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Optimal speed of a machine in an assembly line using the continuous time Markov chain rate matrix

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ABSTRACT

The optimal speed of a machine in an assembly line is determined using a Markov decision process type model. We develop the rate matrix that represents the inter-event time of a machine, either repair time or time to breakdown, as a function of speed. We consider the rate of time to breakdown with a variety of functions of speed. We find limiting probabilities and express profit in terms of these probabilities. We then find the optimal speed to maximize profit. Further, we assume an underlying function of speed and simulate data using R. From the simulated data, we estimate parameters of the speed function, and finally estimate the optimal speed.

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Chapter 1

Introduction

According to various historical accounts, the industrial revolution began in the late 1700's. In 1913, Henry Ford introduced the assembly line to build automobiles. Assembly lines are an important area of study. An assembly line consists of a sequence of machines or people who complete a restricted task on an item before an item moves to the next stage. Assembly lines are important since they create efficiencies and save time and effort. Major goals are to maximize profit/revenue and to minimize waste or time. A machine working alone, or within an assembly line, can break down. Such equipment failure can result in lost revenue and repair costs, and even health and safety risks. Bialy and Ruzbarsky (2018) address the issue of cause and effect of breakdown. There are a variety of reasons for breakdown of a machine, including the speed of the machine, the lack of maintenance, improper use, and age.

In this paper, we consider the speed of a machine in an assembly line. The speed of the machine can be adjusted but the higher the speed, the more likely the machine is to breakdown. When the machine is broken, the manufacturer loses productivity due to downtime. So, there is no reward. When the machine is working, there is productivity and rewards. A slower speed means lower productivity and lower profit. This paper discusses finding the optimal speed of the machine in an assembly line to maximize the profit.

We build a continuous time Markov model. A rate matrix is introduced as a function of speed to maximize the profit. There are number of options to build the rate matrix as a function of speed. First we discuss the expression for the speed function. The options for the rate function include linear, square root of linear, square root of quadratic, quadratic, cubic and square root of cubic. These six cases are considered and we graph profit vs speed (θ). The linear, square root of linear and square root of quadratic cases are not reasonable models because the functions are increasing and hence there is no finite speed which gives the maximum profit. Although these models are possible, the maximum profit would occur when we run the machine as fast as possible. We thus consider quadratic and cubic rate functions since they result in the profit function having maximum at some finite value of speed. The rate matrix is initially 2×2 (two states 0 and 1, 0 represents the working state and 1 represents the broken state). Later we consider a 3×3 matrix (making two sub states of the broken state 1_1 and broken state 1_2 from the original broken state 1). Similarly, we can extend this up to a $n \times n$ matrix. By considering larger sizes of a rate matrix, the repair time can be approximated by a deterministic distribution, rather than the implied exponential distribution when using a 2×2 matrix. The expected repair time approaches a constant and the variance of the repair time approaches zero.

We perform a simulation assuming an underlying quadratic function of speed for the breakdown rate. We use regression in order to try to identify the breakdown rate as a function of speed. Finally, we discuss some issues for further study.

1.1 Research Objective

The main objective of this paper is find the optimal speed of a machine in an assembly line by using a continuous time Markov process and a rate matrix (which models inter-event times, either repair time or time to break down). We simulate data using R software. We use Maximum Likelihood Estimators (MLE) and regression polynomials to estimate parameters. As a byproduct of this study, optimal speed will reduce the high amount of energy consumed by the machine and will increase the motor life of the machine.

1.2 Uses of the study

This work can be used for a manufacturing company which has automatic filling, capping or labelling machines which are used in the food and beverage industry, pharmaceutical industry, chemical industry, e-liquid industry, and many more to maximize their rewards/profit.

This work is related to the issue of 'speed vs accuracy.' Thus there would be applications to many types of tasks: writing a report, taking an exam, creating a computer program, cooking a meal, building a house, etc.

Chapter 2

Literature review

Durrett (1999) has good insight of the limiting behaviour and stationary distribution of continuous time Markov chains. Albert (1962) discussed the problem of estimating Q (rate matrix) when it is not known. In particular, he derives a maximum likelihood estimate for Q and its large sample properties were investigated. Q (also called the infinitesimal generator for the process) is a square matrix, whose values are fixed.

Ke et al. (2016) considers an M/M/r machine repair problem with warm standbys, switching failures, reboot delays, and a repair pressure coefficient. A birth-and-death process was used to establish the steady-state equations. A matrix-analytic method was then adopted to obtain the steady-state analytical solutions that are used to calculate various systems performance measures, such as the expected number of failed machines, the expected number of idle servers, machine availability, and operative utilization. A function of expected profit per unit time was derived. Probabilistic Global Search Lausanne (PGSL) method was employed to determine the optimal values that maximize this function.

Bhuniya et al. (2019) find that in long-run production systems, unusual energy consumption and machine failures occur frequently. The unit production cost is dependent on the production rate of the machine and its failure rate. The aim of this study was to obtain the optimum profit with a reduced failure rate, under the optimum advertising costs and the optimum sale price. The total profit of the model becomes a complex, non-linear function, with respect to the decision variables. For this reason, the model is solved numerically by an iterative method. However, the global optimality is proved numerically, by using a Hessian matrix. The numerical results obtained show that for smart production, the maximum profit always occurs at the optimum values of the decision variables. Using stochastic optimal control and Markov processes, Tan (2019) concludes that the production rate can be controlled optimally to benefit from producing in advance when the production cost is low while meeting the supply and demand in the best way.

Bartkowiak and Pawlewski (2016) demonstrate the use of a Discrete Event Simulation tool to reduce the negative impact of machine failures on the performance of a filling line. The subject of the study is a filling and packaging production line which consists of seven machines connected by conveyors. Machine failures are registered by a maintenance Data Acquisition system. Those data are used to derive statistical distributions for Time To Repair and Time Between Failures. The model is built using "FlexSim" simulation software and different allocation scenarios are considered. The introduction of buffers results in an increase in mean line through output by 15%. The initial results indicate that the proposed approach may lead to the reduction of negative effects of machine failures.

Widyadana and Wee (2011) discuss deteriorating items production inventory models with random machine breakdown and stochastic repair time. The model assumes the machine repair time is independent of the machine breakdown rate. This study showed that management should give more attention to the production rate. There are some causes for machine breakdown while working- mainly caused by humans, the machine itself, management method and materials. Among those potential causes Biały and Ružbarskỳ (2018) concluded that material affected the proper operation of the machine. Zhang and Li (2015) designed an intelligent control system (programmable logic controller(PLC)) for machine speed in an automated production line of beer. As a result of their work, they proved that they could improve the efficiency of the production line.

Widyadana and Wee (2011), using sensitivity analysis and an exponential distribution, developed deteriorating items production inventory models with random machine breakdown and stochastic repair time. (The model assumes the machine repair time is independent of the machine breakdown rate.) A Markov decision process is used to identify the energy control action and estimate the potential capacity of demand reduction (Sun and Li (2014)). Eboli and Cozman (2008) proposed a methodology for automotive manufacturing line scheduling using Markov decision processes. Ao et al. (2019) scheduled a dynamic maintenance plan based on Markov decision process. In a smart robot production line, robot enabled teaching problem was formulated into an Markov decision process (Cheng and Chen (2013)).

We generally assume exponentially distributed X_i (inter event times) with failure rate α_i. Results of a similar type also happen when component lifetimes follow the Erlang distribution (a special case of the Gamma distribution with integer shape parameter), since an Erlang r.v. can be expressed as a sum of i.i.d. exponential r.v.(Akkouchi (2008)).

When we are dealing with multiple models or shapes we have to choose the best model or shape for the related study among all other competing models. Archontoulis and Miguez (2015) showed how to find the best model using Akaike information criterion (AIC), Bayesian information criterion (BIC). Aho et al. (2014) also discussed the AIC and related tools and BIC and related tools, which are appropriate to the statistical model selection.

