facility could be one of two types: either used tires that have not been retreaded before or used tires that have been retreaded before. The figure, also, shows that new tires customers only feed the CLSC without being part of the loop. Once the used tires are received by the remanufacturing facility, each tire will be inspected and then assigned a quality grade. If the return has a higher quality grade than the optimal minimum quality, then it will be advanced to the remanufacturing process without being stored. Otherwise, it will be rejected from the remanufacturing process and assigned back to the hauler. Redirecting rejected tires to another waste stream or to another processor is the responsibility of the hauler. This is due to the fact that the Canadian Government enforces end users to pay a collection fee with each scraped tire turned in. This fee is handed to the haulers for their collection and distribution services. The remanufacturing facility will bear no cost associated with any rejected tire including the transportation cost, because it is the haulers responsibility to deliver the appropriate tire to the appropriate processor. Also, the remanufacturing facilities offer the haulers the highest acquisition price among all processors. As a result, the haulers are better off satisfying remanufacturers' demand as conveniently as possible. Moreover, the paper studies both a base case where demand is deterministic and a generalized case where demand is stochastic. The system include post-remanufacturing inventory in the case of uncertain demand as production might exceed actual demand. Due to the same reason, over-stocking cost is introduced to the model only in the case of uncertain demand. Moreover, under-stocking cost is considered in both demand cases as it might be optimal not to satisfy demand even if it is predetermined.

Model Assumptions:

- Each return is tested and then quality graded. Based on the return's quality grade, it is decided whether to accept the return or not, the acquisition price for the return and the remanufacturing cost of the return. Therefore, the testing and quality grading processes are assumed to be very precise and the speculation of return's remanufacturing cost is always correct.
- It is assumed that the acquisition price vs. quality follows an increasing linear relationship, while the remanufacturing cost vs. quality follows a decreasing linear relationship. Since it is more profitable to remanufacture higher quality returns compared to lower quality returns, the summation of the previous two relationships produces a decreasing linear relationship and it is called total spending vs. quality. As a result, the slope of the acquisition price vs. quality linear relationship is always less than the slope of the remanufacturing cost vs. quality linear relationship.
- Quality is assumed to be stochastic with either normal or exponential distribution.
- Demand is assumed to be either deterministic or stochastic with normal distribution.
- Return is assumed to be deterministic. This assumption works very well with the tire retreading industry especially if off-the-road (OTR) or passenger cars' tires are considered. As discussed earlier, only 30% of OTR and 2% of passenger cars' tires are

retreaded. Thus, the hauler is, most probably, capable of providing such supply to the remanufacturer. Indeed, variation in the amount of tires collected by the haulers will not considerably affect, if it would, the amount of returns needed by remanufacturers. As a result, assuming return to be deterministic is still realistic.

- Rejected tires are not associated with any cost as far as the remanufacturing facility is concerned.
- Inspection cost is assumed to be zero. Therefore, the more the returns, the better it is for the remanufacturing facility and the worse it is for the hauler and vice versa.
- The system does not have a pre-remanufacturing inventory, because of two main issues. First, tires are combustible and could attract mosquitoes. Thus, they should be stored in a controlled, shaded, and dry environment. In addition, they are very bulky and storing those tires requires a spacious warehouse. Thus, the remanufacturing facility might be better off avoiding a pre-remanufacturing inventory system, due to the high cost associated with storing. Also, if we assume that the facility stores tires before remanufacturing and grades all returns before either accepting or rejecting them. Intuitively, the facility will remanufacture higher quality returns first and store lower quality returns for later use. If the next period has arrived and the lower quality returns have not been remanufactured yet, then the facility will tend store them for another period until they are needed due to the presence of higher quality returns. As a result, those lower quality returns could face aging before they are even directed

to the remanufacturing process. Therefore, it could be better for the facility not to include a pre-remanufacturing inventory system.

- Post-remanufacturing inventory or sometimes called serviceable inventory as well as the over-stocking cost exist in the stochastic demand case only, because it is not wise to exceed demand if it is deterministic.
- Under-stocking cost is considered in both demand cases as it could be more profitable not to satisfy all demand in certain occasions.
- The model assumes single item, single period and no perfect substitution.

3.1 Models Formulation for Single Facility and Single Market Setting

In this subsection we consider an enterprise that operates only one facility to satisfy its own market's demand. This implies that demanded items could only be supplied from that facility only. For such a setting, transportation cost is not considered. Also, the facility should be operating for the enterprise to make a profit.

3.1.1 Parameters and Variables

Parameters

R Quantity of returns

D Forecasted demand, used for the base model when demand is deterministic

U Under-stocking Cost

O Over-stocking Cost

P Selling price

Cs Setup cost

a Acquisition price when quality is zero

b Slope of acquisition price vs. quality linear relationship

α Remanufacturing cost when quality is zero

β Slope of remanufacturing cost vs. quality linear relationship

μ_q Average quality, used when the system is modeled with both exponential and normal quality distributions

σ_q Quality standard deviation, used when the system is modeled with normal quality distribution

$f_q(.)$ PDF for the variables following the distribution assigned for quality

$F_q(.)$ CDF for the variables following the distribution assigned for quality

d Actual demand, used for the generalized model when demand is stochastic

μ_d Average demand, used when the system is modeled with stochastic or normally distributed demand

σ_d Demand standard deviation, used when the system is modeled with stochastic or normally distributed demand

$f_d(.)$ PDF for the variables following the distribution assigned for demand

$F_d(.)$ CDF for the variables following the distribution assigned for demand

$a + b * q$ Acquisition price vs. quality linear relationship

$\alpha - \beta * q$ Remanufacturing cost vs. quality linear relationship

Stochastic Variables

$\boldsymbol{\pi}$ System profit, which is the objective function to be maximized

q Quality of Returns graded between 0 and 100

Decision Variables

Q Optimal minimum quality that should be accepted to the remanufacturing facility

3.1.2 Mathematical Models

If demand is certain, then the profit function will be as the following:

$$\pi = \left\{\int_{Q}^{\infty} P * R\, f_{q}(q)\, dq\right\} - \left\{\int_{Q}^{\infty} [a + b * q] * R\, f_{q}(q)\, dq\right\}$$

$$- \left\{\int_{Q}^{\infty} [\alpha - \beta * q] * R\, f_{q}(q)\, dq\right\} - \left\{U * \left[D - \int_{Q}^{\infty} R\, f_{q}(q)\, dq\right]\right\} - Cs \quad (1)$$

Subject to:

$$D \geq \int_{Q}^{\infty} R * f_{q}(q)\, dq \quad (2)$$

If demand is uncertain, then the profit function will be as the following:

$$\pi = \left\{\int_{-\infty}^{\int_{Q}^{\infty} R f_{q}(q)\, dq} P * \mathrm{d}\, f_{\mathrm{d}}(\mathrm{d})\, \mathrm{dd}\right\} + \left\{\int_{\int_{Q}^{\infty} R f_{q}(q)\, dq}^{\infty} P * \int_{Q}^{\infty} R\, f_{q}(q)\, dq\, f_{\mathrm{d}}(\mathrm{d})\mathrm{dd}\right\}$$

$$- \left\{\int_{Q}^{\infty} [a + b * q] * R\, f_{q}(q)\, dq\right\} - \left\{\int_{Q}^{\infty} [\alpha - \beta * q] * R\, f_{q}(q)\, dq\right\}$$

$$- \left\{O * \int_{-\infty}^{\int_{Q}^{\infty} R f_{q}(q)\, dq} \left[\int_{Q}^{\infty} R\, f_{q}(q)\, dq - \mathrm{d}\right] f_{\mathrm{d}}(\mathrm{d})\, \mathrm{dd}\right\}$$

$$- \left\{U * \int_{\int_{Q}^{\infty} R f_{q}(q)\, dq}^{\infty} \left[\mathrm{d} - \int_{Q}^{\infty} R\, f_{q}(q)\, dq]\right] f_{\mathrm{d}}(\mathrm{d})\, \mathrm{dd}\right\} - Cs \quad (3)$$

The meaning of each term in the above models is given as follows:

$\left\{\int_{Q}^{\infty} R\, f_{q}(q)\, dq\right\}$: In equations (1) to (3), is the quantity of returns accepted and remanufactured by the facility as the integration of $\left\{f_{q}(q)\, dq\right\}$ will give a percentage and the multiplication in R will give a quantity.

$\left\{\int_{Q}^{\infty} P * R\, f_{q}(q)\, dq\right\}$: In equation (1), is the revenue generated from remanufacturing and selling all accepted returns. Equation (1) was built for the deterministic demand, thus the company will not produce more than the forecasted demand as this will add unnecessary holding cost. As a result, all accepted returns are assumed to be sold by the end of each period.

$\left\{\int_{Q}^{\infty}[a + b * q] * R\, f_{\,q}\,(q)\; dq\right\}$: In equation (1) and (3), is the accepted returns' acquisition cost and it is quality dependant.

$\left\{\int_{Q}^{\infty}[\,\alpha - \beta * q] * R\, f_{\,q}\,(q)\; dq\right\}$: In equation (1) and (3), is the accepted returns' remanufacturing cost and it is quality dependant.

$\left\{U * \left[D - \int_{Q}^{\infty} R\, f_{\,q}\,(q)\; dq\right]\right\}$: In equation (1), is the under-stocking cost if there is any present in the system. As mentioned before, equation (1) was built for the deterministic demand, thus the company will not exceed the forecasted demand as this will add unnecessary holding costs. As a result, all accepted and remanufactured returns are assumed to be sold by the end of each period. Also, the company might choose not to satisfy all forecasted demand as this might hurt the optimality of the system.

$\left\{D \geq \int_{Q}^{\infty} R * f_{\,q}\,(q)\; dq\right\}$: In equation (2), is the demand constraint and it forces the system not to have over-stocking scenario.

$\left\{\int_{-\infty}^{\int_{Q}^{\infty} R\, f_{\,q}\,(q)\, dq} P * \mathrm{d}\, f_{\,\mathrm{d}}\,(\mathrm{d})\; d\mathrm{d}\right\}$: In equation (3), is the revenue generated from satisfying demand. We can notice in this term that all demanded items could be satisfied and an over-stocking situation might occur, because demand is less than the accepted and remanufactured returns $\left\{\int_{Q}^{\infty} R\, f_{\,q}\,(q)\; dq\right\}$.

$\left\{\int_{\int_Q^{\infty} R\, f_{q}(q)\, dq}^{\infty} P * \int_Q^{\infty} R\, f_{q}(q)\, dq\, f_{\mathrm{d}}(\mathrm{d}) d\mathrm{d}\right\}$: In equation (3), is the revenue generated from selling all the accepted and remanufactured returns $\left\{\int_Q^{\infty} R\, f_{q}(q)\, dq\right\}$. Also, we can notice in this term that demand could exceed what have been sold to the customers and an under-stocking scenario might occur.

$\left\{O * \int_{-\infty}^{\int_Q^{\infty} R\, f_{q}(q)\, dq} [\int_Q^{\infty} R\, f_{q}(q)\, dq - \mathrm{d}]\, f_{\mathrm{d}}(\mathrm{d})\, d\mathrm{d}\right\}$: In equation (3), is the over-stocking cost encountered by the system. As mentioned before, since demand is uncertain, there is a possibility that the over-stocking scenario occur.

$\left\{U * \int_{\int_Q^{\infty} R\, f_{q}(q)\, dq}^{\infty} [\mathrm{d} - \int_Q^{\infty} R\, f_{q}(q)\, dq]\, f_{\mathrm{d}}(\mathrm{d})\, d\mathrm{d}\right\}$: In equation (3), is the under-stocking cost encountered by the system. Again, since demand is uncertain, there is a possibility that the under-stocking scenario occur.

3.2 Models Formulation for Multiple Facilities and Multiple Markets Setting

Many enterprises in the remanufacturing business have a network of multiple remanufacturing facilities. Each facility has its own characteristics. For example, each facility faces different quality distribution and different quality parameters such as mean and standard deviation. Also, each facility could experience different amount of returns and could be located in such a way that transportation cost should be controlled. At the same time, markets could, also, behave differently. For example, demand in each market might be different. All of those factors affect the way the system is optimised and the optimality values to maximize the profit. Thus, this section will address the possible involvement of a network and the model associated with it.

Therefore, in this subsection we consider an enterprise that operates multiple facilities to satisfy multiple markets' demand. This implies that demanded items by each market could be supplied from one or multiple facilities. Also, the enterprise might not operate all facilities to reach optimality. Therefore, transportation cost should be considered.

3.2.1 Indices, Parameters and Variables

Indices

i	Set of facilities (1,...,F)
j	Set of markets (1,...,M)

Parameters

U	Under-stocking Cost
O	Over-stocking Cost
P	Selling price
a	Acquisition price when quality is zero
b	Slope of acquisition price vs. quality linear relationship
α	Remanufacturing cost when quality is zero
β	Slope of remanufacturing cost vs. quality linear relationship
R_i	Quantity of returns assigned for facility i
D_j	Forecasted demand for market j, used for the base model when demand is deterministic
Cs_i	Setup cost for facility i
Ca_i	Capacity limit for facility i

T_{ij} Transportation cost per item from facility i to market j

$\mu_{q\,i}$ Average quality of returns delivered to facility i

$\sigma_{q\,i}$ Quality standard deviation, for the normal quality distribution, experienced by facility i

$f_q(.)_i$ PDF for the variables following the distribution assigned for quality in facility i

$F_q(.)_i$ CDF for the variables following the distribution assigned for quality in facility i

d_j Actual demand for market j, used for the generalized model when demand is stochastic

$\mu_{d\,j}$ Average demand for market j, used when the system is modeled with stochastic or normally distributed demand

$\sigma_{d\,j}$ Demand standard deviation for market j, used when the system is modeled with stochastic or normally distributed demand

$f_d(.)_j$ PDF for the variables following the distribution assigned for demand in market j

$F_d(.)_j$ CDF for the variables following the distribution assigned for demand in market j

$a + b * q$ Acquisition price vs. quality linear relationship

$\alpha - \beta * q$ Remanufacturing cost vs. quality linear relationship

Stochastic Variables

$\boldsymbol{\pi}$ System profit, which is the objective function to be maximized

q_i Actual quality of returned item to facility i

Decision Variables

ω_i Binary variable (0,1): 0 = facility i is not operating

1 = facility i is operating

Q_i Optimal minimum quality to accept into facility i

V_{ij} Number of items supplied by facility i to market j

3.2.2 Mathematical Models

If demand is certain, then the profit function will be as the following:

$$\pi = \left\{\sum_{i=1}^{F}\sum_{j=1}^{M} V_{ij} * P\right\} - \left\{\sum_{i=1}^{F}\int_{Q_i}^{\infty}\left[a + b * q_i\right] * R_i\, f_q\left(q_i\right)_i dq_i\right\}$$

$$- \left\{\sum_{i=1}^{F}\int_{Q_i}^{\infty}[\alpha - \beta * q_i] * R_i\, f_q\left(q_i\right)_i dq_i\right\} - \left\{U * \left(\sum_{j=1}^{M} D_j - \sum_{i=1}^{F}\sum_{j=1}^{M} V_{ij}\right)\right\}$$

$$- \left\{\sum_{i=1}^{F} \omega_i * Cs_i\right\} - \left\{\sum_{i=1}^{F}\sum_{j=1}^{M} T_{ij} * V_{ij}\right\} \tag{4}$$

