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Behaviour of prestressed lattice structures.

Mokkarala Venkata Prakash

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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS REÇUE
BEHAVIOUR OF PRESTRESSED LATTICE STRUCTURES

by

Mokkarala Venkata Prakash

A Dissertation
Submitted to the Faculty of Graduate Studies
through the Department of
Civil Engineering in partial fulfillment
of the requirements for the Degree of
Doctor of Philosophy
at
The University of Windsor

Windsor, Ontario, Canada
1986
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ABSTRACT

Prestressed lattice structures are those structures in which certain components are pretensioned to improve the performance of the overall structure. Pretensioning in slender structures, where buckling is the prime mode of failure, is beneficial in increasing the load carrying capacity. This study deals with steel structures only. Preliminary research on stayed columns and on lattice frames showed an increase in strength due to prestressing.

In tall slender structures, lateral support to the compression member is provided by the diagonals and horizontals. Pretensioned diagonals, when used in lattice structures, increase the load carrying capacity over that obtained for a similar structure with nonpretensioned diagonals. This increase in strength depends upon various factors which include the magnitude of pretension in the diagonals, the size of the diagonals, slenderness ratio of panel length compared to the overall structure, size of the crossbars, etc. An attempt was made to identify structures in which prestressing the diagonals will bring significant increase in strength. Prestressing in this way is not only economical but essential in certain types of structures.
Single-crossarm stayed columns were tested to identify the failure criteria and the same criteria has been applied to prestressed lattice structures in this study. Offsetting the diagonals on the crossarms and on vertica ls was found to be an effective method for increasing the strength of the lattice frames. There is a definite increase in the strength of a structure with offset diagonals as opposed to a structure with intersecting diagonals. This is a result of the increased rotational restraint.

In structures subjected to axial load only, pretensioning proved less beneficial in single-bay three-storey frames where failure occurred by buckling of the vertica ls in a panel than in slender towers where buckling occurs by Euler type buckling of the entire tower. In a tall slender tower where failure takes place by column type buckling of the complete structure, prestressing the diagonals resulted in a significant increase in strength when subjected to axial load as well as combined axial and horizontal load.
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NOMENCLATURE

\( A \) = cross-sectional area
\( A_c \) = cross-sectional area of vertical member
\( A_{ca} \) = cross-sectional area of crossarm member
\( A_b \) = cross-sectional area of batten member
\( A_d \) = cross-sectional area of diagonal

\( \{ d \} \) = displacement matrix in local coordinates

\( \{ d \}^t \) = transpose of displacement matrix

\( [ D ] \) = displacement matrix in global coordinates

\( [ D ]^t \) = transpose of global displacement matrix

\( E \) = modulus of elasticity
\( E_c \) = modulus of elasticity of the vertical member
\( E_{ca} \) = modulus of elasticity of crossarm member
\( E_d \) = modulus of elasticity of the diagonal member
\( E_b \) = modulus of elasticity of the batten member
\{ F \} = \text{load matrix}

I = \text{moment of inertia}

K = \text{axial stiffness of the column}

K_{ca} = \text{axial stiffness of crossarm member}

K_d = \text{axial stiffness of the diagonal}

[K_e] = \text{master elastic stiffness matrix}

[K_g] = \text{master geometric stiffness matrix}

[k_e] = \text{element elastic stiffness matrix}

[k_g] = \text{element geometric stiffness matrix}

k = \sqrt{P/EI}

L = \text{length of the column}

L_{ca} = \text{length of the crossarm member}

L_d = \text{length of diagonal}

P = \text{axial load on the column}

P_{cr} = \text{buckling load}

P_{\text{max}} = \text{maximum theoretical buckling load of stayed column}
\[ \mathbf{R} \] = load vector

\( T_i \) = initial pretension force in the stay

\( T_f \) = final pretension force in the stay

\( U_e \) = internal strain energy

\( u_1, u_2 \) = element displacements in axial direction

\( V_e \) = potential energy of the applied loads

\( v_1, v_2 \) = element displacements in the transverse direction

\( \Delta D \) = increment of displacement

\( \theta_{162} \) = element rotations

\( \alpha \) = angle between crossarm and diagonal

\( \sigma \) = normal stress

\( \varepsilon \) = axial strain

\( \varepsilon_L \) = linear strain due to axial load and bending

\( \varepsilon_{nl} \) = nonlinear strain component

\( \lambda \) = eigenvalue

\( \Pi_{e} \) = element potential energy

\( \Pi_p \) = global potential energy
Chapter I

INTRODUCTION

1.1 General

This study deals with prestressed lattice structures in which the diagonal members are pretensioned. A latticed structure consists of vertical, horizontal and diagonal members. The horizontals (crossarms) and diagonals provide translational and rotational restraint to the vertical compression members and thus help to increase the overall buckling load of the structure. Under the action of vertical loads and/or lateral loads the diagonals will be subjected to compressive and tensile forces. Slender diagonals will buckle under compressive loads and thereby reduce the lateral stiffness of the structure. Therefore, pretensioning of the diagonals is considered in this study as a means of increasing the stiffness, as well as the strength, of the lattice structure.

When slender diagonals are used in these structures they have the advantage of possessing a relatively small profile area and small weight for a given load carrying capacity. This results in a slender and light structure. The small profile area is desirable since, for example, the Canadian Armed Forces are interested in the development of lattice
masts for the radar antennae for their warships which have a minimum resistance to bomb blast and wind forces. Light masts are desirable in order to reduce the contribution of their weight to the roll of warships [6]. In addition to their applications as ship masts, they can be used as masts for space exploration, antennae towers, oil derricks, etc.

In the past few years, there has been some interest in understanding the behaviour of prestressed metallic structures. Most of the work has concentrated upon the investigation of stayed columns. A stayed column is defined as a column with rigidly attached crossarm members and pretensioned stays (Figure 1.1). A considerable increase in strength was obtained by using pretensioned stays [7]. Some discrepancies in the experimental and theoretical results, however, necessitated a careful examination of the failure criterion.

Limited work on prestressed lattice frames, similar to a single-bay three-storey frame, also indicated an increase in the load carrying capacity. To increase the rotational restraint of the compression member, diagonals were offset slightly on the horizontals and on verticals (Figure 1.2). Previous studies on offset frames [6] showed an increase in strength when compared to frames with diagonals intersecting at panel points. A detailed analytical and experimental study was needed, however, to understand the behaviour of

---

1 The numbers in the brackets are the references listed in Bibliography.
prestressed latticed frames of the type studied.

A relatively slender tower (Figure 1.3), which is expected to have a different mode of buckling than that obtained for the single-bay three-storey frame, was also of interest in order to determine any benefits due to prestressing.

An attempt has been made herein to understand the behaviour of prestressed lattice structures under different loading conditions, prestressing conditions, and different configurations.

1.2 Objectives

Previous research on stayed columns and prestressed lattice frames indicated areas of further research. This research is aimed at determining the behaviour of prestressed lattice frames which includes: (i) the load-deflection relationship; (ii) the load-stay tension relationship; and (iii) the effect of pretension in the diagonals. Single-bay three-storey prestressed lattice frames with different diagonal configurations, i.e., diagonals intersecting, offsetting on horizontals and offsetting on verticals (Figure 1.2), are studied under the category of lattice structures. A prestressed lattice tower with a mode of buckling different from the single-bay three-storey lattice frame was also investigated. The objectives of this research can be summarized as:
(a) to determine the failure criteria of the stayed columns in relation to the results obtained by previous researchers and to apply the same criteria in the case of prestressed lattice structures;

(b) to determine, theoretically, the variation of buckling load with offset distances and an optimum offset;

(c) to determine the effect of axial load on the buckling load of prestressed lattice frames with (i) intersecting diagonals, (ii) offsetting diagonals on crossarms, and (iii) offsetting diagonals on verticals;

(d) to investigate, in collaboration with the Department of Electrical Engineering at the University of Windsor, the feasibility of electrical prestressing; and

(e) to determine the effect of axial load and a combination of axial load and horizontal load upon a relatively slender prestressed lattice tower.

In all the above investigations the effect of out-of-straightness was considered. This research is limited to elastic buckling only. Thus, the main emphasis is to understand the behaviour of prestressed lattice structures and to come up with a theoretical model which can predict the experimental behaviour.
Chapter II

LITERATURE REVIEW

2.1 General

Literature on prestressed lattice structures is very limited. A brief historical review, however, is presented here in two parts. In the first part research on stayed columns will be considered. In the second part prestressed lattice frames will be discussed.

2.2 Stayed Columns

Over the last ten years, there has been considerable interest in understanding the behaviour of stayed columns. In 1963, Chu and Berge [3] developed a general solution for the elastic buckling load of a slender, pin-ended stayed column with tension ties and multiple ideally pin-ended slender crossarms. The solution indicated that the maximum buckling load of the column with intermediate supports arranged symmetrically about the midpoint will be four times the Euler buckling load of the column and will occur when the initial tension in the ties is reduced to zero at the instant of buckling. Tests were performed on a few models that seem to verify the above conclusion. They also found that the ultimate load of the column was dependent upon the initial pretension in the stays.
In 1967, Mauch and Felton [9] presented an analytical foundation for rational design of columns with tension ties, which provide intermediate elastic supports. It was concluded that when the structural index (i.e. $P/L^2$, where $P$ is axial load and $L$ is the length of the column) is low, the supported columns offer potential savings of up to 50% of the weight of the optimum simple tubular columns.

In 1970, the Royal Military College of Canada, Kingston, Ontario, Canada, started work on stayed columns as a design-build-test project [5], assigned to the Fourth Year Civil Engineering undergraduates. Instead of the crossarm members being pin-connected to the columns and relatively flexible, as in Chu and Berge's study, they were relatively rigid and welded to the compression core. The aim of the rigidly connected crossarm members was to provide restraint against rotation of the column. As a result, the buckling load was increased to seven times that of its Euler load.

In 1971, Pearson [11] examined experimentally the behaviour of a single-crossarm stayed columns. Stay slopes and pretensions were varied to determine the effect of the buckling load of the stayed column. No analytical work was attempted to determine the influence of these parameters. It was concluded that the buckling load of the stayed column is directly proportional to the initial pretension in the stays and to the eccentricity of the stays (crossarm lengths).
In 1972, Clarke [4], after extensive testing on similar lines to Pearson's work, concluded that the relationship between the buckling load of the stayed column and the tie pretension is not a simple straight line relationship. Similarly, a complex relationship was observed between the tie eccentricity and column buckling load. Two types of buckling modes were observed from experimental results.

In 1975, Smith, McCaffrey, and Ellis [14] investigated the stability of a pin-ended single-crossarm stayed column. The maximum theoretical buckling load of a pin-ended single-crossarm stayed column was found to be 8.18 times the Euler buckling load. The two modes of buckling were single curvature (symmetrical) and a double curvature (antisymmetrical) deflected shape of the column. A differential calculus approach was adopted to derive theoretical solutions for each assumed buckling mode. A numerical example was studied in order to determine the effects of various stayed column parameters on the mode of instability and the buckling strength of a pin-ended single-crossarm stayed column. The derivation of theoretical buckling solutions for stayed column requires that complete relaxation of the tension in any of the stays must not occur prior to the instant of buckling. One of the important conclusions was that the maximum applied load will be obtained only when the initial tension in the stays is reduced to zero at the instant of buckling. In a discussion
to this paper, Temple [16] pointed out that this conclusion is not compatible with the derived equations and suggested the need for modifications. It was also pointed out that one of the buckled modes for stayed column is not of single curvature but of triple curvature. The analytical method suggested in this paper is only applicable to single-crossarm stayed column and with certain assumed buckling modes.

In 1977, Temple [17] presented a more general method to analyze any type of stayed column. The elastic buckling mode for stayed columns was determined by two different approaches. The first was based on the finite element method using the geometric stiffness matrix, while the second was a more refined approach using stability functions. Results from these methods showed excellent agreement with the solution obtained by Smith et al. [14]. It has been proven that complex buckling shapes can be easily predicted with these methods.

In 1979, Haferz et al. [7] presented the effect of the initial pretension in the stays on the buckling load of a single-crossarm stayed column. An attempt was made to identify minimum effective pretension, the optimum pretension, and the maximum possible pretension for stayed columns. A linear relationship between the initial pretension and the corresponding buckling load was predicted and derived theoretically when the initial pretension is
between the minimum effective pretension and the optimum pretension. The effect of varying several of the stayed column parameters was also examined. Experimental results tend to agree well with theoretical results except when the initial pretension is relatively large.

Williams and Howson [20,21,8] used a substitute column method to determine the buckling loads of stayed columns. A plane-frame computer program was used to determine the buckling load.

In 1983, Wong and Temple [22] investigated the effect of initial out-of-straightness on the buckling strength of a single-crossarm stayed columns. The geometrical nonlinear behaviour was included in the finite element method with a mixed iterative and incremental method. Numerical examples were used to demonstrate the influence of varying the stayed column parameters like the crossarm length, stay size, modulus of elasticity of the stay, initial imperfections etc., on the buckling load, deflection rate, as well as on the minimum effective pretension, optimum and maximum possible pretension. Good agreement was obtained between theoretical and experimental results. The results indicated that the presence of an initial out-of-straightness changes the deflection rate and can significantly reduce the buckling load.
2.3 Prestressed Lattice Frames

In 1957, Chamberlain [2] used prestressed wind bracing in the Queen Elizabeth Hotel, Montreal. Shear walls were replaced by pretensioned bracing bars because of construction problems. In a unique pretensioning technique, both bars in a panel were heated simultaneously and the free end of each one was clamped when the required elongation was obtained. Essentially, the wind bracing was placed in narrow partitions and pretensioned to a point where no bracing bar would go into compression with maximum wind acting on the structure.

In 1980, Ellis [6] reported on prestressed latticed beam-columns with offset diagonals. Experiments were conducted on two-dimensional frames consisting of longitudinals, horizontals, and diagonals. Several parameters were investigated which included the importance of prestressing, and offsetting the diagonals. It was shown that pretensioning the diagonals significantly increases the lateral rigidity of the structure. According to tests conducted on four models of prestressed lattice cantilevers subjected to lateral load at their ends, frames with offset diagonals showed a 20% strength increase over the frames with intersecting diagonals. The need to develop a theoretical approach was outlined.

In 1981, Mohan [10] investigated the effect of initial tension on the critical load of a lattice frame. The
influence of various parameters such as the crossarm length, size and the modulus of elasticity of the diagonals on the critical load were studied. Experiments were conducted to compare the theoretical observations on an axially loaded three panel, single-bay lattice frame. A linear relationship was observed between the initial pretension and the critical load when the initial pretension was greater than the minimum effective pretension. Higher critical loads were obtained experimentally than those theoretically predicted when the pretensions were low, and lower critical loads when the pretensions were high (Figure 2.1). This investigation was limited to comparison of buckling load only.
Chapter III
THEORETICAL FORMULATION

3.1 General

To achieve the objectives of this investigation, it was essential to use a computer program to predict the nonlinear behaviour of prestressed lattice structures. Thus, an existing displacement based finite element method of analysis for beam-columns was used to determine the theoretical response of the structure. This method is based on a mixed incremental-iterative scheme. The same program was modified to handle an eigenvalue problem in order to determine the critical loads for ideal structures. The displacement method of analysis applied to discrete element systems is used and it is assumed that strains are small although displacements may be large. In the following analysis, only the nonlinearity due to geometry was considered. Changes in member length due to bowing were not included. This analysis applies only to cases in which the material remains elastic.

Two sources of nonlinearity exist in large deflection problems. The first is associated with the strain-displacement equations. Even if the strains remain small, rotation of the element adds nonlinear terms to the
strain-displacement equations. The second source of nonlinearity exists with respect to the equilibrium equations. It is necessary to consider the deformed geometry while writing the equilibrium equations. This causes the nonlinearity in the equations. In the stiffness method this is taken into account by the incremental procedure.

3.2 Derivation of Element Stiffness Matrix

Consider a prismatic element (Figure 3.1), which is assumed to sustain only axial and flexural deformation. Shear deformation is disregarded. Thus only axial strains are present and these are described by the following strain-displacement equation [22],

\[ \varepsilon = \varepsilon_x + \varepsilon_{nl} = \frac{du}{dx} - y \frac{dv}{dx^2} \frac{1}{2} \left( \frac{dv}{dx} \right)^2 \]  

[3.1]

where \( \varepsilon \) is axial strain; \( \varepsilon_x \) is strain in the linear portion consisting of axial strain due to compression and flexural strain, \( \frac{du}{dx} - y \frac{dv}{dx^2} \); \( x \) and \( y \) are the coordinate axes; \( u \) and \( v \) are the displacements in \( x \) and \( y \) directions; \( \varepsilon_{nl} \) is the strain in the nonlinear portion, \( \frac{1}{2} \left( \frac{dv}{dx} \right)^2 \) which couples the flexural and axial action.

