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Using Counterexamples to Estimate Degrees of Support

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A very crucial skill in the evaluation of an argument in natural language is the construction of counterexamples to assess the support of its premises for its conclusion. Of course some argumentation and critical thinking textbooks mention counterexamples, but they offer superficial suggestions as to their construction. The field of argumentation can be an odd discipline because it sometimes discovers what needs to be investigated *after* its textbooks have been published. This paper represents my attempt to further explore the logic of counterexamples in natural language. I will first contrast two different kinds of counterexamples, and then use one of them to assess the degrees of support of premises for their conclusion.

Since an argument is valid when and only when it is *impossible* for all its premises to be true and its conclusion false, it is very easy to show that an argument is invalid: we simply imagine a *possible* situation where all the premises are true and the conclusion false. Such a possible situation is a counterexample against the sufficiency of premises, in contrast to counterexamples against the truth of claims.

However, this technique is inadequate against arguments that are not intended to be valid. In fact most everyday arguments are not intended to provide conclusive support. In other words, for most everyday arguments, there is only a certain degree of *improbability* that all the premises are true and the conclusion false. Given the general ease of inventing counterexamples against the validity of an argument, I will explore the logic of such counterexamples in order to find a way of using them to estimate a degree of support that is less than conclusive.

Since there are two basic kinds of counterexamples against the validity of arguments, and my investigation will apply to only one of them, I will first clarify the distinction between them. The counterexamples whose logic I will be examining are very different from counterexamples by analogy. No textbook author describes in any detail how they differ, but a few do present them as being different (Wilson, 1999, 256; Cederblom and Paulsen, 2001, 95; Feldman, 1999, 77-78; Rudinow and Barry, 1994, 207-208). We can see their differences by comparing and contrasting them when they are advanced against the same invalid argument. Let that argument be:

(A) (1) Santa Claus will pass the logic course only if he registers for the course.
(2) He has registered for the course.
So, (3) Santa Claus will pass the logic course.

This argument has the form, (1) P only if Q. (2) Q. So, (3) P. The fact that this is an example of the formal fallacy of affirming a consequent, and that we would typically quickly reject the argument without using any kind of counterexample, is irrelevant. I am just using it as an example against which both kinds of counterexamples can be advanced, and thus both can be appropriately compared. Once we have identified the logical form of an argument, a counterexample by analogy against that argument must have the same form, but have true premises and a false conclusion. I acknowledge that an argument can have more than one logical form, and that there can be controversy regarding the logical form that an argument actually has, but I will not be exploring these issues in this paper. However, if the validity of an argument does depend on a particular form, and if it is possible to construct a different argument that is identical in form but has true premises and a false conclusion, then they are both invalid. The more obviously true the premises and obviously false the conclusion, the more persuasive is a counterexample by analogy in showing the invalidity of a particular form. In this particular case we can advance the following counterexample by analogy against argument (A):

CE1[1] against argument (A):

(1) There's a fire in this room only if there's oxygen in this room.
(2) There's oxygen in this room.
So, (3) there's a fire in this room.

Let us now contrast it to the next counterexample:

CE2 against argument (A)
It is possible that:

(1) Santa Claus will pass the logic course only if he registers for the course. AND
(2) He has registered for the course. AND
What if Santa Claus does not do adequate studying. AND
Not- (3): *It is not the case that* Santa Claus will pass the logic course.

Differences

Both counterexamples successfully show that argument (A) is invalid, in other words, they both show that its premises are not sufficient for its conclusion. However, there are some logically significant differences between them.[2]

1. A counterexample by analogy is an *argument* analogous in form to the argument against which it is advanced. But a counterexample such as CE2 is *not* an argument, and so such a counterexample cannot have the form of the argument against which they are advanced. The kind of counterexample illustrated by CE2 is a possible conjunction of propositions. Accordingly, I propose that we name it a *"counterexample by possible conjunction."* I invite anyone to propose a better descriptive label that will clearly differentiate this kind of counterexample from counterexamples by analogy.

2. In a counterexample by possible conjunction each premise of an argument is granted and *un*changed (all the given reasons *as stated* are assumed to be true), and the argument's conclusion is negated. These two characteristics are necessary because the goal of a counterexample by possible conjunction is to show that all the given premises are not jointly sufficient for the truth of their conclusion. In contrast to these two characteristics,

counterexamples by analogy, as illustrated by CE1, alter some of the content of the premises and conclusion, and they do not negate the conclusion.

