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Commercial Banking and Interbank Insurance: The Diamond-Dybvig Model Revisited

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Abstract

The goal of this paper is to investigate the possibility of incorporating interbank insurance among commercial banks. This is done by building upon the Diamond-Dybvig model. We extend the Diamond-Dybvig model in three ways. First we abandon the single intermediary environment by introducing a banking sector that is made up of a continuum of banks of mass 1; we assume that a small proportion (exogenously given) of banks experience runs. Next, we suppose that all banks have access to and invest in interbank insurance. Lastly, it is assumed that when banks withdraw illiquid funds prematurely they must pay a mandatory transaction cost. In a banking system with liquidity shocks we show that, by designing an optimal interbank insurance contract, the possibility of a bank becoming illiquid during a bank run is zero.

Keywords: Interbank insurance, Liquidity shocks, Bank runs.

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Contents

1 Introduction

Financial crises have been widely discussed and documented by economists over the last half-century. From the banking panics of 1930-31 to the 2008 financial crisis, we have learned that history tends to repeat itself; we cannot rule out financial crises from happening. However, in recent years banking panics rarely occur.

It is important to understand the role of banks and why they are crucial to our economy. A core function of banks is to create liquidity (Berger et al., 2015). That is, banks use liquid liabilities to fund long-term illiquid assets (Bryant, 1980). Banks do this by offering demand-deposit contracts which allow its customers to withdraw funds at any point in time while providing them access to profitable long-term investments (Goldstein and Pauzner, 2005).

Liquidity creation plays a significant role in any economy (Berger et al., 2015). Diamond and Dybvig (1983) showed that the level of utility achieved via demand-deposit contracts offered by banks is unattainable under autarky. That is, agents can be made better off when banks are introduced. However, when banks create liquidity they leave themselves exposed to risks such as bank runs (Diamond and Dybvig, 1983). A bank run is a situation where all agents, concerned about a bank's solvency, rush to the bank to withdraw funds; this may cause a bank to run out of funds and be unable to serve all of its customers (Goldstein and Pauzner, 2005).

Many different remedies for bank runs have been put in place so that banking panics do not occur. For instance, suspension of convertibility can be applied so that agents lose incentive to run on the bank.[1] Diamond and Dybvig (1983) show that if the number of agents withdrawing are randomly distributed then suspension of convertibility does not eradicate

[1] Suspension of convertibility works in the following way: those who want to withdraw early get what is promised by the contract as long as the fraction of agents withdrawing early is below a certain threshold; if this fraction is beyond that threshold then agents must wait until the next period to withdraw funds (Diamond and Dybvig, 1983).

bank runs. It is shown that if the number of agents running on a bank is random, then the best policy to implement is government deposit insurance; this will rule out panic based bank runs (Diamond and Dybvig, 1983).[2]

The model created by Diamond and Dybvig (1983) was used as the foundation for this paper. It was shown, using the results obtained by Diamond and Dybvig (1983), existence of multiple Nash equilibria when banks offer demand-deposit contracts. One Nash equilibrium is known as a "truth-telling" equilibrium. That is, if all agents tell the truth about their types, and this is common knowledge, then they withdraw funds based on these types and this constitutes a Nash equilibrium.[3] Another equilibrium, that is known as a bank run equilibrium, also arises. A bank run equilibrium occurs when agents believe banks will become insolvent. Agents withdraw in the current period and even those who do not wish to claim early consumption have incentive to run since they will be left with nothing if they do not. That is, if agents wait until the next period then they get zero with certainty since the bank would become insolvent.

The goal of this paper is not to find another way to eradicate bank runs but, instead, to design a contract in which the possibility of banks becoming illiquid (during a bank run) is zero. First we abandon the single intermediary environment by introducing a banking sector that is represented by a large number of banks.[4] Introducing a large number of banks opens the door to linkages amongst banks. We assume that a mass α of banks experience runs while a mass $1-\alpha$ of banks do not (banks do not know if their location will be hit with a bank run, they just know that bank runs occur). The next extension we make is that banks

[2] Government deposit insurance promises a fixed amount to those who withdraw early and is financed by imposing taxes on impatient agents (Diamond and Dybvig, 1983). These taxes are proportional to agents' wealth. A key assumption made by Diamond and Dybvig (1983) is that the decision on how much to tax may be decided after observing the number of agents withdrawing early.

[3] There are two types of agents: impatient and patient. Impatient agents withdraw funds early while patient agents withdraw funds late.

[4] We assume that there is a continuum of banks of mass 1. Note that each bank serves a continuum of agents of mass 1.

put aside an amount for interbank insurance. All of these amounts are pooled and evenly distributed among those banks experiencing runs. This insurance is put into place so that banks that experience runs are able to cover all of its customers. Lastly we assume that when banks liquidate assets (during a panic) they must pay a liquidation fee.[5]

The optimal interbank insurance contract is designed in the following way: with probability α an agent is located at a bank that experiences runs, and with probability $1 - \alpha$ an agent is located at a healthy bank. We want to choose the amount each banks pays into interbank insurance such that all agents withdrawing early get what is promised by the contract whether they are located at a healthy bank or not. By construction we have designed an optimal interbank insurance contract that rules out the possibility of banks, who opt for this insurance, becoming illiquid during a bank run. Again, we are not in pursuit to find a remedy that rules out bank runs completely; instead we use interbank insurance so that banks do not default in times of crisis.

Much like how deposit insurance leads banks into investing in speculative projects, interbank insurance could potentially have this same effect on banks. That is, since banks cannot become illiquid under the optimal interbank insurance contract (no matter the state they are in) this could lead to banks engaging in activities that could have a negative impact on the economy. To ensure this does not happen, a regulator is introduced in the model. The role of the regulator is to watch over all banking activities within the banking system (Cooper and Ross, 2002).

We suppose that the regulator imposes a restriction on the banks' ability to offer agents the same amount of funds when experiencing a crisis and when they are not. That is, the amount a bank experiencing a run offers its customers[6] is equal to a proportion of what

5 Diamond and Dybvig (1983) assume that banks can prematurely withdraw the illiquid technology at no cost.

6 Note here that agents receive the same amount during a crisis, no matter their type since they all rush to withdraw funds early.

healthy banks may offer its impatient customers. We show, through a numerical example, that it is socially optimal for agents who are located at banks that experience runs to earn the same amount as impatient agents do at healthy banks.

2 Literature Review

There has been a considerable amount of research done in the past four decades that has helped us understand bank runs. Bryant (1980) along with Diamond and Dybvig (1983) were among the first economists to show how bank runs occur and what can be done to eradicate them. For instance, Diamond and Dybvig (1983) showed that a panic (about a bank's solvency) arises among depositors when a bank mixes its liquid liabilities with illiquid assets. Diamond and Dybvig (1983) showed the existence of multiple equilibria: an equilibrium where all agents get what is desired and another where all depositors withdraw early and cause the bank to become illiquid. Diamond and Dybvig (1983) showed that through the use of government deposit insurance bank runs can be ruled out completely.

The vast majority of the literature on bank runs was built upon the work done by Diamond and Dybvig (1983). Cooper and Ross (2002), who used an extension of the Diamond-Dybvig model, show that full deposit insurance can lead to a moral hazard problem where intermediaries invest in speculative projects and the first-best contract cannot be achieved. It is shown that after imposing an additional capital requirement on banks the first-best contract is obtained (Cooper and Ross, 2002).

Allen and Gale (2000) extend the Diamond-Dybvig model by replacing the single intermediary environment with four identical banking regions; their goal was to investigate the possibility of financial contagion when banks are interlinked. These four regions are interlinked in the sense that banks hold claims on other banks to provide insurance when they experience liquidity shocks (Allen and Gale, 2000). Each region is represented by a single bank and it is shown that if one bank experiences a bank run then this will become contagious and spread into the other regions if the banking system is not complete; however, if the structure is complete the banking system might survive (Allen and Gale, 2000).