The strain energy \( U_e \) may be expressed as
\[ U = \frac{1}{2} \int_{\text{vol}} \sigma \, d(\text{vol}) \]  

where \( \sigma \) is the normal stress. The stress and strain relationship \( (\sigma = E\varepsilon) \) will be substituted in Eq. (3.2) and the result is

\[ = \frac{1}{2} \int B \varepsilon \, dx \, dy \, dz \]  

where \( B \) is the modulus of elasticity of the material. Substituting Eq. (3.1) in Eq. (3.4) gives,

\[ = \frac{1}{2} \int \left( \frac{du}{dx} - y \frac{d^2v}{dx^2} + \frac{1}{2} \frac{dv}{dx} \frac{d^2z}{dx^2} \right) E \, dA \, dx \]

where \( dA \) is the elemental area. Expanding Eq. (3.4) gives,

\[ = \frac{1}{2} \int \left[ \left( \frac{du}{dx} \right)^2 + 2y \frac{du}{dx} \frac{d^2v}{dx^2} + y^2 \left( \frac{d^2v}{dx^2} \right)^2 \right. \]

\[ + \left. \frac{1}{4} \frac{dv}{dx} \frac{d^2z}{dx^2} - y \left( \frac{dv}{dx} \right) \left( \frac{d^2z}{dx^2} \right) \right] dA \, dx \]

where \( y \) is the distance from the centroid. By integrating across the depth of the member, and substituting the following relationships,
\[ \int A \, dA = A ; \int y \, dA = 0 ; \int y^2 \, dA = I \quad [3.6] \]

and where \( A \) is the area of the cross-section and \( I \) is the moment of inertia.

\[ U = \frac{1}{2} e \int \left[ \frac{du^2}{dx} + \frac{d^2v}{dx^2} + A \left( \frac{dv}{dx} \right)^2 \right] dx \quad [3.7] \]

The higher order term \( A \left( \frac{dv}{dx} \right)^2 \) can be ignored since its contribution to the potential energy is negligible. The axial load, \( P \), is related to the axial deformation by the linear relationship.

\[ P = E A \frac{du}{dx} \quad [3.8] \]

Then Eq. (3.7) becomes,

\[ U = \frac{1}{2} e \int \left[ E A \left( \frac{du}{dx} \right)^2 + EI \left( \frac{d^2v}{dx^2} \right)^2 + P \left( \frac{dv}{dx} \right)^2 \right] dx \quad [3.9] \]

Selecting the displacement functions for the beam-column as

\[ u(x) = a_0 + a_1 x \quad [3.10] \]

and

\[ v(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 \]
where $a_0, a_1, b_0, b_1, b_2,$ and $b_3$ are constants. Integrating the above equations,

\[
\frac{du}{dx} = a_1
\]

\[
\frac{dv}{dx} = \theta = b_1 + 2b_2x + 3b_3x^2 \quad \text{[3.11]}
\]

\[
\frac{d^2v}{dx^2} = 2b_2 + 6b_3x
\]

Also writing $u$, $v$, and $\theta$ at each node from Eqs. 3.10 and 3.11, and by solving algebraically for the constants in Eq. 3.10 provides:

\[
a_0 = u_1
\]

\[
a_1 = \frac{u_2 - u_1}{L}
\]

\[
b_0 = v_1
\]

\[
b_1 = \theta_1
\]

\[
b_2 = \frac{3(v_2 - v_1)}{L^2} + \frac{(2\theta_1 + \theta_2)}{L}
\]

\[
b_3 = \frac{-2(v_2 - v_1)}{L^3} + \frac{(\theta_1 + \theta_2)}{L^2} \quad \text{[3.12]}
\]
where \( u_1, v_1, \theta_1, u_2, v_2, \) and \( \theta_2 \) are the displacements at the member ends respectively. Substituting Eqs. 3.11 and 3.12 in Eq. 3.9, integrating and putting in a matrix form,

\[
U = - \begin{bmatrix} d \end{bmatrix} \begin{bmatrix} k_e \end{bmatrix} \begin{bmatrix} d \end{bmatrix} + - \begin{bmatrix} d \end{bmatrix} \begin{bmatrix} k_g \end{bmatrix} \begin{bmatrix} d \end{bmatrix} \tag{3.13}
\]

where \( [d] \) is \( \{u_1, v_1, \theta_1, u_2, v_2, \theta_2\} \), displacements in local coordinates; \( (d)^T \) is transpose of \( [d] \); \( [k_e] \) is the elastic stiffness matrix in local coordinates; and \( [k_g] \) is the geometric stiffness matrix in local coordinates (see Appendix C for the matrices). The potential of the applied loads is given by,

\[
V = - \begin{bmatrix} d \end{bmatrix}^T \begin{bmatrix} F \end{bmatrix} \tag{3.14}
\]

where \( V_e \) is the external work done by the applied loads, where \( \begin{bmatrix} F \end{bmatrix} \) is the load vector \( \{F_1, F_2, F_3, F_4, F_5, F_6\} \). The element potential energy \( \Pi_{pe} \) is represented by,

\[
\Pi_{pe} = U + V_e \tag{3.15}
\]

Substituting Eqs. (3.13) and (3.14) in (3.15) gives,

\[
\Pi_{pe} = \frac{1}{2} \begin{bmatrix} d \end{bmatrix}^T \begin{bmatrix} k_e \end{bmatrix} \begin{bmatrix} d \end{bmatrix} + \begin{bmatrix} d \end{bmatrix}^T \begin{bmatrix} k_g \end{bmatrix} \begin{bmatrix} d \end{bmatrix} - \begin{bmatrix} d \end{bmatrix}^T \begin{bmatrix} F \end{bmatrix} \tag{3.16}
\]
The element potential energy in Eq. 3.16 will be written in nodal displacements with the use of the following equation:

\[ \{ d \} = [ T ] [ D ] \]  \hspace{1cm} [3.17]

where \( [ D ] \) = displacements of nodes in global coordinates, \( \{ U, V, \theta \} \), and \( [ T ] \) = transformation matrix (given in Appendix C). The global potential energy is given by the summation of the potential energies of all the elements:

\[ A_P = \sum A_{PE} \]  \hspace{1cm} [3.18]

After substituting Eq. 3.17 to Eq. 3.16, Eq. 3.18 can be written as:

\[ A_P = \left( \frac{1}{2} \right) \left( [ D ]^T [ K_e ] [ D ] + [ D ]^T [ K_g ] [ D ] \right) \cdot [ R ] \]

\[ \{ 0 \} \]  \hspace{1cm} [3.19]

where \( [ K_e ] \) is the global elastic stiffness matrix, \( [ T^T ] [ k_e ] [ T ] \), and \( [ K_g ] \) is the global geometric stiffness matrix, \( [ T^T ] [ k_g ] [ T ] \), \( [ R ] \) is the global load vector, \( [ T^T ] [ F ] \). The summation was achieved using a variable correlation table. For conditions of stable equilibrium, i.e., where the axial load is less than the critical value, application of the principle of stationary potential energy in the form of first variation of \( \delta A_P \) to Eq. (3.19) results in the stiffness matrix;
\[
[R] = [K_e][D] + [K_g][D] \quad \text{[3.20]}
\]

For simplicity Eq. [3.20] can be written as

\[
[K][D] = [R] \quad \text{[3.21]}
\]

where \([K] = [K_e] + [K_g]\). To determine the deflected shape, Eq. (3.21) was solved using a Newton-Raphson procedure as discussed in the next section.

3.3 Mixed Incremental-Iterative Method

Eq. (3.21) represents a system of nonlinear algebraic equations which must be solved for any generalized load vector \([R]\). The incremental and iterative solution is carried out by applying the external load in small increments and iterations are performed until Eq. (3.21) is satisfied. For iteration cycle \(j\) at load level \(i\) the process may schematically be written as

\[
\Delta [D]_{i,j} = [R]_{i,j} - [R]_{i,j}^{\text{ext}} \quad \text{[3.22]}
\]

where \([R]_{i,j}^{\text{ext}}\) is the external load vector and \([R]_{i,j}^{\text{int}}\) is the load calculated from the internal deformations. Solution of Eq. 3.22 yields an improved solution vector.
\[ [D]_i,j = [D]_i,j-1 + (\Delta D)^i,j \]  \[ \text{[3.23]} \]

This process can be easily understood from Figure 3.2. 
\([K]^{i,j-1}\) is the updated global stiffness matrix at the end of the \((j-1)\)th cycle. In order to determine the convergence during the iterations the following criterion was used[1].

\[
\text{Tolerance} = \left[ 1 - \frac{1}{N} \sum_{k=1}^{j} \left( \frac{\Delta D_k}{D_k,\text{ref}} \right)^2 \right]^{1/2} \quad \text{[3.24]}
\]

where \(N\) is the number of non-zero degrees of freedom and \(\Delta D\) is the change in displacement component \(k\) during iteration cycle \(j\). \(D\) is the largest displacement component of the corresponding category. The iteration was terminated when the tolerance becomes smaller than 0.001.

3.4 Eigenvalue Problem

If \([K]_g\) in Eq. (3.20) is based on a unit load and \(\lambda\) is an arbitrary scalar multiplier, then \(\lambda[K]_g\) is the geometric stiffness matrix for the whole structure. It is possible to determine \(\lambda\) such that the reference configuration and an infinitesimally close \{buckled\} configuration are both equilibrium configurations. If displacements \([D]\) of the reference configuration are augmented by virtual displacements \((\Delta D)\) while applied loads \([R]\) remain constant, Eq. [3.20] can be written as
\[
\begin{align*}
&\begin{cases}
[K] + \lambda[K] & \{D\} = \{B\} \\
\text{where } e & \quad \text{(3.25)} \\
[K] + \lambda[K] & \{D + \Delta D\} = \{B\} \\
\text{where } e & \quad \text{(3.26)} \\
\end{cases}
\end{align*}
\]

Subtracting Eq. 3.25 from Eq. 3.26 yields the eigenvalue problem:

\[
\begin{align*}
&\begin{cases}
[K] + \lambda[K] & \{\Delta D\} = \{0\} \\
\text{where } e & \quad \text{(3.27)} \\
\end{cases}
\end{align*}
\]

where the critical (buckling) load is associated with \(\lambda\), the lowest magnitude eigenvalue of Eq. (3.27). Displacements \(\{\Delta D\}\) represent the relative value of displacement in the buckled shape and not the magnitude of the displacements.

### 3.5 Computer Program

A computer program was written based on the above theory to obtain the failure load of the prestressed lattice structure. The program incorporates both beam and truss elements. The effect of the axial load on the beam elements was considered where necessary. Diagonal members were considered as truss members because of their slenderness relative to the main load carrying members. Pretension in the diagonals was considered by determining the change of force at every increment and adding that to the initial pretension. Diagonals with compressive force were considered as ineffective. The computer program with the flow chart is given in Appendix D.
3.6 Preliminary Theoretical Study

In this study, three types of pretensioned structures were theoretically modelled. One of them is a two-dimensional single-crossarm stayed column. Although some studies were conducted before [14,16,17,7,22], a new failure criterion was identified in this study which needs some explanation. This will be discussed briefly in later chapters.

The second type of structure is a single-bay three-storey prestressed lattice frame. In this category three types of configurations were investigated. They are i) intersecting diagonals, ii) offsetting diagonals on the crossarms, and iii) offsetting diagonals on the verticals (Figure 1.2). These frames were studied under the effect of axial force alone.

The third type is a relatively slender prestressed lattice tower with intersecting diagonals. This was selected after observation of the results of the above structures.

3.6.1 Parametric Study

3.6.1.1 Effect of the Number of Storeys

Since most codes allow a maximum slenderness ratio of 200 for a compression member, this value was selected initially to be the limiting slenderness between the panel points of a prestressed lattice frame. From a practical point of view, i.e., fabrication, economy, etc., the panel length was fixed
at 12 in. A rectangular cross-section of 1 x 1 1/4 in. was arbitrary chosen for the verticals and horizontals. With the distance between the panel points fixed at 12 in., a slenderness ratio of 167 was obtained with the selected cross-section. Four types of frames were initially selected (Figure 3.3) for a theoretical study, with the height between the panel points fixed at 12 in. for all frames. An eigenvalue analysis was performed and the results are summarized for the frames in Table 1. Figure 3.4 (a) and (b) show the buckling modes for batten frames and lattice frames respectively. Batten frames had no diagonals. Mode A is the first buckling mode for the batten frame and Mode B is the second buckling mode which occurs as lateral restraint increases. In a similar way, Mode C is the first buckling mode for the lattice frame and Modes D and E, the second and third buckling modes occur as the lateral restraint increases. Batten frames were considered to represent lattice frames with zero pretension in the diagonals. Figure 3.5 shows the variation of buckling loads for these frames. There is not much variation in the buckling load as the number of storeys is increased. Therefore, consideration was given to selecting a model with a bay size of 12 in. and a distance of 12 in. between the panel points with a total of three panels.
3.6.1.2 Effect of Offset Distance

It was also necessary to find the optimum offset distance for the lattice frames [6]. Therefore, a numerical example was selected and the relationship between the buckling load and the offset distance was determined by the eigenvalue analysis. The numerical example has a total length of 72.0 in., a width of 12 in., with three panels. The modulus of elasticity of all the members was taken as 29000 ksi. The diameter of verticals and horizontals is 0.625 in. solid and a 0.1875 in. diameter steel rod was used as a diagonal. A comparison of critical loads is presented in Table 2 for various offsetting configurations. Figure 3.6 (a) and (b) show the variation of critical load with offset distance on crossarms and verticals respectively. From these figures, it can be observed that the critical load at the optimum offset distance is the same for both types of offset frames, namely offset on the horizontals and the verticals. The optimum offset distances are, however, different. An optimum distance of 1.08 in. on the crossarm was obtained, where as for the verticals an optimum offset of 1.45 in. was obtained. To understand the behaviour that is causing an increase or decrease in strength as the offset is increased, four cases were compared closely for modes C and E at an offset distances of 0.5 and 2 inches on the crossarm (Figures 3.7 to 3.10) From Figures 3.7 and 3.8 where the eigenvectors are compared, it can be clearly seen that an
increase in offset distance increases the rotational restraint (compare the values of rotation at joints (1) and (2)) which results in an increase in load carrying capacity. From Figures 3.9 and 3.10 no change in load carrying capacity takes place when the offset on the crossarm changes from 0.5 in. and 2 in. A close look at the eigenvectors indicates very little difference between Figures 3.9 and 3.10. It can also be observed that the crossarm bends in double curvature in Mode E, while in Mode C it bends in single curvature. This type of bending of the crossarm in double curvature does not result in any elongation of the diagonals, and hence no change in the load carrying capacity when the offset distances increase. The same reasoning can be applied to offsetting on verticals where a similar behavior can be observed. From Table 2, with an offset distance of zero in, the buckling load of the frame is 13.44 kips, (twice the buckling load of the vertical) while with an optimum offset the buckling load is 19.44 kips. Thus, an increase of 45% (for this particular example) was observed by offsetting the diagonals on crossarms when compared to intersecting.

Therefore it was decided to check this observation by building a model with a rectangular cross-section of 1 x 1/4 in. for both verticals and horizontal. However, from the consideration of the yielding of the verticals, it was necessary to increase the length of the distance between the
panel points to 18 inches. The angle of the diagonal with the horizontal member was still kept at 45 degrees by increasing the crossarm length to 18 in. The buckling load for the battened frame was 1.75 kips and for the lattice frame it was 4.22 kips. Thus, the dimensions of the experimental model for intersecting model was determined from a theoretical study. Figure 3.11 shows the variation of the buckling load with offset on crossarm and on vertical for this problem. An offset distance of 2 in. on crossarms and on verticals was selected from this to give the maximum load-carrying capacity.

3.6.1.3 Effect of Mode of Buckling

The third type of prestressed lattice structures was a relatively slender tower-like structure. The mode of buckling for this structure is different from a single-bay three-storey frame. A relatively slender lattice tower is formed by placing the verticals close to each other and the panels are designed in such a way that buckling of the overall frame takes place. The analysis for this is similar to built-up columns and is given in Ref. [19]. The maximum load carrying capacity for a given tower-like structure can be determined by the following approximate formula.

The buckling load of a lattice column is given by [19],
\[
\begin{align*}
P_{cr} &= \frac{\pi^2 EI}{L^2} \left[ 1 + \frac{\pi^2 EI}{L^2} \left( \frac{1}{A_b E \sin \theta \cos^2 \theta} + \frac{a}{A_b E} \right) \right].
\end{align*}
\]

where \( P_{cr} \) is the critical load of the laced column; \( I \) = moment of inertia of the cross-section of the strut; \( A_b \) = area of the battens; \( A_d \) = area of the diagonal; \( E, E_b, E_d \) = modulus of elasticity of the vertical, horizontal and the diagonal, respectively; \( \theta \) = the angle between the diagonal and the horizontal; \( a \) = distance between the panel points; and \( b \) = distance between the verticals. In Eq. (3.28), \( A_d \) can be changed to simulate the nonprestressed case as well as the prestressed cases. A preliminary study showed that prestressing the diagonals can increase the load carrying capacity up to 18-25%. This will be discussed in the following chapters.

