Initiation of bed deformation in meandering channels: A theoretical and numerical study.

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INITIATION OF BED DEFORMATION IN MEANDERING CHANNELS:
A THEORETICAL AND NUMERICAL STUDY

by

Dipanneeta Banerjee

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ABSTRACT

It is attempted to investigate whether the initial flow patterns computed by the vertically averaged hydrodynamic model of Silva 1995, 1999 for “small sinuosity” ($\theta_0 < \approx 30^\circ$) and “large sinuosity” ($\theta_0 > \approx 70^\circ$) channels, are able to predict the expected erosion-deposition patterns for both small and large sinuosity channels, by themselves, without invoking cross-circulation. The significance of cross-circulation in producing the characteristic bed deformation patterns in meandering channels is also studied. Moreover, the function relating the “variable resistance factor” introduced by Silva 1995, 1999, to the channel curvature and the width-to-depth ratio (which, for a flat bed, is a function of only the deflection angle $\theta_0$ and the width-to-depth ratio $B/h_m$), is investigated.

For this purpose, two numerical models are developed and used along with the hydrodynamic model of Silva 1995: the sediment transport model, for the computation of the bed deformation; and the meander path model, which facilitates post-processing of the results. A framework to run the three distinct models sequentially, is presented. Some numerical simulations are then performed, making extensive use of experimental conditions and field measurements from the works of other authors.

The results of these numerical experiments clearly illustrate that the initial flows computed by the hydrodynamic model of Silva 1995, 1999 can, indeed, produce the characteristic bed deformation patterns for both small and large sinuosity channels, without invoking cross-circulation. It is also shown that cross-circulation plays an insignificant role in determining the overall “erosion-deposition” patterns on the bed. Finally, the function defining the “variable resistance factor”, as estimated by Silva 1995, 1999, is modified, and a new form of this function is suggested.
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LIST OF SYMBOLS

(a) General

$A$  cross-sectional area of the flow
$B$  flow width
$C$  dimensional Chezy friction coefficient
$B_R$  regime flow width
$D$  typical grain size
$E$  elevation (z-value) of the free surface
$T$  resultant of the bed shear stress and the curvature stresses
$g$  acceleration due to gravity
$h$  local flow depth
$h_R$  regime flow depth
$h_m$  channel average flow depth
$i$  unit vector
$J_0(\theta_0)$  Bessel function of the first kind and zero-th order (of $\theta_0$)
$J_r$  transverse water surface slope
$k_s$  effective bed roughness
$K_l$  correlation coefficient in the longitudinal equation of motion Eq. (5.3)
$K_r$  correlation coefficient in the radial equation of motion Eq. (5.4)
$K_{ui,ui}$  correlation coefficient in the equations of motion which appear due to vertical averaging
$l$  longitudinal coordinate
$L$  length of a meandering channel over one meandering period
$l_c$  longitudinal coordinate along channel centreline with origin at the cross-over
$n$  radial coordinate with origin at the centreline
$P$  porosity of the bed material
$Q$  flow rate
$q_{ij}$ scalar components of the stress tensor $\Pi_s (= \tau_{ij})$
$q_s$ specific volumetric sediment transport rate
$q_{sb}$ specific volumetric bed-load rate
$q_{ss}$ specific volumetric suspended load rate
$q_{sbl}$ specific volumetric bed-load transport rate in the longitudinal direction
$q_{sbr}$ specific volumetric bed-load transport rate in the radial direction
$R$ centreline radius of curvature of the channel as illustrated in Fig. 1.5
$R_e$ effective radius of curvature of a streamline as given in Eq. (2.25)
$R_O$ centreline radius of curvature at the crossovers
$R_a$ centreline radius of curvature at the apex
$\mathcal{R}$ flow region of the length $L/2$
$S_b$ bed slope of a meandering channel at any $n$ or $r$
$S_{bc}$ centreline slope of a meandering channel ($n = 0$ or $r = 0$
$S_R$ regime slope
$S_v$ valley slope
$S_0$ initial slope
$t$ time
$T$ time taken by a channel to reach its equilibrium bed topography
$T_R$ time needed for a channel to achieve its regime state
$T_\delta$ time at the end of a sand wave development in Fig. 2.1
$U$ local flow velocity vector
$U$ magnitude of $U$
$U_{av}$ cross-sectional average velocity
$U_b$ velocity of flow at the bed
$u$ scalar component of $U$ along $l$ or $l_c$
$u_b$ longitudinal velocity of flow at the channel bed
$u$ scalar component of $U$ along $r$ or $n$
\( v_\alpha \) component of the flow due to cross-circulation

\( v_\alpha * \) cross-circulation velocity at the bed

\( v_* \) shear velocity

\( w \) scalar velocity of \( U \) along \( z \)

\( x \) average direction of flow

\( y \) direction perpendicular to \( x \) to the average direction

\( z \) vertical coordinate

\( z_b \) elevation of the bed surface (measured from any horizontal reference plane)

\( \alpha_q \) dimensionless coefficient in the expression of \( c_q^{-2} \)

\( \gamma \) specific weight of the fluid

\( \Gamma \) cross-circulation

\( \gamma_s \) specific weight of grains in the fluid

\( \gamma' \) parameter in Yalin's bed-load formula, as given in Eq. (2.39)

\( \theta \) deflection angle between the direction of \( l_c \) and the \( x \)-axis at a cross-section at a distance \( l_c \) from a cross-over

\( \theta' \) Shield's parameter in the equations expressing the direction of sediment motion

\( \epsilon \) height of the bed-load sediment layer

\( \theta_0 \) meander angle, i.e., value of \( \theta \) at the cross-over where \( l_c = 0 \)

\( \kappa \) Von Karman constant

\( \Lambda \) meander wavelength

\( \nu \) kinematic viscosity

\( \nu_t \) turbulent kinematic viscosity

\( \Pi_s \) stress tensor

\( \rho \) fluid density

\( \rho_s \) grain density

\( \tau_0 \) local shear stress acting on the bed
\( \tau_S \)  
skin friction bottom stress in Eq. (2.38)

\( \tau_{CH} \)  
shear stress due to dune or ripple form drag, as given in Nelson and Smith 1989a, b

\( \tau_D \)  
shear stress associated with topographic features such as the point bar as given in Nelson and Smith 1989a, b

\( \delta \)  
angle of deviation of the streamlines from the centreline of a circular channel, due to cross-circulation

\( \alpha' \)  
angle between the direction of the bed shear stress, and the \( x \)-axis, as given in Eq. (2.27)

\( \alpha \)  
angle between the transverse velocity and the longitudinal velocity \( \tan \alpha = \omega \)

\( \varphi \)  
direction of sediment motion with respect to the \( x \)-axis, as in Eq. (2.27)

\( \beta \)  
coefficient in Bagnold's bed load formula in terms of the bed velocity \( U_b \)

\( \beta' \)  
coefficient in Bagnold's bed load formula in terms of the vertically averaged velocity \( \bar{U} \)
(b) Dimensionless combinations

- $c$  
  resistance factor at a location $(l_c; n)$ of flow

- $c_s$  
  part of $c$ due to bed friction

- $c_q$  
  part of $c$ due to the stresses $q_{ij}$

- $N'$  
  dimensionless counterpart of $n(= n/R)$ in Eq. (2.36)

- $Fr$  
  Froude number ($= u^2/gh$)

- $Re$  
  flow Reynolds number ($= u_m h_m/\nu$)

- $Re_s$  
  flow Reynolds number in terms of shear velocity ($= u_s k_s/\nu$)

- $X$  
  grain size Reynolds number ($= u_s D/\nu$)

- $Y$  
  mobility number ($= \rho u_s^2/\gamma_s D$)

- $\eta_s$  
  relative flow intensity ($= Y/Y_{cr} = \tau_0/\tau_{0cr}$)

- $\zeta$  
  dimensionless counterpart of $z(= z/h)$

- $\zeta_m$  
  dimensionless counterpart of $z(= z/h_m)$

- $\eta$  
  dimensionless counterpart of $n(= n/B)$

- $\xi$  
  dimensionless counterpart of $\lambda$

- $\Pi_A$  
  dimensionless counterpart of a quantitative flow property $A$

- $\sigma$  
  channel sinuosity ($= L/\Lambda$)

- $\phi_h$  
  dimensionless counterpart of $h(= h/h_m)$

- $\phi_u$  
  dimensionless counterpart of $u(= u/u_m)$

- $\psi_c$  
  dimensionless counterpart of $c(= c/c_m)$

- $\psi_z$  
  dimensionless counterpart of $z_b(= z_b/h_m)$

- $\Phi$  
  dimensionless coordinate in the dimensionless equations of motion
(c) Superscripts and Subscripts

$au$: cross-sectional average

$cr$: value corresponding to the initiation of sediment transport, i.e., the critical stage of sediment transport

$m$: channel-average value

$l$: value along $l$

$r$: value along $r$

"bar" over a quantity signifies its vertically-averaged value.
1.0 INTRODUCTION

1.1 General

i) Meandering is perhaps one of the most striking phenomena found in alluvial streams, and has inspired innumerable researchers in hydraulics, physical geography and geology. Of fundamental importance to hydraulic engineers and scientists is the understanding of the mechanics of turbulent flow in alluvial meanders, as well as of the associated bed and bank deformation. The ability to fully simulate the natural or man-made changes occurring in river meanders is the first step towards the design and execution of most river management projects, especially where navigation, land use and protection, mitigation of floods, and water intake are concerned. In recent times, ecological concerns particularly the preservation of the fish and plant species typical of a stream, have also gained in significance. In meanders, the bed exhibits pool-riffle sequences, whose ecological importance in fish habitat is extensively explained e.g., in Stuart 1953. The installation of structures to enhance fish habitat is also gaining popularity, and the assessment of their impact, too, requires simulation of aspects of bed and bank deformation in meanders. The rapid development of high-speed computers and numerical methods in the latter half of the twentieth century led to an impressive amount of theoretical literature being produced on the mechanics of turbulent flow in meanders and the deformation of their alluvial (movable) boundary. However, a complete understanding of the phenomenon and a satisfactory numerical model simulating the development in time of the alluvial boundary (bed and banks) still continue to elude us.

ii) Both field and laboratory measurements (see Literature Review) show that streams with different sinuosities behave differently. If the sinuosity of the meander stream is “small”, i.e., if the deflection angle $\theta_0$ is “small” ($\theta_0 \approx 30^\circ$), then the bed topography is as shown in Fig. 1.1a: the zones of the most pronounced erosion and deposition are near the cross-overs $O$ and $O'$. If on the other hand, the sinuosity is “large”, i.e., the deflection angle is “large” ($\theta_0 \geq 70^\circ$), these zones are near the apex (Fig. 1.1b). The
bed deformation patterns are closely related to the bank deformation. Indeed, the banks are eroded mostly at those locations where the bed adjacent to them is eroded, whereas a bank “built-up” occurs at those locations where there is deposition at the bed (Yalin and Silva 1993). Hence, the erosion-deposition pattern in Fig. 1.2a leads mainly to the downstream migration of the meandering channel (although the channel does exhibit a weak expansion); the pattern in Fig. 1.2b leads mainly to its expansion.

Figure 1.1: Variation in the location of erosion-deposition zones in natural meandering streams with deflection angles (from Silva 1995)

In the above discussion, the focus has been on the limiting cases of “small” and “large” $\theta_0$. For intermediate values of $\theta_0$ (i.e., $\approx 30^\circ < \theta_0 < \approx 70^\circ$), the location of the erosion-deposition zones are known to be “intermediate” with regard to those of Fig. 1.1a and Fig. 1.1b; and the meandering channel exhibits both, downstream migration and lateral expansion.

It follows that in nature both the bed and the banks of a meandering stream are
Figure 1.2: Migration and expansion of meandering streams (from Yalin and Silva 1993)

deforming\(^1\). However, the time-scale associated with the bank deformation is much larger than that of the bed deformation, and therefore, the deformations of the bed and the banks are usually studied by considering them separately. In other words, the bed deformation, as well as the mechanics of flow in the meandering channel, are studied by treating the banks as rigid (Shimizu and Itakura 1989, Nelson and Smith 1989, Silva 1995, Jia and Wang 1999, etc.).

iii) Because of the difficulties and costs associated with extensive field measurements, laboratory experiments on the mechanics of flow and bed deformation in meandering streams have proved extremely useful. Conventionally, laboratory studies on bed deformation have been carried out in (rigid) channels of a given deflection angle \((\theta_0)\), and by using an initially flat movable bed. An experimental run is then carried out by making a flow rate \(Q\) to enter the channel (at \(t = 0\)), and by keeping \(Q\) constant throughout

\(^1\)It should thus be clear that natural meanders are, in general, very dynamic, changing features- although some meanders are rather stable (unchanging). An explanation for why some meanders are actively expanding (unstable), whereas others are stable, resting on the “regime theory” is reviewed in Section 2.1.
the duration of the run. At the time $t = 0$, the flow past the flat initial bed (which, following Silva 1995, will henceforward be termed initial flow) is not in equilibrium with the channel bed. Thus, this flow will start by eroding sediment from some regions and depositing it in others. With the passage of time, the erosions and the depositions become more and more pronounced; eventually, at a time $t = T_b$, a stage is reached where although the sediment is being transported, the bed is no longer changing. This bed is then said to be in “dynamic equilibrium” (Nelson and Smith 1989, Yalin 1992, Whiting and Dietrich 1993), and the bed topography at this stage has been termed “equilibrium bed topography”. It should be pointed out that as the bed is changing (i.e., as “shoals” emerge at some locations, and “deeps” in others), the flow pattern itself changes. For the “shoals” “steer” the flow away as explained in Dietrich and Whiting 1989, Yalin 1992, etc.

However, the location (in plan view) of the erosion-deposition zones remains practically unchanged throughout the course of an experimental run (i.e., for the time $t \in [0; T_b]$). To put it in other words, the evolution of the bed topography consists of the growth in amplitude of “shoals” and “deeps”, without any significant shift in their location. It follows that the final destiny of the bed is “in-built” into (i.e., predetermined by) the mechanical structure of the flow. Therefore, since the bed topography of “small” and “large” sinuosity channels are rather different, it is only natural to expect the mechanical structure of the initial flows in small and large $\theta_0$ channels to be also rather different. It appears that Yalin 1992 was the first to realize this fact; however, this author deduced it entirely from theoretical considerations - without presenting any experimental evidence. This evidence can be found in the works of Whiting and Dietrich 1993, Silva 1995, Termini 1996. From the measurements of the aforementioned researchers, it follows that the flow patterns in small and large sinuosity channels are indeed rather different. In a small $\theta_0$ channel (see Fig. 1.3a), the initial flow is convectively accelerated near the inner bank in the region between the cross-over $O$ and the apex $a$ (and of course, it is convectively decelerated near the outer bank between $O$ and $a$). This flow exhibits the maximum velocity at the inner bank of the apex $a$ (where the spacing between adjacent streamlines is the smallest).
This type of flow has been termed an “ingoing flow”, by Silva 1995, 1999. In a large $\theta_0$ channel (see Fig. 1.3b), the initial flow is convectively decelerated near the inner bank throughout the entire loop $OaO'$ (and accelerated near the outer bank). This flow exhibits the maximum velocity at the inner bank near the cross-over $O$. This type of flow has been termed an “outgoing flow” by Silva 1995.

![Diagram](image)

**Figure 1.3:** Convergence-divergence zones of meandering streams (from Silva 1995)

iv) The conventional way of simulating the development of the bed topography for $t\in[0; T_b]$ comprises alternately using a “vertically-averaged” hydrodynamic model and a sediment transport model, the former predicting the flow-field, and the latter enabling the determination of the erosions and depositions on the bed. It is generally agreed that the hydrodynamic model should be able to reproduce the flow field accurately at every development stage of the bathymetry, beginning with the initial flow. In other words, the models should be able to predict an ingoing flow for small $\theta_0$ and outgoing flows for large $\theta_0$ channels. Yet, existing (vertically-averaged) hydrodynamic models (e.g., Smith and McLean 1984, Shimizu and Itakura 1989, Jia and Wang 1999) yield ingoing flows for all channels, irrespective of the value of $\theta_0$. Fig. 1.4 shows the initial flows computed
by Smith and McLean 1984: the maximum velocity lies always at the inner bank of the apex. It is interesting to note that, since the “ingoing flow” cannot produce the deposition at the inner bank around the apex which is found in large $\theta_0$ channels, the aforementioned researchers have introduced an “artificial bump” at the inner bank at the apex; to force the solution. The appearance of this bump was attributed to the sediment brought from the outer bank to the inner bank due to the cross-circulation or (see Section 3.3 for aspects of cross-circulatory motion in curved channels). In fact, in these works, the cross-circulation is assumed to have an important role at any stage of development of bed topography. Yet, many field researchers have pointed out much earlier on, that for rivers with a large width to depth ratio, the effects of cross-circulation are negligible.

Figure 1.4: Vertically-averaged flow velocities (from Smith and McLean 1984)

It should be noticed that, (as has been discussed extensively in the Literature Review) in the hydrodynamic models of Smith and McLean 1984, Shimizu and Itakura 1989, etc., the turbulence stresses are usually neglected, or they are approximated by straight channel relations. Silva 1995 (see also Silva 1999) has argued that, with a proper representation of the turbulence stresses the vertically-averaged hydrodynamic model should be able to produce ingoing and outgoing initial flows. In her model, this author considered the effect of the turbulence stresses by means of an increased (or decreased) bed shear stress (which, in dimensionless form, appears as a variable “resistance factor”). The
increment or decrement of the bed shear stress is a function of location in the flow plan and of the channel curvature. This model succeeded in predicting an outgoing initial flow for large sinuosity channels (channels with large $\theta_0$), as well as an ingoing flow for small sinuosity channels. Silva 1995, 1999 went on to argue on the basis of some theoretical considerations that the vertically-averaged initial flows predicted by her model should produce initial bed deformations in agreement with the equilibrium bed topography for channels of all sinuosities, without any need to introduce an “artificial bump” or invoke cross-circulation. However, in Silva 1995, 1999, no attempt was made to use the computed (vertically-averaged) initial flows to determine the associated erosions and depositions. Therefore, it has not been conclusively proved that the accurate reproduction of the vertically-averaged initial flows is sufficient for the accurate prediction of the bed topography. This thesis, which is an extension of the works by Silva 1995, 1999, concerns the bed topography generated by the initial flows. An attempt is made to reveal the significance of the cross-circulation in relation to the bed topography.

v) In accordance with reality, it is assumed that the width-to-depth ratio of the stream is large ($B/h > 10$, say) the curved channel flow is sub-critical ($Fr < 1$) and rough turbulent ($Re_\infty = u_* k_x/\nu > \approx 70$). The meandering channel is assumed to be regular, (in the sense that “the meander loops are symmetrical with respect to the axis of the bend” Yalin 1992) as illustrated in Fig.1.5a; following Silva 1995, 1999, the plan shape of the channel is sine-generated (see Literature Review). The flow width $B$ is treated as constant (in the sense that it does not vary in the flow direction $l_c$). The location in flow plan of the space-point $P$ (Fig.1.5b) is specified by the distance $l$ measured along the longitudinal direction from the cross-over section $O$, and by the distance $r$ to the center of the channel curvature; the elevation of the point is given by the distance $z$ measured from a (horizontal) reference datum (i.e., $P = P(l, r, z)$).
Figure 1.5: Plan shape and pertinent geometric characteristics of a meandering channel (from Silva 1995)
1.2 Objectives and Methodology

The primary objectives of the present study may be outlined as follows:

1. To investigate whether the ingoing and outgoing initial flows (for small and large $\theta_0$, respectively) computed by the hydrodynamic model of Silva 1995, 1999 by themselves, are able to produce the erosion-deposition zones in the expected locations for small and large $\theta_0$ streams, without invoking cross-circulation.

2. To perform some numerical experiments in order to reveal the extent to which the cross-circulatory flow is significant in producing the bed deformation patterns in meandering rivers with different sinuosities and width to depth ratios.

The following point also proved to be of interest while conducting the present research (secondary objective): as mentioned earlier, Silva 1995, 1999 introduced a variable "resistance factor" which would accurately reflect the turbulence stresses produced by the curvature of meandering rivers. However, this factor is completely known (through theoretical considerations and laboratory experiments) only for channels with $\theta_0 = 110^\circ$ and $\theta_0 = 30^\circ$, for the specific conditions used by Silva 1995, 1999 in her experiments. An attempt has been made to investigate the variation of this factor with different parameters ($\theta_0$ and $B/h$).

For the above purposes, two numerical models are developed in this thesis: the sediment transport model counterpart of the hydrodynamic model of Silva 1995, 1999 for the computation of bed erosiions and depositions (Chapter 6); and a meander path model for easy visualization of the results (Chapter 4). A framework to run sequentially the three distinct models (hydrodynamic, sediment transport, and meander path), and thus to determine the bed topography, is also developed (Section 7.1).

Experimental data from other authors have been used extensively in this investigation in order to study the various aspects of the initial bed deformation in meandering flows.
1.3 Layout

The present thesis is divided into eight chapters, which are then further subdivided into sections and subsections. A brief discussion of the contents of each of these Chapters is given below:

Chapter 1 introduces the topic of research, along with its practical relevance. The objectives of the thesis are outlined here.

Chapter 2 contains the Literature Review, and discusses the contributions of previous researchers in this field. A chronological sequence is followed while presenting the works, and an attempt is made to trace the entire history of developments in this field, beginning with the earliest studies, to the present day research. The main focus is on those researchers (theoretical and experimental), whose works have influenced this thesis significantly.

Chapter 3 describes the mechanics of meandering flows. Some fundamental properties of the flow, which are being investigated in this study (e.g., cross-circulation, convective acceleration and deceleration), are discussed at length.

Chapter 4 contains a description of the meander path model (MEANDERPATH). Some mathematical relations pertaining to the geometry of a sine-generated meandering stream are discussed. Some of these relations are used to develop the meander path model, others have been used in the hydrodynamic and sediment transport models.

Chapter 5 presents a brief summary of the vertically-averaged hydrodynamic model (MEANDERFLOW), originally developed by Silva 1995.

Chapter 6 is concerned with the development of the two versions of the sediment transport model (MEANDERBED-I and MEANDERBED-II), which are used to calculate the erosions and depositions on the bed after a given time interval, as well as the actual bed topography at this time.

Chapter 7 describes the “numerical experiments” performed by simulating the experimental conditions of various researchers. The interrelations among the various models which are used to build up the entire framework, is also explained with the help of a flowchart. The predictions from the numerical model are presented, and compared with
the available measured data. A brief discussion on each of the results is presented, with relevance to the Objectives of this thesis.