Investigating the possibility of systemic bank runs has been widely studied and discussed ever since the 2008 financial crisis. Uhlig (2010) considers a three period model in which there are a fixed number of core banks, a continuum of local banks of mass 1, and a continuum of agents of mass 1. The interesting difference in this paper compared to others previously mentioned is that the local banks run on the core banks rather than having agents run on local banks (Uhlig, 2010). Core banks invest in asset backed securities (ABS) which are sold to outside investors (Uhlig, 2010).[7] It is shown that under the case of adverse selection the probability of a systemic bank run occurring is low; under the assumption of uncertainty averse outside investors bank runs become systemic if the market share of core banks experiencing runs is too large (Uhlig, 2010).

Diamond and Dybvig (1983) show that bank runs are completely avoidable in the presence of government deposit insurance. Since commercial banks know that their depositors' investments are safe, this could potentially lead to banks engaging in 'shadow banking' activities (Luck and Schempp, 2015). That is, a moral hazard problem could arise from the use of deposit insurance. Luck and Schempp (2015) show that if commercial banks actively engage in shadow banking activity (which is not backed by deposit insurance), then a run on those banks susceptible to runs could become contagious and impact the commercial banks if the size of the shadow banking sector is large. A key result provided by Luck and Schempp (2015), in the context of commercial banking, is that if a proportion of the commercial banks are susceptible to runs (given that the commercial banks are interlinked) and the size of this

[7] Uhlig (2010) investigates two different circumstances: the first case is when outside investors are uncertainty averse which means they follow certain rules to avoid uncertainty about the long term asset they are purchasing; the second case is when investors do not know the quality of the term long asset that is being sold to them (Uhlig, 2010).

proportion is beyond a certain threshold then the run can become systemic and even impact those banks that are not susceptible to runs (Luck and Schempp, 2015).

One way that the Diamond-Dybvig model is extended in this paper is by relaxing the assumption of having one unique bank that represents the entire banking system. That is, the banking system is made up of many banks. This opens the door for interbank insurance and trades. Brighi (2002) considers a static 3-period model where two banks make up the entire banking system. Banks have an option to invest excess liquidity in the interbank market (at a positive interest rate); it is shown that when banks have access to interbank liquidity the probability that a bank becomes illiquid decreases and banks' expected profits increases (Brighi, 2002).

3 Model Outline

We will follow the model used in a seminal paper written by Diamond and Dybvig (1983), also known as the Diamond-Dybvig model.[8] Consider an economy that runs through three periods, $t = 0, 1, 2$. The model economy is populated with a continuum of agents of mass 1 and 1 commercial bank (This will be extended to a continuum of banks of mass 1 in Section 4.3).

Consumers

Agents are born in period $t = 0$ and are endowed with one unit of a consumption good. Upon birth, each agent is identical and does not know their own type; it is assumed that there are two types of agents: impatient and patient. The proportion of agents that are impatient is denoted as π while the proportion of patient agents is denoted as $1 - \pi$. Impatient

[8] Section 3 contains a description of the Diamond-Dybvig model; Sections 4.1-4.2 contain explanations of some major results obtained by Diamond and Dybvig (1983). Section 4.3 is where we generalize the Diamond-Dybvig model.

agents only care about consumption in period $t = 1$ while patient agents only gain utility from consumption in period $t = 2$.

Each agent has the option to hoard their endowment or deposit their endowment at a bank.[9] Diamond and Dybvig (1983) show that agents never choose to hoard their endowment so we only consider the case where agents deposit their funds into a bank. Initially, agents face uncertainty about their preferred date to withdraw funds. In period 1 each agent learns their type (this is an agent's private information); this can be viewed as a liquidity shock (Luck and Schempp, 2015). After observing their own types, agents then decide if they want to withdraw funds in period 1 or wait until period 2 to withdraw funds. It is assumed that an agent's utility function is of the form $u(c_t) = \frac{1}{1-\eta} c_t^{1-\eta}$.

Let c_1 and c_2 denote the amount received by an impatient and patient consumer, respectively. Then the consumption profile denoted as (c_1, c_2) induces the following expected utility:

$$U(c_1, c_2) = \pi u(c_1) + (1 - \pi)\beta u(c_2). \tag{1}$$

Equation (1) represents an agent's expected utility at date $t = 0$. Here β is the discount factor by which utility is discounted in period 2.

Commercial Banks

Commercial banks cannot observe the agents' types. Each bank offers two types of technologies: storage and productive technologies. Storage yields no return on deposits. If the agents invest in the productive technology they get 1 if they withdraw in period $t = 1$ and $R > 1$ if they withdraw in period $t = 2$.

It is assumed that the agents do not have direct access to the productive technology so they must deposit their funds into a commercial bank to gain access. The commercial bank-

[9] If an agent decides not to deposit in a bank and hoard their endowment instead, then this is the agent's private information. For the purpose of this paper, we say that agents have an 'outside' option which is just hoarding their endowment period to period.

ing system works in the following way: agents deposit their endowments into a bank, then the bank takes these deposits and leaves a fraction of them aside (for on-demand liquidity) then invests the rest in the productive technology.

Technology	$t = 0$	$t = 1$	$t = 2$
Storage (withdrawal in $t = 1$)	-1	1	0
Storage (withdrawal in $t = 2$)	-1	0	1
Productive (withdrawal in $t = 1$)	-1	1	0
Productive (withdrawal in $t = 2$)	-1	0	R

Table 1: Description of the technologies offered by banks

Table 1 shows how the investment-return mechanism works. From a societal point of view a little in storage with the rest in the productive technology is the best option. However, agents cannot do this without banks; this is why banks are so important.

4 Financial Intermediation and Bank Runs

First we will establish the first best allocation by solving the social planner's problem. Then we will investigate how commercial banks can offer a contract to its depositors such that the first best allocation is achieved.

4.1 The Social Planner's Problem

Consider the case where there is a benevolent planner who knows the types of all agents. Since all agents are identical upon birth we may consider the representative agent problem; hence the planner solves the following problem:

$$\max_{(c_1,c_2)\in\mathbb{R}^2_+}\left[\pi\frac{1}{1-\eta}c_1^{1-\eta}+\beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right] \tag{2}$$

subject to

$$(1-\pi c_1)R=(1-\pi)c_2 \tag{3}$$

where (2) represents the agent's expected utility and equation (3) is the planner's resource constraint.

The resource constraint states that the amount of resources available in period 2 must be equal to the amount consumed in period 2. Impatient agents consume c_1 in $t=1$ and what remains is $1-\pi c_1$; this amount matures in period 2 and grows by a factor of R. So, the amount of resources available in $t=2$ is $(1-\pi c_1)R$. Since the proportion of patient agents is equal to $1-\pi$ and each of whom consumes c_2, we have that the total amount of consumption in $t=2$ is equal to $(1-\pi)c_2$. Note that the amount of endowment collected is equal to $\displaystyle\int_0^1 1\cdot di=1$.

Assumption 1. $\beta R>1$.

Lemma 1 (First-best Allocation)**.** *The first-best allocation, denoted as* $(c_1^{\text{FB}},c_2^{\text{FB}})$, *is given by*

$$(c_1^{\text{FB}},c_2^{\text{FB}})=\left(\frac{(\beta R)^{-\frac{1}{\eta}}R}{\pi(\beta R)^{-\frac{1}{\eta}}R+(1-\pi)},\frac{R}{\pi(\beta R)^{-\frac{1}{\eta}}R+(1-\pi)}\right). \tag{4}$$

Proof. See Appendix A.

