Modelling the viscoelastic properties of low-density polyethylene.

Michelle Marie. Karl
University of Windsor

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MODELLING THE VISCOELASTIC PROPERTIES OF LOW DENSITY POLYETHYLENE

by

Michelle Marie Karl

A Thesis
Submitted to the Faculty of Graduate Studies and Research through the Department of Mechanical and Materials Engineering in Partial Fulfillment of the Requirements for the Degree of Masters of Applied Science in Engineering Materials at the University of Windsor

Windsor, Ontario, Canada
1998

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To My Husband,
Karsten
and my parents,
Clare and Barbara Renaud
ABSTRACT

As polymers become more widely used by design engineers, it is important that the viscoelastic mechanical behaviour of these materials is properly taken into account. It is common that textbooks written on the viscoelastic properties of polymers include a section on the use of spring-dashpot models as a useful tool for the characterization of these properties. This thesis is an investigation into the ease of use, the reliability and the flexibility of three spring-dashpot analog models to represent the viscoelastic response of a polymer under different constraints of imposed loads and strains. In particular, this thesis describes attempts to take results from constant stress creep tests and constant strain stress relaxation tests and apply these results to a more varied four zone tensile test regimen. This anticipates a scenario where a design engineer might use creep and stress relaxation data in a numerical modelling program (e.g. finite element modelling) to predict behaviour in bending or some other loading-unloading sequence.

In the present study, four different spring dashpot models were considered. One of these, commonly called the Burger’s model, which is widely discussed in the literature, proved to be completely unsuitable for this application. A second, the Maxwell-Weichert model was somewhat more successful. Two new models were developed by the author that proved to be more useful. However, the general conclusion was that the models which could successfully simulate constant stress creep and constant strain stress
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I. INTRODUCTION

A fundamental characteristic of polymer materials is that they are viscoelastic in their response to stress and strain. This characteristic leads to the time dependent nature of their response to loading or deformation. This dependence on time is quite different from the elastic response of metals and ceramics to applied stress and strain, which obey Hooke’s law, which is time independent. In contrast, liquids demonstrate a complete time dependence in response to stress and strain, where the stress is proportional to the rate of straining. Thus, polymers behave somewhere between solids and liquids in their response to applied mechanical stress or strain. The response of polymers to stress and strain is dependent on several factors such as temperature, residual stress, time of loading, loading rate, and molecular structure.

A common way of simulating the behaviour of polymer materials subjected to an applied stress or strain is to use analogous mechanical elements. Viscoelastic behaviour suggests that the deformation of the polymer material can be divided into an elastic component and a viscous component. The spring can represent the linear elastic component of deformation. Its characteristic equation is given by Hooke’s Law and is dependent on the stiffness of the spring and the deformation that it undergoes. Its fundamental characteristic is that it can deform without plastic strain, such that it is capable of returning to its original shape upon unloading. The dashpot represents the time dependent plastic loading. Its fundamental equation is dependent on the viscosity of the
dashpot (the inverse of its fluidity) and the rate at which it is strained. Its characteristic 
equation is given by Newton’s law which describes the linear viscous behaviour.

In the nineteenth century, James Maxwell was apparently one of the first 
researchers who proposed an analog model to represent material behaviour. He designed 
a model consisting of a linear elastic element, a spring, in series with a linear viscous 
element, a dashpot. This model was originally derived to explain the mechanical 
behaviour of materials such as tar or pitch. Its derivation and limitations will be discussed 
further in Chapter 2.

The Voigt Kelvin model another nineteenth century analog that was derived to 
predict the behaviour of polymer materials. This system is a spring arranged in parallel 
with a dashpot. The parallel arrangement of the spring and the dashpot means that the 
strain in each element is equal and the stress on the element is equal to the sum of the 
stresses in the respective components. The derivation and limitations of the Voigt Kelvin 
element will be discussed later as well as its application in more complex models.

At an introductory level, various texts have demonstrated that the Maxwell model 
is accurate in describing the stress relaxation phenomenon of polymeric materials, while 
the Voigt Kelvin model describes the behaviour of creep. Therefore, the next logical step 
is to find a model that can accommodate both of these stress/strain constraints as well as 
others that may be placed on the system. Many complex models have been devised which
give a better representation of the viscoelastic nature of polymers. Unfortunately, the mathematical models that are derived can only describe the mechanical response of polymers. They can not give much insight as to the details of how the molecules respond.

The present study involves the development of models that can accommodate various types of stress-strain-time schemes with relative accuracy. The project is different from the standard approach in that it applies each of three models to several different stress or strain test procedures, rather than choosing a different equivalent model for each set of stress-strain-time constraints. In addition, it will simulate stress and strain over a history of different stress-strain-time constraints. This history includes a testing sequence that involves four zones. These zones were derived to represent the polymer behaviour that occurs in the clinical insertion of LDPE intra uterine devices (IUDs), and also in a commercially standard memory test. The memory test is an international quality standard for the retentive memory of an IUD. The IUD is in the form of a T shaped object. Thus, the memory test involves deformation of the T. The top is referred to as the lateral arms of the T whereas the base is referred to as the stem. The memory test is such that the lateral arms of the T are bent down parallel to the stem of the T section. The IUD is then placed in a cavity for a period of time where the arms are released but confined to their bent position by the wall of the cavity. The T is then removed from the cavity and allowed to relax, and a measurement of recovery of shape is read from a defined scale after a specified relaxation time.
The arms of the T device were too small to accurately measure bending loads, so it was decided to do extensive tensile tests to mimic the load, hold and release cycle. It is assumed that one dimensional tensile test data are representative of bending stresses and strains at different positions along a long slender arm.

The tensile test apparatus used was a Rheometrics Scientific MiniMat 2000 modified to meet the specifications of an IUD memory test. This machine loaded the sample at constant strain rate to a prescribed load, then the device is held at a constant strain while measuring the stress relaxation, then it is unloaded at constant strain to a zero load, and then the reverse creep occurring at zero load for a prescribed time period is measured.

In order to model this viscoelastic behaviour of polymer materials, we have proposed three mechanical analog models. They will be evaluated individually with respect to their response to some fundamental polymer characteristics such as stress relaxation and creep, and then finally, applied to the four zone tensile regimen described above.
II. LITERATURE REVIEW

The application of models to represent the stress-strain-time domain of polymer materials is not a new practice. Rather, many attempts have been made to find a constitutive equation for stress-strain-time behaviour using spring dashpot analogs.

2.1 Linear/Non-linear Behaviour

Polymers are viscoelastic at all temperatures. They never behave as simply elastic solids; rather, their response is always time dependent. Their viscoelastic properties are also highly temperature dependent such that in testing polymers the maximum temperature must be specified. The major difference between elastic solids, such as metals, and polymers can be illustrated by relating their response to molecular movement. As a result of the long molecular chains of polymeric materials, polymers cohere even when small sections of their chains undergo motion. In a liquid polymer, the entire chain undergoes some motions as polymer chains slide past each other. In a glassy polymer, only limited motion is possible since the chain is essentially immobile. A mechanical stress applied to polymers leads to time dependent movement of sections of the molecular chains. When the mechanical stress is removed, the molecules move to reposition themselves. Stachurski\(^1\), following Doi and Edwards\(^2\), and de Gennes\(^3\), describes this behaviour using the reptation theory. Stachurski describes the flow of polymers as being very unusual and complicated. The mobility of polymer macromolecules is quite different from the mobility of atoms in metal and ceramics. The polymer chain cannot move freely because chains
cannot simply pass through or across other adjacent polymer chains. Thus, polymers have no sideways translations. A simple explanation can be done by imagining that the polymer chain is confined to a “tube like” region. This is shown below in Figure 2.1. Within the polymer “tube” the flexibility of the chain allows motion of the molecules along the center line of the chain but not across other chains. The two exceptions are the chain ends. The ends can move in random directions within the polymer melt. This snake-like wriggling motion is called reptation. It describes the basic nature of diffusion and the sliding of chains past each other in polymer materials.

![Figure 2.1 - Polymer molecules in a tube-like region as described by reptation theory. (from Ref [1])](image)

Creep is described as the time dependent strain that results from a constant stress being applied to a material. The results of linear viscoelastic creep experiments can be seen in Figure 2.2. If a constant stress is applied to a viscoelastic material, then the strain is observed to be time dependent (Figure 2.2(a)). If a larger stress is applied to the same material, the time dependence of the strain resulting from the second stress application can be seen in Figure 2.2(b). If the strains at any particular time $t$, plotted against the stress result in a straight line, then the material is defined to be “linearly viscoelastic”. Therefore, for any time $t$, the strains at stresses $\sigma_1$ and $\sigma_2$ are related by the equation:
\[
\frac{\varepsilon_1(t)}{\sigma_1} = \frac{\varepsilon_2(t)}{\sigma_2}
\]  

(2.1)

Since the strains in the two experiments are proportional to the stresses, this leads to the definition of the creep compliance \(J(t)\) such that we get the equation:

\[
J(t_1) = \frac{\varepsilon_1(t_1)}{\sigma_1} = \frac{\varepsilon_2(t_1)}{\sigma_2}
\]  

(2.2)

Polymers demonstrate this linear response to creep at very low values of stress.

![Figure 2.2 - Linear viscoelastic creep: (a) constant stress \(\sigma_1\) applied leads to time-dependent strain \(\varepsilon_1(t)\); (b) a higher stress \(\sigma_2\) applied leads to time dependent strain \(\varepsilon_2(t)\); (c) linear relationship between stress and strain at different time \(t_1\) and \(t_2\). (from Reference [4])](image)

The creep curves plotted where strain is a function of the stress at a specific time, Figure 2.2(c), are known as isochronal plots. Isochronal plots are used to determine where the viscoelastic behaviour changes from linear to non-linear. The non-linear behaviour leads the material to creep faster as shown by the full lines in Figure 2.3.
Figure 2.3 - Isochronal plots taken after the initiation of the creep experiment demonstrating the transition from the linear to the non-linear region. (from Ref [4])

It is noted that in the linear range the creep compliance \( J(t_i) \) is independent of the stress.

The value of stress is not definitive since the same value of \( J(t_i) \) is found at time \( t_i \), whether the stress used is \( \sigma_1 \) or \( \sigma_2 \). However, in the non-linear range this is clearly not the case, since the compliance \( J(t_i) \), which is the slope of the \( \varepsilon(t) \) vs. \( \sigma \) line, is now a higher order function of the stress.

A similar analysis can be done using a typical stress relaxation test. Stress relaxation is the time dependent response of stress to a constant strain value. Stress is observed to decrease with time as shown in Figure 2.4. Suppose a constant strain value \( \varepsilon_1 \) is applied to a linear viscoelastic material. The resulting stress is observed to decrease with time as seen in Figure 2.4(a). If a higher strain is then imposed on the same material, a higher initial stress will result but will also decrease with time. As low strains, as observed with creep, the isochronal plots are found to be linear. Therefore, at an any time \( t_i \), the stresses in the experiments are described by the equation:
\[
\frac{\sigma_1(t_i)}{\varepsilon_1} = \frac{\sigma_2(t_i)}{\varepsilon_2}
\] (2.3)

This leads to the definition of a relaxation modulus at time \(t_i\):
\[
E(t_i) = \frac{\sigma_1(t_i)}{\varepsilon_1} = \frac{\sigma_2(t_i)}{\varepsilon_2}
\] (2.4)

Figure 2.4 - Linear viscoelastic stress relaxation: (a) constant strain \(\varepsilon_1\) applied leads to time-dependent stress \(\sigma_1(t)\); (b) a higher strain \(\varepsilon_2\) applied leads to time dependent stress \(\sigma_2(t)\); (c) linear relationship between stress and strain at different time \(t_a\) and \(t_b\). (from Ref [4])

It is relatively simple to determine the strain range at which the viscoelastic stress relaxation changes from linear to non-linear. In order to do so, it is necessary to consider several isochronals such as those in Figure 2.4(c). The transition from linear to non-linear behaviour is demonstrated in Figure 2.5 below.
2.2 - Mechanical Models

A common way of simulating the behaviour of polymer materials subjected to an applied stress or strain is to use mechanical elements or combinations of mechanical elements. Since the fundamental behaviour of polymer materials is viscoelastic, it can be assumed that the deformation of a polymer can be divided into an elastic component and a viscous component. In terms of mechanical elements, these two phenomena can be represented by an elastic spring and a viscous dashpot. Various models have been proposed to represent viscoelasticity in terms of these two mechanical elements. The two most common seen in the literature are the Maxwell model and the Voigt Kelvin model. These two models are composed of a spring and a dashpot in series and parallel respectively.

The linear elastic behaviour is represented by the spring (shown in Figure 2.6). Its behaviour is described by Hooke's Law
\[ \sigma = E \varepsilon \]  

(2.5)

where \( E \) is the modulus of elasticity of the material.

The viscous component is represented by a viscous dashpot (Figure 2.7), which follows Newton's law of linear viscous behaviour

\[ \sigma = \eta \frac{d \varepsilon}{d t} \]  

(2.6)

where \( \eta \) is the viscosity and \( \frac{d \varepsilon}{d t} \) is the strain rate.

Considering the Maxwell and Voigt model individually, we can compare their response to applied stress and strain by their constitutive equations.

**A. The Maxwell Model**

The Maxwell model was apparently proposed in the last century by Lord Maxwell to explain the time-dependent mechanical behaviour of viscous materials. The most
commonly quoted reference makes no mention of springs or dashpots, nor does it propose the equations representing a "Maxwell" model. An extensive review of his work (1967) similarly has no evidence that he proposed such a model.

The Maxwell model consists of a spring and dashpot in series as shown in Figure 2.8.

![Figure 2.8 - The Maxwell Model](image)

The fundamental equations for the stress and strain in a Maxwell model are

\[ \sigma = \sigma_{\text{spring}} = \sigma_{\text{dashpot}} \]  
(2.7)

and \[ \varepsilon_{\text{tot}} = \varepsilon_{\text{spring}} + \varepsilon_{\text{dashpot}} \]  
(2.8)

Differentiation of equation (2.8) leads us to

\[ \frac{d\varepsilon_{\text{tot}}}{dt} = \frac{d\varepsilon_{\text{spring}}}{dt} + \frac{d\varepsilon_{\text{dashpot}}}{dt} \]  
(2.9(a))

and when (2.5) and (2.6) are substituted gives

\[ \frac{d\varepsilon_{\text{tot}}}{dt} = \frac{d\sigma_{\text{spring}}}{dt} \frac{1}{E} + \frac{\sigma_{\text{dashpot}}}{\eta} \]  
(2.9(b))
The stress-strain-time relationships can be determined from Equation (2.9(b)) as one considers the model subjected to different stress-time and strain-time constraints.

For example, applying a constant stress $\sigma = \sigma_c$ at $t = 0$, Equation (2.9(b)) becomes a first order differential equation of strain such that the relationship of strain with respect to time is

$$\varepsilon(t) = \frac{\sigma_c}{E} + \frac{\sigma_c}{\eta} t \quad (2.10)$$

The result can be seen in the Graph of Figure 2.9 below.

![Creep of Maxwell Model](image)

**Figure 2.9** - Creep at constant stress of the Maxwell Model.

If at any time the stress is removed, the strain in the elastic spring will reduce to zero but a permanent deformation will remain in the dashpot.

If the Maxwell model is subjected to an instantaneously applied constant strain $\varepsilon = \varepsilon_c$, the relationship of the stress response with time can be obtained by solving the differential Equation (2.9(b)) yielding

$$\sigma(t) = C_1 \exp\left(\frac{-E}{\eta} t\right) \quad (2.11(a))$$
where $C_1$ is the constant of integration which can be solved at $t=0$. Since the dashpot is time dependent, at $t = 0$, it cannot have undergone any strain at this instant, and all the initial strain is in the spring so that

$$C_1 = E\varepsilon_o$$

(2.11(b))

where $E$ is the elasticity of the spring. The equation for stress with respect to time becomes

$$\sigma(t) = E\varepsilon_o \exp\left(\frac{-E}{\eta} t\right)$$

(2.11(c))

Equation (2.7(c)) describes the stress relaxation equation for the Maxwell model under constant strain. This phenomenon is shown in Figure 2.10 below.

![Stress Relaxation of the Maxwell Model](image)

Figure 2.10 - Stress Relaxation of the Maxwell Model held at constant strain.

In equation (2.11(c)), the quantity $\frac{\eta}{E}$ is commonly referred to as the relaxation time $\tau$.

Thus, it is common to see equation (2.11(c)) represented by:

$$\sigma(t) = E\varepsilon_o \exp\left(-\frac{t}{\tau}\right)$$

(2.11(d))
According to Equation (2.9(b)), extrapolation of the initial rate of stress relaxation will intersect the time scale at \( t \) which is equal to \( t = \frac{\eta}{E} \). The relaxation time is one of the properties that characterizes a material. Most of the stress relaxation of a material occurs at a time before this value as can be seen in Figure 2.10. Therefore, at a time \( t=t_1 \), only 37% of the original stress remains.

In the text by Flugge\textsuperscript{6} (1967), the Maxwell model is subjected to what Flugge describes as a “standard test” consisting of a creep phase at constant stress followed by a stress relaxation phase at constant strain. Flugge uses the same creep derivation seen above but stops this stage of loading at some time \( t=t_1 \). It is noted that if the creep phase were to extend beyond \( t=t_1 \), then the strain would increase beyond a reasonable range. In the physical sense, this would mean that we would be extending the strain beyond its linear region and therefore a more complicated model would be necessary to describe true polymeric behaviour. Flugge terms the Maxwell model, the “Maxwell Fluid” model because in the ranges observed, the material shows a typical property of a fluid in that it has unlimited deformation under applied stress.

Following the creep phase of the “standard test”, the same stress relaxation equation is seen by Flugge

\[
\sigma(t) = C_2 \exp\left(\frac{-E}{\eta} t\right)
\]  

(2.11(a))
However, in this case, there is a loading history and it cannot be assumed that the strain was applied instantaneously. To solve for $C_2$, it must be assumed that at the time the stress relaxation phase was applied, the stress was equal the constant stress of the creep phase. To assume otherwise would result in a discontinuity in the curve. Therefore, equation (2.11(a)) becomes

$$\sigma(t) = \sigma_0 \exp\left(\frac{-E}{\eta} (t - t_1)\right) \quad (2.12)$$

The Maxwell model subjected to Flugge's "standard test" is then shown graphically in Figure 2.11. This "standard test" does not have much of an effect when applied to a Maxwell model but it is useful when applied to the Voigt-Kelvin Model discussed next.

![Figure 2.11 - The Maxwell model under the constraints of the two phase standard test described by Flugge. The two phases are: the creep at constant stress followed by the stress relaxation at constant strain.](image)

B. The Voigt-Kelvin Model

Another simple model to represent viscoelasticity is the Voigt-Kelvin Model. The Voigt-Kelvin Model is made up of an elastic spring in parallel with a viscous dashpot. It can be seen in Figure 2.12 below. Its arrangement means that the elongation of the spring
must always be equal to the elongation of the dashpot and the total stress on the model is divided into the sum of the stress in the spring and the stress in the dashpot.

Fundamentally, the spring is still described by Hooke’s Law, and the dashpot by Newton’s law such that

\[ \sigma = E \varepsilon \]  
\[ \sigma = \eta \frac{d\varepsilon}{dt} \]

(2.5)  
(2.6)

The total stress and strains are described by

\[ \sigma = \sigma_{\text{spring}} + \sigma_{\text{dashpot}} \]

(2.13)

\[ \varepsilon_{\text{tot}} = \varepsilon_{\text{spring}} = \varepsilon_{\text{dashpot}} \]

(2.14)

Substituting the relationships given by Hooke and Newton, gives

\[ \sigma = E \varepsilon + \eta \frac{d\varepsilon}{dt} \]

(2.15)

where \( \varepsilon \) is the strain in each of the spring and the dashpot.

![Figure 2.12 - The Voigt-Kelvin Model](image)

If the Voigt Kelvin model is considered subjected to constant stress creep, the differential equation for strain with respect to time can be solved to give Equation (2.16).
\varepsilon(t) = \frac{\sigma_0}{E} \left(1 - \exp\left(-\frac{E}{\eta} t\right)\right) \tag{2.16}

As shown in Figure 2.13, the strain is seen to increase exponentially with time until reaching an asymptotic value at \( \frac{\sigma_0}{E} \). At the time of instantaneous loading to a constant stress \( \sigma = \sigma_0 \), the stress is initially supported by the dashpot since the spring cannot extend. Over time, the load is transferred to the spring. As the time approaches infinity (\( t \to \infty \)), the spring carries the entire stress. This behaviour is sometimes called delayed elasticity.

![Creep of the Voigt Kelvin Model](image)

**Figure 2.13 - Creep at constant stress of the Voigt-Kelvin Model. [From Ref. 6]**

The strain rate for the Voigt Kelvin Model in creep at constant stress can be found by differentiating Equation (2.16).

\[
\frac{d\varepsilon}{dt} = \frac{\sigma_0}{\eta} \exp\left(-\frac{E}{\eta} t\right) \tag{2.17}
\]

Thus, the strain rate at \( t=0 \) is finite and equal to \( \frac{\sigma_0}{\eta} \). As shown in Figure 2.13, as the time approaches infinity, the strain approaches a constant value such that the strain rate approaches zero. If the strain rate were to be extrapolated at its initial rate \( \frac{\sigma_0}{\eta} \), then
this line would intersect the asymptotic value of strain \( \varepsilon = \frac{\sigma_o}{E} \) at time \( \frac{t}{\eta} \). This value is called the relaxation time, and is a very useful indication of the viscoelastic response. It is commonly represented as \( \tau \) in creep and stress relaxation equations.

It is very common but unrealistic to assume an instantaneous constant strain of the Voigt Kelvin model, because the dashpot is time dependent in deformation and the spring cannot extend independently. Therefore, stress relaxation is better considered as a second phase application, following a creep phase.

This "standard test" of the Voigt-Kelvin model is applied by Flugge. Since in the creep phase, the strain does not grow indefinitely, this behaviour is almost indicative of an elastic solid. However, in this case, the strain does not instantly attain a final value but rather approaches this value gradually. As a result, Flugge terms this the Kelvin solid or the Voigt solid.

In the relaxation phase of the experiment at \( t > t_1 \), we keep the strain equal to the final strain of the creep phase \( \varepsilon = \varepsilon_1 \) and thus we get from Equations (2.15) and (2.16):

\[
\sigma(t) = E\varepsilon_1 = \sigma_o \left( 1 - \exp \left( -\frac{E}{\eta} t \right) \right)
\]  \hspace{1cm} (2.18)

If the strain has already reached its asymptotic value, then the stress will remain constant at the value of \( \sigma_o = E\varepsilon_1 \). There is no stress relaxation because the dashpot senses no load. However, if the strain has not yet reached the asymptotic value, then the strain in
the spring will immediately relax by a certain amount but the strain in the dashpot will remain constant and forever at this value. Flugge considers the latter case to be incomplete stress relaxation. These two cases can be shown in Figures 2.14 and 2.15.

Figure 2.14 - The Voigt Kelvin Model subjected to the standard test. The relaxation phase is initiated after the strain reaches the asymptotic value. [from Ref 6]

Figure 2.15 - The Voigt-Kelvin Model subjected to the standard test. The relaxation phase is initiated before the strain reaches the asymptotic value. [from Ref 6]

2.3 The Four Parameter Model

It has been suggested by many authors that the Maxwell and Voigt Kelvin model be combined in order to provide an accurate representation of both stress relaxation and creep phenomena in the same model. The four parameter model is reviewed in the texts by Progelhof and Throne (1993)\textsuperscript{7}, Carreau, DeKee and Chabra (1997)\textsuperscript{8}, and Findley, Lai
and Onaran (1989)⁹ to name but a few. Authors Findley, Lai and Onaran in their text call this combination the Burger’s model.

The Burger’s model can be seen in the figure below. It is configured such that a Maxwell element is placed in series with a Voigt Kelvin element.

![Figure 2.16 - The Burger’s four element model](image)

The authors derive the constitutive equation for a Burger’s model by considering the strain response of the model to an applied constant stress. The values of elasticity of the springs are $E_1$, and $E_2$ and the values of the viscosities are $\eta_1$ and $\eta_2$ as shown in the figure above. The authors further separate the Burger’s model into three elements. The total strain will be the sum of the strains of the three elements where the spring and the dashpot in the Maxwell model are considered to be two separate elements.

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$  \hspace{1cm} (2.14)

where $\varepsilon_1$ is the strain in the spring
\[ \varepsilon_1 = \frac{\sigma}{E_1} \]  \hspace{1cm} (2.1)

\( \varepsilon_2 \) is the strain in the dashpot such that

\[ \frac{d\varepsilon_2}{dt} = \frac{\sigma}{\eta_1} \]  \hspace{1cm} (2.2)

and \( \varepsilon_3 \) is the strain in the Voigt Kelvin element

\[ \frac{d\varepsilon_3}{dt} + \frac{E_2}{\eta_2} \varepsilon_3 = \frac{\sigma}{\eta_2} \]  \hspace{1cm} (2.10)

where the stress is assumed to be constant. The authors then combine the strains of each individual defined element, and use Laplace Transformations to yield the following result

\[ \sigma + \left( \frac{\eta_1}{E_1} + \frac{\eta_2}{E_2} \right) \frac{d\sigma}{dt} + \frac{\eta_1 \eta_2}{E_1 E_2} \frac{d^2 \sigma}{dt^2} = \eta_1 \frac{d\varepsilon}{dt} + \eta_2 \frac{d^2 \varepsilon}{dt^2} \]  \hspace{1cm} (2.19)

The creep behaviour of the Burger's model under constant stress \( \sigma_0 \) can then be obtained by solving the differential equation above using two initial conditions. At \( t=0 \) (i.e. after the instant of loading), it is assumed that all strain is initially in the spring since the other two elements are time dependent.

\[ \varepsilon = \varepsilon_1 = \frac{\sigma_0}{E_1} \]  \hspace{1cm} (2.20(a))

\[ \varepsilon_2 = \varepsilon_3 = 0 \]  \hspace{1cm} (2.20(b))

However, the strain rate of the other two elements can be solved at \( t=0 \) and can be combined to give

\[ \frac{d\varepsilon}{dt} = \frac{\sigma_0}{\eta_1} + \frac{\sigma_0}{\eta_2} \]  \hspace{1cm} (2.20(c))

Therefore, the equation 2.21 results in
\[ \varepsilon(t) = \frac{\sigma_o}{E_1} + \frac{\sigma_o}{\eta_1} t + \frac{\sigma_o}{E_2} \left( 1 - \exp \left( -\frac{E_1 t}{\eta_2} \right) \right) \] (2.21)

If at any time the stress is removed, the recovery behaviour can also be obtained by Equation (2.20) by applying the Boltzmann superposition principle that the moment the stress is removed, an equal and opposite stress is placed on the system \((\sigma = -\sigma_o)\), while the original stress is maintained.

The stress relaxation of the Burger's model is derived from Equation (2.19). For relaxation resulting from a step strain of \(\varepsilon_o\) at \(t=0^+\), the strain \(\varepsilon\) is equal to the constant multiplied by the Heaviside function

\[ \varepsilon = \varepsilon_o H(t) \] (2.22)

\[ \frac{d\varepsilon}{dt} = \varepsilon_o \delta(t) \] (2.23)

where \(\delta(t)\) is the Dirac Delta function, and

\[ \frac{d^2\varepsilon}{dt^2} = \varepsilon_o \frac{d\delta(t)}{dt} \] (2.24)

Substituting 2.21-2.23 in Equation (2.17), gives

\[ \sigma + \left( \frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} \right) \frac{d\sigma}{dt} + \frac{\eta_1\eta_2}{E_1 E_2} \frac{d^2\sigma}{dt^2} = \eta_1 \varepsilon_o \delta(t) + \frac{\eta_1\eta_2}{E_2} \varepsilon_o \frac{d\delta(t)}{dt} \] (2.25)

Using the same notation as authors Findley, Lai and Onaran, we can simplify the constants by the expressions
\[ p_1 = \frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} \]

\[ p_2 = \frac{\eta_1 \eta_1}{E_1 E_1} \]

\[ q_1 = \eta_1 \]

\[ q_2 = \frac{\eta_1 \eta_2}{E_2} \]

The expression (2.25) can be written as

\[ \sigma + p_1 \frac{d\sigma}{dt} + p_2 \frac{d^2 \sigma}{dt^2} = q_1 \varepsilon_o \delta(t) + q_2 \varepsilon_o \frac{d\delta(t)}{dt} \quad (2.26(a)) \]

Solving the differential equation for stress with respect to time, yields

\[ \sigma(t) = \frac{\varepsilon_o}{\sqrt{A}} \left[ (q_1 - q_2 r_1) \exp(-r_1 t) - (q_1 - q_2 r_2) \exp(-r_2 t) \right] \quad (2.27) \]

where

\[ A = \sqrt{p_1^2 - 4 p_2} \]

\[ r_1 = \frac{(p_1 - A)}{2 p_2} \]

\[ r_2 = \frac{(p_1 + A)}{2 p_2} \]

### 2.4 The Standard Linear Model

The standard linear solid has also been defined in some of the literature (Flugge, Findley, Lai and Onaran, Young\(^{10}\)). However, the configuration of the springs and
dashpots differ. Flugge describes this model as a spring in series with the Voigt Kelvin
element (Figure 2.17), while Young shows the standard linear solid to be represented by a
spring in parallel with a Maxwell model (Figure 2.18).

![Figure 2.17 - Flugge's Standard linear solid.]

![Figure 2.18 - Young's Standard Linear Solid]

The above models are similar in that they both possess solid like characteristics
with two types of elasticity: an instantaneous elasticity and a delayed elasticity.

In Flugge's derivation and application to the "standard test", a general equation is
derived through the use of Laplace transforms to yield
\[
\sigma + \left( \frac{\eta_1}{E_1 + E_2} \right) \frac{d\sigma}{dt} = \left( \frac{E_1 E_2}{E_1 + E_2} \right) \varepsilon + \left( \frac{E_1 \eta_1}{E_1 + E_2} \right) \frac{d\varepsilon}{dt}
\]

(2.28)

where \( \varepsilon \) is the total strain of the model and the coefficient \( E_1 \) refers to the single spring, \( E_2 \) to the Voigt-Kelvin spring and \( \eta_1 \) to the dashpot in parallel with \( E_2 \).

Applying the "standard test" to the model, the equations for creep followed by stress relaxation are derived as follows.

For the creep phase, a constant stress \( \sigma = \sigma_o \) is applied and the strain is solved with respect to time to give

\[
\varepsilon(t) = \frac{\sigma_o (E_1 + E_2)}{E_1 \eta_1} \left( \frac{E_2}{\eta_1} \left( 1 - \exp \left( -\frac{E_2}{\eta_1} t \right) \right) + \frac{\eta_1}{E_1 + E_2} \exp \left( -\frac{E_2}{\eta_1} t \right) \right)
\]

(2.29)

Flugge notes that the material has instantaneous elasticity by solving for the strain at \( t=0 \) to get

\[
\varepsilon(0) = \frac{\sigma_o}{E_1}
\]

In addition, Flugge shows that the strain also reaches an asymptotic value as \( t \to \infty \) of

\[
\varepsilon(\infty) = \frac{\sigma_o (E_1 + E_2)}{E_1 E_2}
\]

Therefore, Flugge concludes that the material qualifies as a solid.

For the relaxation phase of the standard test, Flugge shifts to a new time scale \( \tau \)

where \( \tau = t - t_1 \). Therefore for \( \tau > 0 \), the strain \( \varepsilon = \varepsilon_1 \) to yield
\[
\sigma(t) = \frac{E_1E_2}{E_1 + E_2} \varepsilon_t \left(1 - \exp\left(-\frac{\tau(E_1 + E_2)}{\eta_1}\right)\right) + \sigma_o \exp\left(-\frac{\tau(E_1 + E_2)}{\eta_1}\right)
\]  

(2.30)

Therefore, the stress decreases gradually as \(t \to \infty\) to the value:

\[
\sigma(\infty) = \frac{E_1E_2}{E_1 + E_2} \varepsilon_t
\]

2.5 - Kelvin Chains and Generalized Maxwell Models

As reviewed by Flugge, there are two ways of systematically building up more complicated models. These are the Kelvin chain and the generalized Maxwell model. The Kelvin chain, suggested in Flugge's text, has an arbitrary number of Kelvin units connected in series. However, Flugge also includes a single spring in series and a single dashpot such that the model is seen to look like Figure 2.19.

![Figure 2.19 - The Voigt-Kelvin chain as described by Flugge.](image)

The presence of the single spring allows the model to have an immediate time independent response to stress and strain. The presence of the single dashpot leads to a more fluid like behaviour, and allows permanent deformation after unloading.
On a critical note, this model is not a true representation of a Kelvin chain because of these additional single elements which are added to conveniently describe viscoelastic polymer behaviour.

A large portion of the literature suggests the use of this extended model to describe the creep phenomenon of polymeric materials. The derivation of the constitutive equations for this model is well known with respect to creep. However, none of the authors reviewed in the present study extend the use of the Voigt Kelvin chain to stress relaxation. This is not surprising since an individual Voigt-Kelvin element cannot model stress relaxation effectively. Rather, general practice is to use the generalized Maxwell model.

Flugge also describes the general Maxwell model. This model, seen in Figure 2.20 below, consists of an arbitrary number of Maxwell elements in parallel. However, this too has a single spring and dashpot element in parallel.

![Figure 2.20 - The generalized Maxwell model as described by Flugge.](image)
Flugge goes on to assume the absence of the single dashpot, thereby assuring an immediate impact response and similarly, the absence of the spring assures the fluid-like behaviour. This leads to discussion of a true generalized Maxwell model, the Maxwell-Weichert Model, which will be used in the present study.