Chapter 8 is concerned with the conclusions from the present work, regarding the various points in the Objectives. It also presents some suggestions for future research into this topic.
2.0 LITERATURE REVIEW

2.1 Meandering and Regime Formation

Rivers, as pointed out by Leopold and Wolman 1957, are seldom straight. Meander formation is the most common plan shape assumed by rivers in nature, the term ‘meander’ itself having been coined from the plan form of the Meander River in Turkey. Following the pioneering works of Bettess and White 1983, Chang 1988, Yalin 1992, there appears to be general agreement among researchers that a river meanders because it “wants” to achieve its “regime state”. The “regime concept” is usually explained with the aid of the following model. Consider an initially straight channel excavated in an unbounded homogeneous alluvium: the channel has the width $B_0$, the depth $h_0$, and the slope $S_0$. At time $t = 0$, let a certain flow rate $Q$ (which is the bankfull discharge) enter the channel, and proceed with the experiment without varying $Q$. It is further assumed that this is a bankfull flow, capable of transporting sediment, and consequently inducing channel deformation. With the passage to time, the flow deforms the channel, until a time $t = T_R$ is reached, when the flow has formed a certain channel “of its own” which then shows little or no tendency to change with time. This channel, at time $t = T_R$, is called “regime channel” or “stable channel”: it is characterized by the regime width $B_R$, the regime depth $h_R$, and the regime slope $S_R$. Fig. 2.1 illustrates schematically the development of the initial channel $[B_0, h_0, S_0]$ into the regime channel $[B_R, h_R, S_R]$. The development of the regime width $B_R$ takes place much faster than that of the regime slope $S_R$ (and also of the regime depth $h_R$). Indeed, the development of $B_0 \rightarrow B_R$ is practically complete, when that of $S_0 \rightarrow S_R$ is still in its very initial stages (Fig. 2.1). The evolution of the regime channel may thus be almost totally identified with the evolution of its slope $(S_0 \rightarrow S_R)$ at a constant width $(B \approx B_R)$, as stated in Yalin 1992. During this process, the slope of the channel is constantly decreasing (from $S_0$ to $S_R$).

Indeed, in a meandering channel which is expanding, the slope is continually decreasing. This is because through meandering, the channel progressively increases its length.
Figure 2.1: Time scales associated with the development of the regime channel parameters (from Yalin 1992)

between the consecutive points \( O, O', O'' \) on its path, as shown in Fig. 2.2.

Since the elevation of these points remains unchanging, the slope is thus continuously decreasing during this process. In fact, meandering is the most efficient way in which a river can decrease its slope (Jansen 1979, Chang 1988, Yalin 1992). Bettess and White 1983, Chang 1988, Yalin 1992, have suggested that the initial slope of the channel \( S_0 \) can be identified with the valley slope \( S_v \); and if \( S_v < S_R \), then the river will “remain as it is”, i.e., straight. The extension of meandering is entirely dependent on the discrepancy between \( S_v \) and \( S_R \) (Silva 1991, Yalin 1992). If \( S_v - S_R \) is “small”, then the river will meander (expand) only as much as to make \( S_v = S_R \), and it will then stop expanding, becoming a stable meandering river. If, on the other hand, the discrepancy \( S_v - S_R \) is “large”, the river may expand up to the points where the loops start “touching each other”. at which point an “oxbow” will occur, and the river becomes straight again (without ever reaching its regime state).

### 2.2 Plan Shape of an Alluvial Meandering Channel

i) The first mathematical formulation of the centreline of an (ideal) alluvial meandering stream was made by Von Schelling 1951, 1964. He based his derivations on a random
walk model and postulated that the most probable path of the centreline of a meandering river should satisfy the condition,

$$\frac{1}{L} \int_0^L \frac{1}{l^2} \, dl \rightarrow \min,$$

where $L$ is the meander length and $R$ is the centreline radius of curvature (see Fig. 1.5, which illustrates these dimensions, as also the coordinates $l_c$ and $\theta$ which have been used below). He then demonstrated that for this path of the highest probability, the distance along the channel centreline $l_c$ and the deflection angle $\theta_0$ are related as

$$l_c = \frac{1}{\epsilon} \int_{\theta_0}^{\theta} \frac{d\theta}{[2(\cos \theta_0 - \cos \theta)]^{1/2}}. \quad (2.1)$$

However, it is difficult to obtain the value of $l_c$ corresponding to this elliptic integral. Leopold and Langbein 1966 showed that the above expression may be approximated as the sine-generated function,

$$\theta = \theta_0 \cos(2\pi l_c/L) \quad . \quad (2.2)$$
Fig. 2.3 shows the agreement of data from the plan shape of natural meandering streams with sine-generated curves.

Silva 1991 derived this sine-generated curve independently of the probabilistic approach above. Consider the channels shown in Figs. 2.4a and b. In both cases, the curves shown are continuous, yet there is a discontinuity in the $(1/R)$ diagram. Nature would never create such curves, for the moving fluid particles would experience jolts at the locations where the value of the of $1/R$ suddenly changes.

Thus it follows that not only the function representing the centreline of the stream itself, $\theta = f(l_c)$ (curve $C_0$ in Fig. 2.5a), but also its derivatives must be continuous. Furthermore, the function $\theta = f(l_c)$, along with its derivatives, must be periodic and antisymmetrical. The curves $C_1$ and $C_2$ in Fig. 2.5b and c), which represent the derivatives of the function, are given by:

$$\frac{d\theta}{dl_c} = \frac{1}{R} \quad \text{and} \quad \frac{d^2\theta}{dl_c^2} = \frac{d}{dl_c} \left( \frac{1}{R} \right)$$ (2.3)

Thus $C_0$ and $-C_2$ must be in phase with one another. The simplest way of achieving this is to consider them to be proportional. Consequently,

$$\theta = -\text{const} \frac{d^2\theta}{dl_c^2}. \quad (2.4)$$

Integrating this linear and second order homogenous differential equation, Silva 1991 obtained the sine-generated curve given by Eq. (2.2). Yalin 1992 succeeded in deriving the sine-generated curve using variational calculus. This author argued, following Leopold and Langbein 1966, that "... meanders are not mere accidents of nature, but the form in which rivers do the least work in turning ...". Hence, for a river turning in a meander loop having a given average curvature (square),

$$\frac{1}{R_{av}^2} = \frac{1}{L} \int_0^L (1/R)^2 dl,$$ (2.5)

the average rate of change of its curvature $(1/R')$ should be minimum:

$$\frac{1}{L} \int_0^L [(1/R')]^2 dl \to \text{min.} \quad (2.6)$$
Figure 2.3: Comparision of sine-generated curves with data from the plan-shape of meandering streams, (from Silva 1991)
Figure 2.4: Discontinuity of curvature in continuous curves (from Yalin 1992)

Figure 2.5: The function $\theta = f(l_c)$ and its derivatives (from Silva 1991)
From this consideration, he then derived the sine-generated relation of Eq. (2.2) using the principles of variational calculus.

ii) Another important feature of natural meandering streams is the meander wavelength $\Lambda$ (which is the distance between crossovers as shown in Fig. 1.5). It has been shown by Yalin 1977 that the meander wavelength $\Lambda$ of an alluvial meandering stream is proportional to the flow width $B$, the proportionality factor being $\approx 2\pi$, i.e.,

$$\Lambda \approx 2\pi B$$

(2.7)

Fig. 2.6 shows the agreement of Eq. (2.7) with extensive field and laboratory data.

![Graph showing the relationship between $\Lambda$ and $B$](image)

Figure 2.6: Relation between meander wavelength $\Lambda$ and flow width $B$ in alluvial meandering streams (from Yalin 1992)
2.3 Experimental Research

2.3.1 Review of Laboratory Experiments

The mechanics of turbulent flows and aspects of the bed deformation in meanders have been the object of intensive research since the middle of the twentieth century.

In the earlier experimental works (see e.g., Einstein and Harder 1954, Ippen and Drinker 1962, Yen and Yen 1971, Francis and Asfari 1971, Varshney and Garde 1975, Kikkawa, Ikeda and Kitagawa 1976, Chaudhry and Narasimhan 1977, de Vriend and Koch 1978, Odgaard 1984, Steffler 1984, Almquist and Holley 1985, etc.) the laboratory “meandering” channels consisted of one single circular bend or of sequences of circular bends connected in reverse directions. An example of a single circular bend is the channel used by Kikkawa, Ikeda and Kitagawa 1976 (shown in Fig.2.7). The center line radius of the channel was 450 cm and the central angle was 306°. A 2m long straight reach was placed before the bend to create an area for adjusting flow conditions. In some cases, the bends were connected by adding a straight portion. An example of this type of channel is shown in Fig. 2.8. This channel, which was used by Yen and Yen 1971, consisted of two 90° bends of 8-ft center line radius connected in reverse direction by a straight “tangent” reach 14 ft long.

![Figure 2.7: Schematic plan of channel used by Kikkawa, Ikeda and Kitagawa 1976](image)

Although the experiments in circular channels have provided useful information on
Figure 2.8: Experimental conditions of Yen and Yen 1971

various aspects of flow in curved channels (Chang 1988), there appears to be general agreement that the flow in such channels is considerably different from that in these sine-generated meandering channels. Hence, the older works bear a limited relevance only to the current topic (Hooke 1974, Silva 1991, Yalin 1992). It seems that Hooke 1974 was the first researcher to realize this fact, and was thus the first to conduct laboratory measurements in sine-generated meandering channels. Hooke’s channel, which is shown in Fig. 2.9, has $\theta_0 = 55^\circ$. The width of the channel was $B = 1m$ and the wavelength was $\Lambda = 10.33m$. In his experimental research, Hooke focused mainly on the shear stress acting on the bed of the channel and the equilibrium bed topography; in particular, he studied the effect of the width-to-depth ratio on the equilibrium bed topography. He conducted four experimental runs, where the initial bed slope was nearly the same, but having different flow depths (and thus, flow rates). Table 2.1 gives Hooke’s experimental conditions, (along with the conditions used by the other researchers whose works are discussed in this section).

Contour maps for the equilibrium bed topography for each of his runs are shown in Figs. 2.10a, b, c, and d. In all of his experiments, the maximum erosion-depositions occur around the apex (or somewhat downstream of it). No significant influence of the width-
Figure 2.9: Plan view of Hooke's channel (from Hooke 1974)

Table 2.1: Summary of the experimental conditions used by some previous researchers

<table>
<thead>
<tr>
<th>Author</th>
<th>$\theta_0$</th>
<th>$B$ (m)</th>
<th>$Q$ (l/s)</th>
<th>$S_{bc}$</th>
<th>$B/h_{sw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hooke 1974</td>
<td>55°</td>
<td>1.00</td>
<td>10-50</td>
<td>$\approx 1/500$</td>
<td>8-20</td>
</tr>
<tr>
<td>Hasegawa 1983</td>
<td>20°</td>
<td>0.22</td>
<td>0.53</td>
<td>1/160</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>0.3</td>
<td>0.75;1.87</td>
<td>1/300;1/70.9</td>
<td>12.34</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>0.22</td>
<td>0.87</td>
<td>1/120</td>
<td>16</td>
</tr>
<tr>
<td>Ikeda &amp; Nishimura 1986</td>
<td>40°</td>
<td>0.3</td>
<td>2.60</td>
<td>1/720</td>
<td>6.0</td>
</tr>
<tr>
<td>Whiting &amp; Dietrich 1993c</td>
<td>10°</td>
<td>0.25</td>
<td>0.97-1.67</td>
<td>1/240-1/210</td>
<td>12-18</td>
</tr>
<tr>
<td></td>
<td>20°</td>
<td>0.25</td>
<td>1.04-1.08</td>
<td>1/230-1/150</td>
<td>14-17</td>
</tr>
<tr>
<td>Whiting &amp; Dietrich 1993a</td>
<td>100°</td>
<td>0.25</td>
<td>0.94-1.18</td>
<td>1/250-1/167</td>
<td>15-17</td>
</tr>
<tr>
<td></td>
<td>115°</td>
<td>0.125-0.520</td>
<td>0.42-1.26</td>
<td>1/250-1/150</td>
<td>5-64</td>
</tr>
<tr>
<td>Whiting &amp; Dietrich 1993b</td>
<td>100°</td>
<td>0.25</td>
<td>1.18</td>
<td>1/250</td>
<td>13</td>
</tr>
<tr>
<td>Silva 1995</td>
<td>110°</td>
<td>0.4</td>
<td>2.0</td>
<td>1/1120</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>0.4</td>
<td>2.0</td>
<td>1/1000</td>
<td>13</td>
</tr>
<tr>
<td>Termini 1996</td>
<td>110°</td>
<td>0.5</td>
<td>6.5</td>
<td>$\approx 1/270$</td>
<td>17</td>
</tr>
</tbody>
</table>
to-depth ratios on the erosion-deposition patterns was noticed. Based on his observations, Hooke was the first laboratory researcher to point out that “the importance of secondary currents in determining bed geometry is often overstated”.

Hasegawa 1983 carried out measurements of equilibrium bed topography in sine-generated channels having $\theta_0 = 20^\circ$ and $30^\circ$ (Table 2.1). Fig. 2.11 shows the equilibrium bed topography for the $\theta_0 = 30^\circ$ channel. Hasegawa carried out measurements also in a $90^\circ$ channel — however, this channel is not symmetric, hence it does not follow a sine-generated curve. Yet, the largest erosion-depositions are around the apex, just as in the case of a sine-generated curve. Hasegawa also reports measurements of velocity fields over the equilibrium bed topography.

Ikeda and Nishimura 1986 measured the equilibrium bed topography for a $40^\circ$ meander angle channel (Table 2.1). The sinuous channel was “… connected to a straight inlet reach of 162 cm” . They studied the development of the equilibrium bed beginning with a flat movable bed. Measurements were taken for the elevation of the free surface and the bed. The contour map of the equilibrium bed is as depicted in Fig. 2.12. The discontinuities in the contour lines probably occurred due to the fact that “The fine sand used inherently generates sand ripples which made it difficult to depict a contour map of the bed topography” (Ikeda and Nishimura 1986). Nevertheless, it is quite clear that the regions of the most pronounced erosions and depositions occur around the apex. In fact, the contour maps of the bed bear a marked resemblance to those of Hooke 1974.

An extensive work on flow and erosion-deposition patterns in sine-generated channels of small and large sinuosity channels has been carried out by Whiting and Dietrich 1993a, b, c (Table 2.1). Whiting and Dietrich 1993c conducted laboratory tests on movable bed sine-generated channels of small sinuosities ($\theta_0 \leq 20^\circ$). Fig. 2.13 shows the equilibrium bed topography for the channel having $\theta_0 = 20^\circ$. The patterns of erosion-deposition are in agreement with that obtained by previous authors (Losiyevski, whose measurements are shown in Fig. 1.1a in Introduction, Hasegawa 1983, etc.): the largest erosion-depositions are around the crossovers.

In Whiting and Dietrich 1993 a and b, an intensive laboratory study on channels of
Figure 2.10: Contour maps of the bed topography as measured by Hooke 1974 for runs 10, 20, 35 and 50 l/s
Figure 2.11: Contour map of the bed topography for a $\theta_0 = 30^\circ$ channel as measured by Hasegawa 1983 (from Shimizu and Itakura 1989)

Figure 2.12: Contour maps of the bed topography as measured by Ikeda and Nishimura 1986
large sinuosities ($\theta_0 = 100^\circ, 115^\circ$) is presented. The equilibrium bed topography is as shown in Fig. 2.14. However, one of the purposes of this work was to investigate the formulation and migration of bars in meandering channels — which explains the large width-to-depth ratios used. The presence of bars superimposed on the topography due to the channel curvature (and concealing the topography) it is quite clear from Fig. 2.14. In all of the above-mentioned works, the measurements were carried out over the equilibrium bed. No effort was made to study the initial flow over the flat bed or to analyze its role in the evolution of the ultimate stable bed morphology. The first study in which there was an effort to investigate the initial flow, is that of Whiting and Dietrich 1993 b, where a set of measurements were carried out in a channel having $\theta_0 = 100^\circ$ and a flat rigid bed. The flow depths were measured and the vertically averaged velocity $\bar{u}$ was computed from an approximate relation derived by decoupling the water surface slope “into a centreline component and a transverse component controlled by the centrifugal acceleration”. The relation is given by:

$$\frac{\partial E}{\partial l_c} = \frac{\partial E_c}{\partial l_c} - \frac{n}{(1 - n/R)g \partial l_c} \left( \frac{\bar{u}^2}{R} \right)$$  \hspace{1cm} (2.8)

In Eq. (2.8), $E$ is the elevation of the water surface at any point in the channel, and $E_c$ is the water surface elevation of the channel centreline; $l_c$ and $n$ are the longitudinal
Figure 2.14: Contour plot of the equilibrium bed for a channel with $\theta_0 = 115^\circ$ (from Whiting and Dietrich 1993a)

and radial coordinates respectively (Fig. 1.5). The vertically averaged flow velocities are assumed to be 0.8 times the surface velocities. Figs. 2.15a and b illustrate the results obtained by the author. The resultant flow is "outgoing" with the high velocity cores lying near the cross-overs. It is evident from Fig. 2.15 a that the "corridor of high velocity" crosses from the inner to the outer bank in the meander loop, and thus the flow is easily discerned as "outgoing".

A more extensive study of the initial flow is presented by Silva 1995, in which experiments were carried out in channels having $\theta_0 = 30^\circ$ and $\theta_0 = 110^\circ$ and a rigid flat bed. The measured vertically averaged flow velocity vectors are shown in Figs. 2.16a and b. Her experimental conditions are described in Table 2.1. As seen from the Figures, the "ingoing" nature of the flow in the 30° channel bears a marked contrast to the "outgoing" nature of the flow in the 110° channel. Termini 1996 extended the work of Silva 1995 by carrying out measurements on a sine-generated channel with $\theta_0 = 110^\circ$ under flow conditions very similar to those of Silva 1995. However a much larger slope and a smaller grain size were used in order to obtain a more significant bed deformation. The "initial
Figure 2.15: Free surface velocities and water surface topography in the flat bed experiment (from Whiting and Dietrich 1993b)

flow” measurements are in agreement with those obtained by Silva 1995. The contours of divergence of the sediment transport rate \( \nabla q_b \) which was computed by the measured values of flow depth and vertically averaged flow velocities over the flat bed, are shown in Fig. 2.17. The equilibrium bed topography (Fig. 2.18) was also measured (see Fig. 2.18). It should also be noted that the erosion-deposition patterns as produced by the initial flow (the zones of erosion coincide with the zones of positive \( \nabla q_b \), and the zones of deposition coincide with those of negative \( \nabla q_b \)) and as seen in the equilibrium bed, seem to be more or less in agreement with each other, i.e., there has not been any major shift in the location of the erosion-deposition zones in the flowplan during the evolution of the bed topography.

Thus, the works of the previous researchers, when viewed together, cover a wide range of values of \( \theta_0 \) and one can obtain a very good idea about the variation in the location of the most pronounced erosion-deposition zones with \( \theta_0 \). Using data from a number of authors, Whiting and Dietrich 1993a, produced the curve shown in Fig. 2.19 which relates the appearance of the location of the first pool with respect to the bend apex, to the deflection angle of the channel. It should be mentioned that in this figure, the origin of the dimensionless coordinate \( \frac{L_c}{L} \) (and thus the origin of \( L_c \)) is not at the cross-over.
Figure 2.16: Flow fields in channels with $\theta_0 = 30^\circ$ and $\theta_0 = 110^\circ$ (from Silva 1995)
Figure 2.17: $\nabla q_s$ over the flat bed of the 110$^\circ$ channel (from Termini 1996)
Figure 2.18: Equilibrium bed as obtained by Termini 1996 (at $t = 400$ minutes)

Figure 2.19: Variation in the position of the initial pool with the meander angle (from Whiting and Dietrich, 1993b)
but at the apex.

2.3.2 Review of Field Measurements

The available field measurements are very few in comparison to the vast amount of laboratory research, due to practical constraints. Among the early field researchers, Matthes 1941 (also 1948), Rozovskii 1961, Jackson 1975 (also 1976), Bridge and Jarvis 1976 (also 1977), Hickin 1978 (also Nanson and Hickin 1983) are singularly insightful and form the pioneering literature on this subject. Rozovskii 1961 worked on data obtained from the Desna River, which is a branch of the Dnieper in the former U.S.S.R. Kinoshita 1961 investigated the topographical features of alluvial meandering streams of Japan, and studied the channel migration of such streams. Jackson 1975 studied the flow field and bed topography of the sand bedded Lower Wabash River (Fig. 1.1b) in Southeastern Illinois. Hickin 1978 (as also Hickin and Jarvis 1983) measured flow components in the Beatton River, Canada. Dietrich, Smith and Dunne 1979, Dietrich and Smith 1983, Dietrich 1987, Nelson and Smith 1989 all refer to a series of measurements carried out in the Muddy Creek (Fig. 1.1b), which is a sand-bedded, lowland tributary of the upper Green River in western Wyoming. Whiting and Dietrich 1989 studied the Solfatara Creek, a gravel bed channel in Yellowstone National Park, Wyoming while investigating the “topographically induced convective acceleration” in meandering streams. Bridge and Jarvis 1982 measured the major flow patterns and bed topography in the South Esk River in Scotland (Fig. 2.20). Forbes 1983 observed the bed topography in the gravel bedded Lower Babbage River in northern Yukon. Geldof and de Vriend 1983 investigated the water depth and vertically averaged flow velocity at a number of bends in the river Dommel in the Netherlands. In all of the above works, the patterns of erosion-deposition are seen to vary with the sinuosity of the meandering river. In channels of low sinuosity, the most pronounced erosion-depositions occur near the crossovers, just as in the case of the laboratory streams described in the previous sub-section. For the measurements taken in bends of large \( \theta_0 \), such as the Muddy Creek or the Lower Wabash River (Fig. 1.1b), the maximum erosion-depositions are found to be near the apex. In summary, the measurements of the
field researchers seem to confirm the results of the laboratory experiments.

