Long-range transmission and distribution planning.

Hosam Kamal Mohamed. Youssef

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LONG-RANGE TRANSMISSION AND DISTRIBUTION PLANNING

by

HOSAM KAMAL MOHAMED YOUSSEF

A Dissertation
Submitted to the
Faculty of Graduate Studies and Research
through the Department of
Electrical Engineering in Partial Fulfillment
of the requirements for the Degree
of Doctor of Philosophy at
the University of Windsor

Windsor, Ontario, Canada
1988
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DEDICATION

TO MY WONDERFUL WIFE AND DAUGHTERS,
BELOVED PARENTS AND UNCLE
ABSTRACT

A large amount of bulk power must be transferred from the generation sites to the major load areas and then distributed to the individual areas of demand. The increasing costs and the need for reliable electric systems require optimum designs of the different sections of the power system. The optimal designs must satisfy an efficient system performance with minimum cost, or within certain budgetary constraints. A new model for long range planning of large distribution systems is developed in this work. The model takes into account both the fixed and the variable costs for all planned facilities (substations and feeders) as well as the cost of the energy losses. The distribution planning model incorporates an accurate formulation of the continuous nonlinear cost function in terms of the planning variables (time and power flow through the planned network). The model also gives the possibility of expanding the power capacity of an existing feeder or substation as an alternative to constructing new facilities.

The formulation of the problem includes constraints on voltage drop in the primary feeders, demand satisfaction, power flow conservation, and feeders and substations overloads.
To enhance the efficiency and the accuracy of the planning process for the overall power system, a new optimization model for the optimal long-range transmission planning is introduced in this work. Given the existing network and anticipating a requirement for future facilities and future loads, an accurate nonlinear cost function for the transmission system is formulated. The cost function includes the fixed, the variable, and the energy loss costs of the different planning facilities. This cost function is minimized subject to demand satisfaction and overloading constraints. The model has the advantage of including security constraints on the bus voltage magnitudes and the swing angles, which are essential for the system stability.

Accurate ac load flow equations are also included in the optimization model. This ensures that the optimal solutions obtained from this model satisfy the load flow requirements, while previously separate solutions were required for each change in the networks.

In the non-integer models developed in this work, the number of the planning variables is reduced significantly compared to the corresponding models used hitherto. Thus the new models are applicable to larger distribution and transmission systems. Also because of the continuous nature of the formulations in the models, they are suitable for different nonlinear applications, such as contingency studies. The reliability test is included in the
transmission planning model in a form of deterministic contingency approach. This ensures that the designs obtained from this model perform satisfactorily under severe outage conditions.

The new models are applied successfully to utility examples in the static and the dynamic modes of planning. The results obtained in the different cases are compared with the results reported previously in the literature. The comparison shows good accuracy, high efficiency, and demonstrates the applicability of the new models.
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NOMENCLATURE

A

bus incidence matrix

A

element-node incidence matrix

a

complex voltage ratio of standard voltage transformer

ij

ij-th element in the node-feeder incidence matrix

\( a \)

a function accounting for the sum of equal payments over the planning time segment \( \tau \)

B

imaginary component of the complex admittance \( Y \)

B

imaginary part of the \( ij \)-th element of the bus admittance matrix (susceptance)

B

a matrix presents a section of the imaginary part of the network bus admittance matrix \( (B_{MM}) \), simulating the incidence of the load buses with each other only in the network

B

the imaginary part of the network bus admittance matrix after eliminating the row and the column of the slack bus

b

a column vector contains the real known quantities in the power network
the total present worth cost function of a distribution or transmission network

the cost of the energy losses in element \( j \) at the time of planning

the present worth value of the fixed cost of line \( i \) at the planning time segment \( \tau \)

the present worth value of the variable cost for element \( j \)

the present worth value of the energy losses cost of line \( i \) at planning time segment \( \tau \)

the present worth value of the energy losses cost of line \( i \), operates in case \( k \) at planning time segment \( \tau \)

the present worth value of the cost of the energy losses in element \( j \)

the present worth value of the variable cost of line \( i \) at the planning time segment \( \tau \)

the present worth value of the variable cost of line \( i \), operates in case \( k \) at the planning time segment \( \tau \)

a continuous function in the power flow in line \( i \) at the planning time segment \( \tau \), represents the capital cost per \( \text{km} \) at the planning time, of adding new paths to line \( i \) at the planning time segment \( \tau \)

the real component of the complex current \( I \) through the \( i \)-th element in the power network
the capital cost per km, at the planning time, of any power capacity added to the first power segment in the right-of-way $i$

the annual variable cost per km of line $i$, at the planning time

the demand at or from node $i$ at the planning time segment $\tau$

the injected active power at bus $j$, at the planning time segment $\tau$

the imaginary component of the complex current $I$ through the $i$-th element in the power network

a continuous decision function in the power flow in line $i$ at the planning time segment $\tau$

a diagonal matrix of the complex bus voltages

the variable cost of element $j$ at the planning time

the total power segments of element $j$

the logarithmic exponent function

the real component of the complex voltage $V$ of the $i$-th element in the power network

the capitalized cost of the new power segment $e$ of feeder $i$, at time of planning

the capitalized cost of the new power segment $e$ of supply $s$, at time of planning

the present worth value of the capitalized cost
for potential, or expansion permitted in, feeder
future

\( F_i \)

the capitalized cost of element \( i \) at the time of
installation

\( PV_i \)

the present worth value of the capital cost of
element \( i \)

\( FS_s \)

the present worth value of the capitalized cost
for potential, or expansion permitted in, substa-
tion \( s \)

\( F_\tau,i \)

a continuous decision function in the power flow
through line \( i \) at planning time segment \( \tau \)

\( F_\tau,i_k \)

the present worth value of the fixed cost of line
\( i \), built for case \( k \) in the planning time segment \( \tau \)

\( F_0i \)

the capitalized cost of element \( i \) at time of pla-
nning

\( F\text{EXP}_i \)

the total power segments permitted for potential
or expandable feeder \( i \)

\( F_1 i,t \)

a decision function for facility \( i \) at the time
segment \( t \)

\( F_2 i,t \)

a decision function for facility \( i \) at the plann-
ing time segment \( t \)

\( f_i \)

the imaginary component of the complex voltage \( V_i \)
of the \( i \)-th element in the power network

\( G_1 \)

the real component of the complex admittance \( Y_i \)
(conductance)

\( G_1j \)

the real part of the \( ij \)-th element of the bus
admittance matrix (conductance)
a matrix presents a part of the Jacobian matrix of the network at iteration (K), introducing the partial derivatives of the network bus active powers w.r.t. the network bus swing angles, an array of the power flow equations

a matrix presents the partial derivatives of the power flow equations (h) w.r.t. the network state variables (x)
a set of network performance indices

the complex current in the i-th element in the power network

the real component of the complex current I

the imaginary component of the complex current I

the magnitude of the complex current I

the rated current of the transmission line t

an integer constant for each new feeder facility

an integer constant for each new substation facility s

a set of outage units at bus k

the Jacobian matrix of the electric power network at iteration (K)
K a large positive weighting factor
KC(Ω) the number of cases of operation to be studied at
time segment Ω
K u a set of contingency cases in the power network
L i the length of line i, km
\( L^{(K)} \) a matrix presents a part of the network Jacobian
matrix at iteration (K), introducing the partial
derivatives of the load bus reactive powers
w.r.t. the load bus voltage magnitudes
M a large positive number
N the total number of network nodes (supply, load,
and transshipment nodes)
NB the total number of existing and future buses in
the transmission network
NC i the number of the power segments (parallel paths)
permitted in the right-of-way i
NF the total number of existing and potential feeders
in the distribution network
NFEP a subset of the potential and the expandable feeders
NL the total number of existing and future right-of-
ways in the transmission network
NS the total number of existing and potential substations in the distribution network
NSEP a subset of all the potential and the expandable
substation

NT the total number of the time-phased planning segments

N_j a subset of all the transmission lines connected to bus j

n the total number of buses in a transmission network, usually denotes the slack bus in the network

n the total number of the load flow equations in a power network

n_G the number of the transmission network generator buses, excluding the slack bus

n_L the number of the load buses in a transmission network

P_i the real part of the complex power \( S \) (active power)

\(|P_{i,t}|\) the absolute value of the power flow in element i at time segment t

\( P_{-M} \) a column vector presents the active powers of the network buses, excluding the slack bus

\( P_{\tau,i} \) the active power through line i at the planning time segment \( \tau \)

\(|P_{\tau,e}| \) the absolute value of the real variable \( P_{\tau,e} \) which represents the power flow in the power segment e of feeder i, at the planning time segment \( \tau \)
A positive real variable represents the power flow in power segment \( e \) of supply \( s \), at the planning time segment \( \tau \).

The lower active power capacity limit of line \( i \), at the planning time segment \( \tau \).

A continuous function in the power flow in line \( i \) at time segment \( \tau \), which simulates the number of parallel paths in the right-of-way \( i \).

The upper active power capacity limit of line \( i \), at the planning time segment \( \tau \).

A large positive constant.

The rated reactive power at generator bus \( q \).

The imaginary part of the complex power \( S \) (reactive power).

The injected reactive power at bus \( j \), at time segment \( \tau \).

The complex residual at iteration \( (K) \), between the complex injected currents of bus \( i \).

The inflation rate at the planning time segment \( \tau \).

The interest rate at the planning time segment \( \tau \).

The complex power of element \( i \) in a power network.

The magnitude of the complex power \( S \).

The conjugate of the complex power \( S \) of the \( i \)-th element in the power network.
$S^{(K)}$ the complex power of element $i$ in the network, at iteration $(K)$

$S_{EXP}$ the total power segments permitted for potential or expandable substation $s$

$T \tau$ the length of the time segment $\tau$, years

$T^{(K)}_{k}$ an adjoint network matrix of coefficient, at iteration $(K)$

$T_{i}^{(K)}$ the operating time of line $i$, in case $k$ at the planning time segment $\tau$

$TY_{i}$ an integer constant equal to zero if line $i$ originally exists in the network, and equal to one for the future lines

$t_{i}$ time variable

$t_{i}$ real variable presents the installation time of the new power segment $e$ of feeder facility $i$

$t_{i}$ real variable presents the installation time of the new power segment $e$ of supply facility $s$

$t_{i}$ the installation time of element $i$

$U_{i}$ a complex vector presents the network complex quantities

$U_{i}$ the power capacity limit of power segment $e$ of feeder $i$

$U_{i}$ the power capacity limit of power segment $e$ of supply $s$

$U_{i}$ the power capacity limit of element $i$
the \( i \)-th complex quantity in vector \( \mathbf{U} \)

the magnitude of the complex quantity \( u_i \)

the real component of the complex quantity \( u_i \)

the imaginary component of the complex quantity \( u_i \)

the volt-ampere base chosen for the power network

the p.u. voltage magnitude of bus \( j \) at the planning time segment \( \tau \)

the lower limit on the p.u. voltage magnitude of bus \( j \), at the planning time segment \( \tau \)

the upper limit on the p.u. voltage magnitude of bus \( j \), at the planning time segment \( \tau \)

the specified voltage magnitude of generator bus \( q \)

the complex voltage across element \( i \) in the power network

the magnitude of the complex voltage \( V_i \)

the conjugate of the complex voltage \( V_i \)

the complex voltage across element \( i \) in the power network, at iteration \( (K) \)

the complex voltage of bus \( i \) at intermediate iteration \( (K+(1/2)) \)

the p.u. voltage magnitude at the sending node of feeder \( i^* \), at time segment \( \tau \)
the p.u. voltage magnitude at the receiving node
of feeder i, at time segment \( \tau \)

the nominal bus voltage magnitude at bus \( m \)

the p.u. lower limit on the voltage magnitude at
node \( n \)

the p.u. upper limit on the voltage magnitude at
node \( n \)

the p.u. voltage magnitude at node \( n \), at time
segment \( \tau \)

the p.u. voltage magnitude across line \( i \), at time
segment \( \tau \)

the real component of the complex voltage \( V_i \)

the imaginary component of the complex voltage \( V_i \)

a weighting factor, less than or equal one, asso-
ciated with the power segment \( e \) of feeder \( i \)

a weighting factor associated with the cost of
the energy loss of line \( i \), in the cost function

a weighting factor for the power received at, or
delivered by, bus \( j \) at the time segment \( \tau \)

a weighting factor associated with equation \( i \)

the synchronous reactance of the synchronous
machine
an array contains all the real unknown variables in the power flow equations

\( \Delta x \)  
\( (k) \)  
an array of the errors in the network state variables \( (x) \), at iteration \( (k) \)

\( Y \)  
the bus admittance matrix

\( Y_{ij} \)  
the p.u. admittance magnitude of a single path in the right-of-way \( i \)

\( |Y_{ij}| \)  
the magnitude of the \( ij \)-th element in the bus admittance matrix \( Y \)

\( P_Y \)  
the primitive admittance matrix

\( Z \)  
the bus impedance matrix

\( Z_{i\zeta} \)  
the p.u. impedance magnitude of feeder \( i \) at time segment \( \zeta \)

\( \alpha_i \)  
the ratio between the capital cost of the first power segment of line \( i \) (including the cost of towers and accessories) to the cost of any additional single power segment

\( p_{\zeta} \)  
a function accounts for the present worth value of the costs at the planning time segment \( \zeta \)

\( \beta_{\zeta,i} \)  
a function accounts for the present worth value of the cost of element \( i \) at planning time segment \( \zeta \)

\( \gamma_{\zeta,i} \)  
a function converts the cost value of line \( i \), operates at time segment \( \zeta \), to its present worth value
the phase angle of the complex admittance \( Y \)

the phase angle of the \( i^j \)-th element of the bus admittance matrix

the phase angle of the complex current \( I \)

the phase angle of the voltage difference across line \( i \), at the planning time segment \( \tau \)

the phase angle of the complex power \( S \)

the phase angle of the complex voltage \( V \)

a column vector presents the swing angles of the network buses, excluding the slack bus

the nominal bus voltage swing angle at bus \( m \)

the swing angle of bus \( j \) at the time segment \( \tau \), radians

the lower limit on the swing angle of bus \( j \), at time segment \( \tau \)

the upper limit on the swing angle of bus \( j \), at time segment \( \tau \)
Chapter I

INTRODUCTION

Since the beginning of the third quarter of the twentieth century there has been considerable awareness of the economical, optimal, and efficient use of energy. With the increase need of electrical energy and with the limited available resources, the efficient transmission and distribution of this energy has become more important.

Accordingly the research has been divided into two main areas, in order to face the dilemma of increasing needs with a limited availability of the supply of energy. The first area has been concerned with finding and developing new energy sources. The second area of research is concerned with optimizing the use of the available energy resources.

Because electrical energy is the most widely used form of energy involving a very large consumption of different kinds of fuel (some of which are subject to short-terms shortages) and requiring tremendous capital investment to meet increasing demand, there has been increasing interest in electric power system planning in recent years.

The electric power system can be divided into three major areas, namely generation, transmission, and distribution systems. The work presented in this thesis is concerned
with the long-range planning of both the transmission and the distribution networks of the power system.

1.1 General Planning Steps

The electric power system planning process can be summarized in the following main steps:

1. A decision on whether the system should be expanded and/or a reconfiguration is needed must be made due to the load growth with time. This decision is based on the maximum and the timing of the load forecast at different load sections.

2. The development of an objective(s) to be satisfied and met by the plan within well defined constraints. The system constraints include operational, security, environmental, political, and economical. Because of the constraints and requirements, some measures of effectiveness of a plan relative to the objectives have to be set. It should be noted here that some of the constraints imposed on the different plans can not be simulated mathematically in order to be handled by a computerized aided design model. There are two ways of handling such problem. Firstly, these constraints can be looked at as a final comparison between the different alternative plans depending upon the planner judgement and experience. Secondly, each of the non-mathematical constraints can be given a
weighting cost factor according to its importance and then be included in any mathematical model. Again these weighting factors will be dependent on the planner experience and on the circumstances.

3. Finally the choice of the overall most effective plan has to be made.

The above steps have been implemented to each section of the power system separately due to the differences between the different sections of the system. The main differences between the transmission and the distribution sections of the power system are given in the following section.

1.2 Nature of Transmission and Distribution Systems

Transmission and distribution networks are different in several ways. Mainly they have widely different voltage magnitude (Fig. 1.1). While the generated voltage is in the range of 10/25 kV, it is increased by step-up transformers to the main transmission voltage level. Transmission voltages are typically 765 kV, 500 kV, and 345 kV in North America, while they are 400 kV and 275 kV in Britain and other parts of the world [1]. Most of the large and efficient stations feed through transformers directly into this network. This network in turn feeds a subtransmission network which operates at lower voltage (115 kV in North America and 132 kV in Britain and the rest of the world). The subtransmission network supplies the distribution
network which operates at much lower voltages feeding the consumers. Typical distribution voltages are 33 kV, 11 kV, 6.6 kV, and 4.16 kV which in turn are stepped down to three-phase 220 V in North America and to three-phase 415 V in Britain and the rest of the world.

![Schematic diagram of a part of a supply network.](image)

Figure 1.1: Schematic diagram of a part of a supply network.

Beside the voltage level difference between the transmission and the distribution networks, the number of
branches and sources is much higher in the distribution networks. In addition the general structure or topology is different.

Review of the literature reveals that there has been more interest in generation and transmission planning than in distribution planning since the energy crisis in the Seventies. The distribution planning has only been considered seriously over the past few years. The reason for that was attributed to the complexity of the dynamic nature of the distribution systems as well as to the very large amount of data required to analyze the distribution systems compared to that required for the generation and transmission systems [2]. Furthermore, it had been felt that there was less urgency to improve the planning of the distribution systems because of the reduced impact of the failure of the distribution equipment on the total network.

In the following sections a summary of the existing techniques and models for both transmission and distribution planning is introduced. The advantages and the shortcomings of these techniques will also be briefly illustrated.

1.3 Literature Survey
1.3.1 Existing Techniques for Long-Range Transmission Planning

There has been a great deal of interest in long range transmission planning over the years (1.3-17, 19-22). Among the early automated methods for transmission expansion is the method proposed by Garver using linear programming (11). The method included two main steps. The first step was called the linear flow estimation in which the power flow equations were formulated as a linear minimization problem. The objective function or the loss function used was the multiplication of the power flow in each link by its guide number. Two different types of links were suggested between the buses. The first type of links was called "circuits" which were considered as effective power carriers having low guide numbers but they were of limited power capacities. Perhaps the most important new feature of the flow-estimation technique was the introduction of the second type of links which was called "overhead paths". These paths existed in every permitted right-of-way. They had unlimited power capacity and high guide numbers. This high guide number of an overload path was given to account for the cost of adding new circuit as well as to penalize overloading until all circuits (first kind of links) capacities had been used. In the flow estimation technique the network was divided into a tree and a subtree. Then all power flows through overload paths in the subtree were set to zeros and
power flows through the subtree circuits were set to either zero or capacity limit. Knowing power flow through the subtree links and to satisfy conservation of power at each bus in the network, the power flow through the tree links could then be determined. Bus guide potentials were then defined and computed. Using these guide potentials, evaluation of power flow through the subtree links could be computed and compared with the previous estimated flows. Adjustment of the wrongly estimated flows had to be made then. Computing the flow pattern and determining whether it minimized the loss function through the disappearance of any overload in the network concluded the first phase of the method. If not all the overloads had been eliminated, the rule of the second phase of the linear programming theory was then employed to make one change at a time in order to move towards the minimum. This presented the second step of the method which dealt with new circuit selection. The selection of a new circuit addition was based on the location of the largest overload in the flow estimation obtained in the first step.

Among the advantages of the use of overload paths method was the ease of handling the study of buses not included in the existing network. In addition the method has the advantages of computational speed and simplicity. On the other hand, the major drawback of the method is that it did not implement any of the true electrical laws except the
conservation of active power at all buses. The method had no control on the number of parallel lines in each right-of-way. In addition the bus voltage magnitudes and angles were not included in the procedure which might lead to infeasible or unreliable solutions. Finally, overload problems would develop in the network under contingency conditions.

At about the same time Peschon et al. [12], had introduced a new approach for long range transmission planning. The model minimized the overall present worth cost of the network while satisfying the outage constraints. In contrast with Garver's method [11], which was only suitable for static (one step) mode of planning, Peschon's method [12] was applicable for both static and dynamic (multi-steps) planning procedures. The method [12] was based on the solution of the dc load flow equations for the existing network under small changes in the node injections (i.e. for small future time segments). Linear programming was then used to minimize the capital cost required to eliminate the overloads produced, subject to the reliability constraints. These reliability constraints included limits on the phase angle difference between each branch end node. The constraints implied the first-order sensitivities of small phase angle differences to small variations in the branch capacities. The method introduced a way of determining economical lower bounds to the capacity
additions necessary to satisfy the system security constraints at a given time. Two different procedures had then been suggested to obtain practical investment decisions and not just economical lower bounds to the capacity additions. The first method was called "Look-Ahead factor" method. It was based on the addition of more capacity in a given year than the minimum determined by the linear programming. This could be achieved by multiplying the vector of capacity changes by a factor larger than one to anticipate economies of scale and prevent the frequent additions of low capacity circuits. The major drawback of the "Look-Ahead factor" method was the assumption that there would be a continuous need for higher capacities. The second approach had been suggested was the use of dynamic programming. Again the linear programming was used to determine the branch overloads over a certain number of years (e.g. p years) and then using the dynamic programming, the expansion of each branch during the p-years was optimized separately. The main limitation of using the dynamic programming was that the choice of p was limited to a small number of years, otherwise the capacity changes required became too large for the linearized sensitivity equations to remain valid. The method presented by Peschon et al., with the suggested approaches for practical decision, suffered from the following drawbacks. First of all, it did use a linear power flow model in which
bus voltage magnitudes and reactive power considerations were omitted. The method depended on linearized sensitivity equations which were only accurate and suitable for small changes, and accordingly over short time periods. Also a linearization of the branch costs had been applied which once again was only valid and applicable for small power flow variations. In addition the method required load flow solution for each year and for each change in the network. Finally, except with the dynamic programming approach the minimum cost was obtained for each individual expansion required separately. This led to a series of sequential but separate single-step planning method which may not produce an optimal solution over the overall planning time period.

To handle the discrete nature of long-range transmission planning as well as the uncertainty involved in the future parameters, Peschon et al. had introduced another approach [13]. The model implemented the stochastic dynamic programming as a powerful tool for handling the planning problem in its probabilistic form. The planning variables were divided into two different kinds, namely the state variables which defined the system topology, equipment ratings, and connected loads at any given time. The second kind of variable was called the decision variable which was responsible for deciding the transition of the system from one state to another. To reduce the number of state variables, the whole system configuration was treated as one
state variable and the unreasonable states were rejected on the basis of the planner experience. The model included the system capital and operating costs as well as penalty costs associated with constraint violations. The model was also capable of handling system security and operational constraints, such as reliability constraints, cost constraints, and stability requirements. Although this approach was very powerful for planning and was capable of handling the probabilistic nature of the system different parameters, it had some major limitations, such as the large number of variables required which made the model incapable of handling large networks. In addition, a great deal of effort was required for preparing the necessary input data.

In a trial to reduce the number of alternatives in the discrete planning problem, Dusonchet and El-Abiad had proposed a new approach using discrete dynamic optimization coupled with a stopping criterion [14]. The approach can be summarized by the random choice of a candidate from the possible plans followed by the generation of a neighborhood consisting of a limited number of strategies, including the randomly chosen plan. A local optimum was calculated for the neighborhood using the forward dynamic programming procedure. The three above steps, random choice, generation of neighborhood, and calculation of local minimum, were repeated until a stopping criterion was satisfied. Although the method implemented the dynamic programming as a powerful
tool along with a stopping criterion to reduce the number of plans to be checked, there had been many questions about the capability and the efficiency of the method. Among the questions raised was how to generate the neighborhoods and the effect of the method of choice on the final solution. In addition the choice of the parameters involved in the stopping criterion had an impact role on the final solution. Furthermore the method presented a sequence of separate optimal expansions from one stage to another which might not necessarily be the global optimum. Beside all the above comments on the method, the question of its applicability to large practical systems was still raised.

Similar to Peschon [12], Puntel et al. [15], had used the sensitivity analysis to introduce a different approach for the planning problem. Instead of using a linear programming as in [12], which limited the effectiveness and the accuracy of the sensitivity approach to small changes, Puntel had used a non-linear static optimization technique to improve the model accuracy. Also the performance index (objective function) used, had included the line overload cost in addition to the cost of line additions. Although both models [12,15] implemented the dc load flow equations and presented one-step planning (static), Peschon's model [12] had the advantage of being amenable to modification to multi-steps (dynamic) planning model. The main advantage of Puntel's model [15] was the use of adjoint network approach to
calculate efficiently the performance index sensitivities with respect to line changes. The planning approach [15] could be summarized in solving the dc load flow for the original system from which the performance index was calculated. The adjoint power injections were then calculated as well as the adjoint network angles. Using the above calculated values, the gradient vector could be calculated. Based on the values of the gradient vector elements, the appropriate line changes were made. The above steps were repeated until the planning was completed. The program also had the option of including the line outages test in the procedure.

Due to the large number of network configurations in practical power systems, the different approaches summarized above which used the dynamic programming procedure were limited to small systems. In addition the linearization assumptions involved in some of the above models resulted in inaccurate solutions. To handle such problems a branch-and-bound integer programming model had been introduced in [16]. In this approach the mathematical burden in addition to the model complexity had been reduced using the concepts of optimal cost-capacity curves and screening algorithms. The sensitivity analysis was the base of the screening schemes suggested. Two different screening algorithms were included in the procedure. The first one was called the coefficients of effectiveness which measured the
effect of the admittance of line additions to the amount of overloads. The second algorithm presented the ratio of the cost of line additions to the overloads and it was called the cost-effectiveness ratio. The optimal cost-capacity curves used in the method was based on the determination of the combination of line types for each value of the capacity of each right-of-way which resulted in the minimum cost. The algorithm [16] could then be summarized in the steps of computing the optimal cost-capacity curves and then the overload lines were identified using dc load flow. The sensitivity coefficients of the angular differences across overload lines to changes in the admittances were calculated. Using these coefficients, the screening algorithms were applied to reject the ineffective right-of-ways before proceeding to the optimization program. Finally, the branch-and-bound integer program was used to obtain a set of line additions and the dc load flow of the network, including the new additions, was solved again to check overloading. The procedure continued until all overloads had been removed from the network. Finally, if the contingency test was not required in the planning, the procedure was completed; otherwise a set of contingencies had to be performed to obtain additional overload constraints and the above steps were repeated. Although the method implemented different screening algorithms along with the branch-and-bound criterion, the applicability of the
model to large systems was still questionable [13]. Further investigation on the effect of the program parameters on the accuracy of the solution was also required. In addition the accuracy of using linearized sensitivity coefficients was also critical. It has to be noted that the method [16] was a static planning approach which might not lead to an optimal or even a feasible solution over the total planning time period.

In order to overcome the inaccuracy in the results due to the approximations involved in the previous linear programming models, and to eliminate the need for large number of alternatives and the use of different sensitive stopping criteria in the integer programming models, Sawey and Zinn introduced an approach based on linear mixed integer programming [17]. This model was suitable for both static and dynamic modes of transmission planning. It was also capable of handling relatively larger systems due to the smaller number of variables involved in the formulation. In the sense of not including any of the electrical laws in the model except that of conservation of power flow, the model [17] was similar to that of Garver [11], with the exception that the method in [17] was applicable for time-phased planning and had more accurate formulation for the cost function than that used in [11]. The objective function presented in [17] included the capital investment cost of constructing new facilities (plants and transmission
lines) as well as the system operating costs and the cost of energy losses. The cost function was to be minimized subject to the demand satisfaction constraints, plant capacity constraints, lines overloading constraints, and the physical restrictions.

In the past few years Meliopoulos et al., had presented a series of publications (19-21) handling the transmission expansion planning problem. The basic model was based on non-linear mixed integer branch- and-bound algorithm. The model in its original form (19) did not differ significantly from the previously summarized models except that, the cost function used was formulated more accurately in a non-linear form and included the system different type of costs. The method was applicable for dynamic mode of planning since the cumulative present worth cost of expanding the system was formulated. The objective function was minimized subject to the demand satisfaction constraints, plant capacity constraints, and line overloading constraints. The algorithm incorporated the use of either the ac or the dc load flow calculations to represent the system operational and security constraints. The method of solving the problem was simply that at each state and stage the different discrete alternatives for expanding the system were enumerated. The branching process was then applied and the accumulated cost due to the chosen alternative was calculated. If this cost was higher than the cost of the
best known expansion plan obtained earlier, then another alternative would be studied. The alternative plans were automatically generated in the program by solving the network at certain stage and state with the demand of the next stage was applied. The overloaded and critically loaded circuits could then be determined. Using the sensitivity analysis and cost effectiveness analysis, the effective right-of-ways suitable for eliminating such network overloads were determined and the alternatives were then constructed. Most of these alternatives were then eliminated by filtering schemes applying the feasibility and optimality conditions. The implementation of what was called the preventive or corrective control action, used for handling system contingencies instead of line construction, had been added to the model as a new feature in [20]. This action was based on the rescheduling of the power output of some power plants in order to satisfy system security. Although rescheduling action appeared to be more economical than construction of new lines, this technique required that the new generation schedule to be as close as possible to the economic generation schedule, otherwise a non-optimal final solution might be obtained. It had been stated implicitly in [20], that the violation of this condition might happen and the only solution in this case was to stop the procedure. The model introduced in [19], with or without its new feature presented in [20], had the same limitations
of the other methods that used the branch-and-bound criteria. These limitations were mainly the sensitivity of the solution to the stopping schemes, as well as the questionable applicability of the method for large networks, due to the large number of alternatives to be studied for each state and stage. In addition there was always a doubt in the accuracy of using the sensitivity analysis in the detection scheme because of the necessity of small changes occurring in the network. Finally in reference [21], Bennon, Joves, and Meliopoulos had a new screening algorithm to prefilter the different alternative plans before proceeding to the optimization step. This modified screening algorithm could be considered as a combination of the two screening approaches previously introduced in [16], namely coefficients of effectiveness and cost effectiveness. The effectiveness ratio vector used in [21] was the multiplication of the coherency ratio times the ratio of the acquisition costs for the overloaded circuit and the candidate line. The coherency ratio was a measurement of the coherency of the suggested line to the power flow in an overloaded line.

An interactive transmission planning model had been introduced in [22]. The method was basically another integer algorithm while the expansion strategies were ranked according to the least-effort index. This index incorporated the use of an overload measurement and the cost
of the new additions (in terms of the susceptance) and it was similar to [15] in that sense, with the difference in performing the index. The expansion of the network was based on the cost effectiveness analysis of the least-effort paths in the network. Finally, the method (22), was a one-step planning technique which again did not necessarily lead to the optimal solution over the overall planning time period.

1.3.2 Existing Techniques for Long-Range Distribution Planning

Among the reasons which lead to more difficulties in distribution planning over generation and transmission planning, as it has been stated earlier, are the dynamic nature of the distribution systems and the very large amount of data required to analyse such systems. This can be more clear by displaying the logical and realistic sequence of the planning steps. First of all, the distribution planner has to investigate and predict the future load densities for the various areas of the system, followed by determining the number of feed points needed to supply such areas with different primary feeder alternatives. These alternatives include, among other parameters, different existing facilities (substations and feeders), all possible sites and capacities for new substations and right-of-ways, as well as the existing expandable facilities. Knowing these parameters and factors, in addition to other factors such as economical factors and system security and reliability constraints, the
planner has to determine the most desirable means of supplying the demand points, satisfying the different economical and operational constraints.

It seems, to the best of the author's knowledge, that the first serious attempts for formulating the distribution system planning steps in mathematical forms were directed towards the determination of the primary feeder points and load area geometry [23, 24]. In both references some simplifying assumptions were applied to reduce the complexity of the subject. Under these assumptions, the area under consideration was assumed to have a uniformly distributed load and of a regular geometric shape, such as rectangular, diamond, or triangle shapes. The relations between voltage drop, load density, circuit voltage, area dimensions, and conductor size, loss, and type were formulated. These relations were based on the principle of specifying a maximum primary voltage drop between the first and the last transformers in the network.

