NOVEL CUTTING DEFORMATION MODES ON AXIALLY LOADED CIRCULAR AA6061-T6 EXTRUSIONS FOR SUPERIOR CRASHWORTHINESS PERFORMANCE

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NOVEL CUTTING DEFORMATION MODES ON AXIALLY LOADED CIRCULAR AA6061-T6 EXTRUSIONS FOR SUPERIOR CRASHWORTHINESS PERFORMANCE

By

Shun Yi Jin

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the Degree of Doctor of Philosophy at the
University of Windsor

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Novel Cutting Deformation Modes on Axially Loaded Circular AA6061-T6 Extrusions for Superior Crashworthiness Performance

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ABSTRACT

The study detailed in this dissertation focuses on the force/displacement and energy absorption performances of circular AA6061-T6 aluminium alloy extrusions subjected to novel cutting deformation modes under both dynamic and quasi-static axial loading conditions.

The experimental investigation of this novel cutting deformation mode on the circular AA6061-T6 extrusions was completed utilizing a specially designed cutter with or without the presence of a deflector. Experimental results showed that the cutting deformation mode exhibited higher crush force efficiency of 94.2% and eliminated the high peak crush force associated with the progressive folding or global bending deformation mode. Factors that influence the cutting deformation mode were investigated. Testing results showed that slight difference of the cutter geometries and extrusion diameters had no significant influence on the load/displacement response of the extrusions. An increasing, almost linear, relationship was observed between the steady-state cutting force and the extrusion wall thickness/number of cutter blades.

Moreover, controlling the load/displacement response through varying instantaneous extrusions wall thickness along the axis of the specimens was investigated. Experimental results showed a direct relationship between the cutting force and instantaneous wall thickness of the extrusion exists.

Additionally, numerical simulations of the axial cutting deformation process employing an Eulerian finite element formulation method and the axial crushing deformation process employing a Lagrangian finite element formulation method were performed. Good predictive capabilities were observed for both configurations.

Finally, a theoretical study of steady-state cutting circular extrusion by a cutter with multiple blades with/without a deflector was conducted. It is assumed that the extrusion will deform similar to the experimental observations and dissipate energies through the following plastic or fracture deformations: (1) far-field moving hinge line with the advance of cutter blade; (2) far-field membrane deformation near the intersection zone between the cutter blade and blade shoulder; (3) near blade tip circumferential membrane stretching; (4) continuous chip formation ahead of the cutter blade; and (5) cut
petalled sidewalls bending outwards. Then the contribution of friction force between the cutter blade and cut petalled sidewalls is included into the proposed model. A good correlation was found between the theoretical prediction and experimental observations.
DEDICATION

To my wife, Hua Rong, and our daughters, Emilia and Julia for your endless love, patience, and inspiration. To my parents, Fankang and Meifang for your love and support.
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CLAIMS TO ORIGINALITY

Aspects of this work constitute, in the author's opinion, new and distinct contributions to the technical knowledge pertaining to axial cutting deformation of circular AA6061-T6 extrusions under quasi-static and impact loading conditions. These include:

(i) Development of a novel cutting deformation for circular AA6061-T6 extrusions under dynamic or quasi-static axial loading condition. This novel cutting deformation mode exhibits extremely high crush force efficiency with least degree of force oscillation during the cutting process. A special designed cutter was used to generate the desired cutting deformation mode with or without the presence of deflector which was used to flare the cut petalled sidewalls and also to save the spatial requirement for the system.

(ii) Use of strain rate insensitive material as AA6061-T6 minimizes the force fluctuation during dynamic cutting process.

(iii) An increasing, almost linear, relationship was observed between the steady-state cutting force and extrusion wall thickness as well as between the steady-state cutting force and the number of cutter blades. Thus, a desired steady-state cutting force could be achieved through varying tube wall thickness and/or cutter blade quantities and/or other parameters discussed in this work.

(iv) The proposed novel cutting deformation was observed to be stable, controllable, and with good repeatability.

(v) Dual-stage cutting deformation is generally a superposition of two single stage cutting processes, which can be used as an adaptive energy absorption device.

(vi) Finite element modeling employing an Eulerian finite element formulation method exhibited good predictability for this novel cutting deformation mode.

(vii) A theoretical model that predicts the steady-state mean cutting resistance force of circular tubes by a cutter with multiple cutter blades with/without the presence of deflector was developed and illustrated good predictability.
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**NOMENCLATURE**

\[ B \]  One-half of the wedge/blade shoulder width

\[ C \]  Mean side width of a square tube

\[ D \]  Mean diameter of a circular tube

\[ D_c \]  Critical damage value

\[ D_o \]  Outer diameter of a circular tube

\[ E \]  Young’s modulus

\[ E_{absorbed} \]  Energy absorbed by a structure through plastic strain

\[ \dot{E}_{b,\text{far\_field}} \]  Rate of energy dissipation for far-field bending (moving hinge line OP)

\[ \dot{E}_{b,\text{axial}} \]  Rate of energy dissipation for cut petalled sidewall bending outward

\[ \dot{E}_{\text{chip}} \]  Rate of energy dissipation due to continuous chip formation

\[ \dot{E}_{m,\text{trans}} \]  Rate of energy dissipation for membrane deformation zone between the transient and stable flaps

\[ \dot{E}_{m,\text{tip}} \]  Rate of energy dissipation for membrane deformation in the vicinity of blade tip

\[ F_{ss} \]  Steady-state force observed during an axial cutting test

\[ H \]  One-half of the initial distance between plastic hinges at the top and bottom of a basic folding element for a square or circular tube

\[ i \]  Index for the \( i^{th} \) data point

\[ L \]  Length of a circular/square tube

\[ L_{cr} \]  Critical length of a circular/square tube that depicts the transition between progressive folding and global bending modes

\[ L_{\text{reduced}} \]  Length of the segment of reduced wall thickness of a circular extrusion

\[ M_o \]  Fully plastic bending moment

\[ n \]  Number of cutter blades

\[ N \]  Number of circumferential lobes for non-axisymmetric progressive folding of a circular tube

\[ P \]  Axial crushing/cutting force

\[ P_m \]  Mean crushing/cutting force

\[ P_{\text{max}} \]  Maximum load observed during an axial crushing/cutting test

\[ R, r_m \]  Mean radius of a circular tube

\[ r_i \]  Inner radius of a circular tube

\[ r_o \]  Outer radius of a circular tube
<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<td>$R_{axial}$</td>
<td>Axial bent radius for cut petalled sidewall</td>
</tr>
<tr>
<td>$R_{deflector}$</td>
<td>Profile radius of the curved deflector</td>
</tr>
<tr>
<td>$R_r$</td>
<td>Rolling radius of curls at the side of the wedge/blade</td>
</tr>
<tr>
<td>$R_{rt}$</td>
<td>Rolling radius of curls at the front of the wedge/blade, $R_{rt} = R_r \cos \theta$</td>
</tr>
<tr>
<td>$S$</td>
<td>Positive material damage parameter</td>
</tr>
<tr>
<td>$t$</td>
<td>Wall thickness of a circular/square specimen</td>
</tr>
<tr>
<td>$T$</td>
<td>Blade tip width of a cutting blade</td>
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<tr>
<td>$Y$</td>
<td>Reduced wall thickness of a circular extrusion</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Coefficient of friction</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>Static material plastic flow stress</td>
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<td>$\sigma_y$</td>
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<td>$\delta$</td>
<td>Crosshead displacement in the axial crushing/cutting direction</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>Effective crushing distance for square tube</td>
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<tr>
<td>$\delta_t$</td>
<td>Total crosshead displacement in the axial direction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Wedge/cutting blade semi-angle</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------------------------------------</td>
</tr>
<tr>
<td>AA</td>
<td>Aluminium Alloy</td>
</tr>
<tr>
<td>ALE</td>
<td>Arbitrary Lagrangian-Eulerian</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>CFE</td>
<td>Crush Force Efficiency</td>
</tr>
<tr>
<td>CNC</td>
<td>Computer Numeric Control</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear voltage differential transformer</td>
</tr>
<tr>
<td>NHTSA</td>
<td>National highway traffic safety administration</td>
</tr>
<tr>
<td>SEA</td>
<td>Specific Energy Absorption</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
</tr>
<tr>
<td>TEA</td>
<td>Total energy absorption</td>
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1 INTRODUCTION

In the past decades, thousands of people have died in motor vehicle traffic crashes in North America. The estimated number of fatalities in 2009 is 33,963 for the United States alone [1]. Vehicle occupant safety is one of the most primary concerns for customers and so for automotive manufactures. In order to improve vehicle safety and to meet more and more stringent government regulations, many crash avoidance systems (such as anti-lock braking system, traction control devices, backup camera, adaptive cruise control, pre-crash system) and crashworthiness devices (such as seatbelts, airbags, crumple zones, padding of the instrument panel, laminated windshield) have been adapted to the recently made vehicles. While the crash avoidance systems prevent and minimize the possibility of crash occurrence, the crashworthiness devices protect the occupant safety and minimize the injury of occupants during a crash.

Another one of the major challenges for the automotive industry is to reduce greenhouse emissions and improve fuel efficiency in vehicle engineering. Carbon dioxide (CO₂) emission associated with road transport is the third largest source of greenhouse gas which accounts for 33% of total emissions in the United States [2]. Material selection is critical to achieve the above goals without compromising occupant safety. Aluminium alloys have been widely applied into vehicular structures recently as a result of favourable material strength to weight ratio, lightweight characteristics, corrosion resistance, recyclability, and relative low cost. When using aluminium alloy in the body structure of a vehicle, weight savings of up to 25% may be attained compared to conventional steel structures [3]. Manufacturing flexibility of aluminium alloys in forms of cast and extruded members makes it possible to produce complex shape structural members in vehicle design, which also optimizes vehicle weight distribution and overall performance.

As the key structures of vehicles, thin-walled structures must dissipate the kinetic impact energy in a controllable manner while maintaining the integrity of occupant compartment during a crash. The impact force transmitted to the occupant compartment has to be in compliance with defined tolerance levels to minimize the potential injury to occupants. Energy absorption devices, such as crash boxes, have been implemented into
vehicle structures to absorb impact energy during a crash and maximize the protection to occupant safety. An ideal energy absorber should absorb the impact energy at a constant steady-state force throughout the entire plastic deformation of structures. No initial peak load should be necessary to activate the device, which minimizes the initial impact to the occupants. In addition, an ideal energy absorber also has to ensure good controllability and repeatability. Depending upon the crash conditions, such as crash speed and crash locations, energy absorption devices may be required to be adaptive or controllable to the amount of energy absorbed with regards to the crash distance/time.

The research presented in this dissertation involves the study of a novel cutting deformation on axially loaded circular cross sectional AA6061-T6 aluminium alloy extrusions as a potential energy absorption device. The objective of this research is to examine the load versus displacement and energy absorption characteristics of the circular AA6061-T6 extrusions under both dynamic and quasi-static loading conditions towards an ideal energy absorber. Special testing apparatus have been designed to achieve this novel cutting deformation mode and factors that influence this deformation mode of the circular extrusions are discussed. Furthermore, controlling of the load/displacement responses of the extrusions have also been investigated as a potential adaptive energy absorber that is stable, controllable, and repeatable. Additionally, finite element modeling of cutting deformations employing an Eulerian element formulation has been developed to predict the cutting behaviour and compared to the experimental results. Finally, a theoretical study of the steady-state cutting of circular extrusions by a cutter with multiple blades has been completed to predict the cutting resistance force. Parametric study on extrusion wall thickness, tube diameter, cutter blade tip width, and cutter blade quantities are conducted and compared to the experimental data.
2 LITERATURE REVIEW

Vehicular passive safety requires energy dissipation devices/structures to effectively absorb the kinetic energy at impact through plastic deformations (bending, folding, twisting or other) in a stable and controlled manner. Good energy absorbing devices require the following features: (1) controlled and constant reactive force, (2) long stroke, (3) stable and repeatable deformation mode, (4) lightweight and high specific energy absorption capacity, and (5) low cost and easy to install. A significant amount of experimental, numerical, and theoretical studies have been conducted on structural crashworthiness of thin-walled structures. In particular, those of square or circular cross section are a common type of energy absorber owing to the wide range of deformations that can be generated, their effectiveness to absorb energy, and low cost.

As related to the present study, the literature review presented in this chapter discusses energy absorption characteristics and crashworthiness performance of axially loaded tubular structures under both dynamic and quasi-static loading conditions. Section 2.1 discusses the collapse modes of axially loaded tubular structures. Section 2.2 discusses factors that influence the collapse modes of axially loaded tubes, including geometrical parameters, extrusion materials, and crush initiators. Section 2.3 discusses some of the analytical models developed by other researchers to predict mean crush force for axial crushing of square and circular tubes. Section 2.4 details the analytical models of wedge cutting of a plain plate. Section 2.5 discusses finite element modeling of the axial crushing/cutting of tubes using different finite element formulations as well as the finite element model validation assessment method. Section 2.6 deals with the strain rate insensitivity of flow stress for the AA6061-T6 aluminium alloy.
2.1 Modes of deformation for axially loaded tubes

The main mechanisms associated with energy absorption of metal structures are plastic deformation and/or fracture. The effectiveness of an energy absorbing device highly depends on its plastic deformation mode under specific loading conditions. A wide range of these modes exist for axially loaded tubes, such as global bending, progressive folding, inversion, splitting and curling, cutting, and so on [4, 5, 6, 7].

2.1.1 Axial plastic buckling

The possible collapse modes available for axial plastic buckling of tubular specimens are progressive folding, global bending, and transition between the progressive folding and global bending, depending on material properties, geometrical parameters, boundary conditions, imperfections, and the loading conditions. Details of how these factors influence the collapse mode will be discussed in section 2.2 of this dissertation. Figure 2.1 illustrates an example of the progressive folding and global deformation modes for the axial crushing of circular tubes. Figure 2.2 presents typical load/displacement responses for progressive folding and global bending deformation modes.

Figure 2.1 Illustration of (a) progressive folding and (b) global bending deformation modes for axial crushing of circular tubes.
Abramowicz and Jones [8, 9, 10] have done extensive experimental crushing tests for the axially loaded square and circular steel tubes under both dynamic and quasi-static loading conditions. Detailed categories of progressive folding deformations for the square tubes (symmetric, asymmetric, and extensional) and the circular tubes (axisymmetric and non-axisymmetric) were identified and defined in references [8] and [9], respectively. Theoretical predictions of the mean crushing forces for the square and circular tubes were then developed based upon different types of progressive folding deformations in references [8] and [9], respectively, and corrected in reference [10] by increasing the range of geometrical parameters and impact energies. According to reference [8], possible symmetric modes of progressive folding deformation in a layer for a square tube include (1) four individual lobes deforming inwards, (2) three lobes inwards and one outwards, and (3) two opposite lobes deforming inwards with the other two opposite lobes deforming outwards; possible asymmetric mode of progressive folding deformation in a layer include (1) a layer of three individual lobes deforming outwards and one inwards and (2) two adjacent lobes deforming outwards with the other two
adjacent lobes deforming inwards; extensional progressive folding deformation in a layer refers to four individual lobes deforming outwards. A transition from asymmetric progressive folding to global bending could occur if sufficient asymmetric lobes developed to produce instability in the sense of Euler. A transition from symmetric progressive folding to global bending may also occur if the symmetry deformations introduce deflections or disturbances into the uncrushed part of a column, which act as imperfections and produce global bending and eventually failure.

Langseth and Hopperstad [11] experimentally investigated the crush behaviour of axially loaded square AA6060 extrusions with T6, T4, and T4* (modified T4) tempered conditions under both quasi-static and dynamic axial loadings. The geometries of the extrusions considered in their study had a tube length ($L$) of 310 mm, width ($C$) of 80 mm and three different wall thicknesses ($t$) of 1.8 mm, 2.0 mm, and 2.5 mm. Testing results showed that while all tubes collapsed in symmetric progressive deformation mode under quasi-static loading conditions regardless of wall thickness and tempered conditions, symmetric, asymmetric, and mixture of the two previous deformation modes were observed for the AA6060-T6 and T4 extrusions under axial impact loading conditions. The mean crush force and total energy absorption for the AA6060-T6 extrusions were reported higher than those for the AA6060-T4 extrusions if the same tube geometry and loading condition were considered. This finding was believed to be contributed to the higher yield strength of the T6 temper material. A quick comparison of the dynamic and quasi-static load/displacement responses of representative AA6060-T6 extrusions with a wall thickness of 2.5 mm is shown in Figure 2.3. It is obvious from Figure 2.3 that dynamic peak crush force and mean crush force were significantly higher than the corresponding static force for the same axial displacement. As the strain rate effects have minor importance, they indicated the observed difference had to be associated with inertia effects set up at the instant of impact due to lateral movement of sidewalls in order to initiate the folding process.
Hsu and Jones [12] performed experimental tests on the cylindrical AA6061-T6 aluminium alloy tubes, where the influences from striking mass, initial impact velocity, and specimen length on the behaviour of the tubes were studied. It was found that the inertia properties of the striker had an important effect on the initiation of buckling for high velocity impacts and that the development of the buckling process was sensitive to the initial velocity and the specimen length.

Jensen et al. [13] experimentally and numerically studied the transition between collapse modes of square cross sectional AA6060-T6 extrusions under quasi-static and dynamic axial loading conditions with respect to the extrusion geometries. Square AA6060-T6 extrusions with four different nominal wall thicknesses ($t$) of 2.0 mm, 2.5 mm, 3.5 mm, and 4.5 mm, a constant nominal side width ($C$) of 80 mm, and tube lengths ($L$) varied from 400 mm to 1919 mm for the quasi-static experimental testing and from 638 mm to 1920 mm for the impact testing. The impact tests were carried out at an impact velocity of 13 m/s and 20 m/s with a corresponding mass of the impact equal to 1400 kg and 600 kg, respectively. Figure 2.4 illustrates the deformation mode and
force/displacement response for a representative specimen (#S32) that experienced transition from progressive to global bending under quasi-static axial buckling. It can be found from Figure 2.4 that the crush force oscillated with the formation of progressive folds and then dropped quickly after transition to the instable global bending mode. In their study, the authors observed that the total energy absorption decreased when increasing impact velocity due to inertia forces preventing direct global bending and the early transition from progressive to global bending. A direct relationship was found to occur between energy dissipation and both $L/C$ and $C/t$ extrusion geometric aspect ratios in the quasi-static tests and dynamic tests with an impact velocity of less than 13 m/s. An inverse relationship was found when the impact velocity was 20 m/s. Anomalous response was observed in the experimental testing for all slenderness ratios, i.e. different collapse modes were found in parallel tests with the same local and global slenderness. The energy absorption of members collapsing in a transition between progressive and global buckling was very dependent on the time of the transition.

![Figure 2.4](image)

(a) Deformation mode and (b) force/displacement response for a representative specimen (#S32) that experienced transition from progressive to global bending under quasi-static axial buckling [13].
Karagiozova and Alves [14] experimentally and numerically studied collapse behaviour of circular aluminium tubes of outer diameter of 50.8 mm and wall thickness of 2 mm under both dynamic and quasi-static axial loading conditions. The tubes, freely supported at both ends, were initially tested in a compression testing machine. By varying the tube length, a critical length \( (L_{cr}) \) of 315 mm was found for the given cross sectional tube under quasi-static loading. The tube lengths considered for the dynamically tests were 360 mm, 500 mm, and 650 mm. It was shown from the experimental tests that the critical length \( (L_{cr}) \) was significantly influenced by the impact velocity, \( V_o \), as shown in Figure 2.5.

![Figure 2.5 Influence of the impact velocity on the dynamic buckling transition of circular aluminium alloy tubes [14].](image_url)

Numerical simulations of the axial impact on tubes having the same cross sectional dimensions as the ones experimentally tested were carried out using the finite element (FE) code ABAQUS/Explicit. Shell elements RS4 (3.9 mm \( \times \) 3 mm) were used to model all the analysed tubes. The load was applied as a point mass attached to the nodes of a rigid body which have an initial velocity, \( V_o \). The contact between the shell and striker and between the distal end of the shell and rigid surface was defined using the
‘surface interaction’ concept together with a friction coefficient of 0.25 at both ends. Any self-contacts of the inner and the outer surfaces of the shell were assumed frictionless. In order to trigger asymmetric buckling patterns, initial imperfection was applied at a magnitude of 0.0005L, where L being the length of the tube. Three different bilinear material models as shown in Figure 2.6 were used in order to explore the influence of the material parameters on the buckling transition, namely the flow stress and strain hardening. It was shown from the numerical simulations that not only the inertia effects but also the material characteristics played a significant role in the occurrence of different buckling mode. Thus, the dynamic buckling transition phenomenon which occurs in circular tubes cannot be analysed assuming a yield stress averaged with respect to the plastic strains when material strain hardening is present. It was observed that the material hardening characteristics had a significant influence on the dynamic collapse mechanisms. The circular extrusions made of ductile alloys with a high yield stress and low strain hardening characteristics had a better energy absorption performance than extrusions with a low yield stress and high strain hardening characteristics.

Figure 2.6 True stress/true strain characteristics of the aluminium alloy (Experimental). Mat1 and Mat2 represented two bilinear models used in the simulations; Mat3 was used as a material model with a low yield stress. [14]
Galib and Limam [15] experimentally and numerically investigated the quasi-static and dynamic axial crushing of circular AA6060-T5 extrusions subjected to variable impact mass and impact velocity values. Tubes considered for the experimental testing had a length of 200 mm, mean diameter of 58 mm, and wall thickness of 2.0 mm. The specimens were fixed at the lower end in both quasi-static and dynamic experimental tests by means of steel pieces (sleeves) embedded a distance of 18 m inside the tube matching the inside periphery of the specimens as illustrated in Figure 2.7. The observed progressive folding deformation modes of the specific circular extrusion under both dynamic and static loading were generally the same. The main difference was related to the first part of the impact, where the dynamic force was approximately 40-60% higher than the static one. The mean dynamic crush forces were about 10% higher than the corresponding values in the quasi-static tests, which indicated the strain rate insensitivity property of this type of material.

![Figure 2.7](image.png)

Figure 2.7  Boundary conditions used for quasi-static and dynamic axial crushing of the circular tubes [15].
2.1.2 Axial inversion

Under certain conditions, circular tubes can show a simple and special type of deformation mode: tube inversion or invertube. A circular tube can be inverted either externally or internally and there are basically two types of tube inversion: free inversion and inversion with a die as shown in Figure 2.8 [16]. The former method requires suitable performing of the tube at one end and employing attachments to fix the formed end [17, 18]. The latter process requires no pre-forming operations, but a conical die with a radius must be used. Free inversion can happen if the material of the tube is ductile enough and is not highly strain hardened, while the occurrence of inversion with a die requires satisfaction of stricter conditions [19]. Tube geometry, the strength and ductility of the tube material, die radius and the condition of the contact surfaces are all influencing factors [20, 21, 22, 23].

![Figure 2.8 Sketch for two types of tube inversion: (a) free inversion and (b) inversion with a die [16].](image)

A simple expression was derived by Guist and Marble [24] to predict the inverting load for free inversion of tubes. Perfectly plastic material and contact thickness
and tube length were assumed. By considering the balance of internal energy and the work done by the external force, the steady-state force is given by:

\[ P_m = 2\pi\sigma_o Rt \left( \frac{b}{R} + \frac{t}{4b} \right) \]  

(1.1)

where \( R \) is the radius of the tube and \( b \) is the knuckle radius. When the value of the knuckle radius \( b \) is set to \( \sqrt{\pi t}/2 \) to minimize the force \( P \), the bending and stretching processes dissipate the same amount of energy. The experimental results of the inversion load agreed well with their analytical predictions.

Their theory was revisited by Reddy [18]. A rigid, linear, kinematic, strain-hardening material model obeying the Tresca yield condition with its associated flow rule was adopted in his analysis. The Bauschinger effect was included. The difference between the experimental and theoretical optimum radius during free external inversion was bridged by realizing the influence of material parameters on the natural knuckle radius. The effect of strain rate and inertia during dynamic free inversion process were further investigated by Colokoglu and Reddy [19]. However, the prediction process is very complicated and agreement between the predictions and experiments is not very good. The predicted quasi-static inversion load is significantly lower than the experimental value while the predicted dynamic mean loads are overestimated.

There are two interesting stages in the tube inversion process with the presence of a die: the first stage is the curling phase when the tube end is forced to conform to the shape of the curved die and begins to curl up; the second stage involves the formation of a second wall after the curling process. The main advantage of this mode of deformation is the constant steady-state load that can be obtained for a uniform tube. A typical load/displacement profile and the external inversion of a circular tube are given in Figure 2.9 and Figure 2.10, respectively [25]. However, tube inversion is limited by die radius. If the die radius is small, progressive buckling of the tube will result and if the radius is larger than some limiting value, tube splitting will occur [21].
Figure 2.9 Load/displacement response for the external inversion of a circular tube under axial compressive load [25].

Figure 2.10 Deformed circular tube that underwent external inversion [25].

Al-Hassani et al. [17] conducted experimental and analytical studies on the inversion of tubes using shaped dies. The external and internal inversion of tubes of different materials, loaded with different speeds and using different die angles were experimentally investigated. In addition, by using power-law-type strain-hardening
material models, expressions for the steady inverting load and the optimum die radius for the inside-out inversion of a tube were given.

Miscow and Al-Qureshi [26] performed an experimental and theoretical study of the invertube process under quasi-static and dynamic axial loading conditions. The specimens used in this investigation were copper and 70:30 brass tubes having an outside diameter of 50.8 mm, wall thickness of 1.58 mm and length of 88.9 mm. The quasi-static tests were carried out using a 200 kN capacity hydraulic testing machine at ram velocity of 20 mm/min. The die assembly was attached to the lower platen of the hydraulic testing machine and the hollow punch was fixed to the movable upper arm. Typical load versus displacement profiles for the external inversion processes of copper and 70:30 brass tubes using a die radius of 4.76 mm under a quasi-static loading condition are presented in Figure 2.11. Figure 2.12 shows the quasi-static external inversion process of copper tubes from initial flaring of the material to the final steady-state inversion stage, where the letters refer to Figure 2.11. Their experiments have shown that materials in the as-received and/or the partially work hardened conditions were more appropriate to this technique than in annealed state, which generally demonstrated premature buckling. They also observed a considerable increase in the overall hardness along the inverted tube, in addition, an increase in wall thickness of approximately 8% throughout the inverted tube.

The authors also developed a theoretical expression for the steady-state inverting force by dividing the problem into the contact zone and the free zone and establishing the radial and tangential stress equilibriums. Experimental results were used to obtain contact pressure which turns the developed analytical model to be somewhat empirical. The author suggested that although this model can be used for estimating a number of design parameters, such as collapse, dynamic load, impact velocity, axial shortening, and circumferential expansions the predicted theoretical results must not be taken as the absolute values. Several variables exist that will modify the estimated values. However, despite these critics, the authors claimed that there was a reasonable agreement between the experimental and the theoretical results.
Reid and Harrigan [22] experimentally and numerically investigated the transient effects in the quasi-static and dynamic internal inversion of metal tubes using a conical die. Details of the deformation processes during internal inversion were examined using the non-linear finite element code ABAQUS. It was found that the early stages of the forced inversion characteristic were very complex with different deformation modes.
dominating the behaviour at different times. The peak impact loads can only be predicted through a full dynamic analysis since inertial effects strongly influence the magnitude of the dynamic load. Both experimentally observed and numerically predicted steady-state inversion forces were lower under dynamic loading conditions than under quasi-static loading.

Leu [27] experimentally and theoretically investigated the curling behaviour of quasi-static external inversion of circular tubes. An energy method technique was used to determine the critical bending radius as a curling criterion to distinguish between curling and flaring on a conical die. It was found theoretically that the strain hardening exponent and half-apex angle of die had very marked effects on the bending radius, however, the friction coefficient was not as great dependence as the strain hardening exponent. In case of external inversion, a half-apex angle of 90° was considered. The load required to cause inversion was derived by equating the incremental work done by the load to the incremental plastic energy absorbed. The range of semicircular curling and the compressing load of inversion in tube inversion with a quarter circle die were then found theoretically. Experimental tests were completed on the circular A1050-H18 aluminium tubes and A5052-H34 aluminium alloy tubes to validate the theoretical predictions. It appeared that the material properties of the aluminum tubes used had little effect on the curling behaviour of inversion. Good agreement between the theoretical prediction and experimental results was reported in terms of the compressing load, except in the case of the sharp fillet die radius for the strictly bending effect to be ignored in the prediction.

![Figure 2.13 Scheme of outward curling and flaring of a tube with a conical die [27].](image-url)
Rosa et al. [23] investigated the influence of interface friction on the material flow and the effect of strain path and material damage on the occurrence of fracture. Theoretical investigation was accomplished by using virtual prototyping modeling techniques based on the finite element method. Experimental work was performed on circular aluminium Al6060 tubes with inner radius of 18 mm, tube length of 70 mm, and four different wall thicknesses of 0.5 mm, 1.0 mm, 1.5 mm, and 2.0 mm. External inversion was accomplished by pressing the tubular specimens onto dies of different fillet radii that ranged from 2 mm to 10 mm. The role of friction in the tube inversion process was studied by invert-forming the specimens with and without lubricant and by utilizing a redesigned tool system as illustrated in Figure 2.14. The redesigned tool system allowed the air trapped inside the upper part of the tube to escape outside and enabled variations in the lubrication regime. Both the experimental and numerical results showed that low values of the ratio between the die fillet radius and the inner radius of the tube originated undesirable buckling modes of deformation while high values of the ratio stimulated the occurrence of cracking around the circumference. Cracking was essentially controlled by thinning of the tube wall as a result of extensive stretching in the circumferential direction at the tube-die contact region. Friction played an important role on the overall success of the tube inversion process. A critical friction value between the tube and the die was found to exist, where friction value larger than the critical one resulted in the progressive folding and friction value smaller than the critical one resulted in the inversion mode. An overall good agreement was found between theoretical and experimental results.

![Figure 2.14](image)

Figure 2.14  (a) Standard and (b) redesigned tool for analyzing the role of lubrication in the external inversion process [23].
2.1.3 Axial splitting

Splitting mode of deformation is a special case of tube inversion where the die radius is large enough to cause splitting instead of inversion [17, 21]. The axial splitting deformation mode has advantages from the viewpoint of energy absorption capabilities. Tube splitting and curling was more efficient than transverse crushing, axial buckling or tube inversion based on specific energy dissipation and had a better stroke to length ratio than any of these alternative deformation modes [28].

Stronge et al. [29] investigated the splitting and curling behaviour of square HE30 aluminium tubes having length \( L \) of 50 mm and wall thicknesses \( t \) of 1.6 mm and 3.2 mm. A saw-cut of 6 mm or 12 mm or 25 mm was introduced at the four corners of the square tubes. Axial splitting mode was achieved in a controlled manner by passage of a mandrel through the square tube. The tubes were pressed against a steel die with a small radius to form curls or had a flat plate normal to the tube axis under both quasi-static and dynamic loading conditions. The tubes were split by fracture at the four corners and four split plates were then free to bend outward. Several energy dissipation mechanisms were identified during this process, including energy associated with tube splitting, plastic deformation associated with the formation of the curls, and friction between the tube and the mandrel. Such an energy absorbing device exhibited a long stroke and operated at a load which increased mildly as the deformation progresses.

Reddy and Reid [30] studied the splitting behaviour of circular cold drawn mild steel and HE30 aluminium tubes in as-received and annealed conditions by compressing them onto a die. Various configurations, such as with/without pre-saw-cut, different die radii, with/without curling prevention stopper plate, different stand-off distance between the stop-plate and the die surface, quasi-static and dynamic loading conditions, axial or oblique loading directions, were considered. The load/displacement profiles for the as-received mild steel tubes with or without the presence of stopper plate are presented in Figure 2.15 and corresponding photographs of the deformed specimens are illustrated in Figure 2.16. They reported that different load levels can be achieved by varying the die radius and friction conditions as well as allowing the strips to curl, or being prevented from doing so. Constant load/displacement profiles after the initial transition period and
stroke efficiency of as high as 95 percent were observed. This mode of deformation was shown to be at least as efficient as other modes of deformation under axial loading (meaning tube inversion and progressive buckling).

Figure 2.15 Load/compression curves of as-received mild steel tubes (S2 and S11 without stop-plate; S7 and S12 with stopper plate) [30].

Figure 2.16 Photographs of the deformed as-received mild steel specimens: (S2) curling and (S7) curls prevented [30].

Lu et al. [31] conducted experimental studies on the axial splitting of square aluminium and mild steel tubes of thicknesses ranging from 0.47 mm to 1.67 mm. The experiments were carried out by driving four rollers, each attached to the side of the tube,
leading to the bending of the wall to a constant curvature and, at the same time, tearing the material along the four corners. By pre-saw-cutting some corners to a different length, the tearing energy involved was determined. They found that the tearing energy per unit torn area may be related to the tube material ultimate stress and the fracture strain.

Huang et al. [32] experimentally and theoretically investigated the axial splitting and curling behaviour of circular tubes by axially pressing the tubes onto a series of conical dies with different semi-angle (\(\alpha\)). All the experimental tests were completed quasi-statically and the experimental set-up scheme is presented in Figure 2.17. The specimens selected for this investigation were 200 mm long circular mild steel and aluminium tubes having \(D/t\) ratios ranged from 15 to 60. In order to establish the splitting and curling mode while preventing other collapse modes, eight initial 5 mm long saw-cuts were made into the specimen and evenly spaced around the lower circumference. The conical die was fixed to the bottom bed of the testing machine and a short cylindrical mandrel was placed inside the tube to prevent the tube from tilting. Three different semi-angles (\(\alpha\)), 45°, 60° and 75°, were selected for the conical die.

It was observed that at the beginning of a typical test, the strips between initial saw-cuts buckled and flared as guided by the die which led to the circumferential stretching of the tube. When this extension reached a certain level, cracks occurred at some initial saw-cut locations and propagated along the axial direction due to continuous ductile tearing. The strips so formed by the cracks rolled up into curls as the end of these strips was free to bend. The curling radius mainly depended on the semi-angle (\(\alpha\)) of the die and the dimensions of the tube. In some tests, when the strips of tubes began to curl up, branching or merging of cracks was observed.

Typical force-displacement curves for circular mild-steel tubes (\(D = 74.0 \text{ mm and } t = 1.8 \text{ mm}\)) against dies with three different semi-angle and corresponding photographs of the deformed specimens are presented in Figure 2.18 and Figure 2.19, respectively. Typical force-displacement curves for circular aluminium tubes (\(D = 77.9 \text{ mm and } t = 1.9 \text{ mm}\)) against dies with three different semi-angle and corresponding photographs of the deformed specimens are presented in Figure 2.20 and Figure 2.21, respectively.
Figure 2.17 Sketch of the experimental set-up, with 8 evenly spaced 5 mm initial saw-cuts around lower circumference [32].

Figure 2.18 Typical load/displacement curves for mild steel tubes with $D = 74.0$ mm and $t = 1.8$ mm against dies with semi-angle $\alpha = 45^\circ$, $60^\circ$ and $75^\circ$, respectively [32].
Figure 2.19 Photographs of typical deformed circular mild steel tubes ($D = 74.0$ mm and $t = 1.8$ mm) after splitting against dies with semi-angle ($\alpha$) of (a) $45^\circ$, (b) $60^\circ$ and (c) $75^\circ$, respectively [32].

Figure 2.20 Typical load/displacement curves for circular aluminium tubes with $D = 77.9$ mm and $t = 1.9$ mm against dies with semi-angle $\alpha = 45^\circ$, $60^\circ$ and $75^\circ$, respectively [32].
Figure 2.21 Photographs of typical deformed circular aluminium tubes ($D = 77.9$ mm and $t = 1.9$ mm) after splitting against dies with semi-angle ($\alpha$) of (a) $45^\circ$, (b) $60^\circ$ and (c) $75^\circ$, respectively [32].

It is obvious from Figure 2.20 and Figure 2.21 that similar deformation modes were observed for both mild steel and aluminium tubes. The axial force initially increased with the crosshead movement until it reached the first peak as shown in Figure 2.18 and Figure 2.19, which corresponded to the onset of inversion of strips from the initial cut. A second peak force then occurred and this corresponded to the initiation of cracks. After approximately another 20 mm of displacement the force reached a steady state and remained almost constant. The decrease in the applied force due to the increasing radius of the next roll was offset by an increase in friction between the tube and the inside mandrel. Three energy dissipation mechanisms were identified: (1) the ‘near-tip’ tearing associated with tube splitting; (2) the ‘far-field’ deformation associated with the plastic bending and stretching of curls; and (3) the friction as the tube interacted with the die.

An approximate analysis was also performed to predict the force at the steady-state stage. As a result, the crack number was predicted independently by minimum energy approach involving a competition between the plastic bending and fracture
energy. Thereby, the curl radius and the applied force were determined using the predicted crack number. The predicted results agreed well with the experimental results. Both experimental and theoretical results showed an increasing relationship between the steady-state axial splitting force and the die semi-angle.

Hung et al. [33] further investigated the energy absorbing behaviour of square mild steel and aluminium tubes under axial splitting deformation modes. Square tubes with a nominal side width (C) of 50 mm, wall thicknesses (t) ranging from 1.6 mm to 3.2 mm and length (L) of 200 mm were selected for this study. Specimens were pressed slowly against rigid pyramid shaped dies having three different semi-angles (α) of 45°, 60° and 75°. The testing set-up scheme was similar to what they had previously used to generate the splitting deformation mode in circular tubes as shown in Figure 2.17 [32]. The square tubes were pre-cut four 5 mm long slits at the four corners of lower end. The tubes were observed to have cracks propagating along the four corners. All four free end sides then rolled up into curls with a certain constant radius. The applied force became almost constant after the initial peak load initiated the cracks. Typical force/displacement profiles for square tubes against dies with three different semi-angles are shown in Figure 2.22 and the corresponding photographs showing the deformed specimens after splitting deformation mode are presented in Figure 2.23. Three energy dissipation mechanisms that are similar to what have been reported in reference [32] were identified. An approximate analysis was performed. By balancing the external work and the energy dissipated through plastic deformation, the steady-state splitting force required to split and curl the square tube by a die of semi-angle (α) was determined. Comparison between the theoretical predictions and the experimental determined steady-state splitting forces showed a good agreement.
Figure 2.22 Typical load versus displacement curves for square mild steel tubes with $t = 2.5$ mm against dies with semi-angle $\alpha = 45^\circ$, $60^\circ$ and $75^\circ$, respectively [33].

![Graph showing load versus displacement for square mild steel tubes with different semi-angles.]

Figure 2.23 Photographs of typical deformed square tubes after splitting against dies with semi-angle ($\alpha$) of (a) $45^\circ$, (b) $60^\circ$ and (c) $75^\circ$, respectively [33].

![Images of deformed square tubes with different semi-angles.]

Figure 2.23 Photographs of typical deformed square tubes after splitting against dies with semi-angle ($\alpha$) of (a) $45^\circ$, (b) $60^\circ$ and (c) $75^\circ$, respectively [33].
2.1.4 Axial cutting

In order to overcome the dependence of extrusion geometry on the progressive folding deformation mode, reduce the high peak load, and improve the crush force efficiency (\(\text{CFE}\), being the ratio of the mean crush load to the peak crush load), Cheng and Altenhof [6] conducted an experimental study on the load/displacement and energy absorption characteristics of square AA6061-T6 aluminium alloy extrusions under a cutting deformation mode utilizing a specially designed cutter. Tube lengths \((L)\) of 200 mm and 300 mm with a wall thickness \((t)\) of 3.15 mm and nominal side width \((C)\) of 38.1 mm were used in their research. Axial cutting deformation was generated at the four corners of the square extrusion. No initial peak cutter force was observed to initiate the cutting deformation mode. An almost constant cutting force was observed after approximately 10 mm cutter penetration. This cutting force was maintained constant until a crosshead of approximately 50mm. After that displacement, the axially cutting force increased, which the authors declared to be a result of cut petalled sidewalls bending outwards, and reached its second plateau. They reported a high \(\text{CFE}\) of 80% for the square tubes experienced the cutting deformation mode, compared to that of 18%-25.4% for the extrusions underwent the global bending mode. The force versus displacement profiles for the square tubes that experienced the cutting deformation and global bending deformation modes are presented in Figure 2.24. Photographs of cutting process for a representative specimen are shown in Figure 2.18. No significant influence of tube length on the force/displacement response of the extrusions which experienced the cutting deformation mode was reported. Two energy dissipating mechanisms were identified, namely, a cutting deformation mechanism and a petalled sidewall outward bending mechanism.
Majumder et al. [7] experimentally investigated the cutting deformation behaviour of circular AA6061-T6 and T4 extrusions under quasi-static loading conditions with two different extrusion wall thicknesses \((t)\) of 3.175 mm and 1.587 mm using cutters from reference [34]. From the cutting phenomena and load/displacement response of the circular extrusions, it was observed that the T6 temper extrusions with both wall thicknesses and T4 temper extrusion with \(t = 3.175\) mm exhibited a typical clean cut, while T4 temper extrusion with \(t = 1.587\) mm showed a braided cut. Photographs showing the cutting characteristics of a clean cut and a braided cut observed for T6 and
T4 temper specimens, respectively, are presented in Figure 2.26. An almost constant cutting force was observed throughout the cutting process after the transient stage for all specimens that experienced a clean cut deformation. Force oscillations were observed for extrusions that underwent a braided cut mode. The cutting deformations were found to be very stable and repeatable and with no length dependence of the extrusions. The steady-state cutting force reduced approximately 50% when the extrusion wall thickness reduced 50% for both temper extrusions. The AA6061-T6 extrusions exhibited a higher total energy absorption capacity compared to the T4 temper extrusions. Much higher crush force efficiency (CFE) was observed for extrusions that experienced the cutting deformation mode than those underwent the progressive folding and global bending deformation modes.

Figure 2.26 Petalled sidewall cutting deformation characteristics of AA6061-T4 and T6 tempered extrusions: (a) entire extrusions; (b) close range image illustrating back and forth folding of sidewalls (solid arrows) for T4 specimen and smooth continuous cut (dashed arrows) for T6 specimen [7].
2.2 Factors that influence collapse modes

The energy absorbing capacity of an axially crushed tube highly depends on the collapse mode that generated. Generally, two basic collapse modes, namely, progressive folding mode and global bending mode, can be generated for the axial crushing of tubes between two parallel plates. Factors that influence collapse modes include tube material properties, geometrical parameters, boundary conditions, imperfections, and the loading conditions.

Bardi et al. [35] experimentally studied the collapse behaviour of circular AA6061-T6, AA6260-T4, and CS1020 tubes with different $L/2R$ and $2R/t$ ratios under quasi-static and dynamic axial compression. In this geometrical range, the tubes should collapse in a global deformation mode according to the predictions by Abramowicz and Jones [36] and Andrews et al. [37]. The experimental setup is shown in Figure 2.27. The tubes were clamped at the lower end and crushed under displacement control. The authors showed that through the change of boundary conditions, the tubes collapsed in an axisymmetric progressive folding mode which otherwise should have collapsed in global bending modes according to predictions from [36, 37].

![Figure 2.27](image)

**Figure 2.27** Experimental setup used to crush tubes under displacement control [35].
In this section, the effect of geometrical parameters, different material properties, material and geometrical imperfections, and different loading conditions will be discussed in details.

2.2.1 Geometrical parameters

Geometrical parameters which govern the deformation mode are the ratios of $L/2R$ and $2R/t$ for circular tubes and $L/C$ and $C/t$ for square tubes, where $C$ is the mean side width of a square tube, $R$ is the mean radius of a circular tube, $L$ is the length of the tube, and $t$ is the tube wall thickness. A tubular column of square ($C/t$) or circular ($2R/t$) cross-section that experiences a stable progressive folding deformation is an efficient energy absorber. However, when the length ($L$) of the column is greater than a critical length ($L_{cr}$), which identifies the transition between progressive folding and global bending, it deforms in the global bending mode. Global bending is an inefficient mode of energy absorption and needs to be avoided in crashworthiness applications. Thus, a significant amount of studies have been done to predict the deformation modes for circular and square tubes during axial buckling.

Abramowicz and Jones [36] performed extensive experimental testing on the quasi-static and dynamic axial crushing of square and circular mild steel columns which buckled mostly in the plastic range in order to investigate the transitions between progressive folding and global bending deformation modes. The columns considered in their research had six different square cross sections ($5.5 \leq C/t \leq 38$) and five different circular cross sections ($9.6 \leq 2R/t \leq 48$), with a range of different lengths sufficient to encompass both progressive folding and global bending ($2.4 \leq L/C \leq 51.2$ and $2.2 \leq L/2R \leq 35.9$). For dynamic crushing, the columns were struck axially at one end by masses travelling with initial impact velocities of up to 12.14 m/s. It was observed that even relatively short columns, which entered the plastic range in a straight configuration and then plastically buckled in the global bending mode, a transition to progressive plastic buckling had been seen later in the collapse process.

The quasi-static experimental results for square and circular cross sectional tubes are summarized in Figure 2.28 and Figure 2.29, respectively. The solid line in Figure 2.28 and Figure 2.29 was obtained by means of a best curve fitting procedure for
the experimental results which approximately separates the progressive folding and global bending regions. The empirical critical length-to-width and length-to-diameter aspect ratios for square and circular tubes are presented in Equations (1.2) and (1.3), respectively. Thus, thin-walled columns having geometries that lie above the solid line will deform in global bending mode while thin-walled columns having geometries that lie below the solid line will deform in progressive folding mode under a quasi-static axial loading condition.

Figure 2.28 The deformation map for square columns subjected to quasi-static axial loading [36].

Figure 2.29 The deformation map for circular columns subjected to quasi-static axial loading [36].
\[
\left( \frac{L}{C} \right)_{cr}^{\text{static}} = 2.482e^{0.0409(C/t)} 
\]

(1.2)

\[
\left( \frac{L}{2R} \right)_{cr}^{\text{static}} = 2.996e^{0.02(2R/t)} 
\]

(1.3)

The dynamic experimental results for square and circular cross sectional tubes are summarized in Figure 2.30 and Figure 2.31, respectively. The two solid lines in both figures obtained by means of a best curve fitting procedure for the experimental results which separate the region of initial instability in which a column is bent without developing a single plastic lobe and the region of progressive folding. The empirical counterparts of Equations (1.2) and (1.3) are given in Equations (1.4) and (1.5), respectively. The empirical expressions for the second transition line in Figure 2.30 and Figure 2.31, which separates the region of classical progressive collapse from the mixed local-global collapse mode region, are presented in Equations (1.6) and (1.7) for square and circular tubes, respectively.