3. In counterexamples by possible conjunction all the given premises of an argument and the negation of its conclusion are conjoined to a finite number of other statements, e.g. "What if Santa Claus does not do adequate studying" in CE2. These statements play the very important role of *making us understand how it is possible for all the given premises to be true and the conclusion false*.

Why is this understanding so important? Though a counterexample by possible conjunction is not *in itself* an argument, it is evidence advanced to show to someone who has presented an argument that his/her premises are not sufficient. If a counterexample is not understood by the person presenting the argument, then s/he will not be convinced that the premises are not sufficient, in other words, s/he will not be convinced that his/her argument is invalid. Thus, understanding the counterexample, which involves understanding how it is possible for the argument's premises to be true and its conclusion false, is a necessary condition to show to an arguer that his/her argument is invalid. This is analogous to the construction of any argument: if the argument is not understood by its intended audience, then it will not be convincing, even if it is impeccably logical and has necessarily true premises. This aspect of the construction of a counterexample is very context/audience dependent: it will be effective generally only when it is sensitive to the level of knowledge, intelligence, and imagination of the person to whom the counterexample is presented. And these three factors affect one's level of understanding.

Given this crucial role of the statements conjoined to the granted premises and negated conclusion to form a counterexample by possible conjunction, I need a convenient way to distinguish them from the granted premises and negated conclusion. I will thus sometimes label them by means of the letter "X". Since CE2 would typically be succinctly presented as "*What if* Santa Claus does not do adequate studying," and this common way of communicating this kind of counterexample focuses *exclusively* on the statements that make us understand how it is possible for all the given premises to be true and the conclusion false, I propose to name them the *"what-if-statements"* of the counterexample. Again, I invite anyone to propose a better descriptive label. In contrast to these counterexamples, no new statement is added to a counterexample by analogy.

4. The conjunction constituting the counterexample, $P_1 \& P_2 \ldots \& P_n \;\&\; X_1 \& X_2 \ldots \& X_n \;\&\; {\sim}C$, is just presented as a logical possibility. However, as illustrated by CE1, a counterexample by analogy can have actually true premises and an actually false conclusion.

5. Counterexample CE2 has the *specific* form, *it is possible that* $P \& X \& {\sim}C$. The *general* form of a counterexample by possible conjunction is, *it is possible that* $P_1 \& P_2 \ldots \& P_n \;\&\; X_1 \& X_2 \ldots \& X_n \;\&\; {\sim}C$. Despite what this general form can be interpreted as suggesting, the number of X-statements does not necessarily correspond to the number of premises. Of course these conjuncts could be in any order, but I present them in this order because it is clearer, and because this order closely parallels the general structure of the argument against which it is advanced: $P_1 \& P_2 \ldots \& P_n$, *so* ${\sim}C$.

In contrast, counterexamples by analogy do not have a common general logical form. For as illustrated by CE1, the form of a counterexample by analogy must correspond precisely to the form of the specific argument against which it is advanced, and of course there is no specific form common to all arguments. For example, not all arguments correspond in form to argument (A).

6. Counterexamples by possible conjunction help us to identify implicit assumptions of an argument. For example CE2 shows that argument (A) rests on the assumption that Santa Claus will do sufficient amount of studying. In other words, argument (A) assumes the *contradictory of the what-if-statement* in counterexample CE2. It must assume it in order to block counterexamples that use that specific what-if-statement. Such counterexamples are blocked because they must grant *all* the premises of the argument against which they are advanced; and if a reconstructed argument contains the negation of a what-if-statement as a premise, no counterexample can use that what-if-statement, and so such counterexamples are automatically eliminated. Counterexamples by analogy, on the other hand, do not identify any implicit assumptions.

7. The consequences of these two types of counterexamples are different. A successful counterexample by possible conjunction shows that the *specific premises,* $P_1\&P_2\ldots.\&P_n$, are not sufficient for, do not guarantee, the truth of a *specific conclusion C*. However, a successful counterexample by analogy shows that *the specific form* it expresses is invalid, and consequently, it proves that *any argument having its form* (and no other form that is valid)[3] is invalid. So, *no* premises of *any* argument having *this specific form* (and no other form that is valid) are sufficient for the truth of conclusion *C*.