In reference [23] the procedure started by dividing the load area into subareas. Applying the predetermined present or future loading factor to the existing transformer capacity within each subarea, the load density of each subarea could then be determined. These subareas could then be redivided into sections with appropriate load densities. Finally, the number of primary feed points were obtained directly from the relationships which had been based on the
specifications required for each of the parameters involved. It was suggested that it would be desirable to implement such method in the overall planning procedure as a first step, since it could give the areas to be supplied by a single feed point as well as the demand to be supplied [26]. The location of the feed points could be also approximately determined and accordingly a subtransmission network, satisfying the system security and operational constraints, could be laid out. A transmission system could be subsequently determined to go with the subtransmission system obtained and the total investment could then be calculated. Finally, this procedure would be repeated for different voltages to obtain the most economical plan. It has to be noted here that the number, the capacity, and the location of the system substations and feeders had not been optimized from the economical and the operational points of view.

Van Wormer [24] attacked the problem in a more interesting and useful way. The area to be fed at a certain voltage drop was maximized, or alternatively the voltage drop was minimized for a specific area of demand. In addition the thermal limitation of the cables was also studied [24]. Different interesting and useful relationships between the system parameters were derived in [23,24]. Finally, it should be mentioned that these studies could only be used as guides, since they were derived for ideal systems [24].
Another comprehensive study for developing different relations between the power distribution system parameters was presented in [25]. This study was similar to those discussed in [23,24] in the sense that the voltage regulation was the basic principle in deriving the relationships.

In order to produce different strategies for distribution system planning, Denton and Reps in [27] used the concept of devising a hypothetical system equivalent to the actual system under consideration for quick and simple analytical studies. The objective of the method was to find the suitable changes in the system parameters required to improve the system quality of service, operation, and economy. Among the parameters which had been considered in the analysis were substation size, primary feeder voltage, number of feeders per substation, and allowable voltage drop in the feeder circuits. In deriving the planning strategies, two different approaches had been followed. The first approach was for constant load-density planning where it could be handled either by increasing the primary feeder voltage, or by keeping it constant. In both cases the plans could be accomplished by varying the number of primary feeders or/and the substation loading. The second approach concerned with the planning under increasing load density conditions. This could be treated by substation constant load plans, or by substation constant service area. Using
the different schemes of planning and relationships derived in the study [27]. Different planning alternatives could then be proposed and qualitative and economical analysis had to be performed to obtain the best plan. However, it was not discussed in the study, how the economical and the service quality analysis could be applied in an efficient and fast way. In addition, the time variables had not been included in any of the relations obtained.

It seems that the first automatic methods for distribution system planning were those introduced in [28,29]. Grimsdale and Sinclar derived an iterative planning procedure, taking into account the practical and economical factors of design [28]. The economical aspect of the method was based on the road-selection procedure, searching for the shortest route between two points. This procedure usually led to the choice of the most economical number of distributor outlets from substations rather than the maximum load or the maximum number of consumers per outlet, a criterion which had been in use before. The planning procedure introduced in this study, [28], could be summarized in six main steps. First, the number of substations required as well as their initial locations, ignoring the right-of-way constraints were calculated. Relaxing the right-of-way constraints might result in unacceptable substation sites. At this stage a decision of interrupting the automatic operation and manual choice of
the closest feasible sites to the initial locations had to be made. Using the criterion of the shortest possible route, an initial feeder design was obtained. An improved design could then be searched for by choosing the most economical combination of cable sizes, wherever saving could be obtained. Another set of substation sites then used and other designs were obtained. Finally, the above steps were repeated, using different number of substations and different cable sizes and materials, producing different alternative designs. The best overall plan was then chosen. The method gained an appreciation at that time, for being almost fully automated. Although the method accounted for many important planning parameters, such as substations number and locations in addition to the number, size, type, and layout of the feeders, this reduced the applicability of the method to solve large distribution systems. In addition, the method only treated the static (one time step) mode of planning, which may not lead to an optimal or even to a feasible plan for the overall planning time period. Beside the limitations of the method, the shortest route criterion used in the model might not lead to optimal road selection [30].

On the other hand, the method presented by Knight in [29] implemented different planning approaches, based on linear programming techniques, to obtain an optimal network design, while satisfying the network constraints. Different
design criteria were suggested in that study including minimum-cost network, minimum circuit length, and minimum apparent power by distant product. Any of these design criteria could be then formulated in a linear equation form and minimized subject to the system security and operational constraints, which again had to be formulated as a set of linear equations. Technical constraints, such as limitation on the number of circuits connected to loads or to substations, or along any given routes could easily be formulated linearly. In addition, load satisfaction constraints as well as supply capacity constraints were also formulated in linear forms. Because of the linearity restrictions imposed on the model, the cost relations of the different facilities had to be linearly approximated. Knight included a part in each circuit cost to represent a portion of the supply cost as well as the cost of the control devices. The method of including the cost of the facilities required the knowledge of the number of circuits connected to each supply prior to the solution. Although Knight correctly realized this fact, it is felt that he incorrectly suggested that this approximation in calculating the costs was unlikely to affect the final design. In addition, the linearity requirements made the method incapable of performing the network stability investigations. Finally, although the method seemed to be fully computerized, it suffered from the drawback which was, the more practical
constraints had to be satisfied, the larger the size of the problem became, thus limiting the applicability of the method to only small systems. Using almost the same formulation as in [29], and using the highly efficient transshipment code, Wall, Thompson, and Northcote-Green had suggested a new method for solving the distribution planning problem [31]. Proposing different forms of objective functions, in terms of feeder distance, resistance, or cost weighted by the feeder load subject to the system load satisfaction and capacity constraints, the optimal network configuration could then be obtained. Although the use of the transshipment model had the advantages of speed and economical memory requirements, as well as the ease of modifying the problem to study the effect of different planning parameters, it also had the major drawback of restricting the problem to a linear formulation. This restriction had made the method suggested in [31] to be inaccurate in presenting the cost function and prevented the inclusion of the system nonlinear requirements, such as the voltage drop constraints.

Trying to improve the accuracy of the methods presented in [29, 31], by including the actual cost of the feeders, Burstall introduced a new integer linear programming technique [32]. The problem was formulated quite similarly to that in [29], with the exception of adding the actual cost of each right-of-way. In this approach, given the
supply capacities and locations, load demands, circuit capacity, and cost of each right-of-way permitted, the problem could be easily formulated after neglecting the ac effects, as well as the losses in the system. The objective was to choose the best feeder layout and the number of standard circuits in each feeder to minimize the system total cost while satisfying the operational and security conditions. The solution procedure was started by a feasible but expensive solution and then a series of small changes, circuit additions and deletions, to this solution would be made, hoping to obtain a feasible and cheaper solution. The process continued until no further improvement could be obtained. In order to reduce the number of possible changes to be studied, the feature of "focus set" had been used. The idea was to limit the changes to a certain area in the network by choosing a set of points, focus set, and changes were only allowed in this set. These changes had to be made without violating the local constraints of the focus set, which acted as a synopsis of the global constraints pertaining to the whole problem. After checking all the possible changes in this focus set, another set had to be studied and so on. In studying the possible changes, and although the problem was formulated in an integer linear form, the changes had been made by exploring a decision tree seeking for reduction in the effort involved and allowing more flexible incorporation of extra restrictions or
suggestions from the designer. It had been admitted by Burstall that this technique [32] was not guaranteed to produce an optimal solution. On the other hand, with all the approximations and the high risk of not developing an optimal solution, the capability of the method for handling large size networks was still limited. Finally, like the above models, the technique ignored the time factor.

Boardman and Hogg introduced an interactive computer method for the distribution planning, [33], which was similar to that in [32]. The optimization of the cost function was carried out through the program by defining two sets of load-substation groups. One of these sets presented capacity surplus groups, where circuits deletion could be made, and the other set was for capacity shortages groups, where circuits addition had to be made. A list of all the required deletions and additions of the circuits, along with their corresponding saving and cost, were obtained and saved. An optimal ranking of the information stored was then made by the computer and presented to the designer to render his decision. In order to reduce the size of the problem, mainly due to the large number of constraints involved in the model, a new scheme was proposed in [33]. The scheme was based on a sampling-analysis plan for the system constraints. The plan consisted of generating a series of random numbers, sampling the constraints and applying a form of quality control. The circuit requirement for each group
was found from the total groups requirement by using a security function.

To account for the non-linearity in the network cost function, to improve the planning accuracy, Carson and Cornfield introduced a new approach in [30]. This approach mainly consisted of two parts. The first part of the procedure was an iterative approach responsible for providing the best radial network for minimizing the cost. To do so, the route-finding algorithm was based on the generalized minimum-work theorem which gave the resistance per unit length of a line in terms of the current flow through this link and the nonlinear cost function of this link. An empirical continuous cost function was assumed for the link in terms of its current. At the start, the data and the initial values of the model parameters were given. Accordingly, the link resistances were computed as well as the corresponding new current flows through the network. If the computed current flows were changed from their initial values, corrections in the link resistances had to be made and the iterations continued until no changes were found. By opening at the nearest null points, radial networks could be obtained. Check on the maximum cable ratings was then applied and additional nodes and links had to be inserted if necessary. By accommodating all the network currents, the first part of the planning scheme was completed. The second part of the scheme employed the design technique for tapered
radial network introduced in (34). The technique was used for determining the best cable size and length to minimize the system cost while satisfying the thermal and voltage constraints. Finally, the overall network cost was computed. At this stage, the whole procedure was then repeated for different values of the parameters in the cost function, if required, and the lowest cost design was chosen. Lastly, if there were different substation combinations to be considered, the procedure had to be repeated. It is to be noted here that the design of the tapered radial network was made for the radial network obtained with relaxed network constraints. This might not lead to the overall optimal design. In addition, the sensitivity of the solution obtained to the initial parameter values was questionable. Other drawbacks of the technique presented in (30), lied in the long time consumed for the iterative process required for computing network current flows, as well as neglecting the effect of the time factor in the planning process.

Since the linear models might result in non-optimal and/or unfeasible solutions, because of the restriction of using linear relationships presenting the system cost and constraints, and due to the large number of decision variables involved in the distribution planning problem, most of the researchers had preferred to solve the planning problem using mixed integer-linear programming approaches (35-39). In such approaches, more accurate cost functions of
some elements could be achieved by including their fixed costs. With the use of integer decision variables, the linear transshipment model can be applied to each of the possible alternative designs. The exhaustive study of all the possible plans becomes dramatically large even with medium size problems. Trying to direct the enumeration process to only those alternatives which lead to the optimal solution, different branch-and-bound criteria had been suggested in the literature. The general principle of any of these criteria was to carry out a search for the optimal solution through a structured enumeration process, that could be represented by a decision tree. At each node in the tree, a subset of the integer variables was fixed, while the integerality constraints were relaxed for the rest of the variables. At each step in the tree search, the decisions of which node to be processed next, which variable to be branched on next, and which branch to be solved first, had to be made. Different authors had set different criteria for the above decisions. The efficiency, the accuracy, and the capability of any branch-and-bound algorithm depended on the bounding method used to eliminate most of the potential branches on the tree. Otherwise, the tree would grow to be so large such that an excessive computer time would be required.

Adams and Laughton suggested a fixed-cost transportation-type model to account for both network
security and costs of network losses as well as the fixed costs of the new elements [35]. The security constraints were imposed on the network by restricting the power flow through the network elements to some values slightly less than their maximum capacities. Although the nonlinear cost of losses was included in the formulation, through a piecewise linearization, this technique increased the number of variables in the model. The problem was then solved by a branch-and-bound criterion. The branching and variable selection was made by different methods. The priority method required a list of variables, placed in order of priority for rounding to an integer value in addition to a preferred branching direction, to be supplied by the user. Although this method gave a kind of interaction between the program and the designer, it required that the user should have a good idea of what the optimal solution might be. The second branching and variable selection method was based on computing the increase in the cost incurred by imposing the new branch constraints, and it required an additional computational effort. The third method proposed was called the integer-infeasibility criterion. It used an empirical function of the fractional part of the current relaxed integer variables to select the next variables. The bounding criterion used was based on terminating the search in any direction at any node which had a solution cost greater than the best solution obtained before it. The model discussed
above [35], seemed to have a great deal of details and it was also capable of handling the dynamic (time-phased) mode of planning. But on the other hand, it failed to represent the electrical requirements of the network, such as the voltage drop constraints. In addition, there was always the question of the accuracy of the branch-and-bound criterion used and the sensitivity of the solution obtained, to it. Hindi et al. [36], introduced a model quite similar to that of Adams and Laughton [35]. The branch-and-bound criterion used was based on the network feasibility, the integrerality requirements, and the overall cost. El-Kady proposed some improvements to the mixed-integer linear programming model used in [35], by including the voltage drop constraints at the demand nodes explicitly [37]. The explicit inclusion of the time-dependent fixed and variable charges for the substations and the primary feeders, as well as the cost of losses were also established. The choice between piecewise linear or step approximations of the feeder power losses with respect to the feeder power flow was also given in the model. Although these improvements increased the accuracy of the model, the model was only applicable for small systems due to the large number of variables involved.

Thompson and Wall modified their previous approach [31], by including the fixed cost of the new substations, to improve the model accuracy [38]. The addition of the fixed costs which should be accounted for if the decision to use that
new substation was taken, made the formulation in the form of mixed-integer linear programming. The problem was then solved using a branch-and-bound algorithm with a transshipment linear programming model as a subroutine in that algorithm. A lower bound on the cost of the decision emanating along each branch was calculated. If that lower bound was greater than the actual cost of the best feasible solution found then, it was not necessary to actually search the decision alternatives given by that branch or any decision subsequent to it. Two bounding methods were suggested to fathom the search tree to limit the size of the problem to a manageable size. A shortest path table was used to define lower bound on the cost of serving a customer in the branch-and-bound algorithm. This table was constructed by relaxing the existing substation capacities to infinity, one of the future substations, and neglecting all fixed costs. In addition, all feeder upper capacities were set to infinity and the lower bounds, on the marginal cost of supplying an extra unit of power, were obtained. Finally, a lower bound on the cost of supplying power to a certain demand location could be obtained by calculating the smallest of the lower bounds of serving that demand location when each of the substations was open separately. Another lower bound on the cost of providing power demands using the existing substations together with any set of potential substations was calculated quickly by adding the fixed costs
of the potential substations to the minimum incremental cost. This minimum incremental cost was computed by setting the capacity of all substations (existing or potential) to maximum capacity and then the linear transportation problem was solved with all fixed costs set to zeros. The branching process depended on the substation which had the smallest lower bound on the cost of providing power demands. On the other hand, the bounding process depended on the feasibility of such station capacity and on the cost compared to the best feasible solution obtained then. It is to be noted here that the fixed costs of only the substations were considered. Further, the voltage requirements as well as the time factor had been ignored in the model. In order to improve the accuracy of the model presented in [38], Sun et al. proposed the inclusion of the fixed cost of the future (new) feeders to the objective cost function [39]. In addition a suggestion to take into account the effect of load growth with time was also made. Different branch-and-bound criteria had been used in [39]. The variable branching criterion used was based on identifying the infeasible or costly patterns. This was achieved through the guidance of some penalty factors. Two different criteria for node selection which were based on the level of enumeration were used. At the start of the branch-and-bound process, the minimum lower bound selection criterion was used. It fathomed the node had have an
objective function value that exceeded the incumbent solution value. As the size of the enumeration tree grew, the "Last-In-First-Out" algorithm was used. The concept of a horizon year had been implemented to include the effect of load growth with time. The horizon year represented a suitable choice of time in the future and served as the end of the study time period. The time-phased problem was solved in two steps. In the first phase, the optimal static solution for the horizon year was obtained. Then in the second phase, the optimal static solution for each intermediate time segment was obtained by selecting the facilities to be used exclusively from the set of equipment determined in phase one. When the entire period of study was viewed as a continuous progression, the collection of the subsystems of all time intervals then constituted a series of system expansions that transformed the base year design to the horizon year design. This technique might not lead to the optimal solution, since it was controlled by the static planning of the horizon year, which might not be an optimal one.

Among the drawbacks of the integer linear programming models is the large size of the search tree in order to have a reasonable degree of accuracy, even with the use of different branch-and-bound criteria to fathom much of the alternative designs before exhaustive study. In order to reduce the size of the search tree and improve the accuracy
of the mixed integer linear programming models, reported earlier [35-39], Fawzi et al., introduced their model [40]. In this model the fixed costs and the concave nonlinearities in the cost functions, of all elements, and in the operational constraints were considered. A concave fixed cost model was used to represent elements with large fixed costs (substations and possibly some feeders), and linear cost function was assumed for the remaining elements. This way, the search tree for the network was decreased to the size of those elements which only had large fixed costs. Branch-and-bound algorithm was used to solve this sub-problem, giving the optimal number, location, and loading of the substations as well as the optimum feeders configuration, based on the approximate cost function. The fixed costs of the remaining elements were then accounted for through an iterative procedure which modified the solution of the first step. This nonlinear iterative approach allowed the nonlinear constraints to be included accurately in the process. Three different bounding criteria were used in the model namely, cost criterion, capacity criterion, and voltage drop criterion. While the branching criterion depended on the difference between the linearized cost, of the free variables, and their actual concave cost. The horizon year concept was also suggested to handle the effect of load growth with time as in [39]. Perhaps one of the early models to realize the importance of including the time
factor in the distribution planning was the model introduced by Adams and Laughton ([41]). They used the dynamic programming procedure along with network flow algorithms for rapid estimation of system security, and to check the feasibility of the system states at each stage of the dynamic programming process. Although the model had the advantages of fast evaluation of the load flow patterns in the network, it contained the major drawback of ignoring the system electrical relationships. In addition, the model capability of handling large size systems was limited because of the large number of alternatives to be studied.

One of the interesting and unique methods for the expansion planning of radial subtransmission systems, was that introduced in ([42]). The method could be used to generate the feasible alternatives in the planning process based on the conductor overloading and voltage drop constraints. Then, cost calculations for each alternative had to be made and finally, these feasible plans were ranked according to their costs. The major advantage of this method was the fast generation of the feasible solutions. The method could be summarized in the steps of generating the system incidence matrix and then formulating only the trees (radial configurations) by manipulating this matrix. A simple load flow was performed on each of these trees and conductor capacities were checked, to determine the validity of each tree. If overloading existed in the network, changes
had to be made where necessary and if possible. A simple cost-capacity function was minimized subject to capacity constraints to determine the most economical expansion required. Finally, voltage drop had to be computed for each valid tree to determine the validity via voltage drop criterion. Lastly, all the valid solutions were listed and ranked. This method was only applicable for radial configuration networks in the static mode of planning. Also, the use of integer programming in solving for the most economical expansion steps, limited the capability of the model to small size networks.

Other areas in the planning of the distribution networks have gained a great deal of interest over the past few years. Different models have been proposed for locating, sizing, timing, and deriving the service area for distribution substation [43,44]. Masud [43], employed linear and integer programming approach to optimize the system's substation capacities subject to, the constraints of cost, load, voltage, and reserve requirements. The model extended the techniques introduced in [27] for forecasting the substation loads and calculating feeder voltage drop. Crawford and Holt [44], addressed the problem of planning for distribution substation locations, sizes, and service boundaries. Their approach used two operations research techniques, the minimum path algorithm and the transportation algorithm, to simultaneously optimize
substation sizes and service boundaries, given alternative locations for substations and reliability constraints. The results would optimize the feeder losses and substation construction costs.

Different other approaches have been suggested in the literature to determine the best cable size, type, and length [34,45,46]. Snelson and Carson presented an analytical method for calculating the optimum economic design of a continuously tapered, branched radial, low voltage distributor under balanced-loading conditions [34]. The assumption that the voltage drop was the limiting factor was applied as well as the linear relationship between the cable cost per unit length and the cable cross-sectional area. This assumption was then applied to the design of the distributor from standard cable sizes. Also, the program proposed, took into account the cable thermal ratings, capitalization of losses, and unbalanced loading. On the other hand, the implicit enumeration of all composition of a certain set of conductors of a predetermined radial network was proposed in [45]. The optimum composition which minimized the total cost of conductors and energy losses over a given time span, while satisfying the voltage drop and the thermal constraints, could be determined. The problem of selecting the optimum route for a single primary distribution feeder radiating from a rural substation at a given location, and feeding a number of locals with known
demands and locations, was presented in [46]. The problem was formulated as an optimization problem of minimizing an objective cost function, subject to voltage regulation, current carrying capacity, and meeting the load constraints. Different approaches had been considered to simplify the problem, such as single straight feeder model, linear programming approach, and nonlinear optimization approach.

1.4 General Objectives

After analyzing the available techniques for both transmission and distribution systems planning, it is obvious that there are serious limitations in both fields. Summaries of the limitations will be given in chapters 3 and 4. There is a requirement for modifying and developing more accurate and efficient techniques for system planning. Accordingly the general objective of this work, from the beginning, was to overcome most of the limitations existed in the available methods of both the transmission and the distribution long range planning. The detailed modifications suggested as objectives to be achieved vary according to whether the distribution planning or the transmission planning model is under development. The specific objectives sought for the former are given in chapter 3, while the corresponding specific objectives for the latter are given in chapter 4.
New models for long range planning for both parts of the power system have been developed in this work and applied to different examples, in order to prove their capabilities and efficiency. Good improvements have been achieved through the new developed models and these will be demonstrated and analyzed in the conclusions presented in chapter 5.
Chapter II

POWER_SYSTEM_ANALYSIS

Electric power system planning is very complicated due to the large number of requirements which must be fulfilled: economical, operational, environmental, and social requirements must be met simultaneously. As has been summarized in the previous chapter, the planning process starts with the forecasting of the load growth with time. This is actually the reason for the need of the planning process, where the system has to satisfy this future demand efficiently, economically, reliably, and at certain level of stability. On the other hand, the importance of having a reliable transmission system to deliver the electric energy to load centers at the proper voltage level is obvious. Planning a reliable network requires the use of simulation tools, such as power flow analysis and contingency analysis.

In the following sections and throughout this chapter, different analysis which are necessary for electric power system planning will be briefly discussed.
2.1 Load Forecasting Analysis

The crucial part of the planning process lies in the forecasting of the future load requirements. Load forecast does not only determine the capacity of generation, transmission, and distribution system additions, but also determines the type of facilities required in the system and when it should be added. Load forecast is also responsible for establishing procuring policies for construction capital. That is why, the need for an accurate and efficient ways of estimating the peak load and the energy are necessary. It is often said that the key to a good planning and financial success is a good forecast. Unfortunately, it is impossible to have one analytical procedure to obtain an accurate forecast at all times, for all systems, and under all conditions. The choice of the best suitable technique for a given utility depends on the forecaster experience and knowledge of the advantages and the disadvantages of the different available methods.

2.1.1 Forecasting Definition and Approaches

Forecasting, as it has been defined in [47], is simply the determination of future load requirements, using a systematic process, in sufficient quantitative detail to permit important system expansion decisions to be made.

Based on the time period of a specific application, where forecasting is required, the forecasting procedures may be classified as short, intermediate, or long term techniques.
For example, the long range power system planners require 15-30 years forecast to establish feasible future generation and transmission plans. On the other hand, the system operators are only interested in the load minutes or hours ahead to be able to commit equipment or schedule an energy interchange with a neighboring utility. While daily forecasts are required to schedule generating units for optimum economy, weekly, monthly, or even one to three years forecasts of weekly peak demand, are required for preparing maintenance schedules and developing power pooling agreements.

There are different questions which have to be answered by the forecaster in order to set the direction to be taken in determining future requirements. For instance, the forecaster has to answer whether the peak demand should be estimated using energy and load factors forecasting, or it should be forecasted separately. Both approaches have their own advantages and disadvantages. While the energy forecasting is easier than the peak demand forecasting, the load factor estimation is as difficult as that of the peak demand. In addition, energy forecast is readily related to the demographic and the economic factors. On the other hand, the peak demand estimation can be related directly to weather variables, such as temperature. Another question has to be answered is whether the total forecast be directly obtained from historical data, or by combining forecasts of
appropriate load components, such as geographic areas. Simplicity, indicative information, and capability of detecting abnormal conditions are some of the factors which differentiate between the two options. Some attempts to combine both approaches had been made in [53-55]. The way of including the effect of weather conditions on the forecast is another question which has to be answered by the forecaster. Finally, the more difficult question of which forecasting technique to be used has to be tackled.

2.1.2 Forecasting Techniques

Since the forecasting technique to be used has a great influence on the operational and the economical aspects of the system under consideration, and since there has been an agreement that there is absolutely no single technique suitable for all systems under all conditions for all types of studies, thus the classification of the different forecasting techniques, and the concept of each, will be given here along with a brief review of their advantages and disadvantages.

In general, the forecasting techniques may be divided into techniques which are based on extrapolation, correlation, or a combination of both. Also these techniques could be either deterministic, probabilistic, or stochastic. The concepts of these classes of forecasting techniques are based on are given in [47], and briefly summarized here.
The extrapolation (trending) methods use curve fitting to historical load data and estimate the load growth by extrapolating the curve. Since the random errors in the data and/or in the analytical model are not accounted for, this technique is classified as a deterministic extrapolation. The technique becomes probabilistic extrapolation if statistical entries are used to account for the uncertainty in the estimated results. The stochastic process on the other hand, is a process developing in time in a manner controlled by probabilistic laws [49]. Accordingly, The stochastic models are used to estimate the future load growth from random inputs derived from historical data. Such inputs to the model tend to be the random change in the trend component, the random slope of the change in seasonal component and associated weighting factor, and a general noise component. The statistics of all the random inputs are determined by matching the statistics of the historical demand data with the corresponding statistics for the output of the model. The resulting time series of the process is the forecast desired.

The other major class of forecasting techniques is the simulation (correlation) methods. The simulation models predict the load growth by simulating the interaction of the causal factors controlling the growth. Among the controlling factors are the population, employment, building permits, appliance saturation, business indicators, and weather data.
This approach has the advantage of forcing the forecaster to understand clearly the interrelationship between load growth patterns and other measurable factors. On the contrary, this approach has a major drawback due to the fact that the need to forecast demographic and economic factors can be more difficult than forecasting system load itself.

Once again, it has to be emphasized that there is no specific forecasting method to be effective and suitable in all situations. Since this work is mainly concerned with transmission and distribution planning, the author feels that more attention has to be given, in this work, to some of the available models for demand forecast.

2.1.3. Review of Some Existing Demand Forecasting Models

Stanton and Gupta reviewed some different approaches to annual peak-demand forecasting [48]. Emphasis was given in that review to the probabilistic monthly or weekly peak-demand forecasting and the derivation of annual peak forecasts from it. Six different schemes were discussed including the use of annual energy forecasts and converting it to annual peak-demand by multiplying by the annual load factor. The accuracy of this method and that of the direct estimate of the peak-demand was almost the same due to the difficulty of forecasting the annual load factor. The difficulty in providing estimates of the variance of the annual peak-demand forecast was also stated as a major limitation of the energy and load-factor methods. The
fitting of a suitable time function to past values of annual peak-demand and extrapolating this trend curve forward to the desired time of the forecast was also discussed. In such procedure, it was assumed that the annual peak-demand always occurred in the same season and accordingly the effect of weather on the peak demand was ignored. In addition to the fact that this assumption was proven to be inaccurate, the model had an unsatisfactory reliability due to the limited amount of historical data. A modified approach of extrapolating the past annual peak-demand values was discussed. The approach was based on increasing the data without resorting to a longer data base. This was done by sampling some days, which were potentially annual peak-demand days, for each year. Assuming that similar weather conditions could be expected to prevail at the time of each demand data point, the weather influence on the observed demand values was again ignored. The reliability of the variance estimation using this approach would increase with the number of data points used for each year, but the value of the variance would start to increase if too many data points were used. This was due to the higher risk of including data points for which peak-demand conditions did not apply. A number of six was suggested as a reasonable number of data points per year. To account for the effect of the weather on peak-demand forecast, the modified extrapolation of the annual peak-demand method, discussed
above, had been adapted for separate treatment of weather-sensitive and non-weather-sensitive components of demand using a probabilistic extrapolation-correlation method. Generally, this method was based on the use of weekly peak-demand and weather data to develop weather-load models year by year, or season by season. These models were then used to separate the annually observed values of high demand (six per year were proposed earlier) into non-weather-sensitive and weather-sensitive components. A trend curve was then fitted to the non-weather-sensitive components of demand and extrapolated to obtain both the mean and the variance of the annual peak-demand at the desired time of the forecast. Growth curves were also fitted to the changing coefficients of the weather-load models and extrapolated forward to obtain the expected values and variances of the coefficients. After that, the historical demand and weather data were used to determine the mean and variance of the weather variance corresponding to annual peak-demand conditions, assuming that the weather variable is normally distributed. At last the forecasted weather-load models obtained, along with the weather statistics determined before, were combined to forecast the mean and variance of the weather-sensitive component of the future peak demand. Finally, the forecasts of the nonweather-sensitive component and peak-demand were combined with the forecasts of the weather-sensitive component to
produce the total peak-demand forecast. Great attention to saturation conditions had been suggested in projecting the growth of the weather-load models. Another method, based on the procedure described above for treating the influence of the weather effect, had been introduced for forecasting monthly or weekly peak-demands, instead of annual peak-demand directly. A procedure was then introduced to derive the annual or seasonal peak-demand forecast from the weekly or monthly demand forecasts. This method was explained in details and with examples in [47, 49]. Another practical application of the weather-sensitive load forecasting techniques to system planning was described in [50]. The method, used in developing the load-weather models, was basically a multiple regression analysis.

An introduction to the use of stochastic methods for forecasting annual peak-demand had been also given in [51]. The procedure had been summarized in major three steps. At first, the form of the stochastic model had to be specified. Historical data was then used to determine the unknown random inputs to the stochastic model. Finally, the response of the model was calculated giving the forecast. Because of the mathematical and computational complexity of the stochastic models, most of the forecasters have been discouraged from using such methods. One of the early stochastic models developed for peak-demand forecasting was that introduced by Gupta [49]. His procedure was based on
the concepts of the prediction theory of stationary stochastic time series, which had been developed further to predict those types of nonstationary series that could be reduced to a stationary series by a finite linear transformation.

Willis and his colleague introduced several models for a small area forecasting (52-54). These models were extrapolating techniques with some new features to improve the accuracy of the trending techniques and at the same time, using their advantages of economical demand on data, manpower, and computer resources. Willis and Northcote-Green reported in (52), two major problems affecting the accuracy of the extrapolating techniques. These two problems were the over extrapolation of load growth in area with a high recent history of load growth, and the inability to forecast growth in a vacant area which had no load history to trend. They introduced a hierarchical trending method to overcome these problems. The method was based on the fact that the future load growth in a presently vacant small area was part of the trend in the load growth of the region containing that small area. Thus, load growth in a historically vacant area could often be inferred by observing that, its region (group of small areas containing it) had a smooth continuous load growth history, while the load in all small areas that had a load history was approaching saturation. Later Willis et al., presented
another new trending method using clustering of historical load at the small area level as a forecasting algorithm [53]. While the growth for a large region (a sizable number of small areas) was a much smoother and more continuous process, the growth was decidedly nonlinear, more a quick-burst from vacant to full density than a lengthy ("S" curve), for the small area level. The small area cells displayed a similarity of overall "S" curve shape but different when their "burst" of growth occurred. The cluster based method used in [53] determined a set of clusters where each cluster contained a group of small areas with similar "S" curves. All small areas belonging to a cluster had a similar growth curve while their years of significant growth might occur at different times. Using a fast clustering algorithm, a set of average growth curve shapes at the small area level was established. Future load growth was then extrapolated for each small area by assigning it to the average curve which seemed to best fit it and by using this curve, extended to the future, as the forecast. Finally, Willis and Tram had combined the above two methods [52,53], and introduced another method in which, each of the previous methods reinforced the other [54].