2.6 The Maxwell-Weichert Model

The Maxwell-Weichert model is also called the generalized Maxwell model without the existence of the degenerated unit spring and dashpot as seen in Figure 2.16. To provide a more realistic simulation of polymer materials, the Maxwell model was extended to an arbitrary number of Maxwell elements in parallel with each other. It is particularly useful in describing the stress relaxation phenomena that occurs in polymer materials in which the model is subjected to constant strain. For constant strain, the derivative of strain with respect to time is equal to zero which simplifies the left side of Equation (2.9(b)) for each Maxwell element and for the Maxwell-Weichert model as a whole.

The constitutive equations of the Maxwell-Weichert model subjected to constant strain are easily derived and are solved in much of the polymer rheology literature.

Considering two Maxwell elements in parallel with each other as in Figure 2.21, the equations are derived below.
\[ \varepsilon_{\text{eq}} = \varepsilon_1 = \varepsilon_2 \]  
\[ (2.31) \]

\[ \sigma_{\text{eq}} = \sigma_1 + \sigma_2 \]  
\[ (2.32) \]

Differentiating equation (2.32) gives

\[ \frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_2}{dt} \]  
\[ (2.33) \]

In the case of stress relaxation at constant strain, the strain rate is zero so

\[ 0 = \frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_2}{dt} \]  
\[ (2.34) \]

But each Maxwell unit will individually obey Equation (2.9(b)) giving

\[ \frac{d\sigma_1}{dt} \frac{1}{E_1} + \frac{\sigma_1}{\eta_1} = 0 \]  
\[ \frac{d\sigma_2}{dt} \frac{1}{E_2} + \frac{\sigma_2}{\eta_1} = 0 \]  
\[ (2.35(a),(b)) \]

Each of these differential equations can be solved such that

\[ \sigma_i = C_i \exp \left( -\frac{E_i}{\eta_i} t \right) \]  
\[ (2.36) \]
Using the conditions at $t = 0$, the constants $C_i$ can be solved. At $t = 0$, the model is equivalent to that made up of only two springs in parallel. Therefore, using statics, the initial stresses are equal to

$$\sigma_i = \varepsilon E_i,$$  \hspace{1cm} (2.37)

where strain is a constant imposed value $\varepsilon$, and so the values for $C_i$ are equal to

$$C_i = \varepsilon E_i.$$

From this, the following equations for the model, representative of the case stress relaxation at constant strain are

$$\sigma_1 = \varepsilon E_1 \exp\left(-\frac{E_1}{\eta_1} t\right)$$  \hspace{1cm} (2.38(a))

$$\sigma_2 = \varepsilon E_2 \exp\left(-\frac{E_2}{\eta_2} t\right)$$  \hspace{1cm} (2.38(b))

A. The Apparent Modulus

The application of the Maxwell-Weichert model to represent the creep of polymer materials, where these are subjected to a constant stress, has generally not been done. The idea of introducing an apparent modulus is seen in several of the texts. The use of an apparent modulus is misleading. By introducing an apparent modulus, this gives the impression that this modulus is applicable for any case of loading. However, this is an inaccurate assumption. If the derivation is completed under constant strain constraints, then the apparent modulus derived is that for the case of constant strain only. A different apparent modulus of the model can be derived for the case of constant stress.

Symbolically, this is shown by the following.
In the case of constant strain stress relaxation, the relationship between stress with time can be shown as

\[ \sigma(t) = E(t)\varepsilon_0 \]  \hspace{1cm} (2.39)

For constant stress creep, the relationship for the way the strain changes with time is not correctly described by the reciprocal of Equation 2.39

\[ \varepsilon(t) \neq \frac{\sigma_0}{E(t)} \]  \hspace{1cm} (2.40)

This assumption was used incorrectly by authors Progelhof and Throne (1993). They extend the use of the apparent modulus to a case of constant stress loading using a two parallel Maxwell model. This example can be seen below with the following constraints and constants.

\[ E_1 = 5 \times 10^3 \text{ lb}/\text{in}^2 \]
\[ \lambda_1 = 6 \text{ sec} \]
\[ E_2 = 7 \times 10^2 \text{ lb}/\text{in}^2 \]
\[ \lambda_2 = 1000 \text{ sec} \]

The apparent modulus of the two element Maxwell-Weichert model is:

\[ E(t) = E_1 \exp\left(\frac{-t}{\lambda_1}\right) + E_2 \exp\left(\frac{-t}{\lambda_2}\right) \]

\[ E(t) = 5 \times 10^3 \exp(-0.1666t) + 7 \times 10^2 \exp(-0.001t) \]

Part b of the example then asks to calculate the deformation of 25s if a stress of 1000lb/in2 is applied.
This is done as follows:

\[ E(25) = 3374 + 683 = 4057 \text{ lb/} \text{ in}^2 \]

\[ \varepsilon(25) = \frac{\sigma}{E(25)} = \frac{1000}{4057} = 0.247 \]

Most of the polymer literature does show a model that would represent the constant stress creep case. However, they do so by abandoning the Maxwell-Weichert model and rather suggest the Kelvin Model in series with each other to represent the constant stress creep case which is described above.

2.7 The Creep Function and The Relaxation Function

Ferry\textsuperscript{11} has reviewed methods to fit viscoelastic models to constitutive equations found under different linear stress-strain-time constraints. To keep the relative stresses and strains in simple form, Ferry considers a simple cubical element in simple shear. This means that two opposite faces of a cubical element are displaced by sliding as seen in Figure 2.22. It is assumed that the deformation is homogeneous, such that the stress and strain components do not vary with position. This is assumed to reduce the stress and strain tensors down to relatively simple form.

![Figure 2.22 - Illustration of simple shear of a cubic element.](image)
Therefore the stresses and strains become

\[
\sigma_y = \begin{pmatrix}
-P & \sigma_{12} & 0 \\
\sigma_{21} & -P & 0 \\
0 & 0 & -P
\end{pmatrix}
\] (2.41(a))

where \(P\) is an isotropic pressure

\[
\gamma_y = \begin{pmatrix}
0 & \gamma_{21} & 0 \\
\gamma_{12} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\] (2.41(b))

The stress \(\sigma_{21}\) and strain \(\gamma_{21}\) are functions of time and are related by the constitutive equation for linear viscoelasticity which is based on the changes of strain being additive such that:

\[
\sigma_{21}(t) = \int_{-\infty}^{t} E(t - t') \frac{d\gamma_{21}(t')}{dt} dt'
\] (2.42)

Where \(E(t)\) is the relaxation modulus and the times \(t'\) and \(t\) refer to past time and current time respectively.

If the relaxation function \(E(s)\) approaches zero as \(s\) approaches infinity then this corresponds to behaviour characteristic of a viscoelastic liquid. The alternative formulation of Equation 2.42 expressed in terms of strain history rather than rate of strain is given by:

\[
\sigma_{21}(t) = -\int_{-\infty}^{t} m(t - t') \gamma_{21}(t, t') dt'
\] (2.43(a))

where \(m(t)\) is the called memory function and is expressed by:
\[ m(t) = -\frac{dE(t)}{dt} \tag{2.43(b)} \]

Similarly, an alternative constitutive equation for strain in terms of the time derivative of stress, can be derived and defined by the creep compliance \( J(t) \) instead of the relaxation modulus \( E(t) \). From this derivation the dependence of strain with respect to time, when the material is subjected to constant stress creep, can be derived exactly from this alternative expression.

Using the above formulation, consider the model subjected to a stress relaxation after a sudden imposition of strain. The strain is imposed during a brief time interval \( \xi \) (from figure 2.23(a)) in the form of a constant strain rate \( \frac{dy}{dt} = \frac{Y}{\xi} \). This is shown below in Figure 2.23. Equation 2.43(a) then becomes

\[ \sigma(t) = \int_{t_0 - t}^{t_0} E(t' - t) \left( \frac{Y}{\xi} \right) dt' \tag{2.43(c)} \]

where \( t_0 \) is the time at which the strain has reached the constant value. For long times compared to the interval of time necessary for loading, the stress is equal to \( \sigma(t) = \gamma E(t) \).
Relating this to mechanical models, Ferry points out that the time dependence of the creep and relaxation function \( J(t) \) and \( E(t) \) can be imitated by the behaviour of a mechanical models with a sufficient number of elastic spring elements and viscous dashpot elements. He applies a constant strain in the case of the generalized Maxwell model to demonstrate the exponential relaxation. Expressed in terms of the modulus analogy, the contribution of the \( i \)th Maxwell unit in the model, \( E_i(t) \) is given by

\[
E_i(t) = E_i \exp \left( -\frac{E_i}{\eta_i} t \right) = E_i \exp \left( -\frac{t}{\tau_i} \right)
\]

(2.44)

Where \( \tau_i \) refers to the relaxation time: \( \frac{\eta_i}{E_i} \)

Therefore, the expression for the modulus of the model is equal to

\[
E(t) = \frac{\sigma(t)}{\varepsilon} = \sum_i E_i \exp \left( -\frac{t}{\tau_i} \right)
\]

(2.45)

To find a corresponding equation to the constitutive equation 2.43 it is given that

\[
\sigma(t) = -\int_{-\infty}^{t} \left\{ \sum_i \frac{E_i}{\tau_i} \exp \left( -\frac{t-t'}{\tau_i} \right) \right\} \varepsilon(t', t') dt'
\]

(2.46)
where the quantity in brackets refers to the memory function. Since the stress will relax to zero as \( t \to \infty \), the model is representative of a viscoelastic liquid.

Ferry goes on to comment that the generalized Maxwell and the Voigt-Kelvin chain are "equivalent with an appropriate assignment of parameters subject to certain requirements which depend on whether the system is a viscoelastic liquid or viscoelastic solid". This equivalency is contingent on the fact that the Maxwell model and the Voigt-Kelvin model have degenerated units. (i.e. an isolated spring or an isolated dashpot) This is further explained below by Alfrey and Doty (1967) in their paper where they compare various methods of viscoelastic characterization. For a liquid, Ferry notes that all the coefficients of viscosity in the Maxwell model must be finite while in the Voigt-Kelvin chain, must all be zero. For a solid material, one of the coefficients of viscosity in the Maxwell model must be infinite while in the Voigt-Kelvin, all the springs must be non-zero. However, as stated by Kuhn, "there are rules for interrelating the parameters" of the Maxwell model and the Voigt-Kelvin chain.

2.8 Methods of Specifying the Properties of Viscoelastic Materials

Alfrey and Doty (1945) specify seven different methods to characterize viscoelastic behaviour. These seven include the Voigt Model, the Maxwell model, the Operator Equation, the Mechanical Impedance Function, the Creep Curve, the Relaxation Curve and the Dynamic Modulus function. They separate the methods into two classes. Class I
methods are more general and fundamental in nature, and include the Voigt-Kelvin models
coupled in series and the Generalized Maxwell model. Class II methods are experimental
curves which "map out" the viscoelastic phenomenon of the material. They state that the
methods defined in Class I are more satisfactory because their equations of stress and
strain consist of differentials that can be solved for a wide variety of transient conditions.
Alfrey and Doty then attempt to relate the methods to one another. Of interest to us is
the way in which they relate the Voigt and Maxwell models to the creep function, the
relaxation function and to each other.

A. The Voigt Kelvin model's relationship to Maxwell model

In Alfrey and Doty's Voigt-Kelvin chain and generalized Maxwell model, they
degenerate a spring and dashpot. These models are seen in Figures 2.24 and 2.25
respectively. They derive the general differential equations for their Voigt-Kelvin model
and the Maxwell model. These are similar to the models Ferry considers which include an
arbitrary number of "Voigt elements" or "Maxwell elements" with the addition of
degenerated units as well.

Figure 2.24 - The Voigt Model described by Alfrey and Doty.
B. The Creep Function

Alfrey and Doty describe the creep function by considering the sudden application of a stress (at t=0) that is held constant. The strain will increase with time according to some function $\varepsilon(t)$. Since for a linear viscoelastic material, the strain at any time is proportional to the stress, then they define the creep function $\Phi(t)$ to be equal to the strain at any time $t$ divided by the constant stress value.

$$\Phi(t) = \frac{\varepsilon(t)}{\sigma} \quad (2.47)$$

They describe the Voigt-Kelvin model as having a practical relationship to the creep function. If the viscoelastic properties and constants of a material have been specified, then it is relatively simple to calculate the response to a simple constant stress since the behaviour of each Voigt-Kelvin unit in the sequence can be calculated individually. The strains are simply additive. Therefore, this makes simulating the creep curve a relatively simple task.
Relating the creep function to the Maxwell model is not as simple as its relation to the Voigt-Kelvin. Simha\textsuperscript{13} (1941) defines a continuous Maxwell model that can be derived from the creep curve of a material using a distribution function. The method used is based on Fourier Transforms.

C. The Relaxation Function

The relaxation function defined by Alfrey and Doty is similar to the creep function. It assumes that at $t=0$, a sample is forced to take on a constant strain $\varepsilon$ which remains constant while the stress will decay according to some function $\sigma(t)$. Therefore, the relaxation function is equal to the stress at any time $t$ divided by the value of the constant strain:

$$\gamma(t) = \frac{\sigma(t)}{\varepsilon}$$  \hspace{1cm} (2.48)

Therefore, Alfrey and Doty conclude that the relaxation function of a material defines its viscoelastic properties in the same fashion as does the creep function.

The Maxwell Model is easily related to the relaxation function since the constant strain is known, and the total stress is equal to the sum of the stresses in each Maxwell unit. Similar to Ferry, Alfrey and Doty also replace the set of $n$ Maxwell elements with a distribution of relaxation times $G(\tau)$ such that the stress follows the relationship:

$$\sigma(t) = \varepsilon \int_0^\infty G(\tau) \exp \left( -\frac{t}{\tau} \right) d\tau$$  \hspace{1cm} (2.49)
D. The Maxwell related to the Voigt Kelvin Model

The translations from the Maxwell model to the Voigt-Kelvin model can be accomplished following a few rules:[from Ref 13]

1. The Maxwell model must contain the same number of springs and dashpots as the equivalent Voigt-Kelvin model.

2. A standard Voigt-Kelvin model (with no incomplete elements) corresponds to a "doubly degenerate" Maxwell model (one with an isolated spring and an isolated dashpot).

3. A standard Maxwell model corresponds to a "doubly degenerate" Voigt-Kelvin model.

4. A singly degenerate Voigt Model with one isolated (unretarded) spring corresponds to a singly degenerate Maxwell model with one isolated (non-relaxing) spring.

5. A singly degenerated Voigt-Kelvin model with one isolated dashpot corresponds to a singly degenerate Maxwell Model with one isolated dashpot.

6. The presence of an isolated element of one kind in a model corresponds to the absence of an isolated element of the opposite kind in the conjugate model. (e.g., the presence of an isolated spring in a Voigt model implies the absence of an isolated dashpot in the equivalent Maxwell model).

The figure below show a number of specific illustrations that correspond to the rules above.

Figure 2.26(a) - Equivalent relationships between the Voigt-Kelvin and the Maxwell models. [From Ref. 13]
The analytical derivation of the equivalency of the models is much more complicated. Alfrey and Doty state that it is "out of the question" to write down the general functional relationships for one set in terms of the other. Because of the complexity, it is suggested that numerical derivation and calculations relating to the comparison be done using the Operator Equation formulation (which is one of the
methods suggested to characterize the material) rather than attempt to directly derive their relationship to each other.

E. The Relaxation Spectrum

It has been stated that the following formulation apparently postulated by Maxwell to describe the exponential decrease of the stress held at constant strain.

\[
\frac{d\varepsilon}{dt} = \frac{1}{E} \frac{d\sigma}{dt} + \frac{1}{\eta} \sigma
\]  

(2.50)

where \(E\) is the modulus and \(\eta\) is the viscosity. According to Equation 2.48, if the load was held for a long period of time, then the elastic component becomes insignificant. Many researchers such as Weber\(^{15}\), who investigated silk threads, Kohlrausch and Weichert\(^{16}\) who considered glass fibers and Taylor\(^{17}\) and collaborators show that Maxwell's relationship is not capable of producing actual results. Thompson and Wiechert\(^{18}\) were the first to consider that the behaviour of polymer materials can be characterized by a set of relaxation times, which give a set of stresses that combine with internal relaxation times. This concept of multiple relaxation times was again investigated by several authors including Thompson\(^{19}\), Gemant\(^{20}\) and Burgers\(^{21}\). Their models include variations of springs and dashpots and attempt to illustrate the visco-elastic nature of polymers.

Ferry also examines the relaxation spectrum. A group of Maxwell elements in parallel represents a discrete spectrum of relaxation times. Since for a parallel
arrangement, the stresses in each Maxwell element are additive, then it can be shown that the viscoelastic function $E(t)$ can be found by simply adding the expression of $n$ elements:

$$E(t) = \sum_{i=1}^{n} E_i \exp\left(-\frac{t}{\tau_i}\right)$$  \hspace{1cm} (2.51)

Since the Maxwell model is effective in describing stress relaxation, it is not surprising that any stress relaxation curve can be fitted with any desired degree of accuracy using a series of terms, by equation 2.51. The general Voigt-Kelvin model is similarly able to fit the creep curves, as described by Ferry.

If the number of Maxwell units in Ferry’s Maxwell model seen above (in Figure 2.25) are increased without limit, then the result is a continuous spectrum in which each contribution to rigidity is associated with relaxation times lying in the range of $\tau$ and $\tau + \frac{\text{d}t}{\text{d}t}$.

F. The Retardation Spectrum

In a manner analogous to above, Ferry defines the retardation spectrum by a series of Voigt-Kelvin units. In turn, the creep function is fit to an arbitrary number of Voigt-Kelvin models in series and the retardation spectrum represents an infinite number of retardation times.
2.9 Determination of the Relaxation Time Spectrum

The work conducted by H. Winter uses rheological models to characterize materials by their properties. Winter (1995) states that polymers relax in a "broad spectrum of relaxation times". The spectrum is derived from a series of stress/strain linear experiments. Although many efforts have been made to quantify this spectrum using computer algorithms, most computer programs focus on the conversion of stress/strain/time data to a relaxation spectrum and not on the actual resulting spectrum itself.

In order to use the relaxation spectrum effectively, it is important to understand how the spectrum describes properties of a material such that the rheological behaviour can be described completely by the spectra $H(\lambda)$. Winter has attempted to relate the shape of the spectra to the molecular structure such that the spectra will change with small changes in the molecules. These changes can then be related to various classes of materials.

Using dynamic mechanical analysis, Winter considers two functions of the material; the storage modulus and the loss modulus denoted by the symbols: $G'(\omega)$ and $G''(\omega)$ respectively. Calculating these as a function of the relaxation spectra gives:

$$G'(\omega) = G_o + \int_0^\infty H(\lambda) \frac{(\omega \lambda)^2}{1 + (\omega \lambda)^2} \frac{d\lambda}{\lambda}$$  (2.52)
\[ G''(\omega) = \int_0^\infty H(\lambda) \frac{\omega \lambda}{1 + (\omega \lambda)^2} \frac{d\lambda}{\lambda} \] (2.53)

where \( G_e \) is the equilibrium modulus and \( \omega \) is the frequency of the dynamic oscillation.

Winter chooses \( G' \) and \( G'' \) experiments over other linear viscoelastic experiments due to the following reasons:

(i) rapid developments of quasi-steady state in a time \( \Delta t \approx 2\pi / \omega \)

(ii) selective probing of relaxation modes near \( \lambda = 1/\omega \)

(iii) no zero drift problem of the transducer

(iv) \( G' \) and \( G'' \) data contain the same information

(v) the limitation of a finite time window

From Winter's evaluation of the relaxation spectrum, the equations 2.52 and 2.53 described above are found to be expressed in terms of discrete moduli

\[ G'(\omega) - G_e = \sum_{i=1}^{N} g_i \frac{\omega \lambda_i}{1 + (\omega \lambda_i)^2} \] (2.54)

\[ G''(\omega) = \sum_{i=1}^{N} g_i \frac{\omega \lambda_i}{1 + (\omega \lambda_i)^2} \] (2.55)

where \( g_i \) is the elasticity and \( \lambda_i \) is the relaxation time of each Maxwell unit \( i \).

These two equations can fit any \( G' \) and \( G'' \) experimental data with a suitable choice of parameters \( g_i, \lambda_i \) with \( i=1,2,\ldots,N \). Therefore, Winter places a large number of Maxwell models with a continuous spectrum over the range of frequencies. The model is
then optimized to the best fit with the least number of models. The number $N$ is found by computer algorithm and varies according to the desired degree of accuracy of the model such that an infinite number of Maxwell units is possible.

In the current study, although an infinite number of Maxwell units may seem ideal, a limited practical number of Maxwell units is used in order to apply the model in Finite Element Analysis (FEA).

Winter tests only in the linear viscoelastic range. The main objective of Winter and his colleagues is to quantify the relationship of $G'$ and $G''$ with as few parameters as possible in order to be used in laboratory and research analysis. In a sequence of well defined steps, Winter creates a way of transferring rheological data into material data. The relaxation spectrum found is then extended into predictions of tests such as: linear stress responses, creep and recovery, and step strains. In comparison to the present study, much higher strains are considered in computer modeling and computer algorithms to allow for extension of the study to finite element analysis, design improvements and material quality criteria.

2.10 Using the Relaxation Time Spectra

Winter uses the relaxation time spectra to model various rheological phenomena such that it provides all the basic information needed for modeling the elasticity of
polymeric materials. In the linear viscoelastic range, the Boltzmann superposition principle is used such that the following constitutive equation for stress is described as:

\[ \sigma(t) = \int_{-\infty}^{t} G(t-t') \frac{d\varepsilon}{dt'} dt' \]  
(2.56)

Where \( \sigma \) is the time dependent stress,

\( \frac{d\varepsilon}{dt} \) is the rate of strain

\( G(t-t') \) is the linear relaxation modulus.

\( t' \) is the time when the relevant strain was applied.

Relating this equation to the discrete spectrum with \( N \) relaxation times (i.e. \( N \) Maxwell elements), the relaxation modulus can be replaced by the equation (2.57) below:

\[ G(t-t') = \sum_{i=1}^{N} g_i \exp \left( -\frac{(t-t')}{\lambda_i} \right) \]  
(2.57)

where coefficients \( g_i \) and relaxation times \( \lambda_i \) are parameters of the discrete relaxation time spectrum.

Maxwell models are a convenient tool in providing a systematic discrete spectrum since they approximate the data well and are easy to use in a computer algorithm.

2.11 Extension of the Relaxation Time Spectrum to Large Strains
The general constitutive equation of linear viscoelasticity discussed above is restricted to small strains for two imperative reasons. Primarily, the equation describes the relaxation modes of the molecules and their structures in the equilibrium state and this equilibrium state would be disturbed by introducing large strains into the material. The other reason is the measurement of the strain focuses on the linear terms and not the quadratic terms of the strain tensor such that at larger strains the strain tensor fails. Larger strains originate from the non-linearity of the strain tensor which does not account for the increase in molecular motion. Experiments by Einaga et al [ref] have shown that the constitutive equations are mostly dependent on strain magnitude and not on the strain rate which was believed for a long time to have effect on the constitutive equation. This was concluded from measurements of the relaxing stress found after a sudden input strain. The relaxation modulus was found by taking this time dependent stress value divided by the strain. At small strains, the calculated relaxation modulus is identical with the relaxation modulus found by the linear viscoelastic experiments. However, at larger strains, the relaxation modulus is shifted while the shape of the curve is maintained. In the past, the models found to successfully simulate large values of strain have been exclusively applied to polymer liquids and are very difficult to use.
III. USE OF THE MINIMAT TESTER TO EVALUATE THE
VISCOELASTIC/VISCOPLASTIC PROPERTIES OF MINERAL FILLED LDPE

The viscoelastic/viscoplastic behaviour of the mineral filled LDPE material used in
Janssen-Ortho's slimline IUD has been systematically quantified through the use of various
tests on the Minimat Data Acquisition system. These tests mimic in one dimension the
sequence of stresses and strains that the IUD experiences from its placement in the
insertion device through to its release in utero. Although the actual stress strain pattern
involves the bending of the material, it is assumed that tensile testing in one dimension
would mimic the response of the material in bending.

The main body of the experimental results has been sent to Janssen Ortho for their
use, and will not be reproduced here. The key to understanding the memory effect
behaviour of the IUD's is to divide the strain that is put into the material during loading
into three components of strain activated upon release of the load. These components are:
the elastic unloading strain which is the immediate recovery of the material upon release,
the creep recovery strain which is recovered over time through the retentive memory
effect, and finally the permanent plastic deformation strain which is the strain remaining at
the end of the test period. Figure 3.1 below demonstrates a four zone memory test. The
omenclature seen for each of the strain components listed above is as follows:

- Strain(1) is the maximum strain reached in the loading period (Zone 1)
- Strain(C) is the permanent plastic deformation strain left at the end of the test
- Strain(Z) is the elastic unloading strain reached as the stress is unloaded.
- The creep recovery strain as defined as Strain(Z)-Strain(C), is the time
dependent shape recovery that occurs at zero load.
4 Zone Memory Test

Figure 3.1 - Four zone memory test identifying the key stress and strain values in each zone.

To get optimum recovery of the material, the plastic deformation should be kept to a minimal value and the elastic and creep recovery should be maximized.

It is interesting to note that all three of these components activated upon release are linearly related to the initial forward strain. Thus, the forward strain is the most important factor in governing the optimum shape recovery.
The elastic strain recovery is the amount of strain that is recovered immediately after unloading of the material. As mentioned above, the elastic strain is linearly related to the forward strain put into the material. This value is also weakly dependent on the initial loading rates of the material (or rate of bending), but it is independent of the relaxation time (length of time held at constant strain). These results can be seen below in Figures 3.2 and 3.3.

**Figure 3.2** - Elastic unloading strain vs. forward strain for 0.1 mm/min loading rate.

**Figure 3.3** - Elastic unloading strain vs. forward strain for 1.0 mm/min loading rate.
The creep recovery strain is the time dependent recovery which occurs after unloading and following the initial elastic recovery. Figure 3.4 demonstrates the linear relationship of the creep recovery to the forward strain. The value of the creep recovery is almost always equal to about 6% of the forward strain irrespective of the magnitude of the forward strain. The magnitude of the creep recovery strain is independent of the loading rate. It is not strongly statistically correlated with relaxation times, however changes in the relaxation time do produce a second order effect in the creep recovery strain, compared to its being considered a constant fraction of the forward strain.

![Creep Recovery vs. Forward Strain](image)

Figure 3.4 - Creep recovery vs. forward strain for varied relaxation periods and 1mm/min loading rate.

The value of the permanent non-reversible deformation strain is almost solely a factor of the magnitude of the forward strain. This result can be seen in Figure 3.5 in the linear relationship between the permanent plastic deformation strain and the forward strain. It varies from zero at very small values of forward strain to about 46% of the forward strain at high (~30%) values of forward strain. It is not correlated with the forward loading rate and is not strongly correlated to the amount of relaxation time. It is
affected at a second order magnitude to the relaxation times compared to the effect of the forward strain magnitude.

Quantifying these relationships has significance in the systematic design of better IUDs and of the insertion devices used to place the IUDs in patients. Since it is known which factors most significantly contribute to the memory effect, the design of the IUD can be improved to enhance the characteristics that would increase the retentive memory and decrease the plastic deformation. Furthermore, the results are also of use to make a critical review of the commonly used standard shape memory effect test parameters. This is a result of the wide range of stress and strain put into the material in the standard memory test. Our interpretation is that different operator methodology for inserting the IUDs into the holes generates great variation in the amount of strain put into the material which is the governing factor in shape recovery. The length of time spent in the test stand has considerably less significance in the amount of shape recovery that occurs in the
material. The taking of reading after a time period of one minute to allow for time
dependent shape recovery is also not supported by the creep recovery results.

In addition, the spring-dashpot mechanical analog models generated as part of this
study together with the results described above, also have the potential to be a great
influence in designing a more effective IUD. The models will essentially mimic creep,
stress relaxation and the experiment standard memory test results. Using the coefficients
found from the modelling analysis, finite element models can be generated to describe
areas of high stress, weak material, plastic deformation and ultimately poor design.
IV. PROPOSED MODELS FOR MODELING THE VISCOELASTICITY OF A LDPE

Historically, as reviewed earlier in this thesis, attempts have been made to develop complex combinations of springs and dashpots to exemplify polymer phenomena. Neither the Maxwell nor the Voigt-Kelvin model can accurately describe all of the characteristics of polymers. The present study has attempted to find a simple model comprised of relatively few elements, that accurately reproduces the physical behaviour of the LDPE used in IUD's, and which can be used in a general numerical analysis (e.g. FEA).

As a preliminary start for modeling, the four element Burger's model was used. The Burger's model, as previously demonstrated, is a Maxwell element connected in series with a Voigt-Kelvin element. This combination was proposed to accommodate both stress relaxation and constant stress creep phenomena in the same model since individually, the Maxwell model and the Voigt-Kelvin model were not capable of representing both of these polymeric characteristics. The Burger's four element model was found to be insufficient in representing the results a two zone constant stress test and the four zone memory test. The difficulties with the Burger's Model are explained in detail in Section 4.2.

For the purposes of this study, three models have been proposed using combinations of springs and dashpots that will simulate the behaviour of mineral filled
Low Density Polyethylene. Each model will be subjected to creep, stress relaxation, monotonic tensile tests and four zone tensile tests.

4.1 Model I - Parallel Maxwell Elements

The first model proposed is a combination of Maxwell elements that are in parallel with each other. The model, shown in Figure 4.1, is commonly called the Maxwell-Weichert Model. It is particularly useful in describing the stress relaxation phenomena that occurs in polymer materials in which the model is subjected to constant strain. In the present study, the use of the Maxwell-Weichert model will be extended to other stress-strain-time constraints, including creep, stress relaxation and the four zone memory test sequence. An approximate analytical solution for creep and stress relaxation was derived for two Maxwell units in parallel using the assumption that the constant stress creep or constant strain stress relaxation is applied instantaneously to the model. (i.e. there is no period of loading). Since this assumption is not entirely accurate, a short algorithm was written using the commercial programming language Watfor 77 in order to simulate the loading behaviour and then the subsequent holding at the required constant for any number of Maxwell units. The computer algorithm method of analysis was also extended to the four zone memory test. The four zone memory test is history dependent, therefore, an algorithm was necessary to track the stress-strain-time history over the course of four zones. The application of computer programming was extremely useful for extending the
analysis of the Maxwell Weichert model to any number of Maxwell models in parallel with each other.

4.2 Model II - A Spring in Series with a Modified Voigt Kelvin Element

The second type of model proposed has a spring in series with a second element comprised of a dashpot in parallel with a Maxwell unit. This configuration is seen in Figure 4.2 below.
This model much more accurate in describing the behaviour of the LDPE material than was the four element Burger's model which was initially considered. One loading case is the one seen below which contains the condition of constant stress creep that has been unloaded to zero load. This test is shown in Figure 4.3.

![Figure 4.3 - Experimental constant stress creep curve.](image)

In attempting to simulate this curve using the Burger's model (shown in Figure 4.4), the model was found to be insufficient to represent both the loading and unloading using the same values of the coefficients.

![Figure 4.4 - The Burger's Model.](image)
This is to state that a set of coefficients that fit the constant stress zone would not simulate the unloaded zone. This shortcoming is mainly the result of the single dashpot used in the Burger's model. Consider the quantity of permanent deformation strain achieved in the unloaded zone of the above test. Referring to the example loading case above, the permanent plastic deformation found at the end of the test period must be the resultant strain in the single dashpot of the Burger's model since the spring and the Voigt-Kelvin model will eventually relax completely. The viscosity of the dashpot must therefore be relatively low to have accumulated this amount of strain. This being the case, if a low value of viscosity was used in that dashpot element in the loading sequence of Figure 4.3, then the strain rate as result of loading and then holding the stress constant will have a slope which is much greater than that shown in the initial constant loading zone. To compensate for this in Model II, a Maxwell element was put in parallel with that dashpot such that a low value of viscosity will allow strain in to accumulate in loading at a constant load but at a limited rate.

Extending this analysis to the four zone test is not quite as simple. Since the elements are time and history dependent, a computer algorithm is necessary to compare the curves. However, based on the argument of the previous test, the same conclusion can be made; the curvature of all four zones could not be simulated by a single set of coefficients of elasticity and viscosity in a Burger's model.
4.3 Model III - Model II in series with a Voigt-Kelvin Element

Finally, the third model (Figure 4.5) was derived from Model II such that it has a stronger retentive memory response than the previous model. Model III is configured such that it has a spring in series with a second element comprised of the Maxwell unit in parallel with a dashpot in series with a third Voigt-Kelvin element.

The derivation of the constitutive equations for loading conditions of constant stress and constant strain for each of the models are elucidated in the following sections for each model respectively. The sections which describe each model will also describe in detail the logic on which the computer algorithms are based as well as a corresponding flow chart.

The results for these models are demonstrated in Section VIII which compares the models to the experimental tests of creep and stress relaxation. Section IX demonstrates the accuracy of comparison found between the model algorithms and the four zone
memory test. Finally, the areas where the models can be applied to other applications such as Finite Element Analysis will be identified as an extension to the current study.
V. MODEL I - PARALLEL MAXWELL ELEMENTS

The Maxwell element is commonly used to simulate the stress relaxation that occurs in polymeric materials. A common practice is to use a model comprised of several Maxwell elements in parallel with each other, commonly called the Maxwell-Weichert Model.

5.1 Stress Relaxation at Constant Strain

The Maxwell-Weichert model is particularly useful in describing the stress relaxation phenomena that occurs in polymer materials when the model is subjected to constant strain. For constant strain, the strain rate is equal to zero for each Maxwell element and for the Maxwell Weichert model as a whole.

Consider the case of two Maxwell elements in parallel with each other. In the equations which follow, the subscript 1 represents Maxwell element on the left acting in parallel with Maxwell element on the right as in Figure 5.1.
The constitutive equations are as follows:

$$\varepsilon_{\text{tot}} = \varepsilon_{1} = \varepsilon_{2} \tag{5.1}$$

$$\sigma_{\text{tot}} = \sigma_{1} + \sigma_{2} \tag{5.2}$$

Differentiating equation (5.1) gives:

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_2}{dt} \tag{5.3}$$

which is a condition that must be obeyed at all times. This is obvious by simple inspection of the physical model.