Figure 2.20: Bed topography of the South Esk River (from Bridge and Jarvis, 1977)

Even though the above mentioned works are of immense scope and significance, it should be remembered that all of the laboratory and field measurements were necessarily undertaken in streams with small $B/h$ ratios ($B/h \leq 20$). Thus, Dietrich 1987 commented that “The necessity of working on relatively small rivers to collect accurate and thorough data has resulted in a biasing of observations to stream channels with low width-depth ratio, typically 7 to 20”. Todate, “No detailed measurements of the kind needed to investigate mechanics and test theory have been collected in bends of high width-depth ratio” (Dietrich 1987). As pointed out in Chapter 3, the intensity of the cross-circulation $\Gamma$ decreases with increase in the $B/h$-ratio. But the exact role and significance of the cross-circulation $\Gamma$ to the development of the bed topography in natural meandering streams (where the $B/h$ ratio is large, i.e., $B/h \gg 20$, say) seems to be not quite known. All field researchers (G.Matthes, S.Leliavskii, etc.) appear to agree that cross-circulation plays a small role in the determination of the overall erosion-deposition patterns. Matthes 1948, based on his observations on the Mississippi River, remarked that “transverse circulation only takes place in distorted experimental models, and in such channels whose width is small compared to its depth ...”. Among the laboratory researchers, Hooke 1979 stated that “the importance of secondary circulation in determining the geometry of river beds in meanders has been over-emphasized for many years, and it will take some time to bring the significance of such flow patterns into proper perspective.” However, Hooke 1974 also pointed out that “... while the existence of the point bar cannot be attributed to secondary currents, the detailed geometry of the bar is at least partly controlled by them".
2.4 Theoretical Research

2.4.1 Hydrodynamic Models

The turbulent flow in meandering open channels is determined by the three equations of motion and the equation of continuity, which form a system of four non-linear differential equations (see, e.g., Schlichting 1968, Milne-Thompson 1968, etc.), the four unknowns being the three scalar components $u, v, w$ of the flow velocity vector $U$ along the $l, r$ and $z$ directions respectively and the flow depth $h$. Because of the complexity of the governing system of equations, “it is not surprising that these equations were solved in the past by introducing a number of drastic simplifications (although less so in recent works)” (Silva 1995).

It was, in fact, Rozovskii 1957, who first attempted to use the equations of motion and continuity to study a curved open-channel flow. Rozovskii limited his analysis to a short channel region of the length $\delta l_c$ ($l_c$ being the longitudinal coordinate measured along the channel centreline - see Fig. 1.5) and within this region, he identified the flow with a “fully developed” circular flow i.e., the flow in an infinitely long circular channel, (spiral flow). He further confined his solution to the neighborhood of the channel centreline where $w \approx 0$, and assumed that $v \gg u$. With these simplifications, Rozovskii arrived at the following reduced forms of the equations of motion, where all convective terms are neglected:

$$-gS_b + \frac{1}{\rho} \frac{\partial \tau_{zl}}{\partial z} = 0$$

$$-gJ_r + \frac{1}{\rho} \frac{\partial \tau_{zr}}{\partial z} + \frac{u^2}{r} = 0$$

where $g$ is the acceleration due to gravity, $J_r$ is the transverse water surface slope, and $S_b$ is the longitudinal slope of the bed. By further assuming a logarithmic velocity distribution, he obtained a parabolic distribution for the kinematic viscosity $\nu_t = f_{\nu_t}(z)$. Thus, he simplified the equations of motion to arrive at an analytical solution for $v$ and $u$ in the neighborhood of the centreline. His radial velocity was completely due to cross-
circulation, i.e., $v = v_\alpha$ (see Chapter 3 for an explanation of the origin of cross-circulation). The expression for the cross-circulation velocity at a certain flow depth, $v_\alpha$ obtained by Rozovskii 1957 (see also Chang 1988) are as follows:

In the case of a “smooth bottom”,

$$
\frac{v_\alpha}{\bar{u}} = \frac{1}{\kappa^2} \frac{h}{R} \left[ F_1(\zeta) - \frac{g^{1/2}}{\kappa C} F_2(\zeta) \right] \tag{2.11}
$$

in the case of a “rough bottom”,

$$
\frac{v_\alpha}{\bar{u}} = \frac{1}{\kappa^2} \frac{h}{R} \left[ F_1(\zeta) - \frac{g^{1/2}}{\kappa C} F_2(\zeta) + 0.81 + ln(\zeta) \right] \tag{2.12}
$$

In these expressions, $\kappa$ is the Von Karman constant, ($\kappa \approx 0.4$), $C$ is the dimensional Chezy friction coefficient, and the functions $F_1(\zeta)$ and $F_2(\zeta)$ are given by

$$
F_1(\zeta) = \int \frac{2 \ln \zeta}{\zeta - 1} d\zeta, \tag{2.13}
$$

and

$$
F_2(\zeta) = \int \frac{\ln^2 \zeta}{\zeta - 1} d\zeta. \tag{2.14}
$$

In these expressions, $\zeta = z/h$ is the “relative depth”, $h$ being the local flow depth.

Following the pioneering work of Rozovskii, several other authors studied the meandering flow with the aid of the “fully developed” circular flow (Yen and Yen 1971, Ikeda 1975, Kikkawa et. al. 1976, also 1974, etc.) . Of particular relevance is the expression produced for $v_\alpha$ by Kikkawa, Ikeda and Kitagawa 1976 (also 1974):

$$
\frac{v_\alpha}{\bar{u}} = F^2 \frac{1}{\kappa^2} \frac{h}{R} \left[ F_A(\zeta) - \frac{v_\alpha}{\bar{u}} F_B(\zeta) \right], \tag{2.15}
$$

where $F$ is the radial distribution of the longitudinal velocity $u$ normalized by the cross sectional average velocity $\bar{u}_{av}$. The functions $F_A(\zeta)$ and $F_B(\zeta)$ are given by

$$
F_A(\zeta) = -15 \left( \zeta^2 \ln \zeta - \frac{1}{2} \zeta^2 + \frac{15}{54} \right) \tag{2.16}
$$

and

$$
F_B(\zeta) = \frac{15}{2} \left( \zeta^2 \ln^2 \zeta - \zeta^2 \ln \zeta + \frac{1}{2} \zeta^2 - \frac{19}{54} \right) \tag{2.17}
$$
Figure 2.21: Graphs of the functions $F_A$ and $F_B$ from Kikkawa, Ikeda and Kitagawa, 1976

The graphs of these functions are given in Fig. 2.21.

Engelund 1974 was the first to appreciate the fact that meander flows are not completely circular flows and should not be studied as such. He was also the first to solve for the “vertically-averaged” flow, which has since then, become the common practice in such studies. He retained the convective acceleration term $u \frac{\partial u}{\partial l}$ in the vertically averaged longitudinal equation of motion (see the full form of the vertically-averaged equations of motion in Chapter 5 - Eq. (5.3) and Eq. (5.4)); however, no convective term was introduced in the radial equation of motion (which, in fact, was but an equilibrium condition between the centrifugal and radial pressure forces). The terms involving the turbulence stresses ($q_t$ and $q_r$ in Eq. (5.3) and Eq. (5.4)) were still neglected.

In Kalkwijk and de Vriend 1980, and de Vriend and Geldof 1983, all the convective acceleration terms in the longitudinal equation of motion were retained. The convective acceleration terms in the radial equation of motion were still neglected. The equations of motion and continuity were solved in their steady state, vertically-averaged forms (expressed in the curvilinear system of coordinates). de Vriend and Geldof 1983 obtained reasonably good agreement with the data collected from the Dommel River in Holland.

It appears that Smith and McLean 1984 were the first to solve the steady state equations
of motion retaining all the convective acceleration terms. (Introductory remarks regarding this model are also to be found in Dietrich, Smith and Dunne 1979, which is primarily an experimental work, and Dietrich and Smith 1983.) They used a perturbation expansion model in which the vertically averaged flow, expressed in the “channel-fitted” system of coordinates (l-n), was solved for the predominant force balance, yielding the “zero-order solution”. Corrections were then added for the introduction of successively smaller terms, the solutions for a given order being always significantly smaller than the solution of the previous order. Calculations were done by a forward marching procedure through the identical loops of the meandering stream until the solution converged to a desired accuracy. Thus, they were the first to utilize the periodic flow conditions that exist in a meandering channel. However, the terms involving the turbulent stresses $q_{ij}$ were still neglected. A number of meandering angles were chosen and the solutions of the initial flow over these flat bed streams were illustrated. As mentioned in the Introduction, the researchers always computed an ingoing flow irrespective of the value of the meander angle (Fig. 1.4).

Struiksma et al. 1985 presented a similar work, but compared the results to semi-circular laboratory channels. “A most promising verification” of the computational results was also obtained with field data from a meandering reach of the Waal River, which is the main Rhine branch in the Netherlands. As in the case of Smith and McLean 1984, the vertically-averaged equations of motion and continuity were used and all the convective acceleration terms were retained. However, the coordinate system and the computational scheme used in this model were different from those of Smith and McLean. Although, according to the authors, the model is applicable only to river geometries which can be modeled by “a series of circular bends”, there is no reason why it should not have predicted similar results to those of Smith and McLean when applied to sine-generated channels.

Shimizu and Itakura 1989 formulated a hydrodynamic model in which not all terms comprising $q_i$ and $q_r$ in Eq. (5.3) and Eq. (5.4) were zero, but there was still considerable simplification. Diffusion terms from the stress tensor were retained in the flow equations.
to achieve better convergence of results. Thus the terms pertaining to \( \frac{\partial}{\partial s} \left( \bar{v}_t \frac{\partial \bar{u}}{\partial t} \right) \) and \( \frac{\partial}{\partial n} \left( \bar{v}_t \frac{\partial \bar{u}}{\partial n} \right) \) were included in the depth averaged equations of motion (both longitudinal and radial) expressed in the \( l - n \) system of coordinates (It should be noted that for Shimizu and Itakura, \( n = 0 \) at the left bank). The “diffusion coefficient”, \( \bar{v}_t \) (which can be identified with the kinematic viscosity), was calculated by relating it to the shear velocity \( \nu_* \), and assuming a linear distribution of the shear stress. The model results were verified against the experimental measurements of the \( \theta_0 = 30^\circ \) channel of Hasegawa 1983, and a good agreement was obtained.

Silva 1995 was the first to consider the effect of all of the turbulent stresses in the equations of motion. However, the direct evaluation of \( q_l \) and \( q_r \) in Eq. (5.3) and Eq. (5.4) is bypassed. The effect of the turbulence stresses is taken into account through the introduction of a “variable resistance factor”. It seems that this variable resistance factor is easier to evaluate than \( q_l \) and \( q_r \) present in the equations of motion. A detailed discussion of this model can be found in Chapter 5.

In Jia and Wang 1999, the vertically-averaged flow equations have been solved by the “efficient element method (EEM)”, which is a finite element numerical scheme developed by the authors. The turbulence stresses have been approximated using the Boussinesq assumption which relates the flow velocities to the stresses using the coefficient of eddy viscosity \( \bar{v}_t \) as

\[
\bar{r}_{ij} = \bar{v}_t \left( \bar{u}_{ij} + \bar{u}_{ji} \right)
\]  \hspace{1cm} (2.18)

Here,

\[
\bar{u}_{ij} = \frac{\partial \bar{u}_i}{\partial j} \quad \text{and} \quad \bar{u}_{ji} = \frac{\partial \bar{u}_j}{\partial i}
\]  \hspace{1cm} (2.19)

Two options for the calculation of the eddy viscosity coefficient have been given in the model — the first is by using the depth-integrated parabolic (kinematic) eddy viscosity formula (same as that used by Shimizu and Itakura 1985) given as,

\[
\bar{v}_t = \frac{A_{xy} \cdot \kappa \nu_* h}{6},
\]  \hspace{1cm} (2.20)

37
Figure 2.22: Flat bed velocities calculated by Jia and Wang 1999 for the $\theta_0 = 30^\circ$ channel

Figure 2.23: Flat bed velocities calculated by Jia and Wang 1999 for the $\theta_0 = 110^\circ$ channel
where $A_{xy}$ "is an adjustable coefficient of eddy viscosity; its default value being set to unity". The second is by employing the depth-integrated mixing length model where $\nu_t$ is expressed as

$$\nu_t = \tilde{l}^2 \sqrt{2 \left( \frac{\partial \bar{u}_x}{\partial x} \right)^2 + 2 \left( \frac{\partial \bar{u}_y}{\partial y} \right)^2 + \left( \frac{\partial \bar{u}_z}{\partial y} + \frac{\partial \bar{u}_y}{\partial x} \right)^2 + \left( \frac{\partial \bar{U}}{\partial z} \right)^2}$$  \hspace{1cm} (2.21)

where

$$\tilde{l} = 0.267 \kappa h$$  \hspace{1cm} (2.22)

and the depth-averaged velocity gradient used in Eq. (2.21) is obtained as

$$\frac{\partial \bar{U}}{\partial z} = C_m \frac{v_*}{\kappa h}$$  \hspace{1cm} (2.23)

In the above equation, $C_m$ is a coefficient, whose value has been set to 2.34375 so that Eq. (2.22) will recover the form of Eq. (2.21) for "a uniform flow in which all horizontal velocity gradients vanish". The model has been verified against the initial flows measured by Silva 1995 (Figs. 2.16a,b). A very good agreement is obtained for the $\theta_0 = 30^\circ$ channel, as shown in Fig. 2.22. However, in spite of the introduction of all of the stresses, the numerical results for the $\theta_0 = 110^\circ$, shown in Fig. 2.23 deviate considerably from the experimental measurements. This is not surprising, for Eq. (2.20) and Eq. (2.21) for $\nu_t$ and $\tilde{l}$ are valid, strictly speaking, for straight uniform flows (see e.g., Schlichting 1968, Milne-Thompson 1968, etc).

It has been the common practice of theoretical researchers to use a vertically-averaged model when studying the flow field and sediment transport characteristics in open channel flows. As pointed out by Jia and Wang 1999, the effect of the vertical motion in most open channel flows is usually insignificant, and thus it is generally accepted that the depth-averaged equations may be used for such studies with reasonable accuracy and efficiency. Especially when dealing with sediment transport and bed deformation processes, which necessitates going back and forth between a flow model and a sediment transport model, the vertically-averaged model saves a lot of computational time and memory. However,
over the years, there have been some very prominent works involving 3D modeling of the phenomenon. In these models, invariably the popular $k - \epsilon$ method of turbulence closure is used. Leschziner and Rodi 1979 appear to be the first to have used the $k - \epsilon$ turbulence model (where $k$ is the turbulent kinetic energy and $\epsilon$ is the rate of dissipation) to calculate the flow in strongly curved channels with rectangular cross-sections. In Demuren and Rodi 1986, the same method was used to calculate the flow and pollutant dispersion in meandering streams with rectangular cross-sections. In Demuren 1989, the work was extended to calculate the suspended sediment transport, and in Demuren 1991, a bed-load transport model was included. The resultant model was validated against the measurements of Odgaard and Bergs 1988, which were carried out in a 180° channel bend. Demuren 1993, further generalized the hydrodynamic model for a meandering channel developed in Demuren and Rodi 1986: a finite volume model was employed to solve the three dimensional equations of motion and continuity in the Cartesian coordinate system. The results were validated against the experiments of Almquist and Holley 1985, which used both a rectangular cross-section as well as for deformed bed conditions. The numerical results compared, in general, favorably with the measurements; however, some discrepancies were observed between them.

Wu, Rodi and Wenka 2000 has developed a 3D numerical model for calculating flow and sediment transport in open channels. The turbulence stresses in the flow equations have been estimated using the Boussinesq assumption, $\nu_t$ being calculated from the principles of $k - \epsilon$ turbulence modeling. The model was then tested using the experiments of Odgaard and Bergs 1988 in a 180° channel bend. The numerical results for the depth-averaged longitudinal velocities seem to show a reasonable agreement with the experimental measurements. However, this model has not yet been tested for meandering channel flow conditions.
2.4.2 Sediment Transport Models: Development of the Equilibrium Bed Topography

The current approach of determination of the equilibrium bed topography is due to Struikema 1985, Shimizu and Itakura 1989, Nelson and Smith 1989, etc. The flow field past the initial (movable) bed as computed by the hydrodynamic model (steady-state in the majority of works), is used as input to the sediment transport model, which then predicts the erosion-depositional patterns on the bed under the action of the flow, after a small interval of time $\Delta t$ has elapsed. Thus, the flow is assumed to remain constant for this small interval of time, during which the bed deformation is taking place. The new bed topography is now used as input to the hydrodynamic model to calculate the flow over the deformed bed. The steps are repeated until the stable bed topography is reached. This is schematically represented by the flowchart in Fig. 2.24. The differences in the works arise because of differences in formulation of both the hydrodynamic and sediment transport models, as well as differences in the numerical schemes adopted. Struikema 1985 considers that sediment moves only as bed load, the bed-load rate $q_{sb}$ being estimated with the aid of the Meyer-Peter and Muller formula (see Yalin 1972, Yalin 1992).

The cross-circulation is considered to be of fundamental importance when dealing with the sediment motion. As originally suggested by Van Bendegom 1947, the resultant direction of the sediment motion is assumed to be due to the bed shear stress with the effect of the spiral motion incorporated. According to the author, the direction of the bed shear stress $\tau_b$, which plays the dominant role in causing the sediment motion, does not coincide with direction of the vertically-averaged streamlines, but gets deflected by the spiral flow (see Chapter 3). Thus the expression for the angle $\alpha'$ between the bed shear stress and the $x$-axis of the Cartesian coordinate system followed by the authors (see Fig. 2.25) was given by:

$$\alpha' = \arctan(\frac{\overline{u_y}}{\overline{u_x}}) - \arctan(A \frac{h}{R_x}). \quad (2.24)$$

Evidently, it is the second term which takes the effect of the cross-circulation into account.
Figure 2.24: Flow chart illustrating the iterative scheme used to simulate the development of bed topography in channels (after Nelson and Smith 1989 b)

Figure 2.25: Direction of bed load motion (from Jia and Wang 1999)
Following Jansen 1979, $A$ is a constant for a given flow, and is related to the friction factor $C$. $R_*$ is the effective radius of curvature of the streamline defined as

$$R_* = \frac{h\sqrt{\overline{u}_x^2 + \overline{u}_y^2}}{I}$$  \hspace{1cm} (2.25)

$I$ being a measure of the “intensity” of cross-circulation, evaluated as

$$\beta Ch \frac{\partial I}{\partial l} + I = \frac{h}{R_*} \sqrt{\overline{u}_x^2 + \overline{u}_y^2},$$  \hspace{1cm} (2.26)

The coefficient $\beta$ is equal to 0.6 in Struiksma’s works. At later stages of bed deformation, when “shoals” and “deeps” are prominent, the radial and longitudinal bed slopes may be significant. In this case, gravity also plays a role (by making it easier for particles to roll down the hills and more difficult to climb them). As a result of the gravity force, the direction of the sediment motion $\varphi$ (with respect to the x-axis) is different from the direction $\alpha'$ of the bed shear stress (see Fig. 2.25). After Van Bendegom 1947 and Koch and Flokstra 1980, $\varphi$ is expressed in Struiksma et al. as

$$\tan \varphi = \frac{\sin \alpha' - \frac{1}{f_s \theta'} \frac{\partial z_b}{\partial y}}{\cos \alpha' - \frac{1}{f_s \theta'} \frac{\partial z_b}{\partial x}},$$  \hspace{1cm} (2.27)

where $f_s$ is the shape factor of the grains, and its value is considered to be between 1 and 2 (The value of this factor was experimentally determined to be 1.7 by Talmon et al. 1995). $\theta'$ is Shield’s parameter given as

$$\theta' = \frac{\overline{u}_x^2 + \overline{u}_y^2}{C^2 g^2 \left( \frac{\tau_s}{\gamma} \right) D_{s0}}$$  \hspace{1cm} (2.28)

Finally, the bed deformations at all points on the grid are computed by invoking the sediment transport continuity equation:

$$\frac{\partial z_b}{\partial t} + \frac{\partial q_{sbx}}{\partial x} + \frac{\partial q_{sby}}{\partial y} = 0.$$  \hspace{1cm} (2.29)

$q_{sbx}$ and $q_{sby}$ are the components of the volumetric rates of transport in the $x-$ and $y-$ directions respectively, expressed as
\[ q_{sbz} = q_{sb} \sin \varphi \]  
\[ q_{sby} = q_{sb} \cos \varphi \]  

(with \( q_{sb} \) computed by Meyer Peter formula, as mentioned earlier). Eq. (2.29) is solved by an explicit finite difference scheme. It should be noted that the suspended load transport was neglected in this work.

The next significant contribution in the field was made by Shimizu and Itakura 1989. The overall scheme for the calculation of the bed topography was the same as that of Struiksmma et al., i.e., an iterative procedure between the flow model and the sediment transport model was followed. In fact, their sediment transport model appears to have been influenced by the works of the aforementioned researchers in other ways, as well. Shimizu and Itakura 1989 expressed the sediment transport equation in channel-fitted coordinates, \( l \) and \( n \) (see Fig. 1.5), i.e., as

\[ \frac{\partial z_b}{\partial t} + \frac{1}{1 - P} \left( \frac{\partial q_{sbl}}{\partial l} + \frac{1}{r} \frac{\partial (r q_{sbn})}{\partial n} \right) = 0 \]  

In this equation, \( P \) is the porosity of the bed material, \( q_{sbl} \) is the volumetric bed-load transport rate in the \( l \)-direction, and \( q_{sbn} \) is the transport rate in the \( n \)-direction. Meyer-Peter-Muller formula was used to estimate the bed-load transport rate in the \( l \)-direction. When computing \( q_{sbn} \), both the effect of the cross-circulation and the gravity force due to the tranverse slope of the bed were again considered to be of importance. The formula suggested by Hasegawa 1984 was adopted for estimating \( q_{sbn} \). Thus,

\[ q_{sbn} = q_{sbl} \left( \tan \alpha - \sqrt{\frac{Y_{cr}}{\mu_s \mu_k Y}} \frac{\partial z_b}{\partial n} \right) \]  

The first term gives the direction of the bed shear stress (just as in Struiksmma 1985), and the second term expresses the effect of gravity due to the transverse slope which would tend to make the grains roll from points at higher elevation to points at lower elevation along the cross-section. In Eq. (2.33), \( \mu_s \) is the coefficient of static friction, and \( \mu_k \) is the coefficient of kinetic friction. The value of \( \mu_s \) has been adopted as 1.0 by the authors,
and the value of $\mu_k$ has been considered as 0.45. Thus even though both Shimizu and Itakura 1989, and Struiksma et al. 1985 consider the sediment transport due to the effect of gravity, their mathematical formulations are different. The same can be said about their expressions for $\alpha$. In the works of Shimizu and Itakura 1989:

$$\tan \alpha = \frac{\bar{v}}{u} + \frac{v_{a^*}}{u_b}$$

(2.34)

Here, $\bar{v}/u$ in Eq. (2.34), is in fact, the deviation angle $\omega$ in the present thesis; $u_b$ is the longitudinal velocity at the bed. The second term expresses the effect of the spiral flow (or cross-circulation) and is formulated according to the expressions originally derived by Rozovskii 1961, Engelund 1974, Zimmerman 1977, Kikkawa et al. 1974 (also 1976), for the deviation angle $\delta$ of the streamlines at the bed of a circular channel from the centreline, due to the cross-circulation or spiral flow (see Section 3.2). According to Shimizu and Itakura 1989,

$$\tan \delta = \frac{v_{a^*}}{u_b} = -N_* \frac{h}{r} f_0(\frac{n}{B})$$

(2.35)

Following the expression for the cross-circulation velocity derived by Rozovskii 1961, Engelund 1974 had proposed that $N_* = 7$. This value of $N_*$ has been adopted in the works of Shimizu and Itakura. $f_0(n/B)$ is a function given by Kikkawa et al. 1974 to take the “bank effect” into account. This function is described in detail in Section 6.2.2. A finite difference scheme was adopted by Shimizu and Itakura 1989. Although the flow equations were solved implicitly, an explicit scheme was used for the sediment transport algorithm. The suspended load transport was neglected as in the case of Struiksma et al., 1985. The results of the computational model were verified against the experiments of Hasegawa 1984. The contours of the computed bed topography are illustrated in Fig. 2.26. Even though the regions of erosion and deposition more or less coincide with the experimental results, there appears to be significant discrepancies between the contours of the computed bed topography and the measured bed topography (see Fig. 2.11).

