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Simplicity and falsifiability a critical examination of Karl Popper's stipulative definition of simplicity.

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SIMPLICITY AND FALSIFIABILITY: A CRITICAL EXAMINATION OF EARL POPPER'S STIPULATIVE DEFINITION OF SIMPLICITY

by

Brian Paul MacPherson

A Thesis submitted to the Faculty of Graduate Studies through the Department of Philosophy in Partial Fulfillment of the requirements for the Degree of Master of Arts at The University of Windsor

Windsor, Ontario, Canada
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ABSTRACT

SIMPLICITY AND FALSIFIABILITY:
A CRITICAL EXAMINATION OF
KARL POPPER'S STIPULATIVE DEFINITION OF SIMPLICITY

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In chapter seven of *The Logic of Scientific Discovery*, (i.e. L.Sc.D.), Karl Popper outlines several problems in epistemology (and in particular induction) connected with the concept of simplicity. For example, given "n" data-points and an infinite number of possible curves which can "fit" these points, which of these curves is the simplest and, for that matter should we prefer the simplest curve? Popper attempts to resolve problems such as the ones just mentioned by equating a theory's degree of simplicity with its degree of falsifiability. Thus, we would desire (or prefer) the simplest curve because it is also the most falsifiable. And, we desire high falsifiability because, as Popper argues in chapter one of L.Sc.D., falsifiability is what distinguishes scientific from non-scientific theories.

In order to see how certain other problems (in epistemology) connected with the concept of simplicity can be resolved by equating simplicity with degree of falsifiability, it will be necessary to outline the two methods which Popper proposes in chapter six of L.Sc.D. for comparing theories' degrees of falsifiability. First, by his "subclass"
method, a theory $T_1$ is more falsifiable and hence simpler than another theory $T_2$ if $T_1$'s empirical content (defined as the class of all those singular statements which can contradict $T_1$, viz., its "potential falsifiers") includes $T_2$'s empirical content. Consequently, $T_1$ will be easier to falsify than $T_2$ since everything that can falsify $T_2$ can also falsify $T_1$, but not vice-versa. Second, using Popper's "dimension" method, if the number of (relatively atomic) statements needed to refute $T_1$ is smaller than the number of statements needed to refute $T_2$ then $T_1$ will be easier to refute and therefore more falsifiable and simpler than $T_2$.

Applying the "dimension" method outlined above to the problem of which curve (from amongst $n$ curves) is to count as the simplest (or where $n=2$, the simpler), a theory expressible as a circle-equation, for example, would be regarded as being simpler than a corresponding ellipse-theory. It will be noted that this agrees with our intuitions. That is, a circle-theory can be more easily falsified than an ellipse-theory. This is so because we only need (at minimum) four statements corresponding to four points, not all lying on the curve, to falsify the former, whereas, we need at least six statements corresponding to six points, not all lying on the curve, to falsify the latter.

The expository material mentioned in the preceding paragraph is outlined in the first chapter of the thesis. In the second chapter, other authors' criticisms of Popper's stipulative definition of simplicity (along with related criticisms) are critically reviewed. For example, Carl Hempel, in his Philosophy of Natural Science, argues that, if we conjoin two unrelated hypotheses, the resulting conjunction "$H_1 \land H_2$" will be more falsifiable than either of the conjuncts and yet, it would be wrong to say that "$H_1 \land H_2$" is simpler than either conjunct.
However, Hempel fails to recognize the stipulative nature of Popper’s definition of simplicity. As Popper states in chapter seven of L.Sc.D., he is not concerned with whether or not the concept of simplicity which he is clarifying is the one which epistemologists normally mean by simplicity. He is merely proposing his clarification of simplicity in order to resolve certain problems in epistemology (briefly mentioned above).

Having critically reviewed other authors’ criticisms of Popper’s definition of simplicity in the second chapter of the thesis, we then propose some of our own criticisms of this definition in the third chapter. First, in chapter seven of L.Sc.D., Popper makes it clear that he is excluding from discussion such pragmatic concerns as the relative ease (or simplicity) of performing certain tasks. Such matters are of no interest to the epistemologist, says Popper. Yet, as we have seen, a circle-theory is simpler than an ellipse-theory because the former is easier (or simpler) to refute than the latter. Consequently, by the force of Popper’s own distinction between matters epistemological and matters practical, it is clear that he has only shown us from a practical viewpoint why, for example, a circle-theory is simpler than an ellipse-theory. That is, the circle-theory is simpler than the ellipse-theory, since the former is easier or simpler to refute than the latter. But, this is uninteresting from an epistemological viewpoint.

Further, in chapter three an alternative definition of simplicity is proposed which, for example, is able to explain the simplicity of a circle-theory over that of an ellipse-theory, without appealing to such practical notions as ease of refutation. This clarification of
(relative) simplicity can be summarized as follows: A theory's simplicity is to be the same as its "ontological" simplicity. Further, a theory's (relative) ontological simplicity will be defined in terms of Ockam's Razor, which dictates that we keep our ontological commitments to a minimum. Thus, $T_1$ is simpler, and hence ontologically simpler, than $T_2$, in which case $T_1$ rules out certain events (or entities) which $T_2$ does not rule out.
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VITA AUCTORIS
GENERAL INTRODUCTION

If we have a data-points and an infinite set of possible curves which would describe these points, should we choose the simplest curve here? And for that matter, what makes one curve simpler than another? In his *The Logic of Scientific Discovery*, (hereafter to be called L.Sc.D.), Karl Popper attempts to answer these questions by equating simplicity with degree of falsifiability. Thus, a curve (or a law expressed as a general equation of this curve) is simpler than another curve (or law) if the former is more falsifiable than the latter. Furthermore, because falsifiability is the distinguishing mark of a scientific theory, it stands to reason that we shall prefer the law which is most highly falsifiable and hence the simplest.

But, what does it mean for one theory to be more falsifiable than another? Popper proposes two methods for comparing theories' degrees of falsifiability. Briefly, if the empirical content of a theory $T_1$ (defined as all those singular (basic) statements which can contradict $T_1$) includes the empirical content of another theory $T_2$ (in the sense that a class includes a subclass) then $T_1$ is more falsifiable than $T_2$. Everything that can falsify $T_2$, can falsify $T_1$, but not everything that can falsify $T_1$, can falsify $T_2$, in which case $T_1$ is easier to falsify (and hence more falsifiable) than $T_2$. This method involves the notion of "subclass relations". The second method which Popper proposes, viz., the "dimension" method, can be characterized as follows:
If it takes fewer (relatively) atomic statements to refute \( T_1 \), than to refute \( T_2 \), we can say that \( T_1 \) is more falsifiable than \( T_2 \). Further, it is clear that \( T_1 \) is more easily falsified than \( T_2 \). For example, a theory expressible as a circle-equation can be refuted by four relatively atomic statements (corresponding to four points on a graph) whereas we need at least six statements (or points on a graph) to refute an ellipse-theory. Thus, the circle-theory is more (easily) falsifiable than the ellipse-theory. Notice that in both the subclass and dimension methods, degree of falsifiability seems to be closely tied with ease of falsification.

Also, in relation to the point that since high falsifiability is desirable so is high simplicity, it might be asked why Popper regards falsifiability as being the criterion which distinguishes science from non-science. In \( L.Sc.D. \), Popper argues that induction cannot be (logically) justified and hence we cannot claim that what distinguishes science from non-science is that in science we are able to argue from the truth of singular statements to the truth and falsity of universal statements. Thus, if we did require that for a statement to be scientific, its truth as well as its falsity must be decidable, then no theory could be admitted as being scientific. To avoid this problem, Popper argues that we simply require that it be possible to argue from the truth of singular statements to the falsity of theories for these theories to be admitted as "scientific". And, for this type of reasoning, we have this modus tollens of classical logic at our disposal. Hence, there is no problem of finding a principle of falsifying inference.

All of the material mentioned above will be covered in the expository chapter of the thesis. In the second chapter of this thesis,
some of the criticisms of Popper's stipulative definition of simplicity along with certain related criticisms will be reviewed and assessed. One of the main criticisms made by Sober, Bunge, Hempel and Post against Popper's dimension method for comparing degrees of falsifiability is the following: It is argued that Popper's dimension method adequately handles laws expressed as general equations (e.g., a circle is simpler than an ellipse) although it leads to counter-intuitive results in the case of equations whose parameters are specified (e.g., \(x^2 + 2x + 3\) is equally simple to \(y = 3x + 4\) which is counter-intuitive).

Also, it has been objected by Barker and Hempel that there will be cases where \(T_1\) is assessed as more falsifiable than \(T_2\) by the subclass method and yet clearly \(T_1\) is not simpler than \(T_2\). Further, both Ayer and Wittgenstein argued against the necessity of finding a principle of induction which in turn casts doubt on the necessity of Popper's falsifiability proposal and hence the desirability of high falsifiability and high simplicity. Finally, from the history of science, it is not clear that Popper's methodology which he proposes in order to save his falsifiability proposal would fare very well. In fact, if applied to certain situations, such as the Copernican Revolution, Paul Feyerabend, in Against Method, argues that progress would have been hindered. But, if this is so, it is not clear that Popper's falsifiability proposal can be saved in which case it is no longer clear that high falsifiability and high simplicity are so desirable. As we shall later see, some of these objections can be dealt with by Popper and some seem to be fairly conclusive.

Finally, in chapter three, it will be argued that by the force
of Popper's own distinction between matters epistemological and matters practical, whether or not the task of falsifying $T_1$ is easier (i.e., simpler) than the task of falsifying $T_2$ is more a practical matter than an epistemological concern. Consequently, it is not clear that Popper's definition of simplicity really resolves problems in epistemology—it only shows why one curve is simpler to falsify than another. Also, an alternative (stipulative) definition of simplicity will be offered which does not make use of the practical notion of ease of refutation: A theory $T_1$ will be said to be simpler than another theory $T_2$ if $T_1$ is ontologically simpler than $T_2$. The notion of ontological simplicity will be spelled out in terms of Occam's razor, which dictates that ontological commitments be kept to a minimum.
CHAPTER ONE

EXPOSITION

Introduction

Before outlining Popper's stipulative definition of the epistemological concept of simplicity in terms of degrees of falsifiability (along with his two methods for comparing theories' degrees of falsifiability) in Parts III - V, we shall in Parts I and II be outlining relevant background material. This background material includes Popper's argument in support of the claim that we should adopt his proposal that falsifiability demarcates science from non-science (along with certain methodological proposals which ensure the applicability of his criterion of demarcation). This material is important because, if we can find difficulties in Popper's arguments in support of his falsifiability proposal, then it is no longer clear that we should desire high falsifiability and therefore, high simplicity. Also, we shall examine Popper's characterization of falsifiability and falsification in terms of the relation between basic statements and theories: Understanding the notions of "basic statement" and "theory" (and their relationships) will be helpful in our examination of Popper's discussion of his methods for comparing theories' degrees of falsifiability. Further, if we can find difficulties in Popper's characterization of falsifiability and falsification, then again it is not clear that high falsifiability and hence, high simplicity are so desirable.
Part I: Popper's Argument in Support of his Falsifiability Proposal

In chapter one of L.Sc.D., Popper notes that an attempt has been made to characterize science as involving the inductive method which enables scientists to make inferences from singular statements to universal statements. However, this characterization of science runs into immediate difficulties, viz., in what sense are we justified in inferring from a finite number of instances (no matter how large "n" is) of A's being B's that all A's are B's? The problem of finding a way of justifying inductive inferences is called by Popper the problem of induction. Alternatively, Popper notes that the problem of induction can also be characterized as follows:

The problem of induction may also be formulated as the question of how to establish the truth of universal statements which are based on experience, such as the hypotheses and theoretical systems of the empirical sciences.

Or, as Popper also says, how can we reduce the truth of universal statements to the truth of singular statements? Also, Popper points out that the implied criterion of demarcation in logical positivism is none other than the demand for an inductive logic, i.e., the demand that the truth of universal statements can be reduced to the truth of singular statements:

Accordingly, they wish to admit, as scientific or legitimate, only those statements which are reducible to elementary (or 'atomic') statements of experience. It is clear that the implied criterion of demarcation is identical with the demand for an inductive logic.

This criterion of demarcation implied by positivism is identical to the criterion of demarcation inherent in the inductivist attempt to find a way of being able to reduce the truth of universal statements to the
truth of singular statements. That is, this criterion of demarcation shared by inductivism and logical positivism is the assumption that the truth of universal statements or theories can be decided, as well as their falsity:

The criterion of demarcation inherent in inductive logic—that is, the positivistic dogma of meaning—is equivalent to the requirement that all the statements of empirical science . . . must be capable of being finally decided, with respect to their truth and falsity; we shall say they must be conclusively decidable.7

However, the plausibility of this shared criterion of demarcation, viz., that statements must be decidable with respect to both their truth and falsity, rests on the success of finding a principle of induction8 which would make possible the reduction of the truth of universal statements to the truth of singular statements. (Or this principle of induction would justify inductive inferences.9) But, as Popper argues, the attempt to formulate a principle of induction ends in failure. First, we might regard the principle of induction as being a tautological or analytic statement. However, the problem here, says Popper, is that inductive inferences would merely be "purely logical or tautological transformations, just like inferences in deductive logic."10 Next, if we regard the principle of induction as a universal synthetic statement, there is a further problem, viz., how do we justify this (universal) synthetic principle? In order to justify this principle, a higher order principle must be formulated which, in turn, must be justified (since it cannot be a tautology) ad infinitum.11

Thus the attempt to base the principle of induction on experience breaks down, since it must lead to an infinite regress.12
Popper does not a possible solution, viz., vis a vis Kant, we might regard the principle of induction as being a priori valid or justified, although Popper finds this solution to be unacceptable. Similar problems arise, says Popper, if we regard inductive inference as probable inference. That is, the principle which would assign probability values to inductively inferred statements would itself (if it is not a tautology) be in need of a probability assignment, ad infinitum. The alternative, viz., "a priorism" (i.e. regarding this principle as being a priori valid), Popper finds unacceptable. And so, he concludes:

In short, like every other form of inductive logic, the logic of probable inference, or 'probability logic', leads either to an infinite regress, or to the doctrine of a priorism.

And so, since the attempts to formulate a principle of induction which would enable us to reduce the truth of universal statements to the truth of singular statements (i.e. to decide the truth of universal statements) have all failed, it is not clear that the truth of universal statements is decidable. But, this requirement of decidability (with respect to truth as well as falsity) is none other than the criterion of demarcation shared by the positivists and the inductivists. Consequently, if we were to adopt this (shared) criterion of demarcation, then no statement could be admitted as scientific, since there is no principle which enables us to decide the truth of a statement by its reduction to other statements.

And it is precisely over the problem of induction that this attempt to solve the problem of demarcation comes to grief: positivists, in their anxiety to annihilate metaphysics, annihilate natural science along with it. For scientific laws, too, cannot be logically reduced to elementary statements of experience.
It is at this point that Popper proposes that we can distinguish or demarcate science from non-science if we simply require that it be possible to decide the falsity of statements in science on the basis of empirical tests or "experience":

I shall not require of a scientific system that it shall be capable of being singled out, once and for all, in a positive sense; but I shall require that its logical form shall be such that it can be singled out, by means of empirical tests, in a negative sense: it must be possible for an empirical scientific system to be refuted by experience.

Further, in only requiring that the falsity and not also the truth of scientific statements be decidable, we avoid the problems encountered by the inductivists and positivists, viz., the problem of trying to find a principle making falsifying inference possible. This is so because in arguing to the falsity of theories from the truth of singular statements we can employ the already available (and uncontroversial) inferences of deductive logic:

Consequently, it is possible by means of purely deductive inferences (with the help of the modus tollens of classical logic) to argue from the truth of singular statements to the falsity of universal statements.

This "falsifying mode of inference" will be briefly outlined in Part II of this exposition.

Before continuing, it will be helpful at this point, to place the material so far discussed into perspective. First, Popper makes it clear that his falsifiability criterion of demarcation is "a proposal for an agreement or convention." He contrasts his view of his own criterion of demarcation to the way the positivists view their criterion of demarcation. That is, the positivist, according to Popper, believes that scientific theories are inherently reducible to singular statements.
and it is this inherent quality that distinguishes scientific statements from non-scientific statements. Popper calls this view "naturalistic". Second, Popper points out that his only argument in support of his falsifiability proposal (i.e. his proposal that we agree to only require falsifiability for a statement to be called scientific) is that this proposal (by means of its "logical consequences") enables us to resolve (and explain) certain difficulties in epistemology:

There is only one way, as far as I can see, of arguing rationally in support of my proposals. This is to analyze their logical consequences: to point out their fertility—they're the power to elucidate the problems of the theory of knowledge.

As we have already seen, Popper's falsifiability proposal supposedly resolves the problem of demarcation, since given the modus tollens of classical logic, we can decide the falsity of theories in science and therefore it is not the case (unlike induction) that no theory (by this falsifiability criterion) is scientific. Further, his falsifiability proposal supposedly resolves the problem of induction by waiving the requirement that it must be possible to decide both a theory's truth and falsity. If we simply require that theories' falsity be decidable, then we no longer have the problem of finding a principle of induction, since we already have "the tautological transformations of deductive logic whose validity is not in dispute." However, Popper notes that ultimately whether or not one adopts his falsifiability proposal depends on one's views of what the goals or purpose of science should be (i.e. one's values). Consequently, being receptive or open to his argument for his falsifiability proposal presupposes certain shared values:
As to the suitability of any such convention opinions may differ; and a reasonable discussion of these questions is only possible between parties having some purpose in common. The choice of the purpose must, of course, be ultimately a matter of decision, going beyond rational argument.\textsuperscript{29}

One of these values is none other than "freedom from dogmatism."\textsuperscript{30}

Further, Popper admits that in the "last analysis" he was guided by values such as freedom from dogmatism in arriving at his proposals.\textsuperscript{31}

Although with the help of the "purely tautological transformations" of deductive logic, we can reason to the falsity of theories, Popper notes that it could still be objected that no theory can ever be conclusively falsified.\textsuperscript{32} This is so because a theory's falsification can always be evaded by the introduction of ad hoc auxiliary hypotheses which would take the brunt of the falsification or we could doubt the reliability of the experimenter, etc:

It might be said that ... it is still impossible for various reasons, that any theoretical system should ever be conclusively falsified. For it is always possible to find some way of evading falsification, for example by introducing ad hoc an auxiliary hypothesis ... \textsuperscript{33}

In chapter four of L.Sc.D., Popper gives a more detailed exposition of this objection to his falsifiability proposal under the name "conventionalism." According to the conventionalist position, scientific laws (or theories) are not falsifiable by experiments or tests since they (i.e. the laws or theories) determine the measurements that are used in these experiments.\textsuperscript{34} Consequently, the conventionalist methodology dictates that such measures as the introduction ad hoc of auxiliary hypotheses be employed in falsifying instances in order to ensure the agreement between theory and "reality".\textsuperscript{35} Popper, admitting the soundness of this objection, notes that there is a moral to be drawn from
all this, viz., that the empirical sciences cannot be characterized by his criterion of demarcation alone. Therefore, he proposes to also define empirical science in terms of its "empirical method", a method which disallows measures which, if employed, would result in a theory's having evaded falsification.

Further, Popper characterizes this method in terms of certain methodological rules which he proposes:

... I tried to define empirical science with the help of the criterion of falsifiability; but as I was obliged to admit the justice of certain objections, I promised a methodological supplement to my definition. Just as chess might be defined by the rules proper to it, so empirical science may be defined by means of its methodological rules.

In light of his promise to characterize the empirical method as a method which prevents theories' evasion of falsification, Popper notes that all of his methodological rules are governed by the following metarule: All rules must ensure that no theory (or statement) can evade falsification.

As an example of a methodological rule, Popper cites the rule that no well-corroborated theory can be abandoned without good reason, a good reason being that a more falsifiable theory has been found or that the established theory is falsified. Note here that this rule requires the abandonment of a theory if it is falsified, which is in agreement with the metarule outlined above. Also, in chapter four, Popper cites several "anti-conventionalist" rules whose function is to counter the conventionalist methodological dictates. For example, one of those "counter-rules" requires that an auxiliary hypothesis can be introduced only if it increases a theory's falsifiability (as opposed to the conventionalist rule that auxiliary hypotheses be introduced to ensure
agreement between theory and "reality"). In chapter two of L.Sc.D., Popper also notes that his methodological proposals function in the following capacity: They ensure the applicability of his criterion of demarcation. That is, it was objected to Popper's requirement, that for a theory to be scientific it must be falsifiable, that in principle, no theory can be conclusively falsified since it is always possible to introduce auxiliary hypotheses, etc. Popper's response to this is to introduce certain methodological proposals which would disallow theories' evasion of falsification thus ensuring that theories can be falsified or are falsifiable.

Finally, it should be noted that Popper regards his methodological proposals like his falsifiability proposal, to be proposals for conventions. As Popper states in chapter one, "Methodological rules are here regarded as conventions." Further, as with the issue of demarcation, Popper rejects the "naturalistic" approach to methodology which involves the use of "the methods of empirical science" to ascertain whether or not scientists use this or that method such as the method of induction. He further chastises those who take this naturalistic approach to methodology for not realizing that "whenever they believe themselves to have discovered a fact, they have only proposed a convention." However, it is interesting to note at this point that in a later work, *Conjectures and Refutations*, Popper makes the "naturalistic" claim that induction is not the procedure of science, but rather the procedure used in science is that of exposing our theories (as conjectures) to potentially refuting tests.
(1) Induction, i.e. inference based on many observations, is a myth. It is neither a psychological fact, nor a fact of ordinary life, nor one of scientific procedure.

(2) The actual procedure of science is to operate with conjectures: to jump to conclusions—often after one single observation. . .

(3) Repeated observations and experiments function in science as tests of our conjectures or hypotheses; i.e. as attempted refutations.45

Yet, Popper could respond here that claim (1) is based on his (and Hume’s) logical criticism of induction, viz., that there is no “rule” which can “guarantee that a generalization inferred from true observations, however often repeated, is true.”47 Also, Popper could point out that claims (2) and (3) are based on purely logical considerations.48 That is, these so-called logical considerations have to do with the logical priority of “interpretations” over the observation of similarities. That is, it is impossible to see one situation as similar to another unless we already expect them to be similar.49 In short, we interpret them as being similar and therefore we impose our interpretations on these two situations.

Popper then extends these considerations to science:

... I thought that it would apply in the field of science also; that scientific theories were not the digest of observations, but that they were inventions—conjectures boldly put forward for trial, to be eliminated if they clashed with observations.50

In any event, in his Conjectures and Refutations, scientific method seems to be more than just a class of methodological conventions. It seems to be the way scientists actually behave.

Part II: Popper’s Characterization of Falsifiability and Falsification

In chapter four of L.Sc.D., Popper notes that he will provide
us with a logical characterization of falsifiability in terms of the relations between "basic statements" and "theories":

We shall attempt to characterize the falsifiability of a theory by the logical relations holding between the theory and the class of basic statements. 51 Consequently, in order to fully appreciate Popper's characterization of falsifiability (and falsification) our first task will be to get clear on what he means by the terms "basic statement" and "theory".

First, in chapter three of L.Sc.D., Popper argues that theories in science are strictly universal statements (as opposed to numerically universal statements):

It is strictly universal statements which I have in mind so far when speaking of universal statements—of theories or natural laws. 52 A universal statement which is "strict" (or "pure") is one which applies to an unlimited number of individuals, since it "claims to be true for any place and any time." 53 On the other hand, a universal statement which is "numerical" applies to a limited number of individuals, since it is restricted to a "finite" spatio-temporal region. 54 (As an example of a numerically universal statement, Popper cites the following: "All human beings now living on the earth do not exceed the height of eight feet." 55) Popper points out that since a numerically universal statement applies to a limited number of individuals (and therefore each individual can be enumerated), it is equivalent to a conjunction of "n" singular statements. 56 Thus, he notes that numerically universal statements can be classed as being "singular" themselves. 57 In contrast to numerically (or singularly) universal statements, strictly universal statements are not equivalent to a conjunction of "n" singular statements.
owing to the fact that they refer to an unlimited number of individuals. Popper is aware that his distinction between numerical (or singular) and strict (or purely) universal statements is not consistent with the treatment given universal statements in classical logic; for in classical logic all universal statements are regarded as being equivalent to a finite or infinite conjunction of "singular" statements. Of course, what Popper neglected to note is that even certain infinite classes are such that their individual members can be enumerated by placing these individuals in one-to-one correspondence with the natural numbers (which itself is infinite). Such a set or class is said to be "countably infinite" as Kleene notes in *Mathematical Logic*:

Collections which can be placed in 1-1 correspondence with the natural numbers we call countably infinite or denumerably infinite.

Thus the classical logician could, referring to Popper's point that all singular universal statements are equivalent to "n" singular statements, since the individuals to which they refer are enumerable, argue that what Popper calls strictly universal statements are themselves equivalent to a conjunction of "n = ∞" singular statements. This is so because even the infinite number of individuals to which the strictly universal statement refers are enumerable by placing these individuals in one to one correspondence with the natural numbers. However, Popper dismisses all arguments for or against his proposal that we regard the theories or laws of science as being strictly universal statements. Popper, instead proposes that as a matter of convention we regard theories or laws as strictly universal statements:

... the question whether the laws of science are strictly
or numerically universal cannot be settled by argument. It is one of those questions which can be settled only by an agreement or a convention.61

Another type of "strict" or "pure" statement which Popper notes is what he calls strict or purely existential statements.62 A purely existential statement is an existential statement which (like strictly universal statements) contains no individual names.63 Thus, the statement, "There are black ravens" is purely existential whereas the statement, "There are now in Windsor black ravens" since it contains the individual names "Windsor" and "Now" is not purely existential. (In fact, as Popper later notes, this latter statement is singular.)64 That is, this latter statement refers to a finite space-time region. As we shall next see, this notion of purely existential statements figures heavily into Popper's characterization of theories' falsifiability.

In agreement with classical logic,65 Popper argues that a strictly universal statement such as "All ravens are black" (or symbolically, "(x) (Rx → Bx)") is equivalent to the negation of the purely existential statement "There are no non-black ravens" (i.e. "\( \neg \exists x (Rx \land \neg Bx) \)).66 Consequently, the laws or theories of science as strictly universal statements are equivalent to (or expressible as) negations of strictly existential statements:

The theories of natural science, and especially what we call natural laws, have the logical form of strictly universal statements; thus they can be expressed in the form of negations of strictly existential statements...67

Further, because a theory is expressible as the negation of a purely existential statement, it can be regarded as denying the existence of a thing in which case it rules this thing out. For example, the theory
that all ravens are black can be written as "There are no non-black ravens." In short, if we express this theory as the negation of a strict existential statement, we can see that it prohibits or rules out non-black ravens by denying their existence:

In this formulation we see that natural laws might be compared to 'proscriptions' or 'prohibitions'. They do not assert that something exists or is the case; they deny it. They insist on the non-existence of certain things or states of affairs: they rule them out.88

Also, Popper points out that a theory's falsifiability lies in the fact that it prohibits or rules out certain states of affairs (or events) by denying their existence (or their occurrence in the case of events).