Miyanaji et al. (2018) discussed the effect of printing speed on quality and integrity of the printed components. It was shown that increasing printing speed reduces the accuracy of the fabricated parts regardless of their orientation in the powder bed due to the enhanced inertia forces.

Chapter 3

Modelling rate function

3.1 Introduction of rate matrix

A manufacturing process in which an arrangement of machines works to configure the finished product is called as an assembly line. Generally, a production line is a semi-automated system in which products go through a production process. Various sections of the line station have human workers who oversee production. An assembly line can be either working 'on' or broken 'off'. If broken 'off', it earns no rewards/profits. If working 'on' it earns rewards/profits proportional to the speed of the machine until the machine reaches its optimum speed.

$$Profit \propto Speed$$

The production line breaks down at a rate that is an increasing function of speed. The broken system is repaired at a rate is which is not a function of speed. This means that repair time is independent of θ (speed).

Assumptions:

- Time spent in any state (working $\rightarrow$ broken and broken $\rightarrow$ working) is exponentially distributed.
- Profit is proportional to the speed of the machine while working.

Definition 3.1. *A transition rate matrix (also known as an intensity matrix or infinitesimal generator matrix) is a square array of numbers describing the instantaneous rate at which a continuous time Markov chain transitions between states. In a transition rate matrix Q, element q_{ij} (for $i \neq j$) denotes the rate departing from state i and arriving in state j. Diagonal elements q_{ii} are defined such that,*

$$q_{ii} = -\sum_{j \neq i} q_{ij}$$

and therefore the rows of the matrix sum to zero.

Our time-dependent Markov chain rate matrix Q is a function of speed. Here $S(\theta)$ is the rate function from working state to the broken state. Where, θ is the speed. This 2×2 matrix is the original matrix with two states '0' and '1'. '0' represents the working machine and '1' represents the broken machine. Without loss of generality we set the repair rate to be 1.

$$\text{States} = \begin{cases} 0, & \text{if working} \\ 1, & \text{if broken} \end{cases}$$

$$Q = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} -S(\theta) & S(\theta) \\ 1 & -1 \end{bmatrix} \end{array} \tag{3.1}$$

Definition 3.2. *The row vector $\vec{\pi}$ is called a stationary distribution of the continuous time Markov chain if $\vec{\pi}$ has entries $(\pi_j : j \in S(state\,space))$ such that:*

a. $\pi_j \geq 0, and \sum_j \pi_i = 1$

b. $0 = \pi \times Q$, *where Q is the rate matrix of the chain. Thus,* $0 = \sum_i \pi_i \times q_{ij}, \forall j.$

Solve, $\vec{\pi} \times Q = 0.$ for limiting probabilities

$$\vec{\pi} = (\pi_0, \pi_1)$$

$$\pi_0 + \pi_1 = 1$$

$$\begin{aligned} \text{Profit} = M &= \text{prop. time working} \times \text{income per unit} \times \text{speed of the machine} \\ &= \pi_0 \times k \times \theta \end{aligned}$$

Theorem 3.1. *For the rate matrix* $Q = \begin{bmatrix} -S(\theta) & S(\theta) \\ 1 & -1 \end{bmatrix}$, *the limiting probability* π_0 *of being in the working state is* $\pi_0 = \dfrac{1}{1+S(\theta)}$.

Proof. We solve $(0,0) = (\pi_0, \pi_1)Q$ subject to $\pi_0 + \pi_1 = 1$. The result follows. □

Corollary 1. *The profit for speed* θ *is* $\dfrac{k\theta}{1+S(\theta)}$.

3.2 Types of rate function in terms of speed.

Rate function $S(\theta)$ can be linear, root of linear, quadratic, root of quadratic, cubic, root of cubic . . . etc. as a function of θ. In this paper we indicate how to determine a reasonable rate function. R software is used to get graphs (Profit(M) vs Speed (θ)). We use the derivative of the profit in order to find the optimal speed to maximize the profit. Six cases are considered in total.

3.2.1 Case I: Linear rate function

In this case, the rate function is linear i.e $S(\theta) = a + b\theta$

$$\vec{\pi} \times Q = 0 \implies (\pi_0, \pi_1) \times \begin{pmatrix} -(a+b\theta) & (a+b\theta) \\ 1 & -1 \end{pmatrix} = 0$$

$$\implies \begin{cases} -(a+b\theta) \times \pi_0 = \pi_1 \\ \pi_0 + \pi_1 = 1 \end{cases}$$

$$\Longrightarrow \pi_0 = \frac{1}{a + b\theta + 1}$$

Hence,

$$\begin{aligned} M &= \pi_0 \times k \times \theta \\ &= \frac{1}{a + b\theta + 1} \times \ k \times \theta \end{aligned}$$

Then, take the derivative of M with respect to θ, we get

$$\begin{aligned} \frac{\partial M}{\partial \theta} &= \frac{(a+1) \times \ k}{(a + b\theta + 1)^2} \\ \frac{\partial M}{\partial \theta} &= 0 \end{aligned}$$

In this case optimal speed cannot be found as graphs 3.1 and 3.2 do not have maximum turning point. So, this speed function will no longer be considered, as the graph indicates the optimal speed is as fast as possible.

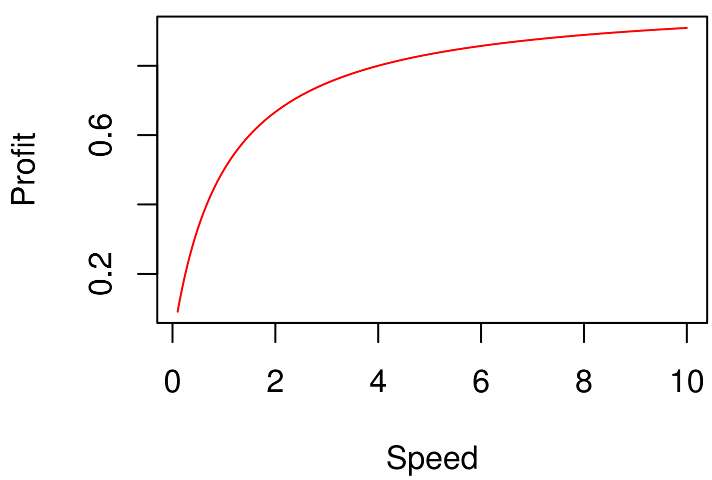

Figure 3.1: Linear rate function with $a = 0, b = 3$

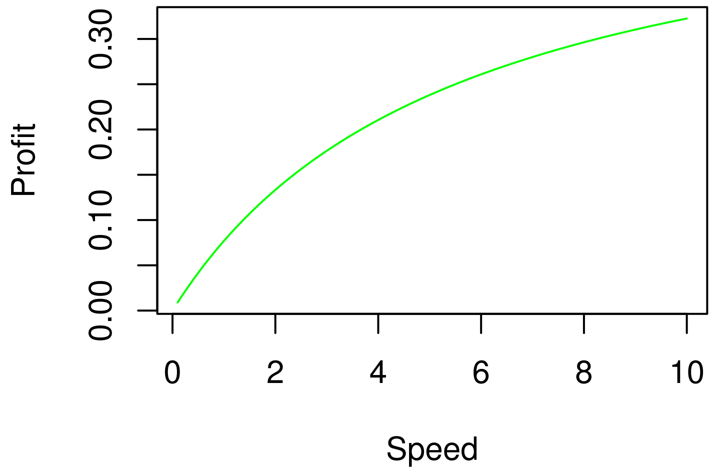

Figure 3.2: Linear rate function with $a = 2, b = 3$

3.2.2 Case II: Root of linear rate function

When the rate function of the speed $= \sqrt{(a + b\theta)}$

$$\vec{\pi} \times Q = 0$$

$$(\pi_0, \pi_1) \times \begin{pmatrix} -\sqrt{(a+b\theta)} & \sqrt{(a+b\theta)} \\ 1 & -1 \end{pmatrix} = 0$$

$$M = Profit = \frac{\theta}{\sqrt{a + b\theta + 1}}$$
$$\frac{\partial M}{\partial \theta} = \frac{b\theta + 2(a+1)}{2(a + b\theta + 1)^{3/2}}$$
$$\frac{\partial M}{\partial \theta} = 0$$
$$\hat{\theta} = \frac{-2(a+1)}{b}$$

Though we get the value for the optimal speed, solution is not a positive value since ‘a’ and ‘b’ are positive values. Graphs 3.3 and 3.4 are not having turning points. So, in this case the optimal speed is as large as possible.