□

Assumption 2. $(\beta R)^{-\frac{1}{\eta}} R > 1$.

The risk sharing between impatient and patient agents is given by the Euler equation (FOC to the problem (2)-(3))

$$R\beta u^{'}(c_2) = u^{'}(c_1). \tag{5}$$

Using the fact that $u^{'}(\cdot)$ is decreasing in c_t and combining equation (5) with Assumptions 1 and 2 we get that $1 < c_1^{\text{FB}} < c_2^{\text{FB}} < R$.[10] Thus, $(c_1^{\text{FB}}, c_2^{\text{FB}})$ is a unique solution to (2).

4.2 Implementing The First-best Allocation

In this section we show that a bank can implement the first-best allocation by means of optimal insurance contracts (OIC). These OIC's are signed by agents in period $t = 0$. It offers c_1^{FB} to impatient agents who withdraw in period 1, and c_2^{FB} to patient agents who withdraw in period 2. It is important to note that banks do not know the agents' types. The following proposition introduces the notion of truth-telling and how it is optimal for agents to withdraw at appropriate dates based on their private information.

Proposition 1. *Under the optimal insurance contract, there exists an equilibrium where truth-telling is optimal for all agents. That is, the consumption profile in equilibrium is* $(c_1^*, c_2^*) = (c_1^{\text{FB}}, c_2^{\text{FB}})$.

Proof.

Clearly the consumption profile obtained under the OIC is feasible and it satisfies equation (5). Since impatient agents do not gain utility from period $t = 2$ consumption, it is obvious that impatient agents will never choose to withdraw funds in period $t = 2$. Next,

[10] Using Assumption 2 we get that $(\beta R)^{-\frac{1}{\eta}} R(1-\pi) > (1-\pi)$. This implies that $(\beta R)^{-\frac{1}{\eta}} R > \pi(\beta R)^{-\frac{1}{\eta}} R +$ $(1-\pi)$. Thus $c_1^{\text{FB}} > 1$. Using the fact that $u^{'}(\cdot)$ is decreasing in c_t and by combining Assumption 1 with Equation (5) we get that $c_1^{\text{FB}} < c_2^{\text{FB}}$. Using Assumption 2 once more we get that $(\beta R)^{-\frac{1}{\eta}} R\pi > \pi$. This implies that $(\beta R)^{-\frac{1}{\eta}} R\pi + (1-\pi) > 1$ and $c_2^{\text{FB}} < R$. Therefore $1 < c_1^{\text{FB}} < c_2^{\text{FB}} < R$.

since $c_1^* < c_2^*$ this implies that a patient agent has no incentive to claim consumption in $t = 1$, provided that everyone is telling the truth. Further, patient agents claim period $t = 2$ consumption only because they know that all patient agents are telling the truth and this is common knowledge. Otherwise, patient agents' behaviour might not be optimal. Therefore, truth-telling constitutes an equilibrium. □

Implementing the first-best contract through optimal insurance contracts allows optimal risk sharing among agents; incentives are such that, in equilibrium, all agents reveal their true types (Luck and Schempp, 2015). Although the first-best outcome is achieved via OICs, another equilibrium outcome arises: this outcome is known as a 'bank run'. In the context of this paper, the situation where all agents claim consumption in period $t = 1$ is called a 'bank run'.

Proposition 2. *Under the optimal insurance contract, a 'bank run' is an equilibrium; an agent gets c_1^* with probability $p = \frac{1}{c_1^*}$ and zero with probability $(1 - p)$.*

Proof.

Suppose that a bank run occurs in period $t = 1$. Then all agents attempt to withdraw funds in period 1. The bank knows that it will eventually run out of liquid funds and needs c_1^* to cover all withdrawing agents.[11] However, not all agents who are withdrawing will receive the amount promised by the OIC. That is, since 1 unit was deposited in to the bank in period 1 only 1 can be gotten. Hence the probability of an agent receiving the promised amount during a bank run is equal to $\frac{1}{c_1^*}$ and the probability of getting zero is $1 - \frac{1}{c_1^*}$.[12] Define $p = \frac{1}{c_1^*}$.

[11] Note here that the bank needs $\int_0^1 c_1^* di = c_1^*$ to cover all agents.

[12] Here we assume the cost of withdrawing the illiquid investment in period 1, τ, is zero. Otherwise, the probability an agent receives the promised amount in period 1 is given by $\frac{1 - \tau i}{c_1^*}$, where i represent the amount a bank invests in the illiquid technology (Ennis and Keister, 2006).

It only suffices to show that no agent has incentive to deviate from withdrawing in period 1 when all others are claiming period 1 consumption. Since the bank will run out of funds in period 1, those who do not run will receive zero with probability 1. On the other hand, if an agent chooses to run they will receive c_1^* with probability p or zero with probability $(1-p)$. Therefore, if everyone runs and claims early consumption, the best response to this is to run. □

4.3 Generalizing the Diamond-Dybvig Model

In this section we extend the Diamond-Dybvig model in three ways. First, we will consider the case where there is a continuum of commercial banks of mass 1 that compete for agents' deposits. Further, suppose a bank run will occur with certainty and only a mass α of banks are subject to runs while a mass $1-\alpha$ are not.[13] Second, we assume that all banks place funds into interbank insurance, i_1, in period $t = 0$; these funds are pooled together and the insurance is only paid out to those banks experiencing runs. Lastly, we assume that liquidating assets in $t = 1$ comes at a cost. That is, when banks withdraw illiquid funds in $t = 1$ they must pay a transaction cost, τ, on each unit invested in the illiquid technology (i_2).[14]

Banks know that a run will occur with certainty but do not know whether a run will

13 Diamond and Dybvig (1983) show existence of multiple Nash equilibria: one outcome where all agents get the promised amount offered in the deposit contract, and another outcome known as a bank run. In this generalized model both outcomes are incorporated in designing the optimal contract. That is, bank runs are assumed to happen where the probability that any of the mass 1 of banks face a run is α (exogenously given). Since there is a continuum of banks of mass 1 this implies that only a fraction α of banks face runs. Goldstein and Pauzner (2005) address the issue of calculating the probability of a run occurring at a bank in the Diamond-Dybvig environment.

14 In the Diamond-Dybvig model the bank, in times of crisis, liquidates assets at no cost. In this model it is assumed that banks must pay a transaction cost when they liquidate assets; this is more realistic because banks might not be able to costlessly liquidate assets. This opens the door to investigate whether there exists some threshold for τ such that investing in the illiquid technology becomes too costly in times of crisis and banks opt for holding more funds in storage (Ennis and Keister, 2006). Diamond and Kashyap (2016) along with Ennis and Keister (2006) introduce a liquidation cost for the illiquid technology in their respective models.

occur at their location or not; the banks only know α (exogenously given). If an agent is located at a bank that does not experience runs then the amount of expected utility they obtain is $\pi u(c_{1,1}) + \beta(1-\pi)u(c_2)$. On the other hand, if the agent is located at a bank that is experiencing a run, the agent earns $pu(c_{1,2}) + (1-p)u(0)$[15] on average. Note that $c_{1,1}$ denotes the amount an impatient agent receives at a healthy bank in $t = 1$, and $c_{1,2}$ is the amount an agent receives in $t = 1$ if they are located at a bank that experiences runs.

Agents do not know if their bank is affected but know that runs occur with certainty. It is assumed that α is sufficiently small so that the amount each bank stores aside for safe-keeping is relatively small. It is also assumed that when a bank experiences a bank run it must liquidate assets in $t = 1$ to attempt to meet the demand of agents withdrawing early. This mandatory liquidation in times of crisis can be viewed as a form of regulation.