That is, a theory which prohibits a certain thing or event runs the risk of being contradicted and hence falsified by a singular statement asserting that this thing (or event) which the theory rules out really does exist:

And it is precisely because they do this that they are falsifiable. If we accept as true one singular statement which, as it were, infringes the prohibition by asserting the existence of a thing (or the occurrence of an event) ruled out by the law, then the law is refuted.89

Thus, if the theory denies P's existence but the singular statement (which has been accepted) asserts P's existence, then the theory has been contradicted, and hence refuted. (Although, as we shall later see, the statement asserting P's existence must actually be deduced from the singular statements.)90 Also, Popper notes that in contrast to their negations (viz., the theories and laws) purely existential statements are, by his falsifiability criterion, "metaphysical".91 That is, they are not falsifiable. A statement which asserts, for any place and
any time that there exists at least one P can never be refuted owing to the fact that it is impossible to search the entire universe past, present and future to prove that there are no P's:

Strict or pure statements, whether universal or existential, are not limited as to space and time. They do not refer to an individual, restricted, spatio-temporal region. This is the reason why strictly existential statements are not falsifiable. We cannot search the whole world in order to establish that something does not exist, has never existed, and never will exist.72

Before examining in more detail the logical form of these singular statements which, if accepted, result in a theory's refutation, we should get clear on just what Popper means by his assertion that a singular statement asserts the occurrence of an event.73

For the purpose of clarification, we shall examine exactly what Popper means by the terms "occurrence" and "event". In chapter four of L.Sc.D., Popper first defines the notion of an occurrence (which he eventually uses to define the term "event"), as follows: A singular statement PK such as "It is now thundering here." and all statements equivalent to it such as, "It is thundering now at Tecumseh and Ouellette," are elements of the class which Popper calls the occurrence PK, viz., "that it is now thundering here".74 Also, Popper notes that we might say that the statement PK represents the occurrence PK:

The realistic formulation 'The statement PK represents the occurrence PK' can then be regarded as meaning the same as the somewhat trivial statement 'The statement PK is an element of the class PK of all statements which are equivalent to it.75

Further, according to Popper, an event is "what may be typical or universal about an occurrence."76 Thus, the event P (i.e. thundering) is what is typical about the occurrence "that it is now thundering here".
In terms of classes, the class of occurrences having as its elements \( p_K \) (that it is now thundering here), \( p_L \) (that it was thundering there), etc. which differ in terms of their spatio-temporal regions is called the event \( P \) (i.e. thundering). In short, the event \( P \) is the class of occurrences \( p_K, p_L, \) etc., each of which in turn is a class of singular statements (such as \( p_K \) and all statements equivalent to it, if we are talking about \( p_K \)). Thus, the statement, \( p_K \), "It now is thundering here" represents (or is an element of the class of) the occurrence \( p_K \) ("that it is now thundering here") which is typified by the event \( P \) (thundering). Or, we can also interpret the singular statement \( p_K \) as asserting the occurrence of the event \( P \) at the space-time region, \( K \). (\( p_L \) would assert the occurrence of the event \( P \) at \( L \), etc.).

Speaking of the singular statement \( p_K \), which represents an occurrence \( p_K \), one may say, in the realistic mode of speech, that this statement asserts the occurrence of the event \( P \) at the spatio-temporal position \( K \).

And so, singular statements can be said to assert the existence of things or states of affairs (at, as we shall later see, various spatio-temporal regions) or as we have just seen, they can also be said to assert the occurrences of events at various \( K \)’s. Further, a singular statement which refutes a theory does so by asserting the existence of a thing (or the occurrence of an event) at \( K \) which is forbidden by this theory. Popper calls these statements which can serve as falsifiers of a theory "basic statements":

What I call a 'basic statement' or a 'basic proposition' is a statement which can serve as a premise in an empirical falsification.?
"basic statements".

In chapter five of L. Sc. D., Popper specifies the logical form of basic statements with reference to the following two formal conditions which all basic statements must satisfy. First, no basic statement is derivable from a theory alone without the aid of other singular statements, or "initial conditions". Second, it must be possible for basic statements to contradict theories. Commenting on these conditions, Popper argues that the second condition can be fulfilled only if it is possible to derive the negations of the basic statements which contradict the theory from this theory without initial conditions. (Thus, from any theory which denies the existence of P, and which can therefore be contradicted by basic statements asserting P's existence, we can deduce such statements as "There are no P's at K", "There are no P's at L" etc. Further, these are negations of the basic statements asserting that there are P's at K, L, etc.) Further, Popper notes that keeping in mind condition one (i.e. that no basic statements are derivable from theories alone) with the requirement that negations of basic statements are derivable from theories, it follows that the logical form of basic statements must be different from that of their negations. Thus, in attempting to specify the logical form of basic statements, we have as a "guideline" or as a "frame of reference" the conclusion that basic statements must have logical forms different from that of their negations.

Keeping this "guideline" in mind, Popper points out that we have already seen such an instance where statements have different logical forms from those of their negations, viz., at the level of
strict or pure statements. That is, a universal statement such as "(x) (Rx → Bx)" has a different logical form from its negation, viz., "(∃ x) (Rx & ∼Bx)." Popper points out that by analogy this is also the case at the level of singular statements. The only difference at this level is that spatio-temporal regions have been specified. Thus, Popper characterizes basic statements as having the following logical form:

A statement of the form 'There is a so-and-so in the region K' or 'such-and-such an event is occurring in the region K' may be called a 'singular existential statement' or a 'singular there-is statement.' ... basic statements have the form of singular existential statements.

Also, a basic statement as a singular existential statement (i.e. an existential statement which specifies a space-time region as, for example, "(∃ x) (Rx & ∼Bx & Kx)" where K is the spatio-temporal region) will have as its negation what Popper calls a "singular there-is-not statement." Thus, if our basic statement is "(∃ x) (Rx & ∼Bx & Kx)" its negation will be the negation of a singular existential statement, viz., "(¬∃ x) (Rx & ∼Bx & Kx):

And the statement which results from negating it, i.e. 'There is no so-and-so in the region K' or 'No event of such-and-such, a kind is occurring in the region K' may be called a singular non-existence statement,' or a 'singular there-is-not statement.'

Now notice here that just as a strict non-existence statement (i.e. "(x) (Rx → Bx)" or "¬(∃ x) (Rx & ∼Bx)") has a different logical form from its negation (i.e. "(∃ x) (Rx & ∼Bx)"") it is also the case that a singular non-existence statement (i.e. "(¬(∃ x) (Rx & ∼Bx) & Kx)"") has a different logical form from its negation, a singular existential statement or basic statement (i.e. (∃ x) (Rx & ∼Bx & Kx)). Further, Popper points out that if we characterize basic statements' logical form as
being that of singular existential statements then all basic statements are able to satisfy both formal conditions outlined above, which all basic statements must satisfy: That is, recalling that basic statements can be expressed as singular existential statements (ex: \( \exists x \) (Rx & \( \neg \exists x \) ) ) and that theories can be expressed as the negation of strict existential statements (ex: \( \neg \exists x \) (Rx & \( \neg \exists x \) )) it is clear that no basic statements can be derived from theories alone. Thus condition one is satisfied.\(^9\) (That is, from "There are no non-black ravens" we obviously cannot conclude that there is a non-black raven at K, l, etc.) Also, Popper says that condition two, viz. that it must be possible for basic statements to contradict theories, can be easily satisfied because from a singular existential statement such as "There is a non-black raven at K" we can deduce "There exists at least one non-black raven" which is a strict existential statement.\(^9\) And, this strict existential statement is the contradictory of the theory as a strict non-existence statement.\(^9\) (Thus, "There exists at least one non-black raven" is the contradictory of "There are no non-black ravens.")

Further, Popper points out that if we can join two or more non-contradictory basic statements, the resulting conjunction will in turn be a basic statement:

It should be noticed that the conjunction of two basic statements, \( d \) and \( r \), which do not contradict each other, is in turn a basic statement.\(^9\)

This is an important point to keep in mind because as we shall see in our exposition of Popper's dimension method for assessing theories' degrees of falsifiability (Part IV), a basic statement's "compositeness" or degree of composition is taken into account. Also, before concluding
our exposition of Popper's treatment of basic statements we should also note that Popper requires basic statements to satisfy one "material condition", viz., they must assert the occurrences of observable events.\textsuperscript{94} Popper, in order to escape the charge of psychologism, points out that the notion of "observable event" need not be given a psychologistic characterization (although he says that such a characterization is possible).\textsuperscript{95} That is, we can regard a statement which asserts an "observable" event's occurrence as making an assertion about the movement and position of macroscopic bodies:

Or we might lay it down, more precisely, that every basic statement must either be itself a statement about relative positions of physical bodies, or that it must be equivalent to some basic statement of this 'mechanistic' or 'materialistic' kind.\textsuperscript{96}

Spelling out the notion of observable event in terms of positions of physical bodies rather than in terms of observation, Popper supposedly avoids the charge of "psychologism." Further, a basic statement, since it asserts the relative position of physical bodies, is according to Popper intersubjectively testable.\textsuperscript{97}

Now that we have clarified what Popper means by the terms "basic statement" and "theory" (in terms of which he characterizes falsifiability and falsification), we are in a position to examine his characterization of falsifiability and falsification. First, a theory is falsifiable according to Popper if it divides all possible basic statements into two non-empty subclasses, viz., the class of basic statements which the theory forbids or which contradict the theory and second, those basic statements which the theory permits or which do not contradict it.\textsuperscript{98} Or, more concisely:
... a theory is falsifiable if the class of its potential falsifiers is non-empty.99

Also, towards the end of chapter four, Popper expresses what it is for a theory to be falsifiable in terms of "events". That is, a theory which is falsifiable is one whose class of potential falsifiers contains at least one non-empty class of "homotypic" basic statements, i.e. basic statements representing various occurrences of one event, (i.e. one forbidden event).

We can then say that every non-empty class of potential falsifiers of a theory contains at least one non-empty class of homotypic basic statements.100

Further, since the theory is not restricted in terms of space and time (and consequently it refers to an infinite number of individuals), it follows that there will be an infinite number of basic statements asserting an infinite number of occurrences of this forbidden event.101 In short, a falsifiable theory is one which rules out "at least one event" (which is described by an infinite number of basic statements).

As we have just noted, a theory which is falsifiable will have a non-empty class of potential falsifiers; i.e. it will have a non-empty class of forbidden basic statements which are inconsistent with the theory. If we actually accept one or more of these basic statements which contradict the theory then we can still not say that the theory in question has been falsified, according to Popper, because these accepted basic statements must also corroborate a "falsifying hypothesis".102 And so, for Popper, there are two conditions, each of which is necessary and jointly sufficient, which must be satisfied for a theory's falsification, viz. there must be accepted basic statements which contradict
the theory and these basic statements must also corroborate a falsifying hypothesis:

If accepted basic statements contradict a theory, then we take
them as providing sufficient grounds for its falsification only
if they corroborate a falsifying hypothesis at the same time.\textsuperscript{103}

We shall comment on each of these conditions, both of which are to be
satisfied for a theory's falsification.

First, with respect to condition one, the basic statements
which contradict the theory in question have been accepted. But, we
might now ask on what basis these statements have been accepted, i.e.
what are Popper's criteria for acceptance? Popper notes in chapter five
of \textsc{ls.
Sc.D.} that even basic statements (like theories) are testable,
since from the basic statement in question in conjunction with some
theory, other basic statements can be derived which are then used to
test this basic statement.\textsuperscript{104} Further, each of these derived basic
statements can themselves be tested, ad infinitum.\textsuperscript{105} Admitting that
logically we can go on testing basic statements ad infinitum, Popper
argues that practically speaking we must eventually agree to accept
certain basic statements. Such statements, says Popper, are "especially
easy to test."\textsuperscript{106}

Thus if the test is to lead us anywhere, nothing remains
but to stop at some point or other and say that we are satis-
\textsuperscript{fied, for the time being.}\textsuperscript{106}

And so, in answer to the question concerning what is involved in the
acceptance of basic statements, Popper would reply that when we are all
satisfied that we need go no further in our testing, then we simply
agree to accept certain basic statements. These basic statements, says
Popper, can always be tested in the future if we cease to be satisfied.\textsuperscript{107}
Second, Popper notes that the basic statements which contradict a theory must also corroborate a "falsifying hypothesis". But what is a falsifying hypothesis and further, what constitutes its corroboration? Popper explains that a falsifying hypothesis will typically be an empirical, i.e. falsifiable hypothesis which has a low level of universality. As an example of a falsifying hypothesis, Popper points out in a footnote that such an hypothesis can be a "very low level of universality" which can be obtained by generalizing from a set of "individual coordinates of a result of observation" as is the case with "There is a family of white ravens at the zoo in New York." This falsifying hypothesis, if corroborated by basic statements which contradict the theory in question, will result in the falsification of the following theory: "All ravens are black" or "There are no non-black ravens." The basic statements which if accepted, contradict this theory would be such statements as: "There is a white raven in the New York Zoo." "There is another white raven at the zoo in New York," etc. Further, these statements corroborate the falsifying hypothesis "There is a family of white ravens at the zoo in New York." But what do we mean by corroboration here? Popper earlier said in chapter one of L.Sc.D. that a statement which has been corroborated is one which has so far stood up to potentially refuting test. That is, it has "proved its mettle". Further, he notes here that corroboration should not be confused with verification (or more precisely conclusive verification) because a statement which has stood up to tests so far could, in the future, be falsified:

It should be noticed that a positive decision can only
temporarily support the theory, for subsequent negative decisions can always overthrow it.112

Of course, the reader might now argue that what if the falsifying hypothesis which has been corroborated today and has resulted in the falsification of a theory is falsified tomorrow? Popper would reply that a theory such as "All ravens are black" rules out for all places and all times non-black ravens and that even if there is no longer a family of white ravens at the zoo in New York, there was at one time (and at one place) a family of non-black ravens at the zoo. Consequently, our theory that all ravens are black has been refuted.

Before concluding our exposition of Popper's characterization of falsifiability and falsification with a word on the "falsifying mode of inference" we might mention the significance of the notion of a "falsifying hypothesis." Popper notes that a falsifying hypothesis corresponds to the notion of a "reproducible effect"113 or in chapter one, what he called a "physical effect."114 In chapter one of L.Sc.D., Popper makes a distinction between "physical effects" on the one hand and "occult effects" on the other. An occult effect is an occurrence which "no serious physicist would offer for publication, as a scientific discovery."115 This occult effect corresponds to the few "stray" basic statements which contradict a theory which Popper talks about in chapter four of L.Sc.D:

\[
\ldots \text{we have seen that non-reproducible single-occurrences are of no significance to science. Thus a few stray basic statements contradicting a theory will hardly induce us to reject it as falsified.}\]

Further, in chapter one Popper defines a physical effect (or reproducible effect in chapter four) as an effect which is reproducible "by
anyone who carries out the appropriate experiment . . . "117 And, according to Popper this physical effect, because it is reproducible, is scientifically significant. Consequently, in chapter four of L.Sc.D. Popper requires not merely a few stray forbidden basic statements (or occult effects) for a theory's falsifications but rather he requires that these effects be reproducible; i.e. that a falsifying hypothesis be corroborated for a theory to be falsified. In short, Popper is paying lip service, as it were, to the requirement of reproducibility in science with his notion of a falsifying hypothesis.

Finally, if we require that for a theory to be scientific it must be falsifiable, then how do we reason from the "truth" of a set of accepted basic statements to the falsity of the theory in question? That is, what will our "falsifying mode of inference"118 be? As we have already seen, Popper in chapter one of L.Sc.D. has noted that the mode of inference involved here will be deductive in nature: In chapter three of L.Sc.D., Popper gives a detailed account of what the falsifying inference will look like: If "t" is a theory plus initial conditions and "p" is derivable from "t" then if we assume "¬p" (i.e. the negation of the derived prediction "p") then we can by modus tollens conclude "¬t" (i.e. t is false) or, symbolically, "[(t → p) & ¬p → ¬t]."118 And so, as Popper states:

Given the relations of deducibility, t → p, and the assumption ¬p, we can then infer t (read 'not-t'); that is, we regard t as falsified.119

Using Popper's "raven" example, we can illustrate this deductive falsifying mode of inference as follows: We have the theory "There are no non-black ravens" or symbolically, "¬(∃x)(Rx & ¬Bx)" where Rx is "x is
a raven" and Bx is "x is black." From our theory, "\( \exists x \) (Rx & \neg Bx)" (and recalling Popper's comments on the derivability of negations of basic statements contradicting a theory in chapter five) we can derive the following negation of the basic statement contradicting the theory, viz., "\( \exists x \) (Rx & \neg Bx & Kx)" where Kx is "x is at the space-time region K". Now, assume the negation of the statement derived from the theory which turns out to be the basic statement which contradicts the theory, viz., "\( \exists x \) (Rx & \neg Bx & Kx)" or "\( \neg \exists x \) (Rx & \neg Bx & Kx)". Thus, by modus tollens we can conclude our theory is false (i.e. \( \neg (\exists x \) (Rx & \neg Bx)) as follows:

1. \( \exists x \) (Rx & \neg Bx) \rightarrow \neg (\exists x \) (Rx & \neg Bx & Kx)
2. \( \neg \exists x \) (Rx & \neg Bx & Kx)
   \[ \neg (\exists x \) (Rx & \neg Bx & Kx) \]
   or \( \exists x \) (Rx & \neg Bx)

Thus, using modus tollens, we have reasoned to the theory's falsity.

Part III: The problem of Simplicity

As we have seen in the first two sections of this exposition, Popper states that a theory is falsified if the class of its potential falsifiers is non-empty.120

Or, more precisely, a theory is falsifiable if the theory rules out at least one event whose occurrence it denies121 in which case the theory's class of potential falsifiers "contains at least one non-empty class of homotypic basic statements."122 That is, its non-empty class of potential falsifiers consists of at least one non-empty class of basic statements asserting various occurrences of a certain event forbidden by the theory. In chapter six of L.Sc.D., Popper proposes
two (related) methods for comparing the degrees of falsifiability of
two (or more) theories. Further, he states that one of the problems
in epistemology which can be elucidated by these methods is none other
than the problem of simplicity:

I believe that these methods can help us to elucidate
epistemological questions, such as the problem of simplicity
which will be our next concern.123

It is Popper's treatment of the problem of simplicity which is the center
(or focus) of concern in this thesis. Also, in this part of the exposition,
the problem of simplicity as outlined by Popper in chapter seven
of L.Sc.D. will be presented. That is, Popper's outline of the diffic-
culties in epistemology having to do with the problem of simplicity
(which will supposedly be resolved with the help of his methods for
comparing degrees of falsifiability) will be presented.

Admitting that the term "simplicity" is used in very many
different senses,124 Popper excludes from discussion those uses of this
term having to do with presentations or expositions or the solutions of
problems. He notes that it can be said of a presentation or exposition125
that it is "not simple but intricate,"126 or of a solution to a problem
that it is "not simple but difficult."127 Also, of a task it can be
said "that one task may be 'carried out by simpler means' than another
that it can be done more easily ..."128 These uses of the term
"simple" and hence the preference for the simplest (or easiest) solution
to a problem or the simplest (or most elegant) exposition are, Popper
claims, of a pragmatic or aesthetic nature and are therefore of little
concern in the theory of knowledge.129 And so, Popper concludes that
"in all such cases the word 'simple' can be easily eliminated; it use
is extra-logical."\textsuperscript{130} Thus, Popper will not deal with such supposedly non-epistemological issues as whether or not we should prefer the easiest way of performing a task or for that matter what would constitute the easiest solution. Such issues would be regarded by Popper as being of a pragmatic or aesthetic nature.

Quoting from Schlick,\textsuperscript{131} Popper states that the epistemological concept of simplicity (or the use of the term simplicity which is of interest in epistemology) has to do with the simplicity of formulas, which because of their simplicity are regarded by the scientist as laws. Thus, the problem of what constitutes the simplest law (or theory) or, the problem of why we should prefer a simple law or theory are difficulties of interest in epistemology. Popper next outlines in more detail just what are some of these difficulties associated with the epistemological concept of simplicity.

First, Popper notes that Schlick attempted to distinguish between "law" and "chance" using the concept of simplicity.\textsuperscript{132} Also, he states that Schlick (along with Feigl\textsuperscript{133}) expected that the concept of simplicity would help in the definition and comparison of degrees of law-likeness or regularity.\textsuperscript{134} However, as Popper points out, Schlick abandoned his attempt to distinguish between law and chance with the help of the concept of simplicity because of the difficulty of defining "simplicity" precisely.\textsuperscript{135}

Next, Popper outlines the difficulties associated with the epistemological concept of simplicity in the various theories of induction. According to Popper, the inductivists hold that a law (or a theory) is a generalization from (a finite number) of observations.\textsuperscript{136}
However, if we are given a set of observations represented as pairs of coordinates on a graph the inductivist is faced with the problem that an infinite number of functional laws can be "inferred" as it were from these observations. This is so because an infinite number of curves can be drawn through the set of plotted co-ordinates. Thus, the inductivist is faced with the problem of deciding which curve (or general functional law) is to be adopted or chosen from the infinity of possible curves. The answer, says Popper, given by most inductivists is that the simplest curve (or general law) should be chosen. And, by the simplest curve (or general law), "it is usually tacitly assumed that a linear function, say, is simpler than a quadratic one, a circle simpler than an ellipse, etc." However, Popper argues that the inductivist is still faced with two difficulties associated with "simplicity". First, on what basis do we say that one law (such as a circle-hypothesis) is simpler than another (such as an ellipse-hypothesis). Second, why should we always choose the simplest law? Or as Popper states:

But, no reasons are given either for choosing this particular hierarchy of simplicities in preference to any other, or for believing that 'simple' laws have advantage over the less simple--apart from aesthetic and practical ones.

Popper then examines an attempt by Natkin to resolve at least the first of these difficulties. Popper notes that Natkin's resolution as characterized by Schlick was to regard one curve as simpler than another "if its average curvature is smaller." He further notes that Natkin's resolution as characterized by Feigl was to regard one curve as simpler than another "if it deviates less from a straight line." Popper then argues that Natkin's resolution, although intuitively acceptable, is
faced with difficulties. For example, parts of a hyperbola would be
considered more simple than parts of a circle by his definition of
simplicity. Yet, a circle is usually regarded as simpler than a hyper-
bola. Further, Popper argues that Natkin's definition of simplicity
does not tell us why we should prefer the simplest theory.

Also, as Popper notes, Weyl made an attempt to show why one
curve (such as a circle) should be regarded as simpler than another
(such as an ellipse) and why we should prefer the simplest curve by
defining simplicity in terms of probability. According to Weyl, given
for example, twenty pairs of observations (plotted as pairs of coordinates
on a graph) which lie almost perfectly on a straight line, it would be
conjectured that the functional law relating "x" and "y" is linear.
(i.e. f(x) = mx + b). Further, a linear law would be conjectured here,
because it is the simplest law in the sense that it would be highly im-
probable that these coordinate pairs lie on a straight line if the func-
tional law relating "x" and "y" were non-linear. However, Weyl
rejects his attempt to define simplicity in terms of probability on the
grounds that it could be said that the same coordinate pairs lie on a
non-linear curve. And so, it could be said that it would be highly
improbable that these points lie on this non-linear curve if the func-
tional law relating "x" and "y" were not non-linear. And so, Weyl,
having abandoned his attempt to define simplicity in terms of probability
argues that any function which is simple must be given as such a priori
by mathematics. Further, in the case of a mathematically simple function,
the number of unspecified constants which when determined, determine the
functional law\textsuperscript{150} are at a minimum. At this point, while noting that his own (stipulative) clarification of simplicity bears similarities to Weyl's\textsuperscript{151} Weyl's clarification does not tell us what this a priori mathematical simplicity is nor why we should prefer simplicity.\textsuperscript{152}

And so, Popper has noted that any attempts to solve the inductivist problem of what constitutes the simplest law or theory and why we should choose the simplest law to represent a given set of observations have been unsuccessful.

Also, earlier in chapter seven,\textsuperscript{153} Popper notes that another difficulty associated with the epistemological concept of simplicity is why in our description of the world, theories are considered to be simpler than singular statements. That is, in what sense are theoretical descriptions simpler than descriptions with the help of singular statements. (Or, why are descriptions of higher universality simpler than descriptions of lower universality?)

Finally, Popper argues that the difficulties associated with the concept of epistemological simplicity outlined above can be resolved by equating the concept of epistemological simplicity with "degree of falsifiability":

The epistemological questions which arise in conjunction with the concept of simplicity can all be answered if we equate this concept with \textit{degree of falsifiability}.\textsuperscript{154}

Thus, a theory $T_1$ is simpler than another theory $T_2$ only if $T_1$ is more falsifiable than $T_2$. But then, what is it for $T_1$ to be more falsifiable than $T_2$? As we have already seen briefly, Popper outlines two independent (though related) methods which make possible the comparison of degrees of falsifiability. Thus, our next task (in Part IV) is to
examine just what these methods for the comparison of degrees of falsifiability are.

Before going on to the next section, we should be clear on the nature of Popper's clarification of the epistemological concept of simplicity in terms of "degrees of falsifiability". In his *Philosophy of Natural Science*, Carl Hempel makes a distinction between descriptive definitions and stipulative definitions. A descriptive definition involves the assertion that the definiendum (the term to be defined) has the same meaning as the definiens (the defining term). On the other hand, a stipulative definition involves the assertion that the definiendum is to have the same meaning as the definiens. And, this is exactly what Popper is doing by equating simplicity with degree of falsifiability. That is, by stipulating that the degree of simplicity of a theory is the same as its degree of falsifiability, certain problems in epistemology connected with the concept of simplicity can be resolved. He is not claiming that he has provided us with an iron-clad (descriptive) definition of simplicity which would stand up to counter-examples (as supposedly the definition of "knowledge" was supposed to do in epistemology): Thus, his (stipulative) definition of simplicity cannot be criticized, for example, by arguing that although by his definition $T_1$ is simpler than $T_2$ we would normally not say this:

It is possible to reject any attempt (such as mine) to make this concept precise by saying that the concept of simplicity in which epistemologists are interested is really quite a different one. To such objections I could answer that I do not attach the slightest importance to the word "simplicity", and he goes on to say:

All I do assert is that the concept of simplicity which I am
going to clarify helps to answer those very questions which have so often been raised by philosophers of science in connection with their 'problem of simplicity'.

As we shall see in Chapter II (i.e. "Other Authors’ Criticisms), Several of the criticisms raised by other philosophers of science, viz. Barker and Hempel, can be handled if we keep in mind the stipulative character of Popper’s definition of simplicity.

Part IV: The Comparison of Degrees of Falsifiability

Given that a theory which is falsifiable is one whose class of potential falsifiers is non-empty, could we say that if we have a theory $T_1$ whose class of potential falsifiers is larger than the class of potential falsifiers of another theory $T_2$ then $T_1$ is more falsifiable than $T_2$? Popper answers this question in the negative on the grounds that "the classes of potential falsifiers are infinite classes." That is, even in the case of a theory which forbids only one event's occurrence, there is an unlimited number of possible basic statements asserting specific occurrences of the forbidden event at various k's. This is so, says Popper, since "a theory does not refer to individuals as such." As a second attempt at comparing degrees of falsifiability, we could propose that a theory $T_1$, which forbids only one event's occurrence is less falsifiable than $T_2$ which forbids two events. However, again Popper responds in the negative to this attempt since for any forbidden event when conjoined with any other event again results in another forbidden event. But, such a process could conceivably be carried out ad infinitum. Consequently, any falsifiable theory forbids an infinite number of events.

Popper notes, however, that if we were without difficulty, able
to say that a theory $T_1$ is more falsifiable than $T_2$ since $T_1$ has more potential falsifiers than $T_2$ or because $T_1$ forbids more events than $T_2$, then the following would be the case: A theory which has a high degree of falsifiability would be easy to falsify (as opposed to a theory with a low-degree of falsifiability). This is so because a highly falsifiable theory rules out the occurrence of more events and hence it forbids more basic statements (as compared to a less falsifiable theory) and therefore there is a greater opportunity for its falsification:

A theory like this would obviously be very easy to falsify, since it allows the empirical world only a narrow range of possibilities; for it rules out almost all conceivable, i.e. logically possible, events. It asserts so much about the world of experience, its empirical content is so great, that there is, as it were, little chance for it to escape falsification.

Also, for Popper, a theory which can easily be falsified because it rules out more events and hence (it rules out) more basic statements (asserting various occurrences of these forbidden events at various $K$'s) is one which has a high empirical content. That is, a theory which rules out more (events and basic statements) than another theory has a higher empirical content than that theory.