3.6.1.4 Effect of Optimum Pretension

Hafez et al. [7] have defined optimum pretension as the initial pretension in the stays which disappears completely just after the load in the column reaches its maximum buckling load. This means that the stays remain effective until the maximum buckling load of the column has been applied. Theoretically, the optimum pretension is the best value for the initial pretension in the stays. Previous
research [7] derived a linear relationship between the initial pretension in the stays and the buckling load of the stayed column. They concluded that the buckling load of a stayed column varies between two limits. The lower limit is the Euler load for the core of the stayed column, which results from a zero or a very small initial pretension in the stays. The upper limit is the buckling load determined for the stayed column from the finite element method. But according to the research done in this study, it was found that only the upper limit was a valid one. This will be discussed in Chapter 5. Therefore, the upper limit which was used in the previous studies to determine the optimum pretension can still be used. This was determined from the force-deformation relationships of an ideal column and only the final result will be given here. (A detailed derivation is given in [7].)

\[ P_{\text{max}} = \frac{T_i - T_f}{2K'(\frac{1}{c} + \frac{2\sin^2\alpha}{K} + \frac{2\cos^2\alpha}{K})} \frac{1}{d\cos\alpha} c \]

where \( T_i, T_f \) = initial pretension in the diagonals prior to the application of the load and final pretension after the application of the maximum load, respectively; \( \alpha \) = angle between the vertical and the diagonal members of the stayed
column; \( K = \frac{AE}{L} \) in which \( A \) = the cross-sectional area of the column, \( E \) = modulus of elasticity of the column, and \( L \) = the total length of the column; \( K = \frac{A_{ca} E_{ca}}{L_{ca}} \) in which \( A_{ca} \) = the cross-sectional area of the crossarm, \( E_{ca} \) = modulus of elasticity of the crossarm, and \( L_{ca} \) = the length of the crossarm; \( K = \frac{A_{d} E_{d}}{L_{d}} \) in which \( A_{d} \) = cross-sectional area of the diagonal, \( E_{d} \) = modulus of elasticity of the diagonal, and \( L_{d} \) = the length of the diagonal; and \( P_{\text{max}} \) = critical load of the stayed column as determined by the finite element method. A similar method was adopted for single-bay three-storey prestressed lattice frames. A detailed derivation was given in [10]. The final formula which was used to find the optimum tension is given by

\[
T = \frac{P \cos \alpha}{\text{max}}
\]

\[
T = \frac{1}{K + \frac{2 \sin^2 \alpha}{2 \cos^2 \alpha}} \left( K_{\text{c}} K_{\text{d}} + \frac{K_{\text{c}}^2}{K_{\text{ca}}} \right)
\]

[3.30]

where \( P_{\text{max}} \) = the critical load of the prestressed lattice frame of the corresponding type, i.e., intersecting diagonals, offsetting etc. The same formula was also applied to a tall slender tower of 23 panels and found applicable.
3.6.2 Load-Deflection Relationship for Slender Lattice Tower

To determine the behaviour of a lattice frame under a loading combination of axial and horizontal loads, experiments were conducted at a constant axial load and increasing horizontal load until failure. Under this situation, i.e., for a given longitudinal force, the deflections of the bar are proportional to the lateral load (Figure 3.12). Because of this fact, the principle of superposition can be applied provided that the axial force does not change [19]. For completeness, the procedure used will be briefly described. The differential equation of the deflection curve is, for \( x < a \),

\[
EI \frac{d^2y}{dx^2} = \frac{Qa}{L} - \frac{Q(L-a)}{L} - P \frac{y}{L} \tag{3.31}
\]

where \( Q \) = Horizontal load acting at a distance \( a \) and \( L-a \); \( P \) = constant axial load; \( y \) = deflection at a distance \( x \) along the length; \( L \) = length of the beam; \( E \) = modulus of elasticity; and \( I \) = moment of inertia. The solution of Eq. 3.31 is as follows [19]:
\[ y = \frac{\sin kx \cdot Q \cdot \sin ka}{P \cdot k \cdot \sin kL} \cdot \frac{x \cdot Q \cdot a}{P \cdot L} + \frac{\sin k(L-x) \cdot Q \cdot \sin ka}{P \cdot k \cdot \sin kL} \cdot \frac{(L-x) \cdot Q \cdot a}{P \cdot L} \] [3.32]

where \( k = \sqrt{\frac{P}{EI}} \). The only difficulty in Equation 3.32 is the determination of \( I \) for the built-up column. However, for comparison purposes approximate values of deflection can be obtained by using an equivalent moment of inertia.
Chapter IV

EXPERIMENTAL PROCEDURE

4.1 General

One of the objectives of this investigation was to
develop a theoretical model capable of predicting results
which have been obtained from experiments. This has
resulted in the fabrication of three types of prestressed
structures. One of them was a two-dimensional
single-crossarm stayed column. The second type consists of
three similar models of similar dimensions. They are
single-bay three-storey lattice frames, one with
intersecting diagonals, one with diagonals offset on the
crossarms, and the third with diagonals offset on the
verticals. The third experimental model is a 23 panel
slender tower-like structure. These structures were tested
under varying amounts of pretension and different load
conditions. In the following paragraphs, the fabricating
and testing procedures, the model dimensions, etc., will be
given in detail.
4.2 Stayed Columns

A two-dimensional, pin-ended, single-crossarm stayed column, shown in Figure 4.1, was used in the experimental program. The main compression member was made from a rectangular section, 1 in. wide and 3/8 in. thick. The stayed column had a central core 32 in. in length, and 3 in. long crossarms on either side of the central core at mid-height. The crossarms are of the same cross-section as the verticals. The stays (diagonals) were 1/8 in. diameter steel rods. From tension tests the modulus of elasticity of the central core and the crossarms was determined to be 29000 ksi. The yield strength of the central core and the stays are 52 ksi and 57 ksi, respectively. Because of the proportions of the cross-section, the stayed column deflected in the plane containing the stays. In other words, no restraints were required in order to force the stayed column to buckle in the plane of the stays. Knife edges were used at top and bottom to provide hinged supports. Four ring beam load cells, one in each stay, were used to set the initial pretension and to measure the tension in the stays as the load was applied. These load cells were fastened to the stays by nuts on the inside of the ring beam load cells which also were used to apply the initial pretension. An equivalent modulus of elasticity, using the axial stiffnesses, with the ring beam load cells was determined, from theoretical formula [7], to be 20000
ksi. A universal flat type load cell was placed beneath the column in order to determine the load which was applied by a hydraulic jack. The tension in the stays and the applied load were determined from the strain readings measured with a strain indicator. Dial gauges were used to measure the lateral deflection at the quarter points and at mid-height.

The main aim of the experimental program was to determine the variation of the failure load with different amounts of pretension and with different magnitudes of initial out-of-straightnesses. An initial series of tests was conducted with increasing initial pretensions and an out-of-straightness of L/1000, where L is the height of the central core. Another series of tests was conducted at an initial pretension of 150 lbs and an out-of-straightness which varied from L/500 to L/3000. To obtain the desired initial out-of-straightness, weights were hung over a pulley and attached to the stayed column at mid-height. These weights and pulleys can be seen in Figure 4.1.

A total of seven stayed columns were tested out of which five had an initial out-of-straightness of L/1000 and an initial pretension which varied from zero to about 300 lbs. Two tests were conducted with a pretension 150 lbs and at an initial out-of-straightness of L/3000 and L/500.
4.3 Single-bay Three-storey Prestressed Lattice Frames

Two-dimensional, pin-ended, single-bay, three-storey frames were used in the experimental program. Three frames were fabricated with the only difference in them being the configuration of diagonals. One frame had intersecting diagonals, while in the other frames the diagonals were offset on the horizontals, and then offset on the verticals. These can be seen in Figures 4.2 to 4.4. The three frames were built of the same material and similar geometric proportions. The frame has two vertical compression members, 54 in. in length, and 18 in. long crossarms connecting the two verticals. The vertical and horizontal members have a cross-section which is 1 in. wide and 1/4 in. thick. The diagonals in each panel are of 1/8 in. diameter steel rods. The modulus of elasticity of both the verticals and horizontals is 29000 ksi. The yield strength of the verticals and horizontals is 60 ksi, while the diagonals have a yield strength of 57 ksi. Two rigid horizontal members, 1 in. wide and 2 in. deep, were used at the ends of the frame, one at the top and the other at the bottom. They were used to distribute the applied load uniformly to the frame. The entire length of 54 in. was divided into three equal 18 in. panels. The proportions were determined from consideration of the limitations of the available equipment and yield strength of the compression members as discussed in Chapter 3. Lateral supports were provided at the third
points as the unsupported length was too long for the buckling to take place in the plane containing the stays. Knife edges were used at top and bottom to provide hinged supports. Axial load on the frame was applied through the knife edges, which were attached to the middle of the top and bottom crossarms. This facilitated the symmetrical application of the load. The end crossarms go through a rigid body rotation about the center point due to this arrangement. Six ring beam load cells, one in each stay, were used to set the initial pretension and to measure the tension in the stays as the load was applied. An equivalent modulus of elasticity for the stays and the ring load cells was determined theoretically to be 23700 ksi. A universal flat load cell was placed beneath the frame in order to determine the load which was applied by a hydraulic jack. The tension in the stays and the applied load were determined from readings obtained from a strain indicator. Dial gauges were used on both the legs of the frame to measure the lateral deflection at several points on the frame. To ensure elastic buckling, the verticals were so proportioned to achieve a slenderness ratio of 200 between the panel points. The diagonals were so slender that they have very little capacity in compression and thus are effective in tension only.

The optimum offset distance for the offset frames was determined from an eigenvalue analysis. The distance of 2
in. was selected for both cases. The offset or eccentricity was measured from the intersection of the centerlines of the verticals and horizontal members.

The main aim of the experimental program in this case was to determine the effect of prestress on the failure load for each type of frame, and to compare any advantages offset may have over intersecting diagonals. The initial out-of-straightness of each frame was measured before testing. This data is necessary for comparison of the theoretical and experimental results. No attempt was made to vary the out-of-straightness. A previous study [10] has already considered the variation of buckling load with initial pretension. It was decided, therefore, to conduct two tests on each type of frame, one simulating a non prestressed lattice frame and the other simulating a prestressed lattice frame with optimum pretension in the diagonals. Thus the results on these frames would show the effect of prestressing as well as the importance of offsetting in pretensioned lattice frames. A total of seven tests were conducted in this category, one battened and two for each type of prestressed lattice frame discussed before.

The tests with non prestressed diagonals had a near zero pretension in all the diagonals whereas the prestressed frame had an optimum tension in all the diagonals. In the case of intersecting diagonals the values of initial pretensions used for non prestressed and prestressed were 10
and 70 lbs. The corresponding values in the offset frames were 10 and 92 lbs. The battened frame was tested without any diagonals. All the tests were taken up to the failure load which occurs when the verticals buckle elastically.

4.4 Relatively Slender Tower

A relatively slender tower was fabricated to determine the effects of prestressing the diagonals. The main reason for this test is that the mode of failure is different from that obtained for the prestressed lattice frames discussed previously. Only intersecting diagonals were used in this type of frame. The difference in the mode of failure is that this frame buckles in a mode similar to that of an Euler column, whereas in the previous frames, as explained above, buckling occurred in each panel. In relatively slender towers the differences in behaviour between prestressed and nonprestressed cases could be significant because of the tendency of the tower to deflect laterally. Therefore, two tests were conducted, one with about 50 lbs. pretension in the diagonals, which is close to the nonprestressed case, and another with approximately 300 lbs. pretension in the diagonals, which is close to 70% of the optimum pretension, which is calculated according to Eq. 3.30.

The frame is a two-dimensional pin-ended one as shown in Figure 4.5. It has a total length of 120 in. with 3 in. long crossarms. Two vertical members, 1 in. wide and 1/4 in.
thick, were connected by horizontals with the same cross-section. A 1/8 in. diameter solid steel rod was used as a diagonal. Two diagonals were used in each panel. To initiate overall buckling, rather than individual member buckling in each panel, a length of 5.13 in. was selected for each panel. Thus, the entire frame has 23 panels. The panel length was arrived from the limitations of the equipment in the laboratory and desired slenderness ratios. The modulus of elasticity of the verticals and horizontals is 29000 ksi. The yield strength of the vertical and horizontal members is 60 ksi and of the diagonals is 57 ksi.

The frame was laterally braced at the sixth points so that buckling was confined to the plane containing the stays. Two thick end members, 1 in. wide and 2 in. deep, were used as top and bottom members. This would allow for a uniform distribution of the axial load to both of the verticals. Two thick plates (6x6 in.) were used to connect the frame to the knife edges. The bottom member of the frame was connected to the thick steel plate by just two screws. The top connection was made in a similar manner. Two ring beam load cells were used in each panel, one in each stay, for a total of 46 load cells. These load cells were used primarily to set the initial pretension and to measure the tension in the stays as the load was applied. These load cells were 1.5 in. outside diameter and 1 in. inside diameter and were made up of aluminum. An equivalent modulus of elasticity
was determined theoretically to be 18680 ksi. An automatic strain indicator was used to monitor the stay tensions, and the axial and horizontal loads. Dial gauges were used to measure the lateral deflection at several points along the length of the frame. Since the legs of the frame were so close to each other, the tower can be analyzed as a built-up column [15]. The slenderness ratio of the whole tower was kept at a larger value than that of the each panel to ensure overall buckling. The slenderness ratio of the tower is approximately 79 and that of the verticals between the crossarms is less than 70. This situation enabled the entire frame to buckle similar to an Euler column. The diagonals were inclined at 60 degrees with horizontal.

The main aim of the experimental program was to determine the behaviour of the prestressed lattice frame under different types of loading, such as axial load and a combination of axial and horizontal loads. To determine the effects of prestressing it was decided to do two tests in each case, one nonprestressed and the other prestressed. This was achieved by having roughly 50 lbs. tension in the diagonals for the nonprestressed case and 300 lbs. for the prestressed case. Five sets of loading cases were investigated to determine an interaction diagram between the axial load and horizontal load. The first case was under the action of pure axial load. To observe the behaviour under lateral load, a horizontal load was applied under a
constant axial load. The specific axial loads used in these four tests are 2, 5, 8, and 10 kips. Thus the behaviour of the slender lattice tower under nonprestressed and prestressed conditions was determined. In all tests, the initial out-of-straightnesses were measured and were used to determine the theoretical response of the prestressed lattice structures to the applied load.

4.5 Electro-thermal Prestressing

The laboratory methods of introducing the prestress in the diagonals are not suitable for large scale lattice towers or for a mass production environment. Therefore, electrical heat, in the form of resistance heating, (not to be confused with direct resistance heating) was used as the means of introducing the prestress in the diagonals. In this method a heating element was placed alongside the diagonal and the heat was controlled manually. This part of the work was performed by Seguin [12] as a Fourth Year Project for the Department of the Electrical Engineering, at the University of Windsor. (The author of this thesis was involved in helping out with the theoretical and experimental part of this work with Mr. Seguin.) The frame used in the experimental program is shown in Figure 4.6. It was constructed of 2 1/2 x 2 1/2 x 1/4 in. angles welded at the corners, representing a typical panel in a lattice frame. The outer dimensions of the square frame are 24
in. The diagonals were 1/4 in. diameter steel rods. The length of the diagonal between the centers of the weld was taken as 30.8 in. (Figure 4.7). Resistance heating was used as a method of prestressing the diagonals. A heating element was placed beside the diagonal. The heat was controlled manually. The diagonal was welded at one end. Then the diagonal was heated until the desired elongation occurred. The far end was then welded. After the weld and the diagonal had cooled, strain gauges were attached to the rod. The load in the diagonal was determined by applying a strain gauge to the diagonal and then cutting it. A temperature of about 200 degrees Fahrenheit was applied in order to elongate the rod by 0.002 in.
Chapter V

DISCUSSION OF RESULTS

This study, as was mentioned before, was divided into three parts. The first part dealt with the failure criteria for stayed columns. The results from this study have affected the second part which was to determine the behaviour of prestressed latticed frames with intersecting diagonals, with diagonals offset on the crossarms, and with diagonals offset on the verticals. These three types of frames were studied under the action of axial load alone. Some of the results of the second part have led to the third and final part of this investigation which was to understand the behaviour of a prestressed slender latticed tower. This structure was subjected to axial load and a combination of axial and horizontal loads. The main aim of the experiments was to determine the effect of prestressing the diagonals on the load-carrying capacity of lattice structures and to understand the behaviour of these structures in general. This has obviously led to the testing of one non prestressed and one prestressed frame for each type of frame studied and for each loading condition except for the stayed column in which case the effect of different pretensions was investigated. All the experimental results were compared with theoretical results. A detailed discussion follows.
5.1 Stayed Columns

In this section results from experimental and theoretical studies will be compared for several stayed columns. As pointed out before, this part of the investigation was done to check the buckling failure mode as well as to determine the failure criteria. Three possible failure modes are:

(a) the stress in the central core reaches the yield stress; (b) the buckling of central core; and (c) the yielding of the stays. These three failure modes have been incorporated into the computer program. With a real stayed column imperfections are unavoidable. An initial out-of-straightness is always present. This initial out-of-straightness is amplified in the presence of an axial load. Thus it was realized that for a real stayed column, any attempt to predict the maximum load carrying capacity must account for the initial out-of-straightness and the increase in lateral deflection caused by the axial load. This lateral deflection affects the magnitude of the tension in the stays, which, in turn, influences the maximum load carrying capacity.