Figure 2.30  The deformation map for square columns subjected to dynamic axial loading [36].
Andrews et al. [37] experimentally investigated the collapse modes of deformation for the circular Ht-30 aluminium alloy tubes with $2R/t = 4.63$ and $L/2R = 0.17-8.75$ under quasi-static axial compressive loading condition. Several deformation modes were observed and a collapse mode classification chart was presented in Figure 2.32 to reveal the relationship between different deformation modes and tube geometries.
Guillow et al. [38] experimentally investigated quasi-static axial crushing of thin-walled circular 6060-T5 aluminium alloy tubes with different geometrical parameters. Circular tubes with $D/t = 10-450$ and $L/D \leq 10$ were considered and five categories of axial collapse deformation were identified as axisymmetric (concertina) buckling, non-symmetric (diamond) buckling, mixed buckling (combination of the two previous modes), global buckling and other deformation behaviour (simple compression, single folds, etc). A collapse mode classification chart was given and presented in Figure 2.33. The empirical expression of the mean crush force for the quasi-statically axial crushing of circular 6060-T5 aluminium alloy tubes was determined by curve fitting the experimental data and is presented in Equation (1.8).

$$P_m = 72.3M_o(2R/t)^{0.32}$$  \hspace{1cm} (1.8)
Hsu and Jones [39] conduct quasi-static and dynamic axial crushing tests on the square and circular stainless steel type 304 specimens in order to investigate the transitions between dynamic progressive folding and global bending of thin-walled tubes. The columns consisted of three different circular ($2R/t = 7.5, 22, 47$) and three different square ($C/t = 7.7, 24, 42$) cross sections and had a range of different lengths, $L$, ($3.38 \leq L/2R \leq 15.45$ and $3.37 \leq L/C \leq 20.8$). Three principal modes of failure were distinguished for the circular tubes: axisymmetric, non-axisymmetric, and global bending. Regular progressive collapse was initiated at either end with the formation of an axisymmetric wrinkle followed by the diamond pattern of which the number of circumferential lobe number, $N$, appeared to be dictated by the $2R/t$ ratio: two for $2R/t = 7.5$ and 22 and three for $2R/t = 47$. Three modes of failure were also identified for the square tubes: symmetric, extensional, and global bending. By contrast to the circular specimens, the first lobes, however, were initiated at different positions alone the length. The empirical critical length-to-diameter and length-to-width aspect ratios resulting in the transition between progressive folding and global bending were derived from curve-fitting the experimental quasi-static and dynamic crushing data points. The critical
length-to-diameter ratios for circular tubes under quasi-static and dynamic axial loadings are presented in Equations (1.9) and (1.10), respectively.

\[
\left(\frac{L}{2R}\right)_{cr}^{static} = 4e^{0.138(2R/t)} \quad (1.9)
\]

\[
\left(\frac{L}{2R}\right)_{cr}^{dynamic} = 4e^{0.0266(2R/t)} \quad (1.10)
\]

The critical length-to-width ratios for square tubes under quasi-static and dynamic axial loadings are presented in Equations (1.11) and (1.12), respectively.

\[
\left(\frac{L}{C}\right)_{cr}^{static} = 3.5e^{0.0399(C/t)} \quad (1.11)
\]

\[
\left(\frac{L}{C}\right)_{cr}^{dynamic} = 4e^{0.0631(C/t)} \quad (1.12)
\]

The critical \( L/2R \) or \( L/C \) ratio in the dynamic loading conditions for both the circular and square cross-sections was observed to be much higher than that in the quasi-static loading conditions.

Hsu and Jones [40] conducted further experimental investigations on the circular thin-walled tubes made of stainless steel 304, 6063-T6 aluminium alloy, and mild steel under quasi-static and dynamic loading conditions to investigate the critical slenderness ratios \( (L/2R) \) at the transition between progressive folding and global bending deformation modes. They reported that the stainless steel tubes absorbed the most energy, but they were the least efficient of the three materials for both quasi-static and impact. The 6063-T6 aluminium alloy tubes were found to be the most efficient energy absorbers. The critical specimen lengths for a transition from an energy efficient progressive folding to a potentially catastrophic global bending behaviour for quasi-static loads were similar for the three materials. However, the transition to a global bending response was more complex under dynamic axial loads. The critical length generally increased with an increase in the impact velocity. Figure 2.34 shows the critical length
versus impact velocity relationship for circular stainless steel tubes under axial impact crushing tests with 9 kJ impact energies.

Figure 2.34  Critical length versus impact velocity relationship for circular stainless steel tubes under axial impact crushing tests with 9 kJ impact energies [40].

Experimental observations from references [13, 36, 37, 38, 39] indicated a critical tube length for buckling transition exists under static loading, for given extrusion material and cross-sectional geometry. Extrusions shorter than this critical length collapsed progressively, while longer extrusions developed a global bending mode. However, for dynamic loading conditions, the collapse mode of the extrusion was no longer dependent only on material properties, boundary conditions and extrusion geometries but also depended on the impact velocity [14, 40]. Furthermore, extrusion imperfections played an important role in dynamic crush conditions [13]. Thus, the collapse behaviour of an extrusion under dynamic loading conditions is very unstable and difficult to control.

2.2.2 Extrusion materials

Extrusion material has a very important role in the specific collapse mode and energy absorption capability for a tubular specimen. A significant amount of studies have been performed on circular and square tubes of commonly used materials including aluminium alloys, stainless steel, mild steel and high strength steel.
As presented in section 2.1.1, Karagiozova and Alves [14] showed that the material hardening characteristics had a significant influence on the dynamic collapse mechanisms. The circular extrusions made of ductile alloys with a high yield stress and low strain hardening characteristics had a better energy absorption performance than extrusions with a low yield stress and high strain hardening characteristics. Langseth and Hopperstad [11] observed that the number of lobes formed during the deformation process as well as the way the different lobes were formed was a function of the hardening properties of the materials.

Gupta and Gupta [41] experimentally investigated different collapse behaviour of quasi-statically loaded circular aluminium and mild steel tubes. The aluminium and mild steel tubes were tested in both as-received and annealed conditions. The engineering stress versus engineering strain curves for the middle steel specimens in both as-received and annealed states obtained from tensile tests are presented in Figure 2.35. The $t/D$ ratios of aluminium and mild steel tubes considered for their tests varied from 0.033 to 0.96 and from 0.034 to 0.096, respectively. The $L/D$ ratios selected for aluminium tube were 2 or 3 and for mild steel tubes were 3, respectively. In this range, Andrews et al. [37] predicted that all tubes with $L/D$ of 2 and $L/D$ of 3 (in range of $t/D = 0.04-0.08$) should deform in concertina mode. Figure 2.36 shows the load versus displacement profiles for 52.6 mm diameter mild steel tubes in both as-received and annealed conditions. Figure 2.37 illustrates the deformed shapes of 52.6 mm diameter mild steel tubes with in annealed and in as-received states. It can be seen from Figure 2.36 and Figure 2.37 that the same geometry mild steel tubes of different material properties exhibited significantly different load/displacement responses and deformation modes (diamond mode for annealed tube and concertina mode for as-received tube). Results from the experimental tests showed that the mode of deformation of the tubes depended on the initial state of work hardening and the subsequent annealing process, as well as on the tube geometry. A highly cold worked as-received aluminium tube deformed in diamond mode and when annealed it deformed in a concertina mode. As-received strain-hardened steel tubes deformed in concertina mode and on annealing they deformed in diamond mode.
Figure 2.35  Engineering stress versus engineering strain curves for tensile test performed on specimens machined from 53 mm diameter mild steel tube in as-received and annealed states [41].

Figure 2.36  Load/displacement curves for 52.6 mm diameter mild steel tube in as-received and annealed conditions [41].
Figure 2.37 Deformed shape of the 52.6 mm diameter steel tube (a) in annealed state and (b) in as-received state after 50% compression [41].

2.2.3 Crush initiators

In order to control and stabilize the collapse mode and improve the energy absorption capability of extrusions under axial loading conditions, crush initiators or triggers are often introduced. Crush initiators are imperfections or stress concentrations existing in energy absorbers in forms of material property variations and geometrical discontinuities. Use of crush initiators can considerably reduce the peak plastic buckling load, improve crush performance parameters, trigger deformations at a specific location, and enhance energy absorption performance of the energy absorbers. Material property variations can be achieved by localized heat-treating at the regions of interest while, geometrical discontinuities, due to their easy implementation, are commonly used to initiate a specific collapse mode and improve the stability of deformation. A recent overview of the characteristics of tubular structures with geometric and material modifications was completed by Yuen and Nurick [42].

Gupta and Gupta [41] studied the influence of length-to-diameter and diameter-to-thickness ratios as well as cut-outs in the form of circular holes on deformation behaviour of round aluminium and mild steel tubes. The discontinuities were introduced through
laterally drilled holes and varied in diameter, number and position. They have shown that in tubes where holes were located in parallel positions, deformation was initiated at the location of the holes at one of the planes. For tubes with holes in cross positions, it was found that the first peak load reduced and the mode of collapse altered.

Krauss and Laananen [43] numerically investigated the effect of crush initiator geometry on peak and mean crush loads as well as energy absorption capabilities of square steel tubes. The initiators were located at a distance from the end equal to \( C/2 \), where \( C \) is the side width of the square tubes. Three different crush initiate shapes were examined, namely, a transverse bead on two sides of the tube cross section, a diamond notch on each of the four corners, and a circular hole on the corners. For each shape, three sizes of initiators with a cross-sectional area reduction of 5%, 10%, and 15% were analyzed and compared with an un-notched baseline specimen. The results of the parametric study showed that the peak crush force was reduced as high as approximately 45% with the introduction of the 15% bead initiator as shown in Figure 2.38. While the total energy absorption was reduced with the introduction of the initiators, the bead initiator demonstrated highest energy efficiency amongst the three types of crush initiators investigated. The total energy absorption was observed to reduce generally with the increase of the size of the initiators except for the bead initiator.

In order to reduce the peak crush load on the load/displacement characteristics and control the collapse mode, Abah et al. [44] conduct experimental and numerical studies on the collapse behaviour of square aluminium tubes with circular hole cut-outs at the four corners under both quasi-static and dynamic loading conditions. While the number of cut-outs was fixed, their dimensions and locations were variable. Quasi-static and dynamic tests results of the aluminium extrusions showed that the introduction of cut-outs significantly reduced the first peak load while the mean crush load remained relatively constant.
Marshall and Nurick [45] experimentally studied the effects of induced imperfections on the collapse behaviour of thin-walled square mild steel tubes. The imperfections were a circular hole, indentations of various shapes, and combinations of a hole positioned centrally in an indentation and located in the opposite walls of the tubes. Figure 2.39 shows the quasi-statically crushed square tubes with two opposing holes of different sizes. Figure 2.40 illustrates the quasi-statically crushed square tubes with different indentation radius. The experimental results showed that as the severity of simple imperfections increased, so the stability of the symmetric buckling mode decreased. The stability of the buckling mode was compromised due to the change in the size of the first buckling lobe. Holes effectively decreased the width of the tube and thus decrease the size of the first buckling lobe until tearing occurred as shown in Figure 2.39. However, indentations increased the size of the first lobe and if the dent was sufficiently large a very large first lobe was created and the tube bent over. Combined imperfections
had a greater effect on reducing the initial peak crush load than either holes or indentations acting individually.

![Figure 2.39 Quasi-statically crushed square tubes with two opposing holes (diameter of the holes increases from left to right) [45].](image)

Lee et al. [46] studied the effect of triggering dents on the energy absorption characteristics of quasi-statically compressed 6063 aluminium alloy extrusions. The tubes had a cross-sectional area of $50 \times 50$ mm$^2$, length of 300 mm, and thickness of 2 mm. Two types of dents, full-dent and half-dent, as shown in Figure 2.41, were introduced at the folding sites pre-estimated by computer simulations. The results showed that the first peak load decreased with the introduction of the dents. The specimens containing half-dents exhibited the same number of plastic hinges as the tubes.

![Figure 2.40 Quasi-statically crushed square tubes with opposing parallel cylindrical indentations (depth of indentations increased from left to right) [45].](image)
with full-dents, as shown in Figure 2.42, while repulsive force required to the formation of each hinge was increased. When the triggering dents of the same interval were introduced without consideration of the peak location of the folding wave, inhomogeneous deformation together with overall bending occurred.

Figure 2.41 Shape and dimensions of (a) full-dent and (b) half-dent introduced into the aluminium extrusions by Lee at al. [46]. (Units are in millimeter)

Figure 2.42 Deformed aluminium tubes with schematic of the types and locations of triggering dents [46]. (Units are in millimeter)

Hosseinipour and Daneshi [47] experimentally and theoretically studied the load/displacement behaviour and energy absorption characteristics of circular mild steel tubes containing circumferential grooves as shown in Figure 2.43. The tubes had a length
(L) of 100 mm, a nominal diameter (D) of 54 mm, and a tube wall thickness (t) of 2 mm. The grooves had a width (w) of 3 mm and a depth (d) of 1 mm and were allocated along the tube axis at different positions. Figure 2.44 shows the deformed tubes with different spacing of the grooved after the axial crushing tests. It was observed that the modes of deformation were altered through the implementation of the groove triggers. Both the experimental and theoretical results showed that the grooved tubes exhibited favourable characteristics as energy absorption members in terms of load uniformity and low deceleration pulse.

Figure 2.43  Details of the specimen design used by Hosseinipour and Daneshi [47].

Figure 2.44  Deformed specimens after axial crushing. From left to right are specimens B1, B8, and B9, respectively. [47]
Arnold and Altenhof [48] experimentally investigated the crush characteristics of square AA6061-T4 and T6 tubes with and without the presence of circular discontinuities under quasi-static axial loading. The tubular geometries had lengths ($L$) of 200 mm and 300 mm, nominal side width ($C$) of 38.1 mm and wall thickness ($t$) of 3.15 mm, which should result in a global bending deformation mode according to the predictions by Abramowicz and Jones [36]. Circular holes were machined into the two opposite walls of the tube at center location to serve as crush initiators. Two different hole diameters of 7.1 mm and 14.2 mm were considered to commence the plastic buckling process. It was reported that collapse modes and energy absorption of the structure depended largely on material properties and to a lesser extent on the diameter of the discontinuity. A reduction of the peak crush load and higher crush force efficiency ($CFE$) were generally found for the extrusions with discontinuities. Furthermore, the energy absorption abilities of the extrusions were greatly improved by altering the deformation mode within the extrusion through the implementation of discontinuities. In addition to progressive buckling, collapse modes involving cracking and splitting were observed in many tests and were characterized using photographs of the experimental process as illustrated in Figure 2.45. The splitting collapse modes observed in the AA6061-T6 specimens with circular hole discontinuities provided a large increase in energy absorption over the AA6061-T6 specimens with no intentional discontinuities, which may be attributed to the decrease in peak buckling load and the high plateau load associated with the sustained cracking and splitting observed during the compression process.

![Figure 2.45 Slitting mode observed during the axial crushing process for the AA6061-T6 extrusions with a tube length of 200 mm and a hole diameter of 14.2 mm [48].](image)
Cheng et al. [49] experimentally studied the crush characteristics and energy absorption capacity of square AA6061-T6 aluminium alloy extrusions with centrally located through-hole discontinuities. The square tube had a length ($L$) of 200 mm, nominal side width ($C$) of 38.1 mm, and wall thickness ($t$) of 3.15 mm. Three different shapes (circular, slotted and elliptical holes) and different sizes (three different major axial lengths of 7.14 mm, 10.72 mm, and 14.29 mm and three different aspect ratios of 1.33, 2.0, and 3.0) were considered as shown in Figure 2.46 to investigate their effects on crush behaviour of the square tubes. A splitting and cutting deformation was initiated as shown in Figure 2.47 rather than a global bending mode through the implementation of hole discontinuities. Comparisons of typical load versus displacement profiles for the square AA6061-T6 extrusions with or without the circular discontinuities are presented in Figure 2.48.

![Figure 2.46 Geometries of the AA6061-T6 extrusion and discontinuities considered by Cheng et al. [49].](image)
The authors also observed that the peak load was reduced by incorporating the through-hole crush initiators within a range of 5.2% to 18.7%; and total energy absorption was increased in the range of 26.6% to 74.7%. The most significant improvement was reported for crush force efficiency in the range of 54.5% to 95.8%. The peak crush load and total energy absorption was to be independent of initiator geometry and aspect ratio for the extrusions with major axis length of 7.14 mm.
However, for specimens with a major axis length of 10.72 mm and 14.29 mm and aspect ratio of 3, a geometrical influence on the peak load and total energy absorption was apparent.

Han et al. [50] experimentally and numerically investigated the quasi-static and dynamic crush response and energy absorption capacity of circular aluminium and steel tubes with and without a square shape cut-out. The influence of location of cut-out on the energy absorption capacity of tubes having various length/radius ratios, subject to various impact loading conditions, was studied. Very good agreements were observed between numerical and experimental results. It was reported that in the quasi-static tests, the tubes with the cut-out located at the mid-height generally collapsed in a global bending mode while the tubes with the cut-out located near the top end generally deformed in progressive folding. Thus, the energy absorption capacity of both aluminium and steel tubes was improved when the cut-out location was moved from mid-height to their top end.

2.3 Analytical models for axial crushing

Analytical models for the axial progressive folding of square and circular tubes have been extensively investigated in order to predict the mean crush forces for the axial crushing processes.

Abramowicz and Jones [8, 10] developed theoretical models to predict the mean crush forces for the symmetric, axisymmetric, and extensional progressive folding deformation modes by equating the total work done by the mean crush force to the energy dissipated in the plastic deformation of all the basic collapse elements, namely, inward lobe (type I) and outward lobe (type II), in a complete layer of formed lobe. These two basic collapse elements are illustrated in Figure 2.49. The energy absorbed in the type I and type II basic folding elements are given in Equations (1.13) and (1.14), respectively.

\[
E_1 = M_o (16HI_1b/t + 2\pi C + 4I_3H^2/b) \tag{1.13}
\]

\[
E_2 = M_o (2\pi H^2/t + 2\pi C + \pi H) \tag{1.14}
\]
where $I_1 = 0.555$, $I_3 = 1.148$, $b$ is the radius of the toroidal shell element, $H$ is one-half of the initial distance between plastic hinges at the top and bottom of a basic folding element, and $M_o = \sigma_o t^2/4$ is the fully plastic bending moment.

By equating the external work and internal work and introducing the concept of effective crushing distance, $\delta_e$ ($1.46H$ for the symmetric progressive folding mode), the mean static crushing load for square tubes which collapsed in the symmetric mode consists of four type I elements in a layer was determined to be:

$$P_m = 52.22 M_o (C/t)^{1/3}$$  \hspace{1cm} (1.15)

Similarly, for squares tubes which collapsed in the axisymmetric mode consists of three type I and one type II basic folding element in a layer and the extensional collapse mode which consists of four type II elements in a layer, the mean static crushing loads are given in Equations (1.16) and (1.17), respectively. Here, the effective crushing distance, $\delta_e$, for the axisymmetric and extensional collapse modes is $1.54H$.

$$P_m = M_o \left[ 42.92 (C/t)^{1/3} + 3.17 (C/t)^{2/3} + 2.04 \right]$$ \hspace{1cm} (1.16)

$$P_m = M_o \left[ 32.64 (C/t)^{1/2} + 8.16 \right]$$ \hspace{1cm} (1.17)

![Figure 2.49 Basic collapse elements proposed by Abramowicz and Jones [8, 10].](image)

As discussed in section 2.1.1, axially crushed circular tubes could collapse in axisymmetric (concertina) and non-axisymmetric (diamond) progressive folding
deformation modes. Relatively thicker tubes generally deform into axisymmetric folds and thinner tubes collapse in diamond folds. Some tubes start deforming into axisymmetric folds but then revert to a diamond mode as collapse progresses. The first fold almost generally forms at one of the two ends of the circular tube and is facilitated by a radially outward movement.

Pugsley and Macaulay [51] was one of the first to develop analytical models for non-axisymmetric progressive folding of circular tubes. The mean crush force under this deformation mode in a quasi-static loading condition is given in Equation (1.18).

\[
P_m = 8\pi R M_o (1.6/R + 0.18/t)
\]

(1.18)

Figure 2.50 Linear segment collapse mechanism proposed by Alexander [52].

Alexander [52] performed an earlier analysis on the axial axisymmetric folding of circular tubes. The assumed collapse mechanism is illustrated in Figure 2.50, which
consists of three plastic hinges (black dots in Figure 2.50) in one layer of axisymmetric folds. The region between the extreme plastic hinges was assumed to move either completely outwards or completely inwards, exhibiting plastic stretching (or compression) in the hoop direction. He then equated the external work done by the mean crush force with the internal energy absorbed by the proposed plastic deformation mechanism and produced an upper bound solution for the mean crush force as given in Equation (1.19).

\[ P_m = M_o \left[ 20.725 \left( \frac{2R}{t} \right)^{1/2} + 6.283 \right] \quad (1.19) \]

Abramowicz and Jones [9, 10] proposed a different collapse mechanism for the axially crushing of circular tubes as shown in Figure 2.51 to develop analytical models for prediction of the mean crush forces. The region between the plastic hinges which undergoes hoop expansion in Alexander’s model [52] will have two equal parts of the same curvature but of opposite sense. This region was assumed to move either completely outwards or completely inwards. By equating the external work and the internal energy dissipated by the proposed folding mechanism and employing the effective crushing distance, \( \delta_e \) (1.5\( H \) and 1.46\( H \) for the axisymmetric and non-axisymmetric modes, respectively), the mean crush forces are determined and given in Equations (1.20) and (1.21) for the axisymmetric and non-axisymmetric modes, respectively, where \( N \) is the number of circumferential lobes when the non-axisymmetric folding mode occurs.

\[ P_m = M_o \frac{25.23 \left( \frac{2R}{t} \right)^{1/2} + 15.09}{0.86 - 0.568 \left( \frac{t}{2R} \right)^{1/2}} \quad (1.20) \]

\[ P_m = \begin{cases} 
M_o \left[ 31.01 \left( \frac{2R}{t} \right)^{1/2} + 17.22 \right]; & \text{For } N = 2 \\
M_o \left[ 28.86 \left( \frac{2R}{t} \right)^{1/2} + 44.74 \right]; & \text{For } N = 3 \\
M_o \left[ 28.23 \left( \frac{2R}{t} \right)^{1/2} + 83.15 \right]; & \text{For } N = 4 \\
M_o \left[ 27.95 \left( \frac{2R}{t} \right)^{1/2} + 132.49 \right]; & \text{For } N = 5 \\
M_o \left[ 27.81 \left( \frac{2R}{t} \right)^{1/2} + 192.80 \right]; & \text{For } N = 6 
\end{cases} \quad (1.21) \]
Wierzbicki et al. [53] introduced a more realistic axisymmetric plastic folding of circular tubes as shown in Figure 2.52. The authors used a model based on the assumption that crushing progresses by virtue of instantaneous formation of three plastic hinges leading to a fold comprised of two elements of equal lengths. As the fold develops, the mechanism allows both inward and outward radial displacements of the tube sidewall according to a certain ratio. This ratio, denoted by $m$, represents a geometric eccentricity factor which has not been introduced in earlier publications. However, the value of $m$ is arbitrary and indeterminate. The mean crush force for a steady-state axisymmetric folding cycle is independent of $m$ and is given by:

$$P_m = M_o[22.27(2R/t)^{1/2} + 5.63]$$

(1.22)
Singace et al. [54] proposed an improved linear segment folding mechanism as shown in Figure 2.53 and further investigated the geometric eccentricity factor on the axisymmetric plastic folding of circular tubes. Friction effects and geometric and material imperfections play an important role in the formation of the initial buckle. The author showed that as the tube is compressed, its wall tends to move radially outwards, due to a Poisson effect and axial shortening, and the first fold will always form with an outward buckle. A critical angle, $\gamma_0$, exists for the formation of the first outward fold. It was noted that once the first fold has been formed ($\gamma_0 \to 0$), different critical angles, $\alpha_o$ and $\beta_o$, necessary for the definition of the inward and outward folds for the second and subsequent folds, respectively, exists in the concertina mode of failure due to the diminish of edge effects. The illustration of the folding processes of the second or subsequent fold is presented in Figure 2.54.

Following the same basic procedure as used by Wierzbicki et al. [53], values for the eccentricity factor, $m$, and the critical angles, $\alpha_o$ and $\beta_o$, for the formation of the inward and outward folds were derived and determined to be 0.65, 49.62°, and 69.38°, respectively. It is worthy to note that these derived values are independent of the tube geometry and material properties. The mean crush force was found to be identical to that given by Wierzbicki et al. [53] and is presented in Equation (1.23) again.

$$P_m = M_o[22.27(2R/t)^{1/2} + 5.63]$$  \hfill (1.23)
Figure 2.53 Improved linear segment folding mechanism proposed by Singace et al. [54].
Figure 2.54 The formation and progression of the first inward fold (phase 1, second fold): (a) completion of the first fold, and (b) first phase of the second fold; and the transition between the first inward and the subsequent outward fold (phase 2, second fold): (c) completion of the first phase of the second fold, and (d) second phase of the second fold. [54]

Quasi-static experimental testing was carried on a set of circular Ht-30 aluminium alloy tubes to measure and validate the derived critical values for $m$, $\alpha_0$, and $\beta_0$, as well as the mean crush force. The theoretical predictions for $m$, $\alpha_0$, $\beta_0$, and $P_m$ were generally in good agreement with those determined experimentally. Figure 2.55 shows one
representative load/displacement profile for the axial crushing of Ht-30 aluminium alloy tube crushed up to the fourth inward fold in an axisymmetric deformation mode. Further experimental work was completed by Singace and Elsobky [55] on the axisymmetric progressive folding of circular aluminium alloy, brass, copper, and mild steel tubes in order to measure the critical values for \( m, \alpha_o, \) and \( \beta_o \) and validate the theoretical derivations developed in reference [54]. The experimentally obtained values for the eccentricity factor ranged from 0.59 to 0.67 for all the materials and tube geometry considered. The measured values for the critical angles \( \alpha_o \) and \( \beta_o \) ranged from 45° to 50° and from 68° to 72°, respectively. These experimentally obtained values were generally in good agreement with the theoretical findings.

Figure 2.55  Load/displacement profile for the axial crushing of a representative circular Ht-30 aluminium alloy tube underwent an axisymmetric deformation mode, where letters ‘I’ and ‘O’ indicate the inward and outward folds, respectively. Insert is the tube compressed up to the fourth inward fold. [54]
Using the same linear segment collapse mechanism proposed in reference [54], Singace [56] developed an analytical model for the non-axisymmetric progressive folding of circular tubes. Instead of all circumferential tube material folding inward or outward in the concertina failure mode, partial of circumferential tube material fold inward while the other partial of the circumferential tube material folds outward in the diamond failure mode. The illustration of the folding processes for the second and the subsequent folds is presented in Figure 2.56 for the diamond failure mode. The simultaneously formation or diminish of the inward and outward folds in one layer of the progressive folding deformation converts the circular tube into \( N \) number of triangles with equal and definitive \( N \) sides as shown in Figure 2.57.

The eccentricity factor \( (m) \), the critical angles for the inward and outward folds \( (\alpha_o \text{ and } \beta_o \text{, respectively}) \), and the mean crush forces were then determined by deriving the internal energies associated with bending, de-curving (the bending energy required to flatten and remove the tube curvature), and membrane deformations and equating them with the work done by the mean crush force. The theoretical derived values for \( m, \alpha_o \), and \( \beta_o \) were found to be 0.642, 50.06°, and 69.02°, respectively. The mean crush force for non-axisymmetric progressive folding of a circular tube with \( N \) number of circumferential lobes was found to be:

\[
P_m = M_o \left[ -\frac{\pi}{3} N + \frac{4\pi^2}{N} \tan \left( \frac{\pi}{2N} \right) \frac{R}{t} \right]
\] (1.24)

Quasi-static axially crushing of circular aluminium alloy, brass, and copper tubes with different geometrical dimensions were experimentally tested to determine the eccentricity factor and critical angles associated with diamond modes. Figure 2.58 represents a typical load/displacement profile for a circular tube that collapsed in a non-axisymmetric deformation mode. Comparison of the theoretically and experimentally determined values for \( m, \alpha_o, \beta_o \), and \( P_m \) was completed and found to be in good agreement.

59
Figure 2.56  The development of the second and subsequent lobes: (a) the critical position of the inward and outward folds; (b) the transition between the inward and outward folds of the lobe; (c) the critical position of the inward and outward folds of the next lobe; (d) the transition between the inward and outward folds of the next lobe. [56]
Figure 2.57 Geometric relationships for a tube deforming into three circumferential lobes: (a) the eccentricity factor and the inclination angle relationship; (b) the plan view of the tube; (c) the development plan view for the three lobes. [56]

Figure 2.58 Load/displacement profile for the axial crushing of a representative circular Brass tube underwent a non-axisymmetric deformation mode. Insert is the tube crushed up to the third inward fold [56].
2.4 Analytical models for wedge cutting of a plain plate

The cutting deformation process of thin-walled plates, as one of the primary energy absorbing mechanisms, has received considerable attention and a thorough literature review dealing with experimental and theoretical analyses of the plate cutting resistance force by a sharp wedge was presented by Lu and Calladine [57], and Simonsen and Wierzbicki [58]. Depending on the deformation behaviour, three categories of the cutting process were identified in references [58, 59], namely, clean curling cut, braided cut, and concertina tearing. Deformation characteristics of the three different cutting modes are shown in Figure 2.59. Though the mechanics of the cutting process is complicated, which involves plastic flow (and fracture) of the plate in the vicinity of the wedge tip, membrane deformation of the plate, and friction between the wedge and plate, the analysis of the cutting process falls into two stages: initial wedge penetration (transient) stage and steady-state cutting stage. The transient stage considers first contact between the wedge tip and plate edge to the state where the resistance force reaches a constant level. If the penetrator has a finite width, the plate reaction force will reach a constant value after a certain penetration depth and the process is then said to be steady-state. Although many empirical and theoretical analyses to predict the cutting resistance force between a plate and wedge are available, only those applicable to this research will be further discussed in this dissertation meaning that the wedge must have a finite shoulder width to reach a steady-state cutting process.

Simonsen and Wierzbicki [58] and Zheng and Wierzbicki [59] developed similar closed form solutions to predict the steady-state cutting resistance force for a clean plate-wedge cutting process considering the contribution of a finite shoulder width \((2B)\) of the wedge. Three major energy dissipation mechanisms [58, 59] were considered in the development of the analytical models, namely, crack tip zone in front of the wedge (ductile fracture and moving hinge line), membrane deformation, and friction. The rate of energy dissipation associated with each mechanism was calculated and balanced with the work done by the steady-state cutting force. The contribution of friction was then added to the closed form solutions.
Figure 2.59  Photographs of different failure modes. (a) Concertina tearing by a blunt wedge, (b) Braided tearing of a plate by a narrow wedge and (c) Center ‘clean cut’ of a plate by a sharp wedge with stable flap buckled [59].

The steady-state wedge cutting force given by Zheng and Wierzbicki [59] is:

\[
F = \frac{\sigma_0 t^2}{4} \left( 1.268 \frac{R_r}{t} \cos \theta + 2 \frac{R_r + B}{R_r} + 1.28 \theta^2 \frac{(R_r + B)^2}{R_r t} \cos(\theta/2) \right) \cdot (1 + \mu \cot \theta) \tag{1.25}
\]

where \( R_r \) is the rolling radius as expressed in Equation (1.26), \( B \) is one-half of the wedge shoulder width, \( \sigma_0 \) is the flow stress, \( t \) is the plate thickness, \( \theta \) is the wedge semi-angle, and \( \mu \) is the friction coefficient.

\[
R_r = B \sqrt{\frac{2(t/B) + 1.28 \theta^2 \cos(\theta/2)}{1.268 \cos \theta + 1.28 \theta^2 \cos(\theta/2)}} \tag{1.26}
\]

The steady-state cutting force derived by Simonsen and Wierzbicki [59] is:

\[
F = \left( \frac{0.64}{\sqrt{3}} \sigma_0 t R_r \cos^2 \theta (1 + 0.55 \theta^2) + \frac{\sigma_0 t^2 (R_r + B)}{\sqrt{3} R_r \cos \theta} + \frac{2}{\sqrt{3}} \sigma_0 t B \theta \right) \cdot \left( 1 - \frac{\mu}{\sin \theta + \mu \cos \theta \cos(\theta/2)} \right)^{-1} \tag{1.27}
\]
where the roll radius $R_r$ is determined through Equation (1.28).

$$R_r = \sqrt{\frac{Bt}{0.64(1 + 0.55\theta^2)\cos^3 \theta}}$$

(1.28)

2.5 Finite element modeling of axial crushing/cutting

Finite element (FE) method is an important design and analysis tool which has been widely used in structural crashworthiness to assess the crash behaviour of individual structural members as well as the entire structures under. Automotive companies today employ numerical simulations as a support in the design process to reduce the number of prototypes, developing time, and cost. Furthermore, finite element method enables new designs and materials to be evaluated without extensive testing and provides a framework for implementing new knowledge gained through experiments and improvement of theory of materials and structures.

A typical finite element analysis process involves five steps: development of a simplified model, formulation of governing equations, discretization of governing equations in space, solving the equations either explicitly or implicitly in time, and interpretation of results. In the axial crushing/cutting process which involves large plastic deformations and is dynamic and non-linear in nature, the explicit solvers, such as LS-DYNA™, are preferable because of both CPU-time efficiency and robustness. There are a number of element formulation techniques available in commercial large deformation FE packages, such as Lagrangian, Eulerian, Arbitrary Lagrangian-Eulerian (ALE), mesh free Lagrangian (SPH), and element free Galerkin (EFG). Lagrangian FE formulation is the most common in the majority of structural numerical simulations employing the FE method. However, in large deformation processes the massive mesh distortion of Lagrangian type elements may lead to significant numerical error. An alternative element selection for large deformation processes is the Eulerian or Arbitrary Lagrangian/Eulerian (ALE) element formulations.
2.5.1 Lagrangian finite element formulation

Langseth et al. [60] numerically investigated the axial crushing of thin-walled square AA6060-T4 and T6 aluminium alloy extrusions under both quasi-static and dynamic loading conditions using non-linear finite element code LS-DYNA™. The validation of the numerical model was accomplished using an experimental database obtained by completion of similar tests to the numerical study. A specimen length of 310 mm with single cosine half-wave trigger located at the top or at mid-section was used in the numerical simulation as shown in Figure 2.60. Due to symmetry deformation mode observed in experimental testing, only one quarter of the specimen was modeled using two symmetry planes. The specimens were modeled using the Belytschko-Lin-Tsay shell element with nine integration points through the thickness and one point in the plane of the elements. An element size of 3 mm × 3 mm was used giving a total of 2500 elements in the quarter model. Full constrains was prescribed at the lower end and the rotational degrees of freedom were fixed at the upper end to avoid unrealistic deformation modes. The quasi-static crushing load was applied at the upper end of the specimens by prescribing a displacement of 250 mm in 25 ms to a rigid block modelled with brick elements, while the projectile was modelled with a rigid body given a specified initial velocity in the dynamic simulations. Contact between the rigid block/projectile and the specimen was modelled using a node-to-surface contact algorithm with a friction coefficient of 0.25. A single surface contact algorithm without friction was used to account for the contact between the lobes. A material model (*MAT 103 within LS-DYNA™) developed by Berstad et al. [61] was utilized for the extrusion model. This material model uses isotropic elastic plastic behaviour, the von-Mises yield criterion, associated flow rule and non-linear isotropic hardening. Mass scaling was used in the quasi-static crushing tests to save computational time and it was controlled by ensuring that the calculated kinetic energy was insignificant when compared to the strain energy absorbed by the specimen.

The quasi-static and dynamic simulations for the axial crushing of aluminium alloy extrusions correlated well with the experimental results from reference [11]. The finite element model predicted the number of lobes as well as the shape of the load/displacement curve quite well. For the quasi-static simulations, the peak load and
the mean load as a function of the axial displacement were predicted within ±10% compared with the experimental data. The simulated results showed that the magnitude of the mass scaling used in the quasi-static simulations influenced the response parameters, such as initial buckling load and mean crush load, due to inertia effects.

Figure 2.60 One quarter finite element model for AA6060 extrusions including trigger position [60].

Jensen et al. [62] conducted numerically simulations using LS-DYNA™ in order to study the transition between progressive and global buckling of axially loaded square aluminium extrusions in alloy AA6060 temper T6. The validation of the numerical model was completed against experimental tests. A full model instead of symmetry model used in reference [60] was used to avoid possible prevention of transition from progressive folding to global bending based upon the preliminary simulations that used symmetry. The bottom row of nodes was fixed in all degrees of freedom while the upper end of the extrusion was unrestrained. Both global and local initial imperfections were modelled to trigger the deformation process. The specimen was modelled with quadratic shell element with the option of ‘membrane strain causing thickness change’ adopted. In
addition, seven integration points through the thickness and one point in the plane of the element were used. An element size of approximately $4 \text{ mm} \times 4 \text{ mm}$ was used. The impacting mass was modelled as a rigid body using brick elements. The contact between the impacting mass and the specimens was modeled using automatic nodes to surface algorithm with a friction coefficient of 1.0 to avoid unrealistic lateral movement of the upper end of the specimen. Isotropic elasticity, the von Mises yield criterion, the associated flow rule and isotropic hardening, i.e. *MAT_103 in LS-DYNA™, were used for the extrusion. The stress/strain curves of the material obtained through tensile testing were utilized and represented in a parametric form as defined in Equation (1.29) to gather the input parameters for the material model.

$$
\sigma_{eff} = \sigma_o + \sum_{i=1}^{2} Q_i [1 - \exp (-C_i \varepsilon_{eff}^p)]
$$

(1.29)

Figure 2.61 Collapse modes for axial crushing of AA6060-T6 extrusions. (a) $v_o = 13 \text{ m/s}$, $L = 1120 \text{ mm}$, and $t = 2.5 \text{ mm}$. (b) $v_o = 13 \text{ m/s}$, $L = 1520 \text{ mm}$, and $t = 2.0 \text{ mm}$. [62]

A good description of the progressive buckling pattern was found as shown in Figure 2.61(a); and a relatively accurate description of the transition from progressive folding to global bending mode is presented in Figure 2.61(b). A relative good agreement found between the numerical simulations and the experimental results was the collapse mode. The FE simulation was found to be able to capture the progressive
folding and the transitions modes. However, the direct global bending mode that was observed in the experimental impact tests for specimens with 4.5 mm wall thickness at 13 m/s was not found in the numerical simulations.

Marsolek and Reimerdes [63] simulated the concertina and diamond folding process of circular aluminium and steel tubes using the explicit FE code LS-DYNA3D. A load introduction device as shown in Figure 2.62 was used in both experimental and numerical crushing tests to initiate the non-axisymmetric folding patterns. The cylindrical shells were idealised with linear 4-node Belytschko-Tsay shell element with reduced integration, which are commonly used in crash simulations. This element type has a lumped mass matrix as required by the explicit calculation scheme and is suitable for the large deformations which occur in the folding process. Five integration points were used in the thickness direction. To capture the details of the folding process, an element size of $1.5 \, \text{mm} \times 1.5 \, \text{mm}$ was chosen for the tubes. For the nodal positions, random imperfections of 2% of the shell wall thickness were utilized. The material behaviour of steel and aluminium was modelled with an elastic-plastic material model using a piecewise linear stress/strain curve obtained from experimental tensile tests. For the dynamic load case the strain-rate dependent material behaviour of steel was taken into account using the Cowper-Symonds model ($D = 3000 \, \text{s}^{-1}, \ p = 4$), the strain-rate dependency for the given aluminium was not taken into account. The load introduction device and the boundary conditions were idealised with rigid walls. Contact algorithms were activated to simulate contact between the load introduction and the shell specimens as well as self-contact of the shell with a friction coefficient of 0.3. Good agreement was found between the numerical simulations and the experiments for the axisymmetric (shown in Figure 2.63) and the non-axisymmetric (shown in Figure 2.64) progressive folding processes of circular tubes.
Figure 2.62 Load introduction device for initiation of non-axisymmetric folding patterns in the circular tubes [63].

Table 2.62 Load introduction device for circumferential wave-number 3

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<tr>
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Figure 2.63 Axisymmetric folding pattern in experimental test and FE simulation [63].

Table 2.63 Axisymmetric folding pattern in experimental test and FE simulation

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<td>δₖ = 54mm</td>
<td>δₖ = 89mm</td>
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Figure 2.64 Non-axisymmetric folding pattern with induced circumferential wave number 3 in experimental test and FE simulation [63].
Arnold and Altenhof [64] experimentally and numerically investigated the energy absorption abilities of square aluminium alloy extrusions with or without the presence of circular hole discontinuities under a quasi-static axial loading condition. Circular holes having diameters of 7.1 mm or 14.2 mm were allocated at the two opposite walls of the extrusions. The extrusion materials considered were AA6061-T4, AA6061-T6, and AA6063-T5, which differ greatly in yield strength and strain hardening properties as shown in Figure 2.65. Due to the relatively large wall thickness of the extrusions considered for their research, two separate models were developed for each specimen using solid hexahedral elements and shell elements to investigate the suitability of element types. Only one quarter of the extrusion was modeled due to symmetry observed in experimental testing. The discretization of the extrusion was carried out using the parametric mesh generation software TrueGrid™. In order to accurately capture the stress distribution resulting from stress concentration due to the presence of discontinuity, the mesh density was finer in the region of the structure surrounding the circular hole discontinuity as shown in Figure 2.66. Four solid elements through the thickness of the tube were utilized. The solid elements used to model the extrusions were selectively reduced hexahedral solid element (solid element formulation #2 in LS-DYNA™), which was selected for elimination of zero energy modes. Belytschko-Tsay shell elements employing a rigid material model were used for the movable platen. Contact between the rigid plate and the specimen was modelled using a surface-to-surface contact algorithm. Contact between the walls of the extrusion was modelled using a single-surface contact algorithm. The axial crushing process of the specimens was modelled by prescribing a constant velocity of 2 m/s to the rigid plate in the axial direction of the tube (the negative Z-direction in Figure 2.66).

Material model #105 in LS-DYNA™ was used to model the extrusion tube materials. This material model allows the direct input of the true stress versus true plastic strain data in the form of a piecewise linear curve. During the simulation, LS-DYNA™ performs a curve fit of the data and determines the strain hardening properties. This material model also allows the implementation of failure mechanism. An iterative calibration process was capable to determine the numerical failure parameters $D_c$ (critical damage value) and $S$ (damage material constant).
Figure 2.65  The engineering stress versus engineering strain curved obtained from tensile test specimens extracted from the AA6061-T4, AA6061-T6, and AA6063-T5 thin walled ($t = 2.38$ mm) and thick walled ($t = 3.15$ mm) extrusions [64].

Figure 2.66  Discretization of specimens with solid elements (discretization of specimens with shell elements was similar). The inset shows a detail of the discretization of the circular hole discontinuity region [64].
Experimentally and numerically observed load/displacement profiles for axial crushing of AA6061-T4 extrusions with a circular hole discontinuity of 14.2 mm are presented in Figure 2.67 and the corresponding crushing process is shown in Figure 2.68. It was observed that the FE model constructed using solid elements over predicted the experimentally determined peak buckling load and crippling point loads but predicted fairly closely the folding loads and folding process. The numerical load/displacement curve resulting from the simulation using shell elements, however, more closely predicted the peak buckling load. Numerical simulation of axial crushing of AA6061-T6 and AA6063-T5 extrusions were completed using shell elements only. Experimentally and numerically observed load/displacement profiles for axial crushing of AA6061-T6 extrusions with a circular hole discontinuity of 14.2 mm are presented in Figure 2.69 and the corresponding crushing process is shown in Figure 2.70. A good correlation was observed between the results of FE simulations and the results of quasi-static crush testing of extrusion absorber structures. It was found that material model 105 in LS-DYNA™, which incorporates non-linear plasticity and employs damage mechanics theory, successfully predicted the cracking and complex splitting collapse modes observed in experimental testing of the AA6061-T6 and AA6063-T5 specimens.

Figure 2.67  Experimentally and numerically observed load/displacement profiles for axial crushing of AA6061-T4 extrusions with a circular hole discontinuity of 14.2 mm [64].
Figure 2.68 Experimentally and numerically observed crushing process for AA6061-T4 extrusions with a circular hole discontinuity of 14.2 mm [64].
Figure 2.69 Experimentally and numerically observed load/displacement profiles for axial crushing of AA6061-T6 extrusions with a circular hole discontinuity of 14.2 mm [64].

Figure 2.70 Experimentally and numerically observed crushing process for AA6061-T6 extrusions with a circular hole discontinuity of 14.2 mm [64].
2.5.2 Eulerian finite element formulation

The use of an Eulerian element in numerical simulations has historically been associated with fluid mechanics problems [65] and recently has seen more implementation in solid mechanics problems associated with high velocity impacts [66, 67, 68, 69] and, more relevant to the axial cutting deformation, metal cutting [70, 71, 72]. Benson [73] provided an overview of the applicability of the Eulerian element formulation for solid mechanics problems and indicated that structural problems involving changing topology may be better suited for this type of element. One of the most important aspects of the Eulerian element formulation is the capability of generating new free surfaces as a result of material transport [69, 70, 71, 72, 73].

In the Eulerian element formulation the material coordinates and spatial coordinates of the FE mesh are dissociated and the material moves through the FE mesh. In the explicit time integration scheme, during every cycle (time step) of the simulation a Lagrangian formulation is first used to determine material and mesh deformation, however, prior to the next cycle the spatial coordinates of the FE mesh are remapped to their original position in a process referred to as advection, and material transport to the remapped mesh occurs. While the FE mesh is remapped to its original position, the material coordinates are not and will move through the FE mesh. Therefore, an airmesh must surround the original material location of the extrusion material for evaluation of the deformed material state. At the start of the simulation, the airmesh contains no material and its only purpose is to accommodate deformed material. Care must be taken to model the airmesh large enough to account for any possible material deformation during the simulation, and yet to allow a fine enough mesh to appropriately predict deformation.