From the preceding differences it follows that counterexamples by possible conjunction and by analogy are two very different kinds of counterexamples.

Consistency in Counterexamples by Possible Conjunction

I will next show that the *mere consistency* among the granted premise(s), the what-if-statement(s), and the negated conclusion in a counterexample by possible conjunction is *not* enough for the counterexample to show us that those premises are not sufficient for their conclusion. Consider the following counterexample against argument (B):

(B) (1) Winds are blowing a rain storm in our direction.
So, (2) it's going to rain here tomorrow.

CE3 against argument (B)
It is possible that:

(1) Winds are blowing a rainstorm in our direction. AND
What-if-statement: "Sirius" is the name of the closest star to our solar system. AND
Not- (2): It is not the case that it's going to rain here tomorrow.

The counterexample has the correct form, *it is possible that P & X & ~C*, and all the propositions are consistent, yet the counterexample fails to show us that the premise is not sufficient for the conclusion.

Contrast it to the next example:

CE4 against argument (B)
It is possible that:

(1) Winds are blowing a rainstorm in our direction. AND
What-if-statement: Strong winds from another direction are going to divert the storm away from us. AND
Not- (2) It is not the case that it's going to rain here tomorrow.

This counterexample is effective in proving to us that the premise is not sufficient for its conclusion. Since a central difference between counterexamples CE3 and CE4 is that it is only in the latter case that the what-if-statement makes us understand how it possible for the premise to be true and its conclusion false, then that understanding is a necessary condition for a counterexample to show us that premises are not sufficient for their conclusion. A discussion of the logic involved in making us understand how it is possible for premises to be true and their conclusion false is beyond the scope of this paper, and is not necessary in order to grasp the practical rudiments of this kind of counterexample.

Counterexamples by Possible Conjunction and Degrees of Support

We have been examining some of the logic of the use of counterexamples by possible conjunction to determine whether an argument is valid. Whenever a counterexample is successful, it proves that an argument's premises are not sufficient for (do not guarantee/necessitate) its conclusion. The serious limitation of this use is that the premises of most everyday arguments are not intended to provide conclusive support. We will now explore a way to use these counterexamples to estimate the degree of support that is less than conclusive.

Elementary probability theory suggests a way to begin examining the logic of this additional role.

(1) Pr(~P or P) = 1.
(2) Pr(~P) + Pr(P) = 1.
We are looking for a substitution of "~P" and "P" that will allow us to assess the degree of *support* of *any* argument, $P_1 \& P_2 \ldots . \& P_n$, so C. Replace both "P's" in (2) by, "$P_1 \& P_2 \ldots . \& P_n$ & ~C":
(3) $Pr({\sim}(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C)) + Pr(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C) = 1$.
Subtract $Pr(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C)$ from both sides of the equation:
(4) $Pr({\sim}(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C) = 1 - Pr(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C)$.
Replace "${\sim}(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C)$" in (4) by the logically equivalent expression, "$(P_1 \& P_2 \ldots . \& P_n \supset C)$", which expresses the support that the premises bring to the conclusion in terms of the probability of the conditionalized inference:
(5) $Pr(P_1 \& P_2 \ldots . \& P_n \supset C) = 1 - Pr(P_1 \& P_2 \ldots . \& P_n \;\&\; {\sim}C)$.

But what does "$Pr(P_1\&P_2….\&P_n \& \sim C)$" represent exactly? In order to determine how to interpret it, let us examine an everyday argument and various counterexamples against it.

C (1) Each student beginning my course is sufficiently intelligent to pass the course.
(2) So, each student beginning my course will pass it.

CE5 against argument (C)
It is possible that:

(1) Each student beginning my course is sufficiently intelligent to pass the course. &
What-if statement: at least one student will be sick too often to do all the necessary work to pass. &
~(2): It is *not* the case that each student beginning my course will pass it.

Consider the following condensed counterexamples against argument (C). Assume that their what-if-statements are conjoined to (1) & ~(2), and that the conjunction of all these statements forming each counterexample falls within the scope of the operator, "it is possible that," just as in CE5.