In the case of stress relaxation at constant strain, the strain rate is zero so:

$$\frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_2}{dt} = 0 \tag{5.4}$$

Recall that the strain rate of an individual Maxwell unit is:

$$\frac{d\varepsilon}{dt} = \frac{d\sigma_{\text{spring}}}{dt} \frac{1}{E} + \frac{\sigma_{\text{dissipate}}}{\eta}$$

(2.9(b))
Each Maxwell unit will individually obey equations (2.9(b)) and (5.4) giving:

\[
\frac{d\sigma_1}{dt} \frac{1}{E_1} + \frac{\sigma_1}{\eta_1} = 0
\]

\[
\frac{d\sigma_2}{dt} \frac{1}{E_2} + \frac{\sigma_2}{\eta_2} = 0
\]

(5.5a,b)

Each of these differential equations can be solved such that:

\[
\sigma_i = C_i \exp\left( -\frac{E_i}{\eta_i} t \right)
\]

(5.6)

If it is assumed that the stress is applied instantaneously (without an initial loading sequence) then at \( t = 0 \), the constant of integration \( C_i \) can be solved by considering the model consisting of only two time independent springs. At the instant after which the strain is applied (\( t = +0 \) using the Heaviside formalism), the model is equivalent to that made up of only two springs in parallel, because the dashpots have not had time to move.

Therefore, using statics, the initial stress is equal to:

\[
\sigma_i = \varepsilon E_i
\]

(5.7)

where strain is a constant imposed value \( \varepsilon \) and so \( C_i \) is equal to:

\[
C_i = \varepsilon E_i
\]

(5.8)

From this, the following equations for the model, representative of the case of stress relaxation at constant strain, are:

\[
\sigma_1 = \varepsilon E_1 \exp\left( -\frac{E_1}{\eta_1} t \right)
\]

(5.9a)

\[
\sigma_2 = \varepsilon E_2 \exp\left( -\frac{E_2}{\eta_2} t \right)
\]

(5.9b)

and the total stress is therefore:
\[ \sigma_w = \sigma_1 + \sigma_2 = \sigma \exp\left(-\frac{E_1}{\eta_1} t\right) + \sigma \exp\left(-\frac{E_2}{\eta_2} t\right) \] 

(5.9c)

However for most practical experimental conditions, it is inaccurate to assume that the stress or strain is applied instantaneously. Rather, an initial loading pattern must also be considered. For each of the Maxwell units in parallel, the following equation remains valid for constant strain stress relaxation, irrespective of the previous history:

\[ \sigma_1 = C \exp\left(-\frac{E_1}{\eta_1} t\right) \] 

(5.6)

If the final values of stress from the loading previous loading zone are used as the initial values of stress for creep at constant strain, then the constant of integration can be found for each of the Maxwell units. This was accomplished using a computer algorithm written in the commercial software Fortran.

The advantage of using a computer algorithm is that the equations for two Maxwell units in parallel are easily extended to five Maxwell elements in parallel. The accuracy of the simulation increases as the number of Maxwell elements increases. For the case of stress relaxation, the resultant stress-time relationship is easily derived by summing the stresses in each Maxwell unit. However, the derivation for constant stress creep behaviour, or other stress-strain-time constraints becomes increasingly complex as additional Maxwell units are added.
The stress-time curve for loading at a constant strain rate and then holding the strain constant while the model undergoes stress relaxation is shown in Figure 5.2 below.

![Stress Relaxation of Maxwell-Weichert Model (with 5 elements)](image)

**Figure 5.2 - Stress relaxation graph demonstrating the stress-strain time relationship of a Maxwell Weichert Model containing five parallel Maxwell elements.**

5.2 Creep at Constant Stress

In much of the existing literature, the derivation of the Maxwell-Weichert model to represent the creep of polymeric materials, where these are subjected to a constant stress, is often incomplete and occasionally incorrect. A significant amount of the polymer literature abandons the use of the Maxwell Weichert model for creep at constant stress and rather employs a series of Voigt Kelvin models in series as discussed in Chapter 2. This means that it is not a valid assumption that the time dependent modulus of the constant stress case is simply equal to the reciprocal of the time dependent compliance from the constant strain case. Symbolically, for stress relaxation:

\[ \sigma(t) = E(t) \varepsilon \]  

(5.10a)

cannot be inverted for use in creep at constant stress:
\[
\varepsilon(t) = \frac{1}{E(t)} \sigma \quad (5.10b)
\]

as has sometimes been seen in the literature.

For the case of creep at constant stress, the equations are not as simple as these for stress relaxation, and they are not generally available in the literature. Compared with the creep behaviour, difficulty arises because the equations are not simplified by the fact that the constant strain constraint makes the strain rate equal to zero. Equation (5.4) is not valid for these boundary conditions, and therefore equation (5.9(c)) is not applicable.

Furthermore, a second common but very weak assumption is that the stress on each individual Maxwell unit stays constant with time. This will be shown in the next two paragraphs to be incorrect. The assumption of constant load is valid for a constant stress enforced on a single Maxwell element. But in the Maxwell Weichert model, the only force requirement is that the sum of the stresses acting on all the Maxwell units remains constant. The stress on each individual Maxwell element will change with time according to the individual constants of elasticity and viscosity according to the equation (2.9(b)),

\[
\frac{d\varepsilon}{dt} = \frac{d\sigma_{\text{spring}}}{dt} \frac{1}{E} + \frac{\sigma_{\text{damping}}}{\eta} \quad (2.9(b))
\]

but the sum of the stresses for all the units must remain equal to the constant applied stress on the system. Due to the fact that the Maxwell units are in parallel, they each must maintain the same strain rate.
Consider a Maxwell Weichert model where each Maxwell unit has the same spring constant \( E_1 = E_2 = E_3 \), but the individual dashpot elements all have different viscosities \( \eta_1 < \eta_2 < \eta_3 \) varying over a wide range. If a load is suddenly applied (in the manner of a Heaviside function), then the dashpots will not have time to move, and so the load will initially be distributed equally among the elements because they each have the same spring constant. But as the dashpots begin to move, the load distribution among the individual units will adjust so that those which would creep most slowly under equal loading will have more load transferred to them so that they can keep up with the faster moving elements. These latter elements will shed load as they do not require an equal share of the load to produce the required strain rate. The mechanism for transferring the load is that the springs attached to the slower dashpots will be stretched more (thereby producing more force). The springs in series with weak dashpots will relax more quickly and will soon be stretched less than average because the dashpots will deliver most of the required strain.

So, in summary there are two surprising common errors in the rheological literature with respect to Maxwell Weichert models. One is that during creep that the stresses acting on the individual Maxwell elements remain constant with time. (Progelho and Thorne, 1993)\(^7\) Secondly, by just inverting the time dependent elastic modulus derived for fixed strain, some authors (Progelho and Thorne) have incorrectly used the inverse modulus for a fixed stress condition to describe creep.
There are various ways to derive the constitutive equations for stress and strain of the Maxwell-Weichert model for the case of constant stress. One method is to use Laplace Transformations to convert the differential equations to algebraic expressions. The stresses and strains on each respective Maxwell element in the model can then be found subject to the condition that the sum of the stresses is equal to the total stress on the model, and the strains in each Maxwell element are equal. This derivation can be seen below.

At any instant in time

$$\varepsilon = \varepsilon_1 = \varepsilon_2$$  \hspace{1cm} (5.1)

and

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} = \frac{d\varepsilon_2}{dt}$$  \hspace{1cm} (5.3)

but

$$\frac{d\varepsilon_1}{dt} = \frac{d\sigma_1}{dt} \frac{1}{E_1} + \frac{\sigma_1}{\eta_1}$$  \hspace{1cm} (2.9(b))

and similarly,

$$\frac{d\varepsilon_2}{dt} = \frac{d\sigma_2}{dt} \frac{1}{E_2} + \frac{\sigma_2}{\eta_2}$$  \hspace{1cm} (2.9(c))

Since it is a property of parallel models that the strain and strain rate in each element must be equal, then it becomes evident that

$$\frac{d\sigma_1}{dt} \frac{1}{E_1} + \frac{\sigma_1}{\eta_1} = \frac{d\sigma_2}{dt} \frac{1}{E_2} + \frac{\sigma_2}{\eta_2}$$  \hspace{1cm} (5.11)

Combine this equation together with the property that the total stress on the model is the sum of the stresses of the individual Maxwell units

$$\sigma_{\text{tot}} = \sigma_1 + \sigma_2$$  \hspace{1cm} (5.2)
and there are two equations for the two unknowns: $\sigma_1$ and $\sigma_2$.

Rearranging and substituting equation (2.9) into equation (5.3), gives

$$\frac{d\sigma_1}{dt} \frac{1}{E_1} \eta_1 + \sigma_1 = \frac{d\sigma_2}{dt} \frac{1}{E_2} \frac{1}{\eta_1} + \frac{\sigma_1}{\eta_2}$$

(5.12a)

Collecting like terms

$$\frac{d\sigma_1}{dt} \frac{1}{E_1} + \frac{d\sigma_2}{dt} \frac{1}{E_2} \frac{\sigma_1}{\eta_1} + \frac{\sigma_1}{\eta_2} = \frac{d\sigma_{se}}{dt} \frac{1}{E_2} + \frac{\sigma_{se}}{\eta_2}$$

(5.12b)

The derivative of constant total stress is equal to zero for fixed load creep leaving the following:

$$\frac{d\sigma_1}{dt} \left( \frac{1}{E_1} + \frac{1}{E_2} \right) + \sigma_1 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) = \frac{\sigma_{se}}{\eta_2}$$

(5.12c)

Using Laplace transformations to change the differential equation into an algebraic expression yields:

$$s \sigma_1 \left( \frac{1}{E_1} + \frac{1}{E_2} \right) - \sigma_1 (t = 0) \left( \frac{1}{E_1} + \frac{1}{E_2} \right) + \sigma_1 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) = \frac{\sigma_{se}}{s \eta_2}$$

(5.13)

The initial conditions are required to use the Laplace transform. Therefore at $t = 0$, it can be assumed that the model is comprised of two parallel springs. By using the original properties of the model for stress and strain gives:

$$\sigma_1 = \sigma_{se} \frac{E_1}{E_1 + E_2}$$

(5.14a)

$$\sigma_2 = \sigma_{se} \frac{E_2}{E_1 + E_2}$$

(5.14b)
Substituting equation (5.14(a)) into (5.13) gives:

\[ s\sigma_1 \left( \frac{1}{E_1} + \frac{1}{E_2} \right) - \frac{\sigma_{\infty} E_1}{E_1 + E_2} \left( \frac{1}{E_1} + \frac{1}{E_2} \right) + \sigma_1 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) = \frac{\sigma_{\infty}}{s\eta_2} \]  

(5.15a)

Rearranging equation (5.15(a)), gives

\[ \sigma_1 \left( \frac{s}{E_1} + \frac{s}{E_2} + \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) - \frac{\sigma_{\infty} E_1}{E_1 + E_2} \left( \frac{E_1 + E_2}{E_1 E_2} \right) = \frac{\sigma_{\infty}}{s\eta_2} \]  

(5.15b)

and then

\[ \sigma_1 = \frac{\sigma_{\infty} \left( \frac{1}{s\eta_2} + \frac{1}{E_2} \right)}{\left( \frac{s}{E_1} + \frac{s}{E_2} + \frac{1}{\eta_1} + \frac{1}{\eta_2} \right)} \]  

(5.15c)

Finally, taking the inverse Laplace Transformation, gives:

\[ \sigma_1 = \frac{\sigma_{\infty} E_2 \eta_1 \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2)}{(E_1 + E_2) \eta_1 \eta_2} t \right)}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} + \frac{\sigma_{\infty} E_1 \eta_2 \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2)}{(E_1 + E_2) \eta_1 \eta_2} t \right)}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \]  

(5.16a)

Similarly, for the second element

\[ \sigma_2 = \frac{\sigma_{\infty} \eta_2}{\eta_1 + \eta_2} + \frac{\sigma_{\infty} E_1 \eta_1 \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2)}{(E_1 + E_2) \eta_1 \eta_2} t \right)}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \]  

(5.16b)

The strain can next be calculated from the values of stress. There are two components of strain for each Maxwell element. The first is the elastic component, comprised of the spring, and which is equal to:
\[ \varepsilon_{\text{spring}} = \frac{\sigma_i}{E_i} \]  \hspace{1cm} (2.8)

This is an instantaneous or non-time history dependent equation: \( \varepsilon \) will vary over time linearly in phase with \( \sigma_i \). The second component is that which is contributed by the dashpot. It is the time history dependent component, and can be calculated for the \( i^{th} \) dashpot as:

\[ \varepsilon_{\text{dashpot}_i} = \int_0^t \frac{\sigma_i}{\eta_i} dt \]  \hspace{1cm} (5.17)

Combining equations (2.9(b)) and (5.17) allows the calculation of the overall strain in one Maxwell unit. This must, of course, be equal to the strain in the other Maxwell element.

\[ \varepsilon_1 = \frac{\sigma_1}{E_1} + \int_0^t \frac{\sigma_1}{\eta_1} dt \]  \hspace{1cm} (5.18a)

Similarly, \[ \varepsilon_2 = \frac{\sigma_2}{E_2} + \int_0^t \frac{\sigma_2}{\eta_2} dt \]  \hspace{1cm} (5.18b)

Using equations (5.16(a) and (b) derived for \( \sigma_1 \) and \( \sigma_2 \) in the expression above gives:

\[
\begin{align*}
\varepsilon_1 &= \frac{1}{E_1} \left[ \frac{\sigma_{\text{ax}1} \eta_1}{\eta_1 + \eta_2} \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2) t}{(E_1 + E_2) \eta_1 \eta_2} \right) \right] + \frac{\sigma_{\text{ax}2} E_2 \eta_1}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2) t}{(E_1 + E_2) \eta_1 \eta_2} \right) \\
&\quad + \int_0^t \left[ \frac{\sigma_{\text{ax}2} \eta_1}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2) t}{(E_1 + E_2) \eta_1 \eta_2} \right) \right] dt \\
&\quad + \frac{\sigma_{\text{ax}1} \eta_2}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2) t}{(E_1 + E_2) \eta_1 \eta_2} \right) + \frac{\sigma_{\text{ax}2} \eta_2}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2) t}{(E_1 + E_2) \eta_1 \eta_2} \right) \\
&\hspace{1cm} + \int_0^t \left[ \frac{\sigma_{\text{ax}1} \eta_2}{E_1 \eta_1 + E_2 \eta_1 + E_1 \eta_2 + E_2 \eta_2} \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2) t}{(E_1 + E_2) \eta_1 \eta_2} \right) \right] dt \\
\end{align*}
\]  \hspace{1cm} (5.19a)

Similarly,
\[ e_2 = \frac{1}{E_2} \left[ \sigma_{\infty} E_2 \eta_2 \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2)}{(E_1 + E_2) \eta_1 \eta_2} \right) \right] + \frac{1}{\eta_2} \int_0^t \left[ \sigma_{\infty} E_2 \eta_1 \exp \left( -\frac{E_1 E_2 (\eta_1 + \eta_2)}{(E_1 + E_2) \eta_1 \eta_2} \right) \right] dt \]

(5.19b)

This method is useful in that it provides an analytical solution, which can be used in modelling (preferably with the aid of simple computer software) such that the constants of elasticity and viscosity can be independently changed, and the results viewed instantly.

However, for this analytical solution, we are limited with respect to the number of Maxwell units that can be placed in parallel with each other. Following a similar derivation, the analytical solution may also be found for three Maxwell models in parallel. However, the derivation becomes increasingly complex as the number of Maxwell models increases.

In order to remedy this complexity, a simple algorithm was written to accommodate any number of Maxwell units. The program included the specification of constant stress and the time allocated for creep to occur while satisfying the fundamental stress-strain behaviour of each Maxwell unit. The logic behind the program is seen in the Creep flow chart, and the Creep algorithm is listed in Appendix 1.
The response of strain to constant stress creep of the Maxwell Weichert model containing five parallel Maxwell units is demonstrated in Figure 5.3.

![Graph showing creep of the Maxwell Weichert model](image)

**Figure 5.3** - The stress-strain-time relationship of the Maxwell Weichert model containing five parallel Maxwell elements subjected to a constant stress creep.

### 5.3 The Four Zone Memory Test

In order to derive the equations for each stress-strain regime in each of the four zones, a computer algorithm was written. The analytical equations for each zone could be derived, as seen for the stress relaxation and creep case above, by considering only two or three Maxwell elements. The computer program was written so that it could accommodate any number of Maxwell elements. Since the accuracy of simulation increases as the number of Maxwell elements increases, the logic for each of the four zones of testing will be discussed. The algorithm constrains were that fundamental equations (5.1) and (5.2) were to be satisfied for the model as a whole, as was equation (2.9(b)) for each individual Maxwell unit, for all four zones. Therefore, it can be assumed that the program is valid.
Recall equations (5.1), (5.2) and (2.9(b)):

\[ \varepsilon_{\text{at}} = \varepsilon_1 = \varepsilon_2 \quad \text{(5.1)} \]

\[ \sigma_{\text{at}} = \sigma_1 + \sigma_2 \quad \text{(5.2)} \]

\[ \frac{d\varepsilon}{dt} = \frac{d\sigma_{\text{spring}}}{dt} \frac{1}{E} + \frac{\sigma_{\text{damper}}}{\eta} \quad \text{(2.9(b))} \]

The four zone tensile test was designed to mimic the stress-strain regimen a LDPE T
shaped IUD device experiences as it undergoes deformation in a standard memory test or in practical use.

The flow chart found in Appendix I represents a step-by-step layout of the way in which the computer algorithm applies the stress-strain constraints in each of the four zones respectively. They are shown as independent processes for each of the respective zones.

Zone one involves stretching at a constant strain rate until a specified load is reached. Once the specified load is reached, the commencement of zone two is initiated such that the strain at that initial stress level and time is held constant, and the stress allowed to relax. The time for relaxation is specified such that once the relaxation time is accumulated, zone three starts to unload at a constant strain rate until zero stress is reached. The zero stress condition at the end of zone 3 represents the onset of zone four, in which creep recovery at zero stress occurs. As a result of the different elastic/viscous time dependent deformations accumulated by the individual Maxwell elements in zones
one through three, the final zone demonstrates a gradual decrease in the individual residual stresses until an asymptotic strain value is reached.

The computer algorithm written in Watfor 77 Fortran programming language can also be seen in Appendix 1.
VI. MODEL II
A SPRING IN SERIES WITH A MODIFIED VOIGT-KELVIN ELEMENT

Model II was originally proposed to be a Maxwell model in series with a Voigt Kelvin element as shown below in Figure 6.1. This four parameter model is known as the Burger's model (as discussed in Chapter II). It was believed that this four parameter model would simulate both of the polymeric characteristics of stress relaxation and of creep since individually, the Maxwell model is representative of stress relaxation and the Voigt Kelvin gives the correct general response for creep.

![Figure 6.1 - The Burger's model](image)

The equations for creep and stress relaxation of the Burger's Model have been solved by Findley, Lai and Onaran, are seen in Chapter II as follows.

For creep at constant stress: \( \sigma = \sigma_0 \)

\[ \varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{\eta_1} t + \frac{\sigma_0}{E_2} \left( 1 - \exp \left( \frac{-E_2 t}{\eta_2} \right) \right) \]

For stress relaxation at constant strain: \( \varepsilon = \varepsilon_0 \)
\[
\sigma(t) = \frac{\varepsilon_0}{A} \left[ \left( \eta_1 - \frac{\eta_1 \eta_2}{E_2} B \right) \exp(-Bt) - \left( \eta_1 - \frac{\eta_1 \eta_2}{E_2} C \right) \exp(-Ct) \right]
\]

Where
\[
A = \sqrt{\left( \frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} \right)^2 - 4 \frac{\eta_1 \eta_2}{E_1 E_2}}
\]
\[
B = \frac{\left( \frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} - A \right)}{2 \left( \frac{\eta_1 \eta_2}{E_1 E_2} \right)}
\]
\[
C = \frac{\left( \frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} + A \right)}{2 \left( \frac{\eta_1 \eta_2}{E_1 E_2} \right)}
\]

The problem with using the Burger's model is that it did not conform to the shape of a constant stress creep test followed by a sequence of unloading. It did not predict properly the permanent deformation. Therefore, the model was modified so that this loading sequence could be simulated.

Model II moves the Maxwell dashpot to be in series with the spring in the Voigt Kelvin element, giving the configuration seen below. This allows both dashpots to contribute to the memory effect recovery, and to the permanent strain after unloading.
The characteristic equations of this model can be found when we consider the model subjected to specific stress-strain-time constraints. The derivation of the equations can be best understood if we break the model up into respective elements shown in Figure 6.3.

The designation of their constants are denoted by the subscript number.

Element 1 is the spring whose elasticity coefficient is denoted $E_1$. Element 2 is the complex parallel arrangement. The coefficients of the Maxwell unit are denoted by the
number 2 which are in parallel with the third dashpot $\eta_1$. This numbering system will then be used consistently with Model III which has similar elements to those in Model II.

Due to the arrangement of the elements, the following equations provide the fundamental basis on which different stress-strain-time conditions are derived.

$$\sigma_{\infty} = \sigma_1 = \sigma_2 \quad (6.1)$$

$$\varepsilon_{\infty} = \varepsilon_1 + \varepsilon_2 \quad (6.2)$$

6.1 Model II Subjected to Constant Stress

When loaded by a constant stress, Element 1 contributes an instantaneous loading which can be represented as a step function. Element 2 is much more complicated contributing an exponential rise in strain until reaching an asymptotic value. Element 2 is also important in its unloading pattern as it unloads exponentially leaving a residual strain or permanent deformation. The equations are derived as follows.

Stress is a constant, therefore

$$\sigma_{\infty} = \sigma_1 = \sigma_2 = \sigma_\circ \quad (6.3)$$

The total strain is the sum of the strains in each element.

$$\varepsilon_{\infty} = \varepsilon_1 + \varepsilon_2 \quad (6.4(a))$$

Consider element 1 first. It is known that Hooke's law is applied such that:

$$\sigma_\circ = E_i \varepsilon_1 \quad (6.5)$$
Rearranging, gives:

\[ \varepsilon_1 = \frac{\sigma_0}{E_1} \quad \text{(6.6)} \]

Element 2 is best understood if we split it into two sides. Side 1 \( (s1) \) denotes the left side whereas side 2 \( (s2) \) is the right. The strain in both sides is equal as shown below:

\[ \varepsilon_2 = \varepsilon_{(s1)2} = \varepsilon_{(s2)2} \quad \text{(6.7)} \]

Similarly, the derivatives with respect to time must also be equal:

\[ \frac{d\varepsilon_2}{dt} = \frac{d\varepsilon_{(s1)2}}{dt} = \frac{d\varepsilon_{(s2)2}}{dt} \quad \text{(6.8)} \]

The strain rate of the Maxwell unit on side 1 must obey the fundamental Maxwell equation:

\[ \frac{d\varepsilon_{(s1)2}}{dt} = \frac{d\sigma_{(s1)2}}{dt} \frac{1}{E_2} + \frac{\sigma_{(s1)2}}{\eta_2} \quad \text{(6.9)} \]

The strain rate of the dashpot on side 2 is equal to:

\[ \frac{d\varepsilon_{(s2)2}}{dt} = \frac{\sigma_{(s2)2}}{\eta_1} \quad \text{(6.10)} \]

Since the two strain rates must be equal, the equations can be combined to get:

\[ \frac{d\sigma_{(s1)2}}{dt} \frac{1}{E_2} + \frac{\sigma_{(s1)2}}{\eta_2} = \frac{\sigma_{(s2)2}}{\eta_1} \quad \text{(6.11(a))} \]

or rearranged:

\[ \sigma_{(s2)2} = \frac{d\sigma_{(s1)2}}{dt} \frac{\eta_1}{E_2} + \sigma_{(s1)2} \frac{\eta_1}{\eta_2} \quad \text{(6.11(b))} \]

The total stress on Element 2 is equal to the sum of the stresses on each side:

\[ \sigma_0 = \sigma_{(s1)2} + \sigma_{(s2)2} \quad \text{(6.12(a))} \]
Combining equation (6.11(b)) in the equation for the sum of the stresses gives:

\[ \sigma_o = \sigma_{(s1)2} + \frac{d\sigma_{(s1)2}}{dt} \frac{\eta_1}{E_2} + \sigma_{(s1)2} \frac{\eta_1}{\eta_2} \]  
\begin{equation}
(6.13)
\end{equation}

Since \( \sigma_o \) is known and is equal to the total stress, the solved differential equation of \( \sigma_{(s1)2} \) yields:

\[ \sigma_{(s1)2} = \sigma_o \frac{\eta_2}{\eta_2 + \eta_1} + C_1 \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) \]  
\begin{equation}
(6.14(a))
\end{equation}

where \( C_1 \) is a constant of integration. If an instantaneous load is assumed, than \( C_1 \) can be solved using this as an initial condition. As such, at \( t = 0 \), the spring in the Maxwell element cannot move. If the spring cannot have any strain, than the stress on the spring must also be equal to zero. Therefore, the stress in side 1 of element 2 is equal to zero at \( t = 0 \). Therefore, the constant of integration is solved to get:

\[ C_1 = -\sigma_o \frac{\eta_2}{\eta_1 + \eta_2} \]  
\begin{equation}
(6.14(b))
\end{equation}

The stress on side one becomes:

\[ \sigma_{(s1)2} = \sigma_o \frac{\eta_2}{\eta_1 + \eta_2} - \sigma_o \frac{\eta_2}{\eta_1 + \eta_2} \exp \left( -E_2 \left( \frac{1}{\eta_1} - \frac{1}{\eta_2} \right) t \right) \]  
\begin{equation}
(6.14(c))
\end{equation}

The stress on side 2 is solved by subtracting the stress on side one from the total:

\[ \sigma_o = \sigma_{(s2)2} + \sigma_{(s1)2} \]  
\begin{equation}
(6.12(a))
\end{equation}

or \[ \sigma_{(s2)2} = \sigma_o - \sigma_{(s1)2} \]  
\begin{equation}
(6.12(b))
\end{equation}

Finally, the stress on side 2 gives:

\[ \sigma_{(s2)2} = \sigma_o \frac{\eta_1}{\eta_1 + \eta_2} + \sigma_o \frac{\eta_2}{\eta_1 + \eta_2} \exp \left( -E_2 \left( \frac{1}{\eta_1} - \frac{1}{\eta_2} \right) t \right) \]  
\begin{equation}
(6.15)
\end{equation}
Substituting this in the equation for $\varepsilon_2$ and solving the integral:

$$
\varepsilon_1 = \sigma_o \frac{\eta_1}{\eta_1 + \eta_2} t - \sigma_o \frac{\eta_1^2 \eta_1}{E_2 (\eta_1 + \eta_2)^2} \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) + \sigma_o \frac{\eta_1^2 \eta_1}{E_2 (\eta_1 + \eta_2)^2}
$$

(6.16(b))

Similarly, one could solve for the strain using the other side of Element 2

$$
\varepsilon_2 = \frac{\sigma_o (t)}{E_2} + \int \frac{\sigma_o (t)}{\eta_2} dt
$$

(6.17(a))

and this would lead to the same result.

The total strain is the sum of the strains in each of the elements.

$$
\varepsilon_{tot} = \frac{\sigma_o}{E_1} + \sigma_o \frac{\eta_1}{\eta_1 + \eta_2} t - \sigma_o \frac{\eta_1^2 \eta_1}{E_2 (\eta_1 + \eta_2)^2} \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) + \sigma_o \frac{\eta_1^2 \eta_1}{E_2 (\eta_1 + \eta_2)^2}
$$

(6.18)

In the experimental results available, the load was not applied instantaneously. Rather, a loading sequence must also be considered. In order to apply a loading sequence prior to a constant stress creep phase, a computer algorithm was written. This algorithm and its logic can be found in Appendix II.

Consider the model subjected to a constant force of 10 N (~5MPa). Assuming some arbitrary values for the constants of elasticity and viscosity, the following stress-strain-time result can be seen graphically.
6.2 Model II Subjected to Stress Relaxation at Constant Strain

Under conditions of constant strain stress relaxation, the stress becomes a function of time. Reference will be made to the same element numbers and coefficient notation as in the preceding section. The fundamental equations (6.1) and (6.2) still must be satisfied. However, the derivative of strain with respect to time is simplified since constant total strain means that the total strain rate is equal to zero. This is shown in the equations below:

\[
\sigma_{tot} = \sigma_1 = \sigma_2 = \sigma(t) \quad (6.18)
\]

\[
\varepsilon_{tot} = \varepsilon_1 + \varepsilon_2 = \varepsilon_o \quad (6.19)
\]

\[
\frac{d\varepsilon_{tot}}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt} = 0 \quad (6.20)
\]
Taking the derivative with respect to time of Equation (6.6) for Element 1 and the derivative of Equation (6.16) for Element 2, and combining these as in (6.20) yields the relationship:

$$\frac{d\sigma}{dt} \frac{1}{E_1} + \sigma \frac{1}{\eta_1 + \eta_2} + \sigma \frac{\eta_2}{\eta_1(\eta_1 + \eta_2)} \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) = 0 \quad (6.21)$$

Using commercial mathematics software MATLAB, this differential equation can be solved yielding the result:

$$\sigma(t) = C_1 \frac{\exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) - \eta_1 t \eta_2 \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) - \eta_2 t \eta_1 \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right)}{E_2 \left( \eta_1 + \eta_2 \right)} \quad (6.22(a))$$

where $C_1$ is once again the constant of integration.

If an instantaneous constant strain is assumed to be placed on the model, then $C_1$ can be easily solved making the following assumptions.

At $t=0$, the only element that is capable of straining is the isolated spring since it is independent of time. Therefore, the spring also defines the amount of stress that is placed on the Model at $t=0$. As such, the value of the constant of integration is:

$$C_1 = \varepsilon_0 E_1 \quad (6.22(b))$$

Putting this back into the equation above gives:
\[ \sigma(t) = E_1 \varepsilon_o \exp \left( \frac{-E_1 \left( E_2 \eta_1^2 t + E_2 \eta_1 \eta_2 t - \eta_2^2 \exp \left( -E_2 \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) t \right) \right)}{E_2 (\eta_1 + \eta_2)^2} \right) \]  

where \( \varepsilon_o \) is the value of constant strain enforced on the system.

Similar to the case of constant stress creep, the available experimental results do not impose in instantaneous strain on the model. Therefore, another computer algorithm was written to accommodate a loading phase and a constant strain stress relaxation phase such that the resultant stress-strain-time relationship is history dependent. This curve can be seen in Figure 6.2 below.

\[ \text{Figure 6.5 - Stress Relaxation curve of Model II} \]

6.3 Model II Subjected to the Four Zone Tensile Test Regime
The application of Model II was also extended to the four zone test. As a result of the history dependent nature of loading, another computer algorithm was necessary to apply Model II to the four zone test. The sequence of operations is demonstrated in the flowchart found in Appendix II accompanied by a copy of the computer algorithm for Model II.

Consider zone one. In zone one, the model is loaded up at a constant strain rate until a specific stress is reached. While the model is loading, the spring strains first and the dashpots in element 2 strain gradually with time.

In zone two, the strain is held constant while the model undergoes stress relaxation. The strain in the spring is gradually unloaded to the lower element where the dashpots move, and therefore, the total stress drops off. This is logical considering the equation of the spring. Recall Hooke’s law:

$$\sigma = E\varepsilon$$

(2.5)

As the strain in the spring is diminished, so is the stress on the spring and since the stress in the spring is equal to the stress on the model, the stress of Model II decreases.

In zone three, the model is unloaded to zero load. This indicates that the top spring is carrying zero load and therefore, will return to its original length. The total strain is not unloaded to zero however, since some permanent strain has accumulated in element two.
Finally, zone four is the retentive memory creep phase of the four zone memory test. In this zone, the stress has already been unloaded to zero. As a result of the time dependency and residual strain of the dashpots, the strain does not equal zero, nor is the stress in the second spring zero. On unloading, this spring (2) cannot relax to its original position until dashpots 1 and 2 have moved. The unrelaxed tensile force of the spring produces a tensile stress in dashpot 2 and an equal compressive stress in dashpot 2. Over time, the shortening of the right hand side of element 2, and the lengthening of dashpot 2 will allow the spring to return to its original length. But, this will leave a permanent deformation in the two dashpots. Hence, the strain continues to decrease over time until it reaches an asymptotic value.

6.4 - Effects of the Coefficients of Elasticity and Viscosity on Model II

The coefficients of elasticity and viscosity have a great effect on the way in which the model responds to applied stress and strain. Changing the values of the coefficients can severely change the shape of the resultant curves or the magnitude of the resultant stress or strain. In order to set values of each of the coefficients of elasticity and viscosity, it is necessary to understand how each of the coefficients contribute to the response of stress or strain.
Consider Model II subjected to a constant stress creep test of 5 newtons. If each of the coefficients are adjusted individually by a factor of two, the curves change in shape and magnitude as seen below. The curves are represented by colour and by letter from a to e.