At time $t = 0$, the probability of an agent being located at a bank which experiences runs and receiving the promised amount in period 1 is αp. If $p = 1$, then all agents running at the α-many banks will receive the promised amount in period $t = 1$. If $p < 1$, then the funds raised from liquidating assets would not be sufficient to cover all withdrawing agents and banks will become illiquid in $t = 1$. We assume that $p < 1$ but then, to meet the demand of the agents withdrawing, all banks (including those experiencing runs) will choose an amount to put into interbank insurance so that all agents earn what is promised by the contract.

The goal is to design an optimal interbank insurance contract. We want to construct the contract in such a way that an agent located at a bank which experiences runs earns the promised amount in period 1 with certainty. Interbank insurance works in the following way: all banks put i_1 into interbank insurance and a mass α of banks receive these funds. That is, the amount each of the α-many banks receives from this interbank insurance is $\frac{i_1}{\alpha}$.

In $t = 0$ the α-many banks each collect endowment of 1, invest in the long term asset, and

[15] Note that p and $(1-p)$ were stated in section 4.2; if a bank run happens and the agent withdraws in period 1, then the agent receives c_1 with probability p and 0 with probability $(1-p)$.

put aside an amount for interbank insurance. In $t = 1$ a bank run occurs and the α-many banks need to liquidate assets which comes at a cost τ. A bank that experiences a run in $t = 1$ has $1 - i_1 - \tau i_2 + \frac{i_1}{\alpha}$ at their disposal.[16] The probability an agent located at one of the α-many banks gets the promised amount in $t = 1$ is $p = \frac{1 - i_1 - \tau i_2 + \frac{i_1}{\alpha}}{c_{1,2}}$. We want to choose i_1 such that $p = 1$. In addition to choosing i_1, we also consider the case where agents withdrawing in $t = 1$ get the same amount at any bank. That is, we impose the condition where $c_{1,1} = c_{1,2}$. Note that this can be generalized to the case where $c_{1,2} = \gamma c_{1,1}$, where γ is the rate set by a bank regulator.

Now we write the social planner's problem to the generalized model:

$$\max_{\{c_{1,1},c_{1,2},c_2,i_1,i_2\}\in\mathbb{R}^5_+} (1-\alpha)\left[\pi\frac{1}{1-\eta}c_{1,1}^{1-\eta} + \beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right] + \alpha\left[\frac{1}{1-\eta}c_{1,2}^{1-\eta}\right] \tag{6}$$

subject to

$$\pi c_{1,1} = 1 - i_1 - i_2 \tag{7}$$

$$(1-\pi)c_2 = i_2 R \tag{8}$$

$$c_{1,2} = 1 - i_1 - \tau i_2 + \frac{i_1}{\alpha} \tag{9}$$

$$c_{1,1} = c_{1,2} \tag{10}$$

Equations (6)-(10) represent the social planner's problem to the generalized model.[17] Equations (7)-(9) are the social planner's resource constraints. Equation (10) ensures that agents who run on the mass α of banks earn the same as impatient agents located at healthy banks.

[16] Note that the banks invest i_2 and put aside i_1 in $t = 0$ then, because of the bank run, they are forced to pull out the long-term investment (which comes at a cost of τ) and receive $\frac{i_1}{\alpha}$ from the interbank insurance. Thus we get $1 - i_1 - i_2 + (1-\tau)i_2 + \frac{i_1}{\alpha} = 1 - i_1 - \tau i_2 + \frac{i_1}{\alpha}$.

[17] The social planner has control over all resources. Both sides of equations (7) and (8) are multiplied by an "invisible" $(1-\alpha)$. These terms cancel each other and we are left with what is shown in equations (7) and (8). Similarly for equation (9), there is an "invisible" α that is multiplied on both sides of the equation.

Equation (7) tells us that the amount received by impatient agents at healthy banks is equal to what is left after subtracting the investment in the illiquid technology and interbank insurance from the collected deposits. Note here that the amount chosen to be put aside for interbank insurance, i_1, is used to help bail out the banks experiencing runs. Equation (8) says that the amount given to patient agents at healthy banks is equal to the return on the investment of the production technology.

Equation (9) represents the condition that all agents withdrawing early get the promised amount in $t = 1$. Cooper and Ross (2002) along with Ennis and Keister (2006) use a variation of Equations (6)-(9) in their studies; they split their models into two cases: if $p = 1$ then all agents running on a bank will get the promised amount, and if $p < 1$ then the bank will be unable to meet the demand for early withdrawals and thus become illiquid.[18]

Assumption 3. $\dfrac{1-\alpha}{1-\alpha+\alpha\tau}\beta R > 1.$

Lemma 2 (Optimal Interbank Insurance Contract). *Given the problem shown in equations (6)-(10), the optimal consumption profile is given by*

$$c_{1,1}^{**} = \frac{\phi(R\beta)^{-\frac{1}{\eta}}R}{[\phi(R\beta)^{-\frac{1}{\eta}}R(\alpha+(1-\alpha)\pi)+(1-\pi)(1-\alpha+\alpha\tau)]}, \tag{11}$$

$$c_2^{**} = \frac{R}{[\phi(R\beta)^{-\frac{1}{\eta}}R(\alpha+(1-\alpha)\pi)+(1-\pi)(1-\alpha+\alpha\tau)]}, \tag{12}$$

and

$$c_{1,2}^{**} = c_{1,1}^{**}, \tag{13}$$

[18] The models used by Cooper and Ross (2002) and Ennis and Keister (2006) differ from the extended model shown in this paper in a number of ways. Here we consider a continuum of banks of mass 1, incorporate an interbank insurance scheme, impose the condition that agents located at banks experiencing runs receive the same as impatient agents do at healthy banks, and force p to be 1.

where $\phi = \left[\dfrac{1-\alpha}{1-\alpha(1-\tau)}\right]^{-\frac{1}{\eta}}$.

Proof. See Appendix B.

□

Assumption 4. $\dfrac{\phi(R\beta)^{-\frac{1}{\eta}}R(1-\alpha)}{1-\alpha+\alpha\tau} > 1.$

The Euler equation for the problem shown in (6)-(10) is given by

$$u^{'}(c_{1,1}) = \frac{1-\alpha}{1-\alpha(1-\tau)}\beta R u^{'}(c_2). \tag{14}$$

That is, the solution lies on the ray $c_{1,1} = \phi(\beta R)^{-\frac{1}{\eta}}c_2$. From Assumptions 3 and 4 we get that $1 < c_{1,1}^{**} < c_2^{**} < R$.[19]

As mentioned before, all banks pay into this interbank insurance and if a bank happens to experience a run then they receive insurance which is strictly used to help pay off all agents running on the bank. According to the contract given in Lemma 2, impatient and patient agents located at healthy banks receive $c_{1,1}^{**}$ and c_2^{**}, respectively. On the other hand, agents located at banks which experience runs receive $c_{1,2}^{**}$, no matter what type they are.[20] By construction, the optimal interbank insurance contract eliminates the possibility of a bank becoming illiquid during a bank run.

[19] See Appendix B for this derivation.

[20] Recall (from Proposition 2) that if everyone else runs and withdraws in $t = 1$, then it is a best response for an agent to run. Hence both types of agents located at the mass α of banks will receive $c_{1,2}^{**}$.

5 Interbank Insurance Contracts Under Regulation

5.1 Bank Regulation

To enrich the model a regulator, who watches over all banking activities, is introduced. Suppose that the regulator imposes the following restriction:

$$c_{1,2} = \gamma c_{1,1}, \quad 0 \leq \gamma \leq 1. \tag{15}$$

That is, the mass α of banks experiencing runs are only able to offer agents withdrawing in $t = 1$ a proportion of what healthy banks do in period $t = 1$. A reason for this might be that if banks know they can provide the same to agents who withdraw in $t = 1$ no matter the state they are in and never become illiquid, then banks might take on more risky projects. This regulation could be seen as a way to keep banks "in check".

