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Stability of cylindrical corrugated sheet shells.

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STABILITY OF CYLINDRICAL CORRUGATED SHEET SHELLS

by

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ABSTRACT

The main objective of this study is to examine the overall buckling of cylindrical shells made of corrugated sheets and supported along the four edges. The need for this study arises from the fact that buckling of this type of shells presents one of the main criteria of failure. To achieve this goal, a system of governing equations for the analysis and stability of corrugated sheet shells is obtained in a more accurate and reliable form than the system available in the literature.

Therefore, this study is composed of:

1. Study of the different formulations of the governing equations.
2. Study of the elastic analysis of this type of shells.
3. Study of the stability problem of this type of shells supported along the four edges.

The proposed formulations are found to lead to improved results, without an increase in the difficulty of obtaining a solution. The buckling loads for shells simply supported on four edges are determined for various cases of loading.

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NOTATIONS

B_x, B_ϕ = bending rigidity in xz- and ϕz planes, respectively;

$B_{x\phi}$ = torsional rigidity;

c = corrugation pitch;

D_x, D_ϕ = axial rigidity in x- and ϕ- directions, respectively;

$D_{x\phi}$ = shear rigidity in the xϕ- plane;

E = modulus of elasticity for isotropic material;

f = half depth of corrugation;

ℓ = half length of corrugation;

L = length of the shell;

M = root of the characteristic equation;

M_x, M_ϕ = bending moment per unit length acting in the xz- and ϕz- planes, respectively;

$M_{x\phi}$, $M_{\phi x}$ = torsional moment per unit length acting about the ϕ- and x- axis, respectively;

N_x, N_ϕ = axial force per unit length acting in x- and ϕ- directions, respectively;

$N_{x\phi}$ = shear force per unit length acting in the x-ϕ plane;

P_x, P_ϕ, P_z = external loading per unit area of the middle surface acting in the x, ϕ, and z- directions, respectively;

Q_x, Q_ϕ = lateral shear force per unit length acting perpendicular to x- and ϕ axis respectively;

R = radius of curvature of the shell;

t = average thickness of corrugated sheet;

u, v, w = displacement in the x-, ϕ-, and z- directions, respectively;

γ = factor of increase in strain rigidity, D_x, for dimpled sheets.

ϵ_x, ϵ_ϕ = axial strain in the x- and ϕ- directions, respectively;

$\epsilon_{x\phi}$ = shear strain in the xϕ- plane;

λ = $\frac{n\pi R}{L}$

μ_1, μ_2 = Poisson's ratios;

ρ = reduction factor of shear rigidity;

σ_x, σ_ϕ = axial stress in the x- and ϕ- directions, respectively;

$\tau_{x\phi}$ = shear stress in the xϕ- plane;

ϕ_e = half central angle of shell.

CHAPTER I

INTRODUCTION

The advantages of using light gage steel sheets in folded plate roofs have been established throughout studies and practical applications in Canada, England and the U.S.A. Although, corrugated sheets with cylindrical curvature are widely available, they are employed mainly in non-structural capacities. These applications include such items as long shell roofs, half barrel utility buildings and grain bins. Therefore, cylindrical shells made of corrugated sheets present an added economical and practical application for the corrugated steel sheets. They can be built as:

a- Simply supported shells with stiffeners along the valleys only.

b- Simply supported shells with stiffeners along the valleys and crown.

c- Shells supported along their four edges.

Reference (13) dealt with the analysis of these shells considering the approximations proposed by Donnell (12). The results reported in this investigation, together with the general concept of the shell theory, indicate that the criteria governing the load carrying capacity differ from one case to another of the above mentioned shells. This can be obtained as follows:

a- Large deflection is the prime criterion for the load carrying capacity of the shells with longitudinal stiffeners in valleys only (13).

b- Longitudinal stiffeners along the crown as well as the valleys reduce the deflection considerably. In this case the corrugated sheets are subjected, mainly, to shear forces. The load carrying capacity of such shells is thus, governed by the local shear buckling of the shell. This problem has been examined in references (1) and (2).

c- The overall buckling is expected to be a prime factor defining the ultimate carrying capacity for shells supported along their four edges. This problem has not been examined before, and is the main objective of the present study.

It is also recognized that a more reliable formulation is required for shells supported along their four edges. Thus, it became essential to, first, examine the different formulations that can be used in the analysis of such shells.

The most precise formulation of the linear theory of isotropic and orthotropic cylindrical shells was established by Flugge back in 1932 (15). The governing equations, of this formulation, were too difficult to be used in solving many practical problems. Therefore, simplified equations have been introduced using different approximations in the conditions of equilibrium and the geometric relationships of the shell (12,14,20,23&28). The solutions obtained by using the simplified formulations were examined in the case of isotropic shells only, to evaluate their degree of accuracy and the factors affecting their results (21). In chapter II, Flugge's formulation of the theory of orthotropic cylindrical shells is presented. Simplified formulations using Donnell, Vlasov and Schorer's theories are applied to the orthotropic shells made of corrugated sheets. Also, alternative simplified formulations are suggested, in order to improve the accuracy of the results without increasing the difficulty of obtaining a solution.

It is also expected that the accuracy of the shell analysis, using the different formulations, is affected by the boundary conditions. Thus, the effect of the boundary conditions is studied in chapter III.

With respect to the stability problem, most of the literature is concerned with the buckling of a cylindrical tube under constant external load (12,16). Cylindrical shells used in structures such as curved panels or shell roofs are mostly of open type. Solutions have been obtained for isotropic curved panels with simply supported or clamped longitudinal edges subject to shear (5,6,12), to uniform axial compression (25), to a combined shear and compression (19), and to extenal normal pressure (7,18). The buckling of isotropic open cylindrical shells with free longitudinal edges subjected to uniform axial compression is also studied (10). The buckling of the same kind of structures subjected to lateral loads is investigated, using a nonlinear finite difference scheme (11). The overall buckling of corrugated sheet shells simply supported along the four edges is studied in chapter IV, using the proposed formulation and a Galerkin approach. Solutions are obtained for various cases of loading.

CHAPTER II

FORMULATION OF THE LINEAR THEORY OF ORTHOTROPIC CYLINDRICAL SHELLS

In this chapter, formulations of the linear theory of orthotropic cylindrical shells are derived. These formulations are applied to orthotropic shells made of corrugated sheets. They include Flugge's formulations, which is considered to be the most presise one, as well as formulations using Donnell, Vlasov and Schorer's assumptions. Alternative simplified formulations are also introduced.

II. 1. FLUGGE'S FORMULATIONS (15)

These formulations are presented first, since they will serve as a basis for comparison with the other simplified formulations which will follow.

Fig. 1 shows the coordinates used: x, ø and z being the longitudinal, angular and radial coordinates, respectively. The differential equations governing the linear behavior of cylindrical shells, are obtained by considering the conditions of equilibrium together with the elastic and geometric relationships of an infinitesimal element, dx.Rdø, of the undeformed shell (Fig. 1).

II. 1. 1. Conditions of Equilibrium

The six conditions of equilibrium, in space, are given by:

$$\sum F_x = 0 \Rightarrow \frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\phi x}}{\partial \phi} + p_x = 0 \qquad (1a)$$

$$\sum F_\phi = 0 \Rightarrow \frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} - \frac{Q_\phi}{R} + p_\phi = 0 \qquad (1b)$$

$$\sum F_z = 0 \Rightarrow \frac{1}{R}\frac{\partial Q_\phi}{\partial \phi} + \frac{\partial Q_x}{\partial x} + \frac{N_\phi}{R} + p_z = 0 \qquad (1c)$$

$$\sum M \circledast p_x = 0 \Rightarrow \frac{1}{R}\frac{\partial M_\phi}{\partial \phi} + \frac{\partial M_{x\phi}}{\partial x} - Q_\phi = 0 \qquad (1d)$$

$$\sum M \circledast p_\phi = 0 \Rightarrow \frac{\partial M_x}{\partial x} + \frac{1}{R}\frac{\partial M_{\phi x}}{\partial \phi} - Q_x = 0 \qquad (1e)$$

$$\sum M \circledast p_z = 0 \Rightarrow R\, N_{x\phi} - R\, N_{\phi x} + M_{\phi x} = 0 \qquad (1f)$$

Eqs. (1) can be reduced to the following four equations, by eliminating the lateral shear components Q_x and Q_ϕ:

$$N_x' + \dot{N}_{\phi x} + R\, p_x = 0 \qquad (2a)$$

$$R\, \dot{N}_\phi + R\, N_{x\phi}' - \dot{M}_\phi - M_{x\phi}' + R^2 p_\phi = 0 \qquad (2b)$$

$$\ddot{M}_\phi + \dot{M}_{x\phi}' + M_x'' + \dot{M}_{\phi x}' + R\, N_\phi + R^2 p_z = 0 \qquad (2c)$$

$$R\, N_{x\phi} - R\, N_{\phi x} + M_{\phi x} = 0 \qquad (2d)$$

in which $R\frac{\partial(\ )}{\partial x} = (\ )'$ and $\frac{\partial(\ )}{\partial \phi} = (\dot{\ })$.

II. 1. 2. Geometric Relationships (Deformations and strains).

The deformation of the cylindrical shell may be described by the three components of the displacement, in the coordinates directions, of an arbitrary point A of the shell (Fig. 2 a,b) together with the following assumptions:

i - The shell is thin.

ii - Normals to the middle surface of the shell remain normal to it and undergo no change in length during deformation.

iii - The transverse normal stress is negligible.

iv - All displacements are small, i.e. they are negligible compared with the radius of curvature of the middle surface and their first derivatives, the slopes, are negligible compared with unity. This keeps the equations linear.

The strain displacement relations of the middle surface are given by:

$$\varepsilon^{o}_{x} = \frac{\partial u}{\partial x} \tag{3a}$$

$$\varepsilon^{o}_{\phi} = \frac{1}{R}\frac{\partial v}{\partial \phi} - \frac{w}{R} \tag{3b}$$

$$\gamma^{o}_{x\phi} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \phi} \tag{3c}$$

Refering to Fig. 2 the displacement of a point A at a distance z from the middle surface are:

$$u_A = u - z\frac{\partial w}{\partial x} \tag{4a}$$

$$v_A = \frac{R-z}{R}v - \frac{z}{R}\frac{\partial w}{\partial \phi} \tag{4b}$$

$$w_A = w \tag{4c}$$

Hence, the strains at A are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{5a}$$

$$\varepsilon_\phi = \frac{1}{R}\frac{\partial v}{\partial \phi} - \frac{z}{R(R-z)}\frac{\partial^2 w}{\partial \phi^2} - \frac{w}{R-z} \tag{5b}$$

$$\gamma_{x\phi} = \frac{R-z}{R}\frac{\partial v}{\partial x} - \frac{z}{R}\frac{\partial^2 w}{\partial x\partial\phi} + \frac{1}{R-z}\frac{\partial u}{\partial \phi} - \frac{z}{R-z}\frac{\partial^2 w}{\partial x\partial\phi} \tag{5e}$$

in which, u, v, and w = the displacements of the middle surface in the x–, ϕ– and z– directions; ε_x, ε_ϕ = axial strains in the x– and ϕ– directions; $\gamma_{x\phi}$ = shear strain in the xϕ– plane.

II. 1. 3. Stress Strain Relations

For the case of an ideal orthotropic shell, the material has three planes of symmetry with respect to its elastic properties. These planes are the coordinate planes, x, ϕ and z. The relation between stress and strain components, for the case of plane stress in the xϕ-plane can be represented by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_\phi \\ \tau_{x\phi} \end{Bmatrix} = \begin{bmatrix} E_1 & E_\mu & 0 \\ E_\mu & E_2 & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\phi \\ \gamma_{x\phi} \end{Bmatrix} \tag{6}$$

in which: $E_1 = \dfrac{E_x}{1-\mu_1\mu_2}$; $E_2 = \dfrac{E_\phi}{1-\mu_1\mu_2}$;

$E_\mu = \mu_1 E_2 = \mu_2 E_1$ for conservative materials;

μ_1 & μ_2 = poisson's ratios.

II. 1. 4. Internal Force Components

The internal force components can be calculated as follows:

$$N_x = \int_{-t/2}^{+t/2} \sigma_x \left(1 - \frac{z}{R} \right) dz \quad , \quad N_\phi = \int_{-t/2}^{+t/2} \sigma_\phi \, dz$$

$$N_{x\phi} = \int_{-t/2}^{+t/2} \tau_{x\phi} \left(1 - \frac{z}{R} \right) dz \quad , \quad N_{\phi x} = \int_{-t/2}^{+t/2} \tau_{\phi x} \, dz$$

$$M_x = \int_{-t/2}^{+t/2} \sigma_x z \left(1 - \frac{z}{R} \right) dz \quad , \quad M_\phi = \int_{-t/2}^{+t/2} \sigma_\phi \, dz \tag{7 a-h}$$

$$M_{x\phi} = \int_{-t/2}^{+t/2} \tau_{x\phi} z \left(1 - \frac{z}{R} \right) dz \quad , \quad M_{\phi x} = \int_{-t/2}^{+t/2} \tau_{\phi x} \, dz$$

Substituting Eq. 6 into Eqs. 7, the internal force components are found to be:

$$N_\phi = \frac{D_\phi}{R}(v^{\bullet} - w) + \frac{D_\mu}{R}u' - \frac{B_\phi}{R^3}(w + w^{\bullet\bullet}) \tag{8a}$$

$$N_x = \frac{D_x}{R}u' + \frac{D_\mu}{R}(v^{\bullet} - w) + \frac{B_x}{R^3}w'' \tag{8b}$$

$$N_{\phi x} = \frac{D_{x\phi}}{R}(u^{\bullet} + v') + \frac{B_{x\phi}}{2R^3}(u^{\bullet} - w'^{\bullet}) \tag{8c}$$

$$N_{x\phi} = \frac{D_{x\phi}}{R}(u^{\bullet} + v') + \frac{B_{x\phi}}{2R^3}(v' + w'^{\bullet}) \tag{8d}$$

$$M_\phi = -\frac{B_\phi}{R^2}(w + w^{\bullet\bullet}) - \frac{B_\mu}{R^2}w'' \tag{8e}$$

$$M_x = -\frac{B_x}{R^2}(w'' + u') - \frac{B_\mu}{R^2}(w^{\bullet\bullet} + v^{\bullet}) \tag{8f}$$

$$M_{\phi x} = -\frac{B_{x\phi}}{R^2}(w'^{\bullet} - \tfrac{1}{2}u^{\bullet} + \tfrac{1}{2}v') \tag{8g}$$

$$M_{x\phi} = -\frac{B_{x\phi}}{R^2}(w'^{\bullet} + v') \tag{8h}$$

in which:

$$D_x = E_1 t, \quad D_\phi = E_2 t, \quad D_\mu = E_\mu t, \quad D_{x\phi} = G t,$$

$$B_x = \frac{E_1 t^3}{12}, \quad B_\phi = \frac{E_2 t^3}{12}, \quad B_\mu = \frac{E_\mu t^3}{12}, \quad B_{x\phi} = \frac{G t^3}{6}.$$

II. 1. 5. Orthotropic Shells Made of Corrugated Sheets

Cylindrical shells made of corrugated sheets can be treated as being made of elastic orthotropic material, in which the mechanical properties are equal to the average properties of the sheets. This approach was proved to be valid, and to adequately consider the main features of response of these shells (1,13).

Cylindrical shells are usually produced with the standard arch-and-tangent type of corrugation. The mechanical properties of this type of corrugation are given in Table No. 1 (13). This table shows also, the mechanical properties of two of the modified sheet configurations. These are the dimpled sheets and the standard corrugated sheets spot welded to plane sheets. The effect of Poisson's ratios ($\mu_2 E_x$ and $\mu_1 E_\phi$) can be considered negligible, and have no effect on the mechanical properties of these sheets. Therefore, the internal force components are simplified to the following:

$$N_\phi = \frac{D_\phi}{R} (\dot{v} - w) - \frac{B_\phi}{R^3} (w + \ddot{w}) \qquad (9a)$$

$$N_x = \frac{D_x}{R} u' + \frac{B_x}{R^3} w'' \qquad (9b)$$

$$N_{\phi x} = \frac{D_{x\phi}}{R}\left(u^{\bullet} + v' \right) + \frac{B_{x\phi}}{2R^3}\left(u^{\bullet} - w'^{\bullet} \right) \tag{9c}$$

$$N_{x\phi} = \frac{D_{x\phi}}{R}\left(u^{\bullet} + v' \right) + \frac{B_{x\phi}}{2R^3}\left(v' + w'^{\bullet} \right) \tag{9d}$$

$$M_{\phi} = - \frac{B_{\phi}}{R^2}\left(w + w^{\bullet\bullet} \right) \tag{9e}$$

$$M_{x} = - \frac{B_{x}}{R^2}\left(w'' + u' \right) \tag{9f}$$

$$M_{\phi x} = - \frac{B_{x\phi}}{R^2}\left(w'^{\bullet} - \frac{1}{2} u^{\bullet} + \frac{1}{2} v' \right) \tag{9g}$$

$$M_{x\phi} = - \frac{B_{x\phi}}{R^2}\left(w'^{\bullet} + v' \right) \tag{9h}$$

II. 1. 6. Governing Differential Equations

It is now appropriate to carry out the reduction of the governing equaions of the cylindrical shell to three equations that relate the displacements u, v and w. This is achieved by substituting the relations (9) into eqs. (2), which gives the following governing equations for the case of no surface loading:

$$D_x u'' + \frac{B_x}{R^2} w''' + D_{x\phi}\left(u^{\bullet\bullet} + v'^{\bullet} \right) + \frac{B_{x\phi}}{2R^2}\left(u^{\bullet\bullet} - w'^{\bullet\bullet} \right) = 0 \tag{10a}$$

$$D_{\phi}\left(v^{\bullet\bullet} - w^{\bullet} \right) + D_{x\phi}\left(u^{\bullet\prime} + v'' \right) + \frac{3B_{x\phi}}{2R^2}\left(v'' + w''' \right) = 0 \tag{10b}$$

$$B_\phi(w^{\cdot\cdot\cdot\cdot} + 2w^{\cdot\cdot} + w) + B_x(w'''' + u''') + B_{x\phi}(2w''^{\cdot\cdot} + \frac{3}{2} v''^{\cdot} - \frac{1}{2} u'^{\cdot\cdot}) - R^2D_\phi(v^{\cdot} - w) = 0 \tag{10c}$$

II. 2. FORMULATIONS USING DONNELL'S APPROXIMATIONS

Donnell's approximations were first applied to isotropic shells with very small ratio of thickness to radius. These approximations are discussed in reference (12) and can be summarized as follows:

i - Approximations in the conditions of equilibrium by neglecting the effect of the shear force Q_ϕ on the equilibrium of forces in the ϕ- direction (i.e. $Q_\phi/R = 0$), and the effect of $M_{x\phi}$ on the equilibrium in the z- direction (i.e. $M_{x\phi}/R = 0$). This assumption can be expected to improve in accuracy as the ratio of the radius to the thickness of the shell increases.

ii - Approximations in the geometric relationships, by neglecting z in the term (R-z) in Eqs. (5). This term represents the fact that the hoop fibers at different levels z have different lengths. Also, the term (z/R) in Eqs. (7) is neglected in comparison with unity. This term appears due to the trapizoidal shape of the faces x = constant of the shell element.

II. 2. 1. Conditions of Equilibrium

Using Donnell's assumptions, these are

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\phi x}}{\partial \phi} + p_x = 0 \qquad (11a)$$

$$\frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + p_\phi = 0 \qquad (11b)$$

$$\frac{1}{R}\frac{\partial Q_\phi}{\partial \phi} + \frac{\partial Q_x}{\partial x} + \frac{N_\phi}{R} + p_z = 0 \qquad (11c)$$

$$\frac{1}{R}\frac{\partial M_\phi}{\partial \phi} + \frac{\partial M_{x\phi}}{\partial x} - Q_\phi = 0 \qquad (11d)$$

$$\frac{\partial M_x}{\partial x} + \frac{1}{R}\frac{\partial M_{\phi x}}{\partial \phi} - Q_x = 0 \qquad (11e)$$

$$N_{x\phi} - N_{\phi x} = 0 \qquad (11f)$$

And, eliminating the shear components Q_x and Q_ϕ they reduce to:

$$N_x' + \dot{N}_{\phi x} + Rp_x = 0 \qquad (12a)$$

$$\dot{N}_\phi + N_{x\phi}' + Rp_\phi = 0 \qquad (12b)$$

$$\ddot{M}_\phi + 2\dot{M}_{x\phi}' + M_x'' + RN_\phi + R^2 p_z = 0 \qquad (12c)$$

$$N_{x\phi} - N_{\phi x} = 0 \qquad (12d)$$

II. 2. 2. Geometric Relationships

Applying Donnell's assumptions together with the assumptions used in Flugge's formulation, the strains of the cylindrical shell are given by:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{13a}$$

$$\varepsilon_\phi = \frac{1}{R}\frac{\partial v}{\partial \phi} - \frac{w}{R} - \frac{z}{R^2}\frac{\partial^2 w}{\partial \phi^2}. \tag{13b}$$

$$\gamma_{x\phi} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \phi} - \frac{2}{R}z\frac{\partial^2 w}{\partial x \partial \phi} \tag{13c}$$

II. 2. 3. Internal Force Components

According to Donnell's assumptions, these will have the following expressions

$$N_\phi = \frac{D_\phi}{R}(v^{\cdot} - w) \tag{14a}$$

$$N_x = \frac{D_x}{R}u' \tag{14b}$$

$$N_{\phi x} = N_{x\phi} = \frac{D_{x\phi}}{R}(u^{\cdot} + v') \tag{14c}$$

$$M_\phi = -\frac{B_\phi}{R^2}w^{\cdot\cdot} \tag{14d}$$

$$M_x = -\frac{B_x}{R^2}w'' \tag{14e}$$

$$M_{\phi x} = M_{x\phi} = -\frac{B_{x\phi}}{R^2}w'^{\cdot} \tag{14f}$$

II. 2. 4. Governing Differential Equations

by substituting the internal forces, Eqs. (14), in the conditions of equilibrium, Eqs. (12), the governing equations, for the case of no surface loads, are obtained in the form:

$$D_x u'' + D_{x\phi} (u^{\cdot\cdot} + v^{\prime\cdot}) = 0 \qquad (15a)$$

$$D_\phi (v^{\cdot\cdot} - w^{\cdot}) + D_{x\phi} (u^{\cdot\prime} + v'') = 0 \qquad (15b)$$

$$B_\phi w^{\cdot\cdot\cdot\cdot} + B_x w'''' + 2B_{x\phi} w^{\prime\prime\cdot\cdot} - R^2 D_\phi (v^{\cdot} - w) = 0 \qquad (15c)$$

Eqs. (15) can be reduced to the following 8th order governing differential equation

$$D_x B_x w^{\prime\prime\prime\prime\prime\prime\prime\prime} + \left(2D_x B_{x\phi} + \frac{D_x D_\phi B_x}{D_{x\phi}} \right) w^{\prime\prime\prime\prime\prime\prime\cdot\cdot} + \left(D_x B_\phi + \frac{2D_x D_\phi B_{x\phi}}{D_{x\phi}} + D_\phi B_x \right) w^{\prime\prime\prime\prime\cdot\cdot\cdot\cdot} + \left(\frac{D_x D_\phi B_\phi}{D_{x\phi}} + 2D_\phi B_{x\phi} \right) w^{\prime\prime\cdot\cdot\cdot\cdot\cdot\cdot} + D_\phi B_\phi w^{\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot}$$
$$+ R^2 D_\phi D_x w'''' = 0 \qquad (16)$$

II. 3. FORMULATIONS USING VLASOV'S APPROXIMATIONS

Vlasov's formulation is based on neglecting the longitudinal moment M_x and the twisting moment $M_{x\phi}$. These assumptions are the same as those used by Finsterwalder (14).

II. 3. 1. Conditions of Equilibrium

$$\frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_{\phi x}}{\partial \phi} + p_x = 0 \qquad (17a)$$

$$\frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} - \frac{Q_\phi}{R} + p_\phi = 0 \qquad (17b)$$

$$\frac{1}{R}\frac{\partial Q_\phi}{\partial \phi} + \frac{N_\phi}{R} + p_z = 0 \qquad (17c)$$

$$\frac{1}{R}\frac{\partial M_\phi}{\partial \phi} - Q_\phi = 0 \qquad (17d)$$

$$N_{x\phi} - N_{\phi x} = 0 \qquad (17e)$$

$$Q_x = 0 \qquad (17f)$$

And eliminating Q_ϕ, the equations of equilibrium reduce to

$$N_x' + \dot{N}_{\phi x} + Rp_x = 0 \qquad (18a)$$

$$R\dot{N}_\phi + RN_{x\phi}' - \dot{M}_\phi + R^2 p_\phi = 0 \qquad (18b)$$

$$\ddot{M}_\phi + RN_\phi + R^2 p_z = 0 \qquad (18c)$$

$$N_{x\phi} - N_{\phi x} = 0 \qquad (18d)$$

II. 3. 2. Internal Force components

In this theory, Vlasov considered the following relations between the internal forces and the displacements

$$N_\phi = \frac{D_\phi}{R}(\dot{v} - w) \qquad (19a)$$

$$N_x = \frac{D_x}{R}u' \qquad (19b)$$

$$N_{\phi x} = N_{x\phi} = \frac{D_{x\phi}}{R}(\dot{u} + v') \qquad (19c)$$

$$M_\phi = -\frac{B_\phi}{R^2}\left(v^{\cdot} + w^{\cdot\cdot} \right) \tag{19d}$$

$$M_x = M_{x\phi} = M_{\phi x} = 0 \tag{19e}$$

II. 3. 3. Governing Differential Equations

The resulting governing equations are

$$D_x u'' + D_{x\phi}(u^{\cdot\cdot} + v'^{\cdot}) = 0 \tag{20a}$$

$$D_\phi(v^{\cdot\cdot} - w^{\cdot}) + D_{x\phi}(u'^{\cdot} + v'') + \frac{B_\phi}{R^2} (v^{\cdot\cdot} + w^{\cdot\cdot\cdot}) = 0 \tag{20b}$$

$$B_\phi(v^{\cdot\cdot\cdot} + w^{\cdot\cdot\cdot\cdot}) - R^2 D_\phi(v^{\cdot} - w) = 0 \tag{20c}$$

II. 4. FORMULATIONS USING SCHORER'S APPROXIMATIONS (23)

In addition to the assumptions of Vlasov and Finsterwalder theories, Schorer assumed that the tangential strain

$$\varepsilon_\phi = \frac{1}{R}\left(\frac{\partial v}{\partial \phi} - w\right),$$

and the shear strain

$$\gamma_{x\phi} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \phi}$$

are both small in comparison with the longitudinal strain, ε_x, and hence can be neglected. This leads to

$$w = \frac{\partial v}{\partial \phi} \tag{21a}$$

$$\text{and} \quad \frac{\partial v}{\partial x} = -\frac{1}{R}\frac{\partial u}{\partial \phi} \tag{21b}$$

Also, the effect of the transverse shear force, Q_ϕ, is neglected when considering the condition of equilibrium in the ø- direction. The transverse moment is assumed to have the same form as in Donnell's theory (Eq. 14d).

II. 4. 1. Governing Differential Equation

The above assumptions used by Schorer lead to the following governing equation for the case of corrugated sheet cylindrical shells.

$$w^{::::} + \frac{D_x R^2}{B_\phi} w^{''''} = 0 \tag{22}$$

II. 5. PROPOSED FORMULATIONS

As mentioned before, reference is made in this research to the orthotropic cylindrical shells made of corrugated sheets. The radius of curvature, R, of such shells is usually large, so that the approximations made by Donnell with regard to the conditions of equilibrium are well justified, and are used in these formulations.

II. 5. 1. Internal Force Components

The membrane and bending rigidities of a standard corrugated sheet shell are substituted in Flugge's formulations for the internal membrane components, Eqs. (8 a-d). This leads to

$$\frac{R}{t} N_\phi = \left(\frac{E\ell}{c}\right)(v^{\cdot} - w) - (.552\ E)\left(\frac{f}{R}\right)^2 (w + w^{\cdot\cdot}) \tag{23a}$$

$$\frac{R}{t} N_x = \frac{Et^2}{6(1-\mu^2)f^2} u' + \frac{cE}{12\ell(1-\mu^2)} (\frac{t}{R})^2 w'' \qquad (23b)$$

$$\frac{R}{t} N_{\phi x} = \rho \frac{Ec}{2(1+\mu)\ell} (u^{\cdot} + v') + \frac{E}{24c(1+\mu)} (\frac{t}{R})^2 (\dot{u} - w'^{\cdot}) \qquad (23c)$$

$$\frac{R}{t} N_{x\phi} = \rho \frac{Ec}{2(1+\mu)\ell} (u^{\cdot} + v') + \frac{E}{24c(1+\mu)} (\frac{t}{R})^2 (v' + w'^{\cdot}) \qquad (23d)$$

Hence, two approximations are proposed.

i - Proposal No. 1

Equations (23) encounter terms multiplied by $(t/R)^2$ in which t is the thickness of the sheet, which usually does not exceed 0.1 inch (GA. 10). therefore, $(t/R)^2$ has negligible weight on these equations. Also, the longitudinal and circumferential displacements, u and v, are usually too small when compared with the lateral displacement, w. Therefore, their derivatives in Eqs. (9) are neglected.

ii - Proposal No. 2

In addition to the approximations considered in proposal No. 1, the depth of corrugation, f, may be considered small when compared with the radius of curvature so that the term $(f/R)^2$, in Eq. (23a), can be neglected.

The relationships between the internal force components and displacements obtained by applying the proposed approximations to Flugge's precise formulations are:

i- Proposal No. 1

$$N_\phi = \frac{D_\phi}{R}(v^{\cdot} - w) - \frac{B_\phi}{R^3}(w + w^{\cdot\cdot}) \tag{24a}$$

$$N_x = \frac{D_x}{R} u' \tag{24b}$$

$$N_{\phi x} = N_{x\phi} = \frac{D_{x\phi}}{R}(u^{\cdot} + v') \tag{24c}$$

$$M_\phi = -\frac{B_\phi}{R^2}(w + w^{\cdot\cdot}) \tag{24d}$$

$$M_x = -\frac{B_x}{R^2} w'' \tag{24e}$$

$$M_{\phi x} = M_{x\phi} = -\frac{B_{x\phi}}{R^2} w^{\cdot\prime} \tag{24f}$$

ii- Proposal No. 2

$$N_\phi = \frac{D_\phi}{R}(v^{\cdot} - w) \tag{25a}$$

$$N_x = \frac{D_x}{R} u' \tag{25b}$$

$$N_{\phi x} = N_{x\phi} = \frac{D_{x\phi}}{R}(u^{\cdot} + v') \tag{25c}$$

$$M_\phi = -\frac{B_\phi}{R^2}(w + w^{\cdot\cdot}) \tag{25d}$$

$$M_x = -\frac{B_x}{R^2} w'' \tag{25e}$$

$$M_{\phi x} = M_{x\phi} = -\frac{B_{x\phi}}{R^2}\, w^{\prime\cdot} \qquad (25f)$$

II. 5. 2. Governing Differential Equations

i- Proposal No. 1

Substitution of Eqs. (24) into the equilibrium Eqs. (12), results in:

$$D_x u^{\prime\prime} + D_{x\phi}(u^{\cdot\cdot} + v^{\prime\cdot}) = 0 \qquad (26a)$$

$$D_\phi(v^{\cdot\cdot} - w^{\cdot}) + D_{x\phi}(u^{\cdot\prime} + v^{\prime\prime}) - \frac{B_\phi}{R^2}(w^{\cdot} + w^{\cdot\cdot\cdot}) = 0 \qquad (26b)$$

$$B_\phi(w^{\cdot\cdot\cdot\cdot} + 2w^{\cdot\cdot} + w) + B_x w^{\prime\prime\prime\prime} + 2B_{x\phi} w^{\prime\prime\cdot\cdot} - R^2 D_\phi(v^{\cdot} - w) = 0 \qquad (26c)$$

and, neglecting the term $\frac{B_\phi}{R^2}(w^{\cdot} + w^{\cdot\cdot\cdot})$ in comparison with $D_\phi w^{\cdot}$, in Eq. (26b), the governing equations take the form

$$D_x u^{\prime\prime} + D_{x\phi}(u^{\cdot\cdot} + v^{\prime\cdot}) = 0 \qquad (27a)$$

$$D_\phi(v^{\cdot\cdot} - w^{\cdot}) + D_{x\phi}(u^{\cdot\prime} + v^{\prime\prime}) = 0 \qquad (27b)$$

$$B_\phi(w^{\cdot\cdot\cdot\cdot} + 2w^{\cdot\cdot} + w) + B_x w^{\prime\prime\prime\prime} + 2B_{x\phi} w^{\prime\prime\cdot\cdot} - R^2 D_\phi(v^{\cdot} - w) = 0 \qquad (27c)$$

Eqs. (27) can be reduced to the following 8 th order governing differential equation(Appendix I):

$$D_x B_x w^{\prime\prime\prime\prime\prime\prime\prime\prime} + \left(2D_x B_{x\phi} + \frac{D_x D_\phi B_x}{D_{x\phi}}\right) w^{\prime\prime\prime\prime\prime\prime\cdot\cdot} + \left(D_x B_\phi + \frac{2D_x D_\phi B_{x\phi}}{D_{x\phi}}\right.$$

$$\left. + D_\phi B_x\right) w^{\prime\prime\prime\prime\cdot\cdot\cdot\cdot} + \left(\frac{D_x D_\phi B_\phi}{D_{x\phi}} + 2D_\phi B_{x\phi}\right) w^{\prime\prime\cdot\cdot\cdot\cdot\cdot\cdot} + D_\phi B_\phi w^{\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot} +$$

$$2D_x B_\phi w^{:!!} + \frac{2D_x D_\phi B_\phi}{D_{x\phi}} w^{::!} + 2D_\phi B_\phi w^{:::} + D_\phi B_\phi w^{::} + (D_x B_\phi + R^2 D_\phi D_x)\, w^{!!} + \frac{D_x D_\phi B_\phi}{D_{x\phi}} w^{:!} = 0 \tag{28}$$

ii- Proposal No.2

Using Eqs. (25) together with Eqs. (12), the governing equations are found to be:

$$D_x u^{''} + D_{x\phi}(u^{\cdot\cdot} + v^{'\cdot}) \tag{29a}$$

$$D_\phi(v^{\cdot\cdot} - w^{\cdot}) + D_{x\phi}(u^{\cdot'} + v^{''}) \tag{29b}$$

$$B_\phi(w^{\cdot\cdot\cdot\cdot} + w^{\cdot\cdot}) + B_x w^{'''} + 2B_{x\phi} w^{''\cdot\cdot} - R^2 D_\phi (v^{\cdot} - w) = 0 \tag{29c}$$

which can be reduced to:

$$D_x B_x w^{!!!!} + \left(2D_x B_{x\phi} + \frac{D_x D_\phi B_x}{D_{x\phi}}\right) w^{!!!:} + \left(D_x B_\phi + \frac{2D_x D_\phi B_{x\phi}}{D_{x\phi}} + D_\phi B_x\right) w^{!!::} + \left(\frac{D_x D_\phi B_\phi}{D_{x\phi}} + 2D_\phi B_{x\phi}\right) w^{!:::} + D_\phi B_\phi w^{::::} +$$

$$D_x B_\phi w^{:!!} + \frac{D_x D_\phi B_\phi}{D_{x\phi}} w^{::!} + D_\phi B_\phi w^{:::} + R^2 D_\phi D_x w^{!!} = 0 \tag{30}$$

II. 6. CHARACTERISTIC EQUATIONS

The solution of any set of governing equations, as given by the above formulations, is usually obtained as a sum of a membrane and bending solutions. The membrane solution satisfies the differential equations considering the surface loading, but without satisfying the boundary conditions. This solution is usually easy to obtain and is the same for each formulation and will be discussed in the following chapter. The bending solution is superimposed in order to satisfy the boundary conditions. This solution deals with the homogeneous governing equations ($p_x = p_\phi = p_z = 0$).