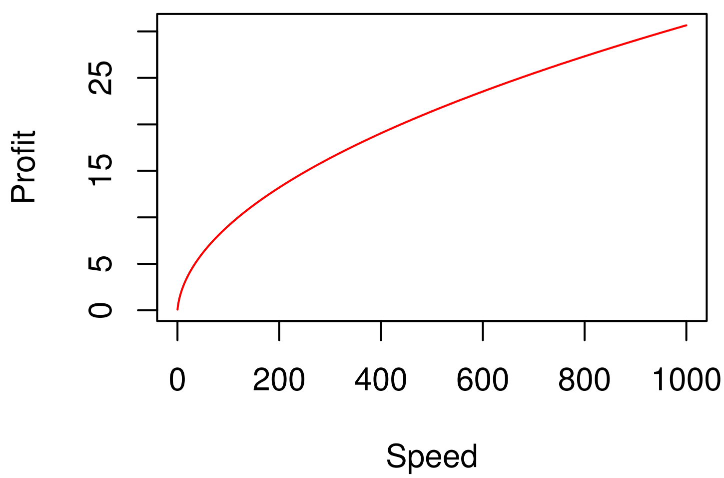

Figure 3.3: Root of linear rate function with $a = 0, b = 3$

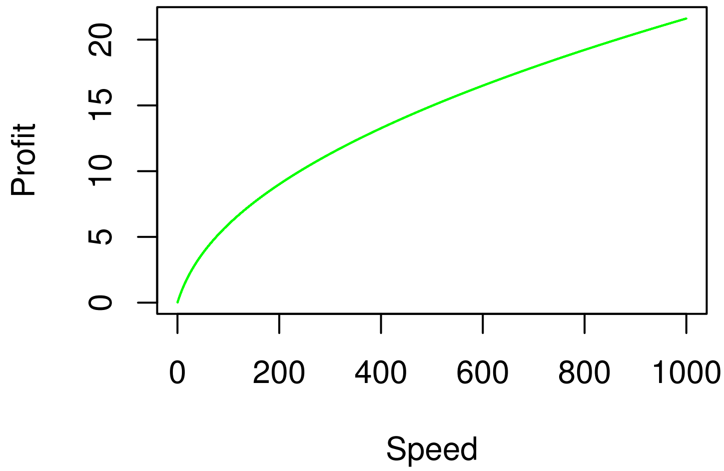

Figure 3.4: Root of linear rate function $a = 2, b = 3$

Henceforth we take $k = 1$ in our expression for profit.

3.2.3 Case III: Quadratic rate function

When rate function $S(\theta) = a + b\theta + c\theta^2$

$$\vec{\pi} \times Q = 0$$

$$(\pi_0, \pi_1) \times \begin{pmatrix} -(a + b\theta + c\theta^2) & (a + b\theta + c\theta^2) \\ 1 & -1 \end{pmatrix} = 0$$

$$M = \text{Profit} = \frac{\theta}{a + b\theta + c\theta^2 + 1}$$

$$\frac{\partial M}{\partial \theta} = \frac{a - c\theta^2}{(a + b\theta + c\theta^2 + 1)^2}$$

$$\frac{\partial M}{\partial \theta} = 0$$

$$0 = a + 1 - c\theta^2$$
$$\hat{\theta} = \sqrt{\frac{a+1}{c}}$$

There is a finite value for the optimal speed in this case and graphs also have turning points. This model is possible.

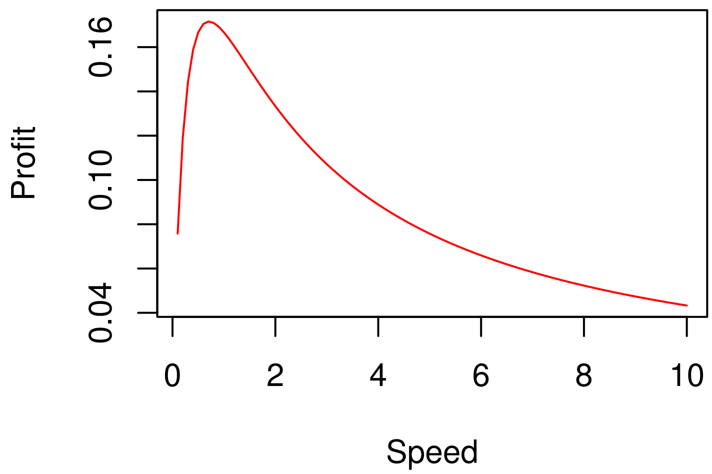

Figure 3.5: Quadratic rate function with $a = 0, b = 3, c = 5$

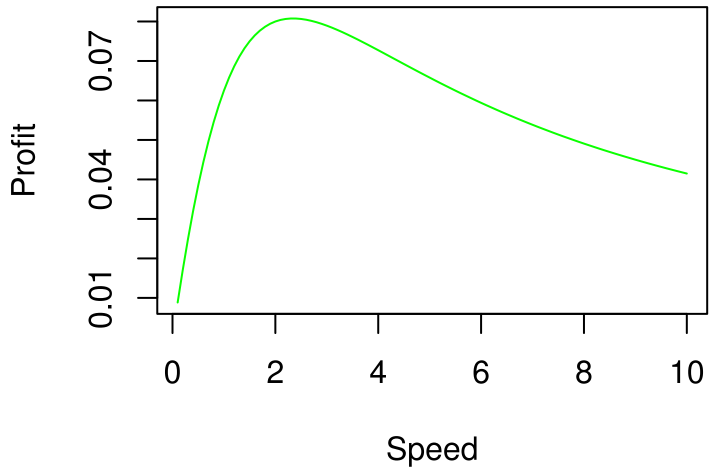

Figure 3.6: Quadratic rate function with $a = 2, b = 3, c = 5$

It is reasonable to assume that when the speed $\theta = 0$, the breakdown rate $S(\theta) = 0$. This implies that $a = 0$.

3.2.4 Case IV: Root of quadratic rate function

When rate function $S(\theta) = \sqrt{a + b\theta + c\theta^2}$

$$\vec{\pi} \times Q = 0$$

$$(\pi_0, \pi_1) \times \begin{pmatrix} -\sqrt{(a + b\theta + c\theta^2)} & \sqrt{(a + b\theta + c\theta^2)} \\ 1 & -1 \end{pmatrix} = 0$$

$$M = Profit = \frac{\theta}{\sqrt{a + b\theta + c\theta^2} + 1}$$

$$\frac{\partial M}{\partial \theta} = \frac{b\theta + 2a + 2}{2(a + b\theta + c\theta^2 + 1)^{3/2}}$$

$$\frac{\partial M}{\partial \theta} = 0$$
$$0 = b\theta + 2a + 2$$
$$\hat{\theta} = \frac{-2(a+1)}{b}$$

In this case, the solution for the optimal speed is a negative value for $a > -1$ and $b > 0$. The result in this case similar to the linear case.

