August 2020

A Warmer, Green Golden Rule

Kyle James Stuart
University of Windsor, stuartk@uwindsor.ca

A Warmer, Green Golden Rule

by

KYLE STUART

A Major Research Paper

Submitted to the Faculty of Graduate Studies

through the Department of Economics

in Partial Fulfilment of the Requirements for

the Degree of Master of Arts at the

University of Windsor

Windsor, Ontario, Canada

2020

A Warmer, Green golden Rule

by

KYLE STUART

Approved By

C. Trudeau

Department of Economics

M. Arbex, Advisor

Department of Economics

June 29, 2020

Author's Declaration of Originality

I hereby certify that I am the sole author of this major paper and that no part of this major paper has been published or submitted for publication.

I certify that, to the best of my knowledge, my major paper does not infringe upon anyone's copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my major paper, published or otherwise, are fully acknowledged in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owner(s) to include such material(s) in my major paper and have included copies of such copyright clearances to my appendix.

I declare that this is a true copy of my major paper, including any final revisions, as approved by my major paper committee and the Graduate Studies office, and that this major paper has not been submitted for a higher degree to any other University or Institution.

Abstract

The goal of this paper is to analyse the impact of temperature changes on the green golden rule. The green golden rule is the maximization of consumer utility based on consumer preference between consumption and environmental stock. The trade-off between environmental stock and consumption which is found to be negative. With temperature change being very prevalent in our era, we look at temperature change in the form of a damage function. By looking at both an increasing and decreasing damage function, along with changing variables in the green golden rule, we see that when the average global temperature deviates from the mean, be it above or below the mean, there is a decrease or increase in the damage function respectively.

Keyword: Environment, temperature anomalies, green golden rule

JEL Classification: Q54, Q56.

Dedication

To everyone who has supported me throughout the tough times

Acknowledgment

I would like to thank all of my professors throughout my time at the University of Windsor who have taken the time to support me through my tenure here. As well to me my family, friends and even old high school teachers who helped me get to where I am.

Special thanks to Dr. Marcelo Arbex who helped me with this paper, none of this would have been possible without his help, patience and care. He helped me more than I could ever express in this. As well to all in the Economics department who helped me get to where I am today.

Contents

List of Tables

List of Figures

1 Introduction

One of the most pressing issues of the day is the uncertainty related to the state of the environment. But finding a compromise to help the environment without compromising our daily and future needs is a very tough task. In order to complete all these we need to focus on sustainable development.

Sustainable development is when the current goals of three pillars are met without compromising the future. These pillars are the environment, the economy, and the social structure. Being able to sustain these three pillars currently while also making a survivable future is the goal of many economists, environmentalists, and politicians. However, the main goal is to create a more sustainable environment without compromising the economy. Ultimately, the goal is to create a larger, stronger economy while limiting the damage done to the environment.

The focus of this paper is the issue of rising temperatures and average global temperatures caused by increases in global emissions from fossil fuels. Major related issues to these changes in the average temperature include droughts and a rising sea level due to ice caps melting.

From NASA (2020), we can see that the global sea level has risen over time by roughly 3.3mm per year up to 2020. If this continues to happen at this rate of increase, not only does this cause damage to the environment due to temperature changing, but this could potentially lead to cities being put under water, full economies being destroyed.

Also, droughts are a major side effect of this temperature change. Drought can be especially dangerous to developing countries since it is very difficult for them to be able to properly irrigate in these droughts. As well, these countries rely heavily on the agriculture industry for their economy and for their own people since they typically cannot afford to purchase imports to feed their people, especially countries that depend on subsistence agriculture. Subsistence agriculture is when the farmers yield enough crops to feed themselves and their families and cannot produce enough to sell.

In these developing nations, climate change will impact them even more than it will impact the countries that can afford to properly irrigate. At the current rates with sea level increasing, the average global temperature increasing, and the increasing possibilities of more droughts because of this, many countries' agricultural sectors will suffer. Developing countries for which agriculture accounts for a majority share of their economy and GDP will be made significantly worse off.

