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**MATHEMATICAL MODELLING OF LOADING PROBLEM
IN FLEXIBLE ASSEMBLY SYSTEMS**

by

Xiaomei Duan

A Thesis
submitted to the
Faculty of Graduate Studies and Research
through the Department of
Industrial Engineering in Partial Fulfilment
of the requirements for the Degree
of Master of Applied Science at
the University of Windsor

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ABSTRACT

Assembly plays an important role in modern industry. Flexible Assembly Systems (FAS), as a new, promising production technology, can provide high productivity and flexibility. The research efforts made on FAS problems are insufficient, especially on the loading problem. This research investigates the loading problem in FAS by means of 0-1 integer programming modelling. Model 1 is formulated to assign operations to work stations so as to minimize the total processing cost. Important assembly realities such as different component sizes, component mating, operation precedence, station workload balance and different work station characteristics are considered. Model 2 is proposed to deal with the system with large number of work stations. The workload unbalance among work stations is considered as a penalty in the objective function in Model 2 in order to reduce problem size. Some examples are solved using these models for computational experience and also to reflect the utility of these models. The effects of workload balance are investigated. Sensitivity analyses of total available time and component feeder capacity are carried out. The effects of the weight of the penalty term in Model 2 are also investigated. Some important issues related to this study are discussed such as the total cost of system, total available time, cycle time and work station reliability. The comparison of the two proposed models are given.

DEDICATION

To my mother

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TABLE OF CONTENTS

ABSTRACT	vi
DEDICATION	vii
ACKNOWLEDGEMENTS	viii
LIST OF TABLES	xi
LIST OF FIGURES	xii
CHAPTER 1. INTRODUCTION	1
1.1 Background	1
1.2 Statement of the Problem	4
1.3 Outline of the Project	5
CHAPTER 2. LITERATURE REVIEW	9
2.1 Review of the Related Literatures	9
2.2 The Limitations of the Existing Literature	17
2.3 Motivation of the Proposed Research	18
2.4 Objectives of the Proposed Research	18
CHAPTER 3. MATHEMATICAL MODELLING	20
3.1 Definitions	20
3.2 Problem Description	21
3.3 Model Formulation	24
CHAPTER 4: COMPUTATIONAL EXPERIENCE: ANALYSIS AND DISCUSSIONS	33
4.1 Examples for Model One	33
4.2 Examples for Model Two	39

CHAPTER 5: DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS	77
5.1 Discussions	77
5.2 Conclusions	81
5.3 Recommendations for Future Research	82
REFERENCES	87
APPENDICES	90
I. Equations of Formulations	91
II. Computer Programs for Data Input	104
VITA AUCTORIS	126

LIST OF TABLES

Table 2.1 Basic differences between FAS and assembly lines	19
Table 3.1 Number of equations of each constraint	32
Table 4.1 Total available times and feeder capacities at work stations	42
Table 4.2 Component size	43
Table 4.3 Processing costs of operations on different work stations	44
Table 4.4 Processing times of operations on different work stations	46
Table 4.5 Component requirements of various assemblies	48
Table 4.6 Optimal work station loading, base case example of Model 1	50
Table 4.7 Optimal work station loading, case 1 of Model 1 ($p=10\%$)	51
Table 4.8 Optimal work station loading, case 2 of Model 1 ($p=20\%$)	52
Table 4.9 Optimal work station loading, case 4 of Model 1 ($p=40\%$)	53
Table 4.10 Optimal work station loading, case 5 and 6 of Model 1 ($p=50\%$ & 60%)	54
Table 4.11 Objective function values with respect to different p values	55
Table 4.12 Work station utilization with different p values	56
Table 4.13 Objective function values with respect to different available times	57
Table 4.14 Optimal work station loading, case 7 of Model 1 ($90\% T_k$)	58

Table 4.15 Optimal work station loading, case 8 of Model 1 (80% T_k)	59
Table 4.16 Optimal work station loading, case 9 of Model 1 (70% T_k)	60
Table 4.17 Optimal work station loading, case 10 of Model 1 (60% T_k)	61
Table 4.18 Actual total processing time at each work station with respect to different available times	62
Table 4.19 Total processing costs with respect to different feeder capacities	63
Table 4.20 Work station utilization with respect to different feeder capacities	64
Table 4.21 Optimal work station loading, case 11 of Model 1 (80% F_k)	65
Table 4.22 Optimal work station loading, case 12 of Model 1 (60% T_k)	66
Table 4.23 Optimal work station loading, case 13 of Model 1 (40% T_k)	67
Table 4.24 Resulting work station loading, base case of Model 2 (without workload balance consideration)	68
Table 4.25 Results of the examples for Model 2	69
Table 4.26 Resulting work station loading with $\mu=300$ in Model 2	70
Table 4.27 Resulting work station loading with $\mu=600$ in Model 2	71
Table 4.28 Resulting work station loading with $\mu=900$ in Model 2	72
Table 5.1 Total processing costs at different parameter values for the two models	85

LIST OF FIGURES

Figure 1.1 General assembly layout	7
Figure 1.2 Classification of assembly systems	8
Figure 4.1 Total processing cost vs. p value (Model 1)	73
Figure 4.2 Total processing cost vs. total available time (Model 1)	74
Figure 4.3 Total processing cost vs. component feeder capacity (Model 1)	75
Figure 4.4 Total processing cost vs. the weight of the penalty (Model 2)	76
Figure 5.1 Cost comparison (two models)	86

CHAPTER ONE

INTRODUCTION

1.1 Background

Recent technical developments and increased competition in modern industry invoke new strategic targets of production management. They include:

- Development of new products with increased frequency (diversification),
- Ability to react to a greater number of variants (customer adaptation),
- Saving in shorter delivery time,
- Reduction of cost by all available means and
- High quality in all phases of the product life cycle

Assembly plays an important role in the modern manufacturing industry. It consumes approximately 30% of the production cost and 60% of the manufacturing time in major industries, such as metal-making, automotive, machinery manufacture, and electronics (Andreasen and Ahm, 1988).

An assembly line, in its basic form, consists of a finite set of work elements or tasks. Each task has an operation processing time and a set of precedence relations, which specify the permissible orderings of the tasks (Ghosh and Gagnon, 1989). An assembly process is made up of handling, composing and checking operations. At a greater level of detail, the assembly process can be divided into storage, transport and positioning operations. It also consists of special operations related to joining, checking and special processes (Andreasen and Ahm, 1988). Figure 1 shows the general layout of an assembly system and Figure 2 shows the classification of assembly systems.

Flexibility is the key concept in future development of assembly systems. The flexibility of an assembly system is its ability to assemble different products by switching over to new tasks (Andreasen and Ahm, 1988). The introduction of flexible assembly systems in manufacturing industries provides significant potential for both productivity improvement and a more rapid response to market demands (Donath and Graves, 1988).

Flexible assembly systems (FAS) are essentially automated material handling systems and flow lines with versatile work stations that can simultaneously assemble a variety of part types. The most important advantages of FAS are high productivity and flexibility (Milberg and Schmidt, 1990). If an FAS is properly designed, the following

benefits can be expected from this new and expensive production technology:

- high output,
- low input, in terms of unit cost,
- short lead time,
- high equipment utilization,
- reduced workforce requirements,
- higher and consistent quality levels.

FAS has been closely associated with the concept of flexible manufacturing systems (FMS). Both FAS and FMS can be broadly described as computer integrated manufacturing systems with some features in common. However, they differ in the following aspects:

- The processing time of each operation in FAS is much less than that in FMS. Thus, the ratio of the transfer time to the processing time in FAS is very high in comparison with that of FMS.
- It is normally necessary to join several components at the same time and/or in the same place in FAS, while the operations at each work station in FMS involve just one part at a time.
- The material handling system in FAS is more complex than that in FMS.

- The cycle time restricts each work station to operate at a fixed pace in FAS.

1.2 Statement of the Problem

FAS planning problems are basically decision making problems. These decisions have to be made before the FAS can begin to assemble parts. Flexible assembly systems are different from traditional assembly lines. Consequently, the planning problems in FAS are more difficult to deal with because:

- each work station can perform many operations,
- each operation may be processed at more than one work station,
- the system can assemble several components simultaneously,
- large number of components and tools are involved,
- job routing is flexible, i.e., jobs may be routed only to the work stations with necessary operations
- the operation times differ from work station to work station.

One of the important production planning problems of FAS is the loading problem, which may be stated as the task of assigning each operation of each assembly to a work station, subject to the technical and capacity constraints of the system (Kasilingam and Lashkari, 1990).

Research to date on the loading problem has generally been focussed on FMS which is concerned with machining operations (for example, Kusiak 1983, Lashkari et al. 1987, Shanker and Srinivasulu 1989). The loading problem in FAS, however, differs from the loading problem in FMS in several aspects. For example:

- a large number of components and tools are involved,
- limited component and tool staging capacity at each work station,
- small processing time for individual operations,
- mating of two or more components involved in one operation.

Due to these characteristics and difficulties in FAS, the planning problem in its general form is hard to tackle directly. In the present study, the loading problem in FAS is investigated by means of mathematical modelling.

1.3 Outline of the Project

The presentation of the study is organised as follows. A review of related literature is given in Chapter 2. Motivation and objectives of the proposed research are also given in Chapter 2. Mathematical models are formulated for loading problem in FAS. The description of these models is presented in Chapter 3. A number of examples are solved using these models for computational experience and also to reflect the utility of these

models. Analysis and discussions of the results of the example problems are presented in Chapter 4. Finally, Chapter 5 provides a summary of the discussions, conclusions and recommendations for future research.

Figure 1.1 General Assembly Layout (Andreasen and Ahm, 1988)

Figure 1.2 Classification of Assembly Systems (Lotter, 1986)

CHAPTER TWO

LITERATURE REVIEW

2.1 Review of the Related Literature

2.1.1 Assembly Line Problem

The topics of assembly line design, balancing and scheduling have been of continuing interest to researchers (Graves and Lamar, 1983; Baybars, 1986; Rachamadugu and Talbot, 1991).

Ghosh and Gagnon (1989) presented a comprehensive literature review of the design, balancing and scheduling of assembly systems. In their review, the literature on assembly line balancing was classified into four categories:

- (1) **Single Model Deterministic (SMD):** It assumed dedicated, single-model assembly lines where the task times were known deterministically and an efficiency criterion was to be optimized.
- (2) **Single Model Stochastic (SMS):** It introduced the concept of task-time variability. The operation times were seldom constant.
- (3) **Multi/Mixed Model Deterministic (MMD):** It assumed deterministic task times, but introduced the concept of an assembly line producing multiple products.

Multi-model lines assembled two or more products separately in batches. The issues of model selection, model sequencing and launching rate(s), and model lot sizes became more critical here than in the single model case.

- (4) Multi/Mixed Model Stochastic (MMS): It differed from its counterpart MMD in that stochastic operation times were allowed. However, factors such as learning effects, worker skill level, job design and worker task time variability became more difficult to analyze because the line was frequently rebalanced for each model assembled.

As pointed out by Ghosh and Gagnon (1989), more attention had been paid to economic objective criteria instead of technical criteria because a solution which maximized efficiency did not necessarily maximize profit. This trend was likely to continue and expand, with the economic measurement criterion perhaps gaining acceptance as the only measure broad enough to encompass the enlarged, strategic view of the assembly system.

In addition, Ghosh and Gagnon (1989) categorised 77 assembly-line considerations into eight levels:

- (i) Output focus

- (ii) Line type
- (iii) Process and equipment considerations
- (iv) Facility considerations
- (v) Work-station considerations
- (vi) Task-related considerations
- (vii) Work-related considerations
- (viii) Schedule-related considerations

Many of the factors had been individually modelled, but few studies had addressed their integration.

2.1.2 Flexible Assembly Problem

Flexible assembly system is a relatively new area of research, and, therefore, only limited literature is available.

Ammons, et al. (1986) discussed a number of issues associated with design and operation of a flexible printed circuit board assembly system including machine configuration, operating policies and their impact on system performance.

Hall and Steckel (1986) addressed different problems in the design phase of developing flexible assembly systems such as determining design objectives, part type selection

during system design, part type design for automatic assembly, work station design and material handling design. These problems were illustrated in the context of a system that was then under development. Some important differences between FAS design and FMS design (such as work station design, demand on the material handling system, tooling and component delivery) were also discussed.

Bullinger and Sauer (1987) reported a modular assembly system for heavy fork lifts. The implemented system was described along with the layout, transportation system, as well as integrated material zone. The planning process was supported by application of a simulation model. Simulation of different ways of operating the system were used for making decision and judgement for a particular system and its operation.

Donath and Graves (1988) proposed a heuristic scheduling and routing procedure, called SCHEDULE, for solving FAS scheduling problem. This procedure recognized multiple products being simultaneously assembled in an FAS environment and provided near real-time decision-making capacity for scheduling and part routing. The heuristic explicitly worked towards high workplace utilizations, while attempting to respect job due dates. It was assumed that the material-handling response time could support these routing decisions. The extended testing conditions and the results associated with the use of SCHEDULE over the test conditions were also discussed (Donath & Graves,

1989).

Ranky (1988) addressed the design rules on how to design flexible assembly and inspection cells. The rules were justified by solid model simulation and the author's industrial and laboratory experience. The solid model based animation and simulation programs had been developed to represent important steps in the solid model based cell design and simulation efforts. The simulation programs had been written for the systems of both conventional assembly line and truly flexible assembly cell. The difference in flexibility and reliability between the dedicated and the truly flexible assembly systems was discussed and shown by the simulation programs.

Das, et al. (1989) presented a cost-based model for capacity planning of Flexible Robotic Assembly Systems capable of handling batch assembly of multiple end products. The non-linear integer programming model was based on a specific cell design procedure that considered two aspects: robot selection and assembly cell configuration in forming the assembly cell. The model incorporates stochastic variations in the batch arrival rate, batch size, and processing times.

Donath, et al. (1989) gave a detailed definition of FAS scheduling process and discussed key differences between this process and the classical simple job shop

scheduling process. A mathematical model was developed for the minimum makespan problem, and it was shown that the problem was NP-complete. The design of heuristic solution procedures was also discussed.

El-Tamimi, et al. (1989) developed a discrete modular simulation program for the assessment of real-time part-sequencing systems for flexible assembly cells. The program was used with different sequencing strategies to study the cell performance and decision lead time. A combined index was used as a measure of the cell performance. The index was a function of throughput, throughput time and cell utilization. Time response of the part sequencing rule was assessed by the decision lead-time index which is a function of the average computation time required by computer module to generate a decision.

Ketcham and Watt (1989) developed a manufacturing logic incorporated in a parametric simulation system called SIMBED to represent the characteristics of flexible manufacturing/flexible assembly systems with multiple products and flexible part routings. The core simulation model incorporated the logic necessary to represent four FMS/FAS subsystems, i.e., assembly work stations, product routing decisions, inventory policies, and material handling. The modular structure of the core model was mirrored in a simulation database and a spreadsheet-like interface.

Udomkesmalee and Daganzo (1989) discussed the advantages of parallel processing in flexible assembly systems and analyzed job sequences after parallel processing and provided strategies for dealing with jobs getting out of sequence. One strategy involved re-sequencing materials using material buffers and another involved re-sequencing jobs using job buffers. An analysis determined the size of material buffers. The size of job buffers was estimated using analytical approximation and formulas fitted to simulation results.

Grasso (1990) introduced a late start policy to launch and transfer the AGVs to improve the system performance and proposed a heuristic sequencing algorithm for an AGV-based flexible assembly line. Different AGV launch policies, buffer capacities and management goals were considered by using the computer-aided planning procedures.

Yano, et al. (1991) studied design and sequencing issues in flexible assembly systems for large products. Two sequencing rules were considered which had a tremendous impact on the design of the system, they were: in sequence (INSEQ) which required that the departure sequence from a queue be the same as the arrival sequence; and first-come-first-served (FCFS). A simulation model was developed using hypothetical but representative data for the proposed system. The capacity planning issues and scheduling strategies to facilitate the component mating process were investigated.

2.1.3 Loading Problem in FAS

Much of the research on the loading problem has been focused on flexible manufacturing systems. Researchers paid little attention to handle the loading problem in FAS. As a result, the literature in this area is quite limited.