Benson and Okazawa [70] studied machining of AISI 4340 steel using the Eulerian finite element formulation and successfully simulated the formation of discontinuous chips. Additionally, Raczy et al. [71] and Akarca et al. [72] have successfully utilized Eulerian element formulations in the study of the continuous chip formation associated with machining of C11000 copper and 1100 aluminum respectively. These efforts illustrated that an Eulerian element formulation was capable of simulating...
large material deformations and predicting chip formation. In references [70, 71, 72] as the rigid cutter penetrates into the workpiece, material separation (of the workpiece) is based upon the location of the cutter within the workpiece and material transport is governed by the advection algorithm. As a result of the disassociation between the FE mesh and the material coordinates as well as the transport of material through the fixed FE mesh free surface formation is possible in penetration type problems without the use of any material damage or failure criteria [73]. Effectively, material separation in the Eulerian FE formulation is based upon kinematic topological changes in the model. Complexities associated with determining parameters for damage and/or material rupture modeling are eliminated when utilizing an Eulerian FE formulation. Disadvantages which may arise through use of an Eulerian FE formulation include larger CPU costs and a greater degree of mesh discretization. However this FE formulation is beneficial in dealing with the large plastic deformation processes and numerical instabilities associated with severe mesh distortion.

Majumder et al. [74] numerically studied the axial cutting deformation of circular AA6061-T6 aluminium alloy tubes with the presence of deflector utilizing an Eulerian element formulation. Due to the symmetry observed in the experimental quasi-static cutting process of the extrusions, only one quarter of the tubular specimen, one quarter of the deflector and one corresponding cutter blade were considered in the FE model. Moreover, only approximately 100 mm length of the tubular specimen was considered during modelling to further reduce the computational cost as the steady-state cutting process was observed in the experimental tests after a cutter displacement of approximately 60 mm. Eight-noded solid elements were utilized for the tubular extrusion, airmesh, straight/curved deflector, and cutter blade. For the extrusion and airmesh a single point quadrature Eulerian element was selected. As shown in Figure 2.71, the mesh density of the extrusion in the vicinity of the region of contact between the cutter and extrusion was finer than other regions in order to ensure an accurate approximation of the stress distribution and deformation near the contact region. The deflector and the cutter blade utilized a constant stress (single point integration) Lagrangian element formulation. Contact between the Eulerian extrusion and airmesh and the Lagrangian cutter blade and deflector was completed through
Eulerian/Lagrangian coupling by employing a single CONSTRAINED_LAGRANGE_IN_SOLID contact algorithm available in LS-DYNA™. A penalty based formulation for contact was employed and the coefficient of friction of 0.15 was specified for both static and dynamic conditions. A hydrodynamic material model (referred to as MAT_ELASTIC_PLASTIC_HYDRO within LS-DYNA™) was selected for the extrusion and airmesh. A rigid material definition was applied to the cutter blade and deflector. Full degrees of freedom were fixed for all nodes at the lower end of the extrusion and nodes lying in the symmetry planes of the tube were constrained to move only within the corresponding symmetry plane. The axial cutting process of the tubular specimens was modeled by prescribing a penetration of 80 mm in the axial direction in 11 ms, which is equivalent to an average axial cutting speed of 7 m/s.

Figure 2.71 Discretization of the AA6061-T6 tubular extrusions, the tube airmesh, the cutter blade, and the (straight) deflector [74].

Results from the numerical simulation showed that the Eulerian FE model predicted the cutting process very well. Important energy dissipation mechanisms associated with axial cutting deformation mode were captured, such as near cutter blade tip plastic deformation, circumferential membrane stretching of the extrusion, cut petalled sidewall bending outward, and chip formation. The predicted cutting force was in a good
agreement with the experimental tests for the cutting deformation with the presence of
the curved deflector. It was generally slightly over-predicted with a maximum over-
prediction of 20% throughout the cutting process. In the case of straight deflector, the FE
under-predicted the peak load occurring during the initial cut sidewall contact with the
deflector and over-predicted the cutting force by approximately 33% over the
displacement range of 35 mm to 60 mm. The numerical predicted steady-state cutting
force (after a displacement of approximately 65 mm) was within 5% of the experimental
observations.

2.5.3 Finite element model validation assessment method

As introduced in the previous sections, finite element analysis has been
extensively used by automotive companies. “Virtual prototyping” and “virtual testing” is
now being used in engineering development to describe numerical simulation for the
design, evaluation, and “testing” of new structures and even entire vehicles due to
increased competition in the automotive market. Furthermore, the safety aspects of the
product represent an important, sometimes dominant element of testing or validating
numerical simulations. Verification and validation are the primary means to assess
accuracy and reliability in computational simulations. Briefly, verification is the
assessment of the accuracy of the solution to a computational model by comparison with
known solutions. Validation is the assessment of the accuracy of a computational
simulation by comparison with experimental data. Verification is primarily a
mathematics issue while validation is primarily a physics issue.

Oberkampf and Trucano [75] proposed a validation metric, $V$, to assess how
accurately the computational results compare with the experimental data with quantified
error and uncertainty estimates. With the assumption of zero experimental measurement
error, the validation metric $V$ is given in Equation (1.30):

$$
V = 1 - \frac{1}{\delta_{\text{end}} - \delta_{\text{start}}} \int_{\delta_{\text{start}}}^{\delta_{\text{end}}} \tanh \left( \frac{N_{\text{result}}(\delta) - E_{\text{result}}(\delta)}{E_{\text{result}}(\delta)} \right) d\delta \quad (1.30)
$$
where $\delta_{start}$ and $\delta_{end}$ are the first and last location, respectively, in the domain of interest. The relative error is given as:

$$\text{Error} = \frac{1}{\delta_{end} - \delta_{start}} \int_{\delta_{start}}^{\delta_{end}} \left| \frac{N_{\text{result}}(\delta) - E_{\text{result}}(\delta)}{E_{\text{result}}(\delta)} \right| d\delta$$

(1.31)

The above validation metric has the following four advantages. First, it normalized the difference between the computational results and the experimental data by computing a relative error norm. Second, the absolute value of the relative error only permits the difference between the computational results and the experimental data to accumulate. Third, when the difference between the computational results and the experimental data is zero at all measurement locations, then the validation metric is unity. And fourth, when the summation of the relative error becomes large, the validation metric approaches zero. Figure 2.72 shows how the validation metric given in Equation (1.30) varies as a function of constant values of the relative error throughout the specified domain. If the summation of the relative error is 100% of the experimental measurement, the validation metric would yield a value of 23.9%.

Figure 2.72  Plot of validation metric, $V$, given in equation (7) as a function of constant values of relative error [75].
2.6 Strain rate insensitivity of AA6061-T6 aluminium alloy

AA6061-T6 aluminium alloy is an attractive material due to its superior mechanical properties such as a high strength-to-weight ratio, good corrosion resistance, excellent weldability and deformability and has been increasingly used in many applications where the structural components are subjected to dynamic loading. A significant amount of work has been carried out using a variety of experimental techniques by various authors [76, 77, 78, 79, 80] on the strain rate dependence of mechanical properties in AA6061 aluminium alloys. The flow stress properties of AA6061-T6 obtained by the abovementioned authors is summarized and plotted as a function of strain rate in Figure 2.73.

Figure 2.73 Flow stress properties of AA6061-T6 obtained by several authors as a function of strain rate.

It is observed from Figure 2.73 that little or no significant strain rate sensitivity exhibits at strain rates in the range $10^{-4} \text{ s}^{-1} - 10^3 \text{ s}^{-1}$, however, a significant positive strain rate sensitivity of flow stress can be observed at strain rates in excess of $10^3 \text{ s}^{-1}$. Similar
observations was also found by Jones [4] and Maiden and Green [81]. Moreover, considering the Cowper-Symonds constitutive equation for AA6061-T6 material:

\[
\frac{\sigma_e}{\sigma_0} = 1 + \left(\frac{\dot{\varepsilon}_e}{D}\right)^{1/q} \tag{1.32}
\]

Values for \(D\) and \(q\) are 1288000 s\(^{-1}\) and 4, respectively [4]. The high value of \(D\) again indicates a low degree of rate sensitivity for this aluminum alloy.
3 FOCUS OF RESEARCH

The reviewed literature indicated that the axial cutting mode is an efficient energy absorption mode with exceptional load/displacement characteristics compared to other deformation modes presented in the literature review section of this dissertation. No initial peak cutting force is necessary to generate the cutting deformation mode within the tubular specimens. An almost constant axial cutting force can be achieved after the transient cutting process. This cutting deformation mode demonstrated high crush force efficiency and long stroke efficiency. Factors that influence the axial cutting deformation mode include extrusion cross sectional geometry, tube material properties, extrusion geometrical parameters, cutter geometries, deflector geometries, and loading conditions. The past investigations [6, 7, 82] have shown that the circular cross sectional extrusions have a favourable load/displacement response and higher crush force efficiency compared to the square cross sectional extrusions when subjected to the axial cutting deformation mode. Moreover, the AA6061-T6 extrusions underwent the axial cutting deformation mode exhibited better energy absorption capability than the AA6061-T4 specimens which experienced the same deformation mode as the T6 temper material [7]. Additionally, AA6061-T6 is a strain rate insensitive material as discussed in section 2.6, which is favourable when subjected to a dynamic loading condition. Thus, the present research will focus on the study of this novel cutting deformation mode on circular AA6061-T6 aluminium alloy extrusions. All factors that influence the axial cutting deformation mode will be discussed and the following investigations will be detailed in this dissertation to explore the load versus displacement and energy absorption characteristics of the circular AA6061-T6 extrusions as potential energy absorbers:

1. Quasi-static axial cutting of circular AA6061-T6 extrusions with or without the presence of deflector will be studied. Two different conical deflectors, namely straight and curved, will be utilized to flare the cut petalled side walls and reduce the spatial requirement of the cutting system.

2. Quasi-static axial cutting of circular AA6061-T6 extrusions by a single cutter only having different multiple cutting blades will be completed.
3. Dynamic and quasi-static axial cutting of circular AA6061-T6 extrusions by a cutter/deflector assembly will be detailed. Two slightly different geometries of cutters and deflectors will be considered.

4. Dynamic and quasi-static axial cutting of circular AA6061-T6 extrusions with different extrusion diameters and wall thicknesses by a cutter/deflector assembly will be studied.

5. Dynamic and quasi-static axial crushing of circular AA6061-T6 extrusion having same geometrical parameters as those used for the axial cutting deformation mode will be completed to compare the progressive folding and global bending deformations modes with the axial cutting deformation mode.

6. Dynamic and quasi-static dual-stage axial cutting of circular AA6061-T6 extrusions, where two cutters are placed in series, will be completed as a potential adaptive energy absorption system.

7. Controlling of the load/displacement response of extrusions under both dynamic and quasi-static loading conditions through varying instantaneous extrusion wall thickness in the axial direction with/without the presence of deflector will be presented.

8. Finite element models of the axial cutting and crushing processes, employing an Eulerian element formulation and Lagrangian element formulation, respectively, will be developed to predict the cutting/crushing behaviour and compare to the experimental results. Different tube geometries and loading conditions will be considered in the finite element models. The finite element models will finally be validated using the experimental data employing validation assessment techniques discussed in section 2.5.3.

9. A theoretical study of the steady-state cutting of circular extrusions by a cutter with multiple blades with/without the presence of deflector will be developed to predict the cutting resistance force. Parametric study of extrusion wall thickness, tube diameter, cutter blade tip width, and cutter blade quantities will be conducted using the proposed theoretical model and compared to the experimental results and predictions for the wedge cutting a plain plate process from other analytical models developed by other researchers.
4 EXPERIMENTAL TESTING METHOD

The experimental testing considered in this research includes the axial crushing and cutting of circular AA6061-T6 aluminium alloy extrusions under both dynamic and quasi-static loading conditions. Test specimens were organized into 111 groups with regards to the extrusion geometries and testing configurations. The identification system for each specimen within each group follows the naming convention $R\alpha-D\beta-t\gamma-\eta-nT\xi-\psi-\zeta$.

Where ‘$R\alpha$’ indicates the tube length of the round extrusion (for extrusions with reduced wall thickness, the length of the extrusion is the length of the reduced section).

‘$D\beta$’ describes the original outer diameter of extrusion.

‘$t\gamma$’ represents the extrusion wall thickness.

‘$\eta$’ represents the version of cutter(s) used (either ‘RevI’ or ‘RevII’ or a combination of them). For the crushing test of the extrusion, this indicator is skipped.

‘$nT\xi$’ indicates the number of cutter blades as well as the blade tip width of the cutter (for example, ‘4T1.0’ indicates the cutter has 4 blades and the blade tip width is 1.0 mm). If crush test was conducted on the extrusion, progressive folding (‘PF’) or global bending (‘GB’) or a combination of both these modes (‘PG’) is utilized.

‘$\psi$’ indicates whether a deflector is presented in the cutting process (‘ND’ for no deflector, ‘SD’ for straight deflector, and ‘CD’ for curved deflector). For the crushing test of the extrusion, this indicator is skipped.

‘$\zeta$’ represents the cutting/crushing test loading condition (‘Dyn’ for dynamic and ‘QS’ for quasi-static).

Material properties of AA6061-T6 were obtained from the tensile testing of specimens extracted from the extrusion stock material completed by Arnold and
Altenhof [48]. An overview of the tensile testing procedure is presented in this section as well as the optical microscopy observation procedure of the AA6061-T6 tensile specimens.

4.1 Tensile testing of aluminium alloy extrusion

4.1.1 Overview of tensile testing procedure

Tensile tests were performed to acquire material properties of the commercially obtained AA6061-T6 aluminium alloy extrusions. Eight tensile specimens were extracted from the sidewalls of the square extrusions with a wall thickness of 3.15 mm in the axial direction of extrusion as shown in Figure 4.1. Although the geometry of these extrusions are not the same as the extrusions considered in the experimental part of present research, through a comparison of the engineering stress/strain response of same material obtained by other researchers [83] good agreement exists for the mechanical material behaviour of the AA6061-T6 extrusions.

Figure 4.1 Extraction of tensile specimen from square AA6061-T6 extrusions [48].
The tensile testing was completed in accordance to ASTM standard E8M [84] on an INSTRON tensile testing machine equipped with a 100 kN load cell. The elongation of the specimen was measured using an extensometer with a gauge length of 25.4 mm. The extensometer was fastened to the specimen in the centre region of the gauge using elastic bands. Figure 4.2 illustrates the arrangement of extensometer, tensile specimen and wedge grips of the testing machine. Data from the load cell and extensometer was acquired using a computer controlled data acquisition system. Load and extension measurements were recorded at a sampling rate of 5 Hz. The tests were conducted at a constant crosshead speed of 5 mm/min at room temperature.

![Figure 4.2 Tensile testing arrangement for the test specimen, the extensometer, and the wedge grips [48].](image)

4.1.2 **Optical microscopy observation procedure of the tensile specimen**

In order to investigate the commercially obtained AA6061-T6 grain structure, optical examination of the tensile specimen was completed prior to and after tensile testing. Optical specimens were prepared by firstly polishing the tensile specimens followed by etching for duration of 5 to 10 seconds. The composition of the etchant included 25 ml of methanol (CH₃OH), 25 ml of nitric acid (HNO₃), 25 ml of hydrochloric acid (HCL) and one drop of hydrofluoric acid (HF).
4.2 Axial crush tests

Dynamic and quasi-static axial crushing tests were performed to evaluate the energy absorption and load/displacement responses of the specimens which underwent progressive folding and global bending deformation modes for future comparison with the same geometry extrusions yet underwent the novel cutting deformation proposed in this dissertation. In this section, specimen preparation and the dynamic and quasi-static crush testing procedures will be presented.

4.2.1 Specimen preparation for axial crushing tests

The extrusions considered in this research are commercially available AA6061-T6 extrusions with a stock length of 6 mm from the supplier. The testing specimens were cut down from the stock extrusion to the appropriate lengths, making sure that both end faces of the cut extrusion were perpendicular to the axial direction of the specimen.

The specimens considered for the axial crushing tests were circular AA6061-T6 aluminium alloy extrusions with a tube length \( L \) of 200 mm or 300 mm, a nominal external diameter \( D_o \) of 50.8 mm, and wall thicknesses \( t \) of 3.175 mm or 1.587 mm as shown in Figure 4.3. In order to accommodate the impact capacity of the droptower testing machine which will be discussed in section 4.2.2.1, specimens with reduced wall thicknesses \( Y \) of 1.0 mm, 1.2 mm, 1.25 mm, or 1.5 mm spanning a length \( L_{reduced} \) of 150 mm or 250 mm were selected as illustrated in Figure 4.4. The extrusions considered for reduced wall thickness had a nominal external diameter \( D_o \) of 44.45 mm, 50.8 mm, or 63.5 mm and an original wall thickness \( t \) of 3.175 mm. Material removal of the extrusions was completed using a CNC lathe machine as illustrated in Figure 4.5. A plastic insert was firstly inserted into the extrusion to ensure axial alignment of both ends of the specimen and avoid undesired deformation during the machining process. Additionally, this process was completed in an attempt to ensure a constant wall thickness throughout the reduced region of the extrusion. The machining process was computer numerical controlled with minimal material removal in the final cut of the specimen.
Figure 4.3  Geometry of AA6061-T6 aluminium alloy extrusion specimens considered in present research. \( L \) is the length of the extrusion specimen, \( D_o \) is the nominal external diameter of the specimen and \( t \) is the wall thickness of the specimen.

Figure 4.4  Geometry of AA6061-T6 aluminium alloy extrusion specimens with reduced wall thickness. \( L \) is the length of the original extrusion specimen, \( L_{\text{reduced}} \) is the length of the reduced wall thickness section of the specimen, \( t \) is the original wall thickness of the specimen, and \( Y \) is the reduced wall thickness of the specimen.

Figure 4.5  Material removal of circular AA6061-T6 extrusions. From left to right: plastic insert, CNC control panel, and lathe machine.
Test specimens were organized into twenty-four groups and three specimens were tested in each group if not indicated otherwise. A summary of grouping and extrusion geometry information for specimens considered for quasi-static and dynamic axial crushing tests are presented in Table 4.1 and Table 4.2, respectively.

Table 4.1 Grouping and extrusion geometry information for specimens considered for quasi-static axial crushing tests.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>$L_{reduced}$ (mm)</th>
<th>$t$ (mm)</th>
<th>$Y$ (mm)</th>
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<tbody>
<tr>
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<td>3.175</td>
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<td>300</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
<tr>
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<td>R200_D50.8_t1.587_PF_QS</td>
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<td>-</td>
<td>1.587</td>
<td>-</td>
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<tr>
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<td>300</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
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<tr>
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<td>150</td>
<td>3.175</td>
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<td>150</td>
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<td>1.2</td>
</tr>
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<td>250</td>
<td>3.175</td>
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</tr>
<tr>
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<td>250</td>
<td>3.175</td>
<td>1.25</td>
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<td>3.175</td>
<td>1.5</td>
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<tr>
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<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
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<td>250</td>
<td>3.175</td>
<td>1.25</td>
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<td>1.5</td>
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<td>3.175</td>
<td>1.0</td>
</tr>
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<td>3.175</td>
<td>1.25</td>
</tr>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
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Table 4.2 Grouping and extrusion geometry information for specimens considered for dynamic axial crushing tests.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>$L_{reduced}$ (mm)</th>
<th>$t$ (mm)</th>
<th>$Y$ (mm)</th>
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<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
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<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>18</td>
<td>R250_D44.45_t1.5_PF_Dyn</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
<tr>
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<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
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<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
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<td>50.8</td>
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<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
<tr>
<td>22</td>
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<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
<td>23</td>
<td>R250_D63.5_t1.25_PF_Dyn</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>24</td>
<td>R250_D63.5_t1.5_PF_Dyn</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
</tbody>
</table>

4.2.2 Crush test methodology

4.2.2.1 Dynamic crush test methodology

Dynamic axial crushing tests of the AA6061-T6 extrusions were performed using a custom built droptower system illustrated in Figure 4.6. A schematic diagram of the dynamic crushing test setup is shown in Figure 4.7(a). The system consisted of a pneumatic accelerator, a dropping entity with a mass of 53.7 kg, two guide posts which the drop mass translated along, and a support device with a three-jaw chuck to constrain the experimental apparatus to properly hold the test specimen. Held by the three-jaw chuck was a flat, round, hardened AISI 4140 steel disk which supported a dynamic piezoelectric load-cell (PCB model # 200C20 with a capacity of 89 kN) and a hardened AISI 4140 cup which supported the extrusion. During testing, an extrusion rested within the cup which had an inside diameter approximately 0.5 mm greater than the outer diameter of the extrusions. No mechanical means of fastening the extrusion to the support cup were employed. A 25.4 mm thick AA6061-T6 plate was fixed to the
dropping entity and acted as the impacting surface. A desktop computer with custom-developed software was used to control the height of the impact mass.

Displacement of the impacting surface of the AA6061-T6 thick plate during the cutting process was measured using a micro-epsilon non-contact laser displacement transducer with a range of 200 mm or an Acuity high accuracy non-contact laser displacement transducer with a range of 300 mm, depending on the displacement range of the impact tests. The model number of the 200 mm laser displacement transducer employed was optoNCDT 1607-200. The model number of the 300 mm laser displacement transducer employed was AR700-12.

Figure 4.6 The droptower system used for dynamic crushing/cutting test under consideration.
Figure 4.7  Schematic diagrams for axial crushing tests. (a) Dynamic test setup and (b) Quasi-static test setup.

Analog voltage output from the laser displacement transducer was measured using a National Instruments NI 9215 4 channel, 16 bit, analog input module which was incorporated into a National Instruments CompactDAQ data acquisition system. The NI 9215 had a capacity of simultaneously measuring at 100 kHz/channel. Output from the piezoelectric load cells was measured using a National Instruments NI 9233 module which incorporated integrated electronic piezoelectric (IEPE) signal conditioning. The NI 9233 had a capacity of simultaneously measuring at 50 kHz/channel. A laptop computer equipped with National Instruments LabVIEW 2010 data acquisition software was used to record the measurements of the laser displacement transducer and the two load cells through the NI 9215 and NI 9233 modules. A consistent data sampling rate of 50 kHz was used for all impact tests. All testing was completed at room temperature.

For the impact testing of the reduced wall thickness extrusions with varied extrusion diameters, a Photron SA4 high speed camera was also used for acquiring visual observations during the impact events. A frame rate of 5000 frames/s and shutter speed of 1/7000 s was utilized. Images obtained had a resolution of 768 x 1008 pixels².
Synchronization between the transducer data acquisition and the high speed camera triggering system was completed using the NI9401 high speed digital input/output module which was also incorporated into the CompactDAQ system. Within the custom developed LabVIEW 2010 software, timing for appropriate triggering, based upon measurements from the laser displacement transducer, was applied for transducer data acquisition and digital signal output to the SA4 high speed camera for triggering of this device.

Prior to impact testing, the dropping entity was raised to the maximum height of approximately 1514 mm supported by the droptower system and the pneumatic accelerator was pressurized to approximately 649.5 kPa. The combination of the pneumatic pressure and dropping height resulted in an approximate impact velocity of approximately 7.0-9.0 m/s. Just prior to impact, the data acquisition system commenced sampling of the signals from the laser displacement transducer and the piezoelectric load cell. Additionally, triggering of the SA4 high speed camera also occurred based upon a trigger set point specified from the laser displacement transducer if it was used. Data acquisition was terminated following completion of the impact crush test.

4.2.2.2 Quasi-static crush test methodology

Quasi-static axial crushing of specimens was performed using a hydraulic Tinius-Olsen compression testing machine as shown in Figure 4.8. A schematic diagram of the quasi-static crushing test setup is shown in Figure 4.7(b). The specimen was placed with its extrusion direction parallel to the direction of cutting at the centre of the fixture of the testing machine. Depending upon the magnitude of the maximum crushing forces, PCB Strain Gauge load cell (PCB model # 1204-02B with a capacity of 89 kN or PCB model # 1204-03B with a capacity of 222 kN) or load cell embedded in the Tinius-Olsen testing machine (with a capacity of 150 kN) was used to determine the axial crush load. The translating plate displacement was measured using a linear voltage differential transformer (LVDT) with a range of 150 mm.

The laptop computer equipped with National Instruments LabVIEW 2010 data acquisition software and National Instruments CompactDAQ data acquisition hardware systems using the voltage measurement module(s) NI 9215 and/or NI 9237 were used to
record the measurements of the displacement and crushing load. A data sampling rate of 60 Hz was used for all quasi-static tests. The specimens were crushed at a translating plate speed of approximately 2.2 mm/s at room temperature, which was considered acceptable to evaluate the deformation behaviour as quasi-static [4]. It is generally accepted and noted in reference [4] that dynamic loads applied at velocities on the order of 10 m/s or lower may be considered quasi-static.

![Figure 4.8 Tinius-Olsen compression testing machine used for quasi-static axial crushing and cutting tests.](image)

**Figure 4.8** Tinius-Olsen compression testing machine used for quasi-static axial crushing and cutting tests.

### 4.3 Axial cutting tests

Dynamic and quasi-static axial cutting tests were performed to evaluate load/displacement responses and energy absorption characteristics of circular AA6061-T6 specimens as potential novel energy absorbers. Specially designed cutters and/or deflectors were used to generate the axial cutting deformation mode within the extrusions. In this section, specimen preparation for axial cutting tests, design and manufacturing process of the cutting tool and deflector, specimen grouping information, and the dynamic and quasi-static cutting testing procedures will be discussed.
4.3.1 Specimens preparation for axial cutting tests

The aluminium alloy extrusions considered in the axial cutting tests were circular AA6061-T6 extrusions of lengths \( (L) \) 200 mm and 300 mm. The original wall thicknesses \( (t) \) of the extrusions selected were 3.175 mm and 1.587 mm (Figure 4.3). The reduced wall thicknesses \( (Y) \) of 1.0 mm, 1.2 mm, 1.25 mm and 1.5 mm and different extrusion external diameters \( (D_o) \) of 44.45 mm, 50.8 mm, and 63.5 mm (Figure 4.4) were selected to investigate the influence of wall thickness and extrusion diameter on the load/displacement responses of the extrusions. Moreover, variable instantaneous wall thicknesses of the extrusion in the axial direction as shown in Figure 4.9 and Figure 4.10 were considered for possible controlling of the load/displacement responses of extrusions which underwent the axial cutting deformation mode. Both the reduced and variable wall thicknesses of the extrusions were completed on a CNC lathe machine from extrusions of original wall thickness of 3.175 mm with minimal material removal in the final cut of the specimen detailed in section 4.2.1.

Figure 4.9 Extrusion geometries considered for dynamic and quasi-static cutting testing for controlling of the load/displacement response of the extrusion (all dimensions in mm).
4.3.2 Cutting tool design and manufacturing

In an effort to generate a cutting mode of deformation within the tubular specimens, two geometries of cutters (referred to as RevI and RevII hereafter) as illustrated in Figure 4.11 and Figure 4.12, respectively, were designed. Both reversion of cutters had an outer outside diameter of 101.6 mm and a block thickness of 20 mm. The RevI cutter had four tapered blades with a nominal blade shoulder width ($2B$) of 3 mm, a nominal blade tip width ($T$) of 1.0 mm, and a nominal blade length ($w$) of 7 mm.
The RevII cutter had 3, 4, 5, or 6 tapered cutting blades with a nominal blade shoulder width ($2B$) of 3 mm, a nominal blade tip width ($T$) of 1.0 mm on one side of the cutter and 0.75 mm on the other side, and a nominal blade length ($w$) of 26.1 mm. The cutting blades were designed with widths that would initiate stresses in a tubular member that should exceed the ultimate stresses of the AA6061-T6 aluminium alloy without deformation or failure of the cutting blades.

The cutters were machined on a computer numeric controlled (CNC) machining centre from AISI 4140 round bar stock followed by a two-stage heat treatment. In the first stage, the cutter was heated to 843°C and held at this temperature for one hour to ensure the completeness of the austenitic transformation. The second stage involved oil quenching to room temperature. Oil quenching provided a fast cooling rate to produce a martensitic structure. After hardening, tempering was completed at a temperature of 225°C for one hour to reduce residual stresses induced during quenching. The cutters were then cleaned using a sand blasting machine for removal of any film from the heat treatment process. The hardness of the cutters after heat treatment was determined to be no less than approximately HRC 53 for both versions of cutters.

**Figure 4.11** Geometries of RevI cutter under consideration (all dimensions in mm), where $w = 7$ mm, $2B = 3$ mm, and $T = 1.0$ mm.
Figure 4.12 Geometries of RevII cutter under consideration (all dimensions in mm), where $w = 26.1$ mm, $2B = 3$ mm, and $T = 1.0$ mm or 0.75 mm.

Although the 6-bladed cutter was manufactured, trial tests using the 6-bladed cutter showed that the cut petalled sidewalls curled significantly and contacted the load cell placed on the translating plate of the test machine before completion of the tests. Thus, only the 3-, 4-, 5-bladed cutters were selected for the cutting tests.

4.3.3 Deflector design and manufacturing

Deflectors were designed and combined with the cutter(s) to simplify the cutting test apparatus by eliminating the need for an additional structural member to push the cutter into the extrusions, resulting in a significant decrease in spatial requirement. The deflectors were designed to be able to fasten to the hub of the cutter and control the motion of the cut petalled sidewalls of the extrusion. Two con-shaped deflectors with a straight and a curved surface profile were considered as shown in Figure 4.13(a) and (b). Moreover, in order to accommodate the small tube diameter, another curved surface profile deflector was designed. This version of curved deflector (as shown in
Figure 4.13(c)) was used in the axial cutting tests for the extrusions in groups 49 through 98, while the other version of curved deflector (as shown in Figure 4.13(b)) was selected for the axial cutting tests for the extrusions other than in groups 49 through 98 and where curved deflector was utilized. All deflectors had an outside diameter of 108 mm and a thickness of 50 mm. The straight deflector had a straight surface profile with an angle of 41.4° to the horizontal and both curved deflectors had curved surface profile with a curvature radius of 50.8 mm as detailed in Figure 4.13.

The deflectors were CNC machined from AISI 4140 round bar stock followed by the same two-stage heat treatment process as described in section 4.3.2 for heat treatment of the cutters.

![Geometry of the straight and curved deflectors](image)

**Figure 4.13** Geometry of the straight and curved deflectors (all dimensions are in mm).

### 4.3.4 Specimens grouping information for quasi-static cutting tests only

The specimens considered for quasi-static cutting test only were circular AA6061-T6 extrusions of lengths ($L$) 200 mm and 300 mm, a nominal external diameter ($D_o$) of 50.8 mm, and wall thicknesses ($t$) of 3.175 mm and 1.587 mm as illustrated in Figure 4.3.
Specimens grouping information for extrusions considered for quasi-static cutting tests only will be presented in this section.

### 4.3.4.1 Specimens grouping for quasi-static cutting tests without/with the presence of deflector

Specimens considered for this configuration had lengths \( L \) of 200 mm and 300 mm, a nominal external diameter \( D_o \) of 50.8 mm, and a wall thickness \( t \) of 3.175 mm. Different tube lengths were selected to investigate its influence on the load/displacement and energy absorption characteristics of the extrusions under the axial cutting deformation mode. The RevI cutter having four cutter blades and a blade tip width \( T \) of 1.0 mm was employed to initiate the axial cutting deformation mode within the specimens. A straight or curved deflector was used to flare the cut petalled sidewalls of the extrusion and reduce the spatial requirement of the energy absorption system. The influence of the deflector on the load/displacement responses of the extrusions is then studied and compared to the axial cutting tests without the presence of deflector.

Test specimens were organized into four groups and three specimens were tested in each group. A summary of grouping and extrusion geometry information are presented in Table 4.3.

Table 4.3 Grouping and extrusion geometry information for specimens considered for quasi-static cutting tests without/with the presence of deflector.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>( D_o ) (mm)</th>
<th>( L ) (mm)</th>
<th>( L_{\text{reduced}} ) (mm)</th>
<th>( t ) (mm)</th>
<th>( Y ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>R200_D50.8_t3.175_RevI_4T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
<tr>
<td>26</td>
<td>R300_D50.8_t3.175_RevI_4T1.0_ND_QS</td>
<td>50.8</td>
<td>300</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
<tr>
<td>27</td>
<td>R200_D50.8_t3.175_RevI_4T1.0_SD_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
<tr>
<td>28</td>
<td>R200_D50.8_t3.175_RevI_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
</tbody>
</table>
4.3.4.2 Specimens grouping for quasi-static cutting tests with different number of blades without the presence of deflector

Specimens considered for this configuration had a length \((L)\) of 200 mm, a nominal external diameter \((D_o)\) of 50.8 mm, and wall thicknesses \((t)\) of 1.587 mm and 3.175 mm. The RevII cutters having multiple cutter blades, namely 3, 4, 5, or 6 blades, and a blade tip width \((T)\) of 1.0 mm were used to investigate the influence of cutter blade quantities \((n)\) on the load/displacement responses of extrusions.

Test specimens were organized into eight groups and three specimens were tested in each group. The detailed specimens grouping and their geometry information are presented in Table 4.4.

Table 4.4 Grouping and extrusion geometry information for specimens considered for axial cutting tests with different number of cutter blades without the presence of deflector under quasi-static loading conditions only.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>(D_o) ((mm))</th>
<th>(L) ((mm))</th>
<th>(L_{\text{reduced}}) ((mm))</th>
<th>(t) ((mm))</th>
<th>(Y) ((mm))</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>R200_D50.8_t1.587_RevII_3T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>R200_D50.8_t1.587_RevII_4T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
<tr>
<td>31</td>
<td>R200_D50.8_t1.587_RevII_5T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
<tr>
<td>32</td>
<td>R200_D50.8_t1.587_RevII_6T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
<tr>
<td>33</td>
<td>R200_D50.8_t3.175_RevII_3T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
<tr>
<td>34</td>
<td>R200_D50.8_t3.175_RevII_4T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
<tr>
<td>35</td>
<td>R200_D50.8_t3.175_RevII_5T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
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<td>36</td>
<td>R200_D50.8_t3.175_RevII_6T1.0_ND_QS</td>
<td>50.8</td>
<td>200</td>
<td>-</td>
<td>3.175</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3.5 Specimens grouping information for dynamic and quasi-static cutting tests

The specimens considered for both dynamic and quasi-static cutting tests were circular AA6061-T6 extrusions of length \((L)\) 300 mm, a nominal external diameter \((D_o)\) of 50.8 mm, and a wall thickness \((t)\) of 1.587 mm as illustrated in Figure 4.3 as well as
extrusions with reduced wall thicknesses ($Y$) of 1.0 mm, 1.2 mm, 1.25 mm and 1.5 mm and different extrusion external diameters ($D_o$) of 44.45 mm, 50.8 mm, and 63.5 mm as shown in Figure 4.4. Moreover, variable instantaneous wall thicknesses of the extrusion in the axial direction as shown in Figure 4.9 and Figure 4.10 were considered for possible controlling of the load/displacement responses of extrusions under both loading conditions. Specimens grouping information for extrusions considered for both the dynamic and quasi-static cutting tests will be presented in this section.

### 4.3.5.1 Specimens grouping for dynamic and quasi-static cutting tests with different cutter geometries with the presence of deflector

Specimens considered for this configuration had length ($L$) of 300 mm, a nominal external diameter ($D_o$) of 50.8 mm, and a wall thickness ($t$) of 1.587 mm. The RevI and RevII cutters having four cutter blades yet different blade tip widths ($T$) of 1.0 mm and 0.75 mm and blade lengths ($w$) of 7 mm and 26.1 mm were employed to investigate the influence of cutter geometries. A straight or curved deflector was used to study the influence of different deflector surface profiles.

Test specimens were organized into twelve groups and two specimens were tested in each group except that three specimens were tested for groups 43 and 44. The detailed specimens grouping and their geometry information are presented in Table 4.5.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>$L_{reduced}$ (mm)</th>
<th>$t$ (mm)</th>
<th>$Y$ (mm)</th>
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</thead>
<tbody>
<tr>
<td>37</td>
<td>R300_D50.8_t1.587_RevI_4T1.0_CD_Dyn</td>
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<td>300</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
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<td>38</td>
<td>R300_D50.8_t1.587_RevI_4T1.0_SD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
<tr>
<td>39</td>
<td>R300_D50.8_t1.587_RevII_4T0.75_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
<tr>
<td>40</td>
<td>R300_D50.8_t1.587_RevII_4T0.75_SD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>-</td>
<td>1.587</td>
<td>-</td>
</tr>
</tbody>
</table>
### 4.3.5.2 Specimens grouping for dynamic and quasi-static cutting tests of extrusions with different outer diameters and tube wall thicknesses cut by a cutter with different number of blades with the presence of the curved deflector

Specimens considered for this configuration had a reduced wall thickness length \((L_{\text{reduced}})\) of 250 mm, nominal external diameters \((D_o)\) of 44.45 mm, 50.8 mm, and 63.5 mm, and reduced wall thicknesses \((Y)\) of 1.0 mm, 1.25 mm and 1.5 mm. The RevII cutters having multiple cutter blades, namely 3, 4, and 5, and a blade tip width \((T)\) of 1.0 mm and the curved deflector were used to generate the cutting deformation modes under both loading conditions and to comprehensively study the effects of cutter blade quantities \((n)\), tube wall thickness, and tube diameter.

Test specimens were organized into fifty groups and three specimens were tested in each group. The detailed specimens grouping and their geometry information are presented in Table 4.6, Table 4.7, and Table 4.8 for extrusions with outer diameter \((D_o)\) of 44.45 mm, 50.8 mm and 63.5 mm, respectively.
Table 4.6  Grouping and extrusion geometry information for specimens of $D_o = 44.45$ mm with different tube wall thickness considered for both axial dynamic and quasi-static cutting tests cut by a cutter with different number of cutter blades.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>$L_{reduced}$ (mm)</th>
<th>$t$ (mm)</th>
<th>$Y$ (mm)</th>
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</thead>
<tbody>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>52</td>
<td>R250_D44.45_t1.25_RevII_5T1.0_CD_Dyn</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
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<td>R250_D44.45_t1.5_RevII_3T1.0_CD_Dyn</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
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<td>R250_D44.45_t1.5_RevII_5T1.0_CD_Dyn</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
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<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
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<td>250</td>
<td>3.175</td>
<td>1.0</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>60</td>
<td>R250_D44.45_t1.25_RevII_4T1.0_CD_QS</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
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<td>62</td>
<td>R250_D44.45_t1.5_RevII_3T1.0_CD_QS</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
<tr>
<td>63</td>
<td>R250_D44.45_t1.5_RevII_4T1.0_CD_QS</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
<tr>
<td>64</td>
<td>R250_D44.45_t1.5_RevII_5T1.0_CD_QS</td>
<td>44.45</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Table 4.7  Grouping and extrusion geometry information for specimens of $D_o = 50.8$ mm with different tube wall thickness considered for both axial dynamic and quasi-static cutting tests cut by a cutter with different number of cutter blades.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>$L_{reduced}$ (mm)</th>
<th>$t$ (mm)</th>
<th>$Y$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>R250_D50.8_t1.0_RevII_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
<td>66</td>
<td>R250_D50.8_t1.0_RevII_5T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
<td>67</td>
<td>R250_D50.8_t1.25_RevII_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>68</td>
<td>R250_D50.8_t1.25_RevII_5T1.0_CD_Dyn</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>69</td>
<td>R250_D50.8_t1.5_RevII_3T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
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<td>70</td>
<td>R250_D50.8_t1.5_RevII_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
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<td>250</td>
<td>3.175</td>
<td>1.0</td>
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<td>R250_D50.8_t1.0_RevII_4T1.0_CD_QS</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
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<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
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<td>77</td>
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<td>50.8</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
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<td>250</td>
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<td>1.5</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Table 4.8 Grouping and extrusion geometry information for specimens of $D_o = 50.8$ mm with different tube wall thickness considered for both axial dynamic and quasi-static cutting tests cut by a cutter with different number of cutter blades.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>$L_{reduced}$ (mm)</th>
<th>$t$ (mm)</th>
<th>$Y$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>81</td>
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<td>250</td>
<td>3.175</td>
<td>1.0</td>
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<td>300</td>
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<td>3.175</td>
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<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
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<td>3.175</td>
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<td>250</td>
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<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
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<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
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<td>250</td>
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<td>1.5</td>
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<td>250</td>
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<td>3.175</td>
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<td>250</td>
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<tr>
<td>92</td>
<td>R250_D63.5_t1.0_RevII_5T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
<td>93</td>
<td>R250_D63.5_t1.25_RevII_3T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>94</td>
<td>R250_D63.5_t1.25_RevII_4T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>95</td>
<td>R250_D63.5_t1.25_RevII_5T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.25</td>
</tr>
<tr>
<td>96</td>
<td>R250_D63.5_t1.5_RevII_3T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
<tr>
<td>97</td>
<td>R250_D63.5_t1.5_RevII_4T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
<tr>
<td>98</td>
<td>R250_D63.5_t1.5_RevII_5T1.0_CD_QS</td>
<td>63.5</td>
<td>300</td>
<td>250</td>
<td>3.175</td>
<td>1.5</td>
</tr>
</tbody>
</table>
4.3.5.3 Specimens grouping for dual-stage cutting tests with the presence of the curved deflector

Specimens considered for this configuration had a reduced wall thickness length \((L_{\text{reduced}})\) of 150 mm, a nominal external diameter \((D_o)\) of 50.8 mm, and reduced wall thicknesses \((Y)\) of 1.0 mm and 1.2 mm. Two cutters (RevI and RevII) and the curved deflector were used to generate the dual stage cutting process.

Test specimens were organized into four groups and two specimens were tested in each group. The detailed specimens grouping and their geometry information are presented in Table 4.9.

Table 4.9 Grouping and extrusion geometry information for specimens considered for dual stage axial cutting tests under both dynamic and quasi-static loading conditions.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>(D_o) (mm)</th>
<th>(L) (mm)</th>
<th>(L_{\text{reduced}}) (mm)</th>
<th>(t) (mm)</th>
<th>(Y) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>R150_D50.8_t1.0_RevI&amp;RevII_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>150</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
<td>100</td>
<td>R150_D50.8_t1.2_RevI&amp;RevII_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>150</td>
<td>3.175</td>
<td>1.2</td>
</tr>
<tr>
<td>101</td>
<td>R150_D50.8_t1.0_RevI&amp;RevII_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>150</td>
<td>3.175</td>
<td>1.0</td>
</tr>
<tr>
<td>102</td>
<td>R150_D50.8_t1.2_RevI&amp;RevII_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>150</td>
<td>3.175</td>
<td>1.2</td>
</tr>
</tbody>
</table>

4.3.5.4 Specimens grouping for controlling load versus displacement responses of extrusions with and without the presence of deflector

In order to control the load versus displacement responses of circular AA6061-T6 extrusions, variation of instantaneous wall thickness along the axial direction of the extrusions were considered. Figure 4.9 shows the extrusion geometries used for both the dynamic and quasi-static axial cutting tests while Figure 4.10 illustrates the tube geometries used only for the quasi-static cutting tests. The \(x\)-coordinate indicates the location and direction of initial cutting (in the axial direction) for all extrusions. Both stepped and tapered variations of the wall thickness profile along the length of the extrusion were considered. Although the tube length had no significant influence on the energy absorption and load/displacement response of the extrusion which experienced
axial cutting deformation mode as reported by Cheng and Altenhof [6] and what will be shown in sections 6.2 and 6.3 of this dissertation, length and wall thickness of each reduced section were carefully selected to obtain the desired cutting deformation modes while to avoid undesired global bending and progressive folding collapse mode.

Test specimens were organized into nine groups and two specimens were tested in each group except that one test was tested for each configuration in group 111. The detailed specimens grouping and their geometry information are presented in Table 4.10.

Table 4.10  Grouping and extrusion geometry information for specimens considered for axial cutting tests under both dynamic and quasi-static loading conditions for controlling of the load/displacement response of the extrusion.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$D_o$ (mm)</th>
<th>$L$ (mm)</th>
<th>Tube Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>103</td>
<td>R300_D50.8_config(a)_RevI_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.9(a)</td>
</tr>
<tr>
<td>104</td>
<td>R300_D50.8_config(b)_RevI_4T1.0_CD_Dyn</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.9(b)</td>
</tr>
<tr>
<td>105</td>
<td>R300_D50.8_config(a)_RevI_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.9(a)</td>
</tr>
<tr>
<td>106</td>
<td>R300_D50.8_config(b)_RevI_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.9(b)</td>
</tr>
<tr>
<td>107</td>
<td>R300_D50.8_config(c)_RevI_4T1.0_ND_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.10(c)</td>
</tr>
<tr>
<td>108</td>
<td>R300_D50.8_config(d)_RevI_4T1.0_ND_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.10(d)</td>
</tr>
<tr>
<td>109</td>
<td>R300_D50.8_config(e)_RevI_4T1.0_ND_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.10(e)</td>
</tr>
<tr>
<td>110</td>
<td>R300_D50.8_config(f)_RevI_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.10(f)</td>
</tr>
<tr>
<td>111</td>
<td>R300_D50.8_config(g)_RevI_4T1.0_CD_QS</td>
<td>50.8</td>
<td>300</td>
<td>Figure 4.10(g)</td>
</tr>
</tbody>
</table>

4.3.6  Cutting test methodology

4.3.6.1  Dynamic cutting test methodology

Dynamic cutting tests of the extrusions were performed using the identical droptower system introduced in section 4.2.2.1. A schematic diagram of the dynamic cutting test setup is shown in Figure 4.14(a). The cutter(s) and the deflector were
fastened together using a standard ¼ inch fastener and manually placed at the top end of the specimen with careful alignment to ensure that the centreline of the cutter(s)/deflector assembly was along the axial centreline of the specimen. The dual-stage cutting process was accomplished by placing two cutters in series. The second cutter was rotated 45° relative to the first cutter so that four new cuts would be made into the cut petalled sidewalls of the extrusion. Prior to inserting the test specimen into the testing machine the extrusion and the cutter(s)/deflector assembly were placed onto a nearby loading frame. A hydraulic jack was used to push the cutter(s)/deflector assembly into the extrusion and pre-cut the extrusion approximately 1 mm in depth. This was completed to avoid any shifting or mis-alignment between the centerlines of the extrusion and the cutter/deflector assembly as a result of slight shaking of the droptower system during the dropping process.

Figure 4.14  Schematic diagrams for axial cutting test. (a) Dynamic test setup and (b) Quasi-static cutting test setup.
Two piezoelectric impact load-cells manufactured by PCB Piezotronics Inc. were incorporated into the experimental apparatus. The first impact load cell, referred to as the upper load cell, was fastened to the top end of the deflector to measure the impact force between the 25.4 mm thick impacting aluminium plate and the deflector. The capacity of this load cell (model # 200C50) is 222 kN. In order to measure the impact cutting force, a second dynamic piezoelectric load cell (model # 200C20) with a capacity of 89 kN was fastened between the flat supporting disc, constrained by the three-jaw chuck, and the steel support cup which the extrusion rested in. This load cell is referred to as the lower load cell. The average masses of the RevI and RevII cutters are 0.71 kg and 0.48 kg, respectively. Both deflectors had a mass of approximately 2.18 kg, and both load cells had a mass of approximately 0.43 kg.