Curve \(a\) is indicative of \(E_1\) multiplied by two. This has a great effect on the magnitude of the strain that results from a constant stress. Curve \(b\) is indicative of \(\eta_1\) multiplied by 2, while keeping the original value for \(E_1\). Increasing \(\eta_1\) appears to decrease the rise time of the slope as it moves towards the asymptotic value of strain. Curve \(c\) is indicative of \(E_2\) multiplied by 2. This has the most effect on the slope of the model taking away the curvature and giving two constant slopes: one for the early loading period and the other slowing increasing once the maximum stress has been reached. Finally, curve \(d\) is the second dashpot \(\eta_2\) multiplied by 2. This does not have much effect on the model other than lowering the end slope. Curve \(e\) was identified to provide the best fit at this stress which results from combining curves \(a\) and \(d\) or was done by adjusting spring \(E_1\) and dashpot \(\eta_2\).
Similarly, the same graph was analyzed for Model II subjected to constant strain stress relaxation. The same adjustments were also found to affect the slope and the magnitude of the curves as they did in the case of creep. In addition, the same combination was found to be the best representation of the experimental curve considered.
Finally, the values found for elasticity $E$ and viscosity $\eta$ are as follows:

$E_1 = 200 \text{ MPa}$

$E_2 = 465 \text{ MPa}$

$n_1 = 95,000 \text{ MPa}\cdot\text{sec}$

$n_2 = 10,400,000 \text{ MPa}\cdot\text{sec}$
Model III was developed as an extension of Model II. Since the LDPE seemed to have a stronger retentive memory than that simulated with Model II, Model III was developed to accommodate this shortcoming. In order to provide a stronger retentive memory, a Voigt Kelvin element was added.

Similar to Model II, it is easiest to understand the derivation of creep and stress relaxation if one breaks the model up into separate elements.
Elements 1 and 2 have the same coefficient notation as Model II. Element 3 has added two additional coefficients denoted by the number 3.

### 7.1 Constant Stress Creep

The derivation of constant stress creep is very similar to that of Model II. By the principle of superposition, the results of Model II can be used and the total strain modified to include the strain of Element 3. The derivation of strain for Element 3 is seen in Chapter III where the Voigt Kelvin element is described. If the Voigt Kelvin unit is split into side 1 (s1) and side 2 (s2) as was done for element 2 in the previous model, then s1 would denote the right side or the spring $E_3$ and s2 would indicate the left side or the dashpot $\eta_3$. The total stress on element 3 is equal to the sum of the stresses on either side:

$$\sigma_3 = \sigma_{(s1)3} + \sigma_{(s2)3}$$  \hspace{1cm} (7.1)

The parallel arrangement means that the strains on sides 1 and 2 are equal.

$$\varepsilon_3 = \varepsilon_{(s1)3} = \varepsilon_{(s2)3}$$  \hspace{1cm} (7.2)

Similarly, if one takes the derivative of the strain:

$$\frac{d\varepsilon_3}{dt} = \frac{d\varepsilon_{(s1)3}}{dt} = \frac{d\varepsilon_{(s2)3}}{dt}$$  \hspace{1cm} (7.3)

Substituting the equations

$$\frac{d\varepsilon_{(s1)3}}{dt} = \frac{d\sigma_{(s1)3}}{dt} \frac{1}{E_3}$$ \hspace{1cm} (7.4(a), (b))

$$\frac{d\varepsilon_{(s2)3}}{dt} = \frac{\sigma_{(s2)3}}{\eta_3}$$

for sides one and two gives:
\[
\frac{d\sigma_{(11)3}}{dt} \frac{1}{E_3} = \frac{\sigma_{(11)3}}{\eta_3} 
\]  
(7.5(a))

Rearranging:
\[
\sigma_{(11)3} = \frac{d\sigma_{(11)3}}{dt} \frac{\eta_3}{E_3} 
\]  
(7.5(b))

and substituting gives:
\[
\sigma_3 = \sigma_{(11)3} + \frac{d\sigma_{(11)3}}{dt} \frac{\eta_3}{E_3} 
\]  
(7.6)

Solving the differential equation results in
\[
\sigma_{(11)3} = \sigma_3 + C_1 \exp\left(\frac{-E_3}{\eta_3} t\right) 
\]  
(7.7)

where \(C_1\) is the constant of integration. To solve \(C_1\), we must consider element 3 at time zero. If we assume instantaneous loading of the material is assumed, then when \(t=0\), the stress on side one must also be zero, since the spring is limited to movement by the parallel dashpot. If the spring cannot move, then it cannot hold any of the load. Therefore, at \(t=0\):
\[
C_1 = -\sigma_3 
\]  
(7.8)

Which leads to the equation:
\[
\sigma_{(11)3} = \sigma_3 - \sigma_3 \exp\left(\frac{-E_3}{\eta_3} t\right) 
\]  
(7.9)

Now, solving for the stress on side 2:
\[
\sigma_{(12)3} = \sigma_3 - \sigma_{(11)3} 
\]  
(7.10)
\[ \sigma_{(1)3} = \sigma_3 \exp \left( -\frac{E_3}{\eta_3} t \right) \] (7.11)

And therefore, the strain is equal to:

\[ \varepsilon_3 = \frac{\sigma_{(1)3}}{E_3} \] (7.12)

\[ \varepsilon_3 = \frac{\sigma_3 - \sigma_3 \exp \left( -\frac{E_3}{\eta_3} t \right)}{E_3} \] (7.13)

Finally, the total strain of model three subjected to constant stress is equal to:

\[
\varepsilon_{\text{model-3}} = \sigma_o E_1 + \sigma_o \frac{\eta_1}{\eta_1 + \eta_2} t - \sigma_o \frac{\eta_1 \eta_2^2}{E_1 (\eta_1 + \eta_2)^2} \exp \left( -\frac{E_3}{\eta_1 + \eta_2} \right) + \frac{\eta_1 \eta_2^2}{E_2 (\eta_1 + \eta_2)^2} + \frac{\sigma_o - \sigma_o \exp \left( -\frac{E_3}{\eta_3} t \right)}{E_3}
\] (7.14)

In considering the experimental data, it is noted that the constant stress value is not achieved instantaneously as has been assumed above. Therefore the value of the integration constant \( C_1 \) is not exact. In order to solve for the exact value of \( C_1 \) one must consider the loading rate and the strain achieved at the end of this loading. Therefore, from the loading zone, the final value of stress and strain will become the initial conditions for the constant stress creep model. The values of stress and strain at the end of loading can be solved for the mathematical model by using a computer algorithm (which will eventually be extended to the Four Zone Tensile test loading). The algorithm and logic for creep of Model III can be found in Appendix III.
Assuming some initial values of elasticity and viscosity and applying the model to a 10 newton (5.13 MPa) creep test results in the following curve found in Figure 7.3.

![Creep of Model III](image)

**Figure 7.3** - The stress-strain-time results for Model III subjected to a constant stress creep test of 5 MPa.

### 7.2 Model III Subjected Stress Relaxation at Constant Strain

Consider the case of Model III subjected to constant strain stress relaxation. Much like the case of creep, similarities are observed between Model II and Model III such that the derivation is simplified. The strain of Model III is equal to the sum of the strains of the respective elements:

\[ \varepsilon_{\text{tot}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_o \]  

(7.15)

Similarly, one can take the derivative of this equation such that:

\[ \frac{d\varepsilon_{\text{tot}}}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt} + \frac{d\varepsilon_3}{dt} = 0 \]  

(7.16)

It is known from earlier derivations in Model II that
\[
\frac{d\varepsilon_1}{dt} = \frac{d\sigma}{dt} \frac{1}{E_1} \tag{6.21}
\]

\[
\frac{d\varepsilon_2}{dt} = \sigma \frac{\eta_1}{\eta_1 + \eta_2} + \sigma \frac{\eta_2}{\eta_1(\eta_1 + \eta_2)} \exp \left( -E_2 \frac{1}{\eta_1} \frac{1}{\eta_2} t \right) \tag{6.21}
\]

And it is possible to solve for \( \frac{d\varepsilon_3}{dt} \) using the properties of the Voigt-Kelvin such that:

\[
\frac{d\varepsilon_3}{dt} = \frac{\sigma_s \exp \left( -\frac{E_3}{\eta_3} t \right)}{\eta_3} = \frac{\sigma_s}{\eta_3} \tag{7.17}
\]

Substituting equations from (6.21) and (7.17) into the sum of equation (7.16) results in the following differential equation for stress relaxation:

\[
\frac{d\sigma}{dt} \frac{1}{E_1} + \frac{\sigma}{\eta_1 + \eta_2} + \frac{\eta_2}{\eta_1(\eta_1 + \eta_2)} \exp \left( -E_2 \frac{1}{\eta_1} \frac{1}{\eta_2} t \right) \sigma \frac{\eta_3}{\eta_3} \exp \left( -\frac{E_3}{\eta_3} t \right) = 0
\]

\[
(7.18)
\]

There are several different ways to solve this differential equation. However, for simplicity, a commercial mathematics solver package MATLAB was employed to give the result:

\[
\sigma(t) = C_1 \exp \left[ \begin{bmatrix}
-\eta_1^2 E_3 \exp \left( -\frac{E_2(\eta_1 + \eta_2)}{\eta_1 \eta_2} t \right) - 2E_2 \eta_1 \eta_2 \exp \left( -\frac{E_3}{\eta_3} t \right) - E_2 \eta_2^2 \exp \left( -\frac{E_3}{\eta_3} t \right) \\
-E_2 \eta_1^2 \exp \left( -\frac{E_3}{\eta_3} t \right) + E_2 \eta_1 \eta_2 t + E_2 \eta_2 t
\end{bmatrix} \right]
\]

\[
(7.19)
\]
Similar to the constant stress creep case above, initial conditions for stress at $t=0$ can be assumed such that if an instantaneous strain is put into the material, $C_1$ becomes:

$$C_1 = \varepsilon_0 E_1$$  \hspace{1cm} (7.20)

Substituting this equation for $C_1$ into (7.19) yields the equation for stress with respect to time:

$$\sigma(t) = \frac{\varepsilon_0 E_1 \exp\left(-\frac{-E_1(\eta_1 + \eta_2)t}{\eta_1 \eta_2}\right) - 2E_2 \eta_1 \eta_2 \exp\left(-\frac{E_2}{\eta_3}t\right) - E_2 \eta_2^2 \exp\left(-\frac{E_2}{\eta_3}t\right)}{E_2 \eta_3^2}\left(\frac{-E_2 \eta_2^2 \exp\left(-\frac{E_2}{\eta_3}t\right) + E_2 \eta_3^2 \exp\left(-\frac{E_2}{\eta_3}t\right) + E_2 \eta_3^2 \exp\left(-\frac{E_2}{\eta_3}t\right)}{(E_2 \eta_3^2(\eta_1 + \eta_2)^2)}\right)$$  \hspace{1cm} (7.21)

It is an inaccurate assumption to assume instantaneous strain. Therefore a short algorithm describing the actual loading conditions and stress relaxation of the material can be found in Appendix III. Using the computer algorithm and assuming some initial values for elasticity and viscosity of the model, the following graph demonstrates Model III applied to a constant strain stress relaxation test.
7.3 Model III Loaded Under the Four Zone tensile test

Model III was also extended to the four zone test application. Once again, the history dependence of the material warranted the use of another computer algorithm to determine the stress-strain-time data.

7.4 The Effects of Elasticity and Viscosity on Model III

Similar to Model II, the values of the coefficients have a strong effect on the response to an induced stress or strain.

Consider Model III subjected to a constant stress creep test at maximum load 5 newtons. Each of the coefficients were multiplied by a factor of 2 (as was done in the case...
of Model II) to determine the effects of the curvature and magnitude of the resultant curve. The legend for each curve can be seen below:

<table>
<thead>
<tr>
<th>Curve</th>
<th>Constant Affected</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$E_1 \times 2$</td>
</tr>
<tr>
<td>b</td>
<td>$\eta_1 \times 2$</td>
</tr>
<tr>
<td>c</td>
<td>$E_2 \times 2$</td>
</tr>
<tr>
<td>d</td>
<td>$\eta_2 \times 2$</td>
</tr>
<tr>
<td>e</td>
<td>$E_3 \times 2$</td>
</tr>
<tr>
<td>f</td>
<td>$\eta_3 \times 2$</td>
</tr>
</tbody>
</table>

It is demonstrated how each of the above conditions affects the curvature and the magnitude of the creep curves for Model III. Curve $g$ is a combination of both curves $a$ and $d$ and was found to be the most representative of the experimental curve.

![Constant Stress Creep Curves of Model III](image)

*Figure 7.5 - Model III subjected to constant stress creep at 5 newtons while adjusting the coefficients of elasticity and viscosity. [a=$E_1 \times 2$, b=$\eta_1 \times 2$, c=$E_2 \times 2$, d=$\eta_2 \times 2$, e=$E_3 \times 2$, f=$\eta_3 \times 2$, g=a+d]*
Consider Model III subjected to the constant strain stress relaxation case. The same type of graph was analyzed using the legend found in the table above. Although there is not much change seen from the original curve, curve g, a combination of both a and d were also found to be the most representative of the experimental test data.

![Stress Relaxation Curves for Model III](image)

*Figure 7.6 - Model III subjected to constant strain stress relaxation at 5 newtons while adjusting the coefficients of elasticity and viscosity. [a=E1X2, b=n1X2, c=E2X2, d=n2X2, e=E3X2, f=n3X2, g=a+d]*

The final values of elasticity and viscosity that were found to simulate both creep and stress relaxation are as follows:

\[
E_1 = 203 \text{ MPa} \quad \eta_1 = 260,000 \text{ MPa}\text{sec}
\]

\[
E_2 = 950 \text{ MPa} \quad \eta_2 = 1,200,000 \text{ MPa}\text{sec}
\]

\[
E_3 = 700 \text{ MPa} \quad \eta_3 = 20,000 \text{ MPa}\text{sec}
\]
VIII. ANALYSIS OF THE CREEP AND RELAXATION CURVES

Experimental curves of creep and stress relaxation at incremental ceiling stress values were performed on LDPE samples. These results were then examined critically, and preliminary values for all of the coefficients of elasticity and viscosity were chosen to approximate these experiments. Whether or not values of elasticity and viscosity derived from this analysis are able to describe the four zone memory test will be evaluated in Section IX.

Constant stress creep and constant strain stress relaxation tests were conducted at different maximum loads. Due to experimental scatter, several tests at each stress level were carried out. The assumption of linear polymeric behaviour and a transition from linear to non-linear behaviour in isochronal plots (as discussed in Section II) was utilized as the analytical model to fit the data from the creep and stress relaxation tests, and to determine which curves at each load level were or were not representative of the true material behaviour.

8.1 Constant Stress Creep Experimental Data

Consider creep tests A, B, and C at each load range. Experimental scatter exists within most of the creep plots. One simple way to extract the extraneous curves is to identify the strains at each load level and those at the next highest load level such that
strains achieved at a lower load cannot exceed those found at a higher load. The outlier curves can be identified by the values achieved in creep or stress relaxation since the initial strains from the loading zone of the test should be comparable. Finally, comparing the curves of constant stress creep isochronal plots, following an analysis method commonly used in the literature, to the experimental creep isochronals, gives an indication which experimental creep curves behave in an unexpected manner with respect to the resultant strains.

The complete set of curves at each load level for constant stress creep are seen in the following graphs.

![Creep at 5 N](image)

**Figure 8.1(a)** - Experimental creep curves at maximum load of 5 newtons (which is equivalent to a maximum stress of 2.57 MPa).
Figure 8.1(b) - Experimental creep curves at maximum load of 10 newtons (which is equivalent to a maximum stress of 5.13 MPa).

Figure 8.1(c) - Experimental creep curves at maximum load of 15 newtons (which is equivalent to a maximum stress of 7.70 MPa).

Figure 8.1(d) - Experimental creep curves at maximum load of 20 newtons (which is equivalent to a maximum stress of 10.3 MPa).
In examining the curves at each load level, it is evident that significant experimental scatter exists. There are a few hypotheses as to why there is such a wide range of data points. It is assumed that the sudden increases in slope found in the loading range (e.g. test A at 15N) are probably the result of the sample slipping within the grips, or sample necking, or local inhomogenieties. The temperature of the room at testing can be a factor in the polymeric behaviour. The molding conditions can also have an effect on how the material responds to applied stress and strain.

A. Applying Creep Data to Isochronal Plots

Recall Figure 2.3, the isochronal creep curve which shows the strain as a function of the stress at a specific time. The isochronal plot determines where the viscoelastic behaviour changes from linear to non-linear. The non-linear behaviour leads the material to creep faster as shown by the full lines in Figure 2.3.
To eliminate the extraneous tests, the isochronal curves were plotted from each of the creep data test sets at 100 seconds, 600 seconds and 5000 seconds of creep time. These isochronal results can be found in Figures 8.2 (a) - (c). All three tests were plotted at each load to yield the graph found in Figure 8.3. From Figure 8.3, the most accurate curves of creep at each respective load were then compiled to create the experimental isochronal plot at each time for the constant stress creep condition. The tests that were chosen as being representative of the material were the same set used in Chapters VI and VII in the model coefficient simulation.
Figure 8.2(a) - Experimental creep isochronal curve indicating the resultant strain after 100 seconds for each respective load level. Note: Points with horizontal slash indicate those chosen as representative.

Figure 8.2(b) - Experimental creep isochronal curve indicating the resultant strain after 600 seconds for each respective load level. Note: Points with horizontal slash indicate those chosen as representative.
Figure 8.2(c) - Experimental creep isochronal curve indicating the resultant strain after 5000 seconds for each respective load level. Note: Points with horizontal slash indicate those chosen as representative.

Graphing these three curves simultaneously yields Figure 8.3 below.

Figure 8.3 - Experimental constant stress creep isochronal of all three tests conducted at each respective load level. One data point indicates the amount of creep time at the respective load which contributes the indicated amount of strain.
As described above, one curve from each load level was then selected to plot single isochronal curves at each time step. This curve then provides a guideline to which the spring-dashpot models can be fit with the least relative error. The values which were in fact chosen constitute the lower envelope of creep values, as these form the only non-irregular set of values, especially at high load levels. These values at each time and load step give the following isochronal results.

![Single Curve Experimental Creep Isochronal Data](image)

*Figure 8.4 - Single curve isochronal plot from constant stress creep data from loads of 5N (2.57 MPa), 10N (5.13MPa), 15N (7.70MPa), and 20N (10.3MPa).*

As evident in the graph, the experimental data represents the ideal isochronal plot shape extremely well. The change from linear to non-linear in each creep time is identified and occurs at strains of less than 5%, which in Figure 8.4 corresponds to a maximum load of 10 newtons. This differs slightly with the literature, which indicates the change from linear to non-linear occurs at small strains of approximately 0.5%. It is possible to calculate a creep compliance $J(t)$ of the material by finding the slope of the strain-stress curve in the isochronal plot. Since LDPE behaves linearly within the 10 Newton load
range (5.13 MPa), the value of the creep modulus (the slope of the graph) is constant in this range. However, as the load increases J(t) also increases thus, becoming a function of the maximum load.

B. Applying Models I, II and III to Simulate the Creep Behaviour of LDPE

Using the spring dashpot analogs, it is possible to simulate the behaviour of the material by estimating values of the coefficients of elasticity and viscosity. Since the models are restricted to the linear range of deformation, the models generated will simulate creep curves that demonstrate linearity in the experimental isochronal plot discussed above. The curves which exemplify this linear behaviour are 5 newtons and 10 newtons. Therefore, for loads greater than 10 newtons, the coefficients of elasticity and viscosity become functions of the maximum load. The linear fit of the models is demonstrated by plotting the isochronal data of the models at the same time steps chosen above. Figure 8.5(a) - (c) shows Models I, II, and III linear isochronal plot.
Figure 8.5(a) - Isochronal plot demonstrating the linear relationship of strain to stress for Model I which consists of five Maxwell elements in parallel with each other.

Figure 8.5(b) - Isochronal plot demonstrating the linear relationship of strain to stress for Model II which consists of a spring in series with a parallel element of a Maxwell unit with a dashpot.
Figure 8.5(c) - Isochronal plot demonstrating the linear relationship of strain to stress for Model II which consists of Model II in series with a Voigt Kelvin element.

Consider the creep curves of the spring dashpot models I, II and III. As described above, the models can accurately simulate the creep curves for maximum loads of 5N and 10N. However, the error between the experimental strain values and those of the models significantly increases as the load increases. Consider Figure 8.6(a) and (b) below which demonstrates the ability of the models to simulate creep curves of constant loads 5 and 10 newtons.
As can be seen by both plots, the models simulate the creep curves accurately but tend to be less accurate at the 10 newton load as expected. If the models are applied to higher loads, the error between the experimental values and the models calculated values will increase because these loads occur in the non-linear range. This is shown graphically in figures 8.6(c)-(e) which demonstrate the models at 15 newtons, 20 newtons and 25...
newtons. It is evident that the error between the experimental curves and the creep curves increase as the load increase.

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**Figure 8.6(c)** - Comparison of the experimental creep curves to the creep curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III. Note: The same values for the coefficients were used at 10 N as at the 5 N.

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**Figure 8.6(d)** - Comparison of the experimental creep curves to the creep curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III. Note: The same values for the coefficients were used at 10 N as at the 5 N.
Figure 8.6(e) - Comparison of the experimental creep curves to the creep curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III. Note: The same values for the coefficients were used at 10 N as at the 5 N.

8.2 Constant Strain Stress Relaxation Experimental Data

Similar to constant stress creep, constant strain stress relaxation curves were also considered for modeling. Experimental stress relaxation curves were generated at each of the maximum loads described by creep. These curves are shown in Figure 8.7 (a)-(e) below.

Figure 8.7(a) - Experimental stress relaxation curves at maximum load 5 Newtons (which is equivalent to a maximum stress of 2.57 MPa).
Figure 8.7(b) - Experimental stress relaxation curves at maximum load 10 Newtons (which is equivalent to a maximum stress of 5.13 MPa).

Figure 8.7(c) - Experimental stress relaxation curves at maximum load 15 Newtons (which is equivalent to a maximum stress of 7.70 MPa).

Figure 8.7(d) - Experimental stress relaxation curves at maximum load of 20 Newtons (which is equivalent to a maximum stress of 10.3 MPa).
There is much less experimental scatter in the way the polymer unloads during stress relaxation. Even though there is great repeatability in the way the stress relaxes, the strains that are achieved at the initial maximum load are not as repeatable. Slightly different strain levels are generated at the same maximum load due to experimental scatter, caused by material variation. Because of this phenomenon, the extraneous curves were eliminated by comparing the strains reached in creep to those reached in stress relaxation at any given load level.

A. Applying Stress Relaxation Data to Isochronal Plots

Recall Figure 2.5 which shows an isochronal plot of stress relaxation. The isochronal plot for stress relaxation shows stress as a function of the strain at various stress relaxation times.
The isochronal plot developed from the stress relaxation tests at each time step is demonstrated below in Figure 8.8(a)-(c). All the test results are combined in Figure 8.9. Using the strains achieved at each stress level in creep, one curve at each stress was extracted to develop an isochronal at each time as demonstrated in Figure 8.10.
Figure 8.8(b) - Experimental stress relaxation isochronal curve indicating the resultant stress after holding constant strain for 600 seconds from each initial maximum load.

Figure 8.8(c) - Experimental stress relaxation isochronal curve indicating the resultant stress after holding constant strain for 5000 seconds from each initial maximum load.
The transition from linear to non-linear behaviour is not as clearly defined in the plot of stress relaxation isochronals as it was for the creep results. A reason for this discrepancy can be explained again by the different strains that are reached at the same maximum stress. In addition, the other possibilities of material related experimental scatter also apply.

As demonstrated in the graph, there are several points that do not fall in the predicted isochronal range of curvature. These data points occur at the 15 newton load (or 7.70 MPa) where the stresses appear to relax at a greater rate at constant strain than the other maximum load values. Since these values seem to fall out of the predicted
isochronal shape the curve is plotted without these points to yield the following single curve isochronal plot:

![Single Curve Experimental Stress Relaxation Isochronal Data](image)

**Figure 8.10 - Single curve isochronal plot from constant strain stress relaxation data using the loads of 5N, 10N, 20N and 25N**

In Figure 8.9 and 8.10 it is also important to note that for the highest stress levels, the 100 sec isochronal could not be considered since the material was still undergoing increase in the load towards the maximum value.

Using the single isochronal curves and eliminating some of the extraneous curves, the transition from linear to non-linear behaviour is much more visible and appears at stresses around 5 MPa which corresponds to loads of approximately 10N. Therefore, the loads used in simulating the stress relaxation curves by the spring dashpot models described in this study are restricted to the linear range of 5N and 10N. Loads greater than 10 Newtons appear to behave non-linearly.
B. Applying Models I, II and III to Simulate the Stress Relaxation Behaviour of LDPE

Similar to the case of creep, isochronal plots are applied to each of the spring dashpot models using the same values for the coefficients of elasticity and viscosity. Each model demonstrates a linear pattern of stress relaxation as they did in creep deformation. This is demonstrated in Figures 8.11(a)-(c). The models were then applied to each load range. Since the evaluation of the experimental data indicated that the linear range was restricted to loads of 5 newtons and 10 newtons, Figures 8.12 (a)-(e) show the decrease in accuracy of the models to fit the loads greater than 10 newtons.

![Model I - Stress Relaxation Isochronal Data](image)

Figure 8.11(a) - Isochronal plot demonstrating the linear relationship of stress to strain in Model I which consists of five parallel Maxwell elements.
Figure 8.11(b) - Isochronal plot demonstrating the linear relationship of stress to strain in Model II which consists of a spring in series with a parallel element of a Maxwell unit with a dashpot.

Figure 8.11(c) - Isochronal plot demonstrating the linear relationship of stress to strain in Model III which consists of Model II in series with a Voigt-Kelvin element.
As expected, all three models exemplify linearity in their isochronal plots. Using the same coefficients of elasticity and viscosity as those applied to the creep condition in each loading case yields the following curves:

![Stress Relaxation from 5 N](image)

**Figure 8.12(a)** - Comparison of the experimental stress relaxation curves to the stress relaxation curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III.

![Stress Relaxation from 10 N](image)

**Figure 8.12(b)** - Comparison of the experimental stress relaxation curves to the stress relaxation curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III.
The models are very accurate at describing the 5 newton load range. At 10
newtons, Model I is less accurate than the other two models. As the load increases, the
accuracy of the models decreases.

Figure 8.12(c) - Comparison of the experimental stress relaxation curves to the stress relaxation
curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III.

Figure 8.12(d) - Comparison of the experimental stress relaxation curves to the stress relaxation
curves generated by fitting the coefficients of elasticity and viscosity to Models I, II and III.
The accuracy of the models applied to the creep curves decreases more dramatically than the accuracy of the models for the stress relaxation curves. This could be a result of the greater repeatability in the stresses of the stress relaxation tests as well as the experimental scatter described previously. The models are very good in describing linear polymeric behaviour. In order to extend them to fit the non-linear load ranges, the values of the coefficients must be altered.
IX. USING MODELS I, II AND III TO SIMULATE THE FOUR ZONE MEMORY TEST

In Section VIII, Models I, II and III were shown to be accurate depictions of polymer behaviour in the linear region of creep deformation and for stress relaxation. The coefficients for elastic moduli and viscosity that were found to describe the linear behaviour must be altered to become functions of the maximum stress in the non-linear range of stress or strain. Thus, in simulating the four zone test, the models were restricted to the ranges identified as behaving linearly, namely stresses less than 10N and maximum strains of 5%. It is of interest to determine whether the models accurately simulate the four zone test, because these tests comprise a different sequence of load/strain variations while using the same material measured in simple creep and stress relaxation. In the four zone test, the model was loaded at a constant rate causing the stress to increase with time, the strain was then held constant while the model underwent stress relaxation, the model was next unloaded at a constant rate to zero load after which creep recovery occurred over time all of which left some remnant permanent deformation.

These tests reflect the methodology in which the IUD’s are tested for their retentive memory effect. The goal of this study is to eventually use the coefficients of the models in Finite Element Analysis and other such numerical model stress-strain applications.
As part of the four zone experimental testing program, seven different test parameters were varied, some at 6 different levels. So, amongst the more than 300 tests, there were a large variety of experimental conditions. A few sets of parameters were chosen to evaluate the fidelity of the three spring dashpot models. These parameters were: the strain rate during loading in zone 1; peak load in zone 1; length of time for stress relaxation in zone 2.

9.1 The Models Applied to a Four Zone Test of 5 newtons

As seen in the Chapter VIII, values of elasticity and viscosity were identified for each of the models in order to mimic constant stress creep and constant strain stress relaxation tests. Using the same values of the coefficients, the models will now be applied to the four zone test using a cutout force of 5 newtons. Several combinations of the experimental tests on real material were completed using various loading rates and stress relaxation times. These tests showed that the forward strain put into the material can be divided into three strains activated upon unloading the material. The strains that result in unloading are the elastic recovery strain, the permanent deformation strain and the creep recovery time dependent strain. In addition, this experimental study found that the forward strain is linearly related to all three of these component strains independent of the amount of relaxation time of the material.
In terms of the models, these results are important because the amount of relaxation time that the model undergoes in zone two should have no effect on the accuracy of the fit of the spring-dashpot models. Consider Figure 9.1 below which is a four zone memory test loaded at 1.0 mm/min to 5 newtons (2.57 Mpa), held for 100 seconds at constant strain, unloaded at 0.5 mm/min to zero load and held for 1200 seconds.

![Figure 9.1 - Experimental results for the four zone memory test loaded at 1.0 mm/min to 5 newtons, held for 100 seconds stress relaxation, unloaded at 0.5 mm/min and held for 1200 seconds undergoing creep recovery. [test 210]](image)

Applying each of the models to this testing regimen yields the following comparison:
The graph in Figure 9.2(a) demonstrates the response of Model I subjected to the same four zone test constraints used in the experimental method. The general shape of the curve is accurate, however, the model has some significant shortcomings. In the loading (zone 1), Model I attains a much higher value of strain than does the experimental test. This is somewhat surprising since the values of creep chosen to represent "typical" behaviour (Figures 8.2(a)-(c)) represented the lower envelope of creep strain for a given load level. In the stress relaxation phase (zone 2), the model does not achieve the same amount of stress relaxation as the experimental value. In zone 3, during which the model is unloaded to zero load, the elastic recovery is slightly greater than the experimental value. Finally in zone 4, the model almost completely recovers its strain, leaving very little permanent deformation. Recall that the creep tests and simple stress relaxation tests had provided no data on unloading behaviour.
Consider Model II subjected to the same four zone test with a 5 newton cutout force.

![Model II - Four Zone Memory Test Loaded to 5 newtons](image)

*Figure 9.2 (b) - The four zone memory test applied to Model II.*

It is demonstrated that Model II is relatively accurate in describing the stress that occurs in the four zone memory test. However, it is not able to accurately describe the strain. In the loading during zone 1, once again the model has a much higher strain at the end of loading. In zone 2, the strain is held constant and the amount of stress relaxation is slightly less than the experimental test. In zone 3, the model is unloaded to zero load. The model has very little elastic recovery in comparison to the elastic recovery of the experimental test, but similar to Model I it recovers almost completely to zero strain in zone 4.

Finally, consider Model III.
Model III due to its similarities to Model II has a similar fit to Model II. However, it is noted that Model III is more accurate in zone 2 for stress relaxation. However, it has even less elastic recovery than Model II in zone 3.

Similarly, to observe the effects of a shorter relaxation time (zone 2), the models are next applied to the experimental results of the Four Zone Test in Figure 9.3 below.
Applying Models I, II and III to the above constraints give the following result:
The same conclusions are made with respect to the models and the fit of the four zone test regimen. The stress response of all three models is accurate in describing the experimental stress. However, the models fail to accurately describe the strain. They load to a higher strain level in zone I than the experimental value, and return completely to zero permanent strain in zone four. In addition, Models II and III demonstrate only a small bit of elastic unloading which does not follow the experimental data at all.
9.2 - The Models Applied to a Four Zone Test of 10 newtons

Extending the linear analysis to the 10 newton load range, similar results are found. Consider the experimental test shown in Figure 9.5. The test included loading at 1 mm/min, holding at constant strain for 100 seconds, unloading to zero load and allowing 1200 seconds of creep recovery.

![Four Zone Memory Test Loaded to 10 Newtons](image)

*Figure 9.5 - Four zone memory test loaded at 1 mm/min, held for 100 seconds stress relaxation, unloaded at 0.5 mm/min and held for 1200 seconds creep recovery. [test 211]*

Applying Models I, II and III to the above experimental method results in the following curves:
Model I demonstrates the expected general shape of the curves. However, as the load increases, the material tends to behave non-linearly. Therefore, it is reasonable to see a less accurate fit, shape and magnitude than in the previous fit at 5 newtons. The strain of Model I is much greater than the experimental strain and, similar to the 5 newton case, there is no residual permanent strain after the 1200 second creep recovery. Rather, the model almost completely recovers its original dimensions. The stress of Model I starts at the same magnitude after zone 1, but the model undergoes significantly less stress relaxation in zone 2 as was also seen for the 5 newton load.
Consider Figure 9.6 (b) which is Model II subjected to the same four zone memory test. The strain of the model in zone 1 is greater than the experimental strain, but it is closer to the experimental strain was Model I. Model II is not very accurate in simulating the unloading pattern of the strain. The elastic unloading of the Model (the linear portion of the unloading strain) is much less than that of the experimental case. This causes a long slow unloading during the zero load creep recovery stage. Although Model II does not appear to unload completely, the magnitude of the permanent strain is much less than that of the experiment test. The stress is however, much more accurately simulated in the case of Model II than with Model I. There is a similar loading pattern, stress relaxation curve and unloading.

Finally, consider Model III subjected to the same test.
Figure 9.6(c) - The four zone memory test applied to Model III.

Model III has similar shortcomings to the previous two models however, it is the most accurate at simulating the experimental test. Once again the strain of the Model is much greater than the experimental strain. However, the elastic recovery strain activated during the unloading sequence is fairly accurate given the predicted magnitude of the strain after loading. In addition, the elastic strain of Model III is much more accurate than that of the elastic strain found in Model II. The stress relaxation of Model III is accurate in simulating the experimental stress relaxation. Model III also is much more accurate in simulating the four zone memory test at 10 newtons than at 5 newtons maximum load. The loading of the single top spring in Model III at 10 newtons must be significantly greater than at 5 newtons resulting in a more accurate fit in elastic loading and unloading. The accuracy of the stress relaxation zone is approximately the same.