Subject to:

1. Quantity constraint

$$\left\{\sum_{j=1}^{M} V_{ij} \leq \int_{Q_i}^{\infty} R_i\, f_q\left(q_i\right)_i dq_i\right\} \quad \text{for each i} \tag{5}$$

2. Capacity constraint

$$\left\{\int_{Q_i}^{\infty} R_i\, f_q\left(q_i\right)_i dq_i \leq Ca_i\right\} \quad \text{for each i} \tag{6}$$

3. Demand constraint

$$\left\{D_j \geq \sum_{i=1}^{F} V_{ij}\right\} \quad \text{for each j} \tag{7}$$

4. Quality constraint # 1

$$\left\{Q_i \geq \left(1 - \omega_i\right) * (Very\ Large\ Number)\right\} \quad \text{for each i} \tag{8}$$

5. Quality constraint # 2

$$\left\{Q_i \leq 100 * \omega_i + \left(1 - \omega_i\right) * (Very\ Large\ Number)\right\} \quad \text{for each i} \tag{9}$$

6. Excess quantity correction constraint

$$\left\{V_{ij} \leq \omega_i * R_i\right\} \quad \text{for each i and j} \tag{10}$$

If demand is uncertain, then the profit function will be as the following:

$$\pi = \left\{ \sum_{j=1}^{M} \int_{-\infty}^{\sum_{i=1}^{F} V_{ij}} P * \mathrm{d}_j f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\} + \left\{ \sum_{j=1}^{M} \int_{\sum_{i=1}^{F} V_{ij}}^{\infty} P * \sum_{i=1}^{F} V_{ij} f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\}$$

$$- \left\{ \sum_{i=1}^{F} \int_{Q_i}^{\infty} \left[a + b * q_i \right] * R_i f_q \left(q_i \right)_i dq_i \right\} - \left\{ \sum_{i=1}^{F} \int_{Q_i}^{\infty} \left[\alpha - \beta * q_i \right] * R_i f_q \left(q_i \right)_i dq_i \right\}$$

$$- \left\{ \sum_{j=1}^{M} O * \int_{-\infty}^{\sum_{i=1}^{F} V_{ij}} \left(\sum_{i=1}^{F} V_{ij} - \mathrm{d}_j \right) f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\}$$

$$- \left\{ \sum_{j=1}^{M} U * \int_{\sum_{i=1}^{F} V_{ij}}^{\infty} \left(\mathrm{d}_j - \sum_{i=1}^{F} V_{ij} \right) f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\} - \left\{ \sum_{i=1}^{F} \omega_i * Cs_i \right\} - \left\{ \sum_{i=1}^{F} \sum_{j=1}^{M} T_{ij} * V_{ij} \right\} \quad (11)$$

Subject to:

All constraints presented in equations (5 – 10) except the demand constraint presented in equation (7).

The meaning of each term in the above models is given as follows:

$\left\{ \sum_{i=1}^{F} \sum_{j=j}^{M} V_{ij} * P \right\}$: In equation (4), is the revenue generated by selling all items supplied from all facilities i to all markets j in the case of deterministic demand.

$\left\{ \sum_{i=1}^{F} \int_{Q_i}^{\infty} \left[a + b * q_i \right] * R_i f_q \left(q_i \right)_i dq_i \right\}$: In equations (4) and (11), is the overall returns' acquisition cost experienced by the enterprise.

$\left\{ \sum_{i=1}^{F} \int_{Q_i}^{\infty} \left[\alpha - \beta * q_i \right] * R_i f_q \left(q_i \right)_i dq_i \right\}$: In equations (4) and (11), is the overall returns' remanufacturing cost experienced by the enterprise.

$\left\{U * \left(\sum_{j=1}^{M} D_{j} - \sum_{i=1}^{F} \sum_{j=j}^{M} V_{ij}\right)\right\}$: In equations (4), is the overall under-stocking cost the enterprise is facing.

$\left\{\sum_{i=1}^{F} \omega_{i} * Cs_{i}\right\}$: In equations (4) and (11), is the overall setup cost the enterprise is facing.

$\left\{\sum_{i=1}^{F} \sum_{j=j}^{M} T_{ij} * V_{ij}\right\}$: In equations (4) and (11), is the overall transportation cost experienced by the enterprise.

$\left\{\sum_{j=1}^{M} V_{ij} \leq \int_{Q_i}^{\infty} R_{i} f_{q}\left(q_{i}\right)_{i} dq_{i}\right\}$: In equations (5), is the quantity constraint and it makes sure that facility i will only satisfy demand from what have been accepted and remanufactured by the facility.

$\left\{\int_{Q_i}^{\infty} R_{i} f_{q}\left(q_{i}\right)_{i} dq_{i} \leq Ca_{i}\right\}$: In equations (6), is the capacity constraint which makes sure that production from facility i does not exceed the capacity limit for that facility.

$\left\{D_{j} \geq \sum_{i=1}^{F} V_{ij}\right\}$: In equations (7), is the demand constraint and it forces the system not to exceed the deterministic demand from market j. Thus, the system will experience no over-stocking scenario in the case of deterministic demand.

$\left\{Q_{i} \geq \left(1 - \omega_{i}\right) * (Very\ Large\ Number)\right\}$: In equations (8), is the first quality constraint and it makes sure that the optimal minimum quality Q_i is meaningful if that facility i is not operating. As a result, if the facility is operating and $\omega_{i} = 1$, then $Q_{i} \geq 0$. Also, if the facility is not operating and $\omega_{i} = 0$, then $Q_{i} \geq (Very\ Large\ Number)$. As Q_{i} become very large or close to infinity, the acquisition cost, remanufacturing cost and quantity produced by the facility will converge to zero. As a result, the facilities that do not operate experience neither production nor cost. Thus, quality reading will be meaningful in both cases.

$\left\{Q_{i} \leq 100 * \omega_{i} + \left(1 - \omega_{i}\right) * (Very\ Large\ Number)\right\}$: In equations (9), is the second quality constraint and it makes sure that the optimal minimum quality Q_i is meaningful if the facility i is operating. Furthermore, if the facility is operating and $\omega_{i} = 1$, then $Q_{i} \leq 100$. Also, if the facility is not operating and $\omega_{i} = 0$, then $Q_{i} \leq (Very\ Large\ Number)$. Indeed, a very high quality reading associated with a certain facility means that both costs and the number of items accepted to that facility converge to zero indicating its disruption. Thus, quality reading will be meaningful in both cases.

$\left\{V_{ij} \leq \omega_{i} * R_{i}\right\}$: In equation (10), is the excess quantity correction constraint. In our model we use distributions that have values between ± infinity. Thus, the previous quality constraints might be not enough to block all returns from entering a non-operating facility. This is true, because we could not choose infinity instead of a very large number while programming the solver program as infinity is unrecognised value. Thus, this constraint is added to compensate for such an error if it exists. As a result, this constraint ensures that no demand is satisfied from a non-operating facility.

$\left\{ \sum_{j=1}^{M} \int_{-\infty}^{\sum_{i=1}^{F} V_{ij}} P * \mathrm{d}_j f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\}$: In equation (11), is the revenue generated by the enterprise, if the uncertain demand in market j is found to be less than all remanufactured items supplied by all facilities $\left\{ \sum_{i=1}^{F} V_{ij} \right\}$ to that specific market.

$\left\{ \sum_{j=1}^{M} \int_{\sum_{i=1}^{F} V_{ij}}^{\infty} P * \sum_{i=1}^{F} V_{ij} \, f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\}$: In equation (11), is the revenue generated by the enterprise, if the uncertain demand in market j is found to be more than all remanufactured items supplied by all facilities $\left\{ \sum_{i=1}^{F} V_{ij} \right\}$ to that specific market.

$\left\{ \sum_{j=1}^{M} O * \int_{-\infty}^{\sum_{i=1}^{F} V_{ij}} \left(\sum_{i=1}^{F} V_{ij} - \mathrm{d}_j \right) f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\}$: In equation (11), is the expected over-stocking cost, if the uncertain demand in market j is found to be less than all remanufactured items supplied by all facilities $\left\{ \sum_{i=1}^{F} V_{ij} \right\}$ to that specific market.

$\left\{ \sum_{j=1}^{M} U * \int_{\sum_{i=1}^{F} V_{ij}}^{\infty} \left(\mathrm{d}_j - \sum_{i=1}^{F} V_{ij} \right) f_{\mathrm{d}} \left(\mathrm{d}_j \right)_j d\mathrm{d}_j \right\}$: In equation (11), is the expected under-stocking cost, if the uncertain demand in market j is found to be more than all remanufactured items supplied by all facilities $\left\{ \sum_{i=1}^{F} V_{ij} \right\}$ to that specific market.

CHAPTER 4: SOLUTION METHODOLOGY

4.1 Solver Approach

4.1.1 A Single Facility and Single Market Setting with Deterministic Demand

We start by rearranging the profit function in equation (1) to get the following:

$$\pi = R * (P + U - a - \alpha) * \int_{Q}^{\infty} f_{q}(q)\, dq + R * (\beta - b) * \int_{Q}^{\infty} q\, f_{q}(q)\, dq - U * D - Cs \tag{12}$$

4.1.1.1 Exponential Behaviour for Quality Uncertainty

Since quality is assumed to be exponentially distributed, then:

$$f_{q}(q) = \frac{e^{\left(-q/\mu_{q}\right)}}{\mu_{q}} \tag{13}$$

By using the following two concepts:

$$\int e^{cx}\, dx = \frac{1}{c} * e^{cx} \tag{14}$$

$$\int x * e^{cx}\, dx = \frac{cx - 1}{c^2} * e^{cx} \tag{15}$$

The following holds true;

$$\int_{Q}^{\infty} f_{q}(q) q = \int_{Q}^{\infty} \frac{e^{\left(-q/\mu_{q}\right)}}{\mu_{q}} dq = \left[-e^{\left(-q/\mu_{q}\right)}\right]_{Q}^{\infty} = \left[-0 - \left(-e^{\left(-Q/\mu_{q}\right)}\right)\right] = e^{\left(-Q/\mu_{q}\right)} \tag{16}$$

And,

$$\int_{Q}^{\infty} q\, f_{q}(q)\, dq = \int_{Q}^{\infty} q * \frac{e^{\left(-q/\mu_{q}\right)}}{\mu_{q}}\, dq = \left[-\left(\mu_{q} + q\right) * e^{\left(-q/\mu_{q}\right)}\right]_{Q}^{\infty}$$

$$= \left\{-0 - \left[-\left(\mu_{q} + Q\right) * e^{\left(-Q/\mu_{q}\right)}\right]\right\} = \left(\mu_{q} + Q\right) * e^{\left(-Q/\mu_{q}\right)} \tag{17}$$

Therefore, the profit function in equation (12) will be:

$$\pi = R * (P + U - a - \alpha) * e^{\left(-Q/\mu_{q}\right)} + R * (\beta - b) * \left(\mu_{q} + Q\right) * e^{\left(-Q/\mu_{q}\right)} - U * D - Cs \tag{18}$$

Also, the constraint inequality in equation (2) will be:

$$D \geq R * e^{\left(-Q/\mu_{q}\right)} \tag{19}$$

4.1.1.2 Normal Behaviour for Quality Uncertainty

In order to use the errorf function that is built in the solver program, we need to express the profit function using the standard normal distribution rather than the normal distribution.

Therefore, we let:

$$Y = \frac{q - \mu_{q}}{\sigma_{q}} \rightarrow q = \sigma_{q} * Y + \mu_{q} \tag{20}$$

$$\therefore\ dq = \sigma_{q}\ dY \tag{21}$$

Also, the integration limits will change based on equation (20):

$$Y = \infty\ (when\ q = \infty)\ and,$$

$$Y = \frac{Q - \mu_q}{\sigma_q} \ (when\ q = Q)$$

Because quality follows a normal distribution, then

$$f_q(q) = \frac{1}{\sigma_q * \sqrt{2\pi}} * e^{\left(-\left(q-\mu_q\right)^2 \Big/ 2*\sigma_q{}^2\right)} \tag{22}$$

By substituting the values of *q, $f_q(q)$ and dq* into equation (12) and changing the integration limits, the profit function can be expressed as:

$$\pi = R * \left[(P + U - a - \alpha) + \mu_q * (\beta - b)\right] * \int_{\left(Q-\mu_q / \sigma_q\right)}^{\infty} \frac{1}{\sqrt{2\pi}} * e^{\frac{-Y^2}{2}} dY$$

$$+ \sigma_q * R * (\beta - b) * \int_{\left(Q-\mu_q / \sigma_q\right)}^{\infty} Y * \frac{1}{\sqrt{2\pi}} * e^{\frac{-Y^2}{2}} dY - U * D - Cs \tag{23}$$

We can notice that the term $\int_{\left(Q-\mu_q / \sigma_q\right)}^{\infty} \frac{1}{\sqrt{2\pi}} * e^{\frac{-Y^2}{2}} dY$ follows a standard normal distribution. Thus, it can be replaced by $\left\{1 - errorf\left(\frac{Q-\mu_q}{\sigma_q}\right)\right\}$ while using the solver.