Ammonset al. (1985) formulated a bi-criterion model for a special case of the loading problem applied to flexible assembly. The objectives were to balance work station workload and to minimize the total number of job-to-work station assignments. Several limitations, however, existed, such as

- components were of identical size.
- an assembly needed only one unit of a particular component.
- the operations performed at work stations do not have any precedence relations.
- work stations have the same operational characteristics, e.g. operation times and component feeder capacities are the same at all work stations.

Kasilingam and Lashkari (1990) proposed a 0-1 integer programming formulation of the loading problem in flexible assembly systems. Several important generalizations were considered in the aggregating stage such as, various component sizes, component

ming and different operational characteristics of work stations. A Lagrangian relaxation based solution approach was also presented.

2.2 Limitations of Existing Literature

The review of the literature indicates the following:

- (1) Many researches focused on the problems of assembly lines; however, the results are not valid in flexible assembly systems because FAS and assembly lines have different system characteristics (see Table 2.1). The problem of FAS is more difficult to handle.
- (2) Problems in FAS, until now, have not received enough attention in the literature. Research work on FAS is limited, making further investigations necessary.

In specific, the following issues need to be further investigated and/or incorporated into any research on FAS:

- Components are not necessarily of identical size.
- Two or more units of a particular component may be involved in an operation.
- Two or more types of components may be involved in an operation.
- Work stations do not necessarily have the same characteristics.

- Some operations have precedence relations

2.3 Motivation of the Proposed Research

- Flexible assembly systems are a new and promising production technology, which represent expensive investment. In an existing FAS, planning decisions are major factors in trying to improve system utilization.
- The research efforts made on the loading problem in FAS are insufficient. Some significant factors are ignored in existing literature, which need to be considered in future investigations.

2.4 Objectives of the Proposed Research

This proposed research focuses on the loading problem of FAS. The objectives are:

- to develop mathematical models to generate optimal solution to FAS loading problem considering the issues raised earlier in Section 2.2.
- to analyze the system sensitivity to variations in the following parameters:
 - (i) total time available in a work station
 - (ii) component feeder capacity
 - (iii) workload unbalance among work stations

Table 2.1 Basic differences between FAS and assembly lines (Udomkesmalee and Daganzo, 1989; Ranky, 1988; Donath, et al., 1989)

Flexible Assembly Systems	Assembly Lines
Jobs are not necessarily know prior to the system design.	Designed for a set of known jobs.
Not paced at a fixed rate or paced at a fixed rate but not necessarily the same rate for all work stations.	All work stations process jobs at a fixed rate
Jobs can stay at the work stations until work is properly completed. Variable processing times are allowed to improve product quality.	Sufficient time may not be always available to complete operations at work stations and to recycle jobs to repair.
Each work station can perform more than one operation type.	Each work station can perform only one operation type.
Each operation can be performed at more than one work station.	Each operation can be performed at only one work station.
Job routing is flexible. (Job may be routed to only the work stations with necessary operations)	Job routing is unique. (Conveyance mechanisms force jobs to go through all work stations)

CHAPTER THREE

MATHEMATICAL MODELLING

3.1 Definitions

Before discussing the proposed problem, some important concepts are introduced:

Component:

An entity on which work is performed at the work stations. This phrase can be used to mean a piece of part or a sub-assembly.

Work station:

A physical device or human capable of executing operations on a component.

Operation:

A physical process being executed by a work station on a component(s) which will change the state of the component(s) on which it is performed.

Assembly:

In this research an assembly means a set of operations which need to be performed to produce a final product.

Relative workload:

Relative workload at a work station is the ratio between the processing time of all the operations assigned to this work station and total available time at this work station.

Planning horizon:

A pre-specified time duration over which some orders of assemblies are planned to be processed.

Total available time:

Total available time at each work station during a planning horizon depends on many factors, such as the planning horizon, expected breakdown rate of the work station, etc.

Component feeder capacity:

The total number of slots of component feeder at the work station.

3.2 Problem Description

The loading problem addressed in the present study is stated as follows: there are K work stations, I assemblies and L types of components. Certain assembly operations J_i

have to be performed in order to produce a final assembly i . Some operations have precedence relations. Each work station can perform different operations. Each operation of an assembly may be processed at more than one work station. For each operation one or more units of various components may be required. There are some restrictions on work station available time and component feeder capacity at each work station. The objective is to assign the operations to the work stations so as to minimize the total assembly cost while trying to achieve a balanced workload among work stations.

The problem is formulated as a 0-1 integer programming model with the following assumptions:

- Data regarding production requirements of the assemblies, total available times at work stations, and component feeder capacities at work stations are available.
- Processing times and costs for various assembly operations are known.
- The operation sequence of an assembly is known and no alternative operation sequences are considered.

The following notation will be used:

Subscripts

l -- index of component, $l=1, \dots, L$

i -- index of assembly, $i=1, \dots, I$

j -- index of operation, $j=1, \dots, J_i$

k -- index of work station, $k=1, \dots, K$

Parameters

a_{lij} -- number of units of component l required by operation j of assembly i

t_{ijk} -- time to perform operation j of assembly i at work station k

c_{ijk} -- cost to perform operation j of assembly i at work station k

d_i -- order size of assembly i during the planning horizon

T_k -- total available time at work station k during the planning horizon

F_k -- maximum number of slots of component feeder at work station k

b_l -- number of slots occupied by one unit of component l

p -- a percentage

μ -- weight of the penalty term in the objective function

Decision variables

$$x_{ijk} = \begin{cases} 1 & \text{if operation } j \text{ of assembly } i \text{ is assigned to work station } k \\ 0 & \text{otherwise} \end{cases}$$

3.3 Model Formulation

The objective of loading problem is to minimize the overall assembly cost, which depends on production requirements of the assemblies and the work station at which a particular assembly operation is performed.

A mathematical model is developed such that, once solved, the following decisions can be made based on the solution:

- operation allocation
- process routing

3.3.1 Model One

The objective function of Model 1 is to minimize the total processing cost:

$$\text{MIN} \quad Z = \sum_i d_i \sum_j \sum_k x_{ijk} c_{ijk} \quad (1)$$

The decisions are constrained as follows:

$$\sum_i d_i \sum_j t_{ijk} x_{ijk} \leq T_k \quad \forall k \quad (2)$$

The total processing time of assemblies at each work station does not exceed the total available time restriction during the planning horizon on that work station.

$$\sum_l b_l \sum_j a_{lij} x_{ijk} \leq F_k \quad \forall i, \forall k \quad (3)$$

The total number of components assigned to a work station, which are involved in an assembly, must not exceed the component feeder capacity at that work station.

$$\sum_k x_{ijk} = 1 \quad \forall i, \forall j \quad (4)$$

Any operation of an assembly is performed at exactly one work station.

$$x_{i(j+1)k} \leq \sum_{h=1}^k x_{ijh} \quad \forall i, \forall k \quad (5)$$

This constraint expresses the requirement that certain operation j is to be processed prior to operation $j+1$. An assumption is made here that the components involved in one operation have no precedence relations. If they have, the operation should be broken into two or more separate operations according to the precedence relations. Then, the operation sequence can be taken into account in this constraint.

$$\left| \frac{\sum_i d_i \sum_j t_{ijk} x_{ijk}}{T_k} - \frac{\sum_i d_i \sum_j t_{ijh} x_{ijh}}{T_h} \right| \leq p \quad (6)$$

$$k=1, \dots, K-1, \quad h=k+1, \dots, K$$

This is a workload balancing constraint, i.e. the relative workload difference between any two work stations must not exceed a given percentage.

$$x_{ijk} \in (0,1) \quad \forall i, \forall j, \forall k \quad (7)$$

The decision variables are integer.

Although constraints (2) and (3) were, in simpler forms, considered in previous work (Ammons, et al., 1985), they are generalized in the present study to handle more complicated real-life situations such as various component sizes and different work station characteristics.

Constraint (5) is introduced in this model because the precedence relation between operations is so common in FAS that it can not be ignored when the loading decisions are made. So far, there is no loading methodology which deals explicitly with precedence in FAS (Ammons, et al., 1985).

Similarly, constraint (6) is introduced here to deal with the important question of balancing the work station utilization to the extent possible. If the workloads are not reasonably well balanced, it will be difficult to operate the system without the many problems associated with large work in process queues and bottlenecks (Ammons, et al, 1985).

Constrain (6) deals with the absolute value of the relative workload difference between any two work stations. It means, for any two work stations k and h, two constraints have to be generated. One is

$$\frac{\sum_i d_i \sum_j t_{ijk} x_{ijk}}{T_k} - \frac{\sum_i d_i \sum_j t_{ijh} x_{ijh}}{T_h} \leq p \quad (8)$$

$$k=1,2,\dots,K-1, \quad h=k+1,\dots,K$$

The other one is

$$\frac{\sum_i d_i \sum_j t_{ijk} x_{ijk}}{T_k} - \frac{\sum_i d_i \sum_j t_{ijh} x_{ijh}}{T_h} \geq -p \quad (9)$$

$$k=1,2,\dots,K-1, \quad h=k+1,\dots,K$$

For an FAS involving I assemblies, J_i operations for assembly i , and K work stations, the total number of constraint equations are listed in Table 3.1. From Table 3.1 we can see that the problem size increases when the number of work stations in the system increases, especially for constraint 6. In order to reduce the problem size, a modified model is proposed as follows.

3.3.2 Model Two

In this model, the workload unbalance is considered as a penalty term in the objective function formulation. The objective function, now, is:

$$\begin{aligned}
 \text{MIN } Z = & \sum_i d_i \sum_j \sum_k x_{ijk} c_{ijk} \\
 & + \mu \sum_{k=1}^{K-1} \sum_{h=k+1}^K \left| \sum_i d_i \sum_j \frac{t_{ijk} x_{ijk}}{T_k} - \sum_i d_i \sum_j \frac{t_{ijh} x_{ijh}}{T_h} \right|
 \end{aligned} \tag{10}$$

Constraint (6) in Model 1 is relaxed here to reduce the problem size. The penalty term in this objective function is an absolute value, which is difficult to handle. A direct way is to form a quadratic function of the penalty term, rendering the objective function quadratic instead of linear. Although integer programming problems involving quadratic terms may be solved by linearizing the quadratic terms, the number of variables and constraints is increased by the linearization process. In large problems,

this represents a considerable increase in the problem size, which may render the solution process quite difficult. A suboptimal solution procedure is instead proposed, which may be used in some cases. The procedure is explained below.

MODEL TWO (SIMPLIFIED)

The objective function is now formulated as:

$$\begin{aligned}
 MIN \quad Z = & \sum_i d_i \sum_j \sum_k x_{ijk} c_{ijk} \\
 & + \mu \sum_{k, k \neq s} \left(\sum_i d_i \sum_j \frac{t_{ijs} x_{ijs}}{T_s} - \sum_i d_i \sum_j \frac{t_{ijk} x_{ijk}}{T_k} \right)
 \end{aligned} \tag{11}$$

where s indicates the bottleneck work station which has the highest workload in the system, and μ is the penalty weight.

$$S.t. \quad \sum_i d_i \sum_j t_{ijk} x_{ijk} \leq T_k \quad \forall k \tag{12}$$

$$\sum_l b_l \sum_j a_{lij} x_{ijk} \leq F_k \quad \forall i, \forall k \tag{13}$$

$$\sum_k x_{ijk} = 1 \quad \forall i, \forall j \tag{14}$$

$$X_{i(j+1)k} \leq \sum_{h=1}^k x_{ijh} \quad \forall i, \forall k \quad (15)$$

$$X_{ijk} \in (0,1) \quad \forall i, \forall j, \forall k \quad (16)$$

$$\frac{\sum_i d_i \sum_j t_{ijs} x_{ijs}}{T_s} - \frac{\sum_i d_i \sum_j t_{ijk} x_{ijk}}{T_k} \geq 0 \quad (17)$$

$$\forall k \quad k \neq s$$

Constraint (17) guarantees that each penalty term in the objective function is non-negative.

The procedure to apply Model 2 is as follows:

1. Solve Model 1 by dropping constraint (6).
2. Determine the work station which has the highest relative workload.
3. Designate this work station as the target work station s and use Model 2 to solve the problem.

In situations where there may be more than one work station with haigh utilizations, better solutions may be obtained by selecting another work station as the target, and reapplying the above procedure.

As was pointed out earlier, the solution of Model 2 can not be guaranteed to be optimum. The solution obtained is only feasible, and achieves an improvement in the workload balance in the system.

Table 3.1 Number of equations of each constraint

Constraint	# of Equations
2	K
3	$I \times K$
4	ΣJ_i
5	$I \times K$
6	$K(K-1)$
# of variables	$K \sum_{i=1}^I J_i$

CHAPTER FOUR

COMPUTATIONAL EXPERIENCE:

ANALYSIS AND DISCUSSIONS

To illustrate the applications of the models developed, a number of examples are presented and analyzed in this chapter.

4.1 Examples for Model One

4.1.1 Base Case

In this example, we consider a flexible assembly system with six work stations. The planning horizon is 80 hours. The total available times and the component feeder capacities for the six work stations are given in Table 4.1. There are eight assemblies and six types of components involved in this example. The component sizes are listed in Table 4.2, and the processing cost and processing time for each operation of each assembly are shown in Table 4.3 and 4.4, respectively. The order sizes for the eight assemblies are assumed to be 30, 20, 20, 50, 70, 20, 50, and 40, respectively.

The component requirements for each operation of each assembly are given in Table 4.5. The sequencing requirements are:

- Operation 3 must be processed prior to operation 4 for assemblies 1, 2, 3, 4, 6 and 8.
- There is no precedence restriction on other operations.

The required percentage of relative workload balance, p , is assumed to be 30%. The model 1 formulation is applied to this problem, which includes 198 0-1 integer variables, and 165 constraints. After eliminating infeasible operation-work station combinations, the problem size reduces to 122 0-1 integer variables, and 135 constraints.

The problem is solved using LINDO on an IBM 4381 machine. The program code appears in Appendix I, and the computer program written to input all the given data is shown in Appendix II. The input time for this problem is about 10 minutes and the running time is 50 seconds. The resulting work station loading is given in Table 4.6. The objective function value in this case is 11950. The solution indicates that, for assembly 1, for instance, operations 1, 2, 3 and 4 are assigned to work stations 4, 3, 1 and 4, respectively. The results for other assemblies can be similarly interpreted.

4.1.2 Effects of the Workload Balance

To investigate the effect of the workload balance on loading decisions, additional

computations are performed on the base case example with different p values. With different workload balance percentages, the loading decisions are totally different from case to case (see Tables 4.7 to 4.10), and consequently, the objective function value changes as shown in Table 4.11 and Figure 4.1. Table 4.12 shows the changes in the work station utilization values when the value of p is changed. According to the results shown in Table 4.6 and Table 4.8, the operation allocation of case 2 ($p=20\%$) is different from that of the base case ($p=30\%$). For example, all the four operations of assembly 4 are assigned to work station 4 in the base case, but in case 1, the four operations are assigned to work stations 4, 1, 2, and 3, respectively, to achieve the more critical workload balance requirement ($p=20\%$).

The objective function of Model 1 is to minimize the total assembly cost. Each operation is supposed to be performed at the work station with the least cost. When the workloads among work stations are required to be balanced within some given limits, some operations may have to be assigned to the work stations with higher processing costs. As a result, the loading decisions are changed and the total assembly cost is increased. Thus, a more balanced workload may be obtained at the expense of an increase in total assembly cost.

Figure 4.1 shows that the total assembly cost decreases when p is increased. However,

there seems to be a critical point beyond which the total assembly cost does not decrease, but somehow levels off; in the example, the critical example is 50%. This implies that, at this point, constraint (6) in Model 1 becomes inactive. On the other hand, it seems impossible to get exactly balanced workloads among work stations, i.e. when $p=0$. In this example, no feasible solution can be found when p is decreased below 5%. Actually, achieving exactly balanced workload in FAS is neither necessary nor desirable; even in an asynchronous assembly line, the greatest throughput is not achieved in an exactly balanced line (Muth and Alkaff, 1987).

4.1.3 Sensitivity Analysis of Total Available Time for Model One

The total available time at each work station has a significant effect on the loading decisions in FAS. The base case still refers to the example in section 4.1. In cases 7 to 10, all the parameters remain the same as in the base case except the total available time at each work station which is now reduced by 10% in each case. Comparing the results of these 5 cases, the objective function values are increased as the total available time at each work station decreases (as shown in Table 4.13 and Figure 4.2). The operation allocations are also different from case to case (see Tables 4.14 to 4.17). It is noted that in cases 8 and 9, the total available times at each work station are, respectively, 80% and 70% of the base case; however, the total processing costs for both cases are the same, although the operation allocations are different; operation 5 of assembly 3 is switched

from work station 4 to work station 2. The actual total processing times at all work stations are listed in Table 4.18 indicating changes in operation allocation at each work station when the total available time is reduced.