In doing so, we have many new models such as the green golden rule which is consumer utility maximization based on environmental stock and consumption. This model is based off of theory written by Beltratti et al (1995); in their paper they look at consumer utility but with consumption and environmental stock. Tthey focus on consumer substitution between environmental stock and consumption. They derive this result by maximizing utility of consumer consumption and environmental stock with a single constraint, that of the environmental reproduction function that I as well am using in this paper. The difference between their environmental reproduction function and mine is that I will be introducing temperature anomalies in the damage function.

In introducing temperature anomalies in the Green Golden Rule (Beltratti et al, 1995), we need to look at all possible angles of climate change which will be represented by a damage function. The damage function will be representing the temperature anomalies in this addition. This function represents the positive, negative, or even negligible impact climate change can have on the model being used in both production and the resource renewal functions.

This damage function was introduced in Nordhaus (1991) where the damage function is used to denote how much output is lost because of climate change. However, we do not want to look at it just as being impacted on output, but we also want to look at it as it has an impact on the resource renewal constraint. Nordhaus (1991) uses this damage function on the production function which is why I as well will be using this on the production function along with the environmental reproduction function. Nordhaus (1991) also talks about how the there is no conventional damage function which is why it is just represented by a constant damage function and not an actual function like the production and environmental reproduction functions.

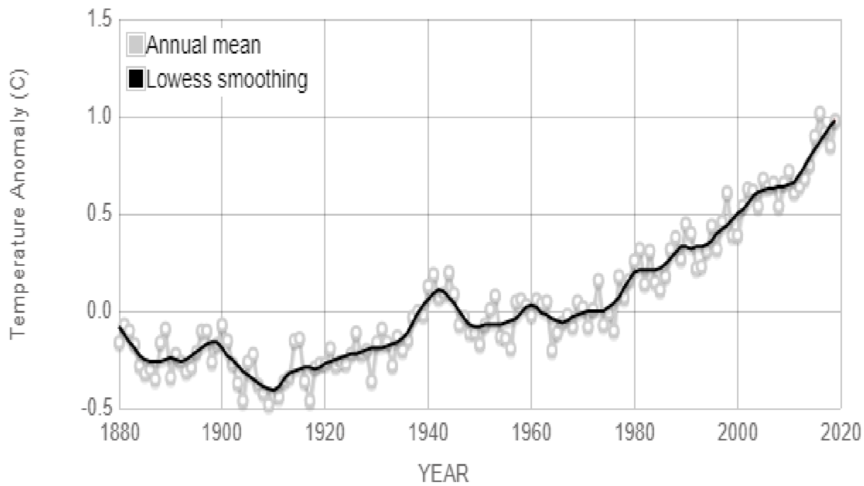
In running the analysis, we can see that when temperature does not deviate from the global mean, we have the original green golden rule, in which the consumer's trade-off between environmental stock and consumption is the negative of the environmental reproduction function derivative. When we see temperature anomalies deviate above the mean, the damage function is decreased to represent the decreased environmental reproduction. When we see this deviation from the mean, it causes the steady state environmental stock to decrease, which causes consumption to decrease based on the household budget constraint. From there, we get a decreased consumer utility.

But when we get cycles of the average temperature deviate below the mean we see the opposite, as this aids the environmental reproduction process and causes the steady state environmental stock to increase. In addition, we see an increase in consumption and utility.

2 Some Stylized Facts

When looking at how climate change can impact these functions we must look at all possibilities where the damage function can be positive, negative, or zero. For the damage function, T is the average global temperature which is analysed as how temperature deviates from said average, as well, the damage function is how these deviations impact the economy. We look at these possibilities because of Figure 1:

Figure 1: Temperature Change from 1880-2020



We can see from Figure 1 that even though the average global temperature has an upward trend, we do see the temperature remaining stable or decreasing year-over-year in some periods, forcing us to include all three possibilities in our model.