Also, the term $\int_{\left(Q-\mu_q / \sigma_q\right)}^{\infty} Y * \frac{1}{\sqrt{2\pi}} * e^{\frac{-Y^2}{2}} dY$ can be integrated using the function below:

$$\int x * e^{cx^2}\, dx = \frac{1}{2C} * e^{cx^2} \tag{24}$$

$$\therefore \int Y * e^{-\frac{1}{2}Y^2}\, dY = -e^{-\frac{1}{2}Y^2} \tag{25}$$

Therefore, equation (23) can be rewritten as the following:

$$\therefore \pi = R * \left[(P + U - a - \alpha) + \mu_q * (\beta - b)\right] * \left(1 - errorf \frac{Q - \mu_q}{\sigma_q}\right)$$

$$+ \frac{\sigma_q * R * (\beta - b)}{\sqrt{2\pi}} \left\{ e^{\left(-\left(Q - \mu_q\right)^2 / 2 * \sigma_q^2\right)} \right\} - U * D - Cs \tag{26}$$

The constrain function in equation (2) will also be reformulated to be:

$$D \geq R * \left(1 - errorf \frac{Q - \mu_q}{\sigma_q}\right) \tag{27}$$

4.1.2 A Single Facility and Single Market Setting with Stochastic Demand

To represent demand stochastically, it was assumed to be normally distributed. The model will be solved with exponentially distributed quality first and then with normally distributed quality. Similar to the deterministic demand model in the previous section, the profit function was analysed.

$$We\ assume:\ Z = \frac{\mathrm{d} - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \rightarrow \mathrm{d} = \sigma_{\mathrm{d}} * Z + \mu_{\mathrm{d}} \tag{28}$$

$$\therefore\ d\mathrm{d} = \sigma_{\mathrm{d}}\ dZ \tag{29}$$

$$\therefore Z = \infty\ (when\ \mathrm{d} = \infty)\ and,$$

$$Z = -\infty\ (when\ \mathrm{d} = -\infty)\ and,$$

$$Z = \frac{\int_Q^\infty R\, f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}\ \left(when\ \mathrm{d} = \int_Q^\infty R\, f_q(q)\, dq\right)$$

Because demand follows a normal distribution, then

$$f_{\mathrm{d}}(\mathrm{d}) = \frac{1}{\sigma_{\mathrm{d}} * \sqrt{2\pi}} * e^{\left(-\left(\mathrm{d}-\mu_{\mathrm{d}}\right)^2 / 2*\sigma_{\mathrm{d}}^{\;2}\right)} \tag{30}$$

Then,

$$\int_{-\infty}^{\int_Q^\infty R f_q(q)\,dq} f_{\mathrm{d}}(\mathrm{d})\,d\mathrm{d} = \int_{-\infty}^{\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}} \frac{1}{\sigma_{\mathrm{d}} * \sqrt{2\pi}} * e^{\left(-\left(\mathrm{d}-\mu_{\mathrm{d}}\right)^2 / 2*\sigma_{\mathrm{d}}^{\;2}\right)} \sigma_{\mathrm{d}}\; dZ$$

$$= \int_{-\infty}^{\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}} \frac{1}{\sqrt{2\pi}} * e^{\frac{-1}{2}Z^2}\, dZ = errorf\left(\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}\right) \tag{31}$$

Also,

$$\int_{-\infty}^{\int_Q^\infty R f_q(q)\,dq} \mathrm{d}\, f_{\mathrm{d}}(\mathrm{d})\,d\mathrm{d} = \int_{-\infty}^{\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}} \frac{\left(\sigma_{\mathrm{d}} * Z + \mu_{\mathrm{d}}\right)}{\sigma_{\mathrm{d}} * \sqrt{2\pi}} * e^{\left(-\left(\mathrm{d}-\mu_{\mathrm{d}}\right)^2 / 2*\sigma_{\mathrm{d}}^{\;2}\right)} \sigma_{\mathrm{d}}\; dZ$$

$$= \frac{\sigma_{\mathrm{d}}}{\sqrt{2\pi}} * \int_{-\infty}^{\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}} Z * e^{\frac{-1}{2}Z^2}\, dZ + \mu_{\mathrm{d}} * \int_{-\infty}^{\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}} \frac{1}{\sqrt{2\pi}} * e^{\frac{-1}{2}Z^2}\, dZ$$

$$= \frac{-\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2}*\left(\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}\right)^2} \right) + \mu_{\mathrm{d}} * errorf\left(\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}\right) \tag{32}$$

In a similar way

$$\int_{\int_Q^\infty R f_q(q)\,dq}^{\infty} \mathrm{d}\, f_{\mathrm{d}}(\mathrm{d})\,d\mathrm{d}$$

$$= \frac{\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2}*\left(\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}\right)^2} \right) + \mu_{\mathrm{d}} - \mu_{\mathrm{d}} * errorf\left(\frac{\int_Q^\infty R f_q(q)\,dq-\mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}}\right) \tag{33}$$

Therefore, equation (3) can be rearranged to be as the following:

$$\pi = P * \left\{ \frac{-\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2} * \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right)^2} \right) + \mu_{\mathrm{d}} * errorf \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right\}$$

$$+ P * \int_Q^\infty R f_q(q)\, dq * \left\{ 1 - errorf \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right\}$$

$$- \int_Q^\infty a * R f_q(q)\, dq - \int_Q^\infty b * q * R f_q(q)\, dq - \int_Q^\infty \alpha * R f_q(q)\, dq + \int_Q^\infty \beta * q * R f_q(q)\, dq$$

$$- O \left\{ \int_Q^\infty R f_q(q)\, dq * errorf \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) + \frac{\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2} * \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right)^2} \right) \right.$$

$$\left. - \mu_{\mathrm{d}} * errorf \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right\}$$

$$- U \left\{ \frac{\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2} * \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right)^2} \right) + \mu_{\mathrm{d}} - \mu_{\mathrm{d}} * errorf \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right.$$

$$\left. - \int_Q^\infty R f_q(q)\, dq * \left[1 - errorf \left(\frac{\int_Q^\infty R f_q(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right] \right\} - Cs \tag{34}$$

Equation (34) can be simplified more to be

$$\pi = (P + O + U) * \left[\left(\mu_{\mathrm{d}} - \int_{Q}^{\infty} R\, f_{q}(q)\, dq \right) * errorf\left(\frac{\int_{Q}^{\infty} R\, f_{q}(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right.$$

$$\left. - \frac{\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2} * \left(\frac{\int_{Q}^{\infty} R\, f_{q}(q)\, dq - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right)^{2}} \right) \right] + (P - a - \alpha + U) * \int_{Q}^{\infty} R\, f_{q}(q)\, dq$$

$$+ (\beta - b) * \int_{Q}^{\infty} R * q\, f_{q}(q)\, dq - U * \mu_{\mathrm{d}} - Cs \tag{35}$$

4.1.2.1 Exponential Behaviour for Quality Uncertainty

For the exponential distribution we apply the results obtained from equations (13), (16) and (17) to further rework equation (35) in order to suit the exponentially distributed quality.

$$\pi = (P + O + U) * \left[\left(\mu_{\mathrm{d}} - R * e^{\left(-Q/\mu_{q} \right)} \right) * errorf\left(\frac{R * e^{\left(-Q/\mu_{q} \right)} - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right) \right.$$

$$\left. - \frac{\sigma_{\mathrm{d}}}{\sqrt{2\pi}} \left(e^{\frac{-1}{2} * \left(\frac{R * e^{\left(-Q/\mu_{q} \right)} - \mu_{\mathrm{d}}}{\sigma_{\mathrm{d}}} \right)^{2}} \right) \right] + R * e^{\left(-Q/\mu_{q} \right)} \left[(P - a - \alpha + U) + (\beta - b) * \left(\mu_{q} + Q \right) \right] - U * \mu_{\mathrm{d}} - Cs \tag{36}$$

4.1.2.2 Normal Behaviour for Quality Uncertainty

Similarly, for the normal distribution we apply the results obtained in section 4.1.1.2 to further rework equation (35) in order to suit the normally distributed quality.

$$\pi = (P+O+U) * \left[\left(\mu_d - R + R * errorf\left(\frac{Q-\mu_q}{\sigma_q}\right)\right) * errorf\left(\frac{R - R * errorf\left(\frac{Q-\mu_q}{\sigma_q}\right) - \mu_d}{\sigma_d}\right) \right.$$

$$\left. - \frac{\sigma_d}{\sqrt{2\pi}} \left(e^{\frac{-1}{2} * \left(\frac{R - R * errorf\left(\frac{Q-\mu_q}{\sigma_q}\right) - \mu_d}{\sigma_d}\right)^2} \right) \right] + \left[R * \left[(P + U - a - \alpha) + \mu_q * (\beta - b) \right] * \left(1 - errorf \frac{Q-\mu_q}{\sigma_q} \right) \right.$$

$$\left. + \frac{\sigma_q * R * (\beta - b)}{\sqrt{2\pi}} \left\{ e^{\left(-\left(Q-\mu_q\right)^2 / 2 * \sigma_q^2 \right)} \right\} \right] - U * \mu_d - Cs \tag{37}$$

4.1.3 A Multiple Facilities and Multiple Markets Setting with Deterministic Demand

To solve the model equations presented for this setting, we will use the same procedures followed in the previous sections of this chapter. Thus, no details are presented while working the equations out.

4.1.3.1 Exponential Behaviour for Quality Uncertainty

$$\pi = \left\{\sum_{i=1}^{F}\sum_{j=1}^{M} V_{ij} * (P+U)\right\} - \left\{\sum_{i=1}^{F}\sum_{j=1}^{M} V_{ij}\, T_{ij}\right\} - \left\{U * \sum_{j=1}^{M} D_j\right\} - \left\{\sum_{i=1}^{F} \omega_i * Cs_i\right\}$$

$$- \left\{\sum_{i=1}^{F} R_i * e^{\left(-Q_i / \mu_{q\ i}\right)} * \left[a + \alpha + (b-\beta) * \left(Q_i + \mu_{q\ i}\right)\right]\right\} \tag{38}$$

The term $\left\{\int_{Q_i}^{\infty} R_i f_q\left(q_i\right)_i dq_i\right\}$ in the first and second quality constraints should be expressed as $\left\{R_i * e^{\left(-Q_i / \mu_{q\ i}\right)}\right\}$.

4.1.3.2 Normal Behaviour for Quality Uncertainty

$$\pi = \left\{\sum_{i=1}^{F}\sum_{j=1}^{M} V_{ij} * (P+U)\right\} - \left\{\sum_{i=1}^{F}\sum_{j=1}^{M} V_{ij}\, T_{ij}\right\} - \left\{U * \sum_{j=1}^{M} D_j\right\} - \left\{\sum_{i=1}^{F} \omega_i * Cs_i\right\}$$

$$- \left\{\sum_{i=1}^{F} R_i \left[\left(1 - errorf\left(\frac{Q_i - \mu_{q\ i}}{\sigma_{q\ i}}\right)\right) * \left(a + \alpha + \mu_{q\ i}(b-\beta)\right) + (b-\beta) * \frac{\sigma_{q\ i}}{\sqrt{2\pi}} * e^{\frac{-1}{2}\left(\frac{Q_i - \mu_{q\ i}}{\sigma_{q\ i}}\right)^2}\right]\right\} \tag{39}$$

The term $\left\{\int_{Q_i}^{\infty} R_i f_q\left(q_i\right)_i dq_i\right\}$ in the first and second quality constraints should be expressed as $\left\{R_i * \left(1 - errorf\ \frac{Q_i - \mu_{q\ i}}{\sigma_{q\ i}}\right)\right\}$.

4.1.4 A Multiple Facilities and Multiple Markets Setting with Stochastic Demand

4.1.4.1 Exponential Behaviour for Quality Uncertainty

$$\pi = \left\{ \sum_{j=1}^{M} \left[(P + O + U) * \left(\left(\mu_{d\ j} - \sum_{i=1}^{F} V_{ij} \right) * errorf\left(\frac{\sum_{i=1}^{F} V_{ij} - \mu_{d\ j}}{\sigma_{d\ j}} \right) - \frac{\sigma_{d\ j}}{\sqrt{2\pi}} * e^{\frac{-1}{2}\left(\frac{\sum_{i=1}^{F} V_{ij} - \mu_{d\ j}}{\sigma_{d\ j}} \right)^{2}} \right) + (P + U) * \sum_{i=1}^{F} V_{ij} - U * \mu_{d\ j} \right] \right\} - \left\{ \sum_{i=1}^{F} \sum_{j=1}^{M} V_{ij} T_{ij} \right\} - \left\{ \sum_{i=1}^{F} \omega_i * Cs_i \right\} - \left\{ \sum_{i=1}^{F} R_i * e^{\left(-Q_i / \mu_{q\ i}\right)} * \left[a + \alpha + (b - \beta) * \left(Q_i + \mu_{q\ i} \right) \right] \right\} \tag{40}$$

Just as before, the term $\left\{ \int_{Q_i}^{\infty} R_i f_q\left(q_i\right)_i dq_i \right\}$ in the first and second quality constraints should be expressed as $\left\{ R_i * e^{\left(-Q_i / \mu_{q\ i}\right)} \right\}$.

4.1.4.2 Normal Behaviour for Quality Uncertainty

$$\pi = \left\{ \sum_{j=1}^{M} \left[(P+O+U) * \left(\left(\mu_{d\ j} - \sum_{i=1}^{F} V_{ij} \right) * errorf \left(\frac{\sum_{i=1}^{F} V_{ij} - \mu_{d\ j}}{\sigma_{d\ j}} \right) - \frac{\sigma_{d\ j}}{\sqrt{2\pi}} * e^{\frac{-1}{2}\left(\frac{\sum_{i=1}^{F} V_{ij} - \mu_{d\ j}}{\sigma_{d\ j}} \right)^2} \right) \right. \right.$$

$$\left. \left. + (P+U) * \sum_{i=1}^{F} V_{ij} - U * \mu_{d\ j} \right] \right\} - \left\{ \sum_{i=1}^{F} \sum_{j=1}^{M} V_{ij} T_{ij} \right\} - \left\{ \sum_{i=1}^{F} \omega_i * Cs_i \right\}$$

$$- \left\{ \sum_{i=1}^{F} R_i * \left[\left(1 - errorf \left(\frac{Q_i - \mu_{q\ i}}{\sigma_{q\ i}} \right) \right) * \left(a + \alpha + \mu_{q\ i} * (b - \beta) \right) + (b - \beta) * \frac{\sigma_{q\ i}}{\sqrt{2\pi}} * e^{\frac{-1}{2}\left(\frac{Q_i - \mu_{q\ i}}{\sigma_{q\ i}} \right)^2} \right] \right\} \qquad (41)$$

Similar as before, the term $\left\{ \int_{Q_i}^{\infty} R_i f_q \left(q_i \right)_i dq_i \right\}$ in the first and second quality constraints should be expressed as $\left\{ R_i * \left(1 - errorf \ \frac{Q_i - \mu_{q\ i}}{\sigma_{q\ i}} \right) \right\}$.

4.2 Analytical Approach

Due to the complexity of the multiple facilities and multiple markets setting, only the single facility and single market setting will be analytically solved in this paper. Future work might include the solution for the multiple setting.

4.2.1 A Single Facility and Single Market Setting with Deterministic Demand

In order to find the optimal minimum quality Q that should be accepted to our remanufacturing facility, we take the derivative of the profit function in equation (12) in terms of the quality (q) and equate it to zero. Thus, we will have the following expression:

$$\frac{d\pi}{dq} = R\, f_q(q) * [(P + U - a - \alpha) + (\beta - b) * q] = 0 \tag{42}$$

At the same time we have to satisfy the demand constraint in equation (2). Therefore, we find what quality satisfies $D = \int_q^{\infty} R * f_q(q)\, dq$ and what quality satisfies equation (42) and then we take the higher quality as the optimal quality Q.