Total available time is one of the important characteristics of the work stations in FAS. It affects loading decisions not only because it is considered as a resource of the system (as reflected in constraint (2)), but also because it plays a major role in the relative workload balance through constraint (6). A small change in total available time may result in different loading decisions when constraint (2) is active. In some cases, work stations may have surplus available time and thus a small change in total available time may not affect constraint (2). However, it alters the relative workload in constraint (6), which may result in a change in total processing cost and the loading decisions. In case 7, for instance, the total available time at each work station is decreased by 10% relative to the base case. The reduced time is still enough for processing the operations assigned to each work station, and thus constraint (2) is not violated. But the relative workload at each work station is now changed. In order to maintain the desired workload balance percentage, the operations have to be reassigned; the total processing cost and the loading decisions are consequently different from the base case.

4.1.4 Sensitivity Analysis of Component Feeder Capacity

To investigate the influence of component feeder capacity, sensitivity analysis is performed on the base case considering variations in the feeder capacity at each work station. The component feeder capacities are reduced to 40% of their base case values in steps of 20% each time. Table 4.19 and Figure 4.3 show that the component feeder capacities do not have a significant effect on the total processing cost. In case 11, the feeder capacity at each work station is decreased by 20%, but the total processing cost and the loading decisions are the same as those in the base case (Tables 4.19 and 4.20). When the feeder capacity is decreased further in cases 12 and 13, the cost and the loading decisions begin to change, but only slightly; for instance, the cost increase in case 12 is 0.17%. The loading results of cases 11 to case 13 are reported in Tables 4.21 to 4.23.

The component feeder capacity restriction is considered for individual assemblies. That means that when the feeder capacity at one work station is enough for a particular assembly, this assembly can be assigned to that work station, no matter what the order size of this assembly is. For example, assembly 4 has 4 operations which need 20 slots of the component feeder in total. In the base case and case 11, these four operations are assigned to work station 4 which has 26 slots of feeder capacity in the base case and 21 slots in case 11. In case 12, the component feeder capacity at this work station reduces to 16 slots. Obviously, in this case, the feeder does not have enough capacity to hold all

the components required by the four operations assigned to work station 4. Consequently, operations 3 and 4 are switched to work stations 2 and 3, respectively, in order to satisfy the feeder capacity constraint at work station 4.

4.2 Examples for Model Two

4.2.1 Base Case

The same data (from Table 4.1 to Table 4.5) are used for Model 2. The program code appears in Appendix I, and the computer program for data input is listed in Appendix II. Before applying Model 2, the problem is solved using Model 1 without constraint (6) in order to find the bottleneck work station which has the highest workload. This is considered as the base case for Model 2. Table 4.24 presents the work station loading for this case. Obviously, the resulting loading decisions are not well balanced, since the workload balance restriction is eliminated. The work station utilizations in this case are shown in Table 4.25. The target work station, station #4, has a utilization rate of 59.4%, while work station #3 has a utilization rate of only 10.4%. Obviously, the system is not working under a well balanced condition with this loading policy. The workload has to be balanced to some extent. Taking work station 4 as the target work station s, Model 2 can then be applied.

4.2.2 Effect of the Weight of the Penalty Parameter

When Model 2 is applied, the workload unbalance is added to the objective function as a penalty. The problem now includes 122 0-1 integer variables, and 110 constraints.

The weight of the penalty term, μ , determines the extent to which workload balance is considered important. When workload balance is a major consideration in loading decisions, μ must be a large number. On the other hand, μ may be given a small value if cost is important. So the choice of μ is subjective and depends on how important the workload balance is. Some computations are carried out with different values of parameter μ , and the results are listed in Table 4.25. The cost increase with respect to increased weight of the penalty term is shown in Figure 4.4.

As the weight of the penalty term is increased, the relative workload at the bottleneck work station is decreased. Some of the operations performed at work station 4 are assigned to other work stations in order to minimize workload disparities among the stations (Tables 4.26 to 4.28). As a result, the workload balance situation can be improved. When the weight $\mu=300$, the workload at work station 4 decreases from 59.4% in the base case to 43.5%. The difference between the highest workload and the lowest is 32.6%; at the same time, the value of the penalty term in the objective function reduces from 1.278 in the base case to 0.667, which is a measure of the total difference

between the workload of the target work station and those of all the other work stations. In the case of $\mu=900$, the difference between the highest workload and the lowest is as low as 14%, indicating a considerable improvement in the workload balance. Although the solution is not guaranteed to be optimal, a feasible loading decision with well balanced workloads can be obtained by using Model 2.

Table 4.1 Total available times and feeder capacities for work stations

WORKSTATION k	AVAILABLE TIME T_k (min.)	FEEDER CAPACITY F_k (slots)
1	3360	24
2	3360	20
3	3840	30
4	3840	26
5	4320	34
6	4320	38

Table 4.2 Component size

# of slots	Component <i>l</i>					
	1	2	3	4	5	6
b_l	2	1	1	1	1	4

Table 4.3 Processing costs of operations at different work stations

ASSEMBLY	OPERATION	PROCESSING COST C_{ijk}					
		WORK STATION k					
		1	2	3	4	5	6
1	1	15	-	17	13	23	19
	2	--	-	24	24	23	23
	3	17	-	19	16	--	25
	4	7	-	7	5	10	9
2	1	-	11	15	-	11	17
	2	-	12	-	-	19	11
	3	-	3	3	-	3	3
	4	-	-	27	-	33	33
	5	-	9	11	-	15	13
3	1	5	4	-	3	6	5
	2	20	20	-	-	19	-
	3	3	3	-	3	3	3
	4	15	13	-	13	23	21
	5	7	6	-	6	-	9
4	1	5	4	4	3	-	-
	2	25	-	31	23	-	-
	3	15	13	17	13	-	-
	4	-	6	7	6	-	-
5	1	-	4	4	3	-	5
	2	-	13	17	13	-	-
	3	-	9	11	-	-	9
6	1	15	-	17	-	23	21
	2	5	-	4	-	6	5
	3	33	-	-	-	32	33
	4	5	-	4	-	5	5
	5	3	-	3	-	3	3
7	1	-	13	23	-	-	-
	2	-	12	20	-	21	-
	3	-	-	6	-	5	-

Table 4.3 (Cont'd)

ASSEMBLY	OPERATION	PROCESSING COST c_{ijk}					
		WORK STATION k					
j	i	1	2	3	4	5	6
8	1	18	-	25	22	-	33
	2	12	-	-	16	-	11
	3	3	-	4	5	-	-
	4	3	-	3	3	-	3

Table 4.4 Processing times of operations at different work stations

ASSEMBLY NO.	OPERATION NO.	PROCESSING TIME t_{ijk} (minute)					
		WORK STATION k					
i	j	1	2	3	4	5	6
1	1	10	-	10	8	12	9
	2	--	-	10	9	12	11
	3	12	-	11	10	--	12
	4	4	-	2	2	5	4
2	1	-	8	9	-	11	8
	2	-	5	--	-	6	5
	3	-	1	1	-	1	1
	4	-	-	16	-	18	16
	5	-	6	6	-	8	6
3	1	2	2	-	1	3	2
	2	8	8	-	-	10	--
	3	1	1	-	1	1	1
	4	10	10	-	8	12	10
	5	4	4	-	3	--	4
4	1	2	2	2	1	-	-
	2	18	--	19	15	-	-
	3	10	10	10	8	-	-
	4	--	4	4	3	-	-
5	1	-	2	2	1	-	2
	2	-	10	10	8	-	-
	3	-	6	6	-	-	7
6	1	10	-	10	-	12	10
	2	2	-	2	-	3	2
	3	15	-	--	-	17	16
	4	2	-	2	-	2	2
	5	1	-	1	-	1	1
7	1	-	10	14	-	--	-
	2	-	9	12	-	11	-
	3	-	--	2	-	2	-

Table 4.4 (Cont'd)

ASSEMBLY NO.	OPERATION NO.	PROCESSING TIME t_{ijk} (minute)					
		WORK STATION k					
i	j	1	2	3	4	5	6
8	1	12	-	15	14	-	16
	2	8	-	-	10	-	12
	3	1	-	2	2	-	-
	4	1	-	1	1	-	1

Table 4.5 Component requirements of various assemblies

ASSEMBLY i	DEMAND d_i unit/week	OPERATION j	COMPONENT I					
			1	2	3	4	5	6
1	30	1	2					
		2			1			
		3					2	
		4		1				1
2	20	1				2		1
		2	1				2	
		3		2				
		4				1	1	
		5			4			
3	20	1	1					
		2		1				
		3				1		
		4						2
		5			1			
4	50	1			2			
		2	1			4		
		3		2			2	
		4						2
5	70	1		1				
		2			2			1
		3					4	
6	20	1					1	
		2	4					
		3				2		
		4			1			1
		5		1			1	
7	50	1		1				1
		2			2		1	
		3	1					

Table 4.5 (Cont'd)

ASSEMBLY <i>i</i>	DEMAND d_i unit/week	OPERATION <i>j</i>	COMPONENT <i>l</i>					
			1	2	3	4	5	6
8	40	1	1			1		
		2					1	2
		3	1		1			
		4				2		

Table 4.6 Optimal work station loading, base case example of Model 1

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1	3		2	1 4		
2		5	3 4		1	2
3		3 5		1 4	2	
4				1 2 3 4		
5		2		1		3
6	1		2 5		3 4	
7		1 2			3	
8	1 3		4			2

Table 4.7 Optimal work station loading, case 1 of Model 1 ($p=10\%$)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1			3	1 4	2	
2			3 4		1	2 5
3				1 3 4 5	2	
4	2	3		1 4		
5				1 2		3
6			1 2 5		3	4
7		1 2			3	
8	1		3 4			2

Table 4.8 Optimal work station loading, case 2 of Model 1 ($p=20\%$)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1				1 3 4	2	
2		5	3 4		1	2
3				1 3 4 5	2	
4	2	3	4	1		
5				1 2		3
6			1 2 5		3	4
7		1 2			3	
8	1 3		4			2

Table 4.9 Optimal work station loading, case 4 of Model 1 ($p=40\%$)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1			2	1 3 4		
2		5	3 4		1	2
3	3	5		1 4	2	
4				1 2 3 4		
5		2		1		3
6	1 3		2 4 5			
7		1 2			3	
8	1 3		4			2

Table 4.10 Optimal work station loading, case 5 and 6 of Model 1 ($p=50\%$ & $p=60\%$)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1				1 3 4	2	
2		5	3 4		1	2
3	3	4		1 5	2	
4				1 2 3 4		
5		2 3		1		
6	1 3		2 4 5			
7		1 2			3	
8	1 3		4			2

Table 4.11 Objective function values
with respect to different p values

	Required percentage	Cost
Case 1	p=10%	12240
Case 2	p=20%	12080
Case 3 (base case)	p=30%	11950
Case 4	p=40%	11920
Case 5	p=50%	11890
Case 6	p=60%	11890

Table 4.12 Work station utilization with different p values

	Work station utilization (%)					
	1	2	3	4	5	6
Case 1	36.7	37.3	27.3	36.2	28.2	28.5
Case 2	37.8	41.8	21.9	40.1	28.2	25.7
Case 3 (base case)	28.7	49.7	19.7	49.5	20.8	24.8
Case 4	27.7	49.2	20.3	57.3	17.4	24.8
Case 5	19.7	58.8	10.4	59.4	20.8	21.1
Case 6	19.7	58.8	10.4	59.4	20.8	21.1

Table 4.13 Objective function values with respect to different available times.

	AVAILABLE TIME	COST
Base case	T_k	11950
Case 7 *	90% T_k	12000
Case 8	80% T_k	12070
Case 9	70% T_k	12070
Case 10	60% T_k	12120

* In cases 7 to 10, it is assumed that $p=30\%$.

Table 4.14 Optimal work station loading, case 7 of Model 1 (90% T_k)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1	3		2	1 4		
2		5	3 4		1	2
3	3	4		1 5	2	
4		3	4	1 2		
5				1 2		3
6	1		2 5		3 4	
7		1 2			3	
8	1 3		4			2

Table 4.15 Optimal work station loading, case 8 of Model 1 (80% T_1)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1			3	1 4	2	
2		5	3 4		1	2
3	3 4			1 5	2	
4		3	4	1 2		
5				1 2		3
6	1		2 5		3	4
7		1 2			3	
8	1 3		4			2

Table 4.16 Optimal work station loading, case 9 of Model 1 (70% T_k)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1			3	1 4	2	
2		5	3 4		1	2
3	3 4	5		1	2	
4		3	4	1 2		
5				1 2		3
6	1		2 5		3	4
7		1 2			3	
8	1 3		4			2

Table 4.17 Optimal work station loading, case 10 of Model 1 (60% T_k)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1			3	1 4	2	
2		5	3 4		1	2
3	3			1 4 5	2	
4	2	3		1 4		
5				1 2		3
6			1 2 5		3	4
7		1 2			3	
8	1 3 4					2

Table 4.18 Actual total processing time at each work station
with respect to different available times.

	Actual total processing time (minutes)					
	Work station					
	1	2	3	4	5	6
Base case	1080	1870	740	1900	900	1070
Case 7	1100	1770	940	1810	900	1070
Case 8	940	1570	970	1810	1220	1110
Case 9	940	1650	970	1750	1220	1110
Case 10	1480	1570	930	1370	1220	1110

Table 4.19 Total processing costs with respect to different feeder capacities.

	FEEDER CAPACITY	COST
Base case	F_k	11950
Case 11 *	80% F_k **	11950
Case 12 *	60% F_k **	11970
Case 13 *	40% F_k **	11990

* In cases 11 to 13 the value of p is 30%.

** The reduced values of component feeder capacity in cases 7, 8 and 9 are rounded to their nearest integer values.

Table 4.20 Work station utilization with respect to different feeder capacities

	Work station utilization (%)					
	Work station					
	1	2	3	4	5	6
Base case	28.7	49.7	19.7	49.5	20.8	24.8
Case 11	28.7	49.7	19.7	49.5	20.8	24.8
Case 12 *	37.2	49.2	18.7	45.6	20.4	24.8
Case 13 *	31.4	47.1	16.7	45.6	28.2	25.7

* In cases 12 and 13, due to round up error in Constraint (6), the workload unbalance between stations 2 and 3 is slightly higher than $p=30\%$.

Table 4.21 Optimal work station loading, case 11 of Model 1 (80% F_1)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1	3		2	1 4		
2		5	3 4		1	2
3		3 5		1 4	2	
4				1 2 3 4		
5		2		1		3
6	1		2 5		3 4	
7		1 2			3	
8	1 3		4			2

Table 4.22 Optimal work station loading, case 12 of Model 1 (60% F_k)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1	3			1 4	2	
2		5	3 4		1	2
3	3	4 5		1	2	
4		3	4	1 2		
5				1 2		3
6	1 3		2 4 5			
7		1 2			3	
8	1 3		4			2

Table 4.23 Optimal work station loading, case 13 of Model 1 (40% F_k)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1	3			1 4	2	
2		5	3 4		1	2
3	3 5	4		1	2	
4		3	4	1 2		
5				1 2		3
6	1		2 5		3	4
7		1 2			3	
8	1 3		4			2

Table 4.24 Resulting work station loading, base case of Model 2 (without workload balance consideration)

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1				1 3 4		2
2		3 5	4		1	2
3				1 3 4 5	2	
4				1 2 3 4		
5		2 3		1		
6	1 5		2		3 4	
7		1 2			3	
8	1 3		4			2

Table 4.25 Results of the examples for Model 2.

	Relative Workload						TOTAL PENALTY VALUE	
	Work Station							
	1	2	3	4*	5	6		
Base case	19.7%	58.8%	10.4%	59.4%	20.8%	21.1%	11890	1.278
$\mu=300$	43.1%	42.8%	10.9%	43.5%	29.2%	24.8%	12090	.667
$\mu=600$	39.4%	39.1%	23.4%	39.5%	28.2%	25.7%	12120	.442
$\mu=900$	38.3%	38.6%	27.1%	38.8%	29.2%	24.8%	12170	.360

* The target workstation with the maximum workload.

