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The Newtonian Revolution as a revolution in scientific reasoning

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Let me begin by setting the focus of this presentation. Newton's *Principia* first appeared in 1687. It underwent two revisions with the final third edition appearing in 1726, one year before the author's death. What I shall do here is the following. First, I will outline the methodological themes, which arise out of a careful reading of the *Principia*. Second, we will see that crucial to the fulfillment of Newton's ideal of empirical success is the solution of a very thorny problem. In celestial mechanics this is known as the Lunar Theory. Third, the solution to this thorny problem appeared in the mid-eighteenth century in the works of Clairaut, d'Alembert, and Euler.

Newton's ideal of empirical success as exemplified in his deductions from phenomena informs the transition from natural philosophy to natural science. Among these themes is the method of answering important theoretical questions empirically by measurement from phenomena. According to Newton's ideal, a successful theory has its parameters measured by the phenomena it purports to explain. The ideal of empirical success is not limited to prediction, but also generates reliable measurements of the parameters of the phenomena.

Newton's *Principia* exemplifies deductions from phenomena where a higher-level theoretical claim is inferred from certain other high level theoretical principles along with phenomena. Newton established that a measure of the rate of orbital precession could sensitively measure the exponent of the force law. In the moon-test, Newton turned the theoretical question regarding the measure of the force holding the moon in her orbit into an empirical question of measurement of the length of a seconds pendulum on earth. Two different phenomena are giving agreeing measurements of the same inverse-square centripetal force field. The agreement in measured values is a higher-level phenomenon, which relates the two phenomena in question. Newton's model of evidential reasoning exploits these agreements in such a way that the stakes are raised for rival theories.

In considering the effect of the sun on the moon in her orbit, Newton was able to account for only half of the observed lunar precession. The solution to this problem came in the late 1740s and early 1750s through the work of Clairaut, d'Alembert, and Euler. The zero precession left over after the effect of the sun on the lunar orbit had been correctly solved removed impediments to the acceptance of Newton's system. The solution exhibited Newton's stronger ideal.

Consider how Newton provides resources to answer the theoretical question of the direction of the force deflecting a planet into its orbit by one that is answered empirically by measurement. Theorem 1 of Book 1 of the *Principia* asserts:

> The areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes, and are proportional to the times in which they are described. (Newton 1934, 40)

If the force deflecting a body into an orbit is directed at a centre, the orbit will lie in a plane and the radius will sweep out equal areas in equal times. Further along in Theorem 2 Newton shows that if a body moves in a plane orbit **and** satisfies the area law (that is, it sweeps out areas proportional to the times) with respect to a centre, then it is deflected into that orbit by a force directed at the centre. There are two important corollaries to this last theorem and these concern the situations of increasing or decreasing areal rates. An increasing areal rate, for instance, corresponds to a deflecting force that deviates from the centre in the direction of motion. The opposite would be true for a decreasing areal rate. Thus a constant areal rate measures the centripetal direction of a force deflecting a body into a plane orbit. Newton's appeal to these sorts of systematic dependencies allowed him to turn the theoretical question of the direction of the force deflecting a planet into its orbit into one that is answered empirically by measurement. Newton can then use this established fact that bodies are being deflected into their orbits by centripetal forces along with theorems about motion under centripetal forces in order to have phenomena measure the inverse square variation of forces. For instance one can use a stable orbit to measure the force to vary inversely with the square of the distance.[1] As we shall see, Newton appeals to Harmonic Law ratios for a system of orbits and to the stability of any particular orbit as phenomena, which measure inverse-square variation.

The biggest challenge to Newton and his theory was the lunar perturbation problem. The inverse-square law offered the natural philosophers of the enlightenment an opportunity to obtain new and more precise quantitative results in celestial mechanics. Through the help of the telescope, motions of heavenly bodies could be measured very precisely. In this respect this new set of data from which the phenomena are generalised provided the ultimate challenges to the inverse-square law. d'Alembert, for instance, once remarked:

> The [Newtonian] system of gravitation can be regarded as true only after it has been demonstrated by precise calculations that it agrees exactly with the phenomena of nature; otherwise the Newtonian hypothesis does not merit any preference over the [Cartesian] theory of vortices by which the movement of the planets can be very well explained, but in a manner which is so incomplete, so loose, that if the phenomena were completely different, they could very often be explained just as well in the same way, and sometimes even better. The [Newtonian] system of gravitation does not permit any illusion of this sort; a simple article or observation which disproves the calculations will bring down the entire edifice and relegate the Newtonian theory to the class of so many others that imagination has created and analysis has destroyed. (Hankins 1985, 37)