Interesting and unique applications of signal processing in power system planning were also introduced by Willis and Aanstoos [55]. Comprehensive discussion of the applications was given in the reference.
2.2 Power-Flow Analysis

In planning new networks and expanding existing ones, it is necessary to investigate various alternative ways in which the generation system can be connected to the load centers. Due to the large number of ways to achieve the desired goals, it may seem impossible to consider all the various alternative ways. Although the complexity of the network can not be minimized, the constraints that must be respected drastically limit the feasible alternatives. With the help of a variety of analytical tools, identifying the one plan that best satisfies the desired performance criteria can be made.

Load flow studies help the planner to ensure that:

1. The load demands and generator capacities constraints are satisfied.
2. The overloading constraints of the network circuits are not violated even in the contingency conditions.
3. The system level of voltage magnitudes and bus swing angles are acceptable.

Before proceeding to the discussion of the different mathematical techniques of load flow solution, an introduction to the power modeling will be given first.
2.2.1 Power Network Modeling

Since the power flow studies are concerned with the performance of the system under steady-state conditions, hence the different mathematical techniques used are based on the per phase models of the different system components. These basic components are the synchronous generators, the power transformers, and the transmission / subtransmission / distribution systems. The mathematical models of the basic system elements are given in details in (1, 47, 56). A brief presentation of the different component models is given in the following subsections.

2.2.1.1 Notation

The following notation will be used in presenting the network complex quantities. A complex vector \( \mathbf{U} \) of \( n \) components \( u_i \), where \( i \) varies from 1 to \( n \), is given by,

\[
\mathbf{U} = \begin{bmatrix}
    u_1 \\
    u_2 \\
    \vdots \\
    u_n
\end{bmatrix}
\]

(2.1)

The component \( u_i \) can be presented in the complex domain, as shown in Fig. 2.1. It can be defined in the rectangular form of representation by its real component \( u_1 \), and its imaginary component \( u_2 \). It can also be defined in the polar form of representation by its magnitude \( |u_i| \) and its phase angle \( \theta_i \). These forms of identification can be written mathematically as follows,
\[ u_i = \mathbf{u}_i + j \mathbf{u}_{i2} \]

\[ = |\mathbf{u}_i| e^{j\theta_i} \]  

\[ (2.2) \]

\[ (2.3) \]

---

Figure 2.1: Representation of a complex quantity in the complex domain.

Equations (2.2) and (2.3) represent the rectangular and the polar forms of representation, respectively. Combining
equation (2.1) with equation (2.2), the complex vector \( U \) can then be written in the form of two separate real vectors as follows:

\[
U = U_1 + jU_2
\]  

(2.4)

The complex conjugate of the quantity is given by \( U^* \), where

\[
U^* = u_1 - j u_2
\]  

(2.5)

\[
= |u| e^{-j\theta}
\]  

(2.6)

2.2.1.2 Power Network Variables (Quantities)

There are four quantities to describe any of the power network components. These four quantities have only two degrees of freedom, i.e. defining any two of them, the other two can be accordingly calculated.

These quantities are given in Table 2.1, with their standard symbols used in the literature along with their rectangular and polar forms of representation.
Table 2.1: The complex quantities defining power network elements.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Standard symbol</th>
<th>Rectangular form of representation</th>
<th>Polar form of representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>I</td>
<td>$I = \frac{I_1}{i_1} + j\frac{I_2}{i_1}$</td>
<td>$</td>
</tr>
<tr>
<td>Voltage</td>
<td>V</td>
<td>$V = \frac{V_1}{i_1} + j\frac{V_2}{i_1}$</td>
<td>$</td>
</tr>
<tr>
<td>Admittance</td>
<td>Y</td>
<td>$Y = \frac{G}{i_1} + j\frac{B}{i_1}$</td>
<td>$</td>
</tr>
<tr>
<td>Apparent power</td>
<td>S</td>
<td>$S = \frac{P}{i_1} + j\frac{Q}{i_1}$</td>
<td>$</td>
</tr>
</tbody>
</table>

$G$ and $B$ are the conductance and the susceptance of element $i$, while $P$ and $Q$ are its active and reactive power, respectively. The element admittance $Y$ can be replaced by its impedance $Z$ which is the reciprocal of the admittance.

The above four quantities, for any component $i$, are governed by the following main relationships:

\[
I = V \cdot Y \quad \text{and} \quad S = V \cdot I
\]  

(2.7)  

(2.8)
and by substitution from equation (2.7) into equation (2.8),

\[
\begin{align*}
S &= V \cdot (V \cdot Y) \\
    &= (|V| \cdot Y) \\
\end{align*}
\]

From equation (2.9), it can be seen that, the angles \( \theta \) and \( \psi \), in Table 2.1 are equal in magnitude and have opposite signs, i.e. the apparent power angle in any element is completely determined by the angle of its admittance.

2.2.1.3 Power Network Element Indices

\[\text{Figure 2.2: Classification of network branches.}\]
In modeling the power network, the system components will be represented by different types of branches. These branches can be classified as source branches and line branches. The source branches will present the load, the generator, and the slack generator components in the network. The line branches will represent the transmission lines, the shunt elements, and the transformers in the network. Throughout this work the source branches will be given the subscript \( i \) or \( b \) (for branch) in general, while the subscripts \( l \), \( q \), and \( n \) might be used in the case of differentiating between the load, the generator, and the slack generator, respectively. The line branches will always be given the subscript \( i \), \( b \), or \( t \).

Figure 2-2 summarises the classification of the different network branches, while the indices used with each specific branch are given in the figure between brackets.

2.2.1.4 Power Network Steady-State Models

The steady-state model for each system component is given along with the quantities which are usually specified for this element.

Load-Branch Modeling:

Usually the load is defined by its complex power requirements, i.e., its active and reactive power components.

\[ S = P + jQ \]

is specified.
Often limits on the load voltage magnitude $|V_1|$ and angle $\delta_1$ are specified on the basis of voltage regulation limits and system stability requirements. The load branch can be modeled as shown in Fig. 2.3.

\[ S_1 = P_1 + jQ_1 \]

Figure 2.3: Load branch model.

**Generator Branch Modeling:**

\[ S_g = P_g + jQ_g \]

Figure 2.4: Generator branch model.
The system generator can be modeled as a constant voltage source with a specific active power. Accordingly $|V|_q$ and $P_q$ of generator $q$, are the known quantities. Specific limits on the generators reactive powers $Q_q$ are usually set according to the characteristics of each individual machine. Generator branch model is shown in Fig. 2.4.

**Slack_Generator_Branch_Model:**

The system power losses magnitude can not be known until the solution of the load flow is completed, which in turn can not be obtained unless one generator branch should have no power constraints to supply these losses to the network. Accordingly one generator branch is always specified by a constant voltage magnitude $|V|_n$ and phase angle $\theta_n$ with no constraints on its complex power. This generator is known as the slack generator. The slack generator branch model is the same as that in Fig. 2.4, with the subscript 'n' replaces the subscript 'q'.

**Modeling of Transmission Line and Shunt Elements:**

![Diagram](https://via.placeholder.com/150)

**Figure 2.5:** Transmission line model.
Each of these elements is usually defined by its admittance $Y$ (or impedance $Z$). The electrical resistance, inductance, conductance, and capacitance of each element can be calculated using expressions determined from evaluating the fields, electric and magnetic, for this element. Such element can be modeled as shown in Fig. 2.5.

**Power Transformer Model:**

(a)

(b)

Figure 2.6: Transformer models. (a) General per phase model of a standard voltage transformer; (b) $\pi$-equivalent per phase model of the transformer.
Finally, the last branch to be modeled is the power transformer. The power transformer is an essential device in the power network since it allows the electric power to be transferred at higher voltage level and accordingly, the network losses can be reduced. The general per phase model of a standard voltage transformer is shown in Fig. 2.6(a). The transformer is defined by its admittance \( Y \) and complex voltage ratio \( a \).

The equations controlling the model shown in Fig. 2.6(a) are:

\[
\frac{V_p}{V_s} = a_{st} \tag{2.10}
\]

\[
I_{pr} \cdot V = I_{psq} \cdot V_s \tag{2.11}
\]

This method of modeling, Fig. 2.6(a), is not suitable for load flow studies and accordingly the equivalent \( \pi \)-model of the transformer is given in Fig. 2.6(b). This equivalent model is controlled by the following equations:

\[
I_{st} - a_{st} \cdot I_{st} = (Y_{st}/a_{st}) \cdot V_{st} - (a_{st} \cdot Y_{st})/a_{st} \cdot V_{st} \tag{2.12}
\]

\[
I_{st} - a_{st} \cdot I_{st} = (Y_{st}/a_{st}) \cdot ((1/a_{st})-1) \cdot V_{st} - (a_{st} \cdot Y_{st}/a_{st}) \cdot ((1/a_{st})-1) \cdot V_{st} \tag{2.13}
\]

\[
I_{st} - a_{st} \cdot I_{st} = -Y_{st} \cdot ((1/a_{st})-1) \cdot V_{st} - a_{st} \cdot Y_{st} \cdot ((1/a_{st})-1) \cdot V_{st} \tag{2.14}
\]
2.2.2 Network Matrices and Power Flow Equations

Figure 2.7: Representation of a simple power network.
(a) Single line diagram of the network;
b) oriented graph of the network. The node
numbers are given inside circles.

The techniques used for performing the power flow studies
are usually different, depending on the type of network
under consideration. For example, radial networks are much
easier to solve than the large closed loops systems. In
radial network, the load flow studies are usually performed
by ignoring the phase shifts, since they are not usually
important. The problem is usually formulated in the form of
the total voltage drop along the feeder in terms of the load.
currents fed by the feeder, and its resistance and reactance. The determination of the cable resistance and reactance is usually based on the feeder type. For example, if it is underground cable, its cross-section is determined on the basis of the thermal limitation and the total voltage drop is then calculated. If the voltage drop along the cable is not within the specific limits, the design has to be modified. On the other hand, if the feeder is an overhead line, the cross-section is usually determined on the basis of a specific voltage drop rather than thermal limitations.

Large closed loops network requires much more effort in calculating its load flow. At the beginning, large networks were solved by what were known as "alternating current analyzers". Some time later, special-purpose electronic analogue computers were used to perform the studies. With the revolution of the digital computer development, a great attention has been directed towards the use of numerical methods for the analysis. Accordingly, the use of the matrix algebra for formulating and solving the complex problems has grown steadily. In the following subsections, the main network matrices will be discussed briefly along with the formulation of the network performance equations in the matrix notation. More details about the subject are given in [56].
2.2.2.1 Element-Node Incidence Matrix \( \hat{A} \)

The simplest way of describing the geometrical interconnection of a network is by using graphs, where each network branch can be replaced by a single line regardless of its characteristics. In such graph the lines and their terminals are called elements and nodes, respectively.

Elements and nodes incidence can be presented by what is called "element-node incidence matrix". It is a rectangular matrix of rows equal to the number of the graph elements, and columns equal to the number of nodes in the graph. The element \( ij \) of the matrix will be \(+1\) or \(-1\), if element \( i \) in the graph is incident to node \( j \) in the graph and oriented away or toward the node, respectively. If the element \( i \) is not incident to node \( j \), the element \( ij \) in the element-node incidence matrix will be set to zero. Figure 2.7 presents the positive sequence network diagram of a simple power network along with its corresponding oriented graph. Orientation of the connected graph of the network is arbitrary.

The resultant element-node incidence matrix \( \hat{A} \) of the network shown in Fig. 2.7, is
2.2.2.2 Bus Incidence Matrix \( \hat{A} \)

The element-bus incidence matrix \( \hat{A} \) can be obtained by deleting a column from the matrix \( \hat{A} \), corresponding to any arbitrary node. This node is referred to as the reference node and the other remaining nodes will be called buses. The variables of the different buses can be measured with respect to the reference.

\[
\hat{A} = \begin{bmatrix}
+1 & -1 & 0 & 0 \\
+1 & 0 & -1 & 0 \\
0 & +1 & -1 & 0 \\
0 & -1 & 0 & +1 \\
0 & 0 & +1 & -1 \\
\end{bmatrix}
\]

(2.15)

\[
A = \begin{bmatrix}
+1 & -1 & 0 \\
+1 & 0 & -1 \\
0 & +1 & -1 \\
0 & -1 & 0 \\
0 & 0 & +1 \\
\end{bmatrix}
\]

(2.16)
The element-bus incidence matrix $A$ of the network shown in Fig. 2.7 is given in equation (2.16), after selecting node 4 as a reference.

2.2.2.3 Primitive Admittance Matrix $Y_p^T$

The primitive admittance matrix $Y^p$ of a network is a square matrix of dimension equal to the number of elements in the network graph. The matrix presents the self-admittance $Y_{ii}$ of element $i$ in the network as well as the mutual-admittance $Y_{ij}$ between elements $i$ and $j$ in the network.

If the mutual-admittances between the elements of the network shown in Fig. 2.7 are neglected, the network primitive admittance matrix will then be a diagonal matrix as given in equation (2.17).

\[
Y^p_{-T} = \begin{bmatrix}
Y_{11} & 0 & 0 & 0 & 0 \\
0 & Y_{12} & 0 & 0 & 0 \\
0 & 0 & Y_{23} & 0 & 0 \\
0 & 0 & 0 & Y_{34} & 0 \\
0 & 0 & 0 & 0 & Y_{45}
\end{bmatrix}
\]
2.2.2.4 Bus Admittance Matrix $Y_T = Y_{bus}$

The bus admittance matrix $Y$ of a network is used to relate the variables of the network elements to the network bus quantities. It can be obtained using the network bus incidence matrix $A$ with the network primitive admittance matrix $Y_P$. The bus admittance matrix is,

$$Y_T = A Y_P A^{-T} \quad (2.18)$$

The detailed derivation of equation (2.18) is given in [56]. As an example, the bus admittance matrix of the three-bus system shown in Fig. 2.7, is

<table>
<thead>
<tr>
<th>Bus number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Y_{12} + Y_{13}$</td>
<td>$-Y_{12}$</td>
<td>$-Y_{13}$</td>
</tr>
<tr>
<td>2</td>
<td>$-Y_{12}$</td>
<td>$Y_{12} + Y_{23} + Y_{24}$</td>
<td>$-Y_{23}$</td>
</tr>
<tr>
<td>3</td>
<td>$-Y_{13}$</td>
<td>$-Y_{23}$</td>
<td>$Y_{13} + Y_{23} + Y_{34}$</td>
</tr>
</tbody>
</table>

The bus impedance matrix $Z$ of the network can be obtained by inverse transformation of the network bus admittance matrix $Y_T$, i.e.

$$Z_T = Z_B \equiv Y_T^{-1} \quad (2.20)$$
2.2.2.5 Network Performance Equations

The performance of an interconnected power network can be studied in different frames of reference. In the bus frame of reference, the performance of the network is described by \( n \) independent nodal equations, where \( n \) is the number of buses in the network (number of network nodes excluding the reference node). The performance can also be described by \( b \) independent branch equations in the branch frame of reference, where \( b \) is the number of network tree branches.

On the other hand, the network performance is described by \( l \) independent loop equations in the loop frame of reference, where \( l \) is the number of links or basic loops in the network. The network loop is simply a closed path in the network produced by adding one of the network links, which is an element not included in the network tree, to the tree.

Since the main concern in the load flow studies in the planning process is to determine the values of the bus variables in the network, the performance equations discussed here will be in the bus frame of reference. The formulations in the other frame of references are given in details in [56].

Based on equation (2.7), the performance of the connected network, in the bus frame of reference, can be written in the form,

\[
I = Y V
\]

(2.21)
where \( I \) is a vector of dimension \( n \), the number of the network buses, presents the injected bus currents. \( V \) is a vector of dimension \( n \), presents the bus voltages, while \( Y \) is the network bus admittance matrix of dimension \( nxn \).

The network bus loading equation can be obtained, on the basis of equation (2.8), in the form:

\[
S = E I \quad \text{(2.22)}
\]

where \( S \) is a vector of dimension \( n \), presenting the bus complex powers, and \( E \) is a diagonal matrix of the complex bus voltages. Combining equations (2.21) and (2.22), the following form of the performance equation, in the matrix notation, can be obtained as:

\[
S = E Y V \quad \text{(2.23)}
\]

For the three-bus example, shown in Fig. 2.7, equation (2.23) is written as:

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3
\end{bmatrix} =
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Y_{12}+Y_{13} & -Y_{12} & -Y_{13} \\
-Y_{12} & Y_{12}+Y_{23}+Y_{22} & -Y_{23} \\
-Y_{13} & -Y_{23} & Y_{13}+Y_{23}+Y_{33}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\]
2.2.2.6 Modes of Formulating Power Flow Equations

There are basically three modes of formulation considered in load flow studies in electric power systems. These modes of formulation are:

1. the complex (basic) form,
2. the rectangular form, and
3. the polar form.

Definitions:

The following definitions will be useful in discussing each of the above modes of formulating the power flow equations.

- \( n \) is the number of buses in the augmented network
  \[ n = n + n + 1 \]
  \[ L \quad G \]

- \( n \) is the number of load buses in the network. The load buses will usually be ranked before the generator buses; i.e., the subscript \( l \) referring to a load bus will be given the values \( l = 1, 2, \ldots, n \).

- \( n \) is the number of the network generator buses excluding the slack bus, which will be ranked as the last bus in the network. So, the subscript \( g \) denoting a generator bus will take the values \( g = (n + 1), (n + 2), \ldots, (n + n) \).

- \( Y \) is the \( ij \)-th element in the network bus admittance matrix
  \[ Y_{ij} = G_{ij} + jB_{ij} \]  
  in the rectangular form, and
\[
 Y_{ij} | \theta_{ij} \quad \text{in the polar form.}
\]

**Basic (Complex) Form of Formulation:**

Equation (2.23) is used to formulate the load flow in the complex form. Since we have decided to work with the bus frame of reference throughout this discussion, the subscript 'Bus' could then be dropped and the basic form of formulation can then be written in the matrix notation, as

\[
 E^* Y_{ij} V_i - S_i = 0
\]

(2.24)

Equation (2.24) presents a set of \( n \) complex equations where the \( i \)-th equation in the set takes the form,

\[
 V_i \sum_{j=1}^{n} (Y_{ij} V_j) - S_i = 0
\]

(2.25)

The above set of equations (2.24), can be organized in terms of known (specified) quantities on one side, and unknown variables on the other, as

\[ f \text{ (unknown variables)} = b \text{ (specific quantities)} \]

The unknown variables, sometimes known as voltage variables, in electric power network include the bus voltage, the generator bus reactive power and voltage phase angle, and the swing bus complex power. The known variables in the network are usually the load bus complex power, the
generator bus active power and voltage magnitude, and the swing bus voltage.

In the complex form, it is more suitable if the unknown quantities chosen to be the set of the bus complex voltage and their conjugate, i.e., \( V \) and \( V^* \). Rearranging the \( i \)-th row of equation (2.24) to separate the specific and the unknown quantities, the form of the obtained equation will be based on the type of bus \( i \). For example, if row \( i \) presents a load bus \( l \), the \( i \)-th complex equation becomes,

\[
\sum_{j=1}^{n} \left( \begin{array}{c} V \cdot Y \cdot V \\ l \cdot l_j \cdot j \cdot l \end{array} \right) = S^* \quad ; \quad l=1,2,\ldots,n \quad (2.26)
\]

Similarly, if row \( i \) presents a generator bus \( g \), the \( i \)-th complex equation then becomes,

\[
\sum_{j=1}^{n} \left[ \begin{array}{c} 1 \\ [-V \cdot V \cdot Y + V \cdot V \cdot Y] \\ q \cdot q_j \cdot q \cdot q_j \end{array} \right] + \left[ \begin{array}{c} V \cdot V^* \left(1/2\right) \\ q \cdot q \end{array} \right] = p + jlv \quad ; \\
q=(n+1),(n+2),\ldots,(n+n) \quad (2.27)
\]

Finally, if row \( i \) presents the slack generator bus \( n \), it can be eliminated since it will be written as,

\[
V = \frac{V}{n} \quad (2.28)
\]
As it can be seen in the basic form, the choice of \( V \) and \( Y \) as the unknown variables in the system has reduced the total number of equations required to solve the load flow from \( n \) to \( n-1 \) complex equations.

**Rectangular Formulation:**

In the rectangular form of formulating the load flow equations, the unknown variables are usually the real and imaginary components of all the bus voltages except that of the slack bus which is usually specified. The complex equation of bus \( i \) can then be broken to two real equations. The form of each of these equations, once again depends on the type of bus \( i \).

For load bus \( l \), the following two equations are used when the specified bus quantities are set on the right hand side of each equation,

\[
\sum_{j=1}^{n} \left[ e_l \left[ \begin{array}{c} e \\ f \end{array} \right] + f \left[ \begin{array}{c} e \\ f \end{array} \right] \right] = P_l ; \quad l=1,2,\ldots,n
\]  

(2.29)

\[
\sum_{j=1}^{n} \left[ f_l \left[ \begin{array}{c} e \\ f \end{array} \right] - e \left[ \begin{array}{c} e \\ f \end{array} \right] \right] = Q_l ; \quad l=1,2,\ldots,n
\]  

(2.30)

For a generator bus, excluding the slack bus, the following two real equations can be used:
\[
\sum_{j=1}^{n} \left( \begin{array}{c}
e^q \\
eg \end{array} \right) \left( \begin{array}{c}e^q \\
eg \end{array} \right) + \left( \begin{array}{c}f^q \\
eg \end{array} \right) = \left( \begin{array}{c}1 \\\vphantom{1} \end{array} \right) \right]
\]

(2.31)

\[
\begin{aligned}
e^q &+ f^q = \left( \begin{array}{c}1 \\
 \end{array} \right) ;

\end{aligned}
\]

(2.32)

From the above set of equations, the load flow is formulated in \(2(n-1)\) real equations in the rectangular form.

**Polar Form of Formulation:**

Choosing the unknown variables to be the load bus voltage magnitudes and phase angles, as well as the phase angles of the generator bus voltages, will reduce the total number of load flow equations to \((2n_L + n_G)\), instead of \((2(n_L + n_G))\), in the rectangular form. The reason for this is the fact that the generator bus voltage magnitudes are usually specified. The equations then can be formulated, in the polar form, as follows.

\[
\sum_{j=1}^{n} \left( \begin{array}{c}|V| \\
\end{array} \right) \left( \begin{array}{c}|V| \\
\end{array} \right) \left( \begin{array}{c}|Y| \\
\end{array} \right) \left( \begin{array}{c}\cos(\theta - \delta + \delta) \\
\end{array} \right) = p ;
\]

\[
\begin{aligned}
L \;
L_g
\end{aligned}
\]

(2.33)
\[ \sum_{j=1}^{n} \left( \begin{vmatrix} V_{1j} & V_{1\bar{j}} & Y_{1j} \\ V_{2j} & V_{2\bar{j}} & Y_{2j} \\ \vdots & \vdots & \vdots \\ V_{n,j} & V_{n\bar{j}} & Y_{n,j} \end{vmatrix} \right) \sin(\theta - \delta_1 + \delta_2) = -q_1 \quad ; \quad l = 1, 2, \ldots, n \]

\[ \sum_{j=1}^{n} \left( \begin{vmatrix} V_{qj} & V_{q\bar{j}} & Y_{qj} \\ V_{gj} & V_{g\bar{j}} & Y_{gj} \\ \vdots & \vdots & \vdots \\ V_{nj} & V_{n\bar{j}} & Y_{nj} \end{vmatrix} \right) \cos(\theta - \delta_1 - \delta_2) = p_q \quad ; \quad g = (n+1), (n+2), \ldots, (n+n) \]

\[ \sum_{j=1}^{n} \left( \begin{vmatrix} V_{Lj} & V_{L\bar{j}} & Y_{Lj} \\ V_{Gj} & V_{G\bar{j}} & Y_{Gj} \\ \vdots & \vdots & \vdots \\ V_{nj} & V_{n\bar{j}} & Y_{nj} \end{vmatrix} \right) = \]
latter were classified as iterative methods \([58, 61-69]\) since the voltage variables were obtained by an iterative approach from the estimated quantities. The review also included some hybrid techniques \([63-70]\), which combined the above two classified methods.

With the development of an efficient sparsity programmed ordered elimination technique in \([71]\), the Newton-Raphson technique has dominated the field of load flow solution. Stott \([73]\) presented a review of specified classes of load flow techniques. Five distinctive classes were discussed in that review. As in \([57]\), the \(Y\)-matrix iterative methods \([58, 61, 62, 65-68]\), were classified as those methods which were based on the calculation of the bus voltages \(V\) from the estimated bus currents \(I\) by the iterative solution of equation \((2.36)\).

\[
I = Y V
\]  

(2.36)

On the other hand, the \(Z\)-matrix methods \([58-60, 72]\), were based on the direct solution of equation \((2.36)\), for the bus voltages in terms of the bus estimated currents using the inverse of the network \(Y\) matrix (\(Z\) matrix). Due to the wide spread and the great interest in the Newton-Raphson method, it had been classified as a separate class \([61, 64, 69, 74-76]\). The approximate Newton methods \([77-80]\), were classified as the decoupled methods, which took advantage of the strong ties between the bus active powers and the bus swing angles.
and between the bus reactive powers and the bus voltage magnitudes. In addition, special class had been reserved for the rest of the methods, which had attracted some attention but could not fit in any of the above classes. These methods were defined as Miscellaneous methods [63,70,81-65]. In recent years, an adjoint network concept has been employed to develop a new method for solving the load flow problems [86]. In addition to the great number of references describing the different techniques of load flow solution, the discussion of the acceleration, convergence, and suitability of the different methods had been also tackled quite often in the literature [57,73,87-90].

In the following subsections, different load flow solution techniques will be described briefly and with different classifications. The different techniques fall into two main categories: namely, the derivative and the non-derivative methods. Only the formulations in terms of bus admittance matrix will be discussed.

2.2.3.1 Non-Derivative Methods

These methods are generally based on the basic complex equation (2.24), which takes the form,

\[
E \quad Y \quad V = S \quad \quad (2.37)
\]

where the i-th row of this set of equations presents the loading equation of bus i, in the form.
\[ \sum_{j=1}^{n} \begin{bmatrix} V * V * Y \\ i \\ j \\ i \end{bmatrix} = S * \quad ; \quad i = 1, 2, \ldots, (n-1) \quad (2.38) \]

Equation (2.38) applies to all types of buses (except the slack bus), where accommodation for generator buses (voltage controlled) is made during the iterative procedure. Different iterative methods can be used to solve equation (2.38) in the network unknown voltage variables.

**Gauss Iterative Method**

Rearranging equation (2.38) in a more suitable form for the iterative procedure, it can then be rewritten in the form,

\[ V * V * Y + \sum_{j=1, j \neq i}^{n} \begin{bmatrix} V * Y * V \\ i \\ i \\ j \end{bmatrix} = S * \quad ; \quad i = 1, 2, \ldots, (n-1) \quad (2.39) \]

For the load bus \( l \), the bus complex power is specified and equation (2.39) can then be written at iteration \((K)\), for this bus, as follows,

\[ V_{l}^{(K+1)} = \frac{1}{S - \sum_{j=1}^{n} \begin{bmatrix} *_{(K)} *_{(K)} \end{bmatrix} \begin{bmatrix} V * V * Y \\ l \\ i \\ j \end{bmatrix}} ; \quad l = 1, 2, \ldots, n \quad (2.40) \]
For the generator bus \( q \), an equation similar to equation (2.40) is derived and it represents an intermediate step \((K^{+}(1/2))\) in iteration \((K)\),

\[
\begin{align*}
V_q^{(K^{+}(1/2))} &= \frac{1}{s(K)} \left[ S \sum_{j=1}^{n} \begin{bmatrix} *y_{qj}^{(K)} \end{bmatrix} \right] \; ; \\
&= \frac{1}{s(K)} \left[ \frac{V_q^{(K)}}{s(qq)} \sum_{j=1}^{n} \begin{bmatrix} *y_{qj}^{(K)} \end{bmatrix} \right] \\
&\quad \text{for } g_q^{(n+1)}, (n+2), \ldots, (n-1) \\
&\quad \text{for } L_q^{(n+1)}, (n+2), \ldots, (n-1)
\end{align*}
\]

where

\[
S_q^{(K)} = p + j \text{Im}(V_q^{(K)}) \sum_{j=1}^{n} \begin{bmatrix} V_q^{(K)} \cdot y_{qj}^{(K)} \end{bmatrix} \\
= \frac{1}{s(K)} \left[ \frac{V_q^{(K)}}{s(qq)} \sum_{j=1}^{n} \begin{bmatrix} *y_{qj}^{(K)} \end{bmatrix} \right] \\
\quad \text{for } g_q^{(n+1)}, (n+2), \ldots, (n-1) \\
\quad \text{for } L_q^{(n+1)}, (n+2), \ldots, (n-1)
\]

Since the generator bus voltage magnitude is specified \((|V_q|)\), iteration \((K)\) will be completed by adjusting the magnitude of the intermediate bus voltage \(V_q^{(K^{+}(1/2))}\) as follows,

\[
\begin{align*}
V_q^{(K+1)} &= V_q^{(K^{+}(1/2))} \cdot \left| \frac{|V_q|}{|V_q^{(K^{+}(1/2))}|} \right| \\
&= V_q^{(K^{+}(1/2))} \cdot \left| \frac{|V_q|}{|V_q^{(K^{+}(1/2))}|} \right| \\
&\quad \text{for } g_q^{(n+1)}, (n+2), \ldots, (n-1) \\
&\quad \text{for } L_q^{(n+1)}, (n+2), \ldots, (n-1)
\end{align*}
\]

Initial values for the bus voltages \( V_q \) are assumed at the
beginning of the iterative procedure. It has been common practice to initiate the procedure with a flat voltage start, i.e., setting the bus voltage magnitudes, where given, to their specified values and the other bus voltage magnitudes equal to one per unit, while the bus voltage angles (bus swing angles) equal to the slack bus angle (usually equal to zero).

**Gauss-Seidel Iterative Method:**

This method is quite similar to the Gauss method, discussed above, except that the updated bus voltages are substituted directly in all followed operations within the same iteration. Accordingly, at iteration \( K \) for load bus \( l \), equation (2.40), becomes,

\[
V^{(K+1)}_l = \frac{1}{K-1} \left[ \sum_{j=1}^{l-1} \left( V^{(K)}_l V^{(K)}_j \right) \sum_{j=l+1}^{n} \left( V^{(K)}_l V^{(K)}_j \right) V^{(K)}_j \right] ;
\]

\[
V_l = \left[ \sum_{j=1}^{l-1} (V_l V_j) \sum_{j=l+1}^{n} (V_l V_j) \right] V_j ;\]

\[
l = 1, 2, \ldots, n \tag{2.44}
\]

while equation (2.41) for generator bus \( g \) becomes

\[
V^{(K+0.5)}_g = \frac{1}{K-1} \left[ \sum_{j=1}^{g-1} \left( V^{(K+0.5)}_g V^{(K+0.5)}_j \right) \sum_{j=g+1}^{n} \left( V^{(K+0.5)}_g V^{(K+0.5)}_j \right) V^{(K+0.5)}_j \right] ;
\]

\[
V_g = \left[ \sum_{j=1}^{g-1} (V_g V_j) \sum_{j=g+1}^{n} (V_g V_j) \right] V_j ;\]

\[
g = (n +1), (n +2), \ldots, (n-1) \tag{2.45}
\]
and equation (2.42) will be
\[ S_{gq} = P + \sum_{q} \text{Im}(V_{gq}) \left[ \sum_{j=1}^{g-1} (V_{gq}) + \sum_{j=q}^{g} (V_{gq}) \right] \]
\[ + \sum_{j=q}^{n} (V_{gq}) \left( (n+1), (n+2), \ldots, (n-1) \right) \]
\[ \text{L} \]
\[ (2.48) \]

and finally equation (2.43) will remain as before
\[ V_{gq} = V_{gq} \left( (V_{gq})^2 / |V_{gq}| \right) \]
\[ \text{sp} \]
\[ \text{q} \]
\[ g=(n+1), (n+2), \ldots, (n-1) \]
\[ \text{L} \]
\[ (2.47) \]

**Relaxation Method:**

Instead of combining the two basic complex performance equations of the network, (2.21) and (2.22), and solving the resultant equation, the relaxation method offers a different procedure for the solution. The approach depends on the separate solution of both equations. Starting with a flat voltage set \( V^{(0)} \), the control variables \( I^{(0)} \) can be roughly estimated using equation (2.22). Using equation (2.21), with the estimated values of the system variables \( V^{(0)} \) and \( I^{(0)} \), will produce a set of residuals \( R^{(0)} \) in the control variables \( I \). The method then depends on eliminating these residuals. The bus voltages and currents, as well as the residuals are updated in an iterative process and when the residual values decrease to zero, the solution is then obtained. The set of equations to be solved in each iteration varies with the bus type. For a load bus 1 the following equations will be used:
\begin{align*}
V^{(K+1)} &= V^{(K)} - (R^{(K)} / Y) \quad (2.48) \\
I^{(K+1)} &= S^{(K+1)} / V \quad (2.49) \\
R^{(K+1)} &= I^{(K+1)} - I^{(K)} \quad (2.50) \\
R_j &= R - Y * (R_j / Y); \quad j=1,2,\ldots,(n-1) \quad (2.51)
\end{align*}

and for a generator bus \( g \), accounting for the voltage magnitude adjustment should be included and accordingly the following set of equations will be used:

\begin{align*}
V^{(K+1/2)} &= V^{(K)} - (R^{(K)} / Y) \quad (2.52) \\
V &= V^{(K+1/2)} - (|V| / |V|) \quad (2.53) \\
S^{*(K+1)} &= P + j \text{Im}(V^{(K+1)} \sum_{j=1}^{n} (V^{(-1)} \cdot Y)) \quad (2.54) \\
I^{(K+1)} &= S^{*(K+1)} / V \quad (2.55) \\
R^{(K+1)} &= I^{(K+1)} - I^{(K)} \quad (2.56) \\
R_j &= R - Y * (R_j / Y); \quad j=1,2,\ldots,(n-1) \quad (2.57)
\end{align*}
It should be noted that updating the residuals of the buses, other than the one under consideration (equations (2.51) and (2.57)) should be made for each updating of each load bus l, where \( l = 1, 2, \ldots, n \), and for each generator bus \( q \), where \( q = (n+1), (n+2), \ldots, (n-1) \).