Table 4.26 Resulting work station loading with $\mu=300$ in Model 2

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1				1 3 4	2	
2		3 5	4		1	2
3		3		1 4 5	2	
4	2	3		1 4		
5				1 2		3
6	1		2 5		3 4	
7		1 2			3	
8	1 3		4			2

Table 4.27 Resulting work station loading with $\mu=600$ in Model 2

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1				1 3 4	2	
2			3 4 5		1	2
3		3		1 4 5	2	
4	2	3	4	1		
5				1 2		3
6	5		1 2		3	4
7		1 2			3	
8	1 3 4					2

Table 4.28 Resulting work station loading with $r_i=900$ in Model 2

ASSEMBLY i	WORK STATION K					
	1	2	3	4	5	6
1				1 3 4	2	
2			3 4 5		1	2
3				1 3 4 5	2	
4	2	3	1 4			
5				1 2		3
6	5		1 2		3 4	
7		1 2			3	
8	1 3		4			2

Fig. 4.1 Cost vs. p value
(Model 1)

Fig. 4.2 Cost vs. total available time

Fig. 4.3 Cost vs. feeder capacity

Fig 4.4 Cost vs. weight of penalty
(Model 2)

CHAPTER FIVE

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

5.1 Discussions

5.1.1 Total Cost of an FAS

The total cost of an assembly system over its expected life consists of fixed costs associated with the equipment and facilities of the system itself and the necessary operating costs of the system (Pinto, et al., 1983). In this study, it is assumed that the equipment and facilities in the system are predetermined during the planning horizon. During this period and at this decision level, the work stations and other facilities involved to accomplish the required assembly tasks are known. The purpose of this research is to assign operations to a set of work stations in order to minimize the operating cost, i.e. total processing cost.

5.1.2 Total Available Time and Work Station Reliability

Total available time at work stations is a major concern in this research. As mentioned in Chapter 3, total available time depends on the planning horizon, work station breakdown rate, etc. In the proposed models, the total available time at each

work station restricts the total operation assignments to that work station. Furthermore, it influences the relative workload at that work station, since the relative workload is defined as the actual processing time of all operations as a percentage of the total available time at that work station. Workload balance is measured in relative terms. Hence, workload balance situation varies with a change in total available time.

In this study, the work station reliability is considered as deterministic. The breakdown rate of a work station is assumed to be a known constant. For example, if the planning horizon is one week, the system runs two shifts a day, and the work station breakdown rate is 20%, then the total available time at this work station is computed as $8 \times 2 \times 5 \times (1-20\%) = 64$ hours. When the work station reliability is reduced, its total available time becomes shorter, and consequently, fewer operations could be assigned to this work station.

5.1.3 Cycle Time vs. Total Available Time

One of the important characteristics of assembly systems is that cycle time restricts a work station to operate at a fixed pace. Cycle time is simply the amount of time available at each work station to complete the assigned tasks for one unit of output (Pinto, et al., 1983). Cycle time is directly related to production rate, or system throughput. The desired production rate is normally based on reasonable aggregate

demand forecast for the planning horizon. If r is the desired production rate per hour, then c , the corresponding cycle time in minutes per unit, is defined as:

$$c = \frac{60}{r}$$

In traditional assembly lines, all work stations process jobs at the same rate. Cycle time is the same at each work station. However, cycle times at work stations can be different in FAS. This could mean either that work is not paced at a fixed rate (i.e. processing times can vary), or that work is paced at a fixed rate but not necessarily at the same rate for all work stations (Udomkesmalee and Daganzo, 1989). Total processing time for each assembly at a work station must be less than the cycle time at that work station. This can be expressed as:

$$\sum_j t_{ijk} x_{ijk} \leq c_k \quad \forall i, \forall k$$

Taking summation over all i on both sides of this equation and multiplying each term by the corresponding order size results in constraint (2) of Model 1. The total processing time at each work station in the system is related to the cycle time. In this study, the relation between the total available time and the cycle time is considered as:

$$T_k \leq \sum_i d_i c_k \quad \forall k$$

If available time restriction is satisfied, cycle time restriction is satisfied automatically. Cycle time restricts the processing time for individual assemblies, while total available time restricts the processing time in aggregating stage.

5.1.4 Comparison of the Two Models

Two models are proposed in Chapter 3. Some computational results are reported in Chapter 4. The optimum loading decision can be made by applying Model 1. As discussed in Chapter 3, the problem size in Model 1 increases when the system consists of more work stations. Constraint (6) generates $K(K-1)$ equations when the system has K work stations. In the examples in Chapter 4, the system has 6 work stations and therefore, the workload balance constraint results in 30 equations.

Model 2 is developed with a view towards reducing the problem size. Constraint (17) in Model 2 generates only $(K-1)$ equations for the problem. For the same number of work stations, the number of equations of constraint (17) is reduced from 30 to 5.

For the same problem, the two models have the same number of decision variables but different numbers of constraints. When the number of work stations is small, the problem size in Model 1 is in a practical range for computation. Model 1 is preferred because it determines optimum loading decisions directly. When the number of work

stations in a system is large. Model 2 is more attractive because it has less constraints than Model 1 even though it is not guaranteed to find optimal solutions. The computational requirements of Model 2 are also less demanding. Table 5.1 and Figure 5.1 show the total processing costs by using the two models applied to the same problem. It is observed that Model 2 provides solutions which are fairly close to those of Model 1.

5.2 Conclusions

As mentioned in Chapter 2, the loading problem in FAS has not attracted a great deal of attention. The existing works on FAS loading have a number of limitations as outlined earlier. In this study, a new formulation of the loading problem in FAS is provided which employs 0-1 integer programming modelling, and which takes into account the following important factors:

- The components may have different sizes.
- Two or more units of a particular component may be involved in an operation.
- Two or more types of components may be involved in an operation.
- Component feeder capacities are limited and different from one work station to another.
- Each work station may have its own cycle time.

- Operations may have precedence relations.
- Workloads are balanced to avoid bottleneck work station.

Obviously, the proposed models can handle more realistic FAS loading problems by considering the above factors.

In order to illustrate the application of the loading models, several examples are solved. The effects of workload balance are investigated, and the sensitivity of the model to variations in the total available time and in the component feeder capacity is studied. The effects of the weight of the penalty term in Model 2 are also investigated. The computational experience with the models leads to the conclusion that the models are highly sensitive to the data. When parameters change, the loading decisions are quite different from case to case; however, the corresponding change in objective function value is relatively small, pointing to the versatility of FAS in providing alternative solutions. From this point of view, the solution of Model 2 is even more acceptable although it is not guaranteed to be optimal. Some important issues in FAS are discussed, such as the total cost of the system, total available time, etc, and the results of the two proposed models are compared.

5.3 Recommendations for future research

The present study is conducted based on certain assumptions. Further efforts may be

worthwhile to extend the study. The following is a list of extensions to be considered in future works:

- (1) There is a need to develop heuristic or approximate solution procedures for large scale problems presented in this study.
- (2) It is implicitly assumed, for the models developed in the present study, that material handling capacity is sufficient or the associated cost is insignificant when compared to the total processing cost. However material handling equipment may become a bottleneck under specific conditions. In such cases, a related constraint on the transportation time of the material handling equipment, should be added to the models. At the same time material handling cost should be considered in the total cost function if it is significant.
- (3) In some circumstances, it is necessary to split an operation of an order into two or more sub-batches. That is, one operation may be simultaneously performed at different work stations. Assigning one operation to more than one work station may balance the work station utilization, and increase throughput.
- (4) When loading decisions are made, it is suggested to consider other factors such

as random machine breakdowns, quality control, alternative precedence among operations, etc.

Table 5.1 Total processing costs at different parameter values,for the two models

Model 1			Model 2		
	BD*	Cost		BD*	Cost
p=30%	30.0%	11950	u=300	32.6%	12090
p=20%	19.9%	12080	u=600	26.6%	12120
p=10%	10.0%	12240	u=900	14.0%	12170

* BD represents the difference between the highest relative workload and the lowest one.

Figure 5.1 Cost comparison (two models)

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APPENDICES

APPENDIX I

EQUATIONS OF FORMULATIONS

1. Model One (Base Case)

MIN $450X_{111}+510X_{113}+390X_{114}+690X_{115}+570X_{116}+720X_{123}+720X_{124}$
 $+690X_{125}+690X_{126}+510X_{131}+570X_{133}+480X_{134}+750X_{136}+210X_{141}+210X_{143}$
 $+150X_{144}+300X_{145}+270X_{146}+220X_{212}+300X_{213}+220X_{215}+340X_{216}+240X_{222}$
 $+380X_{225}+220X_{226}+60X_{232}+60X_{233}+60X_{235}+60X_{236}+540X_{243}+660X_{245}$
 $+660X_{246}+180X_{252}+220X_{253}+300X_{255}+260X_{256}+100X_{311}+80X_{312}+60X_{314}$
 $+120X_{315}+100X_{316}+400X_{321}+400X_{322}+380X_{325}+60X_{331}+60X_{332}+60X_{334}$
 $+60X_{335}+60X_{336}+300X_{341}+260X_{342}+260X_{344}+460X_{345}+420X_{346}+140X_{351}$
 $+120X_{352}+120X_{354}+180X_{356}+250X_{411}+200X_{412}+200X_{413}+150X_{414}+1250X_{421}$
 $+1550X_{423}+1150X_{424}+750X_{431}+650X_{432}+850X_{433}+650X_{434}+300X_{442}$
 $+350X_{443}+300X_{444}+280X_{512}+280X_{513}+210X_{514}+350X_{516}+910X_{522}+1190X_{523}$
 $+910X_{524}+630X_{532}+770X_{533}+630X_{536}+300X_{611}+340X_{613}+460X_{615}+420X_{616}$
 $+100X_{621}+80X_{623}+120X_{625}+100X_{626}+660X_{631}+640X_{635}+660X_{636}+100X_{641}$
 $+80X_{643}+100X_{645}+100X_{646}+60X_{651}+60X_{653}+60X_{655}+60X_{656}+650X_{712}$
 $+1150X_{713}+600X_{722}+1000X_{723}+1050X_{725}+300X_{733}+250X_{735}+720X_{811}$
 $+1000X_{813}+880X_{814}+1320X_{816}+480X_{821}+640X_{824}+440X_{826}+120X_{831}$
 $+160X_{833}+200X_{834}+120X_{841}+120X_{843}+120X_{844}+120X_{846}$

ST

$300X_{111}+360X_{131}+120X_{141}+40X_{311}+160X_{321}+20X_{331}+200X_{341}+80X_{351}$
 $+100X_{411}+900X_{421}+500X_{431}+200X_{611}+40X_{621}+300X_{631}+40X_{641}+20X_{651}$
 $+480X_{811}+320X_{821}+40X_{831}+40X_{841} \leq 3360$
 $160X_{212}+100X_{222}+20X_{232}+120X_{252}+40X_{312}+160X_{322}+20X_{332}+200X_{342}$
 $+80X_{352}+100X_{412}+500X_{432}+200X_{442}+140X_{512}+700X_{522}+420X_{532}+500X_{712}$
 $+450X_{722} \leq 3360$
 $300X_{113}+300X_{123}+330X_{133}+60X_{143}+180X_{213}+20X_{233}+320X_{243}+120X_{253}$
 $+100X_{413}+950X_{423}+500X_{433}+200X_{443}+140X_{513}+700X_{523}+420X_{533}+200X_{613}$
 $+40X_{623}+40X_{643}+20X_{653}+700X_{713}+600X_{723}+100X_{733}+600X_{813}+80X_{833}$
 $+40X_{843} \leq 3840$
 $240X_{114}+270X_{124}+300X_{134}+60X_{144}+20X_{314}+20X_{334}+160X_{344}+60X_{354}$
 $+50X_{414}+750X_{424}+400X_{434}+150X_{444}+70X_{514}+560X_{524}+560X_{814}+400X_{824}$

$+80X834+40X844 \leq 3840$
 $360X115+360X125+150X145+220X215+120X225+20X235+360X245+160X255$
 $+60X315+200X325+20X335+240X345+240X615+60X625+340X635+40X645$
 $+20X655+550X725+100X735 \leq 4320$
 $270X116+330X126+360X136+120X146+160X216+100X226+20X236+320X246$
 $+120X256+40X316+20X336+200X346+80X356+140X516+490X536+200X616$
 $+40X626+320X636+40X646+20X656+640X816+480X826+40X846 \leq 4320$
 $4X111+2X131+5X141 \leq 24$
 $4X113+X123+2X133+5X143 \leq 30$
 $4X114+X124+2X134+5X144 \leq 26$
 $4X115+X125+5X145 \leq 34$
 $4X116+X126+2X136+5X146 \leq 38$
 $6X212+4X222+2X232+4X252 \leq 20$
 $6X213+2X233+2X243+4X253 \leq 30$
 $6X215+4X225+2X235+2X245+4X255 \leq 34$
 $6X216+4X226+2X236+2X246+4X256 \leq 38$
 $2X311+X321+X331+8X341+X351 \leq 24$
 $2X312+X322+X332+8X342+X352 \leq 20$
 $2X314+X334+8X344+X354 \leq 26$
 $2X315+X325+X335+8X345 \leq 34$
 $2X316+X336+8X346+X356 \leq 38$
 $2X411+6X421+4X431 \leq 24$
 $2X412+4X432+8X442 \leq 20$
 $2X413+6X423+4X433+8X443 \leq 30$
 $2X414+6X424+4X434+8X444 \leq 26$
 $X512+6X522+4X532 \leq 20$
 $X513+6X523+4X533 \leq 30$
 $X514+6X524 \leq 26$
 $X516+4X536 \leq 38$
 $X611+8X621+2X631+5X641+2X651 \leq 24$
 $X613+8X623+5X643+2X653 \leq 30$
 $X615+8X625+2X635+5X645+2X655 \leq 34$
 $X616+8X626+2X636+5X646+2X656 \leq 38$
 $5X712+3X722 \leq 20$
 $5X713+3X723+2X733 \leq 30$
 $3X725+2X735 \leq 34$
 $3X811+9X821+3X831+2X841 \leq 24$
 $3X813+3X833+2X843 \leq 30$
 $3X814+9X824+3X834+2X844 \leq 26$

$3X816+9X826+2X846 \leq 38$
 $X111+X113+X114+X115+X116=1$
 $X123+X124+X125+X126=1$
 $X131+X133+X134+X136=1$
 $X141+X143+X144+X145+X146=1$
 $X212+X213+X215+X216=1$
 $X222+X225+X226=1$
 $X232+X233+X235+X236=1$
 $X243+X245+X246=1$
 $X252+X253+X255+X256=1$
 $X311+X312+X314+X315+X316=1$
 $X321+X322+X325=1$
 $X331+X332+X334+X335+X336=1$
 $X341+X342+X344+X345+X346=1$
 $X351+X352+X354+X356=1$
 $X411+X412+X413+X414=1$
 $X421+X423+X424=1$
 $X431+X432+X433+X434=1$
 $X442+X443+X444=1$
 $X512+X513+X514+X516=1$
 $X522+X523+X524=1$
 $X532+X533+X536=1$
 $X611+X613+X615+X616=1$
 $X621+X623+X625+X626=1$
 $X631+X635+X636=1$
 $X641+X643+X645+X646=1$
 $X651+X653+X655+X656=1$
 $X712+X713=1$
 $X722+X723+X725=1$
 $X733+X735=1$
 $X811+X813+X814+X816=1$
 $X821+X824+X826=1$
 $X831+X833+X834=1$
 $X841+X843+X844=1$
 $-X131+X141 \leq 0$
 $-X131-X133+X143 \leq 0$
 $-X131-X133-X134+X144 \leq 0$
 $-X131-X133-X134+X145 \leq 0$
 $-X131-X133-X134-X136+X146 \leq 0$