The empirical challenge of the return of Halley's comet was, in d'Alembert's words, "more tedious than difficult." (Hankins 1985, 40) The answer to the theoretical question regarding the shape of the earth was derived through measurements of the length of a seconds pendulum— measurements of a phenomenon on earth which yield measurements of the length of a degree and offers the same answer as a "bird's eye view" from space of the shape of the earth. Between these two challenges is nestled a problem in which we find Europe's foremost mathematicians heavily committed to solving what was considered the most important challenge.

[1] Corollary I of Proposition 45 of Book 1 states that the centripetal force *f* is as the $\left(\frac{360}{360+n}\right)^2 - 3$ power of the distance if n is the number of degrees of precession per revolution. For a stable orbit — "If the body after each revolution returns to same apse, and the apse remains unmoved" (*Principia* page146) — the value of n is 0. Thus, Newton uses a stable orbit to measure the force to vary as the -2 power of the distance.

The moon moves much more erratically than the planets and Newton was keenly aware of this problem.[2] The calculations of the moon's motion required solving the problem of three mutually gravitating bodies. As early as 1744, Euler in Berlin, and in 1747 with Clairaut and d'Alembert, a solution to this three body problem was set as the goal. All three came to the same conclusion: the observed monthly motion of the lunar apogee differed on average form the predicted value by a factor of two. In 1749 Clairaut proclaimed he had solved the problem without giving his solution. By 1752, it was generally claimed that Newton's theory is right after all—the work completed by Clairaut, d'Alembert, and Euler helped vindicate the theory.

These challenges to universal gravitation shed light on various methodological themes. One may argue that their resolutions merely confirm Newton's theory and, in fact, this is generally how these historical events have been read. On the other hand, we claim that the developments by Newton's successors on perturbation theory helped realise Newton's ideal of empirical success. Clairaut, for instance, was able to solve the lunar precession problem by completely accounting for the known precession through recalculating the action of the sun on the lunar orbit (Wilson 1980; Waff 1975). From Clairaut and Euler to Laplace the Newtonian corrections of Keplerian phenomena "became increasingly precise projectable generalisations that accurately fit open ended bodies of increasingly precise data." (Harper 1997, 62) Crucial to these developments were increasingly accurate measurements of relative masses of bodies in the solar system. In terms of earth masses, Newton's estimate of the mass of the sun in the 2nd edition of the *Principia* was 32% less than the value accepted in 1976 (the 2nd edition was published in 1713). Some 83 years later Laplace gave estimates that turn out to be within 2 percent of the 1976 value (Harper 1997, 62).

The Rules

In the *Principia* Newton's methodology is expressed in his use of Rules of Reasoning in Experimental Philosophy. The first two rules are concerned with the sort of conclusions we should be looking for in natural philosophy. Rule I states a principle of parsimony by claiming that we are to admit no more causes than "such as are both true and sufficient to explain their appearances." In Rule II Newton enjoins us to assign the same causes to the "same natural effects."[3]

The application of Rule I is more than just an *a priori* appeal to simplicity. Rule I helps underwrite inferences from motion phenomena to forces by demanding not only sufficiency of explanation but an appeal to truth. We will be arguing that Newton's inferences from phenomena are backed up by systematic dependencies that make the phenomena into measurements of theoretical parameters. The appeal to truth suggests that Newton is demanding something more

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[2] Hankins (1985), page 39: "Newton has told John Machin that calculating its motion was the only problem that ever made his head ache, and he understood why: It was because the moon is attracted strongly by two bodies—the earth and the sun, pulling at different angles to one another—whereas the planets are attracted strongly only by the sun." The "lunar problem," from Newton's day to the present, has been cast as the "linchpin of the entire argument for universal gravitation." Westfall (1973), page 754.

[3] Koyré, in his *Newtoninan Studies*, discusses the various ancestors to this and the other rules. Newton's firm stance against hypotheses is illustrated by Koyré's discussion.

other than a metaphysical commitment to simplicity. In this sense, Rule I is not simply an "Occamisation" of sorts, for it also requires some stronger sort of empirical evidence.