Equation (15) changes the constraint given in Equation (10). The problem in equations (6)-(10) then becomes:

$$\max_{\{c_{1,1},c_{1,2},c_2,i_1,i_2\}\in\mathbb{R}^5_+} (1-\alpha)\left[\pi\frac{1}{1-\eta}c_{1,1}^{1-\eta} + \beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right] + \alpha\left[\frac{1}{1-\eta}c_{1,2}^{1-\eta}\right] \tag{16}$$

subject to

$$\pi c_{1,1} = 1 - i_1 - i_2 \tag{17}$$

$$(1-\pi)c_2 = i_2 R \tag{18}$$

$$c_{1,2} = 1 - i_1 - \tau i_2 + \frac{i_1}{\alpha} \tag{19}$$

$$c_{1,2} = \gamma c_{1,1} \tag{20}$$

where $\gamma \in [0, 1]$.

Assumption 5. $\dfrac{(1-\alpha)[\pi(1-\alpha)+\alpha\gamma]}{[\pi(1-\alpha)+\alpha\gamma^{1-\eta}](1-\alpha+\alpha\tau)}\beta R > 1.$

Lemma 3. *In the presence of banking regulation, the optimal consumption profile is given by*

$$c_{1,1}^{***} = \frac{\psi(R\beta)^{-\frac{1}{\eta}}R}{[\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma+(1-\alpha)\pi)+(1-\pi)(1-\alpha+\alpha\tau)]}, \tag{21}$$

$$c_{2}^{***} = \frac{R}{[\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma+(1-\alpha)\pi)+(1-\pi)(1-\alpha+\alpha\tau)]}, \tag{22}$$

and

$$c_{1,2}^{***} = \gamma c_{1,1}^{***}, \tag{23}$$

where $\psi = \left[\dfrac{(1-\alpha)[\pi(1-\alpha)+\alpha\gamma]}{[(1-\alpha)\pi+\alpha\gamma^{1-\eta}][1-\alpha(1-\tau)]}\right]^{-\frac{1}{\eta}}.$

Proof. See Appendix B.

□

Assumption 6. $\dfrac{\psi(R\beta)^{-\frac{1}{\eta}}R(1-(1-\alpha)\pi+\alpha\gamma)}{(1-\pi)(1-\alpha+\alpha\tau)} > 1.$

From Assumptions 5 and 6, along with the fact that $u^{'}(\cdot)$ is decreasing in c_t, we can conclude that $1 < c_{1,1}^{***} < c_2^{***} < R$.[21]

[21] See Appendix B for this derivation.

5.2 Socially Optimal Level of Regulation

Introducing banking regulation opens the door to investigating which policy is socially optimal. That is, we are looking to find the value of γ that maximizes the representative agent's expected utility (shown in equation (16)). From Lemma 3 we obtain the optimal consumption profile $((c_{1,1}^{***}, c_2^{***}), c_{1,2}^{***})$, each of which are a function of γ. When we combine equations (21), (22), and (23) with equation (16) we get the following expected utility as a function of γ, denoted as $EU^*(\gamma)$:

$$EU^*(\gamma) = (1-\alpha)\left[\pi u(c_{1,1}(\gamma)^{***}) + \beta(1-\pi)\frac{1}{1-\eta}u(c_2(\gamma)^{***})\right] + \alpha\left[u(c_{1,2}(\gamma)^{***})\right]. \quad (24)$$

The social planner's problems is to choose $\gamma^* \in [0,1]$ so that social welfare is maximized. That is, the social planner's problem is:

$$\max_{0\leq\gamma\leq 1} EU^*(\gamma). \quad (25)$$

Given the complexity of the expected utility function shown in Equation (24) we will solve the maximization problem shown in (25) numerically. From our assumptions we have that the proportion of banks that experience runs is relatively low. We choose $\alpha = 0.02$. Next we choose $\beta = 0.9$ and $R = 1.2$ (Assumption 1 is satisfied). Next we chose the transaction cost of withdrawing the productive investment in $t = 1$ to be $\tau = 0.1$. Suppose that the proportion of impatient and patient agents are equal. The relative risk aversion parameter is chosen to be $\eta = 0.5$. All of these parameters are chosen so that Assumptions 1 through 6 hold.

Now we take the derivative of Equation (24) with respect to γ:

$$\frac{\partial EU^*(\gamma)}{\partial\gamma} = (1-\alpha)\pi\frac{\partial u(c_{1,1}^{***})}{\partial c_{1,1}^{***}}\frac{\partial c_{1,1}^{***}}{\partial\gamma} + \beta(1-\alpha)(1-\pi)\frac{\partial u(c_2^{***})}{\partial c_2^{***}}\frac{\partial c_2^{***}}{\partial\gamma} + \alpha\frac{\partial u(c_{1,2}^{***})}{\partial c_{1,2}^{***}}\frac{\partial c_{1,2}^{***}}{\partial\gamma}. \quad (26)$$

Once we plug the aforementioned parameter values into Equation (26) it turns out that $EU^*(\gamma)$ is an increasing function on the interval $[0,1]$. Since $EU^*(\gamma)$ is continuous $\forall\gamma \in [0,1]$, by the extreme value theorem, the function has a maximum and a minimum on the interval $[0,1]$. Further, since EU^* is increasing on $[0,1]$, the maximum is obtained when $\gamma = 1$. That is, $\gamma^* = 1$ is the solution to the problem showed in (25) when $\alpha = 0.02$, $\pi = 0.5$, $\beta = 0.90$, $R = 1.2$, $\eta = 0.5$, and $\tau = 0.1$.

When we plug $\gamma^* = 1$ into Equations (21), (22), and (23), we get $c_{1,1}^{***} = 1.01491$, $c_2^{***} = 1.17897$, $c_{1,2}^{***} = 1.01491$, respectively. This yields an expected utility of $EU^*(1) = 1.98525$.

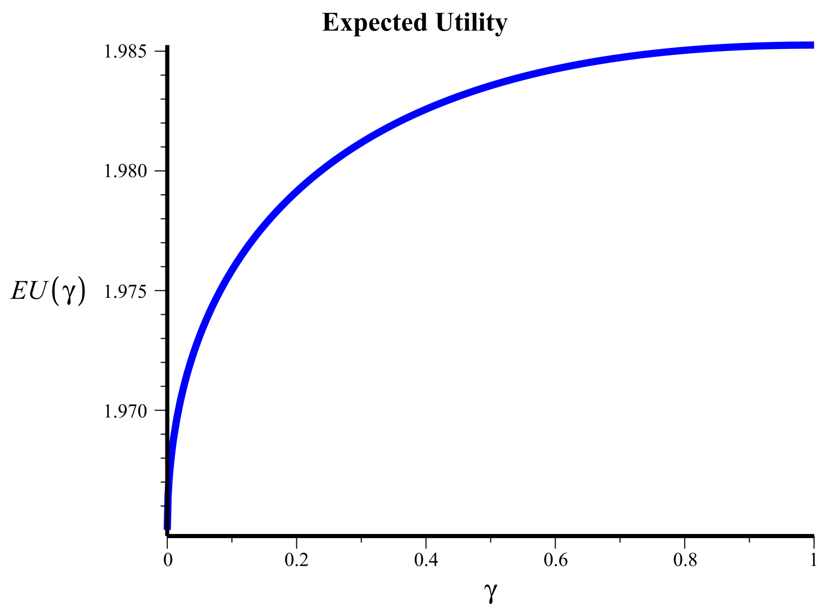

Figure 1: Welfare function for all values of $\gamma \in [0,1]$.