$-X_{232}-X_{233}+X_{243}\leq 0$
 $-X_{232}-X_{233}-X_{235}+X_{245}\leq 0$
 $-X_{232}-X_{233}-X_{235}-X_{236}+X_{246}\leq 0$
 $-X_{331}+X_{341}\leq 0$
 $-X_{331}-X_{332}+X_{342}\leq 0$
 $-X_{331}-X_{332}-X_{334}+X_{344}\leq 0$
 $-X_{331}-X_{332}-X_{334}-X_{335}+X_{345}\leq 0$
 $-X_{331}-X_{332}-X_{334}-X_{335}-X_{336}+X_{346}\leq 0$
 $-X_{431}-X_{432}+X_{442}\leq 0$
 $-X_{431}-X_{432}-X_{433}+X_{443}\leq 0$
 $-X_{431}-X_{432}-X_{433}-X_{434}+X_{444}\leq 0$
 $-X_{631}+X_{641}\leq 0$
 $-X_{631}+X_{643}\leq 0$
 $-X_{631}-X_{635}+X_{645}\leq 0$
 $-X_{631}-X_{635}-X_{636}+X_{646}\leq 0$
 $-X_{831}+X_{841}\leq 0$
 $-X_{831}-X_{833}+X_{843}\leq 0$
 $-X_{831}-X_{833}-X_{834}+X_{844}\leq 0$
 $-X_{831}-X_{833}-X_{834}+X_{846}\leq 0$
 $.08X_{111}+.1X_{131}+.03X_{141}-.04X_{212}-.03X_{222}-.01X_{232}-.03X_{252}+.01X_{311}$
 $-.01X_{312}+.04X_{321}-.04X_{322}+.01X_{331}-.01X_{332}+.05X_{341}-.05X_{342}+.02X_{351}$
 $-.02X_{352}+.03X_{411}-.03X_{412}+.24X_{421}+.13X_{431}-.13X_{432}-.05X_{442}-.04X_{512}$
 $-.19X_{522}-.11X_{532}+.05X_{611}+.01X_{621}+.08X_{631}+.01X_{641}+.01X_{651}-.13X_{712}$
 $-.12X_{722}+.13X_{811}+.09X_{821}+.01X_{831}+.01X_{841}\leq .3$
 $.08X_{111}-.08X_{113}-.08X_{123}+.1X_{131}-.09X_{133}+.03X_{141}-.02X_{143}-.05X_{213}$
 $-.01X_{233}-.08X_{243}-.03X_{253}+.01X_{311}+.04X_{321}+.01X_{331}+.05X_{341}+.02X_{351}$
 $+.03X_{411}-.03X_{413}+.24X_{421}-.25X_{423}+.13X_{431}-.13X_{433}-.05X_{443}-.04X_{513}$
 $-.18X_{523}-.11X_{533}+.05X_{611}-.05X_{613}+.01X_{621}-.01X_{623}+.08X_{631}+.01X_{641}$
 $-.01X_{643}+.01X_{651}-.01X_{653}-.18X_{713}-.16X_{723}-.03X_{733}+.13X_{811}-.16X_{813}$
 $+.09X_{821}+.01X_{831}-.02X_{833}+.01X_{841}-.01X_{843}\leq .3$
 $.08X_{111}-.06X_{114}-.07X_{124}+.1X_{131}-.08X_{134}+.03X_{141}-.02X_{144}+.01X_{311}$
 $-.01X_{314}+.04X_{321}+.01X_{331}-.01X_{334}+.05X_{341}-.04X_{344}+.02X_{351}-.02X_{354}$
 $+.03X_{411}-.01X_{414}+.24X_{421}-.2X_{424}+.13X_{431}-.10X_{434}-.04X_{444}-.02X_{514}$
 $-.15X_{524}+.05X_{611}+.01X_{621}+.08X_{631}+.01X_{641}+.01X_{651}+.13X_{811}-.15X_{814}$
 $+.09X_{821}-.1X_{824}+.01X_{831}-.02X_{834}+.01X_{841}-.01X_{844}\leq .3$
 $.08X_{111}-.08X_{115}-.08X_{125}+.1X_{131}+.03X_{141}-.03X_{145}-.05X_{215}-.03X_{225}$
 $-.08X_{245}-.04X_{255}+.01X_{311}-.01X_{315}+.04X_{321}-.05X_{325}+.01X_{331}+.05X_{341}$
 $-.06X_{345}+.02X_{351}+.03X_{411}+.24X_{421}+.13X_{431}+.05X_{611}-.06X_{615}+.01X_{621}$
 $-.01X_{625}+.08X_{631}-.08X_{635}+.01X_{641}-.01X_{645}+.01X_{651}-.13X_{725}-.02X_{735}$

+.13X811+.09X821+.01X831+.01X841<=.3
 .08X111-.06X116-.08X126+.1X131-.08X136+.03X141-.03X146-.04X216
 -.02X226-.07X246-.03X256+.01X311-.01X316+.04X321+.01X331+.05X341
 -.05X346+.02X351-.02X356+.03X411+.24X421+.13X431-.03X516-.11X536
 +.05X611-.05X616+.01X621-.01X626+.08X631-.07X636+.01X641-.01X646
 +.01X651+.13X811-.15X816+.09X821-.11X826+.01X831+.01X841-.01X846<=.3
 -.08X113-.08X123-.09X133-.02X143+.04X212-.05X213+.03X222+.01X232
 -.01X233-.08X243+.03X252-.03X253+.01X312+.04X322+.01X332+.05X342
 +.02X352+.03X412-.03X413-.25X423+.13X432-.13X433+.05X442-.05X443
 +.04X512-.04X513+.19X522-.18X523+.11X532-.11X533-.05X613-.01X623
 -.01X643-.01X653+.13X712-.18X713+.12X722-.16X723-.03X733-.16X813
 -.02X833-.01X843<=.3
 -.06X114-.07X124-.08X134-.02X144+.04X212+.03X222+.06X232+.03X252
 +.01X312-.01X314+.04X322+.01X332-.01X334+.05X342-.04X344+.02X352
 -.016X354+.03X412-.01X414-.2X424+.13X432-.10X434+.05X442-.04X444
 +.04X512-.02X514+.19X522-.15X524+.11X532+.13X712+.12X722-.15X814
 -.1X824-.02X834-.01X844<=.3
 -.08X115-.08X125-.03X145+.04X212-.05X215+.03X222-.03X225+.01X232
 -.08X245+.03X252-.04X255+.01X312-.01X315+.04X322-.05X325+.01X332
 +.05X342-.06X345+.02X352+.03X412+.13X432+.05X442+.04X512+.19X522
 +.11X532-.06X615-.01X625-.08X635-.01X645+.13X712+.12X722-.13X725
 -.02X735<=.3
 -.06X116-.08X126-.08X136-.03X146+.04X212-.04X216+.03X222-.02X226
 +.01X232-.07X246+.03X252-.03X256+.01X312-.01X316+.04X322+.01X332
 +.05X342-.05X346+.02X352-.02X356+.03X412+.13X432+.05X442+.04X512
 -.03X516+.19X522+.11X532-.11X536-.05X616-.01X626-.07X636-.01X646
 +.13X712+.12X722-.15X816-.11X826-.01X846<=.3
 .08X113-.06X114+.08X123-.07X124+.09X133-.08X134+.02X143-.02X144
 +.05X213+.01X233+.08X243+.03X253-.01X314-.01X334-.04X344-.02X354
 +.03X413-.01X414+.25X423-.2X424+.13X433-.1X434+.05X443-.04X444
 +.04X513-.02X514+.18X523-.15X524+.11X533+.05X613+.01X623+.01X643
 +.01X653+.18X713+.16X723+.03X733+.16X813-.15X814-.1X824+.021X833
 -.02X834+.01X843-.01X844<=.3
 .08X113-.08X115+.08X123-.08X125+.09X133+.02X143-.03X145+.05X213
 -.05X215-.03X225+.01X233+.08X243-.08X245+.03X253-.04X255-.01X315
 -.05X325-.06X345+.03X413+.25X423+.13X433+.05X443+.04X513+.18X523
 +.11X533+.05X613-.06X615+.01X623-.01X625-.08X635+.01X643-.01X645
 +.01X653+.18X713+.16X723-.13X725+.03X733-.02X735+.16X813+.02X833
 +.01X843<=.3

+.08X113-.06X116+.08X123-.08X126+.09X133-.08X136+.016X143-.03X146
 +.05X213-.04X216-.02X226+.01X233+.08X243-.07X246+.03X253-.03X256
 -.01X316-.05X346-.02X356+.03X413+.25X423+.13X433+.05X443+.04X513
 -.03X516+.18X523+.11X533-.11X536+.05X613-.05X616+.01X623-.01X626
 -.07X636+.01X643-.01X646+.01X653+.18X713+.16X723+.03X733+.16X813
 -.15X816-.11X826+.02X833+.01X843-.01X846<=.3
 +.06X114-.08X115+.07X124-.08X125+.08X134+.02X144-.03X145-.05X215
 -.03X225-.08X245-.04X255+.01X314-.01X315-.05X325+.01X334+.04X344
 -.06X345+.02X354+.01X414+.2X424+.1X434+.04X444+.02X514+.15X524
 -.06X615-.01X625-.08X635-.01X645-.13X725-.02X735+.15X814+.1X824
 +.02X834+.01X844<=.3
 .06X114-.06X116+.07X124-.08X126+.08X134-.08X136+.02X144-.03X146
 -.04X216-.02X226-.07X246-.03X256+.01X314-.01X316+.01X334+.04X344
 -.05X346+.02X354-.02X356+.01X414+.2X424+.1X434+.04X444+.02X514
 -.03X516+.15X524-.11X536-.05X616-.01X626-.07X636-.01X646+.15X814
 -.15X816+.1X824-.11X826+.02X834+.01X844-.01X846<=.3
 .08X115-.06X116+.08X125-.08X126-.08X136+.03X145-.03X146+.05X215
 -.04X216+.03X225-.02X226+.08X245-.07X246+.04X255-.03X256+.01X315
 -.01X316+.05X325+.06X345-.05X346-.02X356-.03X516-.11X536+.06X615
 -.05X616+.01X625-.01X626+.08X635-.07X636+.01X645-.01X646+.13X725
 +.02X735-.15X816-.11X826-.01X846<=.3
 .08X111+.1X131+.03X141-.04X212-.03X222-.01X232-.03X252+.01X311
 -.01X312+.04X321-.04X322+.01X331-.01X332+.05X341-.05X342+.02X351
 -.02X352+.03X411-.03X412+.24X421+.13X431-.13X432-.05X442-.04X512
 -.19X522-.11X532+.05X611+.01X621+.08X631+.01X641+.01X651-.13X712
 -.12X722+.13X811+.09X821+.01X831+.01X841>=-.3
 .08X111-.08X113-.08X123+.1X131-.09X133+.03X141-.02X143-.05X213
 -.01X233-.08X243-.03X253+.01X311+.04X321+.01X331+.05X341+.02X351
 +.03X411-.03X413+.24X421-.25X423+.13X431-.13X433-.05X443-.04X513
 -.18X523-.11X533+.05X611-.05X613+.01X621-.01X623+.08X631+.01X641
 -.01X643+.01X651-.01X653-.18X713-.16X723-.03X733+.13X811-.16X813
 +.09X821+.01X831-.02X833+.01X841-.01X843>=-.3
 .08X111-.06X114-.07X124+.1X131-.08X134+.03X141-.02X144+.01X311
 -.01X314+.04X321+.01X331-.01X334+.05X341-.04X344+.02X351-.02X354
 +.03X411-.01X414+.24X421-.2X424+.13X431-.10X434-.04X444-.02X514
 -.15X524+.05X611+.01X621+.08X631+.01X641+.01X651+.13X811-.15X814
 +.09X821-.1X824+.01X831-.02X834+.01X841-.01X844>=-.3
 .08X111-.08X115-.08X125+.1X131+.03X141-.03X145-.05X215-.03X225
 -.08X245-.04X255+.01X311-.01X315+.04X321-.05X325+.01X331+.05X341

-.06X345+.02X351+.03X411+.24X421+.13X431+.05X611-.06X615+.01X621
 -.01X625+.08X631-.08X635+.01X641-.01X645+.01X651-.13X725-.02X735
 +.13X811+.09X821+.01X831+.01X841>= -.3
 .08X111-.06X116-.08X126+.1X131-.08X136+.03X141-.03X146-.04X216
 -.02X226-.07X246-.03X256+.01X311-.01X316+.04X321+.01X331+.05X341
 -.05X346+.02X351-.02X356+.03X411+.24X421+.13X431-.03X516-.11X536
 +.05X611-.05X616+.01X621-.01X626+.08X631-.07X636+.01X641-.01X646
 +.01X651+.13X811-.15X816+.09X821-.11X826+.01X831+.01X841-.01X846>= -.3
 -.08X113-.08X123-.09X133-.02X143+.04X212-.05X213+.03X222+.01X232
 -.01X233-.08X243+.03X252-.03X253+.01X312+.04X322+.01X332+.05X342
 +.02X352+.03X412-.03X413-.25X423+.13X432-.13X433+.05X442-.05X443
 +.04X512-.04X513+.19X522-.18X523+.11X532-.11X533-.05X613-.01X623
 -.01X643-.01X653+.13X712-.18X713+.12X722-.16X723-.03X733-.16X813
 -.02X833-.01X843>= -.3
 -.06X114-.07X124-.08X134-.02X144+.04X212+.03X222+.06X232+.03X252
 +.01X312-.01X314+.04X322+.01X332-.01X334+.05X342-.04X344+.02X352
 -.016X354+.03X412-.01X414-.2X424+.13X432-.10X434+.05X442-.04X444
 +.04X512-.02X514+.19X522-.15X524+.11X532+.13X712+.12X722-.15X814
 -.1X824-.02X834-.01X844>= -.3
 -.08X115-.08X125-.03X145+.04X212-.05X215+.03X222-.03X225+.01X232
 -.08X245+.03X252-.04X255+.01X312-.01X315+.04X322-.05X325+.01X332
 +.05X342-.06X345+.02X352+.03X412+.13X432+.05X442+.04X512+.19X522
 +.11X532-.06X615-.01X625-.08X635-.01X645+.13X712+.12X722-.13X725
 -.02X735>= -.3
 -.06X116-.08X126-.08X136-.03X146+.04X212-.04X216+.03X222-.02X226
 +.01X232-.07X246+.03X252-.03X256+.01X312-.01X316+.04X322+.01X332
 +.05X342-.05X346+.02X352-.02X356+.03X412+.13X432+.05X442+.04X512
 -.03X516+.19X522+.11X532-.11X536-.05X616-.01X626-.07X636-.01X646
 +.13X712+.12X722-.15X816-.11X826-.01X846>= -.3
 .08X113-.06X114+.08X123-.07X124+.09X133-.08X134+.02X143-.02X144
 +.05X213+.01X233+.08X243+.03X253-.01X314-.01X334-.04X344-.02X354
 +.03X413-.01X414+.25X423-.2X424+.13X433-.1X434+.05X443-.04X444
 +.04X513-.02X514+.18X523-.15X524+.11X533+.05X613+.01X623+.01X643
 +.01X653+.18X713+.16X723+.03X733+.16X813-.15X814-.1X824+.021X833
 -.02X834+.01X843-.01X844>= -.3
 .08X113-.08X115+.08X123-.08X125+.09X133+.02X143-.03X145+.05X213
 -.05X215-.03X225+.01X233+.08X243-.08X245+.03X253-.04X255-.01X315
 -.05X325-.06X345+.03X413+.25X423+.13X433+.05X443+.04X513+.18X523
 +.11X533+.05X613-.06X615+.01X623-.01X625-.08X635+.01X643-.01X645

```

+.01X653+.18X713+.16X723-.13X725+.03X733-.02X735+.16X813+.02X833
+.01X843>=-.3
+.08X113-.06X116+.08X123-.08X126+.09X133-.08X136+.016X143-.03X146
+.05X213-.04X216-.02X226+.01X233+.08X243-.07X246+.03X253-.03X256
-.01X316-.05X346-.02X356+.03X413+.25X423+.13X433+.05X443+.04X513
-.03X516+.18X523+.11X533-.11X536+.05X613-.05X616+.01X623-.01X626
-.07X636+.01X643-.01X646+.01X653+.18X713+.16X723+.03X733+.16X813
-.15X816-.11X826+.02X833+.01X843-.01X846>=-.3
+.06X114-.08X115+.07X124-.08X125+.08X134+.02X144-.03X145-.05X215
-.03X225-.08X245-.04X255+.01X314-.01X315-.05X325+.01X334+.04X344
-.06X345+.02X354+.01X414+.2X424+.1X434+.04X444+.02X514+.15X524
-.06X615-.01X625-.08X635-.01X645-.13X725-.02X735+.15X814+.1X824
+.02X834+.01X844>=-.3
.06X114-.06X116+.07X124-.08X126+.08X134-.08X136+.02X144-.03X146
-.04X216-.02X226-.07X246-.03X256+.01X314-.01X316+.01X334+.04X344
-.05X346+.02X354-.02X356+.01X414+.2X424+.1X434+.04X444+.02X514
-.03X516+.15X524-.11X536-.05X616-.01X626-.07X636-.01X646+.15X814
-.15X816+.1X824-.11X826+.02X834+.01X844-.01X846>=-.3
.08X115-.06X116+.08X125-.08X126-.08X136+.03X145-.03X146+.05X215
-.04X216+.03X225-.02X226+.08X245-.07X246+.04X255-.03X256+.01X315
-.01X316+.05X325+.06X345-.05X346-.02X356-.03X516-.11X536+.06X615
-.05X616+.01X625-.01X626+.08X635-.07X636+.01X645-.01X646+.13X725
+.02X735-.15X816-.11X826-.01X846>=-.3
END
INTE 122
LEAVE
TERSE
GO
QUIT