That Newton begins Rule II with "therefore" suggests that it follows from or is implied by Rule I. Taken together with Rule I, Rule II is telling us to choose explanations with common causes, *as far as possible,* whenever we are able to find them. We will see a clear application of these two rules taken together when Newton, in the moon test, infers that the force maintaining the moon in her orbit is terrestrial gravity. The inference is from agreeing measurements of a single centripetal inverse-square acceleration field directed towards the centre of the earth by two different phenomena: the centripetal acceleration exhibited by the lunar orbit and the length of a seconds pendulum at the surface of the earth.

Rule III, on the other hand, is of a different sort. Of the four Rules the third is provided with the greatest discussion. The discussion begins with an interesting defence of universalization, which is in line with ideas of simplicity and consonance of nature. At issue in the third rule are questions that are of concern when one is justified in drawing inferences from observation reports to claims about unobservable entities. This type of reasoning, which entitles us to generalise some of our knowledge about things "within the reach of experiments" so as to be able to make claims about things which (because they are too small or too remote) are outside the reach of experiments, has been called "transduction."[4] Newton will use this rule to argue (in Proposition VI, Corollary II, Book III) that there is an equivalence of the ratio of a body's weight toward the earth and its quantity of matter (i.e., inertial mass) for all bodies universally at equal distances from the centre of the earth. Newton goes on in the final paragraph of his discussion of Rule III to indicate how this rule is to be used in his argument for universal gravitation.

We must, in consequence of this rule, universally allow that all bodies whatsoever are endowed with a principle of mutual gravitation. For the argument from the appearances concludes with more force for the universal gravitation of all bodies than for their impenetrability; [emphasis added] of which, among those in the celestial regions, we have no experiments, nor any manner of observation. Not that I affirm gravity to be essential to bodies: by their *vis insita* I mean nothing but their inertia. This is immutable. Their gravity is diminished as they recede from the earth (Newton 1934, 399).

This passage is interesting for a couple of reasons. First, since we do not have experimental reason to establish the impenetrability of heavenly bodies, universal gravitation has greater support from phenomena. But impenetrability was considered to be an essential quality of

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[4]"Transdiction" was the term used by Mandelbaum in his 1964 book *Philosophy, Science, and Sense Perception* (Baltimore: The Johns Hopkins Press). Mandelbaum argued that Rule III was heavily laden with ontological and epistemological import and that it was through this rule that Newton was able to resolve a tension in his thought. That is, on the one hand Newton demanded that all scientific laws be "deduced from the phenomena," but on the other hand Newton spoke often of corpuscles which were not only unobserved, but in principle unobservable. In his "Atoms and 'Analogy of Nature': Newton's Third Rule of Philosophizing" (*Studies in in the History and Philosophy of Science* I (1970): 3-58) McGuire called the third rule "transduction" because of its reference to seventeenth century characterization of the problem underlying it. McGuire claims: "Now transduction as conceived in the seventeenth century was not simply a type of trans-categoric inference. Induction exemplifies the latter, since all inductive inferences go beyond the evidence from which we begin. The distinctive feature of transductive inferences, I wish to maintain is that the conclusion not only goes beyond the evidence we already have, but goes beyond any evidence we might possibly acquire. Put otherwise, it is not that transductive inferences go beyond the initial evidence, but that the evidence is appropriated to a problem to which it strictly does not apply."

bodies.[5] Newton claims that even though the argument from phenomena supports universal gravitation and not the impenetrability of celestial objects it does not follow that gravity is an essential quality of all bodies. This is suggested by the condition of not admitting intensification or remission of degrees specified in the body of Rule III and seemingly violated by gravity, which is diminished as bodies "recede from the earth." We will see an application of this rule in Proposition VI, Corollary II that at equal distances from the centre of the earth the weights of bodies are proportional to their inertial mass.[6]

The last of the rules, the fourth, was introduced in the third and final edition of the *Principia*. It might be considered to be the most important rule of all. The scope of the rule is to show the confidence one can place in the results from induction. Inductions are to be regarded as "accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined" until new phenomena may make them more accurate or liable to exceptions. Rule IV has the briefest comment associated with it in the text. The Motte-Cajori translation reads as follows:

> This rule we must follow, that the argument of induction may not be evaded by hypotheses.