For shells simply supported along their curved edges, the bending solution of the homogeneous governing equations can be taken as:

$$w = k_1 \, e^{m\phi} \cos \frac{\lambda x}{R} \tag{31a}$$

$$v = k_2 \, e^{m\phi} \cos \frac{\lambda x}{R} \tag{31b}$$

$$u = k_3 \, e^{m\phi} \sin \frac{\lambda x}{R} \tag{31c}$$

in which k_1, k_2 and k_3 = constant, $\lambda = \frac{n\pi R}{L}$, $n = 1,2,3\ldots$

Substitution of Eqs. (31) into the governing equations yields the characteristic equations for each formulaton as follows:

II. 6. 1. Characteristic Equation Using Flugge's Formulations

The simultaneous Eqs. (10), after substituting Eqs. (31), can be put in the following matrix equation:

$$\left[\, Q \,\right] \left\{ K \right\} = \left\{ 0 \right\} \tag{32}$$

in which

$$\{K\} = \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix}$$

$$q_{11} = \frac{B_x}{R}\left(\frac{\lambda}{R}\right)^3 + \frac{m^2 B_{x\phi}}{2R^3}\left(\frac{\lambda}{R}\right)$$

$$q_{12} = -\frac{D_{x\phi}}{R}\, m \left(\frac{\lambda}{R}\right)$$

$$q_{13} = \frac{D_{x\phi}}{R^2} m^2 - D_x\left(\frac{\lambda}{R}\right)^2 + \frac{B_{x\phi}}{2R^4} m^2$$

$$q_{22} = D_\phi m^2 - D_{x\phi} R^2\left(\frac{\lambda}{R}\right)^2 - \frac{3B_{x\phi}}{2}\left(\frac{\lambda}{R}\right)^2$$

$$q_{23} = D_{x\phi} Rm \frac{\lambda}{R}$$

$$q_{31} = -D_\phi - \frac{B_\phi}{R^2} m^4 - \frac{2B_\phi}{R^2} m^2 - \frac{B_\phi}{R^2} - B_x R^2\left(\frac{\lambda}{R}\right)^4 + 2B_{x\phi}\left(\frac{\lambda}{R}\right)^2 m^2$$

$$q_{32} = D_\phi m + \frac{3B_{x\phi}}{2} m\left(\frac{\lambda}{R}\right)^2$$

$$q_{33} = B_x R\left(\frac{\lambda}{R}\right)^3 + \frac{B_{x\phi}}{2R} m^2\left(\frac{\lambda}{R}\right)$$

A non-trivial solution of Eq. (32) requires that the determinant of Q , equals to zero. This leads to the characteristic equation which, after neglecting terms multiplied by $\frac{1}{R^2}$ with respect to unity, has the form:

$$m^8 + \left[- \lambda^2(\frac{D_x}{D_{x\phi}} + \frac{2B_{x\phi}}{B_\phi}) + 2\right] m^6 + \left[\lambda^4(\frac{D_x}{D_\phi} + \frac{B_x}{B_\phi} + \frac{2B_{x\phi}D_x}{B_\phi D_{x\phi}}) - \lambda^2(\frac{2D_x}{D_{x\phi}} + 4 \frac{B_{x\phi}}{B_\phi}) + 1\right] m^4 + \left[- \lambda^6(\frac{D_x B_x}{D_{x\phi}B_\phi} + \frac{2D_x B_{x\phi}}{B_\phi D_\phi}) + \lambda^4(\frac{2D_x}{D_\phi} - \frac{2B_x}{B_\phi} + \frac{3D_x B_{x\phi}}{D_{x\phi}B_\phi}) - \lambda^2(\frac{2B_{x\phi}}{B_\phi} + \frac{D_x}{D_{x\phi}})\right] m^2 + \left[\lambda^8(\frac{B_x D_x}{B_\phi D_\phi}) + \lambda^4(\frac{D_x R^2}{B_\phi} + \frac{3D_x B_{x\phi}}{2D_{x\phi}B_\phi} + \frac{D_x}{D_\phi})\right] = 0 \tag{33}$$

II. 6. 2. Characteristic Equation Using Donnell's Formulations

This can be found in the same way as given in item II. 6. 1, or by substituting Eq. (31a) into Eq. (16) thus,

$$m^8 + \left[- \lambda^2(\frac{D_x}{D_{x\phi}} + \frac{2B_{x\phi}}{B_\phi})\right] m^6 + \left[\lambda^4(\frac{D_x}{D_\phi} + \frac{B_x}{B_\phi} + \frac{2B_{x\phi}D_x}{B_\phi D_{x\phi}})\right] m^4 + \left[- \lambda^6(\frac{D_x B_x}{D_{x\phi}B_\phi} + \frac{2B_{x\phi}D_x}{B_\phi D_\phi})\right] m^2 + \left[\lambda^8(\frac{B_x D_x}{B_\phi D_\phi}) + \lambda^4(\frac{D_x R^2}{B_\phi})\right] = 0 \tag{34}$$

II. 6. 3. Characteristic Equation Using Vlasov's Formulations

A matrix equation as Eq. (32) is obtained in which

$q_{11} = 0$

$q_{12} = - \frac{D_{x\phi}}{R} m (\frac{\lambda}{R})$

$$q_{13} = D_{x\phi}\frac{m^2}{R^2}$$

$$q_{21} = \frac{B_\phi}{R^2}m^3 - D_\phi m$$

$$q_{22} = D_\phi m^2 - D_{x\phi}R^2\left(\frac{\lambda}{R}\right)^2 + \frac{B_\phi}{R^2}m^2$$

$$q_{23} = D_{x\phi}\lambda m$$

$$q_{31} = \frac{B_\phi}{R^2}m^4 + D_\phi$$

$$q_{32} = \frac{B_\phi}{R^2}m^3 - D_\phi m$$

$$q_{33} = 0$$

Thus, the characteristic equation is given by:

$$m^8 + \left[-\lambda^2\left(\frac{D_x}{D_{x\phi}}\right) + 2\right]m^6 + \left[\lambda^4\left(\frac{D_x}{D_\phi}\right) - \lambda^2\left(\frac{2D_x}{D_{x\phi}}\right) + 1\right]m^4$$

$$+ \left[-\lambda^2\left(\frac{D_x}{D_{x\phi}}\right)\right]m^2 + \left[\lambda^4\frac{R^2D_x}{B_\phi}\right] = 0 \qquad (35)$$

II. 6. 4. Characteristic Equation Using Schorer's Formulations

$$m^8 + \frac{D_xR^6}{B_\phi}\left(\frac{\lambda}{R}\right)^4 = 0 \qquad (36)$$

II. 6. 5. Characteristic Equation Using Proposal No. 1 Formulations

$$m^8 + \left[- \lambda^2 \left(\frac{D_x}{D_{x\phi}} + 2\frac{B_{x\phi}}{B_\phi} \right) + 2 \right] m^6 + \left[\lambda^4 \left(\frac{D_x}{D_\phi} + \frac{B_x}{B_\phi} + 2\frac{B_{x\phi} D_x}{B_\phi D_{x\phi}} \right) - 2\lambda^2 \frac{D_x}{D_{x\phi}} + 1 \right] m^4 + \left[- \lambda^6 \left(\frac{D_x B_x}{D_{x\phi} B_\phi} + 2 \frac{D_x B_{x\phi}}{D_\phi B_\phi} \right) + 2\lambda^4 \frac{D_x}{D_{x\phi}} - \lambda^2 \frac{D_x}{D_{x\phi}} \right] m^2 + \left[\lambda^8 \left(\frac{D_x B_x}{D_\phi B_\phi} \right) + \lambda^4 \left(\frac{D_x}{D_\phi} + \frac{D_x R^2}{B_\phi} \right) \right] = 0 \qquad (37)$$

II. 6. 6. Characteristic Equation Using Proposal No. 2 Formulations

$$m^8 + \left[- \lambda^2 \left(\frac{D_x}{D_{x\phi}} + 2\frac{B_{x\phi}}{B_\phi} \right) + 1 \right] m^6 + \left[\lambda^4 \left(\frac{D_x}{D_\phi} + \frac{B_x}{B_\phi} + 2 \frac{B_{x\phi} D_x}{B_\phi D_{x\phi}} \right) - \lambda^2 \frac{D_x}{D_{x\phi}} \right] m^4 + \left[-\lambda^6 \left(\frac{D_x B_x}{D_{x\phi} B_\phi} + 2 \frac{D_x B_{x\phi}}{D_\phi B_\phi} \right) + \lambda^4 \frac{D_x}{D_\phi} \right] m^2 + \left[\lambda^8 \left(\frac{D_x B_x}{D_\phi B_\phi} \right) + \lambda^4 \left(\frac{D_x R^2}{B_\phi} \right) \right] = 0 \qquad (38)$$

II. 7. COMPARISON OF DIFFERENT FORMULATIONS

A comparison between the expressions of the coefficients of the different characteristic equations, shows that the coefficients of Eq.(37) are the closest to those of the exact Flugge's characteristic equation. The numerical values of these coefficients are shown in Appendix II, for shells with $R = 120$ inches and made of the sheets reported in table No. 1. Also, the case of an isotropic steel shell is presented for comparison.

The roots of the characteristic equations are complex and have the form:

$$M_1 = \pm \alpha_1 \pm i\beta_1 \quad , \quad M_2 = \pm \alpha_2 \pm i\beta_2 \qquad (39)$$

which lead to a displacement function in the form

$$\begin{aligned} w = \Big\{ & e^{\alpha_1 \phi}(A \cos\beta_1\phi + B \sin\beta_1\phi) + e^{-\alpha_1\phi}(C \cos\beta_1\phi \\ & + D \sin\beta_1\phi) + e^{\alpha_2\phi}(E \cos\beta_2\phi + F \sin\beta_2\phi) \\ & + e^{-\alpha_2\phi}(G \cos\beta_2\phi + H \sin\beta_2\phi)\Big\} \cos \frac{\lambda x}{R} \end{aligned} \qquad (40)$$

in which A, B, C,..., H are integration constants calculated by satisfying the boundary conditions along the longitudinal edges. Similar expressions for the displacements u and v are also obtained.

These roots of the characteristic equations are determined using the IBM subroutine POLRT, which computes the real and complex roots of a real polynomial using the Newton-Raphson iterative technique. Tables No. 2, 3, 4 & 5 are computer outputs showing the roots $M_1=\alpha_1-i\beta_1$ and $M_2=\alpha_2-i\beta_2$ for shells with radius $R = 240$ inches, with different ratios of L/R, and made of the types of corrugation shown in table No. 1. The roots for an isotropic steel shell with R = 240 inches and thickness = 4 inches are given in Table No. 5. The different roots for shells with R = 120 inches are reported in Appendix II.

Considering Flugge's roots as a basis for the determination of errors, the maximum percentage of errors in α_1, α_2, β_1 & β_2, and also the average errors in the absolute values of the roots are calculated and tabulated in the last two columns of these tables. Figures 3 to 10 show the distribution of the maximum errors with the change of the ratio of the length to radius of the shell, for the different cases.

Since the bending solution is required to satisfy the boundary conditions along the longitudinal edges, this means that it is a correction to the conditions along these edges. Therefore, the errors in the roots of the characteristic equations have different effects on the overall accuracy of the shell analysis depending on the boundary conditions. This is presented in the next chapter.

CHAPTER III

ANALYSIS OF CYLINDRICAL CORRUGATED SHEET SHELLS

The analysis of cylindrical shells may be regarded as, a mebrane solution, or a bending solution. The membrane state of stresses is characterized by the neglection of the bending and torsional moments. In this case there are only three unknown internal forces (N_x, N_ϕ and $N_{x\phi} = N_{\phi x}$) and three equations of projection for the shell element Fig.1., hence the shell is statically determined in the interior. This solution is usually easy to obtain and is the same for each of the formulations mentioned in chapter II. In the bending theory, the solution reduces to a set of four equations (Eqs.2), which contain eight unknown stress resultants. Thus, the problem is statically indeterminate, and it is necessary to study the deformation of the shell as have been done by the various theories.

Flugge (15) separated the two solutions from each other, he considered the membrane theory as an approximate one and stated that "the bending theory,as an exact one, is to be used in certain cases when the bending stiffness of the shell can not be disregarded". The solution of the governing equations of Flugge is far from simple, and solutions have been obtained for only a few of the simplest types of isotropic shells.

Accordingly, approximations have been introduced (12, 23) in order to reduce the governing equations to a single differential equation in w. This facilitates the bending solution. A solution of the single governing equation of Donnell is presented by Gibson (17). The solution in this case consists of a particular integral plus a complementary function. Lundgren (20) and Ramswamy(22) indicated that if the load, as a function of x and ϕ, is sufficiently smooth (for example the own weight), the membrane solution may be used as a particular integral.Donnell's formulations assume that the behaviour of a thin cylindrical shell is similar to that of a thin plate. This is evident since the changes in curvature and twist of a thin cylindrical shell are identical to those of the theory of plates. The main exception being the presence of N_ϕ/R in the equilibrium equations and w/R in expression of the circumferential normal strain. In the proposed equations more terms which take into consideration the curved shape of the cylindrical shell are included.

The analysis of corrugated sheet shells presented in this chapter is carried out usihg the governing equation of proposal No. 1 (Eq. 28), as well as Donnell's equation. Two cases of boundary conditions are analyzed, namely, shells simply supported on four edges, Fig. 11a & b, and shells simply supported on traverses with longitudinal stiffeners in valleys only, Fig 11c. With the increase of the shell length, the analysis of the first case reduces to full arch action, whereas the second case reduces to a beam solution.

III. 1. MEMBRANE SOLUTION

Considering the membrane state of stress, the following equations of equilibrium are easily derived with the aid of Fig. 1.

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\phi}}{\partial \phi} + p_x = 0 \tag{41a}$$

$$\frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + p_\phi = 0 \tag{41b}$$

$$N_\phi + Rp_z = 0 \tag{41c}$$

Accordingly:

$$N_\phi = - Rp_z \tag{42a}$$

$$N_{x\phi} = - \int \left(\frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + p_\phi \right) dx + C_1(\phi) \tag{42b}$$

$$N_x = - \int \left(\frac{1}{R}\frac{\partial N_{x\phi}}{\partial \phi} + p_x \right) dx + C_2(\phi) \tag{42c}$$

Shells made of corrugated sheets are mainly subjected to snow loads. A snow load p per horizontal projection may be developed in the form of a Fourier series:

$$p = \frac{4}{\pi} p \left(\cos\frac{\pi x}{L} - \frac{1}{3}\cos\frac{3\pi x}{L} + \frac{1}{5}\cos\frac{5\pi x}{L} - \dots \right) \qquad (43)$$

It is usually adequate to consider the first term of this series, when examining the case of a load constant in the x- direction, thus:

$$p_x = 0 \qquad (44a)$$

$$p_\phi = \frac{-4p}{\pi}\cos^2(\phi_e - \phi)\sin(\phi_e - \phi)\cos\frac{\pi x}{L} \qquad (44b)$$

$$p_z = \frac{4p}{\pi}\cos^2(\phi_e - \phi)\cos\frac{\pi x}{L} \qquad (44c)$$

substituting for p_x, p_ϕ and p_z into Eqs. (42), the stress resultants are found to be:

$$N_x = -\frac{12p}{R\pi k^2}\cos 2(\phi_e - \phi)\cos kx \qquad (45a)$$

$$N_\phi = -\frac{4pR}{\pi}\cos^2(\phi_e - \phi)\cos kx \qquad (45b)$$

$$N_{x\phi} = \frac{6pL}{\pi^2}\sin 2(\phi_e - \phi)\sin kx \qquad (45c)$$

in which $k=\pi/L$, ϕ_e= half the central angle of shell. Neglecting the bending rigidities, Eqs.(9a-c) reduce to

$$N_\phi = \frac{D_\phi}{R}\left(\frac{\partial v}{\partial \phi} - w\right) \qquad (46a)$$

$$N_x = D_x\frac{\partial u}{\partial x} \qquad (46b)$$

$$N_{x\phi} = D_{x\phi}\left(\frac{1}{R}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\right) \qquad (46c)$$

from which,

$$u = \frac{1}{D_x}\int N_x\,dx + C_3(\phi) \tag{47a}$$

$$v = \frac{1}{D_{x\phi}}\int N_{x\phi}dx - \frac{1}{R}\int\frac{\partial u}{\partial \phi}\,dx + C_4(\phi) \tag{47b}$$

$$w = -\frac{RN_\phi}{D_\phi} + \frac{\partial v}{\partial \phi} \tag{47c}$$

Substituting Eqs. (45) into Eqs. (47), the displacements of the shell is given by:

$$u = -\frac{12p}{Rk^3\Pi D_x}\cos 2(\phi_e - \phi)\sin kx \tag{48a}$$

$$v = -\frac{6p}{\Pi}\left(\frac{1}{D_{x\phi}k^2} + \frac{4}{R^2k^4D_x}\right)\sin 2(\phi_e-\phi)\cos kx \tag{48b}$$

$$w = \frac{12p}{\Pi}\left(\frac{1}{D_{x\phi}k^2} + \frac{4}{R^2k^4D_x}\right)\cos 2(\phi_e-\phi)\cos kx + \frac{4pR^2}{\Pi D_\phi}\cos^2(\phi_e - \phi)\cos kx \tag{48c}$$

III. 2. BENDING SOLUTION

As mentioned before the bending solution starts with calculating the roots of the characteristic equation which are given by Eq. (39). The displacement function of the radial displacement w is represented by Eq. (40). In the following analysis the proposed equation No. 1 will be utilized. The underlined terms are those which drop if Donnell's formulations are used. In order to proceed easily with the solution, all the stress resultants and displacements are expressed in terms of the radial displacement w, as follows:

$$M_\phi = -\frac{B_\phi}{R^2}(\underline{w} + w^{\cdot\cdot}) \tag{49a}$$

$$M_x = -\frac{B_x}{R^2} w'' \tag{49b}$$

$$M_{x\phi} = M_{\phi x} = -\frac{B_{x\phi}}{R^2} w'^{\cdot} \tag{49c}$$

and, substituting into Eqs. (11d&e)

$$Q_\phi = -\frac{B_\phi}{R^3}(\underline{w^{\cdot}} + w^{\cdot\cdot\cdot}) - \frac{B_{x\phi}}{R^3} w''^{\cdot} \tag{49d}$$

$$Q_x = -\frac{B_x}{R^3} w''' - \frac{B_{x\phi}}{R^3} w'^{\cdot\cdot} \tag{49e}$$

from Eq. (11c),

$$N_\phi = \frac{1}{R^3}(\underline{B_\phi w^{\cdot\cdot}} + B_\phi w^{\cdot\cdot\cdot\cdot} + 2B_{x\phi} w''^{\cdot\cdot} + B_x w'''') \tag{49f}$$

using Eq. (11b),

$$N'_{x\phi} = -\frac{1}{R^3}(\underline{B_\phi w^{\cdot\cdot\cdot}} + B_\phi w^{\cdot\cdot\cdot\cdot\cdot} + 2B_{x\phi} w''^{\cdot\cdot\cdot} + B_x w''''^{\cdot}) \tag{49g}$$

Eqs. (24b) and (11a) gives

$$u''' = \frac{R}{D_x} N_x'' \quad \text{and} \quad N_x'' = -N_{x\phi}'^{\cdot} \text{, hence}$$

$$u''' = \frac{1}{D_x R^2}\left(\underline{B_\phi w^{\cdot\cdot\cdot\cdot}} + B_\phi w^{\cdot\cdot\cdot\cdot\cdot\cdot} + 2B_{x\phi} w^{''\cdot\cdot\cdot\cdot\cdot} + B_x w^{''''\cdot\cdot} \right) \tag{49h}$$

from Eq. (24c),

$$v'''' = \frac{R\, N'''_{x\phi}}{D_{x\phi}} - u^{\cdot'''} \quad \text{, hence}$$

$$v'''' = -\frac{1}{R^2 D_{x\phi}}\left(\underline{B_\phi w^{\cdot\cdot\cdot''}} + B_\phi w^{\cdot\cdot\cdot\cdot\cdot\cdot''} + 2B_{x\phi} w^{''''\cdot\cdot\cdot} + B_x w^{''''''\cdot} \right) - \frac{1}{R^2 D_x} \cdot \left(\underline{B_\phi w^{\cdot\cdot\cdot\cdot\cdot}} + B_\phi w^{\cdot\cdot\cdot\cdot\cdot\cdot\cdot} + 2B_{x\phi} w^{''\cdot\cdot\cdot\cdot\cdot} + B_x w^{''''\cdot\cdot\cdot} \right) \tag{49i}$$

Now, the bending solution reduces to finding the integration constants A, B, C,....H of Eq. (40), by satisfying the boundary conditions along the longitudinal edges.

III. 3. SHELLS SIMPLY SUPPORTED ON FOUR EDGES

The boundary conditions along the straight edges of this shell, shown in Fig. 11 a & b, are:

At $\phi = 0$ and $\phi = 2\phi_e$

i - $w = 0$

ii - $M_{\phi} = 0$

iii - $u = 0$

iv - $v = 0$

which means that the shell is hinged along these edges.

These boundary conditions are transfered, using Eqs. (49), into equations in w only. The expression of w, equation 40, is then substituted yielding a system of equations. These equations can be written, for the case of a uniform snow load acting on half barrells (ϕ_e = 90) supported along the four edges, in the following matrix equation

$$\left[M \right] \left\{ U \right\} = \left\{ N \right\} \tag{50}$$

in which

$$\left\{ U \right\} = \left\{ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix} \right\}$$

and the elements of M and N are given in Appendix III

Solution of Eq.(50) yields the constants, A.... H, from which the displacements and the internal force components can be determined utilizing Eqs. (49)

Tables 6 to 10 show the results of the analysis of corrugated sheet shells with R = 120 inch, using proposal No. 1 formulations. In this program the roots of the characteristic equation are calculated using the IBM subroutine POLRT, as well as the equations suggested in reference (9). The two methods showed good agreement. The same analysis using Donnell's equations are reported in tables 11 to 15 .

Figures 12 to 16 show the distribution of the internal force components and the maximum deflection of these shells for different values of length to radius ratio. It shows that the values of N_x obtained using the proposed equations No. 1, reach a constant value for large values of L/R, thus, revealing the fact that with the increase of the ratio of L/R the portion of the load carried in the curved direction increases until reaching the limiting case in which the tranverse rings act as arches and no additional load is carried in the longitudinal direction. Moreover, the proposed equations No. 1 indicate that the traverses are relieved from the shearing force $N_{x\phi}$ with the increase of the shell length, also that N_ϕ decreases at the crown and increases at the valley with the increase of L/R. Whenever the value of N_ϕ

at the edge of the shell equals the reaction of an equivalent arch (r), full arch action of the shell is anticipated (Fig. 15).

Figures (17 to 20) show the internal force distribution for shells with $L/R = 4.0$ as obtained from Donnell's equations and the proposed equations No. 1.

III. 4. SHELLS WITH LONGITUDINAL STIFFENERS IN VALLEYS.

These shells are made of corrugated sheets, and are provided with longitudinal stiffeners in the valleys to resist the high tensile forces along the edges. In this case (Fig. 11c) the following boundary conditions are to be satisfied:

At $\phi = 0$ and $\phi = 2\phi_e$

i- $M_\phi = 0$

ii- $\overline{Q}_\phi = 0$

iii- $N_\phi = 0$

iv- $u_{shell} = u_{stiffener}$

in which

$$u_{stiffener} = \frac{\sigma_{st}}{Ek} \sin kx$$

σ_{st} = the tensile stress in the stiffener

$$= \frac{N_{\phi x} \text{ at valley}}{A_{st} k}, \quad A_{st} = \text{area of stiffener}$$

and $\bar{Q}_\phi$ is the effective shear at the edge. It is obtained by combining Q_ϕ and the shear contributed by the twisting moment $(\partial M_{x\phi}/\partial x)$.

The first condition means that the torsional resistance of the edge stiffener is neglected. the second and third mean that the bending rigidity of the edge stiffener is also neglected. The fourth condition can be replaced by $u_{shell} = 0$ in case of very rigid stiffeners without any significant error.

These boundary conditions reduce to the following equation:

$$\left[\bar{M}\right]\left\{U\right\} = \left\{\bar{N}\right\} \tag{51}$$

in which U is the same as in Eq. (50), and the elements of $\bar{M}$ and $\bar{N}$ are given in Appendix III.

After calculating the constants, the solution follow the same steps as in the previous case.

Tables 16 to 25 show the results of the analysis of this type of shells. These shells undergo larger deflections than those supported along the four edges. Solutions are obtained using Donnell's formulation and the proposal No.1. Both formulations show close results since both solutions converge to a beam analysis. Fig.(21-a) shows the maximum deflection for different L/R ratios.

Fig.(21-b) shows a comparison between the theoretical and experimental load deflection curves for the shell tested in reference (13). Good agreement between the solution using Proposal No. 1 and the experimental results is observed.

As a result of this analysis and in view of the foregoing error analysis of the roots, reliable results are always expected by using the proposed equations No.1. The stability problem of shells supported along the four sides can now be studied.

CHAPTER IV

BUCKLING OF CYLINDRICAL CORRUGATED SHEET SHELLS SUPPORTED ON FOUR EDGES

In this chapter the buckling of corrugated sheet shells simply supported along the four edges is studied, using the linear formulations of proposal No.1 . Van Der Neut (27) concluded "that linear theory is adequate for the investigation of general instability of stiffened shells". A review of the rigidities of corrugated sheets, reported in table No.1, shows that they have similar features to stiffened sheets. Thus, the linear theory is expected to furnish reliable results for such shells.

IV. 1. DIFFERENTIAL EQUATIONS GOVERNING THE STABILITY PROBLEM.

The method used here, commonly called the bifurcation or Eigenvalue method, is based on the equilibrium equations of the deformed geometry. Fig. (22) shows an element of the middle surface of the shell after deformation. Assuming small deformations, the equilibrium equations (11a,b,d,e & f) will remain the same. Substituting the stress deformation relations of proposal No.1, Eqs. (24), into the equilibrium equations (11 a & b), equations (a) and (b) of appendix I are obtained as:

$$D_x\frac{\partial^2 u}{\partial x^2} + D_{x\phi}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\,\partial y}\right) = 0 \quad (p_x = 0) \tag{52}$$

and

$$D_\phi\left(\frac{\partial^2 v}{\partial y^2} - \frac{1}{R}\frac{\partial w}{\partial y}\right) + D_{x\phi}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) + p_\phi = 0$$

in which $y = R\phi$ (53)

In considering the projection of the forces in the z direction, the z- components of the forces shown in Fig. 22 have to be added to those shown in Fig. 1.

The z- components of the force $N_x R d\phi$ (fig 22a) are:

$$N_x\frac{\partial^2 w}{\partial x^2}\, dx\, Rd\phi + \frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x}\, dx\, Rd\phi + \frac{\partial N_x}{\partial x}\,\frac{\partial^2 w}{\partial x^2}\, dx^2 Rd\phi$$

The z- components of the force $N_\phi dx$ (fig 22b) are:

$$N_\phi dx\, d\phi + N_\phi\frac{\partial^2 w}{R\partial\phi^2}\, dx\, d\phi + \frac{\partial N_\phi}{\partial\phi}\frac{\partial w}{R\partial\phi}\, dx\, d\phi + \frac{\partial N_\phi}{\partial\phi}\frac{d\phi^2}{2}\, dx$$

$$+ \frac{\partial N_\phi}{\partial\phi}\frac{\partial^2 w}{R\partial\phi^2}\, d\phi^2 dx$$

The z- components of the force $N_{x\phi} Rd\phi$ (fig. 22c) are:

$$N_{x\phi}\frac{\partial^2 w}{\partial\phi\partial x}\, dx\, d\phi + \frac{\partial N_{x\phi}}{\partial x}\frac{\partial w}{\partial\phi}\, d\phi\, dx + \frac{\partial N_{x\phi}}{\partial x}\frac{\partial^2 w}{\partial x\partial\phi}\, dx^2 d\phi$$

The z- components of the force $N_{\phi x} dx$ (fig. 22c) are:

$$N_{\phi x}\frac{\partial^2 w}{\partial\phi\partial x}\, dx\, d\phi + \frac{\partial N_{\phi x}}{\partial\phi}\frac{\partial w}{\partial x}\, d\phi\, dx + \frac{\partial N_{\phi x}}{\partial\phi}\frac{\partial^2 w}{\partial x\partial\phi}\, d\phi^2 dx$$

The z- components of the forces $Q_\phi dx$, $Q_x R d\phi$ and p_z (fig. 1) are:

$$\frac{\partial Q_\phi}{\partial \phi} d\phi \, dx + \frac{\partial Q_x}{\partial x} dx \, R d\phi + p_z dx \, R d\phi$$

Adding the z- components after dividing by $Rd\phi \, dx$ and neglecting small terms, the following equation is obtained

$$\frac{\partial Q_\phi}{\partial y} + \frac{\partial Q_x}{\partial x} + \frac{N_\phi}{R} + N_\phi \frac{\partial^2 w}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{x\phi} \frac{\partial^2 w}{\partial x \partial y} + p_z = 0 \tag{54}$$

Substituting for Q_ϕ & Q_x from equations (11d,e), yields (11):

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{x\phi}}{\partial x \partial y} + \frac{\partial^2 M_\phi}{\partial y^2} + \frac{N_\phi}{R} + N_\phi \frac{\partial^2 w}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{x\phi} \frac{\partial^2 w}{\partial x \partial y} + p_z = 0 \tag{55}$$

Using the relations (24) Of proposal No. 1, the following governing differential equation is obtained:

$$B_x \frac{\partial^4 w}{\partial x^4} + 2B_{x\phi} \frac{\partial^4 w}{\partial x^2 \partial y^2} + B_\phi \left(\frac{\partial^4 w}{\partial y^4} + \frac{2}{R^2} \frac{\partial^2 w}{\partial y^2} + \frac{w}{R^4} \right) - \frac{D_\phi}{R} \left(\frac{\partial v}{\partial y} - \frac{w}{R} \right) - N_x \frac{\partial^2 w}{\partial x^2} - 2 N_{x\phi} \frac{\partial^2 w}{\partial x \partial y} - N_\phi \frac{\partial^2 w}{\partial y^2} - p_z = 0 \tag{56}$$

Equations (52), (53) and (56) are the governing equations of the deformed element.

The governing differential equations of the stability can be written with u, v & w and the corresponding N_x, N_ϕ & $N_{x\phi}$ replaced by $u + u^*$, $v + v^*$ & $w + w^*$, and the corresponding $N_x+N_x^*$, $N_\phi+N_\phi^*$ & $N_{x\phi}+N_{x\phi}^*$, respectively, where the first set refers to values prior to buckling and the stared quantities refer to changes in these values that occur during buckling (24). This approach follows directly from the linear bifurcation response at the moment of neutral equilibrium, since bifurcation buckling is the result of the existance of alternate equilibrium positions in the vicinity of the unbuckled position. Thus the governing equations can be satisfied by:-

$$D_x\frac{\partial^2(u+u^*)}{\partial x^2} + D_{x\phi}\frac{\partial^2(v+v^*)}{\partial x\,\partial y} + D_{x\phi}\frac{\partial^2(u+u^*)}{\partial y^2} = 0 \tag{57}$$

$$D_\phi\left[\frac{\partial^2(v+v^*)}{\partial y^2} - \frac{1}{R}\frac{\partial(w+w^*)}{\partial y}\right] + D_{x\phi}\left[\frac{\partial^2(v+v^*)}{\partial x^2} + \frac{\partial^2(u+u^*)}{\partial x\,\partial y}\right] + p_\phi = 0 \tag{58}$$

$$B_x\frac{\partial^4(w+w^*)}{\partial x^4} + 2B_{x\phi}\frac{\partial^4(w+w^*)}{\partial x^2\,\partial y^2} + B_\phi\frac{\partial^4(w+w^*)}{\partial y^4} + \frac{2B_\phi}{R^2}\frac{\partial^2(w+w^*)}{\partial y^2}$$

$$+ \frac{B_\phi}{R^4}(w+w^*) - \frac{D_\phi}{R}\left[\frac{\partial(v+v^*)}{\partial y} - \frac{(w+w^*)}{R}\right] - (N_x+N_x^*)\frac{\partial^2(w+w^*)}{\partial x^2}$$

$$- 2(N_{x\phi} + N^*_{x\phi})\frac{\partial^2(w+w^*)}{\partial x\,\partial y} - (N_\phi + N^*_\phi)\frac{\partial^2(w+w^*)}{\partial y^2} - p_z = 0 \qquad (59)$$

Subtracting Eq. (52) from Eq. (57) and Eq. (53) from Eq. (58) the following governing equations are obtained

$$D_x \frac{\partial^2 u^*}{\partial x^2} + D_{x\phi} \frac{\partial^2 v^*}{\partial x \partial y} + D_{x\phi} \frac{\partial^2 u^*}{\partial y^2} = 0 \qquad (60)$$

$$D_\phi \left(\frac{\partial^2 v^*}{\partial y^2} - \frac{1}{R}\frac{\partial w^*}{\partial y}\right) + D_{x\phi}\left(\frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial x \partial y}\right) = 0 \qquad (61)$$

Subtracting Eq. (56) from Eq. (59) the third governing equation is obtained as:

$$B_x\frac{\partial^4 w^*}{\partial x^4} + 2B_{x\phi}\frac{\partial^4 w^*}{\partial x^2 \partial y^2} + B_\phi\left(\frac{\partial^4 w^*}{\partial y^4} + \frac{2}{R^2}\frac{\partial^2 w^*}{\partial y^2} + \frac{w^*}{R^4}\right) - \frac{D_\phi}{R}\left(\frac{\partial v^*}{\partial y} - \frac{w^*}{R}\right) - N_x\frac{\partial^2 w^*}{\partial x^2} - N^*_x \frac{\partial^2 w}{\partial x^2} - N^*_x\frac{\partial^2 w^*}{\partial x^2} - 2N_{x\phi}\frac{\partial^2 w^*}{\partial x \partial y} - 2N_{x\phi}\frac{\partial^2 w}{\partial x \partial y}$$

$$- 2N^*_{x\phi} \frac{\partial^2 w^*}{\partial x \partial y} - N_\phi \frac{\partial^2 w^*}{\partial y^2} - N^*_\phi \frac{\partial^2 w}{\partial y^2} - N^*_\phi \frac{\partial^2 w^*}{\partial y^2} = 0 \qquad (62)$$

Or:

$$B_x \frac{\partial^4 w^*}{\partial x^4} + 2B_{x\phi} \frac{\partial^4 w^*}{\partial x^2 \partial y^2} + B_\phi\left(\frac{\partial^4 w^*}{\partial y^4} + \frac{2}{R^2} \frac{\partial^2 w^*}{\partial y^2} + \frac{w^*}{R^4} \right)$$

$$- \frac{D_\phi}{R} \left(\frac{\partial v^*}{\partial y} - \frac{w^*}{R} \right) - N_x \frac{\partial^2 w^*}{\partial x^2} - 2\, N_{x\phi} \frac{\partial^2 w^*}{\partial x \partial y} - N_\phi \frac{\partial^2 w^*}{\partial y^2}$$

$$= 0 \qquad (63)$$

In obtaining equation 63 the following assumptions are used (24):

i- $N_x^* \dfrac{\partial^2 w}{\partial x^2}$ and $N_x^* \dfrac{\partial^2 w^*}{\partial x^2}$ are small compared to $N_x \dfrac{\partial^2 w^*}{\partial x^2}$.

ii- $N_{x\phi}^* \dfrac{\partial^2 w}{\partial x \partial y}$ and $N_{x\phi}^* \dfrac{\partial^2 w^*}{\partial x \partial y}$ are small compared to $N_{x\phi} \dfrac{\partial^2 w^*}{\partial x \partial y}$.

iii- $N_\phi^* \dfrac{\partial^2 w}{\partial y^2}$ and $N_\phi^* \dfrac{\partial^2 w^*}{\partial y^2}$ are small compared to $N_\phi \dfrac{\partial^2 w^*}{\partial y^2}$.

The buckling limit is reached if a solution of equations 60, 61 and 63 can be obtained, for which the displacements u^*, v^* and w^* are not all zero, are compatible, and satisfy the boundary conditions.

Equations (60) and (61) are written:

$$\frac{\partial}{\partial x}\left(D_x \frac{\partial u^*}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{x\phi} \frac{\partial v^*}{\partial x} + D_{x\phi} \frac{\partial u^*}{\partial y}\right) = 0 \tag{64}$$

$$\frac{\partial}{\partial x}\left(D_{x\phi} \frac{\partial v^*}{\partial x} + D_{x\phi} \frac{\partial u^*}{\partial y}\right) + \frac{\partial}{\partial y}\left(D_\phi \frac{\partial v^*}{\partial y} - D_\phi \frac{w^*}{R}\right) = 0 \tag{65}$$

Equations (64) and (65) are satisfied identically by the introduction of a stress function Φ, such that:

$$D_x \frac{\partial u^*}{\partial x} = -\frac{\partial^2 \Phi}{\partial y^2} \tag{66a}$$

$$\left(D_{x\phi} \frac{\partial v^*}{\partial x} + D_{x\phi} \frac{\partial u^*}{\partial y}\right) = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{66b}$$

$$D_\phi \frac{\partial v^*}{\partial y} - D_\phi \frac{w^*}{R} = \frac{\partial^2 \Phi}{\partial x^2} \tag{66c}$$

The displacements u^*, v^* are next eliminated between equations (66a) to (66c) and between equations (63) and (66c) as follows:

Applying $D_\phi \frac{\partial^2}{\partial y^2}$ to Eq. (66a), $-\frac{D_x D_\phi}{D_{x\phi}} \frac{\partial^2}{\partial x \partial y}$ to Eq. (66b), $D_x \frac{\partial^2}{\partial x^2}$ to Eq. (66c) and adding the compatibility equation is obtained as

$$\nabla^4 \Phi + \frac{D_x D_\phi}{R} \frac{\partial^2 w^*}{\partial x^2} = 0 \tag{67}$$

in which

$$\nabla^4 = D_x \frac{\partial^4}{\partial x^4} + \frac{D_x D_\phi}{D_{x\phi}} \frac{\partial^4}{\partial x^2 \partial y^2} + D_\phi \frac{\partial^4}{\partial y^4}$$

and Eq. (63) becomes:

$$B_x \frac{\partial^4 w^*}{\partial x^4} + 2B_{x\phi} \frac{\partial^4 w^*}{\partial x^2 \partial y^2} + B_\phi \left(\frac{\partial^4 w^*}{\partial y^4} + \frac{2}{R^2} \frac{\partial^2 w^*}{\partial y^2} + \frac{w^*}{R^4} \right) - \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2}$$

$$- N_x \frac{\partial^2 w^*}{\partial x^2} - 2N_{x\phi} \frac{\partial^2 w^*}{\partial x \partial y} - N_\phi \frac{\partial^2 w^*}{\partial y^2} = 0 \tag{68}$$

Eqs. (67) and (68) are the basic governing differential equations of stability.

IV. 2. EXPRESSION OF THE MEMBRANE STRESS RESULTANTS.

Under lateral loads, the membrane stress resultants N_x, N_ϕ and $N_{x\phi}$ will vary from one point, on the middle surface of the shell, to another. The expressions of these membrane forces originating from the bending solution can be found using the following relations:

from (49h)

$$N_x = \iint \left(\frac{B_\phi}{R^5} w^{\bullet\bullet\bullet\bullet} + \frac{B_\phi}{R^5} w^{\bullet\bullet\bullet\bullet\bullet\bullet} + \frac{2B_{x\phi}}{R^3} w^{''\bullet\bullet\bullet\bullet} + \frac{B_x}{R} w^{''''\bullet\bullet} \right) dx\, dx \tag{69a}$$

from (49f)

$$N_\phi = \frac{B_\phi}{R^3} w^{\bullet\bullet} + \frac{B_\phi}{R^3} w^{\bullet\bullet\bullet\bullet} + 2\frac{B_{x\phi}}{R} w^{''\bullet\bullet} + B_x R w^{''''} \tag{69b}$$

from (49g)

$$N_{x\phi} = \int \left(- \frac{B_\phi}{R^4} w^{\bullet\bullet\bullet} - \frac{B_\phi}{R^4} w^{\bullet\bullet\bullet\bullet\bullet} - 2\frac{B_{x\phi}}{R^2} w^{''\bullet\bullet\bullet} - B_x w^{''''} \right) dx \tag{69c}$$

in which the previous notations are modified to:

$$\frac{\partial(\ )}{\partial x} = (\ )' \quad , \quad \frac{\partial(\ )}{\partial \phi} = (\ )^{\bullet} \quad , \quad y = R\phi$$

and

$$w = \Big[e^{\alpha_1 \frac{y}{R}} \left(A \cos\beta_1 \frac{y}{R} + B \sin\beta_1 \frac{y}{R} \right) + e^{-\alpha_1 \frac{y}{R}} \left(C \cos\beta_1 \frac{y}{R} + D \sin\beta_1 \frac{y}{R} \right) + e^{\alpha_2 \frac{y}{R}} \left(E \cos\beta_2 \frac{y}{R} + F \sin\beta_1 \frac{y}{R} \right) + e^{-\alpha_2 \frac{y}{R}} \left(G \cos\beta_2 \frac{y}{R} + H \sin\beta_2 \frac{y}{R} \right) \Big] \sin \frac{\pi}{L} x \tag{70}$$

Substitution of equation (70) into equations (69) gives the expressions for the stress resultants due to the bending effect, to which the membrane stress resultants should be added to get the total stress resultants as given in Appendix IV. Thus, for a uniform snow load Eqs.(44)

$$N_x = P\Big[-\frac{12p}{R\pi}\left(\frac{L}{\pi}\right)^2 \cos 2(\phi_e-\phi) + e^{\alpha_1\phi}(A_{N_x}\cos\beta_1\phi + B_{N_x}\sin\beta_1\phi) + e^{-\alpha_1\phi}(C_{N_x}\cos\beta_1\phi + D_{N_x}\sin\beta_1\phi) + e^{\alpha_2\phi}\cdot(E_{N_x}\cos\beta_2\phi + F_{N_x}\sin\beta_2\phi) + e^{-\alpha_2\phi}(G_{N_x}\cos\beta_2\phi + H_{N_x}\sin\beta_2\phi)\Big]\sin\frac{\pi x}{L} \qquad (71)$$

$$N_\phi = P\Big[-\frac{4Rp}{\pi}\cos^2(\phi_e-\phi) + e^{\alpha_1\phi}(A_{N_\phi}\cos\beta_1\phi + B_{N_\phi}\sin\beta_1\phi) + e^{-\alpha_1\phi}(C_{N_\phi}\cos\beta_1\phi + D_{N_\phi}\sin\beta_1\phi) + e^{\alpha_2\phi}(E_{N_\phi}\cos\beta_2\phi + F_{N_\phi}\sin\beta_2\phi) + e^{-\alpha_2\phi}(G_{N_\phi}\cos\beta_2\phi + H_{N_\phi}\sin\beta_2\phi)\Big]\sin\frac{\pi x}{L} \qquad (72)$$

$$N_{x\phi} = -P\Big[\frac{6Lp}{\pi^2}\sin 2(\phi_e-\phi) + e^{\alpha_1\phi}(A_{N_{x\phi}}\cos\beta_1\phi + B_{N_{x\phi}}\sin\beta_1\phi) + e^{-\alpha_1\phi}(C_{N_{x\phi}}\cos\beta_1\phi + D_{N_{x\phi}}\sin\beta_1\phi) + e^{\alpha_2\phi}(E_{N_{x\phi}}\cos\beta_2\phi + F_{N_{x\phi}}\sin\beta_2\phi) + e^{-\alpha_2\phi}(G_{N_{x\phi}}\cos\beta_2\phi + H_{N_{x\phi}}\sin\beta_2\phi)\Big]\cdot\cos\frac{\pi x}{L} \qquad (73)$$

in which

p = intensity of lateral load;

P = multiplier to account for different values of the load.