```

2. Model Two ($\mu=900$)

MIN 378.1915X111+439.6875X113+671.25X114+615X115+513.75X116
+649.6875X123+1036.406X124+615X125+621.25X126+423.8298X131
+492.6563X133+831.5625X134+675X136+181.2766X141+195.9375X143
+220.3125X144+268.75X145+245X146+181.7021X212+257.8125X213
+174.1667X215+306.6667X216+216.0638X222+355X225+199.1667X226
+55.21277X232+55.3125X233+55.83334X235+55.83334X236+465X243
+585X245+593.333X246+151.2766X252+191.875X253+266.6667X255+235X256
+90.42553X311+70.42554X312+83.4375X314+107.5X315+91.66667X316
+361.7021X321+361.7021X322+338.3333X325+55.21277X331+55.21277X332
+83.4375X334+55.83334X335+55.83334X336+252.1277X341+212.1277X342
+447.5X344+410X345+378.3333X346+120.8511X351+100.8511X352
+190.3125X354+163.3333X356+226.0638X411+176.0638X412+176.5625X413
+208.5938X414+1034.574X421+1327.344X423+2028.906X424+630.3192X431
+530.3192X432+732.8125X433+1118.75X434+252.1277X442+303.125X443
+475.7813X444+246.4894X512+247.1875X513+292.0313X514+320.8333X516
+742.4468X522+1025.938X523+1566.25X524+529.4681X532+671.5625X533
+527.9167X536+252.1277X611+293.125X613+410X615+378.3333X616
+90.42553X621+70.625X623+107.5X625+91.66667X626+588.1915X631
+569.1667X635+593.3333X636+90.42553X641+70.625X643+91.66667X645
+91.66667X646+55.21277X651+55.3125X653+55.83334X655+55.83334X656
+530.3192X712+985.9375X713+492.2873X722+859.375X723+935.4167X725
+276.5625X733+229.1667X735+605.1064X811+859.375X813+1536.25X814
+1186.667X816+403.4043X821+1108.75X824+340X826+110.4255X831
+141.25X833+293.75X834+110.4255X841+110.625X843+166.875X844+111.6667X846
ST
-.07978723X111+.0625X114+.0703125X124-.09574468X131+.078125X134
-.03191489X141+.015625X144-.0106383X311+.005208334X314-.04255319X321
-.005319149X331+.005208334X334-.05319149X341+.041667X344-.0212766X351
+.015625X354-.02659575X411+.01302083X414-.2393617X421+.1953125X424
-.1329787X431+.1041667X434+.0390625X444+.01822917X514+.1458333X524
-.05319149X611-.0106383X621-.07978723X631-.0106383X641-.005319149X651
-.12766X811+.1458333X814-.08510638X821+.1041667X824-.0106383X831
+.02083333X834-.0106383X841+.01041667X844>=0
.0625X114+.0703125X124+.078125X134+.015625X144-.04255319X212-.02659575X222
-.005319149X232-.03191489X252-.0106383X312+.005208334X314-.042553X322
-.005319149X332+.005208334X334-.05319X342+.041667X344-.0212766X352

+.015625X354-.02659575X412+.01302083X414+.1953125X424-.1329787X432
 +.1041667X434-.05319149X442+.0390625X444-.03723404X512+.01822917X514
 -.18617X522+.1458333X524-.1117021X532-.1329787X712-.1196809X722
 +.14583X814+.1041667X824+.02083X834+.01041667X844>=0
 -.078125X113+.0625X114-.078125X123+.0703125X124-.0859X133+.078125X134
 -.015625X143+.015625X144-.046875X213-.005208334X233-.08333334X243
 -.03125X253+.0052X314+.0052X334+.041667X344+.0156X354-.02604X413
 +.01302083X414-.2473958X423+.1953125X424-.1302083X433+.1041667X434
 -.05208333X443+.0390625X444-.03645833X513+.01822917X514-.1822917X523
 +.1458333X524-.109375X533-.05208333X613-.01041667X623-.01041667X643
 -.0052X653-.1822917X713-.15625X723-.02604167X733-.15625X813+.1458333X814
 +.1041667X824-.02083333X833+.02083333X834-.01041667X843+.01041667X844>=0
 .0625X114-.08333334X115+.0703125X124-.08333334X125+.078125X134+.015625X144
 -.03472222X145-.05092593X215-.02777778X225-.00462963X235-.08333334X245
 -.037037X255+.005208334X314-.01388889X315-.0462963X325+.0052X334
 -.00462963X335+.04166667X344-.05555556X345+.015625X354+.01302083X414
 +.1953125X424+.1041667X434+.0390625X444+.01822917X514+.1458333X524
 -.05555556X615-.013889X625-.0787037X635-.009259259X645-.00463X655
 -.1273148X725-.02314815X735+.1458333X814+.1041667X824+.02083X834
 +.01041667X844>=0
 .0625X114-.0625X116+.0703125X124-.076389X126+.078125X134-.08333334X136
 +.015625X144-.02777778X146-.03703704X216-.02314815X226-.00462963X236
 -.074074X246-.02777778X256+.005208334X314-.009259259X316+.0052X334
 -.00462963X336+.04166667X344-.0462963X346+.015625X354-.01851852X356
 +.01302083X414+.1953125X424+.1041667X434+.0390625X444+.01822917X514
 -.03240741X516+.1458333X524-.1134259X536-.0462963X616-.009259259X626
 -.074074X636-.009259259X646-.00462963X656+.1458333X814-.148148X816
 +.1041667X824-.1111111X826+.02083333X834+.01041667X844-.009259259X846>=0
 300X111+360X131+120X141+40X311+160X321+20X331+200X341+80X351
 +100X411+900X421+500X431+200X611+40X621+300X631+40X641+20X651
 +480X811+320X821+40X831+40X841<=3360
 160X212+100X222+20X232+120X252+40X312+160X322+20X332+200X342+80X352
 +100X412+500X432+200X442+140X512+700X522+420X532+500X712+450X722
 <=3360
 300X113+300X123+330X133+60X143+180X213+20X233+320X243+120X253
 +100X413+950X423+500X433+200X443+140X513+700X523+420X533+200X613
 +40X623+40X643+20X653+700X713+600X723+100X733+600X813+80X833
 +40X843<=3840
 240X114+270X124+300X134+60X144+20X314+20X334+160X344+60X354+50X414

$+750X424+400X434+150X444+70X514+560X524+560X814+400X824+80X834$
 $+40X844 \leq 3840$
 $360X115+360X125+150X145+220X215+120X225+20X235+360X245+160X255$
 $+60X315+200X325+20X335+240X345+240X615+60X625+340X635+40X645$
 $+20X655+550X725+100X735 \leq 4320$
 $270X116+330X126+360X136+120X146+160X216+100X226+20X236+320X246$
 $+120X256+40X316+20X336+200X346+80X356+140X516+490X536+200X616$
 $+40X626+320X636+40X646+20X656+640X816+480X826+40X846 \leq 4320$
 $4X111+2X131+5X141 \leq 24$
 $4X113+X123+2X133+5X143 \leq 30$
 $4X114+X124+2X134+5X144 \leq 26$
 $4X115+X125+5X145 \leq 34$
 $4X116+X126+2X136+5X146 \leq 38$
 $6X212+4X222+2X232+4X252 \leq 20$
 $6X213+2X233+2X243+4X253 \leq 30$
 $6X215+4X225+2X235+2X245+4X255 \leq 34$
 $6X216+4X226+2X236+2X246+4X256 \leq 38$
 $2X311+X321+X331+8X341+X351 \leq 24$
 $2X312+X322+X332+8X342+X352 \leq 20$
 $2X314+X334+8X344+X354 \leq 26$
 $2X315+X325+X335+8X345 \leq 34$
 $2X316+X336+8X346+X356 \leq 38$
 $2X411+6X421+4X431 \leq 24$
 $2X412+4X432+8X442 \leq 20$
 $2X413+6X423+4X433+8X443 \leq 30$
 $2X414+6X424+4X434+8X444 \leq 26$
 $X512+6X522+4X532 \leq 20$
 $X513+6X523+4X533 \leq 30$
 $X514+6X524 \leq 26$
 $X516+4X536 \leq 38$
 $X611+8X621+2X631+5X641+2X651 \leq 24$
 $X613+8X623+5X643+2X653 \leq 30$
 $X615+8X625+2X635+5X645+2X655 \leq 34$
 $X616+8X626+2X636+5X646+2X656 \leq 38$
 $5X712+3X722 \leq 20$
 $5X713+3X723+2X733 \leq 30$
 $3X725+2X735 \leq 34$
 $3X811+9X821+3X831+2X841 \leq 24$
 $3X813+3X833+2X843 \leq 30$

$3X814+9X824+3X834+2X844 \leq 26$
 $3X816+9X826+2X846 \leq 38$
 $X111+X113+X114+X115+X116=1$
 $X123+X124+X125+X126=1$
 $X131+X133+X134+X136=1$
 $X141+X143+X144+X145+X146=1$
 $X212+X213+X215+X216=1$
 $X222+X225+X226=1$
 $X232+X233+X235+X236=1$
 $X243+X245+X246=1$
 $X252+X253+X255+X256=1$
 $X311+X312+X314+X315+X316=1$
 $X321+X322+X325=1$
 $X331+X332+X334+X335+X336=1$
 $X341+X342+X344+X345+X346=1$
 $X351+X352+X354+X356=1$
 $X411+X412+X413+X414=1$
 $X421+X423+X424=1$
 $X431+X432+X433+X434=1$
 $X442+X443+X444=1$
 $X512+X513+X514+X516=1$
 $X522+X523+X524=1$
 $X532+X533+X536=1$
 $X611+X613+X615+X616=1$
 $X621+X623+X625+X626=1$
 $X631+X635+X636=1$
 $X641+X643+X645+X646=1$
 $X651+X653+X655+X656=1$
 $X712+X713=1$
 $X722+X723+X725=1$
 $X733+X735=1$
 $X811+X813+X814+X816=1$
 $X821+X824+X826=1$
 $X831+X833+X834=1$
 $X841+X843+X844=1$
 $-X131+X141 \leq 0$
 $-X131-X133+X143 \leq 0$
 $-X131-X133-X134+X144 \leq 0$
 $-X131-X133-X134+X145 \leq 0$

```

-X131-X133-X134-X136+X146<=0
-X232-X233+X243<=0
-X232-X233-X235+X245<=0
-X232-X233-X235-X236+X246<=0
-X331+X341<=0
-X331-X332+X342<=0
-X331-X332-X334+X344<=0
-X331-X332-X334-X335+X345<=0
-X331-X332-X334-X335-X336+X346<=0
-X431-X432+X442<=0
-X431-X432-X433+X443<=0
-X431-X432-X433-X434+X444<=0
-X631+X641<=0
-X631+X643<=0
-X631-X635+X645<=0
-X631-X635-X636+X646<=0
-X831+X841<=0
-X831-X833+X843<=0
-X831-X833-X834+X844<=0
-X831-X833-X834+X846<=0
END
INTE 122
LEAVE
TERSE
GO
QUIT

```

APPENDIX II

COMPUTER PROGRAMS FOR DATA INPUT

1. Computer Program for Model 1

```
100 REM INPUT ORIGINAL DATA
110 CS="COST.DAT"
120 TS="TIME.DAT"
130 PS="PART.DAT"
135 AS="ASSY.DAT"
140 SLS="SLOT.DAT"
150 WSS="WKSTN.DAT"
155 NUMS="NUMBER.DAT"

160 PRINT "IF YOU WANT TO INPUT NEW DATA FOR PRECEDENCE"
161 PRINT "OR BALANCE ECT., TYPE Y. IF NOT, TYPE N"
165 INPUT YS
166 IF YS="Y" THEN 175
170 IF YS="N" THEN 230 ELSE 160
175 OPEN "o", 1, NUMS
180 PRINT "INPUT NUMBER OF ASSEMBLY" : INPUT JOB
185 PRINT "INPUT NUMBER OF WORKSTATION" : INPUT STN
190 PRINT "INPUT NUMBER OF COMPONENT" : INPUT COMP
195 PRINT "INPUT OPERATION # HAVING PROCEDENCE RELATIONS IN ORDER"
200 INPUT PRR, BEH
205 PRINT "INPUT PERCENTAGE OF WORKLOAD BALANCE": INPUT PCT
210 WRITE #1, JOB, STN, COMP, PRR, BEH, PCT
215 CLOSE #1
217 DIM OP(JOB), D(J)
220 GOTO 250

230 OPEN "i", 1, NUMS
235 INPUT #1, JOB, STN, COMP, PRR, BEH, PCT
240 DIM OP(JOB), D(JOB)
245 CLOSE #1

250 PRINT "IF YOU WANT TO INPUT NEW DATA FOR OPERATION #"
251 PRINT "OR DEMAND, TYPE Y. IF NOT, TYPE N"
255 INPUT YS
257 IF YS="Y" THEN 265
```



```

260 IF YS="N" THEN 330 ELSE 250
265 PRINT " NOW INPUT NUMBER OF OPERATIONS FOR EACH ASSEMBLY"
270 FOR J=1 TO JOB
275 INPUT OP(J)
280 NEXT J
285 PRINT "INPUT DEMAND FOR EACH ASSEMBLY"
290 FOR J=1 TO JOB
295 INPUT D(J)
300 NEXT J
305 OPEN "o", 1, AS
310 FOR J=1 TO JOB
315 WRITE #1, OP(J), D(J)
320 NEXT J
325 CLOSE #1: GOTO 350

330 OPEN "i", 1, AS
340 FOR J=1 TO JOB
345 INPUT #1, OP(J), D(J)
348 NEXT J: CLOSE #1

350 PRINT "IF YOU WANT TO INPUT NEW PROCESSING COST,"
351 PRINT "TYPE Y, OTHERWISE, TYPE N"
355 INPUT YS
357 IF YS="Y" THEN 365
360 IF YS="N" THEN 450 ELSE 350
365 PRINT "NOW INPUT PROCESSING COST"
370 DIM CST(JOB,STN,STN)
375 FOR J=1 TO JOB
380 FOR I=1 TO OP(J)
385 FOR K=1 TO STN
390 PRINT "COST("J,I,K,")="
395 INPUT CST(J,I,K)
400 NEXT K: NEXT I
401 PRINT "ARE YOU SURE DATA FOR THIS ASSEMBLY ARE CORRECT?"
402 PRINT "TYPE Y FOR NEXT ASSEMBLY; TYPE N TO RE-INPUT"
403 INPUT YS
404 IF YS="Y" THEN 406
405 IF YS="N" THEN 380 ELSE 402
406 NEXT J
407 OPEN "o", 1, CS
410 FOR J=1 TO JOB
415 FOR I=1 TO OP(J)
420 FOR K=1 TO STN
425 WRITE #1, CST(J,I,K)