Newton made it clear that there is an important difference between reasoning from the truth of consequences to the truths of premises from which they are drawn and reasoning from phenomena to forces. That is, the point of Rule IV, for Newton, is to protect the results of induction from being undercut by hypothetical reasoning. Hypotheses are identified as propositions that are not deduced from phenomena.

These four rules reflect what Newton took to be requirements for natural philosophy. But there are remarkably few explicit references to these rules in the *Principia*. Newton used the first and second rule, usually together, to justify the claim that the "same sort of cause" must be responsible for the behaviour of heavy terrestrial objects, for the moon's orbit around the earth, (Newton 1934, Proposition IV, Theorem IV, Book III) for the six planets and earth orbiting the sun, and for the satellites of Jupiter and Saturn orbiting their planets. (Newton 1934, Proposition V, Theorem V, Book III) The third rule is invoked in support of the claim that we should count gravity among the universal properties of bodies. (Newton 1934, *Corollary* II, Proposition VI, Theorem VI, Book III) Newton's illustrating paragraph suggests that the third rule allows us to infer that the inverse-square law is a universal law. Likewise, the fourth rule is used in conjunction with the first two rules, to support the general proposition that whatever causes the moon to retain its orbit will also affect all planets (Newton 1934, *Principia,* Scholium, Proposition V, Theorem V, Book III).

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[5] See, for instance, Newton's *De Gravitatione.* Koyré, in *From a Closed World to an Infinite Universe*, has shown the similarity of Newton's thought on impenetrability to the writings of the Cambridge Platonists (especially Henry Moore). In *De Gravitatione* Newton argues for impenetrability in response to the Cartesian argument for extension as an essential quality of all bodies. According to Koyré, Moore and Ralph Cudworth argued that non-corporeal entities can be said to have extension and what is more properly essential to corporeal bodies is impenetrability.

[6] As Koyré points out the reservation that gravity is diminished as we recede from the earth seems beside the point once one notes the universal agreement in the ratio of the inverse-square adjusted weight towards the earth with inertial mass.

On the Path to Universal Gravitation

Newton's deductions from phenomena are used ultimately to show universal gravitation. Newton's initial volley is to establish two propositions which are very similar: 1. That the moons of Jupiter and of Saturn are kept in their respective orbits by an inverse-square force directed toward Jupiter and Saturn respectively. 2. Likewise, the primary planets are kept in their orbits by an inverse-square centripetal force directed toward the sun.

In Proposition III Newton turns to the moon. Here, Newton argues that the moon is held in her orbit around the earth by an inverse-square centripetal force in the direction of the earth's centre.[7] In this proposition Newton pointed out that given that the lunar apogee precesses 3°3' per revolution (*in consequentia*), the measure of the centripetal force would not be inverse-square but inversely as $2\frac{4}{243}$ power of the distance. Furthermore, Newton suggested that we might neglect this precession as being due to the action of the sun on the moon in her orbit around the earth. He further suggested that he will show this further on. It turns out, as we will show in our discussion of Clairaut, d'Alembert, and Euler, that this lunar theory posed a serious empirical challenge to universal gravitation. This doesn't undermine the task at hand. What is of value here is that Newton is clearly remarking that we can use precession to measure an inverse square relationship. That the moon's orbit does not exemplify this relationship is due to a perturbative effect. In short, if precession is to measure inverse-square variation[8] one must show that the precession is due to perturbations. Newton suggests that this is the case with the moon but it was not successfully shown until Clairaut's address in 1749.

Proposition IV further carries the discussion of the moon and confirms the previous proposition's assertion of inverse-square variation in the earth-moon system. Newton argues, here, that the moon is continually deflected from rectilinear motion by the force of gravity and by this force is retained in her orbit around the earth. Newton has thus inferred that the force holding the moon in her orbit is the same force, which we count as terrestrial heaviness, i.e., gravity. Prior to the publication of the *Principia* “gravitas” literally meant "terrestrial heaviness." To identify the centripetal force that pulls the moon off of tangential motion with gravity, terrestrial heaviness, was a radical departure. It was an admission that what causes heavy objects to fall to the earth also kept the moon from (naturally) following her tangential motion. Historically, the planets had been considered to be of a different sort than terrestrial objects. The "Newtonian" revolution was a revolution that resulted in seeing terrestrial bodies and celestial bodies as of the same kind. Newton infers this "same cause" by appealing to Rules I and II. Newton computes how far the moon, at 60 earth radii from the earth's centre, would fall in one minute if it were deprived of all forward motion.