Figure 1 shows the expected utility function, as seen in Equation (24), for values of $\gamma \in [0,1]$. The function is increasing on the interval $[0,1]$ and the maximum is achieved when $\gamma^* = 1$. Therefore, in the context of this numerical example, the social planner chooses $\gamma^* = 1$ so that social welfare is optimized. The intuition behind $\gamma^* = 1$ can be explained in the following way: since, in this model, banks do not take on an excessively risky investment this implies that the introduction of regulation makes agents worse off.

6 Conclusions

In this paper we start out with the Diamond-Dybvig model in which the banking sector is represented by a single bank. There is an equilibrium where all agents tell the truth about their types and withdraw accordingly, and there exists a second equilibrium known as a bank run; in the context of this paper, a bank run is the situation where all agents, motivated by fear of a bank becoming insolvent, withdraw funds and claim consumption in period 1. The bank, even after liquidating assets, might not be able to meet the demand of all withdrawing agents thus becoming illiquid in period 1. A bank run is considered to be a self-fulfilling prophecy (Allen and Gale, 2000).

We generalize the Diamond-Dybvig model in three ways. The first extension is that now we assume the banking sector is represented by a continuum of banks of mass 1 where a mass α of banks are subject to a bank run while a mass $1-\alpha$ of banks are not. We do not try to eliminate the possibility of a bank run occurring, instead we assume that a bank run occurs with certainty but only a small proportion of banks are affected. Second, we assume each bank puts aside an amount into interbank insurance; all of the funds placed into interbank insurance are pooled together and evenly distributed among those banks experiencing runs. Lastly, we assume banks experiencing runs must liquidate assets in $t = 1$ and pay a mandatory liquidation fee per unit invested in the productive technology.

In this model of bank runs an optimal interbank insurance contract was designed in which all agents get the promised amount in $t = 1$, even during a bank run. The contract, by construction, also rules out the possibility of banks becoming illiquid when a bank run occurs. To enhance the model a banking regulator was introduced. A banking regulator's objective is to watch over all banking operations to ensure that all banks are operating within the rules and regulations (Cooper and Ross, 2002). We suppose that this regulator imposes a restriction on banks experiencing bank runs in an attempt to stop banks from investing in speculative projects. That is, if a bank experiences a run then it cannot offer agents withdrawing in $t = 1$ the same amount as healthy banks do in $t = 1$. It was shown, through a numerical example, that the socially optimal level of regulation was such that healthy banks and those banks experiencing runs offer the same amount to agents who withdraw in $t = 1$.

In the future it would be interesting to investigate the possibility of turning this model into one of shadow banking and systemic bank runs. That is, if some banks perform regulatory arbitrage then they would find themselves outside of the perimeter of regulation which could lead to a moral hazard problem and an emergence of a shadow banking sector (Luck and Schempp, 2015). It would be interesting to see if the size of the shadow banking sector plays a role in determining whether or not bank runs can become contagious.

Appendix A Proof of Lemma 1

We have the following problem:

$$\max_{(c_1,c_2)\in\mathbb{R}_+^2} \left[\pi\frac{1}{1-\eta}c_1^{1-\eta}+\beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right] \tag{27}$$

subject to

$$(1-\pi c_1)R=(1-\pi)c_2. \tag{28}$$

We will solve this problem using the Lagrangian Multiplier Method. We write the Lagrangian and first-order conditions:

$$\mathcal{L}=\left[\pi\frac{1}{1-\eta}c_1^{1-\eta}+\beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right]-\lambda\Big[(1-\pi c_1)R-(1-\pi)c_2\Big] \tag{29}$$

$$\frac{\partial\mathcal{L}}{\partial c_1}=0: \qquad \pi c_1^{-\eta}+\lambda\pi R=0 \tag{30}$$

$$\frac{\partial\mathcal{L}}{\partial c_2}=0: \qquad \beta(1-\pi)c_2^{-\eta}+(1-\pi)\lambda=0 \tag{31}$$

$$\frac{\partial\mathcal{L}}{\partial \lambda}=0: \qquad (1-\pi c_1)R-(1-\pi)c_2=0. \tag{32}$$

Combining Equations (30) and (31) gives us the following Euler equation:

$$c_1^{-\eta}=R\beta c_2^{-\eta}. \tag{33}$$

Simplifying Equation (33) yields

$$c_1=(R\beta)^{-\frac{1}{\eta}}c_2. \tag{34}$$

Combining Equation (32) and (34) gives us

$$(1-\pi(R\beta)^{-\frac{1}{\eta}}c_2)R = (1-\pi)c_2. \tag{35}$$

Simplifying Equation (35) and rearranging for c_2 yields

$$c_2^{FB} = \frac{R}{\pi(\beta R)^{-\frac{1}{\eta}}R + (1-\pi)}. \tag{36}$$

If we plug Equation (36) into Equation (34) we get our value for c_1^{FB}. That is,

$$c_1^{FB} = \frac{(\beta R)^{-\frac{1}{\eta}}R}{\pi(\beta R)^{-\frac{1}{\eta}}R + (1-\pi)}. \tag{37}$$

Therefore, the first-best allocation is given by

$$(c_1^{\mathrm{FB}}, c_2^{\mathrm{FB}}) = \left(\frac{(\beta R)^{-\frac{1}{\eta}}R}{\pi(\beta R)^{-\frac{1}{\eta}}R + (1-\pi)}, \frac{R}{\pi(\beta R)^{-\frac{1}{\eta}}R + (1-\pi)}\right). \tag{38}$$

Note that $1 < c_1^{\mathrm{FB}} < c_2^{\mathrm{FB}} < R$.

$\square$

Appendix B Proof of Lemma 2 and Lemma 3

First we prove Lemma 3. Lemma 2 follows immediately after since it is a special case of Lemma 3. So the problem is written as follows:

$$\max_{\{c_{1,1},c_{1,2},c_2,i_1,i_2\}\in\mathbb{R}^5_+}(1-\alpha)\left[\pi\frac{1}{1-\eta}c_{1,1}^{1-\eta}+\beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right]+\alpha\left[\frac{1}{1-\eta}c_{1,2}^{1-\eta}\right] \tag{39}$$

subject to

$$\pi c_{1,1} = 1 - i_1 - i_2 \tag{40}$$

$$(1-\pi)c_2 = i_2 R \tag{41}$$

$$c_{1,2} = 1 - i_1 - \tau i_2 + \frac{i_1}{\alpha} \tag{42}$$

$$c_{1,2} = \gamma c_{1,1}. \tag{43}$$

First we write the Lagrangian and first-order conditions (note that we plug Equation (43) into Equations (39) and (42) from the start to reduce the dimensionality of the problem):

$$\begin{aligned}\mathcal{L} = {} & (1-\alpha)\left[\pi\frac{1}{1-\eta}c_{1,1}^{1-\eta}+\beta(1-\pi)\frac{1}{1-\eta}c_2^{1-\eta}\right]+\alpha\left[\frac{1}{1-\eta}\gamma^{1-\eta}c_{1,1}^{1-\eta}\right] \\ & -\lambda_1\Big[\pi c_{1,1}-1+i_1+i_2\Big]-\lambda_2\Big[(1-\pi)c_2-i_2R\Big]-\lambda_3\Big[\gamma c_{1,1}-1+i_1+\tau i_2-\frac{i_1}{\alpha}\Big]\end{aligned} \tag{44}$$