In view of the difficulty that a falsifiable theory's class of potential falsifiers and its class of forbidden events are both infinite, Popper initially proposes three methods whereby we can compare degrees of falsifiability in terms of either forbidden events or classes of potential falsifiers. Popper notes that these methods will hopefully render precise the intuitive notions of "more" and "fewer" in comparing classes of potential falsifiers or classes of forbidden events (and hence degrees of falsifiability) thus avoiding the problem of infinite classes.
Now as we have already seen, Popper argues that the more falsifiable theory is one which has high empirical content and which is easily falsified. Thus, as we shall later see, Popper endeavours to show that a theory $T_1$ which is deemed more falsifiable than another theory, $T_2$, by his proposed methods is easier to falsify than $T_2$ and (at least in the case of "subclass relations") $T_1$ has a higher empirical content than $T_2$.

The first method which Popper presents would involve the comparison of classes of potential falsifiers in terms of their cardinality (i.e. the number of potential falsifiers) in order to determine whether or not one theory is more falsifiable than another theory. However, as he has already pointed out, the class of potential falsifiers of any theory is infinite in which case the comparison of these classes of potential falsifiers in terms of their cardinality is not possible. And so, Popper abandons this method "since it can easily be shown that the classes of potential falsifiers have the same cardinal number for all theories."

The second method which Popper proposes takes into account the "degree of composition" of the potential falsifiers (and the permitted statements) of two (or more) theories when comparing their degrees of falsifiability:

Thus it may be possible to compare theories as to their degree of testability by ascertaining the minimum degree of composition which a basic statement must have if it is to be able to contradict the theory.

As we shall later see this second method Popper is proposing will be especially applicable in cases where we are comparing theories (with respect to their degrees of falsifiability) which are expressible as
mathematical formulae. In the case of a circle-theory for example, we know from elementary mathematics that it takes three points to determine (or specify) a circle (since three parameters, viz., h, k, and r in 
\[(x - h)^2 + (y - k)^2 = r^2\] must be specified). Now on the basis of three coordinates (corresponding to three pairs of observations) we cannot refute a theory such as "The orbit of this planet is circular" since any three points (assuming they do not coincide) will determine some circle. In short, we do not have enough coordinates (or observation pairs) to say that we do not have a circle here. In fact, we would need some fourth point which does not satisfy our circle equation (specified by the other three points) in order to falsify our theory that the orbit of the planet is circular. And so, in the case of our circle-theory, any basic statement asserting three (non-coincident) positions of the planet, (sighted at separate times) will be permitted by the theory.

For example, the following 3-tuple would be permitted by this circle-theory: "There is a point at \(x = 7, y = 5\) at time "1"; a second point at \(x = -1, y = 1\) at time "2"; and a third point at \(x = 0, y = -6\) at time "3". Such a statement is permitted according to Popper, owing to its "low degree of composition." That is, it is only a 3-tuple (i.e. a conjunction of three statements), each conjunct asserting the existence of a point. And as we have seen three points are not sufficient to refute a circle theory. But, certain basic statements asserting the existence of four points (corresponding to four positions of the planet), one of which does not satisfy the circle-equation determined by the other three, will result in the theory's falsification. An example of such a statement would be the following 4-tuple: "There is a point at \(x = 7, \)
y = 5, at 1; a second point at x = -1, y = 1 at m; a third point at x = 7, y = -5 at n and a fourth point at x = 4, y = 5 at 0." This statement falsifies the theory in part because its degree of composition has reached the "requisite minimum" and in part because the point (4,5) does not satisfy the circle-equation). That is, it is a 4-tuple, with each conjunct asserting the existence of one point, and, as we have seen, certain sets of four points can falsify a circle-theory. And so, as Popper states:

All basic statements, whatever their content, whose degree of composition does not reach the requisite minimum, would be permitted by the theory simply because of their low degree of composition.

Further, if we attempt to compare the circle-theory just mentioned with a corresponding ellipse-theory, viz., "This planet's orbit is elliptical" with respect to their degrees of falsifiability then we might go through the following process: We could compare the minimum degree of composition which any potential falsifier of a circle-theory must have with the minimum degree of composition which any potential falsifier of an ellipse-theory must have. Now, we already know that in the case of the circle-theory that a potential falsifier of this theory must be at least a 4-tuple. Since it takes five points to determine an ellipse (i.e. the ellipse equation has five unspecified constants), we would therefore need at least six points to refute our ellipse theory, where one of these six points does not satisfy the equation determined by the other five. Consequently, any potential falsifier of an ellipse-theory must be at minimum a 6-tuple (with each of the six conjuncts asserting the existence of one point.) Finally, as we shall later see, Popper
would reason at this point that the circle-theory is more falsifiable
than the ellipse-theory because we only need certain four points (ex-
pressed as a 4-tuple) to falsify the circle-theory.\textsuperscript{178} In short, the
circle-theory whose potential falsifiers must at least be 4-tuples will
be easier to falsify than an ellipse-theory whose potential falsifiers
must at least be 6-tuples.\textsuperscript{179}

As promising as this method of comparison may seem initially,
Popper cautions the reader that "any such program is faced with diffi-
culties."\textsuperscript{180} Being able to say that one statement has more conjuncts
and hence, it is more composite than another presupposes that these
conjuncts are themselves non-composite. That is, such a comparison
presupposes that there are elementary statements or building blocks
from which all other statements are composed. However, Popper argues
that any statement such as the seemingly non-composite basic statement
"There is a glass of water at the place K" contains universal names
(such as "water" in this example) which can be "analyzed" (eg. "Water
is a fluid") thus enabling us to break the statement down into two or
more conjuncts.\textsuperscript{181} In the case of the statement "There is a glass of
water at the place K" Popper suggests that we can further break this
down into the following two conjuncts viz., "There is a glass containing
a fluid at the place K" and "There is water at the place K".\textsuperscript{182} Further,
each of these conjuncts can themselves be broken down, if necessary, by
the introduction of new universal names for this very purpose, ad
infinitem:

There is no hope of finding any natural end to the dissection
of statements by this method, especially since we can always
introduce new universals defined for the purpose of making a
further dissection possible.\textsuperscript{183}
If Popper is right here, there are no elementary statements, which we can use for the purpose of comparing statements' degrees of composition and hence theories' degrees of falsifiability.

Popper makes note of the attempt by philosophers, such as Wittgenstein, in his Tractatus, to designate a certain class of statements as being elementary or "absolutely atomic." According to Popper, if we really could vis a vis these attempts regard a certain class of statements as being "absolutely atomic" then all other statements could be regarded as being built up from or composed of "n" of these elementary statements. These composite statements are, for example, in Wittgenstein's Tractatus regarded as truth-functions of elementary statements, says Popper. (ex.: if p and q are elementary statements, then "p & q" is a truth-function of these two elementary statements.) Thus, for example, we could say that "p & q & r" is more composite than "p & q" because the former is a conjunction of three elementary statements whereas "p & q" is a conjunction of only two elementary statements. However, Popper argues that if we recall his earlier argument that any statement can be "dissected" into two or more conjuncts, then regarding statements of a certain class as being absolutely atomic would restrict the use of scientific language:

But for the reason given above, such a procedure would have to be regarded as highly unsuitable; it would impose serious restrictions upon the free use of scientific language.

In an appendix, Popper elaborates on this argument: If we designate for example, the term "blue" as being elementary or absolutely atomic then we will be restricted from analyzing this term any further in science. And so Popper points out, in certain contexts the term "blue"
is further analyzed or is "explicable in terms of atomic theory."

But, if we had designated this term as elementary, then we would have been restricted from making such an analysis.

Having rejected any attempts at selecting a class of absolutely atomic statements, Popper proposes that we arbitrarily select a class of relatively atomic statements which would be obtained by a generating matrix or schema which is akin to a propositional function (eg. Fx) whose values of x would be obtained by (an infinite number of) various substitutions (eg. Fa, Fb . . .). In an appendix, Popper gives as an example of a generating matrix the following: "At the time x the planet y was in the position z." Further, by filling in values of x, y and z we would obtain an infinite number of substitution instances, viz.: "At 5:15 p.m. the planet Saturn was in the position given by our telescope," etc. These relatively atomic statements obtained by the same generating matrix would be "equicomposite"; i.e. they would have the same degree of matrix composition:

We can then define as relatively atomic and thus as equicomposite, the class of all statements obtained from this kind of matrix (or statement function) by the substitution of definite values.

Further, other statements can be formed by conjoining two or more of these relatively atomic statements. And any such conjunction of n relatively atomic statements, Popper calls an "n-tuple." Also, the class of these relatively atomic statements obtained by a particular generating matrix and their conjunctions Popper calls a "field."

The class of these statements, together with all conjunctions which can be formed from them may be called a "field."

(Note that this leaves open the possibility, which Popper himself
notes of there being different fields. That is, we could have as many different fields as we do generating schemata or matrices for any given theory. Each field would consist of relatively atomic statements (along with their conjunctions) obtained from a particular schema. The fields would differ with respect to the matrices used to obtain these statements. Since all statements obtained from the same type of matrix or schema will be equi-composite, we will then be in a position to do the following: We could make comparisons to see if a statement is compositive and for that matter if it is more or less composite than another statement from the same field (or from a field whose generating schema is the same as that of the original field, as would be the case when comparing potential falsifiers of two theories). For example, using Popper's "planet" example, the statement "At 5:15 the planet Saturn was in the position given by our telescope" is less composite than the statement "At 5:15 the planet Saturn was in the position given by our telescope and at 6:00 the planet Saturn was in the position given by our telescope." Further, Popper is not making any claims that the statements obtained from a generating matrix are absolutely atomic and hence they cannot be analyzed any further. Thus, he is not restricting the use of scientific language. All that he is supposedly saying is that for the purpose of comparison, we will regard these statements as atomic.

And so, now that he has supposedly made possible the comparison of statements with respect to their degrees of composition, Popper sets up his "dimension" method for comparing theories' degrees of falsifiability in terms of the minimum degree of composition of their potential falsifiers. First, Popper makes precise the notion that for a statement
to be a theory's potential falsifier, it must have a minimum degree of composition: For a theory T there is a field of relatively atomic statements (which are singular "but not necessarily basic") such that any d-tuple (i.e., any conjunction of d relatively atomic statements) cannot falsify T though certain d+1-tuples of this field can falsify this theory.¹⁹⁸ (In our earlier example of the circle theory, if we take as a relatively atomic statement any statement obtained by filling in the values of x, y and z for "At the time x the planet y was in the position z" then any 3-tuple of such statements in that theory's field (or in one of its fields) cannot falsify it though certain 3+1-tuples can. Such 3+1-tuples would contain as one of its conjuncts a statement asserting a position which does not satisfy the circle equation determined by the other three. We will have more to say on this example later.) Popper calls the number "d" that theory's "characteristic number" in relation to one of its fields.¹⁹⁹ Also, Popper notes that in fact any n-tuple (of that field) where n < d will be compatible with the theory.²⁰⁰

Before outlining his method for comparing theories' degrees of falsifiability in terms of their respective characteristic numbers (where d for a theory in relation to its field is such that certain d+1-tuples can falsify the theory), Popper notes that there is one further problem to be dealt with. That is, he recognizes the possibility of there being several fields (having different classes of relatively atomic statements obtained from different generating matrices) for any theory.²⁰¹ Thus, even though for one field of a theory T, viz., F₁, certain 4-tuples of F₁ can falsify T, it may also be the case that for
a second field of $T_1$, viz. $F_2$, whose relatively atomic statements are 2-tuples with respect to $F_1$. Certain 2-tuples of $F_2$ can falsify $T$.

And so, in order to alleviate such inconsistencies, Popper introduces the "narrower" concept of a theory's field of application:

But in order to avoid inconsistencies which might arise through the use of different fields, it is necessary to use a somewhat narrower concept than that of a field, namely the field of application.

A field $F$ of a theory $T$ will be called $T$'s field of application, according to Popper, if and only if $T$'s characteristic number "$d$" exists in relation to this field $F$ such that any $d$-tuple of $F$ does not contradict $T$ and a second condition. This second condition runs as follows:

Any $d$-tuple (of $F$) in conjunction with $T$ divides $F$'s remaining relatively atomic statements into two infinite subclasses, $A$ and $B$. All statements of $A$ are such that when any one of these is conjoined with any $d$-tuple of $F$ it will result in $T$'s falsification. And, class $B$ can be further divided into a finite number of infinite subclasses such that any number of statements from any one of these infinite subclasses when conjoined with any $d$-tuple of $F$ will not contradict $T$. The upshot of all this is that we are "zeroing in" as it were on one field of a theory $T$ which we then call $T$'s field of application. Thus, when we say that certain 4-tuples can falsify $T$ (although any 3-tuple cannot) we are talking about certain 4-tuples of $T$'s field of application. The number "3" in this case is $T$'s characteristic number with respect to its field of application. Or, more simply, we are concerned here with the smallest $n$-tuple which can falsify $T$ with respect to $T$'s field of application.

Now that Popper has cleared up any inconsistencies associated
with a theory's various fields by focusing our concern on that theory's field of application, he sets up his dimension method for comparing theories' degrees of falsifiability as follows: First, Popper defines a theory's "dimension" as that theory's characteristic number, \( d \), with respect to its field of application.

The characteristic number \( d \) of a theory \( T \), with respect to a field of application, I call the **dimension** of \( T \) with respect to this field of application.\(^{207}\)

Recalling the significance of a theory \( T \)'s characteristic number or dimension, "\( d \)". i.e. that any \( d \)-tuple of \( T \)'s field of application cannot falsify \( T \) but certain \( d+1 \)-tuples can, there is, says Popper the following relationship between a theory's dimension and its degree of falsifiability:

The smaller the dimension \( d \), the more severely restricted is the class of those permitted statements which, regardless of their content, cannot contradict the theory, owing to their low degree of composition; and the higher will be the degree of falsifiability of the theory.\(^{208}\)

Thus, if \( T_1 \)'s \( d=3 \) then any \( n \)-tuples where \( n \leq 3 \) will not be able to falsify \( T_1 \) and we would say that it is more falsifiable than another theory \( T_2 \) whose \( d=5 \) since \( T_2 \) not only permits 1-, 2- and 3-tuples, but also any 4- and 5-tuples as well. Of course, we can't say here that \( T_2 \) permits more statements than \( T_1 \) since in both cases we are dealing with infinite classes of statements. Also, it should be noted here (although Popper didn't make this point explicit) that if we are to compare the characteristic numbers or dimensions of any two (or more) theories, it would be assumed that the relatively atomic statements of their respective fields of application will have been obtained by the same type of generating matrix. Otherwise we would be faced with the same kinds of difficulties that we encountered in the case of two or more fields of
the same theory. Thus, if we have the following two theories, viz.,
T₁ "This planet's orbit is 'circular'" and T₂ "This planet's orbit is
elliptical" then to say that no 1-, 2-, or 3-tuples can falsify T₁ but
no 1-, 2-, 3-, 4- or 5-tuples can falsify T₂ and hence T₁ is more falsi-
ifiable than T₂, we are making the following assumption: We are assuming
that the relatively atomic statements of which these various non-falsi-
fying n-tuples are composed (for both T₁ and T₂) are obtained from the
same kind of generating matrix such as "At the time x the planet y was
in the position z".

Finally, before examining how Popper's "dimension" method is
especially applicable to theories which can be expressed as mathematical
formulae, we should mention one further difficulty with Popper's dimen-
sion method which he tries to deal with. That is, he earlier noted that
the relatively atomic statements of a theory's field of application need
not be basic, though they must be singular as in not strict; they
specify some space-time region k. (For example, if our theory T is
"This particle travels in a straight line" then one of the relatively
atomic statements in its field of application might be a substitution
instance of the negation of the statement function "At time x the particle
y was at the point z" where z is any point on the straight line. That
is, this relatively atomic statement denying that the particle at such-
and-such a time was at a certain point z on the straight line would be
a singular there-is-not statement.) But isn't one of Popper's aims
here to assess the minimum degree of composition of a theory's poten-
tial falsifiers which he defined in chapter four of L.Sc.D. as being
basic statements? Popper responds to this point by introducing the
assumption that to every highly composite singular statement there
corresponds a highly composite basic statement:

We assume that to highly composite singular statements there correspond highly composite basic statements.\(^{213}\)

Thus, if the falsifying \(d+1\)-tuple of a theory \(T\) is highly composite then that theory's potential falsifier (as Popper defined this term in chapter four) will also be highly composite.

Popper notes in chapter six of *L.Sc.D.* that sometimes a theory's field of application is identical with that theory's field of graphic representation. Also, if this is the case, then every relatively atomic statement of this theory's field of application will correspond to a coordinate pair or point on the graph. And in this case, the theory's dimension, "\(d\)" is identical with the number of points needed to determine the curve which is the theory's graphic representation. Or as Popper states,

\[\ldots each \ point \ of \ this \ field \ of \ graphic \ representation \ can \ be \ taken \ to \ correspond \ to \ one \ relatively \ atomic \ statement. \ The \ dimension \ of \ the \ theory \ with \ respect \ to \ this \ field \ \ldots \ \text{is then identical with the dimension of the set of curves corresponding to the theory.}^{214}\]

For example, the statement, "All orbits of planets are circles"\(^{215}\) has as its graphic representation a circle whose general equation can be written as: \[x^2 + y^2 + Dx + Ey + F = 0^{216}\] (which is a specific instance of the general equation for a conic section, viz., "\(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\")\(^{217}\) Now if we obtain any three points or coordinate pairs, we can solve for the unspecified constants (or parameters) "\(D\)", "\(E\)" and "\(F\)", thus determining the circle equation. For example, if we obtain the following three points, \((7, -5), (-1, 1)\) and \((0, -6)\) then we
can solve for D, E and F by solving for the following system of equations:

\[
\begin{align*}
(1) \quad 49 + 25 + D(7) - 5E + F &= 0 \\
(2) \quad 1 + 1 - D + E + F &= 0 \\
(3) \quad 36 - 6E + F &= 0
\end{align*}
\]

Having solved this system of equations, we obtain \( D = -6, \ E = 4, \ F = -12 \).

We can now, having specified these constants, determine the circle-equation which describes the curve on which our three points fall:

\[
\begin{align*}
x^2 + y^2 + Dx + Ey + F &= 0 \\
x^2 + y^2 - 6x + 4y - 12 &= 0
\end{align*}
\]

Expanding as follows:

\[ (x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 \]

We now obtain the following equation:

\[ (x - 3)^2 + (y + 2)^2 = 12 \]

Also, if we obtain a fourth point or coordinate pair which does not satisfy this equation (and hence which does not lie on the curve) then we may say that these four points do not lie on the curve described by the equation "\((x - 3)^2 + (y + 2)^2 = 12\)". Now, for any circle, it is determined by three points and it can be "falsified" as it were by these three points conjoined with certain other fourth points. (Though still other fourth points will satisfy the equation). At this point, we can see the correspondence between the theory "All orbits of planets are circles" and its graphic representation. That is, each point on the graph corresponds to a relatively atomic statement in the theory's field of application. Also, the three points which define the circle restrict any further values of \( x \) and \( y \) so that some \((x,y)\) pairs satisfy
the equation describing the circle and others are incompatible with it. In the case of the circle-theory, the three relatively atomic statements allow us to make predictions whereby certain $3 + 1$-tuples of the theory's field of application will be compatible with these predictions and other $3 + 1$-tuples will falsify these predictions and hence the theory. (Recall the "falsifying mode of inference" described in Part II of this exposition.) Thus, since for a circle $d = 3$, therefore, the theory "All orbits of planets are circles." will have as its characteristic number (or dimension) $d = 3$. Likewise, in the case of the following ellipse-hypothesis: "All orbits of planets are ellipses". Since it takes five points to determine an ellipse whose equation is $Ax^2 + Cy^2 + Dx + Ey + F^2 = 0$ (where we obtain values for $A, C, D, E$ and $F$ by obtaining five points coordinate pairs), thus the dimension of the above mentioned ellipse-theory is "5". Further, since certain $5 + 1$-tuples will be incompatible with a specified ellipse-equation, certain $5 + 1$-tuples in an ellipse-theory's field of application will falsify that theory. Also, Popper would say that the circle-theory with $d = 3$ is more falsifiable than the ellipse-theory with $d = 5$. This is so, because in the case of the circle-theory only 1-, 2-, and 3-tuples in its field of application are compatible with it, whereas in the case of the ellipse-theory 1, 2, 3, 4 and 5 tuples are compatible with it. But this really seems to amount to the following: The circle-theory is easier to falsify than the ellipse-theory. This is so because it is easier to obtain a potential falsifier for a circle-theory than for an ellipse theory. In short, the ellipse theory is more difficult to falsify than the circle-theory. Of course, this is what Popper probably had in mind given his earlier observation that the highly
falsifiable theory is the one which is easy to falsify:

the theory which is refutable by fewer atomic statements would also be the one which is more easily refutable or testable and thus the one with the greater content. 220

Also, in discussing Kepler's attempt at characterizing the orbits of planets (also, in connection with a theory's "precision") Popper points out that Kepler's initial circle-hypothesis was easier to falsify than his later ellipse-theory:

But I do believe that Kepler owed his success in part to the fact that the circle-hypothesis with which he started was relatively easy to falsify. 221

We shall also see in Part V of this exposition that Popper has corresponding remarks on the simplicity of Euclidean geometry. 222

We earlier mentioned that Popper also observed that a highly falsifiable theory is one whose empirical content is high. However, it is not clear from examining the dimension method that this is the case. Yet, in an appendix, Popper first proposes that a theory's content is inversely proportional to (or the reciprocal of) the minimum number of atomic statements needed to refute a theory:

... we might introduce, as a measure of the content of a theory the reciprocal of the minimum number of atomic statements needed for refuting the theory. 223

Popper argues here that since degree of empirical content is the same as degree of falsifiability, then a highly falsifiable theory which is refutable by a small number of atomic statements will have a high (degree of) empirical content. However, Popper refines his proposal to regard (degree of) empirical content as the reciprocal of the minimum number of atomic statements needed to falsify a theory as follows: The atomic statements which can be used to measure the (empirical) content of a
theory must be the relatively atomic statements of the theory's field of application which are obtained by a generating matrix. And so, a theory's content will be inversely proportional to the minimum number of relatively atomic statements which are needed to falsify the theory.

Before examining Popper's third proposed method for "giving a precise meaning, even in the case of infinite classes, to the intuitive 'more' and 'fewer'" which would enable us to compare degrees of falsifiability, a few "loose ends" having to do with his dimension method should first be cleared up. Recall in our examples of the circle-theory and the ellipse-theory that the equations which describe their graphic representations contain in their most general form (i.e. before initial conditions or points are specified) unspecified constants or parameters. Popper notes that there is a relation between a theory's degree of falsifiability and the number of parameters present in the (general) equation which describes that theory's graphic representation:

We can therefore say that the number of freely determinable parameters of a set of curves by which a theory is represented is characteristic for the degree of falsifiability (or testability) of that theory.

Recall that the equation describing a circle-theory's graphic representation has three parameters which, when specified (by three points) in turn specify the circle equation. A fourth point which does not satisfy the circle-equation results in the falsification of the corresponding theory. On the other hand, the equation describing an ellipse-theory's graphic representation has five parameters which must be specified (by five points) for this equation to be determined. And a sixth point not satisfying the ellipse-equation results in the theory's falsification.
Notice that the circle-theory (whose \( d = 3 \)) is easier to falsify and hence more falsifiable than the ellipse-theory (whose \( d = 5 \)) since the parameters needed to determine the circle-theory's equation are fewer in number, than those needed to determine the ellipse-theory's equation. Or more simply, from these two examples, it would seem that the fewer the parameters needed to determine the equation of a theory's graphic representation, the more (easily) falsifiable that theory will be. As another example, a theory relating two variables "x" and "y" linearly will have as its graphic representation a straight line whose general equation is "\( Ax + By + C = 0 \)" or if \( m = \frac{-A}{B} \) and \( b = \frac{-C}{B} \), \( y = mx + b \). 227 Also, with two points, we can specify the slope "m" and the y-intercept "b". In so doing, we will have also specified the equation. Further, a third point which does not satisfy this equation will result in the theory's falsification. Thus, a linear theory whose equation has two parameters is more (easily) falsifiable than either a circle-theory (whose equation has three parameters) or an ellipse-theory (whose equation has five parameters.) And this is so because, as Popper states, "the dimension of the set of curves depends upon the number of parameters whose values we freely choose." 228 In the case of a straight-line equation it takes two points to determine it (i.e. \( d = 2 \)) because there are two parameters, "m" and "b" which must be specified for the equation to be specified. To characterize a circle, we require three points (i.e. \( d = 3 \)) since the circle-equation has three unspecified constants and in the case of the ellipse, \( d = 5 \) since the ellipse-equation has five parameters.

Finally, in connection with his dimension method, Popper distinguishes between the "material reduction" 229 of a curve's dimension
(or more precisely for a set of curves) and the "formal reduction" of its dimension. In a somewhat preliminary fashion, Popper characterizes the material reduction of a curve's dimension and of the corresponding theory's dimension as involving the specification of one or several points and hence "initial conditions";

Now the stipulation that a curve should pass through a certain singular point (or small region) will often be linked up with, or correspond to, the acceptance of a certain singular statement, i.e. of an initial condition. 231

For example, in the case of a straight line equation describing the graphic representation of a linear theory we need two points to determine this equation (since it has two parameters). But if we specify one of its points (and hence one of its parameters) then we need only one point to determine this less general equation. In specifying this one point (and hence one parameter) we have materially reduced the dimension of the equation (and the corresponding theory) from d = 2 to d = 1. On the other hand, a formal reduction of the dimension of the curve (and its corresponding theory) does not involve the specification of one or more points. Rather a formal reduction involves changing the "form" of the curve thus passing, for example, from an ellipse whose d = 5 to a circle whose d = 3:

The other method, in which the form or shape of the curve becomes more narrowly specified, as for example, when we pass from ellipse to circle, or from circle to straight line, etc., I will call the method of formal reduction of the number of dimensions. 232

Popper also notes that in both formal and material reduction of a curve's dimension, certain parameters of that curve's equation are specified:

Reducing the dimensions of a theory means, in algebraic terms, replacing a parameter by a constant. 233
For example, in the material reduction of the dimension of a straight line equation from \( d = 2 \) to \( d = 1 \), we might specify the following point, viz., \((0, -5)\). The coordinate "-5" is of course the y-intercept "b". Thus we have in specifying a point also specified the parameter "b" and we have further effected a material reduction from \( d = 2 \) to \( d = 1 \). That is, we only need one more point to determine the straight line equation. In the formal reduction from an ellipse-equation to a circle-equation, we could equate the parameters "A" and "C" to "1" thus giving us the following circle-equation, i.e. \( 1x^2 + 1y^2 + Dx + Ey - F = 0 \). Further, whereas for the ellipse-equation \( d = 5 \), for the circle equation \( d = 3 \).

And so, in a material reduction where we specify a point on an ellipse, for example, we still have a less general ellipse-equation (which is not invariant for all coordinate systems)\(^{235} \) whose \( d = 4 \). But in a formal reduction, we move for example from an ellipse-equation to a circle equation without reducing their generality (since in both cases these equations are invariant with respect to all coordinate systems.)\(^{236} \)

In fact, Popper attempts to make more precise his distinction between material and formal reduction by using this notion of "generality":

A reduction of the dimension of a set of curves may now be called formal if the reduction does not diminish the generality of the definition; otherwise it may be called material.\(^{237} \)

Further, Popper notes that in comparing two theories with respect to their degrees of falsifiability, we must not only take into account their dimensions but also "we shall clearly have to take into account their generality."\(^{238} \) For example, we may have two circle theories, both represented by circle-equations. By taking into account their dimension only, then in both cases \( d = 3 \) and hence we would say that both theories
are equally falsifiable. But, if one of the circle-equations has one of its three parameters specified then clearly it is less general (i.e., it is not invariant with respect to all coordinate systems) than the circle-equation which has none of its parameters specified. Consequently, taking into account the lower generality of the equation with the specified constant, it will only require two points for its falsification (or its \( d = 2 \): it has been materially reduced) and hence it is more (easily) falsifiable than the more general equation. As a final note on the notion of a curve's generality, Popper points out that in some cases an equation's generality need not be considered. That is, such considerations are "irrelevant". For example, a linear function relating pressure to temperature cannot be invariant for all sets of coordinates. This is so because if the \( x \)-coordinates are temperature readings and the \( y \)-coordinates are pressure readings, and our theory is that pressure is a linear function of temperature, then it would make no sense to allow pressure readings to be \( x \)-coordinates and temperature readings to be our \( y \)-coordinates.