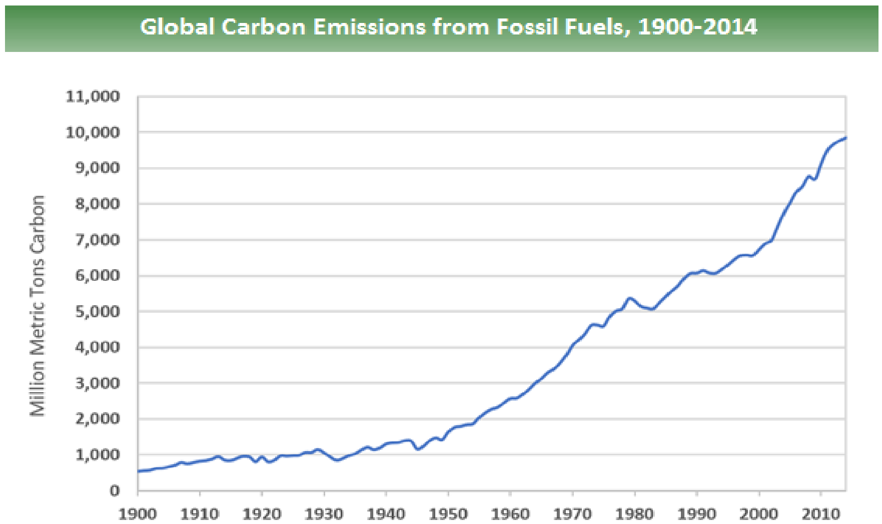
The three possibilities are how temperature deviates from the average global temperature. So when temperature does not deviate from the mean and stays the same over years, the damage function is equal to 1. When we see temperature deviate above the mean, the damage function decreases below 1. As well, when temperature deviates below the mean, the damage function is increased above 1.

In following through with this, we analyse the temperature anomalies through both our production function and our resource production function as well. The reason these temperature anomalies are on these two functions is that our production function is a standard Cobb-Douglas production function. However, instead of the function depending on capital and labour, we follow Beltratti (1995), and replace labour with the environmental stock.

In using environmental stock instead of labour, temperature changes, whether be it through an increase or decrease, will have an impact on the production in an economy. For example, in the agricultural sectors, with the increase of the global temperature as shown in figure 1, we see an increase in green house gas (GHG) emissions, an increase in droughts, and increases in sea levels (Cho, 2018). These changes that were created by an increase in the average global temperature, make it more difficult for the agricultural sector to produce more consistent and quality food, meaning that climate change hinders their production.

From these impacts of an increasing average temperature, we can assume that when temperature increases, it hinders production, whereas when the global temperature decreases, we can then assume that this would help production and make it more productive. The reason these assumptions are made is because of the already significant increases in global temperature that have shown to hurt mass production.

Figure 2: Carbon Emissions Change from 1900-2014



As Figure 2 shows, globally, we are emitting much more carbon and other GHGs from fossil fuels which is still the leading cause of GHG emissions in the world.

The other impact that temperature anomalies has on this model is on the assumption that the environment can renew itself through the environmental stock. The reproduction function is also impacted through these temperature anomalies be it through a positive or negative manner.

As seen in Figure 1, temperature anomalies have increased and have become a major issue. With no deviation, we obtain the regular green golden rule where our damage function is equal to one.

But, since as discussed previously, we can have periods where the average temperature increases or decreases. To model these, we have to analyse when temperature deviates below the mean causing an increased damage function.

The reason that the damage function becomes greater than one and leads to an increase in production is as mentioned earlier, that we assume that decreases in average temperature are better for production.

Finally, when we see temperature deviate above the mean, we see a decreased damage function. In analysing this deviation, it leads to a decrease in environmental reproduction. The damage function however cannot be equal to zero or be negative because we cannot have production stop altogether just because the average global temperature has increased.

3 Theoretical Model

This model is based off of theory used in The Green Golden Rule written by Beltratti et al (1995) where they focus on utility maximization based on the environmental resource constraint.

Consider the following utility maximization problem:

$$U(C_t, A_t) \tag{1}$$

subject to

$$D(T)R(A_t) = A_{t+1} - A_t + \alpha C_t \tag{2}$$

where we have a utility function comprised of consumption and environmental stock. As well as our constraint of the environmental reproduction function impacted by the damage function, along with the difference between periods t and t+1 environmental stock and discounted consumption.