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[7]In "Newton" scholarship the first three propositions of Book III are often dealt with quickly. It is with Propisition IV, the famous moon test, which historians of science have noted that Newton's system stands apart from previous scientific thinking. Proposition IV is taken to be the crucial link in the argument to universal gravitation. As we will see later, it was Proposition III of Book III that offered a major empirical challenge to universal gravitation. It was due to the work of Clairaut, d'Alembert, and Euler that this particular challenge was resolved.

[8]In a sense, we have non-Keplerian orbits. That is, Kepler's ellipse law seems to be violated. This may be why Newton does not include the ellipse law in his list of phenomena.

And now if we imagine the moon, deprived of all motion, to be let go, so as to descend towards the earth with the impulse of all that force by which it is retained in its orb, it will in the space of one minute of time, describe in its fall 15 1/12 *Paris* feet. (Newton 1934, 408)

Assuming, as in Proposition III, that the centripetal force holding the moon in orbit obeys an inverse-square law then we can calculate the centripetal accelerative force operating at the distance of the moon. Newton imagines what would happen if the moon were brought down to the surface of the earth. The moon is at a distance of 60 earth radii from the earth's centre. At just above the earth's surface it would be at one earth radius away from the centre. The ratio of the accelerative force at the current orbit to what it is at the surface is as 1 to 60x60 on the assumption of inverse-square variation. Therefore the force at the earth's surface would be 3600 times greater. On the assumption that gravity is an inverse-square force that extends to the moon, it follows that a heavy object on the earth's surface would freely fall, in one minute, 3600 of these 15 1/12 *Paris* feet. In the increment of one second this heavy object, then, would freely fall 15 1/12 *Paris* feet, or 15 feet, 1 inch, and 1 line 4/9, which Newton claims to be more accurate. (Newton 1934, 408) These calculations are made on the assumption that the lunar distance is sixty earth radii away.

Huygens had shown that, at the latitude of Paris, a seconds pendulum will be 3 Paris feet, 8 lines 1/2 inch in length (i.e., 3.06 Paris feet).

And the space which a heavy body describes by falling in one second of time is to half the length of this pendulum in the duplicate ratio of the circumference of a circle to its diametre (as Mr. *Huygens* has also shown), and is therefore 15 *Paris* feet, 1 inch, 1 line 7/9. (Newton 1934, 408)

Thus

$$\frac{d}{l/2} = \left(\frac{circumference}{diametre}\right)^2 = \left(\frac{2\pi r}{2r}\right)^2 = \pi^2.$$

Finally, $d = \frac{l}{2}\pi^2$, where l=3.06 *Paris* feet.

d=15.09 *Paris* feet.

These results agree so well that, by Rules I and II, "the force by which the moon is retained in her orbit is the very same force which we commonly call gravity." (Newton 1934, 409) Here we have two phenomena that yield agreeing measurements of the same inverse-square force—gravity—toward the earth's centre. The length of a seconds pendulum and the centripetal acceleration of the lunar orbit are two phenomena which measure the same force. The agreement in measured values is another phenomenon, which relates the two phenomena in question. By Rule I if we do not claim that the same force accounts for the centripetal acceleration of the moon and the length of the seconds pendulum at Paris, then we will have to claim that there are two separate causes for these phenomena. The centripetal acceleration of the moon and the length of a seconds pendulum each measure a force resulting in accelerations at a distance of one earth-radius (i.e., at the surface of the earth). The moon-test shows that these accelerations are not only equally directed toward the centre of the earth, but that they are equal in value. Now we have this higher order phenomenon of the agreement in measurements of the two phenomena.

The parsimony invoked by the use of the first rule informs us not to infer another cause for this agreement.

Notice Newton's use of Rule II, to the same effect assign the same cause. That is, we note that something attracts the moon toward the earth and something attracts heavy objects to the earth. By Rule II, that "something" is the same. We note that it is not just a qualitative effect that is generally the same but that it is the same as close as experiments show that they are the same (in this case, Huygens' pendulum experiment). It is Newton's ideal of empirical success which drives his reasoning to show that what we call terrestrial gravity reaches to the moon and, therefore, does not discriminate between terrestrial objects and, thus far, the moon. That is, we have agreeing measurements of the same inverse-square acceleration field from the length of the seconds pendulum and the moon-test.