CE6(C) What if at least one student will not study material that must be studied to pass it.
CE7(C) What if at least one student has family responsibilities that very seriously interfere with his/her academic performance.
CE8(C) What if at least one student has personal problems that very seriously interfere with his/her academic performance.
CE9(C) What if the teacher will grade unfairly.
CE10(C) X_1: What if there is a personality conflict between the teacher and at least one student. &
X_2: What if that student drops the course.

Regardless of the actual probability of any *specific* counterexample by possible conjunction, it is significantly smaller than the Pr[CE5(C) or CE6(C) or CE7(C) or CE8(C) or CE9(C) or CE10(C)]. So, if we were to use the probability of only one counterexample to estimate the degree of support, and discard the probability of this disjunction of counterexamples, then we would significantly *under*estimate the objections against the support of the premises, and consequently, we would significantly *over*estimate the degree of support of the premises. Each counterexample must be included in our estimation of the degree of support because each one exposes a different weakness in the support of the premises. In order to determine precisely the degree of support, we would have to take into consideration *all* counterexamples against the argument. But this is not psychologically possible. So in our less than ideal intellectual world we must take into account as many probable counterexamples as possible. Since this example is representative of everyday non-deductive arguments, it follows that "$Pr(P_1\&P_2….\&P_n \& \sim C)$" in step (5) of the proof represents *ideally* the probability of the disjunction of *all* counterexamples by possible conjunction, but it represents *in practice* the probability of only the disjunction of counterexamples that we do invent. Hence, this use of counterexamples by possible conjunction to assess the degree of support gives us only a rough estimate.

Despite this practical limitation, step (5) of the proof reflects our intuitions. First, when the combined probability of all the counterexamples is in fact zero, when $Pr(P_1\&P_2….\&P_n \&{\sim}C) = 0$, then the probability of the conditionalized inference equals one, $Pr(P_1\&P_2….\&P_n \supset C) = 1$. In other words, when there are in fact truly no counterexamples, an argument is valid. Secondly, step (5) entails that (a) the greater the combined probability of the counterexamples, the smaller the probability of the conditionalized inference $Pr(P_1\&P_2….\&P_n \supset C)$, that is, the weaker the support of the premises; and (b) the smaller the combined probability of the counterexamples, the greater the $Pr(P_1\&P_2….\&P_n \supset C)$, that is, the stronger the support of the premises. This inverse relation between the combined probability of the counterexamples and the strength of the support is precisely what we would expect.

Given that "$Pr(P_1\&P_2….\&P_n \&{\sim}C)$" represents more than one counterexamples by possible conjunction, the probability of the conditionalized inference at step (5) can be stated as, (6) $Pr(P_1\&P_2….\&P_n \supset C) = 1 – Pr(CE1 \text{ or } CE2 \text{ or } CE3… \text{ or } CEn)$. (I will address one of the challenges of estimating such a disjunction of probabilities later.) However, this added precision does not necessarily give us an accurate degree of the support of premises, for as I have already stated, we very rarely have all the counterexamples against the support an argument, and consequently our estimation of the support is very rarely final and complete.

Step (6) can be simplified. In any counterexample by possible conjunction all the given premises and the negation of the conclusion are assumed to be true. So,

$Pr(P_1\&P_2….\&P_n) = Pr({\sim}C) = 1.$
Since the probability of a typical counterexample is,
$Pr(P_1\&P_2….\&P_n \& X_1\&X_2….\&X_n \& {\sim}C) = Pr(P_1\&P_2….\&P_n) \times Pr(X_1\&X_2….\&X_n) \times Pr({\sim}C),$
then $Pr(P_1\&P_2….\&P_n \& X_1\&X_2….\&X_n \& {\sim}C) = Pr(X_1\&X_2….\&X_n).$

Hence, when talking about the probability of a counterexample by possible conjunction, we are in fact talking about the probability of the conjunction of its what-if-statements. Therefore, (6), which includes more than one counterexample against the support of premises, can be more fully stated as:
(7) $Pr(P_1\&P_2….\&P_n \supset C) = 1 – Pr(X1_1\&X1_2…\&X1_n \text{ or } X2_1\&X2_2...\&X2_n \text{ …or } Xn_1\&Xn_2…\&Xn_n).$