The theoretical load-stay tension relationship was obtained by the mixed incremental-iterative approach using the geometric stiffness method explained previously. During the experimental portion of this study stay tensions were monitored so that the same relationship could be determined experimentally.
The experimental and theoretical load versus stay tension curves for all specimens, with an initial out-of-straightness of L/1000, are shown in Figures 5.1 to 5.5. It will be noted that there is a good agreement between the experimental and theoretical results. Some of the differences between the experimental and theoretical results can be attributed to the problems associated with obtaining an accurate determination of the initial out-of-straightness. This quantity is small and difficult to measure with any great degree of accuracy, but has a very significant effect on the results. As will be pointed out later, any small changes in the initial out-of-straightness have a much greater effect at a high than at a low pretension.

It has been mentioned previously that in prior research on stayed columns [7], the experimental buckling load was defined as the load at which the tension in the stays on the concave side approached zero. It is evident from Figures 5.1 to 5.5 that the instant the tension in the stays on the concave side goes to zero does not, at least in some cases, correspond to the maximum load carrying capacity of the stayed column. This is clearly evident when the initial pretension is less than the optimum pretension. Hafez et al. [7] have defined the optimum pretension for an ideal stayed column as the "initial pretension in the stays that disappears completely just after the load in the column..."
reaches its maximum buckling load. Table 3 summarizes the experimental failure loads as determined by
(a) the old criterion that stated that the experimental buckling load is the load at which the tension in the stays on the concave side goes to zero; and
(b) the actual maximum load carrying capacity as determined experimentally.

A typical load-deflection curve is shown in Figure 5.6. The results presented are for the stayed column with an initial out-of-straightness of $L/1000$ and an initial pretension of 150 lbs. The deflection was measured at mid-height of the stayed column. The stayed column at failure can be seen in Figure 4.1. It can be observed that the top diagonal on the left side had failed and thus the column reached its failure load. There is reasonably good agreement between the theoretical and experimental results for all stayed columns except for the specimen with an initial pretension of 300 lbs. Figure 5.7 indicates that the theoretical results predict a mode that involves a type of triple curvature (Mode 1) while the stayed column actually buckled in a double curvature (Mode 2) when tested in the laboratory. The failure modes are shown in Figure 5.8. The reason for this discrepancy can be determined by examining the same figure. Figure 5.9 shows the relationship between the critical load and the ratio of half column height to crossarm length ($I/L_{oa}$, where $I = \text{half central core length}$;
\( L_{ca} \) = crossarm length, the distance from the centroid of the compression core to the intersection of the crossarm and stays. The critical loads for the two failure modes, labelled Modes 1 and 2, are shown. At small \( l/L_{ca} \) ratios, the translational restraint provided by the stays is large enough to force the central core to buckle by Mode 2, a mode that involves double curvature. At large \( l/L_{ca} \) ratios, the rotational restraint is much greater than the translational restraint and Mode 1 buckling governs. For the stayed column tested the \( l/L_{ca} \) ratio is 5.3. As can be seen in Figure 5.8, this ratio is very close to the point at which both modes of buckling are equally possible. It is interesting to note in Figure 5.7 that even though the theoretical and experimental buckled shapes did not correspond, the theoretical and experimental failure loads are virtually the same. Figure 5.8 indicates that such a result is expected.

All the experimental load-deflection curves for stayed columns with an initial out-of-straightness of \( L/1000 \) are shown in Figure 5.9. Another failure mode which could be considered here is excessive deflection. If it is desirable to control lateral deflections to a certain value, the advantage of an initial pretension, and particularly larger values of initial pretension, is obvious from this figure.

All of the failure loads, theoretical predictions and experimental results have been summarized in Figure 5.10. For the seven stayed columns tested in association with this
research, the maximum load was reached when the column became unstable and, in turn, caused the stays to yield. The experimental results indicate that the failure loads are approximately the same with initial pretensions that vary from zero to about 150 lbs. As the initial pretensions are increased beyond 150 lbs., the failure loads increase quite rapidly with increased pretensions. The reason for this can be understood when the tension in the stays is examined. At the lower pretensions, less than about 150 lbs., the tension in the stays on the concave side goes to zero before the maximum load is reached. Thus only the tensions in the stays on the convex side are effective in resisting lateral deflections. Another reason for the rapid increase in failure loads is that the experimental failure mode has changed from Mode 1 to Mode 2. As mentioned previously, the theoretical predictions indicated that the failure mode should have been Mode 1.

When the initial pretensions are greater than 200 lbs, all stays are effective in resisting lateral deflections until the maximum load is obtained. This can be verified by examining the load-stay tension curves in Figures 5.4 and 5.5.

With an initial pretension of 150 lbs, the initial out-of-straightness was varied by changing the lateral load. Figure 4.1 shows the weights hung from the mid-height of the column to obtain this lateral load.
out-of-straightness of L/1000, the failure load was 5.8 kips. Reducing the initial out-of-straightness to L/3000 the failure load was increased to 7.2 kips. Increasing the initial out-of-straightness to L/500 resulted in a failure load of 5.6 kips. This clearly indicates the significant effect that a large change in the initial out-of-straightness has on the failure load. The failure mode for the two specimens with an initial out-of-straightness of L/3000 and L/500 was Mode 1. These results are also shown in Figure 5.10.

The results obtained at zero pretension warrant further comment. With zero pretension the stayed column behaves very much like an ordinary column (no stays). At the Euler load for the central core, 1.27 kips, a significant lateral deflection, as shown in Figure 5.9, occurs. This lateral deflection does not indicate that the maximum load has been reached, but rather results in tension being created in the stays on the convex side. The tension in the stays results in translational restraint which increases the load carrying capacity of the stayed column. The experimental failure load obtained was actually 5.2 kips. This failure load is about four times the Euler load, the load previously taken as the maximum load when the criteria used was zero tension in the stays (Figure 5.7).

Also shown in Figure 5.10 is the theoretical results for failure loads for all specimens with an initial
out-of-straightness of L/1000. These failure loads were obtained from the load-deflection curves which were determined by the finite element mixed incremental-iterative procedure mentioned previously. It will be noted that there is good agreement between the experiments and theory up to the optimum pretension. Beyond this point there is a considerable discrepancy between the experimental and theoretical results, which can, at least in part, be attributed to the values of initial out-of-straightness used in the theoretical determination of the failure load. At the higher pretensions the theoretical failure loads are very sensitive to the magnitude of the initial out-of-straightness. If the initial out-of-straightness measured for the stayed column with an initial pretension of 300 lbs. is reduced by 20%, the theoretical failure load increases from 6.8 to 7.5 kips, while a similar change when the initial pretension is 150 lbs leaves the theoretical failure load virtually unchanged.

Figure 5.10 shows the relationship between the failure load and the initial pretension previously formulated by Hafez et al. [7]. This relationship is based on expressions derived from a geometrical study of an ideal single-crossarm stayed column. The effect of the lateral displacements on the tension in the stays was not considered. In the zones labelled as 1 and 2 in Figure 5.10, the failure load, once again, was taken as the load at which the tension in the
stays goes to zero. In Zone 3, however, the tension force in the stays at the critical load has a nonzero value. Hafez et al. [7], obtained good agreement between this theoretical relationship and experimental results because of the criterion used to define the experimental buckling load. If the experiments had been continued to determine the maximum load carrying capacity, the agreement would not have been as good.

5.2 Single-bay Three-storey Lattice Frame

In this part of the investigation, three frames were built with different diagonal configurations, i.e., intersecting diagonals, offset diagonals on the crossarms, and offset diagonals on verticals. These were tested both in a nonprestressed (near zero pretension in the diagonals) and in a prestressed condition (an optimum pretension in all the diagonals) until failure occurred. Test results are described in detail below.

5.2.1 Intersecting Diagonals

An ideal frame is a theoretical frame which does not contain any imperfections. Figure 5.11 shows a typical buckling mode for an ideal prestressed lattice frame with intersecting diagonals. Typical load-deflection curves are shown in Figures 5.12 and 5.13 for the nonprestressed and prestressed cases, respectively. The initial deflection at zero load is the out-of-straightness measured in the laboratory. The zero pretension case has reasonable
agreement between theoretical and experimental results whereas the prestressed frame showed excellent agreement till failure. A battened frame, without any diagonals, was tested prior to this. Figure 5.14 shows the load-deflection relationship. This frame failed at a load of 1.6 kips whereas the lattice frame with the prestressed and nonprestressed diagonals failed at 4.2 kips. Thus prestressed diagonals which constituted 1.7% of the weight of the whole structure, increased the axial load by a factor of 2.6. The buckled mode for the battened frame was shown in Figure 3.4 (a), whereas the corresponding one for the lattice frame was shown in Figure 3.4 (b). In the frame with intersecting diagonals, the shear force is resisted by diagonals. Therefore, the verticals in lattice frames can be considered to have both translational and rotational restraints at each panel point, the intersection between horizontal, vertical and diagonal members. Depending on the rigidity of the diagonals, the panel points act in between a rigid support and a flexible support. When the frame is subjected to axial load only, for the intersecting diagonals case, the frame buckled between the panel points. Figure 5.15 shows the experimental buckling mode in which buckling of the vertical member can be seen. Figures 5.16 to 5.18 show the relationship between the load and stay tension of one of the panels in the lattice frame. Figure 5.16 shows the stay tension for a pretension of 8 lbs. in the
diagonals. It must be noted that stay 'B' went into compression shortly after the application of the axial load and became slack. This has caused the frame to deflect more and as a result the remaining diagonal went into tension. This can be observed from the change of slope in the experimental curve in Figure 5.16 at an axial load of 0.6 kips. The structure continues to carry axial load until the maximum load of 4.2 kips is reached. Figures 5.17 and 5.18 show the load versus stay tension for a pretension of 70 lbs. in the diagonals. There is a gradual loss of the pretension in stay 'A' (Figure 5.18) until it lost all of its pretension at which point the structure reaches the maximum load. The stay 'B' (Figure 5.17), however, loses some of its pretension due to the application of the axial load, but the tension increases when stay 'A' becomes slack. The failure load has been defined as the load at which there is a sudden increase in deflection without much of an increase in load. A comparison of Figures 5.16 and 5.17 indicates no difference in the failure loads of the nonprestressed and prestressed frames. This can be explained by examining the buckling modes of these frames shown in Figures 5.19 and 5.20. Experimental and theoretical values of the buckling modes were obtained by plotting the deflections along the length of the frame. Deflections were plotted for an axial load of 3.8 kips which is 93% of the failure load. A noticeable feature of both
these buckling modes is their similarity in shape. The frame buckled in each panel length as expected because of the slenderness ratio of the vertical member which is close to 250. Experimental and theoretical deflections are in close agreement for the case of optimum pretension in the diagonals. For the zero tension case the same quantities are in reasonable agreement. A close look at the imperfection data showed that the panel points in both types of frames did not move. This indicates that the horizontals and diagonals provided equal lateral restraint for both frames. Therefore, these intersection points act as if they are rigid supports. In other words, the lateral support given by one diagonal in nonprestressed case is just enough for the frame to reach a maximum failure load in this mode. This may be the reason why there is no increase in failure load due to prestressing for this type of loading and frame. The theoretical buckling mode for an ideal frame is shown in Figure 5.11. The experimental result showed a similar mode except that it is not symmetric because of the imperfections. Therefore, it seems that, in this case, it is necessary for the panel points to move if there is to be an advantage in prestressing. To check the validity of the theoretical and experimental results, load-deflection curves were compared for pretensions of zero and 70 lbs. for a point in the top panel in Figures 5.21 and 5.22.
It can also be observed from Figures 5.19 and 5.20 that the middle panel length was the critical one which initiated failure. Specifically, on the right-hand side the vertical in the middle panel deflected considerably, indicating there could have been a slight eccentricity in the load application causing more compression in the right side verticals than on the left side. This could have been caused by the imperfections. Another reason for this can be that the use of thick horizontals at the ends might have reduced the effective length of the verticals in top and bottom panels. This might have forced the middle member to buckle.

A careful examination of load-stay tension curves for both the nonprestressed and prestressed cases reveals the behaviour of these structures. Figures 5.23 and 5.24 give the load-stay tension relationships for near zero pretension case. The theoretical curve change its slope in the initial stages of loading. This occurred because of the low magnitude of tensions in the stays which cannot be measured accurately by ring beam load cells. Figures 5.25 to 5.28 shows the load-stay tension relationships for the optimum pretension case. Overall there was good agreement between experimental and theoretical stay tensions. The only difference, which is near the failure load, is explained below. Figure 5.26 shows that theoretically the diagonal 'A' loses all of its pretension at an axial load of 4.0 kips whereas Figure 5.25 clearly shows how the diagonal 'B' in
the panel increases its tension rapidly at this point. This is because of the sudden drop in stiffness caused by slackening of one of the diagonals. Another cause of discrepancy between theoretical and experimental results is the magnitude of imperfections. In the laboratory, the smallest resolution of 0.01 in. can be measured through the transit, which might have caused some error.

5.2.2 Offset Frames

These frames are similar to the intersecting frames described above except that the diagonals were attached two in. from panel points (Figure 1.2). This offset distance was determined by a preliminary eigenvalue analysis as described in Sec. 3.6.1.2 (Figure 3.11). Two types of offset frames were considered in this study, one with offset on crossarms, and the other with the offset on verticals. Two tests, one with very little pretension (near zero) in the diagonals and the other with optimum pretension were conducted with each type of offset frame.

5.2.2.1 Offset on Crossarms

Load-deflections and load-stay tensions were plotted to compare the theoretical and experimental results. Ideal offset frame indicated a maximum load carrying capacity of 5.5 kips can be reached with the buckling mode shown (Figure 5.29(a)). The mode is similar to Mode E in Figure 3.4(b). This maximum can be obtained only when all the diagonals
were effective at the buckling load. Figures 5.30 and 5.31 show the load-deflection relationship for the pretensions of zero and 92 lbs., respectively. A maximum axial load of 4.0 kips was obtained experimentally in the case of zero pretension and 4.9 kips in the case of optimum pretension. Only one of the diagonals was effective in each panel in the frame with zero pretension and therefore the lower load carrying capacity. Good agreement can be seen between theoretical and experimental results. Here again, the reason for the reduction in buckling load from the ideal case is the ineffectiveness of some diagonals at loads close to the buckling load. This indicates that the theoretically obtained optimum pretension is slightly less than the actual optimum pretension. However, an increase in the maximum load carrying capacity is obtained when the pretension is changed from zero to the optimum. In Ref.[6] Ellis reported the engineering concept of offsetting the diagonals. In Figure 5.32(a) Euler's equation gives the buckling load, $P_{cr}$, for the slender column where $E = \text{Young's modulus}$, $I = \text{the relevant second moment of area}$ and $L$ is the half column length. In Figure 5.32 (b) and (c) a rigid bar is firmly attached to the column at its middle length and the end of the rigid bar is connected with two springs which have a spring constant $k$. When the column rotates through an angle $\theta$ a resisting moment is developed equal to the expressions given in the figures. The greater the length $D$ of the rigid
bar, the greater the restraining moment. It is to be noted 
that the spring constant $k$ of Figure 5.32 (b) and (c) is 
equivalent to the vertical stiffness of the diagonals. Thus 
by offsetting the diagonals a beneficial restraining moment 
is introduced to the compression chord of a lattice frame. 
Figures 5.33 and 5.34 show the typical load-stay tension 
relationships for both zero and optimum pretension 
respectively. The discrepancies between the theoretical 
predictions and experimental results can be attributed to 
the stage at which one of the stays becomes ineffective. 
Since it is a function of deflection, a complex interaction 
results. Figures 5.35 to 5.37 show the load-stay tension 
relationship for several of the stays at zero pretension 
and Figure 5.38 shows the deflection pattern for the zero 
pretension case. From this, it follows that the crossarms 
might have bent in a single curvature as opposed to the 
double curvature in Mode E of Figure 3.4 (b). The buckling 
mode is governed by imperfections of the frame which 
resulted in a buckling mode which is some combination of 
that shown in Figures 5.11 and 5.29. Figures 5.39 to 5.44 
show the relationship between the load and stay tension for 
several stays at optimum pretension. From these figures one 
can notice that the stay tensions, in general, in both 
prestressed and nonprestressed cases increased in the middle 
panel whereas they decreased in both the top and bottom 
panels. The differences between the theory and experiment,
are more pronounced near maximum loads. Figure 5.45 shows the deflection pattern close to the maximum load. The sudden change of slope in Figures 5.42 and 5.43, close to the maximum load, is caused by the slackening of one of the stays in each panel. Figure 5.46 shows the experimental frame at the maximum load. It can be observed from the figure that the buckling of the right leg is much more visible than that of the left leg. This might be caused by the imperfections of the frame or any eccentricity caused by the axial load. Ideal frame analysis for an offset on crossarms show a 30% increase in strength over intersecting diagonals whereas experimentally an 18% increase was obtained. However, this is not a fair comparison because two different frames were tested and both contain different sets of imperfections. Still, one can see the improvement in strength due to offsetting for the reasons mentioned above. A measurement of stay tensions up to 10 lbs may not be accurate with the ring load cells used. As mentioned before, the increased rotational restraint in the offsetting frame is caused by the bending of the crossarms which in turn change the stay tensions. Thus, the frame with offset diagonals is different in behaviour than the frame with intersecting diagonals in which the diagonals only provide the translational restraint. The experimental buckling mode obtained was similar to that of intersecting whereas an ideal frame shows a different one. This might have been
caused by ineffectiveness of one diagonal in both zero and optimum pretension cases.