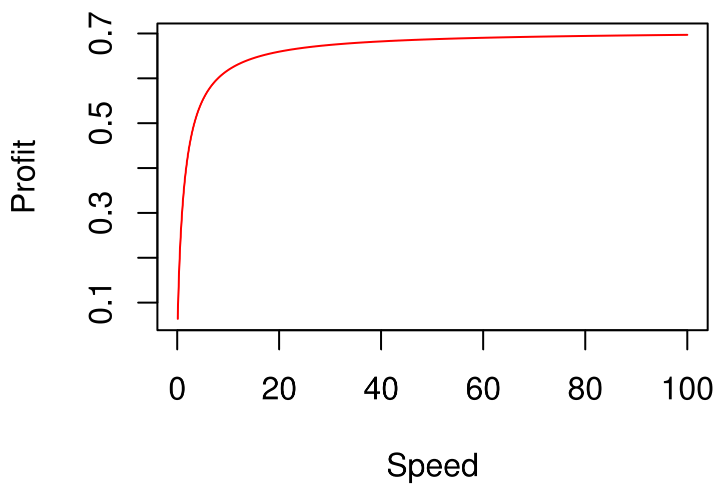

Figure 3.7: Root of quadratic rate function with $a = 0, b = 3, c = 5$

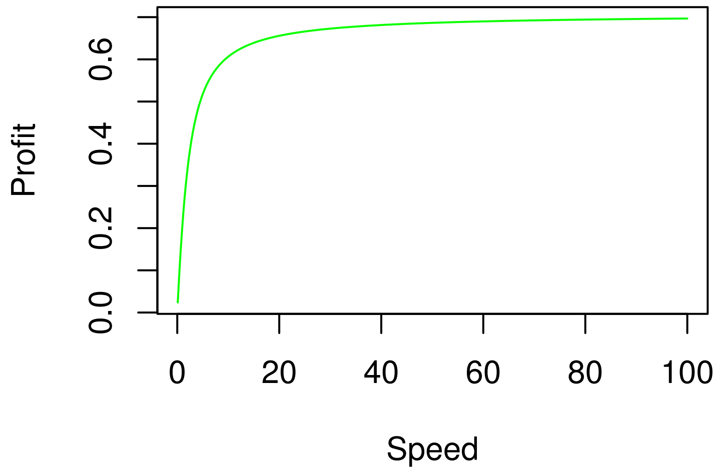

Figure 3.8: Root of quadratic rate function with $a = 2, b = 3, c = 5$

3.2.5 Case V: Cubic rate function

When rate function $S(\theta) = a + b\theta + c\theta^2 + d\theta^3$

$$\vec{\pi} \times Q = 0$$

$$(\pi_0, \pi_1) \times \begin{pmatrix} -(a + b\theta + c\theta^2 + d\theta^3) & a + b\theta + c\theta^2 + d\theta^3 \\ 1 & -1 \end{pmatrix} = 0$$

$$M = Profit = \frac{\theta}{a + b\theta + c\theta^2 + d\theta^3 + 1}$$

$$\frac{\partial M}{\partial \theta} = \frac{-2d\theta^3 - c\theta^2 + a + 1}{(a + b\theta + c\theta^2 + d\theta^3 + 1)^2}$$

$$\frac{\partial M}{\partial \theta} = 0$$

$$0 = -2d\theta^3 - c\theta^2 + a + 1$$

In this case we have a cubic equation to solve to get the optimal value. Since $S(0) = 0$, we get $a = 0$ when $c > 0$, $d > 0$ we can find a solution for the optimal speed.

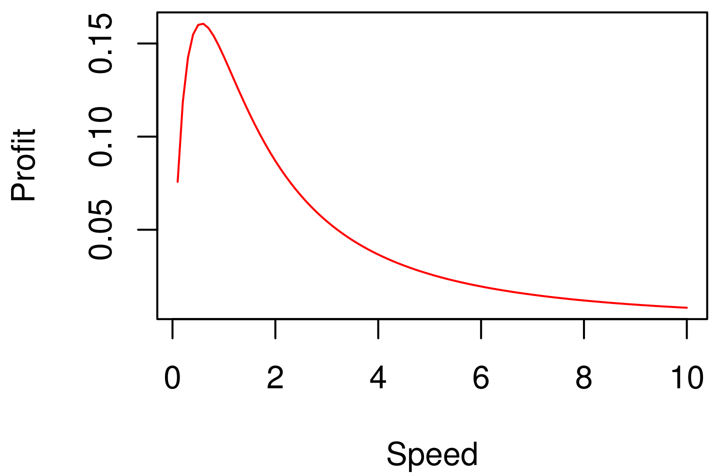

Figure 3.9: Cubic rate function with $a = 0, b = 3, c = 5, d = 7$

3.2.6 Case VI: Root of cubic rate function

When rate function $S(\theta) = \sqrt{a + b\theta + c\theta^2 + d\theta^3}$

$$\vec{\pi} \times Q = 0$$

$$(\pi_0, \pi_1) \times \begin{pmatrix} -\sqrt{a + b\theta + c\theta^2 + d\theta^3} & \sqrt{a + b\theta + c\theta^2 + d\theta^3} \\ 1 & -1 \end{pmatrix} = 0$$

$$M = Profit = \frac{\theta}{\sqrt{a + b\theta + c\theta^2 + d\theta^3 + 1}}$$
$$\frac{\partial M}{\partial \theta} = \frac{-d\theta^3 + b\theta + 2a + 2}{(a + b\theta + c\theta^2 + d\theta^3 + 1)^{3/2}}$$
$$\frac{\partial M}{\partial \theta} = 0$$
$$-d\theta^3 + b\theta + 2a + 2 = 0$$

Solving this cubic for θ will give us the optimal speed to maximize the profit. As before $S(0) = 0$ implies that $a = 0$

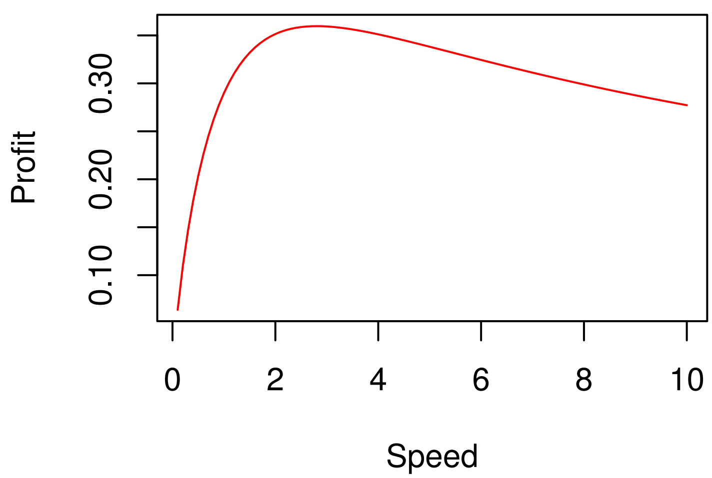

Figure 3.10: Root of cubic rate function with $a = 0, b = 3, c = 5, d = 7$

3.3 Summary of the types of rate function

According to our analysis in the linear case θ should be as large as possible. The same as true in the square root of linear case and in the square root of the quadratic case. In the quadratic case and cubic case and square root of cubic case we can solve for a finite optimal speed θ.

Table 3.1: Summary of the selected rate functions.