Nelson and Smith 1989a, b extended the flow model of Smith and McLean 1984 and incorporated the sediment transport algorithm. The sediment transport models employed in the two studies were identical. However, in the former work, the model was validated
Figure 2.26: Stable bed topography computed by Shimizu and Itakura 1989 for the experimental conditions of Hasegawa 1984

against the data collected in the Muddy Creek, whereas in Nelson and Smith 1989b, comparisons between the numerical results and the measured values from the laboratory experiments of Hooke 1974 ($\theta_0 = 55^\circ$), and Whiting and Dietrich 1983 ($\theta_0 = 10^\circ$) were provided. The algorithm followed by these authors was, again, the same as that followed by Struiksma et al. 1985, and Shimizu and Itakura 1989. The sediment transport continuity equation was expressed in their works as

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-P} \left( \frac{1}{1-N'} \frac{\partial q_{sbl}}{\partial l_c} + \frac{\partial q_{sbn}}{\partial n} - \frac{q_{sbn}}{(1-N')R} \right) = 0 \quad (2.36)$$

In Eq. (2.36), $N' = n/R$. The bed load transport rate was estimated using Yalin’s equation (Yalin 1972). Thus,

$$q_{sbl} = 0.635 \left[ \frac{(\tau_l)_S}{\rho} \right]^{1/2} DS \left[ 1 - \frac{1}{\gamma'S} \ln(1 + \gamma'S) \right] \quad (2.37)$$

$$q_{sbn} = 0.635 \left[ \frac{(\tau_n)_S}{\rho} \right]^{1/2} \left[ \frac{(\tau_n)_S}{(\tau_s)_S} \right]^{1/2} DS \left[ 1 - \frac{1}{\gamma'S} \ln(1 + \gamma'S) \right] \quad (2.38)$$

where $(\tau_l)_S$ and $(\tau_n)_S$ are the $l-$ and $n-$ components of the “skin friction bottom stress” which, in the work of these authors is different from the bed shear stress $\tau_0$ directly computed from the flow model. The authors argued that $\tau_S$, which is the part of the bed shear stress responsible for the sediment motion can be obtained only after subtracting $\tau_D$, the shear stress equivalent to the dune or ripple form drag and $\tau_{CH}$, which represents the form drag with other topographic features such as the point bar. Expressions for $\tau_D$ and $\tau_{CH}$ were also provided by these authors. $S$ is the shear stress at a point in excess
of the critical shear stress $\tau_{cr}$ required for the sediment motion. $\gamma'$ is defined by

$$\gamma' = 2.45(\rho/\rho_s)^{0.4}(\tau_{cr}/(\rho_s - \rho)gD)^{0.5}$$

(2.39)

The gravitational force was also introduced in the model in the form of a stress. However, according to Nelson and Smith, the gravitational force is not only responsible for sediment transport in the transverse direction, but also in the longitudinal direction. The gravity stress $\tau_g$ may be added to (or subtracted from) the bed shear stress. The authors verified their computational results against Hooke’s experimental runs. The computational contours of bed topography as computed by them are shown in Fig. 2.27. The computed bed topography shows favorable agreement with the experimental measurements, which have been shown in Fig. 2.10.

Figure 2.27: Contours of bed topography as computed by Nelson and Smith 1989 for Hooke’s experiment with a) 20 l/s discharge, and b) 50 l/s discharge

In Jia and Wang 1999, the equilibrium bed topography has been simulated using the same method as that used by previous researchers. It should be mentioned that, in the case of the work under discussion, the hydrodynamic model itself is an unsteady-state model. However, the researchers, have still used a much larger time step for calculations of the bed deformation than that used for the flow simulation. The flow has been assumed to be
"quasi-steady" when the bed deformations were being computed. The bed load transport rate has been calculated by using Van Rijn’s bed load formula, while Van Bendegom’s formula was used to calculate the direction of sediment transport Eq. (2.27). The effect of cross-circulation has been considered by defining the angle $\delta$ between the direction of the sediment motion, and the direction of the vertically-averaged streamlines, according to Engelund’s formula:

$$\tan \delta = 7 \frac{h}{R}$$  \hspace{1cm} (2.40)
Evidently, this expression for $\delta$ is rather similar to that adopted by Shimizu and Itakura 1989. The authors have simulated the equilibrium bed topography under the experimental conditions of the 110° channel of Silva 1995, though the experiment itself was never performed with a movable bed. The computational results have been reproduced in Fig. 2.28. The most prominent erosion-deposition zones lie around the apex, which agrees with the other experiments performed in large $\theta_0$ channels. The pools have been shown in black in Fig. 2.28.

Wu, Rodi and Wenka 2000 has provided scope for simulating both the bed load and suspended load transport. The bed load formula of Van Rijn has been used and the suspended load transport has been calculated with the help of a convection-diffusion equation. The model has been then tested using the experiments of Odgaard and Bergs 1988. A 180° channel bend with a mobile bed was studied and the entire process of evolution of the bed was simulated. A good agreement with the experimental measurements was obtained, in the overall pattern of the erosion-deposition zones. However, this work is not being discussed in greater detail because, as mentioned before, it is yet to be validated against the experimental data from a meandering channel.

---

As mentioned before, the relation in Eq. (2.40) had been originally derived for the streamlines of a circular channel. However, both Shimizu and Itakura 1989, and Jia and Wang 1999, have used it for the case of a sine-generated channel.
3.0 MECHANICS OF MEANDERING FLOWS

Consider the steady state turbulent flow in a sine-generated channel: for the sake of simplicity, the (non-deforming) cross-section is assumed to be rectangular. Following Yalin 1992, the flow can be considered to be the result of superimposition of a helicoidal flow (usually referred to as the “cross-circulation” \( \Gamma \)), which is caused by the channel curvature \( 1/R \), on a laterally oscillating fluid mass, which is due to the variation of the channel curvature \( d(1/R)/dl \). Thus, the laterally oscillating flow may be regarded as the basic flow structure, upon which a cross-circulation is superimposed. The next two sections deal extensively with the nature of the basic flow structure and of the cross-circulation. In the last section, the periodic properties of meandering flows are discussed.

3.1 Basic Flow Structure

The basic flow structure of a meandering flow “... is formed by a fluid mass, which shifts (in all its thickness, i.e. flow depth) as it moves downstream” (Yalin 1992), accompanying the (periodic) variation of the channel curvature. This type of motion is illustrated in Fig. 3.1: as the flow, while moving downstream, shifts to the left in this Figure, the vertical 1–2 deforms into 1′–2′. The radial velocities \( v \) of this “shifting” fluid mass have all the same sign, i.e., they are everywhere positive (as shown in Fig. 3.1) or negative.

![Figure 3.1: Convective streamlines of meandering flows (after Yalin 1992)](image)
Note that the amplitude of the lateral oscillation of the streamlines decreases as the bed is approached (implying that in Fig. 3.1, \( \omega_1 > \omega_2 \)). However, since we are dealing with relatively shallow flows (i.e., \( B/h \)-ratio is "large"), \( \omega_1 \) and \( \omega_2 \) do not differ appreciably. and the flow can be regarded and treated by means of its vertically averaged counterpart. It should thus be clear that the vertically averaged streamlines of a meandering flow are neither parallel to each other, nor to the centreline of the channel, but either converging or diverging (Fig. 3.2). The amount of divergence of the convective streamlines from the

![Graph showing streamlines converging and diverging](image)

Figure 3.2: Cross-circulatory flow superimposed on a convective base, (from Silva 1995)

streamline of parallel flow (i.e. the centreline of the channel) is given by the angle of deviation \( \omega \) (Fig. 3.2). In the regions of flow where the streamlines are converging, the flow is convectively accelerated, and in the regions of flow where they are diverging, the flow is convectively decelerated.

As should be evident from Chapters 1 and 2, the basic flow structure (i.e., the convective nature of flows) in small and large sinuosity channels (having a flat bed) is different. In small sinuosity channels, the flow is like in Fig. 1.3a ("ingoing flow"); in large sinuosity channels, it is like in Fig. 1.3b ("outgoing flow").

As also mentioned in Section 1.1, in the case of the ingoing flow in Fig. 1.3a, the flow, as indicated by the vertically-averaged streamlines, is convectively accelerated near
the inner bank between the crossover $O$ and the apex $a$ (and decelerated near the outer bank between $O$ and $a$). Therefore, the deviation angle $\bar{\omega}$ is negative throughout the loop-region between the crossover $O$ and the apex $a$. This angle becomes positive in the loop-region between $a$ and $O'$ (Fig. 1.3a).

In the case of the outgoing flow in Fig. 1.3b, the flow is convectively decelerated near the inner bank throughout the entire loop $OaO'$ (and accelerated near the outer bank - see Section 1.1). In this case, $\bar{\omega}$ is positive throughout the entire loop (i.e. in the region $OaO'$). Fig. 3.3 represents (schematically) the variation of $\omega_c$ (i.e., $\bar{\omega}$ at the centreline of the channel) with the longitudinal distance along the centreline, $l_c$: the region illustrated is between two consecutive cross-overs.

![Diagram](image)

**Figure 3.3: Schematic representation of $\omega_c$ for ingoing and outgoing flows**
3.2 Cross-circulation

All curved channel flows exhibit a super-elevation $\Delta_h$, that is, a difference in the free-surface elevations between the outer and inner banks (see Fig. 3.4): the largest free-surface elevation, i.e., $(\Delta_h)_{\text{max}}$ occurs near the outer bank. Consider now the (highly exaggerated

![Diagram showing cross-circulation](image)

Figure 3.4: Superelevation between inner and outer banks of meandering channels

in the vertical) cross-section of a curved channel flow shown in Fig. 3.5a and any vertical in this cross-section ($AC$, say) as shown in Ackert 2000. Since the pressure distribution is "very nearly hydrostatic" (Henderson 1966) for any vertical in the flow, the transverse pressure gradient $F_p = -\gamma \frac{\partial h}{\partial r}$, acting along the vertical $AC$ in the midst of the fluid element $DEFG$ having the width $\delta_R$ in Fig. 3.5b, is nearly the same at all points in the vertical section. In other words, the force $F_p$ (which points towards the inner bank) can, for all practical purposes, be considered constant along $AC$. On the other hand, the centrifugal force $F_c = \rho u^2/r$, increases along the vertical (i.e., with $z$) since $u$ increases with $z$, and always points towards the outer bank. Consider now a point $B$ located in the midst of the vertical $AC$. Below the point $B$, $F_p < F_c$, resulting in an inward flow, whereas above this point, $F_p > F_c$, resulting in an outward flow (Henderson 1966). Consequently, there occurs a spiral or helical motion of the fluid, known as the cross-
circulation $\Gamma$. The radial velocities $v_\alpha$ due to this cross-circulatory motion are positive towards the free surface (above the point $B$), and negative towards the bed (below $B$) — see Fig. 3.4. In this case, the streamlines near the channel bed are directed towards the inner bank, whereas those at the free surface are directed towards the outer bank, as shown in Fig. 3.6.

![Diagram](image)

Figure 3.5: Origin of cross-circulatory flow (after Ackert 2000)

The cross-circulatory motion described above is superimposed on the convective base as illustrated schematically in Fig. 3.2. Therefore, the radial velocities, $v_\alpha$ of the flows in sinuous channels are the sum of the “shifting” (convective) component and the cross-circulatory component. As should be clear from the Literature Review (see Section 2.4.1), various aspects of the cross-circulatory motion have been the object of intensive research. In particular, several expressions have been proposed for $v_\alpha$, $v_\alpha^*$ (which is the cross-circulation velocity at the bed) as well as for the angle $\delta$. With regard to $v_\alpha$, several
expressions have been reviewed in Section 2.4.1, e.g., Eqs. (2.11), (2.12), and (2.15). The expression for the angle of deviation $\delta$ of the streamlines near the bed from the channel centreline due to the cross-circulation (as shown in Fig. 3.7), which has also been introduced in Section 2.4.2 has been given by several researchers (Rozovskii 1961, Engelund 1974, Zimmermann 1977, etc) as

$$\tan \delta = \frac{h}{R^r}. \quad (3.1)$$

Figure 3.7: Bottom and surface streamlines of meandering flows (from Chang 1988)

From the above considerations, it should be clear that a fully developed circular flow (which would occur in an infinitely long circular channel as that shown in Fig. 3.8)
does not exhibit any convective behavior. The streamlines of such a flow would be simply parallel to each other. So, in the context of the present discussion, the practice of previous researchers of studying a meandering flow as a fully developed circular flow does not seem to be entirely justifiable.

![Diagram](image)

Figure 3.8: Flow in an infinite circular channel (from Silva 1995)

Moreover, since any radial component of a fully developed circular flow is entirely due to its cross-circulation, this type of approach led the earlier researchers to attribute the bed deformation patterns in meandering rivers (which is a radially varying phenomenon) to the cross-circulation solely (Yalin 1992). If the above hypothesis was true, then the bed deformation would always be maximum where the cross-circulation $\Gamma$, and consequently $v_{\alpha}$ was maximum. From the equations for $v_{\alpha}$ in Chapter 2 (see Eqs. (2.11) and 2.15, it should be clear that the maximum $v_{\alpha}$ occurs at the apex, where the ratio $h/R$ attains its maximum. Therefore, according to the above reasoning, the erosion-deposition zones in a meandering river would always be the most pronounced at the apex, and follow the standard pattern shown in Fig. 3.9, irrespective of the values of $\theta_0$ and $h/B (= h/R)(R/B)$.

However, experiment shows that this is not the case, and that the bed deformation pattern in a meandering river is very much affected by $\theta_0$ and $B/h_m$. Thus, the above theory again fails to offer a satisfactory explanation for this behavior.

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3.3 Periodic Property of Meandering Flows

Consider the flow in a very long, sine-generated meandering channel. The flow in such a channel acquires a periodic character, sufficiently far from the channel entrance. Thus, in Fig. 3.10, the flow in Zone I is not periodic, since it is affected by the conditions at the channel entrance. However in Zone II (which is at a large distance from the entrance), the flow will exhibit a periodic behavior, i.e., it will possess the same properties (e.g., velocity profile, bed deformations, etc.) in consecutive loops. The current literature (as most of the other works on the topic) deals with this periodic flow in Zone II.

The flow in the two consecutive meander loops are “mirror-image identical”, as illustrated in Fig. 3.10. Thus, the flow in this part of the channel is periodic with $L/2$ as the effective period of the flow. This implies that the study of the flow actually reduces to the study of any one of its loops.
4.0 MEANDER FLOW PATH

This chapter presents the meander path model developed by the candidate. This model, named MEANDERPATH (FORTRAN 77), generates the coordinates of the channel centerline, banks and interior points, in the cartesian system. The results of this model are used to draw the channel, and for post processing of the results, i.e., enabling visualization of the flow-field determined by the hydrodynamic model (see Chapter 5), and of the bed topography determined by the sediment transport model (see Chapter 6).

4.1 Pertinent Geometric Characteristics of a Sine-generated Channel

Several expressions related to the geometry of sine-generated channels are used in this thesis in the meander path model, as well as in the hydrodynamic and sediment transport models. In this section, these expressions are presented. As mentioned in Chapter 2, the centerline of a sine-generated channel is given by

\[ \theta = \theta_0 \cos \left( \frac{2\pi l_c}{L} \right) \]  

All the coordinates in Eq. (4.1) are as explained in Fig. 1.5. The channel curvature at a cross-section of the channel is defined as the curvature of the channel center-line at that section. According to the channel-fitted system of cylindrical coordinates adopted in this thesis, the radial coordinate of a space-point \( P \) is measured form the centre \( (C) \) of the channel curvature at the section containing the space point \( P \). The centre of curvature of a section at one of its points \( P \) is the centre of the osculating circle (at that point of the section). Thus the center of curvature of the radial co-ordinate \( r \) is not a fixed point, but actually differs from section to section. This is demonstrated in Fig. 4.1 with the help of the points \( P_1 \) and \( P_2 \), whose radial coordinates \( r_1 \) and \( r_2 \) are measured from different origins \( C_1 \) and \( C_2 \). Evidently, the curvature of such a channel is not constant along the channel centerline, but varies from section to section. From Eq. (4.1), the curvature of
the channel centerline is expressed as

\[ \frac{1}{R} = \frac{d\theta}{dl_c} = \theta_0 \frac{2\pi}{L} \sin \left( 2\pi \frac{l_c}{L} \right) \]  

(4.2)

Thus, at the apex \( a \) (where \( l_c/L = 0.25 \)),

\[ \frac{1}{R_a} = \frac{2\pi}{L} \theta_0, \]  

(4.3)

and at the cross-over \( O \) (where \( l_c/L = 0 \)),

\[ \frac{1}{R_O} = 0. \]  

(4.4)

Silva 1991 has shown that the sinuosity of the channel (\( \sigma = L/\lambda \)) is uniquely defined by the deflection angle \( \theta_0 \) as

\[ \sigma = \frac{L}{\lambda} = \frac{1}{J_0(\theta_0)}, \]  

(4.5)

where \( J_0(\theta_0) \) is the Bessel function of the first kind and zero-th order of \( \theta_0 \). In this thesis, and following Silva 1995, \( J_0(\theta_0) \) will be evaluated with the aid of the following
polynomial approximation (which can be found in Allen 1954)

\[ J_0(\theta_0) = 1 - 2.2499997(\theta_0/3)^2 + 1.2656208(\theta_0/3)^4 - 0.3163866(\theta_0/3)^6 
+ 0.0444479(\theta_0/3)^8 - 0.0039444(\theta_0/3)^{10} + 0.0002100(\theta_0/3)^{12} 
+ \epsilon \text{ (with } \epsilon < 5 \times 10^{-8} \) \]  

(4.6)

Using the alluvial inter-relation for the meander wavelength in Eq. (2.7), given by Yalin 1977, viz

\[ \Lambda = 2\pi B \]  

(4.7)

in Eq. (4.5), one determines for the dimensionless channel length,

\[ \frac{L}{B} = \frac{2\pi}{J_0(\theta_0)} \]  

(4.8)

Using Eq. (4.8) in Eq. (4.2) and (4.3), one obtains

\[ \frac{B}{R} = \theta_0 J_0(\theta_0) \sin \left(2\pi \frac{i_e}{L} \right) \]  

(4.9)

and

\[ \frac{B}{R_a} = \theta_0 J_0(\theta_0) \]  

(4.10)

Fig. 4.2a gives the graphical representations of Eq. (4.5) and Eq. (4.10). As seen from the graph, the function \( B/R_a = \theta_0 J_0(\theta_0) \) attains a maximum at \( \theta_0 \approx 70^\circ \). From Figs. 4.2a and b, one can infer that, in practice, the sinuosity \( \sigma \) of a channel cannot exceed \( \approx 8.5 \) (which corresponds to \( \theta_0 \approx 126^\circ \)), for at this stage, the meander loops come into contact, and the meander pattern is destroyed. Fig. 4.3 illustrates the agreement of the relation in Eq. (4.5) with field data.

### 4.2 Determination of Coordinates of Space Points

The following method is used in the program MEANDERPATH to determine the coordinates of a space-point in the channel. Consider Fig. 4.4 and Fig. 4.5 where \((x_1, y_1)\)
Figure 4.2: Variation of channel sinuosity $1/\sigma$ and relative channel curvature at the apex $B/R_a$ with the deflection angle $\theta_0$ (from Yalin 1992)

Figure 4.3: Channel sinuosity versus deflection angle in large Russian rivers (from Yalin 1992)
Figure 4.4: Calculation of coordinates for points along a cross-section

Figure 4.5: Calculation of coordinates for points along the centreline
represent the coordinates of the point on the centerline of the channel at the cross-over. The cross-section through this point - $OO'$ (all cross sections referred to here are orthogonal to the centreline of the channel through the section) is divided into $N$ no. of points (Fig. 4.4). Introducing

$$
\frac{N-1}{2} + 1 = M',
$$

(4.11)

and considering the Cartesian coordinates of the $k$-th point on the cross-section, where $k = 1$ at the inner bank, and $k = N$ at the outer bank,

$$
x_1^k = x_1 + B/2 \sin(\theta_0)(M' - k)/(N/2)
$$

(4.12)

$$
y_1^k = y_1 - B/2 \cos(\theta_0)(M' - k)/(N/2).
$$

(4.13)

Consider now another cross-section (cross-section 2, say) at a "small" distance $\Delta l_c$ along the centerline of the channel, following which the centerline may be approximated as a straight line along this distance (Fig. 4.5). The deflection angle $\theta$ at this cross-section is given by Eq.( 4.1). The Cartesian co-ordinates of the point on the centerline at this section are then:

$$
x_2 = x_1 + \cos(\theta_0)(\Delta l_c)
$$

(4.14)

$$
y_2 = y_1 - \sin(\theta_0)(\Delta l_c)
$$

(4.15)

(The computational model actually uses an average of the two angles $\theta$ and $\theta_0$ in this calculation for greater accuracy). The relation between the coordinates of the centreline at the section and the other points on the section is, again, given by Eq. (4.12) and Eq. (4.13) with $\theta$ (i.e., the deflection angle of this section) replacing $\theta_0$. Of course, for the coordinates of the centerline at the next cross-section, Eq. (4.14) and Eq. (4.15) are used again (with $\theta$ replacing $\theta_0$). The method is repeated at each section (of a pre-determined number of sections) along the channel. How fine the grid should be for the above calculations is evidently governed by the number of points required to produce a smooth curve in the pictorial representation of the channel.