Different formulations for the relaxation methods exist in the literature.

**Use of Acceleration Factors:**

All iterative methods, described above, have to be accelerated to decrease the number of iterations required to reach the solution. The development of different accelerating techniques presents a wide area of research in the field of numerical analysis. As an example, only one method based on updating the formula for each bus voltage in the iteration process will be given below, and for more information about the subject, the reader is referred to references (87-89) and others.

Choosing acceleration factors \( \lambda \) and \( \xi \), the load bus voltage equations (2.40), (2.44), and (2.46) can then be updated as follows,

\[
(K+(3/2)) (K) (K+1) (K) * (K+1) * (K) \\
V^l = V^l + \lambda (V^l - V^l) + \xi (V^l - V^l) \\
1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \\
1 = 1, 2, \ldots, n \\
L
\]
while the generator bus voltage equations (2.43), (2.47), and (2.53) can be updated using the following two equations,

\[
V^{(K+3/2)} = V^{(K)} + \lambda(V^{(K)} - V^{(K-1)}) + j(V^{(K)} - V^{(K-1)}); \quad g = (n+1), (n+2), \ldots, (n-1)
\]

then

\[
V^{(K+7/4)} = V^{(K+3/2)} + j(V^{(K+3/2)} - V^{(K+1)}); \quad g = (n+1), (n+2), \ldots, (n-1)
\]

The acceleration factors can be either real or complex quantities. As an example, if \( \lambda \) is a real quantity, not equal to zero, while \( j \) equal to zero, both the real and the imaginary parts of the complex voltage will be accelerated by the same factor. While if both factors are real non-zero quantities, the real part of the complex voltage will be accelerated by the factor \( \lambda + j \) and the imaginary part will be accelerated by the factor \( \lambda - j \).

**D.C. Load Flow Technique:**

This technique employs an approximate power flow model and it is only used when an approximate solution for the network is required. This method finds wide range of applications in the long range planning and/or in finding a starting solution for other techniques applied to large networks. The technique has the following characteristics,
1. The dc power model is based on the relationship between the bus active powers and the bus voltage phases (swing angles), hence it discards the reactive power flow in the network.

2. The technique reduces the number of equations to be solved to about half of those considered in the ac models. It also employs a constant matrix of coefficients and one direct (non-iterative) solution of a set of linear equations is required.

3. The method treats both load and generator buses in a similar manner, since only the active power flow is considered.

Method Description:

The method is based on the approximation of the active power equations of the buses in the polar form ((2.33) or (2.35)) which can be written for bus \( i \), in the form

\[
P = \sum_{j=1}^{n} \left[ V_i V_j |l_{ij} \cos(-\theta_{ij} + \delta_{ij}) \right]; \quad i = 1, 2, \ldots, (n-1)
\]

Because of the weak ties between bus voltage magnitudes and bus active powers, and since the elements of the network admittance matrix are mostly susceptance, the following assumptions can be made:

(a) set all bus voltage magnitudes to one per unit, i.e.

\[
|V_i| = 1; \quad i = 1, 2, \ldots, n
\]
(b) set the angles of all the admittance matrix elements to \(-\pi/2\) radians, i.e.

\[
\theta_{ij} = -\pi/2 \quad i = 1, 2, \ldots, n \quad j = 1, 2, \ldots, n
\]

(c) since the phase angle differences between the network buses are usually small, \(\sin(\delta - \delta)\) can be approximated to \((\delta - \delta)\) for \(i = 1, 2, \ldots, n\) and \(j = 1, 2, \ldots, n\), where \(\delta_i\) and \(\delta_j\) are in radians.

Based on these assumptions, equation (2.61) can then be written as,

\[
P_i = \sum_{j=1}^{n} B_{ij} \cdot \sin(\delta_i - \delta_j) \quad (2.62)
\]

\[
P_i = \sum_{j=1}^{n} B_{ij} \cdot (\delta_i - \delta_j) \quad (2.63)
\]

\[
= \sum_{j=1}^{n} B_{ij} \cdot (\delta_i - \delta_j) \quad (2.64)
\]

\[
= \delta_i \cdot \left[ \sum_{j=1}^{n} (B_{ij})_{j \neq 1} \right] - \sum_{j=1}^{n} (B_{ij} \cdot \delta_j) \quad (2.64)
\]
Neglecting the shunt ground capacitors in the network, the following relation is satisfied:

\[ B_{ii} = - \sum_{j=1}^{n} B_{ij} \quad (2.65) \]

Substituting equation (2.65) into equation (2.64), then

\[ P_i = -B_{ii} \delta_i - \left( \sum_{j=1}^{n-1} (B_{ij} \cdot \delta_j) \right) ; \quad i = 1, 2, \ldots, (n-1) \quad (2.66) \]

Since the slack bus swing angle \( \delta \) is usually set to zero as a reference. In matrix notation, equation (2.66) is written as

\[ P = -B \delta \]

\[ \begin{pmatrix} \mathbf{B} \\ \mathbf{-M} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{E} \end{pmatrix} \]

where

- \( P \) is a column vector of \( n = n-1 \) elements representing the bus active powers,
- \( \delta \) is a column vector of \( n \) elements representing the bus swing angles in radians, and
B is the imaginary part of the network bus admittance matrix after eliminating the row and the column of the slack bus.

Equation (2.67) can be solved in one step for the unknowns.

Modified Technique:

When specified and/or approximate values of the bus voltage magnitudes in the network are available, equation (2.63) can be modified to the form

\[ P = \sum_{i=1}^{n} \left( |V_i| |V_j| B_{ij} \right) \left( \delta_i - \delta_j \right) ; i=1,2,\ldots,(n-1) \]  \hspace{1cm} (2.68)

then

\[ P = \sum_{i=1}^{n} \left( K_{ij} \delta_j \right) ; i=1,2,\ldots,(n-1) \]  \hspace{1cm} (2.69)

or in a matrix form as

\[ P = K \delta \]

\[ -M \hspace{1cm} -MM \hspace{1cm} -M \]

where

\[ K = \left[ -|V_i| |V_j| B_{ij} \right]_{i,j=1}^{n} \]

and
2.2.3.2 Derivative Methods

In describing the derivative methods in this work, only the real form of variables is used, since the recent more general complex mode of formulations does not concern this work.

The general form of power flow equations is,

\[ h(\text{voltage variables}) = U(\text{known quantities}) \]

or

\[ h(x) = U \quad (2.71) \]

where

- \( x \) is an array contains all the unknown variables in the power flow equations (state variables),
- \( U \) is a column vector contains the known quantities in the network (control variables).

The elements of the vectors \( U \) and \( x \) depend on the form of formulation used. For example,
in the polar form

\[ U = \begin{bmatrix} P \\ -1 \\ P \\ -q \\ Q \\ -1 \end{bmatrix} \]  \hspace{1cm} (2.72)

while in the rectangular form

\[ U = \begin{bmatrix} P \\ -1 \\ P \\ -q \\ Q \\ -1 \end{bmatrix} \left( ||V||^2 \right) \]  \hspace{1cm} (2.73)

where \( P \) and \( P \) are arrays containing the load bus and the generator bus active powers, respectively, \( Q \) is an array of the load bus reactive powers, and \( ||V|| \) is an array with the square of the generator bus voltage magnitudes as its elements.

On the other hand, the array \( \chi \) may contain the real parts of both load bus \( l \) and generator bus \( q \) voltages, as well as their imaginary parts \( f \), in the rectangular form, i.e.

\[ \chi = \begin{bmatrix} e \\ -1 \\ e \\ -q \\ f \\ -1 \\ f \\ -q \end{bmatrix} \]  \hspace{1cm} (2.74)

In the polar form the vector \( \chi \) will contain the swing angles \( \delta \) of both load and generator buses, along with the load bus voltage magnitudes \( ||V|| \), i.e.
\[ x = \begin{bmatrix} \delta_1 \\ -1 \\ \delta_q \\ IV I \end{bmatrix} \] (2.75)

**Newton-Raphson (NR) Method:**

When this method was first described and employed to load flow problems, it was found that the computer time and memory requirements increased rapidly with the problem size. Later it was found that the elimination procedure, used for solving the simultaneous equations, was the main reason for that, and not the method itself. After the implementation of highly efficient sparsity-programmed ordered elimination techniques, the method became the backbone and the most widely used method for solving load flow problems. It has the advantage of powerful convergence characteristics, moderate computer storage, and short computing time. The method also does not need acceleration factors and it is more robust in handling ill-conditioned systems.

**General Newton-Raphson (NR) Iteration:**

If an approximate solution of the voltage variables in the system at iteration \((K)\) is given by \(x^{(K)}\) with error of \(\Delta x^{(K)}\), then equation (2.71) can be written as,

\[ h(x^{(K)} + \Delta x^{(K)}) = U \] (2.76)

and by Taylor's theorem, it becomes
\[
\begin{align*}
\text{(K)} & \quad \left[ \frac{\partial h}{\partial x} \right]^T \quad \text{(K)} \\
- - \quad \left[ \frac{\partial h}{\partial x} \right]^T \quad \text{(K)} \quad \text{+ higher order terms} = U \\
\end{align*}
\] (2.77)

where \[ \left[ \frac{\partial h}{\partial x} \right]^T \] stands for the value of \[ \left[ \frac{\partial h}{\partial x} \right] \] at iteration \( \text{(K)} \) which is called the Jacobian matrix of NR method calculated at iteration \( \text{(K)} \). By eliminating the higher order terms in equation (2.77) and rearranging it, the equation can then be written as,

\[
J \Delta x = U - h(x)
\]

\[
\begin{align*}
\text{(K)} & \quad \text{(K)} \\
= U - U \\
= \Delta U \\
\end{align*}
\] (2.78)

In the iteration process, the vector \( \Delta x \) is defined as,

\[
\Delta x = x - x
\]

and accordingly,

\[
x^{(K+1)} = x^{(K)} + \Delta x
\] (2.79)
From equations (2.78) and (2.79), the following general iteration in the method can be obtained,

\[
\begin{align*}
\begin{pmatrix} (k+1) & (k) & (k) - 1 & (k) \\ (k) & (k) & (k) - 1 & (k) \\ \vdots & \vdots & \vdots & \vdots \\ (k) & (k) & (k) - 1 & (k) \\
\end{pmatrix} \Delta U \\
\end{align*}
\]

(2.80)

where the initial values of the voltage variables \( x \) are assumed. The method can be started with a flat voltage or with the solution of any other approximate method for solving load flow (e.g., dc load flow solution).

Elements of Newton-Raphson Jacobian Matrix:

Using the expressions of the power flow equations, derived above for various modes of formulation, we have in the rectangular mode

\[
\begin{bmatrix}
\partial (P) & \partial (Q) & (1) & (0) & (1) \\
-1 & -g & -1 & -g & -1 \\
\end{bmatrix}
\]

(2.61)
where

\[
J = \begin{bmatrix}
T \\
\frac{\partial h}{\partial x}
\end{bmatrix}
= \begin{bmatrix}
\begin{pmatrix}
T & T & T & 2T \\
-1 & -q & -1 & -q
\end{pmatrix} \\
\begin{pmatrix}
e & -1 \\
\frac{e}{-q} & f \\
\frac{-1}{f} & \frac{-q}{f}
\end{pmatrix}
\end{bmatrix}
\]

then

\[
J = \begin{bmatrix}
T & T & T & 2T \\
\begin{pmatrix}
ap & ap & aq & a(|v|) \\
-1 & -q & -1 & -q
\end{pmatrix} \\
\begin{pmatrix}
\frac{ae}{-q} & \frac{ae}{-q} & \frac{ae}{-q} & \frac{ae}{-q}
\end{pmatrix}
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
T & T & T & 2T \\
\begin{pmatrix}
ap & ap & aq & a(|v|) \\
-1 & -q & -1 & -q
\end{pmatrix} \\
\begin{pmatrix}
af & af & af & af \\
-1 & -1 & -1 & -1
\end{pmatrix}
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
T & T & T & 2T \\
\begin{pmatrix}
ap & ap & aq & a(|v|) \\
-1 & -q & -1 & -q
\end{pmatrix} \\
\begin{pmatrix}
af & af & af & af \\
-1 & -1 & -1 & -1
\end{pmatrix}
\end{bmatrix}
\]

The elements of the Jacobian matrix in equation (2.82) are as follows:
\[ \frac{1}{\delta e} = \frac{e}{i} \cdot \frac{G}{i} + \frac{f}{i} \cdot \frac{B}{i} \quad ; \quad j \neq l \]
\[ \frac{1}{\delta f} = \frac{e}{i} \cdot \frac{G}{i} + \frac{f}{i} \cdot \frac{B}{i} + c \quad ; \quad j \neq l \]
\[ \frac{1}{\delta f} = \frac{-e}{i} \cdot \frac{B}{i} + \frac{f}{i} \cdot \frac{G}{i} \quad ; \quad j \neq l \]
\[ \frac{1}{\delta f} = \frac{-e}{i} \cdot \frac{B}{i} + \frac{f}{i} \cdot \frac{G}{i} + d \quad ; \quad j \neq l \]
\[ \frac{1}{\delta e} = \frac{-e}{l} \cdot \frac{B}{l} + \frac{f}{l} \cdot \frac{G}{l} \quad ; \quad j \neq l \]
\[ \frac{1}{\delta e} = \frac{f}{l} \cdot \frac{G}{l} - \frac{e}{l} \cdot \frac{B}{l} - d \quad ; \quad j \neq l \]
\[ \frac{1}{\delta f} = \frac{-e}{l} \cdot \frac{G}{l} - \frac{f}{l} \cdot \frac{B}{l} \quad ; \quad j \neq l \]
\[ \frac{1}{\delta f} = \frac{-e}{l} \cdot \frac{G}{l} - \frac{f}{l} \cdot \frac{B}{l} + c \quad ; \quad j \neq l \]
\[
\begin{align*}
\frac{\partial v^2}{\partial q} &= 0 \quad ; \quad j \neq g \\
\frac{\partial v^2}{\partial \delta} &= 2e \\
\frac{\partial v^2}{\partial \delta} &= 2f \\
\end{align*}
\]

where both subscripts \(i\) and \(j\) can be either a load bus \(l\) or a generator bus \(q\). The definitions of the different terms in the above equations were given before in Table 2.1.

In the polar form of representation, equation (2.80) can be written as,

\[
\begin{bmatrix}
\frac{\partial (P \quad P \quad Q)}{\partial \delta} \\
\frac{\partial v}{\partial \delta} \\
\frac{\partial v}{\partial q} \\
\end{bmatrix} \begin{bmatrix}
T \\
-P \\
Q \\
\end{bmatrix} \begin{bmatrix}
\frac{\delta}{\delta - 1} \\
\frac{\delta}{\delta - q} \\
\frac{v}{v - 1} \\
\end{bmatrix} \begin{bmatrix}
\delta \\
-q \\
-1 \\
\end{bmatrix} = \begin{bmatrix}
\frac{\delta}{\delta - 1} \\
\frac{\delta}{\delta - q} \\
\frac{v}{v - 1} \\
\end{bmatrix} \begin{bmatrix}
\frac{\delta}{\delta - 1} \\
\frac{\delta}{\delta - q} \\
\frac{v}{v - 1} \\
\end{bmatrix} \begin{bmatrix}
\Delta P \\
\Delta Q \\
\Delta v \\
\end{bmatrix}
\]

(2.83)
where

\[
J = \begin{bmatrix}
\frac{T}{aT} \\
\frac{ah}{aT} \\
\frac{ax}{aT}
\end{bmatrix} = \begin{bmatrix}
\omega(P & P & Q) \\
-1 & -q & -1
\end{bmatrix}
\begin{bmatrix}
\omega \\
\delta \\
\delta \\
\delta
\end{bmatrix}
\begin{bmatrix}
-1 & -1 & -1 & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\omega P & \omega P & \omega Q \\
-1 & -q & -1 \\
\omega \delta & \omega \delta & \omega \delta \\
-1 & -1 & -1
\end{bmatrix}
\begin{bmatrix}
\omega P & \omega P & \omega Q \\
-1 & -q & -1 \\
\omega \delta & \omega \delta & \omega \delta \\
-1 & -1 & -1
\end{bmatrix}
\begin{bmatrix}
\omega P & \omega P & \omega Q \\
-1 & -q & -1 \\
\omega \delta & \omega \delta & \omega \delta \\
-1 & -1 & -1
\end{bmatrix}
\begin{bmatrix}
\omega \delta & \omega \delta & \omega \delta \\
\delta & \delta & \delta \\
\delta & \delta & \delta \\
-1 & -1 & -1
\end{bmatrix}
\]

The elements of the Jacobian matrix in equation (2.84) are,
\[ \frac{\partial P}{\partial \theta} = \sum_{i,j=1}^{n} \left[ \nu \cdot \nu \cdot \nu \cdot \nu \cdot \sin(-\theta + \delta - \delta) \right] \]

\[ \frac{\partial P}{\partial \delta} = -\sum_{i,j=1}^{n} \left[ \nu \cdot \nu \cdot \nu \cdot \nu \cdot \sin(-\theta + \delta - \delta) \right] \]

\[ \frac{\partial P}{\partial l} = \nu \cdot \nu \cdot \nu \cdot \nu \cdot \cos(-\theta + \delta - \delta) \]

\[ \frac{\partial P}{\partial \nu} = 2\nu \cdot \nu \cdot \nu \cdot \nu \cdot \cos(\theta) + \sum_{i,j=1}^{n} \left[ \nu \cdot \nu \cdot \nu \cdot \nu \cdot \cos(-\theta + \delta - \delta) \right] \]

\[ \frac{\partial Q}{\partial \theta} = \nu \cdot \nu \cdot \nu \cdot \nu \cdot \cos(-\theta + \delta - \delta) \]

\[ \frac{\partial Q}{\partial \delta} = \sum_{i,j=1}^{n} \left[ \nu \cdot \nu \cdot \nu \cdot \nu \cdot \cos(-\theta + \delta - \delta) \right] \]

\[ \frac{\partial Q}{\partial l} = \nu \cdot \nu \cdot \nu \cdot \nu \cdot \sin(-\theta + \delta - \delta) \]

\[ \frac{\partial Q}{\partial \nu} = 2\nu \cdot \nu \cdot \nu \cdot \nu \cdot \sin(-\theta) + \sum_{i,j=1}^{n} \left[ \nu \cdot \nu \cdot \nu \cdot \nu \cdot \sin(-\theta + \delta - \delta) \right] \]
where the subscript $j$ can be either a load bus $l$ or a generator bus $q$.

It should be noted here that the Jacobian matrix elements are functions in the voltage variables (unknown variables), hence these elements have to be updated every iteration. It should be mentioned also that the quadratic convergence of NR method is faster than that of any other load flow approach and the process reaches the solution very rapidly if the updated values of the variables are close to the solution.

**Decoupled Newton Load Flow Method:**

This technique is only applicable in the polar mode of formulation since it depends on the fact that the relationships between bus active powers $P$ and the bus swing angles $\delta$, and between the bus reactive powers $Q$ and the bus voltage magnitudes $|V|$, are stronger than that between $P$ and $|V|$, and between $Q$ and $\delta$. Accordingly, the technique ignores the elements:

$$
\begin{pmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} & \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \\
-1 & -\delta & -1 & -\delta
\end{pmatrix}
$$

in the NR Jacobian matrix.

Hence the resulting form of equation (2.84) becomes:
and accordingly equation (2.83) can be written, at iteration (K), as

\[
\begin{bmatrix}
H & 0 \\
0 & L
\end{bmatrix}
\begin{bmatrix}
\delta^{(K)} \\
\Delta V^{(K)}
\end{bmatrix}
= 
\begin{bmatrix}
\Delta P^{(K)} \\
\Delta Q^{(K)}
\end{bmatrix}
\]

where the subscript \( M \) denotes a subset of the bus indices, starting by the load buses followed by the generator buses.

The set of equations represented by (2.86) can be rewritten as two separate sets of equations as follows.
\[ H \Delta \delta_M = \Delta P \]
\[ L \Delta |V|_1 = \Delta Q \]

where

\[
\begin{pmatrix}
\Delta \delta \\
\Delta |V|_1 \\
\Delta \delta_q
\end{pmatrix} =
\begin{pmatrix}
\Delta \delta \\
\Delta \delta_q
\end{pmatrix},
\text{ and }

\[
\begin{pmatrix}
\Delta P \\
\Delta Q
\end{pmatrix} =
\begin{pmatrix}
\Delta P \\
\Delta Q
\end{pmatrix}
\]

Both \( \Delta P \) and \( \Delta Q \) are still functions in both bus voltage magnitudes \( |V| \) and bus swing angles \( \delta \), i.e., the evaluation of the bus active and reactive powers is still exact and the approximations are only applied for updating the voltage variables. Although this technique distorts the quadratic convergence characteristics of the NR method, it saves considerable computer memory and computational time per iteration.

**Fast Decoupled Load Flow Method:**
This method is basically a valid approximation to the NR decoupled load flow technique described above. Starting with the decoupled form, equations (2.87) and (2.88), where the \( i \)-th equation in (2.87) is,

\[
\mathbf{P}_i - \mathbf{P}_i = \Delta \mathbf{P}_i \quad ; \quad i = 1, 2, \ldots, (n-1) \tag{2.89}
\]

where

\[
\mathbf{P}_i = \sum_{j=1 \atop j \neq i}^{n-1} \left[ \mathbf{V}_i \cdot \mathbf{V}_j \cdot Y_{ij} \sin \left( -\theta_{ij} + \delta_i - \delta_j \right) \cdot \Delta \delta_{ij} \right], \quad \text{and}
\]

\[
\mathbf{P}_i = \sum_{j=1 \atop j \neq i}^{n} \left[ \mathbf{V}_i \cdot \mathbf{V}_j \cdot Y_{ij} \sin \left( -\theta_{ij} + \delta_i - \delta_j \right) \cdot \Delta \delta_{ij} \right]
\]

and the \( l \)-th equation in (2.88) is,

\[
\mathbf{Q}_l - \mathbf{Q}_l = \Delta \mathbf{Q}_l \quad ; \quad l = 1, 2, \ldots, n \tag{2.90}
\]

where

\[
\mathbf{Q}_l = \sum_{j=1 \atop j \neq l}^{n-1} \left[ \mathbf{V}_l \cdot Y_{lj} \sin \left( -\theta_{lj} + \delta_l - \delta_j \right) \cdot \Delta |\mathbf{V}_j| \right], \quad \text{and}
\]

\[
\mathbf{Q}_l = 2 \mathbf{V}_l \cdot Y_{ll} \sin(-\theta) + \sum_{j=1 \atop j \neq l}^{n} \left[ \mathbf{V}_l \cdot Y_{lj} \sin(-\theta + \delta_l - \delta_j) \right] \cdot \Delta |\mathbf{V}_j|, \quad \text{and}
\]

\[
\mathbf{Q}_l = 2 \mathbf{V}_l \cdot Y_{ll} \sin(-\theta) + \sum_{j=1 \atop j \neq l}^{n} \left[ |\mathbf{V}_l \cdot Y_{lj} \sin(-\theta + \delta_l - \delta_j)| \right] \cdot \Delta |\mathbf{V}_j|, \quad \text{and}
\]
where $\Delta V_j = 0$, in equation (2.90) for $j = g = (n+1),(n+2), \ldots$, $(n-1)$. Then the sum in the first term in equation (2.90) can be made for the load buses only, up to $n$, only instead of $(n-1)$. From equation (2.34) and equation (2.90), after multiplying and dividing it by $|V|$, and replacing the subscript $j$ by $1'$ for load buses, the following expression can be obtained:

$$QV' + QV = \Delta Q \sum_{l=1,2,\ldots,n} \frac{\Delta V_{1'}}{V_{1'}}$$

(2.91)

where

$$QV' = \sum_{l'=1}^{n} \left( |V| \cdot V_{1'} \cdot Y_{1l'} \cdot \sin (\theta_{1l'} + \delta - \delta_{1l'}) \cdot \frac{\Delta V_{1l'}}{V_{1l'}} \right),$$

and

$$QV = \left[ Q + |V|^2 \cdot Y_{1l} \cdot \sin (\theta_{1l}) \right] \cdot \frac{\Delta V_{1l}}{V_{1l}}$$

Noting that the expression for the reactive power for any bus $i$, is

$$Q_i = \sum_{j=1}^{n} \left[ |V| \cdot V_{ij} \cdot Y_{ij} \cdot \sin (\theta_{ij} + \delta - \delta_{ij}) \right]$$

$$= |V|^2 \cdot Y_{ii} \cdot \sin (-\theta_{ii}) \sum_{j=1}^{n} \left[ |V| \cdot V_{ij} \cdot Y_{ij} \cdot \sin (\theta_{ij} + \delta - \delta_{ij}) \right]$$

$$= |V|^2 \cdot Y_{ii} \cdot \sin (-\theta_{ii}) \sum_{j=1, j \neq i}^{n} \left[ |V| \cdot V_{ij} \cdot Y_{ij} \cdot \sin (\theta_{ij} + \delta - \delta_{ij}) \right]$$

(2.92)

then equation (2.39) can be written as,
\[ \mathbf{P}_J + \mathbf{P}_I = \Delta \mathbf{P}_i \quad ; \quad i=1,2\ldots,(n-1) \] (2.93)

where

\[
\mathbf{P}_J = \sum_{j=1}^{n-1} \left[ \left| \mathbf{V}_{ij} \cdot \mathbf{Y}_{ij} \right| \sin \left( -\theta^i + \delta - \delta^j \right) \Delta \delta \right] , \text{ and} \\
\mathbf{P}_I = (Q + \left| \mathbf{V}_{ii} \right|^2 \mathbf{Y}_{ii} \left| \sin \left( -\theta^i \right) \right|) \Delta \delta
\]

The following assumptions are then employed to equations (2.91) and (2.93), which are:

(i) in all "sin" arguments, \( \delta^i = 0 \); \( i=1,2\ldots,(n-1) \), and

(ii) \( |Q| \ll \left| \mathbf{V}_{ii} \right|^2 \mathbf{Y}_{ii} \equiv \left| \mathbf{V}_{ii} \right|^2 \mathbf{Y}_{ii} \sin \left( -\theta^i \right) \)

then equation (2.93) becomes,

\[ \mathbf{P}_J + \mathbf{P}_I = \Delta \mathbf{P}_i \quad ; \quad i=1,2\ldots,(n-1) \] (2.94)

where

\[
\mathbf{P}_J = \sum_{j=1}^{n-1} \left[ \left| \mathbf{V}_{ij} \cdot \mathbf{Y}_{ij} \right| \sin \left( -\theta^i \right) \Delta \delta \right] , \text{ and} \\
\mathbf{P}_I = \left( \left| \mathbf{V}_{ii} \right|^2 \mathbf{Y}_{ii} \right) \left( -\theta^i \right) \Delta \delta
\]

and equation (2.91) becomes,
\[ Q_L^t - Q_L = 0 \quad l = 1, 2, \ldots, n \quad \text{(2.35)} \]

where

\[ Q_L^t = \sum_{l^t=1}^{n} \left[ |V| \cdot V_{l^t} \cdot Y_{l^t, l} \cdot \sin (-\theta_{l^t}) \cdot \frac{\Delta V_{l^t}}{|V|_{l^t}} \right] \quad \text{and} \]

\[ Q_L = |V| \cdot Y_{1} \cdot \sin (-\theta_{1}) \cdot \frac{\Delta V_{1}}{|V|_{1}} \]

Since \( B_{ij} = |Y_{ij}| \cdot \sin (\theta_{ij}) \); then equation (2.94) and (2.95) can be rewritten in the form of equations (2.96) and (2.97) respectively.

\[ \Delta P = \sum_{i=1}^{n-1} \left[ -|V| \cdot V_{i} \cdot B_{ij} \cdot \Delta \delta_{ij} \right] \quad i = 1, 2, \ldots, (n-1) \quad \text{(2.96)} \]

\[ \Delta Q = \sum_{l^t=1}^{n} \left[ -|V| \cdot V_{l^t} \cdot B_{l^t} \cdot \frac{\Delta V_{l^t}}{|V|_{l^t}} \right] \quad l^t = 1, 2, \ldots, n \quad \text{(2.97)} \]

Due to the weak ties between bus active powers and bus voltage magnitudes, it can be assumed further, in equation (2.96), that

\[ |V| \cdot B_{ij} \approx B_{ij} \quad \text{leading to} \]

\[ j \quad l^t \quad 1^t \]
\[
\sum_{j=1}^{n-1} \left[ -B_{ij} \cdot \Delta \delta_{ij} \right] = \Delta P / |V_i| \quad ; \quad i = 1, 2, \ldots, (n-1) \tag{2.98}
\]

and

\[
\sum_{l'=1}^{n} \left[ -B_{l'l} \cdot \Delta |V_l| \right] = \Delta Q / |V_l| \quad ; \quad l = 1, 2, \ldots, n \tag{2.99}
\]

In matrix notation, equations (2.98) and (2.99) can be presented as

\[
\begin{bmatrix}
-B_{ij} \\
-M_{ij}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_{ij} \\
\Delta |V_i|
\end{bmatrix}
= \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} \tag{2.100}
\]

where \( \Delta P \) and \( \Delta Q \) are vectors of components \( (\Delta P / |V_i|) \) and \( (\Delta Q / |V_l|) \), respectively, and \( B \) is the imaginary part of

the network bus admittance matrix, after excluding the slack bus row and column, and

\[
\begin{bmatrix}
B_{ij} & -1l_i \\
-1l_j & -1q_i
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_{ij} \\
\Delta |V_i|
\end{bmatrix}
= \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} \tag{2.101}
\]
The solution of the decoupled sets of \((2n-2-n)_i\) linear
\(G\) equations \((2.100)\) and \((2.101)\), presents a very fast
computational scheme with constant LU factors of the sparse
matrices \(\mathbf{A}_{\text{MM}}\) and \(\mathbf{B}_{11}\). Although the convergence of the method
is not quadratic, Stott and Alsac in [79] showed that a very
accurate solution can be obtained for systems ranging from
13 to 205 buses in 4 to 11 iterations.