Eqs. (71),(72) and (73) can be written in the following form:

$$N_x = P\ (\ \overline{N}_x\ )\ \sin \frac{\pi}{L} x \tag{74a}$$

$$N_\phi = P\ (\ \overline{N}_\phi\ )\ \sin \frac{\pi}{L} x \tag{74b}$$

$$N_{x\phi} = -P\ (\ \overline{N}_{x\phi})\ \cos \frac{\pi}{L} x \tag{74c}$$

in which $\overline{N}_x$, $\overline{N}_\phi$ and $\overline{N}_{x\phi}$ are functions of ϕ only. Also the origin of the axes x, ϕ and z in Fig. 11 is transferred to coincide with one corner of the shell.

IV. 3. SOLUTION OF THE BASIC STABILITY EQUATIONS

For a shell simply supported on four edges, the deflection due to buckling w^* may be represented by the following series expression:

$$w^* = \sin \frac{m\pi x}{L} \sum_{n=1,2..}^{\infty} a_{mn} \sin \frac{n\pi y}{2\beta} \quad , \ m= 1,2,3.. \tag{75}$$

for which the origin is taken at the corner of the shell and 2β is the length of the curved side of the shell.

In expression 75, the distribution of w^* in the x direction is assumed to agree with the mode of vibration of shells (4).

Substitution of expression 75 into Eq. (67) yields:

$$\nabla^4\Phi - \frac{D_x D_\phi}{R} \left(\frac{m\Pi}{L}\right)^2 \sin\frac{m\Pi x}{L} \sum_{n=1}^{\infty} a_{mn} \sin\frac{n\Pi y}{2\beta} = 0 \qquad (76)$$

The particular solution of (76) is

$$\Phi = \frac{D_x D_\phi}{R} \left(\frac{m\Pi}{L}\right)^2 \sum_{n=1}^{\infty} \frac{a_{mn} \sin\frac{n\Pi y}{2\beta} \sin\frac{m\Pi x}{L}}{\left(\frac{n\Pi}{2\beta}\right)^4 D_\phi + \frac{D_x D_\phi}{D_{x\phi}}\left(\frac{n\Pi}{2\beta}\right)^2\left(\frac{m\Pi}{L}\right)^2 + D_x\left(\frac{m\Pi}{L}\right)^4} \qquad (77)$$

The homogeneous solution can be obtained by substituting $\Phi = e^{\lambda y} \sin\frac{m\Pi x}{L}$ into the homogeneous equation, thus

$D_\phi \lambda^4 - \frac{D_x D_\phi}{D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 \lambda^2 + D_x\left(\frac{m\Pi}{L}\right)^4 = 0$, putting $Z = \lambda^2$, then

$D_\phi Z^2 - \frac{D_x D_\phi}{D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 Z + D_x\left(\frac{m\Pi}{L}\right)^4 = 0$, from which

$$Z = \frac{D_x}{2D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 \pm \frac{D_x}{2D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 \sqrt{1 - \frac{4\, D_{x\phi}^2}{D_x D_\phi}} \qquad (78)$$

putting $\chi = \frac{2\, D_{x\phi}}{\sqrt{D_x D_\phi}}$, then

$$Z = \lambda^2 = \frac{D_x}{2D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 \left(1 \pm \sqrt{1 - \chi^2} \right)$$

and,

$$\lambda = \pm\sqrt{\frac{D_x}{2D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 \left(1 \pm \sqrt{1 - \chi^2}\right)}$$

For corrugated sheet shells χ is less than one (1), hence

$\chi^2 < 1$, and $\lambda = \pm n_1$ or $\pm n_2$, in which

$$n_1 = \sqrt{\frac{D_x}{2D_{x\phi}} \left(\frac{m\pi}{L}\right)^2 \left(1 + \sqrt{1 - \chi^2}\right)} \tag{79a}$$

$$n_2 = \sqrt{\frac{D_x}{2D_{x\phi}} \left(\frac{m\pi}{L}\right)^2 \left(1 - \sqrt{1 - \chi^2}\right)} \tag{79b}$$

and thus:

$$\Phi = \left(A\, e^{n_1 y} + B\, e^{-n_1 y} + C\, e^{n_2 y} + D\, e^{-n_2 y} \right) \sin \frac{m\pi x}{L} \tag{80a}$$

or

$$\Phi = \left(A \cosh n_1 y + B \sinh n_1 y + C \cosh n_2 y + D \sinh n_2 y \right) \cdot \sin \frac{m\pi x}{L} \tag{80b}$$

in which A, B, C, D are arbitrary constants to be determined by satisfying the boundary conditions.

Adding the particular and homogeneous solutions of Φ yields

$$\Phi = \left[\frac{D_x D_\phi}{R} \left(\frac{m\pi}{L}\right)^2 \sum_{n=1}^{\infty} \frac{a_{mn} \sin \frac{n\pi y}{2\beta}}{\left(\frac{n\pi}{2\beta}\right)^4 D_\phi + \frac{D_x D_\phi}{D_{x\phi}} \left(\frac{n\pi}{2\beta}\right)^2 \left(\frac{m\pi}{L}\right)^2 + D_x \left(\frac{m\pi}{L}\right)^4} + A \cosh n_1 y + B \sinh n_1 y + C \cosh n_2 y + D \sinh n_2 y \right] \cdot \sin \frac{m\pi x}{L} \tag{81}$$

Boundary Conditions

For a simply supported shell on the longitudinal edges, the boundary conditions are:

1- at $y = 0$ and $y = 2\beta$: $w^* = \dfrac{\partial^2 w^*}{\partial y^2} = 0$ (82)

2- at $y = 0$ and $y = 2\beta$: $u^* = v^* = 0$ (83)

Boundary condition (1) is already satisfied by expression 75. It is necessary next to express the remaining boundary condition (2) in terms of Φ and w^*. This is achieved by using Eqs. (66).

In order that condition (2) be satisfied, the following conditions should be satisfied (7):

$$\frac{\partial u^*}{\partial x} = 0 \text{ at the boundary} \tag{84}$$

and

$$\frac{\partial^2 v^*}{\partial x^2} = 0 \text{ at the boundary} \tag{85}$$

Eq. (66a) and Eq. (84) yield:

$$\frac{\partial^2 \Phi}{\partial y^2} = 0 \text{ at } \; y = 0 \text{ and } \; y = 2\beta \tag{86}$$

Applying $D_{x\phi}\dfrac{\partial}{\partial y}$ to (66a) gives

$$D_x D_{x\phi}\frac{\partial^2 u^*}{\partial x \partial y} = D_{x\phi}\frac{\partial^3 \Phi}{\partial y^3} \tag{87}$$

Applying $D_x \dfrac{\partial}{\partial x}$ to (66b) gives

$$D_x D_{x\phi}\frac{\partial^2 v^*}{\partial x^2} + D_x D_{x\phi}\frac{\partial^2 u^*}{\partial x \partial y} = - D_x\frac{\partial^3 \Phi}{\partial x^2 \partial y} \tag{88}$$

Subtracting (87) from (88) and using (85), the following condition is obtained:

$$D_x \frac{\partial^3 \Phi}{\partial x^2 \partial y} + D_{x\phi} \frac{\partial^3 \Phi}{\partial y^3} = 0 \quad \text{at } y = 0 \text{ and } y = 2\beta \tag{89}$$

The boundary condition, given by Eq. (86), yields the following two equations:

$$A\, n_1^2 + C\, n_2^2 = 0 \tag{90}$$

and

$$A\, n_1^2 \cosh 2n_1\beta + B\, n_1^2 \sinh 2n_1\beta + C\, n_2^2 \cosh 2n_2\beta + D\, n_2^2 \sinh 2n_2\beta = 0 \tag{91}$$

Putting:

$$\bar{T}_3 = \sum_n - \frac{a_{mn} \left(\frac{n\pi}{2\beta}\right)^3}{\left(\frac{n\pi}{2\beta}\right)^4 D_\phi + \frac{D_x D_\phi}{D_{x\phi}} \left(\frac{n\pi}{2\beta}\right)^2 \left(\frac{m\pi}{L}\right)^2 + \left(\frac{m\pi}{L}\right)^4 D_x}$$

$$T_1 = \sum_n \frac{a_{mn} \left(\frac{n\pi}{2\beta}\right)}{\left(\frac{n\pi}{2\beta}\right)^4 D_\phi + \frac{D_x D_\phi}{D_{x\phi}} \left(\frac{n\pi}{2\beta}\right)^2 \left(\frac{m\pi}{L}\right)^2 + \left(\frac{m\pi}{L}\right)^4 D_x}$$

then, the boundary condition, given by Eq. (89), gives at $y = 0$ the following :

$$\left(\frac{m\pi}{L}\right)^2 \frac{D_x D_\phi}{R} \left[D_{x\phi} T_3 - \left(\frac{m\pi}{L}\right)^2 D_x T_1 \right] + B \left[D_{x\phi} n_1^3 - \left(\frac{m\pi}{L}\right)^2 n_1 D_x \right] + D \left[n_2^3 D_{x\phi} - \left(\frac{m\pi}{L}\right)^2 n_2 D_x \right] = 0 \tag{92}$$

At $y = 2\beta$, the following equation is obtained

$$\left(\frac{m\Pi}{L}\right)^2 \frac{D_x D_\phi}{R} \left[D_{x\phi} T_3 - \left(\frac{m\Pi}{L}\right)^2 D_x T_1\right] + A \left[D_{x\phi} n_1^3 - \left(\frac{m\Pi}{L}\right)^2 D_x n_1\right]$$

$$\cdot \sinh 2n_1\beta + B \left[D_{x\phi} n_1^3 - \left(\frac{m\Pi}{L}\right)^2 D_x n_1\right] \cosh 2 n_1\beta + C \left[D_{x\phi} n_2^3\right.$$

$$\left. - \left(\frac{m\Pi}{L}\right)^2 D_x n_2^3\right] \sinh 2n_2\beta + D \left[D_{x\phi} n_2^3 - \left(\frac{m\Pi}{L}\right)^2 D_x n_2\right] \cosh 2n_2\beta = 0 \tag{93}$$

Equations (90), (91), (92) and (93) are to be solved for A, B, C and D, then substituted into the expression of Φ (Eq. 81) giving:

$$\Phi = \frac{D_x D_\phi}{R} \left(\frac{m\Pi}{L}\right)^2 \sum_{n=1} \frac{a_{mn}}{T(n)} \sin \frac{n\Pi}{2\beta} y \sin \frac{m\Pi}{L} x + \left[- \frac{n_2^2}{n_1^2} \cdot \right.$$

$$\bar{C}\left(\frac{m\Pi}{L}\right)^2 \frac{D_x D_\phi}{R} \cosh n_1 y \left\{\sum_n D_{x\phi} \left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x \left(\frac{m\Pi}{L}\right)^2 \left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)}\right\}$$

$$+ \bar{B}\left(\frac{m\Pi}{L}\right)^2 \frac{D_x D_\phi}{R} \sinh n_1 y \left\{\sum_n D_{x\phi} \left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x \left(\frac{m\Pi}{L}\right)^2 \left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)}\right\}$$

$$+ \bar{C}\left(\frac{m\Pi}{L}\right)^2 \frac{D_x D_\phi}{R} \cosh n_2 y \left\{\sum_n D_{x\phi} \left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x \left(\frac{m\Pi}{L}\right)^2 \left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)}\right\}$$

$$\left. + \bar{D}\left(\frac{m\Pi}{L}\right)^2 \frac{D_x D_\phi}{R} \sinh n_2 y \left\{\sum_n D_{x\phi} \left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x \left(\frac{m\Pi}{L}\right) \left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)}\right\} \right]$$

$$\cdot \sin \frac{m\Pi}{L} x \tag{94}$$

in which

$$n_{\substack{1\\2}} = \sqrt{\frac{D_x}{2D_{x\phi}} \left(\frac{m\Pi}{L}\right)^2 \left(1 \pm \sqrt{1 - \mathcal{K}^2}\right)} \quad , \quad \mathcal{K} = \frac{2\, D_{x\phi}}{\sqrt{D_x D_\phi}} \quad ,$$

$$T(n) = \left(\frac{n\Pi}{2\beta}\right)^4 D_\phi + \frac{D_x D_\phi}{D_{x\phi}}\left(\frac{n\Pi}{2\beta}\right)^2\left(\frac{m\Pi}{L}\right)^2 + \left(\frac{m\Pi}{L}\right)^4 D_x \ ,$$

$$\overline{D} = \frac{(1 - \cosh 2n_1\beta)n_2^2\, Y_3 + \left(\frac{n_1^2}{Y_1} \sin 2n_1\beta\right) Y_4}{Y_2 n_2^2 Y_3^2 - Y_4 Y_5} \ ,$$

$$Y_1 = D_{x\phi}\, n_1^3 - \left(\frac{m\Pi}{L}\right)^2 D_x n_1 \ ,$$

$$Y_2 = D_{x\phi}\, n_2^3 - \left(\frac{m\Pi}{L}\right)^2 D_x n_2 \ ,$$

$$Y_3 = \cosh 2n_2\beta - \cosh 2n_1\beta \ ,$$

$$Y_4 = Y_2 \sinh 2n_2\beta - \frac{n_2^2}{n_1^2} Y_1 \sinh 2n_1\beta \ ,$$

$$Y_5 = n_2^2 \sinh 2n_2\beta - n_1^2 \frac{Y_2}{Y_1} \sinh 2n_1\beta \ ,$$

$$\overline{C} = -\frac{n_1^2 \sinh 2n_1\beta}{n_2^2\, Y_1 Y_3} - \overline{D}\,\frac{Y_5}{n_2^2\, Y_3} \ ,$$

$$\overline{B} = \frac{1}{Y_1}\left(1 - Y_2\overline{D}\right) \ .$$

The expressions for w^* and $\bar{\Phi}$ given by equations (75) and (94) satisfy all the boundary conditions and all the basic equations except equation (74). The Galerkin method is applied by expanding the unknown functions in terms of a suitable set of functions in x and y, each satisfying the boundary conditions but not in general the equation of equilibrium. Then, a simultaneous set of linear algebraic equations in the unknown coefficients a_{mn} is obtained using the original expansion functions as weighting functions, as will be shown in the following derivation. A solution of any desired degree of accuracy may be obtained by the Galerkin method. Therefore, the expressions of w^* and Φ are substituted in equation (68) giving:

$$
\begin{aligned}
& B_x\left(\frac{m\Pi}{L}\right)^4 \sin\frac{m\Pi x}{L} \sum_n a_{mn} \sin\frac{n\Pi y}{2\beta} + 2B_{x\phi}\left(\frac{m\Pi}{L}\right)^2 \sin\frac{m\Pi x}{L} \sum_n a_{mn}\left(\frac{n\Pi}{2\beta}\right)^2 \sin\frac{n\Pi y}{2\beta} \\
& + B_\phi \sin\frac{m\Pi x}{L} \sum_n a_{mn}\left(\frac{n\Pi}{2\beta}\right)^4 \sin\frac{n\Pi y}{2\beta} - B_\phi \frac{2}{R^2} \sin\frac{m\Pi x}{L} \sum_n a_{mn}\left(\frac{n\Pi}{2\beta}\right)^2 \sin\frac{n\Pi y}{2\beta} \\
& + \frac{B_\phi}{R^4} \sin\frac{m\Pi x}{L} \sum_n a_{mn} \sin\frac{n\Pi y}{2\beta} + \frac{D_x D_\phi}{R^2}\left(\frac{m\Pi}{L}\right)^4 \sin\frac{m\Pi x}{L} \sum_n \frac{a_{mn}}{T(n)} \sin\frac{n\Pi y}{2\beta} - \frac{n_2^2}{n_1^2} \cdot \\
& \overline{C}\left(\frac{m\Pi}{L}\right)^4 \frac{D_x D_\phi}{R^2} \cosh n_1 y \; \sin\frac{m\Pi x}{L} \left\{ \sum_n D_{x\phi}\left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x\left(\frac{m\Pi}{L}\right)^2\left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)} \right\} \\
& + \overline{B}\left(\frac{m\Pi}{L}\right)^4 \frac{D_x D_\phi}{R^2} \sinh n_1 y \; \sin\frac{m\Pi x}{L} \left\{ \sum_n D_{x\phi}\left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x\left(\frac{m\Pi}{L}\right)^2\left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)} \right\} \\
& + \overline{C}\left(\frac{m\Pi}{L}\right)^4 \frac{D_x D_\phi}{R^2} \cosh n_2 y \; \sin\frac{m\Pi x}{L} \left\{ \sum_n D_{x\phi}\left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x\left(\frac{m\Pi}{L}\right)^2\left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)} \right\} \\
& + \overline{D}\left(\frac{m\Pi}{L}\right)^4 \frac{D_x D_\phi}{R^2} \sinh n_2 y \; \sin\frac{m\Pi x}{L} \left\{ \sum_n D_{x\phi}\left(\frac{n\Pi}{2\beta}\right)^3 \frac{a_{mn}}{T(n)} + \sum_n D_x\left(\frac{m\Pi}{L}\right)^2\left(\frac{n\Pi}{2\beta}\right) \frac{a_{mn}}{T(n)} \right\} \\
& + N_x \left(\frac{m\Pi}{L}\right)^2 \sin\frac{m\Pi x}{L} \sum_n a_{mn} \sin\frac{n\Pi}{2\beta}y + 2N_{x\phi}\left(\frac{m\Pi}{L}\right) \cos\frac{m\Pi x}{L} \sum_n a_{mn}\left(\frac{n\Pi}{2\beta}\right) \cdot \\
& \cos\frac{n\Pi}{2\beta}y + N_\phi \sin\frac{m\Pi x}{L} \sum_n a_{mn}\left(\frac{n\Pi}{2\beta}\right)^2 \sin\frac{n\Pi}{2\beta}y = 0
\end{aligned}
\tag{95}
$$

Eqs. (74) are substituted into Eq. (95), and in order to drop the functions of x and y, the equation is multiplied by $\sin\frac{m\pi x}{L}$ and integrated from x=0 to x=L. Then, multiplied by $\sin\frac{s\pi}{2\beta}y$ (s=1,2,3,...) and integrated from y=0 to y=2β, using the following integrations:

$$\int_0^L \sin^2 \frac{m\pi x}{L}\, dx = \frac{L}{2} , \quad \int_0^L \sin\frac{m\pi x}{L} \cos\frac{\pi x}{L} \cos\frac{m\pi x}{L}\, dx = \frac{2\, m\, L}{\pi(4m^2-1)} ,$$

$$\int_0^L \sin\frac{\pi x}{L} \sin^2 \frac{m\pi x}{L}\, dx = \frac{4\, m^2\, L}{\pi(4m^2-1)} ,$$

$$\int_0^{2\beta} \sin\frac{n\pi}{2\beta}y \sin\frac{s\pi}{2\beta}y\, dy = \begin{matrix} 0 & \text{for } n \neq s \\ \beta & \text{for } n = s \end{matrix} ,$$

$$\int_0^{2\beta} \cosh n_1 y \sin\frac{s\pi}{2\beta}y\, dy = \frac{(\frac{s\pi}{2})(1 - \cos s\pi \cosh 2n_1\beta)}{n_1^2 + (\frac{s\pi}{2})^2} ,$$

$$\int_0^{2\beta} \cosh n_2 y \sin\frac{s\pi}{2\beta}y\, dy = \frac{(\frac{s\pi}{2})(1 - \cos s\pi \cosh 2n_2\beta)}{n_2^2 + (\frac{s\pi}{2})^2} ,$$

$$\int_0^{2\beta} \sinh n_1 y \sin\frac{s\pi}{2\beta}y\, dy = \frac{-\frac{s\pi}{2} \cos s\pi \sinh 2n_1\beta}{n_1^2 + (\frac{s\pi}{2})^2} ,$$

$$\int_0^{2\beta} \sinh n_2 y \sin\frac{s\pi}{2\beta}y\, dy = \frac{-\frac{s\pi}{2} \cos s\pi \sinh 2n_2\beta}{n_2^2 + (\frac{s\pi}{2})^2}$$

(96a-g)

Noting that $\bar{N}_x$, $\bar{N}_\phi$ and $\bar{N}_{x\phi}$ are functions of y, the following simultaneous equations, for s=1,2,3,..., are obtained.

$$\Big\{ B_x\left(\frac{m\pi}{L}\right)^4 + 2B_{x\phi}\left(\frac{m\pi}{L}\right)^2\left(\frac{s\pi}{2\beta}\right) + B_\phi\left(\frac{s\pi}{2\beta}\right)^4 - \frac{2}{R^2} B_\phi\left(\frac{s\pi}{2\beta}\right)^4 + \frac{B_\phi}{R^4}$$

$$+ \frac{D_x D_\phi}{R^2 T(s)} \left(\frac{m\pi}{L}\right)^4 \Big\} \frac{L}{2} \beta a_{ms} + \sum_{n=1}^{\infty} a_{mn} \Big[\frac{D_x D_\phi}{R^2 T(n)} \frac{L}{2} \left(\frac{m\pi}{L}\right)^4 \cdot$$

$$\Big\{ - \frac{n_2^2}{n_1^2} \bar{C} \frac{\left(\frac{s\pi}{2}\right)}{n_1^2 + \left(\frac{s\pi}{2\beta}\right)^2} (1 - \cos s\pi \cosh 2n_1\beta) +$$

$$\bar{B} \frac{1}{n_1^2 + \left(\frac{s\pi}{2\beta}\right)^2}\left(- \frac{s\pi}{2\beta} \cos s\pi \sinh 2n_1\beta\right) + \bar{C} \frac{\left(\frac{s\pi}{2\beta}\right)}{n_2^2 + \left(\frac{s\pi}{2\beta}\right)^2}(1 -$$

$$\cos s\pi \cosh 2n_2\beta) + \bar{D} \frac{1}{n_2^2 + \left(\frac{s\pi}{2\beta}\right)^2}\left(- \frac{s\pi}{2\beta}\cos s\pi \sinh 2n_2\beta\right)\Big\}$$

$$\cdot \Big\{ D_{x\phi}\left(\frac{n\pi}{2\beta}\right)^3 + D_x\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{2\beta}\right)\Big\} \Big] + P\left[\left(\frac{m\pi}{L}\right)^2 \frac{4m^2 L}{\pi(4m^2-1)}\right] \cdot$$

$$\sum_{n=1}^{\infty} \left(\int_0^{2\beta} \bar{N}_x \sin \frac{n\pi}{2\beta}y \sin \frac{s\pi}{2\beta}y \, dy \right) a_{mn} + P\Big[2\left(\frac{m\pi}{L}\right) \cdot$$

$$\frac{2mL}{\pi(4m^2-1)} \Big] \sum_{n=1}^{\infty} \left(\int_0^{2\beta} \bar{N}_{x\phi} \cos \frac{n\pi}{2\beta}y \sin \frac{s\pi}{2\beta}y \, dy\right)\left(\frac{n\pi}{2\beta}\right) a_{mn}$$

$$+ P \left[\frac{4m^2 L}{\pi(4m^2-1)}\right] \sum_{n=1}^{\infty} \left(\int_0^{2\beta} \bar{N}_\phi \sin \frac{n\pi}{2\beta}y \sin \frac{s\pi}{2\beta}y \, dy\right)\left(\frac{n\pi}{2\beta}\right)^2 a_{mn}$$

$$= 0 \qquad (97)$$

$s = 1,2,3,\ldots\ldots$

$m = 1,2,3,\ldots\ldots$

IV. 4. DETERMINATION OF THE CRITICAL LOAD.

The simultaneous set of equations (97), after evaluating the involved integrals, can be written in the form:

$$\left[S \right] \left\{ a_{mn} \right\} + P \left[T \right] \left\{ a_{mn} \right\} = \left\{ 0 \right\} \tag{98}$$

Multiplying each side by $\left[T \right]^{-1}$, hence

$$\left(\left[U \right] + P \left[I \right] \right) \left\{ a_{mn} \right\} = \left\{ 0 \right\} \tag{99}$$

in which

$$\left[U \right] = \left[T \right]^{-1} \left[S \right] \quad , \quad \left[I \right] = \text{identity matrix}$$

A non trivial solution of Eq. (99) is obtained if:

$$\text{Det.} \left(\left[U \right] + P \left[I \right] \right) = 0 \tag{100}$$

Equation (100) is an eigenvalue problem, in which P is the negative of the eigenvalues of the matrix U .

The simultaneous set of equations (97) is infinite. It has to be truncated to a finite number of equations (N), according to the desired degree of accuracy.

A program was written using Fortran IV, G level and was run on the IBM 360/65 of the University of Windsor. This program evaluates the stability matrices and finds the eigenvalues, and eigenvectors of Eq. (100), using subroutines available from Argonne National Laboratory, Michigan. The eigenvalues are the critical loads and the eigenvectors indicate the corresponding relative values of the indetermined coefficients a_{mn}, from which the buckling mode can be determined.

The analysis of this type of shells shows that the limit of full arch action is not a simple function of its length to radius ratio as could be seen from Fig. 23. This limit is determined from the reaction of an equivalent arch (r) as mentioned before (Fig.15.). A suggested function is reported in Fig.24. in which, the limit of full arch action is reached at a single value of this function. The reported value of the function ($\eta = 0.5$) is valid for standard corrugation with gage 22. The linear buckling solution should be considered unvalid beyond this limit.

Tables 26-28 gives the buckling loads, in which the solution is carried out for different numbers of half waves in the longitudinal direction, m = 1,2,3,4&5. The solutions are carried out for half barrels with R=60 inchs and 120 inchs and for a shell having $\phi_e=67.46^\circ$ (in which the rise equals one third of the transverse span).

Fig.25. shows the buckling loads, it also shows the effect of change of radius of curvature of shell. It is observed that at full arch action the values of the buckling loads are inversily proportional to R^3, a result which agree with the expressions given in reference (25) for the buckling of circular arches.

The effect of the change of the central angle ϕ_e is shown in Fig.26. in which both sets of shells have the same horizontal projection.

The values of the critical loads are found to be convergent at a value of N=10, as shown in Fig.27.

The buckling mode under a uniform snow load is also shown in Fig.28.

IV. 5. BUCKLING LOADS FOR VARIOUS LOADING CONDITIONS.

To find the buckling load for any case of loading, the corresponding values of $\overline{N}_x$, $\overline{N}_\phi$, $\overline{N}_{x\phi}$ have to be substituted in equations (97). the expressions of the stress resultants given in App. IV include the following cases of loading:

i- Uniform load:

This type of loading is to be attributed to a uniformly distributed snow load.

ii- Radial one wave pressure:

This type of loading is that to be attributed to radial wind loading (17).

iii- Radial half wave pressure.

iv- Constant tangential load.

Other cases of loading can be studied by superposing the above loading conditions. Since the equations of the elastic analysis are linear, the expressions of the stress resultants can be superimposed and substituted in Eqs. (97). After evaluating the involved integrals the buckling loads can be determined. Tables 29, 30 and 31 include the buckling loads for half barrels made of the kinds of corrugated sheets given in table No.1 . They include the following cases of loading

i- Uniform Snow Load

ii- Wind Load

iii- Radial Pressure

iv- Unsymmetric Radial Pressure:

This is obtained by superposition of cases (ii)&(iii).

v- Snow Load plus Wind Load:

A factor of 0.75 was encountered in this case, according to the Canadian code of practice.

iv- Wind Load plus Tangential Load:

This is supposed to simulate the case of non uniform load (sine curve) on the horizontal projection. The reported values of the buckling load show insignificant changes due to the presence of the tangential load.

CHAPTER V

OBSERVATIONS AND CONCLUSIONS

In this thesis, proposed formulations for the determination of the linear behavior of cylindrical shells, made of corrugated sheets, are introduced. These formulations are examined in view of the other formulations, which result from using Flugge, Donnell, Vlasov and Schorer's formulations. They encounter more terms, which take into account the curved shape of the shell, than those encountered in the simplified formulations. Yet, it is observed that they are much more simpler than the precise formulations of Flugge.

A theoretical technique is also developed for the determination of the buckling loads of shells simply supported along the four edges and subjected to various types of loading.

From this study the following can be observed:

1. a. The error in the roots of the characteristic equation obtained using Donnell's approximations increase with the increase in the ratio of the length to radius of the shell L/R. On the contrary, Vlasov's approximations are valid, in

general, only for long shells, since the errors in the roots decrease with the increase of L/R. Schorer's approximations are found to be valid for a narrow range of relatively long shells. (Figs. 7,8,9,10 & Tables No. 4,5). However, for the case of shells made of standard corrugated sheets and dimpled sheets, the errors in the roots of the characteristic equation using Vlasov's approximations are very small. The errors using Donnell's and Schorer's approximations for these sheets, are close to each other and increase with the increase of L/R (Figs. 3,4,5,6 & Tables No. 2,3).

1. b. The errors in the roots of the characteristic equations, obtained using the proposed formulations, maintain negligible small values even with the increase in the ratio L/R. Accordingly, reliable analysis can be obtained by using the proposed governing equations. They also have the advantage of being easy to solve since they can be reduced to one differential equation of the 8th order in the deflection w, or to two simultaneous equations of the 4th order in w and a stress function as in the case of buckling solution.

2. An increase in the ratios of the membrane and bending rigidities in the circumferential direction, to those in the longitudinal direction is equivalent to the effect of a decrease in the ratio of L/R. The errors in the roots decrease for both cases, as could be seen by comparing Figs. 3 to 10.

3. The degree of accuracy required in the formulation of the linear theory of cylindrical shells should be established on the basis of the boundary conditions of the problem in hand. Higher degree of accuracy is needed when analysing shells supported along their four edges, while approximate formulations are adequate for shells with free longitudinal edges (Figs. 12 to 21 & Tables 6 to 25).

4. The overall buckling loads for shells simply supported along their four edges are calculated using the Galerkin method, (Figs. 25,26). For shells having the same ratio of length to width of the horizontal projection, the critical load of a uniform snow loading is found to increase with the decrease of the radius of shell. Also it decreases with the decrease of the central angle. The buckling mode under a uniform snow load tends to be unsymmetric with a single longitudinal

half wave (Fig. 28). The mode of buckling in the transverse direction approaches that of arches, near the limit of full arch action of the shell. However, for shorter shells the mode of buckling is not far from symmetry, due to the restraint provided by the end traverses. The assumed mode of buckling is found to provide a rapidly converged solution (Fig. 27).

5. The buckling loads of shells made of dimpled sheets are higher than those of shells made of standard corrugated sheets. Higher buckling loads are obtained for the case of shells made of standard corrugated sheets spot welded to plane sheets (Tables 29, 30&31). Accordingly, higher axial rigidity D_x , which can be reached by using the modified sheet corrugations reported in Table No.1, leads to more pronounced shell action.

In conclusion, it can now be stated that the problem of evaluating the overall buckling loads of cylindrical corrugated sheet shells supported along the four edges is now much more clearly defined. The method reported herein, provides a rapidly converged approach for the determination of such loads. These shells are usually thin and flexible, their ultimate load capacity is mainly governed by their overall buckling load before the onset of any plastic behaviour. With the aid of this method, curves similar to those reported here for the determination of buckling loads, can be drawn for shells made of a certain gage and type of corrugation with different radii. These curves can be used directly in the design of such shells.

The natural extension of this work, in the author's opinion, is the study of the nonlinear behaviour and the dynamic response of this type of structures.

REFERENCES

REFERENCES.

1. Abdel-Sayed, G., "Critical Shear Loading of Curved Panels of Corrugated Sheets", Journal of E.M. Division, ASCE, Dec. 1970, pp 895-912.

2. Abdel-Sayed, G., "Critical Shear Loading of Curved Panels of Corrugated Sheets with Restained Edges", Proc. of the 1st Specially Conference on Cold-Formed Steel Structures, Rolla, Missouri, Aug. 1971.

3. Abdel-Sayed, G. and El-Atrouzy, M.N., "Cylindrical Shells Made of Corrugated Sheets", Internat. Assoc. for Bridge and Struct. Engn., Vol. 32, 1972.

4. Arya, A.S., and Thakkar, S.K., "Free Vibration Characteristics of Shell Roofs", Journal of E.M. Division, ASCE, Vol. 97, No. EM1, February, 1971.

5. Batdorf, S.B., "A Simplified Method of Elastic Stability Analysis for Thin Cylindrical Shells", Report No. 874, NACA, 1947.

6. Batdorf, S.B., Stein, M., and Schilderout, M., "Critical Shear Stress of Curved Rectangular Panels", Technical Note No. 1348, NACA, 1947.

7. Brown, E.H., and Hopkins, H.G., "The Initial Buckling of Long and Slightly Bowed Panel Under Combined Shear and Normal Pressure", R. & M. 2766, British A.R.C, 1949

8. Bryan, E.R. and El-Dakhakhni, W.M., "Shear Flexebility and Strength of Corrugated Decks", Journal of Structural Division, ASCE, Vol. 94, No. ST 11, Nov. 1968, pp 2549.

9. Carbone, A., and Ramaswamy, G.S., "Design and Construction of Cylindrical Shells", National Building Organisation, Governement of India, New Delhi, Dec. 1960

10. Chu, K., and Krishnamoorhy, G., "Buckling of Open Cylindrical Shells", Journal of E.M. Division, ASCE, Vol. 93, No. EM2, April, 1967.

11. Chu, K., and Turula, P., "Buckling of Open Cylindrical Shells Under Lateral Loads", Journal of E.M. Division, ASCE, Vol. 96, No. EM6, December, 1970.

12. Donnell, L.H., "Stability of Thin Walled Tubes Under Torsion", NACA, Technical Report No. 479 (1933).

13. El-Atrouzy, M.N., "Cylindrical Shell Roofs Made of Corrugated Sheets", Ph. D. Thesis, University of Windsor, Windsor, Ontario, 1972.

14. Finsterwalder, U., "Die Theorie der Zylindrischen Schalengewolble System Zeiss-Dywidag und ihre Anwendung ouf die Grossmarkthalle in Budapest", Internat. Assoc. Bridge and Struct. Engn., Publ. 1, 127-52, 1932.

15. Flugge, W., "Die Stabilitat der Kreiszylinderschate", Ing., - Arch., Vol. 3, 1932, pp. 463-506.

16. Gerard, G., and Becker, H., "Buckling of Curved Plates and Shells", Handbook of Structural Stability, Part III, TN 3783, NACA, 1957.

17. Gibson, J.E., "The Design of Shell Roofs", 3rd. Edition, Spon's Civil Engineering Series, London, 1968.

18. Kornishin, M.S., and Mushtari, Kh.M., "The Stability of an Infinitly Long Shallow Cylindrical Strip Under the Action of Uniform Normal Pressure", Izv. KFAN SSSR, Seriya Fiz-mat. ; Tekhn. Nauk, No. 7, 1955.

19. Leggett, D.M.A., "The Initial Buckling of Slightly Curved Panels Under Combined Shear and Compression", R. & M. 1972, British A.R.C., 1943.

20. Lundgren, H., "Cylindrical Shells", Vol. 1, The Danish Technical Press, Copenhagen 1960.

21. Moe, J., and Trondheim, O.E., "ON the Theory of Cylindrical Shells, Explicit Solution of the Characteristic Equations and Discussion of the Accuracy of Various Shell Theories", Int. Assoc. for Bridge and Str. Engrg., Vol. 13, 1953, pp 230-295.

22. Ramaswamy, G.S., "Design and Construction of Concrete Shell Roofs", McGraw-Hill Book Company, 1968.

23. Schorer, H., "Line Load Action on Thin Cylindricals Shells", Proceedings of the ASCE, Vol. 61, p. 281, 1935.

24. Stein, M., and Mayers, J., "A Small-Deflection Theory for Curved Sandwich Plates", NACA Report 1008, 1952.

25. Timoshenko, and ~~Gere~~, "Theory of Elastic Stability", McGraw-Hill, 2nd. ed., 1961.

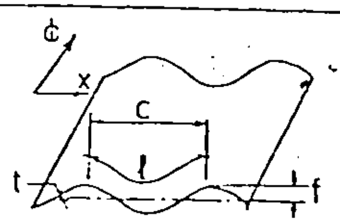
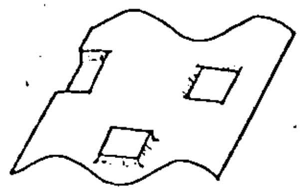
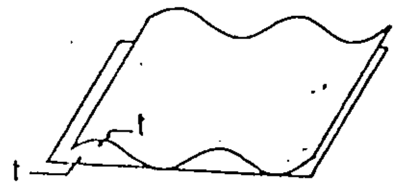
26. Timoshenko, S., and Woinowsky-Krieger, S., "Theory of Plates and Shells", McGraw-Hill, 2nd. ed., New York, 1959.

27. Van Der Neut, A., "General Instability of Orthogonally Stiffened Cylindrical Shells", NASA TND-1510, Dec. 1962, pp. 309-319.

28. Vlasov, V.Z., "General Theory of Shells and its Applications in Engineering", NASA Technical Translation F-99, April 1964.

TABLES

Table No. 1 - Rigidities of Corrugated Sheets

	 Standard Corrugated Sheets	 Dimpled Corrugated Sheets	 Corrugated Sheets Spot Welded To Plane Sheets
D_x	$\frac{E}{6(1-\mu^2)}(\frac{t}{f})^2\ t$	$\gamma\ \frac{E}{6(1-\mu^2)}(\frac{t}{f})^2\ t$	$\frac{E}{6(1-\mu^2)}(\frac{t}{f})^2\ t + Et$
D_ϕ	$\frac{\ell}{c}\ t\ E$	$\frac{\ell}{c}\ t\ E$	$E(\frac{\ell}{c}\ t + t)$
$D_{x\phi}$	$\rho\ \frac{Et}{2(1+\mu)}\ \frac{c}{\ell}$	$\rho\ \frac{Et}{2(1+\mu)}\ \frac{c}{\ell}$	$\rho\ \frac{E(\frac{c}{\ell}\ t + t)}{2(1+\mu)}$
B_x	$\frac{c}{\ell}\ \frac{Et^3}{12(1-\mu^2)}$	$\frac{c}{\ell}\ \frac{Et^3}{12(1-\mu^2)}$	$\frac{E(\frac{c}{\ell}\ t + t)^3}{12(1-\mu^2)}$
B_ϕ	$0.522\ Ef^2t$	$0.522\ Ef^2t$	$0.522\ Etf^2 + 1.044(.125)^2\ Et + (0.125)^2\ Et$
$B_{x\phi}$	$\frac{\ell}{c}\ \frac{Et^3}{12(1+\mu)}$	$\frac{\ell}{c}\ \frac{Et^3}{12(1+\mu)}$	$\frac{1.33\ Et\ f^2}{(1+\mu)}$

γ = factor of increase in strain rigidity, D_x, for dimpled sheets.

ρ = reduction factor of shear rigidity.