After pre-cutting of the extrusion was completed the specimen assembly, consisting of the extrusion, cutter(s)/deflector assembly, and the upper load cell were placed into the steel support cup. No mechanical means of fastening the extrusion to the support cup were employed.

Identical testing apparatus, data acquisition hardware and software, and data sampling rate to what were used in the impact crushing tests were used to measure the displacement of the AA6061-T6 crush plate from the laser transducer and the cutting loads from the two piezoelectric load cells. An impact speed of approximately 7.0-9.0 m/s, consistent with the crushing tests, was implemented for all the impact cutting tests. All tests were completed at room temperature.

4.3.6.2 Quasi-static cutting test methodology

Quasi-static axial cutting tests of the circular AA6061-T6 extrusions were completed using the identical testing apparatus and data acquisition hardware and software, as to the quasi-static crushing tests detailed in section 4.2.2.2. A schematic diagram of the quasi-static cutting test setup with the presence of the deflector is shown in Figure 4.14(b). The specimen was placed at the centre of the testing machine’s platen with the extrusion direction parallel to the direction of cutting. For the cutting tests employing only the cutter, the deflector in Figure 4.14(b) was replaced by a round steel rod with a diameter of 25.4 mm in order to push the cutter into the extrusion specimens.
For the cutting tests with the presence of deflector, the cutter(s) and the deflector were fastened together, using a manual approach and placed at the top end of the specimen. The dual-stage cutting process was accomplished by placing two cutters in series. The second cutter was rotated 45° relative to the first cutter so that four new cuts would be made into the cut petalled sidewalls of the extrusion. Careful alignment of the centreline of the cutter or cutter(s)/deflector assembly and the axial centreline of the specimen was manually ensured. A data sampling rate of 60 Hz was used for all quasi-static tests. The specimens were cut at a translating plate speed of approximately 2.2 mm/s at room temperature.

4.3.7 Scanning Electron Microscope (SEM) observation methodology

Scanning Electron Microscope (SEM) observation for the extrusions with wall thicknesses of 3.175 mm and 1.587 mm after axial cutting tests was carried out to examine the microstructure of the deformed regions. Specimen surfaces were firstly cleaned using acetone and precise cut was made in order to image the specific region of the specimen. The cut specimens were then imaged by a JEOL JSM-5800LV scanning electron microscope (SEM).
5 PARAMETERS USED TO EVALUATE THE CRUSH/CUTTING CHARACTERISTICS OF THE EXTRUSIONS

Different crush performance parameters developed by a number of researchers are used to quantify the load/displacement and energy absorption characteristics of the extrusions. Magee and Thornton [85] used the peak buckling load and mean crush load to characterize the crush behaviour of axially loaded square tubes that collapsed in symmetric mode. Mahmood and Paluszny [86] developed the concept of the crush force efficiency to compare the performance of energy absorbers of different shapes, sizes and strength. Different crush/cutting parameters, including the total energy absorbed (TEA), peak crush/cutting load ($P_{\text{max}}$), mean crush/cutting force ($P_m$), crush force efficiency (CFE), and specific energy absorption (SEA), are described in this section as an assessment of crush/cutting behaviour of different specimens and will be used in subsequent sections to characterize the load/displacement and energy absorption performances of the testing specimens.

5.1 Total energy absorption

The total energy absorbed (TEA) by a specimen is determined experimentally as the work done by the crushing/cutting force and is calculated using Equation (5.1).

$$TEA = E_{\text{absorbed}} = \int P \, d\delta$$

(5.1)

where $P$ is the crushing/cutting force in the axial direction and $\delta$ is the crosshead displacement in the axial direction. This quantity is represented as the area under the axial force versus axial displacement curve. In order to calculate the energy absorbed based on the experimental data, a numerical integration scheme is employed. The scheme presented in Equation (5.2) is the rectangular rule which was utilized in this research to calculate the total energy absorbed. Other numerical integration techniques, such as trapezoidal or Simpson rules can also be implemented.

$$TEA = E_{\text{absorbed}} = \sum_{i=2}^{N-1} P_i \left( \frac{\delta_{i+1} - \delta_{i-1}}{2} \right)$$

(5.2)
5.2 Peak crush/cutting load

The peak crush/cutting load, \( P_{\text{max}} \), is the maximum load experienced by the structure in the axial direction observed throughout the crushing/cutting process.

5.3 Mean crush/cutting force

Based on the total energy absorption defined in Equation (5.2), the mean crush/cutting force, \( P_m \), is defined by dividing Equation (5.2) by the total crushing/cutting displacement, \( \delta \), in the axial direction as presented in Equation (5.3).

\[
P_m = \frac{TEA}{\delta} = \frac{\sum_{i=2}^{N-1} P_i \left( \frac{\delta_{i+1} - \delta_{i-1}}{2} \right)}{\delta}
\]  

(5.3)

5.4 Crush force efficiency

The crush force efficiency (CFE), which is defined as the ratio of the mean crush/cutting force to the peak crush/cutting load as presented in Equation (5.4). A value of unity represents the most desirable value of the CFE, corresponding to a constant load versus displacement profile.

\[
CFE = \frac{P_m}{P_{\text{max}}}
\]  

(5.4)

5.5 Specific energy absorption

The specific energy absorption (SEA) of a structure is the total energy absorbed (TEA) by a structure divided by its mass as defined in Equation (5.5), where, \( m \) is the mass of the absorber. This is a useful parameter that provides a method for comparing energy-absorbing structures with different masses.

\[
SEA = \frac{TEA}{m}
\]  

(5.5)
6 EXPERIMENTAL RESULTS AND DISCUSSION

Results of the experimental tests conducted in this research are presented in this chapter. An overview of the tensile tests results which were completed by Arnold and Altenhof [48] is given in section 6.1 to obtain the material properties of AA6061-T6 extrusion material. Section 6.2 details the dynamic and quasi-static crush testing results for the extrusions with different wall thicknesses and diameters. Section 6.3 presents the dynamic and quasi-static cutting tests results for different cutting configurations. Section 6.4 provides the Scanning Electron Microscope (SEM) observations for the cut specimens.

6.1 Tensile testing results

6.1.1 Material properties

The engineering stress versus engineering strain curve of one representative AA6061-T6 tensile specimen is illustrated in Figure 6.1. It can be seen that AA6061-T6 has a minimal level of strain hardening and an approximate mean strain to failure of 14% over the eight tensile specimens was observed. The material properties of the AA6061-T6 over the eight tensile specimens are summarized in Table 1.

Figure 6.1 The engineering stress versus the engineering strain curve of AA6061-T6 specimen obtained from tensile testing [48].
Table 6.1 Material properties of the AA6061-T6 extrusions from tensile testing [48].

<table>
<thead>
<tr>
<th>Properties</th>
<th>AA6061-T6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$ (GPa)</td>
<td>68.1</td>
</tr>
<tr>
<td>Yield stress, $\sigma_y$ (MPa)</td>
<td>277.5</td>
</tr>
<tr>
<td>Ultimate stress, $\sigma_u$ (MPa)</td>
<td>320.2</td>
</tr>
<tr>
<td>% elongation</td>
<td>14.1</td>
</tr>
</tbody>
</table>

6.1.2 Optical microscopy observations

As discussed in section 4.1.2, optical microscopy observations of the tensile specimens prior to and after tensile testing were completed. Figure 6.2 illustrates the microstructure of the AA6061-T6 extrusion material in its as-received condition. The average grain dimensions, over twenty measurements, in the axial ($y$-axis) and the transverse ($x$-axis) directions were found to be 72.86 $\mu$m and 61.43 $\mu$m, respectively, using the line intercept method. Although no x-ray diffraction or energy dispersive spectroscopy (EDS) analyses were completed the particles within the grains illustrated in Figure 6.2 are extrapolated to be compounds of Al$_7$Cu$_2$Fe and Al$_{12}$(FeMn)$_3$Si [87, 88]. Average grain dimensions obtained from the microstructure of the AA6061-T6 material after tensile testing were measured to be 109.76 $\mu$m in the axial direction and 59.83 $\mu$m in the transverse direction. Figure 6.2 illustrates the elongated grains in the axial ($y$-axis) direction of the tensile specimen. The location of the image presented in Figure 6.3 was along the centerline of the specimen, no further than 2 mm in the axial direction from the fracture location.
Figure 6.2 Microstructure of AA6061-T6 extrusion prior to tensile testing.

Figure 6.3 Microstructure of AA6061-T6 extrusion after tensile testing.
6.2 Crush testing results and discussion

The results are presented in the form of load/displacement profiles and collapse modes for each group of specimens. Although three experimental tests (if not indicated otherwise) were completed for each group the load/displacement observations for all the specimens within each group were fairly consistent if not indicated otherwise. For this reason and for greater clarity, only one representative specimen from each group was selected for illustration and discussion purpose. The load/displacement profiles of all the tests within each group that exhibited the same mode of deformation are presented in Appendix A to demonstrate the repeatability of the tests. A qualitative and quantitative examination of crush testing observations for each specimen group was completed through analysis of photographs and crush parameters.

6.2.1 Quasi-static crush test results for the specimens in groups 1 through 6

The axial compressive crush tests of circular AA6061-T6 tubes with a wall thickness \( t \) of 3.175 mm were performed for the five specimens in groups 1, which had a length of 200 mm, and five specimens in group 2, which had a length of 300 mm. The axial crushing tests of circular tubes with a wall thickness \( t \) of 1.587 mm and tube lengths of 200 mm and 300 mm were also performed for three specimens in groups 3 and 4, respectively. Moreover, axial crushing tests of circular AA6061-T6 extrusions with reduced wall thicknesses \( Y \) of 1.0 mm and 1.2 mm were completed for three specimens in groups 5 and 6, respectively, for future comparison to the same geometry extrusions which underwent the novel cutting deformation modes.

The observed load/displacement profiles of each specimen in group 1 are shown in Figure 6.4. It can be seen that the second, fourth, and fifth specimens in group 1 collapsed in progressive folding mode as predicted by reference [36]. The first specimen initially deformed in a similar manner, however, after approximately 28 mm a switch to global bending deformation occurred. The third specimen collapsed within a combination of progressive folding followed by a switch to global bending after a crosshead displacement of approximate 100 mm.
All specimens in group 1 illustrated an approximate peak crush load of 146 kN after approximately 8 mm crosshead displacement. For the majority of specimens in group 1, a variable crush force corresponding to the development of material folding was observed following the peak crush load. Specimens in group 1 had $L/D$ and $D/t$ ratios of 3.94 and 16 respectively, which were approximately equal to the critical $L/D$ value of 4.07 for a $D/t$ ratio of 16 as indicated in reference [36]. Experimental testing illustrated that specimens with geometries very similar to the critical geometrical dimensions from reference [36] may experience very unstable deformation during axial crush. Minor variations in specimen geometry and/or material characteristics could also contribute to the transition of progressive folding into a global bending mode of deformation.

All specimens in group 2 collapsed in the global bending mode and illustrated very similar load/displacement profiles. Specimens in groups 3 through 6 all demonstrated progressive folding deformation mode and consistent load/displacement responses within each group. The observed load/displacement responses of the representative specimens in groups 1, 2, 3 and 4 and the representative specimens in groups 5 and 6 are shown in Figure 6.6 and Figure 6.7, respectively.

For all the specimens in group 2, as the bending of the specimens progressed cracking occurred within the region of the kink near the mid-span of the extrusion. Global bending and cracking caused the force versus displacement profiles to have a large negative slope after the peak crush load. An approximate average peak crush load of 137 kN was observed for specimens in group 2. After the development of a mid-span kink, which occurred after approximately 40 mm crosshead displacement, the magnitude of the crush force was approximately 8 kN.

Photographs illustrating the progressive folding and global bending deformation modes for the representative specimens from group 1 and group 2 are presented in Figure 6.5(a)-(b) and Figure 6.5(c)-(d), respectively.

Specimens in groups 3 through 6, which collapsed in progressive folding deformation mode, exhibited similar collapse behaviour to the specimens in group 1 which collapsed in the same deformation mode. Specimens in groups 3 and 4 which had the same wall thickness of 1.587 mm but different tube lengths of 200 mm and 300 mm
illustrated consistent load/displacement profiles as shown in Figure 6.6. An approximate peak crush load of 70 kN was observed after approximately 3 mm crosshead displacement. For the specimens in groups 5 and 6, peak crush forces of 41 kN and 50 kN was observed, respectively, after approximately 3 mm crosshead displacement.

Figure 6.4 The load/displacement responses of the five circular AA6061-T6 specimens in group 1.

Figure 6.5 Photographs illustrate the progressive folding and global bending deformation modes for the representative specimens in group 1 and 2. (a) and (b) illustrate the progressive folding mode; and (c) and (d) show the global bending deformation mode.
Figure 6.6 The load/displacement responses of circular AA6061-T6 specimens for representative specimens in groups 1 through 4.

Figure 6.7 The load/displacement responses of circular AA6061-T6 specimens for representative specimens in groups 5 and 6.
6.2.2 Quasi-static crush test results for the specimens in groups 7 through 15

Specimens in groups 7, 10 and 13, groups 8, 11 and 14, and groups 9, 12 and 15 had reduced wall thickness of 1.0 mm, 1.25 mm, and 1.5 mm, respectively, but different tube diameters. All specimens in groups 7, 10 through 15 collapsed in progressive folding modes and illustrated consistent load/displacement profiles within each group as shown in Appendix A. For the three specimens in group 8 and three specimens in group 9, one specimen within each group exhibited a progressive folding mode and the other two specimens within each group illustrated global bending mode, which should all deformed in the global bending modes according to the predictions in reference [36]. The experimental results show that specimens with geometries very similar to the critical geometrical dimensions from reference [36] may experience very unstable deformation during axial crush. Minor variations in specimen geometry and/or material characteristics could also contribute to the transition of progressive folding into a global bending mode of deformation.

The deformed extrusions after axial crush tests for each extrusion geometry considered for this section is presented in Figure 6.8. The observed load/displacement profiles for all specimens in groups 8 and 9 are shown in Figure 6.9 and Figure 6.10, respectively. The observed load/displacement profiles for representative specimens (which collapsed in progressive folding deformation modes) in groups 7, 10 and 13, groups 8, 11 and 14, and groups 9, 12 and 15 are illustrated in Figure 6.11, Figure 6.12, and Figure 6.13, respectively. It is obvious from Figure 6.11, Figure 6.12, and Figure 6.13 that for the extrusions which had the same reduced wall thickness and collapsed in the progressive folding modes, consistent load/displacement responses were observed. The crush force oscillated with the formation of plastic folds. Axisymmetric lobe formation was typically observed for the first and second folds, however, a switch to a non-axisymmetric (diamond) mode was observed for all specimens. Significant material fracture was observed for all the extrusions geometries considered except for the extrusions with a tube diameter of 44.45 mm and a reduced wall thickness of 1.0 mm as shown in Figure 6.8.
Figure 6.8 Deformed extrusions after axial crush tests. (a), (b), and (c) are for the extrusions with tube outer diameters of 44.45 mm, 50.8 mm, and 63.5 mm, respectively.
Figure 6.9  The load/displacement responses of circular AA6061-T6 specimens in group 8.

Figure 6.10  The load/displacement responses of circular AA6061-T6 specimens in group 9.
Figure 6.11  The load/displacement responses of circular AA6061-T6 specimens for representative specimens in groups 7, 10 and 13.

Figure 6.12  The load/displacement responses of circular AA6061-T6 specimens for representative specimens in groups 8, 11 and 14.
6.2.3 Dynamic crush test results for the specimens in groups 16 through 24

Specimens in groups 16, 19 and 22, groups 17, 20 and 23, and groups 18, 21 and 24 had reduced wall thickness of 1.0 mm, 1.25 mm, and 1.5 mm, respectively, but different tube diameters. All specimens in groups 16 through 15 collapsed in progressive folding modes as predicted in reference [36] and illustrated consistent load/displacement profiles within each group as shown in Appendix A.

The observed load/displacement profiles for representative specimens in groups 16, 19 and 22, groups 17, 20 and 23, and groups 18, 21 and 24 are illustrated in Figure 6.14, Figure 6.15, Figure 6.16, respectively. It is obvious from Figure 6.14, Figure 6.15, Figure 6.16 that for the extrusions which had the same reduced wall thickness, the crush force increased with the increase of extrusion wall thickness. The crush force oscillated with the formation of plastic folds. Similar to the quasi-static crushing tests, axisymmetric lobe formation was typically observed for the first and second folds, however, a switch to a non-axisymmetric (diamond) mode was observed for all specimens.
Figure 6.14 The load/displacement responses of circular AA6061-T6 specimens for representative specimens in groups 16, 19 and 22.

Figure 6.15 The load/displacement responses of circular AA6061-T6 specimens for representative specimens in groups 17, 20 and 23.
6.2.4 Crush test results comparison amongst all specimens

The load/displacement responses for the representative extrusions an outer diameter of 50.8 mm and wall thicknesses of 3.175 mm (tube length of 200 mm), 1.587 mm, and 1.0 mm that collapsed in progressive folding mode and a representative specimen with a wall thickness of 3.175 mm (tube length of 300 mm) that collapsed in global bending mode are shown in Figure 6.17. It is obvious from Figure 6.17 that the crush force response of specimens underwent global bending mode is significantly different from the specimens which experienced progressive folding mode. While the crush force oscillated with the development of the plastic folds for the specimens underwent progressive folding mode, it dropped significantly after the initiation of mid-span kink for the specimens experienced the global bending mode. The crush force generally increased with the increase of the tube wall thickness for the specimens collapsed in the progressive folding modes under both quasi-static and dynamic loading conditions as shown in Figure 6.17, Figure 6.18, and Figure 6.19. Similar observation was found for extrusions with the other two different diameters.
Figure 6.17  The load/displacement observations for the representative extrusions with an outer diameter of 50.8 mm and wall thicknesses of 3.175 mm (tube length of 200 mm), 1.587 mm, and 1.0 mm that collapsed in progressive folding mode and a representative specimen with a wall thickness of 3.175 mm (tube length of 300 mm) that collapsed in global bending mode.

Figure 6.18  The load/displacement observations for the representative extrusions with a tube outer diameter of 44.45 mm and reduced wall thicknesses of 1.0 mm, 1.25 mm and 1.5 mm under quasi-static loading.
Figure 6.19  The load/displacement observations for the representative extrusions with a tube outer diameter of 44.45 mm and reduced wall thicknesses of 1.0 mm, 1.25 mm and 1.5 mm under dynamic loading.

For extrusions with the same geometry which subjected to different loading conditions, the dynamic peak crush force was observed to be slightly higher than the quasi-static peak crush force as illustrated in Figure 6.20 through Figure 6.22.

Figure 6.20  Load/displacement comparison for the representative extrusions with a tube outer diameter of 44.45 mm and reduced wall thicknesses of 1.0 mm which subjected to the quasi-static and dynamic loading conditions.
Figure 6.21  Load/displacement comparison for the representative extrusions with a tube outer diameter of 50.8 mm and reduced wall thicknesses of 1.25 mm which subjected to the quasi-static and dynamic loading conditions.

Figure 6.22  Load/displacement comparison for the representative extrusions with a tube outer diameter of 63.5 mm and reduced wall thicknesses of 1.5 mm which subjected to the quasi-static and dynamic loading conditions.
6.2.5 Comparison of crush performance parameters

The experimental results in terms of the energy absorption and crush performance parameters for each specimen group are compared in this section. Crush performance parameters including the peak crush load ($P_{\text{max}}$), mean crush force ($P_m$), crush force efficiency ($CFE$), total energy absorption ($TEA$) and the specific energy absorption ($SEA$) were calculated from the experimental observations. The mean values of crush parameters for each group are presented in Table 6.2 and Table 6.3.

Table 6.2 Calculated mean values of crush parameters for groups 1 through 15.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{\text{max}}$ (kN)</th>
<th>$CFE$ (%)</th>
<th>$TEA$ (kJ)</th>
<th>$SEA$ (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R200_D50.8_t3.175_PF_QS</td>
<td>87.48</td>
<td>146.05</td>
<td>59.9</td>
<td>12.17</td>
<td>47.42</td>
</tr>
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<td></td>
<td></td>
<td>98.7(PF)</td>
<td>145.7(PF)</td>
<td>67.7(PF)</td>
<td>14.0(PF)</td>
<td>54.6(PF)</td>
</tr>
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<td>2</td>
<td>R300_D50.8_t3.175_GB_QS</td>
<td>28.13</td>
<td>137.82</td>
<td>20.4</td>
<td>4.00</td>
<td>10.38</td>
</tr>
<tr>
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<td>R200_D50.8_t1.587_PF_QS</td>
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<td>67.55</td>
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<td>3.70</td>
<td>27.96</td>
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<td>R300_D50.8_t1.587_PF_QS</td>
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<td>67.77</td>
<td>40.3</td>
<td>3.78</td>
<td>19.05</td>
</tr>
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<td>5</td>
<td>R150_D50.8_t1.0_PF_QS</td>
<td>16.12</td>
<td>40.96</td>
<td>39.3</td>
<td>1.77</td>
<td>7.09</td>
</tr>
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<td>6</td>
<td>R150_D50.8_t1.2_PF_QS</td>
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<td>43.8</td>
<td>2.39</td>
<td>9.13</td>
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<td>46.0</td>
<td>1.97</td>
<td>23.74</td>
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<tr>
<td>8</td>
<td>R250_D44.45_t1.25_PF_QS</td>
<td>10.35</td>
<td>41.15</td>
<td>25.2</td>
<td>1.33</td>
<td>12.75</td>
</tr>
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<td></td>
<td></td>
<td>19.7(PF)</td>
<td>41.4(PF)</td>
<td>47.7(PF)</td>
<td>2.8(PF)</td>
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<td></td>
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<td>52.3(PF)</td>
<td>3.9(PF)</td>
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<td>1.95</td>
<td>20.27</td>
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<td>17.47</td>
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<td>4.43</td>
<td>23.75</td>
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Table 6.3  Calculated mean values of crush parameters for groups 16 through 24.

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<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
</tr>
</thead>
<tbody>
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<td>R250_D44.45_t1.25_PF_Dyn</td>
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<td>46.08</td>
<td>44.83</td>
<td>1.74</td>
<td>16.69</td>
</tr>
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<td>18</td>
<td>R250_D44.45_t1.5_PF_Dyn</td>
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<td>56.89</td>
<td>52.95</td>
<td>1.56</td>
<td>12.35</td>
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<tr>
<td>19</td>
<td>R250_D50.8_t1.0_PF_Dyn</td>
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<td>48.82</td>
<td>29.51</td>
<td>1.88</td>
<td>19.54</td>
</tr>
<tr>
<td>20</td>
<td>R250_D50.8_t1.25_PF_Dyn</td>
<td>23.24</td>
<td>56.97</td>
<td>40.86</td>
<td>1.77</td>
<td>14.64</td>
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<td>21</td>
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<td>43.56</td>
<td>1.55</td>
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<td>34.40</td>
<td>1.68</td>
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<tr>
<td>23</td>
<td>R250_D63.5_t1.25_PF_Dyn</td>
<td>25.49</td>
<td>65.77</td>
<td>38.82</td>
<td>1.57</td>
<td>12.74</td>
</tr>
<tr>
<td>24</td>
<td>R250_D63.5_t1.5_PF_Dyn</td>
<td>36.07</td>
<td>86.61</td>
<td>41.69</td>
<td>1.51</td>
<td>8.08</td>
</tr>
</tbody>
</table>

6.2.5.1 The peak crush load and the mean crush force

Table 6.2 clearly illustrates that the difference of the peak load and mean crush force for specimens with the same wall thickness of 3.175 mm that experienced progressive folding and global bending deformation modes. The peak load for specimens underwent progressive folding mode was observed to be slightly higher than that for specimens experienced global bending mode. However, the mean crush force for the specimens which experienced progressive folding mode was observed to be approximately 350% higher compared to the specimens that experienced global bending modes. The peak crush load and mean crush force for the specimens with a wall thickness of 1.587 mm, which underwent progressive folding modes, were determined to be 46% and 27%, respectively, of that for the extrusions with a wall thickness of 3.175 mm which experienced the same deformation modes.

For specimens in groups 7, 10, and 13 which had the same reduced wall thickness of 1.0 mm and subjected to quasi-static loading, the averaged peak crush loads were found to be 30.75 kN, 38.97 kN, and 46.88 kN, respectively. The averaged mean crush
forces were determined to be 14.01 kN, 13.69 kN, and 15.18 kN, respectively. It was observed that the peak crush load was generally increased considerably with the increase of the extrusion diameter while the mean crush force was fairly consistent for different extrusion diameters under quasi-static loading. Similar observations were observed for the specimens in groups 8, 11, and 14 and in groups 9, 12, and 15 which collapsed in the same progressive folding modes.

For specimens in groups 16, 19, and 22 which had the same reduced wall thickness of 1.0 mm and subjected to impact loading, the averaged peak crush loads were found to be 39.07 kN, 48.82 kN, and 54.84 kN, respectively. The averaged mean crush forces were determined to be 15.07 kN, 14.42 kN, and 18.58 kN, respectively. Similar to the quasi-static loading condition, the peak crush load was generally increased considerably with the increase of the extrusion diameter while the mean crush force was fairly consistent for different extrusion diameters under dynamic loading. Consistent observations were observed for the specimens in groups 17, 20, and 23 and in groups 18, 21, and 24.

For specimens which had the same extrusion geometry, the dynamic peak crush load was observed to be 8%-27% higher than the quasi-static peak crush load; the dynamic mean crush force were observed to be 3%-25% higher than the quasi-static mean crush force.

6.2.5.2 Total energy absorption and crush force efficiency

The averaged $TEA$ for the specimens in group 1 which experienced a progressive folding mode and for specimens in group 2 which underwent a global bending mode was determined to be 14.0 kJ and 4.0 kJ, respectively. The averaged $CFE$ was found to be 67.7% and 20.4%, respectively. For the specimens in groups 3 and 4, which had the same wall thickness of 1.587 mm but different tube lengths of 200 mm and 300 mm, respectively, the calculated $TEA$ and $CFE$ were found to be very similar to each other. For specimens in groups 4 and 5, the average $TEA$ was determined to be 1.77 kJ and 2.39 kJ, respectively; the average $CFE$ was determined to 39.3% and 43.8%, respectively.
For the specimens in groups 7, 10, and 13 and in groups 16, 19 and 22 which had the reduced wall thicknesses of 1.0 mm and subjected to the quasi-static loading and dynamic loading, respectively, the $TEA$ was found to be fairly consistent to each other while the $CEF$ was observed to be slightly decreased with the increase of tube diameter. Similar relationship was observed for the specimens in other groups which had the same wall thicknesses.

6.3 Dynamic and quasi-static cutting tests results and discussion

Although three experimental tests (if not indicated otherwise) were completed for each group the load/displacement observations for all the specimens within each group were fairly consistent. For this reason and for greater clarity, only one representative specimen from each group was selected for illustration and discussion purpose. The load/displacement profiles of all the cutting tests within each group are presented in Appendix A to demonstrate the repeatability of the tests. A qualitative and quantitative examination of cutting testing observations for each specimen group was completed through analysis of photographs and cutting parameters. Section 6.3.1 details the cutting tests results for the circular AA6061-T6 specimens which were subjected to the quasi-static loading only. In section 6.3.1, the influence of tube length, deflector, and cutter blade quantities on the cutting characteristics of the extrusions will be discussed. Section 6.3.2 presents the dynamic and quasi-static cutting test results for the circular extrusions cut by different versions of cutter and cone-shape deflectors. Section 6.3.3 comprehensively discusses the dynamic and quasi-static cutting test results for the reduced wall thickness extrusions with different tube diameters cut by a cutting with multiple cutter blades. Sections 6.3.4 and 6.3.5 discuss the dynamic and quasi-static testing results for the dual-stage cutting and controlling load/displacement response cutting tests, respectively.

6.3.1 Quasi-static cutting tests results and discussion for the specimens in groups 25 through 36

In this section, the effects of tube length, deflector, cutter geometries, and cutter blade quantities on the energy absorption and load/displacement response characteristics of the extrusions are discussed.
6.3.1.1 Quasi-static cutting test results for the specimens in groups 25 through 28

The observed load/displacement responses for the representative specimens from groups 25 through 28 are illustrated in Figure 6.23. It is obvious that the 200 mm length specimens in group 25 and the 300 mm long specimens in group 26 exhibited almost identical load/displacement responses. Photographs of the cutting process without the presence of deflector for a representative specimen from group 25 are illustrated in Figure 6.24(a) through (d). The corresponding load/displacement values at which the photographs of the representative specimen in Figure 6.24(a) through (d) were taken are also presented in Figure 6.25. Photographs of the cutting process with the use of straight deflector for a representative specimen from group 27 are shown in Figure 6.26(a) through (d). Load/displacement observations for the corresponding images in Figure 6.26(a) through (d) are presented in Figure 6.27.

Photographs of the cutting deformation illustrate that the cutter can penetrate through the side wall of the specimens and develop highly localized plastic deformation in the vicinity of the cutting blades. Cutting chips were observed to be formed during the cutting process as shown in Figure 6.24(d) and Figure 6.26(b) through (d). No crack propagation was observed in any tests. As the cutting progressed, petalled sidewalls bent slightly outwards for the cutting tests without the use of deflector, which was mostly likely due to the interaction between the cutter blade shoulder and the tube sidewalls. For the cutting tests with the presence of the deflector, as the cutting process proceeded, the petalled sidewalls contacted the deflector and flared outward and finally formed a continuous region of contact with the deflector. Circumferential stretching of the tube was observed to occur for the axial cutting tests without the presence of deflector. Circumferential stretching of the tube was also observed for the axial cutting tests with the use of deflector after initiation of cutting deformation mode but prior to contact with the deflector. After contact between the deflector and petalled sidewalls commenced, a combination of circumferential stretching and large bending was observed to occur within the petalled sidewalls. All the cutting tests were observed to be stable, repeatable, and controllable with regard to the axial cutting deformation mode.
It is evident from the force versus displacement curves that the cutting phenomena for the specimens in groups 25 through 28 can be referred to clean cut [58, 59]. The axial cutting process can be divided into two cutting stages, namely, the transient cutting stage and the steady-state cutting stage, as discussed in section 2.4.

For the axial cutting test without the presence of deflector, the cutting resistance force continued to increase at the transient cutting stage, occurring from the point of initial contact between the blade tip and tube sidewall to the point where the resistance force reaches a constant level. After an approximate 20 mm penetration of the cutter blade, the cutting process transferred to a steady-state cutting stage with an approximate resistance force of 45 kN for all tests in this group. The cutting force in this stage was maintained constant until testing was completed.

For the axial cutting test with the use of straight deflector, the first transient cutting phase was observed to be consistent with the cutting tests without the presence of deflector, which exhibited a nonlinear increase in the cutting load from zero to approximately 45 kN in the displacement range from 0 to approximately 20 mm. As the petalled sidewalls contacted the straight deflector the load surged to approximately 52 kN and resulted in a second transient cutting phase which was observed to occur with displacements in the range of approximately 25 mm to 60 mm. This increase in load in the second transient cutting phase, which was observed to be within the range of 5 kN to 12 kN for all specimens in group 27, is a result of the additional force necessary to crush the vertical cut petalled sidewalls. Experimental observations from all specimens indicated that this load increase was not consistent but the sharp increase in load repeatedly occurred at a crosshead displacement of approximately 27 mm and significantly decreased with increasing displacement up to approximately 35 mm. The sharp reduction in load was believed to occur as a result of the flaring of the cut petalled sidewalls and hence a reduction in the vertical component of the contact force between the deflector and the tube. The cutting force was observed to increase slightly after a crosshead displacement of approximately 35 mm until 60 mm, which was believed to be due to large plastic bending occurring within the petalled sidewalls near the contact region of the extrusion and deflector. Finally, the deformation process reached the steady-state cutting phase after a crosshead displacement of approximately 60 mm with
an approximate resistance force of 38 kN for all specimens in group 27. The cutting force in this phase was maintained constant until testing was completed. The reduction in steady state cutting force from approximately 45 kN to 36 kN was a result of the stretching imposed on the petalled sidewalls of the extrusion from the deflector.

The specimens in group 28 cut with the presence of curved deflector exhibited similar load/displacement responses and cutting phenomena to the specimens in group 27. However, it was observed that the significant increase in cutting force previously observed in the specimens in group 27 when contact with the deflector was initiated no longer existed with the use of the curved deflector. The elimination of the sharp increase in cutting force was caused by the curvature associated with the curved deflector. In addition, the reduction in cutting force after initial contact with the deflector occurred over a longer displacement with the curved deflector compared to the findings for the specimens within group 27. Flaring of the specimens within group 28 was more gradual than observed for specimens within group 27. This observation explains why the reduction in cutting force occurs over a longer displacement. Finally, the cutting process reached a steady-state phase after a crosshead displacement of approximately 70 mm with an approximate resistance force of 38 kN for all specimens within the group. The cutting force in this phase was maintained constant until testing was completed.

![Figure 6.23](image)

Figure 6.23 Experimentally observed load/displacement profiles for the representative extrusions from groups 25 through 28.
Figure 6.24 Photographs illustrating the cutting process without the presence of a deflector for a representative specimen from group 25.

Figure 6.25 Experimentally obtained load/displacement curve for a representative specimen in group 25, positions a, b, c, and d correspond to photographs in Figure 6.24.

Figure 6.26 Photographs illustrating the cutting process with the use of a straight deflector for a representative specimen from group 27.
6.3.1.2 Quasi-static cutting test results for the specimens in groups 29 through 36

The observed load/displacement responses for the representative specimens from groups 29 through 32 and from groups 33 through 36 are illustrated in Figure 6.28 and Figure 6.29, respectively. The cutting behaviour for the specimens in groups 29 through 36 cut by a cutter with multiple blades was similar to the behaviour observed for the specimens in groups 24 and 25 which were cut by a cutter with four blades without the use of deflector. The cutter blade penetrated through the sidewall of the specimen and developed a large localized plastic deformation zone in the vicinity ahead of the cutting blades. This localized plastic deformation zone moved along the extrusion as the cutting process continued. The deformed material rolled away from the two sides of the cutter blade and cutting chips were formed ahead of the cutting blade tip during the cutting process. Furthermore, circumferential membrane stretching of the tube specimens was observed which caused an increased radius of the extrusion. In addition, petalled sidewalls were observed to be bent outwards in all experimental tests due to the eccentric pushing force generated by the interaction between the cutter blade and the tube.
sidewalls. The extent of petalled sidewalls outward bending was observed to be increased with the increase of number of the blades.

A significantly different finding was associated with material fracture which occurred on the petalled sidewalls of the extrusions with a wall thickness of 1.587 mm cut by the 5- or 6-blade cutter (groups 31 and 32). Slight material fracture was also observed for the extrusions with a wall thickness of 1.587 mm cut by the 3- or 4-blade cutter (groups 29 and 30), which typically occurred at the end of the cutting process. Little or no material fracture was observed for the extrusions with a wall thickness of 3.175 mm cut by the 3-, 4-, or 5-blade cutters (groups 33 through 35). However, a slight degree of material fracture was also observed on the petalled sidewalls of extrusions with a wall thickness of 3.175 mm cut by the 6-blade cutter (group 36).

No initial peak cutting force was observed to initiate the cutting deformation mode. After the transient cutting state, the cutting force for the specimens in groups 31 and 32 oscillated as shown in Figure 6.28, which was mostly due to material fracture. However, an almost constant cutting force was observed after the transient cutting stage for the specimens in groups 33 through 36. Examining the load/displacement curves and the cutting deformation observations revealed that the cutting deformation of the circular extrusions in groups 29 through 36 falls into the category of clean cut [58, 59].

![Displacement vs. Axial Load](image)

Figure 6.28 Experimentally observed load/displacement profiles for the representative extrusions from groups 29 through 32.
6.3.1.3 Quasi-static cutting test results comparison amongst the specimens from groups 25 through 36

Figure 6.28 and Figure 6.29 illustrate the load/displacement responses for the specimens with wall thicknesses of 1.587 mm and 3.175 mm cut by a cutter with multiple blades, respectively. It is obvious that the steady-state cutting force for the specimens with a wall thickness of 3.175 mm increased with the increase of cutter blade quantities. Similar observations were found for the specimens with a wall thickness of 1.587 mm cut by the 3-, 4-, and 5-blade cutters. For the specimens with a wall thickness of 1.587 mm and cut by the 6-blade cutter, the cutting force surpassed the load necessary to initiate the cutting deformation for the same geometry extrusion cut by the 5-blade cutter and then dropped and fluctuated significantly due to the large degree of material fracture observed on the specimens.

Figure 6.30 presents the observed relationship between the steady-state mean cutting force and the number of blades of the cutter for the specimens in groups 29 through 36. It can be seen that an almost linear relationship between the steady-state mean cutting force and the number of cutter blades exists for the extrusions with wall thicknesses of 1.587 mm and 3.175 mm cut by a cutter with number of blades less than 6.
For the extrusions cut by a 6-blade cutter (groups 32 and 36), the steady-state cutting forces were observed to be slight below the linear trend which was valid for a cutter with less than 6 blades. The drop of the steady-state mean cutting force for the extrusions was believed to be associated to the material fracture observed on the petalled sidewalls of the extrusion.

![Steady-state mean cutting force versus cutter blade quantities from experimental cutting tests for the AA6061-T6 extrusions with wall thicknesses of 1.587 mm and 3.175 mm.](image)

**Figure 6.30** Steady-state mean cutting force versus cutter blade quantities from experimental cutting tests for the AA6061-T6 extrusions with wall thicknesses of 1.587 mm and 3.175 mm.

**6.3.1.4 Comparison of cutting performance parameters**

This section compares the cutting performance parameters for the specimens in groups 25 through 36. For each specimen tested, the axial cutting force and crosshead displacement were recorded. Post-testing data analysis was completed to determine the peak cutting load \( P_{\text{max}} \), mean cutting force \( P_{\text{m}} \), crush force efficiency \( \text{CFE} \), total energy absorption \( \text{TEA} \) and the specific energy absorption \( \text{SEA} \). The mean values of the cutting performance parameters for each group are summarized in Table 6.4 and Table 6.5. While Table 6.4 compares the cutting performance parameters for specimens with same geometries cut with/without the use of a straight/curved deflector, Table 6.5 presents the cutting parameters for two identical specimen geometries cut by a cutter with multiple blades.
Table 6.4  Calculated mean values of cutting performance parameters for groups 25 through 28.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
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<td>25</td>
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<td>6.05</td>
<td>23.58</td>
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<td>44.29</td>
<td>47.05</td>
<td>94.13</td>
<td>6.30</td>
<td>16.37</td>
</tr>
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<td>27</td>
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<td>36.26</td>
<td>52.25</td>
<td>69.40</td>
<td>5.21</td>
<td>20.30</td>
</tr>
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<td>28</td>
<td>R200_D50.8_t3.175_RevI_4T1.0_CD_QS</td>
<td>36.60</td>
<td>44.86</td>
<td>81.57</td>
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<td>20.58</td>
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Table 6.5  Calculated mean values of cutting performance parameters for groups 29 through 36.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
</tr>
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<td>85.97</td>
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<td>10.37</td>
</tr>
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<td>R200_D50.8_t1.587_RevII_4T1.0_ND_QS</td>
<td>18.44</td>
<td>21.73</td>
<td>84.77</td>
<td>2.47</td>
<td>12.45</td>
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<td>R200_D50.8_t1.587_RevII_5T1.0_ND_QS</td>
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<td>85.13</td>
<td>3.05</td>
<td>15.36</td>
</tr>
<tr>
<td>32</td>
<td>R200_D50.8_t1.587_RevII_6T1.0_ND_QS</td>
<td>22.41</td>
<td>27.34</td>
<td>81.97</td>
<td>3.09</td>
<td>15.53</td>
</tr>
<tr>
<td>33</td>
<td>R200_D50.8_t3.175_RevII_3T1.0_ND_QS</td>
<td>39.30</td>
<td>42.36</td>
<td>92.80</td>
<td>5.45</td>
<td>14.16</td>
</tr>
<tr>
<td>34</td>
<td>R200_D50.8_t3.175_RevII_4T1.0_ND_QS</td>
<td>45.01</td>
<td>48.60</td>
<td>92.60</td>
<td>6.18</td>
<td>16.07</td>
</tr>
<tr>
<td>35</td>
<td>R200_D50.8_t3.175_RevII_5T1.0_ND_QS</td>
<td>50.37</td>
<td>54.61</td>
<td>92.27</td>
<td>6.92</td>
<td>18.00</td>
</tr>
<tr>
<td>36</td>
<td>R200_D50.8_t3.175_RevII_6T1.0_ND_QS</td>
<td>54.30</td>
<td>59.64</td>
<td>91.03</td>
<td>7.53</td>
<td>19.58</td>
</tr>
</tbody>
</table>

6.3.1.4.1  The peak cutting load and the mean cutting force

The peak cutting load and the mean cutting force for the specimens with a wall thickness of 3.175 mm cut by the 4-blade cutter with/without the presence of the deflector ranged from 44.86 kN to 48.60 kN and from 36.26 kN to 45.01 kN, respectively. The maximum peak cutting load was observed for the the specimens cut
employing the straight deflector. Lower mean cutting force was observed for the extrusion cut with the presence of the deflector. Minor difference in the peak cutting load and the mean cutting force was found for the same geometry specimens cut by the RevI and RevII cutters with four blades. For both wall thicknesses extrusions, the peak cutting load and the mean cutting force generally increased with the increase of cutter blade quantities. However, the increment of the mean cutting force for the specimens with a wall thickness of 1.587 mm cut by the 6-blade cutter was significantly decreased due to significant material fracture observed in the experimental tests.

### 6.3.1.4.2 Total energy absorption and crush force efficiency

The TEA and CFE for the specimens with a wall thickness of 3.175 mm cut by the 4-blade cutter with/without the use of the deflector ranged from 5.21 kJ to 6.30 kJ and from 69.4% to 94.2%, respectively. The maximum TEA and CFE were observed for the specimens employing the cutter only and minor difference was observed for the same geometry specimens cut by the RevI and RevII 4-blade cutters. For both wall thickness extrusions, the TEA increased with the increase of cutter blade quantities excepted for the extrusions with a wall thickness of 1.587 mm cut by the 6-blade cutter. The CFE observed for both wall thickness extrusions cut by multiple blades ranged from 82.0% to 92.8%.

### 6.3.2 Dynamic and quasi-static cutting test results for the specimens in groups 37 through 48

In this section, the effects of cutter geometries, in terms of cutter blade length and cutter blade tip width, and deflector geometries, in terms of surface profiles of the deflector, as well as loading conditions on the energy absorption and load/displacement response characteristics of the extrusions are discussed.

### 6.3.2.1 Dynamic cutting test results for the specimens in groups 37 through 42

The dynamic tests for the axial cutting of the circular AA6061-T6 extrusions with a wall thickness of 1.587 mm typically lasted 28-40 ms depending on the configurations. The cutter blades penetrated into the circular extrusions and chips were formed. No crack propagation was observed for any of the tests. Figure 6.31 shows the
load/displacement profiles recorded from the upper load cell and the lower load cell for the AA6061-T6 specimen cut by the RevI cutter/curved deflector assembly. Similar observations were found for other different configurations.

Figure 6.31 Representative load/displacement profiles from the upper load cell and the lower load cell for the circular AA6061-T6 extrusions cut by the RevI/curved deflector assembly.

A delay of approximately 0.1 ms between the instants when the forces in the lower load cell and upper load cell were non-zero was observed. This time is in agreement to the time needed for the impacting stress wave to travel from the top to the bottom of the extrusions. The forces recorded from the upper load cell were much greater than those from the lower load cell at the beginning of the dynamic cutting process. This is due to the difference in the nature of the events which are occurring at the upper and lower load cells. An impact event is occurring in between the dropping entity and the upper load cell while support of the extrusion and cutter/deflector assembly is occurring at the lower load cell. Conducting an impact analysis between the dropping entity and the cutter/deflector assembly justifies the observed high impact forces of approximately 250 kN. Moreover, the upper load cell recorded zero force at certain
intervals, which was due to intermittent contact between the upper load cell and the drop mass.

The recorded load from the lower load cell for all tests was observed to be approximately 5 kN higher than that from the upper load cell at the steady-state cutting stage, which was due to the deceleration of the cutter/deflector assembly and the dropping mass at the steady-state cutting stage. This deceleration was estimated to be approximately 9.0g.

Since the focus of this study is to investigate the cutting behaviour of the extrusions and the influence of different cutter geometries and deflector surface profiles, only the interested cutting resistance load which was recorded from the lower load cell will be presented for discussion.

Figure 6.32 shows the load/displacement response for the extrusions cut by the RevI cutter ($T = 1.0$ mm) and straight/curved deflector. Figure 6.33 and Figure 6.34 illustrate the load/displacement profiles for the extrusions cut by the RevII cutter with $T = 0.75$ mm and $T = 1.0$ mm and straight/curved deflector, respectively.

Observations from Figure 6.32 through Figure 6.34 indicate similar load/displacement responses for extrusions experiencing a dynamic cutting mode of deformation with straight or curved deflector profiles. A high peak cutting load at the initiation of the transient cutting stage was observed followed by slight oscillation of the cutting force until the first steady-state cutting stage was reached at approximately 15 mm displacement. As cutting progressed, the petalled sidewalls interacted with the deflector (at a displacement of approximately 25-30 mm) resulting in a reduction of the cutting load. Then the cutting load reached its second steady stage after a displacement of approximately 32-35 mm, which was consistent with the observations detailed in section 6.3.1.1 for the circular AA6061-T6 extrusions cut quasi-statically. The fluctuation of the load/displacement profiles after reaching the second steady-state cutting stage were mostly due to localized material fracture that occurred on the petalled sidewalls after interaction with the deflector. It was observed that material fracture occurred more often for the combination of RevII cutter/straight deflector as shown in Figure 6.35(b). Minor or no material fracture was observed for extrusions cut with the
RevI cutter/curved deflector as shown in Figure 6.35(a), which was mostly due to the geometrical design of the RevI cutter preventing it from shifting off the centerline of the extrusion. As a result of the longer uniform blade geometry associated with the RevII cutter the need for appropriate alignment of the centerlines of the cutter/deflector and extrusion, prior to impact, was diminished. However, the massive shifting of the cutter/deflector assembly during impact significantly decreased the functionality of the deflector and caused material fracture to be more prevalent. The curved deflector was seen to be more efficient in its function of outward bending of the petalled sidewalls than the straight deflector, which led to less material fracture and less overall displacement of the cutter/deflector assembly (Figure 6.32 through Figure 6.34). However, the occurrence of material fracture was more common and the efficiency of the system depended on the combination of the impact velocity, extrusion material property, cutter geometry, and deflector profile.