From here, we obtain the following first order equations:

$$U_{A,t} = -\lambda_t[D(T)R_{A,t} + 1] \tag{3}$$

$$U_{A,t+1} = \frac{\lambda_t}{\beta} - \lambda_{t+1}[D(T)R_{A,t+1} + 1] \tag{4}$$

$$U_C = \lambda_t \alpha \tag{5}$$

From equations (3) and (5) we can obtain the following condition

$$\frac{U_{A,t}}{U_C} = \frac{-D(T)R_{A,t}}{\alpha} \tag{6}$$

As you can see from equation (6), we can see the consumer trade-off between environmental stock and consumption. This is shown as the negative of the first derivative of the environmental reproduction function divided by the discounted consumption factor (α).

Proposition 1. *The damage function impacts the consumer trade-off between environmental stock and consumption negatively when* $D(T) > 1$ *and positively when* $D(T) < 1$ *as shown in equation (6).*

4 Functional Form Model

In the previous section the theoretical consumption problem was discussed when we have a general form. But now if we add functional forms to these equations, we can obtain the optimal solution.

Using the Optimal Solution

Now, we extend Proposition 1 made earlier using these functional forms, so we obtain the following maximization problem:

$$U(C_t, A_t) = lnC_t + \gamma lnA_t \tag{7}$$

subject to

$$D(T)\left[rA_t - \frac{rA_t^2}{A^s}\right] = A_{t+1} - A_t + \alpha C_t \tag{8}$$

We then obtain the following first order conditions

$$\frac{1}{C_t} = \lambda_t \alpha \tag{9}$$

$$\frac{\gamma}{A_t} = -\lambda_t[D(T)(r - \frac{2rA_t}{A^s}) + 1] \tag{10}$$

$$\frac{\gamma}{A_{t+1}} = \frac{\lambda_t}{\beta} - \lambda_{t+1}[D(T)(r - \frac{2rA_{t+1}}{A^s}) + 1] \tag{11}$$

From these first order conditions we see that we obtain the following intertemporal condition:

$$\frac{\gamma/A_t}{1/C_t} = \frac{-[D(T)(r - \frac{2rA_t}{A^s}) + 1]}{\alpha} \tag{12}$$

Equation (12) shows the household trade-off between consumption and the stock of environmental goods. A negative relationship is obtained meaning that they are willing to give up consumption for an increased stock of environmental goods. This is the functional form of Proposition 1 which is shown to be the negative of the first derivative of the of the environmental reproduction constraint (equation (8)), divided by the discounted consumption factor (α).

We can also then contain the following intratemporal condition:

$$\frac{A_{t+1}}{A_t} = \frac{\lambda_t[D(T)(r - \frac{2rA_t}{A^s}) + 1]}{\lambda_{t+1}[D(T)(r - \frac{2rA_{t+1}}{A^s}) + 1] - \frac{\lambda_t}{\beta}} \tag{13}$$

This intratemporal condition shows the consumers trade-off for future environmental stock versus' today's environmental stock. This is represented by the ratio of today's first derivative of the environmental reproduction function impacted by the damage function, divided by the first derivative of tomorrow's environmental reproduction.

5 Numerical Example

5.1 Parameters

In order to solve this model we assume that this model is in the steady-state. In the steady-state we assume that all time dependent variables are equal. So we have $A_t = A_{t+1} = A$, $C_t = C_{t+1} = C$, and $\lambda_t = \lambda_{t+1} = \lambda$. These steady-state assumptions make it easier to solve the model. We start to solve by setting the model into the steady state and rearranging equation (13) to get:

$$\beta\gamma\frac{1}{A} + \beta\lambda\Big[D(T)\Big(r - \frac{2rA}{A^s}\Big) + 1\Big] = \lambda \tag{14}$$

Then, taking equation (9) in the steady-state and rearranging for λ we have $\lambda = 1/\alpha C$. Then, we substitute λ into equation (14) to get:

$$\beta\gamma\frac{1}{A} + \beta\frac{1}{\alpha C}\left[D(T)\left(r - \frac{2rA}{A^s}\right) + 1\right] = \frac{1}{\alpha C} \tag{15}$$