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4.3 Utilization of the Model

It should be apparent from the above discussions that the program MEANDERPATH uses as its input data, the values of the width of the channel $B$, and its meander angle $\theta_0$. The user must also provide the number of grid points $(M, N)$ at which the Cartesian coordinates are required. The program calculates the sinuosity and wavelength of the channel from Eq. (4.5) and Eq. (4.7) respectively, and the length of the channel, from Eq. (4.8). The required Cartesian coordinates are then computed from the relations given above, and output files, containing the $x-$ and $y-$ coordinates of the selected points, are generated. The algorithm followed in the program has been schematically described in the flowchart in Fig. 4.6. The results file from MEANDERPATH (containing the Cartesian coordinates of the point) can then be used in a spreadsheet to draw channels of different sinuosities: the sine-generated curves in Fig. 4.7 have been generated in MS EXCEL. The results files may also be properly formatted to be used as input data for any commercial package for post-processing (like TECPLOT, which was used in the present thesis) in order to obtain contours of the bed topography, the bed load transport rate, etc. along the
channel. A printout of the computer program MEANDERPATH is also given in Appendix A.

\[ \theta_0 = 110^\circ \]

\[ \theta_0 = 30^\circ \]

Figure 4.7: Sine-generated curves drawn by using the results from MEANDERPATH
5.0 THE HYDRODYNAMIC MODEL

This chapter gives a brief summary of the vertically averaged hydrodynamic model used in the present research. It was originally developed by Silva 1995 (see also Silva 1999). In this thesis, the model was renamed as MEANDERFLOW.FOR (FORTRAN 77).

5.1 Vertically-averaged Equations of Motion and Continuity

The model solves the vertically averaged equations of motion and the equation of continuity. Following Smith and McLean 1984, Shimizu and Itakura 1989, Nelson and Smith 1989, etc., the “channel-fitted system” of cylindrical coordinates is adopted. Before introducing the governing equations themselves, a brief clarification of the coordinate system is in order. As mentioned in the Introduction, in this system of coordinates, the location in flow plan of a point P is determined by the distance l measured along the longitudinal direction from the crossover section O and is positive in the direction of the flow (see Fig. 1.5b), and the distance r to the centre of curvature C (Fig. 1.5b); the elevation of the point P is given by the distance z measured from a reference datum: P = P(l, r, z).

However, “when dealing with meandering streams, it is more convenient to locate a point P (in flow plan) by means of the distance le of its section to the crossover measured along the channel centreline, and the distance n to the channel centreline” (Silva 1995). The coordinate pairs (l; r) and (le; n) are interrelated as:

\[ \frac{dl_e}{R} = \frac{dl}{r} \quad \text{and} \quad r = n + R \]  \hspace{1cm} (5.1)

From Eq. (5.1), it follows that

\[ \frac{r}{R} = \frac{R + n}{R} = 1 + \frac{n}{R} = 1 + \frac{n B}{B R}. \]  \hspace{1cm} (5.2)

---

1It should be clear from Section 4.1 that the centre of channel curvature C at a section is not a fixed point in space, its location differing from section to section.
The vertically averaged equations of motion and continuity in the \((l; r)\) coordinates described above are as follows:

\[
\frac{\partial \overline{u}^2 h}{\partial l} + \frac{\partial \overline{u} \overline{v} h}{\partial r} + 2 \overline{u} \frac{\partial \overline{v} h}{\partial r} + K_l = -gh \frac{\partial E}{\partial l} - \frac{1}{\rho} [((\tau_0)_l + q_l] \tag{5.3}
\]

\[
\frac{\partial \overline{u} \overline{v} h}{\partial l} + \frac{\partial \overline{v}^2 h}{\partial r} + \overline{u}^2 - \overline{v}^2 h + K_r = -gh \frac{E}{\partial r} - \frac{1}{\rho} [((\tau_0)_r + q_r] \tag{5.4}
\]

\[
\frac{\partial \overline{u} h}{\partial l} + \frac{\partial \overline{v} h}{\partial r} + \frac{\overline{v} h}{r} = 0 \tag{5.5}
\]

In these equations, \(K_l\) and \(K_r\) are the correlation coefficients (which come into being because of the vertical averaging). These coefficients are expressed as:

\[
K_l = \frac{\partial (K_{uu} h)}{\partial l} + \frac{\partial (K_{uv} h)}{\partial r} + 2 \frac{K_{uv} h}{r} \tag{5.6}
\]

\[
K_r = \frac{\partial (K_{uu} h)}{\partial l} + \frac{\partial (K_{uv} h)}{\partial r} - \frac{h}{r} (K_{uu} - K_{uv}), \tag{5.7}
\]

where any of the coefficients \(K_{uu}, K_{uv}, \text{etc.}\), may be given by the definition formula:

\[
K_{AB} = (A - \overline{A})(B - \overline{B}) \tag{5.8}
\]

In all of the theoretical studies using the vertically averaged equations, \(K_l\) and \(K_r\) were invariably neglected (eg: Smith and Mclean 1984, Struiksma et al. 1985, Shimizu and Itakura 1989, Nelson and Smith 1989, etc.). The terms \(q_l\) and \(q_r\) are the turbulence stresses acting in the fluid. They are determined by the components of the stress tensor as follows:

\[
-q_l = \frac{\partial (\sigma_{uh})}{\partial l} + \frac{\partial (\tau_{rh})}{\partial r} + 2 \frac{\tau_{rh}}{r} \tag{5.9}
\]

\[
-q_r = \frac{\partial (\tau_{uh})}{\partial l} + \frac{\partial (\tau_{rr})}{\partial r} - \frac{h}{r} (\sigma_{rr} - \sigma_{uu}), \tag{5.10}
\]

In order to take the effect of these turbulent stresses into account, but bypass their direct evaluation, Silva 1995 has considered the combined effect of the bed shear stress \(\tau_0\) and the turbulence stresses with the aid of the resultant resistance vector \(\mathbf{T}\) which is:

\[
\mathbf{T} = \tau_0 + \mathbf{q} \tag{5.11}
\]

where \(\mathbf{q} = q_l \mathbf{i}_l + q_r \mathbf{i}_r\). Fig. 5.1 illustrates schematically the stresses acting within the unit prism of the fluid at their respective levels. "One would expect that the stresses within
the body of flow are organized in such a way that their resultant \( T \) opposes the flow in the most efficient manner; that is, by acting in the direction that is exactly opposite to that of the flow velocity \( \bar{U} \)” (Silva 1995).

![Diagram of stresses on a unit prism of water](image)

Figure 5.1: Schematic representation of the stresses on a unit prism of the water at their respective levels (from Silva 1995)

This collinear relation between \( T \) and \( \bar{U} \) is schematically shown in Fig. 5.2. The vertically averaged deviation angle \( \bar{\omega} \approx \tan \bar{\omega} = \bar{v}/\bar{u} \) as shown in Fig. 5.3 is then introduced into the flow model. In reality, \( 0 < \bar{\omega} < \approx \frac{1}{10} \). From the above two considerations, it follows that the vector sum of Eq. (5.11) can be replaced by the following algebraic sum

\[
T = \tau_0 + q
\]

(5.12)

\( T \) can be expressed in dimensionless form as

\[
\frac{1}{c^2} = \frac{T}{\rho \bar{U}^2},
\]

(5.13)

which will henceforth be referred to as the local resistance factor.
Figure 5.2: Schematic representation of the collinearity between $T$ and $U$ (from Silva 1995)

Figure 5.3: Schematic representation of the vertically averaged deviation angle $\omega$ (from Silva 1995)
For small $\bar{\omega}$,

$$ \bar{U}^2 = \bar{u}^2 + \bar{v}^2 = \bar{u}^2(1 + \bar{\omega}^2) \approx \bar{u}^2 \approx \bar{U}^2 $$  \hspace{1cm} (5.14)

and

$$ \bar{u}^2 - \bar{v}^2 = \bar{u}^2(1 - \bar{\omega}^2) \approx \bar{u}^2 $$  \hspace{1cm} (5.15)

Moreover, the water surface elevation $E$ is given by

$$ E = z_b + h $$  \hspace{1cm} (5.16)

Using Eqs. (5.1), (5.14), (5.15) and (5.16), the system of Eqs. (5.3) - (5.5) may be rewritten as

$$ \frac{\partial \bar{u}^2 h}{\partial l} + \frac{\partial \bar{u} \bar{v} h}{\partial r} + \frac{2 \bar{u} \bar{v} h}{r} = - gh \left( \frac{\partial z_b}{\partial l} + \frac{\partial h}{\partial r} \right) - \frac{\bar{u}^2}{c^2} $$  \hspace{1cm} (5.17)

$$ \frac{\partial \bar{u} \bar{v} h}{\partial l} + \frac{\partial \bar{v}^2 h}{\partial r} - \frac{\bar{u}^2}{r} h = - gh \left( \frac{\partial z_b}{\partial r} + \frac{\partial h}{\partial r} \right) - \frac{\bar{u} \bar{v}}{c^2} $$  \hspace{1cm} (5.18)

$$ \frac{\partial \bar{u} h}{\partial l} + \frac{\partial \bar{v} h}{\partial r} + \frac{\bar{v} h}{r} = 0 $$  \hspace{1cm} (5.19)

Using $\bar{\omega} = \bar{v}/\bar{u}$ instead of $\bar{v}$, and introducing the $(l_c; n)$ coordinates, Eqs. (5.17) - (5.19) can be rewritten as (Silva, 1995):

$$ \bar{u}^2 \frac{\partial \bar{\omega}}{\partial n} = g \left( \frac{R}{R + n} \left( \frac{\partial z_b}{\partial l_c} + \frac{\partial h}{\partial l_c} \right) + \frac{\bar{u}^2}{hc^2} \right) $$  \hspace{1cm} (5.20)

$$ \bar{u}^2 \frac{R}{R + n} \frac{\partial \bar{\omega}}{\partial l_c} - \frac{\bar{u}^2}{R + n} = - g \left( \frac{\partial z_b}{\partial n} + \frac{\partial h}{\partial n} \right) - \frac{\bar{u}^2}{hc^2} \bar{\omega} $$  \hspace{1cm} (5.21)

$$ \frac{R}{R + n} \frac{\partial \bar{u} h}{\partial l_c} + \frac{\partial \bar{v} h}{\partial n} + \frac{\bar{v} h}{R + n} = 0 $$  \hspace{1cm} (5.22)

### 5.2 Evaluation of the Resistance Factor

From Eq. (5.12), it follows that

$$ \frac{T}{\rho U^2} = \frac{r_0}{\rho U^2} + \frac{q}{\rho U^2} $$  \hspace{1cm} (5.23)

i.e.,

$$ c^{-2} = c_s^{-2} + c_q^{-2}. $$  \hspace{1cm} (5.24)
Here, $c_s^{-2}$ is the bed friction factor which can be evaluated by using the well known logarithmic expression

$$c_s^{-2} = \left[ \frac{1}{k} \ln(11h_m/k_s) \right]^{-2}$$  \hspace{1cm} (5.25)

$c_q^{-2}$ is the resistance factor due to the turbulence stresses induced by the channel curvature. For the case of a rough turbulent flow past a flat bed in a sine-generated channel, Silva 1995 has suggested the following expression for $c_q^{-2}$:

$$c_q^{-2} = \alpha_q c_s^{-2} \left[ (R/r)^2 - 1 \right] - 4((R/B)^2 - 1)^{-1}$$  \hspace{1cm} (5.26)

Here, $R/r$ is given by Eq. (5.2), where $B/R$ is as expressed in Eq. (4.9). $\alpha_q$ is expected to vary (at most) with $\theta_0$ and $B/h_m$ and can only be determined experimentally (see also Section 7.2 for discussion on the nature of $\alpha_q$).

### 5.3 Transformation of Basic Equations into their Dimensionless Forms

It seems that dimensionless forms of the Eqs. (5.20) - (5.22) were preferred in Silva 1995. Hence, the coordinates were non-dimensionalized as

$$\lambda = \frac{L}{B} ; \quad \eta = \frac{R}{B} ; \quad \zeta_m = \frac{z}{h_m}$$  \hspace{1cm} (5.27)

The following dimensionless functions were also introduced

$$\phi_u(\lambda, \eta) = \frac{\bar{u}}{u_m} ; \quad \phi_h(\lambda, \eta) = \frac{h}{h_m} ; \quad \psi_z(\lambda, \eta) = \frac{\bar{z}}{h_m} ; \quad \psi_c(\lambda, \eta) = \frac{c}{c_m}.$$  \hspace{1cm} (5.28)

(where for a given experiment, the channel-average quantities $u_m$, $h_m$, $c_m$ have certain constant values). Using these dimensionless variables, the system of Eqs. (5.20) - (5.22) can now be expressed in dimensionless form. Furthermore, a "combined equation" is derived by eliminating $\phi_h$ from the dimensionless forms of the longitudinal and radial equations of motion. This is a second-order linear equation for $\bar{\nu}$ and is used in the flow model in place of the longitudinal equation. Thus one can now write the following system of equations:
\[ A_I \Phi \frac{\partial \omega}{\partial \eta} + A_{II} \frac{\partial \omega}{\partial \eta} + A_{III} \frac{\partial^2 \omega}{\partial \eta^2} + A_{IV} \overline{\omega} + A_V \Phi^2 \frac{\partial^2 \overline{\omega}}{\partial \lambda^2} = A_{VI} \]  

(5.29)

\[ \Phi \frac{\partial \omega}{\partial \lambda} + \frac{\beta_c}{\phi_h \psi_c} \overline{\omega} = -\frac{1}{F_r m} \frac{1}{\phi_u^2} \left( \frac{\partial \psi_z}{\partial \eta} + \frac{\partial \phi_h}{\partial \eta} \right) + \Phi \frac{B}{R} \]  

(5.30)

\[ \Phi \frac{\partial \phi_u \phi_h}{\partial \lambda} + \frac{\partial \omega \phi_u \phi_h}{\partial \eta} + \Phi \frac{B}{R} \omega \phi_u \phi_h = 0 \]  

(5.31)

Here,

\[ \Phi = \frac{1}{\eta \frac{B}{R} + 1}, \quad \beta_c = \frac{B}{h_m c_m^2}, \quad (F_r)_m = \frac{u_m^2}{g h_m} \]  

(5.32)

The coefficients \( A_I, A_{II}, A_{III}, A_{IV}, A_V \) and \( A_{VI} \) in the combined equation, Eq. (5.29), are

\[ A_I = \Phi \frac{\partial \phi_u^2}{\partial \lambda} + \phi_u^2 \frac{\partial \Phi}{\partial \lambda} + \beta_c \frac{\phi_u^2}{\phi_h \psi_c} \]  

(5.33)

\[ A_{II} = \frac{\partial \phi_u^2}{\partial \eta} \]  

(5.34)

\[ A_{III} = A_V = \phi_u^2 \]  

(5.35)

\[ A_{IV} = \Phi \beta_c \frac{\partial}{\partial \lambda} \left( \frac{\phi_u^2}{\psi_c^2 \phi_h} \right) \]  

(5.36)

\[ A_{VI} = \frac{1}{F_r m} \frac{\partial}{\partial \eta} \left( \Phi \frac{\partial \psi_z}{\partial \lambda} \right) + \beta_c \frac{\partial}{\partial \eta} \left( \frac{\phi_u^2}{\psi_c^2 \phi_h} \right) + \Phi \frac{\partial}{\partial \lambda} \left( \phi_u^2 \Phi \frac{B}{R} \right) \]  

\[ + \frac{\Phi}{F_r m} \frac{\partial}{\partial \lambda} \left( \frac{\partial \psi_z}{\partial \eta} \right). \]  

(5.37)

### 5.4 Determination of \( \phi_u, \phi_h \) and \( \overline{\omega} \)

#### 5.4.1 Boundary Conditions

Since any dimensionless property \( \Pi_\overline{X} \) of the vertically-averaged periodic (along \( l_c \)) flow in the semi-infinite zone \( II (x_* < x < \infty) \) shown in Fig. 3.10 satisfies

\[ \Pi_\overline{X} = \phi_\overline{X}(l_c/L, \eta) = \phi_\overline{X}(l_c/L + 1, \eta), \]  

(5.38)

the determination of the meandering flow in the zone \( II \) can be reduced to its determination within any channel region of the length \( L/2 \) (such as the shaded region \( \mathcal{R} \) within the
curvilinear rectangle $abcd$ in Fig. 5.4). Since $\mathbf{v}$ vanishes at the banks, we have $\mathbf{w} = 0$ along the longitudinal boundaries $bc$ and $ad$ of $\mathcal{R}$. However, the absence of knowledge on $\mathbf{w}$, $\phi_u$ and $\phi_h$ for all the radial boundaries $\overline{ab}$ and $\overline{cd}$ within the zone II implies that no particular region $\mathcal{R}$ can be regarded as preferable. Thus, in the present case, the region $\mathcal{R}$ between two consecutive crossovers $O_n$ and $O_{n+1}$ was selected as the “solution domain”.

The knowledge of only $\mathbf{w} = 0$ along the flow banks is sufficient to compute by the present method $\phi_u$, $\phi_h$ and $\mathbf{w}$ for any point $[\lambda; \eta] \in \mathcal{R}$. The absence of prescribed $\phi_h$ and $\phi_u$ along the longitudinal boundaries of $\mathcal{R}$, i.e. along the banks, is compensated by the introduction of the following integrals

$$\int_{-0.5}^{0.5} \phi_h d\eta = 1 \quad (5.39)$$

$$\int_{-0.5}^{0.5} \phi_u d\eta = 1 \quad (5.40)$$

which ensure that the average flow depth, $h_{av}$, and velocity, $u_{av}$, at any cross section of the flow remain the same (see Silva 1995).

### 5.4.2 Numerical Scheme

As should be clear from the contents of the previous sections, the flow model solves the system of equations Eqs. (5.29) - (5.31) for the functions $\phi_u \sim \mathbf{v}$, $\phi_h \sim h$ and $\mathbf{w}$.

These equations are solved using a finite difference method on a computational grid $(i \Delta \lambda ; j \Delta \eta)$, as shown in Fig. 5.5. At any step of calculation, one proceeds from the cross-section $i$ to $i + 1$; $(\phi_u)_i$, $(\phi_h)_i$, $\mathbf{w}_i$ in the cross-section $i$ being known for
all \( j = 1, \ldots, N \), the system is solved for the unknowns \( (\phi_u)^j_{i+1}, (\phi_h)^j_{i+1}, \overline{\omega}^j_{i+1} \) in the cross-section \( i + 1 \) immediately downstream. The solution domain is "swept", in this forward marching manner, iteratively so as to yield mirror-image-identical conditions at consecutive cross-overs (until the predetermined convergence criterion has been fulfilled).

The combined equation is discretized using backward differencing for the \( \lambda \)-derivatives and central differencing for the \( \eta \)-derivatives. \( \overline{\omega} \) is calculated implicitly at all grid-points from the resultant system of equations with a tridiagonal solver (Thomas' algorithm). The discretized form of the radial equation is obtained by using backward differencing for both \( \lambda \)- and \( \eta \)- derivatives. Finally, the continuity equation is then discretized using central differencing for the \( \eta \)-derivative and backward differencing for the \( \lambda \)-derivative. The discretization scheme for each of these equations is described in detail in Silva 1995, pages 91-94.
5.5 Initial Condition

The initial condition adopted for the initial flow is

\[(\phi_u)_0 = 1; \quad (\phi_h)_0 = 1; \quad \bar{\omega}_0 = 0, \quad \text{for } j = 1, \ldots, N, \quad (5.41)\]

which corresponds to the uniform flow in a straight channel. However, a straight channel (having \(B/R = 0\) and \(d(B/R)/d\lambda = 0\)) cannot be connected to any cross-section of a sine-generated channel without introducing some undesirable discontinuities in the functions \((B/R)\) and/or \(d(B/R)/d\lambda\). Hence, the solution domain \(\mathcal{R}\) between two consecutive cross-overs is modified for the first "sweep": its plan geometry is the same as that of a sine-generated channel only for \(l_c/L \geq 1/4\); within the interval \(0 \leq l_c/L < 1/4\) an approach channel, which ensures a smooth transition from a straight channel to the sine-generated channel, is provided.
6.0 THE SEDIMENT TRANSPORT MODEL

In this chapter, the sediment transport model developed by the author is presented. The model is named MEANDERBED (FORTRAN 77). There are actually two versions of this model – namely, MEANDERPATH-I and MEANDERPATH-II. The differences in these two versions will be explained in the later sections of this Chapter. Printouts of both versions are provided in Appendix C. We start this chapter by introducing the pertinent dimensionless variables of the two-phase motion (i.e., of the simultaneous motion of the transporting fluid and the transported sediment), the sediment transport continuity equation, and the evaluation of the sediment transport rate, \( q_s \). Numerical details of the model are presented in later sections.