![Graph showing load/displacement profiles](image)

**Figure 6.32** Load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 1.587 mm cut by the RevI cutter ($T = 1.0$ mm) and straight/curved deflector under impact loading.

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Figure 6.33  Load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 1.587 mm cut by the RevII cutter ($T = 0.75$ mm) and straight/curved deflector under impact loading.

Figure 6.34  Load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 1.587 mm cut by the RevII cutter ($T = 1.0$ mm) and straight/curved deflector under impact loading.
Figure 6.35 (a) Lack of significant material fracture on the specimens dynamically cut by the RevI cutter/curved deflector, and (b) material fracture on the extrusions dynamically cut by the RevII cutter/straight deflector.

6.3.2.2 Quasi-static cutting test results for the specimens in groups 43 through 48

The representative load/displacement curves for the extrusions quasi-statically cut by the RevI cutter ($T = 1.0$ mm) and straight/curved deflector are presented in Figure 6.36. The representative load/displacement curves for the extrusions quasi-statically cut by the RevII cutter with the cutter blade tip widths $T$ of 0.75 mm and 1.0 mm and the straight/curved deflector are presented in Figure 6.37 and Figure 6.38, respectively.

It was observed that the cutter blades penetrated through the extrusion and chips formed ahead of cutter blade tip. Similar to observations from dynamic testing, no crack propagation was observed for any of the quasi-static tests. Localized material fracture was also observed in some tests, which correspondingly resulted in the fluctuation of the load/displacement responses (Figure 6.37 and Figure 6.38). However, material fracture was observed to be much less frequent, under quasi-static loading, compared to what was observed in the dynamic tests. The cutting load was observed to increase and reached its first steady-state stage at approximately 10 mm displacement. As cutting progressed, the cutting load increased when the petalled sidewalls contacted with the deflector at approximately 30 mm displacement. The cutting force increase was very significant for extrusions cut by the RevI cutter/straight deflector configuration and was not as
significant for other configurations, which is similar to the observations for the axial cutting of the extrusions with a wall thickness of 3.175 by a cutter/straight deflector assembly as detailed in section 6.3.1.1. Finally, with the outward flaring of the petalled sidewalls, the cutting load reduced due to flaring and fluctuated corresponding to the extrusion deformation.

Figure 6.36  Load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 1.587 mm cut by the RevI cutter \((T = 1.0 \text{ mm})\) and straight/curved deflector under quasi-static loading.

Figure 6.37  Load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 1.587 mm cut by the RevII cutter \((T = 0.75 \text{ mm})\) and straight/curved deflector under quasi-static loading.
Figure 6.38  Load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 1.587 mm cut by the RevII cutter \( (T = 1.0 \, \text{mm}) \) and straight/curved deflector under quasi-static loading.

6.3.2.3 Cutting test results comparison amongst the specimens in groups 37 through 48

Comparisons of representative load/displacement profiles for dynamic and quasi-static cutting tests for various configurations are presented in Figure 6.39 through Figure 6.44. As can be seen, the main difference is related to the initial part of the impact cutting test where the dynamic forces are significantly higher. As strain-rate effects are assumed to be of minor importance as discussed in section 2.6, the observed difference is attributed to either stress wave propagation, which is only significant when displacements are close to zero, and/or to inertia effects that develop at the instant of impact in order to initiate the cutting process. The displacement needed to reach the steady-state cutting process under impact was observed to be slightly less than that needed for the quasi-static tests. After this initial cutting process, the dynamic cutting forces were typically lower than the quasi-static cutting forces. Dynamic cutting forces were generally consistent with the observed quasi-static loads during the majority of the displacement. As the process continued to maximum displacement, dynamic cutting loads were typically greater than the quasi-static cutting forces. This finding was more evident for the extrusions cut with the RevII cutter. This finding was mostly due to the occurrence of
material fracture on the cut petalled sidewalls, generally away from the cutting zone, and shifting of the cutter in the dynamic test. Cutting force fluctuations were also observed in quasi-static tests for specimens cut by both the RevI and RevII cutters, however, these fluctuations were more significant for the RevII cutter.

Figure 6.39  Load/displacement profiles for specimens with a wall thickness of 1.587 mm cut by the RevI cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic and quasi-static loading conditions.

Figure 6.40  Load/displacement profiles for specimens with a wall thickness of 1.587 mm cut by the RevI cutter ($T = 1.0$ mm) and straight deflector assembly under dynamic and quasi-static loading conditions.
Figure 6.41 Load/displacement profiles for specimens with a wall thickness of 1.587 mm cut by the RevII cutter ($T = 0.75$ mm) and curved deflector assembly under dynamic and quasi-static loading conditions.

Figure 6.42 Load/displacement profiles for specimens with a wall thickness of 1.587 mm cut by the RevII cutter ($T = 0.75$ mm) and straight deflector assembly under dynamic and quasi-static loading conditions.
Figure 6.43  Load/displacement profiles for specimens with a wall thickness of 1.587 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic and quasi-static loading conditions.

Figure 6.44  Load/displacement profiles for specimens with a wall thickness of 1.587 mm cut by the RevII cutter ($T = 1.0$ mm) and straight deflector assembly under dynamic and quasi-static loading conditions.
6.3.2.4 **Comparison of cutting performance parameters**

This section compares the cutting performance parameters for the specimens in groups 37 through 48. For each specimen tested, the axial cutting force and crosshead displacement were recorded. Post-testing data analysis was completed to determine the peak cutting load ($P_{\text{max}}$), mean cutting force ($P_m$), crush force efficiency ($\text{CFE}$), total energy absorption ($\text{TEA}$) and the specific energy absorption ($\text{SEA}$). The mean values of the cutting performance measures for each group are summarized in Table 6.6.

**Table 6.6** Calculated mean values of cutting performance parameters for groups 37 through 48.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{\text{max}}$ (kN)</th>
<th>$\text{CFE}$ (%)</th>
<th>$\text{TEA}$ (kJ)</th>
<th>$\text{SEA}$ (kJ)</th>
</tr>
</thead>
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<tr>
<td>37</td>
<td>R300_D50.8_t1.587_RevI_4T1.0_CD_Dyn</td>
<td>17.14</td>
<td>26.01</td>
<td>66.45</td>
<td>1.27</td>
<td>6.38</td>
</tr>
<tr>
<td>38</td>
<td>R300_D50.8_t1.587_RevI_4T1.0_SD_Dyn</td>
<td>16.90</td>
<td>27.67</td>
<td>61.35</td>
<td>1.30</td>
<td>6.55</td>
</tr>
<tr>
<td>39</td>
<td>R300_D50.8_t1.587_RevII_4T0.75_CD_Dyn</td>
<td>15.94</td>
<td>25.02</td>
<td>63.70</td>
<td>1.44</td>
<td>7.25</td>
</tr>
<tr>
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<td>16.19</td>
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<td>65.25</td>
<td>1.74</td>
<td>8.77</td>
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<tr>
<td>41</td>
<td>R300_D50.8_t1.587_RevII_4T1.0_CD_Dyn</td>
<td>17.05</td>
<td>24.69</td>
<td>70.25</td>
<td>1.31</td>
<td>6.60</td>
</tr>
<tr>
<td>42</td>
<td>R300_D50.8_t1.587_RevII_4T1.0_SD_Dyn</td>
<td>15.15</td>
<td>25.25</td>
<td>60.25</td>
<td>1.49</td>
<td>7.48</td>
</tr>
<tr>
<td>43</td>
<td>R200_D50.8_t1.587_RevI_4T1.0_CD_QS</td>
<td>18.87</td>
<td>21.65</td>
<td>87.13</td>
<td>2.62</td>
<td>19.79</td>
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<tr>
<td>44</td>
<td>R200_D50.8_t1.587_RevI_4T1.0_SD_QS</td>
<td>20.55</td>
<td>25.06</td>
<td>82.00</td>
<td>2.85</td>
<td>21.49</td>
</tr>
<tr>
<td>45</td>
<td>R200_D50.8_t1.587_RevII_4T0.75_CD_QS</td>
<td>16.18</td>
<td>22.90</td>
<td>70.95</td>
<td>2.29</td>
<td>11.49</td>
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<td>46</td>
<td>R200_D50.8_t1.587_RevII_4T0.75_SD_QS</td>
<td>17.38</td>
<td>21.69</td>
<td>80.45</td>
<td>2.45</td>
<td>12.34</td>
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<td>47</td>
<td>R200_D50.8_t1.587_RevII_4T1.0_CD_QS</td>
<td>16.83</td>
<td>20.83</td>
<td>80.80</td>
<td>2.38</td>
<td>11.99</td>
</tr>
<tr>
<td>48</td>
<td>R200_D50.8_t1.587_RevII_4T1.0_SD_QS</td>
<td>15.92</td>
<td>18.53</td>
<td>85.65</td>
<td>2.26</td>
<td>11.37</td>
</tr>
</tbody>
</table>

6.3.2.4.1 **The peak cutting load and the mean cutting force**

It was observed in the dynamic axial cutting tests that the average peak cutting load was consistent for extrusions cut by the same cutter but different deflector with a
maximum deviation of approximately ± 11%. The average peak cutting loads observed for extrusions dynamically cut by the RevI cutter \((T = 1.0 \text{ mm})\), the RevII cutter \((T = 1.0 \text{ mm})\) and the RevII cutter \((T = 0.75 \text{ mm})\) were determined to be 26.8 kN, 25.0 kN and 25.3 kN, respectively. Observations presented in Table 6.6 indicated that, generally, the average of the mean cutting force for extrusions dynamically cut by the same cutter and straight/curved deflector were very similar with a maximum deviation of 1.97 kN. The mean cutting force for specimens cut by the RevI cutter was determined to be higher than that for specimens cut by the RevII cutter with the same nominal blade tip width \(T = 1.0 \text{ mm}\). This was due to the limited blade length \((w)\) of the RevI cutter which caused the petalled sidewalls to interact with the ‘tapered zone’ that lies between the blade and the circular beam that supports it and forced the petalled sidewalls back towards to the centre, resulting in an increased vertical load. The decrease of the cutter blade tip width from \(T = 1.0 \text{ mm}\) to \(T = 0.75 \text{ mm}\) seemed to have a minor effect on either the peak cutting load or the mean cutting force.

Observations provided in Table 6.6 indicate that the averaged \(P_m\) and \(P_{max}\) for the extrusions with \(t = 1.587 \text{ mm}\) quasi-statically cut by the RevII cutter \((T = 1 \text{ mm} \text{ or } T = 0.75 \text{ mm})\) and straight/curved deflectors were very similar. However, the \(P_{max}\) for the extrusions with \(t = 1.587 \text{ mm}\) cut by the RevI cutter and straight deflector were slightly higher than the same cutter with the curved deflector, resulting in a slight higher \(P_m\). The different results between the extrusions cut by the RevI/RevII cutter and straight/curved deflector were believed to be attributed to the longer blade length \((w)\) in the RevII cutter. The \(P_m\) and the \(P_{max}\) for specimens cut by the RevI cutter was determined to be higher than those cut by the RevII cutter with the same nominal blade tip width of \(T = 1.0 \text{ mm}\). The decrease of the RevII cutter blade tip width from \(T = 1.0 \text{ mm}\) to \(T = 0.75 \text{ mm}\) seemed to have a minor effect on either \(P_m\) or \(P_{max}\) for the specimens.

The ratio of dynamic to quasi-static values of \(P_m\) ranged from 0.82 to 1.01 for the extrusions with \(t = 1.587 \text{ mm}\). The ratio of dynamic to quasi-static values of \(P_{max}\) ranged from 1.09 to 1.39 for extrusions with \(t = 1.587 \text{ mm}\).
### 6.3.2.4.2 Total energy absorption and crush force efficiency

For the dynamic axial cutting tests, although the height of the dropping mass and the pressure associated with the pneumatic assist were identical for all dynamic tests, the \( TEA \) was different depending on the travelling distance of the drop entity. With respect to the impact energy absorption, the RevI cutter appeared to be more efficient in terms of the \( CFE \) and \( SEA \). However, the RevII cutter (with a larger blade length, \( w \)) was observed to be more adaptable, meaning that the RevII cutter can generate the designed cutting deformation mode in spite of a slight misalignment or shifting of the specimen, or even when extrusions do not have perfectly square end faces.

For the quasi-static axial cutting tests, the RevI cutter appeared to be more efficient in terms of the \( TEA \) and \( SEA \). No significant difference on the \( CFE \) was observed between extrusions cut by the RevI and RevII cutters.

### 6.3.3 Dynamic and quasi-static cutting test results for the specimens in groups 49 through 98

In this section, the effects of extrusion diameter, extrusion wall thickness, cutter blade quantities, and loading conditions on the energy absorption and load/displacement response characteristics of the extrusions are discussed.

#### 6.3.3.1 Dynamic cutting test results for the specimens in groups 49 through 55, groups 65 through 71, and groups 81 through 89

The dynamic tests for the axial cutting of the circular AA6061-T6 extrusions with a reduced wall thickness typically lasted 25-50 ms depending on the configuration of the extrusion and cutter. The cutter blades penetrated into the circular extrusions and similar observations were found to the extrusion in groups 37 through 42 which experienced same dynamic cutting deformation mode.

Figure 6.47, Figure 6.48 and Figure 6.49 show the representative load versus displacement responses of the extrusions with an original outer diameter of 44.45 mm and wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the multi-blade RevII cutter (\( T = 1.0 \) mm) and curved deflector assembly under impact loading, respectively. The representative load/displacement curves for the extrusions with an
original outer diameter of 50.8 mm and wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm dynamically cut by the multi-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly are presented in Figure 6.50 through Figure 6.52, respectively. The representative load/displacement curves for the extrusions with an original outer diameter of 63.5 mm and wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm dynamically cut by the multi-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly are presented in Figure 6.53 through Figure 6.55, respectively. It is important to note that within some of the these figures illustrating the observed load/displacement responses of the tests, results from only 4 or 5 blade cutters are presented. In a significant number of tests, but not all, with the three blade cutter an observed instability was found to exist in the cutting process. Correspondingly, in a selected number of dynamic tests only results of extrusions with either four or five blades are presented. Rationale for the instability will be discussed in subsequent paragraphs.

It was observed in all dynamic axial cutting tests that chips formed ahead of cutter blade tip and no crack propagation was observed for any tests. Localized material fracture and kinking of cut petalled sidewalls was observed in the cutting tests as illustrated in Figure 6.45, which resulted in the oscillation of axial cutting force as shown in Figure 6.47 through Figure 6.55. Material fracture was observed to be more prevalent for the extrusions with the largest outer diameter and the extrusions with the smallest wall thickness depending upon the axial bent curvature of the petalled sidewalls before contacting with the deflector.

![Figure 6.45](image-url)  
Figure 6.45  Localized material fracture and kinking of cut petal sidewalls from representative specimens.
As a result of the geometry of the cutting blades, which were developed with \( w = 26.1 \text{ mm} \) to accommodate a variety of extrusion diameters, it was observed in impact tests that shifting of the cutter/deflector assembly and extrusion centerlines often occurred during impact tests. The degree of shifting, in the radial direction during cutting, was often on the order of \( \frac{1}{2} \cdot w \), however, it was not always consistent and varied due to the random nature of the exact contact location between the top load cell and the impacting plate. Additionally, it is hypothesized that any slight variations in the extrusion geometry would also influence the degree of radial shifting. As a result of the shifting of the cutter/deflector assembly differences in the curvature of cut petalled sidewalls, within a given specimen, was often observed. This variation in petalled sidewall curvature was observed to be more severe with the decrease of number of cutter blades and with the increase of tube diameter. For the dynamic cutting tests with extrusions having \( D_o = 63.5 \text{ mm} \) and the 3-blade cutter/deflector assembly, offset of the centerlines would become so severe that a switch in the deformation mode of cutting to global bending was observed in some cases. As illustrated in Figure 6.46, which presents photographs from the high speed camera in the case where the extrusion diameter was equal to 63.5 mm and the cutter contained 3-blades, the cutting process was as expected prior to interaction between the cut sidewalls and the curved deflector. However, as a result of the shifting, differences in the circumferential length occurred and correspondingly the degree of sidewall bending would vary, especially at the free ends of the cut side walls. The presence of the 3-blade cutter would allow for less kinematic (rotational) constraint thus resulting in the pivoting of the cutter/deflector assembly about the centerline of the extrusion. As the impacting plate continued to drive into the cutter/deflector assembly, rotations of this entity would become larger and eventually result in a change in the deformation behaviour from a cutting mode to a global bending response with significant bend occurring at or near the pivot point for the cutter/deflector assembly. This change in the mode of deformation resulted in a significantly increase of the cutter force at a displacement of approximately 70 mm as shown in Figure 6.53 through Figure 6.55. However, the mean axial cutting force generally increased with the increase of cutter blades for all the extrusions considered.
Figure 6.46 Switch of cutting deformation to global bending for the dynamic cutting of a representative extrusion ($D_o = 63.5$ mm and $Y = 1.5$ mm) by a 3-blade RevII cutter/deflector assembly.

Comparison of load/displacement profiles for the extrusions ($D_o = 44.45$ mm, 50.8 mm, and 63.5 mm) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly were presented in Figure 6.56 through Figure 6.58, respectively. It was observed from Figure 6.56 that after the transient cutting stage, the axial cutting force reached its first steady-state cutting stage at a displacement of approximately 20 mm. At a displacement of approximately 30 mm, where the cut petalled sidewalls started to interact with the deflector, the axial cutting forces increased. Due to different curvature of cut petalled sidewalls for different outer diameter extrusions, the flaring ‘quality’ of the cut petalled sidewalls was different. For the specimens ($D_o = 50.8$ mm and $Y = 1.0$ mm), the cut petalled sidewalls conformed well with the deflecting surface of the deflector, thus axial cutting force increased linearly up to a displacement of approximately 65 mm. After that, the axial cutting force dropped quickly and reached its second steady-state cutting stage.
For the specimens \((D_o = 63.5 \text{ mm and } Y = 1.0 \text{ mm})\), the curvature of the cut petalled sidewalls was smaller than the curvature of the deflecting surface, thus the axial cutting force climbed significantly after a displacement of approximately 65 mm due to an effect of ‘buckling’ or ‘kinking’ of the cut petalled sidewalls. For the specimens \((D_o = 44.5 \text{ mm and } Y = 1.0 \text{ mm})\), the curvature of the cut petalled sidewalls was much greater than that of the deflecting surface, thus the axial cutting force dropped earlier than the other two diameter specimens at a displacement of approximately 40 mm. Similar observations can be found for the extrusions with reduced wall thicknesses of 1.25 mm and 1.5 mm as shown in Figure 6.57 and Figure 6.58, however, the change of the axial cutting force and where this change took place were different depending on the curvatures of the cut petalled sidewalls. Oscillation of the axial cutting force after the second steady cutting state was mostly due to localized material fracture of the cut petalled sidewalls. Generally, the axial cutting force slightly increased with the increase of tube diameter as shown in Figure 6.56 through Figure 6.58. Similar observations can be found for the extrusions \((D_o = 44.45 \text{ mm, } 50.8 \text{ mm, and } 63.5 \text{ mm})\) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 3-blade/5-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly.

Comparison of load/displacement profiles for the extrusions \((Y = 1.0 \text{ mm, } 1.25 \text{ mm, and } 1.5 \text{ mm})\) with an original extrusion outer diameters of 44.45 mm, 50.8 mm, and 63.5 mm cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly were illustrated in Figure 6.59 through Figure 6.61, respectively. It can be seen that the axial cutting force increased with the increase of extrusion wall thickness. Similar observations can be found for the extrusions \((D_o = 44.45 \text{ mm, } 50.8 \text{ mm, and } 63.5 \text{ mm})\) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 3-blade/5-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly.
Figure 6.47  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 44.45 mm and a reduced wall thickness of 1.0 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.

Figure 6.48  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 44.45 mm and a reduced wall thickness of 1.25 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.
Figure 6.49  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 44.45 mm and a reduced wall thickness of 1.5 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.

Figure 6.50  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 50.8 mm and a reduced wall thickness of 1.0 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.
Figure 6.51 Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 50.8 mm and a reduced wall thickness of 1.25 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.

Figure 6.52 Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 50.8 mm and a reduced wall thickness of 1.5 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.
Figure 6.53  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 63.5 mm and a reduced wall thickness of 1.0 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.

Figure 6.54  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 63.5 mm and a reduced wall thickness of 1.25 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.
Figure 6.55  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 63.5 mm and a reduced wall thickness of 1.5 mm cut by the RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under dynamic loading.

Figure 6.56  Comparison of load/displacement profiles for the AA6061-T6 extrusions \((Y = 1.0 \text{ mm})\) with different outer diameters cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under dynamic loading.
Figure 6.57  Comparison of load/displacement profiles for the AA6061-T6 extrusions \((Y = 1.25 \text{ mm})\) with different outer diameters cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under dynamic loading.

Figure 6.58  Comparison of load/displacement profiles for the AA6061-T6 extrusions \((Y = 1.5 \text{ mm})\) with different outer diameters cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under dynamic loading.
Figure 6.59  Comparison of load/displacement profiles for the AA6061-T6 extrusions ($D_o = 44.45$ mm) with reduced wall thickness of 1.0 mm, 1.25 mm and 1.5 mm, cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.

Figure 6.60  Comparison of load/displacement profiles for the AA6061-T6 extrusions ($D_o = 50.8$ mm) with reduced wall thickness of 1.0 mm, 1.25 mm and 1.5 mm, cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic loading.
6.3.3.2 **Quasi-static cutting test results for the specimens in groups 56 through 64, groups 72 through 80, and groups 90 through 98**

The representative load/displacement curves for the extrusions with an original outer diameter of 44.45 mm and wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm quasi-statically cut by the multi-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly are presented in Figure 6.62 through Figure 6.64, respectively. The representative load/displacement curves for the extrusions with an original outer diameter of 50.8 mm and wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm quasi-statically cut by the multi-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly are presented in Figure 6.65 through Figure 6.67, respectively. The representative load/displacement curves for the extrusions with an original outer diameter of 63.5 mm and wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm quasi-statically cut by the multi-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly are presented in Figure 6.68 through Figure 6.70, respectively.
It was observed in all quasi-static axial cutting tests that the cutter blades penetrated through the extrusion. Chips formed ahead of cutter blade tip and no crack propagation was observed for any tests. Localized material fracture of cut petalled sidewalls was also observed in the cutting tests which resulted in the oscillation of axial cutting force as shown in Figure 6.62 through Figure 6.70. Material fracture was observed to be more prevalent for the extrusions with the largest outer diameter and the extrusions with the smallest wall thickness depending upon the axial bent curvature of the petal sidewalls before contacting with the deflector.

It can be seen from Figure 6.62 through Figure 6.70 that the axial cutting force generally increased with the increase of cutter blade quantities for all the extrusions considered. However, for the extrusions \((D_o = 44.45 \text{ mm}, 50.8 \text{ mm}, \text{ and } 63.5 \text{ mm})\) with a reduced wall thickness of 1.0 mm the axial cutting forces observed for the 4-blade and 5-balde cutters were very similar after contact between the cut petalled sidewalls and the deflector occurred, which was due to the large degree of localized material fracture occurred on the petalled sidewalls when a 5-blade cutter was utilized.

Comparison of load/displacement profiles for the extrusions \((D_o = 44.45 \text{ mm}, 50.8 \text{ mm}, \text{ and } 63.5 \text{ mm})\) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly were presented in Figure 6.71 through Figure 6.73, respectively. It was observed from Figure 6.71 that the axial cutting forces were very similar up to a crosshead displacement of approximately 40 mm for the extrusions \((D_o = 44.45 \text{ mm}, 50.8 \text{ mm}, \text{ and } 63.5 \text{ mm})\) with a reduced wall thickness of 1.0 mm. With the progress of the cutting process, the cutting force oscillated for the extrusions with \(D_o = 44.45 \text{ mm} \text{ and } 50.8 \text{ mm} \text{ and } Y = 1.0 \text{ mm}\) due to localized material fracture. For the extrusions with \(D_o = 63.5 \text{ mm} \text{ and } Y = 1.0 \text{ mm}\), the curvature of cut petalled sidewalls did not conform well with the surface profile of the deflector and buckling of the cut petalled sidewalls was observed, which resulted in a significant increase of axial cutting force from a crosshead displacement of approximately 40 mm to 60 mm. The axial cutting force reduced rapidly after approximately 60 mm displacement and fluctuated significantly due to the ‘buckling’ and material fracture of the cut petalled sidewalls. Similar observations were observed for the extrusions with \(D_o = 63.5 \text{ mm} \text{ and } Y = 1.25 \text{ mm} \text{ and } 1.5 \text{ mm}\). Generally, the axial cutting
force slightly increased with the increase of tube diameter as shown in Figure 6.72 and Figure 6.73. Similar observations can be found for the extrusions ($D_o = 44.45$ mm, 50.8 mm, and 63.5 mm) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 3-blade/5-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly.

Comparison of load/displacement profiles for the extrusions ($Y = 1.0$ mm, 1.25 mm, and 1.5 mm) with an original extrusion diameters of 44.45 mm, 50.8 mm, and 63.5 mm cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly were illustrated in Figure 6.74 through Figure 6.76, respectively. It can be seen that the axial cutting force increased with the increase of extrusion wall thickness. Similar observations can be found for the extrusions ($D_o = 44.45$ mm, 50.8 mm, and 63.5 mm) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 3-blade/5-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly.

![Figure 6.62 Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 44.45 mm and a reduced wall thickness of 1.0 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.](image-url)
Figure 6.63  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 44.45 mm and a reduced wall thickness of 1.25 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.

Figure 6.64  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 44.45 mm and a reduced wall thickness of 1.5 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.
Figure 6.65  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 50.8 mm and a reduced wall thickness of 1.0 mm cut by the RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under quasi-static loading.

Figure 6.66  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 50.8 mm and a reduced wall thickness of 1.25 mm cut by the RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under quasi-static loading.
Figure 6.67 Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 50.8 mm and a reduced wall thickness of 1.5 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.

Figure 6.68 Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 63.5 mm and a reduced wall thickness of 1.0 mm cut by the RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.
Figure 6.69  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 63.5 mm and a reduced wall thickness of 1.25 mm cut by the RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under quasi-static loading.

Figure 6.70  Load/displacement profiles for the AA6061-T6 extrusions with an original outer diameter of 63.5 mm and a reduced wall thickness of 1.5 mm cut by the RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under quasi-static loading.
Figure 6.71 Comparison of load/displacement profiles for the AA6061-T6 extrusions ($Y = 1.0$ mm) with different outer diameters cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.

Figure 6.72 Comparison of load/displacement profiles for the AA6061-T6 extrusions ($Y = 1.25$ mm) with different outer diameters cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.
Figure 6.73 Comparison of load/displacement profiles for the AA6061-T6 extrusions ($Y = 1.5$ mm) with different outer diameters cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.

Figure 6.74 Comparison of load/displacement profiles for the AA6061-T6 extrusions ($D_o = 44.45$ mm) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.
Figure 6.75 Comparison of load/displacement profiles for the AA6061-T6 extrusions ($D_o = 50.8$ mm) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.

Figure 6.76 Comparison of load/displacement profiles for the AA6061-T6 extrusions ($D_o = 63.5$ mm) with reduced wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under quasi-static loading.
6.3.3.3 Cutting test results comparison amongst the specimens in groups 49 through 98

Comparisons of representative load/displacement profiles for dynamic and quasi-static cutting of extrusions ($Y = 1.25$ mm) with an outer diameter of 44.45 mm, 50.8 mm and 63.5 mm are presented in Figure 6.77 through Figure 6.79. As can be seen, the main difference is related to the initial part of the impact cutting test where the dynamic forces are significantly higher. As strain-rate effects are assumed to be of minor importance as discussed in section 2.6, the observed difference is attributed to either stress wave propagation, which is only significant when displacements are close to zero, and/or to inertia effects, mostly, associated with the cutter/deflector assembly, that is most significant at the instant of impact just prior to the cutting process. After this initial cutting process, the dynamic cutting forces were typically lower than the quasi-static cutting forces. This is consistent with theoretical expectations due to the deceleration of the cutter/deflector assembly. The cutting force oscillated slightly during the majority of the displacement due to (i) the occurrence of material fracture on the cut petalled sidewalls, generally away from the cutting zone, and (ii) shifting of the cutter in the dynamic test. These phenomena were more prevalent during the dynamic cutting tests which resulted in the dynamic mean cutting force typically to be greater than the quasi-static mean cutting force. It is noted that the rapid increase of the dynamic cutting force shown in Figure 6.78 was due to the ‘buckling’ or ‘kinking’ effect of the cut petalled sidewalls as discussed in section 6.3.3.1.
Figure 6.77  Load/displacement profiles for specimens with an outer diameter of 44.45 mm and a reduced wall thickness of 1.25 mm cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under dynamic and quasi-static loading conditions.

Figure 6.78  Load/displacement profiles for specimens with an outer diameter of 50.8 mm and a reduced wall thickness of 1.25 mm cut by the 4-blade RevII cutter \((T = 1.0 \text{ mm})\) and curved deflector assembly under dynamic and quasi-static loading conditions.
Figure 6.79  Load/displacement profiles for specimens with an outer diameter of 63.5 mm and a reduced wall thickness of 1.25 mm cut by the 4-blade RevII cutter ($T = 1.0$ mm) and curved deflector assembly under dynamic and quasi-static loading conditions.

6.3.3.4 Comparison of cutting performance parameters

This section compares the cutting performance parameters for the specimens in groups 49 through 98. For each specimen tested, the axial cutting force and crosshead displacement were recorded. Post-testing data analysis was completed to determine the peak cutting load ($P_{\text{max}}$), mean cutting force ($P_m$), crush force efficiency ($CFE$), total energy absorption ($TEA$) and the specific energy absorption ($SEA$). The mean values of these cutting performance measures for each group are summarized in Table 6.7 through Table 6.9.
Table 6.7 Calculated mean values of cutting performance parameters for groups 49 through 64.

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<th>Group</th>
<th>Specimen ID</th>
<th>( P_m ) (kN)</th>
<th>( P_{\text{max}} ) (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
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Table 6.8 Calculated mean values of cutting performance parameters for groups 65 through 80.

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<td>2.98</td>
<td>20.37</td>
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Table 6.9  Calculated mean values of cutting performance parameters for groups 81 through 98.

<table>
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<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{\text{max}}$ (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
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<td>60.60</td>
<td>1.84</td>
<td>14.92</td>
</tr>
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<td>83</td>
<td>R250_D63.5_t1.0_RevII_5T1.0_CD_Dyn</td>
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<td>13.71</td>
</tr>
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<td>77.27</td>
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<td>12.18</td>
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<tr>
<td>94</td>
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<td>17.23</td>
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<tr>
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<td>98</td>
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<td>24.92</td>
<td>26.93</td>
<td>90.10</td>
<td>3.44</td>
<td>18.43</td>
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</table>
6.3.3.4.1 The peak cutting load and the mean cutting force

It can be observed from Table 6.7 through Table 6.9 that the peak cutting load and the mean cutting force generally increased with the increase of number of cutter blades except for the extrusions with an outer diameter of 63.5 mm whose axial cutting load significantly increased due to the switch in deformation mode from cutting to global buckling. The peak cutting load and the mean cutting force increased with the increase of extrusion wall thickness and extrusion diameter.

For the extrusion geometries and cutter blade quantity considered, the dynamic peak cutting load and the mean cutting force ranged from 14.45 kN to 31.31 kN and from 8.28 kN to 20.56 kN, respectively. The quasi static peak cutting load and the mean cutting force ranged from 9.07 kN to 26.93 kN and from 7.06 kN to 24.92 kN, respectively. The dynamic to quasi-static values of $P_{max}$ and $P_m$ ranged from 1.01 to 1.50 and from 0.84 to 1.28, respectively, exclude the specimen with an outer diameter of 63.5 mm cut by the 3-blade RevII cutter, where the increase of axial cutting force due to ‘buckling’ of cut petalled sidewalls was prevailing during the cutting process.

6.3.3.4.2 Total energy absorption and crush force efficiency

For the dynamic cutting tests conducted, the $TEA$ ranged from 1.50 kJ to 2.18 kJ, which was mostly due to the different cutting deformation characteristics observed as detailed in Table 6.7 through Table 6.9. It was found that shifting of the cutter/deflector assembly typically reduced the $TEA$ of the extrusion in the axial direction. For the cutting tests associated with ‘buckling’ or ‘kinking’ of cut petalled sidewalls; the mean axial cutting force was generally increased, which resulted in a significant increase in the calculated total energy absorption in the axial direction. Good flaring of the cut petalled sidewalls also decreased the $TEA$ compared to a pure cutting process without flaring by reducing the ‘clamping’ force near the cutter blade tip by adding normal force between the cut petalled sidewalls and the deflector. More detail discussion on how the axial cutting resistance force was affected by adding a deflector can be found in Section 9. Although the travelling distance of the drop entity varied depending on the actual cutting
displacement, the influence to the $TEA$ was observed to be minor. The $CFE$ for all the dynamic axial cutting tests ranged from 35.8% to 75.1%. For the quasi-static cutting tests conducted, the $TEA$ and $CFE$ ranged from 1.01 kJ to 3.44 kJ and from 70.2% to 90.8%, respectively.

6.3.4 Dynamic and quasi-static cutting test results for the specimens utilizing dual cutters configuration

In this section, load/displacement and energy absorption characteristics of the extrusions utilizing dual cutters configuration will be examined under both impact and quasi-static loading conditions.

6.3.4.1 Dynamic cutting test results for the specimens in groups 99 and 100

The dual-stage axial cutting of the circular AA6061-T6 extrusions with wall thicknesses of 1.0 mm and 1.2 mm under impact loading typically lasted 27-36 ms. The cutter blades penetrated into the circular extrusions and chips were formed. No crack propagation was observed for any of the tests. Although both the upper load cell and the lower load cell were used in the experimental testing, only the recorded load from the lower load cell will be presented for discussion and comparison purpose for the same reasons discussed in section 6.3.2.1.

The observed force versus displacement responses for the specimens with wall thicknesses of 1.0 mm and 1.2 mm under impact loading using dual cutters are presented in Figure 6.80. The load/displacement responses in Figure 6.80 demonstrate that dual-stage cutting is the superposition of two single stage cutting processes. The photograph in Figure 6.81(a) shows that all the cut petalled sidewalls as a result of the first stage cutting passed the second cutter. However, Figure 6.81(b) and Figure 6.81(c) clearly illustrate that, in some dual-state cutting tests, some cut petalled side walls failed to pass through the second stage cutter and formed multiple folds after passing through the first stage cutter. The fluctuation of cutting force during the second stage cutting process may be due to the folding formation of some cut side walls throughout the second stage cutting.
Figure 6.80  Load/displacement profiles for the specimens with wall thicknesses of 1.0 mm and 1.2 mm considered for the dual-stage cutting configuration under impact loading.

Figure 6.81  Photographs captured after completion of tests. (a) Cut petalled side walls passed through the second stage cutting process, (b) reduced number of cut side walls passed through the second cutter, and (c) formation of folds of the cut sidewalls which failed to pass through to the second cutter.

6.3.4.2 Quasi-static cutting test results for the specimens in groups 101 and 102

Quasi-static dual stage cutting tests were completed on the extrusions with wall thicknesses of 1.0 mm and 1.2 mm. Similar observations to the dynamic testing were
observed for the quasi-static tests. The observed load/displacement profiles are presented in Figure 6.82. The load/displacement responses of the extrusions at the second cutting stage were observed to be a superposition of two single cutting processes.

![Load/displacement profiles](image)

Figure 6.82 Load/displacement profiles for the specimens with wall thicknesses of 1.0 mm and 1.2 mm considered for the dual-stage cutting configuration under quasi-static loading.

### 6.3.4.3 Cutting test results comparison amongst the specimens in groups 99 through 102

The comparison of load/displacement responses between dynamic and quasi-static cutting tests for representative test specimens with wall thicknesses of 1.0 mm and 1.2 mm are presented in Figure 6.83 and Figure 6.84, respectively. Similar to the single stage cutting process, the main difference is related to the initial part of the impact cutting test where dynamic forces are significantly higher. The dynamic cutting force was consistent with observations from the quasi-static tests during the first cutting stage. However, dynamic tests, the extrusions with \( t = 1.2 \) mm experienced approximately 6% higher cutting force compared to the quasi-static cutting force during the second cutting stage. For specimens with \( t = 1.0 \) mm wall thickness, the dynamic cutting force was observed to be consistent to quasi-static tests during the second cutting stage. These observations were attributed to the degree of localized material fracture within the specimens and the formation of folds occurring on some cut petalled sidewalls.
Penetration of the second cutter to initiate the second cutting stage for extrusions under dynamic loading was observed to occur approximately 2~4 mm earlier than that for specimens under quasi-static loading. This result was observed due to the nature of the displacement measurement technique used in the impact tests. Displacements were measured for the 25.4 mm thick impacting aluminum plate which slightly lagged behind in displacements compared to the cutter as a result of the impact process between the plate and cutter/deflector assembly.

**Figure 6.83** Load/displacement profiles for specimens with a wall thickness of 1.0 mm considered for the dual-state cutting configuration under dynamic and quasi-static loading conditions.
6.3.4.4 Comparison of cutting performance parameters

This section compares the cutting performance parameters for the specimens in groups 99 through 102. For each specimen tested, the axial cutting force and crosshead displacement were recorded. Post-testing data analysis was completed to determine the peak cutting load \( P_{\text{max}} \), mean cutting force \( P_m \), crush force efficiency \( CFE \), total energy absorption \( TEA \) and the specific energy absorption \( SEA \). The mean values of the cutting performance measures for each group are summarized in Table 6.10.

For the dynamic dual-stage cutting tests, the AA6061-T6 extrusions with 1.2 mm wall thickness experienced approximately 35% higher cutting force compared to extrusions with a wall thickness of 1.0 mm. A 0.2 mm reduction of wall thickness increased the displacement of the cutter/deflector assembly by approximately 13 mm. The total energy absorption was observed to be 1.37 kJ and 1.36 kJ for specimens with wall thicknesses of 1.2 mm and 1.0 mm, respectively.

The observed average total energy absorption for all specimens with wall thicknesses of 1.2 mm and 1.0 mm under quasi-static loading was determined to be
2.87 kJ and 2.35 kJ, respectively, which surpassed the total energy absorbed for the same geometry extrusions that underwent the progressive folding deformation mode.

The ratio of dynamic to quasi-static values of $P_m$ was observed to be 0.94 and 1.01 for extrusions with $t = 1.0$ mm and $t = 1.2$ mm, respectively, using the dual-cutter configuration. The ratio of dynamic to quasi-static values of $P_{max}$ was found to be 1.1 and 1.06 for extrusions with wall thickness of 1.0 mm and 1.2 mm, respectively.

Table 6.10 Calculated mean values of cutting performance parameters for groups 99 through 102.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
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<td>T6_R150_D50.8_t1.0_RevI&amp;RevII_4T1.0_CD_Dyn</td>
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<td>20.81</td>
<td>70.55</td>
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<td>5.43</td>
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<td>1.37</td>
<td>5.24</td>
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<td>2.02</td>
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<td>9.45</td>
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6.3.5 Dynamic and quasi-static cutting tests for controlling the load/displacement response of the extrusion

In this section, controlling of load/displacement responses of the extrusions with variable instantaneous wall thickness in the axial direction under both impact and quasi-static loading conditions will be discussed as potential adaptive energy absorbers.

Dynamic and quasi-static cutting tests were carried out with a single cutter/deflector assembly or a single cutter only. For all cutting tests, the cutter blades penetrated into the circular extrusions and chips were formed. No crack propagation was observed for any test.

6.3.5.1 Dynamic and quasi-static cutting test result for the specimens in groups 103 through 106 utilizing a cutter and deflector assembly

Although both the upper load cell and the lower load cell were used in the impact testing, only the recorded load from the lower load cell will be presented for discussion and comparison purpose for the same reasons detailed in section 6.3.2.1. The
load/displacement responses for the extrusions with geometries as shown in Figure 4.9(a) and Figure 4.9(b) dynamically and quasi-statically cut by a cutter/deflector assembly are shown in Figure 6.85 and Figure 6.86, respectively.

6.3.5.1.1 Dynamic cutting test results for the specimens in groups 103 and 104

It can be seen from Figure 6.85 and Figure 6.86 that the dynamic cutting force exhibited a higher peak load at the initial transient cutting stage. Then the cutting force slightly oscillated and reached the first steady-state cutting stage. With the progress of the cutting process, the petalled sidewalls interacted with the deflector (at a displacement of approximately 25-30 mm) resulting in a slight drop of the cutting force. Then the cutting force reached its second steady-state cutting stage after a displacement of approximately 30-35 mm. After that, for extrusions with geometry as shown in Figure 4.9(a), the cutting force started to climb at a displacement of approximately 72 mm and reached its third steady-state cutting stage at an approximate displacement of 85 mm. For extrusions with geometry as shown in Figure 4.9(b), the cutting force started to ramp at a displacement of approximately 58 mm and reached its third steady-state cutting stage at an approximate displacement of 95 mm. The occurrence of the cutting force climbing or ramping, before reaching the third steady-state cutting stage, was observed to be approximately 2-3 mm ahead of the extrusion wall thickness change, which is mostly due to the intermittent contact between the upper load cell and the drop mass (displacement is measured on the drop mass). Vibration of the droptower may also contribute to the error of displacement measurement. Generally, the load/displacement responses followed the variation of the extrusion’s wall thicknesses.
Figure 6.85  Load/displacement profiles for the extrusions with geometries as shown in Figure 4.9(a) cut by a single cutter/deflector assembly under both impact and quasi-static loading conditions.

Figure 6.86  Load/displacement profiles for the extrusions with geometries as shown in Figure 4.9(b) cut by a single cutter/deflector assembly under both impact and quasi-static loading conditions.
For both geometries of extrusion after the third steady-state stage had been reached, the cutting force oscillated slightly due to localized material fracture that occurred on the petalled sidewalls after interacting with the deflector. The final surge of the cutting force in some cases was due to the shifting of the cutter, resulting contact between the extrusion side walls and the cutter outer rim or inner hub.

At the second steady-state cutting stage, the steady-state cutting force was observed to be approximately 6.5 kN for the extrusions with wall thickness of 0.794 mm. At the third steady-state cutting stage, the steady-state cutting force was observed to be approximately 13.5 kN for the extrusions with wall thickness of 1.587 mm. The third steady-state cutting force was observed to be slightly more than double that of the second steady-state cutting force, while the wall thickness was exactly twice as large at the third steady-state cutting stage compared to the second steady-state cutting stage. This difference is mostly due to the shifting of the cutter/deflector assembly at the third steady-state cutting stage.

6.3.5.1.2 Quasi-static cutting test results for the specimens in groups 105 through 106

It can be seen from Figure 6.85 and Figure 6.86 that the quasi-static cutting load increased and reached its first steady-state stage after an approximately 8 mm displacement. With the progress of the cutting process, the cutting load increased when the petalled walls made contact with the deflector at approximately 30 mm displacement. Then, with the outward flaring of the petalled sidewalls, the cutting load dropped to some extent and reached the second steady-state cutting stage after a displacement of approximately 35-42 mm. After that, for extrusions with geometry as shown in Figure 4.9(a), the cutting force started to climb at a displacement of approximately 75 mm and reached its third steady-state cutting stage. For extrusions with geometry as illustrated in Figure 4.9(b), the cutting force started to ram at a displacement of approximately 60 mm and, then, reached its third steady-state cutting stage at a displacement of approximately 90 mm. The occurrence of the cutting force climbing or ramping was observed to generally match the variation of the extrusions wall’s thickness. For both geometries of extrusion after the third steady-state stage had been reached, the
cutting force oscillated slightly due to localized material fracture that occurred on the petalled sidewalls after interacting with the deflector. The material fracture observed here was much less extent compared to what had been observed in the dynamic testing.

At the second steady-state cutting stage, the steady-state cutting force was observed to be approximately 7 kN for the extrusions with wall thickness of 0.794 mm. At the third steady-state cutting stage, the steady-state cutting force was observed to be approximately 16.5 kN for the extrusions with wall thickness of 1.587 mm. Similar to the dynamic loading condition, the third steady-state cutting force was observed to be slightly more than twice as large as the second steady-state cutting force while the wall thickness was exactly twice as large at the third steady-state cutting stage compared to the second steady state cutting stage.

6.3.5.1.3 Cutting test results comparison amongst the specimens in groups 103 through 106

It can be seen from Figure 6.85 and Figure 6.86 that the main difference is related to the initial part of the impact cutting test where the dynamic force was approximately 1.08-1.74 times higher than that under the quasi-static cutting test. The displacement needed to reach the first steady-state cutting process under impact was observed to be slightly less than that needed for the quasi-static tests. After this initial cutting process, the dynamic cutting forces were typically lower than the quasi-static cutting forces. Since the AA6061-T6 is strain-rate insensitive and has the lowest degree of material hardening characteristics, the measured dynamic cutting force was slightly lower than the quasi-static cutting force due to the lower value of the coefficient of friction between the cutter blades and the sidewalls under dynamic loading.

Dynamic cutting forces were generally consistent with the observed quasi-static loads during the majority of the displacement. At the third steady-state cutting stage, the dynamic cutting force fluctuated significantly due to the occurrence of material fracture, generally away from the cutting zone.

The cutting performance measures for the specimens in groups 103 and 111 were calculated and the mean values for each group summarized in Table 6.11. It can be found from Table 6.11 that the average mean cutting forces for the extrusions with geometries
as illustrated in Figure 4.9(a) and Figure 4.9(b) under impact loading were determined to be 10.61 kN and 10.70 kN, respectively. Though the drop entity height and the pneumatic assist setup were the same for all dynamic testing, the total energy absorption was different depending on the total travelling distance of the drop entity. The average $P_m$ for both the extrusions under quasi-static loading was calculated to be 11.52 kN and 10.95 kN, respectively. The ratio of dynamic to quasi-static values of $P_m$ was observed to be 0.92 and 1.09 for extrusions with geometries as shown in Figure 4.9(a) and Figure 4.9(b) under single stage cutting processes, which show the strain rate insensitivity property of AA6061 material again.