Then by multiplying equation (15) by αCA, we obtain:

$$\beta\gamma\alpha C + \beta\left[D(T)\left(rA - \frac{2rA^2}{A^s}\right) + A\right] = A \tag{16}$$

As well, the household budget constraint is:

$$D(T)\left(rA - \frac{rA^2}{A^s}\right) = \alpha C$$

Which by substituting into (16) we get:

$$\beta\gamma\left[D(T)\left(rA - \frac{rA^2}{A^s}\right)\right] + \beta\left(D(T)\left(rA - \frac{2rA^2}{A^s}\right) + A\right) = A$$

$$\beta(1+\gamma)\left[D(T)\left(rA - \frac{rA^2}{A^s}\right)\right] - \beta D(T)r\frac{A^2}{A^s} - (1-\beta)A = 0 \tag{17}$$

To fully solve the model, we assume certain values for the variables involved. We assume; $\beta = 0.99, \gamma = 0.8, r = 1.10, A^s = 1.0$, and $\alpha = 1.0$. To see the regular green golden rule which is the solution to the consumer maximization problem shown to be the negative of the first derivative of the environmental reproduction constraint divided by the consumption discount factor (α). With these assumptions we set the damage function ($D(T)$) to 1.0, then change it to see how it impacts the model. In doing so, we obtain the following steady state equilibrium values for environmental stock and consumption as shown in Table 1:

Table 1: Benchmark Case

$\gamma^* = 0.8, \beta^* = 0.99,$ $\alpha^* = 1.0,$ $r^* = 1.10, A^{s*} = 1.0$					
Damage function: D(T)=					
	0.5	0.75	1.0	1.5	2.0
Environmental Stock	0.6363	0.6385	0.6396	0.6407	0.6412
Consumption	0.1273	0.1904	0.2536	0.3798	0.5061
Utility	-2.4230	-2.0174	-1.7297	-1.3242	-1.0365

As you can see in Table 1, we have our benchmark parameters as discussed earlier. With these parameters, we obtain the values for the environmental stock through equation (17) by substituting the parameters into said equation. With this we can obtain the optimal level of environmental stock in the steady state. From there, we can substitute the value obtained for the environmental stock and the parameters, and substitute them into the household budget constraint as defined earlier to obtain steady state consumption. Finally, these values of consumption and environmental stock are substituted into the utility equation (equation (7)) to obtain our steady state utility. All of these represent the equilibrium of this model.

Table 1 shows exactly how these variables interact with each other. As the damage function is increased above 1, or temperature deviates below the mean, we can see that it slightly increases the environmental stock as well. Since both of these are positive impacts in the consumption function, it then leads to an increase in consumption by a larger amount compared to the environmental stock increase. Then, as we can see from equation (7), our utility is the log-linear sum of environmental stock and consumption with environmental stock being discounted. So we can see the impact that the effect of changing one steady state variable has on the others.

We want to run a sensitivity analysis on the model to analyse how these parameters will impact the model. In order to see the full effects of changing variables, we will be changing all five of the main variables; utility preference factor (γ), period preference factor (β), discounted consumption relation factor (α), the rate of environmental renewal (r), and the biocapacity reserve (A^s). We begin with the biocapacity reserve, A^s.

Table 2: Effects of changes in biocapacity reserve A^s

$\gamma^* = 0.8, \beta^* = 0.99, \alpha^* = 1.0, r^* = 1.10$

$A^s = 0.5$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental stock	0.3181	0.3198	0.3203
Consumption	0.0636	0.1268	0.1899
Utility	-3.671	-2.9773	-2.5718
$A^s = 2.0$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental stock	1.2726	1.2792	1.2813
Consumption	0.2546	0.5071	0.7597
Utility	-1.1754	-0.4820	-0.0765

When looking at the biocapacity reserves effect when changing its value on the model which is shown in Table 2, we can see that when we cut the biocapacity reserve in half, we see a significant decrease in the environmental stock in that it is reduced by about 50%. Oppositely, when we double A, we see the environmental stock roughly double as well.