It has been shown experimentally that the unloading behaviour of the material is independent of the amount of time spent during zone 2, the stress relaxation period.
Therefore, in considering simulation of an experimental test with a shorter stress relaxation period, the same results should occur.

Consider the following load case which has the same sequence and magnitudes of loading except a shorter relaxation time:

![Four Zone Test Loaded to 18 Newtons](image)

Figure 9.7 - Four zone memory test loaded at 1 mm/min, held for 1 second stress relaxation, unloaded at 0.5 mm/min and held for 1200 seconds creep recovery. [test 250]

Applying Models I, II and III to the loading case of test 250 above yields the following curves:
Model I - Four Zone Test Loaded to 10 Newtons

Figure 9.8(a) - The four zone memory test applied to Model I.

Model II - Four Zone Test Loaded to 10 Newtons

Figure 9.8(b) - The four zone memory test applied to Model II.
Figures 9.8 (a)-(c) show the same simulation problems as those with a higher relaxation time. The same general observations are found concerning the accuracy of the model behaviour for stress and strain.

9.3 - Four Zone Memory tests Loaded at a Higher Strain Rate

There are various factors of loading that can change the way in which the material responds to the applied stress or strain. In Chapter VIII, it was demonstrated that the maximum load changes the way in which the material behaves by altering its stress-strain pattern from linear to non-linear as the load increases. In Sections 9.1, and 9.2 above as well as in another study, it has been shown that the amount of stress relaxation time has little effect on the stress and strain found after unloading. Now consider the case of increasing the strain rate of loading. Due to the time dependent nature of the material, the
strain rate found in loading could have a significant effect on the way in which the material behaves.

Consider experimental test 208 which loads to 10 newtons maximum force. In this test, the material was loaded at a rate of 5 mm/min (5 times the tests considered above in Sections 9.1 and 9.2), held for 100 seconds stress relaxation, unloaded at 0.5 mm/min and then held for 1200 seconds creep recovery.

![Four Zone Memory Test Loaded to 10 Newtons](image)

*Figure 9.9 - Four zone memory test loaded to 5mm/min, held for 100 seconds stress relaxation, unloaded at 0.5 mm/min and held for 1200 seconds creep recovery.* [test 208]

Applying each of the models to this test gives the following curves:
The loading rate has an effect on the way in which Model I loads with respect to the previous loading rate which occurred at 1 mm/min. Although the strain of the Model is greater than the experimental strain, it is not as great as the strain produced by the same model at the lower loading rate. In addition, the strain found during unloading is very accurate, but it overpredicts strain recovery during the zero load creep phase. The stress relaxation simulation is more accurate than in the slower loading case presented earlier. This is because the faster loading rate does not allow the time dependent dashpots to take on load. Therefore, the experimental curve has a lesser slope than it did for the previous loading rate. This condition is good for the model because the model has a shallow slope for stress during zone 2.

Consider Model II:
Model II has similar shortcomings to the slower loading rate. The strain is greater than the experimental strain but not as great is the strain of Model II at the previous slower loading rate. The elastic recovery during unloading is still not accurate with the experimental test however, it much better than Model II at the lower loading rate. The creep recovery is the same as in the case of the slower loading rate. The stress relaxation curve seems to start at a higher load than the experimental value. This is a result of the computer algorithm that overshoots the input load parameter. The curvature of the stress relaxation is adequate. The faster loading rate corresponds to greater stress relaxation accuracy.

Finally, consider Model III subjected to the same test:
The curve fit of Model III is very similar to that of the slower loading rate. However the increased loading rate seems to have a negative effect on the way in which the strain unloads. The good fit of elastic recovery is diminished with the faster strain rate and therefore, the fit of creep recovery is much poorer than with the other loading rate.

The Figures 9.9 - 9.10 (a)-(c) focus on an increased loading rate. Now consider loading at a rate much slower than the initial loading condition. Given the results above, it is assumed that a slower loading rate will result in a greater overshoot of the strain in loading for all of the models. The stress relaxation phase will have a much poorer fit in Model I and II since the increase in strain rate appeared to increase the accuracy of the fit of zone 2 with these two models. The stress relaxation phase of Model III does not appear to be affected by the loading rate. Rather, the unloading strain components are affected. Therefore, a slower loading rate will have a positive effect on the unloading component strains of Model III.
Consider Figure 9.11. The experimental method includes loading at a strain rate of 0.1 mm/min, holding for 100 seconds stress relaxation, unloading at 0.5 mm/min and allowing 1200 seconds for creep recovery.

![Graph of Four Zone Memory Test Loaded to 10 Newtons](image)

**Figure 9.11** - Four zone memory test loaded at 0.1 mm/min, held for 100 seconds stress relaxation, unloaded at 0.5 mm/min and held for 1200 seconds creep recovery. [test 220]

Applying Model I to the above test yields the following graph:

![Graph of Model I - Four Zone Memory Test Loaded to 10 Newtons](image)

**Figure 9.12(a)** - The four zone memory test applied to Model I.
As seen in Figure 9.12(a), the loading rate has significant effect on the slope of loading in both stress and strain. The strain reached in Model I is much higher than the experimental strain and the length of time required to reach the peak load is much greater. The slope of the stress relaxation is much greater for the experimental test than that of Model I, as has been the case for every application to four zone tests. However, the elastic recovery upon unloading is comparable to the experimental case. The model is also seen to fully recover in the creep recovery phase of zone 4 which is a problem with all these simulations.

Consider Model II subjected to the same test:

![Graph](image)

Figure 9.12(b) - The four zone memory test applied to Model II.

In the case of Model II, the slope of loading is more accurate for Model II than for Model I, but is not quite as accurate as the cases where the strain rate is higher. The peak strain is higher for Model II than the experimental strain as was expected. The stress relaxation
in zone 2 is a good fit considering the offset from the loading zone. The elastic recovery simulation is poor as expected and the model once again creeps towards almost full recovery.

Finally consider Model III subjected to the same test:

![Figure 9.12(e) - The four zone memory test applied to Model III.](image)

Model III also has some shortcomings in its simulation to the test at a slower strain rate. The slope of loading for stress is greater for the experimental test than the Model however, the strain is at the same slope. Once again, the peak strain of Model III is much greater than the experimental result. The stress relaxation simulation is adequate. The magnitude of elastic recovery is not great enough and as expected, the simulation of the data in this zone is worse than both of the higher strain rates. Finally, the simulation of the creep recovery is also poor and the model tends to recover completely.
X. DISCUSSION OF THE MODELS AND THEIR SIMULATION

Models I, II and III were developed in order to simulate the behaviour of mineral filled LDPE after being molded as a T-shaped IUD. This material is different from pure LDPE because it includes a barium sulfate filler to which the polymer molecules do not bond. These inclusions affect the way in which the polymer material behaves. They can act as local void nucleation agents.

The models were initially applied to the common polymeric phenomena test procedures; constant stress creep and constant strain stress relaxation. After finding the most appropriate $E$ and $\eta$ values to represent these tests, the models were then applied to the four zone memory test. The parameters in the four zone memory test that were varied in the models in order to test their fidelity were: the peak load within the linear behaviour range; the amount of time spent in stress relaxation in zone 2; and the strain rate used in loading zone 1.

10.1 Model I

Model I consists of parallel Maxwell units. This model is commonly found in polymer literature to describe the stress relaxation phenomena found in polymeric materials. However, it is rarely seen applied to creep, and when applied to constant stress creep it is often derived incorrectly or incompletely. Therefore, the present study attempts
to use parallel Maxwell elements in a broader manner than that to which is ordinarily subjected.

A. Model I Applied to Constant Stress Creep and Constant Strain Stress Relaxation

Values of elasticity and viscosity were proposed for Model I using five parallel Maxwell elements. It was found that as the number of Maxwell elements increases, so does the accuracy of the fit of the curves to constant stress creep and constant strain stress relaxation. Model I was applied to constant stress creep and constant strain stress relaxation at loads of 5N, 10N, 15N, 20N, and 25N. Within the linear range of loading, the same 10 values of elasticity and viscosity were very accurate in describing the creep and stress relaxation behaviour. In order to fit the model to the experimental creep and stress relaxation curves, values of elasticity and viscosity were assigned such that the appropriate shape of the curves was developed. Then, using the ratio of the relaxation/retardation times between each individual Maxwell unit, the magnitude of the curve was adjusted to fit the appropriate magnitude of the experimental values.

B. Model I Applied to the Four Zone Memory Test

i. Four zone memory test at different peak loads

As Model I was extended to the four zone memory test several shortcomings were determined. Primarily, only tests where the peak load within the linear range of loading were considered. The two peak loads which qualified were the 5 newton and the 10 newton experimental tests. In these tests, the loading pattern of the stress simulated the
experimental test in zone 1. However, the peak strain reached after zone 1 was much higher for Model I than the experimental results. At the 5 newton load, the strain reached by Model I was approximately 1.5 times the amount of experimental strain. At the 10 newton load, the strain reached by the model was approximately 2 times the amount of the experimental strain. This overprediction of strain will effect the unloading strain values in zone 3. Zone 3 is observed to unload into three component strains. Experimentally, the greatest of these is the elastic unloading strain. Given the magnitude of the peak strain, the model is observed to have tremendous elastic recovery. This is the result of the low elasticity of some of the springs. In zone 1, it is easy for the springs to load up. Zone 2 shows a limited unloading of the springs as the dashpots creep forward during stress relaxation. This is demonstrated by the slope of this curve. In zone 3, the model has a very large elastic unload.

In zone 2, the stress relaxation response was similar in both the 5 newton and the 10 newton loading case. However, the slope of the stress relaxation was much lower than the slope of the experimental case and therefore, resulted in a poor fit of the curve. This underestimation of the rate of short term stress relaxation can also be seen in Figures 8.12(a) and 8.12(b), although the long term stress relaxation was very good. In terms of the elements of the model, the strain does not unload from the springs to the dashpots fast enough. This indicates that the springs are too weak and that the dashpots may be too viscous.
Zone 3, as discussed above, unloads from the springs producing too large an elastic recovery again indicating that the dashpots have not moved far enough.

Finally, zone 4 was the creep recovery strain. In zone 4, the strain is observed to almost completely recover. This indicates that not enough permanent strain has accumulated in the dashpots. Permanent strain can occur when, on unloading, residual strains with opposite sign in the springs provide a gross zero load, but these will still produce a net overall displacement in the dashpots when the individual loads attempt to relax to a zero load condition.

ii. **The four zone memory test applied with different stress relaxation periods**

It has been shown experimentally that the amount of time spent in stress relaxation has little effect on the resultant strains of the material. All of the models simulated this behaviour. Changing the amount of time spent in stress relaxation had very little effect on the distribution of unloading strains.

iii. **The four zone memory test applied to different strain rates**

The simulation by Model I was greatly affected by the strain rate used in zone 1. Three rates of loading were considered. A strain rate of 1mm/min was initially used to evaluate the fit of the curves at different loads and different stress relaxation times. Then, for peak loads of 10 N, the strain rate was increased to 5mm/min and decreased to 0.1mm/min to determine the effects of changing the rate.
As the strain rate in zone 1 increases, the accuracy of the curvature of the stress was found to increase to a value closer to the experimental results. The simulation of the strain curve remained the same as the case loaded at the lower strain rate which is that it overpredicted the peak strain. The peak strain reached by the model remained about 2 times the peak strain of the experimental data. However, increasing the strain rate in zone 1 in the model caused the slope of the experimental curve during stress relaxation to decrease. This condition was advantageous in terms of the model since the slope of zone 2 for Model I was already shallow, compared to the experimental values, at the lower rate of loading. This effect was further reinforced by observing the curves at 0.1mm/min strain rate. The slope of the stress relaxation for Model I is almost flat after an initial decrease in this zone. The same shortcomings found previously on unloading, namely the elastic unloading is too high and the remnant plastic strain is far too low, were not changed.

To summarize the effects of loading rate, the Table 10.1 below was composed.

**The Factor Between the Strains in Model I and the Experimental Strain**

<table>
<thead>
<tr>
<th>Loading Rate</th>
<th>0.1mm/min</th>
<th>1.0mm/min</th>
<th>5.0mm/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Loading Strain</td>
<td>2.1</td>
<td>2.3</td>
<td>1.6</td>
</tr>
<tr>
<td>Elastic Unloading</td>
<td>2.5</td>
<td>3.0</td>
<td>1.65</td>
</tr>
<tr>
<td>Creep Recovery</td>
<td>4.0</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Plastic Strain</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

*Table 10.1 - Comparison of the factors between the magnitude of the strain components of Model I with the experimental strain components.*
The table above uses the curves found in Figures 9.6(a), 9.10(a), and 9.12(a). The values found for the factors are calculated from the magnitudes of each of the strains indicated. (i.e. at 1 mm/min, the peak strain of Model I is 2.1 times the peak strain of the experimental curve). It is interesting to note the comparison of the factors between each loading rate. The overprediction of peak strain is within a comparable range at all three loading rates. The magnitude of elastic recovery is relatively comparable between the 0.1 and 1.0 mm/min loading rates and falls short at the 5 mm/min loading rate. The creep recovery factor seems to improve as the loading rate in increased. This means that the creep recovery of Model I is much more accurate at the 5 mm/min loading rate than the 0.1 mm/min loading rate. Finally, there is no remnant plastic strain in the model which is indicated by the small values for the factors of the experimental plastic strain.

C. General Discussion with respect to Model I

The model suffered from essentially two problems in simulating four zone behaviour using the elasticity and viscosity coefficients in the spring-dashpot models obtained from single creep and stress relaxation tests.

The first problem was the overprediction of strain at the end of Zone 1. This result is puzzling, and can only be attributed to the inconsistencies in the material. The loading of the material for Zone 1 in the four zone test is identical to that in the simple creep test and in the long term stress relaxation tests. Spring and dashpot coefficients chosen to fit the loading phase of the simple creep and stress relaxation tests should have
reproduced the peak load-peak strain behaviour of the four zone test. Instead, the strain found in the experimental four zone test at 5 newtons was about 0.9% (Figure 9.1) and 1.3% (Figure 9.3), compared with strains of 1.4 to 2.0% in the creep tests (Figure 8.1(a)) and 1.9 to 2.4% in the stress relaxation samples.

What is curious is that the values chosen from the combined creep isochronals (Figures 8.2 (a) to (c)) represented the lower envelope of strain for a given stress value. Based on this, if any systematic error were to be expected, it would be that the models should underpredict the strain in the four zone tests.

The creep and stress relaxation tests were performed about 6 months after the four zone tests, so the polymer material may have aged in that interval.

The second serious shortcoming of Model I is that it underpredicts the short term four zone stress relaxation (less than 200 seconds) and also the permanent deformation remaining at the end of the test. Both of these errors are associated with too little movement of the dashpots. The $E$ and $\eta$ values were chosen to fit creep and stress relaxation tests of 7000 seconds duration. Lowering the viscosities of the dashpots decreases the relaxation times \( \frac{\eta}{E} \), which increases the short term curvature and increases the total creep strains and stress relaxation values. However, reducing these viscosities has the effect that the long term creep and stress relaxation tests do not reach a
fixed asymptote, but rather, they reach a fixed creep strain rate or stress relaxation rate at long times, and these rates are too high to represent the long term (7000 sec) results.

The conclusion is that the best efforts to find $E$ and $\eta$ values to represent the long term creep and stress relaxation tests did not produce a Maxwell-Weichert model that faithfully predicted four zone behaviour, even when loads were restricted to the linear region.

10.2 Model II

Model II consists of a spring in series with a parallel element made up of a dashpot and a Maxwell element. A characteristic of this model is the limited number of elements that can be adjusted which simplifies the derivation of the mathematics describing their behaviour.

A. Model II Applied to Constant Stress Creep and Constant Strain Stress Relaxation

Model II is found to simulate the creep and stress relaxation curves within the linear region of loading very well. Values of elasticity and viscosity were determined by examining the main effects of altering each of the values individually. The two elements having greatest effect on the Model are the spring $E_1$ and the dashpot found in the Maxwell element $\eta_2$. Using the best values for $E$ and $\eta$ in creep and stress relaxation, the model was then applied to the four zone test analysis load sequence.
B. Model II Applied to the Four Zone Memory Test

1. The four zone memory test at different peak loads

To begin examining Model II applied to the four zone test, first consider the different peak loads of 5 newtons and 10 newtons. The difference in peak load did not appear to have much effect on the fit of the Model.

In zone 1, the model is loaded up to its peak force. The loading of the stress is accurate. The peak stress of the model appears to be slightly greater than the experimental test at both load levels as a result of the computer algorithm but in both load cases, it is within 2% of the experimental value. The computer calculates the achieved stress of the model in discrete increments of time. Therefore, there is some error due to the way in which this is calculated. The peak strain of the model is significantly greater than the peak experimental strain, about 1.25 times the experimental strain. This is better than Model I which was between 1.5 and 2 times the experimental strain. Once again considering the unloading pattern, there is a short elastic recovery and a large, slow, time dependent creep recovery strain. This indicates that the single spring does not hold enough strain and that the second parallel element is too weak. It accumulates strain too easily, and therefore results in the large strain and long time for creep recovery.

The stress relaxation in zone 2 fits very well to the experimental strain. The slope of the model is only slightly less than the experimental test. This is the result of the ability
of the parallel element to easily take on strain, relaxing the single spring and thus, unloading the stress.

Finally in zone 4, there is no permanent deformation. This means that the dashpot in the Maxwell unit in element 2 is too strong. It accumulates almost zero strain causing the spring coupled immediately above it to completely unload its deformation into the single dashpot with which it is in parallel. Note that if the unloading were instantaneous, then before the dashpots had time to move, this spring would still be in tension, as would the dashpot below it. But the condition for zero gross load puts the parallel dashpot in compression. During zero load creep, if the parallel dashpot is very much less viscous than the series dashpot, then the parallel dashpot is squeezed back to relax the spring leaving no residual strain.

**ii. The four zone memory test applied with different relaxation periods**

As previously noted, the amount of time spent in stress relaxation has no effect on the resultant experimental strains activated upon unloading. This was simulated correctly by Model II which showed no difference between a relaxation period of 100 seconds and a relaxation period of a single second.

**iii. The four zone memory test applied to different strain rates**

Loading the model at different strain rates had large effects on the resultant stress-strain-time curves. In zone 1, the slope of the stress time curve decreased as the strain
rate of loading decreased. Since one problem with the model was overpredicting the
strain at a given peak load, the most accurate stress-strain loading curve was found at the
5 mm/min loading rate. The excess strain accumulates in the double dashpot parallel
element, in effect unloading the spring as the model is loading up. The slope of the strain-
time curves was approximately equal, as it should be, since this is preset in the tensile
tester in the experimental method, and is prescribed in the computer algorithm in the
model. The peak strain found in loading was once again found to be greater than the
experimental peak strain. However, at the higher strain rate, the peak strain of Model II is
only 1.2 times greater than the experimental strain which is quite different from the 1.5
times greater found at the lowest strain rate. A similar amount of elastic recovery was
found in all three cases, namely one which was not nearly enough to provide a good
simulation to the experimental curves. The values of the coefficients in the parallel
element of Model II need adjusting in order to provide a stronger element, but this may
well adversely affect the fit with the long term creep and stress relaxation.

In zone 2, the slope of the stress relaxation was found to be most accurate at the
lowest loading rate. The magnitude of the peak stress is slightly greater due to the
computer program but overall the stress relaxation corresponds well to the experimental
graph.

In zone 4, a very poor simulation of creep recovery is seen again. As previously
discussed, this is the result of element 2 (the parallel element) being too weak and the
second (series) dashpot being too strong such that the model unloads completely over a long period of time leaving little permanent deformation.

Similar to the case of Model I, the effect of loading rate is summarized in the following table:

<table>
<thead>
<tr>
<th>Loading Rate</th>
<th>0.1mm/min</th>
<th>1.0mm/min</th>
<th>5.0mm/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Loading Strain</td>
<td>~1.5</td>
<td>~1.33</td>
<td>~1.2</td>
</tr>
<tr>
<td>Elastic Unloading</td>
<td>~0.85</td>
<td>~0.75</td>
<td>~0.66</td>
</tr>
<tr>
<td>Creep Recovery</td>
<td>~9.0</td>
<td>~6.5</td>
<td>~10.0</td>
</tr>
<tr>
<td>Plastic Strain</td>
<td>~0.0001</td>
<td>~0.0001</td>
<td>~0.0001</td>
</tr>
</tbody>
</table>

Table 10.2 - Comparison of the factors between the magnitude of the strain components of Model II with the experimental strain components.

For Model II, the table is found from the graphs in Figures 9.6(b), 9.10(b) and 9.12 (b). In examining the factors, some anomalies are observed. The peak strain of the model overpredicts the experimental peak strain. The factors are seen to slightly decrease as the loading rate increases. This indicates that the model is slightly more accurate at predicting the peak strain at the highest loading rate. A trend of decreasing factor is also seen in the comparison of the elastic unloading strains. However, in the case of the elastic recovery of the model, the simulation of the magnitude of elastic recovery is seen to get worse because the factor moves further away from 1. Thus, at 0.1mm/min, the elastic
recovery of Model II is 0.85 times the elastic recovery of the experimental results. This
time slips to 0.66 times the experimental elastic recovery at the higher loading rate
5mm/min. The creep recovery predicted by the model is much greater in all three loading
cases than the experimental results, and there is no trend or reproducibility in the factor.
Finally, it is observed that no permanent deformation strain remains and as such, this
factor results in a very low value.

10.3 Model III

Model III is made up of Model II in series with a Voigt-Kelvin element. It was
developed to have a stronger retentive memory response than Model II.

A. Model III Applied to Constant Stress Creep and Constant Strain Stress Relaxation

In extending Model II, the values of the coefficients of elasticity and viscosity
started out to be equal to those of Model II, since the same two elements have the greatest
effect on the model ($E_1$ and $\eta_2$). However, with the addition of a Voigt-Kelvin element,
the values of the initial coefficients were adjusted. Model III was found to provide the
most accurate fit out of all of the models to long term creep and stress relaxation tests.
Therefore, it was anticipated that this accuracy would extend to the four zone memory
tests as well.
B. Model III Applied to the Four Zone Memory Test

i. Four zone memory test at different peak loads

When Model III was applied to the four zone memory test, it too had some shortcomings with regard to the accuracy of the curve simulation. The difference in peak loads had a great effect on both the input and resultant strains of the four zone test. For tests loaded at 1mm/min, and 5 newtons maximum load, the strain is greater than the experimental strain by approximately 1.25 times. At 10 newtons, the peak strain of Model III is about 1.75 times the experimental strain. In addition, there is wide variety between the elastic unloading strains. Although both loading ranges result in too much strain accumulating in elements 2 and 3, the amount of elastic recovery is more accurate at the 10 newton load than at the 5 newton load while loaded at the same strain rate. In addition, the amount of creep recovery and the slope of the creep recovery curve is the most accurate at the 10 newton maximum load out of all curves and all of the models considered. However, even these values (factors of 8.0 and 0.01) are unacceptable.

In zone 2, the stress relaxation is very accurate. The model loads up to a peak stress comparable to that of the experimental test and unloads, as a result of stress relaxation, at the same slope and time as the experimental curve.

In zone 3, the elastic unloading strain is more accurate at the 10 newton test simulation compared with the 5 newton range, as well as the creep recovery being more accurate. This indicates that at the 5 newton range, the single spring unloads its
deformation into the parallel element of Model II and the Voigt-Kelvin element very quickly during zones 1 and 2. These latter elements appear to reach a peak strain during loading, whereas the isolated spring element continues to load. Therefore upon unloading, the spring is holding a large elastic strain which is given back in zone 3, while the other two elements are holding a limited strain which completely unloads during creep recovery.

ii. The four zone memory test applied with different stress relaxation periods

Similar to the above two cases, the resultant stress-strain-time curves for this model show that the amount of time spent in stress relaxation has no effect on any of the other parameters.

iii. The four zone memory test applied at different strain rates

Finally, Model III was also considered at different strain rates. The results are very similar to those of Model II.

In zone 1, the strain rate affects the slope of the stress-time curve. As the strain rate is decreased, so is the slope of the stress-time graph, but more than proportionately to the strain rate change, and more than the experimental results. This is a result of strain accumulating in elements 2 and 3 and therefore, decreasing the strain in the single spring. If the strain in the spring is decreased, then so is the stress, and thus the slope of the stress-time curve is lowered. Conversely, at the higher rate of loading, there is little difference between the stress-time curve of Model III and that of the experimental data.
In zone 2, the stress relaxes at a rate very comparable to the experimental values. The zone 1 strain rate has little effect, except at the lowest strain rate, where the slope of the model is even greater than the slope of the experimental stress relaxation. This indicates that elements 2 and 3 in Model III accumulate a lot of strain in the slow loading sequence and then even more in the stress relaxation sequence. Although the single spring still has some strain, this increased accumulation in elements 2 and 3 will greatly affect the creep recovery curvature.

The magnitude of the peak strain is affected in that at the lower strain rate the peak strain is about 1.75 times the experimental strain and at the highest strain rate, the peak strain of Model III is about 1.3 times the experimental strain.

In zone 3, the magnitude of the elastic strain that is unloaded is greatest for the 1 mm/min loading rate. This is the middle loading rate analyzed. At the lowest strain rate, it is known that the spring has unloaded strain to the other elements such that the curvature of the creep recovery is greatly skewed. This would then indicate that the fastest loading rate would give the best results for elastic recovery and creep recovery. However, the model demonstrates a higher degree of stress relaxation in zone 2 for the 5 mm/min strain rate than at the 1 mm/min loading rate. This must mean that the spring has unloaded to a greater degree during stress relaxation for the higher loading rate than at 1 mm/min, and therefore the creep recovery curves deviate from what the 1 mm/min strain
rate curve produced. To summarize the results at a quick glance, refer to the Table 10.3 below:

**The Factor Between the Strains in Model III and the Experimental Strain**

<table>
<thead>
<tr>
<th>Loading Rate</th>
<th>0.1mm/min</th>
<th>1.0mm/min</th>
<th>5.0mm/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Loading Strain</td>
<td>~1.75</td>
<td>~1.75</td>
<td>~1.3</td>
</tr>
<tr>
<td>Elastic Unloading</td>
<td>~0.05</td>
<td>~1.50</td>
<td>~0.2</td>
</tr>
<tr>
<td>Creep Recovery</td>
<td>~8.0</td>
<td>~8.0</td>
<td>~8.0</td>
</tr>
<tr>
<td>Plastic Strain</td>
<td>~0.01</td>
<td>~0.01</td>
<td>~0.01</td>
</tr>
</tbody>
</table>

Table 10.3 - Comparison of the factors between the magnitude of the strain components of Model III with the experimental strain components for 10 N peak load.

The table above uses the curves found in Figures 9.6(c), 9.10(c), and 9.12(c). It is interesting to note the comparison of the factors between each loading rate. The overprediction of peak strain is 1.75 times the experimental strain in both the 1 mm/min and 0.1 mm/min loading rates while it is 1.3 times the experimental strain in the 5 mm/min loading rate case. The magnitude of elastic recovery is highest at the intermediate loading rate. The factor of creep recovery in the model to the experimental creep recovery is very reproducible such that in each loading rate, the model overpredicts the magnitude of the creep recovery by 8 times the experimental creep recovery. Finally, there is almost no remnant plastic strain in the model which is indicated by the small values for the factors of the experimental plastic strain.
C. General Discussion with respect to Model II and III

Models II and III exemplified essentially the same problems simulating the
experimental four zone memory test data as Model I. The models severely overpredicted
the strain at the end of loading in zone 1. In addition, the models had problems in
simulating the stress relaxation and permanent deformation.

10.4 - The Material Variation Phenomena

The poor simulation of the models with respect to the forward strain is curious.
The magnitude of the strain at the end of zone 1 in the experimental four zone memory
test should essentially be equal to the magnitude of the strain in the loading zones of the
experimental creep and stress relaxation tests. However, this is not the case. Material
inconsistencies were found in the attempts to correlate the various parameters of stress
and strain in the four zone tests with each other. As was discussed in Chapter 3, the only
simple relationship proven to be experimentally reproducible was the linear relationship
between the three strain components activated upon unloading, as a function of the
forward loading strain. Attempts to correlate with the stress demonstrate a large scatter in
the results.

The general reproducibility of the experimental data can be seen below in Figure
10.1. This graph shows the material variation in two sets of samples received. As
denoted by the legend, the red squares indicate an “Original” set of LDPE IUDs received
in September 1996. The green diamonds represent “Commercial” samples of the LDPE IUDs received in August 1997. It is believed that most of the experimental scatter is the result of real sample to sample variations in the material properties.

![Stress (1) vs Strain (1) Diagram](image)

**Figure 10.1** - The scatter of data in “Original” and “Commercial” samples.

A. Material Effects on Plastic Strain

In examining the different factors that affect the amount of remnant plastic strain in zone 4, this plastic strain (Strain (C)) has been plotted against the stress in zone 1 (Stress(1)) in figure 10.2 below, and also against the strain in zone 1 (forward strain) in figure 10.3. Figure 10.3 illustrates the linear relationship between the forward strain and the plastic strain, which is independent of loading rates and relaxation times in zone 2. As the Stress(1) and Strain(1) increase, so does the amount of plastic strain, Strain(C). This
is not surprising since higher levels of stress are expected to cause an increase in the permanent molecular slippage. Although the loading rates affected the relationship between Strain(C) and Stress(1) shown in Figure 10.2, they do not affect the behaviour of the permanent residual Strain(C) with respect to the forward Strain(1). Figure 10.3 illustrates the linear relationship between forward strain and plastic strain, that is also independent of loading rates.

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**Figure 10.2** - Plastic strain remaining after creep for different relaxation periods.

**Figure 10.3** - Plastic strain remaining after creep for different relaxation periods.
The result seen above is not easily explained. The stress drops during the stress relaxation because molecules locally move past each other, driven by the applied stress. This is equivalent to plastic strain in the direction of the applied stress. The tensile stress drops because the sample has effectively become longer. Therefore, the greater stress relaxation at a given Strain(1), the greater the permanent Strain(C) should be. The data shown in Figure 10.2 and 10.3 does not seem to support this theory.

B. Material Effects on Creep Recovery

Similar to the permanent strain, the same correlation exercise was executed using the creep recovery strain (the difference between the strain when the sample is first unloaded to zero stress: Strain(Z) and the permanent remnant strain left at the end of zone 4: Strain(C)). As the stress in zone 1, Stress(1), increases, the amount of creep recovery strain also increases. This can be seen in Figure 10.4 below.

![Creep Recovery vs. Load](image)

Figure 10.4 - Creep recovery vs. predetermined load for 5 seconds relaxation period.
The amount of creep recovery, however, is independent of the relaxation period. This is observed below in Figure 10.5. This clearly defined non-dependence of the creep recovery on stress relaxation time needs to be explained.

![Creep Recovery vs. Load](image)

**Figure 10.5** - Creep recovery vs. load at different relaxation times.

The effects of creep recovery are considered with respect to the forward strain. As the amount of forward strain increases, the creep recovery strain also increases. This result is seen in Figure 10.6 below.

![Creep Recovery vs. Forward Strain](image)

**Figure 10.6** - Creep recovery vs. forward strain for varied relaxation periods and 1 mm/min loading rate.
The resultant graph above shows a slight dependence of the amount of time spent in stress relaxation. Note that the strain levels here are much smaller (< 2.5%) than those of the plastic strain (< 18%) and the elastic unloading strain (< 14%). As different loading rates are considered, the same linear relationship results.

10.5 - Material Relationships and Multiple Regression Analysis

The most striking and significant results are the linear relationships between the types of strain: the forward strain, Strain(1), the elastic unloading strain, Strain(EU), the permanent residual strain, Strain(C), and the creep recovery strain, Strain(Z-C). No corresponding simple relationship exists between these three strain components and the stress reached at the end of zone 1. The width of scatter in Figure 10.1 is the variation in the terminal value of strain for loading individual samples to the respective terminal load. This demonstrates the wide material variation and helps to explain the essentially irreproducibility of the peak strains at the end of the loading zones in simple creep and stress relaxation tests, and the four zone memory test.

The correlation of the unloading strains to the other experimental factors appears to be linear, and can be treated statistically. A more rigorous way to analyze the relative importance of the contributions is to treat the problem as a set of linear multivariate equations, and then examine the coefficients and their statistical fit. For the permanent strain ε(C), the equation can be written:
\[ \varepsilon(C) = B_0 + B_1 x_1 + B_2 x_2 + B_3 x_3 \]  

(10.1)

where:  
\( x_1 \) is the loading rate (mean value = 0.623)
\( x_2 \) is the relaxation time (mean value = 98.129)

and  
\( x_3 \) is the forward strain (Strain(1)) (mean value = 9.018).

The coefficients \( B_i \) (i=0,1,2,3) are determined by multiple regression analysis.

Similar equations can be written for the elastic unloading strain, \( \varepsilon(EU) \) and the creep recovery strain, \( \varepsilon(CR) \).

The results are given in Table 10.4 below. The values of \( B_i \) are given in the table. Also given is the probability that these coefficients \( B_i \) are zero and this is registered on a scale of 0 to 1, where a rating of 1 means that the variable \( x_i \) has no effect on the dependent \( \varepsilon(C), \varepsilon(EU) \) or \( \varepsilon(CR) \).

<table>
<thead>
<tr>
<th>Strain Component</th>
<th>( B_0 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_0 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon(C) )</td>
<td>-0.821</td>
<td>-0.234</td>
<td>+4.9e-4</td>
<td>+0.481</td>
<td>1</td>
<td>0.9818</td>
<td>0.1289</td>
<td>0</td>
</tr>
<tr>
<td>( \varepsilon(EU) )</td>
<td>+0.860</td>
<td>+0.261</td>
<td>-9.4e-4</td>
<td>+0.456</td>
<td>0</td>
<td>0.0097</td>
<td>0.9842</td>
<td>0</td>
</tr>
<tr>
<td>( \varepsilon(CR) )</td>
<td>-0.040</td>
<td>-0.027</td>
<td>+4.5e-4</td>
<td>+0.063</td>
<td>0.9786</td>
<td>0.9206</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 10.4 - Results of multiple regression analysis.