We rewrite equation (42) to suit the assumption that quality is exponentially distributed. Thus, we have the following equation:

$$\frac{R}{\mu_q} e^{\left(-Q/\mu_q\right)} * [(P + U - a - \alpha) + (\beta - b) * Q] = 0 \tag{43}$$

And if quality is assumed to be normally distributed, then equation (42) will be rewritten as the following:

$$\frac{R}{\sigma_q * \sqrt{2\pi}} e^{\left(-\left(Q-\mu_q\right)^2 / 2*\sigma_q^{\,2}\right)} * [(P + U - a - \alpha) + (\beta - b) * Q] = 0 \tag{44}$$

4.2.2 A Single Facility and Single Market Setting with Stochastic Demand

In a similar way we need to take the derivative of the profit function in equation (3) in terms of the quality (q) and equate it to zero to find the optimal minimum quality Q. Due to the complexity of the model, we will use *Leibniz's* rule which states the following:

$$\frac{d}{dy}\int_{x=g(y)}^{x=h(y)} r(x,y)dx = \int_{x=g(y)}^{x=h(y)} \frac{\partial r(x,y)}{\partial y}dx + r(h(y),y)\frac{dh(y)}{dy} - r(g(y),y)\frac{dg(y)}{dy}$$

Thus, first term in equation (3) will be:

$$\frac{d}{dq}\left\{\int_{-\infty}^{\int_q^\infty R\, f_{\,q}\,(q)\,dq} P * \mathrm{d}\, f_{\,\mathrm{d}}\,(\mathrm{d})\, d\mathrm{d}\right\}$$

$$= P * \left(R * f_{\,q}\,(q)\right) * \left(\int_q^\infty R\, f_{\,q}\,(q)\,dq\right) * \left(f_{\,\mathrm{d}}\left(\int_q^\infty R\, f_{\,q}\,(q)\,dq\right)\right)$$

Second term in equation (3) will be:

$$\frac{d}{dq}\left\{\int_{\int_q^\infty R\, f_{\,q}\,(q)\,dq}^{\infty} P * \int_q^\infty R\, f_{\,q}\,(q)\,dq\, f_{\,\mathrm{d}}\,(\mathrm{d}) d\mathrm{d}\right\}$$

$$= P * \left(R * f_{\,q}\,(q)\right) - P * \left(R * f_{\,q}\,(q)\right) * \left(F_{\,\mathrm{d}}\left(\int_q^\infty R\, f_{\,q}\,(q)\,dq\right)\right)$$

$$- P * \left(R * f_{\,q}\,(q)\right) * \left(\int_q^\infty R\, f_{\,q}\,(q)\,dq\right) * \left(f_{\,\mathrm{d}}\left(\int_q^\infty R\, f_{\,q}\,(q)\,dq\right)\right)$$

Third term in equation (3) will be:

$$\frac{d}{dq}\left\{\int_q^\infty [a + b * q] * R\, f_{\,q}\,(q)\,dq\right\} = (a + b * q\,) * R\, f_{\,q}\,(q)$$

Fourth term in equation (3) will be:

$$\frac{d}{dq}\int_{q}^{\infty}[\alpha-\beta*q]*R\,f_{q}(q)\,dq=(\alpha-\beta*q)*R\,f_{q}(q)$$

Fifth term in equation (3) will be:

$$\frac{d}{dq}\left\{O*\int_{-\infty}^{\int_{q}^{\infty}R\,f_{q}(q)\,dq}\left[\int_{q}^{\infty}R\,f_{q}(q)\,dq-\mathrm{d}\right]f_{\mathrm{d}}(\mathrm{d})\,d\mathrm{d}\right\}$$

$$=O*\left(R*f_{q}(q)\right)*\left(F_{\mathrm{d}}\left(\int_{q}^{\infty}R\,f_{q}(q)\,dq\right)\right)$$

Sixth term in equation (3) will be:

$$\frac{d}{dq}\left\{U*\int_{\int_{q}^{\infty}R\,f_{q}(q)\,dq}^{\infty}\left[\mathrm{d}-\int_{q}^{\infty}R\,f_{q}(q)\,dq]\right]f_{\mathrm{d}}(\mathrm{d})\,d\mathrm{d}\right\}$$

$$=U*\left(R*f_{q}(q)\right)*\left(F_{\mathrm{d}}\left(\int_{q}^{\infty}R\,f_{q}(q)\,dq\right)\right)-U*\left(R*f_{q}(q)\right)$$

By rearranging all the previous terms we can find that $\frac{d\pi}{dq}$ has the following expression:

$$R\,f_{q}(q)*\left\{(P+U-a-\alpha)+(\beta-b)*q-(P+O+U)*\left(F_{\mathrm{d}}\left(\int_{q}^{\infty}R\,f_{q}(q)\,dq\right)\right)\right\} \tag{45}$$

As a result, if quality is exponentially distributed, then the optimal quality Q can be found by:

$$\frac{R}{\mu_{q}}e^{\left(-Q/\mu_{q}\right)}*\left\{(P+U-a-\alpha)+(\beta-b)*Q-(P+O+U)*\left(F_{\mathrm{d}}\left(R\,e^{\left(-Q/\mu_{q}\right)}\right)\right)\right\}=0 \tag{46}$$

And if quality is normally distributed, then optimal quality can be found by:

$$\frac{R}{\sigma_q * \sqrt{2\pi}} e^{\left(-\left(Q-\mu_q\right)^2 / 2*\sigma_q^2\right)} * \left\{(P+U-a-\alpha)+(\beta-b)*Q-(P+O+U)*\left(F_{\mathrm{d}}\left(R-R*F_q(Q)\right)\right)\right\} = 0 \tag{47}$$

CHAPTER 5: NUMERICAL EXAMPLE AND RESULTS

5.1 Data Collection

Due to the difficulty in collecting real data, improvising data from several reports and internet websites was the optimal approach. Thus, a type of tires that is used for off the road (OTR) application has been selected to conduct our study on. An example of (OTR) tires is the tire type 1400R24 Radial. The table below briefly represents the data generation process.

Parameters	Logic of Data Generation	Resources
R_i	• Number of tires generated has a one to one relationship with the population. • Approximately, 6.1% of those tires are OTR tire type. • Approximately, 25% of those OTR tires are retreadable.	Found or calculated from (Ontario Tire Stewardship, 2004).
$P = 250$	• Since the price of remanufactured tires are 30% to 70% less than new tires and the cost of a new 1400R24 radial tire is about \$395, then the selling price P is chosen to be \$250 which is about 36.5% less than a new tire.	(Tire Retread & Repair Information Bureau)
$\alpha = 169$ & $\beta = 0.68$	• Average number of nail holes in an ORT is about 20 which is assumed to occur in the middle of the quality spectrum. • Average cost of retreading a tire is \$135 and 25% of it is assumed to be contributed by the repair stage. Thus, 20 repairs are associated with \$34.	(State of North Carolina, Department of Administration, 2011) and (Ontario Tire Stewardship, 2009)
$a = 90$ & $b = 0.2$	• Average cost of purchasing a retreadable tire is \$100. Thus, it is associated with a tire that has 20 nail holes in it. • It is assumed that each tire with no repair needs will be purchased by \$110. Then, we can work out acquisition cost associated with each quality.	(Ontario Tire Stewardship, 2009)
$[\mu_{q\ i}]_{Normal}$ & $[\mu_{q\ i}]_{Expo}$	• When quality is normally distributed, most of the returns are assumed to have quality readings that are close to the center of the quality spectrum. • When quality is exponentially distributed, most of the returns are assumed to have bad quality readings which indicate an abusive working environment. • In this study we have assumed that the values of ($\mu_{q\ i}$) depend on the type of industries exist in the province. For example, the more a province is involved in mining or industrial activities the worse the average quality will be. This is due to the fact that harsh industries cause more damage to the tires in service.	Wikipedia

$[\sigma_{q\ i}]_{Normal}$	• The quality range (0 - 100) should cover as many σ under the normal quality curve as possible. Indeed, this indicates that the quality of returns is correctly represented.	
D_j or $[\mu_{d\ j}]_{Normal}$	• Due to the lack of data, demand on OTR tires is assumed to be one percent of the province population. Indeed, the higher the population the higher the number of units will be sold from any commodity. Thus, relating demand to the population is realistic.	
$[\sigma_{d\ j}]_{Normal}$	• Following the same principle, a higher population contributes to a higher variance in demand. Thus, σ_d will be calculated by dividing average demand by a hundred.	
U	• Since we are applying a newsvendor like model, under-stocking cost is considered to be the profit lost in case of shortage. Thus, it will be calculated based on the total cost of the return at 50 quality reading and the selling price as; U = price –total cost of return at 50 quality reading	
O	• Similarly, since we are applying a newsvendor like model, over-stocking cost is considered to be the profit lost in case of overage. Thus, it will be calculated based on the total cost of the return at 50 quality reading and the salvage value (assuming half of what it is for the price) as; O = total cost of return at 50 quality reading – salvage value	
Cs_i	• It is very difficult to measure the set up cost associated with each plant without the presence of real data. In any case, the setup costs are assumed to be equal in all facilities. This assumption is supported by the fact that all considered facilities are located in the same country.	
Ca_i	• Will be chosen to be very high, because it is more important to study the effect of other vital parameters.	
T_{ij}	• Trailer's dimensions are 99", 111", and 52' in width height and length respectively. • The approximate tires' diameter is 1400 mm and tires' depth is 315 mm. • Thus, a truck can carry as many as 176 tires. • A truck cost per mile, which is equal to $1.06, is calculated based on the figure presented in (Siebert). • Thus, the tire transportation cost per mile is found to be 0.6023 cents.	(Siebert) and (Alibaba.com)
Index (i and j)	• Since each Canadian province has its own waste tire management program and retreading plants, then this study will consider facilities in the capital of Ontario, Quebec and Manitoba.	

Table 2: Data Generation Logic

To represent the different costs vs. quality linear relationships, the figures below were constructed. We can notice here that the last relationship is the total spending or cost on returns with respect to each quality grade if they were to be accepted into the facility. Such a relationship is formed by adding both the acquisition cost and the remanufacturing cost. We can also, notice that it follows a decreasing pattern. Based on the literatures reviewed before and during the production of this work, this should be the case. In any case, if this relationship was an increasing one, which this model does not support, then the facility is better of remanufacturing the worse quality returns rather than the better quality returns.

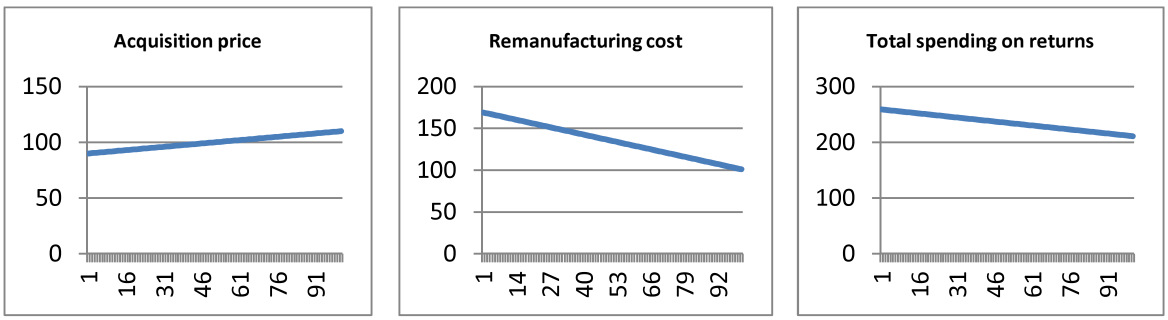


Figure 3: Acquisition Price, Remanufacturing Cost and Total Spending vs. Quality Linear Relationships Constructed for the Numerical Example

Also, based on Table-2, the following numerical example parameters were selected;

Parameter	Value	Parameter	Value	Parameter	Value
$i \in$	{ON, QC, MB}	$[(\sigma_q)_{ON}]_{Normal}$	10	Cs_{MB}	10,000
$j \in$	{ON, QC, MB}	$[(\sigma_q)_{BQ}]_{Normal}$	9	Ca_{ON}	500 K
$R_{Ontario}$	205,875	$[(\sigma_q)_{MB}]_{Normal}$	8	Ca_{QC}	500 K
R_{Quebec}	122,000	$D_{ON}/[(\mu_d)_{ON}]_{Normal}$	135,000	Ca_{MB}	500 K
$R_{Manitoba}$	19,825	$D_{ON}/[(\mu_d)_{QC}]_{Normal}$	80,000	P	250
$[(\mu_q)_{ON}]_{Normal}$	50	$D_{ON}/[(\mu_d)_{MB}]_{Normal}$	13,000	U	15
$[(\mu_q)_{QC}]_{Normal}$	45	$[(\sigma_d)_{ON}]_{Normal}$	1,350	O	110
$[(\mu_q)_{MB}]_{Normal}$	40	$[(\sigma_d)_{QC}]_{Normal}$	800	α	169
$[(\mu_q)_{ON}]_{Expo}$	15	$[(\sigma_d)_{MB}]_{Normal}$	130	β	0.68
$[(\mu_q)_{QC}]_{Expo}$	13	Cs_{ON}	10,000	a	90
$[(\mu_q)_{MB}]_{Expo}$	11	Cs_{QC}	10,000	b	0.2

Table 3: Numerical Example Parameters

To find the total tire transportation cost (T_{ij}) from a capital to another, the tire transportation cost per mile is multiplied by the distance between the two capitals.

	Toronto (ON)	Quebec City (QC)	Winnipeg (MB)
Toronto (ON)	0	490	1270
Quebec City (QC)	490	0	1550
Winnipeg (MB)	1270	1550	0

Table 4: Distances Between Provinces' Capitals

5.2 Mathematical Model and Solver Results

Based on the data and parameters given in Tables-3 & 4, the decision variables and profit for all four scenarios – namely: exponential quality and deterministic demand, exponential quality and normal demand, normal quality and deterministic demand, and normal quality and normal demand– related to both single and multiple settings were calculated and depicted in Tables-5, 6, 7 & 8.

	Exponential q and Deterministic D	Exponential q and Normal D	Normal q and Deterministic D	Normal q and Normal D
Total Profit π	$ 157,180	$ 128,290	$ 2,378,800	$ 2,307,113
Optimal Quality Q_{ON}	6.3	6.6	46.0	46.3
Number of Items Remanufactured by ON factory	All demanded items 135000	Less than average demanded items 132,340	All demanded items 135000	Less than average demanded items 133,060

Table 5: Numerical Example Results for ON's Single Facility and Single Market Case

	Exponential q and Deterministic D	Exponential q and Normal D	Normal q and Deterministic D	Normal q and Normal D
Total Profit π	$ -20,140	$ -36,600	$ 1,192,026	$ 1,152,130
Optimal Quality Q_{QC}	5.5	5.8	41.4	41.6
Number of Items Remanufactured by QC factory	All demanded items 80,000	Less than average demanded items 78,409	All demanded items 80,000	Less than average demanded items 78,815

Table 6: Numerical Example Results for QC's Single Facility and Single Market Case

	Exponential q and Deterministic D	Exponential q and Normal D	Normal q and Deterministic D	Normal q and Normal D
Total Profit π	$ -29,394	$ -31,961	$ 150,626	$ 144,580
Optimal Quality Q_{MB}	4.6	4.9	36.8	37.0
Number of Items Remanufactured by MB factory	All demanded items 13,000	Less than average demanded items 12,739	All demanded items 13,000	Less than average demanded items 12,802

Table 7: Numerical Example Results for MB's Single Facility and Single Market Case

		Exponential q and Deterministic D	Exponential q and Normal D	Normal q and Deterministic D	Normal q and Normal D
Total Profit π		$ 107,644	$ 59,733	$ 3,721,452	$ 3,603,820
Optimal Quality	$Q_{ON\ Factory}$	6.3	6.6	46.0	46.3
	$Q_{QC\ Factory}$	5.5	5.8	41.4	41.6
	$Q_{MB\ Factory}$	4.6	4.9	36.8	37.0
Number of Items Remanufactured by Factor i	ON Factory	135,000	132,340	135,000	133,060
	QC Factory	80,000	78,409	80,000	78,815
	MB Factory	13,000	12,739	13,000	12,802
Number of Items Delivered by Factory i to Market j –(V_{ij})	ON - ON	135,000	132,340	135,000	133,060
	ON - QC	–	–	–	–
	ON - MB	–	–	–	–
	QC - ON	–	–	–	–
	QC - QC	80,000	78,409	80,000	78,815
	QC - MB	–	–	–	–
	MB - ON	–	–	–	–
	MB - QC	–	–	–	–
	MB - MB	13,000	12,739	13,000	12,802
Operation of Facility i (ω_i) 0 = facility i is not operating 1 = facility i is operating	ON Factory	1	1	1	1
	QC Factory	1	1	1	1
	MB Factory	1	1	1	1

Table 8: Numerical Example Results for Multiple Facilities and Multiple Markets Case

We can notice from the results presented in the tables above that the settings in which quality is distributed normally are more profitable than the settings in which quality is distributed exponentially and has higher optimal quality readings. This is caused by the fact that in the setting where quality is normally distributed, most of the returns are in better shape compared to the setting where quality is exponentially distributed. Thus, the enterprise's total spending on remanufacturing will be enormous in the later case.