So far, Popper has proposed two methods which would hopefully make more precise the notions of "more" and "fewer" in the case of infinite classes of potential falsifiers (and events) and hence which would enable us to compare degrees of falsifiability. However, he abandoned one of these proposed methods viz., the method involving the use of the "concept of the cardinality (or the power) of a class" since all classes of potential falsifiers, being infinite, will have the same number of potential falsifiers. The other proposed method for comparing degrees of falsifiability involves the use of the concept of
"dimension" as we have just seen. We shall now examine the third (and final) method for comparing degrees of falsifiability, which Popper proposes, viz., the subclass relation method.

If we have two infinite classes of potential falsifiers and the one class A belonging to a theory $T_1$ contains or includes the other class $B$ belonging to another theory $T_2$ as its subclass (i.e. $B \subset A$) then $T_1$ is more falsifiable than $T_2$. This method of comparing degrees of falsifiability involves the use of the notion of subclass relation.241

Further, Popper notes that although this method goes a long way in making precise the notions of "more" and "less" in the case of infinite classes of potential falsifiers, such a method has one obvious limitation: We can compare two (infinite) classes of potential falsifiers only if the one class is a subclass of the other:

The subclass relation corresponds very well to the intuitive "more" and "fewer", but it suffers from the disadvantage that this relation can only be used to compare the two classes if one includes the other.242

Thus, two mutually irrelevant theories such as the theory of relativity and a geological theory on the origin of the earth's crust are "non-comparable" since their classes of potential falsifiers are mutually exclusive. And so, we could not by the subclass method determine whether or not the geological theory is more falsifiable than the theory of relativity. (However, as Popper notes243 there are some theories which though non-comparable by the subclass relation, can be compared by the dimension method. We shall have more to say on this matter later.)

Popper gives some meat, as it were, to his subclass relation method for comparing degrees of falsifiability by proposing three pro-
visional definitions. That is, these three provisional definitions which Popper proposes will characterize his subclass relation method. First, $T_1$ is more falsifiable than $T_2$ if and only if $T_1$'s class of potential falsifiers includes as a subclass $T_2$'s class of potential falsifiers. Second, $T_1$ is equally falsifiable to $T_2$ if $T_1$'s class of potential falsifiers is identical to $T_2$'s class of potential falsifiers, $^{245}$ i.e., $B \subseteq A$ and $A \cap B)$. Third, if $T_1$'s class of potential falsifiers and $T_2$'s class of potential falsifiers are mutually exclusive, then $T_1$ and $T_2$ "have non-comparable degrees of falsifiability." $^{246}$ Popper next comments on these provisional definitions (which characterize his subclass relation method). He first notes that if the first definition applies, then for both $T_1$ and $T_2$ there will be non-empty (and infinite) complement classes of permitted basic statements. $^{247}$ This is so because a falsifiable theory must be consistent in that it divides all possible basic statements into two classes, viz. those which it contradicts (and which are not derivable from the theory) and those with which it is consistent (and which can be derived from the theory):

A consistent system, on the other hand, divides the set of all possible statements in two: those which it contradicts and those with which it is compatible. (Among the latter are the conclusions which can be derived from it.) $^{248}$

An inconsistent system would, according to Popper, be such that any statement can be derived from it and hence we cannot say that any such statement is either inconsistent or compatible with the theory. Thus, a falsifiable theory will have a non-empty (infinite) class of permitted statements. Further, Popper notes that a theory's complement class will be infinite (if it is non-empty) and therefore, we cannot compare degrees
of falsifiability by saying that $T_1$ forbids more than $T_2$.\footnote{249} (That is, we cannot say that since $T_1$ permits fewer statements than $T_2$ that $T_1$ forbids more than $T_2$.) Also, on the matter of complement classes of potential falsifiers, Popper later points out that if $T_1$'s class of potential falsifiers includes as its subclass $T_2$'s class of potential falsifiers then $T_1$'s class of permitted statements is a subclass of $T_2$'s permitted statements.\footnote{250} (But using the notion of subclass we still cannot say that if $T_1$ forbids more than $T_2$ then $T_1$ permits fewer statements than $T_2$.)

A second comment which Popper makes on his definitions characterizing his subclass method has to do with his second definition (i.e. $T_1$ has the same degree of falsifiability as $T_2$ if $T_1$'s class of potential falsifiers is identical with that of $T_2$.) That is, metaphysical statements (such as strictly existential statements) and tautological statements are such that their classes of potential falsifiers are empty.\footnote{251} Therefore, given Popper's second definition mentioned above, we can say that the classes of potential falsifiers of metaphysical statements and tautologies are identical.\footnote{252} Further, since all metaphysical and tautological statements have empty classes of potential falsifiers, they are therefore non-falsifiable or we can say that such statements have "zero" falsifiability.\footnote{253} That is, they are identical in their degrees of falsifiability, viz. "zero". Thus, any statement or theory which is falsifiable has a non-empty class of potential falsifiers and hence its degree of falsifiability, will be greater than zero.\footnote{254} Further, Popper notes that a self-contradictory statement is a statement which has as its class of potential falsifiers all possible basic statements:
A self-contradictory statement (which we may denote by "c") may be said to have the class of all logically possible basic statements as its class of potential falsifiers.\textsuperscript{255} However, didn't Popper already state (in chapter four of \textit{L.Sc.D.}) that a self-contradictory theory is such that we cannot say either that any basic statement is compatible or incompatible with that theory?\textsuperscript{256} Also, it's not clear here with regard to statements of a high level of universality (such as strictly universal statements or theories) in what sense a self-contradictory statement has as its class of potential falsifiers all possible basic statements. We shall have more to say on this latter point in Part V of Chapter III, where it will be argued that Popper's remark that self-contradictory statements have as their classes of potential falsifiers all possible basic statements, leads to difficulties. In any case, these difficulties having been briefly noted, we shall now continue in our exposition of Popper's subclass method. Popper points out that since a self-contradictory statement has as its class of potential falsifiers the class of all possible basic statements, we can arbitrarily assign to such a statement (with respect to its degree of falsifiability) a value of "1". Further, we can say that a statement which is both consistent and falsifiable can be characterized as falling somewhere between the value "1" (i.e. contradiction) and the value "0" (i.e. tautology):\textsuperscript{257}

If we arbitrarily put $Fsb(c) = 1$; i.e. arbitrarily assign the number 1 to the degree of falsifiability of a self-contradictory statement, then we may even define an empirical statement $e$ by the condition $1 > Fsb(e) > 0$.

At this point, one might be tempted to suggest that we could perhaps quantify any theory's degree of falsifiability as a fraction lying
somewhere between 0 (tautology) and 1 (contradiction). However, Popper rejects this suggestion on the grounds that such an attempted quantification would make comparable theories which at least by the subclass method are incomparable:

Clearly, we cannot possibly order all statements in this way; for if we did, we should be arbitrarily making the non-comparable statements comparable.  

For example, going back to our earlier example of the two non-comparable theories, viz., the theory of relativity and the geological theory of the origin of the earth's crust, we may find that one can be assigned a value of \( \frac{1}{4} \) and the other (let's say relativity) can be assigned the value of \( \frac{1}{3} \). But then, by arbitrarily assigning values to these two theories which are non-comparable by the subclass method we have made them comparable in the following sense: We could say that the theory of relativity is more falsifiable than the geological theory since relativity has a value of \( \frac{1}{4} \) whereas the geological theory has a value of \( \frac{1}{3} \). However, Popper does allow for the arbitrary assigning of numerical values in the case of theories which are comparable by the subclass method.  

That is, if \( T_1 \)'s class of potential falsifiers includes \( T_2 \)'s class of potential falsifiers we might arbitrarily assign the value \( \frac{1}{3} \) to \( T_1 \) and \( \frac{1}{4} \) to \( T_2 \).

As with his dimension method, Popper next attempts to show that by his subclass method a theory which is highly falsifiable is also one which has a high degree of empirical content. He shows this by making precise the notion of "empirical content" using the notion of "derivability." Popper starts out clarifying the notion of empirical content by distinguishing it from Carnap's notion of "logical content".
He will first show that the notion of logical content can be spelled out in terms of derivability and that the results we obtain for comparing logical content using derivability are the same for comparing empirical content given certain conditions \(^{262}\) (which we shall shortly specify.)

First, Popper points out that his notion of empirical content is closely related to Carnap's concept of "logical content" though as we shall next see these two concepts are not identical. Popper next defines empirical content as a theory's class of potential falsifiers:

I define the empirical content of a statement \(p\) as the class of its potential falsifiers.\(^ {263}\)

Using the concept of derivability, Popper spells out the notion of logical content as follows: A theory's (or statement's) logical content is the class of all those non-tautological statements which are derivable from that theory:

The logical content is defined, with the help of the concept of derivability, as the class of all non-tautological statements which are derivable from the statement in question.\(^ {262}\)

Further, this class of non-tautological statements which are derivable from the theory is called the "consequence class" of that theory.\(^ {265}\)

Popper then notes that we can compare (degrees of) logical content of two theories as follows: If \(T_1\) and \(T_2\) are mutually derivable (i.e. \((T_1 \rightarrow T_2) \& (T_2 \rightarrow T_1)\) or \(T_1 \leftrightarrow T_2\)') in which case their "consequence classes" would be identical then \(T_1\) and \(T_2\) have the same degree of logical content:

If the derivability is mutual (in symbols, \(p \leftrightarrow q\')) then \(p\) and \(q\) are said to be of equal content.\(^ {266}\)

Also, if \(T_2\) is derivable from \(T_1\) but \(T_1\) is not derivable from \(T_2\) (i.e. \((T_1 \rightarrow T_2) \& \neg(T_2 \rightarrow T_1)\)') in which case \(T_2\) 's consequence class is a
subclass of \( T_1 \)'s consequence class then \( T_1 \) has a greater logical content than \( T_2 \): 

If \( q \) is derivable from \( p \), but not \( p \) from \( q \), then the consequence class of \( q \) must be a proper sub-set of the consequence class of \( p \); and \( p \) then possesses the larger consequence class, and thereby the greater content (or logical force). 267

Notice at this point the difference between Carnap's concept of logical content and Popper's notion of empirical content: A statement's logical content is the class of all non-tautological statements (which would therefore include such "metaphysical" statements as "(\( \exists x \) (\( Rx \& \neg \exists x \))") which are derivable from it whereas a statement's empirical content is the class of its potential falsifiers. The class of a statement's potential falsifiers is the class of singular existential statements contradicting the theory by asserting specific occurrences of the forbidden event \( P \) at various \( k \)'s. These statements can be derived from the theory but only with the aid of initial conditions. But given these differences between the concepts of empirical content and logical content, can we use the results for comparing (degrees of) logical content for the comparison of (degrees of) empirical content? Popper answers in the affirmative to this question provided that the two statements being compared contain no "metaphysical elements" such as strictly existential statements:

It is a consequence of my definition of empirical content that the comparison of the logical and of the empirical contents of two statements \( p \) and \( q \) leads to the same result if the statements compared contain no metaphysical elements. 269

However, it's not clear that the results for comparing empirical contents of two statements will be the same as for the comparison of logical content simply by eliminating any "metaphysical elements". This is so
because the consequence class of a "pure" empirical statement containing no metaphysical elements will still include as its elements certain permitted basic statements.\textsuperscript{270} That is, the logical content of a "pure" empirical statement will be its class of permitted and forbidden basic statements (and for that matter its singular there-is not statements) whereas the empirical content of this same "pure" empirical statement will be its forbidden basic statements only. Nonetheless, these difficulties aside, Popper spells out how the results for comparing logical contents can be applied for the comparison of empirical contents. First, if \(T_1\) has the same logical content as \(T_2\) then \(T_1\) has the same empirical content as \(T_2\).\textsuperscript{271} Second, if \(T_1\)'s logical content is greater than that of \(T_2\) then \(T_1\)'s empirical content is greater than or equal to \(T_2\)'s empirical content.\textsuperscript{272} Popper notes that he added the disjunct "or equal to" in the case of empirical content to cover cases where \(T_1\) has more logical content than \(T_2\) by virtue of metaphysical statements.\textsuperscript{273} This seems strange, however, given that Popper has already noted that we can use the results obtained from comparing logical contents (using derivability) provided that the statements being compared contain no metaphysical elements.\textsuperscript{274} Third, if \(T_1\) has a higher empirical content than \(T_2\) then \(T_1\)'s logical content will be greater than \(T_2\)'s or at least their logical contents are non-comparable.\textsuperscript{275} Popper notes that he has added the disjunct "or at least their logical contents are non-comparable" on the basis of considerations similar to those for adding "or equal to" in comparing empirical contents given that \(T_1\)'s logical content is greater than that of \(T_2\).\textsuperscript{276} That is, provided that neither \(T_1\) nor \(T_2\) contain any metaphysical elements then their logical contents will be the same.
as their empirical contents (though as we have already seen, this is not really the case). Therefore, if $T_1$'s empirical content is greater than that of $T_2$, then it follows that $T_1$'s logical content is greater than that of $T_2$. But if $T_2$ contains some metaphysical elements (whereas $T_1$ does not) then $T_1$ and $T_2$ are non-comparable with respect to their logical content.

Popper points out that in distinguishing between a statement's logical content and its empirical content we have also made a distinction between non-tautological or synthetic statements on the one hand and empirical statements on the other. That is, a synthetic statement would be a statement which has a non-empty consequence class in which case even a metaphysical statement such as "$(\exists x) (\neg x \& \forall x\neg x)$" would be synthetic since a class of non-tautological statements such as "$(\exists x) (\neg x \& \forall x \neg x)$" can be derived from it. Further, the class of empirical or falsifiable statements would be a subclass of the class of synthetic statements. Also, Popper notes that similar to falsifiable statements, we could regard synthetic statements as lying "in the open interval between self-contradiction and tautology."

And so, Popper has supposedly shown that a theory $T_1$ has a higher degree of empirical content than $T_2$ if $T_2$ is derivable from $T_1$ but $T_1$ is not derivable from $T_2$ in which case $T_2$'s class of potential falsifiers is a subclass of $T_1$'s class of potential falsifiers. This comparison using derivability assumes that both $T_1$ and $T_2$ contain no metaphysical elements. Further, if $T_1$ has higher empirical content than $T_2$ in which case $T_2$'s class of potential falsifiers is a subclass of that of $T_1$ then $T_1$ is also more falsifiable than $T_2$. And so, as Popper
Thus I regard the comparison of the empirical content of two statements as equivalent to the comparison of their degrees of falsifiability.\textsuperscript{281}

In comparing degrees of falsifiability by the subclass method and in comparing degrees of empirical content we consider whether $T_2$'s class of potential falsifiers are a subclass (or identical with) $T_1$'s class of potential falsifiers. And as we have seen (in the case of comparing empirical contents) we can use derivability relations to compare classes of potential falsifiers and therefore to compare degrees of falsifiability:

Thus it will be possible to base the comparison of degrees of falsifiability to a large extent upon derivability relations.\textsuperscript{282}

Popper next uses the notion of derivability used to spell out degrees of empirical content (and hence degree of falsifiability) to show that the methodological demands for theories which are either highly precise or which have a high degree of universality are reducible to the demand for high empirical content (and hence high degree of falsifiability).

Thus, Popper attempts to show that if $T_1$ is either more precise or more universal than $T_2$ then $T_1$ has a higher degree of empirical content (and is more falsifiable) than $T_2$. In order to support this claim, Popper once more uses the notion of derivability. First, Popper introduces four strictly universal statements (or theories) which differ from one another either in terms of degree of precision or in terms of degree of universality or both:

\begin{align*}
p & : \text{All orbits of heavenly bodies are circles.} \\
q & : \text{All orbits of planets are circles.} \\
r & : \text{All orbits of heavenly bodies are ellipses.} \\
s & : \text{All orbits of planets are ellipses.}\textsuperscript{283}
\end{align*}
Further, using the concept of derivability, Popper notes that these statements are related as follows: From p all the others (q, r, s) are derivable and s is derivable from all the others (p, q, r). And as Popper has already noted, there is a close relationship between derivability and logical or for that matter empirical content. That is, if q is derivable from p but not vice-versa, then p's class of potential falsifiers includes that of q as a subclass in which case p has higher empirical content and is more falsifiable than s. Popper sums up these derivability relations as follows:

```
P  q  r
  |  /
  v  v
 ```

Having noted these derivability relations, Popper next shows that the degree of universality and precision of p, q, r and s correspond to their derivability relations. First, Popper notes that in fact p's level of universality is higher than that of q since all orbits of planets are a subclass of all orbits of heavenly bodies. Popper notes that his notion of "universality" corresponds to the notion of "extension of subject" in classical logic. That is, given two statements p and q of the following form: p; "(x) (ϕpx → fpx)" and q; (x) (ϕqx → f qx)", we can say that p has a higher degree of universality than q if "ϕq x" logically implies "ϕpx" but not vice-versa. In short, p is more universal than q because the subject of p, viz "ϕpx" is derivable from the subject of q, viz "ϕqx" but not vice- versa. That is, everything that can be said about the orbits of planets ("ϕ qx") can be said about the orbits of heavenly bodies ("ϕ px") but not everything that is true of
orbits of heavenly bodies is true of orbits of planets. ("opl" is more extended or broader than "oph".) Further, because the orbits of planets are a subclass of the orbits of heavenly bodies, if p is falsified so is q but not vice-versa.290 In short "p is more easily falsified than "q".291 And this was also our conclusion from our desirability relations, viz., since "p→q", p has a higher empirical content and is more falsifiable than q. Second, Popper notes that r which is also derivable from p is less precise than p. (That is, corresponding to the notion of "restriction of predicate" in classical logic, 292 and given that p and r are of the following general form, p: "(x) (ϕx → fx)") and r: "(x) (ψx → frx)" we can say that p is more precise than r. This is so because "fx" logically implies "frx" but not vice-versa.293 In short, everything that can be said of p's predicate "fx" can be said of r's predicate "frx" but not vice-versa. Thus, p's predicate is narrower294 (or more restricted) than that of r and hence we can say that p is more precise than r). Thus, since all circles are a subclass of ellipses, if r is falsified (i.e. an ellipse theory) then so is p (a circle-theory), but not vice-versa.295 And so we can say that p is more (easily) falsifiable than r. Once again there is a correspondence between our derivability relations, viz. "p → r" (in which case p is more falsifiable than r) and degree of precision. That is, "p → r" and hence p is more falsifiable than r and also p is more precise than r. Also, Popper points out that "q → s" (in which case q is more falsifiable and has a higher empirical content than s) and that in fact q is more precise296 than s (i.e. circles are a subclass of ellipses).297 He further notes that "r → s" (in which case r is more falsifiable than s) and that in fact
Finally, he notes that "p → s" (in which case p is more falsifiable than s) and that p is more precise and more universal than s, (i.e. p's subject is more extended than s's and p's predicate is narrower than s's). And so, Popper concludes from his comparison of p, q, r and s that there is a correspondence between degree of falsifiability (and empirical content) on the one-hand and degree of precision or universality on the other:

To a higher degree of universality or precision corresponds a greater (logical or) empirical content, and thus a higher degree of testability.

Before completing our exposition of Popper's subclass method for comparing degrees of falsifiability, several points should be noted.

First, notice that ease of falsification enters into Popper's subclass method (as with his dimension method) for comparing degree of falsifiability. That is, the theory $T_1$ which is either more precise or more universal (or both) than another theory $T_2$ is also easier to falsify than $T_2$. If $T_1$ is more universal than $T_2$ then if $T_2$ is falsified so is $T_1$ but not vice-versa. Thus, $T_1$ is more easily falsified than $T_2$. Also, if $T_1$ is more precise than $T_2$ then if $T_2$ is falsified so is $T_1$ but not vice-versa. Again, $T_1$ is more easily falsifiable than $T_2$. And, since Popper has supposedly shown that the more precise or universal theory is also the one with the highest empirical content (and the highest degree of falsifiability), therefore, the theory which by his subclass method is the most falsifiable theory is also the one which is easiest to falsify. Recall that if "p → q" and hence q's class of potential falsifiers is a subclass of p's class of potential falsifiers then p is more falsifiable (and has a higher empirical content)
than q. Also, since (by his subclass method) we can say that p has "more" potential falsifiers than q in the sense that q's class of potential falsifiers is a subclass of that of p, p can be more easily falsified than q. It would seem that this practical notion of ease of falsification lies behind his subclass method as it does in the case of his dimension method. Thus, in the case of the subclass method, Popper is being consistent with his earlier observations that the most falsifiable theory is the one which can be most easily falsified (and which has the highest empirical content).

Second, notice that by the subclass method, the circle-theory p, "All orbits of heavenly bodies are circles" is more precise and hence more falsifiable than the ellipse-theory x "All orbits of heavenly bodies are ellipses." This result is consistent with the result of the dimension method (as Popper himself points out) viz., the dimension for a circle-theory is "3" whereas the dimension for an ellipse-theory is "5" in which case a circle-theory is more (easily) falsifiable than an ellipse-theory. Further, Popper makes some general comments or remarks having to do with the compatibility between the results obtained from his dimension method and the results obtained from his subclass method. He first notes that "There will be cases in which neither, or only one, of the two methods is applicable." As we have already seen, Popper's subclass method is only applicable in cases where one theory's class of potential falsifiers includes (or is identical with) the other theory's class of potential falsifiers. Recall our example of the theory of relativity versus a geological theory. It's also not immediately clear in this example that these two theories could be compared by Popper's
dimension method either. As Popper later notes,\textsuperscript{307} two theories which may be incomparable by his subclass method may be comparable by his dimension method such as in the case with a circle-theory versus a parabola-theory. They are comparable because the circle-theory can be assigned, "3" as its characteristic number and the parabola-theory can be assigned "4" as its dimension. (Thus, the circle-theory is more (easily) falsifiable than the parabola-theory.) Popper also notes that it could sometimes happen that (when both methods are applicable) by the dimension method two theories will be assessed as being equally falsifiable whereas by the subclass method the one theory will be assessed as being more falsifiable than the other. In such a case, Popper gives the following decision procedure: We should accept the results of the subclass method since it is more "sensitive":

But if in a particular case both methods are applicable, then it may conceivably happen that two theories of equal dimensions may yet have different degrees of falsifiability if assessed by the method based upon the subclass relation. In such cases, the verdict of the latter method should be accepted, since it would prove to be the more sensitive method.\textsuperscript{308}

For example, the following "circle-theories" would be assessed as being equally falsifiable by the dimension method since in both cases, \( d = 3 \); p "All orbits of heavenly bodies are circles" and q "All orbits of planets are circles". However, using Popper's subclass method, we would assess p as being more falsifiable than q since p has a higher degree of universality than q. That is, Popper's subclass method is "sensitive" as it were to the fact of p's higher level of universality. Finally, Popper argues that in all other cases where both methods are applicable their results should agree.\textsuperscript{309} Recall that p "All orbits
of heavenly bodies are circles" and r "All orbits of heavenly bodies are ellipses" are given the following assessment by Popper's subclass method: p is more falsifiable than r because p is more precise than r. And, by Popper's dimension method, p is a circle-theory with d = 3 whereas r is an ellipse-theory where d = 5 in which case p is more falsifiable than r. Thus, the two results agree.

Third, Popper points out that the dimension method and the subclass method are related (vis a vis his comments on possible conflicts between their results)\textsuperscript{310} and also in the following way: An issue which is closely connected with the problem of assessing a theory's degree of falsifiability, says Popper, is the issue of how many initial conditions (and hence parameters) have to be specified before the theory can be in a position to be falsified. That is, the fewer the coordinate pairs and hence parameters to be specified the easier it will be to falsify the theory or at least the easier it will be to make the theory capable of being falsified. Further, as Popper points out, the subclass relation method does not take into account the number of parameters to be specified:

The question of the number of parameters which have to be ascertained, and to be substituted in the formulae, cannot be elucidated with the help of the subclass relation, in spite of the fact that it is evidently closely connected with the problem of testability and falsifiability and their degrees.\textsuperscript{311}

Now as we have already seen, the dimension method does approach the problem of how we can assess a theory's degree of falsifiability from the angle of seeing how many parameters (and coordinates) have to be specified. In short, the subclass method and the dimension method approach the problem of assessing a theory's degree of falsifiability from
different angles. In the case of the subclass method we consider such
things as a theory's degree of universality or precision whereas in the
dimension method we are concerned with the number of freely occurring
parameters in a formula.\textsuperscript{312}

Finally, in his discussion of his subclass method, Popper notes
that a theory's \textit{logical} probability (which he says, is not to be confused
with numerical probability,\textsuperscript{313} i.e. \( P(A) = \frac{n(A)}{n(W)} \) where \( A \) is a subset of
\( W \))\textsuperscript{314} is related inversely to its degree of falsifiability. Thus a
theory which has a low degree of falsifiability will have a high logical
probability and vice-versa:

\begin{quote}
The logical probability of a statement is complementary
to its degree of falsifiability it increases with decreasing
degree of falsifiability.\textsuperscript{315}
\end{quote}

Further, Popper notes that a statement with "0" degree of falsifiability
will have a logical probability of "1". Thus, a tautology or metaphysi-
cal statement as we have seen have a falsifiability of "0" and hence
their logical probability would be "1". (For that matter, a contradic-
tion with falsifiability of "1" would have a logical probability of "0").

The logical probability 1 corresponds to the degree of
falsifiability and \textit{vice-versa}.\textsuperscript{316}

Also, in chapter ten\textsuperscript{317} of \textit{L.Sc.D.}, Popper argues that theories which
have "many opportunities of clashing with basic statements"\textsuperscript{318} are those
which he regards as being logically improbable.\textsuperscript{319} Thus, if a theory
has little chance of being refuted then Popper would regard it as being
highly (logically) probable by virtue of this fact.

\textbf{Part V: The Problem of Simplicity Resolved}

\begin{quote}
In order to resolve certain difficulties associated with the
epistemological concept of simplicity, Popper has stipulated that a theory's degree of simplicity is the same as its degree of falsifiability.\textsuperscript{320} Further, Popper has outlined two related though independent methods for comparing theories' degrees of falsifiability. First, T\textsubscript{1} is more falsifiable (and hence simpler) than T\textsubscript{2} if T\textsubscript{1}'s class of potential falsifiers includes T\textsubscript{2}'s class of potential falsifiers as its subclass.\textsuperscript{321} This method based on subclass relations has one limitation: It is applicable only in situations where one theory's class of potential falsifiers is a subclass of that of the other theory.\textsuperscript{322} Second, T\textsubscript{1} is more falsifiable (and hence simpler) than T\textsubscript{2} if T\textsubscript{1}'s characteristic number or dimension is lower than T\textsubscript{2}'s dimension.\textsuperscript{323} A theory's dimension is the number of relatively atomic statements (in that theory's field of application) which, when conjoined to form a d-tuple are not able to falsify that theory owing to the d-tuple's low degree of composition.\textsuperscript{324} But, certain d + 1-tuples can falsify the theory. (That is, these d + 1-tuples which can falsify the theory would be a composite basic statement asserting a specific occurrence of an event (at K) forbidden by the theory.)\textsuperscript{325} Further, the lower a theory's dimension the easier it is to falsify that theory since if for example d = 3 for T\textsubscript{1} then it would only take certain 4-tuples to falsify it whereas if for T\textsubscript{2}, d = 5, then it would take certain 6-tuples to falsify it. Popper regards the dimension method as being less "sensitive" than the method using subclass relations since by the dimension method we may regard two theories as being equally falsifiable although by the subclass method it may become clear that this is not the case.\textsuperscript{327} (This is assuming that both methods are applicable.)
In the second half of chapter seven of L.Sc.D., Popper tries to show how his stipulative definition of simplicity in terms of degree of falsifiability clears up the problems (associated with the epistemological concept of simplicity) which he outlined in the first half of the chapter. Having outlined precisely what Popper means by "degree of falsifiability" we are now in a position to examine his argumentation here.