Consumption and utility follow the same pattern as the environmental stock. Since, as mentioned under the benchmark case, when the environmental stock is increased, we then get an increase in consumption given the positive relation, which leads to an increase in equilibrium utility.

Table 3: Effects of changes in resource renewal rate r

$\gamma^* = 0.8, \beta^* = 0.99, \alpha^* = 1.0, A^{s*} = 1.0$

$r = 1.25$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.6371	0.6399	0.6409
Consumption	0.1445	0.2880	0.4315
Utility	-2.295	-1.6018	-1.1963
$r = 1.50$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.6380	0.6405	0.6413
Consumption	0.1732	0.3454	0.5176
Utility	-2.1127	-1.4195	-1.0140

From Table 3, we look at an exaggerated change in the environmental renewal rate factor (r) so that we can see how it impacts the model. We can see that this does follow the first two Tables in that as the damage function increases, it then creates an increase in the environmental stock ever so slightly, which as well creates an increase in consumption, then finally creates the increase in utility.

However, consumption and utility see significant changes with an increased environmental renewal rate (r) with the increased damage function, showing us that the environmental renewal rate has a more significant impact on the household budget constraint than the equilibrium environmental stock equation.

Table 4: Effects of changes in the preference relation γ

$\beta^* = 0.99, \alpha^* = 1.0, r^* = 1.10, A^{s*} = 1.0$

$\gamma = 0.6$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.6083	0.6119	0.6130
Consumption	0.1310	0.2612	0.3914
Utility	-2.3301	-1.6371	-1.2316
$\gamma = 1.2$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.6818	0.6846	0.6856
Consumption	0.1193	0.2375	0.3557
Utility	-2.5856	-1.8922	-1.4867

Now we look at the preference factor (γ) and how some people very much favour consumption and how some people today do in fact prefer the environment to their own consumption. When we change the preference factor (γ), we see another small increase in A, which then, with the increase in the damage function, creates an increase in consumption that is prominent. From there, we get the significant increase in equilibrium consumption as we do in the first examples.

Table 5: Effects of changes in discounted consumption α

$\gamma^* = 0.8, \beta^* = 0.99, r^* = 1.10, A^{s*} = 1.0$			
$\alpha = 0.7$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.6363	0.6396	0.6407
Consumption	0.1818	0.3622	0.5426
Utility	-2.0663	-1.3729	-0.9675
$\alpha = 1.3$			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.6363	0.6396	0.6407
Consumption	0.0979	0.1951	0.2922
Utility	-2.6854	-1.9920	-1.5865

Next, we look at the discounted consumption factor of α. Unlike the variables previously changed, the discounted consumption factor has no impact on the environmental stock since as represented in equation (17), the discounted consumption factor does not appear in it. Which means that the environmental stock acts the same as it does in the benchmark case as the damage function increases. Then, given these increases in the environmental stock and the damage function, we get a fairly significant increase in consumption with these increases from the household budget constraint. From there, we again get the increase in utility as we have before.

Table 6: Effects of changes in β

$\gamma^* = 0.8, \alpha^* = 1.0, r^* = 1.10, A^{s*} = 1.0$			
$\beta = 0.80$,			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.4805	0.5617	0.5887
Consumption	0.1373	0.2708	0.3995
Utility	-2.5719	-1.7678	-1.3413
$\beta = 0.90$,			
Damage function: D(T)=			
	0.5	1.0	1.5
Environmental Stock	0.5707	0.6068	0.6188
Consumption	0.1348	0.2625	0.3892
Utility	-2.4530	-1.7373	-1.3276

The final variable to be changed now is our time preference factor β. As you can see in Table 6, the preference factor being decreased negatively impacts the environmental stock in equilibrium. However we still obtain the same pattern as all previous numerical examples with environmental stock increasing as the damage function does. Again, form there, consumption increases with the increased environmental stock and damage function, which again leads to the increase in equilibrium utility.