Table 6.11 Calculated mean values of cutting performance parameters for groups 103 through 111.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
<th>SEA (kJ)</th>
</tr>
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<tbody>
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<td>52.10</td>
<td>1.47</td>
<td>5.62</td>
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<td>105</td>
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<td>7.87</td>
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<td>10.82</td>
</tr>
<tr>
<td>111</td>
<td>T6_R300_D50.8_config(g)_RevI_4T1.0_CD_QS-1</td>
<td>33.94</td>
<td>44.18</td>
<td>76.80</td>
<td>4.88</td>
<td>14.22</td>
</tr>
<tr>
<td></td>
<td>T6_R300_D50.8_config(g)_RevI_4T1.0_CD_QS-2</td>
<td>29.90</td>
<td>44.36</td>
<td>67.40</td>
<td>4.31</td>
<td>13.36</td>
</tr>
<tr>
<td></td>
<td>T6_R300_D50.8_config(g)_RevI_4T1.0_CD_QS-3</td>
<td>39.72</td>
<td>45.38</td>
<td>87.50</td>
<td>5.85</td>
<td>16.09</td>
</tr>
<tr>
<td></td>
<td>T6_R300_D50.8_config(g)_RevI_4T1.0_CD_QS-4</td>
<td>41.59</td>
<td>44.04</td>
<td>94.40</td>
<td>5.90</td>
<td>15.35</td>
</tr>
</tbody>
</table>

For the extrusions under both dynamic and quasi-static loading conditions, the cutting load responses generally agreed with the variation of the extrusion’s wall
thickness. The cutting forces were slightly more than doubled when the wall thickness was doubled from 0.794 mm to 1.587 mm under both loading conditions. Moreover, the implementation of the deflector seemed to have a minor influence on the relation between the cutting force and the extrusion instantaneous wall thickness (there was a slight drop of the cutting force due to the interaction between the petalled side walls and the deflector). For this reason, further quasi-static cutting tests, without the presence of a deflector, for the extrusions with more variation in the wall thicknesses will be presented in the following section in order to further investigate the controllability of the load/displacement responses as well as the relationship between the cutting force and the wall thickness.

6.3.5.2 Quasi-static cutting test results for the specimens in groups 107 through 111 utilizing a single cutter only

Five geometries of extrusions with variation in wall thicknesses as shown in Figure 4.10(c) through Figure 4.10(g) were considered for the cutting tests without the presence of a deflector. Tapered and stepped changes of the wall thickness were implemented into these geometries.

Figure 6.87 illustrates the load/displacement profiles for the extrusions with geometries as shown in Figures 3(c)-(f) cut quasi-statically by a single cutter. Figure 6.88 illustrates the load/displacement responses for the extrusions with geometries as shown in Figure 4.10(g) \((Y = 0.794 \text{ mm}, 1.587 \text{ mm}, 2.381 \text{ mm}, \text{ or } 3.175 \text{ mm})\) cut quasi-statically by a single cutter. No material fracture was observed throughout these cutting tests. The changes of the cutting load followed the changes of the wall thickness of the extrusion for both tempers. The observed steady-state cutting forces for the extrusions with an instant wall thickness of 0.794 mm, 1.587 mm, 2.381 mm, and 3.175 mm were observed to be 6.0 kN, 18.2 kN, 30.9 kN, and 43.3 kN, respectively. The relationship of the steady state-cutting force and the extrusion wall thickness is presented in Figure 6.89. As can be seen from Figure 6.89, an almost linear relationship between the steady-state cutting force and the extrusion wall thickness for both temper extrusions was observed. The calculated mean values of the cutting performance measures are presented in Table 6.11.
Figure 6.87  Load/displacement profiles for the extrusions with geometries as shown in Figure 4.10(c) through Figure 4.10(f) cut by a single cutter only under quasi-static loading.

Figure 6.88  Load/displacement profiles for the extrusions with geometries as shown in Figure 4.10(g) cut by a single cutter only under quasi-static loading.
6.3.5.3 Discussion on the control of load/displacement and energy absorption

Sections 6.3.5.1 and 6.3.5.2 of this dissertation have shown that control of the load/displacement of circular AA6061-T6 extrusions under both dynamic and quasi-static loading conditions can be accomplished through the variation of instantaneous wall thickness of the extrusion in the axial direction. Although the total energy absorption of an extrusion experiencing a single cutting deformation mode is usually not as efficient as the same extrusion undergoing a progressive folding deformation mode, it is much more efficient than a global bending deformation mode [34]. When a dual-stage cutting mode is applied, the total energy absorption of an extrusion surpasses that of a progressive folding mode as shown in section 6.3.4.4. Knowing the relationship between the mean cutting force and the extrusion wall thickness for the cutting deformation mode and the relationship between the peak cutting force and the extrusion wall thickness for the progressive folding or global bending mode, an adaptive energy absorption system - axial cutting of an extrusion - can be designed through the control of the desired load/displacement profiles under different axial loading conditions. Material fracture will
slightly reduce the efficiency of the adaptive energy absorption system. The present research shows that the AA6061-T6 material is a good candidate for desired constant mean cutting force response under quasi-static loading. When dynamic loading is applied to this material, the mean cutting force will be reduced due to material fracture occurring on the petalled sidewalls, which will reduce the energy absorption efficiency of the system. Relative thicker wall thickness and appropriate design of the cutter will significantly reduce the occurrence of material fracture of the extrusions.

6.4 Scanning Electron Microscope (SEM) observation for the cut extrusion

As discussed in section 4.3.7, SEM observation of the cut extrusions with wall thicknesses of 3.175 mm and 1.587 mm were completed and observed results are presented in Figure 6.90 through Figure 6.94.

Figure 6.90 illustrates the chip formation at the inside of the extrusion with a wall thickness of 1.587 mm. It can be seen from Figure 6.90 that material fracture occurred on the formed chip as well as at the base of the chip. However, no crack propagation was observed ahead of the chip formation zone. The width of the formed chip was estimated to be the same size as the width of cutting blade tip. Similar observations were found at the outside of the extrusion as well as for the extrusion with a wall thickness of 3.75 mm.

Figure 6.91 and Figure 6.92 show the chip formation and the plastic/fracture zone in the vicinity of the cutting blade tip for the extrusions with wall thicknesses of 1.587 mm and 3.175 mm, respectively. It can be observed from Figure 6.91 and Figure 6.92 that material in the vicinity of the cutting blade tip rolled up and formed cylindrical transient flap starting from the cracked chip base. Localized material fracture was observed on the boundary of the formed transient flap. Again, no material crack propagation was observed ahead of the cutting blade tip. It can be estimated from Figure 6.91 and Figure 6.92 that the lengths of plastic zone ahead of the blade tip were approximately 2.4 mm and 2.8 mm for the extrusions with wall thicknesses of 1.587 mm and 3.175 mm, respectively. The lengths of plastic zone in the circumferential direction were estimated to be 1.6 mm and 3.2 mm at one side the blade for the extrusions with wall thicknesses of 1.587 mm and 3.175 mm, respectively.
Figure 6.93 and Figure 6.94 illustrate the SEM observations of the transient and stable cut surfaces for the extrusion with wall thicknesses of 1.587 mm and 3.175 mm, respectively. It is obvious that the transition between the transient and stable cut surfaces was very smooth and no crack was observed in this transition zone. Roll-up of the transient and stable flaps and localized material fracture of the flaps were observed. The streamline of the tube material at both transient and stable surfaces followed the tensile direction of the extrusion material, which indicated that the dominant stresses were tension/compression.

Figure 6.90  SEM observation of the chip formation at the inside of the extrusion with a wall thickness of 1.587 mm.
Figure 6.91 SEM observation of the chip formation and plastic/fracture zone in the vicinity of the cutting blade tip for the extrusion with a wall thickness of 1.587 mm.

Figure 6.92 SEM observation of the chip formation and plastic/fracture zone in the vicinity of the cutting blade tip for the extrusion with a wall thickness of 3.175 mm.
Figure 6.93  SEM observation of the transient and stable cut surfaces for the extrusion with a wall thickness of 1.587 mm.

Figure 6.94  SEM observation of the transient and stable cut surfaces for the extrusion with a wall thickness of 3.175 mm.
7 FINITE ELEMENT MODELLING AND SIMULATION METHOD

Finite element (FE) models were developed to simulate the axial cutting of circular AA6061-T6 extrusions in order to better understand the deformation mechanisms that lead to energy absorption. In these models, axial cutting tests under impact and quasi-static loadings with or without the presence of deflector were considered for both single and dual-cutter configurations. Additionally, dynamic and quasi-static axial crushing of circular AA6061-T6 extrusions was also completed for comparison purpose with the cutting deformation. The explicit non-linear FE code LS-DYNA\textsuperscript{TM} [89, 90] was used to predict the response of the axial cutting or crushing of the thin-walled circular AA6061-T6 extrusions by employing an Eulerian FE formulation, in the case of the cutting deformation mode, or a Lagrangian FE formulation for progressive folding of the aluminum extrusions.

Extrusion geometries considered for FE modeling of the quasi-static axial cutting of the circular extrusions \((t = 3.175 \text{ mm as shown in Figure 4.3})\) by a cutter with multiple blades without the presence of deflector is discussed in this section. Moreover, FE simulations for the dynamic axial cutting of circular extrusions by a cutter(s)/deflector assembly are also presented for the single-cutter and dual-cutter configurations. The extrusions considered for the single-cutter and dual-cutter configurations were circular tubes with wall thicknesses of 1.587 mm as shown in Figure 4.3 and 1.2 mm as shown in Figure 4.4, respectively. Additionally, quasi-static and impact axial cutting simulations of the extrusions with a wall thickness of 1.5 mm and outer diameters of 44.45 mm, 50.8 mm, and 63.5 mm (as shown in Figure 4.4) incorporating a curved deflector were also completed to illustrate the predictive capability of the Eulerian FE models. Finally, FE modeling of the axial crushing processes under both dynamic and quasi-static loading conditions is presented for the circular tubes with wall thicknesses of 1.587 mm (as shown in Figure 4.3), 1.2 mm (as shown in Figure 4.4), and 1.0 mm (as shown in Figure 4.4).

7.1 Eulerian FE formulation for axial cutting tests

As discussed in section 2.5, although Lagrangian FE formulation is the most common in the majority of structural numerical simulations employing the FE method, in
large deformation processes the massive mesh distortion of Lagrangian type elements may lead to significant numerical error. An alternative element selection for large deformation processes is the Eulerian or Arbitrary Lagrangian/Eulerian (ALE) element formulations. As discussed in section 2.5, the Eulerian element formulation allows dissociation of material coordinates and spatial coordinates of the FE mesh and the material moves through the FE mesh. In the explicit time integration scheme, during every cycle (time step) of the simulation a Lagrangian formulation is first used to determine material and mesh deformation, however, prior to the next cycle the spatial coordinates of the FE mesh are remapped to their original position in a process referred to as advection, and material transport to the remapped mesh occurs. While the FE mesh is remapped to its original position, the material coordinates are not and will move through the FE mesh. Therefore, an airmesh must surround the original material location of the extrusion material for evaluation of the deformed material state. At the start of the simulation, the airmesh contains no material and its only purpose is to accommodate deformed material. In the literature, it has shown that the Eulerian FE formulation has the capability of generating new free surfaces as a result of material transport and can predict the axial cutting process with important energy dissipation mechanisms very well. Additionally, the SEM observation of the cut petalled sidewalls in section 6.4 showed that no crack propagation was observed ahead of the cutter blade for the axial cutting deformation, which allows proper usage of the Eulerian FE formulation for the axial cutting tests.

Disadvantages which may arise through use of an Eulerian FE formulation include larger CPU costs and a greater degree of mesh discretization. However this FE formulation is beneficial in dealing with the large plastic deformation processes and numerical instabilities associated with severe mesh distortion, which is the case for the axial cutting process.

7.1.1 Model geometry and discretization

Generation of the FE mesh for simulation of the axial cutting of the circular AA6061-T6 tubular extrusions was carried out using FEMB (finite element Model Builder).
7.1.1.1 Model geometry and discretization for quasi-static axial cutting tests

Due to the symmetry observed in the experimental quasi-static cutting process of the extrusions, only a portion of the tubular specimen and one corresponding cutter blade were considered in the FE model. In order to further save computational costs, only a 56 mm length of the tubular specimen was modeled for the cutting simulations without the presence of deflector since it was observed in the experimental tests that a steady-state cutting process was achieved after a cutter displacement of approximately 20 mm; a 100 mm length of the tubular specimen was modeled for the cutting tests with the presence of deflector since it was observed in the experimental tests that a steady-state cutting process was achieved after a cutter displacement of no more than 70 mm.

Eight-noded solid elements were utilized for the tubular extrusion, the airmesh, the cutter blade, and the deflector, as shown in Figure 7.1. For the axial cutting tests without the presence of the deflector, the deflector was removed from Figure 7.1. A single point quadrature Eulerian element was selected for the extrusion and the airmesh. The mesh density of the tube in the vicinity of the region of contact between the cutter and extrusion was finer than all other regions. Higher discretization was completed to ensure an accurate approximation of the stress distribution and deformation near the contact region. Chip formation of the extrusion material was observed in all experimental tests with an approximate thickness of 1 mm. In an attempt to appropriately predict the deformation behaviour and chip formation, the Eulerian mesh of the extrusion and airmesh were discretized with a smallest dimension of 0.27 mm employing an aspect ratio of 1.6 in the region of contact between the extrusion and cutter. Twelve Eulerian elements through the thickness of the tube near the contact region were utilized. Transition elements were introduced between the finer mesh and coarser mesh in three directions. The airmesh was modeled with an 8 mm and 12 mm radial offset from the inner and outer surfaces of sidewall of the tube in the contact region, respectively. In all other regions, a 2.2 mm radial offset from the inner and outer surfaces of the tube sidewalls was employed. The airmesh in the axial direction was offset 1.3 mm from the top surface of the tube. The dimensions of the airmesh were estimated based upon the extent of extrusion deformation observed in the experimental tests.
7.1.1.2 Model geometry and discretization for dynamic axial cutting tests

This model incorporated the extrusion, airmesh, cutter blade, deflector, upper load cell, and impacting plate. Figure 7.2(a) and Figure 7.2(b) illustrate the discretization of the apparatus utilized for impact loading under single- and dual-cutter configurations, respectively.

Due to the symmetry of the problem, one quarter of the tubular specimen and one corresponding cutter blade, deflector, upper load cell, and impacting plate were considered in the FE model. For extrusions dynamically cut by dual cutters, one eighth of the extrusion was considered for further reduction of the CPU time. All other apparatus was modeled with one quarter geometry, however, a reduced density of one-half for these parts was employed. In order to further save computational time, only 100 mm length of the tubular specimen was modeled since it was observed in the experimental tests that the maximum cutting displacement of the extrusions was no greater than 80 mm.
Eight-noded solid elements were utilized for the tubular extrusion and the airmesh. A single-point quadrature Eulerian element was selected for both entities. The mesh density of the tube in the vicinity of the region of contact between the cutter and extrusion was finer than all other regions. Higher discretization was completed near the vicinity of large extrusion deformation to ensure an accurate approximation of the stress distribution and deformation near the cutting region. Transition elements were introduced between the finer mesh and coarser mesh in three directions. At least four layers of elements were considered through the tube thickness (in the region of coarse discretization) in order to capture the bending deformation. Typically the aspect ratio of the elements in the extrusion and the airmesh was less than 3.

Eight-noded solid elements were used to model the cutter blade, the deflector, the load cell and the impacting plate. The degree of discretization was selected such that similar mesh densities between the extrusion, cutter blade and deflector were maintained. This modeling approach assists with ensuring a more appropriate numerical treatment of contact. Relatively larger mesh sizes, with respect to the extrusion, were applied to the load cell and the impacting plate. An under-integrated Lagrangian FE formulation was selected for the impacting plate, deflector, cutter(s) and load cell. Experimental evidence indicated that deformation occurred on the impacting plate. Thus the impacting plate was modeled as a combination of a deformable (solid entity) and a rigid plate (employing shell elements). The rigid portion of the impact plate was used to simplify the numerical model and ensure an appropriate mass distribution in the impact plate while not added to any addition computation requirements. The rigid portion of the impacting plate was modeled using Belytschko-Tsay shell elements which were constrained to the upper layer of nodes farthest from the impacting surface.
Figure 7.2  Discretization of the circular AA6061-T6 extrusion, extrusion airmesh, cutter blade, curved deflector, upper load cell, impacting plate for the impact cutting tests employing (a) single-cutter and (b) dual-cutter configurations.

7.1.2  Modeling contact

7.1.2.1  Modeling contact for quasi-static axial cutting tests

Contact between the Eulerian extrusion and airmesh and the Lagrangian FE cutter blade (or cutter blade and deflector) was completed through Eulerian/Lagrangian coupling by employing a single ‘CONSTRAINED_LAGRANGE_IN_SOLID’ contact definition available within LS-DYNA™. A penalty type contact formulation was employed in the normal direction through a $3 \times 3 \times 3$ point grid representing virtual nodes.
located at the Gauss points of the extrusion/airmesh. Contact forces at the interfaces were utilized a coefficient of friction specified as 0.22.

7.1.2.2 Modeling contact for dynamic axial cutting tests

Contact between Eulerian and Lagrangian FE meshes was completed through Eulerian/Lagrangian coupling. A penalty type contact formulation was employed in the normal direction through a $3 \times 3 \times 3$ point grid representing virtual nodes located at the Gauss points of the extrusion/airmesh. A coefficient of friction of 0.10 was specified for this contact definition which was estimated based upon sliding experiments between the extrusion and the cutter material. Within this contact algorithm only a single constant value of the coefficient of friction may be defined. Rigid bodies, consisting of the cutter blade(s), deflector, and upper load cell, were merged together such that their kinematics was coupled: this was consistent with the experimental testing apparatus. Contact between the Lagrangian FE meshes consisting of the impacting plate and load cell was completed using a single automatic surface-to-surface contact definition. Relative motion between the Lagrangian elements was modeled using static and dynamic coefficients of friction of 0.3 and 0.15, respectively.

7.1.3 Application of boundary conditions

7.1.3.1 Boundary conditions for quasi-static axial cutting tests

The quasi-static axial cutting process of the tubular specimens without the presence of deflector was modeled by prescribing a penetration of 35 mm in axial direction in 5 ms, which is equivalent to an average axial cutting speed of 7 m/s. The quasi-static axial cutting process of the tubular specimens with the presence of the curved deflector was modeled by prescribing a penetration of 70 mm in axial direction in 10 ms, which is also equivalent to an average axial cutting speed of 7 m/s. Motion of the cutter was constrained to the axial direction. Jones [4] noted that crushing speeds on the order of 10 m/s or less can be considered quasi-static. This facilitates the comparison of the FE results to the experimental quasi-static cutting test results. As discussed in section 2.6, AA6061-T6 material is strain-rate insensitive. A comparison of impact and quasi-static experimental observations also clearly indicated that for the impacting speeds considered...
in this research, significant rate effects were not generally observed. Furthermore, the negligible ratio between the kinetic energy and the internal (strain) energy during the axial cutting simulations indicated that the simulations were quasi-static in nature. Therefore, comparisons between the experimental and numerical testing methods are appropriate even though the numerical simulation of the cutting process occurred at a higher speed.

At the lower end of the extrusion, full boundary constraints were applied to all nodes. To ensure symmetry, nodes lying in the symmetry planes of the tube were constrained to move only within the corresponding symmetry plane.

### 7.1.3.2 Boundary conditions for dynamic axial cutting tests

Dynamic axial cutting of the tubular specimens was modeled by prescribing an initial velocity of 7.0 m/s to the impacting plate in a direction parallel with the axis of the extrusion. As a result of symmetry, motion of the impacting plate, load cell, deflector and cutter were constrained to the axial direction.

At the lower end of the extrusion, full boundary constraints were applied to all nodes. To ensure symmetry, nodes lying in the symmetry planes of the tube were constrained to move only within the corresponding symmetry plane.

### 7.1.4 Material models

An elastic plastic hydrodynamic material model was utilized to represent the material behaviour associated with the Eulerian AA6061-T6 aluminum extrusion. The selected material model incorporates the von Mises yield criterion and the equivalent von Mises stress is computed in terms of the deviatoric stress tensor as expressed in Equation (7.1). Furthermore, the effective plastic strain is calculated based upon the time integration associated with the plastic component of the rate of deformation tensor as expressed in Equation (7.2).

\[
\bar{\sigma} = \left( \frac{3}{2} S_{ij} \cdot S_{ij} \right)^{1/2}
\]  

(7.1)
Material yield behaviour, as a function of effective plastic strain, was specified in the material model through input of sixteen data points selected from the stress/plastic strain response of the AA6061-T6 material behaviour assessed from the information provided in Figure 6.1 and Table 6.1. Since the strain-rate sensitivity of aluminum is negligible for the impact speed considered as discussed in section 2.6, strain-rate effects were not considered in this material model. Comparison of the experimentally observed force/displacement responses during cutting deformation under the impact and quasi-static loadings also indicates little or no rate sensitivity.

An equation of state was utilized to describe the pressure/volume relationship associated with the AA6061-T6 material. A linear polynomial equation of state considering only the first order term associated with the volumetric strain was implemented. The first order term was specified as the elastic bulk modulus of the AA6061-T6 material (66.8 GPa).

In order to investigate the rate sensitivity of the AA6061-T6 alloy in the numerical simulations of the cutting deformation process, a piecewise linear plasticity material model, utilizing identical expressions for the effective stress and effective plastic strain as indicated in Equations (7.1) and (7.2), respectively, was implemented. A von Mises yield criteria is also used in this material model. Moreover, the Cowper-Symonds constitutive relationship as shown in Equation (1.32) was used to take in account any elevated rate loading effects. This material model was used in the dynamic axial cutting tests with the presence of straight deflector to illustrate the rate insensitivity of the AA6061-T6 extrusion material.

A rigid material definition was applied to the cutter, the deflector, the load cell, and the rigid portion of the impacting plate (or the crushing plate) as no apparent deformations were observed on these entities during the experimental testing. For the deformable portion of the impacting plate a similar material model to that used for the AA6061-T6 extrusion was implemented.
7.1.5 Simulation procedure

7.1.5.1 Simulation for quasi-static axial cutting tests

Simulations of the quasi-static axial cutting tests of the circular extrusions were completed using LS-DYNA™ version 971s R4.2 on a personal computer with quad-core 2.0 GHz Intel Xeon processors with 12 GB of dynamic random access memory. The simulation time for the FE model was approximately 22 hours.

7.1.5.2 Simulation for dynamic axial cutting tests

Simulations of the dynamic cutting tests were completed using LS-DYNA™ version 971 release 7600 on a personal computer with dual 2.0 GHz AMD Opteron processors with 4 GB of dynamic random access memory. Typical computation times were approximately 130 hours for the single- and dual-cutting simulations.

To investigate the influence of mass scaling on the results two impact simulations were performed with an increase in the density of the extrusion and airmesh of twenty times. Implementing mass scaling resulted in an approximate computational time of 38 hours for the single stage dynamic cutting simulations. A thorough examination of numerical results, with and without mass scaling, showed very minor differences in predictions of the cutting forces prior to petalled wall contact with the deflector. It was observed, and will be discussed further in section 8.1.2, that the increase in mass associated with the extrusion resulted in a significant force fluctuation occurring between the cut petalled sidewall and the deflector surface. However, the approximate mean values of these fluctuations were good estimates of the forces observed when mass scaling was not implemented.

7.2 Lagrangian FE formulation for axial crushing tests

Literature in section 2.5.1 shows that the Lagrangian FE formulation has been successfully used by other researchers to predict the crush behaviour of thin-walled tubular members. Therefore, this formulation was used in the present research for simulation of the dynamic and quasi-static crushing tests.
7.2.1 Model geometry and discretization

The AA6061-T6 circular tubular extrusion with a length of 300 mm was modeled using 10,000 Belytschko-Tsay shell elements to simulate the axial dynamic or quasi-static crushing tests. The aspect ratio of these elements was approximately 2. The crushing plate was modeled with eight-noded solid elements with an aspect ratio of 1.7. The discretization of the circular tube and the crushing plate is illustrated in Figure 7.3.

![Discretization of the circular AA6061-T6 extrusions](image)

Figure 7.3 Discretization of the circular AA6061-T6 extrusions ($L = 300$ mm, $D_o = 50.8$ mm, $t = 1.587$ mm, 1.2 mm, and 1.0 mm) and the crushing plate for the dynamic and quasi-static crushing tests.

7.2.2 Modeling contact

Contact between the crushing plate and the extrusion was implemented using a penalty based nodes to surface contact algorithm. Contact within the extrusions was specified using a single surface contact definition with static and dynamic coefficients of friction of 0.3 and 0.15, respectively.

7.2.3 Application of boundary conditions

Dynamic axial crushing of the tubular specimens was modeled by prescribing an initial velocity of 7.0 m/s to the crushing plate in a direction parallel with the axis of the extrusion. Quasi-static axial crushing of the specimens was completed by prescribing a 150 mm displacement to the crushing plate toward the extrusion. Full boundary constraints were applied to all nodes lying at the lower end of the extrusion.
7.2.4 Material models

Material model that is identical to that was used for the axial cutting tests was implemented in the Lagrangian simulations of axial crushing under quasi-static and dynamic loading.

7.2.5 Simulation procedure

Simulations of the axial crushing tests were completed using LS-DYNA™ version 971 release 7600 on a personal computer with dual 2.0 GHz AMD Opteron processors with 4 GB of dynamic random access memory. Axial crush simulations took approximately 20 minutes to complete.
FINITE ELEMENT SIMULATION RESULTS AND DISCUSSION

The results of the FE simulations of the axial cutting and crushing tests of the circular AA6061-T6 extrusions described in chapter 7 are presented and discussed in this chapter. The numerical results of quasi-static axial cutting tests of the circular extrusions without/with the use of deflector are presented in section 8.1.1. The FE results of the dynamic axial cutting tests of the extrusions for the single and dual-cutter configurations with the presence of a straight or curved deflector are discussed in section 0. Section 8.2 details the simulated results for the dynamic and quasi-static axial crush tests for the selected tube geometries. The results of the FE simulations will be presented in the form of load/displacement profiles, which are overlaid with the experimental load/displacement responses in order to illustrate the predictive capabilities of the FE models. In addition, deformed geometry plots are shown for selected simulations along with photographs taken for the corresponding experimental tests in order to illustrate the ability of the FE models of predicting the deformation of the extrusion. Comparisons of the cutting/crushing performance parameters from the numerical predictions and experimental observations are also presented in each section. Finally, validation assessments of the FE models are completed using the techniques introduced in section 2.5.3 and presented in section 8.3.

8.1 FE simulation results and discussion for axial cutting tests

8.1.1 FE simulation results and discussion for quasi-static axial cutting tests

The load/displacement profiles for the AA6061-T6 extrusions with a wall thickness of 3.175 mm cut experimentally and numerically by a cutter with different number of blades are presented in Figure 8.1. The load/displacement profiles for the AA6061-T6 extrusions \( t = 1.5 \text{ mm}, D_o = 44.45 \text{ mm}, 50.8 \text{ mm}, \text{ or } 63.5 \text{ mm} \) cut experimentally and numerically by the 4-blade RevII cutter with the presence of a curved deflector are shown in Figure 8.2 through Figure 8.4.

Numerical integration, employing a rectangular rule, of the force/displacement relationships for both experimental and numerical testing procedures was completed to determine the energy absorbed during the axial cutting process. Figure 8.5 shows the energy absorbed versus displacement relationships for the circular extrusions cut.
experimentally and numerically by a multiple bladed cutter. Images in Figure 8.6 illustrate the cutting test processes of an AA6061-T6 extrusion with a wall thickness of 3.175 mm cut numerically by a 5-blade cutter. The approximate cutter penetration is cited below each image. Similar cutting behaviour was observed for the other AA6061-T6 extrusions cut by a cutter with multiple blades.

It can be seen from Figure 8.6 that similar cutting behaviour was observed in the numerical simulations for the extrusions as that observed in the experimental testing detailed in section 6.3.1.2. The FE model predicted the major energy dissipation mechanisms very well, including the large localized plastic deformation zone in the vicinity ahead of the cutting blades, continuous chip formation, and circumferential membrane stretching of the extrusion, as well as cut petalled sidewalls bending outward. Consistent observations were also found in the experimental and numerical simulations for the quasi-static axial cutting of the circular AA6061-T6 extrusions ($t = 1.5$ mm, $D_o = 44.45$ mm, 50.8 mm, or 63.5 mm) cut by the 4-blade RevII cutter and curved deflector assembly.

Load/displacement curves shown in Figure 8.1 illustrate that the FE model predicted the transient and steady state cutting process very well for most cases. An over prediction of approximately 10% of the experimental steady-state constant cutting force was found for the cutting simulation of the AA6061-T6 extrusions cut by a 6-blade cutter, which is mostly due to the lack consideration of material fracture in the Eulerian FE model.

Load/displacement curves shown in Figure 8.2 through Figure 8.4 illustrate that the FE model generally predicted the cutting process very well with a under or over prediction of approximately 7% for the first steady-state cutting process. For the second steady-state cutting process, the numerical model over predict the cutting force approximately 12%-23%, which is mostly due to the lack consideration of material fracture in the Eulerian FE model. For the significant difference in the axial cutting force after approximately 45 mm displacement as shown in Figure 8.4, it was mostly due to less degree of cut petalled side wall bending was observed during the simulation, which resulted in the greater degree of ‘buckling’ of the cut petalled sidewalls.
Figure 8.1  Load versus displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion with a wall thickness of 3.175 mm cut by a cutter with multiple blades under quasi-static loading.

Figure 8.2  Load/displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion \((t = 1.5 \text{ mm}, D_o = 44.45 \text{ mm})\) cut by the 4-blade RevII cutter with the presence of a curved deflector under quasi-static loading.
Figure 8.3 Load/displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion ($t = 1.5$ mm, $D_o = 50.8$ mm) cut by the 4-blade RevII cutter with the presence of a curved deflector under quasi-static loading.

Figure 8.4 Load/displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion ($t = 1.5$ mm, $D_o = 63.5$ mm) cut by the 4-blade RevII cutter with the presence of a curved deflector under quasi-static loading.
Figure 8.5  Energy absorbed versus displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion with a wall thickness of 3.175 mm cut by a cutter with multiple blades under quasi-static loading.

Figure 8.6  Numerical axial cutting of the circular AA6061-T6 extrusions with a wall thickness of 3.175 mm cut by a 5-blade cutter under quasi-static loading.
Table 8.1 Cutting performance measures of the specimens from experimental and numerical testing under quasi-static loading. The prefix ‘Exp’ and ‘Sim’ in front of the name of specimens indicate the cutting measures are from experimental and numerical testing, respectively.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>(P_m) (kN)</th>
<th>(P_{\text{max}}) (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp_T6_R200_D50.8_t3.175_RevII_3T1.0_ND_QS</td>
<td>39.30</td>
<td>42.36</td>
<td>92.80</td>
<td>5.45</td>
</tr>
<tr>
<td>Exp_T6_R200_D50.8_t3.175_RevII_4T1.0_ND_QS</td>
<td>45.01</td>
<td>48.60</td>
<td>92.60</td>
<td>6.18</td>
</tr>
<tr>
<td>Exp_T6_R200_D50.8_t3.175_RevII_5T1.0_ND_QS</td>
<td>50.37</td>
<td>54.61</td>
<td>92.27</td>
<td>6.92</td>
</tr>
<tr>
<td>Exp_T6_R200_D50.8_t3.175_RevII_6T1.0_ND_QS</td>
<td>54.30</td>
<td>59.64</td>
<td>91.03</td>
<td>7.53</td>
</tr>
<tr>
<td>Exp_T6_R250_D44.45_t1.5_RevII_4T1.0_CD_QS</td>
<td>15.91</td>
<td>18.15</td>
<td>87.67</td>
<td>2.26</td>
</tr>
<tr>
<td>Exp_T6_R250_D50.8_t1.5_RevII_4T1.0_CD_QS</td>
<td>18.07</td>
<td>19.90</td>
<td>90.83</td>
<td>2.57</td>
</tr>
<tr>
<td>Exp_T6_R250_D63.5_t1.5_RevII_4T1.0_CD_QS</td>
<td>20.54</td>
<td>24.46</td>
<td>83.97</td>
<td>2.93</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_3T1.0_ND_QS</td>
<td>35.69</td>
<td>41.07</td>
<td>86.90</td>
<td>1.25</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_4T1.0_ND_QS</td>
<td>44.69</td>
<td>51.48</td>
<td>86.80</td>
<td>1.57</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_5T1.0_ND_QS</td>
<td>51.47</td>
<td>58.54</td>
<td>88.78</td>
<td>1.80</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_6T1.0_ND_QS</td>
<td>58.35</td>
<td>64.16</td>
<td>90.90</td>
<td>2.04</td>
</tr>
<tr>
<td>Sim_T6_R250_D44.45_t1.5_RevII_4T1.0_CD_QS</td>
<td>16.35</td>
<td>18.22</td>
<td>89.72</td>
<td>1.28</td>
</tr>
<tr>
<td>Sim_T6_R250_D50.8_t1.5_RevII_4T1.0_CD_QS</td>
<td>19.48</td>
<td>21.59</td>
<td>90.20</td>
<td>1.52</td>
</tr>
<tr>
<td>Sim_T6_R250_D63.5_t1.5_RevII_4T1.0_CD_QS</td>
<td>22.25</td>
<td>30.36</td>
<td>73.3</td>
<td>1.74</td>
</tr>
</tbody>
</table>

The corresponding energy absorbed versus displacement profiles shown in Figure 8.5 illustrate almost linear energy absorption versus displacement profile for the majority of the displacement domain. This should be expected due to the observed constant steady-state cutting force after approximately 20 mm cutter displacement. For both numerical and experimental cutting tests, the absorbed energy increased with the increase of number of blades. However, this increment decreased when increased the number of blades from 5 to 6.
Comparisons of the cutting performance parameters for both experimental and numerical cutting tests are presented in Table 8.1. The numerical observed CFE was determined to be slightly lower than the values from experimental observations, which was mostly due to the oscillations of the cutting force associated with the FE model. The numerical observed TEA was much lower than the values from experimental observations since only a total displacement of 35 mm was prescribed in the numerical simulations.

8.1.2 FE simulation results and discussion for dynamic axial cutting tests

It was observed from the simulated impact cutting tests with both single- and dual-cutter configurations that the cutter blades penetrated through the sidewall of the specimen and developed a large localized plastic deformation zone just ahead of the cutting blades. This localized plastic deformation zone moved along the extrusion as the cutting process continued. The deformed material rolled away from the sides of the cutter blade and chips were formed ahead of the blade tip during the cutting process. As the cutting process proceeded, the petalled sidewalls contacted the deflector and flared outward and formed a continuous region of contact with the deflector. In addition, circumferential membrane stretching of the tube specimens was also observed.

Figure 8.7 illustrates the differences observed when considering mass scaling and rate effects into the single stage cutting process. In these simulations, extrusions with $t = 1.587$ mm and a RevI cutter geometry with a straight deflector profile was utilized. Simulated results from Figure 8.7 showed that the strain-rate had minor effect on the cutting force. Cutting forces for displacements in the range of 0 mm to approximately 25 mm were in good agreement with each other, with the simulation using mass scaling predicting a higher cutting force. This finding should be expected as a result of the mass increase associated with the extrusion. At the approximate displacement of 25 mm contact between the cut petalled sidewalls occurred with the deflector surface. Force oscillations as a result of this contact are small when mass scaling was not implemented in the numerical model. However, when mass scaling was used, significant force fluctuations on the order of 8 kN between the estimated mean value and peak value were observed. However, regardless of the amplitude of the oscillations, the local mean force was approximately the same as that predicted without mass scaling.
Load/displacement profiles for the experimental and numerical impact tests for extrusion \( (t = 1.587 \text{ mm}, \ D_o = 50.8 \text{ mm}) \) and the RevI cutter geometry with straight and curved deflector profiles are presented in Figure 8.8 and Figure 8.9, respectively. The predictions in Figure 8.8 and Figure 8.9 are from numerical models which incorporated mass scaling. Load/displacement responses between numerical simulation predictions and experimental observations for extrusion \( (t = 1.2 \text{ mm}, \ D_o = 50.8 \text{ mm}) \) with a dual-cutter configuration under impact loading are presented in Figure 8.10. In this numerical model no mass scaling was utilized. Load/displacement profiles for the experimental and numerical impact tests of extrusions with \( t = 1.5 \text{ mm} \) and \( D_o = 44.45 \text{ mm}, \ 50.8 \text{ mm}, \) and \( 63.5 \text{ mm} \) utilizing the RevII cutter geometry and the curved deflector profiles are presented in Figure 8.11 through Figure 8.13, respectively. The predictions in Figure 8.11 through Figure 8.13 are from numerical models which incorporated mass scaling. The calculated energy absorption measures from the experimental and the numerical axial cutting tests under impact loading are presented in Table 8.2. The mean cutting forces in Table 8.2 indicate that the FE model over-predicted the mean cutting force by 38.1% and 62.3% for the dynamic cutting of extrusion \( (t = 1.587 \text{ mm}, \ D_o = 50.8 \text{ mm}) \) with the straight and curved deflector profiles, respectively. The FE model over-predicted the mean cutting force by 22.3%, 44.2% and 19.7% for the dynamic cutting of extrusion \( (t = 1.2 \text{ mm}) \) with \( D_o = 44.45 \text{ mm}, \ 50.8 \text{ mm} \) and \( 63.5 \text{ mm} \), respectively. The over-prediction of the FE simulation observations were most likely a result of the use of mass scaling and the influence of model symmetry. It would appear that the change in symmetry is more significant, as predictions in the first stage of a dual stage cutting simulation were generally in very good agreement (as will be discussed later in this section).
Figure 8.7 Cutting simulation results considering the influence of mass scaling and strain rate effects for the circular AA6061-T6 extrusions ($t = 1.587$ mm, $D_o = 50.8$ mm).

Figure 8.8 Load/displacement observations from experimental and numerical testing for the circular AA6061-T6 extrusion ($t = 1.587$ mm, $D_o = 50.8$ mm) under impact loading using single RevI cutter and straight deflector.
Figure 8.9 Load/displacement observations from experimental and numerical testing for the circular AA6061-T6 extrusion ($t = 1.587$ mm, $D_o = 50.8$ mm) under impact loading using single RevI cutter and curved deflector.

Figure 8.10 Load/displacement profiles from numerical simulation and experimental testing for the circular AA6061-T6 extrusion ($t = 1.2$ mm, $D_o = 50.8$ mm) under impact loading with dual-cutter configuration.
It was noticed from the load/displacement profiles in Figure 8.11 through Figure 8.13 that no high initial cutting force related to the dynamic cutting process was observed in numerical simulations. After a displacement of approximately 5 mm, the first steady-state cutting stage was reached. With the progress of the cutting process, the sidewall contacted with the deflector and the cutting force oscillated significantly at a displacement of approximately 35-45 mm. This is due to the curvature difference between the cut petalled sidewalls and the deflector surface profile. For the extrusion ($t = 1.2$ mm, $D_o = 44.45$ mm), the curvature difference was the least among the three diameter extrusions considered, thus the oscillation of the cutting force was observed to be the least significant. For the extrusion ($t = 1.2$ mm, $D_o = 50.8$ mm), the curvature difference was the most, so was the oscillation of the simulated cutting force.

Figure 8.11 Load/displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion ($t = 1.5$ mm, $D_o = 44.45$ mm) cut by the 4-blade RevII cutter with the presence of a curved deflector under impact loading.
Figure 8.12  Load/displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion ($t = 1.5$ mm, $D_o = 50.8$ mm) cut by the 4-blade RevII cutter with the presence of a curved deflector under impact loading.

Figure 8.13  Load/displacement observations from experimental and numerical cutting tests for the circular AA6061-T6 extrusion ($t = 1.5$ mm, $D_o = 63.5$ mm) cut by the 4-blade RevII cutter with the presence of a curved deflector under impact loading.
Table 8.2  Cutting performance measures of the specimens from both experimental and numerical testing under impact loading. The prefix ‘Exp’ and ‘Sim’ in front of the name of specimens indicate the cutting measures are from experimental and numerical testing, respectively.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$P_m$ (kN)</th>
<th>$P_{max}$ (kN)</th>
<th>$CFE$ (%)</th>
<th>$TEA$ (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp_T6_R300_D50.8_t1.587_RevI_4T1.0_SDDyn</td>
<td>16.90</td>
<td>27.67</td>
<td>61.35</td>
<td>1.30</td>
</tr>
<tr>
<td>Exp_T6_R300_D50.8_t1.587_RevI_4T1.0_CDDyn</td>
<td>17.14</td>
<td>26.01</td>
<td>66.45</td>
<td>1.27</td>
</tr>
<tr>
<td>Exp_T6_R150_D50.8_t1.2_RevIRevII_4T1.0_CDDyn</td>
<td>19.09</td>
<td>24.67</td>
<td>77.30</td>
<td>1.37</td>
</tr>
<tr>
<td>Exp_T6_R250_D44.45_t1.5_RevII_4T1.0_CDDyn</td>
<td>13.66</td>
<td>22.22</td>
<td>61.69</td>
<td>1.48</td>
</tr>
<tr>
<td>Exp_T6_R250_D50.8_t1.5_RevII_4T1.0_CDDyn</td>
<td>15.19</td>
<td>20.24</td>
<td>75.11</td>
<td>1.64</td>
</tr>
<tr>
<td>Exp_T6_R250_D63.5_t1.5_RevII_4T1.0_CDDyn</td>
<td>17.48</td>
<td>25.64</td>
<td>69.21</td>
<td>1.69</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_RevI_4T1.0_SDDyn</td>
<td>23.22</td>
<td>27.37</td>
<td>84.84</td>
<td>1.14</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_RevI_4T1.0_SD_Dyn_rate</td>
<td>22.84</td>
<td>29.52</td>
<td>77.40</td>
<td>1.20</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_RevI_4T1.0_CDDyn</td>
<td>27.76</td>
<td>32.30</td>
<td>85.96</td>
<td>1.16</td>
</tr>
<tr>
<td>Sim_T6_R250_D50.8_t1.2_RevIRevII_4T1.0_CDDyn</td>
<td>20.98</td>
<td>36.40</td>
<td>57.60</td>
<td>1.13</td>
</tr>
<tr>
<td>Sim_T6_R250_D44.45_t1.5_RevII_4T1.0_CDDyn</td>
<td>16.71</td>
<td>19.59</td>
<td>85.28</td>
<td>1.13</td>
</tr>
<tr>
<td>Sim_T6_R250_D50.8_t1.5_RevII_4T1.0_CDDyn</td>
<td>21.90</td>
<td>34.90</td>
<td>62.75</td>
<td>1.13</td>
</tr>
<tr>
<td>Sim_T6_R150_D63.5_t1.5_RevII_4T1.0_CDDyn</td>
<td>20.92</td>
<td>26.29</td>
<td>79.58</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Figure 8.8 through Figure 8.13 also show that the simulated penetration of the cutter blades was much less than that observed in the experimental testing. However, higher cutter forces were observed in the numerical testing and the $TEA$ was approximately 14% and 24-33% less than in the experimental testing for the dynamic cutting of extrusions ($t = 1.587$ mm, $D_o = 50.8$ mm) and extrusions ($t = 1.5$ mm, $D_o = 44.45$ mm, 50.8 mm, and 63.5 mm), respectively. The difference of the $TEA$ was attributed to the difference in potential energy of the dropping entity as a result of the lower degree of cutter penetration.
Figure 8.10 shows that the simulation predicted the experimental cutting force very well during the first cutting stage; however, over-prediction of the cutting force by approximately 12% in the second stage cutting was noted. The average cutting force was observed to be 20.98 kN consistent with that observed in experimental tests. The total energy absorption from numerical simulation was observed to be approximately 12.5% lower compared to the TEA observed for experimental tests as displacement for numerical simulation was approximately 12 mm less than that observed in experimental tests. Another noteworthy difference between the numerical model predictions and the experimental observations was the degree of bending associated with the cut sidewalls. At displacements of approximately 30 mm, the excessive bending associated with the cut petalled sidewalls causes contact between the sidewalls and the inner flat region associated with the deflector. This results in significantly higher force predictions in the numerical model after this displacement.

8.2 FE simulation results and discussion for dynamic and quasi-static axial crushing tests

It was observed from simulated axial crushing tests under both dynamic and quasi-static loading that axisymmetric progressive folding lobes were developed starting from the top end of the extrusion as the crushing process proceeded. After complete formation of approximately two to three lobes in the axisymmetric manner, formation of non-axisymmetric (more specifically 3-edge diamond) fold was observed all the way to the end of crush simulation. The switch from an axisymmetric to diamond shaped deformation mode is generally consistent with the experimental observations.

Load/displacement behaviour for the axial crushing of specimens with $t = 1.0$ mm, 1.2 mm and 1.587 mm under impact and quasi-static loading conditions are presented Figure 8.14, Figure 8.15, and Figure 8.16, respectively. The calculated energy absorption measures for the crush testing are presented in Table 8.3.

As indicated in the data from Table 8.3, the ratio of the simulated mean crush force under dynamic and quasi-static loading conditions ranges from 1.13 to 1.21 for all extrusion wall thicknesses considered, which indicates an insignificant strain-rate effect
for the AA6061-T6 material. The FE model over-predicted the crush forces by approximately 1%-16% under quasi-static loading.

Figure 8.14  Load/displacement profiles from numerical simulation and experimental testing under impact and quasi-static loading for axial crushing of the extrusions with a wall thickness of 1.0 mm.

Figure 8.15  Load/displacement profiles from numerical simulation and experimental testing under impact and quasi-static loading for axial crushing of the extrusions with a wall thickness of 1.2 mm.
Figure 8.16  Load/displacement profiles from numerical simulation and experimental testing under impact and quasi-static loading for axial crushing of the extrusions with a wall thickness of 1.587 mm.