5.2.2.2 Offset on Verticals

The main objective of offsetting the diagonals on verticals is to increase the rotational restraint at the panel points. This is possible when the column bends about the panel points (Figure 5.47). At the point of attachment of the diagonals with the verticals, the horizontal stiffness of diagonals behave in a manner similar to a spring. If the reversal of curvature takes place around the panel point, there will be forces acting in opposite direction which will cause a net restraining moment. This will benefit the frame to increase its load carrying capacity when compared to the intersecting diagonal frame.

The magnitude of restraining moment depends on the contraflexure points which depend on the amount of translational restraint available from the diagonals at the attached points. Figure 5.48 shows the experimental frame when it reached the maximum load. It can be observed that a mode C buckling took place. This will be discussed below.

Two tests were conducted on this frame, one with near zero pretension and the other with an optimum pretension. Figures 5.49 to 5.51 show the load-stay tension relationship for an offset frame with near zero pretension in the diagonals. These show that there is a good agreement between the theoretical and experimental results. Ideal
frame analysis for an offset on verticals show a 21% increase in strength over the intersecting diagonals for fully prestressed case (buckling mode can be seen in Fig. 5.29 (b)), whereas experimentally for zero pretension a maximum of 4.9 kips was reached which is 17% more than a similar intersecting frame. Therefore, offsetting the diagonals on verticals increases the load carrying capacity when compared to intersecting diagonals. It is interesting to note that the diagonals in the upper and lower panels are effective whereas the stays in the middle panel are ineffective. This may be caused by the pattern of the initial imperfection which has forced the frame to bend accordingly. Similar behaviour was observed for the frame with prestressing at optimum level. Figures 5.54 and 5.55 show the typical behaviour of diagonals in a panel. A maximum load of 5.24 kips was reached for the case with optimum pretension. The ideal buckling load for this frame was predicted to be 5.1 kips whereas experimentally a slightly higher value was obtained. This could be due to a restraint available from the knife edges. Figures 5.56 to 5.59 show the load-deflection behaviour of the nonprestressed and prestressed frames. As loads near failure deflections increase rapidly which results in change in stay tensions. This will cause a change in the slope of the load-deflection curves near the maximum load in Figures 5.57 and 5.59. The theoretical and experimental results indicate
the final experimental buckling mode is different to that predicted theoretically. Figures 5.60 and 5.61 show the deflected shapes of both prestressed and nonprestressed frames at a load of 4.7 kips. The experimental buckling mode is similar to Mode C in Figure 3.4 whereas the theoretical one is similar to that of Mode E. Imperfections and the influence of the neighbouring modes are the main reason for this discrepancy. However, the failure loads in Figures 5.56 to 5.59 indicate a good agreement between the theoretical and experimental results. There is a good correlation of the deflection behaviour up to the buckling load. This is also indicated in Figures 5.49 to 5.55.

Overall, from these tests, an important conclusion can be drawn. Offsetting the diagonals on the verticals results in a rotational restraint that is not present in the case of intersecting diagonals frame. Also, there is a significant increase of strength due to prestressing.

5.3 Relatively Slender Lattice Tower

After a careful analysis of the above results on the three-panel single-bay frames subjected to axial loads, it was discovered that there is not much benefit due to prestressing in the case of intersecting diagonals. Although there may be some benefit under lateral loading, this was not investigated until the behaviour under the axial load is completely understood. From the tests
conducted so far it was observed that during the loading process of the lattice frame, the diagonals essentially change their pretension by losing their initial pretension and providing the needed lateral restraint. The slenderness ratios of the verticals enabled the frame to buckle in each panel with alternating curvatures. The crossarms and diagonals provided the needed lateral support. It was of interest, then, to investigate how a relatively slender lattice tower will behave if the diagonals are prestressed. Here, the slender structure, if allowed to buckle over its entire length would be similar to a pin-ended column. This means that the panel points will displace laterally and, in turn, diagonals will be subjected to compressive and tensile forces. The diagonal with compressive force will buckle at some stage during the loading in which case the lateral restraint will be reduced by one-half. For this reason, prestressing is thought to be beneficial in a tower-like structure.

For simplicity, a two-dimensional tower was studied. The bracing system consists of diagonals and battens. The purpose of the bracing is to reduce the effective length of the verticals and to provide a mechanism for the transfer of shear. This tower is a two-dimensional framework and may be analysed as a built-up column because the legs are reasonably close to one another and because the vertical length of a panel in the braced planes is small compared
with the tower height [15]. The critical buckling load of a built-up column is always lower than that of a solid column, for the same cross-sectional area and the same slenderness ratio. This is due primarily to the fact that the effect of shear on lateral deflection is much greater for the built-up column than for a solid column. The tower was designed in such a way that no member of the structure yields locally before the critical load for the whole column is reached.

The slenderness ratios of the individual lengths of the chords were determined such that they are smaller than the overall slenderness ratio of the tower. This enabled the tower to buckle in a manner similar to an Euler column. The effective length of the tower which is from center-to-center of top and bottom knife edges is 124 in. The distance between the verticals is 3 in. and the panel length is 5.1 in. The diagonals are of 7/8 in. diameter solid steel rods. Preliminary theoretical studies showed a definite increase of 18% in strength for prestressed over nonprestressed.

Physical limitations in the laboratory was the deciding factor in determining the height of the model.

Several tests were performed on the tower. The main purpose was to determine the behaviour of the tower under different loads. Therefore, an interaction type of diagram was thought to serve the purpose of identifying the behaviour of the nonprestressed tower as compared to that of prestressed tower. The tests consist of (a) determining the
critical axial load, and (b) determining the maximum horizontal load under a constant axial load. The second part was conducted at axial loads of 2, 5, 8, and 10 kips. In each case two tests were performed, one with a pretension of approximately 50 lbs., and the other with 300 lbs. pretension. The higher pretension should enable both diagonals in each panel to remain effective till failure. The lower pretension of 50 lbs. is similar to a nonprestressed case. In the following the test results will be discussed.

5.3.1 Pure Axial Load

For the frame used in this study, a theoretical buckling load was obtained for a battened frame and it was determined to be 9.3 kips. The battened frame does not contain diagonal members. In a fully prestressed tower both diagonals in each panel will be effective till the failure load is reached whereas in a nonprestressed case one of the diagonals in a panel will become slack during the loading process. Therefore, analytical work was performed on a tower with one diagonal as well as with two diagonals in a panel. An approximate theoretical buckling load of 15.5 kips was obtained for a lattice frame with one diagonal in each panel and a 18.3 kips for two diagonals (using Eq. 3.23). The theoretical formula used in this calculation involves the tower being considered as an integral unit. The above load considers, as well, the reduction due to shear. An exact
eigenvalue analysis was conducted as a plane frame which gave a buckling load of 17.3 kips for one diagonal in each panel and 19.5 kips for two diagonals. It is likely that only one diagonal will be effective in the case of small pretensions. However, when two diagonals were considered in each panel, which is likely for a fully prestressed case, a 13% increase in strength was obtained. These buckling loads are theoretical and valid only for ideal frames. An exact nonlinear analysis based on Newton-Raphson method was used to obtain the theoretical behaviour of the actual frame. Two cases were tested for comparison. One with about 50 lbs pretension and another with about 300 lbs pretension were conducted and will be used for comparison. Figure 5.62 shows the experimental setup of the lattice frame. Figures 5.63 to 5.66 show experimental and theoretical load-deflection curves. The experiments were stopped very close to the failure load, which is presumed to have occurred when the central deflection reached was 0.7 in. (L/175). The tower at this stage was in elastic range. It can be pointed out that a maximum load of 12.0 kips and a 14.3 kips were obtained for a nonprestressed and a fully prestressed case, respectively. From Figure 5.66 it can be seen that at a deflection of 0.35 in. (L/350), an increase of 40% in strength was obtained experimentally due to prestressing. However, theoretical results show a 29% increase for the same case. The main reason for this discrepancy is the
imperfections. Although the finite element models are supposed to give an upper bound, in this particular case theoretical results are lower than experimental. It is likely that the magnitudes of imperfections were taken slightly larger than they should have been. This could be possible because of the dimensions of the tower, the equipment used, and the alignment of the tower. The change in the magnitude of imperfections was shown in Figure 5.66 for the case of 300 lbs pretension. The imperfections were uniformly reduced or increased to 10% and the effect was determined theoretically. In a pretensioned structure, the effect of imperfection can be considerable because they affect the stiffnesses of the diagonal members which provide the main lateral load resisting system. Therefore, this cumulative effect may be the main reason for the discrepancy between the theoretical and experimental results. For comparison, a few panels were selected and Figure 5.67 shows the location of the stays used in the comparison of theoretical and experimental results. Figures 5.68 to 5.71 show the load-stay tension relationship for both 50 lbs and 300 lbs pretension in the diagonals. In Figure 5.68 the diagonals in the middle panel show that they lose their initial pretension due to axial load application. This is expected because the slope of the elastic curve is nearly zero at the middle of the tower and therefore diagonals do not elongate. In Figure 5.69, the nonprestressed and
prestressed cases are similar. However, in the fully prestressed case, theoretical result diverges close to the failure load. This is because one of diagonals has become slack at about 11 kips whereas experimentally both the diagonals are effective. This discrepancy may be due to the inaccurate measurements of the imperfections. From Figure 5.70 it can be observed that a considerable discrepancy occurs between the theory and experiment. As indicated above, the stay tension depends on the deflection at that point which depends upon the stiffness of the structure. Figure 5.71 indicates a reasonable agreement between theory and experiment in the case of fully pretensioned case. Therefore, one can conclude that prestressing the diagonals in this type of structures is beneficial even if it is subjected only to an axial load.

5.3.2 Combination of Horizontal and Axial load

The final objective of this research program was to determine the effect of horizontal load on prestressed lattice frames. Four test cases were performed on the tower described above. In each case, a certain axial load was applied and held constant under an increasing horizontal force at the mid-height of the tower. The experiment was stopped when the central deflection reached a maximum of 0.7 in. (L/175). This deflection limit is the same as the one in the pure axial load case. The four loading cases are with a constant axial force which is equal to 2, 5, 8, and 10
kips. Each loading case was again tested with two cases of pretension, one near 50 lbs and the other with 300 lbs. In the analysis the effect of pretension on the structure before the application of any external loading is neglected. One of the reasons for this is the changes in geometry are insignificant because of the small values of the prestressing force. The other reason is the difficulty of measuring these changes experimentally. Theoretically it was computed to be less than 0.5%.

Figure 5.72 shows the experimental setup for the application of the horizontal load. This load was applied at panel points near the mid-height. Figures 5.73 and 5.74 show a typical load-deflection relationship for an axial load of 2 kips and a pretension of 50 lbs and 300 lbs, respectively. The theoretical and experimental results show excellent agreement. One can conclude from these figures, the structure essentially behaves as a linear one. As long as the axial load is constant, and it is much lower than the buckling load, this linear relationship holds as can be seen in the following. Figures 5.75 and 5.76 show the deflection at 36.5 in. from bottom and they confirm the above conclusions. Figures 5.77 to 5.80 show the load-stay tension relationship for two sets of panels. The change in slope in Figures 5.77 and 5.78 follows from the fact that one of the diagonals became slack around a load of one-half the maximum load, which changes the stiffness of the
structure. The change in the load-deflection is not severe because of this. For ease of comparison these same quantities will be presented in each of these cases. Note that for the case of 300 lbs. pretension in Figures 5.78 and 5.80 that both the diagonals are effective till the maximum load is reached, which is the load at a central deflection of 0.7 in., whereas in Figures 5.77 and 5.79 only one diagonal is effective. This is the main difference for a low pretension case and a high pretension case. The stay tensions at zero horizontal load are obtained after the application of axial load of 2 kips. It must be pointed out that attempts were made to obtain a pretension of 300 to 400 lbs before the experiment started. However, it was very difficult to obtain this pretension. Therefore, analysis was conducted with the exact tensions as measured from the experiment. Comparing Figures 5.73 and 5.74 there is a slight benefit due to prestressing, that is at a deflection of 0.6 in. experimentally the total horizontal load carried by fully prestressed tower is 8% more than nonprestressed.

Similar results were obtained for an axial load of 5 kips. Figures 5.81 and 5.82 show typical load-deflection relationship for 50 lbs. and 300 lbs. respectively. At 0.6 in. deflection, there is an increase in its experimental load carrying capacity of 20% for the fully prestressed tower as compared to the nonprestressed tower. Figures 5.83 and 5.84 show the typical load-stay tension relationship. As
pointed out before, in Figure 5.83 the change of slope occurs at the point where one of the diagonals become slack.

Figures 5.85 and 5.86 show the load-deflection for an axial load of 8 kips. It can be observed that the initial offset between the theoretical value and the experimental one at zero horizontal load is caused by the imperfections. This can be seen by the parallel relationship between these two curves. There is a 37% increase experimentally due to prestressing in the load carrying capacity at a deflection of 0.6 in. Figures 5.87 and 5.88 show the load-stay tension relationship for an axial load of 8 kips. It can be observed that from Figure 5.87 there is essentially only one effective stay throughout the loading process. The initial 50 lbs. of pretension was lost due to application of axial load of 8 kips.

Figures 5.89 and 5.90 show the load-deflection relationship for an axial load of 10 kips. The initial offset on these figures for zero horizontal load is obtained by the application of the axial load of 10 kips. The offset between theory and experiment is due to the imperfections. There is a 58% increase in strength at a deflection of 0.6 in. experimentally for the prestressed tower over the nonprestressed. Figure 5.91 and Figure 5.92 show the load-stay tension for an axial load of 10 kips. They show a similar behaviour as described above.
Figure 5.93 summarizes the results in a manner similar to an interaction diagram. The theoretical points show a linear relationship whereas the experimental results were scattered around this. The horizontal load carrying capacity at zero axial load was obtained from a general purpose program called STRUDL. Although it is not fair to make this comparison, because of non-uniformity in pretensions for each case, this diagram gives a clear picture concerning the effect of prestressing for a failure criteria of maximum lateral deflection equal to L/200. There is a 28% increase in axial load due to prestressing for a horizontal load of 150 lbs. There is a 33% increase in horizontal load due to prestressing for a constant axial load of 7 kips. As the horizontal load increases, the axial load capacity increases because of prestressing. As the axial load increases, the horizontal load capacity increases considerably due to prestressing. This is because near the buckling load deflections are much higher in nonprestressed frames when compared to prestressed frames. Figure 5.94 compares various load-deflection curves obtained theoretically. Notice the difference between the slopes of load-deflection curves between a fully prestressed and partially prestressed one. The change is significant at higher loads because of lower stiffness for the partially prestressed structure.

In summary, there is good improvement of the strength and stiffness due to prestressing in all the models tested especially when the lateral load is applied.
Chapter VI.

CONCLUSIONS AND RECOMMENDATIONS

In this research program the behaviour of prestressed lattice structures was studied. A special emphasis was placed on determining the effect on the load carrying capacity of prestressing the diagonals. The failure criterion which has been traditionally accepted has been modified. Structures with different modes of buckling were tested to check if prestressing changes the load carrying capacity. Different load conditions were also applied to determine the effects of prestressing.

6.1 Conclusions

In summary, these final conclusions were drawn from this research:

(a) In the past the experimental buckling load of a stayed column has been defined, somewhat arbitrarily, as the load at which the tension in the stays on the concave side goes to zero. It has been shown that many stayed columns, especially those with lower values of pretensions in the stays, possess a strength much greater than this arbitrary definition would suggest. The strength of the stayed columns, of the type
tested, is controlled by yielding of the stays which occurs once the column becomes unstable [18].

(b) Three configurations of frames were tested to see if prestressing the diagonals would improve the strength. These are intersecting diagonals, offsetting diagonals on crossarms, and offsetting diagonals on verticals. Prestressing in offset frames showed a good increase in strength and stiffness especially for offsetting on the crossarms when an axial load is applied.