Newton proceeds to say that

> were gravity another force different from that, [from the centripetal accelerative force on the moon] then bodies descending to the earth with the joint impulse of both forces would fall with a double velocity, and in the space of one second of time would describe 30 1/6 *Paris* feet; altogether against experience. (Newton 1934, 408)

This indirect argument reinforces the appeal, via Rules I and II, to the unification of the measure of the centripetal acceleration of the moon with the measure of the length of a seconds pendulum. Notice that the indirect argument would not count against an alternative hypothesis, say, one which posited a force maintained the moon in her orbit (an inverse-square force to boot) but which did not act on terrestrial bodies. So Newton's appeal to Rules I and II help rule out just this sort of alternative. This alternative hypothesis would demand a separate account of cause for each of the two (or whatever number) basic phenomena.[9] Not only would this alternative hypothesis require a separate cause for each of the phenomena, it would need to accommodate or explain the agreement of the measurements of these phenomena. Furthermore, according to Rule II we are to assign to the same effects the same cause. Until we have evidence to the contrary, there is nothing to indicate that the phenomena are of sufficiently different kinds to warrant us to claim that they have different causes.

Placed in the context of the proof for universal gravitation, Proposition IV carries a new constraint for Newton's theory and for any alternative to the theory. The unification of these phenomena in order to identify the lunar centripetal force with terrestrial gravity forces Newton to constrain systematically the development of the theory. Newton transformed the notion of terrestrial gravity, heaviness, to count as varying inversely with the square of the distance from the earth's centre. Gravity applies not only to terrestrial objects but to the moon as well. After this proposition Newton is committed to counting any phenomena, which measure gravity as also measuring the centripetal force on the moon. Rules I and II impose systematic constraints on theory development such that the measures of the parameters of phenomena which are to be explained by a theory count as accurate measurements of the theory. A rival hypothesis to the claim that the moon is held in her orbit by the very same force, which accounts for terrestrial

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[9] As we will see later, Buffon used a similar line to argue against Clairaut's initial lunar theory which postulated a force which varies according to $\frac{1}{r^2} + \frac{1}{r^4}$.

heaviness, would have to account for the equivalence between the centripetal force on the moon and the length of a seconds pendulum. Newton raised the stakes considerably for theory choice. Before moving on we should also note that Proposition IV emphasises the “empirical” aspect of Newton’s ideal of empirical success. That is, in answering the theoretical question regarding the force holding the moon in her orbit and drawing her away from tangential motion, Newton drew our attention to two phenomena, which give us agreeing measurements of the same theoretical parameter.

Euler on the Motion of the Lunar Apse

The problem of the apsidal motion admitted an algebraic solution by approximation. Newton did not have the analytical tools necessary to develop the algorithmic solution required. He, as has been well documented, employed a geometric technique which was not suited to capture properly the motion of the lunar apse. (Wilson 1995) There was not any significant development with respect to the problem surrounding the motion of the lunar apse until the 1740s when Euler and other continental philosophers applied their analytical techniques to the solution of this problem. Recently, Curtis Wilson, (Wilson 1995) has shown that differentiation and integration of trigonometric functions had not been part of the standard procedures in the solution of differential equations prior to 1739 when it was Euler who incorporated them.

Euler had grave doubts regarding the exactitude of the inverse-square law. In both “*Recherches sur le mouvement des corps célestes en générale*” and “*Recherches sur la question des inégalités du mouvement de Saturne et de Jupiter*,” with the latter winning the Paris Academy prize of 1748, Euler expressed his reluctance in accepting the inverse-square law. Like Clairaut and d’Alembert, Euler was able to recover only half of the motion of the lunar apse. This discrepancy between theory and observation preoccupied Euler, Clairaut, and d’Alembert until the problem was solved.[10] Euler, I believe, offers an interesting case to see whether he adopted a hypothetico-deductivist stance or whether he accepted what we have been calling the entrenchment of Newton’s ideal of empirical success.

By 1748 Euler had publicly called into question the inverse-square law on the evidence that it yielded only half of the motion of the lunar apse. What is interesting about Euler is his metaphysical belief in an aether theory. He maintained that the transmission of all forces had to be carried out in some medium.