Types of rate function	$\hat{\theta}$	Shape of the graph	Conclusion
$a + b\theta$	not exist	increasing no turning point	$\hat{\theta}$ not exist.
$\sqrt{(a + b\theta)}$	$\frac{-2(a+1)}{b}$	increasing no turning point	$\hat{\theta}$ is not positive.
$a + b\theta + c\theta^2$	$\sqrt{\frac{a+1}{c}}$	has a turning point	Both $\hat{\theta}$ and the shape of the graph fullfill the requirements and have to consider this model.
$\sqrt{a + b\theta + c\theta^2}$	$\frac{-2(a+1)}{b}$	no turning point	$\hat{\theta}$ is not positive.
$a + b\theta + c\theta^2 + d\theta^3$	solution exist	has a turning point	Consider this model
$\sqrt{a + b\theta + c\theta^2 + d\theta^3}$	solution exist	has a turning point	Consider this model.

Chapter 4

Repair rate with phases

4.1 Expected repair time of a machine

4.1.1 Original rate matrix

As a specific example, consider a rate matrix

$$Q = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} -0.1 & 0.1 \\ 1 & -1 \end{bmatrix} \end{array} \tag{4.1}$$

with states 0 and 1, where 0 = working state and 1 = broken state. This assumes that the time to leave a state is exponentially distributed. This is reasonable for the time to breakdown, but not necessarily for repair which could be closer to a deterministic value. Recall that an exponential random variable with pdf $f(x) = \lambda e^{-\lambda x}$, $(x > 0)$, has mean $\frac{1}{\lambda}$ and variance $\frac{1}{\lambda^2}$.

The expected time change states is reciprocal of the rate. The expected time in state 0 is $\frac{1}{0.1} = 10$, with variance $\frac{1}{0.1^2} = 100$. The expected time in state 1 is 1 with variance 1.

We use 0.1 as the rate to change from the working state to the broken state. It is reasonable to that the rate from working to broken state should be less than rate from the broken to working state.

We simulate data to give the times to change states using R. Here we use 5 change times for each states because we use too many time points the graphs become cluttered.

```
a1=rexp(5,0.1)
a2=rexp(5,1)
A=matrix(c(a1,a2),5,2)
```

'$a1'$ generates 5 times to change from state 0 to state 1 and 5 times to change from state 1 to state 0. Values for the matrix A as below.(these values change over the simulation)

```
          [,a1]      [,a2]
[1,] 17.765007 0.24840448
[2,]  1.988278 0.87098097
[3,] 10.232093 1.17863528
[4,]  1.378704 0.06740564
[5,] 37.432592 0.82259876
```

To get the time values to alternate between state 0 and state 1 we using the following R code. b represents the inter-event time of a machine (it can be either repair time or time to breakdown.)

```
b=c(t(A))
17.76500743  0.24840448  1.98827828  0.87098097 10.23209251  1.17863528  1.37870380
0.06740564 37.43259178  0.82259876
```

d represents the cumulative sum of the vector b and d holds times for events starting at time 0 with a working machine. The values for d appear below. The vector f indicates the state at the corresponding time.

```
d= cumsum(b),d=c(0,d)
0.00000 17.76501 18.01341 20.00169 20.87267 31.10476 32.28340 33.66210 33.72951
71.16210 71.98470
```

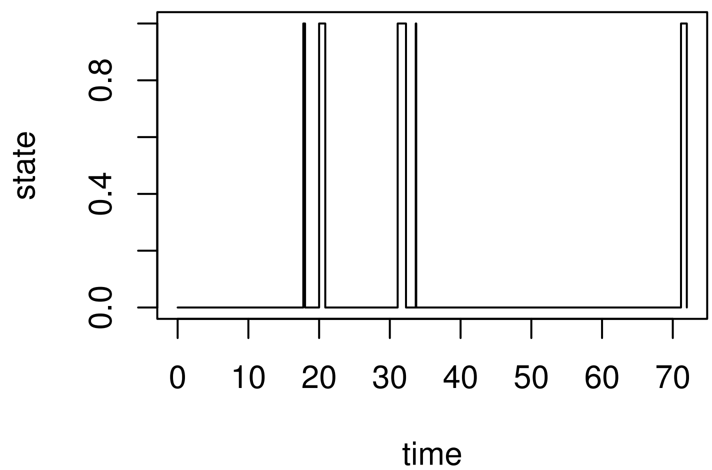

Figure 4.1: The time values to alternate between state 0 and state 1

```
f=c(rep(c(0,1),5),0)

 0 1 0 1 0 1 0 1 0 1 0

plot(d,f,"s", xlab="time", ylab="state")
```

In our diagram we see that the system spends most of the time in state 0 (working) and only spends a small amount of time in state 1 (broken).

B indicates state and time of entry to that state

```
B=matrix(c(f,d),11,2)

      [,1]     [,2]

 [1,]    0  0.00000

 [2,]    1 17.76501

 [3,]    0 18.01341

 [4,]    1 20.00169

 [5,]    0 20.87267

 [6,]    1 31.10476
```

```
[7,]    0 32.28340
[8,]    1 33.66210
[9,]    0 33.72951
[10,]    1 71.16210
[11,]    0 71.98470
```

4.1.2 Rate matrix with two broken levels

To get make the service time become closer to a deterministic distribution we divide the broken state into two or more levels. (e.g. broken level A, broken level B, Broken level C, . . .).This changes the distribution of the amount of time that we are in the broken state, which becomes the sum of exponential (Erlang/Gamma distribution) Akkouchi (2008).

One method to make the time to repair (time in state 1) closer to a deterministic random variable is to use phases/stages. So we initially replace state 1 by two stages, say 1_1 (broken state 1) and 1_2 (broken state 2). We double the rates so the expected time in each stage is half of the previous value but the total expected time is still the same. Consecutive states would be $0 \to 1\_1 \to 1\_2 \to 0 \to 1\_1 \ldots$

The amount of time to move to the next state is the reciprocal of the rate. Our new matrix becomes

$$Q = \begin{array}{c} \\ 0 \\ 1\_1 \\ 1\_2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1\_1 & 1\_2 \end{array} \\ \begin{bmatrix} -0.1 & 0.1 & 0 \\ 0 & -2 & 2 \\ 2 & 0 & -2 \end{bmatrix} \end{array} \tag{4.2}$$

The expected time in state 0 is $\frac{1}{0.1}$. The expected time in state 1_1 is $\frac{1}{2}$ and the expected time in state 1_2 is $\frac{1}{2}$. So total expected time in states $\{(1\_1), (1\_2)\}$ is $\frac{1}{2} + \frac{1}{2} = 1$, just as in the original 2×2 matrix. But the variance of the total time in the broken state is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$, half of the previous variance.

In here we add one more column than original 2×2 matrix to make two broken states.

R codes and results for the two broken levels/states as below.

```
a1=rexp(5,0.1)

a2=rexp(5,2)

a3=rexp(5,2)

A=matrix(c(a1,a2,a3),5,3)

A =


             [,a1]      [,a2]      [,a3]
   [1,] 13.3714614 0.149629528 0.2225692
   [2,] 21.3659530 0.121731688 0.2694744
   [3,]  5.7229672 0.057782232 0.1090097
   [4,]  0.6516988 0.002657414 0.4162677
   [5,]  4.5065944 0.351905711 0.2354946


b=c(t(A))

    b=

13.371461350  0.149629528  0.222569184 21.365953031  0.121731688  0.269474377
5.722967191  0.057782232  0.109009721  0.651698830  0.002657414  0.416267699
4.506594366  0.351905711  0.235494593


d= cumsum(b)

d=

0.00000 13.37146 13.52109 13.74366 35.10961 35.23134 35.50082 41.22379 41.28157
41.39058 42.04228 42.04493 42.46120 46.96780 47.31970 47.55520


f=c(rep(c(0,1,1),5),0)

f=

0 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0
```

```
plot(d,f,"s", xlab="time", ylab="state")
```