```

```

430 NEXT K: NEXT I: NEXT J
440 CLOSE #1

450 PRINT "IF YOU WANT TO INPUT NEW PROCESSING TIME."
451 PRINT "TYPE Y, OTHERWISE, TYPE N"
460 INPUT YS
462 IF YS="Y" THEN 470
465 IF YS="N" THEN 605 ELSE 450
470 PRINT "NOW INPUT PROCESSING TIME"
475 DIM TM(JOB,STN,STN)
480 FOR J=1 TO JOB
490 FOR I=1 TO OP(J)
500 FOR K=1 TO STN
510 PRINT "TIME("J,I,K,")="
520 INPUT TM(J,I,K)
530 NEXT K: NEXT I
531 PRINT "ARE YOU SURE DATA FOR THIS ASSEMBLY ARE CORRECT?"
532 PRINT "TYPE Y FOR NEXT ASSEMBLY; TYPE N TO RE-INPUT"
533 INPUT YS
534 IF YS="Y" THEN 536
535 IF YS="N" THEN 490 ELSE 532
536 NEXT J
540 OPEN "o", 3, TS
550 FOR J=1 TO JOB
560 FOR I=1 TO OP(J)
570 FOR K=1 TO STN
580 WRITE #3, TM(J,I,K)
590 NEXT K: NEXT I: NEXT J
600 CLOSE #3

605 PRINT "IF YOU WANT TO INPUT NEW AVAILABLE TIME"
606 PRINT "OR FEEDER CAPACITY, TYPE Y, OTHERWISE, TYPE N"
607 INPUT YS
608 IF YS="Y" THEN 610
609 IF YS="N" THEN 760 ELSE 605
610 PRINT "NOW INPUT AVAILABLE TIME OF EACH WORK SATION"
615 DIM AT(STN), F(STN)
620 FOR K=1 TO STN
630 PRINT "AT(",K,")="
640 INPUT AT(K)
650 NEXT K
660 PRINT "NOW INPUT FEEDER CAPACITY OF EACH WORK STATION"
670 FOR K=1 TO STN
680 PRINT "F(",K,")="

```

```

690 INPUT F(K)
700 NEXT K
710 OPEN "o", 1, WSS
720 FOR K=1 TO STN
730 WRITE #1, AT(K), F(K)
740 NEXT K
750 CLOSE #1

760 PRINT "IF YOU WANT TO INPUT NEW SLOT NUMBER,"
765 PRINT "TYPE Y, OTHERWISE TYPE N"
770 INPUT YS
771 IF YS="Y" THEN 780
775 IF YS="N" THEN 885 ELSE 760
780 PRINT "NOW INPUT SLOT NUMBER OF EACH COMPONENT"
790 DIM B(COMP)
800 FOR L=1 TO COMP
810 PRINT "B(",L,")="
820 INPUT B(L)
830 NEXT L
840 OPEN "o", 1, SLS
850 FOR L=1 TO COMP
860 WRITE #1, B(L)
870 NEXT L
880 CLOSE #1

885 PRINT "IF YOU WANT TO INPUT NEW REQUIRED COMPONENT #,"
886 PRINT "TYPE Y, OTHERWISE, TYPE N"
887 INPUT YS
888 IF YS="Y" THEN 890
889 IF YS="N" THEN 990 ELSE 885
890 PRINT "NOW INPUT NUMBER OF EACH COMPONENT FOR EACH OPERATION"
895 DIM A(JOB,STN,COMP)
900 FOR J=1 TO JOB
905 FOR I=1 TO OP(J): FOR L=1 TO COMP
910 PRINT "A(",J,I,L,")="
920 INPUT A(J,I,L)
930 NEXT L: NEXT I
931 PRINT "ARE YOU SURE DATA FOR THIS ASSEMBLY ARE CORRECT?"
932 PRINT "TYPE Y FOR NEXT ASSEMBLY; TYPE N TO RE-INPUT"
933 INPUT YS
934 IF YS="Y" THEN 936
935 IF YS="N" THEN 905 ELSE 932
936 NEXT J
940 OPEN "o", 1, PS

```

```

950 FOR J=1 TO JOB: FOR I=1 TO OP(J): FOR L=1 TO COMP
960 WRITE #1, A(J,I,L)
970 NEXT L: NEXT I: NEXT J
980 CLOSE #1
990 END

```

```

100 REM MAIN PROGRAM FOR CALCULATING ALL THE COEFFICIENTS
110 CS="COST.DAT"
120 TS="TIME.DAT"
130 PS="PART.DAT"
140 AS="ASSY.DAT"
150 SLS="SLOT.DAT"
160 WSS="WKSTN.DAT"
170 NUMS="NUMBER.DAT"

```

```

175 REM CALCULATE NUMBERS OF COLUMNS AND ROWS
180 OPEN "i", 1, NUMS
190 INPUT #1, JOB, STN, COMP, PRR, BEH, PCT
195 DIM OP(JOB), D(JOB), TM(JOB,STN,STN)
200 OPEN "i", 2, AS
210 FOR J=1 TO JOB
220 INPUT #2, OP(J), D(J)
230 NEXT J
232 OPEN "i", 3, TS
234 FOR J=1 TO JOB: FOR I=1 TO OP(J): FOR K=1 TO STN
236 INPUT #3, TM(J,I,K)
238 NEXT K: NEXT I: NEXT J
240 CLOSE #1, #2, #3
245 TJ=0: COL=0
250 FOR J=1 TO JOB
252 TJ=TJ+OP(J)
255 FOR I=1 TO OP(J)
260 FOR K=1 TO STN
265 IF TM(J,I,K)=0 THEN 275
270 COL=COL+1
275 NEXT K: NEXT I: NEXT J
280 ROW=TJ+STN*(STN+2*JOB)
285 PRINT "NUMBER OF VARIABLES =", COL
287 PRINT "NUMBER OF CONSTRIANTS =", ROW

290 DIM COF(COL), CST(JOB,STN,STN)
295 COEFS="COEF.DAT"
300 REM COEFFICIENTS OF OBJECTIVE FUNCTION

```

```

310 OPEN "i", 1, CS
320 FOR J=1 TO JOB
330 FOR I=1 TO OP(J)
340 FOR K=1 TO STN
350 INPUT #1, CST(J,I,K)
360 NEXT K: NEXT I: NEXT J
362 CLOSE #1: OPEN "o", 3, COEFS
365 H=1: HANG=0
370 FOR J=1 TO JOB
380 FOR I=1 TO OP(J)
390 FOR K=1 TO STN
395 IF CST(J,I,K)=0 THEN 420
400 COF(H)=D(J)*CST(J,I,K)
410 H=H+1
420 NEXT K: NEXT I: NEXT J
430 FOR H=1 TO COL
440 WRITE #3, HANG, H, COF(H)
450 NEXT H
460 HANG=HANG+1

470 REM COEFFICIENTS FOR CONSTRIANT #2
480 FOR JJ=1 TO STN
490 H=1
500 FOR J=1 TO JOB
510 FOR I=1 TO OP(J)
520 FOR K=1 TO STN
530 IF TM(J,I,K)=0 THEN 590
540 IF K<>JJ THEN 570
550 COF(H)=D(J)*TM(J,I,K)
560 GOTO 580
570 COF(H)=0
580 H=H+1
590 NEXT K: NEXT I: NEXT J
600 FOR H=1 TO COL
610 IF COF(H)=0 THEN 620
615 WRITE #3, HANG, H, COF(H)
620 NEXT H
630 HANG=HANG+1
640 NEXT JJ

650 REM COEFFICIENTS FOR CONSTRIANT #3
655 DIM B(COMP), A(JOB,STN,COMP)
660 OPEN "i", 1, PS
670 FOR J=1 TO JOB

```

```

680 FOR I=1 TO OP(J)
690 FOR L=1 TO COMP
700 INPUT #1, A(J,I,L)
710 NEXT L: NEXT I: NEXT J
712 OPEN "i", 2, SLS
714 FOR L=1 TO COMP
716 INPUT #2, B(L)
718 NEXT L
720 CLOSE #1, #2
725 FOR JJ=1 TO JOB
730 FOR KK=1 TO STN
740 H=1
750 FOR J=1 TO JOB
760 FOR I=1 TO OP(J)
770 FOR K=1 TO STN
772 IF TM(J,I,K)=0 THEN 820
775 COF(H)=0
777 IF J<>JJ THEN 810
778 IF K<>KK THEN 810
780 FOR L=1 TO COMP
790 COF(H)=COF(H)+B(L)*A(J,I,L)
800 NEXT L
810 H=H+1
820 NEXT K: NEXT I: NEXT J
830 FOR H=1 TO COL
835 IF COF(H)=0 THEN 850
840 WRITE #3, HANG, H, COF(H)
850 NEXT H
860 HANG=HANG+1
870 NEXT KK: NEXT JJ

880 REM COEFFICIENTS FOR CONSTRAINT #4
890 FOR JJ=1 TO JOB
895 FOR II=1 TO OP(JJ)
900 H=1
905 FOR J=1 TO JOB
910 FOR I=1 TO OP(J)
915 FOR K=1 TO STN
920 IF TM(J,I,K)=0 THEN 950
925 IF J<>JJ THEN 940
930 IF I<>II THEN 940
935 COF(H)=1: GOTO 945
940 COF(H)=0
945 H=H+1

```

```

950 NEXT K: NEXT I: NEXT J
955 FOR H=1 TO COL
960 IF COF(H)=0 THEN 970
965 WRITE #3, HANG, H, COF(H)
970 NEXT H
975 HANG=HANG+1
980 NEXT II: NEXT JJ

982 REM COEFFICIENTS FOR CONSTRAINT #5
984 FOR JJ=1 TO JOB
985 FOR KK=1 TO STN
987 H=1
990 FOR J=1 TO JOB
1000 FOR I=1 TO OP(J)
1010 FOR K=1 TO STN
1011 IF TM(J,I,K)=0 THEN 1050
1012 IF J<>JJ THEN 1030
1014 IF I=BEH THEN 1024
1016 IF I<>PRR THEN 1030
1018 IF K>KK THEN 1030
1020 COF(H)=-1
1022 GOTO 1040
1024 IF K<>KK THEN 1030
1026 COF(H)=1
1028 GOTO 1040
1030 COF(H)=0
1040 H=H+1
1050 NEXT K: NEXT I: NEXT J
1060 FOR H=1 TO COL
1070 IF COF(H)=0 THEN 1080
1075 WRITE #3, HANG, H, COF(H)
1080 NEXT H
1090 HANG=HANG+1
1100 NEXT KK: NEXT JJ

1110 REM COEFFICIENTS FOR CONSTRIANT #6
1115 DIM F(STN), AT(STN)
1120 OPEN "i", I, WSS
1130 FOR K=1 TO STN
1140 INPUT #1, AT(K), F(K)
1150 NEXT K: CLOSE #1
1155 FOR DBL=1 TO 2
1160 FOR KK=1 TO STN-1
1170 FOR HH=KK+1 TO STN

```

```

1180 H=1
1190 FOR J=1 TO JOB
1200 FOR I=1 TO OP(J)
1210 FOR K=1 TO STN
1215 IF TM(J,I,K)=0 THEN 1255
1220 IF K<>KK THEN 1230
1225 COF(H)=D(J)*TM(J,I,K)/AT(K)
1227 GOTO 1250
1230 IF K<>HH THEN 1245
1235 COF(H)=-D(J)*TM(J,I,K)/AT(K)
1240 GOTO 1250
1245 COF(H)=0
1250 H=H+1
1255 NEXT K: NEXT I: NEXT J
1260 FOR H=1 TO COL
1270 IF COF(H)=0 THEN 1280
1275 WRITE #3, HANG, H, COF(H)
1280 NEXT H
1290 HANG=HANG+1
1300 NEXT HH: NEXT KK: NEXT DBL

1310 REM RELATION BETWEEN BOTH SIDES FOR EACH CONSTRIANT
1320 DIM COE(ROW): H=1
1370 FOR K=1 TO STN
1380 COE(H)=1
1390 H=H+1
1400 NEXT K
1410 FOR J=1 TO JOB
1420 FOR K=1 TO STN
1430 COE(H)=1
1440 H=H+1
1450 NEXT K: NEXT J
1460 FOR J=1 TO JOB
1470 FOR I=1 TO OP(J)
1480 COE(H)=0
1490 H=H+1
1500 NEXT I: NEXT J
1510 FOR J=1 TO JOB
1520 FOR K=1 TO STN
1530 COE(H)=1
1540 H=H+1
1550 NEXT K: NEXT J
1560 FOR K=1 TO STN-1
1570 FOR KK=K+1 TO STN

```



```

1580 COE(H)=1
1590 H=H+1
1600 NEXT KK: NEXT K
1610 FOR K=1 TO STN-1
1620 FOR KK=K+1 TO STN
1630 COE(H)=-1
1635 H=H+1
1640 NEXT KK: NEXT K
1650 FOR HANG=1 TO H-1
1655 X=COL+HANG
1660 WRITE #3, HANG, X, COE(HANG)
1670 NEXT HANG

1680 REM VALUE OF RIGHT HAND SIDE FOR EACH CONSTRIANT
1690 H=0
1700 FOR K=1 TO STN
1710 H=H+1
1720 COE(H)=AT(K)
1730 NEXT K
1740 FOR J=1 TO JOB
1750 FOR K=1 TO STN
1760 H=H+1
1770 COE(H)=F(K)
1780 NEXT K: NEXT J
1790 FOR J=1 TO JOB
1800 FOR I=1 TO OP(J)
1810 H=H+1
1820 COE(H)=1
1830 NEXT I: NEXT J
1840 FOR J=1 TO JOB
1850 FOR K=1 TO STN
1860 H=H+1
1870 COE(H)=0
1880 NEXT K: NEXT J
1890 FOR K=1 TO STN-1
1900 FOR KK=K+1 TO STN
1910 H=H+1
1920 COE(H)=PCT
1930 NEXT KK: NEXT K
1940 FOR K=1 TO STN-1
1950 FOR KK=K+1 TO STN
1960 H=H+1
1970 COE(H)=-PCT
1980 NEXT KK: NEXT K

```

```
1990 FOR HANG=1 TO H
2000 WRITE #3, HANG, 0, COE(HANG)
2010 NEXT HANG
2020 CLOSE #3

2030 END
```

2. Computer Program for Model 2

```
100 REM INPUT ORIGINAL DATA
110 CS="COST.DAT"
120 TS="TIME.DAT"
125 PNTS="PENALTY.DAT"
130 PS="PART.DAT"
135 AS="ASSY.DAT"
140 SLS="SLOT.DAT"
150 WSS="WKSTN.DAT"
155 NUMS="NUMBER.DAT"

160 PRINT "IF YOU WANT TO INPUT NEW DATA FOR PROBLEM SIZE"
161 PRINT "OR PRECEDENCE ECT., TYPE Y. IF NOT, TYPE N"
165 INPUT YS
166 IF YS="Y" THEN 175
170 IF YS="N" THEN 230 ELSE 160
175 OPEN "o", 1, NUMS
180 PRINT "INPUT NUMBER OF ASSEMBLY" : INPUT JOB
185 PRINT "INPUT NUMBER OF WORKSTATION" : INPUT STN
190 PRINT "INPUT NUMBER OF COMPONENT" : INPUT COMP
195 PRINT "INPUT OPERATION # HAVING PROCEDENCE RELATIONS IN ORDER"
200 INPUT PRR, BEH
205 PRINT "INPUT PERCENTAGE OF WORKLOAD BALANCE": INPUT PCT
210 WRITE #1, JOB, STN, COMP, PRR, BEH, PCT
215 CLOSE #1
217 DIM OP(JOB), D(J)
220 GOTO 250

230 OPEN "i", 1, NUMS
235 INPUT #1, JOB, STN, COMP, PRR, BEH, PCT
240 DIM OP(JOB), D(JOB)
245 CLOSE #1

250 PRINT "IF YOU WANT TO INPUT NEW DATA FOR OPERATION #"
251 PRINT "OR DEMAND, TYPE Y. IF NOT, TYPE N"
255 INPUT YS
257 IF YS="Y" THEN 265
260 IF YS="N" THEN 330 ELSE 250
265 PRINT " NOW INPUT NUMBER OF OPERATIONS FOR EACH ASSEMBLY"
270 FOR J=1 TO JOB
275 INPUT OP(J)
280 NEXT J
```

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285 PRINT "INPUT DEMAND FOR EACH ASSEMBLY"
290 FOR J=1 TO JOB
295 INPUT D(J)
300 NEXT J
305 OPEN "o", 1, AS
310 FOR J=1 TO JOB
315 WRITE #1, OP(J), D(J)
320 NEXT J
325 CLOSE #1: GOTO 350