6.1 Sediment Transport Continuity Equation

The sediment transport equation, also known as the Exner-Polya equation, may be written as

\[
\frac{\partial z_b}{\partial t} + \frac{1}{1 - P} \nabla q_s = 0, \tag{6.1}
\]

where \( z_b \) is the bed elevation measured to any arbitrary datum, \( t \) is the time, \( P \) is the porosity of the bed material, and \( q_s \) is the specific volumetric sediment transport rate, which is the total volume of sediment passing through a cross-section per unit width of the channel per unit time. This is a simple equation, intuitively understood as it follows directly from the principle of conservation of mass. From Eq. (6.1), it is evident that wherever \( \nabla q_s \) is negative, deposition occurs (for \( \partial z_b/\partial t > 0 \)) and wherever \( \nabla q_s \) is positive, erosion occurs (for \( \partial z_b/\partial t < 0 \)). Moreover, \( q_s \) is always an increasing function of the vertically averaged flow velocity vector \( \overline{U} \), irrespective of the formulae employed for its estimation. Thus the above relation clearly reveals that the convergence-divergence zones of the flow-field (i.e., zones of convective acceleration-deceleration) must coincide with the location of the erosion-deposition zones in the channel.
6.2 Evaluation of Sediment Transport Rate

6.2.1 Dimensionless variables of the Two-phase Motion

It can be shown that any dimensionless property $\Pi_A$ of the two-phase motion can be expressed as a function of three dimensionless variables (Yalin 1972, 1992):

$$\Pi_A = \phi_A(\Xi, Y, Z)$$

(6.2)

where,

$$\Xi = \left(\frac{\gamma_s D^3}{\rho \nu^2}\right)^{1/3} \text{ (material number)}$$

(6.3)

$$Y = \frac{\rho u_*^2}{\gamma_s D} \text{ (mobility number)}$$

(6.4)

$$Z = \frac{h}{D} \text{ (dimensionless flow depth)}$$

(6.5)

Here, $\gamma_s$ is the submerged specific weight of the sediment, $\rho$ is the density of the fluid, $D$ is the characteristic grain size diameter $D_{50}$, $\nu$ is the kinematic viscosity, $u_*$ is the shear velocity, and $h$ is the flow depth. However, it is not necessary to express a property $\Pi_A$ literally in terms of $\Xi, Y$ and $Z$. Any one of $\Xi, Y$ and $Z$ can be replaced by another variable which is a function of that variable and one or both of the other two (Yalin 1972, 1992). Thus the variable $\eta_*$, which is defined as

$$\eta_* = \frac{Y}{Y_{cr}} = \frac{\tau_0}{(\rho_0)_{cr}} = \frac{u_*^2}{v_{cr}^2}$$

(6.6)

(where $\tau_0$ is the bed shear stress, and the subscript "cr" indicates the value of a quantity at the stage of initiation of sediment transport), is commonly used in place of $Y$–number.

6.2.2 Modes of Sediment Transport Rate; Bagnold’s Equation

Sediment transport actually occurs in two modes: “bed-load transport ” and “suspended-load transport ”. Bed-load transport consists of the motion of the grains in a region near the bed. This region has the height $\epsilon$ – which is a few times larger than the grain
size (see Fig. 6.1), i.e., the magnitude of $\varepsilon$ is of the same order as the magnitude of $D$. Suspended-load transport consists of the motion of grains which are picked up from the bed and spread throughout the flow depth. Following van Rijn 1984, the thickness of the bed-load layer $\varepsilon$, is given by

$$\frac{\varepsilon}{D} = 0.3\varepsilon^{0.7}(\eta_* - 1)^{0.5}. \quad (6.7)$$

![Diagram showing bed-load and suspended load layers](image)

Figure 6.1: Bed-load and suspended load layers (from Yalin 1992)

Thus, the total rate of sediment transport ($q_b$) through a cross-section is the sum total of the bed-load transport rate ($q_{sb}$) and the suspended-load transport rate ($q_{ss}$):

$$q_s = q_{sb} + q_{ss}. \quad (6.8)$$

Before proceeding further, it is important to mention that at lower values of $\eta_*$, the contribution of the suspended-load transport is small (i.e., $q_{ss} \approx 0$). It gains significance only when $\eta_*$ becomes very large. In the present thesis, the suspended-load transport has been assumed to be negligible (thus simplifying the model) \(^1\). Therefore, Eq. (6.8) will henceforth be considered in this thesis as

$$q_s = q_{sb}. \quad (6.9)$$

\(^1\)However, the same procedure may be followed even when the effect of the suspended-load is considered, except that a convection-diffusion equation to estimate the suspended-load must also be included in the sediment transport algorithm before solving the continuity equation.
6.2.3 Bagnold’s Equation

In the vector form, Bagnold’s equation, which is probably the most popular equation for determination of the bed-load transport rate, is given by

\[ q_{sb} = U_b[\beta(\tau_0 - (\tau_0)_{cr})/\gamma_s], \]  

where \( U_b \) is the flow velocity vector at the bed, and \( \beta \) is a coefficient which varies according to the grain size and the regime of the turbulent flow. The graph in Fig. 6.2 shows the variation of this function (\( \beta = 0.5 \) for a rough turbulent flow for all grain sizes).

![Graph showing the variation of \( \beta \) with grain size (from Yalin 1972)](image)

Figure 6.2: Variation of \( \beta \) with grain size (from Yalin 1972)

However, the vertically averaged hydrodynamic model solves for the vertically averaged flow velocities. Hence, it is necessary to express Bagnold’s bed-load formula in terms of the vertically averaged flow velocity vector \( \bar{U} \). Following Yalin and Silva 2000, Bagnold’s equation in terms of the vertically averaged flow velocity is written as:

\[ q_{sb} = \bar{U}[\beta'(\tau_0 - (\tau_0)_{cr})/\gamma_s], \]

where,

\[ \beta' = \beta \frac{U_b}{\bar{U}} \]  

For straight channels with uniform flows, \( \beta' \) is given by

\[ \beta' = \beta \frac{B_s + \frac{2.5}{2} \ln \left( \frac{\epsilon}{2D} \right)}{B_s + 2.5 \ln \left( \frac{0.368h}{2D} \right)} \]  

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It is assumed that this relation holds good for the case of a meandering channel as well. In Eq. (6.13), \( B_s = \phi(u_* K_s/\nu) \) is the well known roughness function (Schlichting 1968, Yalin 1972, Yalin 1992) given by the graph in Fig. 6.3 (where \( Re_s = u_* K_s/\nu \)), \( k_s \) being the roughness height given by

\[
k_s = 2D
\]  
(6.14)

Taking into account that,

\[
\overline{U} = \overline{u} i_l + \overline{v} i_r,
\]

and

\[
q_{sb} = q_{sb_l} i_l + q_{sb_r} i_r
\]  
(6.16)

and also that \( \overline{v} = \overline{w} \overline{u} \), Eq. (6.15) can be expanded as

\[
q_{sb} = \frac{\beta'(\tau_0 - (\tau_0)_{cr})/\gamma_s}{q_{sb_l}} i_l + \overline{w} \overline{u} \left[ \frac{\beta'(\tau_0 - (\tau_0)_{cr})/\gamma_s}{q_{sb_r}} \right] i_r
\]

From Eq. (6.16), it is evident that

\[
q_{sb_r} = (q_{sb_l}) \overline{w}
\]  
(6.18)

Using the dimensionless \( Y (= \tau_0/\gamma_s D) \), instead of \( \tau_0 \) (and, of course, \( Y_{cr} (= (\tau_0)_{cr}/\gamma_s D) \) instead of \( (\tau_0)_{cr} \)) in Eq. (6.18), one obtains

\[
q_{sb} = \overline{u} \left[ \beta' D(Y - Y_{cr}) \right] i_l + \overline{w} \overline{u} \left[ \beta' D(Y - Y_{cr}) \right] i_r.
\]  
(6.19)

Considering Eq. (5.23) and Eq. (5.24), together with it \( Y = \tau_0/\gamma_s D \), one can write

\[
\tau_0 = \frac{\rho \overline{U}^2}{c_s^2} = Y(\gamma_s D),
\]

(6.20)

according to which Eq. (6.19) can be written as

\[
q_{sb} = \overline{u} \left[ \beta' D \left( \frac{\rho \overline{U}^2}{c_s^2 \gamma_s D} - Y_{cr} \right) \right] i_l + \overline{w} \overline{u} \left[ \beta' D \left( \frac{\rho \overline{U}^2}{c_s^2 \gamma_s D} - Y_{cr} \right) \right] i_r
\]

(6.21)

\( Y_{cr} \) or \( \tau_{cr} \) depends only on the properties of the sediment, i.e., there is only one \( Y_{cr} \) for one particular type of sediment. In this thesis, \( Y_{cr} = \Psi(\Xi) \) is evaluated from the following formula of Yalin and Silva 2000:

\[
Y_{cr} = 0.13 \Xi^{-0.392} e^{-0.015 \Xi^2} + 0.045 \left[ 1 - e^{-0.068 \Xi} \right].
\]  
(6.22)
6.2.4 Effect of Cross-circulation

In order to investigate to what extent the cross-circulation $\Gamma$ influences the bed topography, the cross circulation velocity at the bed $v_{\alpha*}$ is now introduced into the sediment transport model. The approach of Shimizu and Itakura 1989 is adopted. According to these authors, $v_{\alpha*}$ is expressed as

$$\frac{v_{\alpha*}}{u_b} = -N_* \frac{h}{r} f_0 \left( \frac{n}{B} \right)$$  \hspace{1cm} (6.23)

where $N_* = 7$, as proposed by Engelund, and the function $f_0(n/B) = f_0(\eta)$ modifies the cross-circulatory radial velocity so as to take into account the presence of the banks (i.e., to make $v_{\alpha*} = 0$ at the banks). This function is adopted from Kikkawa et al. 1974 as

$$f_0(\eta) = -100 (\eta + 0.5)^2 + 20 (\eta + 0.5) \hspace{1cm} \text{for} \hspace{1cm} -0.5 \leq \eta < -0.4$$  \hspace{1cm} (6.24)

$$f_0(\eta) = 1.0 \hspace{1cm} \text{for} \hspace{1cm} -0.4 \leq \eta < 0.4$$  \hspace{1cm} (6.25)

$$f_0(\eta) = -100 (0.5 - \eta)^2 + 20 (0.5 - \eta) \hspace{1cm} \text{for} \hspace{1cm} 0.4 \leq \eta < 0.5$$  \hspace{1cm} (6.26)

Thus $q_{abr}$, under the combined effect of the main flow and the cross-circulation, is now expressed as

$$q_{abr} = (q_{abt}) \left( \frac{v_{\alpha*}}{u_b} \right)$$  \hspace{1cm} (6.27)
In this equation, \( u_b \) is expressed in terms of \( \bar{u} \) as

\[
    u_b = \frac{B_s + \frac{2.5}{2} \ln \left( \frac{e}{2D} \right)}{B_s + 2.5 \ln \left( \frac{0.368h}{2D} \right)} \bar{u} \tag{6.28}
\]

which follows from Eq. (6.12) and Eq. (6.13). Introducing \( \beta' / \beta = \gamma_b \), the cross-circulation velocity in terms of the vertically averaged flow velocity, may be expressed as

\[
    q_{sb_r} = (q_{sb}) \left( \bar{w} - \frac{v_{a_s}}{u} \frac{1}{\gamma_b} \right) \tag{6.29}
\]

Finally, from Eqs. (6.23) and (6.29), one can write

\[
    q_{sb_r} = (q_{sb}) \left( \bar{w} - 7.0\Phi \frac{h}{R} f_0(\eta) \frac{1}{\gamma_b} \right) \tag{6.30}
\]

where,

\[
    \Phi = \frac{1}{\eta \frac{B}{R} + 1}
\]

as shown in Chapter 5.

### 6.3 Non-dimensionalization of the Continuity Equation

Expressing Eq. (6.1) in the channel-fitted system of cylindrical coordinates \( l \) and \( r \), where (as should be clear from the contents of the previous sub-sections) \( q_s \) is to be identified with \( q_{sb} \), one arrives at

\[
    \frac{\partial z_b}{\partial t} + \frac{1}{1 - P} \left[ \frac{\partial q_{sb_l}}{\partial l} + \frac{\partial q_{sb_r}}{\partial r} + q_{sb_r} \right] = 0. \tag{6.31}
\]

The two different expressions for \( q_{sb_r} \), given by Eq. (6.18) and Eq. (6.30) are now used to develop the two different versions of the sediment transport model mentioned earlier, so that the same experiment may be simulated in two ways: one disregarding the cross-circulation completely (using MEANDERBED-I), and the other taking its effect into account when calculating the transverse bed load transport (MEANDERBED-II). The
results may then be compared to provide a deeper insight into the role of cross-circulation in producing the characteristic erosion-deposition patterns in meandering channels.

For the case where cross-circulation is neglected, using the dimensionless coordinates introduced in Section 5.1, Eq. (6.31) can be finally written as:

\[
B \frac{\partial z_b}{\partial t} + \frac{1}{1 - P} \left[ \Phi \frac{\partial q_{sb,i}}{\partial \lambda} + \tilde{\omega} \frac{\partial q_{sb,i}}{\partial \eta} + q_{sb,i} \frac{\partial \tilde{\omega}}{\partial \eta} + q_{sb,i} \tilde{\omega} \Phi \left( \frac{B}{R} \right) \right] = 0. \tag{6.32}
\]

For the case where cross-circulation is taken into account, i.e., \(q_{sb,r}\) is evaluated from Eq. (6.30), Eq. (6.31) can be written as:

\[
B \frac{\partial z_b}{\partial t} + \frac{1}{1 - P} \left[ \Phi \frac{\partial q_{sb,i}}{\partial \lambda} + \left( \tilde{\omega} - 7.0 \Phi \frac{h}{R} f_0(\eta) \frac{1}{\gamma_b} \right) \frac{\partial q_{sb,i}}{\partial \eta} \right] + \\
+ \frac{1}{1 - P} \left[ q_{sb,i} \left( \tilde{\omega} - 7.0 \Phi \frac{h}{R} f_0(\eta) \frac{1}{\gamma_b} \right) \Phi \left( \frac{B}{R} \right) \right] = 0 \tag{6.33}
\]

### 6.4 Discretization Scheme

Eq. (6.32) and Eq. (6.33) are discretized using explicit finite difference formulations. The numerical grid is the same as that used in the flow model. Backward differencing is used in the \(i-\) or \(\lambda-\) direction and central differencing is used in the \(j-\) or \(\eta-\) direction. Such a scheme seems to produce numerically stable solutions and smooth contour lines in plots of the bed topography.

Using the above scheme, one obtains the discretized version of Eq. (6.32) as

\[
\left[ (z_b^j)_{t+\Delta t} - (z_b^j)_{t} \right] \frac{B}{\Delta t} = \frac{1}{1 - P} \left[ \Phi^j_{i-1/2} \left( \frac{q_{sb,i-1}^j - q_{sb,i}^j}{\Delta \lambda} \right) + \tilde{\omega}^j_i \left( \frac{q_{sb,i}^{j-1} - q_{sb,i}^{j+1}}{2 \Delta \eta} \right) \right] + \frac{1}{1 - P} \left[ (q_{sb,i}^j) \left( \tilde{\omega}^{j-1} - \tilde{\omega}^{j+1} \right) \right] - (q_{sb,i}^j) \left( \omega^j_i \Phi^j_i \left( \frac{B}{R} \right)_i \right). \tag{6.34}
\]
Similarly, the discretized version of Eq. (6.33) is
\[
\begin{align*}
\left( z_{ei}^j \right)_{t+\Delta t} - \left( z_{ei}^j \right)_{t} & = \frac{1}{1-P} \left[ \Phi_{i-1/2}^j \left( \frac{q_{ebi}^{j-1} - q_{ebi}^{j+1}}{\Delta \lambda} \right) \right] + \\
& + \frac{1}{1-P} \left[ \left( \omega_i^j - 7.0(\Phi)_i^j \left( \frac{h}{R} \right)_i^j f_0(\eta)_i^j \left( \frac{1}{\eta_{0}} \right)_i^j \right) \left( \frac{q_{ebi}^{j-1} - q_{ebi}^{j+1}}{2\Delta \eta} \right) \right] \\
& - \frac{1}{1-P} \left[ (q_{ebi})_i^j \left( 7.0(\Phi)_i^{j-1} \left( \frac{h}{R} \right)_i^{j-1} f_0(\eta)_i^{j-1} \left( \frac{1}{\eta_{0}} \right)_i^{j-1} - 7.0(\Phi)_i^{j+1} \left( \frac{h}{R} \right)_i^{j+1} f_0(\eta)_i^{j+1} \left( \frac{1}{\eta_{0}} \right)_i^{j+1} \right) \right] \\
& - \frac{1}{1-P} \left[ (q_{ebi})_i^j \left( \omega_i^j - 7.0(\Phi)_i^j \left( \frac{h}{R} \right)_i^j f_0(\eta)_i^j \left( \frac{1}{\eta_{0}} \right)_i^j \right) \right] \left( \Phi_i^j \left( \frac{B}{R} \right)_i^j \right) \right]
\end{align*}

(6.35)

For both the above cases, the use of central differencing along a cross-section necessitates extrapolating for the 2 extreme points \( j = 1 \) and \( j = N \) along a grid-line \( i \). Here, a four point extrapolation scheme is followed. Thus if \( \Delta z_1, \Delta z_2, \Delta z_3, \Delta z_4 \), are the bed deformations at the points \( j = 1, 2, 3, 4 \) for a given \( i \), then we need to extrapolate for \( \Delta z_1 \) at the inner boundary, after having calculated the deformations at the other three points directly from the finite difference Eq. (6.34) and Eq. (6.35). This is done by assuming a linear variation in the difference of the bed deformation values between consecutive points, which is formulated as

\[
(\Delta z_1 - \Delta z_2) - (\Delta z_2 - \Delta z_3) = (\Delta z_2 - \Delta z_3) - (\Delta z_3 - \Delta z_4)
\]

(6.36)

Similarly, at the outer boundary, we have to extrapolate for \( \Delta z_N \), the deformations \( \Delta z_{N-1}, \Delta z_{N-2}, \Delta z_{N-3} \), being known. Following the extrapolation scheme described above,

\[
(\Delta z_N - \Delta z_{N-1}) - (\Delta z_{N-1} - \Delta z_{N-2}) = (\Delta z_{N-2} - \Delta z_{N-3}) - (\Delta z_{N-3} - \Delta z_{N-4})
\]

(6.37)

Therefore, in terms of the unknown deformations,

\[
\Delta z_1 = 3(\Delta z_2 - \Delta z_3) + \Delta z_4 \quad (6.38)
\]

\[
\Delta z_N = 3(\Delta z_{N-1} - \Delta z_{N-2}) + \Delta z_{N-3} \quad (6.39)
\]
It is also evident from Eq. (6.34) and Eq. (6.35) that at the grid-line $i = 1$, one needs the values of $q_{sb_i}$ at the preceding cross-section $i = 0$, which is actually outside the solution domain. These are estimated by imposing periodic boundary conditions (as in the case of the hydrodynamic model) on the longitudinal bed-load rate. Hence, the boundary condition adopted is

$$q_{sb_i}(0, j) = q_{sb_i}(M - 1, N + 1 - j).$$

(6.40)
7.0 RESULTS AND DISCUSSION

7.1 General

Following the works of most theoretical researchers in recent times (see Section 2.4.2), the flow model and the sediment transport model are decoupled in the present study. Thus, the determination of the bed topography produced by the initial flows is carried out in two steps: first, the flow field is computed with the aid of the vertically averaged hydrodynamic model MEANDERFLOW (see Chapter 5), and then the related bed erosions-depositions are computed with either of the two versions of the sediment transport model – MEANDERBED-I or MEANDERBED-II (see Chapter 6). It should be noted that, as mentioned earlier in this thesis, the evolution of the bed topography is an unsteady state phenomenon. The flow field, the sediment transport rates and the bed topography interact with each other, and as a result of this inter-action, they change constantly with time. However, the time-scales associated with the flow are much shorter than those associated with the slow evolution of the bed topography. Therefore, it can be assumed that the flow is steady during the time interval $\Delta t$, and that the associated $\nabla q_s$ remain invariant during this interval, so that the bed deformations can be computed from the sediment transport continuity equation, Eq. (6.1). Once the bed topography is computed, MEANDERPATH facilitates post-processing of the results, which are then used by TECPLOT to draw the contours of bed topography, $\nabla q_s$ etc. The Flowchart in Fig. 7.1 gives a schematic representation of the various models of this framework, and also of how they are interconnected. The model INITIALBED in the flowchart, generates the bed elevation data of the initial bed at every grid point, which is then used as input data for MEANDERFLOW.
Figure 7.1: Flowchart illustrating the sequence of models in the framework
7.2 Bed Deformation caused by the Initial Flow (without Cross-circulation)

As mentioned in the Introduction, the first objective of this thesis is "to investigate whether the ingoing and outgoing initial flows (for small and large $\theta_0$ channels, respectively) computed by the hydrodynamic model of Silva 1995, 1999, by themselves (i.e., without invoking cross-circulation), are capable of producing the erosion-deposition zones in the expected locations (i.e., as in Fig. 1.2a for small $\theta_0$ and as in Fig. 1.2b for large $\theta_0$ channels). As mentioned in Section 5.2, the model makes use of a variable resistance factor, which is defined by Eq. (5.23) and Eq. (5.24), viz.,

$$c^{-2} = c_s^{-2} + c_q^{-2}$$  \hspace{1cm} (7.1)

Here, $c_s^{-2}$, which is the part of $c^{-2}$ due only to the bed friction, is given (for the case of a sine-generated channel with a flat bed) by Eq. (5.25); $c_q^{-2}$, which takes into account the turbulence stresses, and which depends on channel geometry and location of a point in the flow plan, is given (for the case of a rough turbulent in a sine-generated channel with a flat bed) by Eq. (5.26), viz.,

$$c_q^2 = \alpha_q c_s^{-2} \left[\left(\frac{R}{r}\right)^2 - 1\right] - \left[4(\frac{R}{B})^2 - 1\right]^{-1}$$  \hspace{1cm} (7.2)

The coefficient $\alpha_q$ in Eq. (7.2) is expected to be a function of $\theta_0$ and $B/h_m$ (Silva 1995, 1999), i.e., $\alpha_q = \phi(\theta_0, B/h_m)$. This implies that if $\alpha_q$ is plotted versus $\theta_0$, then the result is a family of curves, where each curve corresponds to a different $B/h_m$ value (see Fig. 7.2). So far, only two points on a curve of this family of curves have been revealed through laboratory experiments. These points, which are indicated in Fig. 7.2, fall on the curve corresponding to $B/h_m = 13$, and correspond to $\theta_0 = 30^\circ$ ($\alpha_q = 0.17$), and $\theta_0 = 110^\circ$ ($\alpha_q = 1.7$). Through some theoretical considerations, Silva 1995 was able to predict that each $B/h_m$ curve on the ($\alpha_q, \theta_0$) plane in Fig. 7.2 should have an S-like shape.

In the following sections, the initial laboratory flows of Silva 1995 and Termini 1996 are first simulated by the hydrodynamic model, and then the computed initial flows are
Figure 7.2: Expected Variation of the Coefficient $\alpha_q$ with $\theta_0$ and $B/h_m$ (from Silva 1995) used to determine the bed topography for channels exhibiting both small and large $\theta_0$. The graph of $\alpha_q$ in Fig. 7.2 is used to estimate $\alpha_q$ in the present computations of the flow field and the bed topography. Since cross-circulation is neglected, the program MEANDERBED-I is used. An attempt is also made to establish the bed deformation patterns of natural streams.