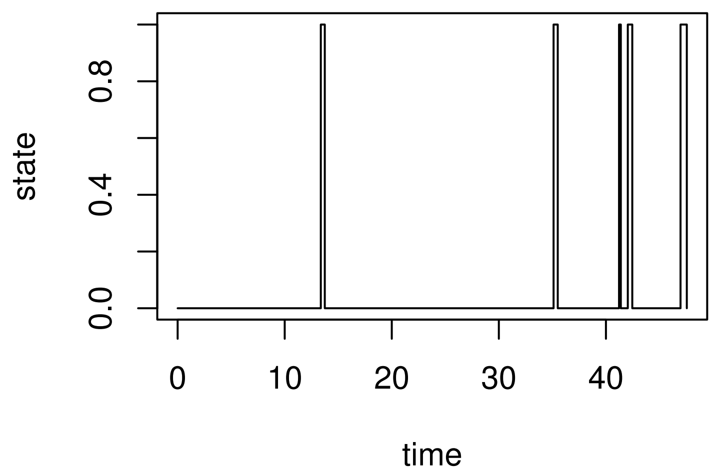

Figure 4.2: The time values to alternate between state 0 and state 1 when repair with two phases

This graph shows only one broken state because using R codes (f) we combined both broken levels together to show the total time in the broken state. We see that the system spends most of the time in state 0 (working) and only spends a small amount of time in state 1 (broken).

```
B=matrix(c(f,d),16,2)

B=

      [,1]     [,2]

 [1,]    0  0.00000

 [2,]    1 13.37146

 [3,]    1 13.52109

 [4,]    0 13.74366

 [5,]    1 35.10961
```

```
 [6,]     1 35.23134
 [7,]     0 35.50082
 [8,]     1 41.22379
[9,]     1 41.28157
[10,]     0 41.39058
[11,]     1 42.04228
[12,]     1 42.04493
[13,]     0 42.46120
[14,]     1 46.96780
[15,]     1 47.31970
[16,]     0 47.55520
```

When it appears with two sub broken states as its, R codes, results and graph as below for the 3×3 matrix.

```
a1=rexp(5,0.1)
a2=rexp(5,2)
a3=rexp(5,2)
A=matrix(c(a1,a2,a3),5,3)
A =
         [,a1]        [,a2]       [,a3]
 [1,]  1.646683 0.27813493 0.07861844
 [2,] 17.184832 0.06623823 0.15299476
 [3,] 33.896511 0.43493146 0.04326425
 [4,]  6.441274 0.61006985 0.16346007
 [5,] 33.052043 0.03905789 1.79278296

b=c(t(A))
b =1.64668340  0.27813493  0.07861844 17.18483196  0.06623823  0.15299476 33.89651105
```

```
0.43493146  0.04326425  6.44127383  0.61006985  0.16346007 33.05204317  0.03905789
1.79278296

d= cumsum(b)
d =  0.000000  1.646683  1.924818  2.003437 19.188269 19.254507 19.407502 53.304013
53.738944 53.782208 60.223482 60.833552 60.997012 94.049055 94.088113 95.880896

f=c(rep(c(0,1,2),5),0)
f = 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0

plot(d,f,"s", xlab="time", ylab="state")
```

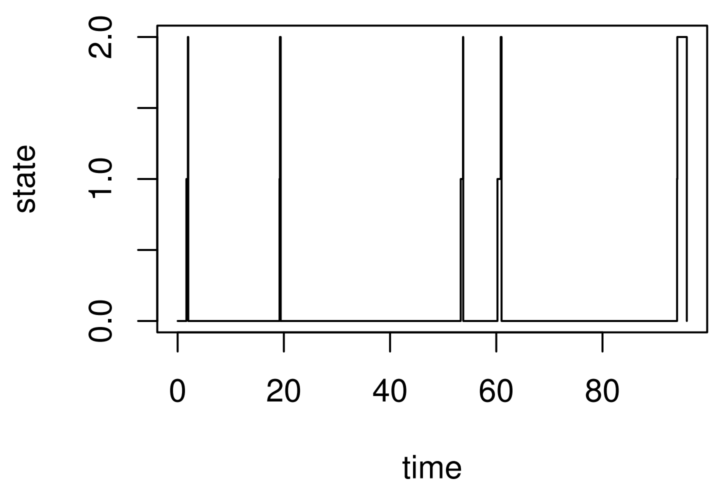

Figure 4.3: The time values to alternate between state 0 and state 1

This graph shows two phases of the broken state where, we separate both broken levels show the amount of time spend in each level. We see that the system spends most of the time in state 0 (working) and only spends a small amount of time in state 1 (broken) and in state 2 (broken).

```
B=matrix(c(f,d),16,2)

B =

    [,1]      [,2]

[1,]    0  0.000000

[2,]    1  1.646683

[3,]    2  1.924818

[4,]    0  2.003437

[5,]    1 19.188269

[6,]    2 19.254507

[7,]    0 19.407502

[8,]    1 53.304013

[9,]    2 53.738944

[10,]    0 53.782208

[11,]    1 60.223482

[12,]    2 60.833552

[13,]    0 60.997012

[14,]    1 94.049055

[15,]    2 94.088113

[16,]    0 95.880896
```

4.1.3 Rate matrix with three broken levels

We can continue in this manner. If we divide the repair state into 3 parts, our new rate matrix becomes

$$Q = \begin{array}{c} \\ 0 \\ 1\_1 \\ 1\_2 \\ 1\_3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1\_1 & 1\_2 & 1\_3 \end{array} \\ \begin{bmatrix} -0.1 & 0.1 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & -3 & 3 \\ 3 & 0 & 0 & -3 \end{bmatrix} \end{array} \tag{4.3}$$

The working state expected time remains as $\frac{1}{0.1}$. The expected time in state 1_1 (broken state 1) is $\frac{1}{3}$, the expected time in state 1_2 (broken state 2) is $\frac{1}{3}$ and the expected time in state 1_3 (broken state 3) is $\frac{1}{3}$ So total expected time in states $\{(1\_1), (1\_2), (1\_3)\}$ is $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$, just as the original value from 2×2 matrix. The expected repair time remains at 1 but the variance of the repair time now becomes $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}$.

Here we added two more columns from the original 2×2 matrix to make three broken phases. R codes and results for the three broken phases appear below.

```
a1=rexp(5,0.1)

a2=rexp(5,3)

a3=rexp(5,3)

a4=rexp(5,3)

A=matrix(c(a1,a2,a3,a4),5,4)

A=

          [,a1]      [,a2]        [,a3]     [,a4]

[1,] 22.320021 0.02203436 0.72015855 0.3100682

[2,] 22.055960 0.76357801 1.64618390 0.2676209

[3,]  8.405908 0.39304455 0.51688823 0.1494886

[4,]  3.081749 0.29536128 0.05784368 0.2329312

[5,] 22.999995 0.46057149 0.48288709 0.3654538
```

```
b=c(t(A))
b=
22.32002139  0.02203436  0.72015855  0.31006825 22.05595967  0.76357801  1.64618390
0.26762090  8.40590784  0.39304455  0.51688823  0.14948858  3.08174868  0.29536128
0.05784368  0.23293124 22.99999529  0.46057149  0.48288709  0.36545375

d= cumsum(b); d=c(0,d)
d=
0.00000 22.32002 22.34206 23.06221 23.37228 45.42824 46.19182 47.83800 48.10563
56.51153 56.90458 57.42147 57.57095 60.65270 60.94806 61.00591 61.23884 84.23883
84.69941 85.18229 85.54775

f=c(rep(c(0,1,1,1),5),0)
f=
0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1 0

plot(d,f,"s", xlab="time", ylab="state")
```

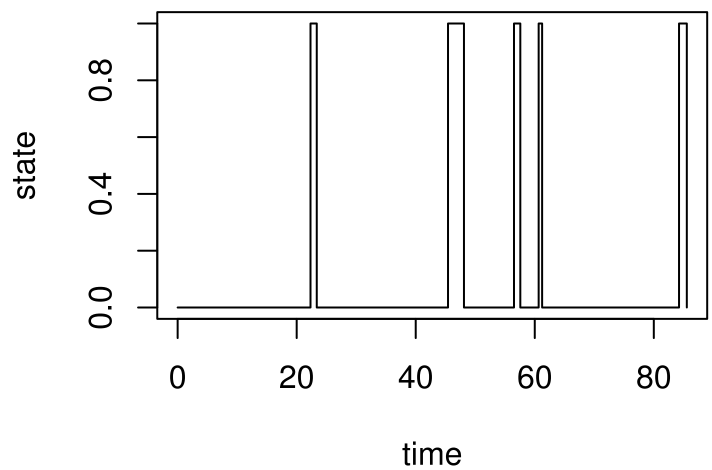

Figure 4.4: The time values to alternate between state 0 and state 1 when three broken phases are combined

This graph shows only one broken state because using R codes we combined three broken levels together to get an idea about the total broken state time. We see that the system spends most of the time in state 0 (working) and only spends a small amount of time in state 1 (broken).