In an essay, "Reflections on Space and Time," which appeared in *The History of the Royal Berlin Academy of Sciences* in 1748, Euler suggests that the relationship between physics and metaphysics should be one where knowledge of the former regulates the principles of the latter. That is, instead of having the concepts of physics regulated by metaphysical principles, Euler reversed this relation so that metaphysical ideas are to be determined by established physical inquiry. Euler concludes that the laws of mechanics are so well confirmed that one cannot deny their veracity. These laws are Newton's three laws and Euler lists them here as unquestionable facts.

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[10] The British Parliament prize went posthumously to Tobias Mayer’s family. In fact they received only £3000. Euler received £300 for his contribution to Mayer’s tables (i.e., Mayer used Euler’s equations).

After the announcement and proof that the Newton's inverse-square law is sufficient for understanding the motion of the lunar apse by Clairaut we find Euler quickly praising this achievement. He wrote to Clairaut in June 1751:

> [T]he more I consider this happy discovery, the more important that it seems to me. … For it is very certain that it is only since this discovery that one can regard the law of attraction reciprocally proportional to the squares of the distances as solidly established; and on this depends the entire theory of astronomy. (Waff 1995, 46)

Euler was quick to agree that the law of attraction is established. Among the phenomena which were discussed in the mid-eighteenth century, (the tides, the shape of the earth, etc.), perturbation theory and especially lunar theory seemed to restrict the acceptance of Newton's inverse-square law on the continent. In fact, the law itself was called into question when, the calculated motion of the lunar apse was not brought into agreement with observation. Aside from the immense success the inverse-square law had, this disagreement between theory and observation had Europe's leading natural philosophers nearly abandoning the law. In the argument for universal gravitation Newton offered as premises the near correspondence of the satellites of Jupiter, and the primary planets to the harmonic and areal laws. With respect to the moon we are told that a measure of the perturbation will measure the deviation from inverse-square. Since the lunar orbit is not quiescent, Newton needed a different argument in support of universal gravitation. Consideration of apsidal motion was seen as crucial for the inverse-square law and for Newton's ideal as well. That is, the inverse-square law undergoes a test in a such a way that support for it is challenged by the examination of apsidal motion which is to support inverse square variation.

The influence Euler exerted with such a pronouncement is remarkable. By 1751, the tide of informed opinion was clearly favouring the agreement of inverse-square with the motion of the lunar apogee. Mayer went on to publish more accurate lunar tables with Euler's help.

The reader will recall that Newton placed a great deal of weight behind his argument for universal gravitation on the proposition that the Moon's motions can be accounted for with an inverse-square variation. Propositions III and IV (the Moon Test) of Book III are important in this respect. That Newton left lunar theory in a state of confusion allowed for non-Newtonians to attack Universal Gravitation. We have seen that Euler, Clairaut, and d'Alembert saw lunar theory as the "jugular" so to speak. Their efforts went from trying to account for the motion of the lunar apogee along with Newton's theory to showing that since the latter was not successful the theory was in need of correction. In the process they discovered that the theory actually did yield the motion of the apogee. In the case of Euler, his metaphysical commitment to aether theory did not detract him from accepting inverse-square variation. His letters to a young princess showed him accepting inverse-square while still claiming he was an impulsionist. He had no scientific grounds for such a belief and he was ready to admit so. He thought conceptually that it made more sense than attraction at a distance but it had no bearing on his acceptance of inverse-square.

In the end then, Clairaut's solution to the lunar precession problem helped buttress universal gravitation. Newton, himself, had placed much weight on such a solution to the problem not only because of its role in supporting his argument for universal gravitation but also for its role in his ideal of empirical success. Euler, as a key player in the tide of informed opinion, certainly appreciated the importance of the solution to the lunar precession problem. The empirical success enjoyed by universal gravitation and the role this solution played in its proof allowed him to bracket his *a priori* commitments to aether. Nonetheless I do not believe he fully realised this ideal of empirical success. He focused, repeatedly, on the agreement between theoretical predictions and observation in his letters to Clairaut, Mayer, and the young princess. We do see, by the mid-1700s, a greater emphasis placed on empirical success and less emphasis on hypothetical thinking. I should say, then, that Euler is on a path toward the entrenchment of Newton's ideal of empirical success.

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