$$\frac{\partial\mathcal{L}}{\partial c_{1,1}} = 0: \quad (1-\alpha)\pi c_{1,1}^{-\eta}+\alpha\gamma^{1-\eta}c_{1,1}^{-\eta}-\lambda_1\pi-\lambda_3\gamma = 0 \tag{45}$$

$$\frac{\partial\mathcal{L}}{\partial c_2} = 0: \quad (1-\alpha)\beta(1-\pi)c_2^{-\eta}-(1-\pi)\lambda_2 = 0 \tag{46}$$

$$\frac{\partial\mathcal{L}}{\partial i_1} = 0: \quad -\lambda_1+\lambda_3\Big(\frac{1-\alpha}{\alpha}\Big) = 0 \tag{47}$$

$$\frac{\partial \mathcal{L}}{\partial i_2} = 0 : \qquad -\lambda_1 + \lambda_2 R - \lambda_3 \tau = 0 \tag{48}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = 0 : \qquad \pi c_{1,1} - 1 + i_1 + i_2 = 0 \tag{49}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = 0 : \qquad (1-\pi)c_2 - i_2 R = 0 \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = 0 : \qquad \gamma c_{1,1} - 1 + i_1 + \tau i_2 - \frac{i_1}{\alpha} = 0. \tag{51}$$

Rearranging Equation (46) gives us

$$\lambda_2 = (1-\alpha)\beta c_2^{-\eta}. \tag{52}$$

Similarly, we rearrange Equation (47) to isolate for λ_1:

$$\lambda_1 = \lambda_3 \Big(\frac{1-\alpha}{\alpha}\Big). \tag{53}$$

If we combine Equation (48) with Equation (53) we get

$$\lambda_3 = \frac{\alpha R}{1-\alpha+\tau\alpha}\lambda_2. \tag{54}$$

Now plug Equation (45) into Equation (53) and simplify to get

$$(1-\alpha)\pi c_{1,1}^{-\eta} + \alpha\gamma^{1-\eta} c_{1,1}^{-\eta} = \lambda_3 \Big[\frac{\pi(1-\alpha)+\alpha\gamma}{\alpha}\Big]. \tag{55}$$

Combining Equations (52) and (54) with Equation (55) yields

$$c_{1,1}^{-\eta}\Big[(1-\alpha)\pi+\alpha\gamma^{1-\eta}\Big]=(1-\alpha)R\beta\left[\frac{\pi(1-\alpha)+\alpha\gamma}{1-\alpha+\alpha\tau}\right]c_2^{-\eta}. \tag{56}$$

Rearranging Equation (56) gives us the following Euler equation:

$$c_{1,1}^{-\eta}=\left[\frac{(1-\alpha)[\pi(1-\alpha)+\alpha\gamma]}{[(1-\alpha)\pi+\alpha\gamma^{1-\eta}][1-\alpha+\alpha\tau]}\right]R\beta c_2^{-\eta}. \tag{57}$$

Raising both sides of Equation (57) to the power $-\dfrac{1}{\eta}$ yields

$$c_{1,1}=\psi(R\beta)^{-\frac{1}{\eta}}c_2. \tag{58}$$

Note that $\psi=\left[\dfrac{(1-\alpha)[\pi(1-\alpha)+\alpha\gamma]}{[(1-\alpha)\pi+\alpha\gamma^{1-\eta}][1-\alpha+\alpha\tau]}\right]^{-\frac{1}{\eta}}$.

Now we combine Equations (49)-(51) and rearrange to get

$$c_{1,1}\Big[(1-\alpha)\pi+\alpha\gamma\Big]R=R-c_2\Big[\alpha\tau(1-\pi)+(1-\pi)(1-\alpha)\Big]. \tag{59}$$

If we plug Equation (58) into Equation (59) then isolate for c_2 we get the equilibrium level of c_2. That is,

$$c_2^{***}=\frac{R}{[\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma+(1-\alpha)\pi)+(1-\pi)(1-\alpha+\alpha\tau)]}. \tag{60}$$

Then we use the expression for c_2^{***} along with Equation (58) to solve for the equilibrium level of $c_{1,1}$ and $c_{1,2}$. That is,

$$c_{1,1}^{***} = \frac{\psi(R\beta)^{-\frac{1}{\eta}}R}{[\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma + (1-\alpha)\pi) + (1-\pi)(1-\alpha+\alpha\tau)]}, \tag{61}$$

and

$$c_{1,2}^{***} = \gamma c_{1,1}^{***}. \tag{62}$$

Now we use the optimal consumption plan and the constraints to solve for the optimal investment (i_2^{***}) and the optimal amount placed in interbank insurance (i_1^{***}). We get that

$$i_1^{***} = 1 - \pi c_{1,1}^{***} - \frac{(1-\pi)}{R}c_2^{***} \quad \text{and} \quad i_2^{***} = \frac{(1-\pi)}{R}c_2^{***}. \tag{63}$$

Therefore, the optimal consumption profile is given by

$$c_{1,1}^{***} = \frac{\psi(R\beta)^{-\frac{1}{\eta}}R}{[\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma + (1-\alpha)\pi) + (1-\pi)(1-\alpha+\alpha\tau)]}, \tag{64}$$

$$c_2^{***} = \frac{R}{[\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma + (1-\alpha)\pi) + (1-\pi)(1-\alpha+\alpha\tau)]}, \tag{65}$$

and

$$c_{1,2}^{***} = \gamma c_{1,1}^{***}. \tag{66}$$

Now we show that $1 < c_{1,1}^{***} < c_2^{***} < R$.[22] From Assumption 6 we have that $(1-(1-\alpha)\pi + \alpha\gamma)\psi(\beta R)^{-\frac{1}{\eta}}R > (1-\pi)(1-\alpha+\alpha\tau)$. This implies that $c_{1,1}^{***} > 1$. Next, using Assumption 5 and Equation (57) together with the fact that $u'(\cdot)$ is decreasing in c_t, we can conclude that $u'(c_{1,1}^{***}) > u'(c_2^{***})$. That is, $c_{1,1}^{***} < c_2^{***}$.

Lastly, to show that $c_2^{***} < R$ it suffices to show that $\psi(R\beta)^{-\frac{1}{\eta}}R(\alpha\gamma + (1-\alpha)\pi) + (1-$

[22] Note that $\frac{(1-\alpha)[\pi(1-\alpha)+\alpha\gamma]}{(1-\alpha+\alpha\tau)[\pi(1-\alpha)+\alpha\gamma^{1-\eta}]} < 1 \; \forall\gamma \in (0,1]$ and $\forall\eta \in (0,1)\bigcup(1,\infty)$.

$\pi)(1-\alpha+\alpha\tau) > 1$. Using the fact that $\dfrac{(1-\alpha)[\pi(1-\alpha)+\alpha\gamma]}{(1-\alpha+\alpha\tau)[\pi(1-\alpha)+\alpha\gamma^{1-\eta}]} < 1\ \forall\gamma \in (0,1]$ and $\forall\eta \in (0,1)\bigcup(1,\infty)$ we can conclude that $\psi > 1\ \forall\gamma \in (0,1]$ and $\forall\eta \in (0,1)\bigcup(1,\infty)$. Further, since $(R\beta)^{-\frac{1}{\eta}}R > 1$ this implies that $\psi(R\beta)^{-\frac{1}{\eta}}R > 1$. Now by expanding and simplifying the denominator of c_2^{***} we get $\alpha\gamma\psi(R\beta)^{-\frac{1}{\eta}}R + (1-\alpha)\pi\psi(R\beta)^{-\frac{1}{\eta}}R + (1-\pi) - \alpha(1-\pi)(1-\tau)$. Expanding and collecting like terms yields the following expression: $1 + \pi(\psi(R\beta)^{-\frac{1}{\eta}}R - 1) + \alpha(\gamma\psi(R\beta)^{-\frac{1}{\eta}}R - (1-\pi)(1-\tau))$. For $\gamma = 1$ the expression is strictly greater than 1. For $\gamma \in (0,1)$ the expression is strictly greater than 1, provided that α is sufficiently small (as assumed in Section 4.3). Thus we conclude that $c_2^{***} < R$. Therefore, $1 < c_{1,1}^{***} < c_2^{***} < R$.