When there are three broken phases the rate matrix Q becomes 4×4. We implement this situation in R code.

```
a1=rexp(5,0.1)

a2=rexp(5,3)

a3=rexp(5,3)

a4=rexp(5,3)

A=matrix(c(a1,a2,a3,a4),5,4)

A=

         [,a1]    [,a2]       [,a3]     [,a4]

[1,]  2.362575 0.1669241 0.40560412 0.4862640
```

```
[2,] 14.217069 0.3873124 0.29534303 0.7850578
[3,]  6.983876 0.2190763 0.09293939 0.3692399
[4,] 23.638095 1.1927417 0.06024340 0.1429057
[5,]  7.031749 0.2439102 0.28933906 2.0446407

b=c(t(A))
b=
2.36257550  0.16692414  0.40560412  0.48626402 14.21706930  0.38731238  0.29534303
0.78505782  6.98387582  0.21907633  0.09293939  0.36923991 23.63809487  1.19274172
0.06024340  0.14290567  7.03174853  0.24391024  0.28933906  2.04464073

d= cumsum(b); d=c(0,d)
d=
0.000000  2.362575  2.529500  2.935104  3.421368 17.638437 18.025749 18.321092
19.106150 26.090026 26.309102 26.402042 26.771282 50.409377 51.602118 51.662362
51.805267 58.837016 59.080926 59.370265 61.414906

f=c(rep(c(0,1,2,3),5),0)
f=
0 1 2 3 0 1 2 3 0 1 2 3 0 1 2 3 0 1 2 3 0

plot(d,f,"s", xlab="time", ylab="state")
```

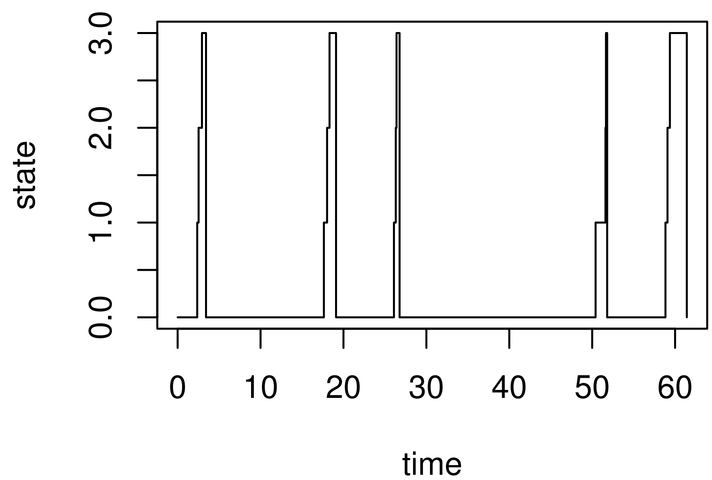

Figure 4.5: The time values to alternate between state 0 and state 1 when repair with three phases

This graph shows three broken phases to indicate the amount of time spent in each of the phases and in the working state 0. We see that the system spends most of the time in state 0 (working) and only spends a small amount of time in state 1 (broken), in state 2 (broken) and in state 3 (broken).

4.2 Summary of the rate matrices with broken phases

We limit the number of times generated because if we generate too many values the graphs get overly cluttered. We see that as we add more phases to the repair state the variance of the total repair time gets smaller and tends to zero. This means that we approximate a deterministic distribution for the repair time. But the expected repair time is kept constant. Thus our result for the optimal speed (which is based on the expected repair time) is valid not just for the initial 2×2 matrix but would be valid for a much larger set of distributions for the repair time.

In our graphs showing the two states (0 and 1), we note that the system only earns rewards when the system is in the state 0.

In summary, in the beginning the repair time is considered as exponentially distributed. But eventually we see that the repair time can be considered as a deterministic value by allowing broken phases.

Table 4.1: Summarize the repair time results by increasing broken phases.

Type of matrix	Expected repair time (Units)	Variance of repair time
2×2	1	1
3×3	$\frac{1}{2} + \frac{1}{2} = 1$	$\frac{1}{2^2} + \frac{1}{2^2} = \frac{1}{2}$
4×4	$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$	$\frac{1}{3^2} + \frac{1}{3^2} + \frac{1}{3^2} = \frac{1}{3}$
5×5	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$	$\frac{1}{4^2} + \frac{1}{4^2} + \frac{1}{4^2} + \frac{1}{4^2} = \frac{1}{4}$
6×6	$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 1$	$\frac{1}{5^2} + \frac{1}{5^2} + \frac{1}{5^2} + \frac{1}{5^2} + \frac{1}{5^2} = \frac{1}{5}$
$\vdots$		
$(n+1) \times (n+1)$	$\frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n} = 1$	$\frac{1}{n^2} + \frac{1}{n^2} + \ldots + \frac{1}{n^2} = \frac{1}{n} = \lim_{n \to \infty} \frac{1}{n} = 0$

Chapter 5

Simulation and parameters estimation

In this chapter, we generate typical data that would be available to the statistical analyst. We illustrate with a model having the breakdown rate $S(\theta)$ as a quadratic function of speed (θ). $S(\theta) = 1.3\theta + 4\theta^2$

Note: There is no intercept term, which is consistent with the fact that if the speed is zero, then the breakdown rate would also be zero.

We work with four values of θ, namely $\theta = 1, 1.3, 1.6, 1.9$. Then $S(\theta) = 5.30, 8.45, 12.32, 16.91$ represent the four rates. We want the repair rate to be faster so we take the repair rate to be 20 for all four cases.

The following R code will generate data.

```
th=c(1,1.3,1.6,1.9)
#th (theta) gives 4 speeds that we choose to examine
fth=1.3*th+4*th^2
#fth=f(theta) is a quadratic function of theta that we use for simulation
n=100;m=n/2
```

```
#n is the number of data points that we generate for each speed, which
#we break into two parts of size m=n/2 each
b1=rexp(n,fth[1])
#b1 generates n values of exponential time-to-breakdown at speed theta[1]
b2=rexp(n,fth[2])
#b2 generates n values of exponential time-to-breakdown at speed theta[2]
b3=rexp(n,fth[3])
b4=rexp(n,fth[4])
b=c(b1,b2,b3,b4)
#b is a vector of size 4*n of simulated breakdown times
r=rexp(4*n,20)
#r simulates 4*n repair times at rate 20
#We next want to alternate between breakdown and repair times.
#We do this using a matrix manipulation.
M=matrix(c(b,r),4*n,2)
c=c(0,c(t(M)))
#c will consist of 8*n inter-event times, alternating between
#time-to-breakdown and repair time.
d=cumsum(c(0,c[1:(8*n)]))
#d will give the actual times of 8*n events plus start at time 0 when
# system is working. d has dimension 8*n+1
e=rep(c(0,1),4*n); e=c(e,0)
#e gives state of system at 8*n times where 0=working; 1=broken
#e has dimension 8*n+1
```

The first line of code generates n random values in vector b (breakdown times) from an exponential random variable with 4 rates corresponding to 4 speeds $1, 1.3, 1.6, 1.9$. These are the times till breakdown. The next line generates $4n$ random values in vector r (repair times) with rate 20. These are the repair times. We want to intersperse these two vectors with $4n$ times each and the 8n interspersed inter-event times are placed in vector c. The vector d gives the actual event times when the system switches states (working (0) and broken (1)). The vector e gives the state of the system for the corresponding time in vector d.