Optimization Methods:

Equation \((2.71)\), representing the load flow equations,
can be written in the form

\[
\mathbf{h}(\mathbf{x}) - \mathbf{u} = \mathbf{0} \quad (2.102)
\]

where any real mode of representation (rectangular or polar)
can be used. This set of equations includes \((2n^2-2)\) equations
in the rectangular form and \((2n-2-n)_G\) in the polar form. All
forms of \(\mathbf{h}\) have been given in the previous subsections.

The criterion of solving the load flow problem by
optimization is based on the fact that, if a solution \(\mathbf{x}'\) of
the unknown variables in the network exists, each of the
equations included in \((2.102)\) should be satisfied, i.e.

\[
\mathbf{h}(\mathbf{x}') - \mathbf{u}_i = \mathbf{0} \quad : \quad i = 1, 2, \ldots, n \quad (2.103)
\]

where \(n = 2n - 2\) in the rectangular mode of formulation, and
\(n = 2n-2-n_G\) in the polar mode of formulation.
The conditions presented by equations (2.103) are equivalent, for example, to

\[ \sum_{i=1}^{n} a \left( h(x') - u \right)_{i} ^2 = 0 ; a > 0 ; i=1,2,\ldots,n \]  

(2.104)

or

\[ \max (B \mid h(x') - u \mid) = 0 ; B > 0 ; i=1,2,\ldots,n \]  

(2.105)

since the left hand side of either (2.104) or (2.105) for any \( x \) is always positive or zero, and hence its minimum is zero if a solution exists.

The optimization technique used for solution depends on the form of equation (2.102). For example, equation (2.104) suggests the use of an unconstrained minimization technique which can be written as,

\[ \text{Minimize } \sum_{i=1}^{n} (a \left( h(x) - u \right)_{i} )^2 \]

(2.106)

w.r.t. \( x \)

On the other hand, the form of equation (2.105) suggests the use of a minimax optimization technique, which can be written as,

\[ \text{Minimize } \max (B \mid h(x) - u \mid) \]

w.r.t. \( x \)

w.r.t. \( i \)
It should be mentioned that most, if not all, the practical solution techniques available are gradient-type iterative methods, where the solution obtained depends on the starting point (initial estimate). Solving the load flow by optimization has a particular value when the power flow equations represent equality constraints within a more general formulation, in which other equality and inequality constraints are included while a certain objective function is optimized (e.g., minimizing the cost).

**Adjoint Network Method:**

The adjoint network method is conventionally used to calculate power network sensitivities. The recent development of this method allowed its application to the exact power network model. Consequently, the methodology for load flow solution has been developed on the basis of the adjoint network concept (86).

The load flow equations have been written earlier, in the real mode of formulation, as

\[ h(x) = 0 \quad (2.106) \]

where \( h \) is a vector of nonlinear functions in the voltage variables \( x \), and \( U \) is a vector of the bus specified quantities in the network. The perturbed form of equation (2.106), at iteration \( (k) \), was also presented earlier, as
\[
\begin{equation}
\begin{bmatrix}
J^{(K)} \\
\Delta x^{(K)} \\
\end{bmatrix}
= \begin{bmatrix}
\Delta U^{(K)} \\
\end{bmatrix}
\tag{2.107}
\end{equation}
\]

where
\[
\begin{align*}
\Delta x^{(K)} &= x^{(K+1)} - x^{(K)}\\
\end{align*}
\]

and
\[
\begin{align*}
\Delta U^{(K)} &= U^{(K)} - U^{(K)} \\
&= \text{specified} \\
\end{align*}
\]

In the Newton-Raphson method, the perturbed equations are solved at each iteration for \( \Delta x \) where \( J \) is updated in each iteration. The computational effort per iteration in this process consists of:

(i) Evaluating the elements of \( J^{(K)} \) since they are voltage dependent.
(ii) Calculating the LU factors of the Jacobian matrix \( J^{(K)} \).
(iii) Performing forward and backward substitutions.

The solution of each iteration for equation (2.107) is given by,
\[
\begin{equation}
\begin{bmatrix}
\Delta x^{(K)} \\
\end{bmatrix}
= \begin{bmatrix}
J^{(K)} \\
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta U^{(K)} \\
\end{bmatrix}
\tag{2.108}
\end{equation}
\]

where the elements of \( [J^{(K)}]^{-1} \) represent the sensitivities of the voltage variables \( x \) with respect to the specified quantities \( U \) at iteration \( (K) \), i.e.
\[
\left[ \begin{array}{c}
J^{(K)} \\
-1
\end{array} \right]^{-1} = \left[ \begin{array}{c}
\frac{\partial x}{\partial U} \\
\frac{\partial u}{\partial U}
\end{array} \right]^{T} (K)
\]

In the adjoint network method, the sensitivities of the i-th network state (unknown) variable at iteration (K), \( x \) with respect to all the network controls \( U \) (i.e., i-th row in equation (2.109)) are calculated by solving a separate set of equations (adjoint equations) of the form

\[
\left( \begin{array}{c}
(\hat{x})^{(K)} \\
\hat{\theta}
\end{array} \right) = \left( \begin{array}{c}
(\hat{x})^{(K)} \\
\hat{\theta}
\end{array} \right) \quad (2.110)
\]

where \( i \) denotes different elements of \( x \), and \( b \) is a simple vector having at most two non-zero elements and it depends on the state \( x \) considered. \( T \) is an adjoint network matrix of coefficients and it is common to all states \( x \). The set of equations (2.110) presents \( n \) equations in general, solved in the adjoint network variables \( \hat{x}^{(K)} \). The required sensitivities of \( x \) with respect to \( U \) are then calculated as linear functions of the corresponding elements of \( \hat{x}^{(K)} \) [56]. The computational effort per iteration in the adjoint network method can be summarized in the following:
(i) Evaluating the only few voltage-dependent elements of \( T^{(K)} \).

(ii) Calculating the LU factors of \( T^{(K)} \).

(iii) Performing forward and backward substitutions for each state considered.

Comparing the computational effort involved in both Newton-Raphson method and the adjoint network method, we find that more effort is involved with NR method than that in the adjoint network method in evaluating the Jacobian matrix for the former and the adjoint network matrix in the latter. On the contrary there is less effort involved in performing the forward and backward substitutions in NR than in the adjoint network method. With respect to the calculation of the LU factors, both methods have almost the same computational effort involved. Finally, it should be noticed that, since the elements of \( \left| J^{(K)} \right|^{-1} \) are calculated in the adjoint network method, based on exact load flow model, the method then has the same rate of convergence as that of the NR method.

After discussing the different techniques available for load flow solution, it should be clear that the choice of a certain technique is mainly based on the network under consideration, the computer facilities available, the accuracy required in the solution, and the objective of performing the solution.
2.3 Reliability and Contingency Analysis

The different power system planning techniques are usually concerned with providing different expansion plans which satisfy the system operational and security constraints under normal operating conditions. One might think that the planning process is completed by selecting the plan which has the best performance and minimum cost. Unfortunately this is not the case, because of the need for testing the network performance under abnormal conditions, since no system ever operates free of disturbances. Faults, lightning strikes, and switching surges are some of the reasons that cause the system to suffer from abnormal conditions, such as loss of system elements. Disturbances are classified based on their time duration into three categories: subcycle, cycle (transient), and steady-state. In this section, due to the scope of this work, we will only discuss briefly the steady state disturbances and the techniques used to determine whether the system is reliable to operate satisfactorily after a disturbance has reached a steady state. Satisfactory or acceptable, as it has been defined in Reference [47] and others, means that no system component is overloaded and that all demands are met at acceptable voltages.

At the beginning, the performance of the power system under contingency has been studied by employing the efficient and effective ac power flow methods. With the growth of the
power networks, the exact ac power flow methods have lost their attractiveness due to the very large computational time involved in studying the large number of outages in the network. Accordingly, faster and approximate techniques have been used instead. Even though there was still too much computational effort and time involved in performing all the required studies. Rather, different fast methods have been suggested to locate potential trouble spots and subsequently only the severe ones could be studied in details, using the ac load flow methods if necessary.

The methods proposed for studying contingency analysis are classified in two major categories, namely deterministic and probabilistic techniques. Those methods which simulate the network element outages by actual removal from the network and do not model the outage probabilities of the elements, are classified to be deterministic methods [47, 91-95].

Instead of exhaustive study of all the possible outages in a network, different performance indices have been proposed in the literature [97-102], to rank the different contingency cases and only those which are considered severe are selected to be studied, by the deterministic methods in details.

In recent years, different probabilistic techniques have been developed to evaluate the relative role, each element in a network plays in determining the reliability of that
system (47, 103-107). These informations can usually be obtained through two indices, namely the loss of load probability (LOLP), and the expected value of demand not served (EDNS).

![Diagram of power system with bus g, generation units P_g1 and P_g2, load at bus l, branches k and m removed.]

**Generation unit loss at bus g**

**Partial generation loss at bus g**

**Load loss at bus l**

**Branch removal across km**

**Figure 2.8:** Different types of single contingencies.

Due to the common use of the deterministic approaches in contingency evaluation, a brief discussion of the basic principles of such techniques is given below.

2.3.1 **Deterministic Concept of Contingency Analysis**

In general, an outage of one of the power network components is known as a single contingency, while a group of single contingencies, occurs at the same time, is referred to as multiple contingency.

In the steady state on-line (or off-line) operation (or planning) of power systems, different types of outages are considered, such as those shown in Fig. 2.8.
Power system contingencies are usually simulated by assigning the appropriate variations (changes) of the corresponding control variables, of the elements involved in the outages, to the power flow model used in the contingency evaluation. Each change \( \Delta U \) in the control variable \( U \) is equal to the outage capacity of the associated component. The effect of single or multiple contingencies on the power network performance is usually estimated, in most approaches, by considering one or more performance indices \( f \) (e.g., overloading, bus voltage level, etc.) and then predict the changes in them based on first order approximations, in most cases. In other approaches efficient algorithms are used to evaluate the exact large change in the network performance indices. This routine of analysis is known as contingency evaluation. The relative severity of power network contingencies is realized via a routine of analysis called contingency ranking.

2.3.1.1 First-Order Approximations of Network Contingencies

Since only changes in control variables \( U \) are considered, the reduced gradient evaluation plays an important role in contingency analysis based on first-order changes. The first-order change of a performance index \( f \); where \( i \) is an element of a set of performance indices \( f \), is given by,
\[ \nabla f = (\frac{\partial f}{\partial x}) \hat{x} + (\frac{\partial f}{\partial u}) \hat{u} \quad ; \quad i \in I \]

\[ \nabla f = (\frac{\partial f}{\partial u}) \hat{u} \quad ; \quad i \in I \quad (2.111) \]

If the contingency set \( K \) is defined as the set of control variables associated with the outages under consideration, e.g. \( K = \{3, 5, 12, \ldots \} \), then equation (2.111) can be rewritten as,

\[ \nabla f = \sum_{k \in K} a_k (\frac{\partial f}{\partial u}) \hat{u} \quad ; \quad i \in I \quad (2.112) \]

where \( a_k \) is set to either zero or one, depending on whether single or multiple contingencies are considered.

The reduced gradients \( (\frac{\partial f}{\partial u}) \) can be evaluated by different techniques such as the sensitivity matrix method, the method of Lagrange multipliers, or the Tellegen's theorem method. The accuracy of the calculated first-order changes is mainly affected by the accuracy of the load flow model used. Hence the more accurate the model used, the more accurate the estimated changes in a performance index is.
In some formulations, specifically those related to ranking power system contingencies, an overall performance index $F$ for the power network may be defined as a function of all performance indices $f_i$, i.e., $F = F(f_i)$, to assess in a general sense, the overall performance of the network. Accordingly, the first-order change of the overall performance index $F$ is given by,

$$
\delta F = \sum_{i \in I} \left( \frac{\partial F}{\partial f_i} \right) \delta f_i \quad (2.113)
$$

and by substituting $\delta f$ from equation (2.112) into equation (2.113), we have

$$
\delta F = \sum_{i \in I} \left( \frac{\partial F}{\partial f_i} \right) \left( \sum_{k \in K} \left[ a_{ik} \frac{df}{dU} \frac{\partial U}{\partial U} \right] \right) \quad (2.114)
$$

or

$$
\delta F = \sum_{i \in I} \sum_{k \in K} \left[ a_{ik} \frac{df}{dU} \frac{\partial U}{\partial U} \right] \quad (2.115)
$$

If all the reduced gradients of the individual performance indices ($df/dU$) are already available, then equation (2.114) can be used, otherwise equation (2.115) can be used where only one forward and backward substitution, in either Lagrange multipliers method or the Tellegen's theorem method can be applied to obtain the reduced gradients of the overall performance index ($dF/dU$) with respect to all outage control variables.
2.3.1.2 Simulation of an Element Loss

When a forced outage of a generator occurs, $\Delta U_k$ denotes the corresponding loss of generation capacity (negative). In practice, usually several generating units are presented at each generator bus, and so the contingency associated with a generator bus $k$ can be substituted by,

$$\Delta U = \sum_{j \in J} \Delta U_{kj}$$

(2.116)

where $j$ denotes the generating unit number which is an element of the set of outages units $J$ at bus $k$.

Similarly, the loss of a transmission line can be simulated, with the exception that changes in three control variables, associated with series and shunt (one at each end of the line) admittances, are assigned simultaneously for each line outage. When several circuits are carried on the same right-of-way $k$, the outage of one circuit $j$ can be simulated by an expression similar to that of equation (2.116).

Simulation of different types of outages are given in more details in [102].
2.3.1.3 Examples of Performance Indices

The performance indices generally denote overloading and/or voltage criteria. The most common types of indices are:

1. line indices, which may be represented by

\[
\begin{align*}
2.117
f &= \frac{I_i}{I_t} (2.117) \\
0 &= \frac{2}{I_t} \\
2 &= \frac{0}{I_t}
\end{align*}
\]

where \( I \) is the rated current of transmission line \( t \).

2. bus voltage indices, which can be introduced in terms of bus voltage magnitude as

\[
\begin{align*}
2.118
f &= \frac{|V_i|}{|V_m|} (2.118) \\
0 &= \frac{|V_i|}{|V_m|} \\
2 &= \frac{|V_i|}{|V_m|}
\end{align*}
\]

or it can be presented in terms of bus swing angle as,

\[
\begin{align*}
2.119
f &= \frac{|\delta_i - \delta_m|}{\delta_m} (2.119) \\
0 &= \frac{|\delta_i - \delta_m|}{\delta_m} \\
2 &= \frac{|\delta_i - \delta_m|}{\delta_m}
\end{align*}
\]

where \( |V_i| \) and \( \delta \) are the nominal bus voltage magnitude and swing angle, respectively at bus \( m \).

3. reactive generation indices, as for example

\[
\begin{align*}
2.120
f &= \frac{|Q_i - Q_g|}{Q_g} (2.120) \\
0 &= \frac{|Q_i - Q_g|}{Q_g} \\
2 &= \frac{|Q_i - Q_g|}{Q_g}
\end{align*}
\]

where \( Q \) is the rated reactive power at generator bus \( g \) and \( Q \).
4. Line voltage indices, which can be presented in terms of the swing angle across the line, as a measure for stability, in the form,

\[ f = \frac{|\delta|}{\delta_0} \]

where \( \delta_0 \) is the nominal voltage phase angle of line \( t \).

On the other hand, the overall system performance index \( F \) is generally formulated as a function of the individual performance indices \( f_i \), e.g.,

\[
( i ) \quad F = \sum_{i \in I_f} w_i (f_i)^2
\]

or

\[
( ii ) \quad F = \max_{i \in I_f} \{ w_i f_i \}
\]

where \( w_i \) are weighting factors accounting for the importance of each individual index.

2.3.1.4 Ranking of Power Network Contingencies

This phase of contingency analysis involves the evaluation of the effects of all possible contingencies on an overall system performance index \( F \), and then listing these effects in a proper order according to the severity of each particular contingency. The change in the index \( F \), \( \delta F \), is estimated using equations (2.114) or (2.115), where \( f_i \) can be any of the forms presented by equations (2.117) to (2.121).
2.4 Stability Analysis

The power system forms a group of interconnected electromechanical elements. The stability of such system can be defined by its capability to return to its normal or stable operation after being subjected to a disturbance. Stability is one of the major requirements in power systems. The electric power system may become unstable either by the loss of synchronism between the system synchronous machines or by the stalling of the asynchronous loads. The former can be divided into transient and steady state instabilities. The transient stability of power system is related to sudden and large changes in the network conditions, while the steady state stability is basically concerned with small disturbances in the system. On the other hand, the asynchronous load stability can be achieved by controlling the voltage across the load.

2.4.1 Stability of a Asynchronous Load

![Diagram of a single line diagram of a line supplying a load](image)

Figure 2.9: Single line diagram of a line supplying a load \( P + jQ \).
An induction motor may become unstable if the voltage across it becomes lower than a certain level. Hence voltage stability is essential for system load stability. To demonstrate the effect of voltage stability on load stability, let us consider the example given in [1], for a conductor line supplying a load as shown in Fig. 2.9.

The relation between the load voltage $V$ and the received power $P$ at different constant power factors is given in Fig. 2.10. The seasonal thermal ratings of the line are shown by broken lines in Fig. 2.10.

![Figure 2.10: Example of load voltage-received power relations at different constant power factors.](image)

It is clear from Fig. 2.10 that there are two load voltage values at each specific received power value. It is also clear that, at low lagging power factors, that the load power might be on that part of the characteristic where small power changes could lead to large voltage changes and
accordingly voltage instability would occur. The voltage then has to be controlled by tap-changing transformers at the receiving end. Since the power factor of transmission plays an important role in voltage stability, it is necessary to maintain the power factor, specially for long lines with heavy amount of power transfers, to a value approaching unity. This can be achieved by reactive power injection at the load buses. For more details, Reference [1] can be consulted.

2.4.2 Steady-State Synchronous Stability

![Diagram](image)

Figure 2.11: Synchronous machine connected to an infinite busbar. (a) Schematic diagram of the connection, (b) corresponding approximate phasor diagram (neglecting the generator resistance).

The simplified concept of steady state synchronous stability can be explained by studying the operation of a generator as if it is connected to an infinite busbar (bus
having constant voltage and frequency), or more accurately by studying the motion equation of the rotating machine.

2.4.2.1 The Concept of a Synchronous Machine Connected to an Infinite Busbar

This concept is very simplified by the assumption that the machine is connected to an infinite busbar, as well as by neglecting the generator resistance and saliency. A schematic diagram of the generator connection to the infinite busbar along with the corresponding phasor diagram are shown in Fig. 2.11.

![Figure 2.12: Approximate synchronous machine P-δ curve.](image)

Since the machine resistance and accordingly the electric losses are neglected, the output power from the turbine is equal to the output power from the generator. Hence changing the turbine output and consequently δ, the generator can
supply any desired load. The angle $\delta$ is known as the load angle and it depends on the input power to the turbine.

The per phase output power from the generator (delivered to the infinite busbar) is,

$$ P = V \cdot I \cdot \cos(\phi) $$

(2.122)

but from Fig. 2.11(b),

$$ E \cdot \sin(\delta) = I \cdot X \cdot \cos(\phi) $$

(2.123)

Substituting the value of $I \cdot \cos(\phi)$ from equation (2.123) into equation (2.122), then

$$ P = \frac{(V \cdot E / X ) \cdot \sin(\delta)}{s} $$

(2.124)

For constant $V$, $E$, and $X$ equation (2.124) is plotted as in Fig. 2.12.

As it can be seen from Fig. 2.12, and from equation (2.124) that, the load angle limit of stability is $(\pi/2)$ radians, since any increase in $\delta$ over this limit to produce more output power than $P_{\text{max}}$ will, on the contrary, lead to a reduction in the output power. Subsequently this will lead to increase in $\delta$ and the machine becomes eventually unstable and loses synchronism.

2.4.2.2 Motion Principles of a Rotating Machine

The equation of motion of a rotating machine has been derived in details in Reference [1], and for a constant mechanical input power, it is,

$$ M \cdot (d^2 \delta / dt^2) + \Delta P = 0 $$

(2.125)

where $M$ is the machine rotor angular momentum, $\delta$ the load
angle, and $\Delta P$ the change in the electrical output power of
the machine. The solution of the generalized form of
equation (2.125) is,

$$\delta = a_1 e^{b_1 t} + a_2 e^{b_2 t} + \ldots + a_n e^{b_n t} \quad (2.126)$$

where the coefficients $a$ are the integration constants,
while $b$ are the roots of the characteristic equation.
Based on equation (2.126), all the roots of the
characteristic equation should have real parts for stability
satisfaction. Different characteristic equations are derived
in [1], for different system operating conditions, showing
the effect of damping and automatic voltage control on the
stability limit. It has been found that the stability limit
is again obtained at $\delta = \pi/2$ in the absence of automatic
voltage control. Effect of governor operation is also
discussed in [1]. Practically, the normal operating load
angle for modern machines is in the range of 60 electrical
degrees, and since the safe stability limit is at 90
electrical degrees, this leaves 30 electrical degrees to
cover the transmission network.
Chapter III

COMPUTER-AIDED DESIGN OF LONG-RANGE DISTRIBUTION PLANNING

3.1 Introduction

The main objective of this work is to overcome some of the limitations which exist in the transmission and distribution planning models. To demonstrate the necessity for developing a new model for distribution planning, a summary of the limitations of the previous models is given in the next section. The objectives to be met in the new model, for overcoming the limitations are also given. The detailed construction of the model is given, with an application to a practical system. A comparison between the results obtained from the proposed model and from Ontario Hydro is also given to demonstrate the capability, the efficiency, and the accuracy of the new model.

3.2 Limitations of The Previously Reported Models for Distribution Planning

The major limitations of the existing long range distribution planning models are summarized below.

1. Due to the complexity of the problem various approximations were necessary in the planning models.
Some of the models relaxed the right-of-way constraints, while others ignored the stability requirements of the network. In some cases, both the right-of-way constraints and the stability constraints were ignored. Some models applied the concept of sampling constraints as a compromise between ignoring the constraints completely (low accuracy) and accounting for some of them (higher accuracy).

2. Many of the available techniques produce impractical expansion plans due to neglecting the important voltage regulation constraints in the network.

3. The assumptions of linearity have been used in almost all the previous models, with different degrees of implementation. Such linear restrictions prevented the inclusion of the nonlinear constraints in the network accurately, resulting in producing unfeasible solutions. On the other hand, the linearity assumptions have also forced the cost functions of the different elements of the network to be simulated approximately, leading to non-optimal plans.

4. Due to the integer form of the decisions to be taken during the planning process, and in order to improve the accuracy of the linear programming models, the research has been directed towards the use of mixed integer linear programming techniques. Although these techniques have improved the accuracy of the results
obtained, they have limited the ability of the models to solve large or even medium distribution systems. Accordingly, different approximations had to be applied to solve the planning problem of large systems. This means that the integer linear programming methods have limited effectiveness in long range planning.

5. The desire for improving the efficiency of the planning techniques and increasing their capabilities of handling large distribution networks more accurately has led to the implementation of different branch-and-bound criteria. The function of these criteria is to fathom a large number of alternatives before exhaustive study of them. In a sense, these approaches have improved the planning techniques, but there is always the question of how sensitive the solution obtained from such model to the branching and bounding criteria used.

6. Another limitation in the existing distribution planning techniques is the inclusion of the time factor. Most of the available techniques solve the planning problem in the static (one time step) mode of planning. Over a long-period of planning, this mode might not lead to an optimal or even to a feasible solution. The optimal installation time and operation schedule of the different facilities in the system
along with their accumulative costs have to be formulated in an accurate dynamic form.

7. Most of the models which have accounted for the load growth with time in the planning process, have used either the horizon year concept or the dynamic programming technique. The former may not produce the optimal solution, while the latter has a very large computational burden and it is usually associated with linear models thus limiting its efficiency and accuracy.

8. The feature of expanding the capacities of some of the existing facilities, has not been included in any of the existing models.

The above are the major limitations of the available techniques for long range distribution planning. It can be stated that the available models have limited accuracy and poor efficiency due to the different approximations and assumptions involved, as well as the large computational effort and large number of variables and constraints, which limit their capabilities.

3.3 Objectives to be Achieved in Long Range Distribution Planning

Based on the above limitations in the distribution planning, the need for a model capable of overcoming these drawbacks is essential. The following are the objectives
sought in developing a model to enhance the efficiency and the accuracy of the distribution planning.

1. A nonlinear model in the planning variables must be capable of simulating accurately the different cost functions of the different components in the distribution system, as well as the network security and operational constraints. Achieving this goal leads to overcoming mainly the first three limitations discussed in the previous section. The decisions can also be formulated in a nonlinear form in terms of the original network parameters (e.g., power flow). This eliminates the need for large number of integer variables, which limits the applicability of the models to small systems only. This form of presenting the decisions in the formulation would also make the use of the different branch-and-bound criteria unnecessary. This will eliminate the above fourth and fifth limitations.

2. The time factor has to be included explicitly and accurately in the new model to allow the accumulated overall cost of the network to be accurately calculated. This will lead also to the determination of the optimal installation time of the new facilities. In this sense, the model has to be flexible with the implementation of both the static (one-step) and the dynamic (multi-steps) modes
of long range planning. Such a model will also allow the investigation of the accuracy of each of the two time modes of planning.

3. The option of possible expansion in some or all capacities of the network existing facilities, wherever and whenever it is possible, has to be also included in the model. This new feature could produce more practical and valuable plans than those obtained under restricted element capacities.

3.4 Model Description

Based on the above discussion, the objective of the distribution system planning is to design systems that can efficiently, economically, and reliably satisfy the load demands, which may grow in time. The sections of the distribution system being planned include the substations and the primary feeder circuits. Expanding from an existing system (if any), new substation sites and feeder routes may be added at certain times, to meet the changing load requirements.

3.4.1 Distribution Planning Variables

The first step in the planning process is to estimate the future load demand in the system. Knowing the spatial load variations in the area under consideration, the distribution planner has to consider the following different variables.
1. The existing system facilities (substations and feeders) have to be considered in order to take advantage, as much as possible, of their capacities. The existing facilities can be useful either by providing additional power capacities without the need for additional capital investment, or by discarding them, if they are not needed in the plan, and using their sale price, if any, as a saving in the capital cost.

2. All the possible sites for new substations and routes for potential right-of-ways should be included in the planning, in order to obtain the most economical and acceptable design. The word "possible" refers to the availability of such sites and routes and satisfying the environmental and social requirements.

3. The possible expansion in the capacity of the existing facilities has to be accounted for, since it might be cheaper and render better performance to expand some of the existing facilities rather than construct new ones.

4. The installation and operation times must be considered explicitly in the planning model to allow for the dynamic formulation. Consequently, accurate timing of the installation decisions and the operation schedule can be made.
5. The voltages at the different network nodes have to be considered in the planning model, since the voltage regulation is important for the system stability.

It seems that there are much details to be included in the planning model in order to have efficient and accurate results.

3.4.2 Objective Function

In this work an accurate and continuous cost model with a dependence on the time and the power flow in each element of the network is presented. This model simulates the actual relation between the planning variables, the time of installation and the power flow in the different elements, and their fixed and variable costs in addition to the cost of the energy losses. The model is formulated in a flexible format which makes it suitable for long range static and dynamic planning.

The overall present worth cost function of the network includes the variable costs of all the feeders and the substations, the capitalized costs of the potential feeders and substations. The maintenance and the operation costs are included in the variable costs. In addition the capitalized cost of the possible expansions permitted in the feeders and the substations are also taken into account. The cost of the energy losses is included in the accepted form of a nonlinear relationship with the planning optimization variables. The objective function $C$, which is the total
present worth cost of the distribution system under consideration is:

\[ C = \sum_{s \in NSEP} P_s + \sum_{i \in NFEP} F_i + \sum_{j=1}^{(NS+NP)} (f_j + L_j) \]  

(3.1)

where \( NSEP \) is the subset of all potential and expandable substations, \( NFEP \) the subset of all potential and expandable feeders, \( NS \) the total number of existing and potential substations forming a complete set, and \( NP \) the total number of existing and potential feeders forming a complete set. \( P_s \) is the present worth value of the capitalized cost for potential, or expansion permitted for, substations \( s \) which is an element of the subset \( NSEP \), \( F_i \) the present worth value of the capitalized cost for potential, or expansion permitted for, feeder \( i \), \( f_j \) the present worth value of variable costs for element \( j \) (feeder or substation) without the cost of its energy losses, and \( L_j \) the present worth value of the cost of energy losses in element \( j \).

Each component in equation (3.1) is a continuous nonlinear function in the power flow of its element. This allows more accuracy and flexibility in presenting the different cost components, as will be demonstrated in the following subsections.
3.4.2.1 Capitalized Substations and Feeders Costs

The capitalized costs, presenting cost of land, equipment, installation, etc., are accounted for in the cost either in the case of substations and/or feeders to be constructed at a future date, or in the case of expanding the power capacity of some existing feeders and/or substations. Since these costs are dependent on the amount of power capacity additions to the network, a certain size and type for each of the future facilities, as well as a certain size for the expansion allowed for different existing elements, are assumed. Since the capitalized costs vary with the changes in economy, so these costs have to be formulated as functions of time, i.e. in a dynamic form. Usually the capital costs of the new facilities at the time of planning are known, but again since they are likely to change by the time of installation, statistical data for the inflation rate has to be employed to these costs. The capital cost of the new facilities can then be formulated as a function of time.

If $P_0$ is the capitalized cost of the new element $i$ at the time of planning, $r$ the inflation rate of the capital cost, and $t$ the installation time of facility $i$, the cost of this new facility at the installation time will be $\{108\}$.
future \[ F_i^t = P_0 \cdot (1+r_i^f)^t \]  
Assuming an interest rate \( r \), the present worth value \( P_i^t \) of future the capital cost \( F_i^t \) can then be formulated as in equation (3.3).

\[ P_i^t = F_i^t \cdot (1+r_i^f)^{-t} \]

This formulation makes it possible to determine the optimal installation time of the new facilities since these times are considered as optimization variables in the model.

To ensure that these costs will only be included in the overall cost if these facilities are needed in service, a decision continuous function is introduced in terms of the power flow in each of the new facilities. Hence

\[ J_{i,t} = 1 - \exp(-K(\frac{|P_{i,t}|}{U})) \]  
where \( J_{i,t} \) is the decision function of facility \( i \) at time \( t \), \( |P_{i,t}| \) the absolute value of the power flow in element \( i \) at time \( t \), \( U \) the power capacity limit of element \( i \), \( K \) a large positive weighting factor, and \( \exp \) the logarithmic exponent function.