The conclusions from the regression analysis are that all three loading strain components are strongly determined by the forward loading strain, Strain(1). In addition, the creep recovery strain is affected by the relaxation time but not at all by the loading.
rate. However, the coefficient B2 multiplied by the mean value (98.129) shows that the relaxation time shifts the ε(C) value by a very small amount; whereas the coefficient B3 times the mean value of the forward strain (9.018) means that changes in the forward strain make a big change in the plastic strain. The plastic strain is mainly affected by the forward strain and secondarily by the relaxation time. The elastic unloading strain is determined by the forward strain as well as the loading rate, but is not affected by the relaxation time. These relationships as functions of the forward strain are well defined. For all other relationships, there is a wide variability in the results, and this has undoubtedly compounded the difficulty of fitting the spring-dashpot models to individual experimental test results.
XI. CONCLUSION

Quantifying the behaviour of polymeric materials is not an easy practice. Polymers are different from metals in that they exhibit time dependency, environmental behavioural changes and molecular inconsistencies. This makes polymer materials difficult to predict in terms of their response to applied stress and strain.

The suitability of four different spring dashpot models to simulate the behaviour of low density polyethylene with barium sulfate filler was examined. The characteristic equations of each model were derived from first principles and then applied using computer algorithms in order to include the history dependent nature of each experimental test. One commonly recommended model, the Burger’s model, was unable to accurately represent the creep at constant stress, and so was abandoned. The other three models were found to simulate both the creep and stress relaxation curves within the linear behaviour range of the material, using a simple set of spring and dashpot coefficients. However, applying these models with the same coefficients to a four zone test which combines the same polymeric phenomena of creep and stress relaxation in a more complicated sequence showed that the models failed to accurately predict the resultant strains from the applied loading conditions, and in some cases, failed to also predict the changes in stress. In Chapter 10, the deficiencies in the three models were identified and discussed with respect to their predictions for the four zone test.
The present study concentrated on a general approach to test the assumption that if coefficients could be identified that would accurately predict behaviour in two different stress-strain schemes, then the models could be applied in any experimental test scheme composed of repetitions of the same general constraints.

The main conclusion of the study is that it is not possible to predict the behaviour of a combination of loading, unloading, creep and stress relaxation sequence by using simple spring-dashpot models that can accurately simulate separate but more simple combinations of these constraints.

Applying the models to a regimen with a more complicated history using coefficients derived to fit more simple loading histories does not work.

This severely limits the applicability of these models for general predictive use, as for example in finite element modelling.
XII. FUTURE WORK

The shortcomings in each of the models provide areas of improvement for polymeric simulation of the four zone test and raises the following question: Would a model with coefficients suitably adjusted to accurately simulate the four zone test, then be capable of predicting the results of the more simple constant stress creep test and the constant strain stress relaxation test?

It has been identified that there could be several combinations or ratios of the values of elasticity and viscosity in each model that could be capable of simulating the tests considered in this study. A more systematic approach to determining and adjusting values of elasticity and viscosity is required in order to develop better fit of the experimental LDPE tested in this study.

The current study concentrates on simulating the behaviour of the linear range of polymer materials. There is still uncertainties as to how to handle the non-linear behaviour of the material.

Other areas of interest that have been identified are the experimental effects of stress and strain on LDPE without the barium sulphate filler, extreme temperature effects, environmental effects and the in utero effects of the IUD material.
XIII. SIGNIFICANT CONTRIBUTIONS TO RESEARCH

In the course of this research, the author of this thesis has done the following unique work:

1. Developed Model II and Model III using springs and dashpots to simulate the simple constant stress creep and constant strain stress relaxation experimental test results.

2. Derived the equations for two parallel Maxwell elements (the Maxwell Weichert model) for both constant stress creep and constant strain stress relaxation.

3. Developed and wrote computer algorithms to calculate the stress-strain-time relationship for constant stress creep and constant strain stress relaxation tests.

4. Developed and wrote computer algorithms to calculate the stress-strain-time relationship for four zone test analysis.

5. Analyzed the applicability of the models developed for constant stress creep and constant strain stress relaxation when applied to the four zone test.
APPENDIX I - FORTRAN ALGORITHMS FOR MODEL I
THIS ALGORITHM SIMULATES PARALLEL MAXWELL ELEMENTS UNDER CONSTANT LOADING RATE AND THEN CONSTANT STRESS CREEP

RRATE is the rate of loading in zone 1
SSTRESSCUT is the stress at which loading in zone 1 is stopped
RRELTIME is the amount of relaxation time in zone 2
DELT is the time step interval (in seconds)
LLo is the original sample length (mm)
EERR is the minimum error constraint
N is the total number of maxwell elements
J is the ID number of the individual maxwell unit
I is the time step counter
E(J) is the elastic modulus of the spring in the Jth maxwell unit
V(J) is the viscosity coefficient of the dashpot in the Jth maxwell unit
SIG(J,I) is the stress of the Jth maxwell element at the time inc I
STN(J,I) is the strain of the Jth maxwell element at the time inc I

Initializing Variables

REAL E(10)
REAL V(10)
REAL RRATE
REAL LLo
REAL DELT
REAL SSTRESSCUT
REAL CCRETIME
REAL EERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE CREEP TIME'
READ*, CCRETIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLo
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, EERR

Input the following parameters:
Number of Maxwell's (N)

PRINT*, 'ENTER THE NUMBER OF MAXWELL ELEMENTS'
READ*, NN

Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: E1 <enter>, n1 <enter>'
ETOT=0
Do J=1,NN
READ*,E(J)
READ*,V(J)
ETOT=ETOT-E(J)
END DO

Write the values of the constants to a file

DO J=1,NN
OPEN(15,FILE='CONSTANTS.DAT')
WRITE(15,*) E(J),V(J)
END DO
CLOSE(15)

Call on subroutines for zone 1 (Loading zone), and zone 2 (stress relaxation)

CALL ZONE1(RRATE,STRESSCUT,DDEL,T,EERR,LO,NN)
CALL ZONE2(CCRETIME,STRESSCUT,DDEL,NN,EERR)

STOP
END

Subroutine for zone 1 - Loading at a constant strain rate

SUBROUTINE ZONE1(RATE,STRESSCUT,DDEL,EERR,LO,N)

Initializing variables

REAL SIG(5,500)
REAL SIGA(5,500)
REAL STN(5,500)
REAL STNA(5,500)
REAL SIGTOT(500)
REAL E(5)
REAL V(5)
REAL VIST(5,500)
REAL DEVSTN(5)
REAL RATE
REAL LO
REAL A
REAL DELT
REAL STRESSCUT

Retrieve the values of the constants for E and V

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J),V(J)
END DO
CLOSE(15)
Calculate the total elasticity

\[ \text{ETOT} = 0.0 \]
\[ \text{DO J}=1, N \]
\[ \text{ETOT} = \text{ETOT} + E(J) \]
\[ \text{END DO} \]

Set all initial values equal to zero

\[ I=1.0 \]
\[ L=1.0 \]
\[ \text{DO J}=1, N \]
\[ \text{STN}(J,I)=0.0 \]
\[ \text{SIG}(J,I)=0.0 \]
\[ \text{VIST}(J,I)=0.0 \]
\[ \text{END DO} \]

\[ \text{SIGTOT}(I)=0.0 \]
\[ \text{DO J}=1, N \]
\[ \text{SIGTOT}(I) = \text{SIGTOT}(I) + \text{SIG}(J,I) \]
\[ \text{END DO} \]

Start of the main loop calculating stress and strain until the total stress is greater than the cut off stress

\[ \text{WHILE (SIGTOT}(I) \geq \text{STRESSCUT)} \]
\[ I=I+1 \]

Calculate the strain at each time step specified by the strain rate

\[ \text{DO J}=1, N \]
\[ \text{STN}(J,I)=((\text{RATE}*(I*\text{DELT}-\text{DELT}))/\text{LO}) \]
\[ \text{SIG}(J,I)=\text{STN}(J,I)*E(J) \]
\[ \text{END DO} \]

Calculate the strain in the dashpots

\[ \text{DO J}=1, N \]
\[ \text{VIST}(J,I)=((\text{SIG}(J,I)+\text{SIG}(J,L))/2*\text{DELT}^2/\text{V}(J)+\text{VIST}(J,L)) \]
\[ \text{STNA}(J,I)=\text{SIG}(J,I)*E(J)+\text{VIST}(J,I) \]
\[ \text{END DO} \]

Correction of stress using the average strain and the individual strains

\[ \text{DO J}=1, N \]
\[ \text{SIGA}(J,I)=\text{SIG}(J,I)*\text{STN}(J,I)/\text{STNA}(J,I) \]
\[ \text{END DO} \]

Calculation of the total stress

\[ \text{SIGTOT}(I)=0.0 \]
\[ \text{DO J}=1, N \]
SIGTOT(I)=SIGTOT(I)-SIGA(J,I)
END DO

Equating the stress to its new corrected value

DO J=1,N
SIG(J,I)=SIGA(J,I)
END DO

Check to make sure that the deviation of strain is not greater than
the error specified in the input.

DO J=1,N
DEVSTN(J)=ABS(STN(J,I)-STNA(J,I))
A=0.0
IF(DEVSTN(J).GE.A) THEN
A=DEVSTN(J)
END IF
END DO
IF (A.GE.ERR) THEN
GO TO 5
ELSE

Write the results at each time increment to a file

OPEN(11,FILE=’CREEP.DAT’)
WRITE(11,*1) I,SIGTOT(I),STN(1,I)
PRINT*,I,SIGTOT(I),STN(1,I)
END IF

Setting the new stress equal to the stress at the previous time step
for a starting point to be iterated and recording the previous strain
in the dashpot to be added to the new strain value.

DO J=1,N
L=1
SIG(J,I)=SIG(J,L)
VIST(J,I)=VIST(J,L)
END DO

END WHILE

Write final results for this zone to a file

DO J=1,N
OPEN(16,FILE=’ZONE1END.DAT’)
WRITE(16,*1) I,SIG(J,I),VIST(J,I),STN(J,I)
END DO
CLOSE(16)
RETURN
END

***********************************************************************
Subroutine for Zone 2 - Creep at constant stress

SUBROUTINE ZONE2(CRETIME, STRESSCUT, DELT, N, ERR)

REAL SIG(10,1500)
REAL SIGA(10,1500)
REAL SIGAA(10,1500)
REAL STN(10,1500)
REAL E(10)
REAL V(10)
REAL VIST(10,1500)
REAL DEVSTN(10)
REAL TOTSTN
REAL A
REAL SIGZ1(10)
REAL STNVIST(10)
REAL STRAIN(10)

Retrieve values of constants for E and V

DO J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

Retrieve the final values of Zone 1 to be used as starting values for zone 2

DO J=1,N
OPEN(16,FILE="ZONE1END.DAT")
READ(16,*) TOT, SIGZ1(J), STNVIST(J), STRAIN(J)
END DO
CLOSE(16)

STEP=CRETIME/DELT
DO I=1,STEP
IF(IEQ.1)THEN

DO J=1,N
SIG(J,I)=SIGZ1(J)
VIST(J,I)=STNVIST(J)
STN(J,I)=STRAIN(J)
PRINT*, I, SIG(J,I), STN(J,I)
END DO
K=I+1
ELSE

DO J=1,N
VIST(J,K)=(SIG(J,K)+SIG(J,L))*2*DELT/V(J)+VIST(J,L)
STN(J,K)=SIG(J,K)/E(J)+VIST(J,K)
END DO
TOTSTN=0
SIGTOT=0
DO J=1,N
  TOTSTN=TOTSTN+STN(J,K)
END DO
STNAVE=TOTSTN/N

DO J=1,N
  SIGA(J,K)=SIG(J,K)*STNAVE/STN(J,K)
END DO

DO J=1,N
  SIGTOT=SIGTOT+SIGA(J,K)
END DO

DO J=1,N
  SIGAA(J,K)=SIGA(J,K)*STRESSCUT/SIGTOT
END DO

DO J=1,N
  SIG(J,K)=SIGAA(J,K)
END DO

DO J=1,N
  DEVSTN(J)=ABS(STNAVE-STN(J,K))
  A=0.0
  IF(DEVSTN(J).GE.A) THEN
    A=DEVSTN(J)
  END IF
END DO
IF (A.GE.ERR) THEN
  GO TO 5
ELSE
  ITOT=ITOT+1
  OPEN(11,FILE='CREEP.DAT')
  WRITE(11,* ITOT,SIGTOT,STNAVE)
  PRINT*,ITOT,SIGTOT,STNAVE
  K=K+1
  END IF
END IF

DO J=1,N
  I=I+1
  SIG(J,K)=SIG(J,L)
  VIST(J,K)=VIST(J,L)
END DO
END DO

CLOSE(11)
RETURN
THIS ALGORITHM SIMULATES PARALLEL MAXWELL ELEMENTS UNDER CONSTANT LOAD RATE AND THEN STRESS RELAXATION

- RRATE is the rate of loading in zone 1
- SSTRESSCUT is the stress at which loading in zone 1 is stopped
- RRELTIME is the amount of relaxation time in zone 2
- DDELT is the time step interval (in seconds)
- LLo is the original sample length (mm)
- ERR is the minimum error constraint
- N is the total number of maxwell elements
- J is the ID number of the individual maxwell unit
- I is the time step counter
- E(J) is the elastic modulus of the spring in the Jth maxwell unit
- V(J) is the viscosity coefficient of the dashpot in the Jth maxwell unit
- SIG(J,J) is the stress of the Jth maxwell element at the time inc I
- STN(J,J) is the strain of the Jth maxwell element at the time inc I

Initializing Variables

REAL E(5)
REAL V(5)
REAL RRATE
REAL LLo
REAL DDELT
REAL SSTRESSCUT
REAL RRELTIME
REAL ERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE RELAXATION TIME'
READ*, RRELTIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DDELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLo
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, ERR

Input the following parameters:
- Number of Maxwells(N)

PRINT*, 'ENTER THE NUMBER OF MAXWELL ELEMENTS'
READ*, NN

Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: E1 <enter>, n1 <enter>'
ETOT=0
Do J=1,NN
READ*(E(J)
READ*(V(J)
ETOT=ETOT+E(J)
END DO

Write the values of the constants to a file

DO J=1,NN
OPEN(15,FILE="CONSTANTS.DAT")
WRITE(15,*) E(J), V(J)
END DO
CLOSE(15)

Call on subroutines for zone 1 (Loading zone), and zone 2 (stress relaxation)

CALL ZONE1(RRATE, STRESSCUT, DDELTA, EERR, LLO, NN)
CALL ZONE2(RREVTIME, DDELTA, NN)

STOP
END

Subroutine for zone 1 - Loading at a constant strain rate

SUBROUTINE ZONE1(RATE, STRESSCUT, DELT, ERR, LO, N)

Initializing variables

REAL SIG(5,200)
REAL SIGA(5,200)
REAL STN(5,200)
REAL STNA(5,200)
REAL SIGTOT(200)
REAL E(5)
REAL V(5)
REAL VIST(5,200)
REAL DEYSTN(5)
REAL RATE
REAL LO
REAL A
REAL DELT
REAL STRESSCUT

Retrieve the values of the constants for E and V

Do J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J), V(J)
END DO
CLOSE(15)
Calculate the total elasticity

ETOT=0.0
DO J=1,N
  ETOT=ETOT+E(J)
END DO

Set all initial values equal to zero

I=1.0
L=1.0
DO J=1,N
  STN(J,I)=0.0
  SIG(J,I)=0.0
  VIST(J,I)=0.0
END DO

SIGTOT(I)=0.0
DO J=1,N
  SIGTOT(I)=SIGTOT(I)+SIG(J,I)
END DO

Start of the main loop calculating stress and strain until the
total stress is greater than the cut off stress

WHILE (SIGTOT(I).LE.STRESSCUT)
  I=I+1

Calculate the strain at each time step specified by the strain rate
Assume all strain is initially in the spring

DO J=1,N
  STN(J,I)=((RATE*((I*DELT)-DELT))/LO)
  SIG(J,I)=STN(J,I)*E(J)
END DO

Calculate the strain in the dashpots

DO J=1,N
  VIST(J,I)=(SIG(J,I)+SIG(J,L)/2*DELT/\V(J)+VIST(J,L)
  STNA(J,I)=SIG(J,I)/E(J)+VIST(J,I)
END DO

Correction of stress using the average strain and the individual
strains

DO J=1,N
  SIGA(J,I)=SIG(J,I)*STN(J,I)/STNA(J,I)
END DO

Calculation of the total stress

SIGTOT(I)=0.0
DO J=1,N
SIGHTOT(I)=SIGHTOT(I)+SIGA(I,J)
END DO

c Equating the stress to its new corrected value
DO J=1,N
SIG(J,I)=SIGA(J,I)
END DO

c Check to make sure that the deviation of strain is not greater than
the error specified in the input.
DO J=1,N
DEVSTN(J)=ABS(STN(J,I)-STNA(J,I))
A=0.0
IF(DEVSTN(J).GE.A) THEN
A=DEVSTN(J)
END IF
END DO
IF (A.GE.ERR) THEN
GO TO 5
ELSE

Write the results at each time increment to a file
OPEN(11,FILE='DATA.DAT')
WRITE(11,'(12)') L,STN(J,I),SIGHTOT(I)
PRINT*,1,STN(1,I),SIGHTOT(1)
END IF

c Setting the new stress equal to the stress at the previous time step
for a starting point to be iterated and recording the previous strain
in the dashpot to be added to the new strain value.
DO J=1,N
L=I
SIG(J,J)=SIG(J,L)
VIST(J,J)=VIST(J,L)
END DO

END WHILE

Write final results for this zone to a file
DO J=1,N
OPEN(16,FILE='ZONE1_END.DAT')
WRITE(16,'(12)') L,SIG(J,J),VIST(J,J),STN(J,I)
END DO
CLOSE(16)
RETURN
END

c ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
Subroutine for Zone 2 - Stress relaxation at constant strain

Initializing Variables

REAL SIG(5,2500)
REAL SIGTOT(2500)
REAL STN(5,2500)
REAL SPRING(2500)
REAL E(5)
REAL V(5)
REAL VIST(5,2500)
REAL ELAS(5,2500)
REAL STRAIN(5)
REAL STNVIST(5)
REAL SIGZ1(5)
REAL DELT

Retrieve values of constants for E and V

Do J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J),V(J)
END DO
CLOSE(15)

Retrieve the final values of Zone 1 to be used as starting values for zone 2

DO J=1,N
OPEN(16,FILE="ZONE1END.DAT")
READ(16,*) ITOT,SIGZ1(J),STNVIST(J),STRAIN(J)
END DO
CLOSE(16)

Start of main loop to calculate stress and strain for the required relaxation time

STEP=RELT/DEL
DO I=1,STEP

DO J=1,N
SIG(J,J)=SIGZ1(J)*EXP(-E(J)*(I*DEL-DEL)/V(J))
END DO

DO J=1,N
ELAS(J,J)=SIG(J,J)*E(J)
VIST(J,J)=STRAIN(J)-ELAS(J,J)
END DO

DO J=1,N
STN(J,J)=ELAS(J,J)+VIST(J,J)
END DO

SIGTOT(I)=0.0
DO J=1,N
SIGTOT(I)=SIGTOT(I)+SIG(J,I)
END DO

ITOT=ITOT+1

c Writing results at each time increment to a file

OPEN(11,FILE="DATA.DAT")
WRITE(11,*) ITOT,STN(I,I),SIGTOT(I)
PRINT*,ITOT,STN(I,I),SIGTOT(I)
END DO

c Writing final results of zone 2 to a file

DO J=1,N
OPEN(14,FILE="ZONE2END.DAT")
WRITE(14,*) ITOT,VI111(I,I),STN(J,J),SIGTOT(I)
END DO
CLOSE(14)
CLOSE(11)

RETURN
END
THIS ALGORITHM SIMULATES PARALLEL MAXWELL ELEMENTS UNDER THE STRESS STRAIN TIME CONSTRAINTS OF A FOUR ZONE MEMORY TEST

RRATE is the rate of loading in zone 1
SSTRESSCUT is the stress at which loading in zone 1 is stopped
RELTIME is the amount of relaxation time in zone 2
UUNRATE is the rate of unloading in zone 3
CCRETIME is the amount of time for viscoelastic creeping to occur
DELT is the time step interval (in seconds)
LLO is the original sample length (mm)
EERR is the minimum error constraint
N is the total number of maxwell elements
J is the ID number of the individual maxwell unit
I is the time step counter
E(J) is the elastic modulus of the spring in the Jth maxwell unit
V(J) is the viscosity coefficient of the dashpot in the Jth maxwell unit
SIG(J,I) is the stress of the Jth maxwell element at the time inc I
STN(J,I) is the strain of the Jth maxwell element at the time inc I

Initializing Variables

REAL E(10)
REAL V(10)
REAL RRATE
REAL LLO
REAL DELT
REAL SSTRESSCUT
REAL RELTIME
REAL UUNRATE
REAL CCRETIME
REAL EERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE RELAXATION TIME'
READ*, RELTIME
PRINT*, 'ENTER THE UNLOADING RATE'
READ*, UUNRATE
PRINT*, 'ENTER THE CREEPING TIME'
READ*, CCRETIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLO
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, EERR

Input the following parameters:
Number of Maxwells(N)
PRINT*, 'ENTER THE NUMBER OF MAXWELL ELEMENTS'
READ*, NN

Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER E1 <enter>, n1 <enter>'
ETOT=0
Do J=1, NN
READ*, E(J)
READ*, V(J)
ETOT=ETOT+E(J)
End Do

Write the values of the constants to a file

Do J=1, NN
OPEN(15, FILE='CONSTANTS.DAT')
WRITE(15,*) E(J), V(J)
End Do
CLOSE(15)

Call on subroutines for zones 1 - 4

CALL ZONE1(RRATE, SSTRESSCUT, DDELTA, EERR, LLO, NN)
CALL ZONE2(RRELTIME, DDELTA, NN)
CALL ZONE3(UUNRATE, DDELTA, LLO, EERR, NN)
CALL ZONE4(DDELTA, EERR, CCRETIME, NN)

STOP
END

*********************************************************************************

Subroutine for zone 1 - Loading at a constant strain rate
*********************************************************************************

SUBROUTINE ZONE1(RRATE, STRESSCUT, DELT, ERR, LO, N)

Initializing variables

REAL SIG(5, 100)
REAL SIGA(5, 100)
REAL STN(5, 100)
REAL STNA(5, 100)
REAL E(5)
REAL V(5)
REAL VIS(5, 100)
REAL DEVS(5)
REAL RATE
REAL LO
REAL A
REAL DELT
REAL STRESSCUT
c Retrieve the values of the constants for E and V

    Do J=1,N
    OPEN(15,FILE="CONSTANTS.DAT")
    READ(15,"(*)") E(J), V(J)
    END DO
    CLOSE(15)

    ETOT=0.0
    DO J=1,N
    ETOT=ETOT+E(J)
    END DO

    I=1.0
    L=1.0
    DO J=1,N
    STN(J,I)=0.0
    SIG(J,I)=0.0
    VIST(J,I)=0.0
    END DO

    SIGTOT=0.0
    DO J=1,N
    SIGTOT=SIGTOT+SIG(J,I)
    END DO

c Start of the main loop calculating stress and strain until the
c total stress is greater than the cut off stress

    WHILE (SIGTOT.LE.STRESSCUT)
    I=I+1
    DO J=1,N
    STN(J,I)=((RATE*((I*DELT)-DELT))/LO)
    SIG(J,I)=STN(J,I)*E(J)
    END DO

    DO J=1,N
    VIST(J,I)=(SIG(J,I)+SIG(J,I))/2*DELT/V(J)+VIST(J,I)
    STNA(J,I)=SIG(J,I)/E(J)+VIST(J,I)
    END DO

c Correction of stress using the average strain and the individual
c strains

    DO J=1,N
    SIGA(J,I)=SIG(J,I)*STN(J,I)/STNA(J,I)
    END DO

    c Calculation of the total stress
    SIGTOT=0.0
    DO J=1,N
    SIGTOT=SIGTOT+SIGA(J,I)
    END DO
c Equating the stress to its new corrected value

    DO  J=1,N
       SIG(J,J)=SIGA(J,J)
    END DO

c Check to make sure that the deviation of strain is not greater than
the error specified in the input.

    DO  J=1,N
       DEVSTN(J)=ABS(STN(J,J)-STNA(J,J))
       A=0.0
       IF(DEVSTN(J).GE.A) THEN
          A=DEVSTN(J)
       END IF
    END DO
    IF (A.GE.ERR) THEN
       GO TO 5
    ELSE

    Write the results at each time increment to a file

        OPEN(11,FILE="DATA.DAT")
        WRITE(11,*) I,STN(J,J),SIGTOT
        END IF

c Setting the new stress equal to the stress at the previous time step
for a starting point to be iterated and recording the previous strain
in the dashpot to be added to the new strain value.

    DO  J=1,N
        L=I
        SIG(J,J)=SIG(J,L)
        VIST(J,J)=VIST(J,L)
    END DO

    END WHILE

c Write final results for this zone to a file

    DO  J=1,N
        OPEN(16,FILE="ZONE\END.DAT")
        WRITE(16,*) I,VIST(J,J),STN(J,J)
    END DO
    CLOSE(16)

    RETURN
    END

Subroutine for Zone 2 - Stress relaxation at constant strain
Subroutine for Zone 2 - Stress relaxation at constant strain
SUBROUTINE ZONE2(RELTIME,DELT,N)

c Initializing Variables

REAL SIG(5,100)
REAL STN(5,100)
REAL SPRING(100)
REAL STNVIST(100)
REAL E(5)
REAL V(5)
REAL VIST(5,100)
REAL ELAS(5,1000)
REAL STRAIN(100)
REAL DELT

c Retrieve values of constants for E and V

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J),V(J)
END DO
CLOSE(15)

c Retrieve the final values of Zone 1 to be used as starting values

c for zone 2

DO J=1,N
OPEN(16,FILE='ZONE1END.DAT')
READ(16,*) ITOT,STNVIST(J),STRAIN(J)
END DO
CLOSE(16)

DO J=1,N
SPRING(J)=STRAIN(J)-STNVIST(J)
END DO

c Start of main loop to calculate stress and strain for the required

c relaxation time

STEP=RELTIME/DELT
DO I=1,STEP

IF (1.EQ.1) THEN

DO J=1,N
VIST(J,I)=STNVIST(J)
ELAS(J,I)=STRAIN(J)-VIST(J,I)
SIG(J,I)=SPRING(J)*E(J)*EXP(-E(J)*(I*DELT-DELT)/V(J))
STN(J,I)=VIST(J,I)+ELAS(J,I)
END DO

SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,I)
END
END DO
ELSE

DO J=1,N
VIST(J,I)=(1/V(J))*(SPRING(J)*E(J)*EXP(-E(J)*
$S (1*DELT-DELT)/V(J))*(V(J)/E(J))-SPRING(J)*E(J)*EXP
$S -(E(J)*(1-1)*DELT-DELT)/V(J))*(V(J)/E(J))+VIST(J,L)
PRINT *, 'VIST(J,I), VIST(J,I)
SIG(J,I)=SPRING(J)*E(J)*EXP(-E(J)*(1*DELT-DELT)/V(J))
ELAS(J,I)=SIG(J,I)/E(J)
STN(J,I)=VIST(J,I)+ELAS(J,I)
PRINT *, 'STRAIN, STN(J,I)
END DO

SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,I)
END DO
END IF

ITOT=ITOT+1

Writing results at each time increment to a file

OPEN(11,FILE="DATA.DAT")
WRITE(11,*) ITOT, STN(J,I), SIGTOT

DO J=1,N
L=1
ELAS(J,I)=ELAS(J,L)
VIST(J,I)=VIST(J,L)
END DO
END DO

Writing final results of zone 2 to a file

DO J=1,N
OPEN(14,FILE="ZONE2END.DAT")
WRITE(14,*) ITOT, VIST(J,I), STN(J,I)
END DO
CLOSE(14)
RETURN
END

Subroutine for Zone 3 - Unloading at a constant strain rate

************************************************************************************
SUBROUTINE ZONE3(UNRATE, DELT, LO, ERR, N)

Initializing variables
Double precision is used as a result of very small deviations of
stress and strain in the iterations of each time step

DOUBLE PRECISION SIG(5,100)
DOUBLE PRECISION DSIG(5,100)
DOUBLE PRECISION SIGA(5,100)
DOUBLE PRECISION DSIGA(5,100)
DOUBLE PRECISION STN(5,100)
DOUBLE PRECISION DSTN(5,100)
DOUBLE PRECISION DSTNA(5,100)
DOUBLE PRECISION STNA(5,100)
DOUBLE PRECISION DSPR(5,100)
DOUBLE PRECISION DEVSTN(15,100)
REAL E(5)
REAL V(5)
REAL DASH(5)
REAL STRAIN(5)
DOUBLE PRECISION VIST(5,100)
DOUBLE PRECISION DVIST(5,100)
DOUBLE PRECISION ELAS(5,100)
DOUBLE PRECISION DELAS(5,100)
REAL Lo
REAL A
REAL DELT

Retrieving values of constants for E and V

DO J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J),V(J)
END DO
CLOSE(15)

Retrieving the final values of zone 2 to become initial values of zone 3

DO J=1,N
OPEN(14,FILE="ZONE2END.DAT")
READ(14,*) ITOT,DASH(J),STRAIN(J)
END DO
CLOSE(14)

I=1.0
L=1.0
DO J=1,N
STN(J,I)=STRAIN(J)
ELAS(J,I)=STRAIN(J)-DASH(J)
VIST(J,I)=DASH(J)
SIG(J,I)=(STRAIN(J)-DASH(J))*E(J)
DVIST(J,I)=0
DELAS(J,I)=0
END DO
SIGTOT=0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,I)
END DO

Start of main loop that calculates stress and strain until the total stress is equal to zero

WHILE (SIGTOT.GE.0)

I=I+1
DO J=1,N
DSTN(J,I)=-UNRATE*(DELT)/LO
STN(J,I)=STN(J,I)+DSTN(J,I)
END DO

DO J=1,N
DELAS(J,I)=-UNRATE*(DELT)/LO
ELAS(J,I)=ELAS(J,I)+DELAS(J,I)
DSIG(J,I)=DELAS(J,I)*E(J)
SIG(J,I)=SIG(J,I)+DSIG(J,I)
END DO

5 DO J=1,N
DVIST(J,I)=((SIG(J,I)+SIG(J,I))/2)*DELT/V(J)
VIST(J,I)=VIST(J,I)+DVIST(J,I)
DSPR(J,I)=(SIG(J,I)-SIG(J,I))/E(J)
DSIG(J,I)=DSPR(J,I)*E(J)
ELAS(J,I)=ELAS(J,I)+DSPR(J,I)
END DO

DO J=1,N
DSTNA(J,I)=DSPR(J,I)+DVIST(J,I)
STNA(J,I)=VIST(J,I)+ELAS(J,I)
END DO

Correcting the deviation of stress

DO J=1,N
DSIGA(J,I)=DSIG(J,I)*DSTN(J,I)/DSTNA(J,I)
END DO

DO J=1,N
SIGA(J,I)=SIG(J,I)+DSIGA(J,I)
END DO

Equating stress to its new corrected value

DO J=1,N
SIG(J,I)=SIGA(J,I)
END DO

SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,I)
END DO
Check to make sure that the deviation of strain is not greater than the error specified in the input.

\[ A = 0.0 \]

\[
\text{DO } J = 1, N \\
\text{DEVSTN}(J, I) = \text{ABS}(\text{STNA}(J, I) - \text{STN}(J, I)) \\
\text{IF}(\text{DEVSTN}(J, I), \text{GE}, A) \text{ THEN} \\
A = \text{ABS}(\text{DEVSTN}(J, I)) \\
\text{END IF} \\
\text{END DO} \\
\]

\[
\text{IF } (\text{ABS}(A), \text{GE}, \text{ERR}) \text{ THEN} \\
\text{GO TO 5} \\
\text{ELSE} \\
\]

Write results at each time step to a file

\[
\text{ITOT} = \text{ITOT} + 1 \\
\text{OPEN}(11, \text{FILE} = \text{"DATA.DAT"}) \\
\text{WRITE}(11, \ast) \text{ ITOT, STN}(I, I), \text{SIGTOT} \\
\text{END IF} \\
\]

Setting the new stress equal to the stress at the previous time step for a starting point to be iterated and recording the previous strain in the dashpot to be added to the new strain value.

\[
\text{DO } J = 1, N \\
\text{L} = I \\
\text{SIG}(J, I) = \text{SIG}(J, L) \\
\text{STN}(J, I) = \text{STN}(J, L) \\
\text{VIST}(J, I) = \text{VIST}(J, L) \\
\text{ELAS}(J, I) = \text{ELAS}(J, L) \\
\text{END DO} \\
\]

END WHILE

Write final results to a file

\[
\text{DO } J = 1, N \\
\text{OPEN}(17, \text{FILE} = \text{"ZONE3END.DAT"}) \\
\text{WRITE}(17, \ast) \text{ ITOT, VIST}(J, L), \text{STN}(J, L) \\
\text{END DO} \\
\text{CLOSE}(17) \\
\text{RETURN} \\
\text{END} \\
\]

Subroutine for zone 4 - Creep at zero load for specified time

Initializing variables
REAL SIG(5,1000)  
REAL SIGA(5,1000)  
REAL SIGAA(5,1000)  
REAL STN(5,1000)  
REAL E(5)  
REAL V(5)  
REAL STRAIN(5)  
REAL STNDA(5)  
REAL VIST(5,1000)  
REAL ELAS(5,1000)  
REAL DEVSTN(5)  
REAL DEVSTRESS(5)  
REAL TOTSTN  
REAL A  
REAL ETOT  
REAL CREATIME  

C Retrieving values of constants for E and V  
DO J=1,N  
OPEN(15,FILE="CONSTANTS.DAT")  
READ(15,*) E(J),V(J)  
END DO  
CLOSE(15)  
  
ETOT=0.0  
DO J=1,N  
ETOT=ETOT+E(J)  
END DO  

C Retrieving final values of zone 3  
DO J=1,N  
OPEN(17,FILE="ZONE3END.DAT")  
READ(17,*) ITOT,STNDA(J),STRAIN(J)  
END DO  
CLOSE(17)  

C Start of main loop to calculate stress and strain for specified  
C creeping time  
STEP=CRETIME/DELT  
DO I=1,STEP  
  
IF(I.EQ.1)THEN  
  
DO J=1,N  
VIST(J,I)=STNDA(J)  
STN(J,I)=STRAIN(J)  
ELAS(J,I)=STN(J,I)-VIST(J,I)  
SIG(J,I)=ELAS(J,I)*E(J)  
END DO  
K=I+1  
ITOT=ITOT+1  
ELSE  
  
DO J=1,N  
VIST(J,I)=STNDA(J)  
STN(J,I)=E(J)*VIST(J,I)  
SIG(J,I)=E(J)*STN(J,I)  
END DO  
K=I  
ITOT=ITOT+1  
ENDIF  
END DO  

C Calculating creep stress and strain for specified  
C creep time  

ELSE

5 DO J=1,N
  VIST(J,1)=(SIG(J,K)+SIG(J,L))/2*DELT/V(J)+VIST(J,1)
  STN(J,1)=SIG(J,K)*E(J)+VIST(J,1)
END DO

TOTSTN=0
SIGTOT=0
DO J=1,N
  TOTSTN=TOTSTN+STN(J,1)
END DO
STNAVE=TOTSTN/N

! Correction of stress using the average strain and the individual strains!