The actual data that we would likely have from a manufacturer would be the state of the system and the time when that state was entered, i.e. We would have vectors d and e. From these, we would have to recover the inter-event times and partition them into times to breakdown times and repair times. So we would try to recover vectors b and r.

```
f=c(); for(i in 1:(8*n)){f[i]=d[i+1]-d[i]}
M1=matrix(f[1:(8*n)],2,4*n)
g=M1[1,]; h=M1[2,]
#Note that h=b and g=r except g has an initial value 0
```

From vector f giving actual event times, we look at pairwise differences to get inter-event times in vector d. We partition vector d into two parts to get time-to-breakdown in vector h and times to repair in vector g.

Let us assume a time-to-breakdown rate $\alpha = S(\theta)$ Our data consist of the times-to-breakdown $x_1, \ldots, x_n$

Theorem 5.1. *The MLE estimator for α for an exponential random variable X with pdf $f_X(x) = \alpha e^{-\alpha x}$, $x \geq 0$, satisfies $\hat{\alpha} = \frac{1}{\bar{x}}$ for fixed θ.*

Proof.

$$L(\alpha) = \prod_{i=1}^{n} \alpha e^{-\alpha x_i} = \alpha^n e^{-\alpha \sum x_i} \text{ so}$$

$$\ln L = n \ln \alpha - \alpha \sum x_i.$$

$$\frac{d \ln L}{d\alpha} = \frac{n}{\alpha} - \sum x_i = set = 0$$

Hence $\hat{\alpha} = \frac{1}{\bar{x}}$ □

In our example we are using 4 different values of θ which are known to the statistical analyst. We have n realizations for each. We divide these into groups of $m = \frac{n}{2}$ each to give more data points for attempted polynomial regression.

```
y11=1/mean(b1[1:m]); y12=1/mean(b1[(m+1):n])
#y11 and y12 give two estimates of the breakdown rate for speed 1
y21=1/mean(b2[1:m]); y22=1/mean(b2[(m+1):n])
#y21 and y22 give two estimates of the rate for speed 1.3
y31=1/mean(b3[1:m]); y32=1/mean(b3[(m+1):n])
#y31 and y32 give two estimates of the rate for speed 1.6
y41=1/mean(b4[1:m]); y42=1/mean(b4[(m+1):n])
#y41 and y42 give two estimates of the rate for speed 1.9
y=c(y11,y12,y21,y22,y31,y32,y41,y42)
x=c(1,1,1.3,1.3,1.6,1.6,1.9,1.9)
plot(x,y)
```

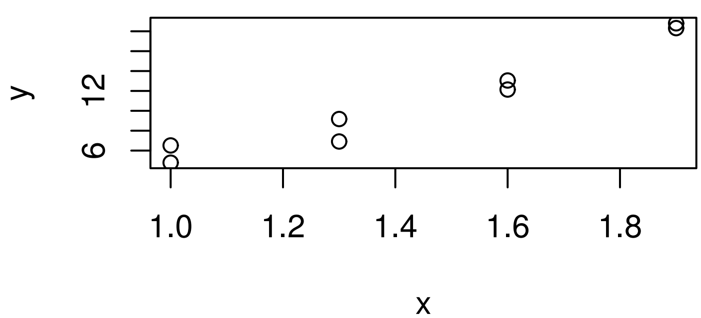

Figure 5.1: Breakdown rate vs Speed

Next we will look at fitting a polynomial to these data, to see how close we can come to recovering the parameter values chosen for the simulation. We are looking for low values of AIC and BIC.

Note: The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) provide measures of model performance that account for model complexity. AIC and BIC combine a term reflecting how well the model fits the data with a term that penalizes the model in proportion to its number of parameters.

```
x1=x; x2=x^2; x3=x^3; x4=x^4
A1=lm(y~0+x1)
# This forces regression through the origin.
AIC(A1)
BIC(A1)
A2=lm(y~0+x1+x2)
AIC(A2)
BIC(A2)
A3=lm(y~0+x1+x2+x3)
AIC(A3)
BIC(A3)
A4=lm(y~0+x1+x2+x3+x4)
AIC(A4)
```

Typical output follows.

```
> AIC(A1)
[1] 37.40917
> BIC(A1)
[1] 37.56805
> A2=lm(y~0+x1+x2)
> AIC(A2)
[1] 16.13386
> BIC(A2)
```

```
[1] 16.37218
> A3=lm(y~0+x1+x2+x3)
> AIC(A3)
[1] 17.42223
> BIC(A3)
[1] 17.73999
> A4=lm(y~0+x1+x2+x3+x4)
> AIC(A4)
[1] 18.6481
```

We want a low value of AIC and BIC so it seems that the quadratic model is best. This matches the fact that we originally chose the quadratic model.

```
> summary(A2)

Call:
lm(formula = y ~ 0 + x1 + x2)
Residuals:
Min      1Q  Median      3Q     Max
-0.4961 -0.3142 -0.1411  0.1570  1.0108

Coefficients:
Estimate Std. Error t value Pr(>|t|)
x1   1.0429     0.6618   1.576    0.166
x2   4.1506     0.4069  10.202 5.17e-05 ***
---
Residual standard error: 0.5264 on 6 degrees of freedom
Multiple R-squared:  0.9985,    Adjusted R-squared:  0.9979
F-statistic:  1935 on 2 and 6 DF,  p-value: 3.712e-09
```

We note that the coefficients of x (1.043) and x^2 (4.151) which correspond to θ and θ^2 are close the the actual values of 1.3 and 4 which we used to generate our data. However the coefficient of x is not significant. However, if we continue with our example, our estimate for $S(\theta)$ is $1.043\theta + 4.151\theta^2$

According to the earlier analysis , the estimated optimal speed for the quadratic model is $\sqrt{\frac{1}{c}} = \sqrt{\frac{1}{4.151}} \approx 0.5$, to maximize the profit.

Chapter 6

Conclusions and further research

In this paper the optimal speed of a machine in an assembly line has been found. Using continuous time Markov chain we developed the rate matrix with the function of speed. That rate matrix represents the inter-event time of a machine, either repair time or time to breakdown.

In order to apply our methodology we assume and underlying function of speed and we simulated data for that speed. To find the best model for the rate function we used polynomial regression AIC and BIC and we found that the quadratic model is the appropriate model for our rate function. Then we determined parameters for the quadratic function using MLE and according to the our analysis , the estimated optimal speed for the quadratic model is $\sqrt{\frac{1}{c}} = \sqrt{\frac{1}{4.151}} \approx 0.5$, to maximize the profit.

Most calculations and simulations were done using readily available R software. Hence our approach can be easily implemented in real-world manufacturing companies to find the optimal speed of their machines using their own data to maximize the companies' profit.

In this study we used only working and broken states to built the model. As for further studies this model can be extended to more than one broken phases as we showed in figures 4.3 and 4.5. We can also extend our model by including other variables besides speed in the rate function. For example, we could include age or temperature in the model. We could also include a learning component in the model.

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