□

We do not need to prove Lemma 2 in the way we proved Lemma 3. All that is needed is to plug $\gamma = 1$ into the solution obtained in Lemma 3. When $\gamma = 1$ we get that $\psi = \left[\dfrac{(1-\alpha)[\pi(1-\alpha)+\alpha * 1]}{[(1-\alpha)\pi + \alpha * 1^{1-\eta}][1-\alpha+\alpha\tau]}\right]^{-\frac{1}{\eta}} = \left[\dfrac{1-\alpha}{1-\alpha+\tau\alpha}\right]^{-\frac{1}{\eta}}$. Put $\phi = \left[\dfrac{1-\alpha}{1-\alpha+\tau\alpha}\right]^{-\frac{1}{\eta}}$.

Then the solution to Lemma 2 is given by:

$$c_{1,1}^{**} = \frac{\phi(R\beta)^{-\frac{1}{\eta}}R}{[\phi(R\beta)^{-\frac{1}{\eta}}R(\alpha+(1-\alpha)\pi) + (1-\pi)(1-\alpha+\alpha\tau)]}, \tag{67}$$

$$c_2^{**} = \frac{R}{[\phi(R\beta)^{-\frac{1}{\eta}}R(\alpha+(1-\alpha)\pi) + (1-\pi)(1-\alpha+\alpha\tau)]}, \tag{68}$$

$$c_{1,2}^{**} = c_{1,1}^{**}, \tag{69}$$

$$i_1^{**} = 1 - \pi c_{1,1}^{**} - \frac{(1-\pi)}{R}c_2^{**}, \tag{70}$$

and

$$i_2^{**} = \frac{(1-\pi)}{R}c_2^{**}. \tag{71}$$

Note that $1 < c_{1,1}^{**} < c_2^{**} < R$.

□

Appendix C Comparing the Extended Model to the Diamond-Dybvig Model

In this section we consider a numerical example where we compare the extended model shown in Equations (16)-(20) with the Diamond-Dybvig model. Suppose that $\pi = 0.4$, $\eta = 0.9$, $\beta = 0.9$, $R = 1.2$, $\tau = 0.2$, and $\alpha = 0.001$ (all assumptions are satisfied). When we combine the aforementioned parameter values with Equation (26) it turns out that $EU^*(\gamma)$ is an increasing function $\forall \gamma \in [0, 1]$. That is, social welfare is maximized when $\gamma^* = 1$.

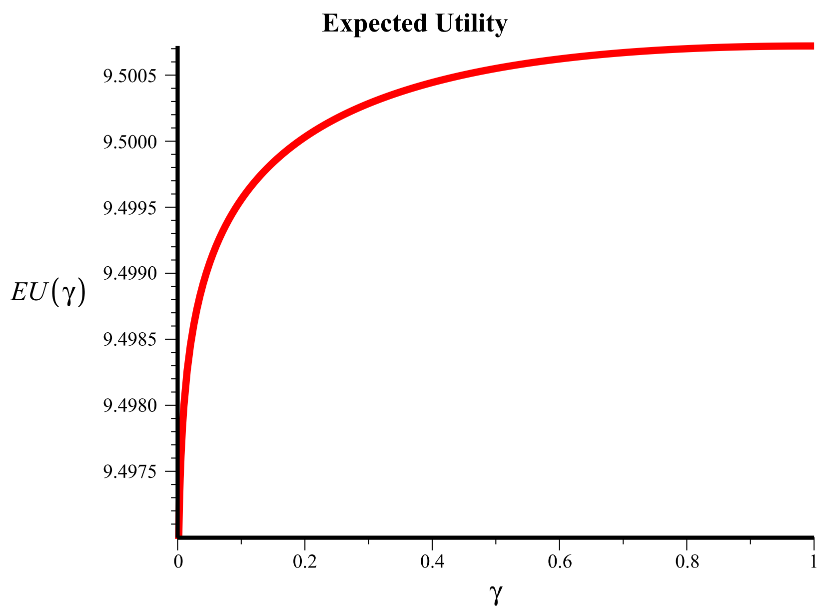

Figure C1: Welfare function for all values of $\gamma \in [0, 1]$.

Figure C1 shows the expected utility function for all values of $\gamma \in [0, 1]$. The function is increasing on the interval $[0, 1]$ and the maximum is obtained when $\gamma^* = 1$. It turns out that $c_{1,1}^{***} = c_{1,2}^{***} = 1.05856$, $c_2^{***} = 1.15281$; this yields an expected utility of $EU^*(1) = 9.50072$. Note that $i_1^{***} = 0.00017$ and $i_2^{***} = 0.57640$.

Now we compare the consumption and expected utility levels obtained in each model. In order to do this we consider the two states of a bank: the first case is where a run does not occur, and the second case is where a run occurs.

Case 1: No run occurs

In this case agents are located at a bank that does not face a bank run. The table below shows the consumption and expected utility levels obtained in both models.

	Extended Model	Diamond-Dybvig Model
Withdrawal in $t = 1$	$c_{1,1}^{***} = 1.05856$	$c_1^{DD} = 1.05861$
Withdrawal in $t = 2$	$c_2^{***} = 1.15281$	$c_2^{DD} = 1.15311$
Expected Utility	9.50016	9.50033

Table C1: Consumption and expected utility levels in both models in the case of no run.

From the results shown in Table C1 we can conclude that the levels of consumption and expected utility are slightly higher in the Diamond-Dybvig model compared to the extended model. Note that the expected utility (in the case that a run does not occur) is calculated using Equation (1).

In the Diamond-Dybvig model the bank collects all deposits and invests it in the productive technology ($i_2^{DD} = 1$). In the extended model a bank invests $i_2^{***} = 0.57640$ in the productive technology. That is, in the extended model a bank decides to invest less in the productive technology and keep more in storage.

Case 2: A run occurs

In this case agents are located at a bank that experiences a run. All agents decide to run on the bank and in the Diamond-Dybvig model an agent receives c_1^{DD} with probability $\frac{1}{c_1^{DD}}$ and 0 with probability $\left(1 - \frac{1}{c_1^{DD}}\right)$. In the extended model agents receive $c_{1,2}^{***}$ with certainty.

	Extended Model	Diamond-Dybvig Model
Withdrawal in $t=1$	$c_{1,2}^{***} = 1.05856$	$\begin{cases} c_1^{DD} = 1.05861 & \text{with probability } 0.94463 \\ 0 & \text{with probability } 0.05537 \end{cases}$
Withdrawal in $t=2$	0	0
Expected Utility	10.05707	9.50033

Table C2: Consumption and expected utility levels in both models in the case of a run.

Table C2 displays the levels of consumption and expected utility obtained in the Diamond-Dybvig model and the extended model. During a run an agent receives 1.05861 with probability 0.94463 and zero with probability 0.05537 in the Diamond-Dybvig model. In the extended model an agent receives 1.05856 with certainty. The consumption levels during a run are higher on average in the extended model. In turn, the level of expected utility obtained during a run is higher in the extended model compared to the Diamond-Dybvig model. In order for banks in the extended model to stay liquid during a run, they must put an amount of $i_1^{***} = 0.00017$ in interbank insurance; this is a small price for banks to pay to stay liquid in times of crisis.

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