First, in response to Schlick's unsuccessful attempt to distinguish between "law" and "chance" using the concept of simplicity, Popper refers the reader to later sections of L.Sc.D. where he will supposedly show that "probability statements about sequences having chance-like characteristics turn out to be of infinite dimension." Consequently, these probability statements about these chance-like sequences will be infinitely complex since the higher a statement's dimensions the lower will be its degree of falsifiability and therefore the higher will be its complexity. Also, recall (from part III of this exposition) that both Schlick and Feigl expected that the concept of simplicity would help in the definition of law-likeness or regularity. Popper notes that since a high degree of universality or precision corresponds to a high degree of falsifiability and hence we can therefore say that a theory's degree of strictness (strictness in the sense of universality) or the degree of law-likeness is the same as its degree of falsifiability and hence simplicity. That is, the more universal or strict or law-like a theory is, the simpler it is.

...we may perhaps identify the degree of strictness of a theory—the degree, as it were, to which a theory imposes the rigour of law upon nature—with its degree of falsifiability, which shows that the latter does just what Schlick...
and Feigl expected the concept of simplicity to do.\textsuperscript{332}

As we shall next see, Popper also tries to show that his stipulative definition of simplicity resolves difficulties associated with the concept of simplicity in inductive logic.

One of the major problems connected with the concept of epistemological simplicity in inductive logic (as we have seen in Part III)\textsuperscript{333} is why we should assume for example\textsuperscript{334} that a theory written as a first degree equation (ex. \( Y = mx + b \)) is simpler than another theory or law written as a second degree equation (ex. \( y = ax^2 + bx + c \)). Popper argues that by his dimension method, the first degree equation has lower dimension (\( d = 2 \)) than the second degree equation (\( d = 3 \); Notice the number of parameters) and therefore it is more (easily) falsifiable and hence \textit{simpler} than the second degree equation:

I have already shown that theories of a lower dimension are more easily falsifiable than those of a higher dimension. A law having the form of a function of the first degree, for instance, is more easily falsifiable than one expressible by means of a function of the second class.\textsuperscript{335}

Popper further notes that a law which can be expressed as an algebraic function of the second degree, however, is the most falsifiable.\textsuperscript{336} However, this seems to amount to an outright contradiction on his part since a second degree function (such as \( y = ax^2 + bx + c \)) is an algebraic function.\textsuperscript{337}

The other major problem having to do with simplicity in inductive logic is the problem of why we should desire the simplest law or theory. Or as Popper states, no reason is given, "for believing that 'simple' laws have advantages over the less simple--apart from aesthetic and practical ones."\textsuperscript{338} However, Popper notes that in stipulating that
(degree of) simplicity is the same as degree of falsifiability we now have ample reason for desiring the simplest theory, viz., the theory which is the simplest will also be the one which is most falsifiable. And since we desire high falsifiability we shall also desire high simplicity:

Above all, our theory explains why simplicity is so highly desirable . . . simple statements, if knowledge is our object, are to be prized more highly than less simple ones because they tell us more; because their empirical content is greater; and because they are better testable.  

But why do we desire high falsifiability (or high empirical content)? As we saw in our exposition of chapters one and two of L.Sc.D. (See Part I of this exposition)  Popper endorses falsifiability as the criterion of demarcation separating science from non-science because it resolves the problems of demarcation and induction. Thus, we will want theories in science which are highly falsifiable given that we all agree (given Popper’s arguments and shared values) that falsifiability is the distinguishing mark of scientific theories.

Recall from Part III of this exposition  that Weyl having abandoned his attempt to base simplicity on probability notes that a function which is a priori mathematically simple must have a low number of unspecified constants or parameters. Popper notes that indeed in his discussion of the dimension method, the theory which is highly falsifiable will have a low number of parameters.  And since he has equated degree of simplicity with degree of falsifiability, it follows that the theory which is highly simple will have (or rather its mathematical expression will have) a low number of parameters.

Another difficulty connected with the epistemological concept
of simplicity which Popper notes in the first half of chapter seven is the problem of why we should regard descriptions with the help of theories to be simpler than descriptions with the help of singular statements. As Popper has already pointed out, the higher a statement's level of universality the more falsifiable and hence the simpler it is.\textsuperscript{343} Also, Popper notes that this agrees with the belief that the "more universal statement can take the place of many less universal ones and for that reason has often been called simpler."\textsuperscript{344}

In addition to solving difficulties connected with the epistemological concept of simplicity which Popper outlined in the first half of chapter seven, Popper further argues that his stipulative definition of simplicity makes possible the resolution of certain contradictions\textsuperscript{345} having to do with simplicity such as the following: It is often assumed that the shape of such curves as logarithmic curves and sine curves is somewhat complex although it is at the same time assumed that laws or theories expressed as sine functions or logarithmic functions are simple:

Few would regard the geometrical shape of, say, a logarithmic curve as particularly simple; but a law which can be represented by a logarithmic function is usually regarded as a simple one. Similarly, a sine function is commonly said to be simple, even though the geometrical shape of the sine curve is perhaps not so very simple.\textsuperscript{346}

However, Popper argues that we can make sense of these two prima facie contradictory assumptions if we keep in mind his earlier comments on number of parameters and corresponding degree of falsifiability (and hence simplicity) and also if we recall his comments (see Part IV of this exposition) on generality (in terms of which he defined formal and
material reduction of dimensions).\textsuperscript{347} That is, a logarithmic curve in its highest state of generality (i.e. it is invariant with respect to all sets of coordinates and hence through various transformations \( x \) and \( y \) coordinates can be interchanged) has, according to Popper, five unspecified constants or parameters.\textsuperscript{348} Now, if we recall that a high dimension (ex: \( d = 5 \)) and hence a low degree of falsifiability and simplicity corresponds to a high number of parameters, then we can make sense of the claim that a logarithmic curve is complex. That is, such a curve is described by a formula with a high number of parameters and hence it is complex.\textsuperscript{349} Popper further notes that we can make sense of the second assumption that a law expressed as a logarithmic function is simple and we shall also see that there is no contradiction here. Recall Popper's earlier comments (again see Part IV) that it is sometimes inappropriate to require high generality or invariance with respect to all coordinate systems. Cases where it would be inappropriate to require invariance are where we have a law which is expressed as in our example here, by a logarithmic function. This is so because it would not make (empirical) sense to interchange \( x \) and \( y \) coordinates if, for example, we are saying that pressure \( (y) \) is a function of (our independent variable) temperature \( (x) \). And so, a law represented by a logarithmic function will have a lower level of generality and hence a lower number of parameters, (since it is restricted to a certain group of coordinate systems).\textsuperscript{350} Further, if a logarithmic function representing a law is less general and hence has a low number of parameters, we would (keeping in mind the relation between number of parameters and degree of falsifiability) say that this function (or the law which it represents) is simple.
Thus, a logarithmic curve by virtue of its generality is complex though a logarithmic function expressing an empirical law owing to its low generality is simple. Popper finally notes that corresponding remarks can be made about sine functions and sine laws.\textsuperscript{351}

Another additional difficulty associated with epistemological simplicity which can be resolved (supposedly) by Popper's stipulative definition of simplicity is the simplicity of Euclidean geometry. As Popper notes, it is often assumed that Euclidean geometry is simpler than various non-Euclidean geometries though no reasons have been given as to why this is so.\textsuperscript{352} Popper argues that if we formulate statements of Euclidean and non-Euclidean geometry as empirical statements (or hypotheses) then we shall see that Euclidean geometry is simpler than non-Euclidean geometries owing to its higher degree of falsifiability:

At first sight the kind of simplicity here involved seems to have little to do with degrees of falsifiability. But if the statements at issue are formulated as empirical hypotheses, then we find that the two concepts, simplicity and falsifiability, coincide in this case also.\textsuperscript{353}

For example, we could, Popper says, identify such geometrical entities as straight lines with light rays.\textsuperscript{354} In so doing, Popper argues that it can be shown that a Euclidean light-ray geometry is simpler than a non-Euclidean light-ray geometry. He notes that the sum of the angles of a light ray triangle in Euclidean geometry should not deviate significantly from 180°.\textsuperscript{355} Otherwise, "any significant deviation from 180 degrees will falsify the Euclidean Hypothesis."\textsuperscript{356} However, the non-Euclidean light-ray hypothesis "would be compatible with any particular measurement not exceeding 180 degrees."\textsuperscript{357} Also, he notes that in order to falsify the non-Euclidean hypothesis it would be necessary to take
measurements of the "(absolute) size" of the triangle involving measure-
of area. In short, given these additional measurements (thus higher
dimension) and the fact that the non-Euclidean hypotheses cannot be fals-
sified by any values under 180°, Popper concludes that the Euclidean
hypothesis can be more easily falsified than the non-Euclidean hypothesis.
(Therefore, the Euclidean hypothesis is simpler than the non-Euclidean
hypothesis).

Thus we see that more measurements are needed for a falsifi-
cation; that the hypothesis is compatible with greater
variations in the results of measurements; and that it is
therefore more difficult to falsify; it is falsifiable to a
lesser degree.

Finally, Popper comments on the complexity of a theoretical system for-
mulated with reference to the conventionalist methodology.

Popper argues that the conventionalist approach to methodology
would lead to highly complex theoretical systems in science. This is
so because the conventionalist approach involves the introduction of ad
hoc auxiliary hypotheses which take the brunt of the falsification in-
stead of the theory and which would also ensure the "agreement" between
theory and reality.

From my point of view, a system must be described as complex
in the highest degree if, in accordance with conventionalist
practice, one holds fast to it as a system established forever
which one is determined to rescue, whenever it is in danger,
by the introduction of auxiliary hypotheses. For the degree
of falsifiability of a system thus protected is equal to zero.

Since a system which is constantly rescued by the introduction of ad hoc
auxiliary hypotheses can never be falsified, Popper assigns a value of
"0" to such a system in terms of its degree of falsifiability. Further,
since such a system is completely unfalsifiable it is therefore infinitely
complex. In chapter ten of L.Sc.D., Popper therefore notes that he
advocated parsimony (or simplicity) of auxiliary hypotheses not for its own sake but to ensure high testability (and hence simplicity as he defines it stipulatively). Thus, Popper states that his "own rule which requires that auxiliary hypotheses shall be used as sparingly as possible" has the following purpose:

I am not interested in merely keeping down the number of our statements: I am interested in their simplicity in the sense of high testability.  

Further, Popper notes that he also advocates that we adopt a small number of statements of higher universality rather than a larger number of statements of lower universality. But once again, he notes his demand for parsimony of statements (preferring fewer highly universal statements over more statements with lower-level universality) has to do with the desire for high degree of falsifiability and hence simplicity. That is, as he has already shown, statements which are highly universal are also highly falsifiable and hence highly simple.

Finally, Popper relates a theory's "degree of corroboration" with its degree of falsifiability and simplicity in chapter ten of _L.Sc.D._ Recall (from Part II) that a statement which is corroborated is one which has so far stood up to potentially refuting tests: Popper (in noting that an appraisal of corroboration is not a hypothesis but can be derived from a theory along with its accepted basic statements) characterizes an appraisal of corroboration as follows:

It asserts the fact that these basic statements do not contradict the theory, and it does this with due regard to the degree of testability of the theory, and to the severity of the tests to which the theory has been subjected, up to a stated period of time.

Note here that Popper notes the dependence of corroboration on that
theory's degree of testability. Consequently a theory's ability to be corroborated will depend upon the severity of the potentially refuting tests to which it is subjected. Further, the severity of the potentially refuting tests to which a theory is subjected is dependent upon that theory's degree of falsifiability and its degree of simplicity. For example, the theory p: "All planets move in circles" when subjected to potentially refuting tests can be refuted by statements of the form "There is a planet, Venus, at k such that Venus moves in an elliptical orbit". However, the more falsifiable and hence simpler theory q: "All heavenly bodies move in ellipses" cannot only be falsified by "Venus moves in an elliptical orbit" but also by such statements as "Hayley's comet moves in a highly eccentric elliptical orbit," etc. Consequently, a test of p will be less severe than a test of q since in a test of q there is a greater opportunity of clashing with basic statements, (and hence it can be more easily falsified). Thus, q can be more highly corroborated than p since the tests which q passes are more severe than those which p passes. And so, Popper sums up this link between degree of corroborability and degree of simplicity (and falsifiability):

But the severity of the test, in its turn, depends upon the degree of testability, and thus upon the simplicity of the hypothesis: the hypothesis which is falsifiable in a higher degree, or the simple hypothesis, is also the one which is corroborable in a higher degree.

This concludes our exposition of Popper's stipulative move of equating (degree of) simplicity with degree of falsifiability and related topics (such as what constitutes a theory's falsifiability and why we should endorse falsifiability as our criterion of demarcation). In the second part of this paper, we shall critically examine other authors' criticisms.
of Popper's stipulative definition of simplicity and in the third part of this paper I shall propose some of my own criticisms.
FOOTNOTES

2. Ibid., p. 27.
3. Ibid., p. 28.
4. Ibid., p. 28
5. Ibid., p. 28.
6. Ibid., p. 35.
7. Ibid., p. 40.
8. Ibid., p. 28.
9. I.e. this principle of induction would put inductive inferences into a logically acceptable form.
11. Ibid., p. 29.
12. Ibid., p. 29.
13. Ibid., p. 29. Also vide p. 294.
15. Ibid., p. 30.
16. Ibid., p. 36.
17. Ibid., vide p. 39 of L.Sc.D.
18. Ibid., pp. 40-41.
19. Note: The inductivist would have no problem with deciding the falsity of statements since they too would have at their disposal the modus tollens of classical logic.
20. Popper, p. 42.
21. Ibid., p. 41.
22. Ibid., p. 76.
23 Ibid., p. 37.
24 Ibid., p. 35.
25 Ibid., p. 35.
26 Ibid., p. 38.
27 Ibid., p. 42.
28 Ibid., p. 37.
29 Ibid., p. 37.
30 Ibid., p. 38.
31 Ibid., p. 38.
32 Ibid., pp. 41-42; p. 50; p. 81.
33 Ibid., p. 42.
34 Ibid., p. 79.
35 Ibid., p. 81.
36 Ibid., pp. 50 and 54.
37 Ibid., p. 42.
38 Ibid., p. 54.
39 Ibid., p. 54.
40 Ibid., pp. 53-54.
41 Ibid., pp. 82-83.
42 Ibid., p. 54.
43 Ibid., p. 53.
44 Ibid., p. 52.

47 Ibid., p. 53.
48. Ibid., p. 46.
49. Ibid., p. 46.
50. Ibid., p. 46.
51. Popper, L.Sc.D., p. 34.
52. Ibid., p. 62.
53. Ibid., p. 62.
54. Ibid., p. 62.
55. Ibid., p. 62.
56. Ibid., p. 62.
57. Ibid., p. 62.
58. Ibid., pp. 62-63.
59. Ibid., p. 63.
62. Ibid., p. 68.
63. Ibid., p. 69.
64. Ibid., p. 102.
67. Ibid., pp. 68-69.
68. Ibid., p. 69.
69. Ibid., p. 69.
70. Ibid., p. 102.
71. Ibid., p. 69.
72. Ibid., p. 70.
73 Ibid., p. 69.
74 Ibid., pp. 88-89.
75 Ibid., p. 89.
76 Ibid., p. 89.
77 Ibid., p. 89.
78 Ibid., p. 89.
79 Ibid., p. 43.
80 Vide p. 60 of L.Sc.D.
81 Ibid., p. 101.
82 Ibid., p. 101.
83 Ibid., p. 101.
84 Ibid., p. 101.
85 Ibid., p. 101.
86 Ibid., pp. 101-02.
87 Ibid., p. 102.
88 Ibid., p. 102.

89 The only difference in the case of singular statements from that of strict statements is the presence of "Kx", i.e. x is at the spatiotemporal region K.
91 Ibid., p. 102.
92 Ibid., p. 102.
93 Ibid., p. 102.
94 Ibid., pp. 102-03.
95 Ibid., p. 103.
96 Ibid., p. 103.
97 Ibid., pp. 102-03.
98 Ibid., p. 86. Also Popper notes that the theory says nothing of its permitted statements. This is so because a theory is a "prohibition"—it rules out certain events and hence the basic statements which assert various occurrences of these events.

99 Ibid., p. 86.
100 Ibid., p. 90.
101 Ibid., p. 90.
102 Ibid., p. 86.
103 Ibid., p. 87.
104 Ibid., p. 104.
105 Ibid., p. 104.
106 Ibid., p. 104.
107 Ibid., pp. 104-05.
108 Ibid., pp. 86-87.
109 Ibid., p. 87.
110 Ibid., p. 33.
111 Ibid., p. 33.
112 Ibid., p. 33.
113 Ibid., p. 86.
114 Ibid., p. 45.
115 Ibid., p. 45.
116 Ibid., p. 86.
117 Ibid., p. 45.
118 Ibid., p. 76.
119 Ibid., p. 86.
120 Ibid., p. 86.
121 Ibid., p. 90.
122 Ibid., p. 90.
123 Ibid., p. 135.
124 Ibid., p. 137.
125 Such as a presentation or exposition of a mathematical proof.
127 Ibid., p. 137.
128 Ibid., p. 137.
129 Ibid., p. 137.
130 Ibid., p. 137.
131 Ibid., p. 137.
132 Ibid., p. 138.
133 Ibid., p. 138.
134 Ibid., p. 138.
135 Ibid., p. 138.
136 Ibid., p. 138.
137 For example, the x-coordinate is a pressure reading and y is a temperature reading.
138 For example, relating pressure to temperature.
140 Ibid., p. 138.
141 Ibid., p. 138.
142 Ibid., p. 138.
143 Ibid., p. 139.
144 Ibid., p. 139.
145 Ibid., p. 139.
146 Ibid., p. 139.
Also, we would prefer the simplest law because of the high improbability.

For example, if we are again given the slope "m" of a linear equation and its y-intercept "b" then, we have precisely determined the equation. Thus, if \( m = 3, b = 5 \), then the equation is \( y = 3x - 5 \).

These similarities are in terms of paucity of parameters and simplicity being mathematically determined.


I.e., a theory rules out a given event or it denies its occurrence. But it does not refer specifically to this or that specific occurrence of this event at a K.

Ibid., p. 114.
Ibid., p. 114.
Ibid., p. 113.
Ibid., p. 113.
Ibid., pp. 122, 124, 131, 141.
Ibid., p. 121.
162 Ibid., p. 114.
170 Ibid., p. 127.
172 Vide p. 36, L.Sc.D., Footnote 72. Popper points out here that a circle theory can be refuted by merely two points coinciding.
173 These values of $x$ and $y$ were taken from an example in Vance, *op cit.* p. 423.
175 Ibid., p. 127.
176 Ibid., p. 127.
177 Ibid., Recall Popper's remarks cited on page 41 of this section.
178 Ibid., pp. 130-31.
179 Ibid., p. 131.
180 Ibid., p. 127.
181 Ibid., p. 127.
182 Ibid., p. 127.
183 Ibid., p. 128.
184 Ibid., p. 126.
185 Ibid., p. 129.
186 Ibid., p. 129.
187 Ibid., p. 128.
188 Ibid., pp. 378-79.
189 Ibid., p. 377.
190 Ibid., p. 128.
191 Ibid., p. 128.
192 Ibid., p. 377.
193. Ibid., p. 128.
194. Ibid., p. 128.
195. Ibid., p. 128.
196. Ibid., p. 128.
197. Ibid., p. 129.
198. Ibid., p. 129.
199. Ibid., p. 129.
200. Ibid., p. 129.
201. Ibid., pp. 129 and 235.
203. Ibid., p. 129.
204. Ibid., pp. 129 and 235.
205. Ibid., p. 235.
206. Ibid., pp. 235-36.
207. Ibid., p. 129.
208. Ibid., p. 129.
210. Ibid., p. 129.
211. Ibid., pp. 102-02.
212. Ibid., p. 102.
213. Ibid., p. 129.
214. Ibid., p. 130.
215. Ibid., p. 122.

216. Vance, op. cit., p. 427. This equation can be obtained from the following circle-equation, viz. "\((x-h)^2 + (y-k)^2 = r^2\)" which is the same as "\(x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = r^2\). Then, let \(D = -2h, E = -2k, F = h^2 + k^2 - r^2\)."
217 Ibid., p. 426.
218 Ibid., pp. 427-29.
221 Ibid., p. 131.
222 Ibid., pp. 143-44.
223 Ibid., p. 378.
224 Ibid., p. 379.
225 Ibid., p. 114.
226 Ibid., p. 131.
227 Vance, op. cit., p. 168.
228 Popper, op. cit., p. 131.
229 Ibid., p. 133.
230 Ibid., p. 133.
231 Ibid., p. 133.
232 Ibid., p. 133.
233 Ibid., p. 133.
234 This is one way of attempting a formal reduction from an ellipse to a circle; Popper outlining other ways, such as making $e = 0$ and specifying the ratio of the axes to be equal to "1".
235 Popper, p. 134.
236 Ibid., p. 134. I.e. the general circle-equation $x^2 + y^2 + Dx + Ey + F = 0$ defines (or describes) all circles regardless of the coordinates by which they are determined.
237 Ibid., p. 134.
238 Ibid., p. 134.
239 This could be achieved, for example, by a 180° rotation of the straight line.
266. Ibid., p. 120.
267. Ibid., p. 120.
268. And also, amongst this class of non-tautological statements there would be permitted basic statements and forbidden statements (which can be derived from a theory only with the help of "initial conditions"), and for that matter singular there is not, statements.
269. Popper.
270. And also, it will contain singular there-is statements.
271. Popper, p. 120.
272. Ibid., p. 120.
273. Ibid., pp. 120-21.
274. Ibid., p. 120.
275. Ibid., p. 121.
276. Ibid., p. 121.
277. Ibid., p. 121.
278. Which in turn must have non-empty consequence classes ad infinitum.
279. Popper, p. 121.
280. We are here using the results obtained in comparing degrees of logical content (using derivability) to compare empirical contents.
281. Ibid., p. 121.
282. Ibid., p. 121.
283. Ibid., p. 122.
284. Ibid., p. 122.
285. Ibid., p. 122.
286. Ibid., p. 122.
287. Ibid., p. 122.
288. Ibid., p. 123, Footnote 2.
289. Ibid., p. 123.
290. Ibid., p. 122.
291. Ibid., p. 122.
292. Ibid., p. 123, Footnote 2.
293. Ibid., p. 123.
294. Ibid., p. 123.
295. Ibid., p. 122.
296. Ibid., p. 122.
297. i.e. the predicate "circle" is narrower and hence more precise than the predicate "ellipse".
298. Popper, p. 122.
299. i.e. the subject "orbits of heavenly bodies" has a greater extension than the subject "orbits of planets".
300. Popper, pp. 122 and 124. (top of page)
301. Ibid., p. 115.
302. Ibid., p. 113.
303. Ibid., p. 131 (top of page)
304. Ibid., p. 130.
305. Ibid., p. 130.
306. Ibid., pp. 115-16. (definition (3) characterizing the subclass method)
307. Ibid., p. 115.
308. Ibid., p. 130.
309. Ibid., p. 130.
310. Ibid., p. 130. Further he notes that "the dimension of a class must be greater than, or equal to, that of its subclasses". Note that statements p and q satisfy this demand.
311. Ibid., p. 127.
312. And also, we could (equivalently) say that the by dimension method assess theories degrees of falsifiability by examining the minimum degree of composition of their potential falsifiers whereas with the subclass method we compare classes of potential falsifiers.

313. Popper, p. 119. Popper does note however that numerical probability can be linked with logical probability, and thus with degree of falsifiability.

314. If \( n(W) = 100 \), \( n(A) = 50 \), then \( P(\text{heads}) = .5 \)

315. Ibid., p. 119 (also see p. 273)

316. Ibid., p. 119.

317. Note also that to highly improbably theory, Popper assigns a low numerical probability (p. 273)


319. i.e. It is improbable because it is 'likely' to be refuted.

320. Popper, p. 140.


322. Ibid., pp. 115-16.

323. Ibid., pp. 125-35.

324. Ibid., p. 129.

325. Ibid., p. 127.

326. Ibid., pp. 131, 144, and 378.

327. Ibid., p. 130.

328. Ibid., pp. 141-45.

329. Popper refers to sections 65, 68, 58 and 59.

330. Ibid., p. 141.

331. Page of this thesis.

332. Popper, p. 141.

333. Note: page 32 of this thesis.
Another example which Popper mentioned is that in inductive logic it is assumed that a circle is simpler than an ellipse. Of course, by his dimension method since $d = 3$ for a circle and $d = 5$ for an ellipse, a circle is a simpler curve than an ellipse (See page 138 of L.Sc.D.)

335 Popper, p. 141.
336 Ibid., p. 141.
337 Vance, op cit., p. 254.
338 Popper, p. 138.
339 Ibid., p. 142.
340 Note: page 9 of this thesis.
341 Note: page 35 of this thesis.
342 Popper, p. 142. "The concept of the dimension of a theory may be said to give precision to Weyl's idea of using the number of parameters to determine the concept of simplicity."
343 Ibid., pp. 141-42.
344 Ibid., p. 142.
345 Ibid., p. 142.
346 Ibid., pp. 142-43.
347 Ibid., p. 143.
348 Ibid., p. 143.
349 Ibid., p. 143.
350 Ibid., p. 143.
351 Ibid., p. 143.
352 Ibid., p. 143.
353 Ibid., p. 143.
354 Ibid., p. 144.
355 Ibid., p. 144.
357 Ibid., p. 144.
358 Ibid., p. 144.
359 Ibid., p. 144.
360 Ibid., p. 145.
361 Ibid., p. 273.
362 Ibid., p. 273.
363 Ibid., p. 273.
364 Ibid., p. 273.
365 Ibid., p. 266.
366 Ibid., p. 267.
CHAPTER TWO

OTHER AUTHORS’ CRITICISMS: A CRITICAL OVERVIEW

Introduction

At this point, we are somewhat familiar with Popper’s stipulative clarification of epistemological simplicity in terms of degrees of falsifiability, and with all the necessary background material. Thus, we are now in a position to critically examine other authors’ criticisms of Popper’s stipulative definition of simplicity (along with other relevant criticisms). First of all, we shall examine and evaluate a criticism of Popper’s dimension method of comparing theories’ degrees of falsifiability and hence, their degrees of simplicity, made by Bunge, Hempel, Sober and Post. Also, several further related criticisms made by Post and Bunge will be considered. Second, two criticisms of Popper’s subclass method, one made by Barker and the other by Hempel will be presented and evaluated. Further, we shall note Ayer’s attempt to “revive” induction by arguing that, since it works in scientific practice, it does not require justification. Also, we shall in connection with this point, mention Wittgenstein’s remarks that induction requires no justification (or, more precisely, to ask for its justification, is not legitimate within a certain language game). The import of these revivals of induction for Popper’s notion of simplicity is that if induction is acceptable, then what makes high falsifiability and hence high simplicity (which is to be regarded as the same as high falsi-
satisfiability) so desirable?

Finally, we shall examine P. K. Feyerabend's attempt (in Against Method) to show the limitations of methodologies, such as Popper's "falsificationist" methodology, by seeing how they fare in actual situations in the history of science. One of the major examples from the history of science which Feyerabend uses to test such methodologies (such as Popper's) is the Copernican Revolution. As we shall see, Popper's methodology, if applied to this period in science, would have hindered "progress". (That is, we would have been forced to abandon the Copernican theory.) However, if Popper's methodology is shown to have limitations, then it is not clear that his falsifiability criterion can and should be endorsed at least in certain situations in science, and hence, it is no longer clear that high falsifiability and high simplicity are always (if ever) desirable.