Table 8.3  Crush measures for the extrusions under both dynamic and quasi-static loading conditions.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>( P_0 ) (kN)</th>
<th>( P_{max} ) (kN)</th>
<th>CFE (%)</th>
<th>TEA (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp_T6_R300_D50.8_t1.587_PF_QS</td>
<td>27.34</td>
<td>67.77</td>
<td>40.33</td>
<td>3.78</td>
</tr>
<tr>
<td>Exp_T6_R150_D50.8_t1.0_PF_QS</td>
<td>16.12</td>
<td>40.96</td>
<td>39.33</td>
<td>1.77</td>
</tr>
<tr>
<td>Exp_T6_R150_D50.8_t1.2_PF_QS</td>
<td>21.76</td>
<td>49.68</td>
<td>43.80</td>
<td>2.39</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_PF_QS</td>
<td>31.84</td>
<td>71.85</td>
<td>44.30</td>
<td>4.77</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_PF_Dyn</td>
<td>38.73</td>
<td>78.19</td>
<td>49.50</td>
<td>1.32</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.0_PF_QS</td>
<td>16.35</td>
<td>44.58</td>
<td>36.70</td>
<td>2.46</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.0_PF_Dyn</td>
<td>18.11</td>
<td>48.53</td>
<td>37.30</td>
<td>1.34</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.2_PF_QS</td>
<td>20.64</td>
<td>53.81</td>
<td>38.40</td>
<td>3.09</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.2_PF_Dyn</td>
<td>23.32</td>
<td>58.70</td>
<td>39.70</td>
<td>1.33</td>
</tr>
</tbody>
</table>
8.3 Finite element model validation assessment

As discussed in section 2.5.3, Oberkampf and Trucano [75] proposed a validation metric to assess how accurately the computational results compare with the experimental data with quantified error and uncertainty estimates. In this section, the validation metric \( V \), as expressed in Equation (1.30), and the relative error \( \text{Error} \), as expressed in Equation (1.31), proposed by Oberkampf and Trucano [75] are used to assess the FE models for prediction of the axial cutting and crushing of the AA6061-T6 extrusions. Moreover, standard error value, as expressed in equation (8.1), for the mean cutting/crushing force for simulations investigating the cutting/crushing deformation mode, is also used to parallel assess the FE models.

\[
SEP_m = \frac{(P_m)_N - (P_m)_E}{(P_m)_E} \cdot 100\%
\]  

(8.1)

where the capital \( N \) and \( E \) in the subscript notation following \( P_m \) represents the numerical and experimental values of the mean cutting force.

The calculated validation metric \( V \) and relative error \( \text{Error} \), as well as the standard error for the mean cutting/crushing force \( SEP_m \) for the FE models considered in this research are presented in Table 8.4. The calculated validation metric \( V \) and relative error \( \text{Error} \) for the quasi-static simulations and for the dynamic simulations were completed in the displacement domain and the time domain, respectively.

8.3.1 Validation assessment for quasi-static cutting/crushing tests

The cutting/crushing simulation under the quasi-static loading is validated throughout the entire displacement domain. The average validation metrics and relative errors for the cutting force for the axial cutting simulations with the use of a multiple bladed cutter were determined to be 91.6% and 8.9%, respectively. The average standard error for the mean cutting force for the axial cutting simulations with the use of a multiple bladed cutter was found to be 4.9%. The average validation metrics and relative errors for the cutting force for the axial cutting simulations with the use of a cutter/deflector assembly were determined to be 85.9% and 17.4%, respectively. The average standard error for the mean cutting force for the axial cutting simulations with the use of a
The cutter/deflector assembly was found to be 6.6%. The average validation metric and relative error for the crush force for the axial crushing simulations of the extrusions with wall thicknesses of 1.587 mm, 1.2 mm, and 1.0 mm were determined to be 66.9% and 50.8%, respectively. The relative low values of the calculated $V$ were due to the development of folds occurred at different displacement observed in the numerical simulation and the experimental testing. The average standard error for the mean crush force for the axial crushing simulations was found to be 7.7%.

### 8.3.2 Validation assessment for dynamic cutting tests

For the dynamic axial cutting tests, the validation metrics and relative errors consider the force and displacement responses in the time domain. These error measures were computed in two different time spans. The span of 7 ms and 16 ms considered only the cutting process for the single-cutter and dual-cutter configurations, respectively. The span of 25 ms considers the entire event incorporating both cutting and sliding of the petalled sidewalls on the deflector surface. The averaged validation metrics of the impact cutting force and displacement for numerical simulations was determined to be 62.2% and 65.2%, respectively, for the entire simulation time. The relative low values of the calculated $V$ for the single-cutter cutter configurations compared to the dual-cutter configuration were due to the use of the mass scaling in the FE models which resulted in unfavourable force oscillations as discussed in section 8.1.2.
Table 8.4  FE model validation assessment by Equations (1.30) and (1.31) [75] and Equation (8.1).

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$SEP_m$ (%)</th>
<th>$V_f$ (%)</th>
<th>Error$_f$ (%)</th>
<th>$V_d$ (%)</th>
<th>Error$_d$ (%)</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_3T1.0_ND_QS</td>
<td>9.2</td>
<td>94.6</td>
<td>5.9</td>
<td>-</td>
<td>-</td>
<td>35 mm</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_4T1.0_ND_QS</td>
<td>0.7</td>
<td>89.6</td>
<td>10.7</td>
<td>-</td>
<td>-</td>
<td>35 mm</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_5T1.0_ND_QS</td>
<td>2.2</td>
<td>91.9</td>
<td>8.9</td>
<td>-</td>
<td>-</td>
<td>35 mm</td>
</tr>
<tr>
<td>Sim_T6_R56_D50.8_t3.175_RevII_6T1.0_ND_QS</td>
<td>7.5</td>
<td>90.1</td>
<td>10.3</td>
<td>-</td>
<td>-</td>
<td>35 mm</td>
</tr>
<tr>
<td>Sim_T6_R250_D44.45_t1.5_RevII_4T1.0_CD_QS</td>
<td>2.8</td>
<td>90.5</td>
<td>11.7</td>
<td>-</td>
<td>-</td>
<td>80 mm</td>
</tr>
<tr>
<td>Sim_T6_R250_D50.8_t1.5_RevII_4T1.0_CD_QS</td>
<td>7.8</td>
<td>85.0</td>
<td>17.0</td>
<td>-</td>
<td>-</td>
<td>80 mm</td>
</tr>
<tr>
<td>Sim_T6_R250_D63.5_t1.5_RevII_4T1.0_CD_QS</td>
<td>8.3</td>
<td>82.1</td>
<td>23.5</td>
<td>-</td>
<td>-</td>
<td>80 mm</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_RevII_4T1.0_SD_Dyn</td>
<td>37.4</td>
<td>62.9</td>
<td>40.9</td>
<td>65.8</td>
<td>35.8</td>
<td>25 ms</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_RevII_4T1.0_CD_Dyn</td>
<td>62.0</td>
<td>47.8</td>
<td>61.5</td>
<td>64.7</td>
<td>37.0</td>
<td>25 ms</td>
</tr>
<tr>
<td>Sim_T6_R150_D50.8_t1.2_RevII_RevII_4T1.0_CD_Dyn</td>
<td>9.9</td>
<td>65.5</td>
<td>42.9</td>
<td>86.0</td>
<td>15.1</td>
<td>25 ms</td>
</tr>
<tr>
<td>Sim_T6_R250_D44.45_t1.5_RevII_4T1.0_CD_Dyn</td>
<td>22.3</td>
<td>76.4</td>
<td>24.2</td>
<td>67.7</td>
<td>33.5</td>
<td>25 ms</td>
</tr>
<tr>
<td>Sim_T6_R250_D50.8_t1.5_RevII_4T1.0_CD_Dyn</td>
<td>44.2</td>
<td>47.4</td>
<td>62.6</td>
<td>51.8</td>
<td>52.6</td>
<td>25 ms</td>
</tr>
<tr>
<td>Sim_T6_R250_D63.5_t1.5_RevII_4T1.0_CD_Dyn</td>
<td>19.7</td>
<td>73.0</td>
<td>29.2</td>
<td>55.3</td>
<td>48.1</td>
<td>25 ms</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.587_PF_QS</td>
<td>16.5</td>
<td>57.5</td>
<td>86.6</td>
<td>-</td>
<td>-</td>
<td>110 mm</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.0_PF_QS</td>
<td>1.4</td>
<td>71.4</td>
<td>33.0</td>
<td>-</td>
<td>-</td>
<td>110 mm</td>
</tr>
<tr>
<td>Sim_T6_R300_D50.8_t1.2_PF_QS</td>
<td>5.1</td>
<td>71.9</td>
<td>32.8</td>
<td>-</td>
<td>-</td>
<td>110 mm</td>
</tr>
</tbody>
</table>

Note: $V_f$ and $V_d$ represent validation assessment for force and displacement respectively. Error$_f$ and Error$_d$ represent error assessment for force and displacement respectively. Domain represents either time domain or displacement domain used to calculate the validation and error assessments for impact and quasi-static cutting test simulations, respectively.
A theoretical study of steady-state cutting of circular tubes by a cutter with multiple blades with/without the presence of deflector will be discussed in this chapter by analyzing experimental observations and developing an analytical model. First, the energy dissipation mechanisms are identified and the energy dissipation rates for each mechanism are calculated. Next, the expression for the steady-state cutting force is derived by employing the principle of virtual power and applying the upper bound theory of plasticity. Afterwards, the effect of the friction force is included in the proposed solution and the total axial cutting resistance force is derived. Finally, a parametric study of the effects of tube wall thickness, cutting blade tip width, cutter blade quantities and extrusion diameter for the proposed analytical model is conducted and the analytical model is validated by comparing the predicted cutting resistance force to the experimental testing data which have been presented in chapter 6.

9.1 Characteristics of steady-state cutter cutting process

When a cutter with ‘n’ blunt cutting blades having finite shoulder widths cuts through a ductile circular tube with an outer radius of \( r_o \) and a tube wall thickness of \( t \), as shown in Figure 9.1, it will pass the transient cutting stage and reach the steady-state cutting process after a certain distance of cutter penetration. Figure 9.2 illustrates the plastic deformation of the circular tube after steady-state cutting is reached. The cutting blades considered in the present study have a blade tip width of \( T \), a blade semi-angle of \( \theta \), and a blade shoulder width of \( 2B \), as illustrated in Figure 9.3. By analyzing experimental observations, the assumed deformations under the steady-state cutting process for the cutter blades cutting through the sidewall of the tube are presented and shown in Figure 9.3 and Figure 9.4. Since the present study is only interested in the steady-state cutting stage, the pictures of the deformation do not change in time. While Figure 9.3 illustrates the assumed mode of deformation for one cutting blade cutting through the sidewall of a circular tube, Figure 9.4 shows the assumed bending deformation for the cut petalled sidewalls. It is assumed that with the presence of blunt cutter blades the circular tube will grow circumferentially but remain circular in cross-
sectional geometry, which leads to the cut petalled sidewalls bending outward (Figure 9.4). Moreover, with the advancing of cutter blades the tube material curls up along two inclined plastic moving hinge lines (OP line in Figure 9.3) on both sides of the tube into cylindrical flaps (transient flaps in Figure 9.3) and then forms stable flaps (Figure 9.3) on both sides of the blade shoulder. Since contact between the cutter blades and the cut petalled sidewalls was observed in all experimental tests, membrane deformation of the tube sidewalls at the transition zone between the transient flaps and the stable flaps (PQT membrane zone in Figure 9.3) exists. In addition to the abovementioned three energy dissipating mechanisms, circumferential membrane stretching in the vicinity of the blade tip (shaded zone in Figure 9.3) and continuous chip formation ahead of the blunt blade (Figure 9.3) are two other major plastic energy dissipating mechanisms. Although material cracking (fracture) was also observed on the cut petalled sidewalls in some experimental tests as discussed in the chapter 6, especially for the thinner wall thickness extrusions, the occurrence of material cracking is random and not necessary a part of a steady-state cutting process. Therefore, it will not be included in the assumed deformation mechanisms. All energy dissipating mechanisms are assumed to be uncoupled to each other and can be treated separately. The effect of friction between the cutter blades and tube sidewalls on plastic flow of the tube material will not be considered initially in order to simplify the preliminary analysis, but will be included to the total axial cutting resistance force after the frictionless cutting force is derived, which has been extensively used by Simonsen and Wierzbicki [58] and Zheng and Wierzbicki [59] for the similar cutting process.
Figure 9.1. Illustration of a cutter with multiple blades cutting through a circular tube.

Figure 9.2. Photograph illustrating the plastic deformation of circular tube after steady-state cutting process is reached.
Figure 9.3. Assumed mode of deformation for one cutting blade cutting through the sidewall of a circular tube.

Figure 9.4. Assumed outward bending deformation for cut petalled sidewalls.
Figure 9.5. Top view of one cutting blade and portion of tube sidewall.

Figure 9.6. Necessary straining illustrated by gap openings.
As illustrated in Figure 9.3 and Figure 9.5, plastic material flow of the circular tube is described in a cylindrical coordinate system \((r, \theta, x)\), where \(x\) is the opposite of blade advancing direction, \(r\) is the radial direction of the circular tube, and \(\theta\) is the circumferential direction of the circular tube. As the material moves along the streamline (\(x\) direction), it experiences bending as it passes the OP-line followed by a continuous increasing shear deformation as it moves towards the blade shoulder. Material that is close to the symmetry line (Ox line) experiences additional tensile deformation in the circumferential direction (\(\theta\) direction) as it passes the zone of tube sidewall separation in front of the cutter blade tip.

### 9.1.1 Moving hinge lines

As discussed in section 9.1, tube material curls up along the plastic hinge line OP on both sides of the tube wall thickness into cylindrical flaps. The moving hinge line OP in Figure 9.3 changes the curvature of the undisturbed outer and inner portion of the tube sidewall from \(-1/r_{m,o}\) to \(1/R_{rt}\) and from \(-1/r_{m,i}\) to \(1/R_{rt}\), respectively, where \(r_{m,o}\) and \(r_{m,i}\) are the mean radii for the outer and inner portion of the circular tube, respectively. The hinge line CD reverts the curvature of curled flaps back to zero so that there is a straight portion of flap (QCD) conforming to the tapered region of the cutting blade. The rolling radius \((R_{rt})\) is kept as a variable in the formulation and taken as the value which gives the lowest rate of energy dissipation. From the geometry of the problem:

\[
R_r = R_{rt}/\cos \theta \tag{9.1}
\]

\[
R_{rt,o} = R_{r,o}\cos \theta = R_r \cos \theta \tag{9.2a}
\]

\[
R_{rt,i} = R_{r,i}\cos \theta = R_r \cos \theta \tag{9.2b}
\]

where \(R_{rt}, R_{rt,o}, \text{ and } R_{rt,i}\) are the rolling radii for the transient flaps; and \(R_r, R_{r,o}, R_{r,i}\) are the rolling radii for the stable flaps.
9.1.2 Far-field membrane deformation

The far-field membrane deformation zone is the transition membrane zone shown in Figure 9.3. For all the experimental cutting tests there exists a smooth transition membrane zone between transient and stable flaps in the steady-state cutting process. Figure 9.6 shows the geometry that is seen if the tube sidewall was cut at the symmetry line (Ox line) and along the edges PT, PQ and folded without membrane deformation of the sidewall. Then the gap, $u_o$, between PT and PQ is an indication of the amount of membrane straining necessary for material continuity during the cutting process. The determination of PT-PQ gap opening is presented in Appendix C.2 and the result is given in Equation (9.3).

$$u_o = B\theta$$  \hspace{1cm} (9.3)

9.1.3 Near blade tip circumferential membrane stretching

When a cutting blade cuts the sidewall of the tube, large stresses and strains in the vicinity of the cutting blade tip cause the tube sidewall to yield and a plastic deformation zone ahead of the cutting blade exists (shaded plastic zone in Figure 9.3). The tube material in front of the cutter blade tip tends to separate in the circumferential direction; however, the ductility of the material holds the material together since no material crack was observed in the experimental cutting tests. This combination of efforts causes the circumferential stretching of the tube material in the vicinity of the cutting blade. Figure 9.6 shows the geometry that is seen if the tube sidewall was cut at the centerline and rolled without membrane deformation of the sidewall. Then the gap, $2\nu_{\theta\theta}$, at the plastic deformation zone is an indication of the amount of circumferential membrane stretching necessary for material continuity. The determination of near blade tip gap opening is presented in Appendix C.1 and the result is given in Equation (9.4).

$$2\nu_{\theta\theta} = 0.317R_c \cos^2 \theta (1 + 0.55\theta^2)$$  \hspace{1cm} (9.4)
9.1.4 Continuous chip formation ahead of cutter blade

Continuous chip formation was observed in front of the blunt cutter blades with the advancing of cutter blades in the experimental axial cutting tests as discussed in chapter 6 and as shown in Figure 9.2.

9.1.5 Cut petalled sidewalls bending outward

As discussed in section 9.1, with the presence of cutter blades the circular tube will grow circumferentially but remain circular in cross-sectional geometry, which leads to the cut petalled sidewalls bending outward. Moving hinge line on each cut petalled sidewall (GH line in Figure 9.4) travels along the x direction and changes the curvature of the undisturbed tube sidewall from zero to \(1/R_{\text{axial}}\), where \(R_{\text{axial}}\) is the radius of axial bending. The axial bent radius of the cut petalled sidewalls can be determined from the geometry of the problem and the detailed development process and the resulting radii for different geometries of tubes considered in this research are presented in Appendix C.3. For the axial cutting tests with the presence of the curved deflector, since the curvature of the curved deflector’s profile considered in this research is greater than \(1/R_{\text{axial}}\), the moving hinge lines (GH line in Figure 9.4) changes the curvature of the undisturbed tube sidewall from zero to \(1/R_{\text{deflector}}\), where \(R_{\text{deflector}}\) is the surface profile radius of the curved deflector.

9.2 Principle of virtual power

When external loads are applied to a deformable structure, the power of these loads must be equal to the incremental energy stored elastically or dissipated in the structure. Assuming a rigid-perfectly plastic material, i.e. no elastic energy is stored in the structure, applying the principle of virtual power:

\[
F \cdot V = \dot{E}_p + \dot{E}_f = \dot{E}_m + \dot{E}_b + \dot{E}_f
\]

where \(F\) is the total cutting resistance force in the direction of \(V\), \(V\) is the velocity of the cutting blade in the advancing direction, \(\dot{E}_p\) is the rate of plastic energy dissipation, \(\dot{E}_f\) is the rate of energy dissipation as a result of friction forces, \(\dot{E}_m\) is the rate of energy
dissipation due to plastic membrane stretching, and $\dot{E}_b$ is the rate of energy dissipation due to plastic bending.

### 9.3 Assumption for internal energy dissipation

In the proposed model, the internal energy dissipation and friction effects are considered separately without coupling. Kinematically admissible displacement fields are constructed according to Figure 9.4 and Figure 9.6 and from the assumed deformation fields. The rates of each plastic energy dissipation mechanisms are calculated with one free parameter, the plastic rolling radius, $R_r$, which is postulated that the actual deformation mode is the one that minimizes the total rate of energy dissipation.

For a plane stress condition, the rate of membrane and bending energies can be expressed in Equations (9.6) and (9.7), respectively, where $N_{\alpha\beta}$, $M_{\alpha\beta}$ are components of the membrane force and bending moment tensors, $\dot{\varepsilon}_{\alpha\beta}$, $\dot{\kappa}_{\alpha\beta}$ are the corresponding generalized strain and curvature rates calculated in the deformation configurations. The material is assumed to be characterized by a flow stress, $\sigma_o$, which is understood as the elevated stress corresponding to an average strain in the cutting process.

\[
\dot{E}_m = \int_A N_{\alpha\beta} \dot{\varepsilon}_{\alpha\beta} \, dA \quad (9.6)
\]

\[
\dot{E}_b = \int_A M_{\alpha\beta} \dot{\kappa}_{\alpha\beta} \, dA \quad (9.7)
\]

In order to simplify the problem, the following assumptions are made:

i. The total cutting resistant force is equally distributed to individual cutting deformation mode generated by one of the cutter blades.

ii. The material is treated as rigid-perfectly plastic with an average flow stress ($\sigma_o$). This flow stress has the same value in both bending and membrane deformation modes.

iii. Plastic in-plane shear strain is neglected.

iv. The out-of-plane displacements in the near-tip membrane deformation zone are neglected in the strain rate calculations.
v. Plastic work in the near-tip zone is predominantly dissipated by the diffused mode. In other words, no local necking is considered and the tube thickness is taken to be constant.

vi. The interactions between each plastic energy dissipation mechanisms are uncoupled and can be treated separately.

vii. The yield behaviour of the tube material obeys a von Mises yield criterion.

For circular tubes with rigid-perfectly plastic material obeying von Mises yield criterion, the plane stress yield condition in a cylindrical coordinate system can be written as:

$$\sigma_{xx}^2 - \sigma_{xx} \sigma_{\theta \theta} + \sigma_{\theta \theta}^2 + 3\sigma_{xx} \sigma_{\theta \theta} = \sigma_o^2$$  \hspace{1cm} (9.8)

The associated flow rule gives three independent equations:

$$\dot{\varepsilon}_{xx} = \dot{\lambda}(2 \sigma_{xx} - \sigma_{\theta \theta})$$
$$\dot{\varepsilon}_{\theta \theta} = \dot{\lambda}(2 \sigma_{\theta \theta} - \sigma_{xx})$$
$$\dot{\varepsilon}_{x \theta} = 6 \dot{\lambda} \sigma_{x \theta}$$  \hspace{1cm} (9.9)

For materials that are in the vicinity of the cutter blade tip, since $$\varepsilon_{x \theta} = 0$$ (No material accumulated ahead of the blade tip beyond the chip formation zone) and $$\varepsilon_{xx} = 0$$ (shear strain rate is neglected in the calculation), from Equations (9.8) and (9.9), stresses in the vicinity of the cutting blade tip are given by:

$$\sigma_{xx} = \frac{1}{2} \sigma_{\theta \theta}$$  \hspace{1cm} (9.10)
$$\sigma_{x \theta} = 0$$  \hspace{1cm} (9.11)
$$\sigma_{\theta \theta} = \frac{2}{\sqrt{3}} \sigma_o$$  \hspace{1cm} (9.12)

For materials that are in the far-field of the cutter blade tip, since $$\varepsilon_{\theta \theta} = 0$$ (No material accumulated in the circumferential direction beside the cutting deformation zone) and $$\varepsilon_{x \theta} = 0$$ (shear strain rate is neglected in the calculation), from Equations (9.8) and (9.9), stresses in the far-field of the cutting blade tip are given by:
\[ \sigma_{\theta\theta} = \frac{1}{2} \sigma_{xx} \quad (9.13) \]
\[ \sigma_{x\theta} = 0 \quad (9.14) \]
\[ \sigma_{xx} = \frac{2}{\sqrt{3}} \sigma_o \quad (9.15) \]

### 9.4 Modification for a steady-state cutting process

In a steady-state cutting process, it is convenient to follow the deformation of a given material element as it goes through the entire deformation path. A material element near the vicinity of the cutter blade tip firstly enters the ‘plastic deformation zone’ in Figure 9.3 where it is subjected to circumferential stretching. Then it passes the bending zone (APQ zone) where it acquires a constant cylindrical curvature. Next, it moves to the transition membrane zone (PQT zone) where it is extended in the axial direction. Finally, on leaving the transition zone (PT line), it buckles (for thin tubes) or it compressed back to its original length (for thick tubes). All tube geometries considered in this research are thin-walled tubes and material buckle in the PQT zone is observed in the experimental tests and is shown in Figure 9.2.

The steady-state cutting process condition is mathematically expressed by:

\[ V = \frac{dx}{dt} \quad (9.16) \]

Using Equation (9.16), the rate term can be re-written in alternative form where the differentiation with respect to time has been replaced by a differentiation with respect to \( x \):

\[ \frac{d}{dt} (\cdot) = \frac{d}{dx} (\cdot) \cdot \frac{dx}{dt} = V \cdot \frac{d}{dx} (\cdot) \quad (9.17) \]
9.5 Rate of internal energy dissipation

9.5.1 Energy rate in far-field bending

As discussed in section 9.1, with the advancing of cutting blade, tube material curled up on both sides of the tube wall thickness into cylindrical flaps and then formed stable flaps on both sides of the blade shoulder. The moving hinge line OP changes the outer and inner portions of sidewalls curvatures differently. The mean radius that differentiates the outer and inner portions of the tube sidewalls is given by:

\[ r_m = \frac{r_o + r_l}{2} \]

(9.18)

And the new mean radii for the outer and inner portion of sidewalls are then determined by:

\[ r_{m,o} = \frac{r_o + r_m}{2} \]

(9.19a)

\[ r_{m,i} = \frac{r_m + r_l}{2} \]

(9.19b)

The new tube wall thickness for the outer and inner portions of sidewalls is given by:

\[ t_o = t_i = \frac{t}{2} \]

(9.20)

From Equation (9.7) and using Equation (9.17), the rates of energy dissipated on both sides of the cutting blade due to the moving hinge line OP are given by:

\[ \dot{E}_{b, far-field} = 2 \int_{L_o} V_n M_{o,o} [\kappa_{nn,o}] dL_o + 2 \int_{L_i} V_n M_{o,i} [\kappa_{nn,i}] dL_i \]

(9.21)

where \( V_n \) is the normal moving speed of plastic hinge line OP; \( M_{o,o} \) and \( M_{o,i} \) is the fully plastic bending moments for the outer and inner portion of the tube sidewall; \( [\kappa_{nn,o}] \) and \( [\kappa_{nn,i}] \) are outer and inner portion of sidewalls’ curvature change, respectively; and \( L_o \) and \( L_i \) are the length of moving hinge lines for the outer and inner portion of the tube sidewall. From geometry of the problem, it can be derived that:
\[ V_n = V \sin \theta \] \hspace{1cm} (9.22)

\[ L_o \approx \frac{B + R_{r,o}}{\sin \theta} = \frac{B + R_r}{\sin \theta} \] \hspace{1cm} (9.23a)

\[ L_i \approx \frac{B + R_{r,i}}{\sin \theta} = \frac{B + R_r}{\sin \theta} \] \hspace{1cm} (9.23b)

\[ [\kappa_{nn,o}] = \frac{1}{R_{rt,o}} + \frac{1}{r_{m,o}} \] \hspace{1cm} (9.24a)

\[ [\kappa_{nn,i}] = \frac{1}{R_{rt,i}} - \frac{1}{r_{m,i}} \] \hspace{1cm} (9.24b)

From the stresses conditions as expressed in Equation (9.12), the fully plastic bending moment is determined to be:

\[ M_{o,o} = \frac{2}{\sqrt{3}} \frac{\sigma_o t_o^2}{4} = \frac{\sigma_o t^2}{8\sqrt{3}} = M_o \] \hspace{1cm} (9.25a)

\[ M_{o,i} = \frac{2}{\sqrt{3}} \frac{\sigma_o t_i^2}{4} = \frac{\sigma_o t^2}{8\sqrt{3}} = M_o \] \hspace{1cm} (9.25b)

Substitute Equations (9.2), (9.19), (9.20), (9.22), (9.23), (9.24), and (9.25) into Equation (9.21) gives:

\[ \dot{E}_{b,\text{far-field}} = \frac{\sigma_o t^2}{2\sqrt{3}} V(B + R_r) \left( \frac{1}{R_r \cos \theta} + \frac{1}{r_o + r_m} - \frac{1}{r_m + r_l} \right) \] \hspace{1cm} (9.26)

### 9.5.2 Energy rate in far-field transition membrane deformation zone

The amount of membrane straining in the far-field transition membrane zone (PQT zone) is indicated by \( u_o \) as explained in section 9.1.2. Assume that the dominated stress is the tensile stress in the streamline of the circular tube. Substituting Equations (9.15) and (9.17) into Equation (9.17), the rates of energy dissipated on both sides of the cutter blade due to membrane stretching of material from transient flap to stable flap are given:


\[
\dot{E}_{m,\text{trans}} = 2 \int_{t_{tt}} V \frac{2\sigma_0 t}{\sqrt{3}} \epsilon_{tt} \, dL_{tt} = \frac{4}{\sqrt{3}} V \sigma_0 (t_o + t_i) u_o = \frac{4}{\sqrt{3}} V \sigma_o t u_o
\]  

(9.27)

where \( tt \) indicates the direction of membrane straining in the transitional zone. Substitute Equation (9.3) into Equation (9.27) simplifies the expression to be:

\[
\dot{E}_{m,\text{trans}} = \frac{4}{\sqrt{3}} V \sigma_o t B \theta
\]  

(9.28)

### 9.5.3 Energy rate in near blade tip circumferential membrane stretching

The amount of membrane stretching in the vicinity of the cutter blade tip is indicated by \( u_{\theta\theta} \) as explained in section 9.1.3. Assume that the dominated stress is the tensile stress in the circumferential direction of the circular tube. Substituting Equation (9.12) into Equation (9.6) and using Equation (9.17) the rate of energy dissipated due to the circumferential membrane stretching in the vicinity of the cutter blade tip is determined and given by:

\[
\dot{E}_{m,\text{tip}} = \int_{L_{\theta\theta}} V \frac{2\sigma_0 t}{\sqrt{3}} \epsilon_{\theta\theta} \, dL_{\theta\theta} = \frac{4}{\sqrt{3}} V \sigma_o t u_{\theta\theta}
\]  

(9.29)

where \( \theta\theta \) indicates the circumferential direction of the membrane stretching. Replacing Equation (9.4) into Equation (9.29) simplifies the expression to be:

\[
\dot{E}_{m,\text{tip}} = 0.366 V \sigma_o t R \cos^2 \theta (1 + 0.55 \theta^2)
\]  

(9.30)

### 9.5.4 Energy rate in continuous chip formation ahead of cutter blade

The material ahead of the blunt cutter blade tip is subjected to compression and the rate of energy dissipated due to the continuous chip formation is given by:

\[
\dot{E}_{\text{chip}} = \frac{2}{\sqrt{3}} V \sigma_o T t
\]  

(9.31)

where \( T \) is the blunt cutting blade tip width.
9.5.5 Energy rate in cut petalled sidewalls bending outward

As discussed in section 9.1.5, with the advancing of cutter blades, the moving hinge line GH changes the curvature of cut petalled sidewalls from 0 to $1/R_{axial}$. The magnitude of axial bent radius, $R_{axial}$, can be determined by the geometry of the problem or by the radius of the curved deflector’s profile as detailed in Appendix C.3. The rate of energy dissipated for one of the cut petalled sidewalls is determined from Equations (9.7) and (9.15) by using Equation (9.17) and is given by:

$$
\dot{E}_{b,axial} = VM_0 \frac{1}{R_{axial}} \frac{2\pi r_m}{n} = V \pi \frac{\sigma_o t^2 r_m}{\sqrt{3} n R_{axial}} \tag{9.32}
$$

9.6 Steady-state cutting resistance force without friction

The steady-state cutting resistance force without considering the effect of friction force is determined from substituting calculated rates of energy dissipation for each mechanism into the expression for principle of virtual power while ignoring the contribution of friction force. Re-writing Equation (9.5) without considering the effect of friction force gives:

$$
F_p \cdot v = \dot{E}_p = \dot{E}_{b, far-field} + \dot{E}_{m, trans} + \dot{E}_{chip} + \dot{E}_{m, tip} + \dot{E}_{b, axial} \tag{9.33}
$$

Substituting Equations (9.26), (9.28), (9.30), (9.31) and (9.32) into Equation (9.33) gives the axial cutting force without the effect of friction force for steady-state axial cutting without the presence of deflector:

$$
F_p = \frac{\sigma_o t^2}{2\sqrt{3}} (B + R_r) \left( \frac{1}{R_r \cos \theta} + \frac{1}{r_o + r_m} - \frac{1}{r_m + r_i} \right) + \frac{4}{\sqrt{3}} \sigma_o t B \theta + \frac{2}{\sqrt{3}} \sigma_o T t \\
+ 0.366 \sigma_o t R_r \cos^2 \theta (1 + 0.55 \theta^2) + \frac{\pi \sigma_o t^2 r_m}{\sqrt{3} n R_{axial}} \tag{9.34}
$$

9.7 Steady-state cutting resistance force including friction

The contribution of the friction force in the cutting process can be found by considering normal and tangential forces at the cutter blade and tube sidewall interface.
The relative velocity, $V_{real}$, between the tube sidewall and the cutting blade at the contact area is assumed to be inclined an angle, $\zeta$, as shown in Figure 9.7. It has been shown by Simonsen and Wierzbicki [58] that it is reasonable to take the value of $\zeta$ to be:

$$\zeta = 0.5\theta \quad (9.35)$$

The normal force, $F_N$, is limited by the plastic and/or fracture resistance of the tube sidewall and thus is treated as known. From the force equilibrium in the $x$ direction as shown in Figure 9.7, the normal force, $F_N$, is given by:

$$F_N = \frac{F_{cut} - F_n}{2 \sin \theta} = \frac{F_P}{2 \sin \theta} \quad (9.36)$$

Figure 9.7. Definition of direction of relative velocity and free body diagram for cutting blade.

One of the components of the tangential friction force, $F_{f1} = \mu F_N \sin \zeta$, which contributes to the cut petalled sidewalls bending outward, is already considered in the solution expressed in Equation (9.34) for the case that no deflector is presented in the axial cutting process. Using the axial bent radii presented in Table C.1 and replacing them into Equation (9.32) and employing the principle of virtual power, it is interesting to find that the axial resistance force necessary to result in one piece of cut petalled sidewall is almost the same for the tube geometries considered, expressed mathematically in Equation (9.37).
Since the radial increment due to the presence of the cutter blades is given by:

$$\Delta r = \frac{nB}{2\pi} \quad (9.38)$$

Observing Equations (9.37) and (9.38) and realizing that the axial bent of cut petalled sidewalls is the result of friction force, we can conclude that the tangential friction force, $F_f$, is an inverse relationship of the number of cutter blades, $n$, that is:

$$F_f = \frac{C_1}{n} \quad (9.39)$$

where $C_1$ is a constant.

For the steady-state cutting process with the presence of a curved deflector, the curvature of the deflector is greater than the axial bent curvature of the cut petalled sidewall if no deflector is present; correspondingly, the cut petalled sidewall is forced to conform to the profile of the deflector. The component of the tangential friction force, $F_{f1} = \mu F_N \sin \zeta$, as well as one component of the friction force between the deflector and the cut petalled sidewalls in the radial direction, which contributes to the cut petalled sidewalls bending outward, has already been included in Equation (9.34). Although the relationship between the friction force and the number of cutter blades is not as obvious as the case if no deflector is present, the relationship as expressed in Equation (9.39) is still assumed to exist.

The other component of the tangential friction force, $F_{f2} = \mu F_N \cos \zeta$, does not affect the assumed deformation mechanisms. In other words, the tube sidewall could resist an arbitrary tangential force without altering the plastic energy dissipation. Projecting $F_{f2}$ on both sides of the cutter blade in the $x$ direction and combining Equations (9.35) and (9.39) gives the tangential friction force:
\[ F_f = \frac{C_2 \left( 2\mu F_N \cos \frac{\theta}{2} \cos \theta \right)}{n} \]  

(9.40)

where \( C_2 \) is another constant. Replacing Equation (9.40) into Equation (9.36) gives:

\[ F_f = \frac{C_2 \mu F_p \cos \frac{\theta}{2} \cot \theta}{n} \]  

(9.41)

Comparing Equations (9.39) and (9.41) gives:

\[ C_1 = C_2 \mu F_p \cos \frac{\theta}{2} \cot \theta \]  

(9.42)

Considering a circular tube cut by a cutter with four blades, the radial direction component of the friction force will be perpendicular to each other acting on one-quarter of the cut petalled sidewall. It is assumed that in this configuration the friction force will not interfere with each other to the rate of energy dissipation as a result of plastic deformation. Configurations other than cut by a cutter with four blades will result in interference between the friction forces on both sides of the cut petalled sidewall. Thus, it is reasonable to take:

\[ C_2 = 4 \]  

(9.43)

Similarly, for the case of axial cutting of circular tubes with the presence of a curved deflector, the relationship between the total friction force and the plastic cutting resistance force as expressed in Equation (9.41) is still valid.

Combining Equations (9.41) and (9.43), the total cutting resistance force for a cutter with \( n \) number of blades cuts through a circular tube without the presence of deflector is determined to be:

\[ F = \left( n + 4 \mu \cos \frac{\theta}{2} \cot \theta \right) F_p \]  

(9.44)
The total cutting resistance force for a circular tube cut by a cutter with ‘n’ cutter blades with/without the use of deflector can be determined by replacing Equation (9.34) into Equation (9.44) and is given by:

\[
F = \left( n + 4 \mu \cos \frac{\theta}{2} \cot \theta \right) \left[ \frac{\sigma_o t^2}{2\sqrt{3}} (B + R_r) \left( \frac{1}{R_r \cos \theta} + \frac{1}{r_o + r_m} - \frac{1}{r_m + r_l} \right) \right. \\
+ \frac{4}{\sqrt{3}} \sigma_o t B \theta + \frac{2}{\sqrt{3}} \sigma_o T t + 0.366 \sigma_o t R_r \cos^2 \theta (1 + 0.55 \theta^2) \\
+ \left. \frac{\pi \sigma_o t^2 r_m}{\sqrt{3} n R_{axial}} \right]
\]  
(9.45)

The rolling radius, \( R_r \), which minimizes the resisting force is determined by:

\[
\frac{dF}{dR_r} = 0
\]  
(9.46)

And the rolling radius, \( R_r \), is found to be:

\[
R_r = \sqrt{\frac{B t}{\cos \theta \left[ \frac{t}{r_o + r_m} - \frac{t}{r_m + r_l} + 1.268(1 + 0.55 \theta^2) \cos^2 \theta \right]}}
\]  
(9.47)

9.8 Parametric study and experimental validation

A parametric study considering the effects of tube wall thickness, cutting blade tip width, cutter blade quantities, and extrusion diameter is completed in this section for the proposed analytical model (Equation (9.45)) and the analytical models from Zheng and Wierzbicki [59] (Equation (1.25)) and Simonsen and Wierzbicki [58] (Equation (1.27)) for sharp wedge cutting through a plain plate. The validation of the proposed theoretical model is also presented in this section by comparison of predicted and experimental results. Experimental steady-state cutting forces are determined by averaging the cutting forces after obvious steady-state behaviour has been reached.

It is also interested to know the effect of each energy dissipation mechanisms to the total cutting resistance force. From the present proposed analytical model, for the
circular AA6061-T6 tube with an outer diameter of 50.8 mm and a tube wall thickness of 3.175 mm cut by a cutter with four blades without the presence of deflector, the predicted cutting resistant force was determined to be 45.77 kN. The circumferential membrane stretching in the vicinity of the cutter blade, the far-field plastic bending, the transition membrane zone between the transient and stable flaps, the continuous chip formation, the cut petalled sidewalls bending outward, and the friction contribute to 5.8%, 13.4%, 4.7%, 9.4%, 2.5% and 64.2% of the total axial cutting resistant force, respectively.

9.8.1 Effect of tube wall thickness

The effect of tube wall thickness on the steady-state cutting forces with/without the presence of the curved deflector is discussed in this section. The input parameters used for the theoretical predictions are presented in Table 9.1. Figure 9.8 shows the comparison of steady-state cutting resistance forces versus tube wall thickness for axial cutting of circular AA6061-T6 extrusions of \( D_o = 50.8 \) mm by a cutter of four blades without the presence of deflector. Figure 9.9 through Figure 9.11 present the comparisons of steady-state cutting resistance forces versus tube wall thickness for axial cutting of circular AA6061-T6 extrusions of \( D_o = 44.45 \) mm, 50.8 mm, and 63.5 mm by a cutter of four blades with the presence of the curved deflector (\( R_{\text{deflector}} = 50.8 \) mm).

It is shown from Figure 9.8 that theoretical predictions for the axial cutting without the use of deflector from the present proposed analytical model agree well with the experimental tests with a maximum relative error of 14.8%. The theoretical predictions from Zheng and Wierzbicki [59] and Simonsen and Wierzbicki [58] under-predict the axial cutting force with maximum relative errors of 36.4% and 24.4%, respectively. For the axial cutting with the presence of the curved deflector, it can be seen from Figure 9.9 through Figure 9.10 that predictions from the proposed model agree well with the experimental data for extrusions with outer diameters of 44.45 mm and 50.8 mm with a maximum relative error of 12.2%. For the extrusions with an outer diameter of 63.5 mm, the theoretical predictions from the proposed model under-predict the axial cutting force, since in the experimental tests the cut petalled sidewalls did not conform well to the profile of the curved deflector due to large diameter of extrusion and resulted in buckling of the cut petalled sidewalls. For all the extrusions considered, an
almost increasing relationship between the steady-state cutting force and the tube wall thickness is observed from the experimental results and the proposed theoretical predictions.

Table 9.1 Input data for comparison of experimental results with theoretical predictions.

<table>
<thead>
<tr>
<th>Input data</th>
<th>AA6061-T6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress, $\sigma_y$ (MPa)</td>
<td>277.5</td>
</tr>
<tr>
<td>Ultimate stress, $\sigma_u$ (MPa)</td>
<td>320.2</td>
</tr>
<tr>
<td>Flow stress, $\sigma_o = 0.92 \sigma_u$ [91] (MPa)</td>
<td>294.6</td>
</tr>
<tr>
<td>Coefficient of friction, $\mu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Axial bent radius, $R_{axial}$ (mm)</td>
<td>As presented in Table c.1 or 50.8</td>
</tr>
<tr>
<td>Blade angle, $2\theta$</td>
<td>8.53°</td>
</tr>
<tr>
<td>Blade tip width, $T$ (mm)</td>
<td>1</td>
</tr>
<tr>
<td>Blade shoulder width, $2B$ (mm)</td>
<td>3</td>
</tr>
<tr>
<td>Number of cutter blades, $n$</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 9.8 Steady-state cutting resistance forces versus tube wall thickness for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm) by a cutter of four blades without the presence of deflector.
Figure 9.9 Steady-state cutting resistance forces versus tube wall thickness for axial cutting of circular AA6061-T6 extrusions ($D_o = 44.45$ mm) by a cutter of four blades with the presence of the curved deflector ($R_{deflector} = 50.8$ mm).

Figure 9.10 Steady-state cutting resistance forces versus tube wall thickness for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm) by a cutter of four blades with the presence of the curved deflector ($R_{deflector} = 50.8$ mm).
Figure 9.11 Steady-state cutting resistance forces versus tube wall thickness for axial cutting of circular AA6061-T6 extrusions ($D_o = 63.5$ mm) by a cutter of four blades with the presence of the curved deflector ($R_{deflector} = 50.8$ mm).

9.8.2 Effect of cutter blade tip width

The effect of cutter blade tip width on the steady-state cutting resistance forces for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm and $t = 1.587$ mm) by a cutter of four blades with different blade tip widths without use of deflector is presented in Table 9.2. The input parameters used for the theoretical predictions are the same as what presented in Table 9.1 except for the value of blade tip width. Although the theoretical predictions from the proposed model over-predict the axial cutting forces, it is shown by the analytical model as well as the experimental data that, for the blade tip widths considered, it has a minor influence on the steady-state cutting forces. A deep look at the analytical model as expressed mathematically in Equation (9.45) reveals that a slowly reducing relationship exists between the blade tip width and the steady-state axial cutting force for the blade tip width greater than 0.75 mm.
Table 9.2 Steady-state cutting resistance forces for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm and $t = 1.587$ mm) by a cutter of four blades with different blade tip widths without use of deflector.

<table>
<thead>
<tr>
<th></th>
<th>Steady-state cutting force, $F$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T = 0.75$ mm</td>
</tr>
<tr>
<td>Experimental</td>
<td>15.97</td>
</tr>
<tr>
<td>Proposed analytical model, Equation (9.45)</td>
<td>19.75</td>
</tr>
<tr>
<td>Zheng and Wierzbicki [59]</td>
<td>15.04</td>
</tr>
<tr>
<td>Simensen and Wierzbicki [58]</td>
<td>16.68</td>
</tr>
</tbody>
</table>

9.8.3 Effect of cutter blade quantities

The effect of cutter blade quantities on the steady-state cutting force for the axial cutting process with/without the presence of the curved deflector is discussed in this section. The input parameters used for the theoretical predictions are the same as what presented in Table 9.1 except that different number of cutter blades are used. Figure 9.12 and Figure 9.13 show the steady-state cutting resistance forces versus number of cutter blades for axial cutting of circular AA6061-T6 extrusions with an outer diameter of 50.8 mm and extrusion wall thicknesses of 3.175 mm and 1.587 mm, respectively, without the presence of deflector. Figure 9.14 through Figure 9.16 present the steady-state cutting resistance forces versus number of cutter blades for axial cutting of circular AA6061-T6 extrusions with the reduced tube wall thickness of 1.5 mm and extrusion outer diameters of 44.25 mm, 50.8 mm, and 63.5 mm, respectively, with the use of the curved deflector ($R_{deflector} = 50.8$ mm).
Figure 9.12 Steady-state cutting resistance forces versus number of cutter blades for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm and $t = 3.175$ mm) without the presence of deflector.

Figure 9.13 Steady-state cutting resistance forces versus number of cutter blades for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm and $t = 1.587$ mm) without the presence of deflector.
Figure 9.14  Steady-state cutting resistance forces versus number of cutter blades for axial cutting of circular AA6061-T6 extrusions ($D_o = 44.25$ mm and $t = Y = 1.5$ mm) with the presence of the curved deflector ($R_{deflector} = 50.8$ mm).

Figure 9.15  Steady-state cutting resistance forces versus number of cutter blades for axial cutting of circular AA6061-T6 extrusions ($D_o = 50.8$ mm and $t = Y = 1.5$ mm) with the presence of the curved deflector ($R_{deflector} = 50.8$ mm).
It can be seen from Figure 9.12 and Figure 9.13 that, for the axial cutting process with the presence of deflector, theoretical predictions from the present proposed analytical model agree well with the experimental tests with a maximum relative error of 8.2%. The maximum relative errors for theoretical predictions from Zheng and Wierzbicki [59] and Simonsen and Wierzbicki [58] are determined to be 41.3% and 33.0%, respectively. For the axial cutting with the presence of the curved deflector, the maximum relative errors for theoretical predictions from the present analytical model are found to be 15.8%, 27.1%, and 28.9% for the circular extrusions with outer diameters of 44.45 mm, 50.8 mm, and 63.5 mm, respectively. The theoretical predictions for the larger diameter extrusions under-predict the axial cutting forces, since in the experimental tests the cut petalled sidewalls did not conform well to the profile of the curved deflector due to large diameter of extrusion and resulted in buckling of the cut petalled sidewalls. For the number of cutter blades considered, a non-linear increasing relationship between the steady-state cutting force and the cutter blade quantities is observed experimentally and theoretically for the extrusions considered.
9.8.4 Effect of extrusion diameter

The effect of extrusion diameter on the steady-state cutting force for the axial cutting process with the presence of the curved deflector is discussed in this section. The input parameters used for the theoretical predictions are presented in Table 9.1. Figure 9.17 through Figure 9.19 show the steady-state cutting resistance forces versus tube diameter for axial cutting of circular AA6061-T6 extrusions with the reduced tube wall thicknesses of 1.0 mm, 1.25 mm, and 1.5 mm, respectively. Theoretical predictions from the proposed model illustrate a minor increasing relationship between the axial cutting force and the extrusion diameter as shown in Figure 9.17 through Figure 9.19. Analytical models from Zheng and Wierzbicki [59] and Simonsen and Wierzbicki [58] show that there is no relationship between them. The experimental results for thicker and larger diameter extrusions are much higher than the predictions from the present analytical model since in the experimental tests the cut petalled sidewalls did not conform well to the profile of the curved deflector and resulted in buckling of the cut petalled sidewalls. The maximum relative error for the proposed analytical model ranges from 0.6% to 22.8% for the extrusion diameter considered.