DO J=1,N
  SIGA(J,K)=(STNAVE-STN(J,1))*E(J)+SIG(J,K)
END DO

! Calculation of the total stress!

DO J=1,N
  SIGTOT=SIGTOT+SIGA(J,K)
END DO

DO J=1,N
  DEVSTRESS(J)=SIG(J,L)-SIGA(J,K)
END DO

IF(SIGTOT.NE.0.0)THEN
  DO J=1,N
    SIGAA(J,K)=SIGA(J,K)-SIGTOT*E(J)/ETOT
  END DO
ELSE
  DO J=1,N
    SIGAA(J,K)=SIGA(J,K)
  END DO
END IF

! Equating the stress to its new corrected value!

DO J=1,N
  SIG(J,K)=SIGAA(J,K)
END DO

! Check to make sure that the deviation of strain is not greater than the error specified in the input.

A=0.0
DO J=1,N
  DEVSTN(J)=ABS(STNAVE-STN(J,1))
IF(DEVSTN(J).GE.A) THEN
  A=DEVSTN(J)
END IF
END DO
IF (A.GE.ERR) THEN
  GO TO 5
ELSE
  ITOT=ITOT+1
END IF

Writing results at each time increment to a file

OPEN(11,FILE="DATA.DAT")
WRITE(11,*) ITOT,STN(J,I),SIGTOT
K=I+1
END IF
END IF

Setting the new stress equal to the stress at the previous time step
for a starting point to be iterated and recording the previous strain
in the dashpot to be added to the new strain value.

DO J=1,N
  L=I
  SIG(J,L)=SIG(J,I)
  SIG(J,K)=SIG(J,I)
  VIST(J,K)=VIST(J,L)
END DO
END DO

DO J=1,N
  OPEN(19,FILE="ZONE4END.DAT")
  WRITE(19,*) ITOT,STN(J,I),SIGTOT
END DO
CLOSE (19)
CLOSE(11)
RETURN
END
PROGRAM START

Enter testing parameters

Enter values spring and dashpot coefficients

Zone 1

Retrieve values of E, a

Is σtot less than σcutoff?

Yes

Stop, End

No

Go to Zone 2

Zone 2

Retrieve values of E, a

Retrieve values of stress and strain from last timestep of zone 1

σspring = σconst - 6dash

Is it the first timestep?

Yes

Write results to file; tstep = 1

For each Maxwell:

σtrue = crate * time / Lo

σ = σtrue * E

εtrue = (σtrue - σ) / E

σtrue = σtrue / εtrue

σtot = Σ σ

Go to t = t+1

No

Does ε = true?

Yes

Write results to file; tstep = 1

For each Maxwell:

εtrue = (1/η) * (σspring * E * exp(-E * time / η) * σ/E

σdash = σspring * E * exp(-E * time / η)

σtrue = σtrue / E

ε = εtrue + σtrue

σtot = Σ σ

Write Results at timestep to a file

Go to t = t+1

Is t greater than relaxation time?

Yes

Go To Zone 3
Zone 3
Retrieve values of \( E, n \)

Retrieve values of stress and strain from last timestep of zone 2

Is strain less than 0?
Yes
Stop, End
No
Go to Zone 4

Zone 4
Retrieve values of \( E, n \)

Retrieve values of stress and strain from last timestep of zone 3

Is it the first timestep?
Yes
Write results to file, tstep=1
No
Set values at timestep to initial values at next timestep

\[ \Delta \varepsilon = \varepsilon(t) - \varepsilon(t-1) \]
\[ \Delta \sigma = \sigma(t) - \sigma(t-1) \]
\[ \sigma(t+1) = \sigma(t) + \Delta \sigma \]

Set values at timestep to initial values at next timestep

Go to \( t+1 \)

For each Maxwell:
set stress and strain at timestep 1 equal to the values from the last timestep of zone 2

\[ \Delta \varepsilon = \varepsilon(t) + \Delta \varepsilon(t) \]
\[ \sigma = \sigma(t) + \Delta \sigma(t) \]

Set values at timestep to initial values at next timestep

Go to \( t+1 \)
APPENDIX II - FORTRAN ALGORITHMS FOR MODEL II
This algorithm simulates creep at constant stress of Model II

*** Note that the program uses STEP WISE calculations and therefore must use a very small timestep interval ***

RRATE is the rate of loading in zone 1
SSTRESSCUT is the stress at which the loading is stopped
RRELTIME is the amount of relaxation time in zone 2
DDEL is the time step interval
LLo is the original sample length (mm) used to calculate strain
EERR is the minimum error constraint
E(I) is the elastic modulus of the Ith spring
V(I) is the viscosity coefficient of the Ith dashpot
SIG(1,I) is the stress in the spring at time interval 1
SIG(2,I) is the stress in the modified Voigt-Kelvin element at time interval 1
STN(1,I) is the strain in the spring at time interval 1
STN(2,I) is the strain in the modified Voigt-Kelvin element at time interval 1

Initializing Variables

REAL E(3)
REAL V(3)
REAL RRATE
REAL LLo
REAL DDEL
REAL SSTRESSCUT
REAL RRELTIME
REAL EERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE AMOUNT OF CREEP TIME'
READ*, CCRETIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DDEL
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLo
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, EERR

NN=2

Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: El <enter>, n1 <enter>'
ETOT=0
Do J=1,NN
READ*,E(J)
READ*,V(J)
END DO

c Write the values of the constants to a file

DO J=1,NN
OPEN(15,FILE="CONSTANTS.DAT")
WRITE(15,*) E(J),V(J)
END DO
CLOSE(15)

c Call on subroutines for zones 1 and 2

CALL ZONE1(RRATE,SSTRESSCUT,DDELT,ERR,LO,NN)
CALL ZONE2(CCRETIME,DDELT,NN)

STOP
END

c Subroutine for zone 1 - Loading at a constant strain rate

SUBROUTINE ZONE1(RATE,SSTRESSCUT,DDELT,ERR,LO,N)

c Initializing Variables

REAL E(3)
REAL V(3)
REAL SIG(3,100)
REAL SIGS1(3,100)
REAL SIGS2(3,100)
REAL STN(3,100)
REAL STNT(100)
REAL STNA(3,100)
REAL STNTOT(100)
REAL STNDF(100)
REAL RATE
REAL DEVSTN
REAL LO
REAL DELT
REAL ERR

c Retrieve the values of the constants for E and V

Do J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J),V(J)
END DO
CLOSE(15)

L=1.0
I=1.0
DO J=1,N
STN(J,1)=0.0
SIG(J,1)=0.0
END DO
SIGS1(2,1)=0.0
SIGS2(2,1)=0.0
SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,1)
END DO

C START OF THE MAIN LOOP

Loop until the cutoff stress is reached

WHILE (SIG(1,1).LE.STRESSCUT)
  I=I+1

STNTOT(I)=RATE*((I*DELT)-DELT)/LO
STNDIF(I)=RATE*DELT/LO

C ASSUME ALL STRAIN IS IN THE SPRING (ELEMENT 1)
STN(1,1)=STN(1,1)+STNDIF(I)
SIG(1,1)=SIG(1,1)*E(1)

C STRESS ON ELEMENT 1 = STRESS ON ELEMENT 2

SIG(2,1)=SIG(1,1)
SIGS2(2,1)=(SIG(2,1)*V(2)/V(1))-SIG(2,1)*
  (V(2)/V(1))*EXP(-E(2)*(I*DELT-DELT)/V(2))
SIGS1(2,1)=SIG(2,1)-SIGS2(2,1)
STN(2,1)=STN(2,1)+(SIGS1(2,1)+SIGS2(2,1))*DELT/V(1)

STNT(I)=STN(1,1)+STN(2,1)
STNA(1,1)=STNTOT(I)-STN(2,1)
STN(1,1)=STNA(1,1)

DEVSTN=ABS(STNTOT(I)-STNT(I))
IF(DEVSTN.GE.ERR) THEN
  GOTO 5
ELSE
  OPEN(11,FILE="MO2DATA.DAT")
  WRITE(11,*)(I,SIG(I,1),STNTOT(I))
END IF
L=I
DO J=1,N
STN(J,1)=STN(J,L)
END DO
SIGS1(2,1)=SIGS1(2,L)
SIGS2(2,i) = SIGS2(2,l)

END WHILE

OPEN(16,FILE='M2SREND.DAT')
WRITE(16,*) LSNI(1,1), LSNI(2,1), STN1I(I), SIG1(I,1)
SIGS2(2,l)
CLOSE(16)

RETURN
END

******************************************************************************************

C subroutine for zone 2 - Holding at constant strain

******************************************************************************************

SUBROUTINE ZONE2(CRETIME,DELT,N)

C Initializing variables

REAL E(3)
REAL V(3)
REAL SIG(2600)
REAL SIGS2(3,2600)
REAL SIGS1(3,2600)
REAL STN(3,2600)
REAL STNS1(3,2600)
REAL STNS2(3,2600)
REAL STNTOT(2600)

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

OPEN(16,FILE='M2SREND.DAT')
READ(16,*) ITOT, STNI(1), STN2, TOTSTN, SIG1, SIGS2L
CLOSE(16)

STEP = CRETIME/DELT
DO I=1,STEP

IF(L.EQ.1) THEN
STN(1,I) = STNI
STN(2,I) = STN2
STNTOT(I) = TOTSTN
SIGS2(2,l) = SIGS2L
SIGS1(2,l) = SIG1 - SIGS2(2,l)
ELSE

C1 = SIGS22L - SIG1 * V(2) / (V(1) + V(2))

SIGS2(2,l) = SIG1 * V(2) / (V(1) + V(2)) + C1 * EXP(-E(2) * (1/V(1) +
$ 1/V(2))*(1*DELT-DELT))
$ STNS2(2,I)=SIGS2(2,I)/E(2)+(1/V(2))*SIG1*V(2)*V(1)*V(2)
$ *(1*DELT-DELT)+C1/V(2)*EXP(-E(2)*(1/V(1)+1/V(2)))*(1*DELT-
$ DELT)*(-V(1)*V(2)/(E(2)*(V(1)-V(2))))

SIGS2(2,I)=SIG1-SIGS2(2,I)
STNS1(2,I)=SIG1*(1*DELT-DELT)/(V(1)-1/V(1))*SIG1*V(2)/
$ (V(1)-V(2))
$ *(1*DELT-DELT)-C1/V(1)*EXP(-E(2)*(1/V(1)+1/V(2)))*(1*DELT-
$ DELT)*(-V(1)*V(2)/(E(2)*(V(1)-V(2))))

STNTOT(I)=SIG1/E(1)+STNS2(2,I)
PRINT*,STNTOT(I)

ITOT=ITOT+1
OPEN(11,FILE='MO2DATA.C.DAT')
WRITE(11,* ITOT, SIG1, STNTOT(I)
END IF
END DO

CLOSE(11)
RETURN
END
This algorithm simulates stress relaxation in Model II

*** Note that the program uses STEP WISE calculations and therefore must use a very small timestep interval ***

RRATE is the rate of loading in zone I
SSTRESSCUT is the stress at which the loading is stopped
RRELTIME is the amount of relaxation time in zone 2
DDELT is the time step interval
LLO is the original sample length (mm) used to calculate strain
ERR is the minimum error constraint
E(J) is the elastic modulus of the Jth spring
V(J) is the viscosity coefficient of the Jth dashpot
SIG(1,J) is the stress in the spring at time interval I
SIG(2,J) is the stress in the modified Voigt-Kelvin element at time interval I
STN(1,J) is the strain in the spring at time interval I
STN(2,J) is the strain in the modified Voigt-Kelvin element at time interval I

Initializing Variables

REAL E(3)
REAL V(3)
REAL RRATE
REAL LLO
REAL DDELT
REAL SSTRESSCUT
REAL RRDELTIME
REAL ERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE RELAXATION TIME'
READ*, RRDELTIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DDELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLO
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, ERR

NN=2

Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: E1 <enter>, v1 <enter>
ETOT=0
Do J=1, NN
READ*,E(J)
READ*,V(J)
END DO

c Write the values of the constants to a file

DO J=1,NN
OPEN(15,FILE='CONSTANTS.DAT')
WRITE(15,*) E(J), V(J)
END DO
CLOSE(15)

c Call on subroutines for zones 1 and 2

CALL ZONE1(RRATE,SSTRESSCUT,DELT,ERR,LO,NN)
CALL ZONE2(RRELTIME,DELT,NN)

STOP
END

c Subroutine for zone 1 - Loading at a constant strain rate

******************************************************************************

SUBROUTINE ZONE1(RATE,SSTRESSCUT,DELT,ERR,LO,N)

******************************************************************************

c Initializing Variables

REAL E(3)
REAL V(3)
REAL SIG(3,1000)
REAL SIGS1(3,1000)
REAL SIGS2(3,1000)
REAL STN(3,1000)
REAL STNT(1000)
REAL STNA(3,1000)
REAL STNTOT(1000)
REAL STNDIF(1000)
REAL RATE
REAL DEVSTN
REAL LO
REAL DELT
REAL ERR

c Retrieve the values of the constants for E and V

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

L=1.0
I=1.0
DO J=1,N
STN(J,J)=0.0
SIG(J,J)=0.0
END DO
SIGS1(2,J)=0.0
SIGS2(2,J)=0.0
SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,J)
END DO

C START OF THE MAIN LOOP

C Loop until the cutoff stress is reached

WHILE (SIG(1,1).LE.STRESSCUT)
I=I+1
STNTOT(I)=RATE*((1*DELT)-DELT)/LO
STNDF(I)=RATE*DELT/LO

C ASSUME ALL STRAIN IS IN THE SPRING (ELEMENT 1)

STN(1,1)=STN(1,1)+STNDF(I)
SIG(1,1)=SIG(1,1)*E(I)

C STRESS ON ELEMENT 1=STRESS ON ELEMENT 2

SIG(2,1)=SIG(1,1)
SIGS2(2,1)=(SIG(2,1)*V(2)/V(1))-SIG(2,1)*
$V(2)*V(1))**EXP(-E(2)*(1*DELT-DELT)/V(2))$
SIGS1(2,1)=SIG(2,1)-SIGS2(2,1)
STN(2,1)=STN(2,1)+(SIGS1(2,1)+SIGS1(2,1))*DELT/V(1)
STNT(I)=STN(I,1)+STN(2,1)
STNA(1,1)=STNTOT(I)-STN(2,1)
STN(I,1)=STNA(1,1)

DEVSTN=ABS(STNTOT(I)-STNT(I))
IF(DEVSTN.GE.ERR) THEN
GOTO 5
ELSE
OPEN(11,FILE="MO2DATA.DAT")
WRITE(11,*) I,SIG(1,1),STNTOT(I)
END IF
L=I
DO J=1,N
STN(J,J)=STN(J,J)
END DO
SIGS1(2,J)=SIGS1(2,J)
SIGS2(2,1)=SIGS2(2,1)

END WHILE

OPEN(16,FILE="M2SREND.DAT")
WRITE(16,*) 1,STN(1,1),STN(2,1),STNTOT(1),SIG(1,1)
CLOSE(16)

RETURN
END

*****************************************************************************

Subroutine for zone 2 - Holding at constant strain
*****************************************************************************

SUBROUTINE ZONE2(RELTIME,DELT,N)

Initializing variables

REAL E(3)
REAL V(3)
REAL SIG(5000)

DO J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

OPEN(16,FILE='M2SREND.DAT')
READ(16,*) ITOT,STN1,STN2,STNTOT,SIG1
CLOSE(16)

STEP=RELTIME/DELT
DO I=1,STEP

C1=SIG1/EXP(E(1)*V(2)**2/(E(2)*V(1)+V(2))**2))
PRINT*, 'C1', C1

SIG(I)=C1*EXP(-E(1)*(-V(2)**2*EXP(-E(2)*((1/V(1))+1/V(2))))
$ ((1*DELT-DELT)+E(2)*V(1)*(((DELT-DELT)+E(2)*V(2)*(1*DELT-
$ DELT))/(E(2)*V(1)+V(2))**2))

PRINT*, SIG(I)
ITOT=ITOT+1
OPEN(11,FILE='MO2DATA.DAT')
WRITE(11,*) ITOT,SIG(I),STNTOT

END DO

CLOSE(11)
RETURN
END
This algorithm simulates Model II under the stress-strain-time constraints of a four zone memory test.

*** Note that the program uses STEP WISE calculations and therefore must use a very small timestep interval ***

RRATE is the rate of loading in zone 1
SSTRESSCUT is the stress at which the loading is stopped
RRETLM is the amount of relaxation time in zone 2
UUNRATE is the rate of unloading in zone 3
CCRETLM is the amount of time for viscoelastic creeping to occur
DDELT is the time step interval
LLo is the original sample length (mm) used to calculate strain
EERR is the minimum error constraint
E(I) is the elastic modulus of the Jth spring
V(I) is the viscosity coefficient of the Jth dashpot
SIG(1,I) is the stress in the spring at time interval I
SIG(2,I) is the stress in the modified Voigt-Kelvin element at time interval I
STN(1,I) is the strain in the spring at time interval I
STN(2,I) is the strain in the modified Voigt-Kelvin element at time interval I

Initializing Variables

REAL E(3)
REAL V(3)
REAL RRATE
REAL LLo
REAL DDELT
REAL SSTRESSCUT
REAL RRETLM
REAL UUNRATE
REAL CCRETLM
REAL EERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE RELAXATION TIME'
READ*, RRETLM
PRINT*, 'ENTER THE UNLOADING RATE'
READ*, UUNRATE
PRINT*, 'ENTER THE CREEPING TIME'
READ*, CCRETLM
PRINT*, 'ENTER THE TIME STEP'
READ*, DDELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLo
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, EERR
NN=2

c Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: E1 <enter>, n1 <enter>'
ETOT=0
DO J=1,NN
READ*, E(J)
READ*, V(J)
END DO

c Write the values of the constants to a file

DO J=1,NN
OPEN(15,FILE="CONSTANTS.DAT")
WRITE(15,*) E(J), V(J)
END DO
CLOSE(15)

c Call on subroutines for zones 1 - 4

CALL ZONE1(RRATE, STRESSCUT, DDELT, ERR, LLO, NN)
CALL ZONE2(RRELTIME, DDELT, NN)
CALL ZONE3(UUNRATE, DDELT, LLO, ERR, NN)
CALL ZONE4(DDELT, CCRETIME, NN)

STOP
END

c Subroutine for zone 1 - Loading at a constant strain rate

c *******************************************************

SUBROUTINE ZONE1(RATE, STRESSCUT, DELT, ERR, LO, N)

c Initializing Variables

REAL E(3)
REAL V(3)
REAL SIG(3,500)
REAL SIGS1(3,500)
REAL SIGS2(3,500)
REAL STN(3,500)
REAL STNT(300)
REAL STNA(3,500)
REAL STNTOT(500)
REAL STNDIF(500)
REAL RATE
REAL DEVSTN
REAL LO
REAL DELT
REAL ERR
Retrieve the values of the constants for $E$ and $V$

Do $J=1,N$
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*)$ E(J), V(J)
END DO
CLOSE(15)

Set all initial values of stress and strain equal to 0.0

L=1.0
I=1.0

Do $J=1,N$
STN(J,I)=0.0
SIG(J,I)=0.0
END DO
SIGS1(2,I)=0.0
SIGS2(2,I)=0.0
SIGTOT=0.0
Do $J=1,N$
SIGTOT=SIGTOT+SIG(J,I)
END DO

Loop until the stress exceed the specified cutoff stress

WHILE (SIG(1,I),$ \leq $ STRESSCUT)
I=I+1

Calculate the total strain from the specified strain rate

STNTOT(I)=RATE*(((I*DELT)-DELT)/LO
STNDIF(I)=RATE*DELT/LO

Assume all the strain is initially in the Spring (Element 1)

STN(1,I)=STN(1,I)+STNDIF(I)
SIG(1,I)=SIG(1,I)*E(1)
The stress on element 1 must equal the stress on element 2
SIG(2,I)=SIG(1,I)

Calculate the stress and strain of side 2 of element 2

SIGS2(2,I)=(SIG(2,I)*V(2)/V(1))-SIG(2,I)*
(V(2)/V(1))**(EXP(-E(2)*(I*DELT-DELT)/V(2))
SIGS1(2,I)=SIG(2,I)-SIGS2(2,I)
STN(2,I)=STN(2,I)+SIGS1(2,I)+SIGS2(2,I))**DELT/V(1)
The total calculated strain is equal to the sum of the strains in each element
STNT(I)=STN(1,I)+STN(2,I)

Find a correction for the strain in element 1

STNA(1,I)=STNTOT(I)-STN(2,I)
STN(1,I)=STNA(1,I)

Check the deviation of the total calculated strain to the total strain specified from the strain rate

DEVSTN=ABS(STNTOT(I)-STNT(I))
IF(DEVSTN.GE.ERR) THEN
GOTO 5
ELSE

Write results to a file

OPEN(11,FILE="D1.DAT")
WRITE(11,*) I,SIG(1,I),STNTOT(I)

END IF
I=I+1
DO J=1,N
STN(J,I)=STN(J,L)
END DO
SIGS1(2,I)=SIGS1(2,L)
SIGS2(2,I)=SIGS2(2,L)
END WHILE

Write final results for this zone to a file

OPEN(16,FILE="M2Z1END.DAT")
WRITE(16,*) I,STN(1,I),STN(2,I),STNTOT(I),SIG(1,I)
CLOSE(16)

CLOSE(11)
RETURN
END

Subroutine for zone 2 - Holding at constant strain

SUBROUTINE ZONE2(RELTIME,DELT,N)

Initializing variables

REAL E(3)
REAL V(3)
REAL SIG(1000)
REAL STN(3,1000)
Read in the values of the coefficients of elasticity and viscosity

```
DO J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
END DO
CLOSE(15)
```

Read in the values of stress and strain from the last time step of previous zone

```
OPEN(16,FILE='M2Z1END.DAT')
READ(16,*) ITOT,STN1,STN2,STNTOT,SIG1
CLOSE(16)
PRINT*,ITOT,STNTOT,SIG1
```

Calculate the stress relaxation using the stress relaxation equation with the initial conditions: the stress at t=0 is equal to that from the last time step of the previous zone

```
I=1.0
L=1.0
STEP=RELTIME/DELT
DO I=1,STEP
```

Calculate the stress at each time step

```
C1=SIG1/EXP(E(1)*V(2)**2/(E(2)*(V(1)+V(2)**2)))
PRINT*,C1,C1
SIG(I)=C1*EXP(-E(1)*(-V(2)**2*EXP(-E(2)*((1/V(1)+1/V(2))**2))
$ (1*DELT-DELT)+E(2)*V(1)*((1*DELT-DELT)+E(2)*V(2)*((1*DELT-DELT))
$ V(1)+V(2))**2))
```

Calculate the strains in each element

```
STN(1,J)=(SIG(I)/V(E(1)))
STN(2,J)=STNTOT-STN(1,J)
PRINT*,SIG(I)
ITOT=ITOT+1
```

Write results to a file

```
OPEN(11,FILE='D2.DAT')
WRITE(11,*) ITOT,SIG(I),STNTOT
L=1
STN(1,L)=STN(1,J)
STN(2,L)=STN(2,J)
SIG(L)=SIG(I)
END DO
```
c Write results at the last time step to a file

OPEN(14,FILE='M2Z2END.DAT')
WRITE(14,*) ITOT,STN(1,L),STN(2,L),STNTOT,SIG(L)

CLOSE(14)
CLOSE(11)
RETURN
END

******************************************************************************
c Subroutine for zone 3 - UnLoading at constant strain rate
******************************************************************************

SUBROUTINE ZONE3(UNRATE,DELT,LO,ERR,N)

c Initializing Variables

REAL E(3)
REAL V(3)
REAL SIG(1000)
REAL STN(3,1000)
REAL STNA(3,1000)
REAL STNTOT(1000)
REAL DSTN(1000)
REAL STNS1(3,1000)
REAL DSTNS1(3,1000)
REAL UNRATE
REAL DELT
REAL ERR
REAL LO
REAL ITOT
REAL STN1
REAL STN2
REAL TOTSTN
REAL SIG1

c Recall values of the coefficients

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
PRINT*,E(J),V(J)
END DO
CLOSE(15)

c Recall the values from the last timestep of the previous zone

OPEN(14,FILE='M2Z2END.DAT')
READ(14,*) ITOT,STN1,STN2,TOTSTN,SIG1
PRINT*,ITOT,STN1,STN2,TOTSTN,SIG1
CLOSE(14)

c Set all initial values to those from the last time step of the
c  previous zone

l=1.0
L=1.0
SIG(1)=SIG1
STN(1,l)=STN1
STNS1(2,l)=STN2
STNTOT(l)=STN(1,l)+STNS1(2,l)

c  Loop while the stress is greater than zero

WHILE(SIG(l).GE.0)
   l=l+1

   c  Calculate the strain from the specified strain rate

   DSTN(l)=-(UNRATE*DELT/LO)
   STNTOT(l)=TOTSTN+(-UNRATE*(1*DELT-DELTY/LO)

   c  Assume all change in strain is initially in the spring

   STN(1,l)=STN(1,l)+DSTN(l)
$ SIG(l)=STN(1,l)*E(1)

   c  Calculate the change in strain of side 1 of element 2

   DSTNS1(2,l)=(SIG(l)-V(2)*SIG(l)V(1)
$ +(V(2)*SIG(L)/V(1))*EXP(-E(2)*(1*DELT-DELTY)
$ V(2)))*DELT/V(1)
   STNS1(2,l)=STNS1(2,l)+DSTNS1(2,l)

   c  Correct the strain in element 1

   STNA(1,l)=STNTOT(l)-STNS1(2,l)
   STN(1,l)=STNA(1,l)

   c  Check the deviation between the calculated total strain and the
   c  total strain specified by the strain rate

   DEVSTN=ABS(STNTOT(l)-STN(1,l)-STNS1(2,l))
   IF(DEVSTN.GE.ERR) THEN
      GOTO 5
   ELSE
      ITOT=ITOT+1

   c  Write results to a file

   OPEN(13,FILE="D3.DAT")
   WRITE(13,*)ITOT,SIG(l),STNTOT(l)
END IF

   L=l
   STN(1,l)=STN(1,l)
   STNS1(2,l)=STNS1(2,l)
END WHILE

c Write final results of this zone to a file

OPEN(14,FILE="M2Z3END.DAT")
WRITE(14,*) ITOT,SIG(I),STN(1,L),STNS1(2,L)
CLOSE (14)
CLOSE (13)
RETURN
END

c *******************************************************************************
c Subroutine for zone 4 - Creeping at constant stress zero

c *******************************************************************************

SUBROUTINE ZONE4(DEL,T,CRETIME,N)

c Initializing Variables

REAL E(5)
REAL V(5)
REAL ITOT
REAL SIG
REAL STN
REAL STNS2
REAL CRETIME
REAL DELT
REAL STNS1(1000)
REAL C1

c Read in the values of the coefficients of elasticity and viscosity

Do J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

c Read in the values of stress and strain from the last time step

c of previous zone

OPEN(16,FILE="M2Z3END.DAT")
READ(16,*) ITOT,SIG,STN,STNS2
CLOSE(16)

c Calculate the creep strain by using the creep equation at

c zero stress conditions. Note this is "step wise".

STEP=CRETIME/DELT
DO I=1,STEP

C1=STNS2/(V(2)*(E(2)*((V(1)+V(2))))

END
STNS1(I) = C1 \times \exp(-E(2) \times ((1/N(1)) - (1/N(2))))^
\times (1*DELT-DELT))^
\times E(2) \times (V(1)+V(2))))^
ITOT = ITOT + 1

Write results to a file

OPEN(15, FILE = 'D4.DAT')
WRITE(15,*) ITOT, SIG, STNS1(I)

END DO
CLOSE(15)
RETURN
END
Model II - Creep Test

PROGRAM START

1. Enter testing parameters
2. Enter values: spring and dashpot coefficients

Zone 1

Retrieve values of E, n

Set all initial values of σ and ε to zero

Is σ_t less than σ_initial?

Yes: σ_true=σ_true*E/L0
Δσ_true=Δσ_true*Δt/L0

espring1(t)=espring1(t-1)+espring(t-1)

σ(t)=espring1(t)+E1
c(t)=σ(t)

σ_side2, elt2(t)=σ(t)*n2/η1
-σ(t)*n2/η1)*exp(-E2*t/η2)

σ_side1, elt2(t)=σ(t)-σ_side2, elt2

e2(t)=e2(t-1)+
[σ_side1, elt2(t)+ σ_side1, elt2(t-1)]/2*Δt/η2

c(t)=Σc

e*(t)=true-ε2(t)

Does σ_t=σ_true?

Yes: Write results to file; tstep=
Go to t=t+1

No: Go to Zone 2

Zone 2

Retrieve values of E, n

Retrieve values of stress and strain from last timestep of zone 1

Is it the first timestep?

Yes: Set all initial values of creep zone to those of last timestep of zone 1
Write results to file; tstep=1

No: Calculate cost of integration

Is t less than creep time?

Yes: σ_side2, elt2(t)=σ_constant*n2/(1/η1+1/η2)*t
+C1*exp(-E2*(1/η1+1/η2)*t)

σ_side2, elt2(t)=σ_side2, elt2/E2
+C1*exp(-E2*(1/η1+1/η2))*(-n1*η2/E2*(η1+η2))
+c(t)=σ_constant/E1+σ_side2, elt2(t)

No: Write results to file;

tstep=tstep+1

t(total)=(final zone)+1

Stop, End
Model II - Four Zone Tensile Test

Program Start

Enter testing parameters

Enter values: spring and dashpot coefficients

Retrieve values of E, n

Set all initial values of σ and ε to zero

Is σtrue less than σcutoff? 

Yes: 

σtrue = σtrue + σt

Actrue = Actrue + Actrue(t-1)

espring1(t) = espring1(t) + E1

σ1(t) = σ1(t)

σside2,Elt2(t) = σ2(t)*η2/η1
-σ2(t)*η2/η1*exp(-E2*t/η2)

σside1,Elt2(t) = σ2(t) - σside2,Elt2

c2(t) = c2(t-1) + (σside1,Elt2(t) + σside1,Elt2(t-1))/2*Δt/η2

εt = εt + \left(\sigma(t) - \sigmatrue\right)

No: 

Go to Zone 2

Retrieve values of E, n

Go to Zone 2

Retrieve values of stress and strain from last timestep of zone 1

Is it the first timestep? 

Yes: 

Calculate constant of integration from results of last zone

Write results to file; tstep = 1

No: 

Stop. End

Is t less than relaxation time? 

Yes: 

Calculate the strains in each element:

ε1(t) = C1*exp(-E1*\left(\frac{\eta1*2*2*exp(-E2*(1/\eta1 + 1/\eta2)*t)}{E2*\eta1*2*2*exp(-E2*(1/\eta1 + 1/\eta2)*t)}\right) + E2*\eta1*2*2*exp(-E2*(1/\eta1 + 1/\eta2)*t)*[E2*(\eta1*2*2*exp(-E2*(1/\eta1 + 1/\eta2)*t))]])

ε2(t) = constant*ε1(t)

Write results to file:

tstep = tstep + 1

t(total) = t(final zone 1) + 1

Go to Zone 3

Stop. End

Does σt = σtrue? 