In his book Simplicity, Elliott Sober agrees with Popper that the dimension method for comparing theories' degrees of simplicity is adequate for cases where we have such general hypotheses as: "The equation relating x and y is linear." and "The equation relating x and y is circular." In short, he would agree that since: for "y = mx+b" is "2", whereas, for "x^2 + y^2 + Dx + Ey + F = 0" is "3", "y = mx+b" is the simpler equation. However, Sober parts company with Popper in the following two cases:

1. \( y = 3x + 6 \)
2. \( y = 5x^4 + 4x^2 - 75x + 168 \)

In their general forms, (1) would clearly be simpler than (2), since
(1) only has two parameters whereas (2) would have four parameters. Yet, with these parameters completely specified in both cases each has zero dimension and hence is falsifiable by one point. And, by the dimension method, they would be regarded as equally simple, which Sober finds unacceptable, and so, Sober comes to the following conclusion:

Popper's theory, although it can handle some conjectures about the general form that an equation will take, cannot handle equations themselves.  

In his *Philosophy of Natural Science*, Carl Hempel, with an inductivist twist, makes a similar criticism of Popper's dimension method. That is, if we have the following four data-points: \((0, 2), (1, 3), (2, 4)\) and \((3, 5)\), then these points can be fitted by the following three hypotheses (or more precisely, by the equations describing the curves which correspond to the hypotheses):  

\[
H_1: v = u^4 - 6u^3 + 11u^2 - 5u + 2 \\
H_2: v = u^5 - 4u^4 - u^3 + 16u^2 - 11u + 2 \\
H_3: v = u + 2
\]

Now, in the absence of any explicit criteria, Hempel notes that \(H_3\) would be favored or preferred "over \(H_1\) and \(H_2\) on the ground that it is a simpler hypothesis than its rivals." Recalling Sober's point to mind, by Popper's dimension method, \(H_3\) (in its most general form) would, of course, be considered to be simpler than either the general forms of \(H_1\) or \(H_2\) owing to the "paucity" of \(H_3\)'s parameters. However, with all parameters specified, \(H_1\), \(H_2\) and \(H_3\), by Popper's method of dimension are equally simple. Further, \(H_1\), \(H_2\) and \(H_3\), as Hempel points out, "can be shown to be false with the same ease--namely, by means of one counter-instance; for example, the data-pair \((4, 0)\) would falsify them all."
And so, since each of these can be falsified with the same ease, they are equally falsifiable and therefore, they are equally simple. Hempel finds this to be unacceptable, since "they do not count as equally simple."\(^8\) Also, R. R. Post, in his article, "A Criticism of Popper's Theory of Simplicity", raises the same criticism as both Hempel and Sober, viz., that "Popper's criteria of falsifiability for measuring simplicity lead to a counter-intuitive result."\(^9\) This counter-intuitive result is, as Sober and Hempel have noted, that two equation-hypotheses which (are in their most general forms assessed by Popper's dimension method as not being equally simple) are, when all of their parameters are specified, assessed as equally simple, even though they are clearly not equally simple. (The examples which Post gives are: A: \(y = 1.50 - 2x\), and B: \(y = 1.50 - 2x + 0.3x^2\) where both A and B are, by Popper's dimension method equally simple, which, according to Post is counter-intuitive.)\(^10\) Finally, in his book The Myth of Simplicity, Runge makes the same point as Hempel, Sober and Post, as can be seen from the following passage:

Besides, the identification of simplicity with paucity of parameters is powerless to gauge the complexity of propositions that cannot be given the form of an equation connecting metrical variables: it assigns the same complexity value (namely, 0) to all expressions in which no adjustable parameters occur.\(^11\)

Runge's criticism here of identifying "simplicity with paucity of parameters" is almost identical with Sober's criticism that Popper's dimension method although it handles general forms of equations, it does not handle cases where all parameters are specified.

In response to the criticism of his dimension method formulated by Sober, Hempel, Runge and Post, Popper could give the following rejoinder. Just as it is often said that a logarithmic curve (described by
an equation with five parameters) is complex, even though a law expressed as a logarithmic function (whose equation contains only one parameter) is simple, it is not counter-intuitive to maintain the following: The curve described by \( y = ax^4 + cx^2 - dx + e \) is obviously more complex than the curve \( y = ax + b \) although the law represented by \( y = 5x^4 + 4x^2 - 75x + 168 \) is no less simple than the law represented by \( y = 3x + 6 \). One could argue that \( y = 3x + 6 \) is easier to solve for \( y \) (we have to do less calculating), but Popper would say this is a practical rather than an epistemological concern. Further, Popper could then argue that just as in the example of a logarithmic function versus a logarithmic curve, we can use his dimension method to make sense of the apparently contradictory claims that, although one curve is simpler than another, the laws represented by their functions are equally simple. That is, in their most general forms in which they are invariant with respect to all coordinate systems, clearly \( y = ax^4 + cx^2 - dx + e \) is less simple than \( y = ax + b \) because the former equation has four parameters (and hence its \( d = 4 \)), whereas the latter has only two parameters (and hence its \( d = 2 \)). On the other hand, the equations \( y = 3x + 6 \) and \( y = 5x^4 + 4x^2 - 75x + 168 \) have each been specified or determined relative to a particular coordinate system and hence in both cases they have no unspecified constants. Consequently, keeping in mind the relation between number of parameters, dimension, degree of falsifiability and degree of simplicity, we would say that they are equally simple.

Another closely related criticism of equating paucity of parameters (along with low dimension) with high simplicity is made by Bunge in *The Myth of Simplicity*. Bunge notes that if we equate paucity
of parameters with (high) simplicity, then the following equations, "y = x" and "y = x^2" are equally simple. But Bunge finds this to be unaccept-
able. (After all, isn't a parabola more complex than a straight line?) However, Popper could easily answer Bunge's criticism as follows: The equation of a parabola can be written alternatively as a specific in-
stance of the general equation of a conic section. Since \( d = 4 \) for a
parabola, (i.e. since it takes four points to specify a parabola) we can write the parabola equation as being a form of "Ax^2 + Bxy + Cy^2 +
Dx + Ey + F = 0", with only four parameters (just as a circle equation
can be written as an instance of the above equation with three parameters).
Further, we can see that, written this way, the equation of a parabola
with four parameters is not equally simple to "y = x" with no parameters.
Also, even if we wish to regard "y = x" as a special instance of the
straight line "y = mx + b", this still has only two parameters and hence,
it is still more simple than a parabola equation.

A third criticism which Bunge, in The Myth of Simplicity,
offers against equating paucity of parameters with simplicity, runs as
follows: The "equation of light absorption", viz., "Ux + \lambda y = 0" has
one parameter (i.e. \( \lambda \)), and therefore is less simple than "Uxx + Uyy +
Uzz - Ut = 0", the "equation of light propagation" which has no para-
meters. But, Bunge says, "this is clearly wrong." Again, Popper
could answer that the more general form of these equations which define
the curves, (or rather the set of curves), regardless of any coordinate
systems (just as "x^2 + y^2 + Dx + Ey + F = 0" defines the set of circles)
are not equally simple. That is, the more general form of the equation
of light absorption would have two parameters and would clearly be
simpler than the general form of the equation of light propagation. Popper's dimension method leads to the counter-intuitive result that the theory $A$: $y = ax + b$ is equal in simplicity to $B$: $y = ax + b + 0.3x^2$. This is not counter-intuitive if we distinguish between complexity of the set of curves described by a general equation, and the complexity of laws represented by less general forms of this equation.

In concluding this section, i.e. criticisms of Popper's dimension method of comparing degrees of simplicity, the general criticism made by Bunge, Sober, Hempel and Post can be characterized (or summed up) as follows: If we equate a theory's simplicity with its dimension (and hence with the number of parameters of the equation which represents it), then we obtain counter-intuitive results, such as two equations with no parameters, but which are "clearly" not equally simple, would be assessed as being equally simple. However, Popper can easily answer this criticism by arguing that it is not counter-intuitive to maintain that, although two curves are not equally simple, two laws described by less general forms of the equations of these curves can be equally simple.

Part II: Two Criticisms of the Subclass Method.

First, Hempel in *Philosophy of Natural Science*, outlines the following criticism of Popper's subclass method: If we have two hypotheses, $H_1$ and $H_2$, which are irrelevant to each other, then we could say (by Popper's subclass method--though it is not clear that Popper would admit that his subclass method is applicable here.) that $H_1$ and $H_2$ conjoined will be more falsifiable (and hence simpler) than either $H_1$ or $H_2$. 
That is, from "H₁ & H₂" we can (by simplification) deduce either H₁ or H₂. But, from either H₁ or H₂ separately, we cannot deduce "H₁ & H₂". Thus, "[(H₁ & H₂) → H₁] & [H₁ → (H₁ & H₂)]" and likewise for H₂. Consequently, H₁'s (and H₂'s) class of potential falsifiers are subclasses of the class of potential falsifiers of "H₁ & H₂" making "H₁ & H₂" more falsifiable and hence, simpler than H₁ or H₂ separately. But, Hempel says, "H₁ & H₂" clearly is not simpler than either H₁ or H₂ separately. The following passage sums up Hempel's counter-argument against Popper's subclass method:

But, Popper alternatively calls one hypothesis more falsifiable and hence simpler, than another if the first implies the second and thus has greater content in a strictly deductive sense. However, greater content is surely not always linked to greater simplicity... for if two unrelated hypotheses (e.g. Hook's ans Snell's laws) are conjoined, the resulting conjunction tells us more, yet it is not simpler, than either component.

In short, Hempel is offering a counter-example to Popper's definition of simplicity in terms of degrees of falsifiability where "H₁ & H₂" is more falsifiable than either H₁ or H₂ separately and yet clearly "H₂ & H₂" is not simpler than either of these hypotheses separately. However, Popper could respond here that his definition of simplicity in terms of degrees of falsifiability is purely stipulative in nature. That is, he is not proposing a descriptive definition of simplicity (whereby T₁ is simpler than T₂ if, and only if T₁ is more falsifiable than T₂) which must cover all situations. Rather, he is stipulating that simplicity is to be the same as degree of falsifiability, because of the "fruitfulness" of such a move, viz., it will enable us to resolve certain problems in epistemology associated with the concept of simplicity:

All I do is assert that the concept of simplicity which I am going to clarify helps to answer those very questions which,
as my quotations show, have so often been raised by philosophers of science in connection with their problem of simplicity.20

Consequently, Hempel's counter-example is inappropriate here (though the method of proposing counter-examples to such descriptive definitions as "knowledge is justified true belief" in epistemology is appropriate).

In fact, Hempel's counter-example does not cast any doubt on the ability of Popper's stipulative definition to resolve the problems in epistemology which were outlined in chapter seven of L.Sc.D. And, Popper proposed his stipulative definition of simplicity in relation to these problems.

Second, C. F. Barker in his article, "On Simplicity in Empirical Hypotheses", presents the following counter-example to Popper's stipulatively defining degree of simplicity in terms of his subclass relation.

Barker sets up the following situation: We have noticed that the spoons are missing and that the maid is acting rather strangely.21 To explain this situation, we might entertain the following hypotheses:

\[
\begin{align*}
H_1: & \text{ The maid has stolen the spoons.} \\
H_2: & \text{ The maid has stolen the spoons and pawned them to raise money which she has used to escape with the butler on the 10:30 plane to Acapulco.}
\end{align*}
\]

As Barker points out, \(H_2\) is more detailed than \(H_1\) and "embraces" \(H_1\).22 That is, \(H_2\) implies \(H_1\) but not vice versa. Thus, \(H_1\)'s class of potential falsifiers are a subclass of \(H_2\)'s class of potential falsifiers. Thus, as Barker points out, both \(H_1\) and \(H_2\) would be falsified if we found that the spoons had simply been misplaced, although finding out that the butler is sick in bed would falsify \(H_2\) but not \(H_1\).23 Consequently, \(H_2\) is more falsifiable (and hence simpler) than \(H_1\). However,
Barker argues that first of all, $H_2$ is a "hasty jumping to conclusions" simply given the skimpy evidence that the spoons are missing and the maid is acting strangely. Thus, he argues that clearly in this example the most falsifiable hypothesis is not the most acceptable one. Second, he notes that $H_2$ is not what we would normally call the simpler hypothesis of the two:

In this example the more falsifiable hypothesis surely is the one which we should normally call the less simple, and it surely is the less acceptable of the two hypotheses.25

However, Popper could fairly easily respond to Barker's two charges (viz., that we have a case where the more falsifiable hypothesis is unacceptable and we would not normally call this hypothesis more simple), as follows: First, Barker's charge that $H_2$ is unacceptable because it is a hasty conclusion from skimpy evidence, smacks of inductivism. That is, Barker seems to presuppose here that there is a principle which would tell us which of two hypotheses is "acceptable", given certain singular occurrences. However, as Popper argued in chapter one of L.Sc.D., there is no principle of induction because such a principle would lead either to an infinite regress or a priorism (since it itself must be a universal synthetic statement and consequently it too must be justified).26 And so, even if one does not accept Popper's critique of induction, it is at the very least clear that induction and the notion of inductive inference are problematic (or problem-laden) and that there is no widely accepted or clearly formulated principle of induction. Thus, Barker's charge that $H_2$ is unacceptable, since it smacks of inductivism, fails (or at the very least it is weakened). Second, Popper could answer Barker's charge that we have a case where $H_2$ is more falsifiable than
\[ H_1 \] and yet clearly \( H_2 \) is more complex than \( H_1 \), in the same way that he answered Hempel's charge. That is, Barker's counter-example is inappropriate because Popper has merely proposed a stipulative definition of simplicity, the purpose of which is not to handle all situations, but rather its purpose is to resolve certain epistemological problems outlined in chapter seven of L.Sc.D. Thus, to criticize Popper's stipulative definition of simplicity, Barker would have to somehow show that Popper's definition does not resolve the problems outlined in chapter seven of L.Sc.D., and, in Chapter III of this paper, we shall attempt to show just that.

Part III: Induction Revived.

A. J. Ayer, in *Language, Truth and Logic*, admits that the problem of justifying inductive inference is for all intents and purposes insurmountable. Like Popper, in L.Sc.D., Ayer argues that if we regard the principle of induction (allowing us to make justified generalizations from the past which will hold for the future) as being a "formal" principle, then "one commits the error of supposing that from a tautology, it is possible to deduce a proposition about a matter of fact". Further, along the same lines as Popper, Ayer argues that we would be begging the question (or assuming what we are trying to prove) if we regarded the principle of induction as being an empirical principle. That is, this empirical principle of induction proving that our generalizations from past experience will hold for the future would assume that "past experience is a reliable guide to the future." But this, says Ayer, is what we were trying to prove in the first place! And so, Ayer concludes that proving induction is an insurmountable task:
Thus it appears that there is no possible way of solving the problem of induction, as it is ordinarily conceived. Yet, unlike Popper, Ayer does not consider the fact that induction cannot be logically justified to be grounds for rejecting it. In fact, he notes that it is absurd to require that something be logically justified which clearly is not capable of ever being justified. Rather, Ayer argues that as long as any method such as induction is reasonably successful in predicting future experience, then we should hold on to it:

We are entitled to have faith in our procedure just so long as it does the work which it is designed to do—-that is, enables us to predict future experience, and so to control our environment.

Further, Ayer points out that what justifies a scientific procedure such as induction is its success in allowing us to predict future experience (given generalizations from past experience). This, of course, as Ayer notes, does not constitute a logical justification.

It should also be mentioned here that Wittgenstein, in his On Certainty, argues along the same lines as Ayer, that there is no need for a justified "law" of induction:

299. The squirrel does not infer by induction that it is going to need stores next winter as well. And no more do we need a law of induction to justify our actions or our predictions.

However, Wittgenstein makes this claim for different reasons than Ayer. That is, we do not even require practical justification (i.e. its success in practice) for our inductive inferences, since the question of their justification, logical or practical, is not a legitimate one within the "language game" which includes believing that the future will be like the past. Doubting or questioning comes to an end at this point:

315. ... the teacher will feel that this is not really a
legitimate question at all.
And it would be just the same if the pupil cast doubt
on the uniformity of nature, that is to say, on the justification
of inductive arguments.
He has not learned the game that we are trying to teach
him. 36

Thus, Wittgenstein goes one step further than Ayer in his remarks on
induction by denying the legitimacy of asking for any kind of justifica-
tion for induction. 37

If both Ayer and Wittgenstein are correct in their remarks on
induction, then what would make high falsifiability and therefore high
simplicity (as Popper defines it) so desirable? That is, Popper proposed
that we adopt falsifiability as our criterion demarcating science from
non-science (and that the scientific method be a method which ensures
falsifiability), because induction cannot be logically justified. (i.e.
if induction cannot be logically justified, then supposedly our criterion
of demarcation, viz., that statements are scientific if, and only if,
their truth as well as their falsity is decidable, fails and must be
abandoned. There is no way of establishing any statement’s truth.)
But, if as Ayer states, induction need not be logically justified or if,
as Wittgenstein says, the question of justification of induction is not
legitimate then why abandon induction and its implied criterion of de-
marcation in favour of falsifiability (and its accompanying method)?
And therefore, why aim for high falsifiability and hence high simplicity,
(as Popper defines it)? At this point, Popper could make the following
response: Why hold on to a method such as induction, which is problem-
laden (for, after all, Ayer and Wittgenstein’s remarks are just two
solutions amongst many) when we can adopt a criterion of demarcation
(and accompanying method) which is at least less problematic? The falsifying mode of inference is, according to Popper, "uncontroversial" (since it employs strictly deductive inference) whereas inductive inference is laden with such problems as the problem of finding a principle of induction. (Ayer says that there is no such solution and therefore practical justification will do. Wittgenstein says that the question or problem of induction is a pseudo-problem. In support of Wittgenstein, it might be argued that he is saying that we can't even legitimately make statements such as "Falsifiability and its method should be adopted over induction." since induction is already part of our activities and our language. That is, there is no choice involved here; this is our language game. To put Wittgenstein's remarks on induction into context, they are after all simply one solution amongst many to the really existent problem of induction in philosophy.) As we shall see in Chapter III, the falsifying mode of inference is no less controversial than inductive inference.

Part IV: Feyerabend on the Limitation of Methodological Rules.

As we saw in part I of the exposition, having conceded the objection to his falsifiability proposal, viz., that no theory can ever be conclusively falsified; Popper proposed certain methodological rules in order to save this proposal. These proposed methodological rules were governed by the metarule stating that any proposed rule must not prevent any theory from being falsified. Further, if it can be shown that Popper's "falsificationist" methodology is in trouble (i.e. it is problematic) then it is no longer clear that his falsifiability proposal can be saved, which in turn casts doubt on the desirability of high
falsifiability and high simplicity.

One way of assessing Popper's methodological proposals is to see how they would have fared if they had been applied to certain situations in the history of science. Popper could argue here that this approach to assessing methodologies is somewhat "naturalistic". And, as he argued in chapter two of L.Sc.D., this is not how methodological disputes are settled. That is, when comparing methodologies, we see which one gives rise to the least number of inconsistencies and which one is most "fruitful" (i.e. useful in resolving problems in epistemology). However, if we view methodological proposals as "prescriptions" for science (and not merely as being fruitful for epistemology or as avoiding inconsistencies), then it is entirely reasonable to assess these proposals by seeing how they would fare if they were actually applied in science, (or if they had been applied to certain episodes in the history of science). When applied to science, do these proposed rules hinder science or do they help it? In Against Method, Paul Feyerabend assesses both induction and falsificationism in this very way. Feyerabend argues in A.M. that if we look to the history of science, we shall see that all methodological rules, whatever they may be have at one time or another been violated. Further, he argues that if these violations had not occurred, then scientific progress (however one wishes to define it) would have been hindered.

We find then, that there is not a single rule, however plausible and however firmly grounded in epistemology, that is not violated at some time or another. It becomes evident that such violations are not accidental events... On the contrary, we see that they are necessary for progress.45

In A.M., Feyerabend further argues that indeed, to ensure "progress" in
science, the most advantageous methodological principle for this point in time (though not necessarily for all times in the philosophy of science) is the principle that "anything goes".

And my thesis is that anarchism helps to achieve progress in any one of the senses one cares to choose.

In short, Feyerabend proposes that we let the method fit the situation rather than attempting to promote some iron-clad rule (or set of rules) which would supposedly cover all situations (and which would, therefore, be violated). Further, in order to support this claim, Feyerabend proposes a counter-method or a set of counter-rules to induction and falsificationism which he calls counter-induction and which would succeed (or fare very well) in certain episodes in the history of science where the other two methods would have failed, (by hindering scientific progress).

This argument in favor of "counter-induction" is intended to persuade those who religiously adhere to either induction or falsificationism that these methods and, in fact, all methods have their limitations. That is, any one method cannot and should not (at this time) be applied to all situations in science or more concisely, anything goes:

My intention is not to replace one set of general rules by another such set: my intention is, rather, to convince the reader that all methodologies, even the most obvious ones, have their limits. The best way to show this is to demonstrate the limits and even the irrationality of some rules which she, or he, is likely to regard as basic. In the case of induction (including induction by falsification) this means demonstrating how well the counter-inductive procedure can be supported by argument.

But, what are some of these "stultifying" methodological rules or requirements whose limitations Feyerabend will attempt to expose? In A.M., Feyerabend argues that both induction and falsificationism require that any new theory be consistent both with established theories (vide
L.Sc.D., pages 276) and with experimental results. He then attempts to argue that cases abound in the history of science where these consistency requirements are not met. Feyerabend then proposes the following counter-inductive rule be applied to these situations:

The 'counterrule' corresponding to it advises us to introduce and elaborate hypotheses which are inconsistent with well-established theories and/or well-established fact. It advises us to proceed counterinductively. He argues that in the very same cases his counterinductive rule, (viz., that we should adopt theories which are inconsistent with experimental results or with established theories), would have fared very well. That is, would not have hindered scientific progress.

Feyerabend's discussion of the requirement that theories must agree (or be consistent) with the experimental results is particularly relevent to Popper's methodological proposals. Consider, for example, Popper's rule that any theory which has been highly corroborated (or it has stood up to severe tests) should not be abandoned unless it is falsified (or one which is better testable comes along). In short, we hold on to a theory that is consistent with experimental results and we abandon it if it ceases to be consistent with these results. This is one of the consistency rules which Feyerabend criticizes in A.M., viz., that theories should be consistent with experimental results. Also, Popper, in chapter four of L.Sc.D., proposed the (anti-conventionalist) rule that no ad hoc auxiliary hypotheses should be introduced which can save a theory from falsification (which, according to Popper, results in the theory ceasing to be falsifiable). This rule is in accordance with the metarule that no rule should protect theories from falsification,
which again means that when a theory ceases to be consistent with experimental results, it should be abandoned.

In *Against Method*, one of the major examples from the history of science which Feyerabend says is a case where the consistency requirement was violated and should have been violated is the Copernican Revolution. Feyerabend argues that not only were certain numerical predictions of the Copernican heliocentric theory inconsistent with obtained results, but also this theory had "qualitative" difficulties as well, meaning that it was inconsistent "with circumstances which are easily noticed and which are familiar to everyone". What were some of these familiar circumstances with which the Copernican theory was inconsistent? According to Feyerabend, one of the refuting arguments made against the Copernican theory was the "tower argument". That is, if the earth is not the stationary center of the universe, but rather it is, like the other planets, rotating on its axis and orbiting around the sun, then if we drop a stone from a tower, it should land away from the tower because of the earth's motion. Yet, what actually happens is that the stone drops in a straight vertical line to the foot of the tower, which would seem to refute the theory that the earth is in motion:

... observation shows that 'heavy bodies ... falling down from on high, go by a straight and vertical line to the surface of the earth. This is considered an irrefutable argument for the earth being motionless. For, if it made the diurnal rotation... the rock ought to strike the earth that distance away from the base of the tower.'

However, Feyerabend notes that Galileo, admitting the "correctness" of the results (or observations) argues that we must be aware of the assumption which influences how we interpret these results, viz., the
assumption that all motion is "operative", i.e. perceptible.65 (Further, as Popper points out in chapter twelve, this assumption arises out of the Aristotelian theory of perception and motion.)66 The moral to be drawn here, according to Feyerabend, is that even supposed neutral observation statements presuppose theories (or certain theoretical assumptions):

From the point of view of 17th century thought and language, the argument is, therefore, impeccable and quite forceful. However, notice how theories ('operative character' of all motion; essential correctness of sense reports), which are not formulated explicitly, enter the debate in the guise of observational terms.67

Popper, in L.Sc.D. also makes this point in several places, viz., that "... observation statements and statements of experimental results, are always interpretations of the facts observed."68 Further, Popper states in this same passage that he regrets that he did not emphasize this point more in L.Sc.D.69 In any event, in A.M., Feyerabend used the view that observation statements presuppose theories (or that they are theory-laden), a "fact" supposedly discovered by Galileo in his defense of Copernicanism, as an argument in support of the counter-inductive rule that we should adopt theories inconsistent with experimental results. That is, if we adopt theories which are inconsistent with experimental results or with the "facts" we will be able to expose the theoretical assumptions or "interpretations" which these experimental results presuppose:

This ... is one of the reasons one can give for retaining, and, perhaps, even for inventing, theories which are inconsistent with the facts. Ideological ingredients of our knowledge and, more especially, of our observations, are discovered with the help of theories which are refuted by them ... 70
And, as we noted earlier, Feyerabend's strategy here is to provide strong argumentation in favor of his counter-method in order to expose the limitations of the much favored methods of induction and falsificationism. That is, our faith in these methods will have been weakened and we shall therefore be more receptive to the idea that anything goes, i.e. that at this point in time there should be no iron-clad method for all situations.

In chapter seven of Against Method, Feyerabend argues that Galileo used "trickery" under the guise of argument, in order to persuade us that there never really was an inconsistency between the "facts" and the Copernican theory and that the "facts" actually agree with or "confirm" this theory. The tower argument actually confirms rather than refutes the Copernican theory. But, what was the nature of this trickery or sleight of hand? According to Feyerabend, the common sense view (i.e. the "ideological ingredients ... of our observations"), of the seventeenth century included the notion of non-operative (or non-perceptible) motion. That is, although all motion is (theoretically) operative, there will be certain situations, viz., in cases of motion, where motion is not operative. According to Feyerabend, Galileo presents the following example of "shared" (non-operative) motion: If we were on a sailing boat which is in motion, we would not notice the motion of the mast (of the boat) because we share with this mast the motion of the boat. Consequently, from the perspective of someone on the deck of the moving boat, the motion of the mast is not perceptible, i.e. it is non-operative. Further, this situation was perfectly consistent with the seventeenth century common-sense view. Feyerabend then
points out that Galileo, exploiting this common sense paradigm of non-operative motion under special circumstances (such as being on a sail boat), attempted to persuade people of that time that the tower example is the same type of situation as the sail boat example. That is, since we share in common with the tower and the stone the earth's motion, this motion will be imperceptible or non-operative. Further, only the motion which we do not share with the tower and the stone will be perceptible, which explains why we only see the stone falling in a straight line to the base of the tower, and not its circular motion. Feyerabend quotes from Galileo, as follows:

But that part of all this motion which is common to the rock, the tower, and ourselves, remains insensible and as if it did not exist. There remains observable only that part in which neither the tower nor we are participants; in a word, that with which the stone, in falling, measures the tower.

And so, the trickery involved here is the attempt to persuade people of the seventeenth century that the tower example is consistent with their common sense views, since it is the same type of situation as the sail boat example. There is, of course, one very important difference between these two situations, viz., in the boat example, the motion of the boat is (from the perspective of someone standing on the shore) perceptible and not in question, whereas in the tower example, the motion of the earth is the very issue in question. In any case, Feyerabend exposes Galileo's "trickery" in the following passage:

Yielding to this persuasion, we now quite automatically start confounding the conditions of the two cases and become relativists. This is the essence of Galileo's trickery.