If we were to assign numbers to the probability of each counterexample, how would we "add" up those numbers. Assume that we are given the argument "P, so C," and that two counterexamples, CE1 and CE2, of equal probability are advanced against the support of P for C:
Assume Pr(CE1) = Pr(CE2) = 0.6. We cannot add these probabilities in the usual way because they would total over 1.0.

$$\begin{aligned} Pr(CE1 + CE2) &= Pr(CE1) + [\mathit{1\text{-}Pr(CE1)}] \times Pr(CE2) \\ &= 0.6 + [\mathit{1 - 0.6}] \times 0.6 \\ &= 0.6 + 0.4 \times 0.6 \\ &= 0.84 \end{aligned}$$

If a third counterexample were constructed, the general procedure is:
Pr(CE1 + CE2 + CE)
$= Pr(CE1) + [\mathit{1\text{-}Pr(CE1)}] \times Pr(CE2) + \{\mathit{1\text{-} (Pr(CE1) + [1\text{-}Pr(CE1)] \times Pr(CE2)}\} \times Pr(CE3).$
For example, if Pr(CE3) = 0.4, then
Pr(CE1 + CE2 + CE3)
$= \quad 0.84 + [1 – 0.84] \times 0.4$

= 0.84 + 0.16 x 0.4
= 0.84 + 0.064
= 0.90

By using this procedure, the "addition" of counterexamples never "adds up" to 1.0, though it always gets closer to 1.0, as it should. Even if we have only a vague estimate of the probability of each counterexample against an argument, the value of this procedure is that it allows us to handle as many counterexamples as we can invent. Without such a procedure it would be very difficult to have an idea how the vague estimates of the probabilities "add," and consequently, it would be difficult to how all the counterexamples affect the intended support of the argument's premises.

Practical/Theoretical Challenges

There are a number of challenges to meet when using counterexamples by possible conjunction to estimate the degree of support of premises. First, how do we go about estimating the probability of a counterexample? Some of my ongoing research is intended to answer this question, and I cannot address it in this paper.

Secondly, how do we determine when to stop constructing counterexamples. For instance, we could have continued inventing more counterexamples against the support the premise of argument (C). But if we wanted to have a reliable estimation of the degree of support, where should we stop? Assuming that time is not an obstacle, we stop once (a) we can only invent extremely unlikely counterexamples, and (b) we have reason to believe that we would continue inventing only extremely unlikely ones. Here is an example of an extremely improbable counterexample:

CE11 against argument (C)
It is possible that:

P: Each student beginning my course is sufficiently intelligent to pass the course. &
What-if statement: at least one registered student is abducted by an extraterrestrial for the duration of the course. &
~C: It is *not* the case that each student beginning my course will pass it.

We stop once we can construct only very unlikely counterexamples because they add nothing significant to the probability of the disjunction of all the realistic counterexamples we have already constructed, and thus subtract nothing significant from 1 in $Pr(P_1 \& P_2 \ldots . \& P_n \supset C) = 1 – Pr(CE_1 \text{ or } CE_2 \text{ or } CE_3 \ldots \text{ or } CE_n)$. It is important to bear in mind that *wherever we stop, it will be due to our limited knowledge, imagination, time, or energy*. So, generally we cannot be certain that we have taken into consideration the best counterexamples. For this reason, it is sometimes important to persist inventing a few wildly imaginative counterexamples because sometimes that creative process can help us to discover more probable (realistic) ones.