330 OPEN "i", 1, AS
340 FOR J=1 TO JOB
345 INPUT #1, OP(J), D(J)
348 NEXT J: CLOSE #1

350 PRINT "IF YOU WANT TO INPUT NEW PROCESSING COST,"
351 PRINT "TYPE Y, OTHERWISE, TYPE N"
355 INPUT Y$
357 IF Y$="Y" THEN 365
360 IF Y$="N" THEN 450 ELSE 350
365 PRINT "NOW INPUT PROCESSING COST"
370 DIM CST(JOB,STN,STN)
375 FOR J=1 TO JOB
380 FOR I=1 TO OP(J)
385 FOR K=1 TO STN
390 PRINT "COST("J,I,K,")="
395 INPUT CST(J,I,K)
400 NEXT K: NEXT I
401 PRINT "ARE YOU SURE DATA FOR THIS ASSEMBLY ARE CORRECT?"
402 PRINT "TYPE Y FOR NEXT ASSEMBLY; TYPE N TO RE-INPUT"
403 INPUT Y$
404 IF Y$="Y" THEN 406
405 IF Y$="N" THEN 380 ELSE 402
406 NEXT J
407 OPEN "o", 1, CS
410 FOR J=1 TO JOB
415 FOR I=1 TO OP(J)
420 FOR K=1 TO STN
425 WRITE #1, CST(J,I,K)
430 NEXT K: NEXT I: NEXT J
440 CLOSE #1

450 PRINT "IF YOU WANT TO INPUT NEW PROCESSING TIME,"
451 PRINT "TYPE Y, OTHERWISE, TYPE N"

```

```

460 INPUT YS
462 IF YS="Y" THEN 470
465 IF YS="N" THEN 605 ELSE 450
470 PRINT "NOW INPUT PROCESSING TIME"
475 DIM TM(JOB,STN,STN)
480 FOR J=1 TO JOB
490 FOR I=1 TO OP(J)
500 FOR K=1 TO STN
510 PRINT "TIME("J,I,K,")="
520 INPUT TM(J,I,K)
530 NEXT K: NEXT I
531 PRINT "ARE YOU SURE DATA FOR THIS ASSEMBLY ARE CORRECT?"
532 PRINT "TYPE Y FOR NEXT ASSEMBLY; TYPE N TO RE-INPUT"
533 INPUT YS
534 IF YS="Y" THEN 536
535 IF YS="N" THEN 490 ELSE 532
536 NEXT J
540 OPEN "o", 3, TS
550 FOR J=1 TO JOB
560 FOR I=1 TO OP(J)
570 FOR K=1 TO STN
580 WRITE #3, TM(J,I,K)
590 NEXT K: NEXT I: NEXT J
600 CLOSE #3

605 PRINT "IF YOU WANT TO INPUT NEW AVAILABLE TIME"
606 PRINT "OR FEEDER CAPACITY, TYPE Y, OTHERWISE, TYPE N"
607 INPUT YS
608 IF YS="Y" THEN 610
609 IF YS="N" THEN 760 ELSE 605
610 PRINT "NOW INPUT AVAILABLE TIME OF EACH WORK SATION"
615 DIM AT(STN), F(STN)
620 FOR K=1 TO STN
630 PRINT "AT(",K,")="
640 INPUT AT(K)
650 NEXT K
660 PRINT "NOW INPUT FEEDER CAPACITY OF EACH WORK STATION"
670 FOR K=1 TO STN
680 PRINT "F(",K,")="
690 INPUT F(K)
700 NEXT K
710 OPEN "o", 1, WSS
720 FOR K=1 TO STN
730 WRITE #1, AT(K), F(K)

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740 NEXT K
750 CLOSE #1

760 PRINT "IF YOU WANT TO INPUT NEW SLOT NUMBER,"
765 PRINT "TYPE Y, OTHERWISE TYPE N"
770 INPUT YS
771 IF YS="Y" THEN 780
775 IF YS="N" THEN 885 ELSE 760
780 PRINT "NOW INPUT SLOT NUMBER OF EACH COMPONENT"
790 DIM B(COMP)
800 FOR L=1 TO COMP
810 PRINT "B(",L,")="
820 INPUT B(L)
830 NEXT L
840 OPEN "o", 1, SLS
850 FOR L=1 TO COMP
860 WRITE #1, B(L)
870 NEXT L
880 CLOSE #1

885 PRINT "IF YOU WANT TO INPUT NEW REQUIRED COMPONENT #,"
886 PRINT "TYPE Y, OTHERWISE, TYPE N"
887 INPUT YS
888 IF YS="Y" THEN 890
889 IF YS="N" THEN 990 ELSE 885
890 PRINT "NOW INPUT NUMBER OF EACH COMPONENT FOR EACH OPERATION"
895 DIM A(JOB,STN,COMP)
900 FOR J=1 TO JOB
905 FOR I=1 TO OP(J): FOR L=1 TO COMP
910 PRINT "A(",J,I,L,")="
920 INPUT A(J,I,L)
930 NEXT L: NEXT I
931 PRINT "ARE YOU SURE DATA FOR THIS ASSEMBLY ARE CORRECT?"
932 PRINT "TYPE Y FOR NEXT ASSEMBLY; TYPE N TO RE-INPUT"
933 INPUT YS
934 IF YS="Y" THEN 936
935 IF YS="N" THEN 905 ELSE 932
936 NEXT J
940 OPEN "o", 1, PS
950 FOR J=1 TO JOB: FOR I=1 TO OP(J): FOR L=1 TO COMP
960 WRITE #1, A(J,I,L)
970 NEXT L: NEXT I: NEXT J
980 CLOSE #1

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990 PRINT "IF YOU WANT TO INPUT THE NEW PENALTY MULTIPLIER,"
1000 PRINT "TYPE Y, OTHERWISE TYPE N."
1010 INPUT YS
1020 IF YS="Y" THEN 1030
1025 IF YS="N" THEN 1100 ELSE 1020
1030 PRINT "INPUT WORK STATION # WITH MAXIMUN WORK LOAD"
1040 INPUT MWL
1050 PRINT "INPUT PENALTY MULTIPLIER"
1060 INPUT LMD
1070 OPEN "o", 1, PNTS
1080 WRITE #1, MWL,LMD
1090 CLOSE #1
1100 END

```

```

100 REM MAIN PROGRAM FOR CALCULATING ALL THE COEFFICIENTS
110 CS="COST.DAT"
120 TS="TIME.DAT"
130 PS="PART.DAT"
140 AS="ASSY.DAT"
150 SLS="SLOT.DAT"
160 WSS="WKSTN.DAT"
170 NUMS="NUMBER.DAT"
180 PNTS="PENALTY.DAT"

```

```

190 REM CALCULATE NUMBERS OF COLUMNS AND ROWS
200 OPEN "i", 1, NUMS
210 INPUT #1, JOB, STN, COMP, PRR, BEH, PCT
220 DIM OP(JOB), D(JOB), TM(JOB,STN,STN), AT(STN), F(STN)
230 OPEN "i", 2, AS
240 FOR J=1 TO JOB
250 INPUT #2, OP(J), D(J)
260 NEXT J
270 OPEN "i", 3, TS
280 FOR J=1 TO JOB: FOR I=1 TO OP(J): FOR K=1 TO STN
290 INPUT #3, TM(J,I,K)
300 NEXT K: NEXT I: NEXT J
310 CLOSE #1, #2, #3
320 TJ=0: COL=0
330 FOR J=1 TO JOB
340 TJ=TJ+OP(J)
350 FOR I=1 TO OP(J)
360 FOR K=1 TO STN
370 IF TM(J,I,K)=0 THEN 390
380 COL=COL+1

```

```

390 NEXT K: NEXT I: NEXT J
400 ROW=TJ+STN*(STN+2*JOB)-(STN-1)^2
410 PRINT "NUMBER OF VARIABLES =", COL
420 PRINT "NUMBER OF CONSTRIANTS =", ROW

430 DIM COF(COL), CST(JOB,STN,STN)
440 COEFS="PCOEF.DAT"
450 REM COEFFICIENTS OF OBJECTIVE FUNCTION
460 OPEN "i", 1, CS
470 FOR J=1 TO JOB
480 FOR I=1 TO OP(J)
490 FOR K=1 TO STN
500 INPUT #1, CST(J,I,K)
510 NEXT K: NEXT I: NEXT J
520 OPEN "i", 2, WSS
530 FOR K=1 TO STN
540 INPUT #2, AT(K), F(K)
550 NEXT K
560 OPEN "i", 3, PNTS
570 INPUT #3, MWS, LMD
580 CLOSE #1, #2, #3
590 OPEN "o", 3, COEFS
600 H=1: HANG=0
610 FOR J=1 TO JOB
620 FOR I=1 TO OP(J)
630 FOR K=1 TO STN
640 IF CST(J,I,K)=0 THEN 700
645 IF MWS=0 THEN 680
650 IF K=MWS THEN 680
660 COF(H)=D(J)*(CST(J,I,K)-LMD*TM(J,I,K)/AT(K))
670 GOTO 690
680 COF(H)=D(J)*(CST(J,I,K)+(STN-1)*LMD*TM(J,I,K)/AT(K))
690 H=H+1
700 NEXT K: NEXT I: NEXT J
710 FOR H=1 TO COL
720 WRITE #3, HANG, H, COF(H)
730 NEXT H
740 HANG=HANG+1

750 REM COEFFICIENTS FOR CONSTRIANT #2
760 FOR JJ=1 TO STN
770 H=1
780 FOR J=1 TO JOB
790 FOR I=1 TO OP(J)

```



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800 FOR K=1 TO STN
810 IF TM(J,I,K)=0 THEN 870
820 IF K<>JJ THEN 850
830 COF(H)=D(J)*TM(J,I,K)
840 GOTO 860
850 COF(H)=0
860 H=H+1
870 NEXT K: NEXT I: NEXT J
880 FOR H=1 TO COL
890 IF COF(H)=0 THEN 910
900 WRITE #3, HANG, H, COF(H)
910 NEXT H
920 HANG=HANG+1
930 NEXT JJ

940 REM COEFFICIENTS FOR CONSTRIANT #3
950 DIM B(COMP), A(JOB,STN,COMP)
960 OPEN "i", 1, PS
970 FOR J=1 TO JOB
980 FOR I=1 TO OP(J)
990 FOR L=1 TO COMP
1000 INPUT #1, A(J,I,L)
1010 NEXT L: NEXT I: NEXT J
1020 OPEN "i", 2, SL$
1030 FOR L=1 TO COMP
1040 INPUT #2, B(L)
1050 NEXT L
1060 CLOSE #1, #2
1070 FOR JJ=1 TO JOB
1080 FOR KK=1 TO STN
1090 H=1
1100 FOR J=1 TO JOB
1110 FOR I=1 TO OP(J)
1120 FOR K=1 TO STN
1130 IF TM(J,I,K)=0 THEN 1210
1140 COF(H)=0
1150 IF J<>JJ THEN 1200
1160 IF K<>KK THEN 1200
1170 FOR L=1 TO COMP
1180 COF(H)=COF(H)+B(L)*A(J,I,L)
1190 NEXT L
1200 H=H+1
1210 NEXT K: NEXT I: NEXT J
1220 FOR H=1 TO COL

```

```

1230 IF COF(H)=0 THEN 1250
1240 WRITE #3, HANG, H, COF(H)
1250 NEXT H
1260 HANG=HANG+1
1270 NEXT KK: NEXT JJ

1280 REM COEFFICIENTS FOR CONSTRAINT #4
1290 FOR JJ=1 TO JOB
1300 FOR II=1 TO OP(JJ)
1310 H=1
1320 FOR J=1 TO JOB
1330 FOR I=1 TO OP(J)
1340 FOR K=1 TO STN
1350 IF TM(J,I,K)=0 THEN 1410
1360 IF J<>JJ THEN 1390
1370 IF I<>II THEN 1390
1380 COF(H)=1: GOTO 1400
1390 COF(H)=0
1400 H=H+1
1410 NEXT K: NEXT I: NEXT J
1420 FOR H=1 TO COL
1430 IF COF(H)=0 THEN 1450
1440 WRITE #3, HANG, H, COF(H)
1450 NEXT H
1460 HANG=HANG+1
1470 NEXT II: NEXT JJ

1480 REM COEFFICIENTS FOR CONSTRAINT #5
1490 FOR JJ=1 TO JOB
1500 FOR KK=1 TO STN
1510 H=1
1520 FOR J=1 TO JOB
1530 FOR I=1 TO OP(J)
1540 FOR K=1 TO STN
1550 IF TM(J,I,K)=0 THEN 1670
1560 IF J<>JJ THEN 1650
1570 IF I=BEH THEN 1620
1580 IF I<>PRR THEN 1650
1590 IF K>KK THEN 1650
1600 COF(H)=-1
1610 GOTO 1660
1620 IF K<>KK THEN 1650
1630 COF(H)=1
1640 GOTO 1660

```

```

1650 COF(H)=0
1660 H=H+1
1670 NEXT K: NEXT I: NEXT J
1680 FOR H=1 TO COL
1690 IF COF(H)=0 THEN 1710
1700 WRITE #3, HANG, H, COF(H)
1710 NEXT H
1720 HANG=HANG+1
1730 NEXT KK: NEXT JJ

1740 REM COEFFICIENTS FOR CONSTRIANT #6
1745 IF LMD=0 THEN 1960
1750 FOR KK=1 TO STN
1755 IF KK=MWS THEN 1950
1760 H=1
1770 FOR J=1 TO JOB
1780 FOR I=1 TO OP(J)
1790 FOR K=1 TO STN
1800 IF TM(J,I,K)=0 THEN 1890
1810 IF K<>KK THEN 1840
1820 COF(H)=-D(J)*TM(J,I,K)/AT(K)
1830 GOTO 1880
1840 IF K<>MWS THEN 1870
1850 COF(H)=D(J)*TM(J,I,K)/AT(K)
1860 GOTO 1880
1870 COF(H)=0
1880 H=H+1
1890 NEXT K: NEXT I: NEXT J
1900 FOR H=1 TO COL
1910 IF COF(H)=0 THEN 1930

1920 WRITE #3, HANG, H, COF(H)
1930 NEXT H
1940 HANG=HANG+1
1950 NEXT KK

1960 REM RELATION BETWEEN BOTH SIDES FOR EACH CONSTRIANT
1970 DIM COE(ROW): H=1
1980 FOR K=1 TO STN
1990 COE(H)=1
2010 H=H+1
2020 NEXT K
2030 FOR J=1 TO JOB
2040 FOR K=1 TO STN

```

```

2050 COE(H)=1
2060 H=H+1
2070 NEXT K: NEXT J
2080 FOR J=1 TO JOB
2090 FOR I=1 TO OP(J)
2100 COE(H)=0
2110 H=H+1
2120 NEXT I: NEXT J
2130 FOR J=1 TO JOB
2140 FOR K=1 TO STN
2150 COE(H)=1
2160 H=H+1
2170 NEXT K: NEXT J
2180 FOR K=1 TO STN-1
2190 COE(H)=-1
2200 H=H+1
2210 NEXT K
2220 FOR HANG=1 TO H-1
2230 X=COL+HANG
2240 WRITE #3, HANG, X, COE(HANG)
2250 NEXT HANG

2260 REM VALUE OF RIGHT HAND SIDE FOR EACH CONSTRIANT
2270 H=0
2280 FOR K=1 TO STN
2290 H=H+1
2300 COE(H)=AT(K)
2310 NEXT K
2320 FOR J=1 TO JOB
2330 FOR K=1 TO STN
2340 H=H+1
2350 COE(H)=F(K)
2360 NEXT K: NEXT J
2370 FOR J=1 TO JOB
2380 FOR I=1 TO OP(J)
2390 H=H+1
2400 COE(H)=1
2410 NEXT I: NEXT J
2420 FOR J=1 TO JOB
2430 FOR K=1 TO STN
2440 H=H+1
2450 COE(H)=0
2460 NEXT K: NEXT J
2470 FOR K=1 TO STN-1

```

```
2490 H=H+1
2500 COE(H)=0
2510 NEXT K
2520 FOR HANG=1 TO H
2530 WRITE #3, HANG, 0, COE(HANG)
2540 NEXT HANG
2550 CLOSE #3

2560 END
```

VITA AUCTORIS

- 1960 Born in Beijing, P.R.China, on July 14th.
- 1983 Graduated from Tsinghua University, Beijing, P.R.China, with a Bachelor's degree of Engineering.
- 1983-1989 Worked in Beijing Design Institute of Posts and Telecommunication Ministry, Beijing, P.R.China, as a mechanical engineer.
- 1990 Worked in Beijing Office of Liebert Co., Beijing, P.R.China, as an application engineer.
- 1992 Currently a candidate for the degree of Master of Applied Science in Industrial Engineering at the University of Windsor, Windsor, Ontario, Canada.