Also, Galileo, in trying to convince us that these two situations are of the same kind, has also attempted to transform an inconsistency
between the tower example and the Copernican theory (which would supposedly result in the refutation of the latter) into a "confirmation" of the Copernican theory.\textsuperscript{82} That is, the Copernican theory predicts that the motion shared by the earth, the tower, the rock and the observer will be imperceptible. And, the tower example confirms this prediction, since we in fact, do not perceive the "circular" motion (resulting from the rotation of the earth) of the rock falling from the tower:

An experience which partly contradicts the idea of the motion of the earth is turned into an experience that confirms it, at least as far as 'terrestrial things' are concerned.\textsuperscript{63}

To sum up, Galileo would have us believe that the tower example, just as the sail boat example, agrees with our common sense, in which case the Copernican theory is confirmed, not refuted.

At this point, Popper could interject with the following remarks: First, he could agree with Galileo that there never really was an inconsistency between the Copernican heliocentric theory and the tower example in which case the Copernican theory was not refuted. He could further remind us that there can be an inconsistency between a theory and a basic statement asserting the occurrence of an event, $P$, only if the theory rules out the event $P$.\textsuperscript{84} And the Copernican theory never did rule out the event of the stone falling in a straight vertical line when dropped from a tower. Further, Galileo did not use trickery or sleight of hand in order to make an inconsistency seem like a confirmation. Rather, he simply articulated the Copernican theory, using the concept of non-operative motion. The result of this "articulation" and not as Feyerabend would have us believe, sleight of
hand, was the realization that the Copernican theory did not rule out events involving non-operative motion, such as the event of a stone dropped from a tower landing at the base of the tower. Consequently, the basic statement asserting that the stone, when dropped, landed at the base of the tower, is not inconsistent with the Copernican theory, and therefore, none of Popper's methodological rules governed by the metarule that no theory should be protected from falsification were violated here. That is, there really was no inconsistency here and therefore there was no refutation which the Copernican theory was protected from. Also, Galileo's argument in favor of non-operative motion was not an ad hoc explanation used to save the Copernican theory, but rather it was an "articulation" of this theory. Further, since there really was no inconsistency between the Copernican theory and the tower example (and other similar examples), it follows that Feyerabend's counter-inductive rule that theories should be adopted which are inconsistent with the facts is not applicable to the Copernican Revolution. In fact, we could say here that it was the articulation of a theory consistent with the "facts" (even though we didn't immediately recognize this consistency) which exposed the ideological or theoretical character of experimental results, which is a reason for endorsing consistency and falsificationism (which requires consistency). Consequently, Feyerabend did not successfully cast doubt on Popper's methodological rules used to save his falsifiability proposal and therefore his falsifiability proposal stands.

However, it seems that Feyerabend is on fairly safe grounds in calling Galileo's handling of the tower argument "trickery" or
sleight of hand. After all, the sail boat example is not the same type of situation as the tower example. In the former case, where the boat is in motion and hence the motion shared by the boat, the person on the deck and the mast of the boat is non-operative, the motion of the boat is not in dispute, but, in the latter case, the assertion that the motion shared by the tower, the rock, the observer and the earth is non-operative presupposes the very claim which is in dispute, viz., the motion of the earth. Consequently, there really was an inconsistency between the Copernican theory and the "facts" (such as the tower example) which Galileo attempted (successfully) to explain away by trickery or sleight of hand, but, by the dictates of Popper's methodology (ensuring his falsifiability proposal's applicability\(^6\)) the Copernican theory, since it was inconsistent with experimental results was refuted and therefore it should have been abandoned. Is this "progress"? Or, as Feyerabend states in chapter twelve of A.M.:

Is it not clear that our beautiful and shining methodologies which implore us to take risks and to take refutations seriously . . . would have given extremely bad advice in the circumstances?\(^7\)

However, even if Popper concedes that in the case of the Copernican Revolution his methodological rules would have hindered progress, there are, as we shall next see, other avenues of defense open to him.

In a sequel to Against Method, viz., Science in a Free Society, Feyerabend notes that it could be objected that on the basis of only one or two cases (such as the Copernican Revolution) where certain methodological rules such as those of Popper are violated, we cannot conclude that these rules are never applicable. Nor can we conclude that in the "long run" these rules would not prove their usefulness. (That is,
eventually, a theory could be made to be consistent with the facts or
barring this, then this theory would be abandoned):

Besides, some critics pointed out, the fact that a rule is
violated in one case does not make it useless in others, or
in the long run. For example, a theory may be in conflict
with facts or ad hoc and may still be retained— but eventually
the conflict will have to be resolved...

Of course, Popper's methodological rules which he proposed in *L.Sc.D.*
would disallow our holding on to a falsified theory (i.e. a theory which
has been contradicted by accepted basic statements which in turn corro-
borate a "falsifying hypothesis"

in the hopes that it could be even-
tually made to be consistent with the "facts" or abandoned if this does
not happen. In fact, this method is none other than Imre Lakatos' method
of "research programmes". A research programme is not a theory, but
rather "it is a succession of theories." It is this series of theories
or research programme which we assess and for one research programme to
be abandoned and replaced by another takes an "indefinitely" long period
of time:

Only an extremely difficult and— indefinitely— long process can
establish a research programme as superseding its rival; and it
is unwise to use the term "crucial experiment" too rashly.

In any event, Popper could adopt the related objection noted by Feyerabend
viz., that just because his methodological rules are not applicable in
the case of the Copernican Revolution, surely this does not exclude the
possibility of their (successful) application in other cases. Therefore,
Popper could argue, his methodological proposals have not been "refuted"
and, consequently, his falsifiability proposal can still be saved by
these rules. However, Feyerabend could easily respond here that all that
he was trying to show in his example of the Copernican Revolution is that
there are limitations with any methodology such as Popper's. That is, Popper's falsificationist methodology is not a set of iron-clad rules which can be applied to all situations. But, if this is so then there will be cases where Popper's methodological rules will be inapplicable in which case his falsifiability proposal will also be inapplicable. That is, if there are some cases where it is appropriate to hold on to a theory despite falsifications, then we will not always be working with falsifiable theories (since as Popper argued in chapter two of L.Sc.D. a falsified theory which is held on to is no longer falsifiable). Consequently, if we were to maintain as Popper does that a theory is scientific only if it is falsifiable, then we would be forced to call the Copernican theory "metaphysical", which is not desirable. Also, if this is so, then there will be cases where high falsifiability and high simplicity are not desirable. Therefore, equating simplicity with degree of falsifiability does not tell us in some cases why simplicity is so desirable. In fact, in some cases, it would seem (if we define simplicity as degree of falsifiability) that simplicity is not desirable.

Part V: Problems with Comparing Relative Simplicity.

Recall Feyerabend's remarks in Against Method, on the ideological or theoretical components of experimental results in the case of the Copernican theory versus the Ptolemaic theory. That is, the "facts" confirming the geocentric theory (and supposedly refuting the heliocentric theory) presupposed certain theoretical assumptions such as the assumption that all motion is operative. However, as Feyerabend further points out in A.M., Galileo constructed a new "observation language" which confirmed the Copernican theory and which contained the assumption
that not all motion is operative (or that not all "real" motion is apparent or perceptible):

Taking this into consideration, it is apparent that Galileo's proposal amounts to a partial revision of our observation language or of our experience.\textsuperscript{92}

We might initially be tempted to say that the following statement is a potential falsifier of both the Copernican theory and the Ptolemaic theory, viz., "The stone, when dropped from the tower fell in a mixed circular-straight fashion (thus landing away from the base of the tower). However, in the case of the former theory, it can be refuted by the statement because of the theoretical assumption that not all "real" motion is operative, in which case the circular motion should not have been perceptible. In the case of the latter theory, (i.e. the Ptolemaic theory), it can be refuted by the above-mentioned statement because of the assumption that all motion is operative (except under certain circumstances) and thus there is motion (i.e. the "circular" motion) where there is supposed to be \textit{none} (and \textit{not} because the motion here, though real, is imperceptible).

And so, as we can see in these two cases, the term "motion" is used differently in the case of the Ptolemaic theory ($T_p$) than in the case of the Copernican theory ($T_c$). Therefore, we really have two potential falsifiers here:

For $T_c$: The stone when dropped from the tower \textit{visibly} displayed both operative and typically non-operative (circular) motion.

For $T_p$: The stone when dropped from the tower \textit{visibly} displayed the expected (straight) motion and an \textit{additional} (unexpected) motion.
From these considerations, it would seem that the empirical contents (i.e. classes of potential falsifiers) for $T_c$ and $T_p$ will be different and also non-comparable. Consequently, we cannot compare $T_c$ and $T_p$ with respect to their degrees of falsifiability and hence simplicity. That is, we cannot, as Popper defines it, say that $T_c$ is more or less simple than $T_p$. In his "Reflections on My Critics", Kuhn notes that theories which Popper attempted to make comparable in terms of their empirical contents are non-comparable because of the fact that "basic statements" are not free of theoretical components:

That is the vocabulary in which Sir Karl's basic statements are framed. He requires it in order to compare the verisimilitude of alternate theories or to show that one is 'rooser' than (or includes) its predecessor. Feyerabend and I have argued at length that no such vocabulary is available.93

Of course, as we have already seen, Popper admits (in several footnotes) the theory-ladeness of basic statements. Thus, in response to Kuhn (and the Feyerabendesque criticism outlined above) Popper could grant the reasonableness of the view that basic statements are theory-laden, pointing out that it is not clear how this will affect the comparison of theories such as the following:

$T_1$ "All planets move in circles."

$T_2$ "All heavenly bodies move in circles."

Surely, independently of the "fact" that basic statements are theory-laden, there is no reason to suppose that $T_2$'s class of potential falsifiers do not include $T_1$'s class of potential falsifiers as a subclass. Consequently, there will be many cases such as these where, regardless of the theory-ladeness of basic statements, we can certainly say that $T_2$ is more falsifiable and hence simpler than $T_1$. And anyway, Popper
could remind us that in chapter six of L.Sc.D., he pointed out that there will be some cases where neither method for comparing degrees of falsifiability and hence simplicity will be applicable.
FOOTNOTES


3. Ibid., p. 68.

4. Ibid., p. 68.

5. Ibid., p. 68.


7. Ibid., p. 45.

8. Ibid., p. 45.


10. Ibid., p. 329.


12. See section (V) of Part I, page 10 and also see L.Sc.D., p. 143.

13. I.e. Their dimensions have been materially reduced.


15. Ibid., p. 61.

16. Ibid., p. 61.


18. Ibid., p. 45.

19. Ibid., pp. 44-45.

20. Popper, p. 140.

22 Popper, p. 169.
23 Ibid., p. 169.
24 Ibid., p. 169.
26 Popper, pp. 27-30.
27 Ayer, p. 49
28 Ibid., p. 49.
29 Ibid., p. 49.
30 Ibid., p. 50.
31 Ibid., p. 50.
32 Ibid., p. 50.
33 Ibid., p. 50.
34 Ibid., p. 50.
36 Ibid., p. 40e.
37 A much more thorough comparison of Popper and Wittgenstein on the issue of induction is made in an unpublished thesis by William Hutchinson, M.A.
38 Popper, p. 42.
39 Ibid., pp. 53-54; pp. 81-84.
40 Ibid., p. 54.
41 Ibid., p. 54. "... theories which we decide not to submit to any further test would no longer be falsifiable.
42 Ibid., p. 52.
43 Ibid., pp. 52-53.
45 Feyerabend, p. 23.
46 Ibid., p. 22.
47 Ibid., p. 28.
48 Ibid., p. 27.
49 Ibid., p. 27.
50 Ibid., p. 22.
51 Ibid., p. 32.
52 Ibid., p. 32.
53 Ibid., p. 29.
54 Ibid., p. 29.
55 Popper, pp. 53-54.
56 Ibid., pp. 82-83.
57 Ibid., p. 54.
58 Ibid., p. 54.
59 Feyerabend, pp. 55-56.
60 Ibid., p. 58.
61 Ibid., p. 58.
62 Ibid., pp. 70-71.
63 Ibid., pp. 70-71.
64 Ibid., p. 71.
65 Ibid., pp. 74-75.
66 Ibid., pp. 148-60.
67 Ibid., p. 75.
68 Popper, p. 107. (footnote #3)
69 Ibid., p. 107. (footnote #3)
70 Feyerabend, p. 77.
71. Feyerabend, p. 81.
72. Ibid., p. 87.
73. Ibid., p. 77.
74. Ibid., p. 83.
75. Ibid., p. 83.
76. Ibid., p. 83.
77. Ibid., p. 84.
78. Ibid., p. 84.
79. Ibid., p. 84.
80. Ibid., p. 84.
81. Ibid., p. 84.
82. Ibid., p. 87.
83. Ibid., p. 87.
84. Popper, p. 69.

85. And also, the consistency rule implicit in Popper's methodology was not violated.

86. Popper, p. 54.
87. Feyerabend, p. 159.
89. Popper, pp. 86-87.
91. Ibid., p. 163.
92. Feyerabend, Science in a Free Society, p. 87.
94. Popper, p. 130.
CHAPTER THREE

A CRITICAL EVALUATION OF POPPER'S STIPULATIVE DEFINITION OF SIMPLICITY

Introduction

As we have seen in our critical overview of other authors' criticisms, Popper can, with some ease, handle some of these. However, Ayer's remark that induction requires no logical justification and Wittgenstein's remarks that induction requires no justification, casts serious doubt on Popper's argument that we should adopt his falsifiability proposal (and hence we should desire high falsifiability and high simplicity) since induction cannot be logically justified, a problem which falsifiability avoids. Yet, Popper could answer that whether or not Ayer and Wittgenstein are right, why adopt a procedure which gives rise to all these problems in the first place, when we can adopt a totally uncontroversial criterion of demarcation, viz., falsifiability (and its accompanying method)? However, as we shall see in Part I, Popper's falsifiability criterion has its share of problems as well and it is therefore no less controversial than induction (and its implied criterion of demarcation). Further, even if we were to accept Popper's arguments in support of his falsifiability proposal, there are still certain problems with his characterization of falsifiability and falsification, which we shall see in Part II. And, if these problems with Popper's characterization of falsifiability and falsification really do exist, then again, why should we desire high falsifiability and therefore high simplicity? In short, one of the claims to be argued for in Parts I
and II is that Popper's falsifiability proposal is laden with problems (like induction). This claim in turn supports the further claim that high falsifiability and therefore high simplicity (as Popper defines it) are not so desirable.

Also, in Parts III to VII, certain difficulties with Popper's dimension and subclass methods for comparing theories' degrees of falsifiability and hence their degrees of simplicity will be outlined. However, even if Popper could somehow resolve these difficulties, both methods ultimately spell out theories' degrees of falsifiability (and simplicity) in terms of ease of refutation. That is, $T_1$ is more falsifiable and hence simpler than $T_2$ if $T_1$ is easier to falsify than $T_2$. Thus, in Part VII, it will be argued that Popper's clarification of simplicity does not really resolve problems in epistemology (including the desirability of simplicity). Rather, his stipulative clarification of simplicity only shows us on a practical level why one theory is simpler or easier to falsify than another theory. Popper would have to concede this point since in chapter seven of L.Sc.D. he designates the use of the word "simple" just noted as being extra-logical and hence practical.

Further, as we shall argue in Part VIII, most of the problems which Popper's clarification of simplicity was intended to resolve are really problems arising in induction. Thus, for example, the problem of whether or not we should use the simplest curve to fit the data is from Popper's viewpoint, a pseudo-problem. It is, therefore, likely that he (unsuccessfully) attempts to resolve such pseudo-problems (from his viewpoint) in order to convince his readers of the fruitfulness of his subclass and dimension methods. However, independent of our methodo-
logical orientation, it is surely desirable to be able to explain why
one law is simpler (or is often regarded as simpler) than another law.
Thus, in Part IX, we shall propose to define theories' degrees of sim-
plicity in terms of their "ontological" simplicity.

Part I: The Alternative to Induction Falsifiability.

In chapter one of *L.Sc.D.*, Popper argues that if we are to
justify our inductive inferences, then we must formulate a principle of
induction. According to Popper, this principle of induction would put
inferences from singular statements to universal statements into a
"logically acceptable form". Further, he argues that this principle
of induction cannot be a tautological statement or principle, since
"just like inferences in deductive logic" inductive inferences put into
a logically acceptable form by this principle would be "purely logical
or tautological transformations". Therefore, he argues, we must regard
the principle of induction as a (universal) synthetic principle. But,
if this is so, then we are faced with an infinite regress, since this
principle will itself be in need of justification ad infinitum. Popper
finds the alternative, viz., regarding the principle of induction as
established a priori to be unacceptable. However, Popper argues that
if we adopt falsifiability as our criterion of demarcation, thus only
requiring that a theory which is scientific is one which can be shown
to be false, then we avoid the problem of trying to find something akin
to a principle of induction to justify our falsifying inferences. After
all, to argue from the falsity of a singular statement to the falsity
of a universal statement or theory, we already have at our disposal the
inferences of deductive logic, such as modus tollens. And who would
doubt these?

If we ... admit as empirical also statements which are decidable in one sense only—unilaterally decidable and, more especially falsifiable—and which may be tested by systematic attempts to falsify them, the contradiction disappears: the method of falsification presupposes no inductive inference, but only the tautological transformations of deductive logic whose validity is not in dispute.

Yet, recalling to mind Ayer's and Wittgenstein's remarks on induction, we could argue against Popper as follows: Why must we assume that it is necessary for induction to be logically justified? According to Ayer, any justification we need for induction will come from its success in allowing us to predict future experience on the basis of past experience. Or, according to Wittgenstein, no justification is required for induction (and one who asks for justification has not "learned" the language game). However, Popper could answer that if we adopt falsifiability (and its accompanying method) then the question as to whether or not we even need a principle putting falsifying inferences into a logically acceptable form would never arise in the first place. After all, both Ayer's and Wittgenstein's remarks arose in a context of problems of induction (such as the fallacy of trying to find a principle of induction). And falsifiability with its deductive techniques avoids these problems.

However, as we shall now see, Popper's falsifiability proposal (and his accompanying methodological proposals) also has its share of problems. First, Popper appears to have contradicted himself in arguing that the falsifying mode of inference is deductive in nature. That is, he argued that the principle of induction cannot be a tautology, since, like the inferences in deductive logic, an inductive inference would be
a purely tautological transformation. (i.e. How can we argue to empirical conclusions through merely logical transformation of statements?) Yet, Popper has also argued that falsifying inferences would presuppose the purely tautological transformations of deductive logic. And so, in what sense can we *not* arrive at empirical conclusions *inductively* through purely tautological transformations although we *can* arrive at empirical conclusions *deductively* through equally tautological transformations? At this point, Popper could try to argue that given the empirical statement "It is now raining." and a conclusion which is derivable from it, viz., "There will be clouds in the sky." we can by *modus tollens* (i.e. by a purely logical transformation) arrive at the following empirical conclusion: "It is not raining." And so, by this example, it is clear that we can *deductively* infer the falsity of empirical conclusions through purely logical transformations. However, if Popper were to argue this, he would also have to admit the *possibility* of being able to arrive at (the truth of) empirical conclusions through purely logical transformations. In any event, there is a problem (which is all that is being argued for here) with characterizing the falsifying mode of inference as involving purely tautological transformations, viz., as in induction, there is the problem of how we can arrive at empirical conclusions through purely logical transformations. In short, Popper's falsifiability proposal is not without its problems.

A further problem for Popper's falsifiability proposal is one which he himself noted in chapters one, two and four, viz., no theory can ever be conclusively refuted since we can always introduce ad hoc auxiliary hypotheses to take the force of any falsifying instances. In
answer to this difficulty, Popper admits that indeed it is at the very least logically possible to always evade falsification. Therefore, he argues, we must characterize the empirical method as a method which excludes ways of evading theories' falsification. (See Part I of the exposition for a more detailed coverage of these matters). Also, in answer to the conventionalist position that we should introduce ad hoc auxiliary hypotheses to ensure agreement between theory and reality, Popper notes that we must simply decide not to employ these conventionalist methods:

The only way to avoid conventionalism is by taking a decision: the decision not to apply its methods. We decide that if our system is threatened we will never save it by any kind of conventionalist strategy.

And so, in order to save his falsifiability proposal from the charge that the evasion of falsification is always possible, Popper in an ad hoc fashion simply decides that theories cannot evade falsification and he therefore proposes certain methodological conventions to ensure the applicability of his falsifiability criterion. But, this creates further problems for Popper. That is, having admitted the soundness of the conventionalist objection to his falsifiability proposal, Popper simply ensures the applicability of his criterion of demarcation. But why ensure the applicability of a criterion of demarcation which has been put into question? For that matter, Popper admits the soundness of the conventionalist stance that theories are simply conventions and that we should attempt to ensure their consistency with reality:

I regard conventionalism as a system which is self-contained and defensible. Attempts to detect inconsistencies in it are not likely to succeed.

But if this is so, then why doesn't he embrace the conventionalist
position which is even freer from difficulties than his falsificationist position? Popper answers that ultimately his reasons for embracing his falsificationist position is a matter of values:

Yet in spite of all this I find it quite unacceptable. Underlying it is an idea of science, of its aims and purposes, which is entirely different from mine.¹⁵

And, as we have seen in Part I of the exposition, one of these values is freedom from dogmatism.¹⁶ That is, conventionalism leads to dogmatism, which Popper finds unacceptable.

And so, as we have seen in the preceding paragraphs, Popper's falsifiability proposal is not without its share of difficulties. Consequently, Popper could not plausibly argue against the implications of Ayer and Wittgenstein's remarks that induction need not be logically justified by pointing out that his falsifiability proposal is not as problem-laden as is induction. But if this is so, then perhaps Ayer and Wittgenstein's comments on induction are worth noting. And also, why should we embrace Popper's falsificationist method over induction, if it is just as problem-laden as induction. For that matter, why not embrace the conventionalist method which is supposedly less problem-laden than either induction or falsificationism? Popper's only answer at this point is that accepting his proposals is a matter of one's purposes or aims. But what if we don't share his purposes? It is therefore by no means clear that high falsifiability and hence high simplicity are so desirable.

Part II: Problems with Popper's Characterization of Falsifiability and Falsification.

There are also problems with how Popper characterizes falsi-
fiability and falsification which also casts doubt on the desirability of high falsifiability and high simplicity. First of all, in Part II of the exposition, it was pointed out that Popper regards purely or strictly existential statements (i.e. existential statements which specify no space-time regions) as being "metaphysical" by his criterion demarcation, since they are non-falsifiable. Also, Popper in his discussion of the formal conditions which any basic statement must satisfy, noted that a basic statement whose logical form is that of a singular existential statement can contradict a theory since from this basic statement a purely existential statement can be deduced. Thus, if "T" is "\(\exists x (Rx \neq \neg Tb)\)" and our basic statement "B" is "\(\exists x (Rx \neq Tb \neq Kx)\)" then "B" can contradict "T" since from "B" we can deduce "S", viz., "\(\exists x (Rx \neq Tb)\). That is, S: (\(\exists x (Rx \neq Tb)\) is the contradictory of T: \(\exists x (Rx \neq Tb)\). But "S" is, according to Popper, a purely existential statement and therefore it is metaphysical. This means that the statement which directly contradicts a theory is itself "metaphysical", (since it is non-falsifiable).

What is strange here is that an empirical (i.e. falsifiable) theory is one which is contradicted by non-empirical (i.e. non-falsifiable) statements. Further, these purely existential statements, since they are the true contradictions of strict non-existence statements or theories, are indispensable for Popper's characterization of falsifiability and falsification.

Second, a more serious difficulty is with Popper's characterization of falsification, specifically with his notion of a falsifying hypothesis. In Part I of the exposition, we noted that in chapter four of L.Sc.D., Popper argues that for a theory to be falsified, the accepted
basic statements which (via strictly existential statements) contradict this theory, must also at the same time corroborate a falsifying hypothesis.\textsuperscript{19} Further, a falsifying hypothesis, says Popper, is typically of a low level of universality.\textsuperscript{20} But if this is so, then a falsifying hypothesis can at best attain a low degree of corroboration given the relation between degrees of corroboration, testability and universality. (See chapter ten of L.Sc.D.\textsuperscript{21}) Thus we will be regarding a theory (such as "All elephants are grey") as falsified on the basis of a weakly corroborated alternative hypothesis (such as "All three elephants in the Detroit Zoo are white.") rather than on the basis of a highly corroborated alternative (such as "All elephants in Southern Africa are white."). Of course, Popper could respond here that a weakly corroborated falsifying hypothesis will be quite sufficient for the refutation of the theory in question.\textsuperscript{22} After all, in the above example, surely the fact that all three elephants at the Detroit Zoo are white will be enough to refute the theory that all elephants are grey. Yet, surely Popper would concede that falsifying a theory on the basis of a highly corroborated alternative hypothesis is preferable to that theory's falsification on the basis of a weakly corroborated alternative. For one thing, in the former case we have a strong alternative (i.e. it is highly falsifiable) to replace the falsified theory. But, in the latter case we would be replacing our falsified theory with a hypothesis whose degree of falsifiability is low, which Popper would find unacceptable. Thus, Popper would have to admit that to falsify a theory it is at least preferable, if not necessary to propose another theory which itself will be highly corroborated by statements contradicting the theory in question. However, Popper could respond
by citing his example of a falsifying hypothesis given in chapter four of L.Sc.D., viz., "There is a family of white ravens at the zoo in New York." He could argue with some plausibility that this low-level hypothesis is highly falsifiable and therefore it is highly corroborable. (We shall see why this is the case in Part VI.) Further, he could propose that we write all falsifying hypotheses in the form of non-strict existential statements as in his example above, thus ensuring their high falsifiability and therefore their high corroborability. But, as we shall see in Part VI, this move leads to further problems.

And so, clearly there are problems with Popper's characterization of falsifiability and falsification. But, if this is so, then it's not clear why high falsifiability and (therefore high simplicity) are so desirable.

As we shall see in the next few parts, there are some rather serious difficulties in the two methods which Popper outlined for the purpose of comparing theories' degree of falsifiability and therefore their degree of simplicity.

**Part III: Dimension, Empirical Content and Infinite Classes.**

As we have noted in Part IV of the exposition, Popper's proposed methods for comparing degrees of falsifiability will make more precise the idea that $T_1$ whose degree of falsifiability is higher than that of $T_2$ has a larger class of potential falsifiers (or has a higher empirical content) than $T_2$ in the case of infinite classes of potential falsifiers (and forbidden events). But, it is not clear that his dimension method fulfills this task. That is, it is not clear how $T_1$
whose $d = 2$ can be said to have "more" potential falsifiers (or higher empirical content) than $T_2$ whose $d = 5$. In an appendix, Popper does make an attempt to relate a theory's content to its dimension, viz., by stipulating that a theory's (empirical) content will be the reciprocal of the minimum number of relatively atomic statements needed for its falsification. Of course, this really doesn't help us much here because what this stipulation amounts to is the following: A theory with low dimension will be said to have a high empirical content. But again, why is this so? Recall Popper's definition of a theory's "field of application" which he presents in appendix 1 of L.Sc.D. The class $A$ of those statements which when conjoined with any $d$-tuple in a theory's field of application will result in a potential falsifier (i.e. a falsifying "$d + 1$ -tuple") is, according to Popper, an infinite class. Consequently for any theory, its class of potential falsifiers, viz. the class of its falsifying $d + 1$ -tuples, will be infinite. And so, in what sense can we say that the class of potential falsifiers of $T_1$ whose $d = 3$ is greater than that of $T_2$ whose $d = 5$? At this point it could be pointed out that "$d + 1$" is the minimum degree of composition of a theory's potential falsifiers. Thus, a circle-theory can be falsified by a fourth point which does not satisfy its equation. But then it can also be falsified by a fifth, sixth, seventh, etc. point which does not satisfy its equation. Thus, this circle-theory can be falsified by certain $d + 1 = 4$ -tuples but also by certain $5, 6, 7 \ldots \infty$ -tuples. More generally, if we have two theories where for $T_1$, $d = 3$ and for $T_2$, $d = 5$ we can say that $T_1$ can be falsified by certain $4, 5, 6, \ldots \infty$ -tuples whereas $T_2$ can only be falsified by certain $6, 7, 8, \ldots \infty$ -
tuples. Thus, we might say that $T_1$ has "more" potential falsifiers than $T_2$. However, there is still a problem here, viz., that the classes of falsifying 4, 5, 6, ..., $\infty$-tuples for $T_1$ and 6, 7, 8, ..., $\infty$-tuples for $T_2$ will both be infinite, given that (as with an axiom schema) an infinite number of relatively atomic statements (corresponding to "points" on a curve) can be generated from a generating schema. (Or, more intuitively, in terms of a theory's graphic representation, the number of points on a curve is infinite.) Again, in what sense can we say that the class of potential falsifiers of $T_1$ is greater than $T_2$? It is therefore by no means clear that a theory with low dimension can be said to have higher empirical content (or have "more" potential falsifiers) than one with a lower dimension.