The formulation in equation (3.4) allows the capital cost of the new facility to be ignored if the power flow through this element is small enough to be considered as zero.
In the time-phased planning, once the decision is taken to add one of the new facilities at certain time period, its capital cost should be included then and only once. To ensure that the capital cost of any new element used at certain time period will not be considered if this element has been used in any of the previous time periods, another function \( F_{i,t} \) for facility \( i \) at time segment \( t \), is included in formulating the capital cost of this element. This function is presented in the form of equation (3.5) as multiplied functions, each corresponding to the power flow in the facility \( i \) in one of the time segments prior to time segment \( t \). Each of these individual functions is responsible for setting the capital cost of one of the new facilities, e.g. \( i \) at time segment \( t \), to zero if the facility has been used previously.

\[
F_{i,t} = \prod_{j=1}^{t-1} \left( \exp(-k(1/P_{i,j})/U) \right) \]  

(3.5)

Based on the above discussion, the terms \( F^S \) and \( F^s \) in equation (3.1) are represented in the cost model in the following forms:

\[
F^S = \sum_{s} \sum_{e=I+1}^{e} \left( \frac{e}{nT} \right)^{t-1} \prod_{j=1}^{c-1} \left[ \exp \left[ -k \cdot \frac{s}{P_{j,e}} \right] \right] \left[ \exp \left[ -k \cdot \frac{s}{U} \right] \right] \]  

(3.6)
\[
F = \sum_{i}^{\text{FEXP}} \sum_{e}^{\text{NT}} \left( \beta_{e} \left[ 1 - \exp \left[ \frac{-K \cdot P_{i}^{t}}{I/U} \right] \right] \prod_{j=1}^{\tau-1} \exp \left[ \frac{-K \cdot P_{j}^{t}}{I/U} \right] \right)
\]

(3.7)

where \( S \) is the total power segments permitted for potential or expandable substation \( s \), \( S \) the total power segments permitted for potential or expandable feeder \( i \), \( I \) an integer constant for each new substation facility; \( I = 0 \) if element \( s \) is potential, and \( I = 1 \) if the element \( s \) is expandable; \( I \) similar to \( I \) but for feeder \( i \), \( NT \) the total number of time-phased planning segments, and \( NT = 1 \) for long range static planning, \( F \) the capitalized cost of the new power segment \( e \) of supply \( s \) at time of planning, \( F \) similar to \( F \) but for feeder \( i \), \( \beta_{e} = \left( \frac{1 + r(\tau)}{1 + r(\tau)} \right) \); where \( r(\tau) \) and \( r(\tau) \) are the inflation and the interest rates at the planning time segment \( \tau \), respectively, \( K \) weighting factor \( >> 0 \); to obtain ramp decision function, \( U \) the power capacity limit of power segment \( e \) of supply \( s \), \( U \) similar to \( U \) but for feeder \( i \), \( t \) real variable presents the installation time of the new power segment \( e \) of supply \( s \) facility \( s \), \( t \) similar to \( t \) but for feeder facility \( i \), \( P \) positive real variable presents the power flow in power.
segment $e$ of supply $s$ at planning time segment $\tau$, and $|P_{e}^{1}{\tau}^{s}|$, the absolute value of the real variable $P_{e}^{1}{\tau}^{s}$ presents the power flow in the power segment $e$ of feeder $i$ at planning time segment $\tau$.

Because of the limitations on computing the exponential functions, some constraints have been used to avoid the very small values of the variables $P_{e}^{1}{\tau}^{s}$ and $P_{e}^{1}{\tau}^{s}$, and then those values have been rounded up or down at the solution.

3.4.2.2 Variable Substations and Feeders Costs

The variable cost of any element in the network presents, operational cost, fuel cost, maintenance cost, etc. The cost of substations and feeders, excluding the cost of their energy losses, is linearly related to the power flow in each of these elements. This can be represented in a simple continuous function, which depends on the power flow in the element, as well as on the period of time in which the element is operated. While the capital costs are only paid once, at the installation time, the variable cost of an element $j$ (substation or feeder) is introduced as the sum of equal payments paid during all the years of the element service. The equivalent present worth value of those payments is formulated as a function of time $[103]$. As in the capital cost, the value of the variable cost at the planning time is considered and the corresponding value at the in-service time is then calculated on the basis of the
inflation rate. Accordingly, the term $f_j$ in equation (3.1) can be formulated as follows:

$$f_j = \sum_{\tau=1}^{NT} \left( \beta_{\tau} \right)^{t_{\tau}-1} \left[ 1 - (a_{\tau})^{-\tau} \right]^{\frac{1}{t}} \left[ \sum_{e=1}^{ESP_j} l_p^{j} \right]$$  \hspace{1cm} (3.8)

where $ESP_j$ is the total power segments of element $j$, $T_{\tau}$ the length of time segment $\tau$ in years, and $t_{\tau}$ the sum of time segment lengths to the end of segment $\tau$, in years, where $t_0$ is equal to zero (starting point of planning). $E$ is the variable cost of element $j$ at the time of planning, and $a_{\tau} = (1 + r(\tau))$.

3.4.2.3 Cost of Energy Losses

Since the cost of the energy losses is related to the power flow in an element in a nonlinear relationship, it is considered separately in formulating the cost function. Although a second order relation has been used in this model, it can be presented by any other differentiable nonlinear form. Similar to both capital and variable costs, the cost of the energy losses at the planning time is used, and then its predicted value at the in-service time for each element is formulated as a function of time. Finally the present worth value of the energy losses cost is formulated.
Hence the term $L$ in equation (3.1) is presented as follows:

$$L = \sum_{j=1}^{NT} \left[ \left( \frac{t \tau^{-1} (1-a_{\tau})^{-T_{\tau}}}{r(\tau)} \right) \left( \sum_{e=1}^{\text{ESP}} \left( \frac{p_{j}}{\tau_{e}} \right)^{2} \right) \right]$$

(3.9)

where $L$ is the present worth value of the cost of the energy losses in element $j$, and $C$ the cost of the energy losses in element $j$ at the time of planning.

After formulating the different cost components, the overall present cost is minimized subject to constraints representing the conservation of the power flow, the capacity constraints on the substations and the feeders, in addition to the constraints on the voltage drop in the network. More detailed description of the system constraints is given below.

3.4.3 Problem Constraints

In the design of the distribution system, the optimal values of the planning variables are determined such that they must satisfy the network operational and security constraints with minimum overall cost for all elements of the system. Hence the optimal design should satisfy the following constraints:

1. The net power received at the load points should be equal to at least the demand at each of these points. Also the dispatched power from any substation should not exceed its capacity limit.
2. The net power at the transshipment nodes should be zero to satisfy the conservation of the power flows.

3. The power flow through a feeder should not exceed the feeder power capacity limit, which is based on the thermal limitations of the feeder.

4. The voltage at the source and the load points should be within certain specific values, for the stability reasons.

The mathematical formulations of the constraints are given in the following subsections.

3.4.3.1 Voltage Constraints

The voltage constraints are included in the model in two different sets. The first set simulates the voltage drop in the feeders, neglecting the phase shifts. Since the future feeder conductor sizes have been assumed prior to the solution, the impedance of each feeder, existing or potential, is known and hence this set of constraints is represented in a linear form as,

\[ V_{i,1} - V_{i,2} - Z_{i} \sum_{e=1}^{ESP} \left[ \frac{P_{i}}{U_{i}} \right] = 0 ; i=1,2,\ldots,NF \]

\[ \tau = 1,2,\ldots,NT \quad (3.10) \]

where \( V_{i,1} \) is the per unit voltage magnitude at the sending node of feeder \( i \) at time segment \( \tau \), \( V_{i,2} \) the per unit voltage magnitude at the receiving node of feeder \( i \) at time segment \( \tau \), and \( Z_{i} \) the per unit impedance magnitude of feeder \( i \) at time segment \( \tau \).
This set of constraints (3.10) relates the node voltage variables to the feeders power flow variables in the network.

The second set of voltage constraints introduces upper and lower values for the voltage magnitudes at all nodes in the network. This set of limits on the network node voltages is essential for network stability, and it is presented in the model in the form,

\[
V_n > V_{\tau} > V_n \quad n = 1, 2, \ldots, N \quad \tau = 1, 2, \ldots, NT (3.11)
\]

where \( V_n \) and \( V_{\tau} \) are the per unit upper and lower limits on the voltage magnitude at node \( n \), \( V_{\tau} \) the per unit voltage magnitude at node \( n \) at time segment \( \tau \), and \( N \) the total number of network nodes, supply, load, and transshipment.

### 3.4.3.2 Conservation of Power Flow

The conservation of power flow law simply states that the amount of power delivered to a node should be equal to the power used at and sent from this node while the amount of power sent from a node should be equal to the power generated at and received by this node. Mathematically this set of constraints can be introduced in the following form,

\[
\sum_{j=1}^{NF} a_{ij} \left[ \sum_{e=1}^{ESP} \sum_{\tau=1}^{j} P_{\tau e} \right] = P_i \quad i = 1, 2, \ldots, N \quad \tau = 1, 2, \ldots, NT (3.12)
\]
where \( a_{ij} \) is the \( ij \)-th element of the node-feeder incidence matrix, and \( D_{ei} \) the demand at or from node \( i \) at time segment \( \tau \), where it is less than zero for load nodes, equal zero for transshipment nodes, and greater than zero for supply nodes.

### 3.4.3.3 Capacity Constraints

To ensure that the feeder thermal limitations will be satisfied in the optimal plan, a set of feeder capacity constraints is included in the new model. This set of constraints presents upper limits on the power flow through the different feeders in order not to exceed their capacity limits. These conditions have to be held for all power segments ESP of feeder \( i \), for all feeders NF, and at all planning time segments NT. Mathematically this set of limits is written as in equation (3.13).

\[
\begin{align*}
\sum_{\tau} \sum_{e} P_{\tau,ei} & < U_{ei} \quad e = 1, 2, \ldots, \text{ESP} \\
\tau, e & \quad e = 1, 2, \ldots, \text{NF} \\
i & = 1, 2, \ldots, \text{NT} 
\end{align*}
\]  
(3.13)

where \( U_{ei} \) is the power capacity limit of power segment \( e \) of feeder \( i \), and \( W_e \) is a weighting factor less than or equal to one associated with power segment \( e \) of feeder \( i \). The function of the factor \( W_e \) is to ensure the existence of some reserve capacity in the feeder to be used if needed in case of emergency.
The proper use of these factors for the feeders and the substations improves the reliability of the system.

Another set of capacity constraints is also used. This set is concerned with setting upper as well as lower limits, \( P_s \), on the power dispatched from any power segment \( e \) of any substation \( s \) at any time segment \( \tau \) during the planning period. The lower limit is set to zero, where the power flow is oriented out of the substation, forcing the substations to only deliver power and not to receive any. The upper bound is set to the power capacity limit \( U_e \) of the power segment under consideration, multiplied by the factor \( w_e \) to ensure the availability of the power even in the emergency cases. This set of constraints are given in equation (3.14).

\[
0 < P_s \leq U_e - \tau_s e - e \\
\tau = 1, 2, \ldots, NT
\]  

(3.14)

3.4.3.4 Radiality Constraints

The model does not include radiality constraints since the practice showed that the optimal solutions of the distribution systems are always radial. Even if a solution has a point or more fed from two or more feeders, the load splitting technique presented in [38] can be applied at these points. This technique is based on the fact that each of the load points is actually an area of customers which can be redivided on power demand bases according to the amount of power delivered by each of the feeders serving this node.
3.4.3.5 Other Necessary Constraints

The inclusion of the new feature of expanding the power capacity of some of the existing facilities has required the satisfaction of some additional practical conditions. It is logically accepted that none of the additional power segments of an element is to be used unless its existing power limit is reached. These conditions are formulated mathematically in the model for both expandable feeders and substations as in equations (3.15) and (3.16), respectively.

\[
M \left[ \begin{array}{c} P \\ \tau \cdot (e-1) \end{array} \right] \frac{1}{U} \left[ \begin{array}{c} P \\ \tau \cdot (e-1) \end{array} \right] + \left[ \begin{array}{c} P \\ \tau \cdot (e-1) \end{array} \right] > 0 ; \quad e=2,3,\ldots \quad \text{FEXP} \\
\tau \cdot e \quad \tau \cdot e \\
\tau = 1,2,\ldots \quad \text{NT} \\
(3.15)
\]

\[
M \left[ \begin{array}{c} S \\ \tau \cdot (e-1) \end{array} \right] \frac{1}{U} \left[ \begin{array}{c} S \\ \tau \cdot (e-1) \end{array} \right] + \left[ \begin{array}{c} S \\ \tau \cdot (e-1) \end{array} \right] > 0 ; \quad e=2,3,\ldots \quad \text{SEXPS} \\
\tau \cdot e \quad \tau \cdot e \\
\tau = 1,2,\ldots \quad \text{NT} \\
(3.16)
\]

where \( M \) is a large positive number.

Equations (3.1) and (3.6) to (3.16), present the mathematical formulation of the new model for long range distribution planning. The objective function (3.1) is minimized subject to the constraints (3.10) to (3.16) producing the optimal choice of the facilities to be used in each of the planning time segments, along with the power flow in each of these facilities. The optimal installation time of each of the required new facilities is also
determined in the solution. In addition the node voltages are given at each planning time period.

3.4.4 Model Advantages

Investigating the formulation of the new model shows that the model has the following advantages over the other available models for distribution planning:

1. There is only minor approximation in accounting for the different planning aspects.

2. The nonlinear nature of the model allows more accurate formulation for the different cost terms as well as for the system constraints.

3. The number of variables required to cover all the planning items is relatively much smaller than the corresponding number in the previous different planning models performing similar studies. This reduction in the number of variables is due to the elimination of the integer constrained variables in the model.

4. The branch-and-bound criteria are not required in the new model and hence the question of the solution sensitivity to the criteria used is no longer relevant.

5. The model includes accurately and practically the stability constraints. These requirements are included in the form of nodes voltage regulation as well as feeder voltage drop constraints.
6. The effect of the time factor has been accounted for in the model in a more accurate way allowing the calculation of the present worth cost value of the system components. The formulation also allows the satisfaction of the network operational and security constraints over the whole period of planning.

7. The new option of increasing the power capacity of some or all existing elements in the network, if possible, has been included in the new model.

The above advantages promise a better accuracy and a higher efficiency of the planning model. These assumptions are validated by applying the model to a practical system and the results are compared with those obtained using other techniques.

3.5 Optimization Routine

Due to the large number of the optimization variables involved in the planning of a distribution system, the nonlinearity of the continuous functions, and to obtain accurate and highly efficient optimal solutions, there is a need for an advanced commercial nonlinear optimization routine. MINOS/Augmented routine has been used, which implements the projected Lagrangian algorithm and has the sparsity facilities (109, 110). The routine successfully dealt with such large scale nonlinear problems. The model is tested on IBM 3031 Computer at the University of Windsor.
3.6 Applications

Figure 3.1: Original 44 Kv distribution network under consideration. ($S_1$ to $S_3$ are existing substations, 4 to 60 are transshipment nodes, 61 to 87 are load points).

To show the applicability, the efficiency, and the accuracy assumed for the new developed model, it has been applied to a practical distribution system of Ontario Hydro.
The detailed description of this system is given in the following subsection followed by the conditions under which the optimal expansion solutions of the system are obtained. The results are presented along with an earlier solution obtained by Ontario Hydro for comparison.

3.6.1 Description of The Typical Distribution System Under Study

The system is a typical 44 Kv subtransmission system in an area located in Eastern Ontario (Fig. 3.1). The network has three existing substations, shown as circles with the substation number inside. Two of the substations (S₁ and S₂) have 140 MVA ratings, while substation S₃ has 200 MVA capacity. There is no new substations to be built within the period of planning. These substations supply 27 load points in the system, shown as solid squares in Fig. 3.1 (nodes 61-87).

Table 3.1 presents the load-time variations at each of the load points. For each load node five parameters are listed: load node number, load at its saturation value, initial load percentage, the first year of load growth, and finally the year at which the load saturates.
The rest of the nodes in Fig. 3.1 (nodes 4 to 60) present transshipment nodes, with zero load and zero generation. The network contains 114 feasible feeders, out of which 39 3-phase existing feeders and 30 3-phase future feeders. The remaining 45 feeders present the load branches which are the means of connecting the 3-phase feeders to the load demand points. These branches do not necessarily represent the actual physical construction.
The typical cost data, at year 1983, of the network under consideration are as follows: the cost for constructing subtransmission feeders, which are typically 556 KCMil aluminum conductors is estimated at $66,000/Km. The cost of constructing primary distribution feeders (336 KCMil aluminum conductors) is estimated at $40,000/Km. The variable cost of substations and feeders, using 1983 prices, is set to 1.0 $/KWh/year, including the cost of energy losses.

The capacity limits of the feeders are 35 MVA for the 556 KCMil aluminum conductors and 25 MVA for the 336 KCMil aluminum conductors.

The inflation rates used are assumed in this application to be constant at 5.0% and 6.7% for the fixed and the variable costs, respectively. The interest rates are also assumed to be constant at 17.0% and 10.0% for the fixed and the variable costs, respectively.

3.6.2 Cases Studied

The objective of the planning study is to determine over a period of twenty years (from the beginning of year 1983 to the end of year 2002) the best configuration of the above subtransmission system (Fig. 3.1). The one time-step optimal solution for the system, which satisfies the load demand at year 2002 (second column in Table 3.1) is obtained. The new facilities are added to the network at the mid-point of the planning period (end of year 1992). The static optimal
solution of the same system under the same conditions has been obtained earlier by Ontario Hydro [111]. A comparison between the solution obtained in [111] and that obtained from the new model is derived, to prove the success of the developed model in terms of its accuracy and efficiency.

Since it has been always said that the static planning solutions are not necessarily optimal, the optimal time-phased expansion (every 5 years) for the same subtransmission system (Fig. 3.1) is obtained to investigate the following:

1. The capability of the new model to handle the dynamic mode of planning accurately and efficiently.
2. The accuracy of the static mode of planning compared to the dynamic mode.

3.5.3 Results and Discussion

The one time-step optimal solution for the system for the period 1983-2002, is introduced in Table 3.2, along with the optimal solution obtained earlier by the Ontario Hydro model, for the same twenty years period. Both power flow patterns through the network feeders as well as the receiving end node voltages are given.
Table 3.2: Comparison between static optimal solutions of earlier Ontario Hydro model and the new introduced model.

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<tr>
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<th>PRESENT MODEL</th>
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Figure 3.2: Optimal network configuration obtained by the present model.
The optimal network configurations, for the present model solution and for Ontario Hydro earlier solution, are shown in Figures (3.2) and (3.3), respectively.

Figure 3.3: Optimal network configuration obtained by earlier Ontario Hydro model.

Comparing the results obtained from both models, it is found that:

1. The variable cost produced by the present method is $3,891,181 and the fixed cost is $3,475,412 producing a total of $7,366,593 for the overall cost. These values are compared to the costs of the earlier solution presented in [11] which are $4,635,793 for the variable cost, $3,013,159 for the fixed cost, and
$7,648,957 for the overall cost. This shows that the overall cost of the plan produced by the new model is less than that of the earlier model by about $282,364. It is noted here that the optimal plans have been assumed to operate for the last ten years of the planning period only and accordingly the variable costs have been calculated for these years (1993-2002) only.

2. In reference (111), there were some feeders which have been slightly overloaded (S3-16, S1-3, S2-36, S3-43, S3-59, S1-44, and S9-40), since the power capacity limits of these feeders are 35 MVA. The present solution on the other hand has no overloaded feeders. This demonstrates the model capability of satisfying the thermal constraints of the network more efficiently.

3. Substation S2 in the case of the solution obtained by the earlier Ontario Hydro model has been overloaded. There was a shortage of about 7.5 MVA in the power required from S2 in that solution. This had to be handled either by adding some new generating units to S2 to cover this shortage, leading to an increase in the capital cost, or by switching some of the loads served by S2 to the other two substations (S1 and S3), leading to reconfiguring the network. In the present model however, none of the substations is
overloaded. Again this shows the higher capability of the new model of satisfying the system operational and security constraints.

4. Load point number 69 is fed from two sources in both solutions, violating the radiality requirements in the network. This problem can be solved by applying the load splitting technique [38] to node 69. In addition, the solution obtained in [111] included one of the transshipment nodes, node number 33, which had been fed from two sources.

5. The voltage regulation constraints are not violated in both solutions. For the nominal 44 kV distribution network, the supply voltage is set to 46.6 kV, while the minimum voltage is set to 41.8 kV.

After validating the efficiency and the accuracy of the newly developed model in handling the distribution planning problem, in its static mode, it is necessary to investigate its capability in dealing with the dynamic mode of planning as well. For the purpose of comparison between the static and the dynamic modes of planning, the dynamic optimal solution (every 5 years) for the same system shown in Fig. 3.1, and for the same twenty years period (1983-2002), is obtained. The optimal network configurations, for the four planning time segments, obtained by the dynamic solution are shown in Fig. 3.4.
It can be seen that the system existing facilities are quite sufficient for operating the system up to the end of year 1992 (the end of the second time segment), since no new feeders are required to be added to the network in the first ten years. Table 3.3 includes the power flow patterns through the network along with the receiving end node voltages, for the four interval time periods.

Figure 3.4: Dynamic optimal solution (every 5 years) of the system given in Figure 3.1.
Table 3.3: Optimal power flow patterns through the network along with the receiving end node voltages for the four time periods.

$P_i$ to $P_4$ are the powers in periods 1 to 4, respectively. $V_i$ to $V_4$ are the node voltages in periods 1 to 4, respectively.

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<th>$V_1^2$</th>
<th>$P_2$</th>
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From examining these results it is found that:

1. Feeder S3–16 becomes slightly overloaded after the first two time segments (i.e. after 10 years). The percentage overloading in this feeder is 3.2% in the third period and 6.0% in the fourth period. If the power capacity of this feeder is increased by 25% after the second time segment, to eliminate any possible violation for the thermal constraints of this feeder, this expansion costs $44,980 present worth value, assuming that the capital cost of the feeder is linearly proportional to its capacity limit.

2. Power demand from each of the substations is held satisfactory below their power capacity limits.

3. The transshipment node number 33 is fed from two sources in the fourth time segment violating the radiality constraints. Although this is not a load node, but a careful look at Fig. 3.4(d) shows that this non-radiality problem can easily be solved by the load splitting technique, applied to load node 84. In addition load point 86 is also fed through two feeders during the last two time periods. The reason for this is due to the fact that the load demand at this node is higher than the capacity limit of a single feeder (35 MVA) during the third time period (35,867 KVA) and in the fourth time period (41,540 KVA). Once again this can be handled by the load splitting technique.
4. The more interesting and valuable observation is that only two new feeders (S1-44 and S2-36) have been added to the network and they have to be in service starting at the third time segment. This is compared to the one-step solution (Fig. 3.2), where 15 new feeders have been used. This dramatic difference is due to the sequential build up of the load demand in the dynamic solution, which accordingly directs the additions of the new facilities towards satisfying the incremental increase in the load demand, period by period. On the other hand the static solution is faced with the load saturation values directly, opening a wide range of alternatives of adding new facilities to satisfy these loads, leading to non-optimal solution in most cases.

5. With respect to the cost comparison, the variable cost over the planning period is $7,047,688, with fixed cost of $181,053, producing $7,228,741 for the overall cost in the dynamic optimal solution. This is less than the overall cost produced for the one-step solution ($7,366,593), although the system costs in the multi-steps solution is calculated for the twenty years operating period, instead of only ten years in the one-step solution. Even if the expansion suggested for feeder S3-16, in the first comment above, is made the total cost will still be less.
6. The voltage regulation constraints are satisfied at all system nodes throughout the planning time periods. From the above results we can see that the new model has proven its capability of handling the dynamic mode of planning. It is also seen that the efficiency and the accuracy of the dynamic mode in obtaining the optimal solution are much better than that of the static mode. This supports the assumed limitation that the static solutions may not lead to the overall optimal designs over the whole planning time period. The efficiency of the dynamic solution can be improved using shorter time segments (e.g., 5 segments of 4 years each). This would make it easier to modify the design should unexpected events occur, such as a downturn in the load growth. It should be noted also that there is a minimum period for each time segment due to the installation time required for each new facility. This is typically 3 years for substations and one year for feeders. In addition, a limitation is necessarily imposed on reducing the time segments within the total period of planning, by the accompanying increase in the requirement for the computer memory.

3.7 Concluding Remarks

The new continuous model for long range distribution planning has been applied successfully in its static and dynamic modes. It has been shown that the model has a high
accuracy and good efficiency in handling the distribution planning problem. It has been also shown that the dynamic solution is preferable to the static solution in terms of lower cost. Despite the continuity nature of the variables in the proposed model, the advanced optimization routine used, with the sparsity facilities available in it, makes the model capable of dealing with large distribution systems. An example of the reduction of the number of variables involved in the new model compared to the number of variables in the previous model, is that the number in the present model in the static case studied is 231, while in the model used by Ontario Hydro for the same study was 261. This is due to the addition of integer variables for the 30 future feeders. This difference, rendering advantage to the new model, increases further in the dynamic mode of planning.

Among the other factors affecting the solution obtained are the interest and the inflation rates which strongly influence the decision of the installation time. For example if the inflation rates are higher than the interest rates, the decision to build the new facilities might be at an earlier point of time during the planning period.
Chapter IV

AUTOMATED_LONG-RANGE_TRANSMISSION_PLANNING

4.1 Introduction

The amount of power that must be transferred from the generation sites to the major load areas has been growing in the past few years. Due to increasing costs and the need for reliable electric systems, optimum designs for the different sections of the power systems are required. The optimal designs must satisfy efficient performances with minimum cost or within certain planning budgetary constraints. In the previous chapter a powerful new nonlinear model for distributing the power delivered from the transmission networks at the substations, to the customers served by the distribution system, has been developed. To enhance the effectiveness of the planning procedure for the overall electric power system, it is necessary to develop a new accurate and efficient model for transmission planning as well. To appreciate the need for a new planning model, a summary of the limitations and drawbacks of the different available techniques for transmission planning is given in the next section. Any proposed model has to overcome most of these limitations. A new transmission planning model is developed satisfying the
objectives given in section 4.3. A description of the new model is then given in details and followed by applications to two different systems to show its capability and advantages.

4.2 Limitations of The Existing Transmission Planning Models

From the review given in the first chapter it can be shown that there are some critical limitations affecting the performance of the existing transmission models. These drawbacks are similar to those found in the distribution planning models and they can be summarized in the following:

1. Linear cost models are assumed in most of the techniques in one way or another. The assumption of linearity usually leads to inaccurate results producing close to but not optimal solutions.

2. Some of the important network constraints are either ignored or relaxed in many of the existing models leading to impractical plans. Among the most common neglected constraints are the limits on the different bus swing angles and the reactive power constraints.

3. The techniques used previously to handle the accuracy problem of the models required a large number of variables and a great deal of effort in formulating the problem. This limited the ability of these models to handle large systems.
4. To improve the capability of handling large transmission systems, different branch-and-bound criteria have been used. The solution accuracy is always dependent on the criterion used, which is questionable in many cases.

5. The sensitivity analysis has been widely used in the screening algorithms, the filtering schemes, and in the different stopping criteria used in searching for the optimal solutions in most of the recent techniques. The sensitivity terms used in such models were usually linearized giving a doubt in the accuracy of the results. In addition, the use of the sensitivity analysis imposes limits on the changes occur in the network to be small.

6. Among the other reasons, which limits the applicability of most of the available transmission planning models, is the continuous need for load flow solutions for every change that occurs in the network. This process is accompanied by the need for long computing time.

7. In almost all previous cases the dynamic mode of planning has been ignored or handled by a sequence of static plans. As has been mentioned and shown in the previous chapter, this may not produce the overall optimal solutions.
8. The reliability tests, simulated by deterministic contingency studies, are usually performed by repeating the planning procedure for each outage case considered.

Overcoming the above limitations requires a derivation of a new model. The technique used in this work is given in terms of objectives to be achieved in the following section.

4.3 Transmission Planning Objectives

To improve the performance of the transmission planning, the development of a more accurate model is required. The accuracy of the results can be enhanced by introducing a nonlinear model capable of including and presenting the different cost components in the system more accurately. Achieving the nonlinear form of modeling not only improves the cost analysis but also allows the important nonlinear constraints of the system to be included accurately. The time consumed in the planning process can be reduced by including the severe contingency cases as a part of the basic planning steps, and not by repeating these steps. Including the contingency analysis will also enhance the reliability of the solutions obtained. Finally the model has to be able to handle the dynamic mode of planning more accurately and not as a series of static build-up plans.

To achieve these objectives a new model is developed for long range transmission planning. The following section
describes the construction and the formulation of the new model.

4.4 New Transmission Planning Model Formulation

In order to attain an optimal design for transmission planning, the model must be capable to furnish the planner with the decisions of where and when to add the new facilities. At the same time the design obtained should satisfy economically and reliably the system constraints.

In the present work an accurate and continuous cost model with a dependence on the time and the lines power flows is presented. The model has the capability of solving the static and the dynamic transmission system planning. It takes into account the fixed and the variable costs of the planned facilities. In addition the cost of the energy losses in the different lines is also accounted for. Operational constraints, presenting demand satisfaction and overloading, are included in the optimization model. The model has the advantage of adding security constraints on the bus voltage magnitudes as well as on the bus swing angles. The accurate ac load flow equations are satisfied within the optimization procedure, since they are included in the constraints. The contingency study, simulating the deterministic reliability evaluation, is also performed during the planning process. The performance of the system under severe outages is presented as a set of constraints to
be satisfied. Finally the model can be rearranged in a form capable of producing the design of a highly efficient network performance within the availability of predetermined budgetary constraints.

4.4.1 Model Cost Function

The optimal selection of the most economical facilities to be used in the system during the different planning time segments has to be made on the basis of a common cost of the different elements at all times. The present worth value of the cost of each of the individual components in the system is considered as the base of the cost selection, where the overall present worth cost function can then be minimized. The cost function includes the variable cost of all the existing facilities and those which may be installed during the planning period, in addition to the cost of their energy losses. The capitalized cost of the future facilities is also included. The model present worth cost function \( C \) is then presented as:

\[
C = \sum_{\tau=1}^{NT} \sum_{i=1}^{NL} (CP_{\tau,i} + CV_{\tau,i} + CL_{\tau,i})
\]  \( (4.1) \)

where \( CP_{\tau,i} \) is the present worth value of the fixed cost of line \( i \) at planning time segment \( \tau \), \( CV_{\tau,i} \) and \( CL_{\tau,i} \) the
corresponding variable and energy loss costs, respectively, 
NT the total number of time-phased planning segments \( NT = 1 \) 
for static planning, and NL the total number of existing 
and future right-of-ways.

The detailed formulations of the different cost terms in 
equation (4.1) are given in the following subsections.

4.4.1.1 Capital (Fixed) Cost \( CF_{\tau,i} \)

![Graphs showing capital cost vs. line length](image)

Figure 4.1: Presentation of the capital cost per unit 
length of line \( i \) at the first planning time 
segment \( (\tau = 1) \).

The present worth value of the installation fixed cost 
\( CF \) is formulated in the model as follows:

\[
CF_{\tau,i} = L \cdot C_1 \cdot \beta^i \quad ; \quad i = 1, 2, \ldots, NL
\]

\[
\tau \cdot i \quad \tau \cdot i \quad \tau \cdot i \quad \tau \cdot i \quad \tau = 1, 2, \ldots, NT
\]

(4.2)
where \( L_i \) is the length of line \( i \) in km, \( C_{t,i} \) a continuous function in the power flow in line \( i \) at the planning time segment \( t \), presents the capital cost per km at the planning time of adding new paths to line \( i \) at planning time segment \( t \), and \( \beta_{t,i} \) accounts for the present worth value of the cost at the planning time segment \( t \).

The formulation of \( C_{t,i} \) depends on the time segment under consideration. Since accounting for the use of element \( i \) in any previous time period should be considered, the function \( C_{t,i} \) for the first time segment \((t=1)\) is different than that of the subsequent time segments. Hence this function is formulated for the first planning time segment as follows:

\[
C_{t,i} = cf \cdot \left(0.5(\text{NC} - 1) + S_{t,i} - SI_{t,i}\right); \text{ } i=1,2,\ldots,\text{NL}_{t,i} \quad \text{ } (4.3)
\]

where

\[
S_{t,i} = TY_a \cdot ((2/M) \cdot \tan^{-1}(-P_{t,i} \cdot (|P_{t,i}|/U_{t,i}))) \quad \text{ } (4.4)
\]

and

\[
SI_{t,i} = \sum_{n=1}^{\text{NC} - 1} \left((1/M) \cdot \tan^{-1}(P_{t,i} \cdot (n - (|P_{t,i}|/U_{t,i})))\right)
\]

where \( cf \) is the capital cost per km, at the planning time, of any power capacity added to the first power segment in the right-of-way \( i \), \( \Gamma \) the ratio between the capital cost of \( i \) the first power segment of line \( i \) (including cost of towers and accessories) to the cost of any additional single power.
segment, \( T_Y \), an integer constant equal to zero if line \( i \) originally exists in the network and equal one for the potential lines, \( U \) the power capacity of single power segment in line \( i \), \( |P| \) the absolute value of the total power flow through line \( i \) at planning time segment \( \tau \), \( N_C \) the number of power segments (parallel paths) permitted in the right-of-way \( j \), and \( P_1 \) a large positive constant. The function \( C_1 \) in equation (4.3) is plotted in Fig. 4.1.