![Figure 9.17](image_url)

Figure 9.17 Steady-state cutting resistance forces versus tube diameter for axial cutting of circular AA6061-T6 extrusions \( (t = Y = 1.0 \text{ mm}) \) by a cutter of four blades with the presence of the curved deflector \( (R_{\text{deflector}} = 50.8 \text{ mm}) \).
Figure 9.18  Steady-state cutting resistance forces versus tube diameter for axial cutting of circular AA6061-T6 extrusions ($t = Y = 1.25$ mm) by a cutter of four blades with the presence of the curved deflector ($R_{\text{deflector}} = 50.8$ mm).

Figure 9.19  Steady-state cutting resistance forces versus tube diameter for axial cutting of circular AA6061-T6 extrusions ($t = Y = 1.5$ mm) by a cutter of four blades with the presence of the curved deflector ($R_{\text{deflector}} = 50.8$ mm).
10 CONCLUSIONS

A significant amount of information regarding the energy absorption capabilities and deformation modes of circular AA6061-T6 aluminum alloy extrusions under dynamic and quasi-static axial compressive loading conditions has been achieved through the experimental tests and numerical simulations conducted in this research. Factors that influence the novel cutting deformation mode, including extrusions geometries, cutter geometries, number of cutters, cutter blade quantities, without/with the use of deflector, deflector surface profiles, of the circular AA6061-T6 extrusions have been studied experimentally, numerically, and theoretically, which provides in-depth knowledge on improving cutting performance and achieving desired force/displacement responses of the extrusions.

10.1 Conclusions for axial crushing tests

Dynamic and quasi-static axial crush tests were completed for the circular AA6061-T6 specimens with various tube lengths, diameters, and wall thicknesses. Based upon the experimental observations and analysis of the experimental data, the following conclusions can be made:

1. A higher peak crush load, with a magnitude of approximately 8%-27% times that of the force necessary under the quasi-static loading, was observed to initiate the progressive folding deformation mode under the impact loading condition for the specimens considered for both loading conditions in this research.
2. The mean crush force from the dynamic tests was determined to be 3%-25% times that from the quasi-static experimental tests for the specimens considered for both loading conditions in this research.
3. The average CFE of the specimens with a wall thickness of 3.175 mm which experienced progressive folding and global bending deformation modes under quasi-static loading were observed to be 67.7% and 20.4%, respectively. The average TEA of the same specimens which underwent progressive folding and global bending deformation modes under quasi-static loading, were found to be 14.0 kJ and 4.0 kJ, respectively.
4. For the specimens with a wall thickness of 1.587 mm and tube lengths of 200 mm and 300 mm, which collapsed in progressive folding mode under the quasi-static loading, fairly consistent crush parameters were observed for both extrusion geometries.

5. For the specimens with same reduced wall thickness and varied extrusion diameter, the mean crush force and TEA were observed to be fairly consistent under the quasi-static loading condition regardless of the extrusion diameter. However, the peak crush load was observed to be an increasing function of the extrusion diameter; and a decreasing relationship was found to exist between the CFE and the tube diameter.

10.2 Conclusions for axial cutting test with/without deflector

Dynamic and quasi-static axial cutting tests employing a single cutter with/without the presence of a straight/curved deflector were completed for the circular AA6061-T6 specimens with various tube lengths, diameters, and wall thicknesses. Based upon the experimental observations and analysis of the experimental data, the following conclusions can be made:

1. The cutting deformation mode initiated by use of the cutter with/without the use of deflector appeared to be stable, repeatable and controllable with regard to the cutting deformation mode.

2. No initial peak cutting force was observed to initiate the quasi-static cutting deformation mode with/without the presence of deflector.

3. For the quasi-static cutting tests of the specimens with a wall thickness of 3.175 mm, a constant load/displacement relationship was observed after a crosshead displacement of approximately 20 mm for the cutting deformation without deflector. A constant load/displacement relationship was observed after a crosshead displacement of approximately 60 mm for the cutting deformation with the straight deflector and a crosshead displacement of approximately 70 mm for the cutting deformation with the curved deflector.

4. The average CFE for the extrusions of a wall thickness of 3.175 mm, which experienced the cutting mode of deformation under the quasi-static loading, with
the straight and curved deflector as well as with no deflector was observed to be approximate 69.4%, 81.6%, and 94.2%, respectively.

5. The slight variation in cutter blade tip width had no significant influence on the load/displacement responses of the specimens for the cutting deformation modes with/without the presence of deflector.

6. Tube length appeared to have no significant influence on the load/displacement responses of the extrusions.

7. For the extrusions which experienced the cutting deformation modes, four major energy dissipation mechanisms were observed, namely, a cutting deformation in the vicinity of the cutter blade tip, a circumferential membrane stretching of the extrusion, a petalled sidewall outward bending, and friction between the cutter blade and the extrusion side wall. In some experimental tests, energy dissipation in the form of material fracture was also observed, especially for the thinner wall thickness extrusions; however, the occurrence of the material fracture on the cut petalled sidewalls is minor.

8. The combination of the RevI cutter with a curved deflector had a lower degree of material fracture on the petalled sidewalls resulting in a lower degree of cutting force fluctuations. The RevII cutter geometry was observed to be more adaptable to any slight misalignment of the centerlines of the cutter(s) and extrusion.

9. An initial high transient cutting force was observed for dynamic tests. This transient force was typically removed from the cutting process in the first 0.1 ms or approximately 1 mm of displacement. The peak cutting force necessary to initiate the cutting deformation under the impact loading was determined to range from 1.01 to 1.98 times the quasi-static cutting force for the extrusions considered in this research. An inverse relationship between extrusion wall thickness and the ratio of the initial dynamic to quasi-static cutting force was found.

10. The dynamic mean cutting force was determined to be from 0.82 to 1.28 times that for the quasi-static cutting tests for the single-cutter configurations considered in this research. The dynamic mean cutting forces for extrusions with wall thicknesses of 1.0 mm and 1.2 mm and considered for the dual-cutter
configuration were determined to be 0.94 and 1.01 times the values under quasi-static loading, respectively.

10.3 Conclusions for axial dual-stage cutting with a curved deflector

Dynamic and quasi-static dual-stage axial cutting tests for the circular AA6061-T6 specimens with reduced wall thicknesses of 1.0 mm and 1.2 mm utilizing the two cutters and curved deflector assembly were completed. Based upon the experimental observations and analysis of the experimental data, the following conclusions can be made:

1. The dual-stage cutting, under both impact and quasi-static loading, is typically a superposition of two single-stage cutting processes with a displacement delay, for the second stage, equal to the cutter thickness.
2. The dynamic cutting force was observed to be generally consistent with the quasi-static cutting force for the extrusion geometries considered.
3. A system incorporating dual cutters can be configured as an adaptive energy absorption system with desired load/displacement profiles if combined with extrusions with different wall thickness.

10.4 Conclusions for controlling load/displacement response of the extrusion

Dynamic and quasi-static cutting tests were completed on the extrusions with variable instantaneous wall thickness in the axial direction in order to investigate the controllability of load/displacement responses of the extrusions. Based upon the experimental observations and analysis of the experimental data, the following conclusions can be made:

1. The load/displacement responses of the extrusions generally varied with the geometric change of the extrusions under both dynamic and quasi-static loadings. The observed linear relationship between the steady-state cutting force and the extrusion wall thickness under quasi-static loading can be used to design a desired energy absorption response. With the presence of the deflector, the mean cutting force was slightly reduced and resulted in slightly lower energy absorption.
2. A system can be configured as an adaptive energy absorption system with desired load/displacement profiles for extrusions having variable wall thickness.

10.5 Conclusions for FE Modeling

FE modeling of the axial cutting and crushing of the circular AA6061-T6 extrusions were completed employing the Eulerian and Lagrangian finite element formulations, respectively. Both quasi-static and dynamic loading conditions were considered for both FE formulations. Moreover, FE models which considered mass scaling and without mass scaling were implemented into the dynamic axial cutting simulations in order to investigate the influence of the mass scaling. Additionally, material model considering the strain rate effects of the AA6061-T6 material was also developed by incorporation of the Cowper-Symonds constitutive relationship into the material model and implemented to one of the dynamic cutting tests to investigate the strain rate sensitivity of the AA6061-T6 material. Based upon the numerical observations and analysis of the numerical data, the following conclusions can be made:

1. Numerical models employing the Eulerian FE formulation method were able to simulate the cutting process of circular extrusions under both impact and quasi-static loadings. Numerical predictions to the quasi-static axial cutting tests by the use of a multi-blade cutter exhibited validation metric and relative error of 91.6% and 8.9%, respectively. For the dynamic cutting simulations with both single-stage and dual-stage configurations, the validation metric and relative error measures in the entire simulation time domain were 62.2% and 43.6%, respectively. The relative low value of validation metric or high value of relative error was mostly due to the use of mass scaling in the FE models.

2. Use of mass scaling in the numerical simulations of the axial cutting test resulted in significant force fluctuations when contact between the cut petalled sidewalls and the deflector occurred, however, the local mean force was approximately the same as that predicted without mass scaling.

3. FE simulation results from the dynamic axial cutting test with/without the consideration of the strain rate effect of the AA6061-T6 material illustrated minor effect on the cutting force.
4. Numerical models employing the Lagrangian FE formulation method were able to simulate the crush process of circular extrusions under both impact and quasi-static loadings. Numerical predictions to the quasi-static axial crushing tests exhibited validation metric and relative error of 66.9% and 50.8%, respectively. The relative low value of validation metric or high value of relative error was due to the development of folds occurred at different displacement observed in the numerical simulation and the experimental testing.

10.6 Conclusions for theoretical study of steady-state cutting process

A theoretical model for the steady-state axial cutting of circular tubes by a cutter with multiple blunt blades with/without the use of deflector was developed based upon the rate of energy dissipation in assumed deformation modes. Five plastic energy dissipation mechanisms were identified by analyzing experimental observations, including the circumferential membrane stretching in the vicinity of the cutter blade, the far-field plastic bending of tube sidewall, the membrane deformation in the transition zone between the transient and stable flaps, the continuous chip formation, and the cut petalled sidewalls bending outward. The steady-state axial cutting resistance force was then determined employing the principle of virtual power. Finally, the effect of friction force was considered and included to the total cutting resistance force.

Comparisons of the proposed analytical model to the experimental results with regard to the effects of tube wall thickness, cutter blade tip width, cutter blade quantities, and extrusion diameter show good agreement between the predictions and the experimental data and the following conclusion can be made:

1. An increasing relationship, almost linear, between the steady-state cutting force and the tube wall thickness exists for the extrusion geometries considered and for the axial cutting process with/without the presence of the curved deflector.
2. The presence of curved deflector has a minor effect on the steady-state cutting resistance force.
3. The blade tip width has a minor effect on the steady-state axial cutting force for a blade tip width greater than 0.75 mm and the extrusion geometries considered.

4. A non-linear increasing relationship between the steady-state cutting force and the cutter blade quantities exists for the extrusions considered.

5. The proposed analytical model shows that the extrusion diameter has minor effect on the axial cutting resistance force, however, the experimental data an increasing relationship between them.

6. The maximum relative errors between the theoretical predictions and the experimental results are found to be 14.8% and 22.8% for the axial cutting processes considered with and without the use of the curved deflector, respectively.

10.7 Future work

Future work in this area may include investigation of the load/displacement and energy absorption characteristics of multi-cell or multi-wall extrusions subjected to this novel cutting deformation mode. Utilization of multi-cell or multi-wall extrusions could potentially improve the total energy absorption of the extrusions significantly and would lead to an adaptive passive energy absorption device through changing the number of walls being cut by adjusting the cutter blade positions. Furthermore, study the responses of single-cell or multi-cell/wall extrusions under varied loading conditions, including but not limited to axial blast loading, axial dynamic impact at high impact speed, and dynamic and quasi-static oblique loading, would be very necessary towards the design of an ideal robust energy absorption device. Finally, study of this novel cutting deformation on other strain rate sensitive materials might be also interested.
REFERENCES


APPENDIX A: EXPERIMENTAL LOAD VERSUS DISPLACEMENT PROFILES

Figure A.1  The force/displacement responses for the circular AA6061-T6 specimens in group 2.

Figure A.2  The force/displacement responses for the circular AA6061-T6 specimens in group 3.

Figure A.3  The force/displacement responses for the circular AA6061-T6 specimens in group 4.
Figure A.4  The force/displacement responses for the circular AA6061-T6 specimens in group 5.

Figure A.5  The force/displacement responses for the circular AA6061-T6 specimens in group 6.

Figure A.6  The force/displacement responses for the circular AA6061-T6 specimens in group 7.
Figure A.7 The force/displacement responses for the circular AA6061-T6 specimens in group 10.

Figure A.8 The force/displacement responses for the circular AA6061-T6 specimens in group 11.

Figure A.9 The force/displacement responses for the circular AA6061-T6 specimens in group 12.
Figure A.10 The force/displacement responses for the circular AA6061-T6 specimens in group 13.

Figure A.11 The force/displacement responses for the circular AA6061-T6 specimens in group 14.

Figure A.12 The force/displacement responses for the circular AA6061-T6 specimens in group 15.
Figure A. 13  The force/displacement responses for the circular AA6061-T6 specimens in group 16.

Figure A. 14  The force/displacement responses for the circular AA6061-T6 specimens in group 17.

Figure A. 15  The force/displacement responses for the circular AA6061-T6 specimens in group 18.
Figure A. 16  The force/displacement responses for the circular AA6061-T6 specimens in group 19.

Figure A. 17  The force/displacement responses for the circular AA6061-T6 specimens in group 20.

Figure A. 18  The force/displacement responses for the circular AA6061-T6 specimens in group 21.
Figure A. 19  The force/displacement responses for the circular AA6061-T6 specimens in group 22.

Figure A. 20  The force/displacement responses for the circular AA6061-T6 specimens in group 23.

Figure A. 21  The force/displacement responses for the circular AA6061-T6 specimens in group 24.
Figure A.22  The force/displacement responses for the circular AA6061-T6 specimens in group 25.

Figure A.23  The force/displacement responses for the circular AA6061-T6 specimens in group 26.

Figure A.24  The force/displacement responses for the circular AA6061-T6 specimens in group 27.
Figure A.25  The force/displacement responses for the circular AA6061-T6 specimens in group 28.

Figure A.26  The force/displacement responses for the circular AA6061-T6 specimens in group 29.

Figure A.27  The force/displacement responses for the circular AA6061-T6 specimens in group 30.
Figure A.28 The force/displacement responses for the circular AA6061-T6 specimens in group 31.

Figure A.29 The force/displacement responses for the circular AA6061-T6 specimens in group 32.

Figure A.30 The force/displacement responses for the circular AA6061-T6 specimens in group 33.
Figure A.31 The force/displacement responses for the circular AA6061-T6 specimens in group 34.

Figure A.32 The force/displacement responses for the circular AA6061-T6 specimens in group 35.

Figure A.33 The force/displacement responses for the circular AA6061-T6 specimens in group 36.
Figure A.34  The force/displacement responses for the circular AA6061-T6 specimens in group 37.

Figure A.35  The force/displacement responses for the circular AA6061-T6 specimens in group 38.

Figure A.36  The force/displacement responses for the circular AA6061-T6 specimens in group 39.
Figure A.37 The force/displacement responses for the circular AA6061-T6 specimens in group 40.

Figure A.38 The force/displacement responses for the circular AA6061-T6 specimens in group 41.

Figure A.39 The force/displacement responses for the circular AA6061-T6 specimens in group 42.
Figure A.40  The force/displacement responses for the circular AA6061-T6 specimens in group 43.

Figure A.41  The force/displacement responses for the circular AA6061-T6 specimens in group 44.

Figure A.42  The force/displacement responses for the circular AA6061-T6 specimens in group 45.
Figure A.43 The force/displacement responses for the circular AA6061-T6 specimens in group 46.

Figure A.44 The force/displacement responses for the circular AA6061-T6 specimens in group 47.

Figure A.45 The force/displacement responses for the circular AA6061-T6 specimens in group 48.
Figure A. 46  The force/displacement responses for the circular AA6061-T6 specimens in group 49.

Figure A. 47  The force/displacement responses for the circular AA6061-T6 specimens in group 50.

Figure A. 48  The force/displacement responses for the circular AA6061-T6 specimens in group 51.
Figure A. 49  The force/displacement responses for the circular AA6061-T6 specimens in group 52.

Figure A. 50  The force/displacement responses for the circular AA6061-T6 specimens in group 53.

Figure A. 51  The force/displacement responses for the circular AA6061-T6 specimens in group 54.
Figure A.52  The force/displacement responses for the circular AA6061-T6 specimens in group 55.

Figure A.53  The force/displacement responses for the circular AA6061-T6 specimens in group 56.

Figure A.54  The force/displacement responses for the circular AA6061-T6 specimens in group 57.
Figure A.55 The force/displacement responses for the circular AA6061-T6 specimens in group 58.

Figure A.56 The force/displacement responses for the circular AA6061-T6 specimens in group 59.

Figure A.57 The force/displacement responses for the circular AA6061-T6 specimens in group 60.
Figure A.58 The force/displacement responses for the circular AA6061-T6 specimens in group 61.

Figure A.59 The force/displacement responses for the circular AA6061-T6 specimens in group 62.

Figure A.60 The force/displacement responses for the circular AA6061-T6 specimens in group 63.
Figure A.61 The force/displacement responses for the circular AA6061-T6 specimens in group 64.

Figure A.62 The force/displacement responses for the circular AA6061-T6 specimens in group 65.

Figure A.63 The force/displacement responses for the circular AA6061-T6 specimens in group 66.
Figure A. 64  The force/displacement responses for the circular AA6061-T6 specimens in group 67.

Figure A. 65  The force/displacement responses for the circular AA6061-T6 specimens in group 68.

Figure A. 66  The force/displacement responses for the circular AA6061-T6 specimens in group 69.
Figure A.67  The force/displacement responses for the circular AA6061-T6 specimens in group 70.

Figure A.68  The force/displacement responses for the circular AA6061-T6 specimens in group 71.

Figure A.69  The force/displacement responses for the circular AA6061-T6 specimens in group 72.
Figure A.70 The force/displacement responses for the circular AA6061-T6 specimens in group 73.

Figure A.71 The force/displacement responses for the circular AA6061-T6 specimens in group 74.

Figure A.72 The force/displacement responses for the circular AA6061-T6 specimens in group 75.
Figure A.73  The force/displacement responses for the circular AA6061-T6 specimens in group 76.

Figure A.74  The force/displacement responses for the circular AA6061-T6 specimens in group 77.

Figure A.75  The force/displacement responses for the circular AA6061-T6 specimens in group 78.
Figure A.76  The force/displacement responses for the circular AA6061-T6 specimens in group 79.

Figure A.77  The force/displacement responses for the circular AA6061-T6 specimens in group 80.

Figure A.78  The force/displacement responses for the circular AA6061-T6 specimens in group 81.
Figure A. 79  The force/displacement responses for the circular AA6061-T6 specimens in group 82.

Figure A. 80  The force/displacement responses for the circular AA6061-T6 specimens in group 83.

Figure A. 81  The force/displacement responses for the circular AA6061-T6 specimens in group 85.
Figure A. 82  The force/displacement responses for the circular AA6061-T6 specimens in group 86.

Figure A. 83  The force/displacement responses for the circular AA6061-T6 specimens in group 88.

Figure A. 84  The force/displacement responses for the circular AA6061-T6 specimens in group 89.
Figure A.85  The force/displacement responses for the circular AA6061-T6 specimens in group 90.

Figure A.86  The force/displacement responses for the circular AA6061-T6 specimens in group 91.

Figure A.87  The force/displacement responses for the circular AA6061-T6 specimens in group 92.
Figure A.88  The force/displacement responses for the circular AA6061-T6 specimens in group 93.

Figure A.89  The force/displacement responses for the circular AA6061-T6 specimens in group 94.

Figure A.90  The force/displacement responses for the circular AA6061-T6 specimens in group 95.
Figure A.91  The force/displacement responses for the circular AA6061-T6 specimens in group 96.

Figure A.92  The force/displacement responses for the circular AA6061-T6 specimens in group 97.

Figure A.93  The force/displacement responses for the circular AA6061-T6 specimens in group 98.
Figure A.94  The force/displacement responses for the circular AA6061-T6 specimens in group 99.

Figure A.95  The force/displacement responses for the circular AA6061-T6 specimens in group 100.

Figure A.96  The force/displacement responses for the circular AA6061-T6 specimens in group 101.
Figure A.97 The force/displacement responses for the circular AA6061-T6 specimens in group 102.

Figure A.98 The force/displacement responses for the circular AA6061-T6 specimens in group 103.

Figure A.99 The force/displacement responses for the circular AA6061-T6 specimens in group 104.
Figure A.100  The force/displacement responses for the circular AA6061-T6 specimens in group 105.

Figure A.101  The force/displacement responses for the circular AA6061-T6 specimens in group 106.

Figure A.102  The force/displacement responses for the circular AA6061-T6 specimens in group 107.
Figure A.103  The force/displacement responses for the circular AA6061-T6 specimens in group 108.

Figure A.104  The force/displacement responses for the circular AA6061-T6 specimens in group 109.

Figure A.105  The force/displacement responses for the circular AA6061-T6 specimens in group 110.
APPENDIX B: PARTIAL INPUT USED IN FE SIMULATIONS

B.1 Partial input for quasi-static cutting of circular AA6061-T6 extrusions by a cutter with multiple cutter blades without the presence of deflector utilizing Eulerian FE formulation

```
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               MATERIAL CARDS                                 $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*MAT_ELASTIC_PLASTIC_HYDRO
$ This material model is for AA6061-T6 extrusion and airmesh
$ MID        RO         G      SIGY        EH        PC        FS
 1   2.7E-06 3.288E+07  271600.0  
$ EPS1      EPS2      EPS3      EPS4      EPS5      EPS6      EPS7      EPS8
 0.0  0.000214  0.001289  0.002246  0.003483  0.0050  0.006797
$ EPS9      EPS10     EPS11     EPS12     EPS13     EPS14     EPS15     EPS16
 0.008873  0.016783  0.019979  0.02246  0.026994  0.0101501  0.117129
$ ES1       ES2       ES3       ES4       ES5       ES6       ES7       ES8
271564.3  276939.4  283788.0  288361.2  291531.0  293903.9  295856.3  297639.3
$ ES9       ES10      ES11      ES12      ES13      ES14      ES15      ES16
299423.7  305662.2  308138.6  313897.1  314386.6  315660.1  316244.6  358802.2
$$
*MAT_RIGID
$ This material model is for the cutter blade
$ MID        RO         E        PR        N    COUPLE         M     ALIAS
 2   2.7E-06   7.2E+07  0.30  
$ CMO      CON1      CON2
 1.0  4.0  7.0
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               SECTION CARDS                                  $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*SECTION_SOLID
$ This section card is for the extrusion and airmesh
$ SECID    ELFORM       AET
 1        12  
$$
*SECTION_SOLID
$ This section card is for the cutter blade
$ SECID    ELFORM       AET
 2         1  
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               EOS CARD                                  $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*EOS_LINEAR_POLYNOMIAL
$    EOSID        C0        C1        C2        C3        C4        C5        C6
 1  0.0 7.563E+07  0.0  0.0  0.0  0.0  0.0
$    E0        V0
 0.0  1.0
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               CONSTRAINED LAGRANGE IN SOLID CARD           $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*CONSTRAINED_LAGRANGE_IN_SOLID
$    SLAVE    MASTER     SSTYP     MSTYP     NQUAD     CTYPE     DIREC     MCOUP
 3         1         0        -3         4         2         0
$    START       END      PFAC      FRIC    FRCMIN      NORM   NORMTYP      DAMP
 0.20  0.22  
$    CQ      HMIN      HMAX     ILEAK     PLEAK   LCIDPOR
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               BOUNDARY PRESCRIBED CARD                            $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*BOUNDARY_PRESCRIBED_MOTION_RIGID_ID
$    ID
 1
$    PID    DOF   VAD    LCID   SF    VID   DEATH   BIRTH
 3   3     2   10    -35.0
```

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**B.2 Partial input for dynamic cutting of circular AA6061-T6 extrusions by a cutter(s)/deflector assembly utilizing Eulerian FE formulation**

```plaintext
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                             LOAD CURVE CARD                                  $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*DEFINE_CURVE_TITLE
$     LCID      SIDR       SFA       SFO      OFFA      OFFO    DATTYP
10         0
$                 A1                  O1
0.0                 0.0
0.0050                 1.0
0.010                 2.0

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               MATERIAL CARDS                                 $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*MAT_ELASTIC_PLASTIC_HYDRO
$ This material model is for AA6061-T6 extrusion and airmesh
$ 20X mass scaling used
$     MID        RO         G      SIGY        EH        PC        FS
1   5.4E-05 3.288E+07  271600.0
$     EPS1      EPS2      EPS3      EPS4      EPS5      EPS6      EPS7      EPS8
0.0  0.000214  0.000611  0.001289  0.002246  0.003483  0.0050  0.006797
$     EPS9      EPS10     EPS11     EPS12     EPS13     EPS14     EPS15     EPS16
0.008873  0.016783  0.019979  0.061336  0.067331  0.086994  0.101501  0.117129
$     ES1       ES2       ES3       ES4       ES5       ES6       ES7       ES8
271564.3  276939.4  283788.0  288361.2  291531.0  293903.9  295856.3  297639.3
$     ES9      ES10      ES11      ES12      ES13      ES14      ES15      ES16
299423.7  305662.2  308138.6  337897.1  341386.6  350660.1  356244.6  358802.2
$$
*MAT_RIGID
$ This material model is for the cutter blade – mass scaling is used
$     MID        RO         E        PR         N    COUPLE         M     ALIAS
3  2.53E-04   7.2E+07      0.30
$     CMO      CON1      CON2
1.0       4.0       7.0
$$
*MAT_RIGID
$ This material model is for the deflector – mass scaling is used
$     MID        RO         E        PR         N    COUPLE         M     ALIAS
4  2.09E-04   7.2E+07      0.30
$     CMO      CON1      CON2
1.0       4.0       7.0
$LCO_OR_A1        A2        A3        V1        V2        V3
$$
*MAT_RIGID
$ This material model is for the upper load cell – mass scaling is used
$     MID        RO         E        PR         N    COUPLE         M     ALIAS
5  6.95E-06   7.2E+07      0.30
$     CMO      CON1      CON2
1.0       4.0       7.0
$LCO_OR_A1        A2        A3        V1        V2        V3
$$
*MAT_PIECEWISE_LINEAR_PLASTICITY
$ This material model is for the deformable crush plate – mass scaling is used
$     MID        RO         E        PR         SIGY      ETAN      EPFF      TDEL
6  2.54E-04 6.807E+07  3.5E-01  2.696E+05         0         0         0
$     C      P      LCSS     LCSR     VP
1
$     EPS1      EPS2      EPS3      EPS4      EPS5      EPS6      EPS7      EPS8
$     ES1       ES2       ES3       ES4       ES5       ES6       ES7       ES8
```
*MAT_RIGID
$ This material model is for the rigid crush plate - mass scaling is used
$ MID RO E PR N COUPLE M ALIAS
   7 3.82E-05 7.2E+07 0.30
$ CMO CON1 CON2
   1.0 4.0 7.0
$LCO_OR_A1 A2 A3 V1 V2 V3

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$ STRESS/STRAIN CURVE CARD
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*DEFINE_CURVE
$ THIS IS THE LOAD CURVE FOR THE YIELD STRESS VS. EFFECTIVE PLASTIC STRAIN
$ FOR THE ALUMINUM MATERIAL 6061-T6 (UNITS ARE BASED UPON BASE SET-KG,MM,SEC)
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$ STRAIN STRESS
  0.0000000000000  271564.3239
  0.0002136327340  276939.4407
  0.0006112091341  283787.9838
  0.0012886340610  288361.2380
  0.0022459075140  291530.9548
  0.0034830294940  293903.9085
  0.0050000000000  295856.3070
  0.0067968190330  297639.3107
  0.0088734865920  299423.7054
  0.0112300026800  301319.8784
  0.0138663672900  303390.4718
  0.0167825804300  305662.2426
  0.0199786421000  308138.6311
  0.0234545522900  310812.0176
  0.0272103110100  313673.5184
  0.0312459182500  316718.0799
  0.0355613740200  319943.4779
  0.0401567832200  323343.4378
  0.0450318311500  326897.1502
  0.0501868325000  330559.4627
  0.0556216823800  334257.2528
  0.0613363807800  337897.1062
  0.0673309277100  341386.5898
  0.0736053231700  344665.5349
  0.0801595671600  347734.8886
  0.0869936596700  350660.0753
  0.0941076007000  353516.6584
  0.1015013903000  356244.6437
  0.1091750284000  358394.7338
  0.1171285150000  358802.2298

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$ SECTION CARDS
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*SECTION_SOLID
$ This section card is for the extrusion and airmesh
$ SECID EIFORM AET
   1 12

$SECTION_SOLID
$ This section card is for the cutter blade, deflector, load cell, and deformable crush plate
$ SECID EIFORM AET
   3 1

$SECTION_SHELL
$ This section card is for the rigid crush plate
$ SECID EIFORM SHRFP NIP PROPT QR/IRID ICOMP SETYP

319
$\begin{array}{cccccccc}
7 & 2 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
\hline
\end{array}$

$\begin{array}{cccccccccccc}
\text{CONSTRAINED RIGID BODIES CARDS} & S \\
\hline
\end{array}$

$\begin{array}{cccccccccccc}
\text{CONSTRAINED_RIGID_BODIES} & PIMD & PIDS \\
\hline
4 & 5 \\
\end{array}$

$\begin{array}{cccccccccccc}
\text{CONSTRAINED RIGID BODIES} & PIMD & PIDS \\
\hline
3 & 4 \\
\end{array}$

$\begin{array}{cccccccccccc}
\text{CONSTRAINED LAGRANGE IN SOLID CARDS} & S \\
\hline
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{CONSTRANDED LAGRANGE IN SOLID CARDS} & SLAVE & MASTER & SSTYP & MSTYP & NQUAD & CTYPE & DIREC & MCoup \\
\hline
2 & 1 & 0 & 0 & -3 & 4 & 2 & 0 \\
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{CONTACT CARDS} & S \\
\hline
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{CONTACT_AUTOMATIC_SURFACE_TO_SURFACE} & SSID & MSID & SSTYP & MSTYP & SOBOXID & MBOXID & SPR & MPR \\
\hline
6 & 5 & 3 & 3 \\
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{INITIAL VELOCITY CARDS} & S \\
\hline
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{INITIAL VELOCITY GENERATION} & ID & STYP & OMEGA & VX & VY & VZ & IVATN \\
\hline
3 & 1 & 0 & 0 & 0 & -6800.0 \\
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{LOAD CURVE CARDS} & S \\
\hline
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{LOAD BODY Z} & LCID & SF & LCIDDR & XC & YC & ZC \\
\hline
2 & 9807.0 \\
\end{array}$

$\begin{array}{cccccccccccccccc}
\text{DEFINE_CURVE} & LCID & SIDR & SCLA & SCLO & OFFA & OFFO \\
\hline
2 & 0 \\
\end{array}$
B.3 Partial input for dynamic crushing of circular AA6061-T6 extrusions using Lagrangian FE formulation

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8$
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8$

*MAT_PIECEWISE_LINEAR_PLASTICITY

This material model is for the 6061-T6 aluminum alloy extrusion

$\text{MID} \quad \text{RO} \quad \text{E} \quad \text{PR} \quad \text{SIGY} \quad \text{ETAN} \quad \text{EPF} \quad \text{TDEL}$
2 2.70E-06 6.807E+07 3.5E-01 2.716E+05 0 0 0

$\text{C} \quad \text{P} \quad \text{LCSS} \quad \text{LCSR} \quad \text{VP}$
1288000.0 4.0 1

$\text{EPS1} \quad \text{EPS2} \quad \text{EPS3} \quad \text{EPS4} \quad \text{EPS5} \quad \text{EPS6} \quad \text{EPS7} \quad \text{EPS8}$

$\text{ES1} \quad \text{ES2} \quad \text{ES3} \quad \text{ES4} \quad \text{ES5} \quad \text{ES6} \quad \text{ES7} \quad \text{ES8}$

$\text{MAT_RIGID}$

This material model is for the crush plate

$\text{MID} \quad \text{RO} \quad \text{E} \quad \text{PR} \quad \text{N} \quad \text{COPPE} \quad \text{M} \quad \text{ALIAS}$
1 1.79E-03 7.2E+07 0.30

$\text{CMO} \quad \text{CON1} \quad \text{CON2}$
1.0 4.0 7.0

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8$

*DEFINE_CURVE

This is the yield curve for the effective plastic strain

$\text{LCID} \quad \text{SIDR} \quad \text{SCLA} \quad \text{SCLO} \quad \text{OFFA} \quad \text{OFFO}$
1 0 1.00 1.00

$\text{STRAIN} \quad \text{STRESS}$
0.0000000000000 271564.3239
0.0002136327340 276939.4407
0.0006120913410 283787.9838
0.0012886340610 288361.2380
0.0022459075140 291530.9548
0.0034830294940 293930.9085
0.0050000000000 295856.3070
0.0067968190330 297639.3107
0.0088734865920 299423.7054
0.0112300028800 301319.8784
0.0138663672900 303390.4718
0.0167825804300 305662.2426
0.0199786421000 308138.6311
0.0234545522900 310812.0176
0.0272103110100 313673.5184
0.0312459182500 316718.0799
0.0355613740200 319943.4779
0.0401566783200 323343.4378
0.0450318311500 326897.1502
0.0501868325000 330559.4627
0.0556216823800 334257.2528
0.0613363807800 337897.1062
0.0673309277100 341386.5898
0.0736053231700 344665.5349
0.0801595671600 347734.8866
0.0869936596700 350660.0753
0.0941076007000 353516.6584
0.1015013903000 356244.6437
0.1091750284000 358394.7338
0.1171285150000 358802.2298

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8$

*SECTION_SHELL
B.4 Partial input for quasi-static crushing of circular AA6061-T6 extrusions using Lagrangian FE formulation

---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               MATERIAL CARDS                                 $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8$
*MAT_PIECEWISE_LINEAR_PLASTICITY
$ This material model is for the 6061-T6 aluminum alloy extrusion
$      MID        RO         E        PR      SIGY      ETAN      EPPF      TDEL
 2  2.70E-06 6.807E+07   3.5E-01 2.716E+05         0         0         0
$        C         P      LCSS      LCSR        VP
 1288000.0       4.0         1
$     EPS1      EPS2      EPS3      EPS4      EPS5      EPS6      EPS7      EPS8
$     ES1       ES2       ES3       ES4       ES5       ES6       ES7       ES8

---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$ This section card is for the extrusion
$      SECID    ELFORM      SHRF       NIP     PROPT   QR/IRID     ICOMP     SETYP
 2         2   0.83333         5
$       T1        T2        T3        T4      NLOC     MAREA
 1.0      1.0      1.0      1.0
*SECTION_SOLID
$ This section card is for the crush plate
$      SECID    ELFORM       AET
 1         1
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                               CONTACT CARDS                                  $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*CONTACT_AUTOMATIC_NODES_TO_SURFACE
$     SSID      MSID     SSTYP     MSTYP    SBOXID    MBOXID       SPR       MPR
 2         3         3         3
$       FS        FD        DC        VC       VDC    PENCHK       BT       DT
 0.10      0.08                          20.0
$       SFS       SFM       SST       MST      SFST      SFMT       FSF       VSF
 1.0       1.0
*CONTACT_AUTOMATIC_SINGLE_SURFACE
$     SSID      MSID     SSTYP     MSTYP    SBOXID    MBOXID       SPR       MPR
 2         0         3
$       FS        FD        DC        VC       VDC    PENCHK       BT       DT
 0.10      0.08                          20.0
$       SFS       SFM       SST       MST      SFST      SFMT       FSF       VSF
 1.0       1.0
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$                                                                              $
$                                B.C. CARDS                                    $
$                                                                              $
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
*BOUNDARY_SPC_SET
$     NSID       CID      DOFX      DOFY      DOFZ     DOFRX     DOFRY     DOFRZ
 1                   1         1         1         1         1         1         1
*INITIAL_VELOCITY_GENERATION
$     ID      STYP     OMEGA        VX        VY        VZ     IVATN
 3         2                                 -7000.0
$     XC        YC        ZC        NX        NY        NZ     PHASE
 100.0
*LOAD_BODY_Z
$     LCID        SF    LCIDDR        XC        YC        ZC       CID
 2    9807.0
*DEFINE_CURVE
$     LCID      SIDR      SCLA      SCLO      OFFA      OFFO
 2         0
$     TIME(S)               VALUE
 0.000               1.000
10.00               1.000
*MAT_RIGID
$ This material model is for the crush plate
$  MID   RO   E    PR    N COUPLE    M ALIAS
1    1.79E-03    7.2E+07    0.30
$  CMO    CON1    CON2
1.0    4.0
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$ FOR THE ALUMINUM MATERIAL 6061-T6 (UNITS ARE BASED UPON BASE SET-KG,MM,SEC)
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$ T1    T2    T3    T4    NLOC    MAREA
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*SECTION_SOLID
$ This section card is for the crush plate
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$ SFS SFM SST MST SFST SFMT FSF VSF
  1.0 1.0
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B.C. CARDS
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APPENDIX C: CUTTING MODEL GEOMETRY

The objective of this section is to find the circumferential gap opening in the front of the cutter blade ($2v_o$) and the gap opening at the blade shoulders ($u_o$), as shown in Figure C.1 and Figure C.2, expressed in terms of the rolling radius $R_r$, the blade shoulder width, $2B$, and the cutter blade semi angle, $\theta$, as well as the axial bent radius for cut petalled sidewalls. The processes of finding the gap openings will follow similar steps as which initially documented by Simonsen and Wierzbicki [58].

![Definitions used to find the gap at the cutter blade tip.](image1)

Figure C.1  Definitions used to find the gap at the cutter blade tip.

![Definitions used to find the gap at the cutter blade sides.](image2)

Figure C.2  Definitions used to find the gap at the cutter blade sides.
C.1 Circumferential gap opening in the front of cutting blade

It is convenient to introduce two coordinate systems, a global coordinate system $(x_G, y_G, z_G)$ and a local coordinate system $(x_L, y_L, z_L)$, both with origin at point O, as shown in Figure C.1. $x_G$-axis is in the opposite direction of the cutter blade advancing direction, while $x_L$-axis is in the bending hinge line OP. The $z_G$- and $z_L$-axes are in the radial direction of the tube. The $y_G$-axis and $y_L$-axis are determined by the other axes. An arc line coordinate, $s$, which follows the curling edge of the tube sidewall, is also introduced as illustrated in Figure C.1. The $s$-axis also has the origin at point O. Then, a point on the curling edge with the arc line coordinate of $s$ has the local coordinates:

\[
\begin{align*}
    x_L &= s \sin \theta \\
    y_L &= -R_{rt} \sin \left( \frac{s \sin \theta}{R_{rt}} \right) \\
    z_L &= R_{rt} \left[ 1 - \cos \left( \frac{s \sin \theta}{R_{rt}} \right) \right]
\end{align*}
\]  
(C.1)

The general relationship between local and global coordinates is given by the transformation:

\[
\begin{align*}
    x_G &= x_L \cos \theta - x_L \sin \theta \\
    y_G &= y_L \sin \theta + y_L \cos \theta \\
    z_G &= z_L
\end{align*}
\]  
(C.2)

Replacing Equation (C.1) into Equation (C.2) gives the global coordinates for a point on the curling edge with the arc coordinate $s$:

\[
\begin{align*}
    x_G &= s \sin \theta \cos \theta + R_t \sin \left( \frac{s \sin \theta}{R_{rt}} \right) \sin \theta \\
    y_G &= s \sin \theta \sin \theta - R_t \sin \left( \frac{s \sin \theta}{R_{rt}} \right) \cos \theta \\
    z_G &= R_t \left[ 1 - \cos \left( \frac{s \sin \theta}{R_{rt}} \right) \right]
\end{align*}
\]  
(C.3)

It is obvious from Figure C.1 that the rolling radius is:

\[
R_r = \frac{R_{rt}}{\cos \theta}
\]  
(C.4)
And the circumferential stretching of the tube sidewall on both sides of the symmetry line is:

\[ 2v_{\theta\theta} = 2r_m \sin^{-1} \frac{y_G}{r_m} \]  

(C.5)

For the tube and cutter blade geometries considered in this research, the circumferential stretching of the tube sidewall can be approximated with:

\[ 2v_{\theta\theta} = 2r_m \sin^{-1} \frac{y_G}{r_m} \approx 2y_G \]

\[ = 2s \sin \theta \sin \theta - 2R_t \sin \left( \frac{s \sin \theta}{R_{rt}} \right) \cos \theta \]  

(C.6)

The exact solution of \( 2v_{\theta\theta} \) can be found by solving Equations (C.3) and (C.6) at the point \( x_G = R_{rt} / \sin \theta = R_r / \tan \theta \). However, the exact solution is too complex and it has to be done numerically. Simonsen and Wierzbicki [58] have shown that the exact solution is very well approximately by the expression:

\[ 2v_{\theta\theta} = 0.317R_r \cos^2 \theta (1 + 0.55 \theta^2) \]  

(C.7)

For the cutter blade geometry considered, the error of Equation (C.7) is typically less than 0.2%.

C.2 Gap opening at blade shoulders

It is convenient to introduce a coordinate system \((X, Y, Z)\) that has the same axes directions as the global coordinate system \((x_G, y_G, z_G)\) yet origins at the intersection of blade shoulder and blade taper regions, as shown in Figure C.2.

The total circumferential opening of the tube sidewall on one side of the symmetry line is:

\[ R_r + r_m \sin^{-1} \left( \frac{B}{r_{mean}} \right) \approx R_r + B \]  

(C.8)
The length of curved part of the stable flap is $\frac{\pi R_r}{2}$, so the length of straight part of the stable flap on one side of blade is:

$$b_s = B - \left(\frac{\pi}{2} - 1\right) R_r \quad (C.9)$$

Likewise, it can be shown that the length of straight part of the initial flap at the tapered region of blade is:

$$b_f = \left(B - \left(\frac{\pi}{2} - 1\right) R_r\right) \cos \theta \quad (C.10)$$

For the assumed tangential contact condition between the deformed flaps and the blade, it is required that:

$$b_s \geq 0 \quad \text{and} \quad b_f \geq 0 \quad (C.11)$$

That is:

$$R_r \leq \frac{B}{\frac{\pi}{2} - 1} \approx 1.75 B \quad (C.12)$$

The position of point T is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_T = \begin{pmatrix} 0 \\ 0 \\ B + \left(2 - \frac{\pi}{2}\right) R_r \end{pmatrix} \quad (C.13)$$

and the position of point Q is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_Q = \begin{pmatrix} B \sin \theta \cos \theta \\ -B \sin^2 \theta \\ \left(B + \left(2 - \frac{\pi}{2}\right) R_r\right) \cos \theta \end{pmatrix} \quad (C.14)$$

Thus, the distance between points T and Q is given by:
It is shown by Simonsen and Wierzbicki [58] that $u_o/B$ is a very weak function of the rolling radius $R_r$, so $u_o$ is well approximated by the following expression for the considered ranges of $\theta$ and $R_r$:

$$u_o \approx B \theta$$  \hspace{1cm} (C.16)

### C.3 Axial bent radius for cut petalled sidewalls

As discussed in the section 9.1, it is assumed that with the presence of blunt cutter blades the circular tube will grow circumferentially but remain circular in cross-sectional geometry, which leads to the cut petalled sidewalls bending outward with a bent radius of $R_{axial}$ as shown in Figure 9.4 and Figure C.3. The increment of tube radius ($\Delta r$) due to the presence of the cutter blades after reaching the steady-state cutting process is given by Equation (C.17), where $n$ is the number of cutter blades and $B$ is half of the blade shoulder width. The axial displacement necessary to reach the steady-state cutting process ($D_{ss}$) is the distance in the $x$ direction from the tip of plastic deformation (point O) to the start of blade shoulder as shown in Figure C.3. This distance can be geometrically determined by Equation (C.18), where $T$ is the blade tip width and $l_b$ is the distance from the blunt blade tip to the beginning edge of finite shoulder as shown in Figure C.1. For the axial cutting process without the presence of deflector, the axial bent radius ($R_{axial}$) can then be determined geometrically by assuming a tangential connection between the undisturbed tube sidewall and the cut petalled sidewalls and free flaring of the cut petalled sidewalls as illustrated in Figure C.3. The geometrically determined axial radii for the tube and cutter blade geometries considered in this research are given in Table C.1. For the axial cutting process with the presence of a curved deflector with a surface profile radius of $R_{deflector}$, the axial bent radius $R_{axial}$ is equal to $R_{deflector}$ if geometrically determined value is bigger than the profile radius of the deflector.
\[
\Delta r = \frac{2nB}{2\pi} 
\]  \hspace{1cm} (C.17)

\[
D_{ss} = \frac{R_r + 0.5T}{\tan \theta} + l_b 
\]  \hspace{1cm} (C.18)

---

**Figure C.3**  Definitions used to determine the axial bent radius of cut petalled sidewalls.

**Table C.1**  Tube and cutter blade geometries considered in this research and axial bent radii of cut petalled sidewalls without the presence of deflector.

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\( \Delta r \), \( R_{axial} \), \( D_{ss} \), \( r_o \), \( t \), \( B \), \( T \), \( n \)
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