TABLE NO. 2 -ROOTS OF A STANDARD CORRUGATED SHELL WITH R = 240 IN

L/R		M1		M2		AVERAGE ERROR %	MAXIMUM ERROR %
1.0	FLUGGE	1.9319	-4.7493	4.6635	-1.9674	0.0	0.0
	PROPOSED(1)	1.9319	-4.7493	4.6635	-1.9674	0.24E-03	0.41E-02
	DONNELL	1.9509	-4.7039	4.7088	-1.9489	0.68E 00	0.99E 00
	VLASOV	1.9313	-4.7502	4.6628	-1.9675	0.13E-01	0.19E-01
	PROPOSED(2)	1.9412	-4.7266	4.6862	-1.9580	0.34E 00	0.49E 00
	SCHORER	1.9485	-4.7065	4.7065	-1.9485	0.64E 00	0.92E 00
2.0	FLUGGE	1.3517	-3.3915	3.2650	-1.4041	0.0	0.0
	PROPOSED(1)	1.3518	-3.3915	3.2649	-1.4041	0.13E-03	0.36E-02
	DONNELL	1.3789	-3.3271	3.3288	-1.3782	0.14E 01	0.20E 01
	VLASOV	1.3515	-3.3918	3.2647	-1.4042	0.62E-02	0.89E-02
	PROPOSED(2)	1.3649	-3.3595	3.2971	-1.3907	0.68E 00	0.98E 00
	SCHORER	1.3785	-3.3280	3.3280	-1.3785	0.13E 01	0.20E 01
3.0	FLUGGE	1.0921	-2.7958	2.6397	-1.1567	0.0	0.0
	PROPOSED(1)	1.0921	-2.7958	2.6397	-1.1567	0.81E-04	0.35E-02
	DONNELL	1.1258	-2.7168	2.7177	-1.1254	0.20E 01	0.31E 01
	VLASOV	1.0920	-2.7960	2.6395	-1.1568	0.41E-02	0.60E-02
	PROPOSED(2)	1.1080	-2.7565	2.6791	-1.1403	0.10E 01	0.15E 01
	SCHORER	1.1255	-2.7173	2.7173	-1.1255	0.20E 01	0.31E 01
4.0	FLUGGE	0.9356	-2.4444	2.2636	-1.0103	0.0	0.0
	PROPOSED(1)	0.9355	-2.4444	2.2636	-1.0104	0.56E-04	0.34E-02
	DONNELL	0.9749	-2.3529	2.3535	-0.9746	0.27E 01	0.42E 01
	VLASOV	0.9356	-2.4445	2.2635	-1.0104	0.31E-02	0.49E-02
	PROPOSED(2)	0.9538	2.3991	2.3091	-0.9913	0.14E 01	0.20E 01
	SCHORER	0.9747	-2.3532	2.3532	-0.9747	0.27E 01	0.42E 01
5.0	FLUGGE	0.8276	-2.2070	2.0046	-0.9112	0.0	0.0
	PROPOSED(1)	0.8275	-2.2070	2.0046	-0.9112	0.62E-04	0.35E-02
	DONNELL	0.8719	-2.1045	2.1050	-0.8718	0.34E 01	0.54E 01
	VLASOV	0.8276	-2.2071	2.0045	-0.9112	0.25E-02	0.43E-02
	PROPOSED(2)	0.8477	-2.1565	2.0557	-0.8899	0.17E 01	0.25E 01
	SCHORER	0.8718	-2.1048	2.1048	-0.8718	0.34E 01	0.53E 01
6.0	FLUGGE	0.7469	-2.0337	1.8118	-0.8384	0.0	0.0
	PROPOSED(1)	0.7469	-2.0337	1.8118	-0.8384	0.46E-04	0.36E-02
	DONNELL	0.7959	-1.9212	1.9216	-0.7958	0.41E 01	0.66E 01
	VLASOV	0.7469	-2.0338	1.8117	-0.8384	0.20E-02	0.38E-02
	PROPOSED(2)	0.7687	-1.9783	1.8678	-0.8150	0.21E 01	0.31E 01
	SCHORER	0.7959	-1.9214	1.9214	-0.7959	0.41E 01	0.66E 01
7.0	FLUGGE	0.6834	-1.9005	1.6607	-0.7821	0.0	0.0
	PROPOSED(1)	0.6834	-1.9005	1.6606	-0.7822	0.0	0.36E-02
	DONNELL	0.7369	-1.7787	1.7790	-0.7368	0.49E 01	0.78E 01
	VLASOV	0.6834	-1.9006	1.6606	-0.7822	0.18E-02	0.35E-02
	PROPOSED(2)	0.7065	-1.8406	1.7213	-0.7569	0.24E 01	0.37E 01
	SCHORER	0.7368	-1.7789	1.7789	-0.7368	0.48E 01	0.78E 01

TABLE NO. 3 -ROOTS OF A DIMPLED CORRUGATED SHELL WITH R = 240 IN

L/R		M1	M2	AVERAGE ERROR %	MAXIMUM ERROR %
	FLUGGE	2.7523 -6.6639	6.6425 -2.7633	0.0	0.0
	PROPOSED(1)	2.7523 -6.6687	6.6425 -2.7633	0.13E-04	0.93E-03
1.0	DONNELL	2.7660 -6.6366	6.6744 -2.7503	0.34E 00	0.50E 00
	VLASOV	2.7520 -6.6694	6.6419 -2.7634	0.67E-02	0.95E-02
	PROPOSED(2)	2.7591 -6.6527	6.6585 -2.7567	0.17E 00	0.25E 00
	SCHORER	2.7570 -6.6560	6.6560 -2.7570	0.14E 00	0.20E 00
	FLUGGE	1.9333 -4.7452	4.6679 -1.9653	0.0	0.0
	PROPOSED(1)	1.9333 -4.7452	4.6679 -1.9653	0.19E-04	0.82E-03
2.0	DONNELL	1.9525 -4.6997	4.7131 -1.9469	0.68E 00	0.99E 00
	VLASOV	1.9332 -4.7454	4.6677 -1.9654	0.34E-02	0.46E-02
	PROPOSED(2)	1.9427 -4.7225	4.6906 -1.9560	0.34E 00	0.49E 00
	SCHORER	1.9495 -4.7065	4.7065 -1.9495	0.58E 00	0.84E 00
	FLUGGE	1.5698 -3.8949	3.7911 -1.6128	0.0	0.0
	PROPOSED(1)	1.5698 -3.8949	3.7911 -1.6128	0.0	0.85E-03
3.0	DONNELL	1.5933 -3.8391	3.8464 -1.5903	0.10E 01	0.15E 01
	VLASOV	1.5697 -3.8950	3.7910 -1.6128	0.22E-02	0.30E-02
	PROPOSED(2)	1.5812 -3.8671	3.8189 -1.6013	0.51E 00	0.73E 00
	SCHORER	1.5917 -3.8428	3.8428 -1.5917	0.95E 00	0.14E 01
	FLUGGE	1.3522 -3.3900	3.2665 -1.4033	0.0	0.0
	PROPOSED(1)	1.3522 -3.3900	3.2665 -1.4034	0.13E-04	0.85E-03
4.0	DONNELL	1.3795 -3.3256	3.3303 -1.3776	0.14E 01	0.20E 01
	VLASOV	1.3522 -3.3901	3.2664 -1.4034	0.17E-02	0.22E-02
	PROPOSED(2)	1.3654 -3.3580	3.2986 -1.3900	0.68E 00	0.98E 00
	SCHORER	1.3785 -3.3280	3.3280 -1.3785	0.13E 01	0.19E 01
	FLUGGE	1.2030 -3.0470	2.9070 -1.2610	0.0	0.0
	PROPOSED(1)	1.2030 -3.0470	2.9070 -1.2610	0.15E-04	0.87E-03
5.0	DONNELL	1.2337 -2.9749	2.9783 -1.2323	0.17E 01	0.25E 01
	VLASOV	1.2030 -3.0471	2.9070 -1.2610	0.13E-02	0.18E-02
	PROPOSED(2)	1.2177 -3.0112	2.9430 -1.2460	0.85E 00	0.12E 01
	SCHORER	1.2330 -2.9766	2.9766 -1.2330	0.17E 01	0.25E 01
	FLUGGE	1.0923 -2.7951	2.6405 -1.1563	0.0	0.0
	PROPOSED(1)	1.0924 -2.7950	2.6405 -1.1563	0.32E-04	0.87E-03
6.0	DONNELL	1.1261 -2.7160	2.7186 -1.1250	0.20E 01	0.31E 01
	VLASOV	1.0923 -2.7951	2.6405 -1.1563	0.11E-02	0.14E-02
	PROPOSED(2)	1.1083 -2.7558	2.6799 -1.1399	0.10E 01	0.15E 01
	SCHORER	1.1255 -2.7173	2.7173 -1.1255	0.20E 01	0.30E 01
	FLUGGE	1.0059 -2.6002	2.4325 -1.0752	0.0	0.0
	PROPOSED(1)	1.0059 -2.6002	2.4325 -1.0752	0.0	0.85E-03
7.0	DONNELL	1.0425 -2.5147	2.5167 -1.0416	0.24E 01	0.36E 01
	VLASOV	1.0058 -2.6002	2.4325 -1.0752	0.96E-03	0.12E-02
	PROPOSED(2)	1.0230 -2.5578	2.4751 -1.0574	0.12E 01	0.18E 01
	SCHORER	1.0420 -2.5157	2.5157 -1.0420	0.24E 01	0.36E 01

TABLE NO. 4 -ROOTS OF A SPOT WELDED CORRUGATED SHELL WITH R = 240 IN

L/R		M1		M2		AVERAGE ERROR %	MAXIMUM ERROR %
	FLUGGE	3.9022	-8.7238	9.4376	-3.6058	0.0	0.0
	PROPOSED(1)	3.9068	-8.7219	9.4357	-3.6107	0.57E-03	0.14E 00
1.0	DONNELL	3.9149	-8.6990	9.4599	-3.5990	0.18E 00	0.33E 00
	VLASOV	3.8115	-8.9515	9.2060	-3.7060	0.18E 01	0.28E 01
	PROPOSED(2)	3.9108	-8.7104	9.4479	-3.6048	0.91E-01	0.22E 00
	SCHORER	3.7603	-9.0781	9.0781	-3.7603	0.28E 01	0.43E 01
	FLUGGE	2.6982	-6.3190	6.5216	-2.6142	0.0	0.0
	PROPOSED(1)	2.7015	-6.3175	6.5202	-2.6176	0.29E-03	0.13E 00
2.0	DONNELL	2.7141	-6.2849	6.5540	-2.6026	0.37E 00	0.59E 00
	VLASOV	2.6663	-6.3995	6.4297	-2.6496	0.90E 00	0.14E 01
	PROPOSED(2)	2.7077	-6.3013	6.5371	-2.6100	0.18E 00	0.35E 00
	SCHORER	2.6589	-6.4192	6.4192	-2.6589	0.11E 01	0.17E 01
	FLUGGE	2.1824	5.2096	5.2746	-2.1554	0.0	0.0
	PROPOSED(1)	2.1852	-5.2084	5.2735	-2.1592	0.18E-03	0.13E 00
3.0	DONNELL	2.2011	-5.1681	5.3146	-2.1404	0.55E 00	0.86E 00
	VLASOV	2.1653	-5.2533	5.2299	-2.1750	0.60E 00	0.91E 00
	PROPOSED(2)	2.1930	-5.1883	5.2941	-2.1492	0.27E 00	0.49E 00
	SCHORER	2.1710	-5.2412	5.2412	-2.1710	0.44E 00	0.72E 00
	FLUGGE	1.8796	-4.5393	4.5404	-1.8781	0.0	0.0
	PROPOSED(1)	1.8810	-4.5383	4.5394	-1.8804	0.15E-03	0.13E 00
4.0	DONNELL	1.8997	-4.4915	4.5867	-1.8603	0.73E 00	0.11E 01
	VLASOV	1.8677	-4.5676	4.5112	-1.8909	0.45E 00	0.69E 00
	PROPOSED(2)	1.8901	-4.5150	4.5631	-1.8702	0.37E 00	0.62E 00
	SCHORER	1.8801	-4.5391	4.5391	-1.8801	0.84E-02	0.11E 00
	FLUGGE	1.6723	-4.0791	4.0420	-1.6876	0.0	0.0
	PROPOSED(1)	1.6744	-4.0782	4.0411	-1.6897	0.98E-04	0.13E 00
5.0	DONNELL	1.6957	-4.0258	4.0939	-1.6675	0.92E 00	0.14E 01
	VLASOV	1.6646	-4.0993	4.0211	-1.6969	0.36E 00	0.55E 00
	PROPOSED(2)	1.6848	-4.0521	4.0676	-1.6784	0.46E 00	0.75E 00
	SCHORER	1.6816	-4.0599	4.0599	-1.6816	0.32E 00	0.56E 00
	FLUGGE	1.5203	-3.7385	3.6751	-1.5465	0.0	0.0
	PROPOSED(1)	1.5223	-3.7377	3.6743	-1.5495	0.11E-03	0.13E 00
6.0	DONNELL	1.5458	-3.6802	3.7320	-1.5244	0.11E 01	0.17E 01
	VLASOV	1.5146	-3.7538	3.6592	-1.5539	0.30E 00	0.47E 00
	PROPOSED(2)	1.5337	-3.7091	3.7033	-1.5361	0.55E 00	0.88E 00
	SCHORER	1.5351	-3.7061	3.7061	-1.5351	0.61E 00	0.97E 00
	FLUGGE	1.4023	-3.4736	3.3903	-1.4368	0.0	0.0
	PROPOSED(1)	1.4041	-3.4728	3.3895	-1.4385	0.64E-04	0.13E 00
7.0	DONNELL	1.4297	-3.4107	3.4519	-1.4127	0.13E 01	0.20E 01
	VLASOV	1.3979	-3.4857	3.3776	-1.4426	0.26E 00	0.41E 00
	PROPOSED(2)	1.4165	-3.4419	3.4208	-1.4252	0.64E 00	0.10E 01
	SCHORER	1.4213	-3.4312	3.4312	-1.4213	0.86E 00	0.13E 01

TABLE NO. 5 -ROOTS OF AN ISOTROPIC STEEL SHELL WITH R = 240 IN, T = 4 IN

L/R		M1		M2		AVERAGE ERROR %	MAXIMUM ERROR %
	FLUGGE	2.8736	-5.5672	6.8483	-2.3290	0.0	0.0
	PROPOSED(1)	2.8869	-5.5602	6.8427	-2.3455	0.86E-03	0.71E 00
1.0	DONNELL	2.8971	-5.5272	6.8786	-2.3280	0.39E 00	0.82E 00
	VLASOV	2.6875	-5.9193	6.5278	-2.4335	0.37E 01	0.63E 01
	PROPOSED(2)	2.8919	-5.5437	6.8607	-2.3367	0.20E 00	0.64E 00
	SCHORER	2.5754	-6.2175	6.2176	-2.5754	0.72E 01	0.12E 02
	FLUGGE	1.8996	-4.2069	4.5870	-1.7393	0.0	0.0
	PROPOSED(1)	1.9097	-4.2022	4.5836	-1.7503	0.90E-03	0.63E 00
2.0	DONNELL	1.9272	-4.1545	4.6332	-1.7281	0.78E 00	0.15E 01
	VLASOV	1.8452	-4.3279	4.4702	-1.7853	0.19E 01	0.29E 01
	PROPOSED(2)	1.9182	-4.1784	4.6085	-1.7390	0.39E 00	0.98E 00
	SCHORER	1.8211	-4.3965	4.3965	-1.8211	0.31E 01	0.47E 01
	FLUGGE	1.5123	-3.5210	3.6624	-1.4524	0.0	0.0
	PROPOSED(1)	1.5207	-3.5173	3.6589	-1.4612	0.87E-03	0.61E 00
3.0	DONNELL	1.5434	-3.4584	3.7191	-1.4352	0.12E 01	0.21E 01
	VLASOV	1.4853	-3.5854	3.5972	-1.4795	0.13E 01	0.19E 01
	PROPOSED(2)	1.5316	-3.4880	3.6892	-1.4479	0.59E 00	0.13E 01
	SCHORER	1.4869	-3.5894	3.5898	-1.4869	0.14E 01	0.24E 01
	FLUGGE	1.2895	-3.0951	3.1269	-1.2755	0.0	0.0
	PROPOSED(1)	1.2970	-3.0920	3.1238	-1.2830	0.88E-03	0.59E 00
4.0	DONNELL	1.3240	-3.0237	3.1930	-1.2538	0.16E 01	0.27E 01
	VLASOV	1.2732	-3.1363	3.0840	-1.2945	0.96E 00	0.15E 01
	PROPOSED(2)	1.3098	-3.0580	3.1587	-1.2679	0.79E 00	0.16E 01
	SCHORER	1.2877	-3.1088	3.1088	-1.2877	0.36E 00	0.96E 00
	FLUGGE	1.1400	-2.7992	2.7666	-1.1527	0.0	0.0
	PROPOSED(1)	1.1467	-2.7964	2.7638	-1.1594	0.87E-03	0.59E 00
5.0	DONNELL	1.1775	-2.7198	2.8409	-1.1273	0.20E 01	0.33E 01
	VLASOV	1.1290	-2.8287	2.7356	-1.1671	0.77E 00	0.12E 01
	PROPOSED(2)	1.1612	-2.7583	2.8028	-1.1427	0.98E 00	0.19E 01
	SCHORER	1.1519	-2.7806	2.7806	-1.1518	0.42E 00	0.10E 01
	FLUGGE	1.0305	-2.5788	2.5026	-1.0613	0.0	0.0
	PROPOSED(1)	1.0366	-2.5763	2.5000	-1.0673	0.85E-03	0.60E 00
6.0	DONNELL	1.0709	-2.4921	2.5843	-1.0327	0.24E 01	0.39E 01
	VLASOV	1.0226	-2.6010	2.4788	-1.0727	0.64E 00	0.11E 01
	PROPOSED(2)	1.0525	-2.5345	2.5426	-1.0492	0.12E 01	0.21E 01
	SCHORER	1.0514	-2.5383	2.5383	-1.0514	0.11E 01	0.20E 01
	FLUGGE	0.9456	-2.4068	2.2981	-0.9899	0.0	0.0
	PROPOSED(1)	0.9514	-2.4045	2.2957	-0.9954	0.86E-03	0.61E 00
7.0	DONNELL	0.9889	-2.3134	2.3865	-0.9585	0.27E 01	0.46E 01
	VLASOV	0.9397	-2.4243	2.2791	-0.9993	0.55E 00	0.95E 00
	PROPOSED(2)	0.9685	-2.3594	2.3417	-0.9759	0.14E 01	0.24E 01
	SCHORER	0.9734	-2.3500	2.3500	-0.9734	0.16E 01	0.29E 01

TABLES (6–15)

ANALYSIS OF HALF BARRELS

SUPPORTED ON FOUR SIDES.

N.B.

The following tables are computer outputs, which led to the following changes in the notations:

BX = B_x ;

BY = B_ϕ ;

BXY = $B_{x\phi}$;

DX = D_x ;

DY = D_ϕ ;

DXY = $D_{x\phi}$;

FY = ϕ ;

MFY = M_ϕ ;

NX = N_x ;

NY = N_ϕ ;

NXY = $N_{x\phi}$.

TABLE. 6— STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 2.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFAI&2 , BETAI&2 BY EQUATIONS & BY POLRT

0.2723647E 01	0.2723637E 01
0.1127078E 01	0.1127119E 01
0.1189391E 01	0.1189375E 01
0.2874085E 01	0.2874099E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	-0.000000	0.0	0.315362	0.0	-0.000003
0	20.00	-0.000000	0.0	0.304616	0.248509	-0.000002
0	40.00	-0.000000	0.0	0.273111	0.490083	-0.000002
0	60.00	-0.000000	0.0	0.222995	0.678939	-0.000002
0	80.00	-0.000000	0.0	0.157681	0.831527	-0.000001
0	100.00	-0.000000	0.0	0.081622	0.927448	-0.000001
0	120.00	-0.000000	0.0	0.000000	0.960165	-0.000000
30	0.0	-0.065653	-0.045782	-0.344143	0.0	-0.820616
30	20.00	-0.063416	-0.044222	-0.332416	0.249061	-0.792654
30	40.00	-0.056857	-0.039648	-0.298036	0.491149	-0.710674
30	60.00	-0.046424	-0.032373	-0.243346	0.680448	-0.580263
30	80.00	-0.032827	-0.022891	-0.172072	0.833375	-0.410309
30	100.00	-0.016992	-0.011849	-0.089071	0.929510	-0.212392
30	120.00	-0.000000	-0.000000	-0.000000	0.962299	-0.000001
60	0.0	0.009912	-0.605858	-0.805297	0.0	-0.326148
60	20.00	0.009574	-0.585214	-0.777857	0.197741	-0.315035
60	40.00	0.008584	-0.524689	-0.697407	0.362688	-0.282453
60	60.00	0.007009	-0.428407	-0.569431	0.512919	-0.230622
60	80.00	0.004956	-0.302930	-0.402649	0.628195	-0.163074
60	100.00	0.002565	-0.156808	-0.208427	0.700660	-0.084414
60	120.00	0.000000	-0.000001	-0.000001	0.725377	-0.000000
90	0.0	0.083106	-1.036896	-0.996184	0.0	0.517583
90	20.00	0.080274	-1.001554	-0.962240	-0.000000	0.499947
90	40.00	0.071971	-0.897970	-0.863721	-0.000001	0.449240
90	60.00	0.058765	-0.733189	-0.704409	-0.000001	0.365986
90	80.00	0.041553	-0.518444	-0.498093	-0.000001	0.258792
90	100.00	0.021509	-0.268367	-0.257832	-0.000002	0.133961
90	120.00	0.000000	-0.000001	-0.000001	-0.000002	0.000001

TABLE.7– STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)

L/R = 3.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06

DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02

ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT

0.2192900E 01	0.2192871E 01
0.9061298E 00	0.9062209E 00
0.9830292E 00	0.9829593E 00
0.2378553E 01	0.2378592E 01

FY	X	W	MX	NY	NXY	MFY
0	0.0	0.000001	-0.000001	0.038128	0.0	0.000006
0	30.00	0.000001	-0.000001	0.036829	0.300353	0.000006
0	60.00	0.000001	-0.000001	0.032020	0.580238	0.000005
0	90.00	0.000000	-0.000001	0.026961	0.820581	0.000004
0	120.00	0.000000	-0.000000	0.019064	1.005001	0.000003
0	150.00	0.000000	-0.000000	0.009868	1.120934	0.000002
0	180.00	0.000000	-0.000000	0.000000	1.160477	0.000000
30	0.0	-0.282958	-0.090127	-0.467692	0.0	-3.246351
30	30.00	-0.273316	-0.087056	-0.451756	0.301085	-3.135735
30	60.00	-0.245048	-0.078052	-0.405033	0.581653	-2.811422
30	90.00	-0.200081	-0.063729	-0.330708	0.822581	-2.295517
30	120.00	-0.141479	-0.045064	-0.233846	1.007452	-1.623178
30	150.00	-0.073235	-0.023327	-0.121048	1.123667	-0.840220
30	180.00	-0.000000	-0.000000	-0.000001	1.163306	-0.000004
60	0.0	0.064548	-1.107421	-0.785655	0.0	-1.096128
60	30.00	0.062349	-1.069686	-0.758885	0.224794	-1.058779
60	60.00	0.055900	-0.959055	-0.680397	0.434269	-0.949275
60	90.00	0.045642	-0.783065	-0.555542	0.614149	-0.775080
60	120.00	0.032274	-0.553711	-0.392828	0.752176	-0.548065
60	150.00	0.016706	-0.286622	-0.203343	0.838943	-0.283700
60	180.00	0.000000	-0.000001	-0.000001	0.869538	-0.000001
90	0.0	0.345030	-1.849008	-0.910370	0.0	0.967576
90	30.00	0.333274	-1.786004	-0.879350	0.000001	0.934606
90	60.00	0.298805	-1.601287	-0.788404	0.000001	0.837945
90	90.00	0.243973	-1.307446	-0.643729	0.000002	0.684180
90	120.00	0.172516	-0.924506	-0.455186	0.000002	0.483789
90	150.00	0.089301	-0.478560	-0.235622	0.000002	0.250428
90	180.00	0.000000	-0.000002	-0.000001	0.000002	0.000001

TABLE.8—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 4.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1872566E 01   0.1872511E 01
0.7722117E 00   0.7723889E 00
0.8612429E 00   0.8610911E 00
0.2087508E 01   0.2087587E 01

FY	X	W	NX	NY	NXY	VFY
0	0.0	-0.000001	0.0	-0.361006	0.0	0.000023
0	40.00	-0.000001	0.0	-0.348705	0.259381	0.000023
0	80.00	-0.000001	0.0	-0.312641	0.501085	0.000020
0	120.00	-0.000001	0.0	-0.255270	0.708642	0.000017
0	160.00	-0.000000	0.0	-0.180503	0.867905	0.000012
0	200.00	-0.000000	0.0	-0.093436	0.968222	0.000006
0	240.00	-0.000000	0.0	-0.000000	1.002171	0.000000
30	0.0	-0.718867	-0.036595	-0.643662	0.0	-7.329872
30	40.00	-0.694373	-0.035348	-0.621730	0.263647	-7.080112
30	80.00	-0.622557	-0.031692	-0.557427	0.509326	-6.347855
30	120.00	-0.508316	-0.025877	-0.455138	0.720296	-5.183002
30	160.00	-0.359434	-0.019298	-0.321831	0.882179	-3.664936
30	200.00	-0.186057	-0.009472	-0.166592	0.983943	-1.897115
30	240.00	-0.000000	-0.000000	-0.000000	1.018653	-0.000002
60	0.0	0.205179	-1.291770	-0.766439	0.0	-3.409294
60	40.00	0.198187	-1.247753	-0.740323	0.201991	-3.293037
60	80.00	0.177690	-1.118705	-0.663755	0.390004	-2.952456
60	120.00	0.145083	-0.913419	-0.541954	0.551549	-2.410670
60	160.00	0.102589	-0.645885	-0.383219	0.675507	-1.704601
60	200.00	0.053104	-0.334336	-0.198369	0.753430	-0.882359
60	240.00	0.000000	-0.000000	-0.000000	0.780008	-0.000001
90	0.0	0.910533	-2.233780	-0.807941	0.0	0.102610
90	40.00	0.879507	-2.157665	-0.780412	0.000001	0.099114
90	80.00	0.788544	-1.934509	-0.699698	0.000001	0.088863
90	120.00	0.643844	-1.579520	-0.571301	0.000002	0.072556
90	160.00	0.455266	-1.116890	-0.403971	0.000002	0.051305
90	200.00	0.235664	-0.578146	-0.209111	0.000002	0.026558
90	240.00	0.000000	-0.000001	-0.000000	0.000002	0.000000

TABLE.9—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING PROPOSAL NO.(I)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 5.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POCRT

0.1651310E 01	0.1651220E 01
0.6791869E 00	0.6794901E 00
0.7788484E 00	0.7785802E 00
0.1891916E 01	0.1892057E 01

FY	X	W	NX	NY	NXY	VFY
0	0.0	0.000001	0.0	-0.692781	0.0	0.000008
0	50.00	0.000001	0.0	-0.669175	0.188775	0.000008
0	100.00	0.000001	0.0	-0.599966	0.364696	0.000007
0	150.00	0.000001	0.0	-0.489870	0.515744	0.000006
0	200.00	0.000000	0.0	-0.346391	0.631654	0.000004
0	250.00	0.000000	0.0	-0.179305	0.704519	0.000002
0	300.00	0.000000	0.0	-0.000001	0.729372	0.000000
30	0.0	-1.435849	0.101764	-0.804161	0.0	-12.618704
30	50.00	-1.386924	0.098296	-0.776760	0.197612	-12.188733
30	100.00	-1.243482	0.088130	-0.696424	0.381758	-10.928119
30	150.00	-1.015299	0.071958	-0.568628	0.539887	-8.922774
30	200.00	-0.717926	0.050882	-0.402081	0.661224	-6.309363
30	250.00	-0.371626	0.026338	-0.208133	0.737500	-3.265968
30	300.00	-0.000002	0.000000	-0.000001	0.763516	-0.000016
60	0.0	0.492388	-1.204910	-0.778093	0.0	-8.653577
60	50.00	0.475610	-1.163854	-0.751580	0.159609	-8.358714
60	100.00	0.426420	-1.043483	-0.673848	0.308342	-7.494218
60	150.00	0.348171	-0.852001	-0.550195	0.436061	-6.119005
60	200.00	0.246194	-0.602456	-0.389047	0.534064	-4.326797
60	250.00	0.127440	-0.311854	-0.201386	0.595671	-2.239716
60	300.00	0.000001	-0.000002	-0.000001	0.616684	-0.000011
90	0.0	1.915908	-2.249645	-0.756681	0.0	-4.476614
90	50.00	1.850625	-2.172990	-0.730898	-0.000001	-4.324077
90	100.00	1.659225	-1.948250	-0.655305	-0.000001	-3.876862
90	150.00	1.354752	-1.590739	-0.535054	-0.000002	-3.165445
90	200.00	0.957956	-1.124825	-0.378341	-0.000003	-2.238311
90	250.00	0.495875	-0.582253	-0.195844	-0.000003	-1.158635
90	300.00	-0.000002	-0.000003	-0.000001	-0.000003	-0.000006

TABLE.10—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 6.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1486066E 01 0.1485933E 01
0.6092232E 00 0.6096983E 00
0.7184690E 00 0.7180457E 00
0.1749811E 01 0.1750038E 01

FY	X	W	NX	NY	MXY	MFY
0	0.0	0.0	0.000044	-0.935850	0.0	-0.000006
0	60.00	0.0	0.000042	-0.903961	0.132570	-0.000006
0	120.00	0.0	0.000038	-0.810470	0.256105	-0.000005
0	180.00	0.0	0.000031	-0.661746	0.362187	-0.000004
0	240.00	0.0	0.000022	-0.467926	0.443586	-0.000003
0	300.00	0.0	0.000011	-0.242216	0.494757	-0.000002
0	360.00	0.0	0.000000	-0.000001	0.512210	-0.000000
30	0.0	-2.597499	0.280869	-0.949239	0.0	-19.902298
30	60.00	-2.508991	0.271298	-0.915928	0.145667	-19.224136
30	120.00	-2.249500	0.243239	-0.821199	0.281407	-17.235886
30	180.00	-1.836709	0.198604	-0.670506	0.397970	-14.073055
30	240.00	-1.298752	0.140435	-0.474120	0.497411	-9.951167
30	300.00	-0.672284	0.072694	-0.245423	0.543637	-5.151176
30	360.00	-0.000003	0.000000	-0.000001	0.562815	-0.000025
60	0.0	1.008183	-1.057768	-0.836274	0.0	-18.449799
60	60.00	0.973831	-1.021725	-0.807778	0.127390	-17.821136
60	120.00	0.873113	-0.916054	-0.724234	0.246099	-15.977006
60	180.00	0.712894	-0.747955	-0.591335	0.348037	-13.045092
60	240.00	0.504093	-0.528885	-0.418138	0.426256	-9.224916
60	300.00	0.260938	-0.273771	-0.216444	0.475428	-4.775171
60	360.00	0.000001	-0.000001	-0.000001	0.492199	-0.000023
90	0.0	3.612938	-2.211154	-0.779625	0.0	-14.735373
90	60.00	3.489830	-2.135811	-0.752094	0.000000	-14.233283
90	120.00	3.128896	-1.914915	-0.674309	0.000001	-12.761213
90	180.00	2.554733	-1.563522	-0.550571	0.000001	-10.419490
90	240.00	1.806472	-1.105578	-0.389313	0.000001	-7.367702
90	300.00	0.935099	-0.572290	-0.201523	0.000001	-3.813806
90	360.00	0.000005	-0.000003	-0.000001	0.000001	-0.000019

TABLE.11—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 2.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.2799465E 01 0.2799465E 01
0.1159710E 01 0.1159720E 01
0.1158854E 01 0.1158865E 01
0.2797401E 01 0.2797400E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	-0.000000	0.0	0.426008	0.0	-0.000006
0	20.00	-0.000000	0.0	0.411492	0.276987	-0.000005
0	40.00	-0.000000	0.0	0.368933	0.535099	-0.000005
0	60.00	-0.000000	0.0	0.301233	0.756744	-0.000004
0	80.00	-0.000000	0.0	0.213004	0.926813	-0.000003
0	100.00	-0.000000	0.0	0.110259	1.033731	-0.000001
0	120.00	-0.000000	0.0	0.000001	1.070198	-0.000000
30	0.0	-0.039591	-0.122811	-0.317485	0.0	-0.383868
30	20.00	-0.038242	-0.118627	-0.306667	0.270383	-0.370788
30	40.00	-0.034287	-0.106358	-0.274950	0.522341	-0.332440
30	60.00	-0.027995	-0.086841	-0.224496	0.738701	-0.271436
30	80.00	-0.019796	-0.061406	-0.158743	0.904720	-0.191935
30	100.00	-0.010247	-0.031786	-0.082171	1.009085	-0.099353
30	120.00	-0.000000	-0.000000	-0.000000	1.044682	-0.000000
60	0.0	0.012642	-0.663095	-0.818008	0.0	-0.462490
60	20.00	0.012211	-0.640500	-0.790135	0.191977	-0.446721
60	40.00	0.010948	-0.574257	-0.708416	0.370871	-0.400519
60	60.00	0.008939	-0.468879	-0.578419	0.524491	-0.327022
60	80.00	0.006321	-0.331548	-0.409005	0.642368	-0.231240
60	100.00	0.003272	-0.171622	-0.211717	0.716468	-0.119699
60	120.00	0.000000	-0.000001	-0.000001	0.741743	-0.000001
90	0.0	0.066005	-1.034300	-1.012046	0.0	0.122998
90	20.00	0.063756	-0.999057	-0.977561	-0.000000	0.118806
90	40.00	0.057162	-0.895730	-0.876457	-0.000000	0.106519
90	60.00	0.046673	-0.731360	-0.715625	-0.000000	0.086972
90	80.00	0.033003	-0.517151	-0.506024	-0.000000	0.061499
90	100.00	0.017083	-0.267697	-0.261937	-0.000000	0.031834
90	120.00	0.000000	-0.000001	-0.000001	-0.000000	0.000000

TABLE.12-STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 3.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.2285496E 01   0.2285496E 01
0.9467272E 00   0.9467354E 00
0.9462618E 00   0.9462700E 00
0.2284372E 01   0.2284372E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	0.000000	-0.000003	0.375694	0.0	0.000001
0	30.00	0.000000	-0.000003	0.362892	0.407413	0.000001
0	60.00	0.000000	-0.000002	0.325360	0.787062	0.000001
0	90.00	0.000000	-0.000002	0.265656	1.113073	0.000001
0	120.00	0.000000	-0.000001	0.187847	1.363230	0.000001
0	150.00	0.000000	-0.000001	0.097237	1.520487	0.000000
0	180.00	0.000000	-0.000000	0.000000	1.574124	0.000000
30	0.0	-0.180733	-0.313065	-0.349590	0.0	-1.471855
30	30.00	-0.174574	-0.302397	-0.337678	0.395522	-1.421702
30	60.00	-0.156519	-0.271122	-0.302754	0.764091	-1.274664
30	90.00	-0.127797	-0.221370	-0.247197	1.090587	-1.040759
30	120.00	-0.090367	-0.156533	-0.174795	1.323443	-0.735929
30	150.00	-0.046777	-0.081027	-0.090481	1.476110	-0.380945
30	180.00	-0.000000	-0.000000	-0.000000	1.528183	-0.000002
60	0.0	0.080003	-1.468690	-0.823559	0.0	-1.610134
60	30.00	0.077277	-1.418645	-0.795497	0.274374	-1.555270
60	60.00	0.069284	-1.271922	-0.713223	0.530051	-1.394417
60	90.00	0.056570	-1.038521	-0.582344	0.749605	-1.138536
60	120.00	0.040001	-0.734346	-0.411780	0.918075	-0.805069
60	150.00	0.020706	-0.380126	-0.213153	1.022980	-0.416734
60	180.00	0.000000	-0.000002	-0.000001	1.060102	-0.000002
90	0.0	0.278981	-2.186926	-1.000074	0.0	-0.880787
90	30.00	0.269475	-2.112408	-0.965998	0.000001	-0.850775
90	60.00	0.241605	-1.893933	-0.866090	0.000001	-0.762784
90	90.00	0.197269	-1.546391	-0.707160	0.000002	-0.622811
90	120.00	0.139491	-1.093465	-0.500038	0.000002	-0.440395
90	150.00	0.072206	-0.566019	-0.258839	0.000002	-0.227965
90	180.00	0.000000	-0.000003	-0.000001	0.000002	-0.000001

TABLE.13—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 4.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1979183E 01   0.1979183E 01
0.8198228E 00   0.8198299E 00
0.8195202E 00   0.8195276E 00
0.1978453E 01   0.1978454E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	-0.000002	-0.000003	0.288711	0.0	0.000004
0	40.00	-0.000002	-0.000003	0.278874	0.520644	0.000004
0	80.00	-0.000002	-0.000002	0.250031	1.005807	0.000004
0	120.00	-0.000001	-0.000002	0.204150	1.422426	0.000003
0	160.00	-0.000001	-0.000001	0.144356	1.742108	0.000002
0	200.00	-0.000000	-0.000001	0.074724	1.943070	0.000001
0	240.00	-0.000000	-0.000000	0.000000	2.011615	0.000000
30	0.0	-0.536345	-0.549295	-0.400636	0.0	-3.878313
30	40.00	-0.518069	-0.530579	-0.386984	0.504791	-3.746162
30	80.00	-0.464488	-0.475704	-0.346961	0.975182	-3.358717
30	120.00	-0.379253	-0.388411	-0.283292	1.379116	-2.742381
30	160.00	-0.268172	-0.274648	-0.200318	1.689065	-1.939157
30	200.00	-0.138817	-0.142168	-0.103692	1.883907	-1.003783
30	240.00	-0.000000	-0.000000	-0.000000	1.950365	-0.000001
60	0.0	0.252131	-2.503832	-0.841400	0.0	-4.739409
60	40.00	0.243540	-2.418515	-0.812730	0.348417	-4.577918
60	80.00	0.218352	-2.168382	-0.728674	0.673089	-4.104448
60	120.00	0.178284	-1.770476	-0.594960	0.951892	-3.351269
60	160.00	0.126066	-1.251916	-0.420700	1.165825	-2.369705
60	200.00	0.065257	-0.648041	-0.217771	1.300308	-1.226652
60	240.00	0.000000	-0.000001	-0.000000	1.346179	-0.000001
90	0.0	0.814178	-3.691792	-1.002770	0.0	-4.097454
90	40.00	0.786436	-3.565996	-0.968602	0.000001	-3.957836
90	80.00	0.705099	-3.197185	-0.868425	0.000002	-3.548499
90	120.00	0.575711	-2.610491	-0.709066	0.000002	-2.897337
90	160.00	0.407089	-1.845896	-0.501385	0.000003	-2.048727
90	200.00	0.210725	-0.955508	-0.259537	0.000003	-1.060501
90	240.00	0.000000	-0.000001	-0.000000	0.000003	-0.000001

TABLE.14— STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 5.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1770173E 01 0.1770173E 01
0.7332377E 00 0.7332444E 00
0.7330213E 00 0.7330279E 00
0.1769650E 01 0.1769650E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	-0.000001	0.000010	0.194136	0.0	0.000054
0	50.00	-0.000001	0.000009	0.187521	0.630800	0.000053
0	100.00	-0.000001	0.000008	0.168126	1.218613	0.000047
0	150.00	-0.000001	0.000007	0.137275	1.723379	0.000039
0	200.00	-0.000000	0.000005	0.097068	2.110700	0.000027
0	250.00	-0.000000	0.000002	0.050246	2.354181	0.000014
0	300.00	-0.000000	0.000000	0.000000	2.437228	0.000000
30	0.0	-1.258820	-0.840833	-0.469834	0.0	-8.358842
30	50.00	-1.215926	-0.812182	-0.453825	0.611282	-8.074021
30	100.00	-1.090169	-0.728183	-0.406889	1.130907	-7.238970
30	150.00	-0.890120	-0.594559	-0.332223	1.670055	-5.910596
30	200.00	-0.629411	-0.420417	-0.234918	2.045391	-4.179428
30	250.00	-0.325807	-0.217624	-0.121602	2.281339	-2.163432
30	300.00	-0.000002	-0.000001	-0.000001	2.361816	-0.000011
60	0.0	0.603814	-3.792326	-0.887999	0.0	-11.418650
60	50.00	0.583240	-3.663106	-0.857742	0.421156	-11.029570
60	100.00	0.522918	-3.284250	-0.769030	0.813612	-9.888842
60	150.00	0.426961	-2.681580	-0.627911	1.150620	-8.074207
60	200.00	0.301908	-1.896166	-0.444001	1.409217	-5.709335
60	250.00	0.156279	-0.981529	-0.229832	1.571778	-2.955371
60	300.00	0.000001	-0.000005	-0.000001	1.627225	-0.000014
90	0.0	1.900995	-5.572399	-1.039529	0.0	-11.408594
90	50.00	1.836220	-5.382524	-1.004107	0.000001	-11.019856
90	100.00	1.646310	-4.825839	-0.900258	0.000001	-9.880133
90	150.00	1.344207	-3.940282	-0.735058	0.000001	-8.067097
90	200.00	0.950499	-2.786204	-0.519765	0.000002	-5.704308
90	250.00	0.492015	-1.442246	-0.269051	0.000002	-2.952768
90	300.00	0.000002	-0.000007	-0.000001	0.000002	-0.000014

TABLE.15– STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON 4-SIDES
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)

L/R = 6.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 05
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02

ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT

0.1615900E 01	0.1615901E 01
0.6693311E 00	0.6693373E 00
0.6691664E 00	0.6691728E 00
0.1615502E 01	0.1615503E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	-0.000004	0.000016	0.077988	0.0	0.000054
0	60.00	-0.000004	0.000016	0.075331	0.744068	0.000053
0	120.00	-0.000003	0.000014	0.067540	1.437429	0.000047
0	180.00	-0.000003	0.000011	0.055146	2.032832	0.000039
0	240.00	-0.000002	0.000008	0.038994	2.489700	0.000027
0	300.00	-0.000001	0.000004	0.020185	2.776901	0.000014
0	360.00	-0.000000	0.000000	0.000000	2.874861	0.000000
30	0.0	-2.555886	-1.195965	-0.572360	0.0	-16.144592
30	60.00	-2.468797	-1.155213	-0.552857	0.720871	-15.594480
30	120.00	-2.213462	-1.035735	-0.495678	1.392615	-13.981629
30	180.00	-1.807284	-0.845675	-0.404719	1.969456	-11.415955
30	240.00	-1.277946	-0.597984	-0.286180	2.412081	-8.072311
30	300.00	-0.661514	-0.309539	-0.148138	2.690328	-4.178538
30	360.00	-0.000003	-0.000002	-0.000001	2.785233	-0.000020
60	0.0	1.236390	-5.367934	-0.978480	0.0	-23.614761
60	60.00	1.194261	-5.185026	-0.945139	0.496261	-22.810104
60	120.00	1.070745	-4.648767	-0.847389	0.958704	-20.450974
60	180.00	0.874260	-3.795704	-0.691890	1.355312	-16.698151
60	240.00	0.618196	-2.683971	-0.489241	1.660523	-11.807403
60	300.00	0.320002	-1.389327	-0.253250	1.852074	-6.111965
60	360.00	0.000002	-0.000007	-0.000001	1.917409	-0.000030
90	0.0	3.851901	-7.875813	-1.124816	0.0	-25.021347
90	60.00	3.720651	-7.607451	-1.086489	0.000000	-24.168762
90	120.00	3.335844	-6.820654	-0.974119	0.000001	-21.669113
90	180.00	2.723706	-5.569042	-0.795365	0.000001	-17.692764
90	240.00	1.925954	-3.937913	-0.562409	0.000001	-12.510697
90	300.00	0.996948	-2.038415	-0.291125	0.000001	-6.476017
90	360.00	0.000005	-0.000010	-0.000001	0.000001	-0.000032

TABLES (16–25)

ANALYSIS OF SHELLS WITH

LONGITUDINAL STIFFENERS IN

VALLEYS ONLY.