In both deterministic cases, demand has been fully satisfied. Intuitively, deterministic demands will be satisfied as long as the total spending on remanufacturing is less than the selling price plus the under-stocking cost (P + U). Also, we can notice the effect of demand uncertainty on facilities' production and its direct relationship with the over and under-stocking costs. As the over-stocking cost (O) exceeds the under-stocking cost (U) in value, the models work out the optimal qualities toward unsatisfying the average demand. Thus, the quality reading will be higher compared to the deterministic case. Vice versa, as the under-stocking cost (U) exceeds the over-stocking cost (O) in value, the models work out the optimal qualities toward satisfying or even exceeding the average demand in certain cases. Consequently, the quality reading could be lower than the quality reading for the deterministic cases.

From Table-9 we can notice that ON, QC and MB facilities are all operating, because they have ω_i values of one. Also, form the V_{ij} values we can observe that each market's demand is satisfied from its facility's production. Thus, if we apply the single facility and single market models for each of the market we should achieve the same optimal quality readings as if we apply the multiple facility and multiple market models. Also, as it is expected, the summation of all profit values for each market calculated by the single facility and single market models will give the same profit value calculated by the multiple facilities and multiple markets model.

Pondering upon the tables above, the reader might wonder how the models would allow for negative profit values. In chapter 6, adequate explanation is presented to clarify such models' behaviour. In any case, the negative values can be avoided by doing one or multiple of the following realistic actions that increase the enterprise's profit:

1. Sell tires at a higher price, if possible, and consequently change the under-stocking cost and over-stocking cost values.
2. Alter the acquisition and the remanufacturing processes in such a way that lower the total spending vs. quality curve and at the same time change its slope so that more profit can be generated.
3. Spread awareness among cars' and trucks' owners on how to maintain tires. For example, over inflating and under inflating are among the many causes that deteriorate tires' quality. Once this is accomplished, the average quality reading should increase and affect the profit positively.

CHAPTER 6: SENSIVITY ANALYSIS

6.1 Sensitivity Analysis for Single Facility and Single Market Setting

To proceed with the sensitivity analysis for the single facility and single market setting, the Ontario market has been chosen to conduct our study on. Ontario's parameters were driven again from Table-3 with the exception that the selling price was reduced to $235. Also, the slope of the remanufacturing cost vs. quality linear relationship has been increased by changing β value to one. Consequently, such a change in β value leads the total spending vs. quality linear relationship to be modified as depicted in Figure-4. Those two alterations were conducted to clearly allow the system to freeze at one point and reject any further decrease in quality as it will come later in this chapter.

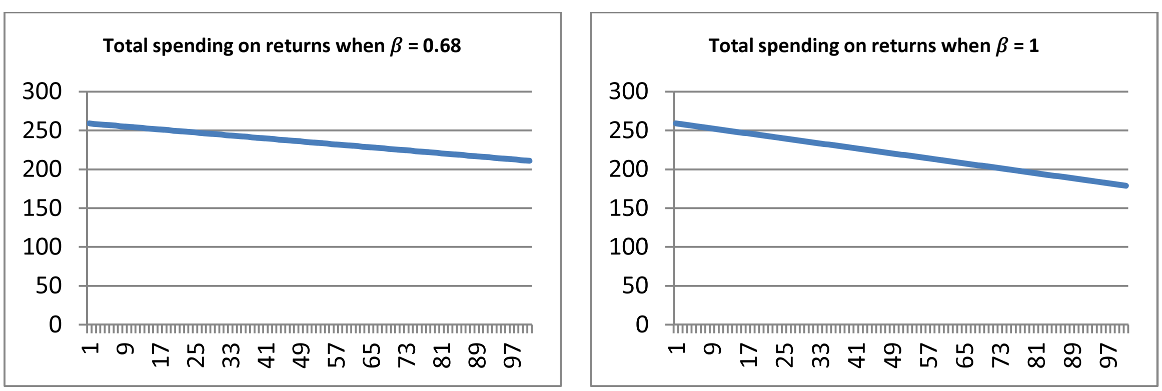


Figure 4: Effect of Changing β Value on the Total Spending vs. Quality Linear Relationship

6.1.1 Return vs. Optimal Quality and Profit

As the return value (R) decreases, more lower quality returns are needed. How much more depends primarily on the quality distribution used. Thus, the optimal quality and profit will decrease, as the value of return (R) decreases. By referring to Figure-4, we can notice that when a returned item has a quality value of lower than 30 its production will affect the profit negatively, because the total spending on remanufacturing will be more than the selling price P = 235. If needed, the model will allow the optimal quality to be less than 30, but not less than the quality value associated with (P + U) in the total spending vs. quality linear relationship which is 11.25 in our case. Thus, the system will freeze at 11.25 allowing for only the under-stocking cost to be acquired with each unsatisfied demand. What have been explained before can be noticed from the optimal quality vs. return curve in Figure-5.

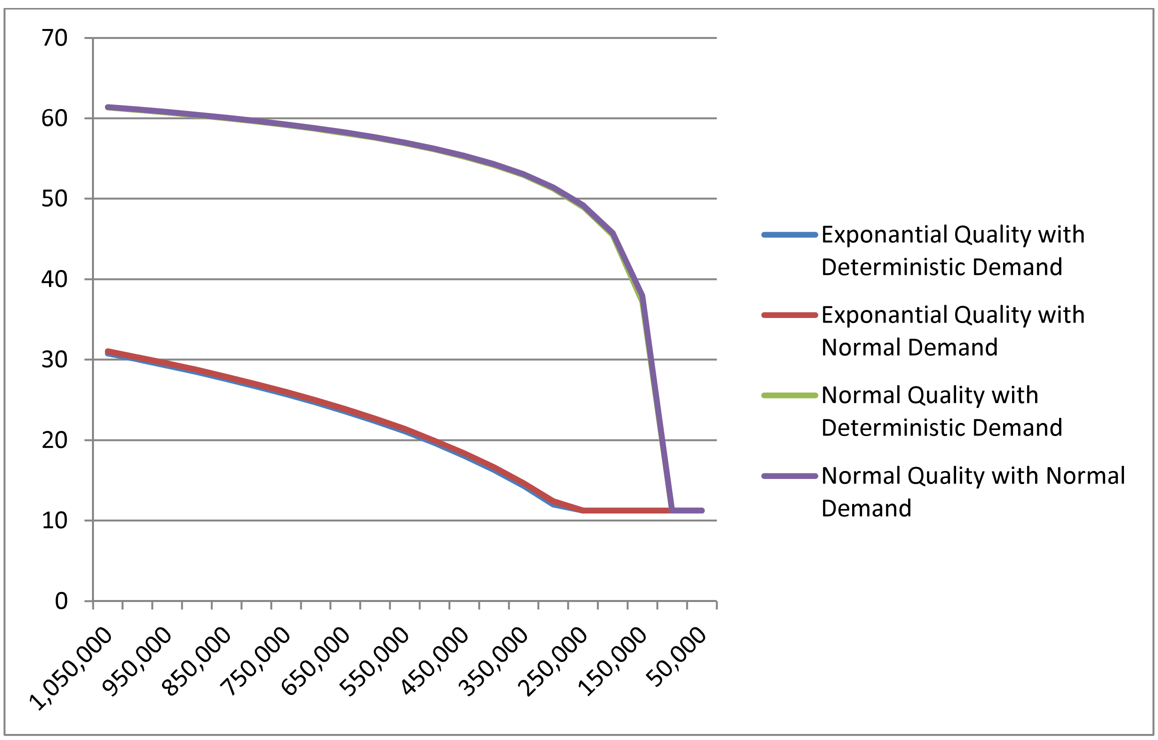


Figure 5: Optimal Quality vs. Return

We observe that the normally distributed system freezes at a lower return value (R) than the exponentially distributed system. This due to the fact that the normal distribution has most of its returns under the bell shape centered at a quality equal to 50. On the other hand, the exponential distribution has most of its returns accumulated towards the lowest quality possible. In general, it is noticed that a system with normal return behaviour is more appealing than a system with exponential return behaviour. Figure-6 shows the general trend in profitability and the superiority of normal distribution. It is, also, important to understand that this superiority could be reversed if the amount of returns is extremely high. This fact is true because each distribution behave differently towards its higher end of quality.

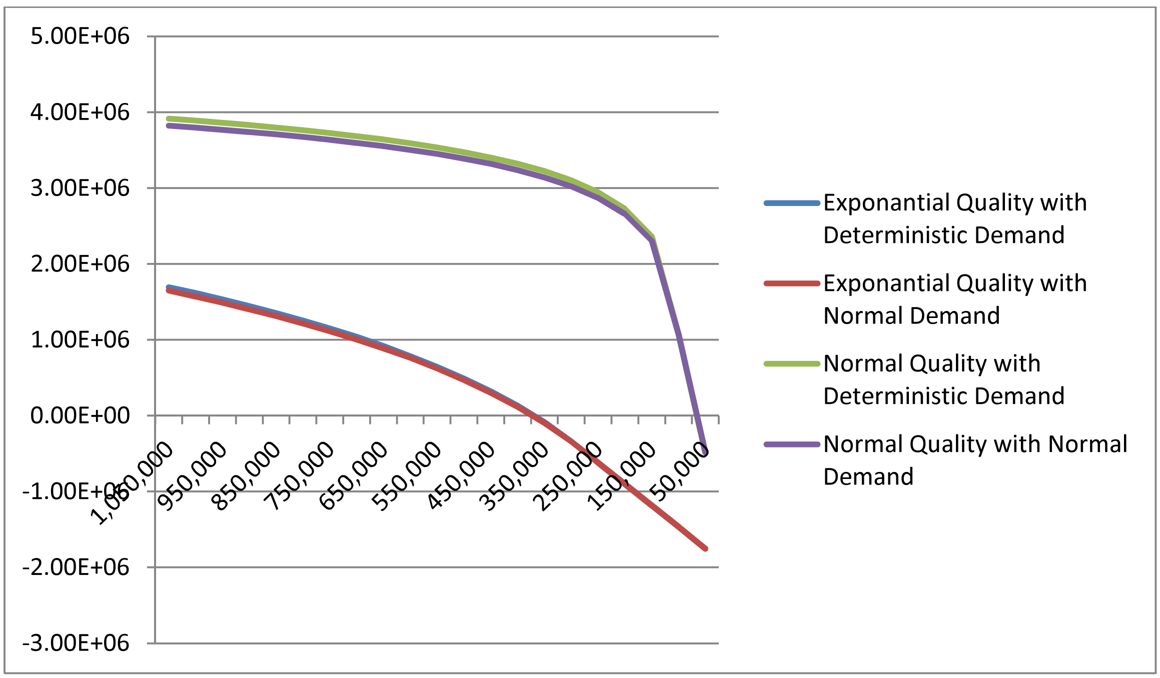


Figure 6: Profit vs. Return

6.1.2 Total Spending Relationship vs. Optimal Quality and Expected Unsatisfied Demand

a, α, b and β work in synchronization to set the total spending on each quality graded return that will be produced by the remanufacturing facility. In another word, they work together to identify the total spending vs. quality linear relationship. Therefore, a change in one of those parameters might be enough to explain all what needs to be explained in this subsection. Thus, (b) value will be changed from 0.1 to 0.9 to decrease the slope of total spending vs. quality linear relationship and consequently our decision variables. Also, as the value of (b) goes up, the quality associated with (P + U) value increases.

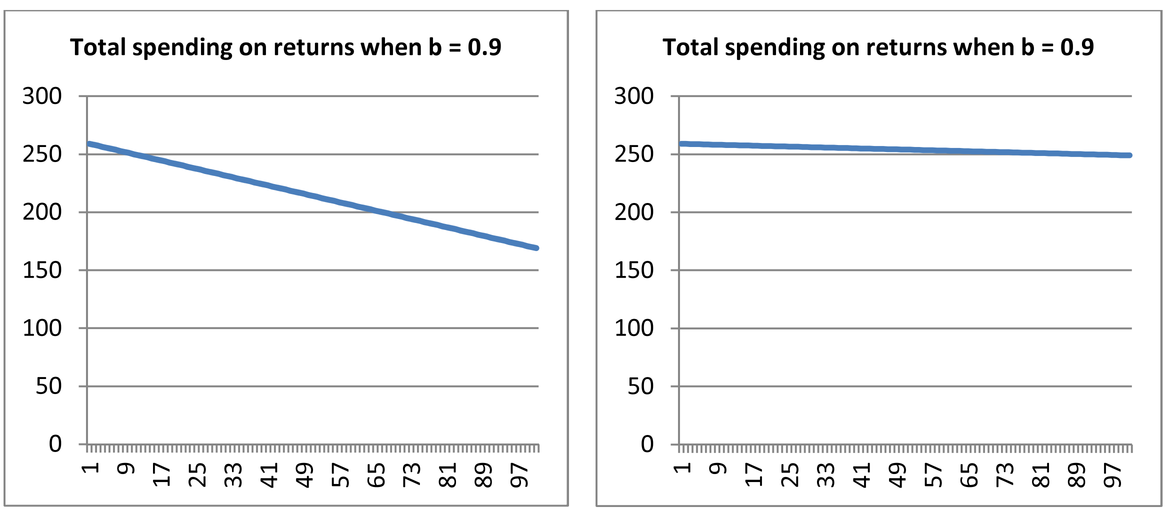


Figure 7: Total Spending vs. Quality Linear Relationships with Different b Values

As explained earlier, the models will behave normally by satisfying all demanded items in the deterministic demand cases and by considering all distribution parameters as well as (U) and (O) values in the stochastic demand cases. Such behaviour exists as long as the optimal quality is larger than the quality associated with (U + P) value in the total spending vs. quality linear relationship. If the quality needed to satisfy demand is less than the quality associated with (U + P) value, then the later quality is considered to be the optimal one. This is due to the fact that the facility is better of incurring under-stocking cost with each unsatisfied demand rather than a higher cost associated with remanufacturing low quality returns.