For any time segment following the first one, the function \( C_1 \) used in equation (4.2) is formulated as in equation (4.5).

\[
C_1 = c_f \cdot \left( C + (0.5) \cdot S_1 - S_2 \right) ; \tau = 2, 3, \ldots, NT \quad \tau, i \quad \tau, i \quad \tau, i \quad \tau, i \quad i = 1, 2, \ldots, NL
\]

where

\[
C = T_Y \cdot a \cdot d \cdot d^{-1} \quad \tau, i \quad \tau, i \quad \tau, i
\]

\[
d = \prod_{j=1}^{\tau-1} \left( \exp(-P_1 \cdot |P| / U) \right)
\]

\[
d = 1 - \exp(-P_1 \cdot |P| / U)
\]

\[
\sum_{n=1}^{\tau} \left( \prod_{\eta=1}^{\tau-1} \left( \left( 1 - (2/n) \cdot \tan (P_1 \cdot (1/n) - (U / |P| \cdot 1)) \right)^{-1} \right) \right)^{i, \eta, i}
\]

\[
S_1 = \sum_{n=1}^{\tau} \left( (0.5) \prod_{\eta=1}^{\tau-1} \left( \frac{1}{1 - (2/\pi) \cdot \tan (P_1 \cdot (1/n) - (U / |P| \cdot 1))} \right)^{-1} \right)^{i, \eta, i}
\]
\[ S_2 = \sum_{n=1}^{\tau - 1} \left( \frac{z_i^j}{\tau j} \right)^{\tau - 1} \]

\[ z_i^j = (0.5/\pi) \tan^{-1}\left(\frac{P_{i1}(n^2 - (|P_{i1}|/|U_{i1}|))}{1+(2/\pi)\tan^{-1}(P_{i1}((1/n^2)-(|U_{i1}|/|P_{i1}|)))} \right) \]

The formulation of \( C_{i1} \) in equation (4.5) only considers the cost of the power segments required in the time segment \( \tau_i \) if they have not been used prior to this segment. The function \( \beta_{i1} \) in equation (4.2) is formulated, as in the distribution planning model, as follows,

\[ \beta_{i1} = \left(\frac{(1+r(\tau_i)/1+r(\tau_i))}{T_i} \right) \]

where \( r(\tau) \) and \( r(\tau) \) are the inflation and the interest rates at the planning time segment \( \tau_i \) and \( T \) the installation time of the new power segment in line 1.

4.4.1.2 Operating and Maintenance Variable Cost \( CV_{i1} \)

The present worth value of the operating and maintenance variable cost \( CV_{i1} \) of line 1 at planning time segment \( \tau_i \) has been treated in a similar way as in the distribution planning model. The sum of equal payments over the operating period of time \( T_{0,i} \) of line 1 in the planning time segment \( \tau_i \) presents the variable cost \( CV_{i1} \), which is formulated mathematically as.
where \( CV_i \) is the annual variable cost per km of line \( i \) at the planning time and \( g_{\tau,i} \) a function converts the cost value of line \( i \) operates at time segment \( \tau \), to its present worth value,

\[
g_{\tau,i} = \frac{1}{r_{\tau} - r} \left( \frac{1 + r_{\tau} - r}{1 + r_{\tau}} \right)^{-\frac{\tau - t}{r}} \left[ 1 - \left( 1 + r_{\tau} \right)^{-\frac{\tau - t}{r}} \right]^{T-\tau,i}
\]

(4.11)

The function \( d_{\tau,i} \) is a continuous decision function in the power flow in line \( i \) at planning time segment \( \tau \). It is equal to zero for a future right-of-way \( i \) at planning time segment \( \tau \), if the absolute value of the power flow through this line \( |P_{\tau,i}| \) is small enough to be considered zero, otherwise \( d_{\tau,i} \) is equal unity.

\[
d_{\tau,i} = 1 - \exp\{-P_{\tau,i}/U\}
\]

(4.12)

The use of the function \( d_{\tau,i} \) ensures that the variable costs of only the existing and the new added lines at any period of time, are considered.

4.4.1.3 Energy Losses Costs

Although there is an argument that the cost of energy losses in transmission lines does not affect the planning design, since it is small compared to the fixed and the variable costs, it has been included in the model to investigate this assumption.
The power losses $E_{i}$ in any line $i$ at planning time segment $\tau_{i}$, is derived first as follows, 

$$E_{i} = (V_{i})^{2} \cdot Y_{i} \cdot P_{i} \cdot \cos(\theta_{i}) \cdot VAB \cdot F_{i} \cdot \tau_{i} \cdot \frac{1}{\tau_{i}} \quad (4.13)$$

where $V_{i}$ is the per unit (p.u.) voltage magnitude across line $i$ at time segment $\tau_{i}$, $Y_{i}$ the p.u. admittance magnitude of a single path in the right-of-way $i$, $\theta_{i}$ the phase angle of $Y_{i}$, $VAB$ the volt-ampere base chosen for the network, $P_{i}$ a continuous function in power flow in line $i$ at time segment $\tau_{i}$, which approximates the number of parallel paths in line $i$ (Fig. 4.2), and $F_{i}$ similar to $\delta$ (equation (4.12)) to ensure that the energy loss in the used lines only is calculated.

The present cost of this power is then calculated using the annual cost of energy losses per unit power at the planning time, $C_{3}$. Finally the inflation and the interest rates are employed to obtain the present worth value of the energy losses cost of this line $i$. So $CL_{i}$ presented in equation (4.1) is introduced in the model in the following form,

$$CL_{i} = W \cdot E_{i} \cdot C_{3} \cdot \delta_{i} \cdot \tau_{i} \cdot \frac{1}{\tau_{i}} \quad (4.14)$$

where $W_{i}$ is a weighting factor usually equal to unity to include the cost of the energy losses of line $i$ in the cost
function. $W_i = 0$ if the cost of the energy losses is not required to be included in the objective cost function, while it may take values higher than unity according to the importance of taking the cost into account.

![Graph of No. of parallel lines vs. $|p_{i}|/CAP$](image)

**Figure 4.2:** Nonlinear continuous approximation of the number of parallel paths required in the right-of-way $i$ at planning time segment $\tau$, to carry the power flow $P_{\tau,i}$.

Developing the above cost terms, fixed, variable, and energy loss costs, concludes the formulation of the cost function which is usually required to be minimized in the procedure subject to the system constraints which are given in the following subsections.
4.4.2 System Operational and Security Constraints

Usually the optimal design, searched for in the planning procedure, is the one which minimizes the overall system costs and at the same time satisfies the system operational and security constraints. Another concept of planning could be optimizing a certain performance index for the network (e.g. stability criterion or energy losses) within a predetermined budget. The performance index required to be optimized, usually simulates one or more of the system constraints. In any case satisfying the system constraints is required in any acceptable design, but we can go further and say that the acceptance of any design is based on satisfying the system constraints. In the following subsections, the transmission system constraints are introduced in the way that they have been formulated in the new planning model.

4.4.2.1 Demand and Power Flow Constraints

The demand constraints are divided into two different sets. The first set is introduced in the form of equation (4.15). It presents the load active power satisfaction constraints for the load buses, and the active power capacity constraints for the generator buses. It ensures that the amount of active power that could be delivered to (or sent from) a load (or a generator) bus, is at least (or at the maximum) equal to the active power required (or available) at this bus.
\[ \sum_{i \in N} \sum_{j} P_{ij} - \sum_{j} P_{j} \quad \tau = 1, 2, \ldots, NT \quad j = 1, 2, \ldots, NB \] (4.15)

where \( W_{ij} \) is a weighting factor for the power received at \( \tau, j \) or delivered by bus \( j \) at the time segment \( \tau \) (\( W_{ij} < 1 \) for generator buses and \( W_{ij} > 1 \) for load buses), \( D_{ij} \) the active power at bus \( j \) at the planning time segment \( \tau \) \( D_{ij} > 0 \) for the generator buses and \( D_{ij} < 0 \) for the load buses), \( NB \) the total number of existing and future buses in the network, \( N \) a subset of all transmission lines connected to bus \( j \), and \( P_{ij} \) the active power through line \( i \) at the planning time segment \( \tau \). The active power flows in the different lines at the different time segments are then formulated in terms of their terminal bus voltage magnitudes and swing angles to satisfy the active power flow constraints in the network. This set of active power flow equations is presented in the model in the polar form as follows,

\[ \frac{V_{i} \cdot Y \cdot P_{i} \cdot \cos(\delta_{i} - \phi_{i} - \theta_{i}) \cdot P_{ij}}{\tau, j} \quad \tau, i \quad i \quad \tau, j \quad \tau, i \quad i \quad \tau, i \quad \tau, i \quad \tau = 1, 2, \ldots, NT \]

\[ i = 1, 2, \ldots, NL \]

\[ j \in N \] (4.16)

where \( VB \) is the \( p \cdot u \) voltage magnitude of bus \( i \) at the time segment \( \tau, i \), \( \delta_{i} \) the swing angle of bus \( i \) at the time segment \( \tau, j \), \( \phi_{i} \) the phase angle of the voltage difference
across line \( i \) at the time segment \( \tau_i \), a continuous decision function to ensure that the left hand side of equation (4.16) is equal to zero if \( P_{\tau_i} \) is equal to zero, and \( NS \) a subset of the sending end buses of the lines.

The second set of the system demand constraints is responsible for satisfying the system reactive power flow equations, the load reactive power, and the reactive power capacity constraints of the generator buses. These constraints can be simulated in two sets of equations similar to equations (4.15) and (4.16). Since the line reactive powers do not affect the cost function and do not have limits imposed on their values, the reactive power conditions are introduced in the combined form shown in equation (4.17).

\[
\sum_{j \in N} V_{V_{\tau_j}} W_{\tau_i} Y_{\tau_i} P_{\tau_i} \sin(\delta_{\tau_i} - \phi_{\tau_i} - \theta_{\tau_i}) \leq W_{\tau_i} Q_{\tau_i} \\
\sum_{j \in N} \sum_{\tau \in \tau_{\tau_j}} Q_{\tau_j} \leq \sum_{\tau \in \tau_{\tau_j}} W_{\tau_i} Q_{\tau_i} \\
(4.17)
\]

where \( Q_{\tau_j} \) is the injected reactive power at bus \( j \) at time \( \tau_j \) segment \( \tau \). Once again we should notice that the use of the weighting factors \( W \) in equations (4.15) and (4.17) increases the reliability of the system. In addition, satisfying equations (4.15) to (4.17) ensures the solution of the ac load flow equations as well as the load and the generator buses demand satisfaction.
4.4.2.2 Lines Capacity Bounds

A set of upper bounds on the power flows in the different right-of-ways is included in the model to impose restriction on the number of parallel paths permitted in each right-of-way. Since the power flow is allowed in the lines in both directions their bounds are presented as upper limits (for the positive directions) and lower limits (for the negative directions). The positive and negative directions of the power flows in the network are arbitrarily chosen. These bounds have to be held for all lines at all times, and they are introduced as follows.

\[
\begin{align*}
\text{PL}_i & < P < \text{PU}_i \quad \text{where } P = 1, 2, \ldots, NT \\
\text{LL}_i & < \text{PU}_i - \text{PL}_i \\
& < \text{LL}_i \quad \text{where } i = 1, 2, \ldots, NL
\end{align*}
\]  

(4.18)

where \( \text{PU}_i \) and \( \text{PL}_i \) are the upper and the lower active power capacity limits of line \( i \) at the planning time segment \( T \).

4.4.2.3 Stability Constraints

The model has the advantage of including security constraints on both the bus voltage magnitudes and the bus swing angles. These constraints are simulated by upper and lower limits on the bus voltage magnitudes (equation (4.19)) and on the bus swing angles (equation (4.20)). These bounds are essential for load and synchronous machines steady state stability, as has been explained in section 2.4.
\[ V_L < V_B < V_U \quad ; \quad \tau = 1, 2, \ldots, NT \]
\[ \tau_j - \tau_{j-1} = \tau_j \quad ; \quad j = 1, 2, \ldots, NB \quad (4.19) \]
\[ S_L < S < S_U \quad ; \quad \tau = 1, 2, \ldots, NT \]
\[ \tau_j - \tau_{j-1} = \tau_j \quad ; \quad j = 1, 2, \ldots, NB \quad (4.20) \]

where \( V_L \) and \( V_U \) are the upper and the lower limits on the p.u. voltage magnitude of bus \( j \) at the time segment \( \tau \).

and \( S_L \) and \( S_U \) the corresponding values for the swing angles.

4.4.3 Advantages of the New Developed Model

Studying the construction of the new model, developed for the transmission planning, it is found that it enjoys the following advantages:

1. A more accurate cost function is introduced in the model, accounting for the nonlinearities in the different cost components in the system.

2. The model includes all the system operational and security constraints in more accurate forms. The model also has the superiority of introducing the stability constraints in terms of bounds on both bus voltage magnitudes and swing angles.

3. The number of the planning variables in the model is small due to introducing the integer decisions in terms of the line power flows without the need for any extra integer variables. In addition, the functions \( PN \) introduced in the model have reduced the number of variables for each right-of-way (e.g., \( i \)) at each
planning time segment by the factor \((1/NC_i)\). This is due to presenting the power flows in all the parallel paths \(NC_i\) permitted in line \(i\) by only one optimization variable.

4. Presenting the planning decisions by continuous nonlinear functions eliminates the need for the sensitive branch-and-bound criteria. In addition, the sensitivity terms used for filtering, screening, and bounding in the previously reported models are no longer needed, and this leads to a higher efficiency and accuracy in the results obtained from the new model.

5. The accurate form of including the time factor allows the model to handle the dynamic mode of planning in a more realistic and accurate way.

6. The inclusion of the exact ac power flow equations as constraints have to be satisfied in the optimization procedure, reduces the computational time. This is because the need for a load flow solution for each change in the network is no longer required.

The above advantages make the new model very useful since they almost solve all the limitations listed for the other available models in the literature.

Due to the approximations involved in presenting the number of parallel paths in each right-of-way by the functions \(PN\), an ac power flow solution for the system has
to be applied with the exact discrete number of parallel paths after obtaining the optimal solution. This step is required for the final adjustments of the different optimization variables.

4.5 Applications

In order to study the suitability of the new developed model for long range transmission planning and to check its capability, the model has been applied to two different transmission systems. The first is the 6-bus system
described in [11], and used in other references as well [16, 22]. A comparison between the results obtained from the present model and that obtained earlier in [11] is given.

The second system used is a fictitious 14-bus system supplied by Ontario Hydro. Different cases have been studied for both systems, to investigate the different features available in the model.

4.5.1 Example of The 6-Bus System

4.5.1.1 System Description

![Diagram of the 6-bus system](image)

Figure 4.4: Future conditions of the 6-bus system.
The 6-bus system described in [11] is shown in Fig. 4.3, before expansion (i.e., existing conditions). The system consists of 3 existing load buses (bus 2 with load demand of 60 MW, bus 4 with load demand of 40 MW, and bus 5 with load demand of 60 MW). The system also has 2 existing generator buses (bus 1 with generating capacity of 150 MW and local load demand of 20 MW, and bus 3 with generation capacity of 120 MW and local load demand of 10 MW). Single 3-phase path exists between each of the buses, 1-2, 1-4, 1-5, 2-3, 2-4, and 3-5.

The conditions for the expanded system are shown in Fig. 4.4, where the loads of the network are increased by a factor of 4. Two additional 120 MW generating units are added at bus 3. A new generating bus 6 with a total generation capacity of 600 MW is also added to the network.

The connection between any two buses is allowed with a limit of 4 parallel paths in each right-of-way. The data of the different lines (existing and future) in the system is given in Table 4.1, where the capacity of each line is determined based on thermal limitations and stability criteria [11].
Table 4.1: Transmission line data for the 6-bus system

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>LENGTH (miles)</th>
<th>REACTANCE in p.u.</th>
<th>RESISTANCE in p.u.</th>
<th>POWER CAPACITY (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>40.0</td>
<td>0.4</td>
<td>0.10</td>
<td>100.0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>38.0</td>
<td>0.38</td>
<td>0.09</td>
<td>100.0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>60.0</td>
<td>0.60</td>
<td>0.15</td>
<td>80.0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>20.0</td>
<td>0.20</td>
<td>0.05</td>
<td>100.0</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>68.0</td>
<td>0.68</td>
<td>0.17</td>
<td>70.0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>20.0</td>
<td>0.20</td>
<td>0.05</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>40.0</td>
<td>0.40</td>
<td>0.10</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>31.0</td>
<td>0.31</td>
<td>0.08</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>30.0</td>
<td>0.30</td>
<td>0.05</td>
<td>100.0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>59.0</td>
<td>0.59</td>
<td>0.15</td>
<td>62.0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>20.0</td>
<td>0.20</td>
<td>0.05</td>
<td>100.0</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>48.0</td>
<td>0.48</td>
<td>0.12</td>
<td>100.0</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>63.0</td>
<td>0.63</td>
<td>0.16</td>
<td>75.0</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>30.0</td>
<td>0.30</td>
<td>0.08</td>
<td>100.0</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>61.0</td>
<td>0.61</td>
<td>0.15</td>
<td>78.0</td>
</tr>
</tbody>
</table>

An assumed practical installation cost of $240,000/km for any new right-of-way (with a single path between its terminal buses), and $150,000/km for any additional path in an existing or future right-of-way is used. An annual operation and maintenance cost of $600/km is also assumed. The cost of the energy losses is assumed to be $0.1/kW/year.
4.5.1.2 Static Expansion Design

The model has been applied to the 6-bus system at first in the static mode of planning. The optimal one-step expansion design of the system is obtained twice. Firstly with neglecting the cost of the energy losses in the cost function, and secondly with minimizing the cost of the energy losses as a part of the objective function. This has been done in order to study the effect of the energy losses cost on the network design. In both cases the objective function (equation (4.1)) was minimized without imposing any constraints on the bus reactive powers (i.e. subject to equations (4.15), (4.16), (4.18) to (4.20) only). The reason for neglecting the reactive power constraints in this case is that the planning procedure used by Garver [11] was based on the active power only, so we tried to keep the same base of comparison between the two models.

The variable cost of the system was calculated for each cases for an assumed 4 years operating time period, while the new facilities were assumed to be added at the time of planning. The optimal network configuration obtained by the present model, with or without minimizing the cost of the energy losses, is shown in Fig. 4.5. In addition the power flow patterns of the network in both cases, along with the power flow pattern reported in [11] are given in Table 4.2, for the sake of comparison. Table 4.3 shows the p.u. bus voltage magnitudes and the bus swing angles in the three
different cases (minimizing the energy losses cost, neglecting the cost of the energy losses, and Garver's solution).

Figure 4.5: Optimal expansion solution of the 6-bus system.
Table 4.2: Optimal power flow patterns of the 6-bus system

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>POWER FLOW FROM GARVER SOLUTION IN (11), MW</th>
<th>POWER FLOW FROM PRESENT MODEL SOLUTION, MW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>neglecting cost of energy loss</td>
<td>minimizing cost of energy loss</td>
</tr>
<tr>
<td></td>
<td></td>
<td>before adjustment</td>
<td>after adjustment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>before adjustment</td>
<td>after adjustment</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>70.0</td>
<td>57.0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.0</td>
<td>32.0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>40.0</td>
<td>54.0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>75.0</td>
<td>57.7</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.0</td>
<td>57.7</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>385.0</td>
<td>57.7</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>200.0</td>
<td>57.7</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>160.0</td>
<td>57.7</td>
</tr>
</tbody>
</table>

Table 4.3: Bus voltage variables at the optimal solution of the 6-bus system

<table>
<thead>
<tr>
<th>BUS NO.</th>
<th>GARVER SOLUTION IN (11)</th>
<th>PRESENT MODEL SOLUTION, neglecting cost of energy loss</th>
<th>PRESENT MODEL SOLUTION, minimizing cost of energy loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p.u. bus voltage</td>
<td>p.u. bus voltage before adjustment</td>
<td>p.u. bus voltage after adjustment</td>
</tr>
<tr>
<td></td>
<td>bus swing angle, radian</td>
<td>before adjustment</td>
<td>after adjustment</td>
</tr>
<tr>
<td>1</td>
<td>1.02</td>
<td>-0.548</td>
<td>-0.532</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
<td>-0.295</td>
<td>1.04</td>
</tr>
<tr>
<td>3</td>
<td>1.04</td>
<td>-0.464</td>
<td>1.04</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
<td>-0.302</td>
<td>0.90</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
<td>-0.651</td>
<td>0.90</td>
</tr>
<tr>
<td>6</td>
<td>1.04</td>
<td>0.0</td>
<td>1.04</td>
</tr>
</tbody>
</table>
Due to the approximations in presenting the number of parallel paths in each right-of-way in the system in the new model, an ac load flow solution is required after the optimal solution is obtained. Since only the active power is included in this case, the parameter adjustments are obtained by solving only the active power portion of the ac load flow equations.

Table 4.4: Optimal power flow patterns of the 6-bus system under different solution conditions

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>ACTIVE POWER FLOW, (MW)</th>
<th>REACTIVE POWER FLOW (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>57.0</td>
<td>55.1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>32.0</td>
<td>33.1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>54.0</td>
<td>58.2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>73.0</td>
<td>74.0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5.0</td>
<td>3.8</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>403.0</td>
<td>376.0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>197.0</td>
<td>196.4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>203.0</td>
<td>191.3</td>
</tr>
</tbody>
</table>

The model has been reapplied to the 6-bus system after including the reactive power constraints, presented by equation (4.17), with minimizing the cost of the energy losses. The optimal configuration obtained in this case was the same as that in Fig. 4.3, while the power flow patterns and the bus voltage variables, obtained at the optimal
solution in this case and the above cases are presented in Tables 4.4 and 4.5, respectively.

Table 4.5: Optimal bus voltage variables of the 6-bus system under different solution conditions

<table>
<thead>
<tr>
<th>BUS NO.</th>
<th>P.U. BUS VOLTAGE MAGNITUDE</th>
<th>BUS SWING ANGLE (RADIANS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARVER SOLUTION PRESENT MODEL WITHOUT Q CONSTRAINTS PRESENT MODEL WITH Q CONSTRAINTS</td>
<td>GARVER SOLUTION PRESENT MODEL WITHOUT Q CONSTRAINTS PRESENT MODEL WITH Q CONSTRAINTS</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>2</td>
<td>0.96</td>
<td>1.04</td>
</tr>
<tr>
<td>3</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
<td>1.004</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
<td>0.90</td>
</tr>
<tr>
<td>6</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>

The values listed in Tables 4.4 and 4.5 are the variable values after the final adjustments in each case.

4.5.1.3 Discussion of the 6-Bus Example

The optimal network configuration obtained by the present model is similar to that obtained earlier [11], whether the reactive power constraints and the cost of the energy losses are accounted for in the model or not. A total of 7 new lines are added to the existing network. The fixed cost of these lines, based on the installation cost assumed, is $35,400,000 in all the cases studied. Also the variable cost (excluding the cost of energy losses) is the same in all
cases and equal to $659,332 over the assumed 4 years operating period. The cost of the energy losses over these 4 years is $324,594 in Garver’s solution [11]. The corresponding energy loss costs calculated based on the optimal power flow patterns obtained from the present model, for the case of not including the energy losses cost in the objective function and for the case of including it, are $315,148 and $313,154, respectively. These results give small savings of $9,446 and $11,443 when using the new model, for the absence and the inclusion of the energy losses, respectively. Even though the cost of the energy losses did not influence the planning decisions, from using the different facilities point of view, it did however play an important role in adjusting the bus voltage variables more suitably. Minimizing the cost of energy losses squeezed the bus voltage magnitudes and swing angles away from their limits towards the permitted regions. The upper and lower limits on the bus voltage magnitudes used in this example are 1.04 p.u. and 0.9 p.u., respectively. The corresponding limits on the bus swing angles are ±0.652 radians. Comparing columns 4 and 7 in Table 4.3 as well as columns 6 and 9 in the same Table shows the effect of minimizing the cost of energy losses on the bus voltage variables. This effect makes the system relatively more stable in addition to being slightly more economical.
Another advantage for the new developed model is its capability of satisfying the overloading constraints more efficiently, since there is no line overloading in the solutions obtained from the new model while there are two lines (6-2 and 6-4) which were slightly overloaded in the solution reported in [11].

Since the bus voltage magnitudes and the bus swing angles in [11] have been obtained from a complete ac power flow equations, there are some differences which are shown in Table 4.3 between these values and those obtained from the active power ac load flow solution used in the present model. These differences have been reduced significantly by presenting the reactive power constraints in the new model. Hence accounting for the reactive power constraints leads to more accurate and informative results as well as providing a simple method for bounding the bus reactive powers which play an important role in the system stability.

4.5.2 Example of 14-Bus System

To check the model in the dynamic mode of planning and its other features, it has been applied to a fictitious test system (14-bus) supplied by Ontario Hydro. The optimal solutions for planning the system over 10 years period (from the end of year 1987 to the end of 1997) are obtained in one-step aiming to 1997, and in 3 time-steps (from the end of year 1987 to the end of 1991, from the beginning of 1992 to the end of 1994, and from the beginning of 1995 to the end
of 1997). A comparison between the two solutions is given at the end of this section.

4.5.2.1 System Description

--- Existing Line

---

Figure 4.6: Existing facilities of the 14-bus system.
The system in its existing conditions at the end of year 1987 is shown in Fig. 4.6.

The system has 7 load buses (1 to 5, 12, and 13), and 6 existing generator buses (6, 8 to 11, and 14). Bus 14 is connected to a large transmission network and is considered as an infinite bus in the system under consideration. The existing load and generation data of the system buses is given in Table 4.6.

Table 4.6: Existing load and generation data of the 14-bus system

<table>
<thead>
<tr>
<th>BUS NO.</th>
<th>GENERATOR TYPE</th>
<th>INITIAL PEAK LOAD (end of year 1987)</th>
<th>DEPENDABLE CAPACITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MW</td>
<td>MVAR</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>36.0</td>
<td>9.8</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>28.8</td>
<td>8.7</td>
</tr>
<tr>
<td>3</td>
<td>Thermal</td>
<td>132.0</td>
<td>41.6</td>
</tr>
<tr>
<td>4</td>
<td>Thermal</td>
<td>81.2</td>
<td>24.2</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>88.4</td>
<td>27.7</td>
</tr>
<tr>
<td>6</td>
<td>Thermal</td>
<td>135.5</td>
<td>53.0</td>
</tr>
<tr>
<td>7</td>
<td>Thermal</td>
<td>120.5</td>
<td>42.0</td>
</tr>
<tr>
<td>8</td>
<td>Hydro</td>
<td>112.5</td>
<td>19.0</td>
</tr>
<tr>
<td>9</td>
<td>Hydro</td>
<td>120.5</td>
<td>42.0</td>
</tr>
<tr>
<td>10</td>
<td>Hydro</td>
<td>112.5</td>
<td>19.0</td>
</tr>
<tr>
<td>11</td>
<td>Synchronous Condenser</td>
<td>182.9</td>
<td>61.7</td>
</tr>
</tbody>
</table>

The system consists of 13 existing lines shown in Fig. 4.6. An annual load growth in the range of 3.0% to 3.4% is
assumed to the different load demands. Accordingly, 3 additional right-of-ways are permitted to be built in the future between buses 3-5, 4-5, and 12-14. An additional generation capacity of 20.0% can also be added to each of the generator buses 8 and 11, as well as 30.0% increase in the generation capacity of bus 6 is also allowed. A new generator bus 7 of rating 170 MW can be in service in the future if needed, with a connection to bus 12. The transmission line data of the system is given in Table 4.7.

Table 4.7: Transmission line data of the 14-bus system

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>NUMBER OF CIRCUITS</th>
<th>CONDUCTOR KC mil</th>
<th>PER CIRCUIT IMPEDANCE (p.u./km)</th>
<th>AMPICITY (Amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R</td>
<td>summer</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>1-double</td>
<td>795</td>
<td>0.000164 0.00103</td>
<td>1000.0</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>1-double</td>
<td>795</td>
<td>0.000164 0.00103</td>
<td>1000.0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>1-double</td>
<td>795</td>
<td>0.000164 0.00103</td>
<td>1000.0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>1-double</td>
<td>477</td>
<td>0.000965 0.00339</td>
<td>500.0</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>1-single</td>
<td>795</td>
<td>0.000164 0.001</td>
<td>1000.0</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>1-double</td>
<td>605</td>
<td>0.000797 0.00335</td>
<td>950.0</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>1-single</td>
<td>477</td>
<td>0.000965 0.00333</td>
<td>500.0</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>1-double</td>
<td>477</td>
<td>0.000965 0.00339</td>
<td>500.0</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>1-double</td>
<td>477</td>
<td>0.000965 0.00339</td>
<td>500.0</td>
</tr>
<tr>
<td>10</td>
<td>13</td>
<td>1-double</td>
<td>605</td>
<td>0.000797 0.00335</td>
<td>950.0</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>1-double</td>
<td>605</td>
<td>0.000797 0.00335</td>
<td>950.0</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>1-single</td>
<td>4x585</td>
<td>0.0000113 0.000134</td>
<td>2400.0</td>
</tr>
</tbody>
</table>

The future conditions of the system are shown in Fig. 4.7.
4.5.2.2  Results of The 14-Bus System Planning

--- Existing Line
--- Future Line

Figure 4.7: Possible future facilities of the 14-bus system.

The optimal network configuration of the system obtained by the static mode of planning, including the reactive power constraints and with minimizing the cost of energy losses, is shown in Fig. 4.8. The solution shows that none of the
new facilities is required to be added to the system during the planning period (1987-1997). The power flow pattern along with the receiving end bus voltage magnitudes and swing angles in the network, before and after adjustment, are given in Table 4.8.