TABLE.16—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 2.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.2723647E 01 0.2723637E 01
0.1127078E 01 0.1127119E 01
0.1189391E 01 0.1189375E 01
0.2874085E 01 0.2874099E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	0.472208	-0.000001	-0.000001	0.0	-0.000003
0	20.00	0.456118	-0.000001	-0.000001	0.623677	-0.000003
0	40.00	0.408944	-0.000001	-0.000001	1.204852	-0.000002
0	60.00	0.333901	-0.000001	-0.000001	1.703918	-0.000002
0	80.00	0.236104	-0.000001	-0.000000	2.086864	-0.000001
0	100.00	0.122217	-0.000000	-0.000000	2.327597	-0.000001
0	120.00	0.000001	-0.000000	-0.000000	2.409706	-0.000000
10	0.0	0.373400	-1.540665	-0.552746	0.0	-0.899944
10	20.00	0.360677	-1.488168	-0.533911	0.565221	-0.869279
10	40.00	0.323374	-1.334254	-0.478692	1.091924	-0.779374
10	60.00	0.264034	-1.089415	-0.390850	1.544214	-0.636356
10	80.00	0.186700	-0.770334	-0.276373	1.891267	-0.449973
10	100.00	0.096643	-0.398754	-0.143061	2.109436	-0.232923
10	120.00	0.000000	-0.000002	-0.000001	2.183849	-0.000001
20	0.0	0.272980	-2.501274	-1.009328	0.0	-2.708171
20	20.00	0.263679	-2.416045	-0.974936	0.418770	-2.615992
20	40.00	0.236408	-2.166166	-0.874104	0.809003	-2.345345
20	60.00	0.193026	-1.768667	-0.713703	1.144102	-1.914966
20	80.00	0.136490	-1.250639	-0.504665	1.401233	-1.354088
20	100.00	0.070653	-0.647379	-0.261234	1.562873	-0.700928
20	120.00	0.000000	-0.000003	-0.000001	1.618006	-0.000003
40	0.0	0.168609	-3.165506	-1.407245	0.0	-4.897391
40	20.00	0.162864	-3.057644	-1.359294	-0.000000	-4.730430
40	40.00	0.146020	-2.741408	-1.218709	-0.000000	-4.241136
40	60.00	0.119225	-2.238351	-0.995072	-0.000000	-3.462914
40	80.00	0.084305	-1.582756	-0.703624	-0.000000	-2.448654
40	100.00	0.043639	-0.819295	-0.364223	-0.000000	-1.267517
40	120.00	0.000000	-0.000004	-0.000002	-0.000000	-0.000006

TABLE.17–STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 3.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.2192900E 01   0.2192871E 01
0.9061298E 00   0.9062209E 00
0.9830292E 00   0.9829593E 00
0.2378553E 01   0.2378592E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	1.055886	0.000001	-0.000023	0.0	-0.000005
0	30.00	1.019907	0.000001	-0.000022	0.922905	-0.000005
0	60.00	0.914424	0.000001	-0.000020	1.782916	-0.000005
0	90.00	0.746625	0.000001	-0.000016	2.521424	-0.000004
0	120.00	0.527944	0.000001	-0.000011	3.088101	-0.000003
0	150.00	0.273284	0.000000	-0.000006	3.444331	-0.000001
0	180.00	0.000001	0.000000	-0.000000	3.565835	-0.000000
10	0.0	1.161193	-3.144538	-0.546632	0.0	-0.906782
10	30.00	1.121626	-3.037438	-0.528006	0.845552	-0.875884
10	60.00	1.005622	-2.723292	-0.473398	1.633481	-0.785296
10	90.00	0.821088	-2.223559	-0.386528	2.310092	-0.641192
10	120.00	0.580598	-1.572296	-0.273317	2.829272	-0.453392
10	150.00	0.300540	-0.813881	-0.141479	3.155644	-0.234693
10	180.00	0.000001	-0.000004	-0.000001	3.265965	-0.000001
20	0.0	1.221675	-5.475224	-1.006465	0.0	-2.741023
20	30.00	1.180047	-5.288660	-0.972171	0.638260	-2.647625
20	60.00	1.058002	-4.741683	-0.871624	1.233024	-2.373796
20	90.00	0.863855	-3.871569	-0.711678	1.743759	-1.938196
20	120.00	0.610839	-2.737617	-0.503233	2.135659	-1.370514
20	150.00	0.316194	-1.417095	-0.260493	2.382020	-0.709431
20	180.00	0.000002	-0.000007	-0.000001	2.466049	-0.000003
40	0.0	1.255556	-7.385571	-1.416451	0.0	-4.980672
40	30.00	1.212773	-7.133915	-1.368186	0.000011	-4.810960
40	60.00	1.087343	-6.396092	-1.226682	0.000021	-4.313399
40	90.00	0.887813	-5.222389	-1.001582	0.000030	-3.521868
40	120.00	0.627779	-3.692792	-0.708227	0.000037	-2.490340
40	150.00	0.324963	-1.911530	-0.366605	0.000041	-1.289096
40	180.00	0.000002	-0.000009	-0.000002	0.000042	-0.000006

TABLE.18—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 4.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1872566E 01 0.1872511E 01
0.7722117E 00 0.7723889E 00
0.8612429E 00 0.8610911E 00
0.2087508E 01 0.2087587E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	2.614399	-0.000003	-0.000028	0.0	-0.000002
0	40.00	2.525315	-0.000003	-0.000027	1.227658	-0.000002
0	80.00	2.264135	-0.000003	-0.000025	2.371654	-0.000002
0	120.00	1.848659	-0.000002	-0.000020	3.354026	-0.000001
0	160.00	1.307199	-0.000002	-0.000014	4.107825	-0.000001
0	200.00	0.676658	-0.000001	-0.000007	4.581683	-0.000001
0	240.00	0.000001	-0.000000	-0.000000	4.743309	-0.000000
10	0.0	3.274212	-5.492307	-0.545587	0.0	-0.908696
10	40.00	3.162645	-5.305161	-0.526997	1.126880	-0.877733
10	80.00	2.835550	-4.756476	-0.472492	2.176965	-0.786954
10	120.00	2.315217	-3.883647	-0.385789	3.078693	-0.642545
10	160.00	1.637106	-2.746154	-0.272794	3.770614	-0.454348
10	200.00	0.847431	-1.421516	-0.141209	4.205573	-0.235198
10	240.00	0.000001	-0.000002	-0.000000	4.353930	-0.000000
20	0.0	3.772824	-9.687250	-1.005978	0.0	-2.747996
20	40.00	3.644268	-9.357165	-0.971700	0.853321	-2.654350
20	80.00	3.267361	-8.389404	-0.871202	1.648490	-2.379834
20	120.00	2.667789	-6.849920	-0.711334	2.331317	-1.943127
20	160.00	1.886412	-4.843626	-0.502989	2.855268	-1.373998
20	200.00	0.976481	-2.507251	-0.260367	3.184638	-0.711236
20	240.00	0.000001	-0.000003	-0.000000	3.296981	-0.000001
40	0.0	4.184194	-13.210126	-1.418027	0.0	-4.996809
40	40.00	4.041620	-12.760002	-1.369708	0.000011	-4.826538
40	80.00	3.623617	-11.440304	-1.228046	0.000021	-4.327355
40	120.00	2.958672	-9.340970	-1.002696	0.000030	-3.533271
40	160.00	2.092097	-6.605064	-0.709014	0.000037	-2.498401
40	200.00	1.082952	-3.419041	-0.367013	0.000041	-1.293270
40	240.00	0.000001	-0.000004	-0.000000	0.000043	-0.000002

TABLE.19—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 5.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1651310E 01 0.1651220E 01
0.6791869E 00 0.6794901E 00
0.7788484E 00 0.7785802E 00
0.1891916E 01 0.1892057E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	5.878137	0.000006	-0.000030	0.0	-0.000047
0	50.00	5.697163	0.000006	-0.000029	1.533584	-0.000045
0	100.00	5.107937	0.000005	-0.000026	2.962657	-0.000040
0	150.00	4.170614	0.000004	-0.000021	4.187830	-0.000033
0	200.00	2.949074	0.000003	-0.000015	5.131472	-0.000023
0	250.00	1.526553	0.000001	-0.000008	5.723416	-0.000012
0	300.00	0.000007	0.000000	-0.000000	5.925318	-0.000000
10	0.0	7.727628	-8.539649	-0.545299	0.0	-0.909881
10	50.00	7.464315	-8.248668	-0.526719	1.408422	-0.879877
10	100.00	6.692323	-7.395554	-0.472243	2.720862	-0.787930
10	150.00	5.464260	-6.038445	-0.385585	3.847981	-0.643383
10	200.00	3.863821	-4.269833	-0.272650	4.712672	-0.454941
10	250.00	2.000062	-2.210229	-0.141134	5.256306	-0.235495
10	300.00	0.000010	-0.000011	-0.000001	5.441730	-0.000001
20	0.0	9.150562	-15.116401	-1.005843	0.0	-2.751057
20	50.00	8.838765	-14.601323	-0.971570	1.067443	-2.657316
20	100.00	7.924620	-13.091188	-0.871086	2.062142	-2.382484
20	150.00	6.470427	-10.688913	-0.711239	2.916310	-1.945292
20	200.00	4.575290	-7.558214	-0.502923	3.571735	-1.375530
20	250.00	2.368345	-3.912422	-0.260332	3.983756	-0.712028
20	300.00	0.000012	-0.000019	-0.000001	4.124289	-0.000003
40	0.0	10.358470	-20.675262	-1.418462	0.0	-5.002796
40	50.00	10.005514	-19.970764	-1.370129	0.000015	-4.832320
40	100.00	8.970698	-17.905304	-1.228424	0.000030	-4.332540
40	150.00	7.324547	-14.619624	-1.003004	0.000042	-3.537504
40	200.00	5.179244	-10.337650	-0.709232	0.000052	-2.501397
40	250.00	2.680976	-5.351165	-0.367126	0.000058	-1.294819
40	300.00	0.000013	-0.000026	-0.000002	0.000060	-0.000006

TABLE.20—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING PROPOSAL NO.(1)

PROPERTIES OF SHELL (LB. & IN.)
L/R = 6.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1486066E 01 0.1485933E 01
0.6092232E 00 0.6096983E 00
0.7184690E 00 0.7180457E 00
0.1749811E 01 0.1750038E 01

FY	X	W	NX	NY	MXY	MFY
0	0.0	11.867392	-0.000041	-0.000029	0.0	0.000111
0	60.00	11.463020	-0.000040	-0.000028	1.839870	0.000107
0	120.00	10.277463	-0.000036	-0.000025	3.554359	0.000096
0	180.00	8.391516	-0.000029	-0.000020	5.026624	0.000078
0	240.00	5.933706	-0.000021	-0.000014	6.156330	0.000055
0	300.00	3.071514	-0.000011	-0.000007	6.866498	0.000029
0	360.00	0.000015	-0.000000	-0.000000	7.108724	0.000000
10	0.0	15.823957	-12.275232	-0.545194	0.0	-0.910863
10	60.00	15.284770	-11.856965	-0.526617	1.690028	-0.879826
10	120.00	13.703950	-10.630664	-0.472152	3.264885	-0.788830
10	180.00	11.189232	-8.679903	-0.385510	4.617246	-0.644077
10	240.00	7.911993	-6.137628	-0.272597	5.654947	-0.455432
10	300.00	4.095551	-3.177072	-0.141107	6.307278	-0.235749
10	360.00	0.000020	-0.000016	-0.000001	6.529776	-0.000001
20	0.0	18.927872	-21.757187	-1.005794	0.0	-2.753177
20	60.00	18.282913	-21.015823	-0.971522	1.281275	-2.659365
20	120.00	16.392014	-18.842270	-0.871043	2.475234	-2.384321
20	180.00	13.384031	-15.384660	-0.711204	3.500510	-1.946790
20	240.00	9.463953	-10.878613	-0.502898	4.287230	-1.376591
20	300.00	4.898906	-5.631188	-0.260319	4.781788	-0.712576
20	360.00	0.000024	-0.000028	-0.000001	4.950472	-0.000003
40	0.0	21.584671	-29.790314	-1.418616	0.0	-5.006370
40	60.00	20.849182	-28.775223	-1.370277	0.000017	-4.835782
40	120.00	18.692871	-25.799164	-1.228558	0.000034	-4.335643
40	180.00	15.262673	-21.064926	-1.003113	0.000048	-3.540039
40	240.00	10.792356	-14.895105	-0.709309	0.000058	-2.503189
40	300.00	5.586537	-7.710320	-0.367166	0.000065	-1.295747
40	360.00	0.000027	-0.000038	-0.000002	0.000067	-0.000006

TABLE.21–STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 2.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 BX = 0.703E 02 BY = 0.294E 05 BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.2799465E 01 0.2799465E 01
0.1159710E 01 0.1159720E 01
0.1158854E 01 0.1158865E 01
0.2797401E 01 0.2797400E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	0.534134	-0.000001	-0.000004	0.0	-0.000004
0	20.00	0.515934	-0.000001	-0.000004	0.625074	-0.000004
0	40.00	0.462574	-0.000001	-0.000004	1.207550	-0.000004
0	60.00	0.377690	-0.000001	-0.000003	1.707734	-0.000003
0	80.00	0.267068	-0.000000	-0.000002	2.091537	-0.000002
0	100.00	0.138244	-0.000000	-0.000001	2.332808	-0.000001
0	120.00	0.000001	-0.000000	-0.000000	2.415102	-0.000000
10	0.0	0.415835	-1.564370	-0.553725	0.0	-0.898751
10	20.00	0.401666	-1.511065	-0.534857	0.565399	-0.868127
10	40.00	0.360123	-1.354794	-0.479540	1.092267	-0.778342
10	60.00	0.294040	-1.106176	-0.391543	1.544700	-0.635513
10	80.00	0.207918	-0.782187	-0.276863	1.891862	-0.449377
10	100.00	0.107626	-0.404890	-0.143315	2.110099	-0.232615
10	120.00	0.000001	-0.000002	-0.000001	2.184536	-0.000001
20	0.0	0.305669	-2.510376	-1.009731	0.0	-2.702866
20	20.00	0.295253	-2.424837	-0.975326	0.417685	-2.610767
20	40.00	0.264717	-2.174049	-0.874453	0.806905	-2.340750
20	60.00	0.216141	-1.775104	-0.713988	1.141138	-1.911215
20	80.00	0.152835	-1.255190	-0.504867	1.397602	-1.351435
20	100.00	0.079113	-0.649735	-0.261338	1.559824	-0.699555
20	120.00	0.000000	-0.000003	-0.000001	1.613813	-0.000003
40	0.0	0.196452	-3.148380	-1.405814	0.0	-4.884265
40	20.00	0.189758	-3.041101	-1.357912	0.000001	-4.717837
40	40.00	0.170132	-2.726577	-1.217470	0.000002	-4.229897
40	60.00	0.138912	-2.226241	-0.994061	0.000002	-3.453697
40	80.00	0.098226	-1.574193	-0.702908	0.000003	-2.442137
40	100.00	0.050846	-0.814863	-0.363852	0.000003	-1.264144
40	120.00	0.000000	-0.000004	-0.000002	0.000003	-0.000006

TABLE.22– STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 3.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.2285496E 01   0.2285496E 01
0.9467272E 00   0.9467354E 00
0.9462618E 00   0.9462700E 00
0.2284372E 01   0.2284372E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	1.372566	-0.000005	-0.000025	0.0	0.000012
0	30.00	1.325797	-0.000005	-0.000024	0.925514	0.000011
0	60.00	1.188677	-0.000005	-0.000022	1.787955	0.000010
0	90.00	0.970551	-0.000004	-0.000018	2.528551	0.000008
0	120.00	0.686284	-0.000003	-0.000013	3.096829	0.000006
0	150.00	0.355247	-0.000001	-0.000006	3.454066	0.000003
0	180.00	0.000002	-0.000000	-0.000000	3.575913	0.000000
10	0.0	1.392646	-3.211251	-0.547879	0.0	-0.905025
10	30.00	1.345192	-3.101830	-0.529210	0.845955	-0.874187
10	60.00	1.206066	-2.781025	-0.474477	1.634260	-0.783775
10	90.00	0.984750	-2.270698	-0.387409	2.311194	-0.639949
10	120.00	0.696324	-1.605628	-0.273940	2.830623	-0.452513
10	150.00	0.360444	-0.831135	-0.141802	3.157151	-0.234238
10	180.00	0.000002	-0.000004	-0.000001	3.268524	-0.000001
20	0.0	1.395040	-5.504073	-1.007014	0.0	-2.733793
20	30.00	1.347505	-5.316526	-0.972701	0.636199	-2.640641
20	60.00	1.208139	-4.766667	-0.872100	1.229022	-2.367535
20	90.00	0.986442	-3.891969	-0.712067	1.739100	-1.933084
20	120.00	0.697521	-2.752042	-0.503508	2.128728	-1.366899
20	150.00	0.361064	-1.424562	-0.260635	2.374290	-0.707560
20	180.00	0.000002	-0.000007	-0.000001	2.458046	-0.000003
40	0.0	1.384583	-7.333877	-1.414590	0.0	-4.963293
40	30.00	1.337404	-7.083982	-1.366388	0.000011	-4.794086
40	60.00	1.199084	-6.351324	-1.225070	0.000021	-4.298261
40	90.00	0.979049	-5.185836	-1.000266	0.000030	-3.509516
40	120.00	0.692293	-3.666945	-0.707296	0.000037	-2.481606
40	150.00	0.358357	-1.898151	-0.366124	0.000041	-1.284575
40	180.00	0.000002	-0.000009	-0.000002	0.000042	-0.000006

TABLE.23—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)

L/R = 4.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06

DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02

ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT

0.1979183E 01	0.1979183E 01
0.8198228E 00	0.8198299E 00
0.8195202E 00	0.8195276E 00
0.1978453E 01	0.1978454E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	3.617338	0.000008	-0.000031	0.0	-0.000076
0	40.00	3.494080	0.000008	-0.000030	1.231254	-0.000073
0	80.00	3.132707	0.000007	-0.000027	2.378601	-0.000066
0	120.00	2.557844	0.000006	-0.000022	3.363850	-0.000054
0	160.00	1.808669	0.000004	-0.000016	4.119858	-0.000038
0	200.00	0.936238	0.000002	-0.000008	4.595104	-0.000020
0	240.00	0.000001	0.000000	-0.000000	4.757203	-0.000000
10	0.0	4.014913	-5.614908	-0.546879	0.0	-0.906458
10	40.00	3.878107	-5.423585	-0.528244	1.127451	-0.875571
10	80.00	3.477015	-4.862653	-0.473611	2.178068	-0.785016
10	120.00	2.838972	-3.970340	-0.386702	3.080254	-0.640963
10	160.00	2.007457	-2.807454	-0.273439	3.772525	-0.453229
10	200.00	1.039138	-1.453248	-0.141543	4.207705	-0.234609
10	240.00	0.000001	-0.000002	-0.000000	4.356137	-0.000000
20	0.0	4.325037	-9.741098	-1.006550	0.0	-2.739706
20	40.00	4.177665	-9.409168	-0.972252	0.850458	-2.646353
20	80.00	3.745591	-8.436029	-0.871698	1.642959	-2.372655
20	120.00	3.058263	-6.887989	-0.711738	2.323495	-1.937264
20	160.00	2.162519	-4.870544	-0.503275	2.845689	-1.369853
20	200.00	1.119404	-2.521185	-0.260515	3.173953	-0.709090
20	240.00	0.000001	-0.000003	-0.000000	3.285919	-0.000001
40	0.0	4.585381	-13.114179	-1.416093	0.0	-4.977539
40	40.00	4.429137	-12.667324	-1.367840	0.000014	-4.807933
40	80.00	3.971055	-11.357211	-1.226372	0.000028	-4.310675
40	120.00	3.242353	-9.273125	-1.001328	0.000039	-3.519651
40	160.00	2.292690	-6.557091	-0.708047	0.000048	-2.488770
40	200.00	1.186787	-3.394207	-0.366513	0.000053	-1.288294
40	240.00	0.000001	-0.000004	-0.000000	0.000055	-0.000002

TABLE.24—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 5.0 R = 120.0 DX = 0.237E 04 DY = 0.950E 06
DXY = 0.328E 06 RX = 0.703E 02 BY = 0.294E 05 RXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1770173E 01 0.1770173E 01
0.7332377E 00 0.7332444E 00
0.7330213E 00 0.7330279E 00
0.1769650E 01 0.1769650E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	8.345741	-0.000043	-0.000030	0.0	0.000031
0	50.00	8.061357	-0.000041	-0.000029	1.538105	0.000030
0	100.00	7.227624	-0.000037	-0.000026	2.971393	0.000027
0	150.00	5.901332	-0.000030	-0.000021	4.202185	0.000022
0	200.00	4.172878	-0.000021	-0.000015	5.146603	0.000016
0	250.00	2.160042	-0.000011	-0.000008	5.740294	0.000008
0	300.00	0.000011	-0.000000	-0.000000	5.942790	0.000000
10	0.0	9.540387	-8.732566	-0.546597	0.0	-0.906829
10	50.00	9.215307	-8.435011	-0.527972	1.409138	-0.875930
10	100.00	8.262218	-7.562624	-0.473367	2.722247	-0.785337
10	150.00	6.746075	-6.174859	-0.386502	3.849839	-0.641226
10	200.00	4.770203	-4.366291	-0.273299	4.715070	-0.453416
10	250.00	2.469239	-2.260159	-0.141470	5.258980	-0.234705
10	300.00	0.000012	-0.000011	-0.000001	5.444498	-0.000001
20	0.0	10.509473	-15.201147	-1.006413	0.0	-2.741448
20	50.00	10.142678	-14.683182	-0.972121	1.063834	-2.648036
20	100.00	9.093677	-13.164580	-0.871580	2.055171	-2.374164
20	150.00	7.424958	-10.748838	-0.711642	2.906451	-1.938497
20	200.00	5.250246	-7.600538	-0.503208	3.559660	-1.370727
20	250.00	2.717729	-3.934356	-0.260480	3.970287	-0.709541
20	300.00	0.000013	-0.000019	-0.000001	4.110345	-0.000003
40	0.0	11.333551	-20.523712	-1.416512	0.0	-4.981909
40	50.00	10.947371	-19.824371	-1.368246	0.000037	-4.812155
40	100.00	9.815145	-17.774048	-1.226735	0.000071	-4.314460
40	150.00	8.014034	-14.512462	-1.001625	0.000101	-3.522742
40	200.00	5.666786	-10.261875	-0.708258	0.000124	-2.490958
40	250.00	2.933346	-5.311940	-0.366621	0.000138	-1.289415
40	300.00	0.000014	-0.000026	-0.000002	0.000143	-0.000006

TABLE.25—STANDARD CORRUGATED SHELL SIMPLY SUPPORTED ON TRAVERSES ONLY
SOLUTION USING DONNELL APPROXIMATION

PROPERTIES OF SHELL (LB. & IN.)
L/R = 6.0   R = 120.0   DX = 0.237E 04   DY = 0.950E 06
DXY = 0.328E 06   BX = 0.703E 02   BY = 0.294E 05   BXY = 0.548E 02
ALFA1&2 , BETA1&2 BY EQUATIONS & BY POLRT
0.1615900E 01   0.1615901E 01
0.6693311E 00   0.6693373E 00
0.6691664E 00   0.6691728E 00
0.1615502E 01   0.1615503E 01

FY	X	W	NX	NY	NXY	MFY
0	0.0	16.950394	-0.000063	-0.000030	0.0	0.000070
0	60.00	16.372818	-0.000061	-0.000029	1.845346	0.000068
0	120.00	14.679473	-0.000055	-0.000026	3.564937	0.000061
0	180.00	11.985743	-0.000045	-0.000021	5.041582	0.000050
0	240.00	8.475212	-0.000031	-0.000015	6.174651	0.000035
0	300.00	4.387095	-0.000016	-0.000008	6.886932	0.000018
0	360.00	0.000021	-0.000000	-0.000000	7.129878	0.000000
10	0.0	19.592484	-12.554819	-0.546506	0.0	-0.907084
10	60.00	18.924861	-12.127025	-0.527884	1.690910	-0.876176
10	120.00	16.967590	-10.872793	-0.473288	3.266589	-0.785558
10	180.00	13.853983	-8.877601	-0.386439	4.619656	-0.641406
10	240.00	9.796260	-6.277421	-0.273254	5.657898	-0.453543
10	300.00	5.070920	-3.249434	-0.141447	6.310569	-0.234771
10	360.00	0.000025	-0.000016	-0.000001	6.533183	-0.000001
20	0.0	21.732544	-21.930814	-1.006381	0.0	-2.742191
20	60.00	20.992020	-21.135239	-0.972090	1.276925	-2.648753
20	120.00	18.820923	-18.949341	-0.871552	2.466830	-2.374807
20	180.00	15.367235	-15.472077	-0.711619	3.488626	-1.939022
20	240.00	10.866292	-10.940427	-0.503191	4.272676	-1.371098
20	300.00	5.624810	-5.663185	-0.260471	4.765553	-0.709733
20	360.00	0.000028	-0.000028	-0.000001	4.933665	-0.000003
40	0.0	23.603256	-29.571045	-1.416656	0.0	-4.983324
40	60.00	22.798996	-28.563431	-1.368383	0.000015	-4.813521
40	120.00	20.441010	-25.609268	-1.226859	0.000029	-4.315685
40	180.00	16.690018	-20.909882	-1.001727	0.000041	-3.523744
40	240.00	11.801650	-14.785550	-0.708329	0.000050	-2.491666
40	300.00	6.108987	-7.653568	-0.366658	0.000055	-1.289782
40	360.00	0.000030	-0.000037	-0.000002	0.000057	-0.000006

TABLES (26-28)

BUCKLING OF SHELLS SIMPLY SUPPORTED ON FOUR SIDES UNDER UNIFORM SNOW LOAD.

Table No.26- Buckling Loads of a Shell Simply Supported on Four Edges (lb/sq.ft).

Load: uniform snow load

Properties of shell:
R = 60.0 in ; $\phi_e = 90.0^0$; standard corrugation ;
gage = 22
a = L/s

a \ m	1	2	3	4	5
2	254.0	552.0	740.0	837.0	906.0
3	146.0	338.0	491.0	669.0	770.0
4	67.8	176.0	273.0	366.0	466.0
5	39.5	99.3	151.0	209.0	266.0
6	23.5	55.9	89.6	123.0	161.0

Table No.27- Buckling Loads of a Shell Simply Supported on Four Edges (lb/sq.ft).

Load: uniform snow load

Properties of shell:
R = 120.0 in ; $\phi_e = 90.0^0$; standard corrugation ;
gage = 22
a = L/s

a \ m	1	2	3	4	5
1	98.5	203.0	280.0	356.0	443.0
2	51.7	104.0	133.0	151.0	165.0
3	29.5	65.3	89.3	101.0	112.0
4	20.3	48.8	67.5	86.8	107.0
5	11.7	29.1	46.1	61.9	78.0
6	7.16	19.3	29.6	40.2	51.0

Table No.28– Buckling Loads of a Shell Simply Supported on Four Edges (lb/sq.ft.).

Load: uniform snow load

Properties of shell:
$R = 130.0$ in ; $\phi_e = 67.46^\circ$; standard corrugation ;
$s = 120.0$ in ; gage = 22
$a = L/s$

a \ m	1	2	3	4	5
2	31.2	73.8	110.0	134.0	155.0
3	17.5	43.1	67.1	91.6	111.0
4	10.9	29.7	45.2	60.5	75.8
5	8.45	20.3	30.6	41.6	52.8
6	6.02	13.2	21.3	28.9	37.3

TABLES (29-31)

CRITICAL LOADS FOR

DIFFERENT

CASES OF LOADING.

Table No.29- BUCKLING LOADS.

Properties of shell:

R = 120.0 in ; $\phi_e = 90.0^0$; gage = 22 ;
standard corrugation ; m = 1 ; N = 10

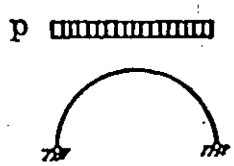
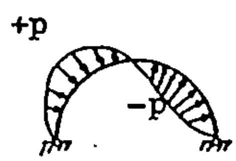
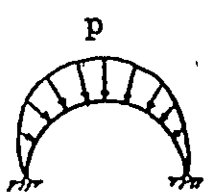
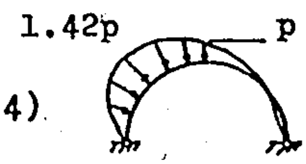
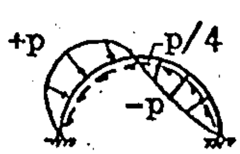
CASE OF LOADING	BUCKLING LOAD (p) in pounds/foot^2					
	L=120	=240	=360	=480	=600	=720in
1) 	98.5	51.7	29.5	20.3	11.7	7.16
2) 	94.4	33.2	18.2	10.0	5.80	3.23
3) 	89.4	47.4	28.0	22.4	14.4	10.1
4)  (3) + (1/2)(2)	64.1	28.2	15.7	9.27	5.70	3.41
5) .75(1 + 2)	82.9	34.4	17.2	9.14	5.24	2.97
6) 	——	48.1	22.8	11.7	6.41	3.41

Table No. 30— BUCKLING LOADS.

Properties of shell:

R = 120.0 in ; $\phi_e = 90.0^0$; gage = 22 ;
dimpled corrugation ; $\gamma = 16$; m = 1 ; N = 10

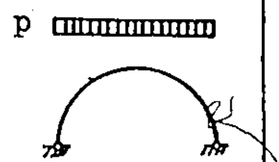
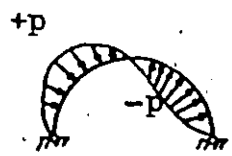
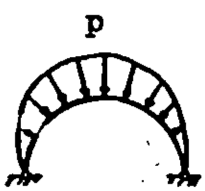
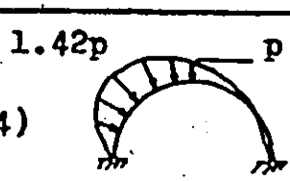
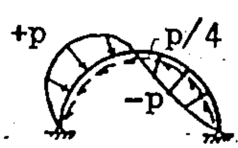
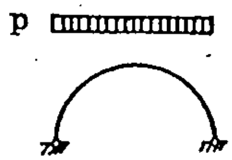
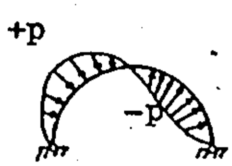
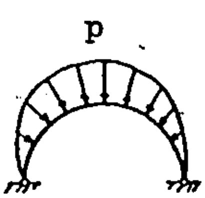
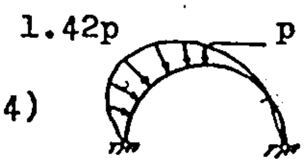
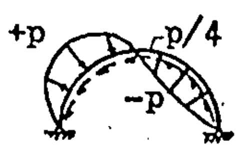
CASE OF LOADING	BUCKLING LOAD (p) in pounds/foot^2					
	L=120	=240	=360	=480	=600	=720in
1) 	173.	95.6	65.1	49.4	37.2	27.6
2) 	188.	85.7	50.6	29.9	23.8	15.7
3) 	162.	86.8	57.4	43.9	33.4	26.1
4)  (3) + (1/2)(2)	124.	61.5	39.0	24.4	19.7	13.5
5) .75(1. + 2)	157.	80.5	49.9	29.6	22.8	14.7
6) 	——	——	——	33.4	27.5	17.5

Table No. 31- BUCKLING LOADS.

Properties of shell:

$R = 120.0$ in ; $\phi_e = 90.0^{\circ}$; gage = 22 ; spot welded plane & corrugated sheets ; $\rho = .8$; $m = 1$; $N = 10$

CASE OF LOADING	BUCKLING LOAD (p) in pounds/foot^2					
	L=120	=240	=360	=480	=600	=720in
1) 	805.	367.	247.	190	148.	127.
2) 	895.	397.	250.	176.	129.	97.7
3) 	730.	343.	229.	174.	135.	112.
4)  (3) + (1/2)(2)	557.	248.	165.	122.	94.3	75.8
5) .75(1 + 2)	719.	319.	213.	158.	122.	96.2
6) 	——	——	——	175.	141.	121.

FIGURES

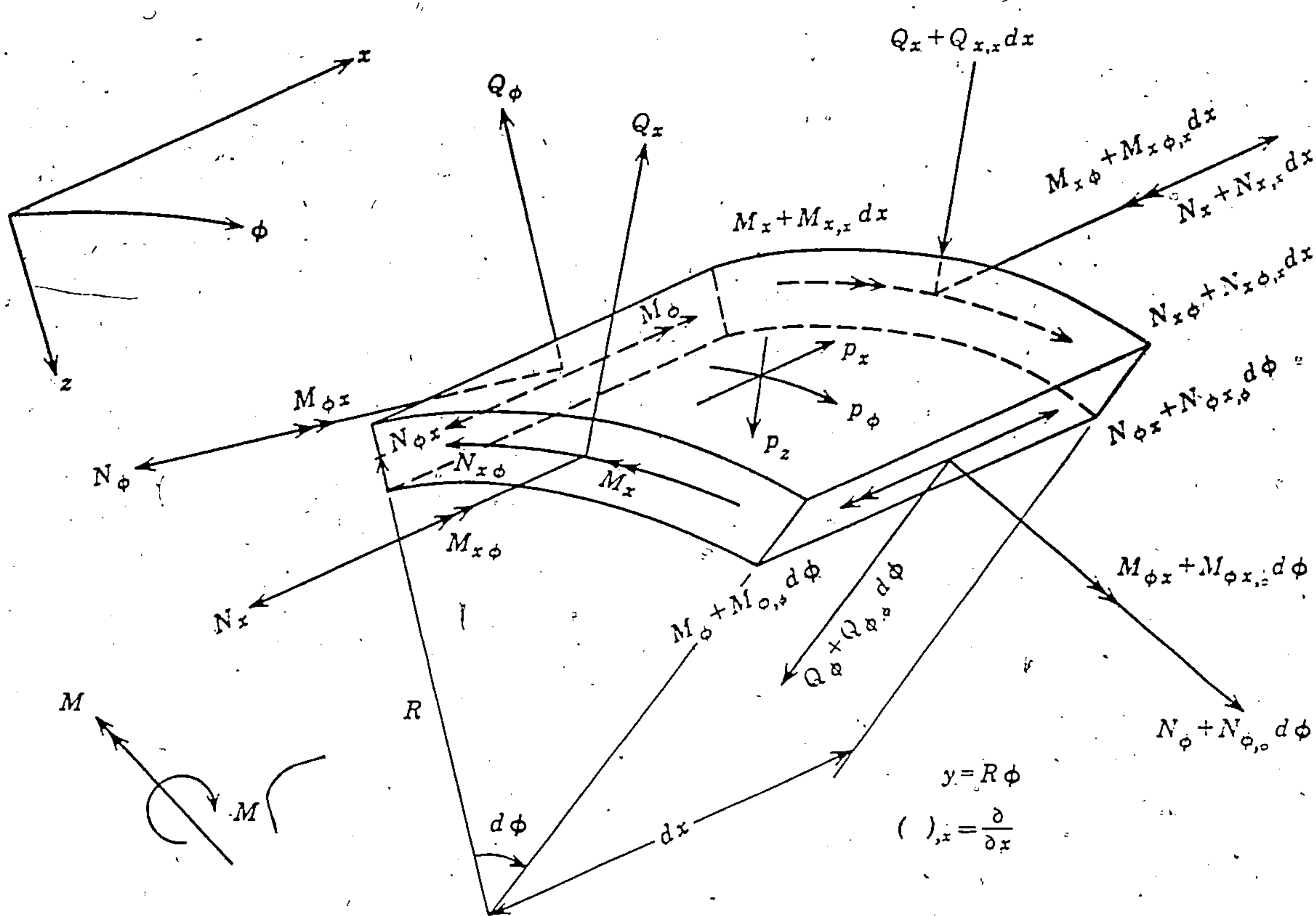


FIG.1. STRESS RESULTANTS AND EXTERNAL LOADS ON SHELL ELEMENT.