In our example, b value starts from 0.1 and the quality, at which the total spending will be U + P = 250, is 10. This is self explanatory when Figure-7 is studied. If quality is exponentially distributed, then the optimal quality is just less than 20 and if it is normally distributed, then the optimal quality is just above 56. We can notice that both optimal qualities are more than the quality associated with (U + P) value. As the value of (b) goes up, those optimal values will stay, roughly, the same until the quality associated with (P + U) exceeds 20 if quality is exponentially distributed and 56 if it is normally distributed. Then quality associated with (P + U) will become the optimal one and all curves will eventually overlap as they will have the same optimal quality. Also, the unsatisfied or expected unsatisfied demand will increase as (b) value increases. Indeed, exponential models get affected first, because they offer lower number of high quality returns. All what have been explained can be observed from Figure-8 & 9 below.

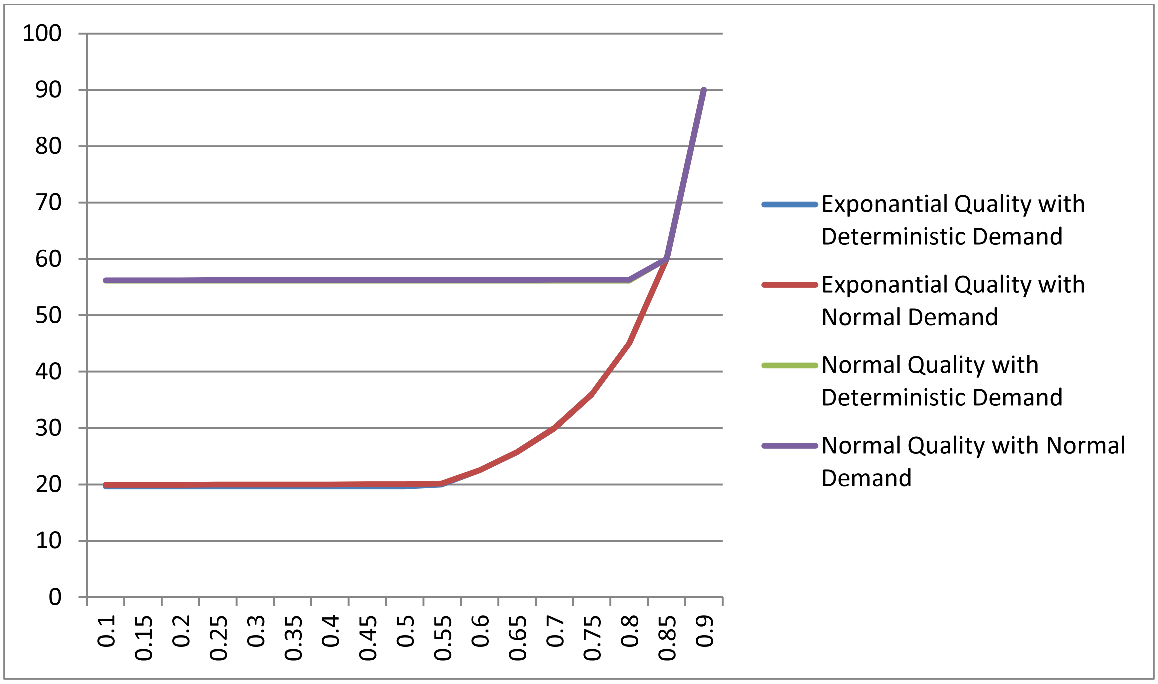


Figure 8: Optimal Quality vs. b Value

The expected unsatisfied demand, in the uncertain demand models, is calculated based on the equation below adapted from (Chopra & Meindl, 2010).

$$\begin{aligned} &Expeced\ understocking \\ &= \left(\mu - R\int_{Q}^{\infty} f(q)\,dq\right) * \left[1 - Fs\left(\frac{R\int_{Q}^{\infty} f(q)\,dq - \mu}{\sigma}\right)\right] + \sigma * fs\left(\frac{R\int_{Q}^{\infty} f(q)\,dq - \mu}{\sigma}\right) \end{aligned} \tag{48}$$

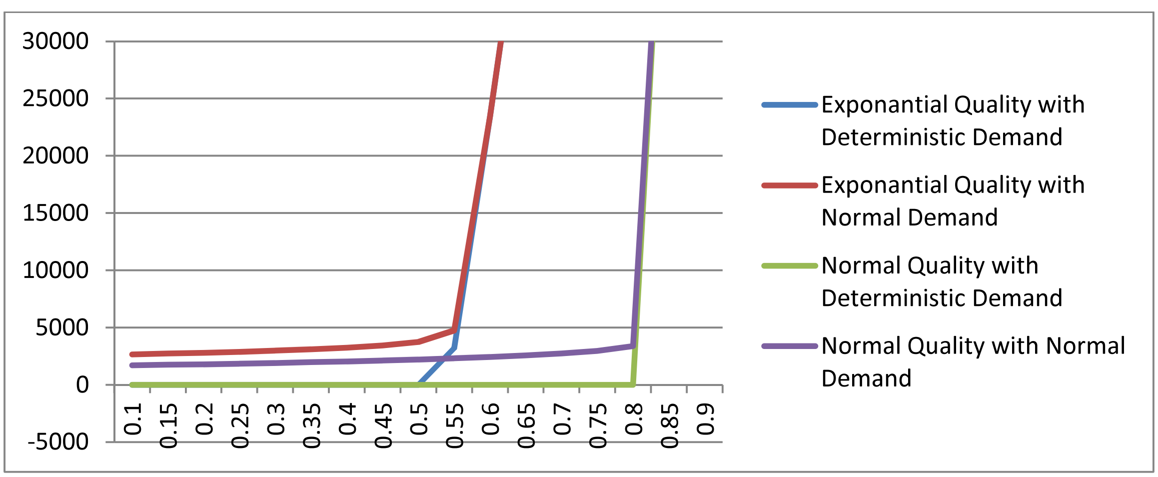


Figure 9: Expected Unsatisfied Demand vs. b Value

6.1.3 Quality Uncertainty vs. Optimal Quality and Profit

Changing quality uncertainty is accomplished by changing the value of σ_q or quality standard deviation whenever quality is normally distributed. To understand the effect of changing σ_q on both optimal quality and expected profit, high (R = 2,000,000) and low (R = 200,000) returns have been selected. When return is high enough, the optimal quality will be higher than the average quality (μ_q = 50). In this case, as σ_q changes from a higher to a lower value, quality uncertainty decreases and the bell shape of the quality distribution shrinks towards the average quality μ_q. As this happen, the model has nothing but to decrease the optimal quality in order to remanufacture enough returns for the optimal demand to be satisfied (Figure-10).

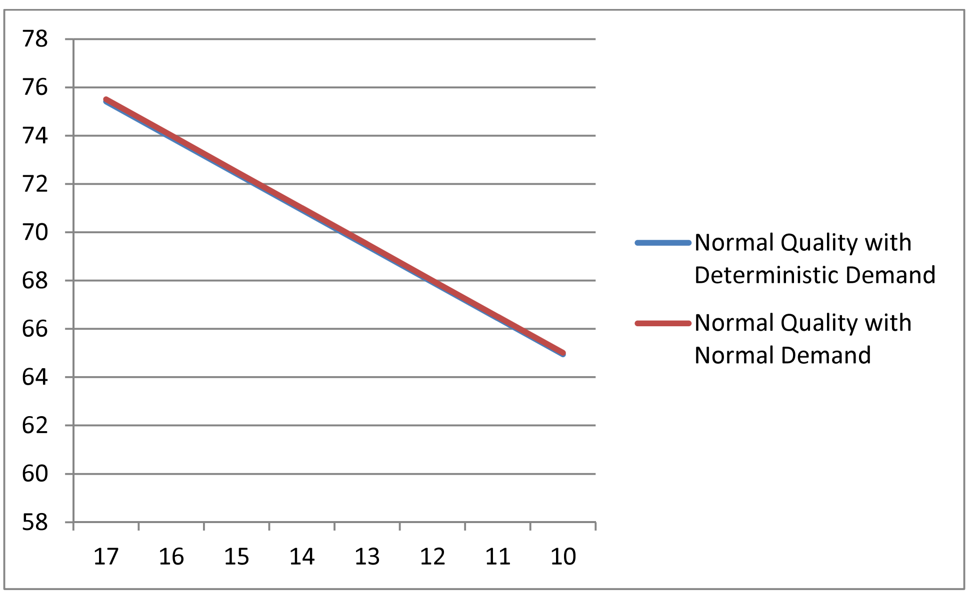


Figure 10: Optimal Quality vs. Quality Uncertainty When Return is High

On the other hand, if return is low, then the optimal quality will be lower than the average quality (μ_q = 50). Thus, as σ_q changes from a higher to a lower value, the model will increase the optimal quality, adapting the new bell shape, in order not to remanufacture more than the optimal demand to satisfy (Figure-11).

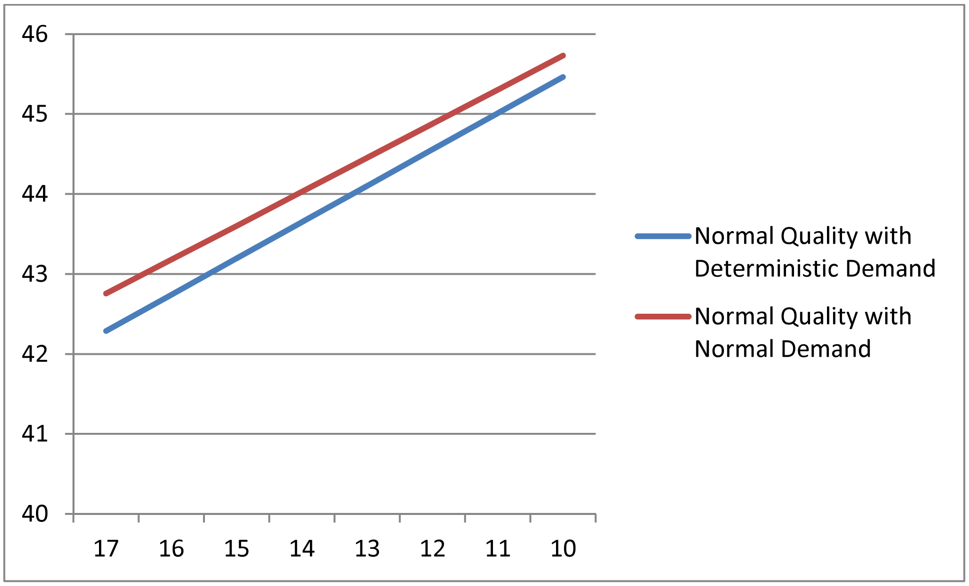


Figure 11: Optimal Quality vs. Quality Uncertainty When Return is Low

In all cases, as uncertainty decreases, the bell shape of the quality distribution will shrink. Therefore, such a process will deprive the system from having higher quality returns originally located at the right end of the quality spectrum. As a result, the profit will decline and the total spending will increase in the system due to the need of remanufacturing lower quality returns (Figure-12). This analysis did not consider a change in μ_q, as the outcome is obvious.

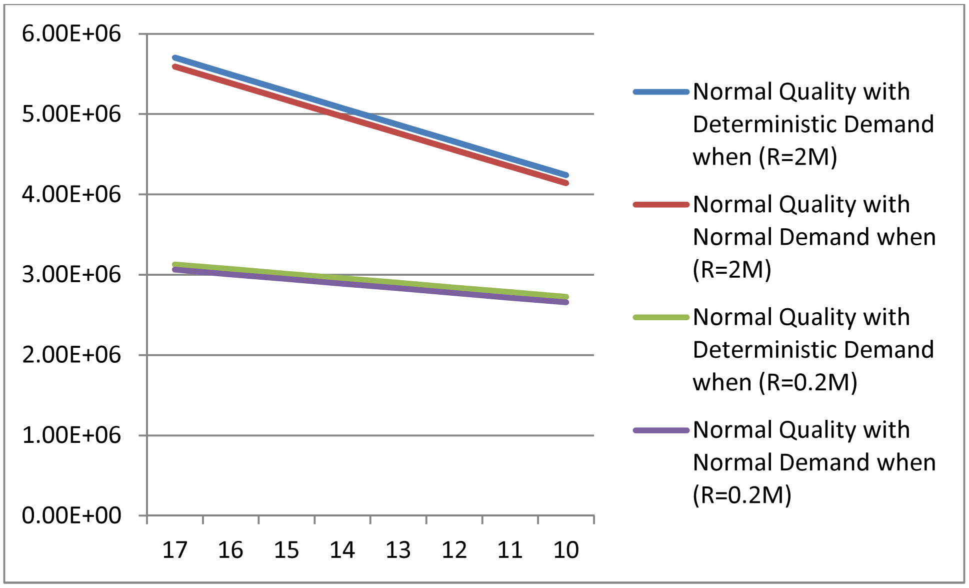


Figure 12: Profit vs. Quality Uncertainty

6.1.4 Demand Uncertainty vs. Optimal Quality and Profit

Demand uncertainty can be changed by changing σ_d or demand standard deviation in the uncertain demand models. As uncertainty increases from 1 to 10,000, the system's expected profit decreases (Figure-13). This is due to the fact that the increase in demand uncertainty increases the probability to experience more over and under-stocking costs and, thus, lower expected profits.

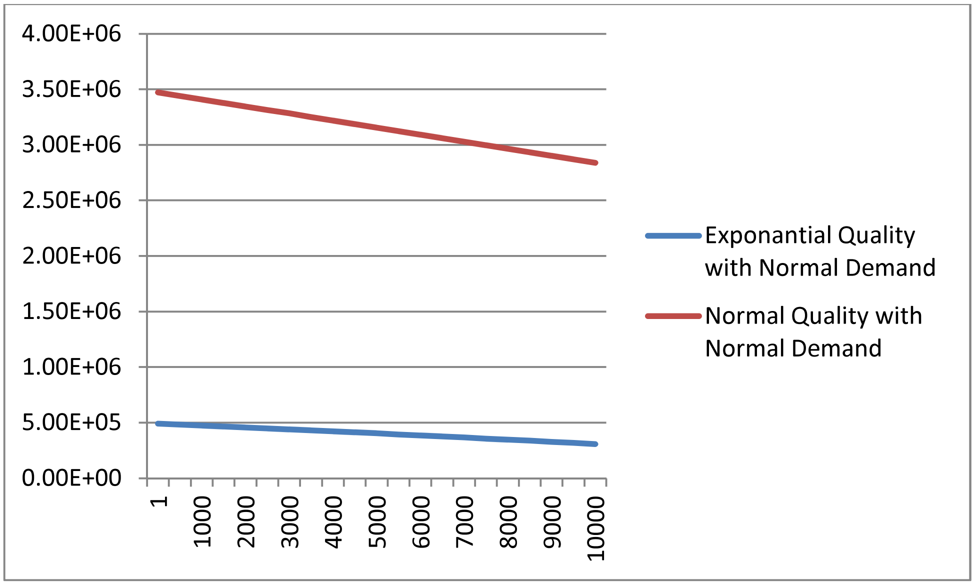


Figure 13: Profit vs. Demand Uncertainty

Also, in our example, the values used for the under-stocking cost (U) and the over-stocking cost (O) are 15 and 110 respectively. We can notice here that (O) is much more than (U). Therefore, as demand uncertainty increases, the optimal quality increases to avoid the increasing chance of bearing high over-stocking cost. We know that the increase in the optimal quality decreases the total returns remanufactured by the facility. Thus, allowing for lower total spending associated with remanufacturing. Both, the declination in total spending associated with remanufacturing and the need to avoid the over-stocking cost help the models to lower the optimal quality of the system. Such system behaviour is depicted by Figure-14 below.