---

**Figure 4.8:** Static optimal network configuration of the 14-bus system.
Table 4.8: Optimal values of the 14-bus network variables obtained by the static solution

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>'TO' BUS VOLTAGE MAGNITUDE (p.u.)</th>
<th>'TO' BUS SWING ANGLE (radians)</th>
<th>ACTIVE POWER FLOW, (MW)</th>
<th>REACTIVE POWER FLOW, (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>before</td>
<td>after</td>
<td>before</td>
<td>after</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>1.09</td>
<td>1.08</td>
<td>-0.15</td>
<td>-0.19</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1.05</td>
<td>1.03</td>
<td>-0.10</td>
<td>-0.12</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1.02</td>
<td>0.99</td>
<td>-0.35</td>
<td>-0.43</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.02</td>
<td>0.98</td>
<td>-0.32</td>
<td>-0.38</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>1.09</td>
<td>1.09</td>
<td>-0.17</td>
<td>-0.22</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>1.07</td>
<td>1.06</td>
<td>-0.20</td>
<td>-0.25</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1.05</td>
<td>1.03</td>
<td>-0.31</td>
<td>-0.39</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>1.13</td>
<td>1.13</td>
<td>-0.23</td>
<td>-0.30</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>1.11</td>
<td>1.11</td>
<td>-0.11</td>
<td>-0.15</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>1.13</td>
<td>1.13</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>1.13</td>
<td>1.13</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>1.09</td>
<td>1.08</td>
<td>-0.14</td>
<td>-0.19</td>
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<tr>
<td>12</td>
<td>14</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

From Table 4.8 it can be seen that all the system overloading and stability constraints are satisfied. Upper and lower limits imposed on the bus voltage magnitudes are 1.13 p.u. and 0.96 p.u., respectively. The corresponding limits on the bus swing angles are ±0.5236 and ±0.5236 radians (±30° electrical degrees). Another remark can be derived from Table 4.8 is that the maximum adjustments made in the different variables are ±20% for the bus swing angle, ±2% for the bus voltage magnitude, ±15% for the active power flow, and ±56% for the reactive power flow.
The dynamic optimal expansion (3 time-steps) for the network is also obtained and the power flow patterns for the different planning time segments are given in Table 4.9. The bus voltage variables at each of the time periods are also given in Table 4.10. The values listed in Tables 4.9 and 4.10 are the final values of the parameters after adjustments. It is found that the network configuration in the dynamic solution is the same as that obtained from the static solution (Fig. 4.8). The reason that the optimal static solution and the optimal solution for each time segment in the dynamic solution are almost the same is due to the large power capacities of the existing lines of the network compared to the network demand, the small number of future alternatives, and the near radial configuration of the network. Nevertheless the example shows the capability of the model of producing the optimal solution in the dynamic mode of planning. In calculating the different costs of the network, different interest and inflation rates have been used due to the continuous change in the economy. Since the fixed cost of the network in the different cases is zero, only the present worth values of the operating and maintenance costs CV and the costs of the energy losses CL are given in Table 4.11.
Table 4.9: Power flow patterns of the 14-bus system at different time segments of the optimal dynamic solution

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>FIRST TIME SEGMENT</th>
<th>SECOND TIME SEGMENT</th>
<th>THIRD TIME SEGMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>active power (MW)</td>
<td>reactive power (MVAr)</td>
<td>active power (MW)</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>-89.2</td>
<td>-12.2</td>
<td>-59.5</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>-50.0</td>
<td>-1.6</td>
<td>-17.2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>90.3</td>
<td>31.6</td>
<td>110.0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>98.5</td>
<td>43.4</td>
<td>106.6</td>
</tr>
<tr>
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<td>6</td>
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<td>-71.1</td>
<td>-198.8</td>
</tr>
<tr>
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<td>2</td>
<td>21.3</td>
<td>13.5</td>
<td>51.8</td>
</tr>
<tr>
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<td>4</td>
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<td>-25.7</td>
<td>-45.4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>-144.7</td>
<td>-53.7</td>
<td>-144.5</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>-264.5</td>
<td>-95.7</td>
<td>-264.5</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>-60.1</td>
<td>-9.3</td>
<td>-88.7</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>-119.5</td>
<td>-8.0</td>
<td>-119.5</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>-62.1</td>
<td>-16.6</td>
<td>-46.1</td>
</tr>
<tr>
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<td>14</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table 4.10: Optimal dynamic values of the bus voltage variables of the 14-bus system

<table>
<thead>
<tr>
<th>BUS NO.</th>
<th>FIRST TIME SEGMENT</th>
<th>SECOND TIME SEGMENT</th>
<th>THIRD TIME SEGMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p.u. voltage</td>
<td>swing angle (radians)</td>
<td>p.u. voltage</td>
</tr>
<tr>
<td>1</td>
<td>1.0567</td>
<td>0.0676</td>
<td>1.0578</td>
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<td>2</td>
<td>1.0746</td>
<td>0.1125</td>
<td>1.0715</td>
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<td>1.0274</td>
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<td>1.0194</td>
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<tr>
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<td>-0.0052</td>
<td>1.0464</td>
</tr>
<tr>
<td>5</td>
<td>1.0360</td>
<td>-0.0460</td>
<td>1.0164</td>
</tr>
<tr>
<td>6</td>
<td>1.0870</td>
<td>0.1403</td>
<td>1.0870</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.1169</td>
<td>-0.2155</td>
<td>1.1411</td>
</tr>
<tr>
<td>9</td>
<td>1.1300</td>
<td>0.3600</td>
<td>1.1300</td>
</tr>
<tr>
<td>10</td>
<td>1.1300</td>
<td>0.3458</td>
<td>1.1300</td>
</tr>
<tr>
<td>11</td>
<td>1.1300</td>
<td>0.0301</td>
<td>1.1300</td>
</tr>
<tr>
<td>12</td>
<td>1.0932</td>
<td>-0.1650</td>
<td>1.0914</td>
</tr>
<tr>
<td>13</td>
<td>1.0985</td>
<td>0.1830</td>
<td>1.0951</td>
</tr>
<tr>
<td>14</td>
<td>1.0440</td>
<td>0.0</td>
<td>1.0440</td>
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</table>

Table 4.11: Present worth values of the variable and the energy losses costs of the 14-bus system at different annual interest and inflation rates

<table>
<thead>
<tr>
<th>MODE OF PLANNING</th>
<th>T To</th>
<th>g =10%, r =3% t</th>
<th>f</th>
<th>g =16%, r =10% t</th>
<th>f</th>
<th>g =16%, r =2% t</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CV, S</td>
<td>CL, S</td>
<td></td>
<td>CV, S</td>
<td></td>
<td>CV, S</td>
<td></td>
</tr>
<tr>
<td>static</td>
<td>1 10</td>
<td>1879,116   53,458</td>
<td></td>
<td>1241,151   35,309</td>
<td></td>
<td>1373,227   39,066</td>
<td></td>
</tr>
<tr>
<td>dynamic</td>
<td>1 4</td>
<td>2097,514   54,739</td>
<td></td>
<td>1706,933   44,540</td>
<td></td>
<td>1935,013   50,491</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3</td>
<td>1985,436   58,557</td>
<td></td>
<td>1455,526   42,928</td>
<td></td>
<td>1630,096   48,076</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 3</td>
<td>1879,116   53,534</td>
<td></td>
<td>1241,151   35,359</td>
<td></td>
<td>1373,227   39,121</td>
<td></td>
</tr>
</tbody>
</table>
where \( r \) and \( r_f \) are the annual interest and inflation rates, respectively. \( T \) denotes the time segment under consideration, and \( T_0 \) the length of the time segment in years.

It can be seen from the above results that the system operational and security constraints are satisfied in the different planning time segments.

### 4.6 Reliability Evaluation

It has been mentioned in discussing the objectives to be achieved in the new model that the reliability test has to be, and can be, included in the model. So far the model and its applications which have been introduced, have ignored the system reliability, with the exception of using the weighting factors \( w \) in equations (4.15) and (4.17). In this section the reliability test in the model is discussed. The reliability criterion used is the deterministic contingency tests. The concept is that the designed network has to operate satisfactorily and without any violation to its constraints even under severe outage of one or more of its components. The network performance under contingencies requires introducing some additional constraints to the model to simulate the different outage cases under consideration. This technique ensures that the network design obtained at the optimal solution will perform satisfactorily under these sets of outages as long as none
of the problem constraints has been violated. The strategy used in this work is that the problem (in its static or dynamic mode of planning) can be simulated as an equivalent dynamic problem with planning segments equal to the sum of the different cases to be studied (normal operation case and the outage cases) in each planning time segment, from the point of view of introducing the system constraints. For example if a dynamic solution of 3 time segments is required where one outage case (single or multiple) in both the first and the third time segments, and two contingency cases in the second time segment are required to be evaluated, then the total number of planning segments is,

\[ N = 1 + 1 + 1 + 2 + 1 + 1 = 7 \]

for the base (normal) case and the outage case of the first time segment,

\[ N = 1 + 2 \]

for the base case and the two outage cases of the second time segment

\[ N = 1 + 1 \]

for the third time segment

So there are 7 planning segments in the problem, each to be presented by a group of constraints similar to equations (4.15) to (4.20).

From the cost and the operating facilities point of view, two different modes of study have been followed. The first mode is referred to as "mode 1", where conditions are set to use the same elements in the base case of each time segment,
in the contingency cases (after eliminating the elements involved in the outages) of this particular time segment. In the second mode, referred to by "mode 0", these conditions are ignored where any set of facilities, even if it includes future facilities, can be used in the contingency conditions. Then the combined facilities used in the base case and in the outage cases for each time segment obtained in this mode (mode "0"), are considered the design of the network in each particular time segment. Accordingly, the capital cost of only the facilities built in the base cases is considered in "mode 1", while the capital cost in all cases (bases and outages) is considered in "mode 0". As it can be seen that introducing the contingency evaluation to the model requires modifications in formulating both the constraints and the objective function. This concept will become more clear after introducing such modifications in the following subsections.

4.6.1 Objective Function Modifications

The objective cost function of the model, presented in equation 4.1, can then be modified to account for the capital cost of all new facilities used in either the base cases and in the contingency cases in "mode 0", or only for those used in the base cases in "mode 1" as follows,
\[ C = \left[ \sum_{\tau=1}^{NT} \sum_{k=1}^{NK} \sum_{i=1}^{NL} \sum_{\tau,i}^{k} CP \right] + \left[ \sum_{\tau=1}^{NT} \sum_{k=1}^{KC(\tau)} \sum_{i=1}^{NL} \sum_{\tau,i}^{k} (CV + CL) \right] \]

\[ \text{NK} = 1 \text{ for "mode 1"}, \text{ and } \quad = KC(\tau) \text{ for "mode 0"}. \]

(4.21)

where \( KC(\tau) \) is the number of cases to be studied at time \( k \) segment \( \tau \) (outage cases plus the base case), \( CP \) the present worth value of the fixed cost of line \( i \) at planning time segment \( \tau \) built in case \( k \) (\( k = 1 \) for the base case), \( CV \) \( \tau,i \) and \( CL \) \( \tau,i \) the variable and the energy losses costs respectively of line \( i \) which operates in case \( k \) at planning time segment \( \tau \).

Modifications in the variable and the energy losses costs are also introduced due to the different operating time periods assumed for each line during the base and the contingency cases. So the formulation of each term in equation (4.21) is as follows,

\[ CP_k = L \beta \cdot C1 \quad \tau,i \quad i \quad \tau,i \]

(4.22)

where

\[ C1 = cf \cdot \left[ 0.5(\text{NC} - 1) - S \right] \quad \tau,i \quad i \quad \tau,i \]

(4.23)
\[
k \sum_{i=1}^{NC-1} \left[ \frac{1}{(2/\pi) \cdot \tan\left(-P_i \cdot |P| \cdot I/U \right)} \right]_{i=1}^{k} \tag{4.24}
\]

\[
S_3 = \sum_{i=1}^{NC-1} \left[ \frac{1}{(2/\pi) \cdot \tan\left(P_i \cdot (n-1) \cdot I/U \right)} \right]_{i=1}^{k}
\]

where \(|P_i|\) is the absolute value of the total power flow \(P_i\) through line \(i\) at the planning time segment \(\tau_i\) during the contingency case \(k\). For the cases other than the base case \((k = 1)\) of the first planning time period \((\tau = 1)\), the \(k\) term \(C_1\) is formulated as,

\[
k \sum_{i=1}^{NC-1} \left[ \frac{1}{(2/\pi) \cdot \tan\left(-P_i \cdot |P| \cdot I/U \right)} \right]_{i=1}^{k} \tag{4.25}
\]

where

\[
k = \prod_{i=1}^{NT} \left[ \frac{1}{(2/\pi) \cdot \tan\left(P_i \cdot (n-1) \cdot I/U \right)} \right]_{i=1}^{k} \tag{4.26}
\]

\[
k \sum_{i=1}^{NC-1} \left[ \frac{1}{(2/\pi) \cdot \tan\left(P_i \cdot (n-1) \cdot I/U \right)} \right]_{i=1}^{k} \tag{4.27}
\]

\[
k = 1 - \exp\left(-P_i \cdot |P| \cdot I/U \right) \tag{4.28}
\]
\[ S_2 = \sum_{n=1}^{NC-1} \left[ 0.5/\pi \cdot \tan \left( P_1 \cdot \left( n-(1/P_{\tau,i}) \right) \right) \right] \cdot S_5 \cdot \left[ 1 - \frac{k}{2} \right] \] (4.29)

where

\[ K_{T,\tau} = \tau - 1 \text{ for "mode 1", and } \]

\[ K_{T,\tau} = \left[ \sum_{m=1}^{\tau} K_0(m) \right] + (k-1) \text{ for "mode 0".} \]

The modified present value of the variable cost is given in equation (4.29) as

\[ CV_{\tau,i} = c_v \cdot L \cdot \delta_{\tau,i} \cdot d_{\tau,i} \cdot \tau - 1, \ldots, NT \]

\[ \tau = 1, 2, \ldots, NL \]

\[ k = 1, 2, \ldots, KC(\tau) \] (4.29)

where

\[ \delta_{\tau,i} = (1/\tau (\tau)) \cdot \left( (1 + \tau (\tau)) / (1 + \tau (\tau)) \right) \cdot \left[ 1 - \left( 1 + \tau (\tau) \right) \right]^{T_{\tau,i}} \] (4.30)

and

\[ d_{\tau,i} = 1 - \exp \left( -P_1 \cdot \left( 1/P_{\tau,i} \right) \right) \] (4.31)
where $T_{oi}^k$ is the operating time of line $i$ in case $k$ at the planning time segment $\tau_i$. It is equal to the overall time segment $\tau$ if $k = 1$ (base case), otherwise a smaller value is set for $T_{oi}^k$ based on the probability of occurrence of outage conditions of case $k$ during the time segment $\tau$.

The present worth value of the cost of the energy losses presented earlier in equation (4.14) is modified as follows:

$$\text{W}_{CL}^k = W \cdot E \cdot C_3 \cdot \delta_{\tau, i} \cdot \tau_{ai} \cdot \tau_{oi}$$  \hspace{1cm} (4.32)

where

$$E_{\tau, i} = \frac{k}{(V_{\tau, i})^2 \cdot Y_{\tau, i} \cdot P_{N,i} \cdot \cos(\theta) \cdot V_{A, B} \cdot P_{V}}$$ \hspace{1cm} (4.33)

4.6.2 **Constraints Modifications Required For Contingency Analysis**

The system demand satisfaction constraints as well as the active and the reactive power flow equations have to be satisfied in each contingency case as in the base case, at every planning time segment. Since the number of parallel paths in each right-of-way varies in the outage cases based on whether "mode 0" is used (free number) or "mode 1" is employed (the number is restricted to those used in the base case of each time segment under consideration), then the above conditions are presented as follows.
\[
\sum_{i \in N} P_{i} \cos(\theta_{i} - \theta_{j}) = \sum_{i \in N} k \cdot F_{i} = W \cdot P_{i} / V_{AB}; \quad j = 1, 2, \ldots, NB
\] (4.34)

\[
\sum_{i \in N} k \cdot k \cdot V \cdot Y \cdot P_{i} \cdot \cos(\delta - \theta_{i} - \theta_{j}) \cdot F \cdot k = W \cdot Q / V_{AB}; \quad j = 1, 2, \ldots, NT
\]

\[
\sum_{i \in N} V \cdot Y \cdot P_{i} \cdot \sin(\delta - \theta_{i} - \theta_{j}) \cdot F \cdot k = W \cdot Q / V_{AB}; \quad j = 1, 2, \ldots, NB
\]

\[
\sum_{i \in N} V \cdot Y \cdot P_{i} \cdot \sin(\delta - \theta_{i} - \theta_{j}) \cdot F \cdot k = W \cdot Q / V_{AB}; \quad j = 1, 2, \ldots, NT
\]

\[
\sum_{i \in N} V \cdot Y \cdot P_{i} \cdot \sin(\delta - \theta_{i} - \theta_{j}) \cdot F \cdot k = W \cdot Q / V_{AB}; \quad j = 1, 2, \ldots, NB
\]

\[
\sum_{i \in N} V \cdot Y \cdot P_{i} \cdot \sin(\delta - \theta_{i} - \theta_{j}) \cdot F \cdot k = W \cdot Q / V_{AB}; \quad j = 1, 2, \ldots, NT
\]

\[
\sum_{i \in N} V \cdot Y \cdot P_{i} \cdot \sin(\delta - \theta_{i} - \theta_{j}) \cdot F \cdot k = W \cdot Q / V_{AB}; \quad j = 1, 2, \ldots, NB
\]

(4.35)

(4.36)

where

\[
m = k \quad \text{if "mode 0" is used, and}
\]

\[
m = 1 \quad \text{if "mode 1" is used.}
\]

The weighting factors \( W \) are functions in the contingency case (k) since slight overloading in the generating units can be allowed under contingency cases as well as certain cut in the load demand, if necessary. In addition, some line overloading and voltage variable violations to their normal stability limits can also be permitted. Then the upper and the lower bounds on the optimization variables are presented as follows,
Modifying the formulation of the model objective function and constraints makes the model capable of handling the performance of the network under the normal and steady state contingency cases. The advantage of the modified model formulation is mainly that it is always sure that the optimal design will perform satisfactory even under the severe outage conditions studied. In addition there are no restrictions on the type of outages to be studied since the model can handle any of the contingency types, such as those given in chapter 2, whether they are single or multiple outages. On the other hand, the larger the number of outages to be studied, the larger the size of the problem becomes. Accordingly, an accurate contingency selection and ranking routine has to be used in order to include only the most severe contingencies in the study.
4.6.3 Application For Planning With Contingency

The new developed transmission planning model has been applied in its modified form (equations (4.21) to (4.39)) to the 14-bus system shown in Fig. 4.6, and described in section 4.5.2.1. The optimal designs of the system for 10 years (1987-1997) using the static mode of planning are obtained with the contingency test, using both "mode 0" and "mode 1".

4.6.3.1 Reliability Criteria

The following reliability criteria are used in the study:

1. Peak load, where winter peak load to be met securely with all transmission are in service and the two largest thermal generating units are out of service. By consulting Table 4.6, the generating unit at bus 6 (200 MW capacity) and one generating unit at bus 8 (150 MW capacity) have been taken out of service in the outage case studied. The choice of these two units simulates most of the different contingency types studied in the literature. For example partial generation loss at bus 8, complete generation unit loss at bus 6, and branch removal between buses 2 and 6, are simulated by the above considered outages.

2. Voltage decline, where the permissible voltage decline on loss of a bulk transmission circuit is 10% of pre-contingency voltage.
3. transient stability, where the pre-contingency bus swing angle limit is 30 electrical degrees.

4.6.3.2 Results

---

Figure 4.9: Static optimal solution of the 14-bus system under contingency conditions using "mode 1".
The optimal network configuration obtained using "mode 1" in handling the contingency case under consideration, is shown in Fig. 4.9. Figure 4.9(a) presents the network configuration in the base case \((k=1)\), while Fig. 4.9(b) shows the network under contingency conditions \((k=2)\).

Table 4.12: Optimal parameter values of the 14-bus system accounting for contingency using "mode 1"

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>NORMAL OPERATION</th>
<th></th>
<th></th>
<th>OUTAGE OPERATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ACTIV-</td>
<td>REAC-</td>
<td>TO'</td>
<td>TO'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>POWER</td>
<td>TIVE</td>
<td>BUS</td>
<td>BUS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MW.</td>
<td>POWER</td>
<td>VOLT-</td>
<td>SWING</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MVAR</td>
<td></td>
<td>AGE.</td>
<td>ANGLE</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>p.u.</td>
<td>rads</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>67.9</td>
<td>-48.7</td>
<td>1.079</td>
<td>-0.12</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>113.4</td>
<td>-36.2</td>
<td>1.053</td>
<td>-0.07</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>119.8</td>
<td>30.9</td>
<td>1.04</td>
<td>-0.25</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>111.8</td>
<td>35.1</td>
<td>1.036</td>
<td>-0.23</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>-200.0</td>
<td>-75.4</td>
<td>1.087</td>
<td>-0.13</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>68.0</td>
<td>1.6</td>
<td>1.075</td>
<td>-0.15</td>
</tr>
<tr>
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<td>4</td>
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<td>-21.7</td>
<td>1.063</td>
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</tr>
<tr>
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<td>5</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>-150.0</td>
<td>-58.4</td>
<td>1.115</td>
<td>-0.17</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>-12.1</td>
<td>-1.3</td>
<td>1.087</td>
<td>-0.09</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>-210.0</td>
<td>-71.8</td>
<td>1.089</td>
<td>-0.11</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>-9.4</td>
<td>0.1</td>
<td>1.112</td>
<td>-0.04</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>64.6</td>
<td>8.0</td>
<td>1.079</td>
<td>-0.12</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>-87.6</td>
<td>-11.1</td>
<td>1.111</td>
<td>-0.04</td>
</tr>
<tr>
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<td>6.2</td>
<td>1.076</td>
<td>-0.12</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>0.0</td>
<td>0.0</td>
<td>1.044</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 4.9 shows that the line facilities used in the contingency case are the same as those used in the base case, with the exception of line 6-2 which is practically involved in the contingency. The optimal power flow patterns
along with the receiving end bus voltage magnitudes and swing angles, with and without outages, are given in Table 4.12.

The corresponding static optimal solution of the 14-bus system under the same contingency test obtained using "mode 0" is given in Fig. 4.10 and Table 4.13.

---

Figure 4.10: Static optimal solution of the 14-bus system using "mode 0" in studying the contingency conditions.
Table 4.13: Optimal parameter values of the 14-bus system accounting for contingency, using "mode 0"

<p>| FROM BUS | TO BUS | NORMAL OPERATION | | OUTAGE OPERATION | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>ACTIVE POWER, MW</th>
<th>REACTIVE POWER, MVAR</th>
<th>VOLTAGE, p.u.</th>
<th>SWING ANGLE, radians</th>
<th>ACTIVE POWER, MW</th>
<th>REACTIVE POWER, MVAR</th>
<th>VOLTAGE, p.u.</th>
<th>SWING ANGLE, radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>49.7</td>
<td>-42.4</td>
<td>1.084</td>
<td>-0.12</td>
<td>131.8</td>
<td>-51.3</td>
<td>1.073</td>
<td>-0.25</td>
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<td>1</td>
<td>95.2</td>
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<td>1.053</td>
<td>-0.08</td>
<td>177.3</td>
<td>-38.9</td>
<td>1.042</td>
<td>-0.14</td>
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<tr>
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<td>3</td>
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<td>1.026</td>
<td>0.26</td>
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<tr>
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<td>-0.0</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>63.0</td>
<td>4.8</td>
<td>1.074</td>
<td>0.17</td>
<td>268.0</td>
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<td>-29.9</td>
<td>1.065</td>
<td>0.26</td>
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<td>-83.9</td>
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<td>0.0</td>
<td>-</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.0</td>
</tr>
<tr>
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<td>0.0</td>
<td>0.0</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
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<td>0.0</td>
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<td>-126.0</td>
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<td>52.3</td>
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<td>115.4</td>
<td>11.5</td>
<td>1.073</td>
<td>-0.25</td>
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<td>-75.3</td>
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<td>1.12</td>
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<td>-154.0</td>
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<tr>
<td>12</td>
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<td>1.086</td>
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<td>116.9</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.044</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 4.10 shows that the line facilities used in the network under contingency are not the same as in the base case. Combining the facilities used with and without contingency in Fig. 4.10 produces the same network configuration obtained by "mode 1". The different costs (fixed, variable, and energy losses costs) of the expansion design obtained using "mode 1" and "mode 0", along with the costs in the static case of planning without contingency test, are listed for comparison in Table 4.14. An annual
interest and inflation rates of 8% and 2%, respectively, are used in calculating the different costs. An operating time period of 3 months in the 10 years planning period is assumed for the contingency conditions. A specific criterion for determining such operating period may be developed in the future, taking into account the probability of occurrence of the different outages as well as their relative severity.

**Table 4.14: Cost comparison for different static design conditions**

<table>
<thead>
<tr>
<th>MODE OF PLANNING</th>
<th>TIME SEGMENT</th>
<th>CASE STUDIED, k</th>
<th>FIXED COST, $</th>
<th>VARIABLE COST, $</th>
<th>ENERGY LOSS COST, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static without outages</td>
<td>1</td>
<td>k=1</td>
<td>0.0</td>
<td>1,373,227</td>
<td>39,066.0</td>
</tr>
<tr>
<td>Static with outages</td>
<td>1</td>
<td>k=1, &quot;mode 1&quot;</td>
<td>303,845.0</td>
<td>1,592,007</td>
<td>42,449.0</td>
</tr>
<tr>
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<td>k=2</td>
<td>0.0</td>
<td>145,485</td>
<td>14,234.0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>k=1, &quot;mode 0&quot;</td>
<td>0.0</td>
<td>1,373,227</td>
<td>39,095.0</td>
</tr>
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<td>k=2</td>
<td>303,845.0</td>
<td>145,485</td>
<td>12,647.0</td>
</tr>
</tbody>
</table>

4.6.3.3 Discussion of The 14-Bus System Planning With Contingency

From the above applications we can see that the model is capable of handling the reliability test efficiently. The contingency evaluation is performed by including different sets of constraints with accompanied modifications in the cost function to ensure that the optimal design obtained
will perform satisfactorily under outage conditions. All
types of single or multiple contingencies can be included at
the same time segment or at different time segments, with
the static or the dynamic mode of planning. Comparing the
network configurations obtained with the reliability test,
using "mode 1" and "mode 0", Fig. 4.9 and Fig. 4.10,
respectively, it is found that the overall final design is
the same in both cases. Future generator bus 7 and two lines
(existing line (9-10) and future line (7-12)) are used in
this study while they have not been used before in the
static design obtained without the contingency evaluation
(Fig. 4.8). An increase in the network overall cost is
resulted (Table 4.14), in this case for the sake of
improving the network reliability. For example, the use of
line (9-10) prevents the isolation of generator bus 9 or bus
10, if line (9-12) or line (10-12) is subjected to outage,
as it would happen in the design shown in Fig. 4.8. Due to
the near radiality construction of the network, it is
noticed that some of the bus swing angles become at their
stability limits under the contingency conditions. To
appreciate the addition of the new facilities to the static
design to improve the network performance under
disturbances, the static solution of the network with the
same outages is obtained using "mode 1", since it is the
practical mode of operation under contingency, but without
imposing any limits on the bus voltage magnitudes and swing
angles in the network. The overall network design in this case is shown in Fig. 4.11.

Figure 4.11: Network configuration under outages, using "mode 1" and without limits on the bus voltage variables.
Table 4.15: Values of the network parameters under contingency, obtained using "mode 1"

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>ACTIVE POWER, MW</th>
<th>REACTIVE POWER, MVAR</th>
<th>'TO' BUS VOLTAGE MAGNITUDE, p.u.</th>
<th>'TO' BUS SWING ANGLE, electrical degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>322.12</td>
<td>-131.29</td>
<td>1.3457</td>
<td>-31.377</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>367.62</td>
<td>-118.94</td>
<td>1.0302</td>
<td>-21.727</td>
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<tr>
<td>2</td>
<td>3</td>
<td>132.19</td>
<td>-32.305</td>
<td>1.1218</td>
<td>-58.356</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>114.65</td>
<td>52.457</td>
<td>1.0141</td>
<td>-55.81</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>263.23</td>
<td>31.202</td>
<td>1.0911</td>
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<tr>
<td>3</td>
<td>4</td>
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<td>1.2241</td>
<td>-57.48</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
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<td>1.3499</td>
<td>-30.684</td>
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<tr>
<td>7</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>-114.5</td>
<td>-270.45</td>
<td>1.162</td>
<td>-38.89</td>
</tr>
<tr>
<td>9</td>
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<tr>
<td>12</td>
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<td>-80.317</td>
<td>1.2039</td>
<td>-41.126</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>152.14</td>
<td>110.5</td>
<td>1.3457</td>
<td>-31.377</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>121.93</td>
<td>-171.28</td>
<td>1.3575</td>
<td>-54.54</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>-</td>
<td>-</td>
<td>1.044</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The power flow pattern and the bus voltage variables (voltage magnitudes and swing angles) in the network under the outage of generator bus 6 and one of the largest generating units of bus 8, are given in Table 4.15 for "mode 1". From Fig. 4.11 it is found that the design obtained is the same as that obtained before in the static solution without contingency evaluation (Fig. 4.8), but on the account of severe violations of the voltage parameters to their stability limits. These violations lead to unstable performance of the network under outage conditions. The advantages of including the reliability test in the planning
model would be more appreciated if the system under consideration has more alternatives to be used (forming more closed loops system), and with more outages to be studied in the design. The lack of practical data prevented so far the further applications for the model.

4.7 Concluding Remarks

The above applications and results show that the new optimization model for long range transmission planning is capable of performing, efficiently and accurately, both the static and the dynamic modes of planning. The nonlinearity nature of the model formulation allows the accurate presentation of the different network cost functions as well as the system operational and security constraints. The model in its final form is capable of handling the different types of outages as a method for a reliability test. This test is very important in the planning process to make sure that the system will perform satisfactorily under disturbances, otherwise unstable system performance may occur. Since the inclusion of the contingency evaluation procedure increases the size of the problem, an accurate and highly efficient contingency ranking routine is required to allow the model to examine only the severe outage cases.
Chapter V

CONCLUSIONS_AND_FUTURE_WORK

The results obtained from the applications of the new models presented and discussed in chapters 3 and 4, suggest the overall conclusions of this work which are summarized in the following sections. The conclusions are followed by the recommendations for future work in the area of power system planning.

5.1 Long_Range_Distribution_Planning_Conclusions

The conclusions derived from chapter 3 concerning the long range planning for distribution systems are listed as follows:

1. An accurate and a continuous model in the variables of long range distribution planning has been successfully developed.

2. The model overcomes most of the limitations existed in the previously reported models in the power distribution system planning.

3. The new model takes into account the exact functions of the running cost, the capitalized cost, and the cost of the energy losses of both feeders and substations in the network.
4. The formulation of the model is suitable for both static (one-time step) and dynamic (time-phased) planning of large distribution networks, and has been applied successfully in both modes.

5. It has been shown that the dynamic solution is preferable to the static solution in terms of lower overall network cost.

6. The continuity nature of the model permits further implementation of other nonlinear applications associated with the planning studies, such as reliability and contingency analysis.

7. The number of variables involved in the new model is much less than the corresponding number in the previously reported models.

8. Despite the continuity and the number of variables in the proposed model, the advanced optimization routine used allows the optimal solutions to be always obtained efficiently.

9. The sparsity facilities available in the routine makes the model capable of dealing with large distribution systems accurately and efficiently.

10. The model permits the implementation of the new feature of possible expansion in some or all of the existing network facilities. This feature would give more economical and practical plans.
5.2 Transmission Planning Conclusions

The following are the conclusions obtained from the applications of the new transmission planning model.

1. A new optimization model for long range transmission planning is developed.

2. The model presents an accurate continuous cost function which includes the fixed cost of any new lines, the operating and maintenance cost, and the cost of the energy losses.

3. The model has the advantage of including accurately the system operational and security constraints.

4. The model has the superiority of accounting for the stability limits on both bus voltage magnitudes and swing angles.

5. The reactive power constraints are included in the model producing more accurate and informative results. In addition they provide a simple mean for bounding bus reactive powers which play an important role in the system stability.

6. Both static and dynamic plans of the transmission systems can be obtained easily and accurately by the model.

7. The advanced optimization routine used overcomes the difficulties that may arise due to the continuity nature of the model.
8. The model can be rearranged to optimize a certain performance index of the system under predetermined budgetary constraints.

9. The contingency analysis is considered in the model as a part of the planning process where the different types of outages can be handled at the same time, with the static or the dynamic modes of planning.

10. Although the effect of the energy losses cost is relatively small and usually has no effect on the planning decisions, it has been shown that it affects the values of the bus voltage variables, leading to relatively more stable systems.

11. Presenting the accurate ac power flow equations as a set of constraints in the model eliminates the continuous need for load flow solutions for each change occurs in the network during the planning process.

12. The model has relatively smaller number of variables compared to the other models which perform similar studies.

5.3 Recommendations for Future Work

The intensive study of the results obtained in this work offers wide ranging possibilities for different areas to be extensively examined in the near future. Among these fields of study are...
1. The careful study of the effect of the different weighting factors, as well as the model parameters on the results obtained from both the transmission and the distribution models. This study is necessary in order to unify the constants in each model to be suitable for any system required to be studied by these models.

2. Further implementation of the models to more practical systems is required in order to establish the full strength and the different points of weakness in the new developed models. The lack of practical data in the literature prevented so far the satisfaction of this point. The availability of such practical data will also allow the determination of the largest system which can be handled by each of the new models.

3. Due to the special format required for presenting the systems data to the optimization routine used, then the development of a routine capable of writing the model data from the system data is necessary for reducing the effort imposed on the user.

4. Including an accurate and efficient load forecasting routine in each model would give additional advantages in the direction of producing a complete planning model in each area of planning in power system.

5. The development of a contingency ranking routine is another area for improving the performance of the developed models.
The idea of producing a complete package for planning the overall electric power system suggests the inclusion of both transmission and distribution planning models in one model. This achievement could lead to the global optimal planning of the power system instead of planning each individual area in the system separately. This package, if it is developed, will not of course be capable of handling large power systems in accurate details due to the computer memory limitations. It can give however outlines for the overall optimal design of the system and then further specific studies can be performed.
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LIST OF PUBLICATIONS FROM THIS THESIS


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