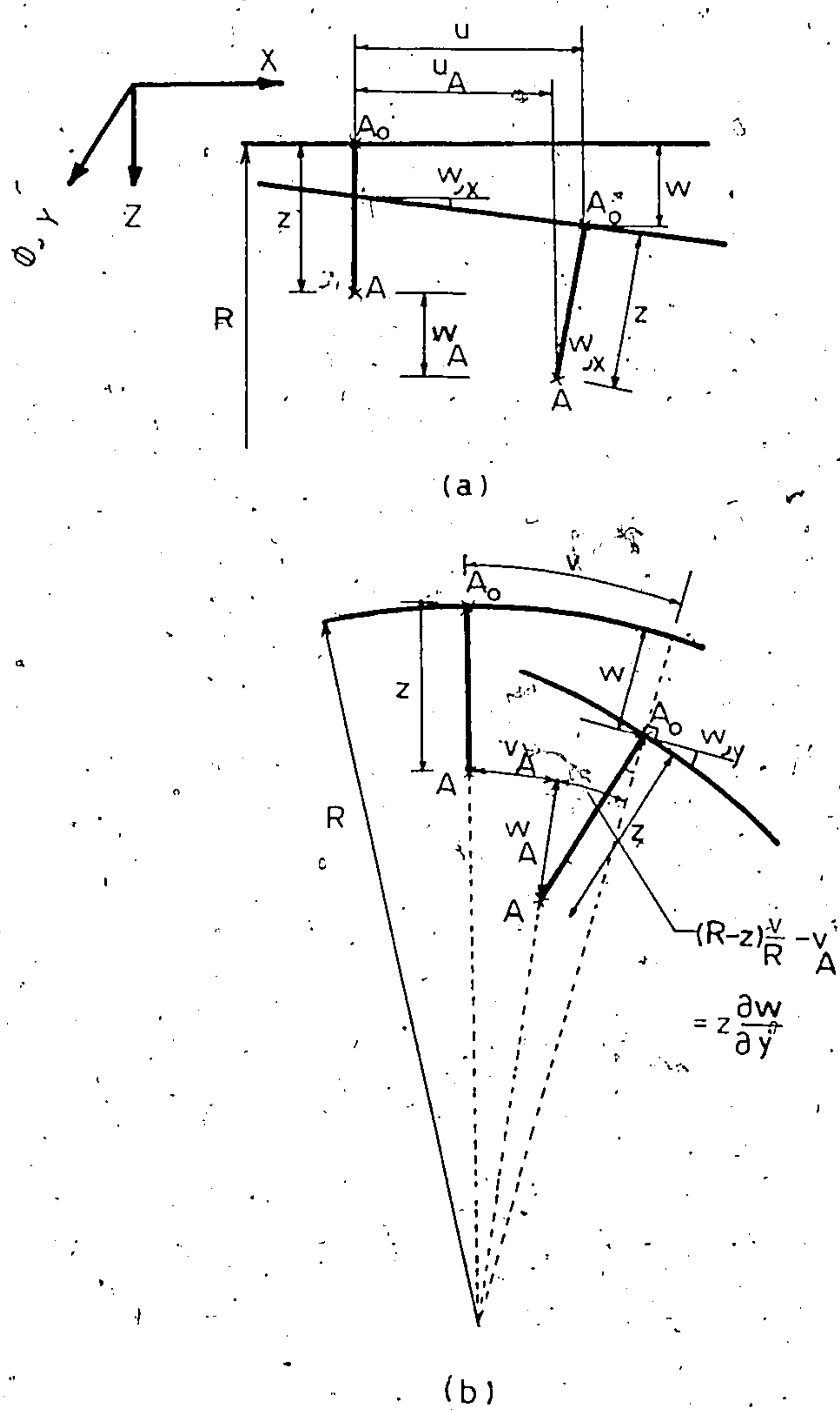


FIG.2. DEFORMATIONS.

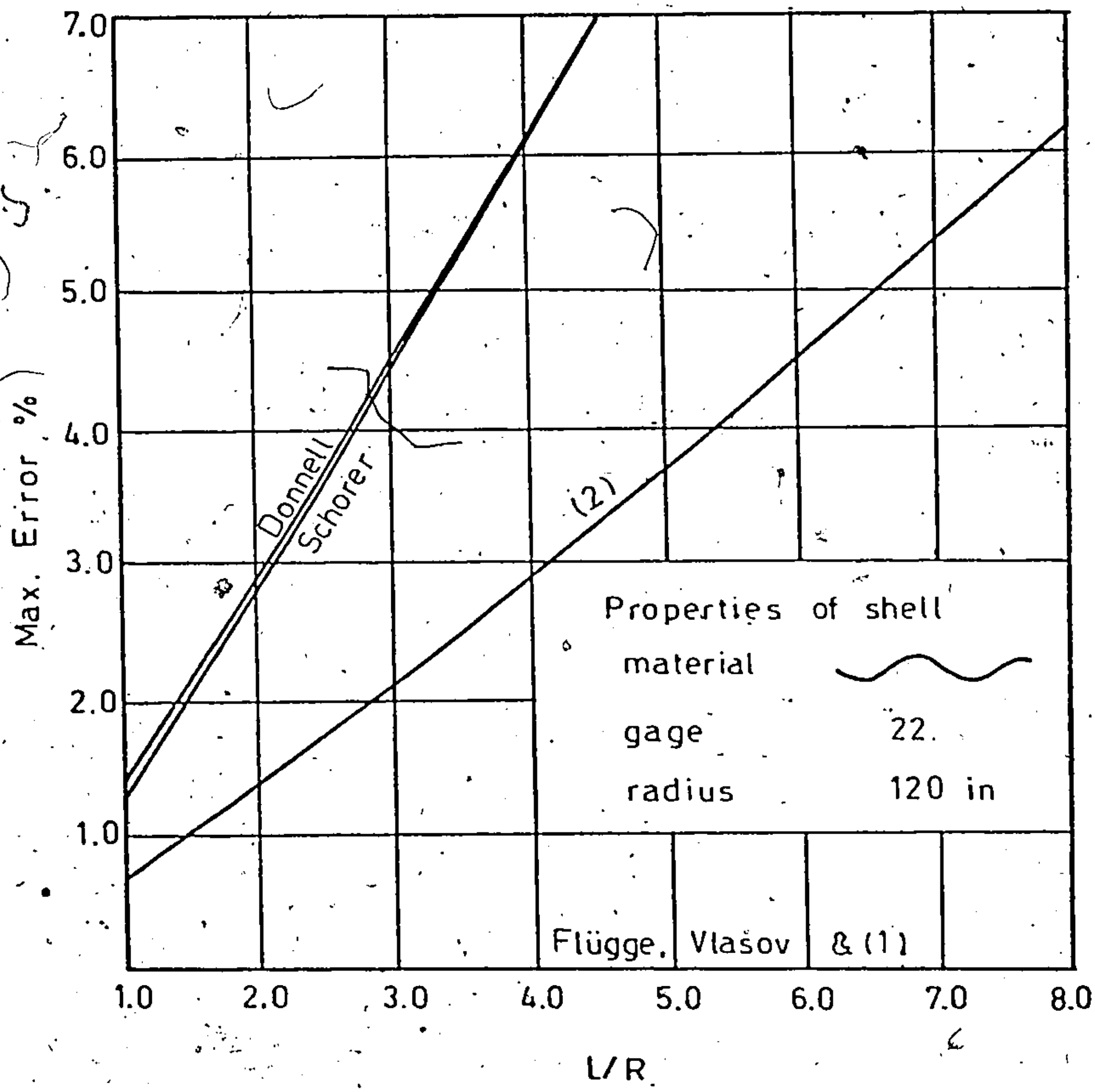


FIG. 3.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR STANDARD CORRUGATIONS WITH R = 120in

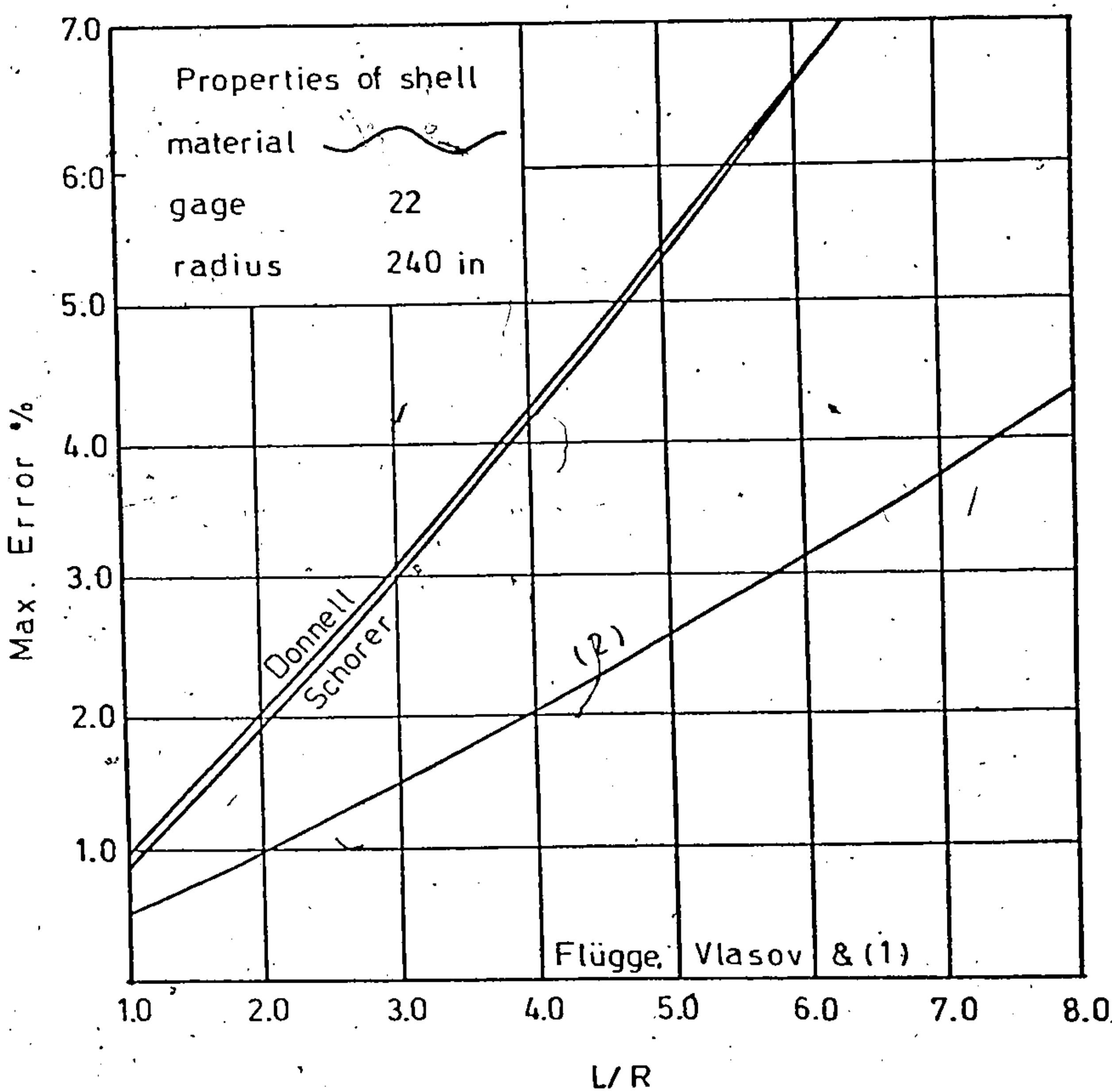


FIG. 4.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR STANDARD CORRUGATIONS WITH R = 240in

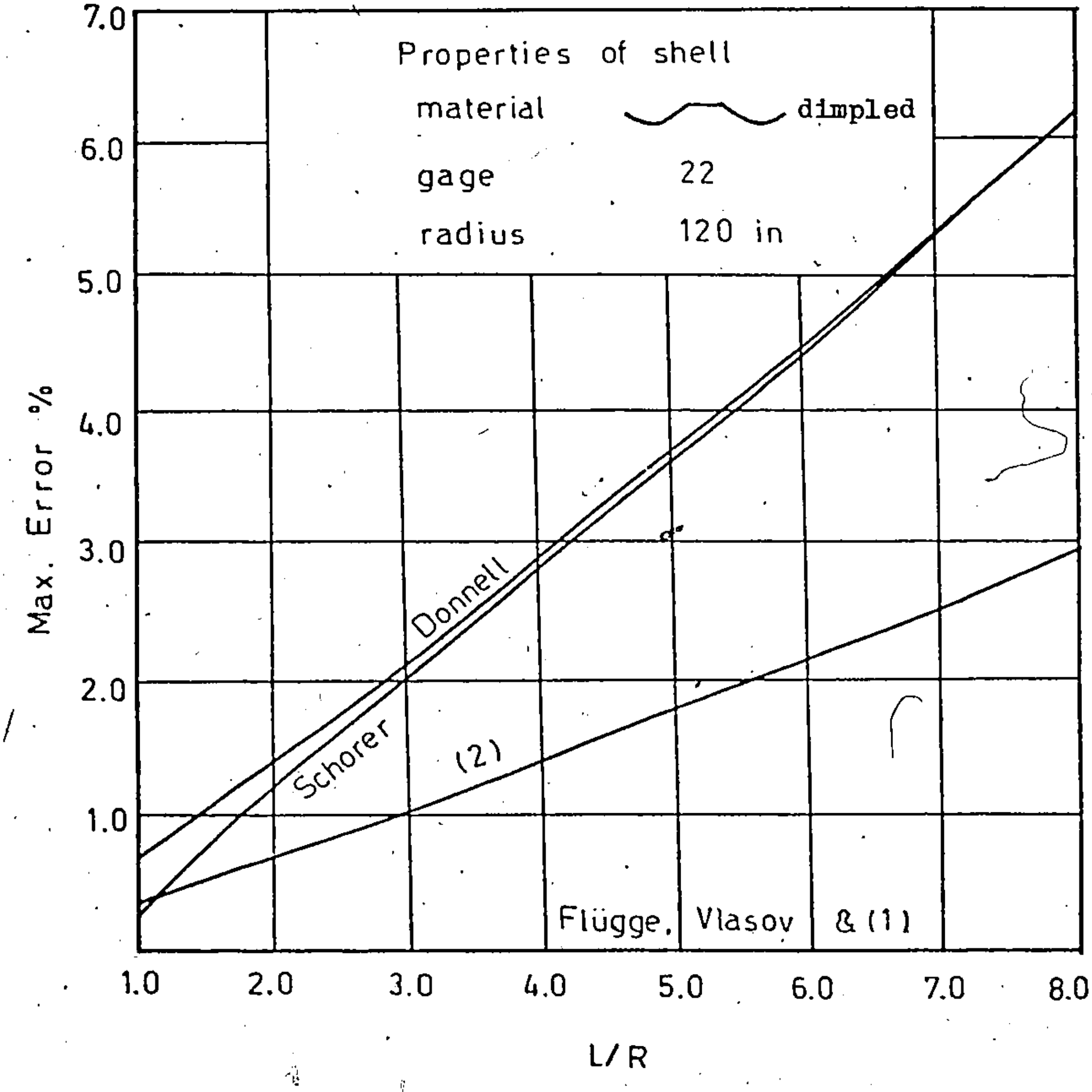


FIG. 5.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR DIMPLED CORRUGATIONS WITH R = 120in

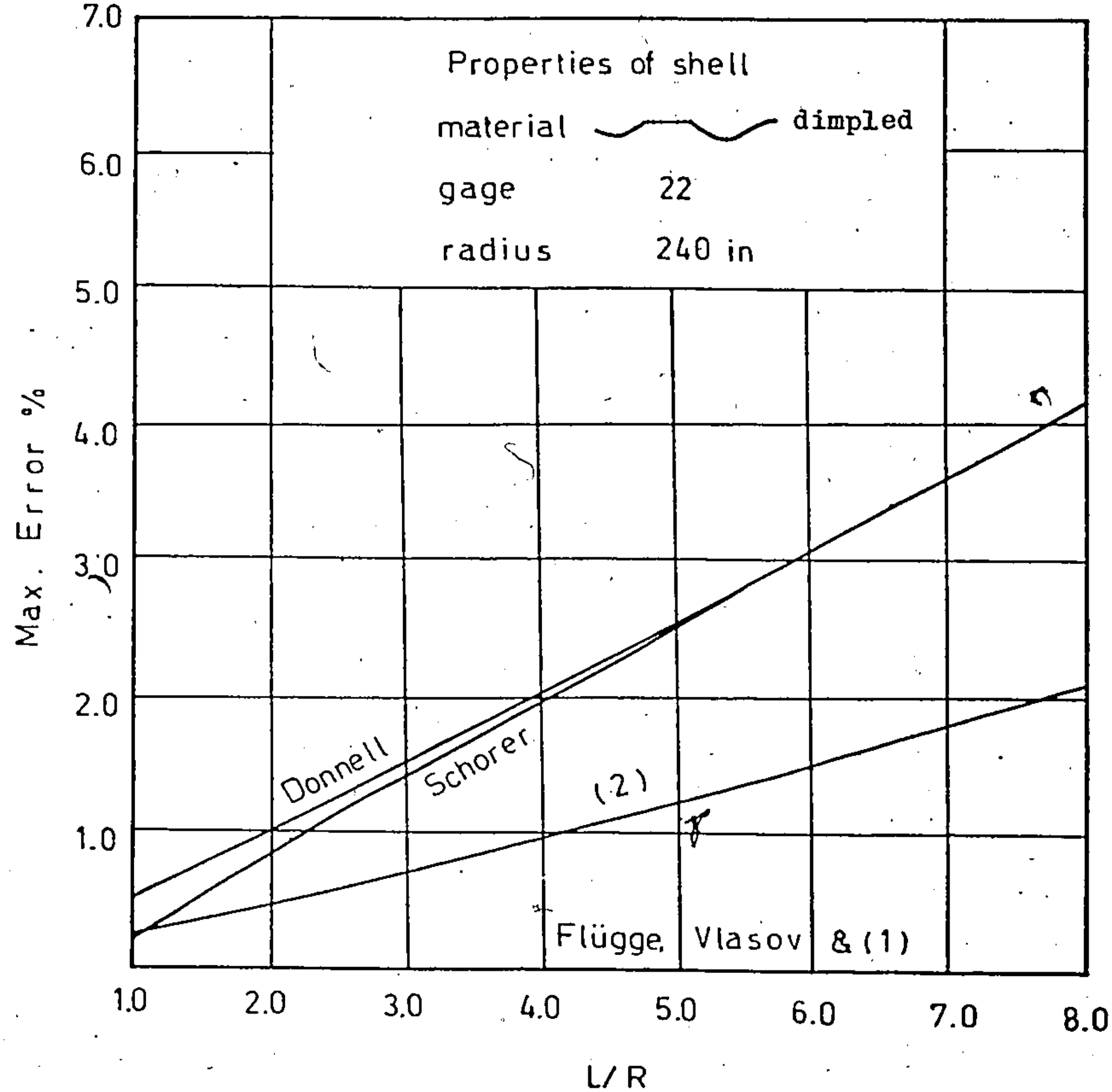


FIG. 6.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR DIMPLED CORRUGATIONS WITH R = 240in

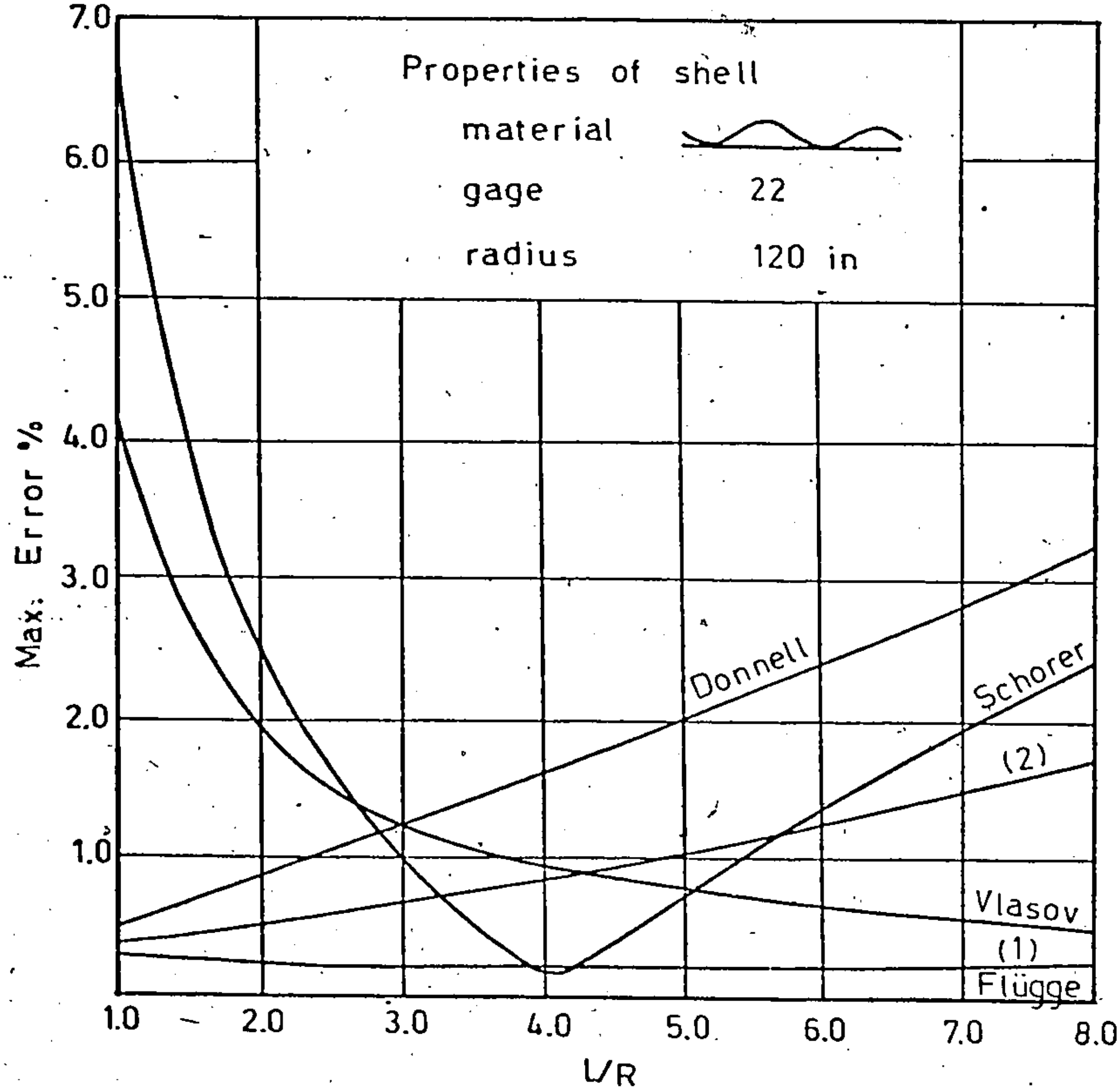


FIG. 7.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR SPOT WELDED CORRUGATIONS WITH R=120in

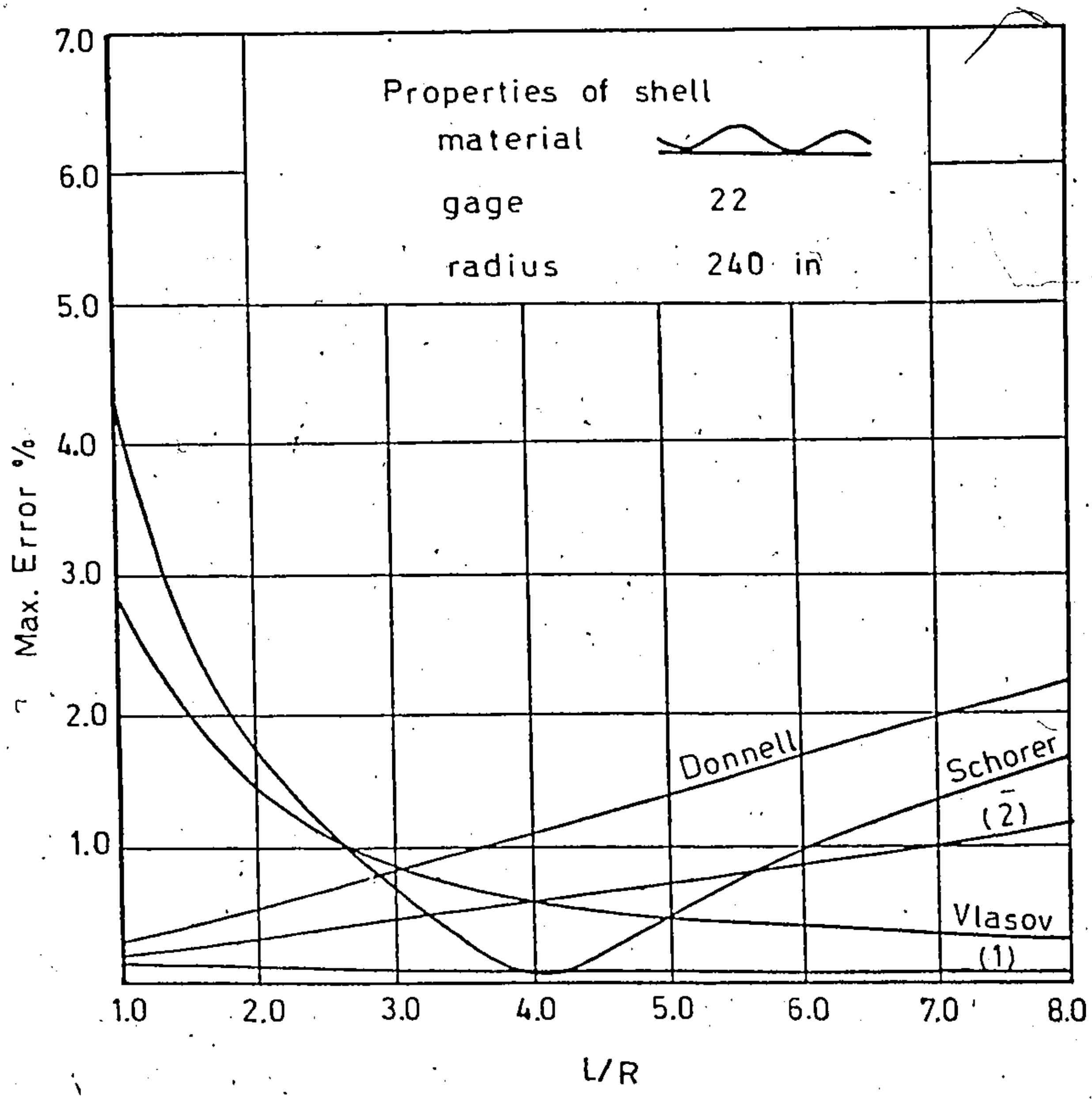


FIG. 8.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR SPOT WELDED CORRUGATIONS WITH R=240in

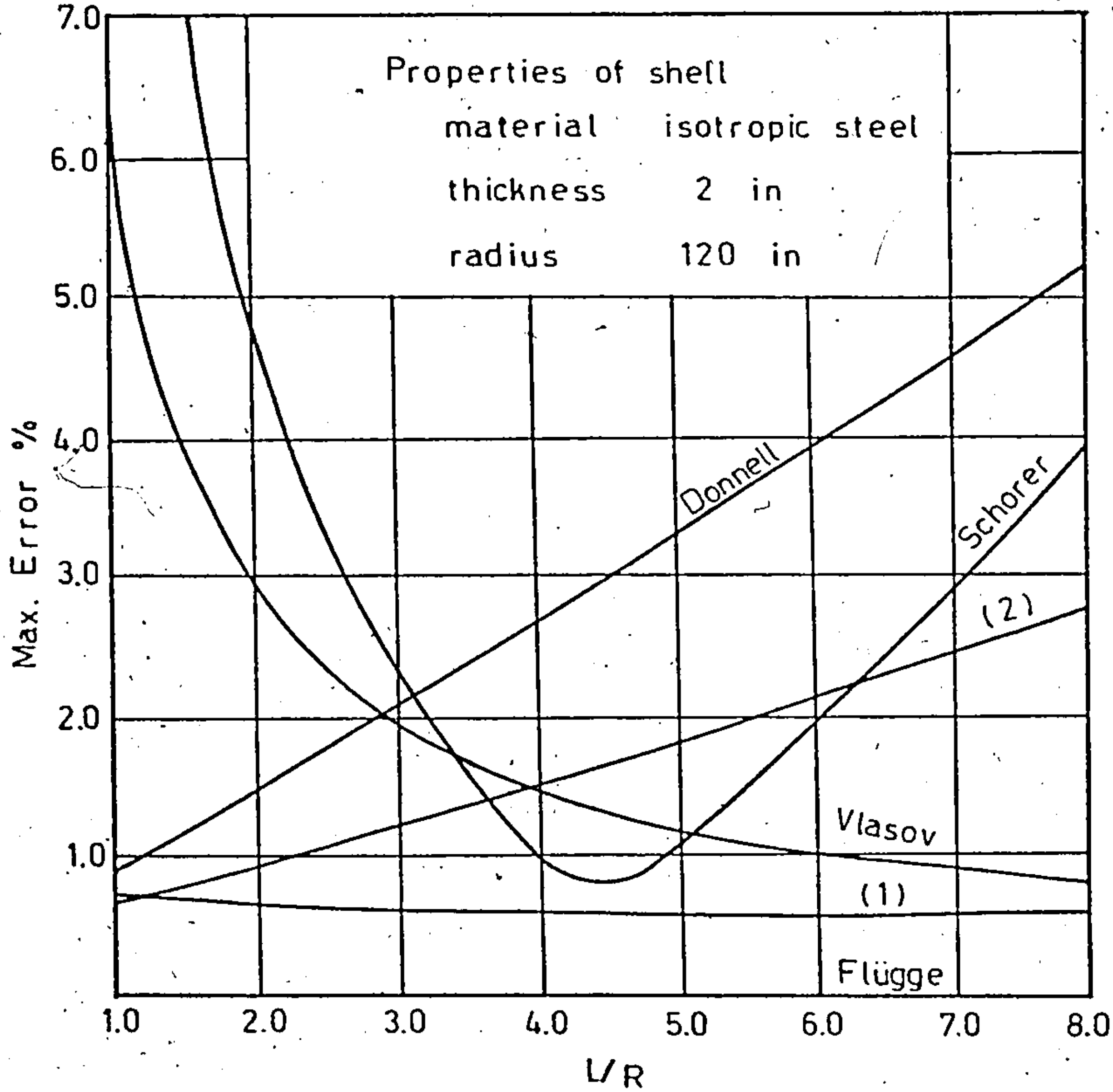


FIG. 9.

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR ISOTROPIC SHELLS WITH R = 120 in

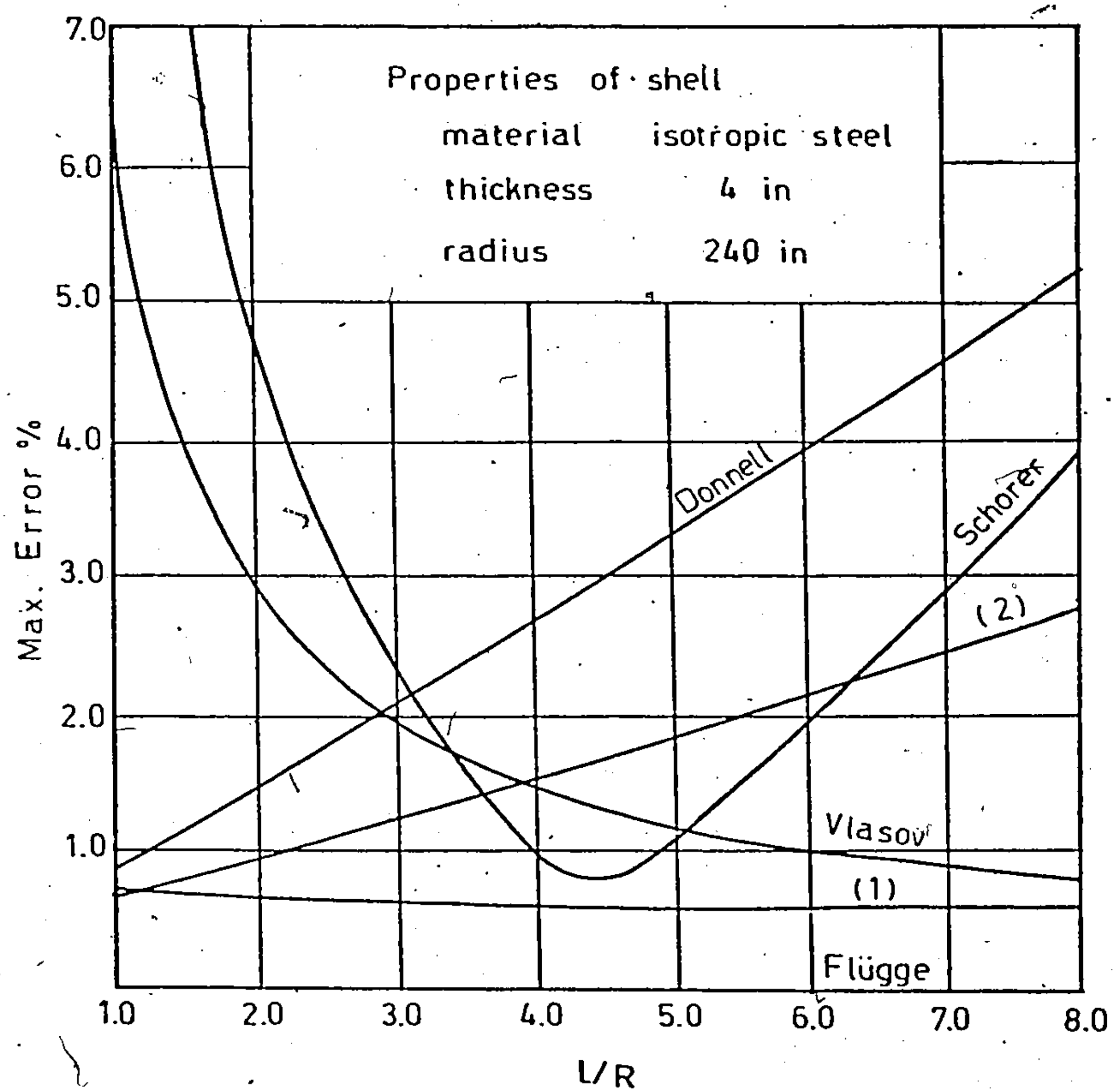
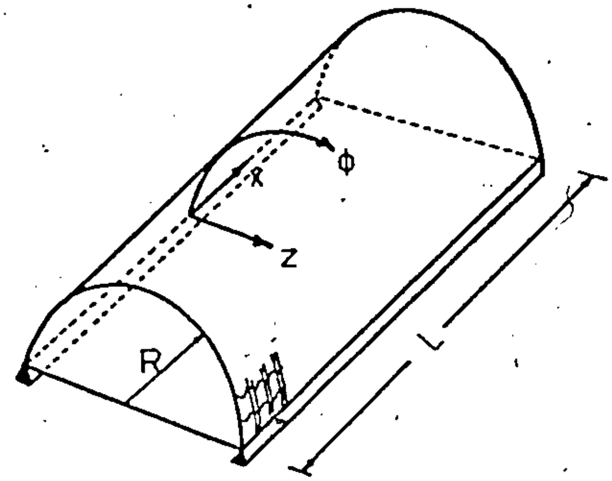


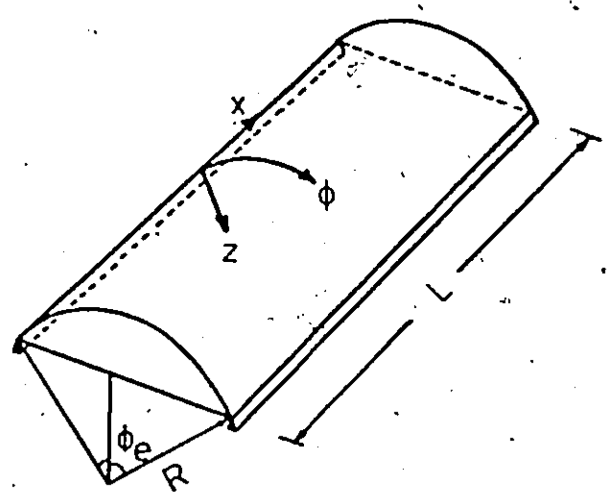
FIG.10

MAXIMUM ERROR IN THE ROOTS OF THE CHARACTERISTIC EQUATIONS FOR ISOTROPIC SHELLS WITH R = 240 in

(a)



(b)



(c)

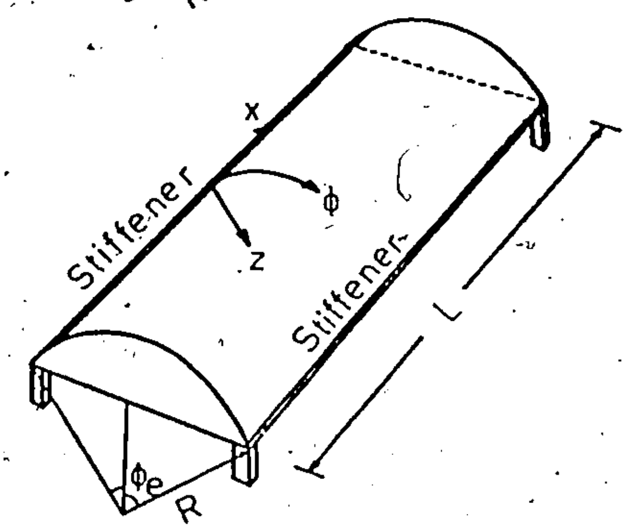


FIG.11.SHELLS CONSIDERED IN EXAMPLES

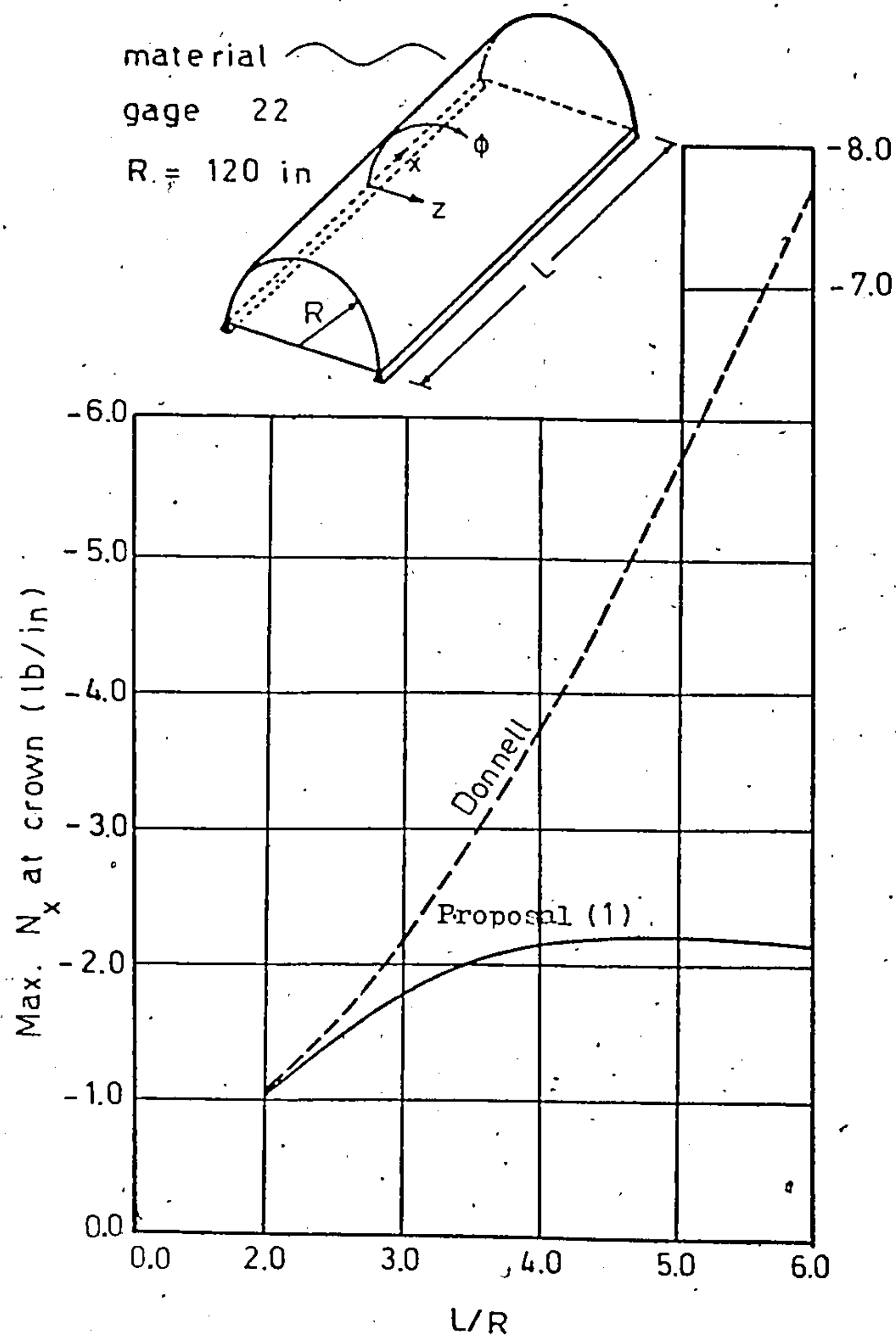


FIG.12.