7.2.1 Results for the Experimental Conditions of Silva 1995

As mentioned in the Literature Review, Silva 1995 measured initial flows in two sine-generated channels, having $30^\circ$ and $110^\circ$. The experimental conditions are as shown in Table 2.1. The initial flows computed by the hydrodynamic model are shown in Figs. 7.3a and 7.4a. Fig. 7.3a shows the flow field for the $30^\circ$ - channel; the value of $\alpha_q = 0.17$ was used. Fig. 7.4a shows the flow-field for the channel. The value of $\alpha_q = 1.7$ was used. It is shown in Silva 1995, that these computed flows are in good agreement with their measured counterparts. The flow is ingoing for $\theta_0 = 30^\circ$ and outgoing for $\theta_0 = 110^\circ$. The associated bed deformations computed by the author with the aid of the sediment transport model MEANDERBED-I are as shown in Figs. 7.3b and 7.4b, respectively.
These "computed bed deformations" are in agreement with experiment — in the sense that the most pronounced erosions-depositions are around the crossovers for $\theta_0 = 30^\circ$, and around the apex for $\theta_0 = 110^\circ$.

The initial flows and bed topographies were also computed for channels having $\theta_0 = 50^\circ, 70^\circ, 90^\circ$ (although Silva 1995 did not investigate these initial flows). The following values of $\alpha_q$ were adopted in the hydrodynamic model: $\alpha_q = 0.17, 1.2$ and $1.6$ for $\theta_0 = 50^\circ, 70^\circ$ and $90^\circ$, respectively. It should be pointed out that, although the values of $\alpha_q$ for $\theta_0 = 70^\circ$, and $\theta_0 = 90^\circ$ (namely $\alpha_q = 1.2$ and $1.6$) are directly read from the graph in Fig. 7.2, the value of $\alpha_q$ assumed for $\theta_0 = 50^\circ$ is not in agreement with the value predicted by this graph. This is because, in the course of conducting the above numerical experiments with the conditions of Silva 1995, it was found that if the value of $\alpha_q$ for the $\theta_0 = 50^\circ$ channel is considered according to the graph in Fig. 7.2, some discontinuities or "kinks" appear in the contours of the bed deformation. After trying several values, it was found that the best results are obtained for $\alpha_q \approx 0.17$, which is the experimentally determined value of $\alpha_q$ for the $\theta_0 = 30^\circ$ channel. Figs. 7.5a, 7.6a, 7.7a, show the computed initial flows, and Figs. 7.5b, 7.6b, 7.7b, show the bed deformations for the $\theta_0 = 50^\circ, 70^\circ, 90^\circ$ respectively. These Figures, along with the Figs. 7.3 and 7.4, illustrate how the location of erosions-depositions "shifts upstream" with the increment of $\theta_0$. This result is in agreement with Fig. 2.19.

It should be noticed that the amount of positive or negative bed deformation in all of these Figures, is nearly the same ($\approx 0.1h_m$) — yet, the time that it takes to reach this same amount of bed deformation varies with $\theta_0$. Fig. 7.8 is a plot of this time versus $\theta_0$. It should be noticed that the process of bed deformation is the fastest for $\theta_0 \approx 70^\circ$. This is not surprising, for at $\theta_0 \approx 70^\circ$, the flow converges and diverges "the most" — as illustrated by Fig. 7.9, where $|\bar{w}_{\text{max}}|$ is plotted versus $\theta_0$. $|\bar{w}_{\text{max}}|$ is defined as the maximum value of $\bar{w}$ (positive or negative) in the entire loop of the channel.
Figure 7.3: Numerical results for the experimental conditions of Silva 1995 for $\theta_0 = 30^\circ$ channel
\[ \theta_0 = 110^\circ ; \ B/h_m = 13 \]

Initial Flow

- \( Q = 2.10 \text{ l/s} \)
- \( D = 2.2 \text{ mm} \)
- \( B = 0.4 \text{ m} \)
- \( h_m = 0.03 \text{ m} \)
- \( S_b = 1/1120 \)
- \( c_s = 10.3 \)
- \( \alpha_q = 1.7 \)

Figure 7.4: a - Computed initial flow field for the experimental conditions of Silva 1995 for \( \theta_0 = 110^\circ \) channel

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\[ \theta_0 = 110^\circ; \ B/h_m = 13 \]

\[ \begin{align*}
Q &= 2.10 \text{ l/s} \\
D &= 2.2 \text{ mm} \\
B &= 0.4 \text{ m} \\
h_m &= 0.03 \text{ m} \\
S_b &= 1/1120 \\
c_s &= 10.3 \\
\alpha_g &= 1.7
\end{align*} \]

**Bed deformation at time = 75 mins**

Figure 7.4: b - Computed bed deformation for the experimental conditions of Silva 1995 for \( \theta_0 = 110^\circ \) channel
Figure 7.5: Numerical results for the experimental conditions of Silva 1995 for $\theta_0 = 50^\circ$ channel
Figure 7.6: Numerical results for the experimental conditions of Silva 1995 for $\theta_0 = 70^\circ$

channel
Figure 7.7: Numerical results for the experimental conditions of Silva 1995 for $\theta_0 = 90^\circ$

channel
Figure 7.8: Time taken to achieve equal deformations in the channels vs. $\theta_0$, for the experimental conditions of Silva 1995

Figure 7.9: $|\omega_{\text{max}}|$ vs. $\theta_0$, for the experimental conditions of Silva 1995
7.2.2 Results for the Experimental Conditions of Termini 1996

Termini measured the initial flow in a 110° sine-generated channel, the experimental conditions of which are given in Table 2.1 (see Literature Review). The corresponding computed initial flow is shown in Fig. 7.10a, and the associated bed deformation (computed by MEANDERBED-I) is shown in Fig. 7.10b. It should be noted that Termini’s flow is in the transitional regime of the turbulent flow, for which Fig. 7.2, strictly speaking, is not valid. However, for lack of better information, the value of \( \alpha_q = 1.7 \) (from Fig. 7.2) was adopted for use in the hydrodynamic model. Nevertheless, the predictions of the “initial deformation” zones (Fig. 7.10b) seem to follow the same patterns of erosion-deposition as shown in Termini’s plot of \( \nabla q_s \) shown in Fig. 2.17, which was computed with the aid of the measured values of the initial flow.

7.2.3 Results for Muddy Creek and Mississippi River

A numerical simulation of the flow conditions observed in the Muddy Creek is also carried out (\( \theta_0 = 70^\circ, B/h_m = 11 \)). The flow conditions, which are assumed to be the same as those of the initial flow, are given in the legend of Fig. 7.11. A value of \( \alpha_q = 1.2 \), was adopted for use in the hydrodynamic model: this value follows from the graph in Fig. 7.2. Fig. 7.11a shows the computed initial flow. The flow is outgoing, as expected from theory. The computed initial bed deformation is shown in Fig. 7.11b. When compared to the field data, there seems to be a small shift in the position of the deepest pool (compare with Fig. 1.1b).

In the field measurements of the equilibrium bed topography however, the deepest pool is at the apex, spreading to its upstream and downstream. But in the numerical results, the position of the deepest pool is downstream of the bend apex. This trend is, in fact, noticed in all of the numerical results discussed so far. It is also pertinent to mention here that all of the above channels have low \( B/h_m \)-ratios. In order to investigate whether the same trend would be observed in the case of rivers with large width-to-depth ratios, a numerical experiment is performed with the Mississippi river data. The computed
\[ \theta_0 = 110^\circ ; \frac{B}{h_m} = 17 \]

Initial Flow

Q = 6.5 l/s  
D = 0.65 mm  
B = 0.5 m  
h_m = 0.03 m  
S_b = 1/270  
c_s = 13.12  
\[ \alpha_q = 1.7 \]

Figure 7.10: a - Computed initial flow field for the experimental conditions of Termini 1996
\[ \theta_0 = 110^\circ; \ B/h_m = 17 \]

**Bed deformation at time = 6mins**

Figure 7.10: b - Computed bed deformation for the experimental conditions of Termini 1996

\[
\begin{align*}
Q &= 6.5 \text{ l/s} \\
D &= 0.65 \text{ mm} \\
B &= 0.5 \text{ m} \\
h_m &= 0.03 \text{ m} \\
S_b &= 1/270 \\
c_s &= 13.12 \\
\alpha_q &= 1.75
\end{align*}
\]
\[ \theta_0 = 70^\circ; \ B/h_m = 11 \]

Figure 7.11: a - Computed initial flow field for the experimental conditions in Muddy Creek

\[ Q = 1680 \text{ l/s} \]
\[ D = 0.7 \text{ mm} \]
\[ B = 5.5 \text{ m} \]
\[ h_m = 0.5 \text{ m} \]
\[ S_b = 1/714 \]
\[ c_s = 7.14 \]
\[ \alpha_q = 1.2 \]
\( \theta_0 = 70^\circ \); \( B/h_m = 11 \)

**Bed deformation at time = 90 mins**

Figure 7.11: b - Computed bed deformation for the experimental conditions in Muddy Creek
initial flow is shown in Fig. 7.12a, and the computed initial bed deformation is shown in Fig. 7.12b. It is evident, that in this case, the discrepancy between the location of the erosion-deposition zones seems to disappear. The erosion and deposition patterns seem to be very much in phase with each other.

7.3 Influence of Cross-circulation on the Initial Bed Deformation

In order to investigate the role of the cross-circulation in producing the characteristic erosion-deposition patterns in a meandering stream, most of the numerical experiments from Section 7.2 were repeated, with the terms, which take the effect of cross-circulation into account, being included in the sediment transport continuity equation. This implies that the sediment transport model MEANDERBED-II is now used in the calculations (see Fig. 7.1). The following is an account of the results obtained.

i) Silva 1995. The initial bed deformations were computed for the experimental conditions of Silva 1995 in the 30°, 70°, 110° channels (with the effect of cross-circulation being taken into account). Fig. 7.13 shows the bed deformation “with cross-circulation” for the 30° channel, which can be compared with the bed deformations shown in Fig. 7.3b, produced by ignoring the cross-circulation completely. Similarly, the computed bed deformation (with cross-circulation) in the 70° channel is presented in Fig. 7.14. This can be compared with Fig. 7.4b, which was obtained by neglecting the cross-circulation. The bed deformation in the 70° channel (with the effect of cross-circulation) is shown in Fig. 7.15, the bed deformation obtained by neglecting the cross-circulation being given in Fig. 7.5b. As is evident from all of these Figures, there does not seem to be any noticeable difference in the overall scheme of the bed topography contours “before” and “after” including the effects of cross-circulation.
\[ \theta_0 = 110^\circ \; ; \; B/h_m = 69 \]

Figure 7.12: a - Initial flow field for the experimental conditions in the Mississippi River
$\theta_0 = 110^\circ ; \frac{B}{h_m} = 69$

**Figure 7.12: b** - Computed bed deformation for the experimental conditions in the Mississippi River
\[ \theta_0 = 30^\circ \ ; \ B/h_m = 12.5 \]

Bed deformation at time = 30 mins

Figure 7.13: Numerical results for the experimental conditions of Silva 1995 for \( \theta_0 = 30^\circ \) channel (taking the cross-circulation terms into consideration)
\[ \theta_0 = 70^\circ ; \text{B/h}_m = 13 \]

DZ(CM)

0.300
0.225
0.150
0.075
0.000
-0.075
-0.150
-0.225
-0.300

Q = 2.10 l/s
D = 2.2 mm
B = 0.4 m
h_m = 0.03 m
S_b = 1/1120
\( c_s = 10.3 \)
\( \alpha_q = 0.17 \)

**Bed deformation at time = 17 mins**

Figure 7.14: Numerical results for the experimental conditions of Silva 1995 for \( \theta_0 = 70^\circ \) channel (taking the cross-circulation terms into consideration)
\[ \theta_0 = 110^\circ ; \frac{B}{h_m} = 13 \]

\begin{itemize}
  \item Q = 2.10 l/s
  \item D = 2.2 mm
  \item B = 0.4 m
  \item h_m = 0.03 m
  \item S_b = 1/1120
  \item c_d = 10.3
  \item \( \alpha_q = 1.7 \)
\end{itemize}

Bed deformation at time = 75 mins

Figure 7.15: Numerical results for the experimental conditions of Silva 1995 for \( \theta_0 = 110^\circ \) channel (taking the cross-circulation terms into consideration)
ii) *Termini 1996.* Fig. 7.16 shows the contours of bed deformation obtained after including the terms which reflect the effect of the cross-circulation, $\Gamma$. Comparing this Figure with Fig. 7.10b, which show the contours of bed deformation obtained by neglecting the effect of cross-circulation, no major differences are noticeable between the two erosion-deposition patterns.

iii) *Muddy Creek and Mississippi River.* The bed deformation obtained by using the field conditions observed in the Muddy Creek after including the effect of the cross-circulation, is given in Fig. 7.17. This is to be compared with Fig. 7.11b, which shows the bed deformation obtained by neglecting the cross-circulation. Finally, the contours of bed deformation in the Mississippi River “with” cross-circulation are shown in Fig. 7.18, and seems to be identical to the contour plot in Fig. 7.12b, which shows the bed deformation for Mississippi “without” cross-circulation.

In all of the above cases, the differences in the trend in bed deformation between “without” cross-circulation and with “cross-circulation” appear to be insignificant, although the $B/h_m$ ratio varies from 11 (for Muddy Creek) to 69 (for the Mississippi River).

### 7.4 Variation in the Coefficient $\alpha_q$ with $\theta_0$ and $B/h_m$

The values of $\alpha_q$ used in the “numerical exercises” described earlier in this Chapter, together with the values known from experiment, are as shown in Table 7.1. The words “experimental” and “numerical” are added to Table 7.1, so as to distinguish between the values of $\alpha_q$ obtained from laboratory experiments and those obtained from the numerical exercises. The values corresponding to $B/h_m \approx 13$ (say) can be taken to characterize the $\alpha_q$ curve corresponding to $B/h_m \approx 13$. These values of $\alpha_q$ are plotted versus $\theta_0$ in Fig. 7.19 (the data points corresponding to each different $B/h_m$ has been shown with a different symbol – see the legend of the graph in Fig. 7.19), together with the original $\alpha_q$ curve suggested by Silva 1995 (Fig. 7.2). Evidently, this curve does not seem to be a good fit for the data points in Fig. 7.2. In fact, it appears that a curve like the one
\( \theta_0 = 110^\circ \); \( B/h_m = 17 \)

Q = 6.5 l/s
D = 0.65 mm
B = 0.5 m
\( h_m = 0.03 \) m
\( S_b = 1/270 \)
c_s = 13.12
\( \alpha_g = 1.75 \)

Figure 7.16: Numerical results for the experimental conditions of Termini 1996 for \( \theta_0 = 110^\circ \) channel (taking the cross-circulation terms into consideration)
\[ \theta_0 = 70^\circ; \frac{B}{h_m} = 11 \]

**Bed deformation at time = 90 mins**

Figure 7.17: Numerical results for the experimental conditions in the Muddy Creek (taking the cross-circulation terms into consideration)
\[ \theta_0 = 110^\circ ; \frac{B}{h_m} = 69 \]

**Bed deformation at time = 625 days**

Figure 7.18: Numerical results for conditions in the Mississippi River (taking the cross-circulation terms into consideration)
Table 7.1: Values of $\alpha_q$ producing the best results in the numerical experiments

<table>
<thead>
<tr>
<th>Experimental conditions of</th>
<th>$\theta_0$</th>
<th>$B/h_m$</th>
<th>$\alpha_q$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silva 1995</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30º</td>
<td>12.5</td>
<td>0.17</td>
<td>Experimental</td>
</tr>
<tr>
<td></td>
<td>50º</td>
<td>13</td>
<td>0.17</td>
<td>Numerical</td>
</tr>
<tr>
<td></td>
<td>70º</td>
<td>13</td>
<td>1.2</td>
<td>Numerical</td>
</tr>
<tr>
<td></td>
<td>90º</td>
<td>13</td>
<td>1.6</td>
<td>Numerical</td>
</tr>
<tr>
<td></td>
<td>110º</td>
<td>13</td>
<td>1.7</td>
<td>Experimental</td>
</tr>
<tr>
<td>Termini 1996</td>
<td>110º</td>
<td>17</td>
<td>1.7</td>
<td>Numerical</td>
</tr>
<tr>
<td>Muddy Creek</td>
<td>70º</td>
<td>11</td>
<td>1.2</td>
<td>Numerical</td>
</tr>
</tbody>
</table>

drawn with a dashed line, would be preferable, as a fit for these data. The new line is a steeper $S-$like curve than the one originally suggested. Hence, it is proposed that this new curve, as suggested by the results of the present research, be adopted for estimating $\alpha_q$ in future.

From theoretical considerations, Silva 1995 also argued that $\alpha_q$ varies with $B/h_m$, which is illustrated by the family of curves in Fig. 7.2. However, no experimental data are available about this variation. The present research also seems to indicate that $\alpha_q$ is not very sensitive to variations of $B/h_m$. This is illustrated through the simulations of the Muddy Creek flow conditions ($B/h_m = 11$), the results of which are as shown in Fig. 7.11b, and much more so by the Mississippi River flow conditions ($B/h_m = 69$), for which the contours of bed deformation are as shown in Fig. 7.12b. Yet, in both cases, the $\alpha_q$ values as predicted by the graph in Silva 1995, which was actually plotted for $B/h_m = 13$, appear to produce sensible results.
Figure 7.19: Expected variation of $\alpha_q$ with $\theta_0$ for $B/h_m = 13$
8.0 CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

8.1 Conclusions

From the results of the numerical simulations performed in Chapter 7, the following conclusions can be drawn with respect to the objectives outlined in Chapter 1:

1. The initial flows computed by Silva 1995 can, indeed produce initial bed deformations, which are in agreement with the characteristic patterns of erosion-deposition in both small and large sinuosity channels.

2. The results also illustrate that cross-circulation plays a rather insignificant role in the determination of the overall “bar-pool” topography of the channel. However, it should be mentioned that only the first stage in the simulation of the equilibrium bed topography has been performed here. On the other hand, the cross-circulation velocity clearly depends on the $h/r$-ratio at each and every grid point. Hence, in subsequent steps, as the bed undergoes more and more deformation, the variation in this ratio from point to point will be much greater. Thus, cross-circulation could, as pointed out by Hooke 1974 (see Literature Review), partially influence the “detailed” geometry of the bed.

3. The coefficient $\alpha_q$, which was introduced by Silva 1995 for the evaluation of the effect of the turbulent stresses with the help of a variable resistance factor, was investigated. It was found that $\alpha_q$ does not vary significantly with variations in $B/h_m$. The graph of the function $\alpha_q = \phi(\theta_0)$ for a given $B/h_m$ as estimated by Silva 1995 was also modified as suggested by the results of the numerical experiments.
8.2 Suggestions for Future Research

It follows from the conclusions of the present thesis that there is need for further research into this field. The framework of models developed should now be extended so as to be able to predict the equilibrium bed topography of sine-generated meandering channels. Through some trials and experimentations performed by the author, it was determined that merely using the developed system of models, is not enough for this purpose. If the flow and sediment transport models are run sequentially, following the procedure depicted in the Flowchart in Fig. 2.24, with the progress in time, the bed will indeed, deform more and more. However, some instabilities are introduced into the system at later stages of development of the bed topography. The predicted flow-field and bed deformation ceases to be continuous functions with smooth variations along the cross-sections and the length of the channel (i.e., the contours of bed topography start developing kinks and the flow acceleration and deceleration zones seem to be completely random). Such discontinuities are not in agreement with experimental findings or field data. These could be due to some discrepancy between the hydrodynamic and sediment transport models. The instabilities could also be caused by the variable resistance factor, the expression for which was determined by Silva 1995, 1999, over a flat bed. It is possible that the form of this function, as suggested by Silva 1995, 1999, does not hold good over a deformed bed, i.e., the variation of this resistance factor over a deformed bed assumes a different form. Evidently, further investigation into these aspects are required.