Yes: 

Go to t = t+1

Stop. End

No: 

Go to Zone 2

Write results to file; tstep = 1

Go to Zone 3
APPENDIX III - FORTRAN ALGORITHMS FOR MODEL III
This algorithm simulates the constant stress creep for Model III

*** Note that this program uses STEP WISE calculations and therefore must use a very small timestep interval ***

RRATE is the rate of loading in zone 1
SSTRESSCUT is the stress at which the loading is stopped
RRTIME is the amount of relaxation time in zone 2
URNATE is the rate of unloading in zone 3
CCRETIME is the amount of time for viscoelastic creeping to occur
DDELT is the time step interval
LLO is the original sample length (mm) used to calculate strain
ERR is the minimum error constraint
E(J) is the elastic modulus of the Jth spring
V(J) is the viscosity coefficient of the Jth dashpot
SIG(J,J) is the stress in element J at time interval 1
STN(J,J) is the strain in element J at time interval 1

Initializing Variables

REAL E(3)
REAL V(3)
REAL RRATE
REAL LLO
REAL DDELT
REAL SSTRESSCUT
REAL CCRETIME
REAL ERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE AMOUNT OF CREEP TIME'
READ*, CCRETIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DDELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLO
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, ERR

NN = 3

Input the Elasticity and Viscosity Coefficients

PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: E1 <enter>, n1 <enter>,'
ETOT = 0
Do J = 1, NN
READ*, E(J)
READ*, V(J)
END DO

C Write the values of the constants to a file

DO J=1,NN
OPEN(15,FILE='CONSTANTS.DAT')
WRITE(15,*) E(J), V(J)
END DO
CLOSE(15)

C Call on subroutines for zones 1 (loading) and 2 (creep)

CALL ZONE1(ERR,STRESSCUT,DELT,EERR,LLO,NN)
CALL ZONE2(ERR,STRESSCUT,DELT,EERR,LLO,NN)

STOP
END

C *****************************************************************************
C Subroutine for zone 1 - Loading at a constant strain rate
C *****************************************************************************

SUBROUTINE ZONE1(ERR,STRESSCUT,DELT,EERR,LLO)

C Initialize variables

REAL E(3)
REAL V(3)
REAL SIG(3,500)
REAL SIGS1(3,500)
REAL SIGS2(3,500)
REAL STN(3,500)
REAL ST(T500)
REAL STNA(3,500)
REAL STNTOT(500)
REAL STNDIF(500)
REAL RATE
REAL DEVSTN
REAL LO
REAL DELT
REAL ERR

C Retrieve the values of the constants for E and V

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

C Set all initial values of stress and strain to 0.0

L=1.0
I=1.0
DO J=1,N
  STN(J,1)=0.0
  SIG(J,1)=0.0
END DO
SIGS1(2,1)=0.0
SIGS2(2,1)=0.0
SIGS1(3,1)=0.0
SIGS2(3,1)=0.0
SIGTOT=0.0
DO J=1,N
  SIGTOT=SIGTOT+SIG(J,1)
END DO

Start of main loop to proceed until stress exceeds the specified cutoff stress
WHILE (SIG(1,1).LE.STRESSCUT)
  I=I+1
Calculate the strain from the specified strain rate
STNTOT(I)=RATE*(I*DELT)-DELT)/LO
STNDIF(I)=RATE*DELT/LO
C Assume all the strain is initially in the spring (Element 1)
STN(1,1)=STN(1,1)+STNDIF(I)
SIG(1,1)=STN(1,1)*E(1)
PRINT*,SIG(1,1)
C The stress in element 1 must equal the stress in element 2
SIG(2,1)=SIG(1,1)
C Calculate the stress and strain of side 2 of element 2
SIGS2(2,1)=(SIG(2,1)*V(2)/V(1))-SIG(2,1)*
  (V(2)/V(1))*EXP(-E(2)*((DELT-DELT)/V(2))
PRINT*,SIGS2(2,1),SIGS2(2,1)
SIGS1(2,1)=SIG(2,1)-SIGS2(2,1)
STN(2,1)=STN(2,1)+(SIGS1(2,1)+SIGS1(2,1))*DELT/V(1)
C The stress on element 1 must also equal the stress on element 3
SIG(3,1)=SIG(1,1)
C Calculate the stress and strain of side 2 of element 3
SIGS2(3,1)=SIG(3,1)
SIGS1(3,1)=SIG(3,1)-SIGS2(3,1)
STN(3,1)=SIGS2(3,1)/E(3)
C Calculate the total strain which is the sum of each element
STNT(I) = STN(1,I) + STN(2,I) + STN(3,I)

c Find correction for strain in element 1

STNA(1,I) = STNTOT(I) - STN(2,I) - STN(3,I)
STN(1,I) = STNA(1,I)

c Check the deviation of the calculated total strain against the
total strain dictated by loading rate

DEVSTN = ABS(STNTOT(I) - STNT(I))
IF(DEVSTN.GE.ERR) THEN
GOTO 5
ELSE

c Write results to a file

OPEN(11,FILE='M3CREEP.DAT')
WRITE(11,*) I,SIG(1,I),STNTOT(I)
END IF
L=I
DO J=1,N
STN(J,I) = STN(J,L)
END DO
SIGS1(2,I) = SIGS1(2,L)
SIGS2(2,I) = SIGS2(2,L)
END WHILE

c Write final results for this zone to a file

OPEN(16,FILE='M3Z1END.DAT')
WRITE(16,*) I,STNTOT(I),SIG(1,I),SIGS1(2,I),SIGS2(2,I)
CLOSE(16)
CLOSE(15)
RETURN
END

c Subroutine for zone 2 - Constant stress creep

c ***************************************************************************

SUBROUTINE ZONE2(CRETIME,DELT,N)

c Initialize Variables

REAL E(3)
REAL V(3)
REAL ITOT
REAL SIG
REAL STN1(5000)
REAL STNS2(3,5000)
REAL SIGS2(3,5000)
REAL C1
REAL C2
DOUBLE PRECISION STNTOT(5000)

Recall values of the coefficients

DO J=1,N
  OPEN(15,FILE="CONSTANTS.DAT")
  READ(15,*) E(J), V(J)
  PRINT*,E(J),V(J)
END DO
CLOSE(15)

Recall the values from the last timestep of the previous zone

OPEN(16,FILE="M3ZIEND.DAT")
READ(16,*) ITOT, TOTSTN, SIG, SIGS22L, SIGS23L
CLOSE(16)

Start of do loop to proceed until the time reaches the specified
amount of creep time

STEP=CRTIME/DELT
DO I=1,STEP

Calculate the stress and strain in side 2 of element 2

C1=SIGS22L-SIG*V(2)/(V(1)-V(2))

SIGS2(2,I)=SIG*V(2)/(V(1)-V(2))-C1*EXP(-E(2)*
$(1/V(1)+1/V(2)))*(1*DELT-DELT))

STNS2(2,I)=SIGS2(2,I)*E(2)-SIG/(V(1)+V(2))*(1*DELT-DELT)
$ +((C1/V(2))*EXP(-E(2)*(1/V(1)+1/V(2)))*(1*DELT-DELT))*(-V(2)*V(1)/
$(E(2)*(V(1)+V(2))))
PRINT*,STN2',STNS2(2,I)

Calculate the stress and strain in side 2 of element 3

C2=SIGS23L-SIG

SIGS2(3,I)=SIG+C2*EXP(-E(3)*(1*DELT-DELT)/V(3))

STNS2(3,I)=SIGS2(3,I)/E(3)
PRINT*,STN3',STNS2(3,I)

Calculate the strain in element 1 by Hooke's law

STN1(I)=SIG/E(1)
PRINT*,STN1(I)

Calculate the total strain by summing the strains in each elt
STNTOT(I) = STN1(I) + STNS2(3.I) + STNS2(2,I)
PRINT*, 'STNTOT', STNTOT(I)

Write results to a file

ITOT = ITOT + 1
OPEN(11, FILE = "M3CREEP.DAT")
WRITE(11, *) ITOT, SIG, STNTOT(I)
END DO

CLOSE(11)
RETURN
END
This algorithm simulates the stress relaxation at constant strain for Model III

Note that this program uses STEP WISE calculations and therefore must use a very small timestep interval

**RRATE** is the rate of loading in zone 1
**SSTRESSCUT** is the stress at which the loading is stopped
**RRELTIME** is the amount of relaxation time in zone 2
**UUNRATE** is the rate of unloading in zone 3
**CCRETIME** is the amount of time for viscoelastic creeping to occur
**DDELT** is the time step interval
**LLO** is the original sample length (mm) used to calculate strain
**EERR** is the minimum error constraint
**E(J)** is the elastic modulus of the Jth spring
**V(J)** is the viscosity coefficient of the Jth dashpot
**SIG(J,J)** is the stress in element J at time interval I
**STN(J,J)** is the strain in element J at time interval I

Initializing Variables

```
REAL E(3)
REAL V(3)
REAL RRATE
REAL LLO
REAL DDELT
REAL SSTRESSCUT
REAL RRELTIME
REAL EERR
```

Input the testing parameters

```
PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE RELAXATION TIME'
READ*, RRELTIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DDELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLO
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, EERR
```

NN=3

Input the Elasticity and Viscosity Coefficients

```
PRINT*, 'ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY'
PRINT*, 'STARTING WITH ELEMENT 1 ENTER: E1 <enter>, v1 <enter>'
ETOT=0
Do J=1,NN
READ*, E(J)
```
READ*, V(J)
END DO

c Write the values of the constants to a file

DO J=1,NN
OPEN(15, FILE='CONSTANTS.DAT')
WRITE(15,*) E(J), V(J)
END DO
CLOSE(15)

c Call on subroutines for zones 1 - 4

CALL ZONE1 (RRATE, S TRESSCUT, DDELT, ERR, LLO, NN)
CALL ZONE2 (RRELTIME, DDELT, NN)

STOP
END

******************************************************************************

c Subroutine for zone 1 - Loading at a constant strain rate

c******************************************************************************

SUBROUTINE ZONE1 (RATE, S TRESSCUT, DDELT, ERR, LLO, N)

REAL E(3)
REAL V(3)
REAL SIG(3,1000)
REAL SIGA(3,1000)
REAL SIGS1(3,1000)
REAL SIGS2(3,1000)
REAL STN(3,1000)
REAL STNT(1000)
REAL STNA(3,1000)
REAL STNTOT(1000)
REAL STNDIF(1000)
REAL RATE
REAL DEVSTN
REAL LO
REAL DELT
REAL ERR

c Retrieve the values of the constants for E and V

Do J=1,N
OPEN(15, FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
END DO
CLOSE(15)

L=1.0
I=1.0

DO J=1,N
STN(I,J)=0.0
SIG(I,J)=0.0
END DO
SIGS1(2,J)=0.0
SIGS2(2,J)=0.0
SIGS1(3,J)=0.0
SIGS2(3,J)=0.0
SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(I,J)
END DO

WHILE (SIG(1,J).LE.STRESSCUT)
I=I+1

STNTOT(I)=RATE*((I*DELT)-DELT)/LO
STNDIF(I)=RATE*DELT/LO

C ASSUME ALL STRAIN IS IN THE SPRING (ELEMENT 1)
STN(1,J)=STN(1,J)+STNDIF(I)
S SIG(1,J)=STN(1,J)*E(1)
PRINT*.SIG(1,J)

C STRESS ON ELEMENT 1=STRESS ON ELEMENT 2
SIG(2,J)=SIG(1,J)
SIGS2(2,J)=(SIG(2,J)*V(2)/V(1))-SIG(2,J)*
S (V(2)/V(1))*EXP(-E(2)*(I*DELT-DELT)/V(2))
PRINT*.SIGS2(2,J)*SIGS2(2,J)
SIGS1(2,J)=SIG(2,J)-SIGS2(2,J)
STN(2,J)=STN(2,J)+SIGS1(2,J)*DELT/V(1)

C STRESS ON ELEMENT 1=STRESS ON ELEMENT 3
SIG(3,J)=SIG(1,J)
SIGS2(3,J)=SIG(3,J)
S -SIG(3,J)*EXP(-E(3)*(I*DELT-DELT)/V(3))
STN(3,J)=SIGS2(3,J)/E(3)

STNT(I)=STN(I,J)+STN(2,J)+STN(3,J)
STNA(I,J)=STNTOT(I)-STN(2,J)-STN(3,J)
STN(1,J)=STNA(1,J)

DEVSTN=ABS(STNTOT(I)-STNT(I))
IF(DEVSTN.GE.ERR) THEN
GOTO 5
ELSE
OPEN(11,FILE='M3S5.DAT')
WRITE(11,*) I,SIG(I,J),STNTOT(I)

END IF
L=I
DO J=1,N
STN(I,J)=STN(I,J)
END DO
SIGS1(2,I)=SIGS1(2,L)
SIGS2(2,I)=SIGS2(2,L)

END WHILE

c Write final results for this zone to a file

OPEN(16,FILE='M3Z1END.DAT')
WRITE(16,*) ITOT,STN1,STN2,STN3,STNTOT,SIG1
CLOSE(16)
RETURN
END

c Subroutine for zone 2 - Holding at constant strain
c

SUBROUTINE ZONE2(RELTIME,DELT,N)

REAL E(3)
REAL V(3)
REAL SIG(2500)
REAL STN(3,2500)
REAL DSTNS1(3,2500)
REALSTNS1(3,2500)

Do J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J),V(J)
END DO
CLOSE(15)
OPEN(16,FILE='M3Z1END.DAT')
READ(16,*) ITOT,STN1,STN2,STN3,STNTOT,SIG1
CLOSE(16)
PRINT*,ITOT,STNTOT,SIG1

STEP=RELTIME/DELT
DO I=1,STEP

C1=SIG1/EXP(-E(1)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)
$ V(1)**2+V(2)**2+V(3)**2)/(E(3)**2+V(1)**2+V(2)**2)
T=I*DELT
SIG1=SIG1*EXP(-E(1)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)
$ +EXP(-E(3)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)
$ V(1)**2+V(2)**2+V(3)**2)/(E(3)**2+V(1)**2+V(2)**2)
$ +EXP(-E(3)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)*E(2)**(2)
$ V(1)**2+V(2)**2+V(3)**2)/(E(3)**2+V(1)**2+V(2)**2)
$ V(1)**2+V(2)**2+V(3)**2)/(E(3)**2+V(1)**2+V(2)**2)
PRINT*,SIG1

ITOT=ITOT+1
OPEN(11,FILE='M3S5.DAT')
WRITE(11,*) ITOT_SIG(I), STNTOT

END DO

CLOSE(11)
RETURN
END
This algorithm simulates Model III under the stress strain time constraints of a four zone memory test

Note that this program uses STEP WISE calculations and therefore must use a very small timestep interval

RRATE is the rate of loading in zone 1
SSTRESSCUT is the stress at which the loading is stopped
RRELTIME is the amount of relaxation time in zone 2
UUNRATE is the rate of unloading in zone 3
CCRETME is the amount of time for viscoelastic creeping to occur
DDELT is the time step interval
LLo is the original sample length (mm) used to calculate strain
EERR is the minimum error constraint
E(J) is the elastic modulus of the Jth spring
V(J) is the viscosity coefficient of the Jth dashpot
SIG(J,1) is the stress in element J at time interval 1
STN(J,1) is the strain in element J at time interval 1

Initializing Variables

REAL E(3)
REAL V(3)
REAL RRATE
REAL LLo
REAL DDELT
REAL SSTRESSCUT
REAL RRELTIME
REAL UUNRATE
REAL CCRETIME
REAL EERR

Input the testing parameters

PRINT*, 'ENTER THE LOADING RATE'
READ*, RRATE
PRINT*, 'ENTER THE CUT OFF STRESS'
READ*, SSTRESSCUT
PRINT*, 'ENTER THE RELAXATION TIME'
READ*, RRELTIME
PRINT*, 'ENTER THE UNLOADING RATE'
READ*, UUNRATE
PRINT*, 'ENTER THE CREEPING TIME'
READ*, CCRETIME
PRINT*, 'ENTER THE TIME STEP'
READ*, DDELT
PRINT*, 'ENTER THE ORIGINAL SAMPLE LENGTH'
READ*, LLo
PRINT*, 'ENTER THE MINIMUM ERROR SPECIFIED'
READ*, EERR

NN=3

Input the Elasticity and Viscosity Coefficients
PRINT*, "ENTER THE COEFFICIENTS OF ELASTICITY AND VISCOSITY"
PRINT*, "STARTING WITH ELEMENT 1 ENTER: E1 <enter> n1 <enter>"

E1 = 0
Do J = 1, NN
READ*, E(J)
READ*, V(J)
END DO

; Write the values of the constants to a file

DO J = 1, NN
OPEN(15, FILE = "CONSTANTS.DAT")
WRITE(15, *) E(J), V(J)
END DO
CLOSE(15)

; Call on subroutines for zones 1 - 4

CALL ZONE1(RRATET, SSTRESS, DDEL, ERR, LLO, NN)
CALL ZONE2(RRELTIME, DDEL, NN)
CALL ZONE3(UUNRATE, DDEL, LLO, ERR, NN)
CALL ZONE4(DDEL, CCRETIME, NN)

STOP
END

; Subroutine for zone 1 - Loading at a constant strain rate

SUBROUTINE ZONE1(RRATET, SSTRESS, DDEL, ERR, LLO, N)

; Initialize Variables

REAL E(3)
REAL V(3)
REAL SIG(3, 500)
REAL SIGS1(3, 500)
REAL SIGS2(3, 500)
REAL STN(3, 500)
REAL STNT(500)
REAL STNA(3, 500)
REAL STNTOT(500)
REAL STNDIF(500)
REAL RATE
REAL DEVS
REAL LO
REAL DELT
REAL ERR

; Retrieve the values of the constants for E and V

Do J = 1, N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(I), V(I)
END DO
CLOSE(15)

C Set initial timestep equal to 1.0 and all initial values for
C stress and strain equal to 0.0

L=1.0
I=1.0

DO J=1,N
STN(J,1)=0.0
SIG(J,1)=0.0
END DO
SIGS1(2,1)=0.0
SIGS2(2,1)=0.0
SIGS1(3,1)=0.0
SIGS2(3,1)=0.0

SIGTOT=0.0
DO J=1,N
SIGTOT=SIGTOT+SIG(J,1)
END DO

C Start of the Main Loop - Loop until stress exceeds specified
C stress cutoff value

WHILE (SIG(1,1).LE.STRESSCUT)
I=I+1

C Calculate the total strain using the strain rate
STNTOT(I)=RATE*((I*DELT)-DELT)/LO
STNDIF(I)=RATE*DELT/LO

C Assume all strain in initially in the spring (Element 1)
STN(1,1)=STN(1,1)+STNDIF(I)
5 SIG(1,1)=SIG(1,1)*E(1)
PRINT*,SIG(1,1)

C Stress on element 1 must equal the stress on element 2
SIG(2,1)=SIG(1,1)

C Calculate the stress on side 2 of element 2 and use it to
C calculate the strain of element 2 (parallel)
SIGS2(2,1)=(SIG(2,1)*V(2)/V(1))-SIG(2,1)*
(V(2)/V(1))*EXP(-E(2)*((I*DELT)-DELT)/V(2))
PRINT*,SIGS2(2,1),SIGS2(2,1)
SIGS1(2,1)=SIG(2,1)-SIGS2(2,1)

STN(2,1)=STN(2,1)+SIGS1(2,1)+SIGS1(2,1)*DELT/V(1)
c Stress on element 1 is also equal to the stress on element 3
SIG(3, l) = SIG(1, l)
c Calculate the stress on side 2 of element 3 and use it to
c calculate the strain of element 3
SIGS2(3, l) = SIG(3, l)
$ -SIG(3, l) \cdot EXP((-E(3) \cdot (I^*DELT-DELT) \cdot V(3)))$
STN(3, l) = SIGS2(3, l) / E(3)
c The total strain is equal to the sum of the strain in each element
STNT(1, l) = STN(1, l) + STN(2, l) + STN(3, l)
c Find correction for strain of element 1
STNA(1, l) = STNTOT(l) - STN(2, l) - STN(3, l)
STN(1, l) = STNA(1, l)
c Check the deviation of the total strain to that found
c from the constant strain rate
DEVSTN = ABS(STNTOT(l) - STNT(l))
IF (DEVSTN.GE.ERR) THEN
  GOTO 5
ELSE
  OPEN(11, FILE="D1.DAT")
  WRITE(11, *) LSIQ(1, l) . STNTOT(l)
END IF
L = 1
DO J = 1, N
  STN(J, l) = STN(J, L)
END DO
SIGS1(2, l) = SIGS1(2, L)
SIGS2(2, l) = SIGS2(2, L)
END WHILE
c Write final results for this zone to a file
OPEN(16, FILE="M3Z1END.DAT")
WRITE(16, *) LSTN(1, l) . STN(2, l) . STN(3, l) . STNTOT(l) . SIG(1, l).
$ SIGS1(2, l) . SIGS2(3, l)
CLOSE(16)
CLOSE(11)
CLOSE(15)
RETURN
END

***************************************************************
c Subroutine for zone 2 - Holding at constant strain
c *******************************************************
SUBROUTINE ZONE2(RELTIME,DELT,N)

  c Initialize Variables

  REAL E(3)
  REAL V(3)
  REAL SIG(1000)
  REAL SIGS2(3,1000)
  REAL SIGS1(3,1000)
  REAL STN(3,1000)
  REAL DSTNS1(3,1000)
  REAL STNS1(3,1000)
  REAL STNT(1000)

  c Recall values of the constants

  DO J=1,N
    OPEN(15,FILE='CONSTANTS.DAT')
    READ(15,*), E(J), V(J)
    END DO
    CLOSE(15)

  c Recall the values of the stress/strain from the last step
  c of the previous zone

  OPEN(16,FILE='M3Z1END.DAT')
  READ(16,*), ITOT,STN1,STN2,STN3,STNTOT,SIG1,SIGS12,SIGS23
  CLOSE(16)
  PRINT*,ITOT,STNTOT,SIG1

  c Calculate the stress relaxation until the time reached the
  c specified relaxation time

  STEP=RELTIME/DELT
  DO I=1,STEP

  c If timestep = 1.0, then set all values equal to those from the
  c last step of the previous zone

  IF (I.EQ.1) THEN
    SIG(I)=SIG1
    SIGS1(2,I)=SIGS12
    SIGS2(3,I)=SIGS23
    STN(1,I)=STN1
    STNS1(2,I)=STN2
    STNS1(3,I)=STN3
  ELSE

  c Calculate the constant of integration

  C1=SIG1/EXP(-E(1)*(-V(2)**2*E(3)-2*E(2)*V(1)*V(2)
     $ -E(2)*V(2)**2-E(2)*V(1)**2)/(E(2)*E(3)*((V(1)+V(2))**2))

  END
\[ T = I^* \text{DELT-DELT} \]

c Calculate the stress at each time step

\[
\text{SIG}(i) = C1^* \exp(-E(1)^*(-V(2)^**2*E(3)^*\exp(-E(2)^*)\]
\[
(1/V(1)+1/V(2))^*T-2^*E(2)^*V(1)^*V(2)^*\exp(-E(3)^*T/V(3))^-
\]
\[
E(2)^*V(2)^**2^*\exp(-E(3)^*T/V(3))^E(2)^*V(1)^**2^*\exp(-E(3)^*T
\]
\[
V(3)^*E(2)^*E(3)^*V(1)^*T+E(2)^*E(3)^*V(2)^*T)/(E(2)^*E(3)^*
\]
\[
(V(1)+V(2))^**2^)
\]

c Calculate the change in strain of element 2

\[
\text{DSTNS1}(2.i)=((\text{SIG}(L)^*\text{SIG}(I))/2)/(V(1)+V(2))^+
\]
\[
(\text{SIG}(L)^*\text{SIG}(I))/2)^*V(2)/(V(1)^*(V(1)+V(2)))
\]
\[
* \exp(-E(2)^*(1^*\text{DELT-DELT})*[(1/V(1)+1/V(2))])*\text{DELT}
\]

c Calculate the strain of element 2

\[
\text{STNS1}(2.i) = \text{STNS1}(2.L)+\text{DSTNS1}(2.i)
\]

c Calculate the change in strain of element 3

\[
\text{DSTNS1}(3.i)=((\text{SIG}(L)^*\text{SIG}(I))/2)^*
\]
\[
\exp(-E(3)^*(1^*\text{DELT-DELT})*V(3))^*\text{DELT}/V(3)
\]

c Calculate the strain of element 3

\[
\text{STNS1}(3.i) = \text{STNS1}(3.L)+\text{DSTNS1}(3.i)
\]

c Calculate the strain in element 1 using Hooke's Law

\[
\text{STN}(1.i) = \text{SIG}(I)/E(1)
\]

c Calculate the total strain (as a check - it should be
c equal to the strain being held constant)

\[
\text{STNT}(i) = \text{STN}(1.i)+\text{STNS1}(2.i)+\text{STNS1}(3.i)
\]
\[
\text{PRINT}*, \text{STNS1}(2.i), \text{STNS1}(3.i)
\]
\[
\text{PRINT}*, 'CALC STN', \text{STN}(i),' CNST STN', \text{STNTOT}
\]
\[
\text{PRINT}*, \text{SIG}(I)
\]

END IF

c Write results to a file

\[
\text{ITOT} = \text{ITOT} - 1
\]
\[
\text{OPEN}(11, \text{FILE} = 'D2.DAT')
\]
\[
\text{WRITE}(11, *) \text{ITOT}, \text{SIG}(I), \text{STNTOT}
\]

L=1
\[
\text{SIG}(L) = \text{SIG}(I)
\]
\[
\text{STN}(1.L) = \text{STN}(1.i)
\]
\[
\text{STNS1}(2.L) = \text{STNS1}(2.i)
\]
STNS1(3,L)=STNS1(3,L)
END DO

c Write results at last timestep to a file

OPEN(14,FILE="M3Z2END.DAT")
WRITE(14,*) ITOT,STN(1,L),STNS1(2,L),STNS1(3,L).
$ STNTOT,SIG(L)
CLOSE(14)
CLOSE(11)
RETURN
END

c Subroutine for zone 3 - UnLoading at constant strain rate

SUBROUTINE ZONE3(UNRATE,DELT,LO,ERR,N)

c Initialize Variables

REAL E(3)
REAL V(3)
REAL SIG(50)
REAL STN(3,50)
REAL STN(3,50)
REAL STNTOT(50)
REAL DSTN(50)
REAL STNS1(3,50)
REAL DSTNS1(3,50)
REAL UNRATE
REAL DELT
REAL ERR
REAL LO
REAL ITOT
REAL STN1
REAL STN2
REAL TOTSTN
REAL SIG1

c Recall the values of the constants

Do J=1,N
OPEN(15,FILE="CONSTANTS.DAT")
READ(15,*) E(J), V(J)
PRINT*,E(J),V(J)
END DO
CLOSE(15)

Recall the values from the final timestep of the previous zone

OPEN(14,FILE="M3Z2END.DAT")
READ(14,*) ITOT,STN1,STN2,STN3,TOTSTN,SIG1
CLOSE(14)
c Set all initial values to those of the last time step of zone 2

I = 1.0
L = 1.0
SIG(I) = SIG1
STN(1,I) = STN1
STNS1(2,I) = STN2
STNS1(3,I) = STN3
STNTOT(I) = STN(1,I) + STNS1(2,I) + STNS1(3,I)

c Loop until the total stress is equal to zero

WHILE(SIG(I) .GE. 0)
I = I + 1

c Calculate the change in strain from the strain rate

DSTN(I) = -(UNRATE * DELT/LO) * 100
STNTOT(I) = TOTSTN + (UNRATE * (I * DELT - DELT)/LO) * 100

c Assume all changes in strain occur initially in the spring

STN(1,I) = STN(1,I) + DSTN(I)
SIG(I) = STN(1,I) * E(1)

c Calculate the change in strain of element 2

DSTNS1(2,I) = (SIG(I) - V(2) * SIG(I)/V(1))
$ + (V(2) * SIG(L)/V(1)) * EXP(-E(2) * (I * DELT - DELT)/
$ V(2))) * DELT/V(1)
STNS1(2,I) = STNS1(2,I) + DSTNS1(2,I)

c Calculate the change in strain of element 3

DSTNS1(3,I) = (SIG(I) * EXP(-E(3) * (I * DELT - DELT)/V(3))) *
$ DELT/V(3)
STNS1(3,I) = STNS1(3,I) + DSTNS1(3,I)

c Find a corrected value of strain in element 1

STNA(1,I) = STNTOT(I) - STNS1(2,I) - STNS1(3,I)
STN(1,I) = STNA(1,I)

c Check the total strain calculated from that dictated by the
unloading strain rate

DEVSTN = ABS(STNTOT(I) - STN(1,I) - STNS1(2,I) - STNS1(3,I))
IF(DEVSTN .GE. ERR) THEN
GOTO 5
ELSE

c Write results to a file
OPEN(13,FILE='D3.DAT')
WRITE(13,*)L,SIG(I),STN(I),STNS1(I),STNS2(I),STNS3(I)
END IF

L=L+1
STN(L)=STN(I)
STNS1(L)=STNS1(I)
STNS2(L)=STNS2(I)
STNS3(L)=STNS3(I)
ITOT=ITOT+1

END WHILE

C Write results at last time step to a file

OPEN(14,FILE='M3Z3END.DAT')
WRITE(14,*) ITOT, SIG(I), STN(I), STNS1(I), STNS2(I), STNS3(I)
CLOSE(14)
CLOSE(13)
RETURN
END

C Subroutine for zone 4 - Creeping at constant stress zero

SUBROUTINE ZONE4(DELT,CRETIME,N)
C Initialize variables

REAL E(3)
REAL V(3)
REAL ITOT
REAL SIG
REAL STN1
REAL STN2
REAL STN3
REAL C1
REAL STNS1(3,1000)
REAL C2
REAL STNIOT(1000)

C Recall values of the coefficients

DO J=1,N
OPEN(15,FILE='CONSTANTS.DAT')
READ(15,*) E(J), V(J)
PRINT*,E(J),V(J)
END DO
CLOSE(15)

C Recall the values from the last time step of previous zone

OPEN(14,FILE='M3Z3END.DAT')
READ(14,*) ITOT, SIG, STN1, STN2, STN3
CLOSE (14)

c  Loop until the time reaches the specified amount of creep time

     STEP=CRETIME/DELT
     DO I=1,STEP

c  Calculate the constant of integration for element 2

     C1=STN2*E(2)*(V(1)+V(2))/V(2)

c  Calculate the strain in element 2

     STNS1(2,I)=C1*EXP(-E(2)*((1/V(1))+(1/V(2)))*
                   (I*DELT-DELT))*
                   (V(2)/(E(2)*(V(1)+V(2)))))

     C2=STN3

c  Calculate the strain in element 3

     STNS1(3,I)=C2*EXP(-E(3)*((I*DELT-DELT)/V(3))

c  Find the total strain value

     STNTOT(I)=STNS1(2,I)+STNS1(3,I)
     ITOT=ITOT+1

c  Write results to a file

     OPEN(15,FILE="D4.DAT")
     WRITE(15,*) ITOT,SIG,STNTOT(I)
     END DO
     CLOSE(15)
     RETURN
END
NOTE TO USERS

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UMI
Model III - Four Zone Tensile Test

1. Enter testing parameters
2. Enter values for spring and dashpot coefficients
3. Retrieve values of $E$, $\eta$
4. Set all initial values of $\sigma$ and $\varepsilon$ to zero
5. Go to Zone 2

**Zone 1**

- Is $\sigma(t)$ not less than $\sigma_{\text{cutoff}}$?
  - Yes: Go to Zone 2
  - No: $\sigma_1(t) = \varepsilon_1(t) + E_1$
    - $\sigma_1(t) = \sigma_2(t)$
    - $\sigma_{\text{side1}}$, $\varepsilon_{\text{side1}}$, $\varepsilon_{\text{elt2}}$
    - $\varepsilon_2(t) = \varepsilon_2(t-1) + (\sigma_{\text{side1}}, \varepsilon_{\text{elt2}} + \sigma_{\text{side1}}, \varepsilon_{\text{elt2}}(t-1)) / 2 \Delta V / \eta_1$
    - $\sigma_1(t) = \sigma_3(t)$
    - $\sigma_{\text{side2}}, \varepsilon_{\text{side2}}, \varepsilon_{\text{elt3}}$
    - $\varepsilon_3(t) = \sigma_{\text{elt2}}(t) \varepsilon_{\text{elt3}} / E_3$
    - $\varepsilon(t) = \Sigma \varepsilon(t)$
    - $\varepsilon_1'(t) = \sigma_{\text{true}} - \varepsilon_2 - \varepsilon_3$
    - $\varepsilon_1'(t) = \varepsilon_1(t)$

6. Retrieve values of stress and strain from last timestep of zone 1
7. Go to Zone 3

**Zone 2**

- Is it the first timestep?
  - Yes: Calculate constant of integration from results of last zone
  - No: Write results to file: $\text{tstep} = 1$

- Is $t$ less than relaxation time?
  - Yes: Calculate $\sigma(t)$ using the constant of integration
  - No: Go to Zone 3

8. Write results to file: $\text{tstep} = \text{tstep} + 1$
9. $t(t_{\text{total}}) = t(\text{final zone}) + 1$

10. Go to $t = t + 1$

11. Stop, End
Model III - Four Zone Tensile Test - Continued

Zone 3
Retrieve values of E, n
Retrieve values of stress and strain from last timestep of zone 1

Is it the first timestep?

Yes
Write results to file: tstep=1
Go to Zone 4

No

Is e not less than 0?

No
strue = e + (ereate * t/Lo)

Δtrue = (ereate * Δt/Lo)

ε1(t) = ε1(t) + Δtrue

σ(t) = ε1(t) * E1

Δcs1ε1(ε1,t) = σ(t) * σ(t) / (η/1 + η/2) * σ(t-1) * exp(-E3 * τ/η3) * Δt/η3

cs1ε1(ε1,t) = ε1(t) + Δcs1ε1(ε1,t)

Δcs1ε1(ε1,t) = σ(t) * exp(-E3 * t/η3) * Δt/η3

cs1ε1(ε1,t) = ε1(t) + Δcs1ε1(ε1,t)

ε1(t) = ε1(t)

cσ = Σε

Does cσ = strue?

Yes
Write results to file

No

tstep = tstep + 1

Go to t = t + 1

Stop. End

Zone 4
Retrieve values of E, n
Retrieve values of stress and strain from last timestep of zone 1

Is it the first timestep?

No

Calculate constants of integration from results of last zone

Is t less than creep time?

Yes

Write results to file: tstep=1

No

Stop. End

Yes

cs1ε1(ε1,t) = C1 * exp((-E2 *(1/η1 + 1/η2) * t) / η2 / (E2 * (η1 + η2)))

cs1ε1(ε1,t) = C2 * exp(-E3 * t/η3)

Write results to file

tstep = tstep + 1

t(total) = t(final zone 1) + 1