Part IV: Infinite Subclasses.

There are corresponding difficulties with Popper's attempt to render precise in the case of infinite classes the idea that $T_1$ has "more" potential falsifiers (or has higher empirical content) than $T_2$ by his subclass method. That is, by the subclass method $T_1$ has higher empirical content than $T_2$ if $T_1$ logically implies $T_2$ but not vice-versa in which case $T_2$'s class of potential falsifiers is a subclass of that of $T_1$. The problem here is that we still cannot say that $T_1$ has "more" potential falsifiers (and in this sense, a higher empirical content) than $T_2$. This is so because even though $T_2$'s class of potential falsifiers is a subclass of that of $T_1$, both classes of potential falsifiers are still infinite. All that we can say is that one class is a subclass of the other--nothing more. Thus, Popper's observations on the level of
finite classes to the effect that $T_1$ has "more" potential falsifiers than $T_2$ if $T_1$ is more falsifiable than $T_2$ cannot be extended to infinite classes either by the subclass method or the dimension method.

Part V: Comparison of Infinite Classes of Events.

As was noted in Part IV of the exposition, at one point in chapter six of L.S.C.D., Popper in an attempt to circumvent the problem that theories' classes of potential falsifiers are infinite, proposes to compare theories' degrees of falsifiability in terms of their forbidden events. That is, $T_1$ is more falsifiable than $T_2$ if $T_1$ forbids more events than $T_2$. However, Popper abandons this attempt to compare theories' degrees of falsifiability since for any falsifiable theory, its class of forbidden events is infinite, given that a forbidden event, when conjoined with any other event results in another forbidden event ad infinitum. But, with the help of Popper's dimension and subclass methods for comparing infinite classes of potential falsifiers, we shall outline a method for comparing theories' infinite classes of forbidden events. However, as we shall see, it will still not be possible to say (in the case of infinite classes of forbidden events) that $T_1$ rules out more events than $T_2$.

First, we shall attempt to compare the degree of falsifiability of the following two theories in terms of their forbidden events:

$T_1$: All heavenly bodies move in circular orbits.

$T_2$: All planets move in circular orbits.

These two theories can also be expressed as follows:

$T_1'$: There does not exist a heavenly body which moves in a non-circular orbit, et cetera.
T₂': There does not exist a planet which moves in a non-circular orbit, et cetera.

The significance of the term "et cetera" is to connote that the theory expressed as a forbidden event can be conjoined with any other event to form a new forbidden event, a process which can be carried out to infinity. If we recall from Popper's discussion of his subclass method, that in the above case, the subject of T₁, viz., "heavenly body" which includes planets, stars, satellites and comets is more extended than the subject of T₂, viz. "planet", we can re-write T₁', as follows:

T₁'': There does not exist a planet which moves in a non-circular orbit, and there does not exist a comet which moves in a non-circular orbit and there does not exist a satellite which moves in a non-circular orbit and there does not exist a star which moves in a non-circular orbit, et cetera.

In comparing T₁'' with T₂' with respect to their degrees of falsifiability we can say that T₁'' is more falsifiable than T₂' because it rules out (at least) three events which T₂' does not, viz., comets, satellites and stars moving in non-circular orbits. This, of course, is in agreement with Popper's subclass method since T₁ is more universal and hence more falsifiable than T₂'. However, given that both T₁'' and T₂' rule out an infinite number of events, we cannot meaningfully say here that T₁'' rules out more events than T₂'.

Also, with the help of Popper's dimension method, we can make the following theories comparable with respect to their degrees of falsifiability in terms of their respective classes of forbidden events:

T₅: All planets orbit in a circular fashion.
T₆: All planets orbit in an elliptical fashion.
which can be re-expressed as:

\[ T_c' : \text{There does not exist a planet which orbits in a non-circular fashion, et cetera.} \]

\[ T_e' : \text{There does not exist a planet which orbits in a non-elliptical fashion, et cetera.} \]

Recall from Popper's discussion of his dimension method that \( T_c' \) can be falsified by certain 4- and 5-tuples (in its field of application) whereas \( T_e' \) cannot be. Thus, we could say that \( T_c' \) rules out the events of four, five, six, etc. points (or positions of the planet) not falling on the curve whereas \( T_e' \) rules out the events of six, seven, etc. points not falling on the curve. Thus, we can re-express \( T_c' \) and \( T_e' \) as follows:

\[ T_c'' : \left( \text{There is not a set of four points not falling on the circle and there is not a set of five points not falling on the circle, et cetera.} \right) \]

\[ T_e'' : \left( \text{There is not a set of six points not falling on the ellipse and there is not a set of seven points not falling on the ellipse, et cetera.} \right) \]

Further, we can say that \( T_c'' \) is more falsifiable than \( T_e'' \) since \( T_c'' \) rules out two events that \( T_e'' \) does not rule out, viz., four points not falling on the curve and five points not falling on the curve. Once more, since we are dealing with infinite classes of forbidden events, we cannot say that \( T_c'' \) rules two more events than does \( T_e'' \). (Also, the above results, viz., that \( T_c'' \) is more falsifiable than \( T_e'' \) is consistent with the dimension method.)

Finally, it should be noted that if we base comparisons of the theories' degrees of falsifiability on comparisons of infinite classes
of forbidden events the following will be the case: $T_1$ which rules out
n events not ruled out by $T_2$ will be easier to falsify than $T_2$ since $T_1$
will have a greater opportunity of clashing with forbidden basic state-
ments than $T_2$.

Part VI: Additional Difficulties with the Subclass Method.

We shall here outline two additional difficulties connected
with Popper's subclass method (besides the one already mentioned, viz.,
that it does not enable us to say that $T_1$ has more potential falsifiers
than $T_2$).

First, in discussing his subclass method, Popper pointed out
that the classes of potential falsifiers of all metaphysical and tauto-
logical statements are empty (and therefore identical) and consequently
we can say that they have a falsifiability of "0".

$^{31}$ In contrast to
metaphysical and tautological statements, self-contradictory statements
can be arbitrarily assigned a falsifiability value of "1" since a self-
contradictory statement has as its class of potential falsifiers all
possible basic statements.

$^{32}$ For example, the self-contradictory state-
ment, "It is raining, but it is not raining," can be falsified either
by, "It is not now raining." or by "It is now raining." As Popper would
say here, "It is falsified by any statement whatsoever." $^{33}$ However, at
the level of theories, it is not so clear that a self-contradictory
statement can be "falsified by any statement whatsoever" as we shall
next see in the following example: We are given the self-contradictory
theory $T_C$: "All ravens are black but not all ravens are black." or
symbolically, "$(x(Rx \to Bx)) \land \neg(x(Rx \to Bx))."$ This self-contradictory theory
can also be written as \( T'_c \). "There are no non-black ravens but there are non-black ravens" or symbolically, \( \forall x (Rx \lor \exists x (Rx \land \lnot Bx)) \)."

Notice that the second conjunct of \( T'_c \), viz., \( \exists x (Rx \lor \lnot Bx) \) is a metaphysical statement since it is a purely existential statement (i.e. it is an existential statement which refers to no space-time regions). Consequently, the class of potential falsifiers of \( T'_c \)'s second conjunct is empty given Popper's comments on the falsifiability of metaphysical and tautological statements. The first conjunct of \( T'_c \) on the other hand, viz., \( \forall x (Rx \lor \lnot Bx) \) has as its class of potential falsifiers the class of potential falsifiers of the consistent theory \( T_e \) "All ravens are black" or symbolically, \( (x) (Rx \lor \lnot Bx) \) which is equivalent to \( T_e \) "\( \exists x (Rx \lor \lnot Bx) \).

In short, the class of potential falsifiers of the first conjunct of \( T'_c \) is identical to the class of potential falsifiers of \( T_e \). Also, since the second conjunct of \( T'_c \) is metaphysical and therefore has an empty class of potential falsifiers, it follows that the class of potential falsifiers of the self-contradictory theory \( T'_c \) is identical to the class of potential falsifiers of the consistent theory \( T_e \). \( T'_c \) can be falsified as a whole by a potential falsifier of its first conjunct since if the first conjunct is falsified, then by the truth table for "conjunction" the whole conjunction \( T'_c \) is falsified. Thus, the potential falsifiers of the first conjunct of \( T'_c \) are also the potential falsifiers of \( T'_c \) as a whole.) Further, both \( T'_c \) and \( T_e \) permit the existence of black ravens. Neither \( T'_c \) nor \( T_e \) would be falsified by \( \exists x (Rx \land Bx) \).

But how can all this be if a self-contradictory statement can be "falsified by any statement whatsoever"? From this example, it seems clear that self-contradictory theories will not be such that all possible basic
statements are their potential falsifiers. The same case made for the "raven" example above could also be made for $T_o$ "All planets move in circles but not all planets move in circles." or $T_o$' "There are no planets which do not move in circles but there are planets which do not move in circles." Clearly, the second conjunct of $T_o$' is a purely existential statement and therefore metaphysical. Thus $T_o$' has the same class of potential falsifiers as $T_n$' "There are no planets which do not move in circles." And so, more generally, we might simply regard a self-contradictory theory as a strict non-existence statement denying the occurrence (or existence) of "P" with a metaphysical statement "added on" which happens to be its contradictory. This metaphysical statement which is added on has no effect on the original theory's class of potential falsifiers (and therefore it has no effect on that theory's degree of falsifiability). But if this is true, then how could Popper answer the question as to why we should prefer consistent theories over inconsistent ones? That is, if we have two theories, the one consistent and the other inconsistent, both having the same class of potential falsifiers and hence being equally falsifiable, then why should we prefer the consistent theory over the inconsistent one?

A second difficulty connected with Popper's subclass method will be a counter-example to Popper's claim that a theory's degree of universality increases with its degree of falsifiability. Recall from Part II of this chapter that Popper could argue that falsifying hypotheses such as his example, "There is a family of white ravens at the New York Zoo", is more falsifiable than the more universal statement "There are white ravens in the U.S.A." This is so because both can be falsified,
if it is discovered that there are no white ravens in the U.S.A. However, the former can be falsified if it is discovered that there are no white ravens at the New York Zoo. But the fact that there are no white ravens at the New York Zoo does not falsify the claim that there are no white ravens in the U.S.A. Consequently, we have an instance where a statement, viz., "There is a family of white ravens at the zoo in New York." has a lower level of universality than a (comparable) statement "There are white ravens in the U.S.A." and yet the former is more falsifiable than the latter. Thus, contrary to Popper's claim that a theory's degree of falsifiability increases with its degree of universality we have a case where a theory's degree of falsifiability decreases as its degree of universality increases.


Even if the difficulties with Popper's subclass and dimension methods outlined in the preceding sections are resolvable, there is a more serious problem in using these methods in order to compare theories' degrees of simplicity. As we shall now see, this problem has to do with the fact that these methods ultimately spell out degree of simplicity in terms of ease of falsification.

As we noted in Part IV of the exposition, Popper in chapter six of L.Sc.D. argues that the theory $T_1$ which rules out "more" events (and therefore it has "more" potential falsifiers) than another theory $T_2$ will be easier to falsify and hence more falsifiable than $T_2$. Further, in spelling out his dimension and subclass methods (which are attempts to make precise in the case of infinite classes, the notion of $T_1$'s
having "more" potential falsifiers than T₂) a theory T₁ which is more falsifiable than another theory T₂ is easier to falsify than T₂. In the case of the subclass method, the more universal theory is the one which is easier to falsify and therefore it is the more falsifiable theory. Popper makes corresponding remarks on a theory's degree of precision as well. (See Part IV, Chapter One) Also, in the case of the dimension method, T₁ whose d = 2 is easier to falsify (and hence more falsifiable) than T₂ whose d = 5. That is, we need fewer relatively atomic statements to falsify T₁ (i.e., d + 1 = 3) than we do for T₂'s falsification (i.e., d + 1 = 6) and therefore T₁ is easier to falsify than T₂.

And so, it would seem that for Popper, T₁ is more falsifiable than T₂ if T₁ is easier to falsify than T₂. Since he has equated degree of simplicity with degree of falsifiability, we can also say that T₁ is simpler than T₂ if T₁ is easier to falsify than T₂. Applying Popper's stipulative definition of simplicity to the problem in epistemology of why one curve (such as a circle) is simpler than another curve (such as an ellipse), we can say that for example a circle-curve is simpler than an ellipse-curve because the former is more easily refuted (by certain sets of four points) than the latter. Further, we desire the simpler curve because it is so easily falsified. However, Popper argued in chapter seven of L.Sc.D. (see Part III of the exposition): that the problem of finding the easiest (or simplest) way of performing a task (or for that matter, why we should prefer the simplest way) is of no concern in epistemology. Rather, problems are of practical concern.

Thus, is it helpful to the epistemologist to suggest that one curve is
simpler to refute than a competing curve and further, that we should endorse this "simpler" curve because of this fact? To be consistent with his claim that ease of performing a task (such as the task of specifying the initial conditions which would make possible a theory's refutation) is a purely practical matter, Popper would have to admit that such an elucidation of simplicity is not helpful to the epistemologist. He would further have to admit that he has really provided us with a stipulative definition of practical simplicity (in terms of ease of refutation), not epistemological simplicity.\textsuperscript{45}

Part VIII: Does Popper Need Simplicity?

Recall from Part III of our exposition that Popper equated simplicity with degree of falsifiability in order to resolve certain problems in epistemology connected with simplicity. However, many of these problems (such as why, given n data-points, we should adopt the simplest curve to "fit" them\textsuperscript{46}) are problems arising in induction. But Popper has rejected induction in favour of his falsifiability proposal (and its accompanying method).\textsuperscript{47} Consequently, from Popper's point of view, most of these problems connected with simplicity are pseudo-problems. That is, if there is (from Popper's point of view) no such thing as induction\textsuperscript{48} then we do not view science as involving the collection of n data-points for which one then tries to find the "simplest" curve. Rather, we view science as involving the proposal of the most falsifiable theory which is then subjected to potentially refuting tests. For Popper, our only concerns in science are high falsifiability of theories\textsuperscript{49} and ensuring that they be subjected to tests.\textsuperscript{50} Therefore,
for the most part, Popper can dispense with the concept of simplicity. However, Popper could point out that the theories which are the most falsifiable also turn out, in many cases, to be the ones that we would usually call the simplest. After all, a circle-theory is more (easily) falsifiable than an ellipse theory and we would usually regard the former to be simpler than the latter. Therefore, why not retain the concept of simplicity and equate it with degree of falsifiability? Still, this definition of simplicity does not explain (except on a practical level) why one curve is simpler than another. It has no explanatory power.

Of course, the bottom line of all this is that Popper was attempting to demonstrate the fruitfulness of his methods for comparing degrees of falsifiability. One way of showing the fruitfulness or utility of these methods is to show that they solve certain problems connected with simplicity. Thus, even if from Popper's point of view, these are pseudo-problems, from the point of view of persuasion, what better way to persuade his readers of his methods for comparing degrees of falsifiability than be convincing them of their utility or fruitfulness.

Part IX: Ockam's Razor and Theoretical Simplicity.

Even though the major methodological concerns of the falsificationist are high falsifiability of theories and that no theory be protected from falsification, it is nonetheless desirable that he be able to explain why more highly falsifiable theories (ex., circle theories) are regarded as simpler than less falsifiable theories (ex., ellipse theories). It will be argued here that if we (stipulatively)
equate theoretical simplicity with "ontological" simplicity we will be in a position to explain why the more falsifiable theory is often the simpler theory without resorting to such practical notions as ease of refutation.

Elliott Sober notes in his book, Simplicity that Ockam's Razor, which dictates that we keep our ontological commitments to a minimum, is an expression of the "desire to minimize the number and kinds of entities admitted by our theory."51 He further argues that Ockam's Razor dictates that we should accept "\( \exists x \text{Fx} \)" in favor of "\( \exists x \text{Fx} \)" (which he argues agrees with his own clarification of simplicity in terms of the "informative"ness of hypotheses52). Perhaps we can spell out relative ontological simplicity in terms of Ockam's Razor as follows: A theory \( T_1 \) which denies the existence of certain (kinds of) entities which a comparable theory \( T_2 \) does not is ontologically simpler than \( T_2 \). Or equivalently, \( T_1 \) is ontologically simpler than \( T_2 \) if \( T_1 \) rules out certain events which \( T_2 \) does not rule out. Further, if we equate theoretical simplicity (or simplicity of laws) with ontological simplicity then we can say that \( T_1 \) is simpler than \( T_2 \) if \( T_1 \) is ontologically simpler than \( T_2 \). To see how this definition of simplicity applies we shall set up three examples, noting in each case how we can determine the relative simplicity of the theories being compared without resorting to the practical notion of ease of refutation.

First, if we are given the following two comparable theories \( T_1 \) and \( T_2 \), we can say that \( T_1 \) is simpler than \( T_2 \):

\[
T_1: \ \exists x (\text{Rx} + \text{Ex}) + \exists x (\text{Dx} + \text{Wx}) \text{ et cetera} \quad 53
\]

\[
T_2: \ \exists x (\text{Rx} + \text{Ex}) \text{ et cetera} \quad 54
\]
T₁ is simpler than T₂ since T₁ is ontologically simpler than T₂. T₁ is ontologically simpler than T₂ because T₁ rules out an event which T₂ does not, viz., "(∃x)(Dx + Wx)". Also note that by Popper's subclass method T₁ → T₂ but it is not the case that T₂ → T₁ in which case T₁ is more falsifiable than T₂. Thus, we have explained why T₁ which is more falsifiable than T₂ is also simpler than T₂ without resorting to the practical notion of "ease of refutation".

As a second example, we are given the following two theories:

T₀: All heavenly bodies have circular orbits, et cetera.

T₀*: All planets have circular orbits, et cetera.

Further, T₀ and T₀* can be re-expressed as the negations of strictly existential statements as follows:

T₀*: "There does not exist a heavenly body whose orbit is non-circular, et cetera."

T₀**: "There does not exist a planet whose orbit is non-circular, et cetera."

At this point, it would seem that T₀* and T₀** do not obviously differ in their ontological simplicity. However, recall from Part V of this chapter that T₀* can be re-expressed as follows (owing to the fact that heavenly bodies include planets, stars, comets and satellites):

T₀*: There does not exist a planet whose orbit is non-circular and there does not exist a comet whose orbit is non-circular and there does not exist a satellite whose orbit is non-circular and there does not exist a star whose orbit is non-circular, et cetera.

Clearly by our clarification of simplicity of laws in terms of ontologi-
cal simplicity $T_h''$ is simpler than $T_p'$. That is, $T_h''$ denies or rules out certain events which $T_p'$ does not, viz., comets orbiting non-circularly, satellite orbiting non-circularly and stars orbiting non-circularly. Also, by Popper's subclass method $T_h''$ is more falsifiable than $T_p'$ (since planets are a subclass of heavenly bodies, i.e. the latter is more universal than the former). Once more, we have explained why the more falsifiable theory is the simpler theory without appealing to the practical notion of ease of refutation.

As a final example, we can compare the following circle-theory to a corresponding ellipse-theory:

$T_c$: All planets have circular orbits.

$T_e$: All planets have elliptical orbits.

$T_c$ and $T_e$ can be re-expressed as follows:

$T_c'$: There does not exist a planet whose orbit is non-circular, et cetera.

$T_e'$: There does not exist a planet whose orbit is non-elliptical, et cetera.

Although at first glance, it would appear that $T_c'$ is equally simple to $T_e'$ (if we define simplicity of laws in terms of ontological simplicity), if we recall from Part V of this chapter that $T_c'$ and $T_e'$ can be re-written, as follows, it will become clear that they are not equally simple:

$T_c'':$ [There does not exist a set of four points not all falling on the curve and there does not exist a set of five points not all falling on the curve, et cetera]
There does not exist a set of six points not all falling on the curve and there does not exist a set of seven points not all falling on the curve, et cetera.

Clearly $T_c$" is ontologically simpler than $T_e"$ since $T_c"$ rules out two events which $T_e"$ does not rule out, viz., the event of four points not all falling on the curve and the event of five points not all falling on the curve. And, since we have stipulated that a theory's (relative) simplicity is to be the same as its (relative) ontological simplicity we can say here that $T_c"$ is simpler than $T_e"$. Further, by Popper's dimension method $T_c"$ whose $d = 3$, is more falsifiable than $T_e"$ whose $d = 5$. Thus, we have again explained why the more falsifiable theory is also the simpler theory without using the notion of "ease of refutation".

Finally, this clarification of simplicity, although in agreement with the major methodological dictates of falsificationism (i.e. it turns out that the more falsifiable theory is often the simpler theory) need not be restricted to a falsificationist methodology. This is so because we do not define a theory's relative simplicity in terms of such falsificationist notions as "ease of refutation". Rather, a theory's relative simplicity is spelled out in terms of its relative ontological simplicity which in turn is spelled out in terms of Ockam's Razor. Further, in, for example, an inductivist context where a justification for preferring the simplest theory is required, Ockam's Razor could be offered as a primitive assumption (i.e. an assumption which is in no need of any justification itself). That is, we should desire the simpler of two theories, since we should keep our ontological commitments to a minimum.
1. Popper, p. 28.
2. Ibid., p. 28.
3. Ibid., p. 28.
4. Ibid., p. 28.
5. Ibid., p. 29.
6. Ibid., p. 29; p. 254.
7. Ibid., p. 42.
8. Ibid., pp. 42, 50, 81.
9. Ibid., pp. 42, 50, 81.
10. Ibid., pp. 42, 50, 81.
11. Page 16 of this thesis.
12. Popper, p. 82.
13. Ibid., p. 54.
15. Ibid., p. 80.
16. Ibid., p. 38.
17. Ibid., pp. 69-70.
18. Ibid., p. 102.
19. Ibid., p. 86.
20. Ibid., pp. 86-87 (footnote #1)
21. Ibid., pp. 267 & 269.
22. Ibid., p. 87 (footnote #1)
23. Ibid., p. 87 (footnote #1)
24. Ibid., p. 114.
26 Ibid., pp. 285-86.
27 Ibid., p. 120.
28 Ibid., p. 114.
29 Ibid., p. 114.

30 Again, each of these forbidden events or any conjunction of them can be conjoined with any other event to form a new forbidden event, a process which can be carried out to infinity.

31 Popper, p. 116.
32 Ibid., p. 116.
33 Ibid., p. 91.
34 Ibid., p. 122.

35 Ibid., p. 87. He notes that this hypothesis is of a "very low level of universality" in footnote 1.

36 Ibid., p. 113.
37 Ibid., p. 114.

38 Also, see Friedman, K.S. "Empirical Simplicity as Testability" British Journal for the Philosophy of Science. 23, 1972, pp. 25-33. Friedman presents an alternative way of spelling out a theory's dimension, although it still involves ease of testing (pp.27-28). That is, a theory's degree of simplicity is the "ease of testing it" which is dependent on the amount of information needed to find out the truth or falsity of a potential falsifier for that theory.

39 Popper, pp. 122 & 124.
40 Ibid., pp. 122 & 124.
41 Ibid., pp. 141 & 378.
42 Ibid., pp. 378-79.
43 Pages 49-50 of this thesis.
44 Popper, p.

45 Note: Bunge in his The Myth of Simplicity makes a similar point to the one being made here, viz., "But formal complexity should not be confused with difficulty of test, although the former may entail the latter", p. 63.
46. Pepper, p. 138.
47. Ibid., p. 40.
48. Ibid., p. 40.
49. Ibid., p. 54.
50. Ibid., p. 56.
51. Sober, p. 40.
52. Ibid., p. 41.

53. "There are no non-black ravens and there are no non-white doves, etc."

54. "There are no non-black ravens, etc."
CONCLUDING REMARKS

And so, Popper stipulates that a theory's degree of falsifiability is to be the same as its degree of simplicity in order to resolve certain problems in epistemology connected with the concept of simplicity. It is because Bunge and Hempel did not recognize the stipulative nature of Popper's definition of simplicity (viz., that it was formulated to help us solve certain problems in epistemology), that their counterexamples are inappropriate (See Chapter II, Part II).¹ One of the problems in epistemology which this definition of simplicity (which Popper proposed) supposedly resolves is whether or not we should prefer the simplest theory or law. That is, if high simplicity is to be the same as high falsifiability then clearly, since high falsifiability is desirable, it follows that high simplicity is also desirable. However, if we take seriously Ayer and Wittgenstein's remarks on induction (viz., that no justification is needed for induction in which case falsifiability is no longer a necessary alternative² and our own remarks that Popper's falsifiability proposal is problem-laden it is no longer clear that high falsifiability and hence high simplicity are so desirable. (Also see Feyerabend's remarks on the limitations of methodologies examined and assessed in Chapter II.³)

Further, as we have seen in Part VIII of Chapter III,⁴ most of the epistemological problems connected with the concept of simplicity which Popper's stipulative definition was intended to solve arise in the context of induction. For example, given six data-points, why should we
choose the simplest curve to "fit" them and for that matter which curve actually is the simplest? However, from Popper's viewpoint (having rejected induction) these are pseudo-problems. It is therefore likely that he went to all this trouble of resolving these pseudo-problems (from his viewpoint) in order to persuade his readers of the fruitfulness of his methods for comparing theories' degrees of falsifiability. (Vide L.Sc.D., page 135). Yet, as we argued in Part VII of Chapter III, these proposed methods do not help us to resolve problems in epistemology connected with the concept of simplicity because they (i.e. the methods) are ultimately spelled out in terms of ease of refutation, clearly (by the force of Popper's own arguments) a practical concern. Thus, being able to determine whether or not T₁ is easier (or simpler) to refute than T₂ does not really tell us why T₁ is simpler (or more complex) than T₁.

As we saw in Part IX of Chapter III, even for the falsificationist, whose major methodological concern is not that we should find the simplest curve to describe n data-points, but rather that we should adopt the most falsifiable theory (which can then be tested by d + 1 data-points), it is desirable that he be able to explain why one curve is simpler than another. This is especially so since it is often the case that the more falsifiable theory (such as a circle-theory) is also typically regarded as the simpler theory (as opposed to an ellipse-theory). Without making the same mistake which Popper made by identifying degree of testability with simplicity (and hence only being able to explain on a practical level why one curve is "simpler" than another) we might give the following stipulative clarification of simplicity:
$T_1$ is simpler than $T_2$ if $T_1$ is ontologically simpler than $T_2$ in which case $T_1$ denies or rules out certain events which $T_2$ does not. Thus, a circle-theory is simpler than an ellipse-theory because the former rules out two events which the latter does not, viz., four points not falling on the curve and five points not falling on the curve. And so, we have explained why the more falsifiable circle-theory is simpler than the corresponding ellipse-theory without equating simplicity with degree of testability and therefore without appealing to the practical notion of ease of refutation.

But what about Feyerabend's remarks on the limitations of falsificationism (or for that matter any methodology)? If falsificationism goes then so does our definition of simplicity in terms of ontological simplicity. However, it can be pointed out here that the clarification of simplicity proposed above need not be restricted to any particular methodology such as falsificationism. It simply explains (using Ockam's Razor) why any given theory $T_1$ is simpler than another theory $T_2$. Further, if any justification is required (such as in an inductivist context, i.e., why should we prefer the simplest theory?) then Ockam's Razor might be used as a primitive assumption (as is so often the case in the philosophy of mind).
FOOTNOTES

1. Pages 150-55.
2. Or in Ayer's, at least no logical justification is required.
3. Pages 161-80.
5. Pages 214-16.
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