6 Conclusion

The goal of this paper is to analyse the impact of temperature changes on the green golden rule. The green golden rule is the maximization of consumer utility based on consumer preference between consumption and environmental stock. From our numerical exercises, we can see that when the damage function increases, with some variables having a larger affect on the environmental stock. When changing the biocapacity reserve we see a rather significant change in the environmental stock with a high biocapacity reserve. Whereas other variables like the time preference factor, utility preference factor, and the environmental renewal rate factor have very small impacts when increased, then there is the discounted consumption factor having no impact on the environmental stock.

With consumption, when increasing the biocapacity reserve and the time preference factor we see a significant increase in consumption when the damage function increases. Other variables like the utility preference factor and the environmental renewal rate factor do not have a significant impact. Then there is the discounted consumption factor where it is its increase that leads to a decrease in consumption, and the factor increases as the damage function increases as well.

Then, from these increases in both environmental stock and consumption, we can see that utility is increased across the board. The variables that have a large impact on either environmental stock or consumption also have a larger impact on utility since it is the sum of the logs of both variables.

The maximum utility yield comes from when we double the biocapacity reserve and have the highest damage function. Whereas the lowest utility yield comes from when we half the biocapacity reserve and have the highest increase in temperature change or decreased damage function.

From this paper, there are no direct links to any policy implications. However, this could be modified to allow for heterogeneous agents over time. We also could set environmental goods to become a public good and have a taxation on the public good. Policy implications could become possible from these possible future adjustments.

As well, this type of analysis involving environmental impact on economies could lead to more normalization of environmental impacts on models. For example, having an environmental factor on models such as the Solow growth model, where we can analyse the impact that the environment has on whole economies instead of just consumer behaviour. We leave this for future research.

References

Beltratti, A., G. Chichilnisky, and G. Heal (1995): The Green Golden Rule. Economic Letters, 49. 175-179.

Boden, T.A., Marland, G., and Andres, R.J. (2017). Global, Regional, and National Fossil-Fuel CO2Emissions. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A.

Chichilnisky, G. (1997): What Is Sustainable Development?. Land Economics, 73(4). 467-491.

Cho, R. (2018, July 25): How Climate Change Will Alter Our Food. https://blogs.ei.columbia.edu/2018/07/25/climate-change-food-agriculture/

Heal, G. (2017): The Economics of the Climate. Journal of Economic Literature, 55(3), 1046-1063.

NASA, (2020, January): Sea Level. https://climate.nasa.gov/vital-signs/sea-level/

NASA/GISS, (2019): Global Temperature. https://climate.nasa.gov/vital-signs/global-temperature/

Nordhaus, W. D. (1991): To Slow or Not To Slow: The Economics of the Grenehouse Effect. The Economic Journal, 101, 920-937

Appendix

A.1. Python Code

```
import math as m
import numpy as np
import matplotlib.pyplot as plt
beta = 0.99
D = 0.5
r = 1.10
X = 1.0
gamma = 0.8
alpha = 2.5
def func(A):
# Function: f(x)
fx = beta*(1 + gamma)*(D*(r*A - r*A**2/X)) - beta * D * r * (A**2/X)
-(1 - beta) * A
return fx
```

```
xmin = 1
xmax = 3
xv = np.arange(xmin, xmax, (xmax - xmin)/200.0)
fxv = np.zeros(len(xv),float) # define column vector
for i in range(len(xv)):
fxv[i] = func(xv[i])
fig, ax = plt.subplots()
ax.plot(xv, fxv)
ax.plot(xv, np.zeros(len(xv)))
# Create a title with a red, bold/italic font
plt.show()
from scipy.optimize import root
guess = 1
print(” ”)
print(” ———— Root ————”)
result = root(func, guess) # starting from x = 2
myroot = result.x # Grab number from result dictionary
print(”The root of func is at ”.format(root))
from scipy.optimize import fsolve
guess = 1
print(” ”)
print(” ———— Fsolve ————”)
result = fsolve(func, guess) # starting from x = 2
myroot = result[0]
# Line above: Grab number from result dictionary
```

print(”Equilibrium Environmental Stock is ”.format(result))

add C(A) and U(C,A) to get equilibirum utility and consumption at once

def C(A):