MAXIMUM N_x UNDER 1 psf UNIFORM SNOW

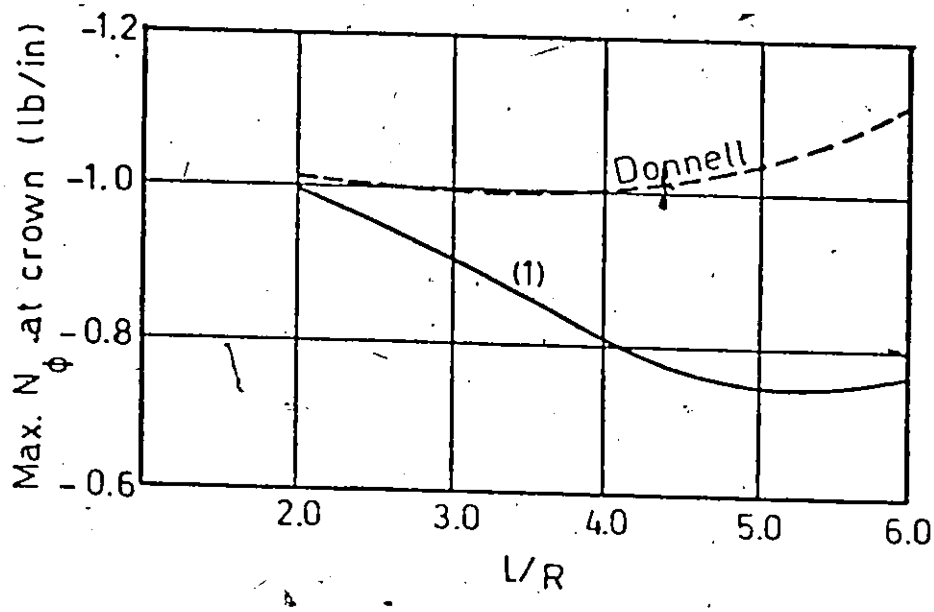


FIG.13.

MAXIMUM N_σ UNDER 1 psf

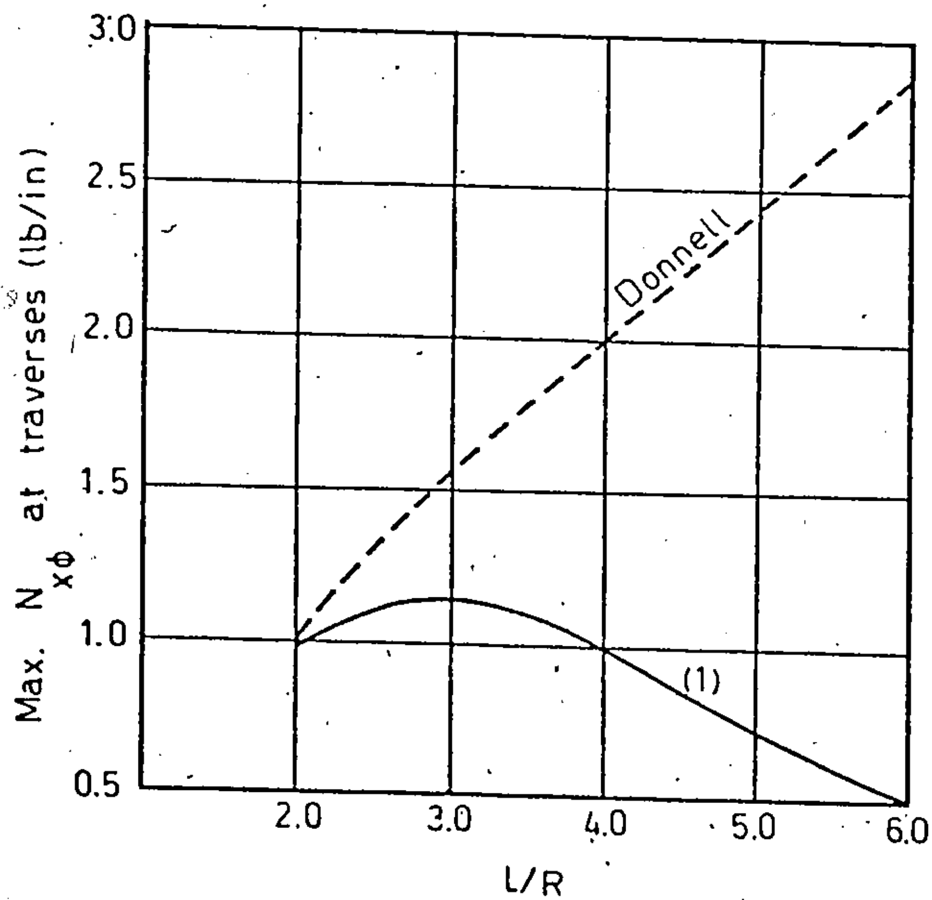
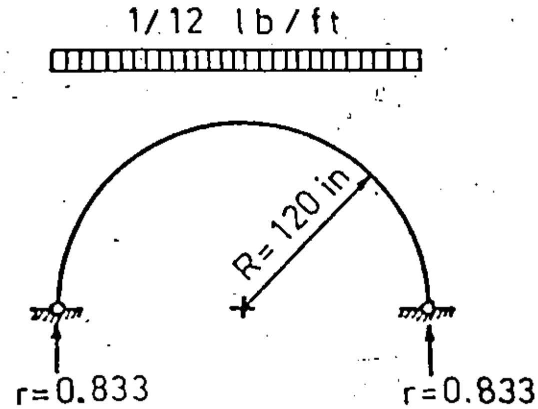


FIG.14.

MAXIMUM $N_{x\sigma}$ UNDER 1 psf



Full arch action

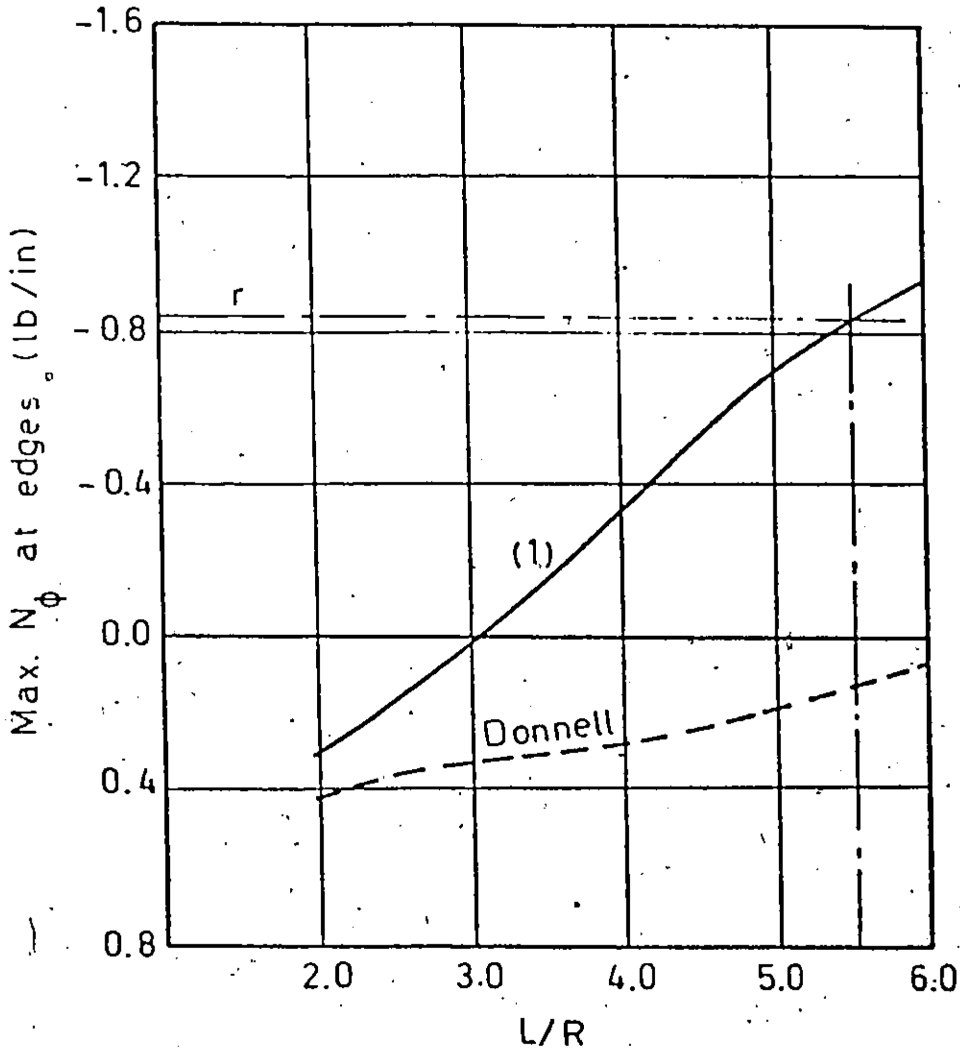


FIG.15.

MAXIMUM N_ϕ AT EDGES

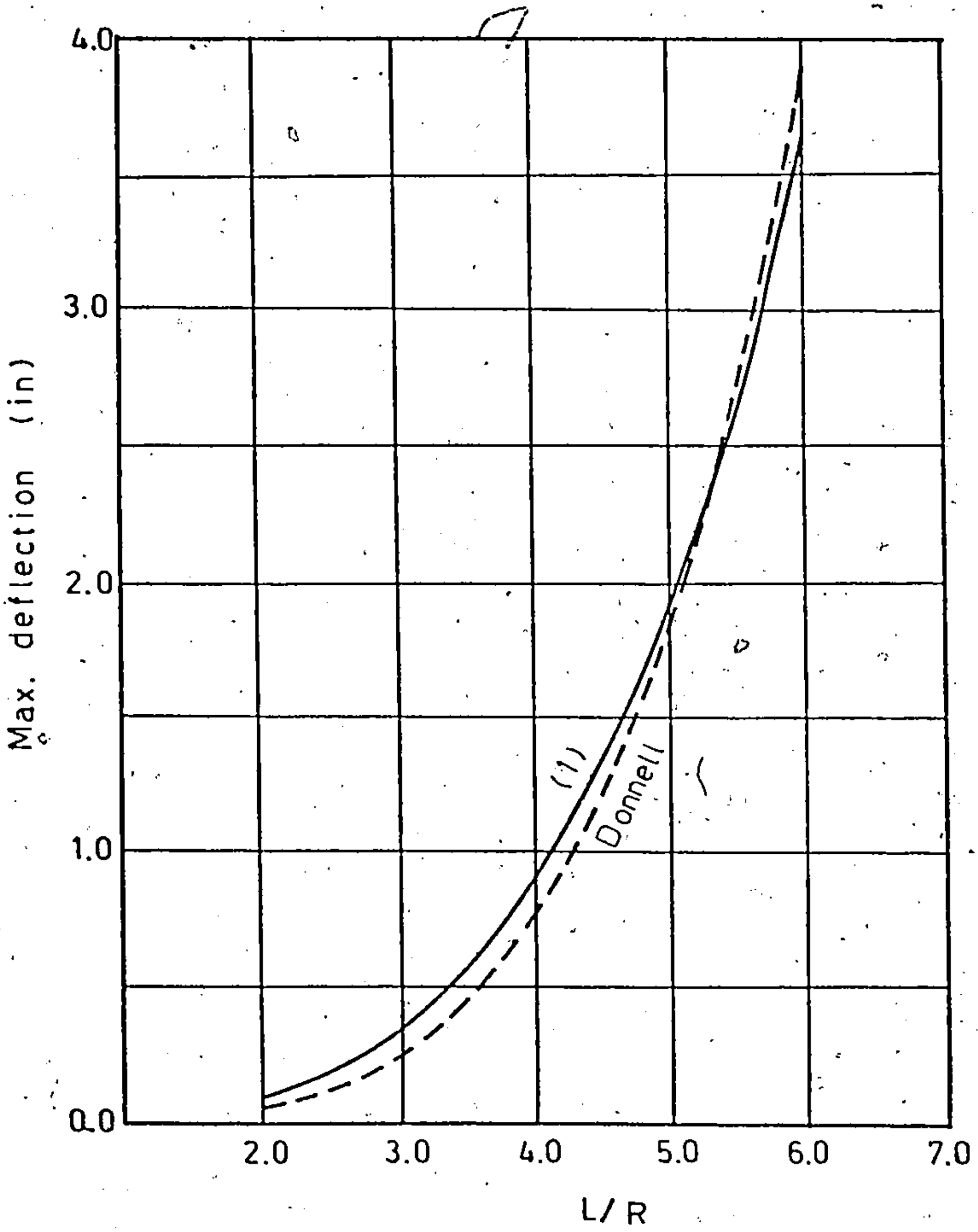


FIG.16.

MAXIMUM DEFLECTION

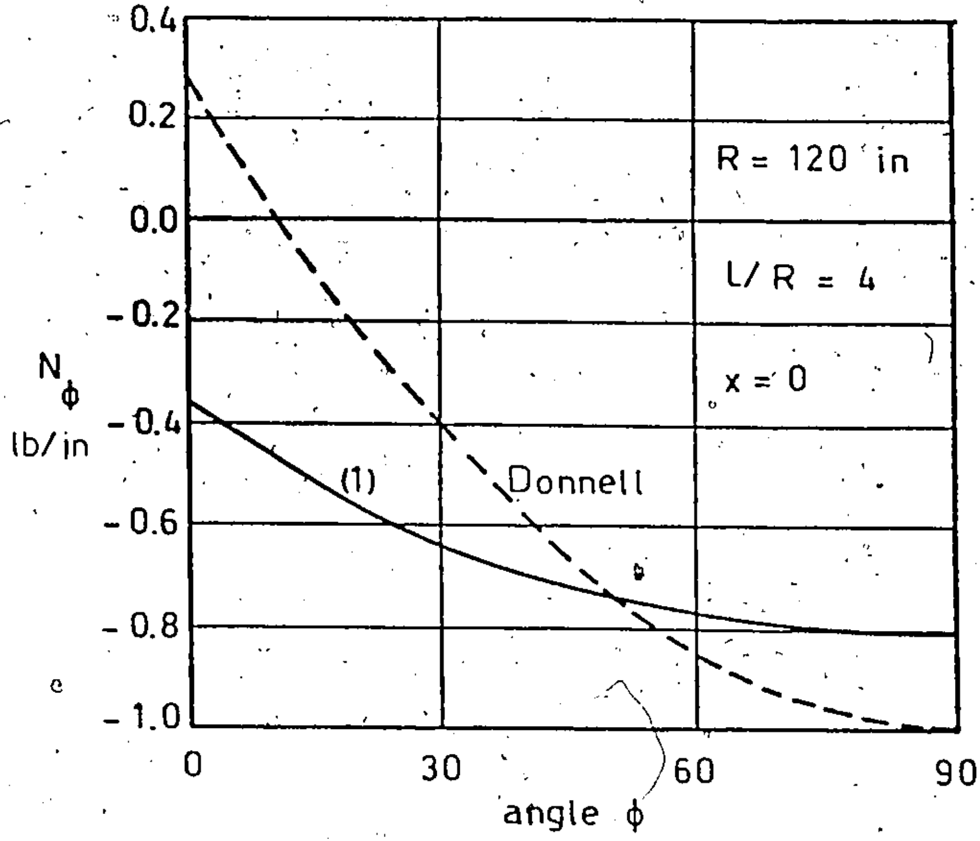


FIG.17.
DISTRIBUTION OF N_ϕ

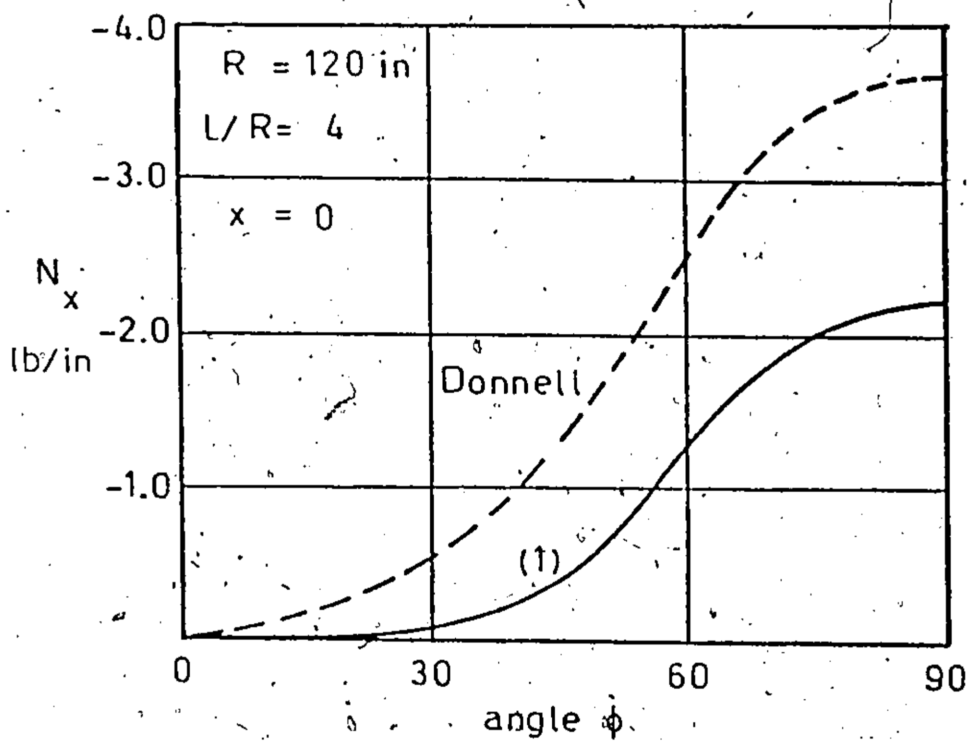


FIG.18.
DISTRIBUTION OF N_x

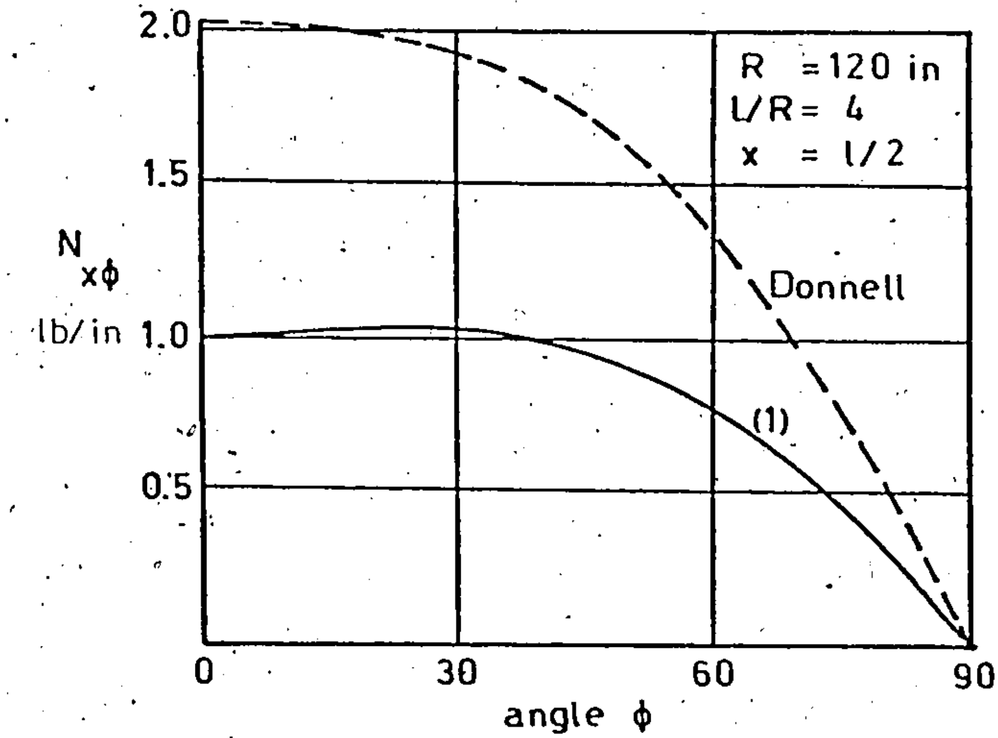


FIG.19.
DISTRIBUTION OF $N_{x\phi}$

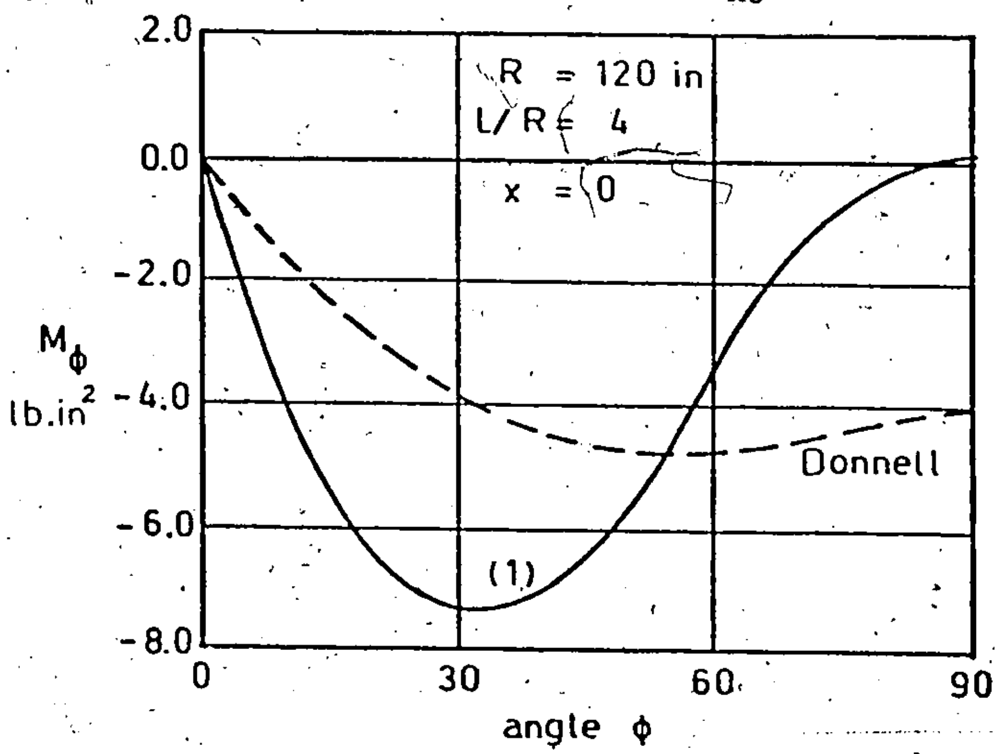


FIG.20.
DISTRIBUTION OF M_{ϕ}

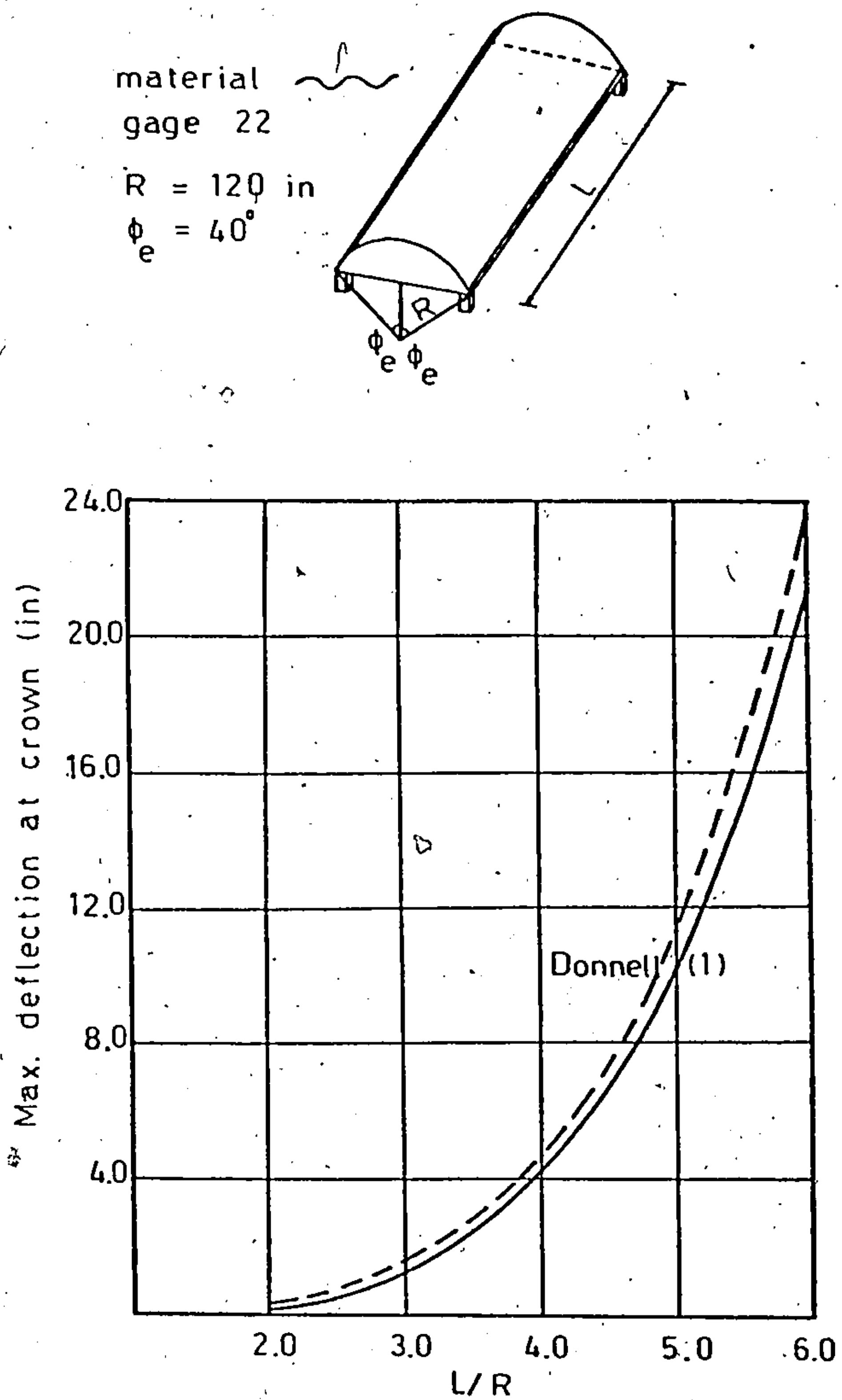


FIG. 21-a

MAXIMUM DEFLECTIONS

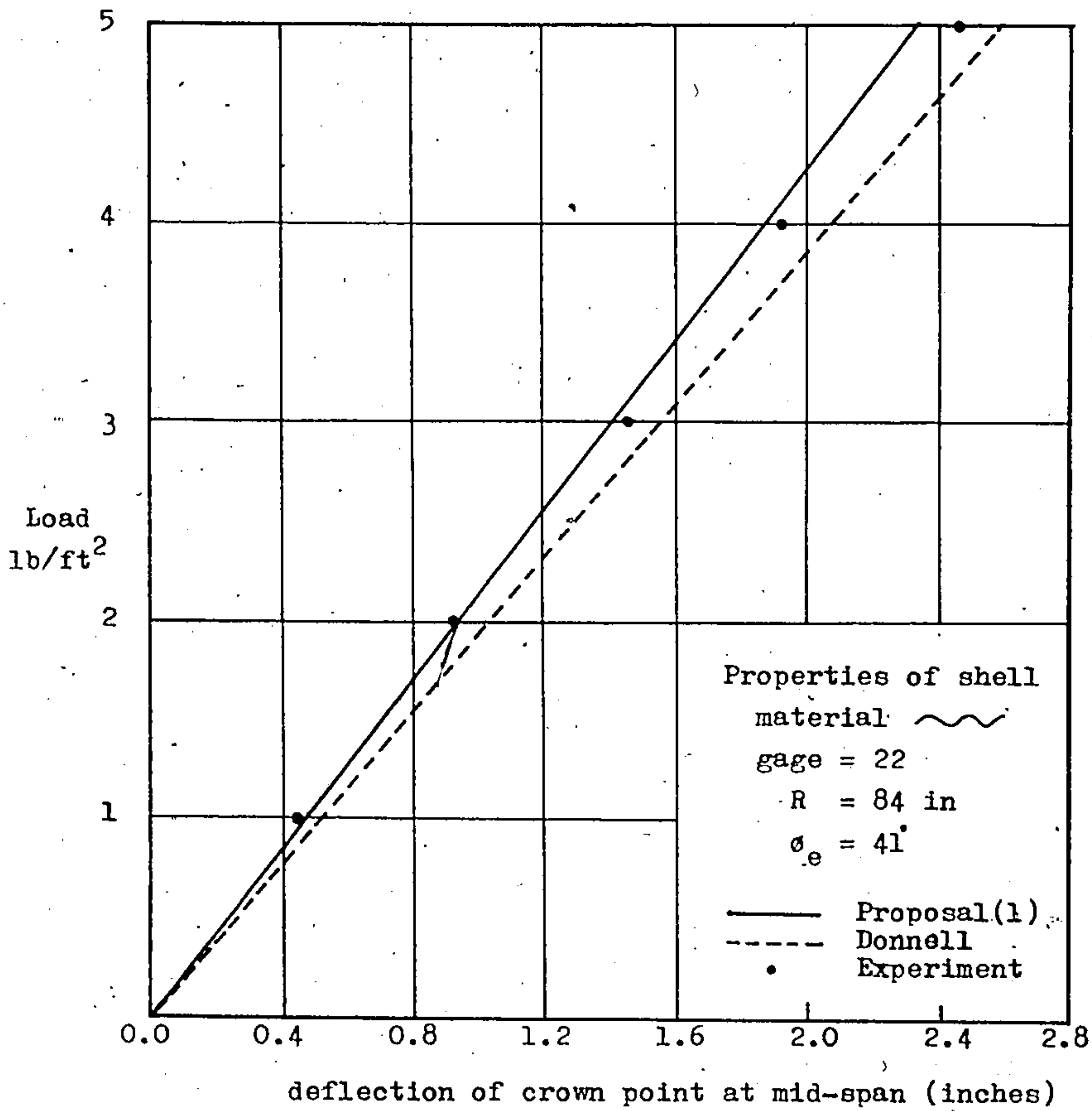


FIG.21-b

MAXIMUM DEFLECTION OF A SHELL WITH LONGITUDINAL STIFFENERS IN VALLEYS

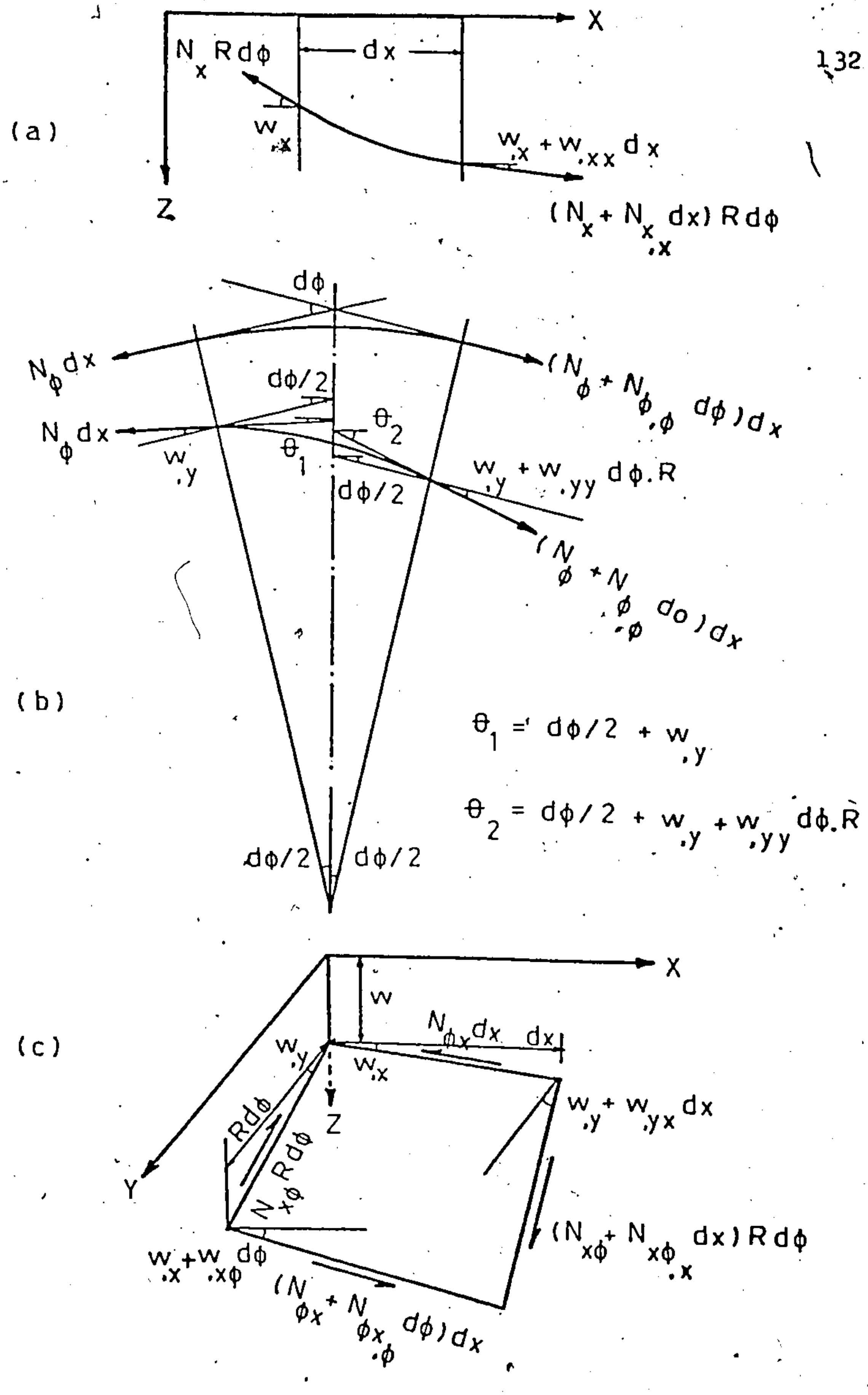


FIG. 22. DEFORMED ELEMENT

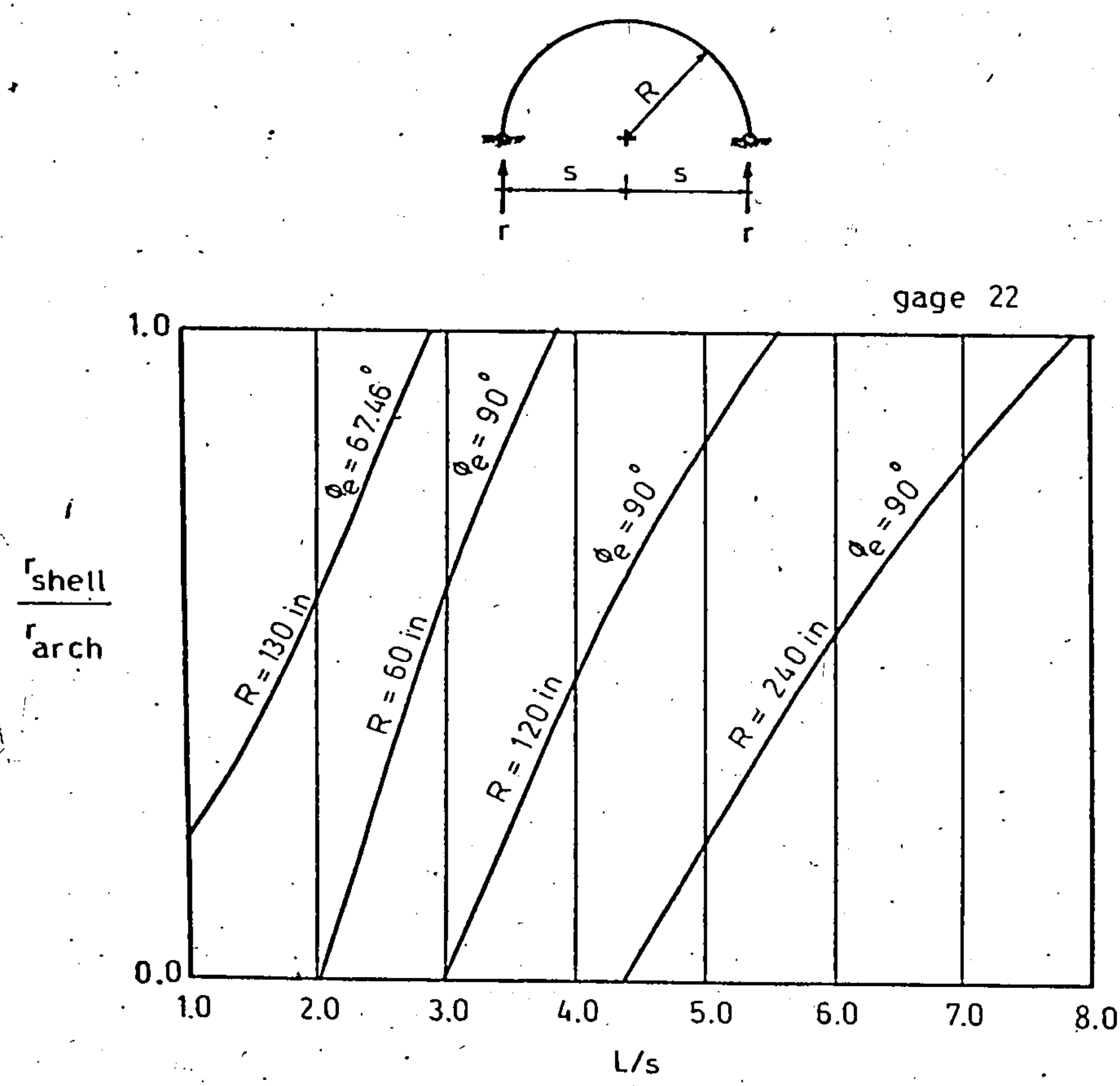


FIG.23. LIMITS FOR FULL ARCH ACTION
(standard sheets)