(c) A relatively slender tower was tested, which has a different mode of buckling than the previously tested frames, and it was found that pretensioning the diagonals is beneficial. When pure axial load was applied, for a 0.35 in. (1/350) experimental deflection at the center, there is a 40% increase in strength. When horizontal load was applied simultaneously with axial load, which is the practical case, there are significant strength increases due to prestressing.

(d) Deflections reduced significantly in the above tower suggesting an improved structure will result in due to an optimum prestress in the diagonals.
6.2 Recommendations

(a) Optimization techniques must be used to obtain a maximum strength increase due to prestressing, taking the above results into consideration.

(b) Further research may be conducted to obtain simplified design methods which will be valid for all types of prestressed lattice structures.
Appendix A

FIGURES
Fig. 1.1 Stayed Column
Fig. 1.2 Prestressed Latticed Frames

(a) Intersecting diagonals

(b) Offset along the crossarms

(c) Offset along the verticals
Fig. 1.3 Prestressed Lattice Tower
Fig. 2.1 Comparison of Theoretical and Experimental Buckling Loads (from ref. 10).
Fig. 3.1 Beam-Column Element
Fig. 3.2 Diagrammatic Illustration of Newton-Raphson Procedure.
**Fig. 3.5(a)** Buckling Loads for Battened Frames.

**Fig. 3.5(b)** Buckling Loads for Latticed Frames.
Fig. 3.6(a) Variation of Critical Load with Offset on Crossarms.
Fig. 3.6(b) Variation of Critical Load with Offset on Verticals.
Fig. 3.7 Mode 'C' - Offset on Crossarm 0.5 in.
Fig. 3.8 Mode 'C' - Offset on Crossarm 2.0 in.
Fig. 3.9 Mode 'E' - Offset on Crossarm 0.5 in.
Fig. 3.10 Mode 'E' - Offset on Crossarm 2.0 in.
Fig. 3.11 Relationship between Ideal Buckling Load and Offset Distance for Lattice Frames.
Fig. 3.12 Beam-Column.
Fig. 4.1 Experimental Set-Up for the Stayed Column.
Fig. 4.2 Lattice Frame with Intersecting Diagonals.
Fig. 4.3 Lattice Frame with Diagonals Offsetting on Crossarms.
Fig. 4.4 Lattice Frame with Diagonals Offset on Verticals.
Fig. 4.5 Experimental Set-Up for Relatively Slender Tower.
Fig. 4.6 Postweld Heating.
Fig. 4.7 Frame used for Electrical Prestressing (ref. 12).
Fig. 5.1 Load versus Stay Tension for Pretension of Zero lbs.

Theoretical Results
Experimental Results

Stay 1
Stay 2
Stay 3
Stay 4

Pretension = Zero lbs.

\[ y_0 = \frac{L}{1000} \]
Fig. 5.2 Load versus Stay Tension for a Pretension of 40 lbs.
Fig. 5.3 Load versus Stay Tension for a Pretension of 150 lbs.
Fig. 5.5 Load versus Stay Tension for a Pretension of 300 lbs.
Fig. 5.6 Load versus Mid-Height Lateral Deflection, $y_0 = L/1000$, $T_i = 150$ lbs.
Fig. 5.7 Load versus Deflection, \( y_0 = L/1000 \), \( T_1 = 300 \) lbs.
Fig. 5.8 Critical Load versus $2/L_{ca}$ Ratio.
Fig. 5.11 Ideal Buckling Mode for Prestressed Lattice Frame with Intersecting Diagonals.

\[ P_{CR} = 4.2 \text{ kips} \]
Fig. 54.12 Load versus Deflection for Pretension of Zero lbs.
Fig. 5.13 Load versus deflection for a pretension of 70 lbs.
Fig. 5.15 Buckling of Prestressed Lattice Frame with Intersecting Diagonals.
Fig. 5.16 Load versus Stay Tension for a Pretension of 8 lbs.

Ideal Buckling Load = 4.2 kips
Ideal Buckling Load = 4.2 kips

Fig. 5.17 Load versus Stay Tension for a Pretension of 70 lbs.
Fig. 5.18 Load versus Stay Tension for a Pretension of 70 lbs.
Fig. 5.19 Theoretical and Experimental Deflections at an Axial Load of 3.8 kips for a pretension of 70 lbs.
Fig. 5.20  Theoretical and Experimental Deflections at an Axial Load of 3.8 kips for a Pretension of zero lbs.
Ideal Buckling Load = 4.2 kips

Fig. 5.21 Load versus Deflection for a Pretension of Near Zero lbs.
Fig. 5.22 Load versus Deflection for a Pretension of 70 lbs.
Fig. 5.23 Load versus Stay Tension for a Pretension of 5 lbs.
Ideal Buckling Load = 4.2 kips

Fig. 5.24 Load versus Stay Tension for a Pretension of 7 lbs.
Ideal Buckling Load = 4.2 kips

Fig. 5.28 Load versus Stay Tension for a Pretension of 70 lbs.
Ideal Buckling Load = 4.2 kips.

Fig. 5.27 Load Versus Stay Tension for a Pretension of 75 lbs.
Ideal Buckling Load = 4.2 kips

Fig. 5.28  Load Versus Stay Tension for a Pretension of 72 lbs.
Fig. 5.29 Ideal Buckling Modes for Offset Frames
Fig. 5.30  Load versus Deflection for Near Zero Pretension.
Fig. 5.31 Load versus deflection for a pretension of 92 lbs.

Ideal Buckling Load = 5.48 kips

Deflection at 'A' (in.)

Axial Load (kips)

0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20

Experimental
Theoretical
Fig. 5.32 Rotational Restraining and Compressive Strength (Ref. 6).

(a) Restraining Moment at Midheight, $M_R = 0$.

(b) $P_{cr} = \frac{\pi^2 EI}{L^2}$

(c) $P_{cr} > \frac{\pi^2 EI}{L^2}$
Fig. 5.33 Load versus Deflection for a Near Zero Pretension
Fig. 5.36 Load versus Stay Tension for a Pretension of 4 lbs.
Fig. 5.37 Load versus Stay Tension for a Pretension of 4 lbs.
Fig. 5.38 Theoretical and Experimental Deflection at an Axial Load of 4 kips for a Near Zero Pretension Case.
Fig. 5.39 Load versus Stay Tension for a Pretension of 85 lbs.
Fig. 5.40 Load versus Stay Tension for a Pretension of 85 lbs.
Fig. 5.41 Load versus Stay Tension for a Pretension of 85 lbs.
Fig. 5.43 Load versus Stay Tension for a Pretension of 90 lbs.
Fig. 5.44 Load versus Stay Tension for a Pretension of 96 lbs.
Fig. 5.45 Theoretical and Experimental Deflections at an Axial Load of 4.5 kips for a Pretension of 90 lbs.
Fig. 5.46 Buckled Shape of the Lattice Frame with Diagonals Offset on Crossarms.
Fig. 5.47 - Rotational Restraint with Diagonals Offset on Verticals.

\[ M_R = F \cdot d \]

Restraining Moment
Fig. 5.48 Buckling of Frame with Offset on Verticals.
Fig. 5.50 Load versus stay tension for a pretension of 6 lbs.

Experimental
Theoretical

Stay tension of 'A' (lbs.)

Axial Load (kips)
Fig. 5.51 Load versus Stay Tension for a Pretension of 8 lbs.
Fig. 5.52 Load versus Stay Tension for a Pretension of 6 lbs.
Fig. 5.53 Load versus Stay Tension for a Near Zero Pretension.
Fig. 5.54 Load versus Stay Tension for a Pretension of 102 lbs.
Fig. 5.56 Load versus deflection for a 'Near Zero Pretension'.
Fig. 5.57 Load versus Deflection for a Pretension of 92 lbs.
Fig. 5.59 Load versus Deflection for a Pretension of 92 lbs.
Fig. 5.60 Theoretical and Experimental Deflections for a Near Zero Pretension and at an Axial Load of 4.7 kips.
Fig. 5.61 Theoretical and Experimental Deflections for a Pretension of 92 lbs, and at an Axial Load of 4.7 kips.
Fig. 5.62 Experimental Setup for Slender Lattice Tower.
Fig. 5.63 Load versus Deflection Under Pure Axial Load.
Fig. 5.66 Load versus Deflection Under Pure Axial Load.

1 Theoretical for 300 lbs. pretension when the magnitude of imperfection was reduced by 10%.

2 Theoretical for 300 lbs. pretension when the magnitude of imperfection was increased by 10%.
Fig. 5.67 Stays 'A', 'B', 'C', 'D' and 'E' of Figs. 5.68 to 5.80.
Fig. 5.68 Load versus Stay Tension for Pure Axial Load.

Stays 'B' and 'C' are shown in Fig. 5.67.
Fig. 5.69  Load versus Stay Tension for Pure Axial Load.
Fig. 5.70. Load versus Stay Tension for Pure Axial Load.

- - - - - Experimental
- - - - - Theoretical

Stays 'D' and 'E' are shown in Fig. 5.67.
Fig. 5.71 Load versus Stay Tension for Pure Axial Load.
Fig. 5.72 Experimental Set-Up for Horizontal Load Application.
Axial Force = 2 kips

Fig. 5.73  Load versus Deflection for a Pretension of Near 50 lbs.
Fig. 5.74 Load Versus Deflection for a Pretension of Near 300 lbs.
Fig. 5.45 Load versus Deflection for a Pretension of near 50 lbs.
Fig. 5.78  Load versus Stay Tension for a Pretension of Near 300 lbs.
Fig. 5.79: Load versus Stay Tension for a Pretension of Near 50 lbs.

Stays 'C' and 'D' are shown in Fig. 5.76.

Axial Force = 2 kips
Fig. 5.80 Load versus Stay Tension for a Pretension of Near 300 lbs.

Axial Force = 2 kips

Experimental

Theoretical

Total Horizontal Load (lbs.)

Stay tension (lbs.)
Fig. 5.81 Load versus Deflection for a Pretension of Near 50 lbs.
Fig. 5.82 Load versus Deflection for a Pretension of Near 300 lbs.
Stays 'A' and 'B' are shown in Fig. 5.76.

Axial Force = 5 kips

Fig. 5.83 Load versus Stay Tension for a Pretension Of Near 50 lbs.
Stays 'A' and 'B' are shown in Fig. 5.76.

Axial Force = 5 kips.

Fig. 5.84 Load Versus Stay Tension for a Pretension of Near 300 lbs.
Fig. 5.85 Load versus Deflection for a Pretension of near 50 lbs.
Fig. 5.86 Load versus Deflection for a Pretension of Near 300 lbs.

Axial Force = 8 kips
Stays 'A' and 'B' are shown in Fig. 5.76.

Axial Force = 8 kips

Fig. 5.87 Load versus Stay Tension for a Pretension of Near 50 lbs.
Fig. 5.89 Load Versus Deflection for a Pretension of Nearly 50 lbs.

Axial Load = 10 kips
Fig. 5.90 Load versus Deflection for a Pretension of Nearly 300 lbs.
Fig. 5.91 Load versus Stay Tension for a Pretension of Nearly 50 lbs.

Stays 'A' and 'B' are shown in Fig. 5.76.

Axial Force = 10 kips
Fig. 5.92 Load Versus Stay Tension for a Pretension of Nearly 300 lbs.
Fig. 5.93 Interaction between Axial Load and Horizontal Load.

- ○ = 50 lbs. Pretension.
- ● = 300 lbs. Pretension.

All the curves are plotted at a maximum deflection = 0.6 in.
Fig. 5.94 Load Versus Deflection for Prestressed and Nonprestressed Tower.
Fig. C.1 Global and Local Axes.
Appendix B

TABLES
Table 1. Buckling Loads for Frames With Different Heights
(see Fig. 3.3)

<table>
<thead>
<tr>
<th>N</th>
<th>( L_{ca} ) (in.)</th>
<th>( P_{(cr)}_B ) (KIPS)</th>
<th>( P_{(cr)}_L ) (KIPS)</th>
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<td></td>
<td>24</td>
<td>1.59</td>
<td>6.32</td>
</tr>
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</table>

\( N \) = No. of panels in a frame.

\( L_{ca} \) = Length of a crossarm.

\( P_{(cr)}_B \) = Critical load of a battened frame.

\( P_{(cr)}_L \) = Critical load of a latticed frame.
**Table 2  Variation of Critical Load \( (P_{cr}) \) With Offset Distance. (KIPS)**

<table>
<thead>
<tr>
<th>Offset Crossarm</th>
<th>Mode C</th>
<th>Mode D</th>
<th>Mode E</th>
<th>Offset Vertical</th>
<th>Mode C</th>
<th>Mode D</th>
<th>Mode E(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>6.72</td>
<td>8.76</td>
<td>9.72</td>
<td>0.0</td>
<td>6.72</td>
<td>8.76</td>
<td>9.72</td>
</tr>
<tr>
<td>0.5</td>
<td>7.69</td>
<td>9.34</td>
<td>9.72</td>
<td>0.5</td>
<td>7.04</td>
<td>8.96</td>
<td>9.72</td>
</tr>
<tr>
<td>1.0</td>
<td>9.42</td>
<td>10.48</td>
<td>9.72</td>
<td>1.0</td>
<td>8.1</td>
<td>9.65</td>
<td>9.68</td>
</tr>
<tr>
<td>1.5</td>
<td>10.57</td>
<td>11.41</td>
<td>9.72</td>
<td>1.5</td>
<td>9.88</td>
<td>10.94</td>
<td>9.68</td>
</tr>
<tr>
<td>2.0</td>
<td>11.06</td>
<td>11.49</td>
<td>9.72</td>
<td>2.0</td>
<td>12.19</td>
<td>11.40</td>
<td>9.67</td>
</tr>
<tr>
<td>2.5</td>
<td>11.17</td>
<td>11.47</td>
<td>9.72</td>
<td>2.5</td>
<td>-</td>
<td>-</td>
<td>9.67</td>
</tr>
</tbody>
</table>

(1) Modes C, D, and E are shown in Fig. 3.4(b).

(2) Not Required.
Table 3: Comparison of Experimental Failure Loads as Determined by Various Criteria

<table>
<thead>
<tr>
<th>Initial Out-of-straightness</th>
<th>Initial Pretension (lbs.)</th>
<th>Old Criteria Failure Loads (lbs.)</th>
<th>Maximum Load Carrying Capacity (lbs.)</th>
<th>Failure Mode²</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/500</td>
<td>150</td>
<td>4000</td>
<td>5600</td>
<td>1</td>
</tr>
<tr>
<td>L/1000</td>
<td>0</td>
<td>1270</td>
<td>5200</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2000</td>
<td>5600</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>5000</td>
<td>5800</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td>6400</td>
<td>6800</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>8600</td>
<td>8600</td>
<td>2</td>
</tr>
<tr>
<td>L/3000</td>
<td>150</td>
<td>5700</td>
<td>7000</td>
<td>1</td>
</tr>
</tbody>
</table>

1. The old criteria stated that the experimental buckling load is the load at which the tension in the stays on the concave side goes to zero.
2. Failure modes are shown in Fig. 5.8.
Appendix C

STIFFNESS MATRICES AND TRANSFORMATION MATRIX

The stiffness matrices given below are for a beam-column element and are obtained from Eq. 3.13.

The elastic stiffness matrix in local coordinate system is given by:

\[
\begin{bmatrix}
\frac{E A}{L} & 12\frac{E I}{L^3} & \text{symmetric} \\
0 & 6\frac{E I}{L^2} & 6\frac{E I}{L} \\
-\frac{E A}{L} & 0 & 0 & \frac{E A}{L} \\
0 & -12\frac{E I}{L^3} & -6\frac{E I}{L^2} & 0 & 12\frac{E I}{L^3} \\
0 & 6\frac{E I}{L^2} & 2\frac{E I}{L} & 0 & -6\frac{E I}{L^2} & 4\frac{E I}{L}
\end{bmatrix}
\]

In the above matrix, \( E \) = modulus of elasticity, \( A \) = cross sectional area, \( I \) = moment of inertia, and \( L \) = length of the element.

The geometric stiffness matrix of the beam-column element in the local coordinate system is given by:
\[
\begin{bmatrix}
  0 & 0 & 6/5L & \text{symmetric} \\
  0 & 1/10 & 2L/15 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & -6/5L & -1/10 & 6/5L \\
  0 & 1/10 & -L/30 & -1/10 & 2L/15 \\
\end{bmatrix}
\]

\[
\begin{array}{cccccc}
  u_1 & v_1 & \theta_1 & u_2 & v_2 & \theta_2 \\
\end{array}
\]

where \( P \) = the initial loading at which the geometric stiffness matrix is calculated.

The elastic stiffness matrix is independent of load level. The geometric stiffness matrix depends not only on the geometry but also on the initial internal force existing in the member at the start of the loading step. This matrix represents the effect of the axial load on the flexural stiffness.