Cv = (D/alpha)*(r*A - r*A**2/X)

return Cv

print(”Equilibrium Consumption is ”.format(C(result)))

def U(C,A):

Uv = m.log(C) + gamma*m.log(A)

return Uv

print(”Equilibrium Utility is ”.format(U(C(result),result)))

A.2. General Market Model

$$U(C_t, A_t) \tag{18}$$

subject to

$$D(T)F(K_t, A_t) = C_t - K_{t+1} + (1-\delta)K_t \tag{19}$$

$$D(T)R(A_t) = A_{t+1} - A_t + \alpha C_t \tag{20}$$

we get the following first order conditions:

$$U_C = \lambda_t^1 + \lambda_t^2\alpha \tag{21}$$

$$U_{A,t} = -\lambda_t^1 D(T)F_{A,t} - \lambda_t^2[D(T)R_{A,t} + 1] \tag{22}$$

$$U_{A,t+1} = \frac{\lambda_t^1}{\beta} - \lambda_{t+1}^1 D(T)F_{A,t+1} - \lambda_{t+1}^2[D(T)R_{A,t+1} + 1] \tag{23}$$

$$F_{K,t} = \frac{-(1-\delta)}{D(T)} \tag{24}$$

$$F_{K,t+1} = \frac{\lambda_t^1}{\beta\lambda_{t+1}^1 D(T)} - \frac{1-\delta}{D(T)} \tag{25}$$

We get the following trade-off:

$$\frac{U_A}{U_C} = \frac{-\left(\lambda_t^1 D(T)F_A + \lambda_t^2[D(T)R_A + 1]\right)}{\lambda_t^1 + \lambda_t^2\alpha} \tag{26}$$

Functional Form

$$U(C_t, A_t) = lnC_t + \gamma lnA_t \tag{27}$$

subject to

$$D(T)\varphi K_t^\sigma A_t^\theta = C_t - K_{t+1} + (1 - /delta)K_t \tag{28}$$

$$D(T)\left[rA_t - \frac{rA_t^2}{A^s}\right] = A_{t+1} - A_t + \alpha C_t \tag{29}$$

We then obtain the following first order conditions:

$$\frac{1}{C_t} = \lambda_t^1 + \alpha\lambda_t^1 \tag{30}$$

$$\varphi K_t^{\sigma-1} A_t^{\theta} = \frac{-(1-\delta)}{D(T)\sigma} \tag{31}$$

$$\varphi K_{t+1}^{\sigma-1} A_{t+1}^{\theta} = \frac{\lambda_t^1 \beta^t}{\lambda_{t+1}^1 \beta^{t+1} \sigma D(T)} - \frac{1-\delta}{D(T)\sigma} \tag{32}$$

$$\frac{\gamma}{A_t} = -\lambda_t^1 D(T)\varphi\theta K_t^{\sigma} A_t^{\theta-1} - \lambda_t^2 [D(T)(r - \frac{2rA_t}{A^s}) + 1] \tag{33}$$

$$\frac{\gamma}{A_{t+1}} = \frac{\lambda_t^2}{\beta} - \lambda_{t+1}^1 [D(T)\varphi\theta K_{t+1}^{\sigma} A_{t+1}^{\theta} - 1] - \lambda_{t+1}^2 \left[D(T)\left(r - \frac{2rA_{t+1}}{A^s}\right) + 1\right] \tag{34}$$

Then, we achieve the following trade off:

$$\frac{\gamma/A_t}{1/C_t} = \frac{-\lambda_t^1 D(T)\varphi\theta K_t^{\sigma} A_t^{\theta-1} - \lambda_t^2 [D(T)(r - \frac{2rA_t}{A^s}) + 1]}{\lambda_t^1 + \alpha\lambda_t^1} \tag{35}$$

Vita Auctoris

Kyle Stuart was born in 1996 in Wallaceburg, Ontario, Canada. He graduated from Ursuline College Chatham in 2014 and went to graduate from the University of Windsor where he obtained a B.A. in Economics in 2019. Currently, he is a candidate for the Master's degree in Economics from the University of Windsor and hopes to graduate in 2020.