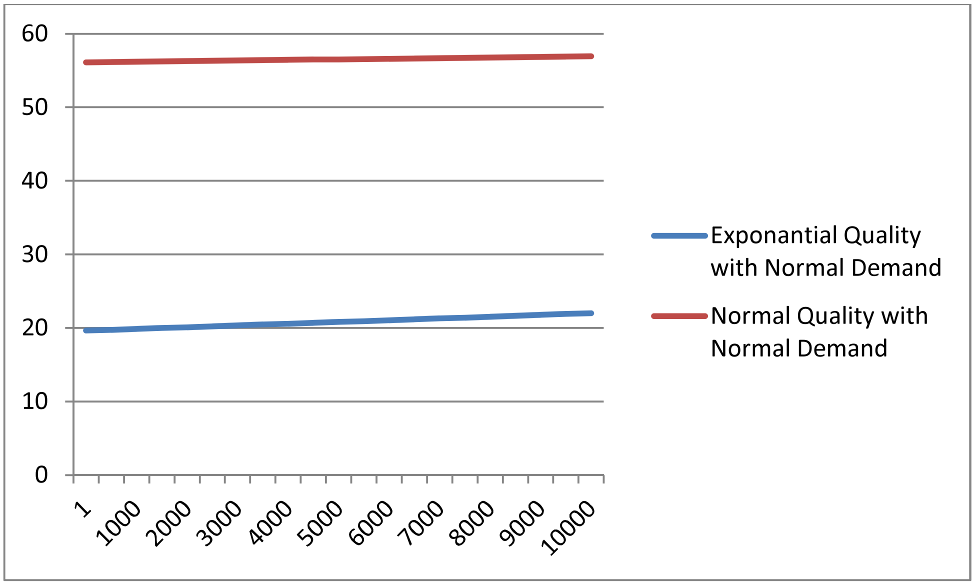


Figure 14: Optimal Quality vs. Demand Uncertainty

If it happens that (U) is more than (O), then there will be a trade off between decreasing the optimal quality to avoid the increasing chance of bearing high under-stocking cost and the high cost associated with remanufacturing lower quality returns. Thus, depending on the total spending vs. quality linear relationship and the difference between (U) and (O), the models could either decrease or increase the optimal quality as demand uncertainty increases.

6.2 Sensitivity Analysis for Multiple Facilities and Multiple Markets Setting

As we know by now that our objective is to maximize the profit of the enterprise. If the enterprise owns many facilities, then the total profit could be increased by managing critical tradeoffs between three different costs. The first cost, is the total spending associated with remanufacturing returns. The more facilities are utilized or opened for remanufacturing purposes, the less the total spending would be. This is due to the fact that with more facilities, the enterprise will be able to utilize more high quality returns from all markets. As a result, the enterprise can satisfy demand by remanufacturing those high quality returns with low total spending. The second cost, is the cost of transportation. The higher the transportation cost, the higher the need for more facilities to satisfy different markets' demand. The importance of this factor depends greatly on the industry and the size of the remanufactured products. As the products become bigger, bulkier and heavier transporting products will become more costly. As a result, the urge to increase the number of remanufacturing facilities becomes higher. The factory setup cost is the third cost that should be considered while managing the critical tradeoffs. High facility setup costs are always associated with the need to aggregate production. The interactions between those three costs while seeking optimality when certain parameters are changed will be discussed in the next two subsections.

The sensitivity analysis that are to be conducted will only consider the deterministic demand cases and will pay no attention to the differences in behaviours between the different quality distributions, because the effect of demand and quality uncertainties have been discussed in the previous section. Also, quality will not freeze before satisfying all demand, thus all demanded tires will be satisfied. All parameters used for this analysis are driven from Table-3. Similar as before, some of those parameters have been changed in order to clearly show certain attributes in the models.

6.2.1 Transportation Cost vs. Facility Production, Optimal Quality and Profit

In this Analysis, we will use the model with normal quality and deterministic demand setting. Also, the parameters that have been altered in this analysis include the amount of returns R_i where they have been doubled. Higher separations between the different markets' average qualities and quality standard deviations have been implemented. To conduct the analysis, transportation cost per mile per tire has been decreased from 0.02 to 0.001. At the higher transportation costs, the enterprise has nothing but to supply each market's demand from its own facility. This trend has continued until the transportation cost became 0.018.

As the transportation cost decline further, we can notice that ON's production increases, while QC's production decreases. This could be achieved by decreasing and increasing ON's and QC's optimal qualities respectively. To understand what is happening here we need to notice that ON's returns average quality is 50, while QC's returns average quality is 40. This implies that if R_{ON} is very high, then ON's facility could possess extra high quality returns that could be cheaply remanufactured and still be transported to QC at a lower cost than producing from QC. Moreover, not all QC's demand is satisfied by ON's facility, because QC's facility still receives many high quality returns that are cheaply remanufactured. Thus, it is not optimal yet to close the facility and save the set up cost. At this stage, the cost associated with remanufacturing ON's high quality returns is not low enough to overcome the high transportation cost between ON and MB. Therefore, MB's facility will completely satisfy its own demand.

When the transportation cost is, as low as, 0.012 and below, ON's facility supply portion of QC's and MB's demand for the same reasons mentioned above. By further decreasing transportation cost to 0.009, the enterprise can shut down MB's facility and satisfy all of its demand from ON's facility. Indeed, MB's demand D_{MB}, amount of return R_{MB} and average quality of returns $\mu_{q\,MB}$ are the lowest among all facilities. Thus, it is a great recipe to close MB's facility and save the setup cost as long as the transportation cost is insignificant. From Figure-17, we can notice the great change in the enterprise profitability as a result of this shut down. All what have been explained above can be easily grasped from Figures-15 and 16 below.

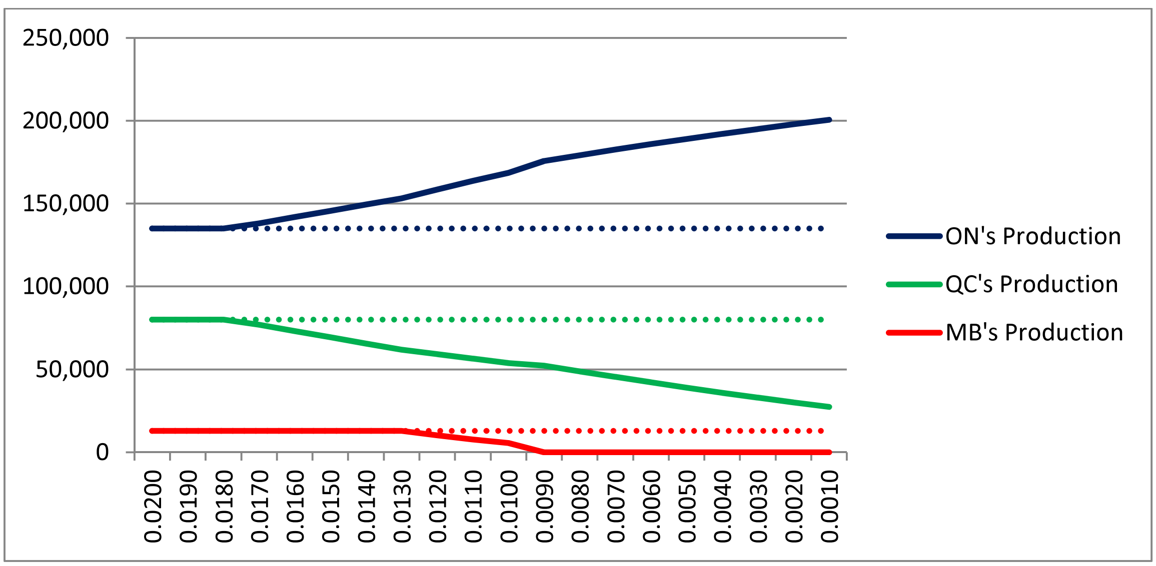


Figure 15: Facilities' Production vs. Transportation Cost

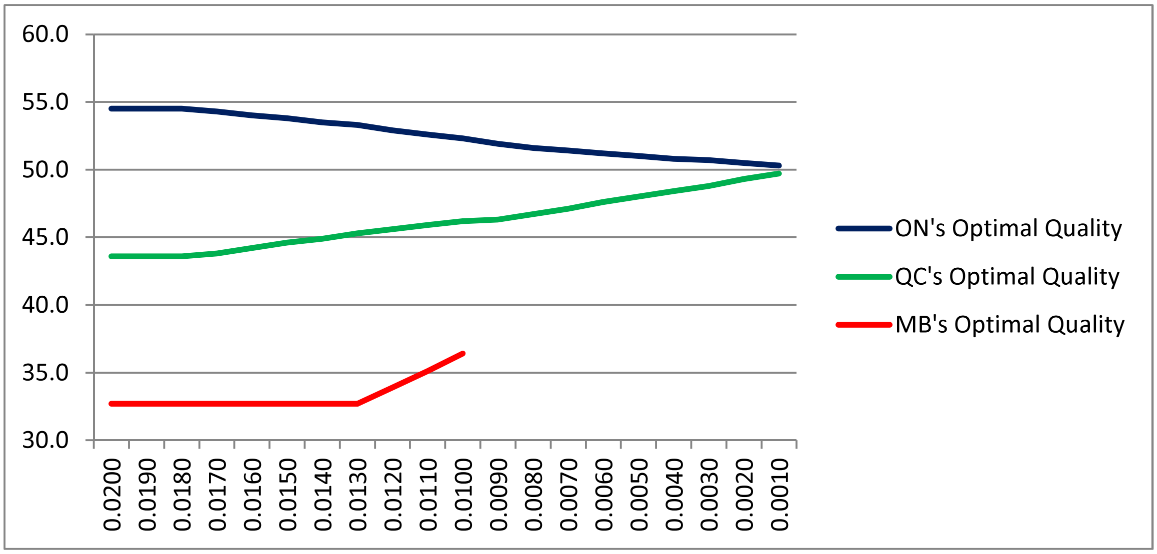


Figure 16: Facilities' Optimal Quality vs. Transportation cost

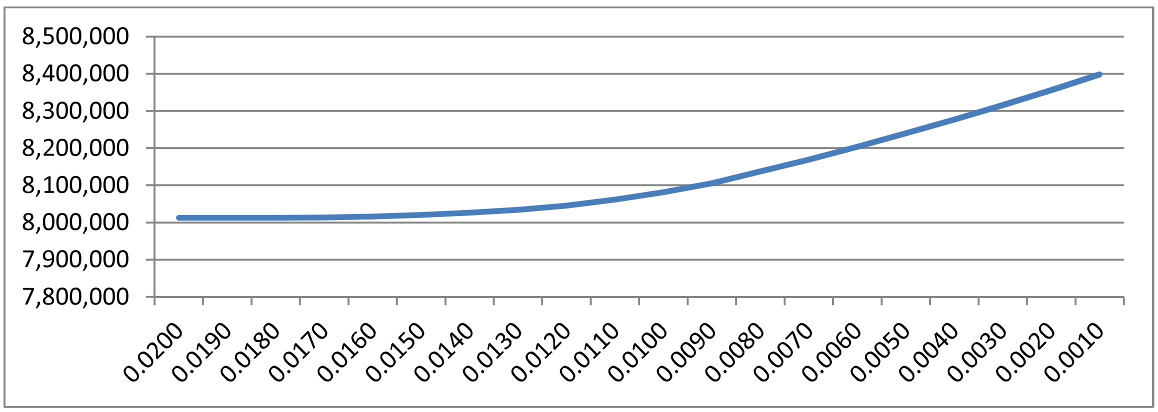


Figure 17: Profit vs. Transportation Cost

6.2.2 Markets' Demand vs. Facility Production, Optimal Quality and Profit

Again, the model with normal quality and deterministic demand will be used in this analysis. Markets' demand will start from the values given in Table-3 for each market. A twenty percent reduction will be performed on markets' demand in order to proceed to the next experiment. In total, 30 experiments have conducted on the specified model. Figuer-18 presents the outcome of all conducted experiments, while Figure-19 presents the outcomes segmented into four stages.

The analysis in this model is very similar to the analysis conducted in the previous subsection. Thus, the detailed analysis will not be presented here. In any case, similar to what have been mentioned before, as markets' demand decrease the facility with high return R_i and average quality will have the chance to possess extra good quality returns and supply other markets that have low returns R_i and average qualities. Figure-19 clearly shows the first stage where each market's demand is satisfied by its own facility. Followed by the stages where ON's facility supplies QC's market only and then by the stage where it supplies both QC's and MB's markets. This goes on until all demanded tires are supplied by ON's facility and every other facility is shutdown to save the setup costs.