### C.1 Transformation Matrix

The following matrix is used to transform the global displacements to local displacements (see Figure C.1):
\[
[T.] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 & 0 & 0 \\
-\sin \theta & \cos \theta & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \theta & \sin \theta & 0 \\
0 & 0 & 0 & -\sin \theta & \cos \theta & 0 \\
u_1 & v_1 & \theta_1 & u_2 & v_2 & \theta_2
\end{bmatrix}
\]

In the above equation, \( \theta \) is the angle between the global and local axes.
Appendix D

DETAILS OF THE COMPUTER PROGRAM

The following computer program is used to determine the nonlinear geometric behaviour of the prestressed lattice structure. As explained in section 3.5 the program uses finite element method to analyze the structure. A flow chart is given below which explains the basic operations involved in the method. A numerical procedure, Newton-Raphson method, was used to arrive at the solution of nonlinear equilibrium equations. The program needs the data card in which the number of members, number of joints, number of degrees of freedom, and member properties are given. This is followed by the load vector, nodal connectivity table, and degrees of freedom for each joint. The program selects a load factor and number of increments the load factor to be increased.

The element stiffness matrices will be computed first in local coordinates and then transformed to global coordinates. The structure stiffness matrix is formed by assembling the element stiffnesses. The nodal displacements will be computed by Gauss-Jordan elimination method. Significant changes in geometry due to load application
necessitates the updating of the joint coordinates after every cycle. The equilibrium position is obtained by iterating using the unbalanced load as the load vector. The convergence of the problem is checked as per sec. 3.3.

The additional load is then applied in the next increment and the process continues till the failure of the structure reached. The failure criteria which was used identifies the instability of the compression member, the yielding of the component members and any excessive deflection. This procedure is further clarified by the following flow chart and the listing of the computer program.
START

Read Member and Joint Input Data

Select the number of load increments (N) and start of the increment.

Read the load vector [R], nodal connectivity table, degrees of freedom

Determine the direction cosines, lengths of the members

Iteration cycle starts

Form the element stiffness matrix in local co-ordinates [k] and transform them into global coordinates [K]

Assemble the global structure stiffness matrix [K]

Solve for nodal displacements [U] = [K⁻¹][F]

Update the joint coordinates, member direction cosines, and member displacements [u]
Find the member forces from the displacements:
\[ [F] = [k] [\{u\}] \]

Calculate the residual force vector:
\[ [R'] = [R] - [F] \]

Is the convergence criteria satisfied?

If no, print the nodal displacements, element stresses.

Is the load increment = \( \mathbf{M} \)?

Yes:

STOP
COMPUTER PROGRAM TO DETERMINE THE BEHAVIOUR OF PRESTRESSED LATTICE STRUCTURES

* THE FOLLOWING VARIABLES DEFINE THE INPUT DATA *

* NM = NUMBER OF MEMBERS *
* NJ = NUMBER OF JOINTS *
* NWG = NUMBER OF MEMBERS WITH SIGNIFICANT AXIAL LOAD *
* NBS = NUMBER OF BENDING MEMBERS *
* NDF = NUMBER OF DEGREES OF FREEDOM *
* NDG = NUMBER OF DIAGONALS *
* NND = NUMBER OF AXIAL LOAD INCREMENTS *
* CN = COORDINATES OF NODES *
* NDV = DEGREES OF FREEDOM AT EACH NODE *
* NV = DEGREES OF FREEDOM AT EACH MEMBER *
* AA = AREA OF MEMBERS *
* ZI = MOMENT OF INERTIA *
* UTOT = CUMULATIVE DISPLACEMENT OF THE NODES *
* UT = INCREMENTAL DISPLACEMENT OF THE NODES *
* E = MODULUS OF ELASTICITY *
* IL = NUMBER OF LOADED NODES *
* P1 = LOADING VECTOR *
* P1 = LOAD FACTOR *
* DLG = STAY TENSIONS *

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION MN(172,2), IVC(172,6), DC(172,2), DG(172), B(303), AD(303)
DIMENSION GLK(303,30), GLL(303,30), PLL(172,6), PGL(6,6), PT(273)
DIMENSION C(102,2), XL(303), IX(102,3), X(6,6), OR(273), A(14(303)
DIMENSION U(303), P(303), UH(172,6), UTL(172,6), T(6,6), TK(273,30)
DIMENSION AA(172), E(172), LGTH(172), ZI(172), C(102,2), P(1(303)
DIMENSION U1(303,30), UD(303), ND(102,2), X1(303), D(303), FT(303)
DIMENSION BKL(6,6), PGL(6,6), ELG(6,6), PP(172), RCM(172), PI(303)
DIMENSION IB(172), S(303,30), TM(303,30), UA(6), U2(303)
DIMENSION U1(303), U1(303), UH(172,6), UTL(172,6), W(6,6)
DIMENSION GP(172,6), PE(303), PR(303), UT(303), GNB(6,6), GNB(6,6)
DIMENSION E(16,1), T2(17,2), TT1(1,16), TT2(2,17), A13(303)
DIMENSION P2(303)
DOUBLE PRECISION AK, LGTH
SORT(X) = DSQRT(X)
ABS(X) = DABS(X)
COS(X) = DCOS(X)
SIN(X) = DSIN(X)

THESE CARDS CONTROL THE DIMENSIONS AND CAN BE CHANGED ACCORDINGLY

NDPD=303
READ THE FOLLOWING INPUT DATA

READ (5,5) NM,HDG,NJ,HRG,NBS,NDF,IND
DO 10 J=1,NJ
  10 CONTINUE
DO 20 J=1,NM
  20 CONTINUE
READ (5,25) I, NM(I,1), NM(I,2), AA(I), ZI(I), E(I)
DO 60 I=NBS,NM
  60 CONTINUE
IF (I.EQ.NM) GO TO 65
READ (5,70) DG(I)
  65 CONTINUE
READ (5,135) IL
DO 140 I=1,IL
  140 CONTINUE
READ (5,145) LJ,P1(LJ)
CONTINUE
READ (5,150) PLFAC

PRINTING THE INPUT DATA

WRITE (6,7) NM, NJ, HDG, HRG, NBS, NDF, IND
DO 35 I=1,NM
  35 CONTINUE
DO 45 I=1,NM
  45 CONTINUE
I1=NM(I,1)
I2=NM(I,2)
DO 50 J=1,3
  50 CONTINUE
IVC(I,J)=IV(I1,J)
IVC(I,J+3)=IV(I2,J)
CONTINUE
IF (I.LE.NBS) GO TO 45
IVC(I,3)=0
IVC(I,6)=0
CONTINUE

DETERMINING THE BAND WIDTH

DO 80 I=1,NM
  80 IB(I)=IABS(NM(I,2)-NM(I,1))
IBJ=IB(1)
DO 85 J=2,NM
IP: IBJ .GE. IB (J) ) GO TO 95
IBJ = IB (J)
95 CONTINUE
IBW = (IBJ + 1) * 3
PRINT 90, IBW
PRINT 160
PRINT 165
PRINT 170
PRINT 175, (I, IL (I), AA (I), ZI (I), RN (I, 1), RN (I, 2), E (I), I = 1, NM)
PRINT 180
PRINT 185, (I, CN (I, 1), CN (I, 2), I = 1, NJ)
PRINT 190
PRINT 195
PRINT 200, (I, (IVC (I, J), J = 1, 6), I = 1, NM)
WRITE (6, 496)
DO 201 I = NBS, NM
II = I + 1
IF (II .EQ. NM) GC TO 202
PRINT 500, I1, DG (I1), AA (I1)
201 CONTINUE
202 CONTINUE
DO 30 I = 1, NDF
IF (I .EQ. IJ) GC TO 30
P (I) = 0.
PE (I) = 0.
F (I) = 0.
30 CONTINUE
DO 40 I = 1, NM
ND (I) = AA (I)
DO 55 I = 1, NM
DO 55 J = 1, 6
UTIL (I, J) = 0.0
55 CONTINUE
DO 75 I = 1, NDF
PTOT (I) = 0.
UTO (I) = 0.
UTOT (I) = 0.
75 U11 (I, 1) = 0.0

DETERMINING OF THE DIRECT COSINES

CALL NEWT (CN, RN, DC, AA, ZI, E, LGTH, NM, HEMP, NJCI)
PLF = 0.0
INDEX = 0

INDEX = 0 GIVES AN ELASTIC ANALYSIS FOR THE GIVEN LOADING AND THEREFORE DETERMINES THE AXIAL LOAD IN THE MEMBERS

PRINT 220
230 PRINT 235
INCREMENTATION OF THE LOAD STARTS HERE

DO 245 I2=1,32
IF(INDEX.EQ.0) GO TO 255
IF(I2.GT.IND) GO TO 257
PLF=PLF+FLFAC

257 CONTINUE
IF(I2.GT.IND) I3=I3+1
IF(I2.LE.IND) GO TO 251
IF((I2.GT.IND).AND.(I3.GT.17)) GO TO 250

251 CONTINUE
PRINT 265
PRINT 270
DO 275 I=1,NDF
UTO(I)=UTOT(I)
P(I)=P(I)+FLFAC
PTOT(I)=PTOT(I)+P(I)
IF(P(I).EQ.0.0) GO TO 275
PRINT 280,1,PTOT(I),PLF

275 CONTINUE
DO 285 I=NB,S,NM
II=I+1
IF(I.EQ.NM) GO TO 290

285 DGT(II)=DG(I)

290 CONTINUE
DO 295 I=1,NJ
DO 295 J=1,2
295 CH1(I,J)=CH(I,J)
PRINT 300,1,I2

255 CONTINUE
DO 355 I=1,NDF
IF(I.EQ.LJ) GO TO 355
IF(INDEX.EQ.0) P(I)=P(I)

355 CONTINUE
KK=1
DO 256 I=1,NDF
256 P2(I)=P(I)
GO TO 305

ITERATION CYCLE STARTS

310 CONTINUE
KK=KK+1
DO 320 I=1,NDF
320 P(I)=PR(I)
305 KK1=KK+1

CHECK TO FIND IF ANY DIAGONAL MEMBERS WENT INTO COMPRESSION
DO 325 I=HB5,HB
     I=I+1
     IF(I.EQ.HB) GO TO 330
     IF(DG(I).GE.0.) AA(I1)=0.0
     IF(DG(I).EQ.0.) DGT(I1)=0.
     IF(LGTH(I1).GE.XL(I1)) AA(I1)=AD(I1)
     IF(AA(I1).EQ.0.) DG(I1)=0.
325 CONTINUE
330 CONTINUE
     IF(INDX.EQ.0) GO TO 336
     PRINT335,KK
     SOLUTION PROCEDURE
     GENERATE THE ELASTIC AND GEOMETRIC STIFFNESS MATRICES
336 CONTINUE
     CALL ELAKM(HM,ZL,AA,E,LGTH,DC,MDF,IVC,GLK1,MBES,MDDF,HB5,IBAND)
     IF (.NOT.(II.EQ.1) .AND. (KK.EQ.1)) GO TO 340
     CALL GEOKM(NKG,LGTH,DC,MDF,IVC,GLK2,MBES,MDDF,HB5,AA,UTIL2,PP,IBM,UM,UTOTL,IND)
     DO 345 I=1,NDF
     DO 345 J=1,IBS
     TM1(I,J)=GLK1(I,J)+(GLK2(I,J)*PLF)
     GO TO 360
     DO 350 I=1,NDF
     DO 350 J=1,IBW
     TM1(I,J)=GLK1(I,J)
360 CONTINUE
     TO FIND THE DISPLACEMENTS
     CALL BANDIN(U,TM1,P,MDF,MDDF,IBW,DE,IBM,S,R)
     DO 385 I=1,NDF
     UT(I)=0.0
     UT(I)=UT(I)+UT(I)
     UT(I)=UT(I)+UT(I)
     DO 385 I=1,NDF
385 CONTINUE
     IF(INDX.EQ.0) GO TO 390
     TO UPDATE THE COORDINATES AND THE DIRECTION COSINES
     CALL COOR(CH,UT,HB5,NA,CH1,AN,IVC,MBES,MDDF,ND,NIJOI)
     CALL NEWT(CH,HB,DC,AA,ZL,E,LGTH,HB,MBES,NIJOI)
390 CONTINUE
     TO TRANSFORM NODAL DISPLACEMENTS TO MEMBER DISPLACEMENTS (GLOBAL AND LOCAL)
     CALL DISPLA(HM,UT,IVC,MBES,MDDF,UM)
     CALL DISPLA(HM,UTOT,IVC,MBES,MDDF,UMT)
     DO,395 K=1,HB
CALL TRAF(K, DC, T, MEMB, NBS)
DO 395 I=1,6
UTOTL(K, I)=0.
UTIL(K, I)=0.0
DO 395 J=1,6
UTOTL(K, I)=UTOTL(K, I) + T(I, J) * UMT(K, J)
UTIL(K, I)=UTIL(K, I) + T(I, J) * UM(K, J)
395 CONTINUE
IF (INDEX.EQ.0) GO TO 400
GO TO 405
400 CONTINUE

TO DETERMINE THE AXIAL LOAD IN THE MEMBERS

WRITE(6, 412)
DO 410 I=1, NKG
PP(I)=E(I) * A(I) * (UTOTL(I-6) - UTOTL(I, 1)) / LGTH(I)
PPNT417, 1, PP(I)
410 CONTINUE
INDEX=1
GO TO 230
405 CONTINUE

DO 415, I=1, NM
DO 420 J=1, 6
420 UA(J)=UTOTL(I, J)
CALL TRAF(I, DC, T, MEMB, NBS)
CALL ELMK(I, E, LGTH, AA, ZL, EKL, MEMB)
IF(I .GT. NKG) GO TO 425
CALL SBO(I, LGTH, MEMB, AA, ELG, E)
425 DO 430 J=1, 6
DO 430 K=1, 6
IF(I .GT. NKG) GO TO 435
FGL(J, K)=EKL(J, K) + (ELG(J, K) * PP(I) * ELF)
GO TO 430
435 FGL(J, K)=EKL(J, K)
430 CONTINUE
CALL TRAF(I, DC, T, MEMB, NBS)
CALL TRAF(T, W, 6, 6)
CALL MULL(W, FGL, V, 6, 6, 6)
CALL MULL(V, T, FGL1, 6, 6, 6)

TO DETERMINE THE INTERNAL MEMBER FORCES (GLOBAL AND LOCAL)

DO 440 J=1, 6
GF(I, J)=0.
DO 440 K=1, 6
440 GF(I, J)=GF(I, J) + FGL(J, K) * UM(I, K)
DO 445 J=1, 6
FLF(I, J)=0.0
DO 445 K=1, 6
FLF(I, J)=FLF(I, J) + T(J, K) * GF(I, K)
445 CONTINUE
450 PE(I)=0.
DO 455 I=1,NDF
455 I=IVC(I,J)
IF(I1.EQ.0) GO TO 455
PE(I1)=PE(I1)-GP(I,J)
CONTINUE

TO FIND THE RESIDUAL FORCE VECTOR

DO 460 I=1,NDF
PE(I)=P2(I)+PE(I)
CONTINUE

TO UPDATE THE STAY TENSIONS

NBS1=HBS+1
DO 470 I=NBS1,NM
470 DG(I)=PGT(I)+PL(I,1)

TO DETERMINE THE CONVERGENCE OF THE SOLUTION

A1=0.
A0=0.
II=0
IZ1=23
IZ2=3
IZ3=30

IZ1,IZ2,IZ3 MUST BE CAREFULLY SELECTED FOR EACH PROBLEM
THESE PARAMETERS ARE IMPORTANT FOR PROPER CONVERGENCE OF THE PROBLEM

NDF11=NDF-IZ3
DO 475 J=IZ1,NDF11,IZ2
II=II+1
475 IF(ABS(U(1,J,KK1)).GT.A0) A0=ABS(U(1,J,KK1))
PRINT480,A0
IF(KK.LE.2) GO TO 310
DO 485 I=IZ1,NDF11,IZ2
A2=ABS(U(I)/A0)**2
A1=A1+A2
CONTINUE
A3=SQRT(A1/II)
IF(A3.LT.0.001) GO TO 490
IF(KK.GT.10) GO TO 515
GO TO 310
CONTINUE
WHITE(6,491)
I1=I2+1
IF(I2.NE.(I1+1)) GO TO 535

515 CONTINUE
WHITE(6,496)
DO 505 I=NBS, NM
I1=I+1
IF(I.EQ.NM) GO TO 495
PRINT500, I1, DG(I1), AA(I1)
505 CONTINUE

495 CONTINUE
PRINT510
DO 520 I=1,NJ
PRINT525, I, (CM(I,J), J=1,2)
520 CONTINUE

IF(EK.GT.10) GO TO 250
IF(I2.EQ.IND) GO TO 530
GO TO 535

530 CONTINUE

C APPLICATION OF HORIZONTAL LOAD AT CONSTANT AXIAL LOAD

C

DO 565 I=1,NDP
P1(I)=0.0
565 CONTINUE.