Thirdly, if we apply probability theory to a disjunction of counterexamples by possible conjunction against an argument in order to estimate the degree of support of its premises, we must address a the fact that not all counterexamples (or more simply, not all what-if-statements of counterexamples) are independent of one another. *Event M is independent of event N if and only if N does not affect the probability of M: if and only if Pr(M, given N) = Pr(M).* For instance, if I am boarding a taxi for a destination that is five miles away, and I infer from my taking the taxi that I will arrive at my destination in less than an hour, there are many interdependent counterexamples against the inference. Here are just three examples: what if there is an accident; what if there is a flat tires; what if the driver becomes sick. These are different physical possibilities that could prevent me from reaching my destination on time, and they are partly interdependent: some accidents are caused by flat tires, and some accidents are caused by a driver's illness. *If events M and N are dependent, then Pr(M or N) = Pr(M) + Pr(N) – Pr(M&N)*, and so the estimation of the probability of my arriving on time would need to include the probability of a flat or an accident, which equals the sum of the probability of a flat and the probability of an accident, minus the probability of the conjunction of a tire having a flat and the taxi driver having an accident. Stated more generally, the greater the interdependence among counterexamples against an argument, and the greater the number of interdependent counterexamples, the smaller the estimated sum of the counterexamples' individual probabilities; and consequently, the stronger the support of the argument's premises. Given the interdependence of many counterexamples by possible conjunction against most everyday arguments, the complete application of probability theory to these counterexamples quickly leads to complicating the assessment of the degree of support of most everyday arguments. The costs in terms of time, mental energy, and possibly even money of this complete application of probability theory would seem to outweigh the benefits.

I propose that we keep considerations of the interdependence of counterexamples in the background. I will use again argument (C) to illustrate my point. If I discard the interdependence of counterexamples CE8 and CE10, I would estimate that probability of the disjunction of the all the counterexamples it is *at least* moderately probable (60% to 70%). Consequently, the probability of the conditionalized inference is *at most* moderately improbable (40% to 30%). What would I change if I were to take into consideration the interdependence? Their overall combined probability would have to diminish, and there would be a corresponding increase in the probability of the strength of the support of premise (1). But what would be the amount of that change? My estimation is that the probability of the conjunction of all six counterexamples would still be *roughly* at least moderately probable. So my consideration of the interdependence makes me only qualify my estimation with "roughly."

If this example is representative of most everyday examples, then for practical everyday purposes, considerations of the interdependence of counterexamples is sometimes not that useful. Secondly, in most situations we don't have the information or the time to figure out the interdependence of counterexamples.

However, I am definitely not suggesting a total disregard of the interdependence of counterexamples. For knowing that some counterexamples against the support of an argument are interdependent makes us aware that the probability of the disjunction of the counterexamples is in fact less than what it would seem at first, thereby indicating from the inverse relation that the support of the premises is stronger than initially estimated. It is also possible that in some cases, the interdependence might be significant and easy to estimate. For these reasons it is prudent, when evaluating most arguments, to keep in the back of our minds the questions, "Are there any interdependent counterexamples? To what degree are they interdependent?", and to deliberately seek to answer them for arguments that are very important.

In this paper I identified two kinds of counterexamples: counterexamples by possible conjunction and counterexamples by analogy; described the logical differences between them; examined some aspects of the logic of counterexamples by possible conjunction. It is a logically possible conjunction of all the premises of an argument (all are assumed true), the negated conclusion, and one or more statements that I named "what-if-statements." I showed that the latter have the very special function of making the proponent of an argument, against which the counterexample is advanced, understand how it is possible for his/her premises to be true and conclusion false; argued that the mere consistency of all the statements constituting these counterexamples is not sufficient for the success of these counterexamples. I used elementary probability theory to justify extending the use of these counterexamples to estimate the degree of premise support that is less than conclusive; showed that the strength of support (i.e. the probability of the conditionalized inference) is inversely proportional to the probability of the disjunction of all the what-if-statements of successful counterexamples against the support; described where we should stop in the construction of counterexamples; and illustrated some of the practical limitation of considering the interdependence of what-if-statements when estimating the probability of their disjunction.

Notes

[1] I will be using special notation to distinguish arguments and their counterexamples: "CE1 against argument (A)" simply means "counterexample number 1 against argument (A)", and "CE2 against argument (A)" means "counterexample number 2 against argument (A)."

[2] The following seven points were presented at the Eleventh NCA/AFA Conference on Argumentation in August 1999, and published in the refereed proceedings of that conference (Gratton, 2000, 109-113).

[3] I include this parenthetical phrase in order to take into account the fact that an argument can have more than one form, and is usually considered valid if it has at least one valid form. For example, the argument, "All philosophers are human. All humans are mortal. So all philosophers are mortal." has at least two forms. If we consider only the propositions, there is the invalid form, "P. Q. So, R". But if we consider the quantifiers within the propositions, there is the valid form "All A are B. All B are C. So all A are C." This argument is valid even though it also has an invalid form.

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