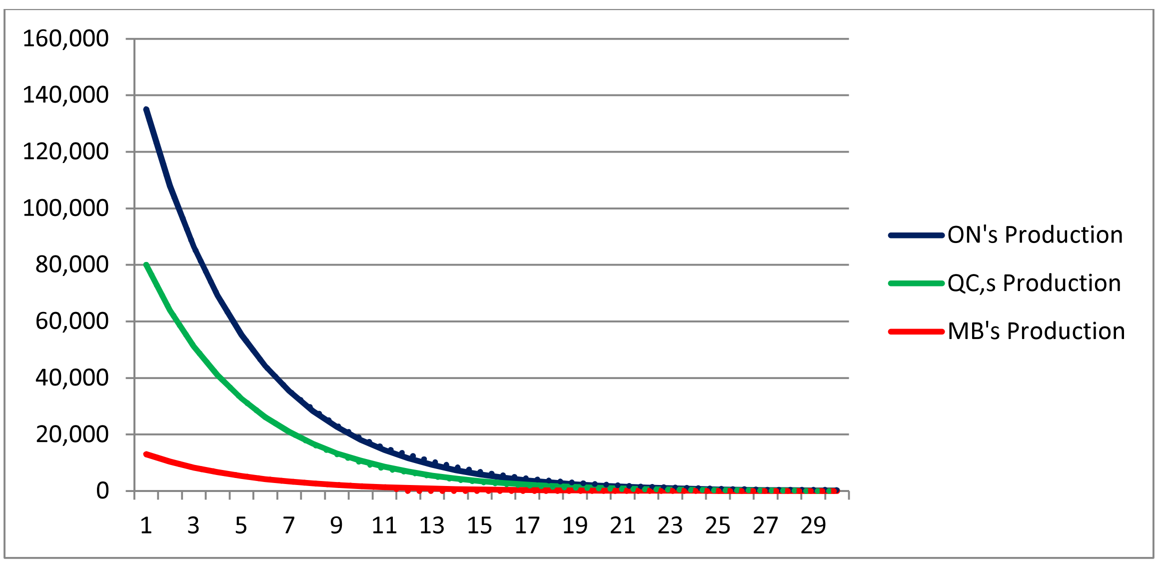
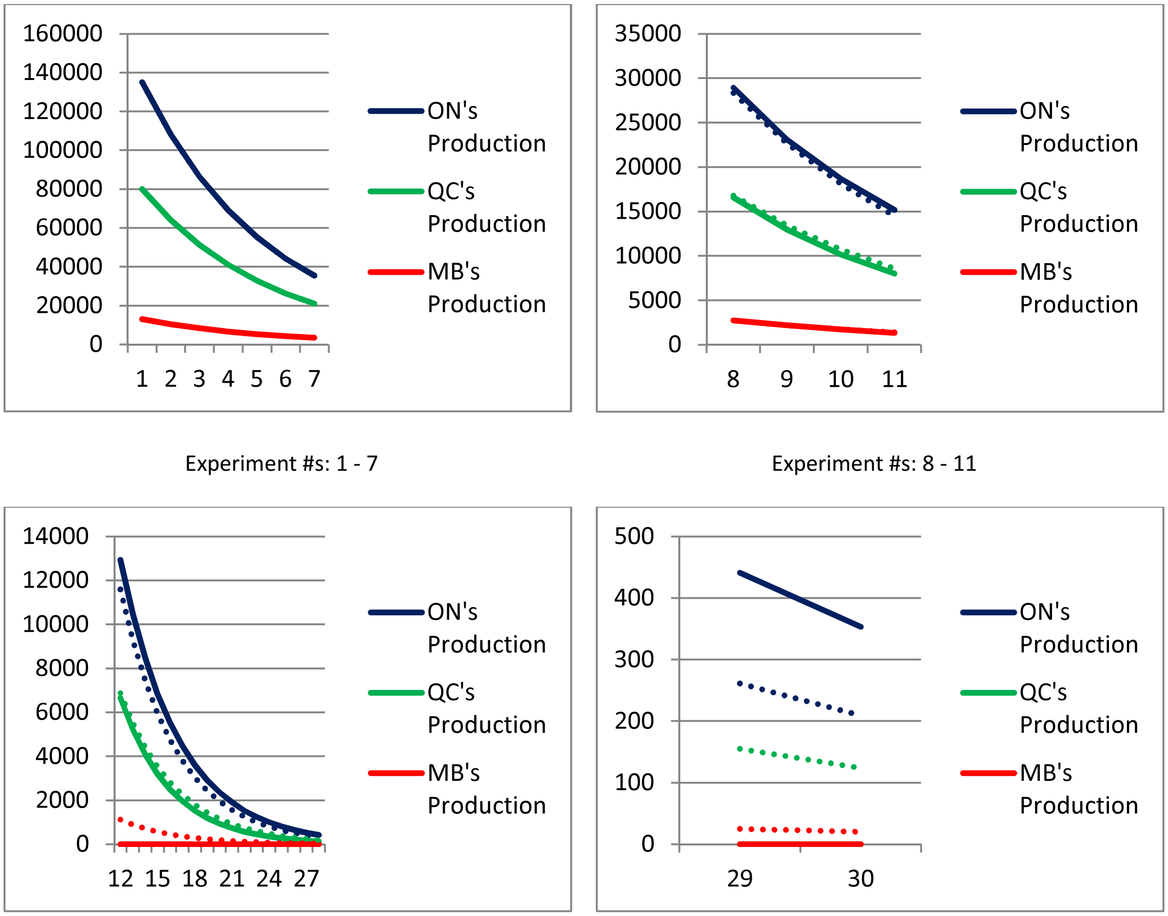


Figure 18: Facilities' Production vs. Markets' Demand in One Graph



Experiment #s: 12 - 28

Experiment #s: 29 - 30

Figure 19: Facilities' Production vs. Markets' Demand in Segmented Graph

In the previous two figures the dotted lines indicate the production needed from each facility if it is to satisfy its own market's demand only.

Indeed, by now we know that the amount of returns remanufactured by each facility is controlled by the optimal quality. From Figure-20 we can notice that as markets' demand decrease the facilities' optimal quality increase implying that the system can avoid some of the low quality returns. With each facility shut down, we can observe that it coincides with a slight drop in other facilities' optimal qualities. This is caused by the urge to satisfy a sudden increase in demand by the facility or facilities in operation.

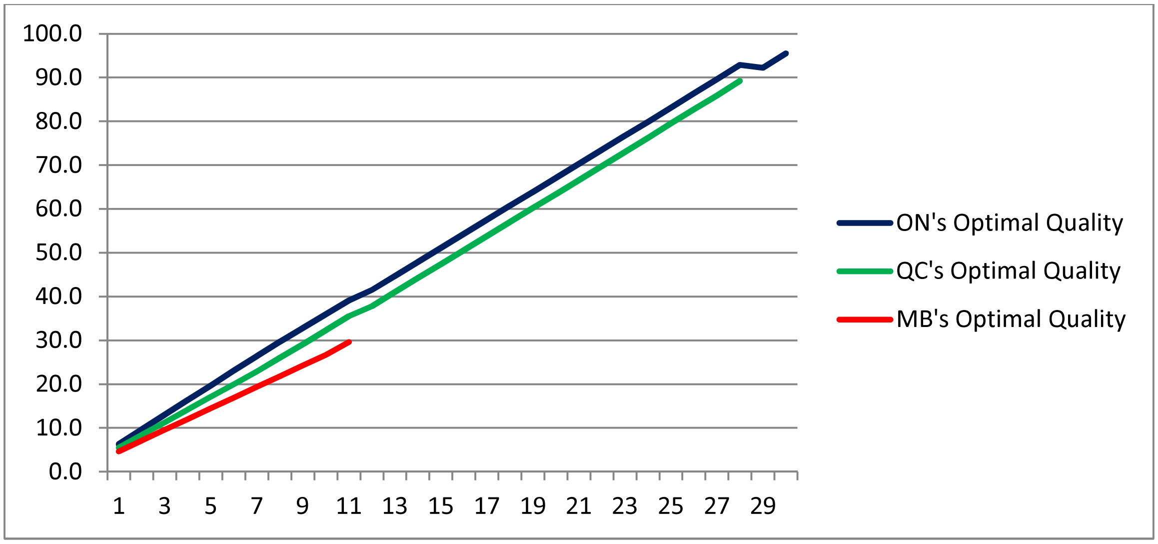


Figure 20: Facilities' Optimal Quality vs. Markets' Demand

In addition, we can notice from Figure-21 that the profit, generally, decreases as demand decreases. This is totally intuitive, but what could be confusing is the increasing pattern at the high markets' demand. As we comprehended from previous sections, the facility is better off remanufacturing returns even if the total spending is more than (P) value, but less than (P + U) value. Thus, when demand is high, the facility had nothing but to satisfy portion of that demand from the unprofitable low quality returns to avoid a higher cost of under-stocking (U). As a result, the profit had been affected negatively.

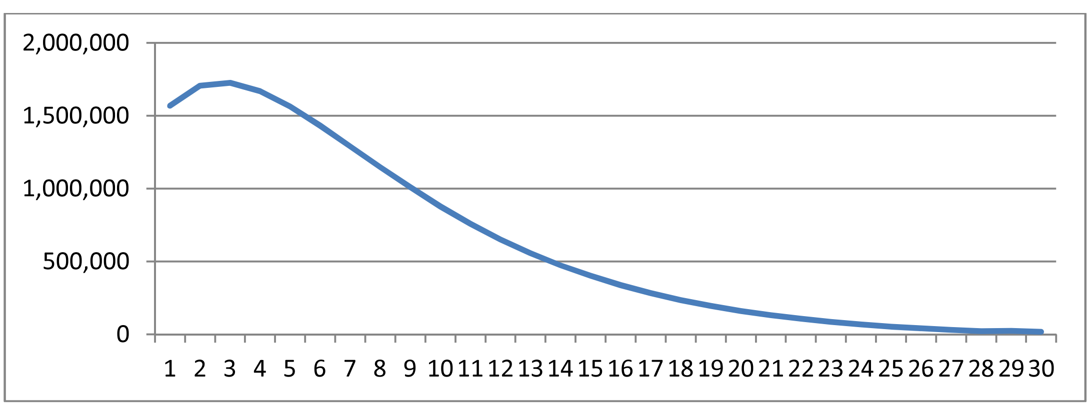


Figure 21: Enterprise Profit vs. Markets' Demand

Indeed, the general definition of the under-stocking cost (U) is the profit lost due to unmet demand and it could be calculated by the difference between the selling price and the manufacturing cost. In contrast, when (U) is applied in a remanufacturing context, its calculation should follow a different approach. Truly, quality of returns should influence the value of (U), because the profit lost associated with missing demand that could be met by remanufacturing very high quality returns is completely different than the profit lost associated with missing demand that would be met by remanufacturing very low quality returns. Such quality influence on (U) could be studied further in later researches.

CHAPTER 7: CONCLUSIONS AND FUTURE RESEARCH

7.1 Conclusion

Quality uncertainty and its effects on remanufacturing systems are among the hot topics that need great attention due to their immense financial impact. Also, the numerous papers published in the remanufacturing literature have abundantly considered perfectly substitutable products, although many products in today's market are not perfectly substitutable. This work, as noticed, has studied system's performance under the impact of both issues. Also, we were able to study system's behaviour when both acquisition price and remanufacturing cost vary linearly with the quality or condition of the returned items. The summation of those two costs forms a decreasing linear relationship with the quality in such a way that producing better quality returns is more profitable than producing worse quality returns. In addition, this work has considered networking when quality is uncertain due to its great influence on system`s optimal values.

In order to apply this study, the remanufacturing enterprise should be able to thoroughly inspect and then quality grade each return based on its condition. This quality grading should be performed before deciding whether to accept or reject returns. Also, the remanufacturing process and cost should be quality dependent either totally or partially. Since remanufacturing cost is quality dependant, acquisition price or return cost should be quality dependant. The dependence of acquisition price on quality could lead suppliers to better select returns or could lower cost associated with remanufacturing. Indeed, safety laws and municipal regulations could be a decision factor whether to apply or not to apply this study. One of those industries that could benefit from our work is the tires retreading industry under the Canadian regulations.

This research developed non-linear programming models to find the optimal minimum quality to accept into the remanufacturing facility if the enterprise posses single facility to satisfy single market's demand. In case the enterprise possesses multiple facilities to satisfy multiple markets' demands, the research developed mixed integer non-linear programming models to select facilities to operate, to find the optimal minimum quality to accept into each operating facility and to find portion of market's demand to satisfy from each operating facility. Quality in both cases is assumed to be either exponentially or normally distributed, while demand is assumed to be either deterministic or normally distributed.

By using GMAS solvers to solve the models, we were able to see how the quality and profit are affected by the amount of items returned to the remanufacturing facility. Indeed, the models with exponential quality behave differently than the models with normal quality due to the differences in concentration of returns along the quality spectrum. Also, the effect of changing linear relationships' parameters (e.g. a, α, b and β) on the system's behaviour has been studied. Understanding such behaviour allows us to better manage the remanufacturing enterprise and increase its profitability.

As far as normally distributed quality is concerned, system's reaction to lowering quality uncertainty is not the same. If the amount of returns is found to be much more than demand, then the optimal quality decreases as uncertainty decreases. Vice versa, if the amount of returns is much less than demand, then the optimal quality increases as uncertainty decreases. In all cases, the increase in quality uncertainty affects the objective function or the profit negatively. Furthermore, when demand is considered to be stochastic, system's reaction to the increasing demand uncertainty is not always increasing nor always decreasing the optimal quality. It depends greatly on the values of under-stocking cost (U), over-stocking cost (O) and the different costs vs. quality relationships. Again, when demand uncertainty increases, the objective function or system's profit decreases.

In the multiple facility and multiple market cases, there are always tradeoffs between setup cost, transportation cost and the total spending associated with remanufacturing. The higher the number of operating facilities, the higher the setup cost, and the lower the transportation cost and the total spending associated with remanufacturing. Therefore, it is imperative to manage those three sources of cost effectively in order to reach optimality. Such management might lead the enterprise to operate a remanufacturing facility in one market, but at the same time satisfy portion of that market's demand from a remanufacturing facility assigned for a different market.

7.2 Future Research

We have noticed that the literature concerning remanufacturing lacks the involvement of quality uncertainty and its effects on systems' behaviour. Thus, this subsection suggests several topics to be further investigated in order to better understand quality in the remanufacturing context. The influence of price on quality could be further investigated. For example, in the literature concerning remanufacturing, higher price is associated with higher returns. After the accomplishment of this work we came to know that higher price could be associated with both higher returns and better returns' condition or quality.

Also, return in our models is assumed to be deterministic. Therefore, a possible extension of this work is to consider return to be stochastic. Furthermore, we have noticed in our model that the more return received by the remanufacturing facility the better it is for the remanufacturer and the worse it is for the supplier or the hauler due to transportation cost. This is supported by the fact that our models attach no cost to any return before it is accepted and remanufactured by the facility. Therefore, return may, also, be considered as a decision variable where the optimal value of return is greatly influenced by the pre-remanufacturing inspection cost and penalty associated with any rejected return.

As has been addressed before, under-stocking (U) and over-stocking (O) costs should be quality related when applied in a remanufacturing context. For example, in a normal production or manufacturing system, (U) is considered to be the profit lost due to leaving one demand unit unsatisfied or the difference between the selling price and the manufacturing cost. Such value is found to be constant. In a remanufacturing environment, the total spending associated with remanufacturing is not constant, but rather quality dependant. Therefore, having (U) and (O) defined based on quality could be a possible extension to this work or to any other work that uses newsvendor like model to resolve remanufacturing issues.

In our work, we have considered only two attributes that are quality dependant. In another problem setting, there could be more. For example, if the enterprise stores returns before the remanufacturing process, then pre-remanufacturing inventory cost should be considered in the model. Such cost is quality dependant, because better quality returns were purchased with higher prices than lower quality returns. Moreover, if remanufacturing lead time is to be considered, then it could be quality dependant too. As a result, higher quality or better condition returns take lower time to remanufacture.

Finally, our models could be, further, extended to include multi-periods, multi products and dependant markets settings.

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VITA AUCTORIS

NAME:	Mohannad Hassan Radhi
PLACE OF BIRTH:	Jeddah, Kingdom of Saudi Arabia
YEAR OF BIRTH:	1979
EDUCATION:	King Fahad University of Petroleum and Minerals, Dhahran, Kingdom of Saudi Arabia B.S.c in Mechanical Engineering University of Windsor, Windsor, Ontario M.A.Sc. in Industrial Engineering