DO 540 I=1,2
READ(5,545) II,P1(II)
PRINT545, II, P1(II)
540 CONTINUE

READ(5,550) PLFAC
I3=0
PRINT550, PLFAC

535 CONTINUE

245 CONTINUE

5 FORMAT(715,F10.4)
7 FORMAT(//' THE NUMBER OF MEMBERS = ',I5,'//')
* NUMBER OF JOINTS = ',I5,'//
* NUMBER OF DIAGONALS = ',I5,'//
** NUMBER OF MEMBERS WITH SIGNIFICANT AXIAL LOAD = ',I5,'//
** NUMBER OF BENDING MEMBERS = ',I5,'//
** NUMBER OF DEGREES OF FREEDOM = ',I5,'//
** NUMBER OF AXIAL LOAD INCREMENTS = ',I5,'//')

15 FORMAT(I5,2F10.4,3I5)
25 FORMAT(I5,2I5,F10.4,2E10.2)
70 FORMAT(F10.5)
90 FORMAT(1 THE SEMI BAND WIDTH IS = ',I5)
135 FORMAT(I5)
145 FORMAT(I5,F10.4)
150 FORMAT(F10.5)
160 FORMAT(//' THE FOLLOWING IS THE INPUT OF THE PROBLEM ',//)
FORMAT ( 'MEMBER DATA (IN KIPS AND INCHES)' )
FORMAT ( 'MEMBER NO.', '3X', 'LENGTH', '9X', 'AREA', '11X', 'E.I.', '5X', 'NEAR END', '5X', 'FAR END', '5X', 'MOD. OF ELASTICITY' )
FORMAT ( 'IN', '5X', 'P10.4', '5X', 'P10.4', '5X', 'P10.4', '5X', 'P10.4', '5X', 'P10.4', '5X', 'P10.4')
FORMAT ( 'NODE', '5X', 'COORDINATES', '9X', 'X', '16X', 'Y' )
FORMAT ( 'IN', '5X', 'P10.4', '10X', 'P10.4')
FORMAT ( 'IVC TABLE', '9X', 'DEGREES OF FREEDOM' )
FORMAT ( 'MEMBER NO.', '7X', '1', '7X', '2', '7X', '3', '7X', '4', '7X', '5', '16X', '6' )
FORMAT ( 'IN', '5X', '10X', '15', '5X', '5X', '5X', '5X', '5X', '5X', '5X', '5X', '5X', '5X', '5X' )
FORMAT ( 'THE FOLLOWING IS THE OUTPUT' )
FORMAT ( 'LOAD VECTOR' )
FORMAT ( '5X', 'D.O.F.', 'AXIAL LOAD (COMPRESSION)' )
FORMAT ( '5X', '10X', 'P10.4', '10X', 'P10.4')
FORMAT ( 'LOAD INCREMENT NO.', 'I5' )
FORMAT ( 'ITERATION CYCLE NO.', 'I5' )
FORMAT ( '5X', '10X', 'P10.5')
FORMAT ( '5X', 'MEMBER NO.', '5X', 'AXIAL LOAD' )
FORMAT ( 'MAXIMUM DISPLACEMENT (LATERAL) =', 'F15.5' )
FORMAT ( 'CONVERGENCE ACHIEVED FOR THIS LOAD INCREMENT!' )
FORMAT ( 'MEMBER NO.', '5X', 'STAY-TENSION', '3X', 'AREA' )
FORMAT ( '10X', 'I5', '10X', '2F10.4')
FORMAT ( '10X', '6X', 'COORDINATES', '5X', '3H X', '9X', '3X', 'Y' )
FORMAT ( 'NONLINEAR GEOMETRIC ANALYSIS BEGINS...!' )
FORMAT ( '11X', 'I3', '20X', 'P10.4', '4X', 'P10.4')
FORMAT ( '15X', 'F10.4')
STOP
SUBROUTINE BANDIN (U, TMK, P, NDFP, NDFP, LEB, DE, IBAND, S, R)
* THIS SUBROUTINE DETERMINES THE DISPLACEMENT 'U', BY INVERTING THE MATRIX 'TMK', AND MULTIPLYING BY THE LOAD VECTOR 'P'.
* IMPlicit REAL*8 (A-H, O-Z)
DIMENSION U(NDFP), P(NDFP), TMK(NDFP, IBAND)
DIMENSION S(NDFP, IBAND), R(NDFP)
IF(NDFP.EQ.1) GO TO 731
DO 730 I=1, NDF
DO 730 J=1, LEB
S(I, J) = TMK(I, J)
DO 740 I=1, NDF
R(I) = P(I)
DO 750 M=1, NDF
DO 750 L=2, LEB
IF(S(M, L).EQ.0) GO TO 780
N=H+L-1
C=S(N, L)/S(N, 1)
J=0
DO 750 K=L,IBW
    J=J+1
    750 S(I,J)=S(I,J)-C*S(N,K)
    S(N,L)=C
    CONTINUE

    DO 51 I=1,NDF
    IF (S(I,1).LT.0.) PRINT50,I,S(I,1)
    50 FORMAT (IS,5X,2E20.7)
    IF (S(I,1).LT.0.) GO TO 52
    51 CONTINUE

    DO 830 N=1,NDF
    DO 820 I=2,IBW
    IF (S(N,L).EQ.0.) GO TO 820
    I=N+L-1
    R(I)=R(I)-S(N,L)*R(N)
    820 CONTINUE

    830 R(N)=R(N)/S(N,1)
    DO 860 N=2,NDF
    N=NDF+1-N
    DO 850 I=2,IBW
    IF (S(N,L).EQ.0.) GO TO 850
    K=N+L-1
    R(N)=R(N)-S(N,L)*R(K)
    850 CONTINUE

    860 CONTINUE
    GO TO 732

    731 R(1)=P(1)/TMK(1,1)

    732 CONTINUE
    GO TO 732

    DO 870 I=1,NDF
    870 U(I)=R(I)
    GO TO 54

    52 PRINT53

    53 FORMAT (' UNSTABLE-MATRIX NOT POSITIVE DEFINITE ')
    STOP

RETURN
END

SUBROUTINE DISPLA(N,U,IVC,MEMBER,NDER,U)

* THIS SUBROUTINE WILL CHANGE THE GLOBAL DISPLACEMENT TO LOCAL *
* DISPLACEMENT ON EACH MEMBERS' ENDS *

DIMENSION U(NDER),IVC(MEMBER,6),UM(MEMBER,6)
DO 300 I=1,M
    DO 300 J=1,6
    K=IVC(I,I)
    IF(K)=40,20,30
    30 UM(I,I)=U(K)
    GO TO 300
20  UM(I, I1) = 0.
  GO TO 300
10  K = IABS(K)
   UM(I, I1) = -U(K)
300  CONTINUE
280  CONTINUE
   RETURN
END

SUBROUTINE ELAKM (H, ZI, AA, E, EL, DC, HDF, IVC, MK, MMB, ND, NBS, IBAND)

******************************************************************************
* THIS SUBROUTINE WILL CALCULATE THE MASTER ELASTIC STIFFNESS             *
* MATRIX                                                                   *
******************************************************************************
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION ZI(MMB), AA(MMB), EL(MMB), E(MMB)
DIMENSION DC(MMB,2), IVC(MMB,6), MK(ND, IBAND), EEG(6,6)
DIMENSION W(6,6), V(6,6), EKL(6,6), T(6,6), TP(6,6), TTP(6,6), V1(6,6)
DIMENSION V2(6,6)
DOUBLE PRECISION MK
M = IBAND
DO 10 I = 1, ND
   DO 10 J = 1, M
10  MK(I,J) = 0.
   PHI = 4. * ATAN(1.0)
   DO 100 K = 1, M
      CALL TRANS(K, DC, T, MMB, NBS)
      CALL ELAK(K, E, EL, AA, ZI, EKL, MMB)
      CALL TRANS(T, W, 6, 6)
      CALL MULT(W, EKL, V, 6, 6, 6)
      CALL MULT(V, T, EEG, 6, 6, 6)
   M = 6
   DO 60 I = 1, M
      IL = IVC(K, I)
      IF (IL .EQ. 0) GO TO 60
      DO 60 J = 1, M
         IN = IVC(K, J)
         IF (IN .EQ. 0) GO TO 60
         IL1 = IABS(IL)
         IN1 = IABS(IN)
         JJ = IL1/IL
         KK = IN1/IN
         IF (IN1 .LE. IL1) GO TO 60
         L = IN1-IL1+1
         MK(IL1, L) = MK(IL1, L) + (JJ*KK*EEG(I, J))
60    CONTINUE
50    CONTINUE
60    CONTINUE
100   CONTINUE
   RETURN
END
SUBROUTINE GEO(I, XL, MEMB, AA, ELG, E)

* THIS SUBROUTINE CALCULATES THE ELEMENT GEOMETRIC STIFFNESS *

* *

INTEGER*8 (A-H, O-Z)
DIMENSION ELG (6, 6), XL (MEMB), AA (MEMB), E (MEMB) /0 (6)
CL=XL (I)
ELG (1, 1) = 0.0
ELG (1, 2) = 0.0
ELG (1, 3) = 0.0
ELG (1, 4) = 0.0
ELG (1, 5) = 0.0
ELG (1, 6) = 0.0
ELG (2, 2) = (6.0 / (5.0 * CL))
ELG (2, 3) = 1/10.
ELG (2, 4) = 0.0
ELG (2, 5) = -ELG (2, 2)
ELG (2, 6) = 1/10.
ELG (3, 3) = (2.0 / 15.0) * CL
ELG (3, 4) = 0.0
ELG (3, 5) = -1/10.
ELG (3, 6) = -CL / 30.
ELG (4, 4) = 0.0
ELG (4, 5) = 0.0
ELG (4, 6) = 0.0
ELG (5, 5) = ELG (2, 2)
ELG (5, 6) = -1/10.
ELG (6, 6) = (2.0 / 15.0) * CL
DO 1 II=1, 6
DO 1 J=1, 6
IF (II.LT. J) GO TO 1
ELG (II, J) = ELG (J, II)
1 CONTINUE
RETURN
END

SUBROUTINE TRANS(W, V, K, L)

* THIS SUBROUTINE WILL TRANPOSE A MATRIX *

* *

IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION W (L, K), V (K, L).
DO 10 I=1, K
DO 10 J=1, L
V (I, J) = W (J, I)
10 RETURN
END

SUBROUTINE GEOKM(NKG, EL, DC, NDF, IYC, MK, MEMB, NDFD, NBS,
AA, N, UTIL, E, PP, IBAND, UK, UTOTL, INDEX)
* THIS SUBROUTINE WILL CALCULATE THE MASTER GEOMETRIC STIFFNESS *

IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION EL(MEMB), DC(MEMB, 2), IVC(MEMB, 6), MK(NDFD, IBAND)
DIMENSION GKL(6, 6), GKL(6, 6), T(6, 6), U(6, 6), V(6, 6), V1(6, 6)
DIMENSION EN1(6, 6), AA(MEMB), UTOTL(MEMB, 6), E(MEMB), PP(MEMB)
DIMENSION UH(MEMB, 6), UTOTL(MEMB, 6), ELG(6, 6), U(6)
DIMENSION GN1B(6, 6), GN2B(6, 6)
DOUBLE PRECISION MK

I1=IBAND
DO 10 I=1, NDF
DO 10 J=1, M1
10 MK(I, J)=0.0
DO 100 K=1, N
IF (K.GT. MNG) GO TO 100
DO 101 J=1, 6
101 U(J)=UTOTL(K, J)
CALL TRANS(K, DC, T, MEMP, NBS)
CALL TRANS(T, U, 6, 6)
CALL GEO(K, EL, MEMB, AA, ELG, E)
CALL MULT(U1, ELG, V, 6, 6, 6)
CALL MULT(V, T, GKL, 6, 6, 6)
IF (INDEX.EQ.1) GO TO 51
PP(K)=E(K)*AA(K)*((UTOTL(K, 4)-UTOTL(K, 1)))/EL(K)
CONTINUE
N1=6
DO 60 I=1, N1
IL=IVC(K, I)
IF (IL.EQ.0) GO TO 60
DO 50 J=1, N1.
IN=IVC(K, J)
IF (IN.EQ.0) GO TO 50
IL1=IABS(IL)
IN1=IABS(IN)
JJ=IL1/IL
KK=IN1/IN
IF (IN1.GT.IL1) GO TO 50
L=IN1-IL1+1
MK(IL1, L)=MK(IL1, L)+(JJ*KK*GKL(I, J)*PP(K))
50 CONTINUE
60 CONTINUE
100 CONTINUE
RETURN
END

SUBROUTINE TRANS(K, DC, T, MEMP, NBS)

* THIS SUBROUTINE WILL CALCULATE THE TRANSFORMATION MATRICES *

IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION DC((memr,2), T(6,6)

DO 20 I=1,6
DO 20 J=1,6

20 T(I,J) = 0.0
CX = DC(K,1)
CY = DC(K,2)
T[1,1] = CX
T[1,2] = CY
T[2,1] = -CX
T[2,2] = CX
T[4,4] = CX
T[4,5] = CY
T[5,4] = -CY
T[5,5] = CX
IF(K.GT.NBS) GO TO 10
T[6,6] = 1.0
T[3,3] = 1.0

10 RETURN
END

SUBROUTINE ELAK(K,E,EL,AA,ZI,EKL,MB)
*************************************************************************
* THIS SUBROUTINE WILL CALCULATE THE ELEMENT STIFFNESS MATRICES *
*************************************************************************
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION E(MEMR),AA(MEMR),EL(MEMR),ZI(MEMR),EKL(6,6)
DO 30 I=1,6
DO 30 J=1,6

30 EKL(I,J) = 0.0
EKL(I,1) = E(K) * AA(K) / EL(K)
EKL(I,4) = -EKL(1,1)
EKL(2,2) = 12.0 * E(K) * ZI(K) / EL(K) ** 3
EKL(2,3) = 6.0 * E(K) * ZI(K) / EL(K) ** 2
EKL(2,5) = -EKL(2,2)
EKL(2,6) = EKL(2,3)
EKL(3,3) = 4.0 * E(K) * ZI(K) / EL(K)
EKL(3,5) = -EKL(2,3)
EKL(3,6) = 2.0 * E(K) * ZI(K) / EL(K)
EKL(4,4) = EKL(1,1)
EKL(5,5) = EKL(2,2)
EKL(5,6) = -EKL(2,3)
EKL(6,6) = EKL(3,3)
DO 40 I=2,6
II = I - 1
DO 40 J=1,II
EKL(I,J) = EKL(J,I)
RETURN
END

SUBROUTINE MUL(A,B,C,M,K,N)
*************************************************************************
* THIS SUBROUTINE WILL MULTIPLY TWO MATRICES *
*************************************************************************
C

IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION A(N, K), B(K, N), C(N, K)
DO 10 I = 1, N
DO 10 J = 1, K
C(I, J) = 0.
10 C(I, J) = A(I, J) + B(I, J)
RETURN
END

SUBROUTINE COOR(CN, U, NBS, N, CN1, NCT, IVC, MBB, NDFD, N, NJOI)
*****************************************************************************
* THIS SUBROUTINE WILL CALCULATE THE NEW COORDINATE OF EACH NODE DUE TO THE APPLIED LOAD
*****************************************************************************
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION CN(NJOI, 2), U(NDFD), NCT(MBB, 2), IVC(MBB, 6)
DIMENSION CN1(NJOI, 2), NJOI(2)
DO 1 I = 1, NBS
N1 = NCT(I, 1)
N2 = NCT(I, 2)
ND(N1, 1) = IVC(I, 1)
ND(N1, 2) = IVC(I, 2)
ND(N2, 1) = IVC(I, 4)
ND(N2, 2) = IVC(I, 5)
1 CONTINUE
DO 2 I = 1, N
N1 = ND(I, 1)
N2 = ND(I, 2)
IF(N1. LT. 0) N1 = IABS(N1)
IF(N2. LT. 0) N2 = IABS(N2)
IF(N1. EQ. 0) GO TO 3
CN(I, 1) = CN1(I, 1) + U(N1)
3 IF(N2. EQ. 0) GO TO 2
CN(I, 2) = CN1(I, 2) + U(N2)
2 CONTINUE
RETURN
END

SUBROUTINE MBBT(CN, M, DC, AA, ZI, ELGTH, N, MBB, NJOI)
*****************************************************************************
* THIS SUBROUTINE CALCULATES THE MEMBER LENGTHS AND DIRECTION COSINES
*****************************************************************************
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION MBB(2), CN(NJOI, 2), DC(MBB, 2), AA(MBB, 2), ZI(MBB)
DOUBLE PRECISION LGTH
SQR(T(X)) = DSQR(X)
DO 250 I = 1, N
LP = MBB(I, 1)
LQ = MBB(I, 2)
SUB = 0.0
255 J = 1, 2
C

255 SUM = SUM + (CN(LQ, J) - CN(LP, J)) ** 2
    LGTH(I) = SQRT(SUM)
  DO 260 J = 1, 2
260     DC(I, J) = (CN(LQ, J) - CN(LP, J)) / LGTH(I)
250     CONTINUE
     RETURN
END
ENTRY
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