Although the present work throws some light on the function, \( \alpha_q = \phi_q(\theta_0, B/h_m) \), over a flat bed, further research is still needed to determine this function completely. Experimental data should be collected from channels with varying sinuosities, so that \( \alpha_q \) can be known discretely at several points on the curve in Fig. 7.19 (as done in the case of the numerical experimentation). In fact, an experimental setup has already been developed with this objective in mind, and is now in place at the Meandering Research Facility of the Great Lakes Institute for Environmental Research.
References


Appendix A: Printout of the computer program MEANDERPATh
**MEANDERPATH**

**PROGRAM FOR CALCULATION OF CHANNEL COORDINATES IN THE CARTESIAN SYSTEM**

REAL*8 XC(400), YC(400), LC, L, THETA1, THETA2, THETA0, THET12
REAL*8 X(50,50), Y(50,50), PI, B, XC1, YC1, DLC, BESST0, LAMBDA
INTEGER*4 M, N, MM, NN, I, J, K, K1, MN, MID, MCC

**OUTPUT FILES**
OPEN (UNIT=11, FILE='X.DAT')
OPEN (UNIT=12, FILE='Y.DAT')
OPEN (UNIT=13, FILE='X-Y.DAT')

**INPUT PARAMETERS**

THETA0=1100. D-1
M=17
N=21
MFULL=33
MM=(M-1)*20
MN-MM+1
MID-MM/2+1
PI=3141592653589.D-12
B=4.D-1
LAMBDA=20.D-1*PI*B

**CONVERT THETA0 TO RADII**
THETA0=(THETA0*PI)/1800.D-1

**CALCULATION OF LENGTH OF CHANNEL**
CALL BESSEL (THETA0, BESST0)
SIGMA=10.D-1/BESST0
L=SIGMA*LAMBDA
PRINT *, L

**CALCULATION OF CO-ORDINATES FOR CENTRELINE**

LC=0.
XC(1)=0.
YC(1)=0.
THETA1=THETA0
DLC=L/(20.D-1*MM)

**DO 50 K=1,10**
X(1,K)=XC(1)+(B/20.D-1)*SIN(THETA0)*((11-K)/100.D-1)
Y(1,K)=YC(1)-(B/20.D-1)*COS(THETA0)*((11-K)/100.D-1)

50 CONTINUE

**DO 60 K=11,21**
X(1,K)=XC(1)-(B/20.D-1)*SIN(THETA0)*((K-11)/100.D-1)
Y(1,K)=YC(1)+(B/20.D-1)*COS(THETA0)*((K-11)/100.D-1)

124
CONTINUE

DO 10 I=2,MN

  LC=LC+DLC
  THETA2=THETA0*COS(20.D-1*PI*(LC/L))
  THET12=(THETA1+THETA2)/20.D-1
  XC(I)=XC(I-1)+DLC*COS(THET12)
  YC(I)=YC(I-1)+DLC*SIN(THET12)

  IF(I.GE.MID) THEN
    MCC=I-MID
    YC(I)=YC(MID-MCC)
    DXC=XC(MID)-XC(MID-MCC)
    XC(I)=XC(MID)+DXC
  END IF

  IF((I/200.D-1).EQ.INT(I/200.D-1)) THEN
    J=INT(I/200.D-1)+1

    DO 20 K=1,10
      X(J,K)=XC(I)+(B/20.D-1)*SIN(THETA2)*(11-K)/100.D-1
      Y(J,K)=YC(I)-(B/20.D-1)*COS(THETA2)*(11-K)/100.D-1
    20 CONTINUE

    DO 30 K=11,21
      X(J,K)=XC(I)-(B/20.D-1)*SIN(THETA2)*(K-11)/100.D-1
      Y(J,K)=YC(I)+(B/20.D-1)*COS(THETA2)*(K-11)/100.D-1
    30 CONTINUE
  END IF

  THETA1=THETA2

10 CONTINUE

DO 120 J=M+1, MFULL
  DO 125 K=1, N
    X(J,K)=XC(MN)+X(J-M+1,N+1-K)
    Y(J,K)=Y(J-M+1,N+1-K)
  125 CONTINUE
120 CONTINUE

WRITE(11,*)'X-COORDINATES FOR CONTOUR PLOTS'
WRITE(12,*)'Y-COORDINATES FOR CONTOUR PLOTS'
WRITE(13,*)'X-Y DATA FOR EXCEL PLOTTING: INSIDE TO OUTSIDE BANK'

DO 40 K=1, MFULL
  WRITE(11,*)
  WRITE(12,*)
    WRITE(11,100) (X(K1,K), K=1,21)
    WRITE(12,100) (Y(K1,K), K=1,21)
40 CONTINUE
100 FORMAT(5(1X,F12.8))
WRITE(13,*)
DO 70 K1=1,MFULL
   WRITE(13,110)X(K1,1),Y(K1,1)
70   CONTINUE
WRITE(13,*)
DO 80 K1=1,MFULL
   WRITE(13,110)X(K1,11),Y(K1,11)
80   CONTINUE
WRITE(13,*)
DO 90 K1=1,MFULL
   WRITE(13,110)X(K1,21),Y(K1,21)
90   CONTINUE
110 FORMAT(2(3X,F12.8))

STOP
END

******************************************************************************

SUBROUTINE BESSEL(ANGLE,K)
******************************************************************************

REAL*8 ANGLE,K

POLYNOMIAL APPROXIMATION:

K=K-39444.D-7*(ANGLE/30.D-1)**(100.D-1)
K=K+2100.D-7*(ANGLE/30.D-1)**(120.D-1)

1000   RETURN
END

******************************************************************************
Appendix B: Printouts of the computer programs MEANDERBED-I and MEANDERBED-II
C******************************************************************************************
C MEANDERBED-I
C PROGRAM FINALIZED ON 29TH MAY, 2000
C CALCULATES BED DEFORMATION IN A SINUOUS CHANNEL
C CROSS-CIRCULATION IS NEGLECTED
C******************************************************************************************

INTEGER*4 M,N,MM,NN
REAL*8 THETA0, SO, CO, BOHO, SIGMA, BESST0, BOL, U0, BORPAR
REAL*8 PI, LAMBDA, LAM, ETA, DLAM, ETA2, L, B, DTIME, ETAS
REAL*8 G, GAMS, RHO, YCR, ABC, CON, SG, D, KS, QSBTOT
REAL*8 ETA2, ETA12, LAM1, LAM12, MIDLAM, BORI1, BORI2, MIDBOR
REAL*8 CPHII12, CPHIIJ, CRIT, SUM, TEMP, CRIT2, ASUM, XI, YAV
REAL*8 Z1(50,50), PHIU(50,50), CHEZ(50,50), Z2(50,50), EP0KS
REAL*8 DELZ(50,50), Y1(50,50), OMEGA(50,50), PHIH(50,50), ERR, QST
REAL*8 VECU(50,50), QSB(50,50), Y(50,50), U(50,50), QSB1(50)
CHARACTER*14 ZDAT, CDAT, UDAT, ZRES, DZ, QFILE

M=33
N=21

C INPUT FILES GENERATED BY CURVE.FOR

WRITE(*,1)
1 FORMAT(1X, 'FILE FOR VELOCITY FIELD (F1.RES) = ')
READ(*,2) UDAT
2 FORMAT(A12)
WRITE(*,3)
3 FORMAT(1X, 'FILE FOR Z1 = ')
READ(*,4) ZDAT
4 FORMAT(A12)
WRITE(*,5)
5 FORMAT(1X, 'FILE FOR FFACTOR=')
READ(*,6) CDAT
6 FORMAT(A12)
WRITE(*,19)
19 FORMAT(1X, 'FILE FOR DZ = ')
READ(*,17) DZ
17 FORMAT(A14)
WRITE(*,89)
89 FORMAT(1X, 'FILE FOR QSB = ')
READ(*,87) QFILE
87 FORMAT(A14)

OPEN(UNIT=11, FILE=UDAT)
DO 15 I=1,M
   READ(11,10) (PHIU(I,J), J=1,N)
   READ(11,10) (OMEGA(I,J), J=1,N)
   READ(11,10) (PHIH(I,J), J=1,N)
15 CONTINUE

OPEN(UNIT=21, FILE=ZDAT)
READ(21,*)
DO 35 I=1,M

128
READ(21,10) (Z1(I,J), J=1,N)
CONTINUE

OPEN(UNIT=31,FILE=CDAT)
READ(31,*)
DO 40 I=1,M
  READ(31,10) (CHEZY(I,J), J=1,N)
40  CONTINUE

FORMAT(5(1X,D12.8))
OPEN(UNIT=77,FILE=QFILE)

C******************************************************************************
**
C RESULT FILE FROM DEF

WRITE(*,7)
FORMAT(1X,'FILE FOR Z2 = ')  READ(*,8)ZRES
8  FORMAT(A12)
OPEN(UNIT=41, FILE=ZRES)
OPEN(UNIT=81, FILE=DZ)

C******************************************************************************
**
C INPUT PARAMETERS (IN SI UNITS)

PI=3.141592653589.D-12
THETA0=1100.D-1

C CONVERT TO RADIUS
THETA0=(THETA0*PI)/1800.D-1

S0=10.D-1/11200.D-1
B=4.D-1
BOH0=4.D-1/300.D-4
C0=10300.D-3
U0=167.D-3
CALL BESSEL(THETA0, BESST0)
LAMBDA=20.D-1*PI*B
SIGMA=10.D-1/BESST0
L=SIGMA*LAMBDA
BOL=B/L
BORPAR=THETA0*BESST0
H0=300.D-4
RHO=10000.D-1
D=220.D-5
YCR=35.D-5
GAMS=161865.D-1
G=981.D-2
SG=165.D-2
NU=1.D-6
C
C CALCULATION OF QSB USING BAGNOLD'S EQUATION:
C
CON=RHO/(D*GAMS)
YAV=CON*G*S0*H0
ETAS=YAV/YCR
KS=20.D-1*D
EP0KS=10.D-1+EP0KS
DO 25 I=1,M
   DO 30 J=1,N
      PHIH(I,J)=HO*PHIH(I,J)
      U(I,J)=PHIU(I,J)*U0
      VECU(I,J)=(U(I,J))**20.D-1
      VECU(I,J)=VECU(I,J)*((OMEGA(I,J))+(OMEGA(I,J))+10.D-1)
      ABC=5.D-1*EP0KS
      ABC=ABC/(1+LOG(368.D-3*(PHIH(I,J)/KS)))/(85.D-1+4.D-1)
      ABC=ABC*D*U(I,J)
      Y(I,J)=(CON*VECU(I,J))/CHEZY(I,J)
      IF(Y(I,J).LT.YCR) THEN
         PRINT *, Y(I,J)
         PAUSE
      END IF
      WRITE(*,444)
      FORMAT(1X,'Y IS LESS THEN YCR. SORRY! WANT TO MAKE A FRESH
            START ?')
      QSB(I,J)=0.
   ELSE IF(Y(I,J).GE.YCR) THEN
      QSB(I,J)=ABC*(Y(I,J)-YCR)
   END IF
30 CONTINUE
25 CONTINUE
QSBTOT=0.
DO 175 J=1,N
   QSBTOT=QSBTOT+QSB(I,J)
   IF(I.EQ.1) THEN
      QST=QSBTOT
      GOTO 25
   END IF
   IF(QSBTOT.LT.QST) THEN
      QST=QSBTOT
      GOTO 25
   END IF
175 CONTINUE
PRINT *, QST
PAUSE
   DO 279 I=1,M
279 CONTINUE
QSBTOT=0.
DO 282 J=1,N
282 QSBTOT=QSBTOT+QSB(I,J)
ERR=QST-QSBTOT
ERR=ERR/N
DO 177 J=1,N
177 QSB(I,J)=QSB(I,J)+ERR
QSBTOT=0.
DO 178 J=1,N
178 QSBTOT=QSBTOT+QSB(I,J)
PRINT *,QSBTOT
PAUSE
DO 197 J=1,N
197 QSB(I,J)=QSB(I,J)+DTIME
279 CONTINUE
DO 81 J=1,N
81 QSB1(J)=QSB(MM,J)

C******************************************************************************

C CALCULATION OF ZB USING EXNER-POLYA

LAMI1=-DLAM
DO 55 I=1,M
55 LAMI2=LAMI1+DLAM
MIDLAM=(LAMI1+LAMI2)/20.D-1
C END IF
MIDBOR=BORPAR*SIN(20.D-1*PI*MIDLAM*BOL)
BORI2=BORPAR*SIN(20.D-1*PI*LAMI2*BOL)
ETA=-45.D-2
DO 45 J=2,N-1
45 CPHII2=10.D-1/(ETA*MIDBOR+10.D-1)
CPHIJ2=CPHII2/B
CPHIJ=10.D-1/(ETA*BORI2+10.D-1)
CPHIJ2=CPHIJ/B
IF (I.EQ.1) THEN
DELZ(I,J)=(QSB1(J)-QSB(I,J))*CPHI22/DLAM
ELSE
DELZ(I,J)=(QSB(I-1,J)-QSB(I,J))*CPHI22/DLAM
END IF
DELZ(I,J)=DELZ(I,J)+(OMEGA(I,J)/(20.D-1*B*DETA))*(QSB(I+
J-1)-QSB(I,J+1))
DELZ(I,J)=DELZ(I,J)+(QSB(I,J)/(20.D-1*B*DETA))*OMEGA(I+
J-1)-OMEGA(I,J+1))
DELZ(I,J)=DELZ(I,J)-QSB(I,J)*CPHIJ2*BORI2*OMEGA(I,J)

ETA=ETA+DETA
CONTINUE

45 DELZ(I,N)=30.D-1*(DELZ(I,N-1)-DELZ(I,N-2))+DELZ(I,N-3)
DELZ(I,1)=30.D-1*(DELZ(I,2)-DELZ(I,3))+DELZ(I,4)

55 LAMI1=LAMI2
CONTINUE
DO 252 I=1,M
DO 251 J=1,N
   Z2(I,J)=Z1(I,J)+DELZ(I,J)
251 CONTINUE
252 CONTINUE

C OUTPUT FILES
WRITE(81,*),'BED DEFORMATION AT 75 MINS'
WRITE(41,*),'CALCULATED DEFORMED BED AT 75 MINS'
WRITE(77,*),'QSBS AT 75 MINS'
DO 50 I=1,M
   WRITE(77,100) (QSB(I,J),J=1,N)
   WRITE(81,100) (DELZ(I,J),J=1,N)
   WRITE(41,100) (Z2(I,J),J=1,N)
50 CONTINUE
100 FORMAT(5(1X,F12.8))

SUM=0.
DO 150 I=1,M
   DO 155 J=1,N
      SUM=SUM+DELZ(I,J)
155 CONTINUE
150 CONTINUE

C IF (SUM.LE.CRIT) THEN
   PRINT *, 'SUM=', SUM
C PRINT *, 'YIPEE, YOU ARE THERE! STOP IF U DO NOT WANT TROUBLE.'
C END IF

STOP
END

C*****************************************************************************
SUBROUTINE BESSEL(ANGLE,K)
C*****************************************************************************

REAL*8 ANGLE, K

C
C POLYNOMIAL APPROXIMATION:
C
K=K-39444.D-7*(ANGLE/30.D-1)**(100.D-1)
K=K+2100.D-7*(ANGLE/30.D-1)**(120.D-1)

RETURN
END

C*****************************************************************************
C*****************************************************************************
C MEANDERBED-II
C CALCULATES BED DEFORMATION IN A SINUOUS CHANNEL
C EFFECT OF CROSS-CIRCULATION IS CONSIDERED
C*****************************************************************************

INTEGER*4 M,N,MM,NN
REAL*8 THETA0, S0, C0, BOHO, SIGMA, BESST0, BOL, U0, BOREAR
REAL*8 PI, LAMBDA, LAM, ETA, DLAM, DELTA, L, B, DTIME, ETAS, BDOB(50,50)
REAL*8 G, GAMS, RHO, YCR, ABC, CON, SG, D, FON(50,50), QSBTOT
REAL*8 ETA2, ETA12, LAM11, LAM12, MIDLAM, BORI1, BORI2, MIDBOR
REAL*8 CPHI12, CPHI13, CRIT, SUM, TEMP, CRIT2, ASUM, XI, YAV
REAL*8 Z1(50,50), PHIU(50,50), CHEZY(50,50), Z2(50,50), EPSKS
REAL*8 DELZ(50,50), Y1(50,50), OMEGA(50,50), PHIH(50,50), ERR, QST
REAL*8 VECU(50,50), QSB(50,50), Y(50,50), U(50,50), QSB1(50)

CHARACTER*14 ZDAT, CDAT, UDAT, ZRES, DZ, QFILE

M=33
N=21

C INPUT FILES GENERATED BY CURVE.FOR

WRITE(*,1)
1 FORMAT(1X, 'FILE FOR VELOCITY FIELD (F1.RES) = ')
READ(*,2) UDAT
2 FORMAT(A12)
WRITE(*,3)
3 FORMAT(1X, 'FILE FOR Z1 = ')
READ(*,4) ZDAT
4 FORMAT(A12)
WRITE(*,5)
5 FORMAT(1X, 'FILE FOR FFACTOR=')
READ(*,6) CDAT
6 FORMAT(A12)
WRITE(*,19)
19 FORMAT(1X, 'FILE FOR DZ = '
READ(*,17) DZ
17 FORMAT(A14)
WRITE(*,89)
89 FORMAT(1X, 'FILE FOR QSB = '
READ(*,87) QFILE
87 FORMAT(A14)

OPEN(UNIT=11, FILE=UDAT)
DO 15 I=1,M
   READ(11,10) (PHIU(I,J), J=1,N)
   READ(11,10) (OMEGA(I,J), J=1,N)
   READ(11,10) (PHIH(I,J), J=1,N)
15 CONTINUE

OPEN(UNIT=21, FILE=ZDAT)
READ(21,*)
DO 35 I=1,M
   READ(21,10) (Z1(I,J), J=1,N)
35 CONTINUE
CONTINUE

OPEN (UNIT=31, FILE=CDAT)
READ (31, *)
DO 40 I=1,M
      READ (31, 10) (CHEY(I,J), J=1,N)
40 CONTINUE
10 FORMAT (5(1X, D12.8))
OPEN (UNIT=77, FILE=QFILE)

**
RESULT FILE FROM DEF
WRITE (*, 7)
FORMAT (1X, 'FILE FOR Z2 = ')
READ (*, 8) ZRES
7 FORMAT (A12)
OPEN (UNIT=41, FILE=ZRES)
OPEN (UNIT=81, FILE=D2)

**
INPUT PARAMETERS (IN SI UNITS)
PI = 3.141592653589.D-12
THETA0 = 1100.D-1
CONVERT TO RADII
THETA0 = (THETA0*PI)/1800.D-1

S0 = 10.D-1/11200.D-1
B = 4.D-1
BOHO = 4.D-1/300.D-4
C0 = 10300.D-3
U0 = 167.D-3
CALL BESSEL (THETA0, BESST0)
LAMBDA = 20.D-1*PI*B
SIGMA = 10.D-1/BESST0
L = SIGMA*LAMBDA
BOL = B/L
BOPAR = THETA0*BESST0
H0 = 300.D-4
RHO = 10000.D-1
D = 220.D-5
YCR = 35.D-5
GAMS = 161865.D-1
G = 981.D-2
SG = 165.D-2
NS = 70.D-1
F = 45.D-2
N = 1.D-6
C
C***********************************************************************
C
C CALCULATION OF QSB USING BAGNOLD'S EQUATION:
C
CON=RHO/(D*GAMS)
YAV=CON*G*SO+H0
ETAS=YAV/YCR
KS=20.D-1*D
EP0KS=10.D-1+EP0KS
DO 25 I=1,M
   DO 30 J=1,N
      PHIH(I,J)=HO+PHIH(I,J)
      U(I,J)=PHIU(I,J)*U0
      VECU(I,J)=(U(I,J))**20.D-1
      VECU(I,J)=VECU(I,J)*((OMEGA(I,J))+(OMEGA(I,J))+10.D-1)
      ABC=EP0KS
      ABC=BDOB(I,J)*D*U(I,J)*5.D-1
      Y(I,J)=(CON*VECU(I,J))/CHEZY(I,J)
      IF(Y(I,J).LT.YCR)THEN
         PRINT *,Y(I,J)
         PAUSE
         WRITE(*,444)
         FORMAT(1X,'Y IS LESS THEN YCR. SORRY! WANT TO MAKE A FRESH
+START ?')
         QSB(I,J)=0.
      ELSE IF(Y(I,J).GE.YCR)THEN
         QSB(I,J)=ABC*(Y(I,J)-YCR)
      END IF
   30 CONTINUE
   QSBTOT=0.
   DO 175 J=1,N
      QSBTOT=QSBTOT+QSB(I,J)
   175 IF(I.EQ.1) THEN
      QST=QSBTOT
      GOTO 25
   END IF
   IF(QSBTOT.LT.QST) THEN
      QST=QSBTOT
   GOTO 25
   END IF
   25 CONTINUE
   PRINT *,QST
C
C CALCULATION OF ZB USING EXNER-POLYA
C
C CALCULATION OF CROSS-CIRCULATION TERM
DO 266 I=1,M
   ETA=-5.D-1
   DO 267 J=1,N
      FON(I, J)=10.D-1
      IF (J.LT.3) THEN
      ELSE IF (J.GE.19) THEN
      END IF
      ETA=-5.D-1+DETA
   CONTINUE
267 CONTINUE
266 CONTINUE

LAMI1=-DLAM
DO 55 I=1,M
   LAMI2=LAMI1+DLAM
   MIDLAM=(LAMI1+LAMI2)/20.D-1
C END IF
   MIDBOR=BORPAR*SIN(20.D-1*PI*MIDLAM*BOL)
   BORI2=BORPAR*SIN(20.D-1*PI*LAMI2*BOL)
   ETA=-45.D-2
   DO 45 J=2,N-1
      CPHII2=10.D-1/(ETA*MIDBOR+10.D-1)
      CPHII2=CPHII2/B
      CPHI=10.D-1/(ETA*BORI2+10.D-1)
      CPHI=CPHI/B
      IF (I.EQ.1) THEN
         DELZ(I, J)=(QSB1(J)-QSB(I, J))*CPHI2/DLAM
      ELSE
         DELZ(I, J)=(QSB(I-1, J)-QSB(I, J))*CPHI2/DLAM
      END IF
DELZ(I,J) = DELZ(I,J) + ((OMEGA(I,J) - 70.0 - 
1*CPII*BOF2*PHI)
+ (I,J) * FON(I,J)) / (20.0 - 1*B*DETA*DBOD(I,J)) * (QB(I,J-1) - 
QB(I,J+1))

DELZ(I,J) = DELZ(I,J) + (QB(I,J) / (20.0 - 
1*B*DETA) * (OMEGA(I,J)
+ 1) - OMEGA(I,J+1)) - 70.0 - 1*CPII*BOF2* (PHI(I,J-1) - FON(I,J-1)
+ DBOD(I,J-1) - PHI(I,J+1) - FON(I,J+1) * BOD(I,J+1)) * QB(I,J)
+ (20.0 - 1*B*DETA)

DELZ(I,J) = DELZ(I,J) - QB(I,J) * CPII*BOF2*(OMEGA(I,J) - 
70.0 + 1*CPII*BOF2*PHI(I,J) * FON(I,J) - BOD(I,J))

ETA = ETA + DETA
CONTINUE

DELZ(I,N) = 30.0 - 1*(DELZ(I,N-1) - DELZ(I,N-2)) + DELZ(I,N-3)
DELZ(I,1) = 30.0 - 1*(DELZ(I,2) - DELZ(I,3)) + DELZ(I,4)

LAM1 = LAM1
CONTINUE

DO 252 I = 1, M
    DO 251 J = 1, N
        Z2(I,J) = S1(I,J) + DELZ(I,J)
    CONTINUE
    252
CONTINUE

C OUTPUT FILES

WRITE(81,*) 'BED DEFORMATION AT 80 MINS'
WRITE(41,*) 'CALCULATED DEFORMED BED AT 80 MINS'
WRITE(77,*) 'QSBS AT 80 MINS'
DO 50 I = 1, M
    WRITE(77,100) (QB(I,J), J = 1, N)
    WRITE(81,100) (DELZ(I,J), J = 1, N)
    WRITE(41,100) (Z2(I,J), J = 1, N)
CONTINUE
100 FORMAT(5(1X,F12.8))

SUM = 0.
DO 150 I = 1, M
    DO 155 J = 1, N
        SUM = SUM + DELZ(I,J)
    CONTINUE
    150
CONTINUE
155
CONTINUE

C IF (SUM.LE.CRIT) THEN
    PRINT *, 'SUM=', SUM
    C PRINT *, 'YIPEE, YOU ARE THERE! STOP IF U DO NOT WANT TROUBLE.'
    C END IF

STOP
END

***********************************************************************
SUBROUTINE BESSEL(ANGLE, K)

REAL*8  ANGLE, K

POLYNOMIAL APPROXIMATION:

K=K-39444.D-7*(ANGLE/30.D-1)**(100.D-1)
K=K+2100.D-7*(ANGLE/30.D-1)**(120.D-1)

RETURN
END
# VITA AUCTORIS

<table>
<thead>
<tr>
<th><strong>NAME</strong></th>
<th>Dipanneeta Banerjee</th>
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<tbody>
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