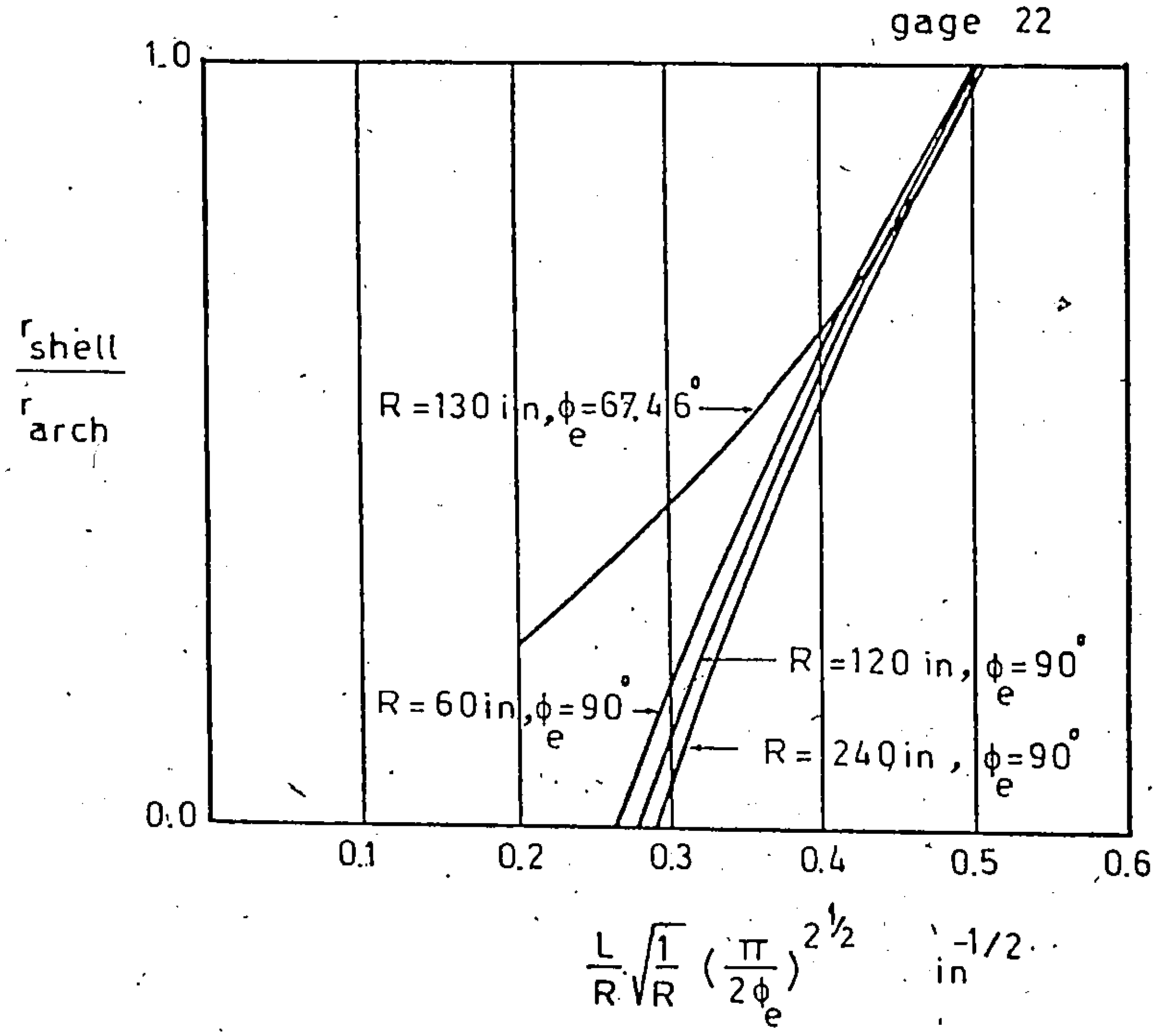


FIG.24. LIMIT FOR FULL ARCH ACTION

(standard sheets)

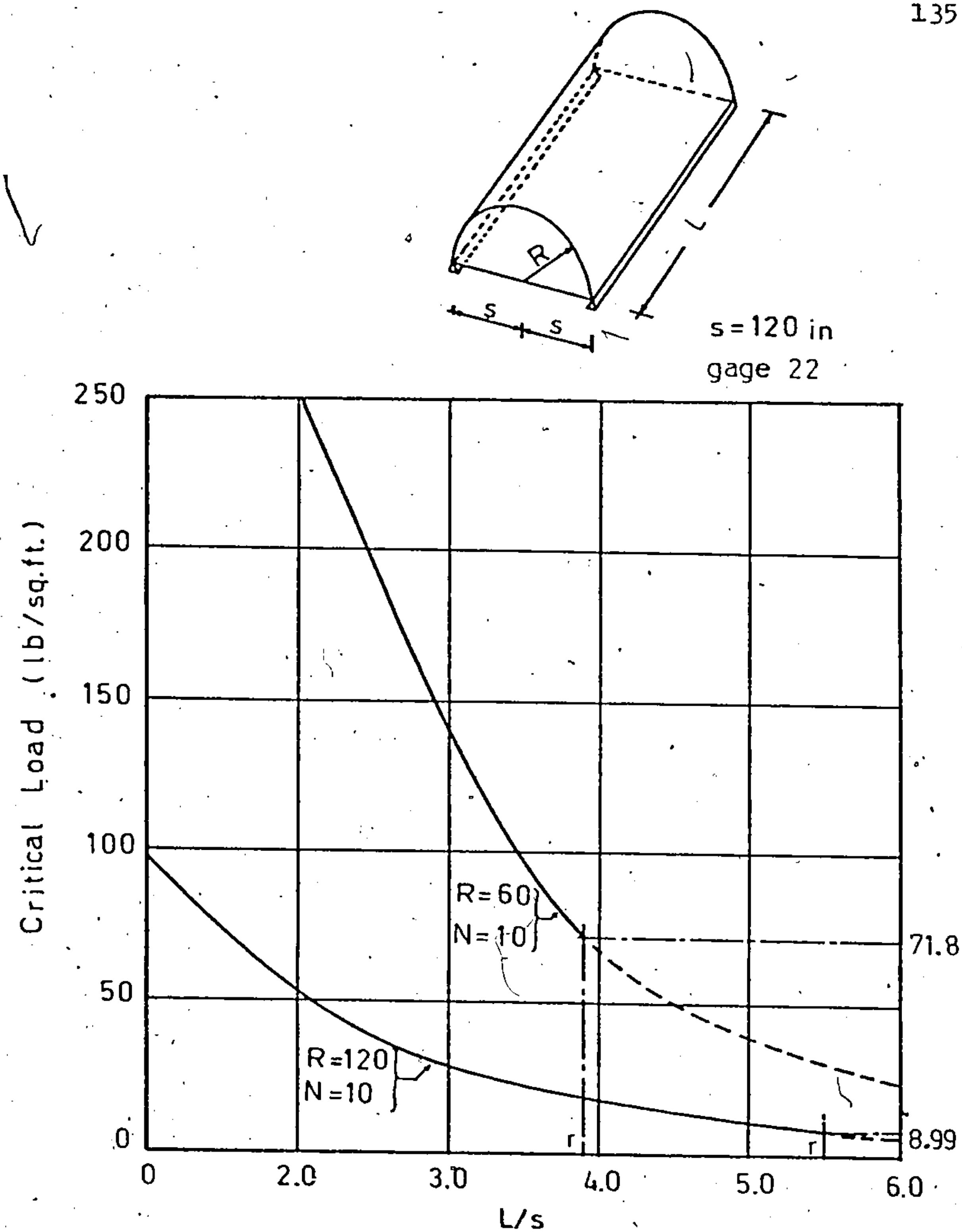


FIG.25. BUCKLING LOADS
(uniform snow load)

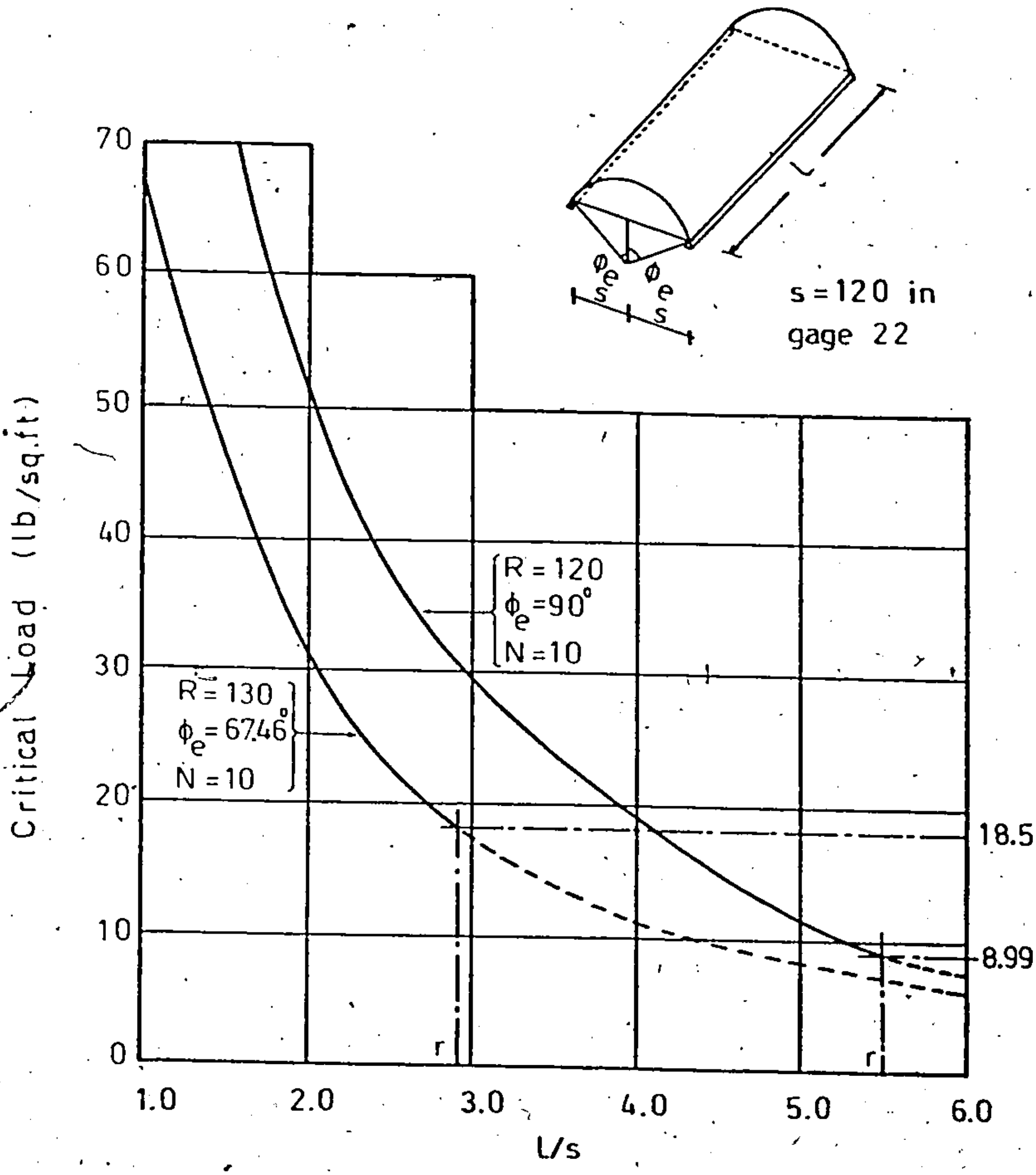


FIG.26. BUCKLING LOADS
(uniform snow load)

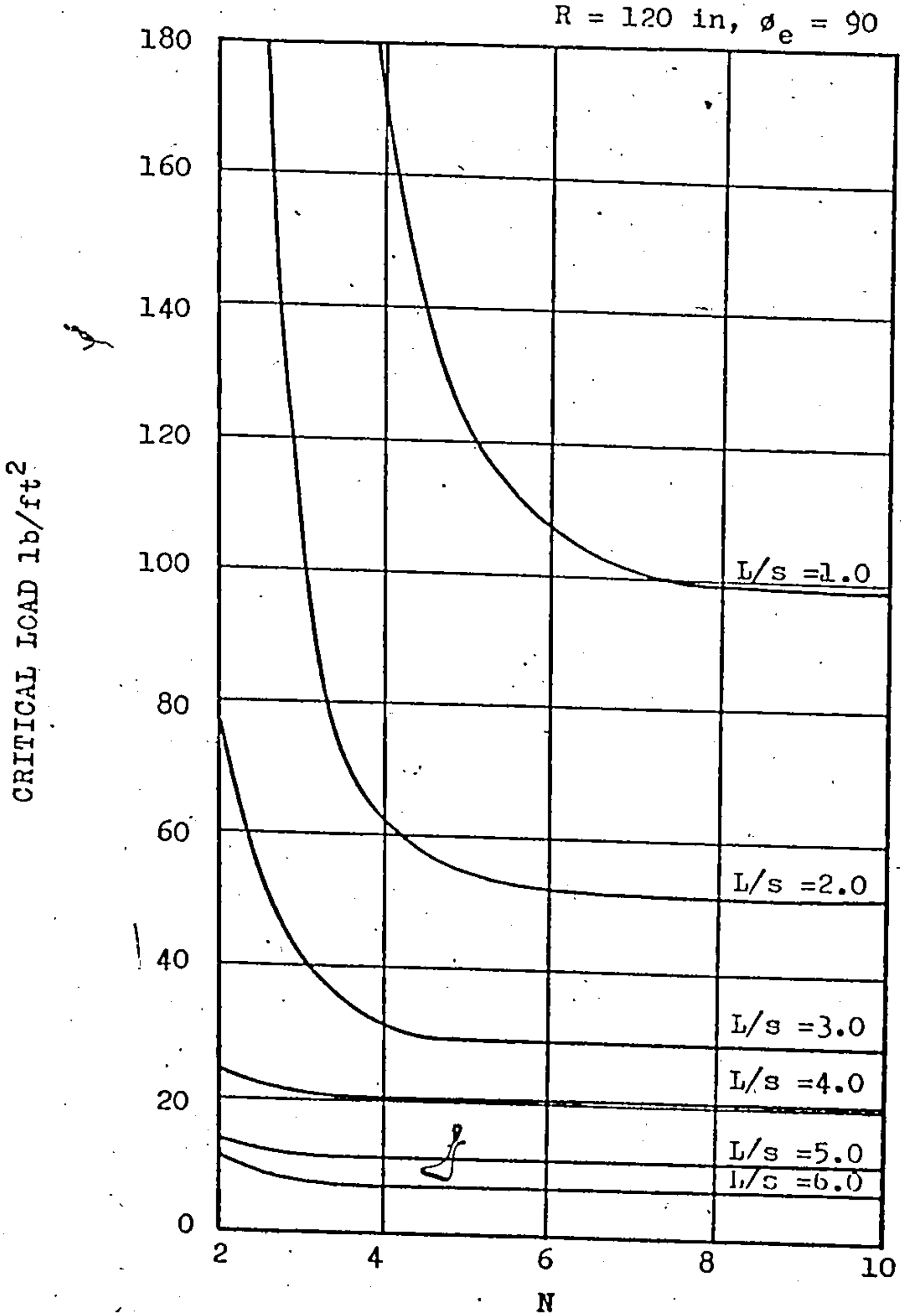


FIG.27. CONVERGENCE OF SOLUTION WITH N

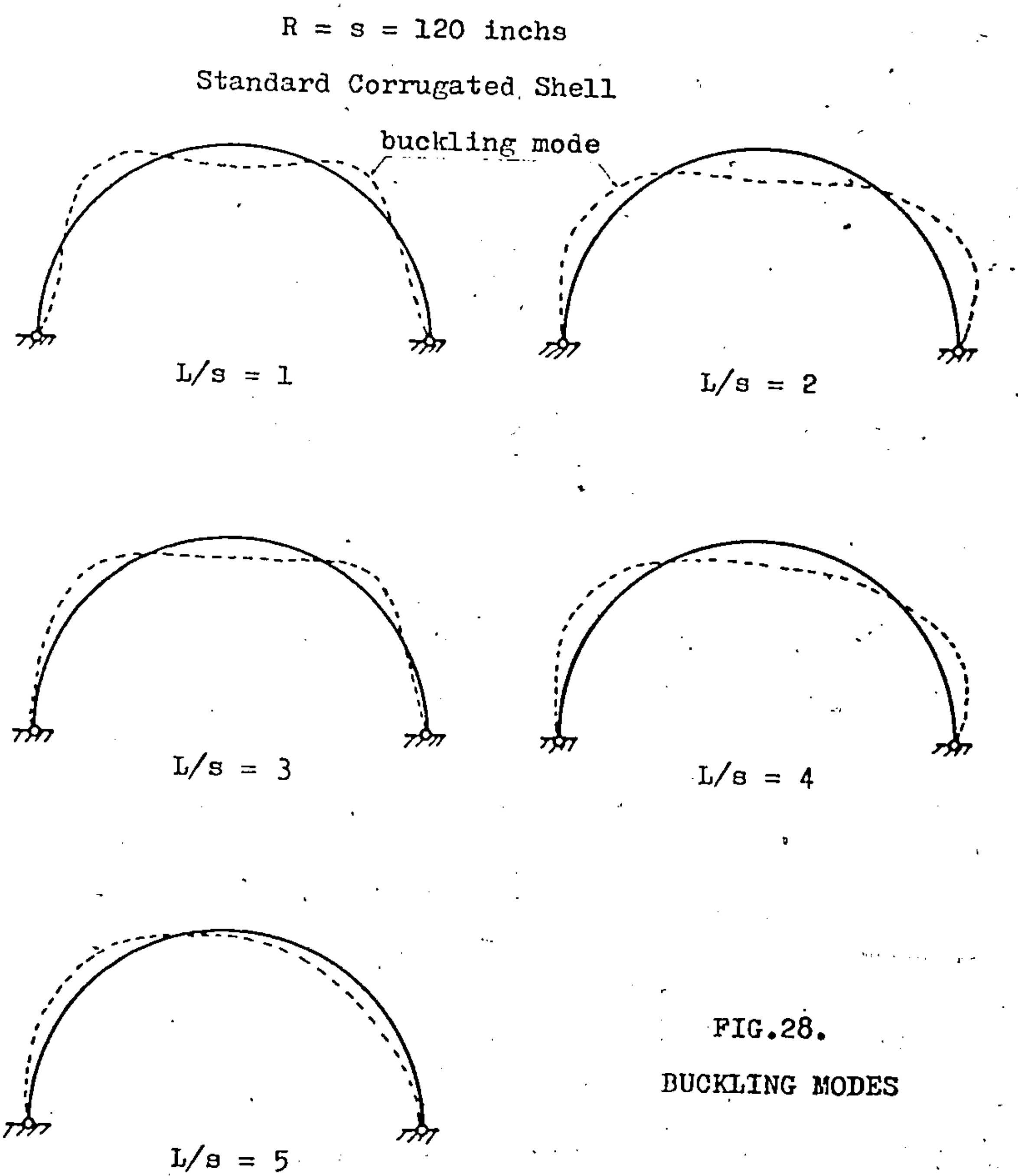


FIG.28.

BUCKLING MODES

APPENDIX (I)

PROPOSED GOVERNING EQUATION No. (1).

The equilibrium equations due to proposal No. 1 are:

$$D_x u^{''} + D_{x\phi}(u^{\bullet\bullet} + v^{'\bullet}) = 0 \qquad (\text{for } p_x = 0) \tag{a}$$

$$D_\phi(v^{\bullet\bullet} - w^{\bullet}) + D_{x\phi}(u^{\bullet '} + v^{''}) + R^2 p_\phi = 0 \tag{b}$$

$$\frac{-B_\phi}{R^2}(w^{::} + 2w^{\bullet\bullet} + w) - \frac{B_x}{R^2} w^{''''} - \frac{B_{x\phi}}{R^2} 2w^{''\bullet\bullet} + D_\phi(v^{\bullet} - w) + R^2 p_z = 0 \tag{c}$$

Applying $(\ )^{''}$ to (a), then

$$v^{'''} = \frac{1}{D_{x\phi}}(-D_x u^{''''} - D_{x\phi} u^{''\bullet\bullet}) \tag{d}$$

Applying $(\ )^{\bullet\bullet}$ to (a), then

$$v^{'\bullet\bullet\bullet} = \frac{1}{D_{x\phi}}(-D_x u^{''\bullet\bullet} - D_{x\phi} u^{\bullet\bullet\bullet\bullet}) \tag{e}$$

And applying $(\ )^{\bullet '}$ to (b), gives

$$D_\phi(v^{\bullet\bullet\bullet '} - w^{\bullet\bullet '}) + D_{x\phi}(u^{\bullet\bullet ''} + v^{'''\bullet}) + R^2 p_\phi^{\bullet '} = 0 \tag{f}$$

Substitute (d) & (e) into (f)

$$\frac{-D_\phi}{D_{x\phi}} D_x u^{''\bullet\bullet} - D_\phi u^{\bullet\bullet\bullet\bullet} - D_\phi w^{\bullet\bullet '} + D_{x\phi} u^{\bullet\bullet ''} - D_x u^{''''} - D_{x\phi} u^{\bullet\bullet ''} + R^2 p_\phi^{\bullet '} = 0 \tag{g}$$

Applying $(\ )^{''}$ to (b), then

$$u^{\bullet '''} = \frac{1}{D_{x\phi}}(-D_\phi v^{\bullet\bullet ''} + D_\phi w^{\bullet ''} - D_{x\phi} v^{''''} - R^2 p_\phi^{''}) \tag{h}$$

Applying $(\ )^{\bullet\bullet}$ to (b), then

$$u^{'\bullet\bullet\bullet} = \frac{1}{D_{x\phi}}(-D_\phi v^{\bullet\bullet\bullet\bullet} + D_\phi w^{\bullet\bullet\bullet} - D_{x\phi} v^{''\bullet\bullet} - R^2 p_\phi^{\bullet\bullet}) \tag{i}$$

And applying $(\ )^{\cdot\prime}$ to (a), yields

$$D_x u'''' + D_{x\phi}(u^{\prime\cdot\cdot\cdot} + v^{\prime\prime\cdot\cdot}) = 0$$

Substituting (h) & (i), then

$$- \frac{D_xD_\phi}{D_{x\phi}} v^{\cdot\cdot\prime\prime} + \frac{D_xD_\phi}{D_{x\phi}} w^{\cdot\prime\prime} - D_x v'''' - \frac{D_x}{D_{x\phi}} R^2 p_\phi'' -$$

$$D_\phi v^{\cdot\cdot\cdot\prime} + D_\phi w^{\cdot\cdot\cdot} - D_{x\phi} v^{\prime\prime\cdot\cdot} - R^2 p_\phi^{\cdot\cdot} + D\ \ v^{\prime\prime\cdot\cdot} = 0$$

Applying $D_\phi(\ )^{\cdot}$, hence

$$- \frac{D_xD_\phi^2}{D_{x\phi}} v^{\prime\prime\cdot\cdot} - D_xD_\phi v^{\prime\prime\prime\prime\cdot} - D_\phi^2 v^{::\cdot} = - \frac{D_xD_\phi^2}{D_{x\phi}} w^{\prime\prime\cdot\cdot} +$$

$$\frac{R^2 D_xD_\phi}{D_{x\phi}} p_\phi^{\prime\prime\cdot} + D_\phi^2 w^{\cdot\cdot\cdot\cdot} - R^2D_\phi p_\phi^{\cdot\cdot\cdot} \qquad \text{(k)}$$

Applying ∇^4 to (c), in which

$$\nabla^4 = D_x(\ )'''' + \frac{D_xD_\phi}{D_{x\phi}} (\ )^{\prime\prime\cdot\cdot} + D_\phi(\ )^{\cdot\cdot\cdot\cdot} \qquad \text{gives,}$$

$$- \frac{D_xB_\phi}{R^2} (w^{::!!} + 2w^{:!!} + w^{!!}) - \frac{D_xB_x}{R^2} w^{!!!!} - \frac{D_xB_{x\phi}}{R^2} 2w^{!!!:}$$

$$+ D_\phi D_x(v^{!!\cdot} - w^{!!}) + D_xR^2p_z^{!!} - \frac{D_xD_\phi B_\phi}{R^2D_{x\phi}} (w^{:::!} + 2w^{::!} + w^{!:})$$

$$- \frac{D_xD_\phi B_x}{R^2D_{x\phi}} w^{!!!:} - \frac{D_xD_\phi B_{x\phi}}{R^2D_{x\phi}} (2w^{!!::}) + \frac{D_xD_\phi^2}{D_{x\phi}}(v^{!:\cdot} - w^{!:})$$

$$+ \frac{R^2D_xD_\phi}{D_{x\phi}} p_z^{!:} - \frac{D_\phi B_\phi}{R^2} (w^{::::} + 2w^{:::} + w^{::}) - \frac{D_\phi B_x}{R^2} w^{!!::}$$

$$- \frac{D_xB_{x\phi}}{R^2} 2w^{!:::} + D_\phi^2(v^{::\cdot} - w^{::}) + D_\phi R^2p_z^{::} = 0$$

Rearranging

$$- \frac{D_x B_x}{R^2} w^{''''} - \left(\frac{2D_x B_{x\phi}}{R^2} + \frac{D_x D_\phi B_x}{R^2 D_{x\phi}} \right) w^{''':} - \frac{D_\phi B_\phi}{R^2} w^{::::}$$

$$- \left(\frac{D_x B_\phi}{R^2} + \frac{2D_x D_\phi B_{x\phi}}{R^2 D_{x\phi}} + \frac{D_\phi B_x}{R^2} \right) w^{''::} - \left(\frac{D_x D_\phi B_\phi}{R^2 D_{x\phi}} + \right.$$

$$\left. \frac{2D_\phi B_{x\phi}}{R^2} \right) w^{':::} - \frac{2D_x B_\phi}{R^2} w^{:''} - \frac{2D_x D_\phi B_\phi}{R^2 D_{x\phi}} w^{::'} - \frac{2D_\phi B_\phi}{R^2} w^{:::}$$

$$- \left(\frac{D_x B_\phi}{R^2} + D_\phi D_x \right) w^{''} - \left(\frac{D_x D_\phi B_\phi}{R^2 D_{x\phi}} + \frac{D_x D_\phi^2}{D_{x\phi}} \right) w^{:'} - \left(D_\phi^2 + \right.$$

$$\left. \frac{D_\phi B_\phi}{R^2} \right) w^{::} + \left(D_\phi D_x v^{''\cdot} + \frac{D_x D_\phi^2}{D_{x\phi}} v^{'\cdot\cdot} + D_\phi^2 v^{::\cdot} \right) + D_x R^2 p_z^{''}$$

$$+ \frac{D_x D_\phi}{D_{x\phi}} R^2 p_z^{':} + D_\phi R^2 p_z^{::} = 0 \qquad \text{(L)}$$

Substituting (k) into (L), yields the following governing equation:

$$- D_x B_x w^{''''} - \left(2D_x B_{x\phi} + \frac{D_x D_\phi B_x}{D_{x\phi}} \right) w^{''':} - \left(D_x B_\phi + \right.$$

$$\left. \frac{2D_x D_\phi B_{x\phi}}{D_{x\phi}} \right) w^{''::} - \left(\frac{D_x D_\phi B_\phi}{D_{x\phi}} + 2D_\phi B_{x\phi} \right) w^{':::} - D_\phi B_\phi w^{::::}$$

$$- 2D_x B_\phi w^{:''} - \frac{2D_x D_\phi B_\phi}{D_{x\phi}} w^{::'} - 2D_\phi B_\phi w^{:::} - D_\phi B_\phi w^{::}$$

$$- \left(D_x B_\phi + R^2 D_\phi D_x \right) w^{''} - \frac{D_x D_\phi B_\phi}{D_{x\phi}} w^{:'} + D_x R^4 p_z^{''}$$

$$+ \frac{D_x D_\phi R^4}{D_{x\phi}} p_z^{':} - \frac{R^4 D_x D_\phi}{D_{x\phi}} p_\phi^{'\cdot} + R^4 D_\phi p_z^{::} - R^4 D_\phi p_\phi^{:\cdot} = 0$$

.....(proposed equation No. 1)

APPENDIX (II)

CHARACTERISTIC EQUATIONS AND ERRORS.

STANDARD CORRUGATED SHEET SHELL

```
CHARACTERISTIC EQUATIONS FOR L/R= 1.0,R=120.0
FLUGGE        (M**8)+0.18917E 01(M**6)+0.12626E 01(M**4)-0.10943E 00(M**2)+0.11339E 06 = 0
PROPOSED(1)   (M**8)+0.18917E 01(M**6)+0.13363E 01(M**4)+0.38974E 00(M**2)+0.11339E 06 = 0
DONNELL       (M**8)-0.10828E 00(M**6)+0.47917E 00(M**4)-0.25621E-01(M**2)+0.11339E 06 = 0
VLASOV        (M**8)+0.19286E 01(M**6)+0.11005E 01(M**4)-0.71433E-01(M**2)+0.11339E 06 = 0
PROPOSED(2)   (M**8)+0.89172E 00(M**6)+0.40773E 00(M**4)+0.21778E 00(M**2)+0.11339E 06 = 0
SCHORER       (M**8)+0.0          (M**6)+0.0          (M**4)+0.0           (M**2)+0.11339E 06 = 0
```

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.6184 -4.0086	3.9066 -1.6606	0.0	0.0
	PROPOSED(1)	1.6185 -4.0085	3.9065 -1.6608	0.61E-03	0.87E-02
1.0	DONNELL	1.6412 -3.9544	3.9603 -1.6388	0.96E 00	0.14E 01
	VLASOV	1.6177 -4.0097	3.9057 -1.6607	0.18E-01	0.28E-01
	PROPOSED(2)	1.6296 -3.9816	3.9335 -1.6495	0.48E 00	0.69E 00
	SCHORER	1.6393 -3.9577	3.9577 -1.6393	0.91E 00	0.13E 01

```
CHARACTERISTIC EQUATIONS FOR L/R= 2.0,R=120.0
FLUGGE        (M**8)+0.19729E 01(M**6)+0.97581E 00(M**4)-0.25940E-01(M**2)+0.70870E 04 = 0
PROPOSED(1)   (M**8)+0.19729E 01(M**6)+0.99423E 00(M**4)+0.12166E-01(M**2)+0.70870E 04 = 0
DONNELL       (M**8)-0.27069E-01(M**6)+0.29948E-01(M**4)-0.40033E-03(M**2)+0.70870E 04 = 0
VLASOV        (M**8)+0.19821E 01(M**6)+0.97950E 00(M**4)-0.17858E-01(M**2)+0.70870E 04 = 0
PROPOSED(2)   (M**8)+0.97293E 00(M**6)+0.12090E-01(M**4)+0.14812E-01(M**2)+0.70870E 04 = 0
SCHORER       (M**8)+0.0          (M**6)+0.0          (M**4)+0.0           (M**2)+0.70870E 04 = 0
```

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.1271 -2.8741	2.7237 -1.1893	0.0	0.0
	PROPOSED(1)	1.1271 -2.8741	2.7236 -1.1894	0.33E-03	0.73E-02
2.0	DONNELL	1.1597 -2.7974	2.7995 -1.1589	0.19E 01	0.29E 01
	VLASOV	1.1269 -2.8745	2.7233 -1.1894	0.88E-02	0.13E-01
	PROPOSED(2)	1.1426 -2.8360	2.7619 -1.1734	0.97E 00	0.14E 01
	SCHORER	1.1592 -2.7985	2.7985 -1.1592	0.19E 01	0.28E 01

STANDARD CORRUGATED SHEET SHELL (CONT.)

CHARACTERISTIC EQUATIONS FOR L/R= 3.0,R=120.0

FLUGGE	(M**8)+0.19880E 01(M**6)+0.98185E 00(M**4)-0.11764E-01(M**2)+0.13999E 04 = 0
PROPOSED(1)	(M**8)+0.19880E 01(M**6)+0.99004E 00(M**4)-0.19624E-02(M**2)+0.13999E 04 = 0
DONNELL	(M**8)-0.12031E-01(M**6)+0.59156E-02(M**4)-0.35145E-04(M**2)+0.13999E 04 = 0
VLASOV	(M**8)+0.19921E 01(M**6)+0.98713E 00(M**4)-0.79370E-02(M**2)+0.13999E 04 = 0
PROPOSED(2)	(M**8)+0.98797E 00(M**6)-0.20214E-02(M**4)+0.29697E-02(M**2)+0.13999E 04 = 0
SCHORER	(M**8)+0.0 (M**6)+0.0 (M**4)+0.0 (M**2)+0.13999E 04 = 0

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	0.9062 -2.3786	2.1929 -0.9829	0.0	0.0
	PROPOSED(1)	0.9062 -2.3786	2.1929 -0.9830	0.21E-03	0.68E-02
3.0	DONNELL	0.9467 -2.2844	2.2855 -0.9463	0.29E 01	0.45E 01
	VLASOV	0.9061 -2.3788	2.1927 -0.9830	0.58E-02	0.85E-02
	PROPOSED(2)	0.9249 -2.3320	2.2398 -0.9633	0.15E 01	0.21E 01
	SCHORER	0.9465 -2.2850	2.2850 -0.9465	0.29E 01	0.44E 01

CHARACTERISTIC EQUATIONS FOR L/R= 4.0,R=120.0

FLUGGE	(M**8)+0.19932E 01(M**6)+0.98834E 00(M**4)-0.66779E-02(M**2)+0.44294E 03 = 0
PROPOSED(1)	(M**8)+0.19932E 01(M**6)+0.99294E 00(M**4)-0.25693E-02(M**2)+0.44294E 03 = 0
DONNELL	(M**8)-0.67672E-02(M**6)+0.18717E-02(M**4)-0.62551E-05(M**2)+0.44294E 03 = 0
VLASOV	(M**8)+0.19955E 01(M**6)+0.99202E 00(M**4)-0.44645E-02(M**2)+0.44294E 03 = 0
PROPOSED(2)	(M**8)+0.99323E 00(M**6)-0.25928E-02(M**4)+0.94451E-03(M**2)+0.44294E 03 = 0
SCHORER	(M**8)+0.0 (M**6)+0.0 (M**4)+0.0 (M**2)+0.44294E 03 = 0

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	0.7723 -2.0876	1.8725 -0.8610	0.0	0.0
	PROPOSED(1)	0.7724 -2.0876	1.8725 -0.8611	0.16E-03	0.68E-02
4.0	DONNELL	0.8198 -1.9785	1.9792 -0.8195	0.39E 01	0.61E 01
	VLASOV	0.7723 -2.0877	1.8724 -0.8611	0.43E-02	0.73E-02
	PROPOSED(2)	0.7936 -2.0338	1.9269 -0.8384	0.19E 01	0.29E 01
	SCHORER	0.8197 -1.9788	1.9788 -0.8197	0.38E 01	0.61E 01

STANDARD CORRUGATED SHEET SHELL (CONT.)

CHARACTERISTIC EQUATIONS FOR L/R= 5.0,R=120.0

```
FLUGGE       (M**8)+0.19957E 01(M**6)+0.99210E 00(M**4)-0.42935E-02(M**2)+0.18143E 03 = 0
PROPOSED(1)  (M**8)+0.19957E 01(M**6)+0.99505E 00(M**4)-0.20801E-02(M**2)+0.18143E 03 = 0
DONNELL      (M**8)-0.43310E-02(M**6)+0.76666E-03(M**4)-0.16397E-05(M**2)+0.18143E 03 = 0
VLASOV       (M**8)+0.19971E 01(M**6)+0.99467E 00(M**4)-0.28573E-02(M**2)+0.18143E 03 = 0
PROPOSED(2)  (M**8)+0.99567E 00(M**6)-0.20906E-02(M**4)+0.38779E-03(M**2)+0.18143E 03 = 0
SCHORER      (M**8)+0.0        (M**6)+0.0        (M**4)+0.0        (M**2)+0.18143E 03 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	0.6794	-1.8921	1.6512	-0.7785	0.0	0.0
	PROPOSED(1)	0.6795	-1.8921	1.6512	-0.7786	0.15E-03	0.71E-02
5.0	DONNELL	0.7332	-1.7697	1.7702	-0.7330	0.48E 01	0.79E 01
	VLASOV	0.6794	-1.8922	1.6511	-0.7786	0.34E-02	0.66E-02
	PROPOSED(2)	0.7027	-1.8319	1.7122	-0.7532	0.24E 01	0.37E 01
	SCHORER	0.7331	-1.7699	1.7699	-0.7331	0.48E 01	0.79E 01

CHARACTERISTIC EQUATIONS FOR L/R= 6.0,R=120.0

```
FLUGGE       (M**8)+0.19970E 01(M**6)+0.99435E 00(M**4)-0.29893E-02(M**2)+0.87494E 02 = 0
PROPOSED(1)  (M**8)+0.19970E 01(M**6)+0.99640E 00(M**4)-0.16092E-02(M**2)+0.87494E 02 = 0
DONNELL      (M**8)-0.30077E-02(M**6)+0.36973E-03(M**4)-0.54915E-06(M**2)+0.87493E 02 = 0
VLASOV       (M**8)+0.19980E 01(M**6)+0.99622E 00(M**4)-0.19842E-02(M**2)+0.87493E 02 = 0
PROPOSED(2)  (M**8)+0.99699E 00(M**6)-0.16145E-02(M**4)+0.18726E-03(M**2)+0.87493E 02 = 0
SCHORER      (M**8)+0.0        (M**6)+0.0        (M**4)+0.0        (M**2)+0.87493E 02 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	0.6097	-1.7501	1.4860	-0.7180	0.0	0.0
	PROPOSED(1)	0.6097	-1.7500	1.4859	-0.7180	0.11E-03	0.74E-02
6.0	DONNELL	0.6693	-1.6155	1.6159	-0.6692	0.58E 01	0.98E 01
	VLASOV	0.6096	-1.7501	1.4859	-0.7180	0.28E-02	0.59E-02
	PROPOSED(2)	0.6346	-1.6841	1.5529	-0.6904	0.29E 01	0.45E 01
	SCHORER	0.6692	-1.6157	1.6157	-0.6692	0.58E 01	0.98E 01

DIMPLED CORRUGATED SHEET SHELL

```
CHARACTERISTIC EQUATIONS FOR L/R= 1.0,R=120.0
FLUGGE        (M**8)+0.82023E 00(M**6)+0.28101E 01(M**4)+0.57958E 01(M**2)+0.18143E 07 = 0
PROPOSED(1)   (M**8)+0.82023E 00(M**6)+0.28837E 01(M**4)+0.62358E 01(M**2)+0.18143E 07 = 0
DONNELL       (M**8)-0.11798E 01(M**6)+0.41696E 01(M**4)-0.40994E 00(M**2)+0.18143E 07 = 0
VLASOV        (M**8)+0.85708E 00(M**6)+0.26085E 01(M**4)-0.11429E 01(M**2)+0.18143E 07 = 0
PROPOSED(2)   (M**8)-0.17977E 00(M**6)+0.30267E 01(M**4)+0.34844E 01(M**2)+0.18143E 07 = 0
SCHORER       (M**8)+0.0         (M**6)+0.0         (M**4)+0.0         (M**2)+0.18143E 07 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	2.3131	-5.6120	5.5808	-2.3262	0.0	0.0
	PROPOSED(1)	2.3132	-5.6120	5.5808	-2.3253	0.79E-04	0.19E-02
1.0	DONNELL	2.3296	-5.5737	5.6187	-2.3110	0.48E 00	0.71E 00
	VLASOV	2.3128	-5.6128	5.5801	-2.3263	0.10E-01	0.14E-01
	PROPOSED(2)	2.3213	-5.5929	5.5998	-2.3185	0.24E 00	0.35E 00
	SCHORER	2.3183	-5.5970	5.5970	-2.3183	0.20E 00	0.29E 00

```
CHARACTERISTIC EQUATIONS FOR L/R= 2.0,R=120.0
FLUGGE        (M**8)+0.17051E 01(M**6)+0.67072E 00(M**4)+0.16025E 00(M**2)+0.11339E 06 = 0
PROPOSED(1)   (M**8)+0.17051E 01(M**6)+0.68914E 00(M**4)+0.19465E 00(M**2)+0.11339E 06 = 0
DONNELL       (M**8)-0.29494E 00(M**6)+0.26060E 00(M**4)-0.64052E-02(M**2)+0.11339E 06 = 0
VLASOV        (M**8)+0.17143E 01(M**6)+0.67193E 00(M**4)-0.28573E 00(M**2)+0.11339E 06 = 0
PROPOSED(2)   (M**8)+0.70506E 00(M**6)-0.25132E-01(M**4)+0.23699E 00(M**2)+0.11339E 06 = 0
SCHORER       (M**8)+0.0         (M**6)+0.0         (M**4)+0.0         (M**2)+0.11339E 06 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.6199	-4.0038	3.9118	-1.6580	0.0	0.0
	PROPOSED(1)	1.6199	-4.0037	3.9118	-1.6580	0.22E-04	0.17E-02
2.0	DONNELL	1.6429	-3.9496	3.9655	-1.6364	0.96E 00	0.14E 01
	VLASOV	1.6198	-4.0040	3.9116	-1.6581	0.51E-02	0.67E-02
	PROPOSED(2)	1.6312	-3.9768	3.9388	-1.6469	0.48E 00	0.69E 00
	SCHORER	1.6393	-3.9577	3.9577	-1.6393	0.82E 00	0.12E 01

DIMPLED CORRUGATED SHEET SHELL (CONT.)

```
CHARACTERISTIC EQUATIONS FOR L/R= 3.0,R=120.0
FLUGGE         (M**8)+0.18689E 01(M**6)+0.78931E 00(M**4)-0.40468E-01(M**2)+0.22398E 05 = 0
PROPOSED(1)    (M**8)+0.18689E 01(M**6)+0.79749E 00(M**4)-0.31398E-01(M**2)+0.22398E 05 = 0
DONNELL        (M**8)-0.13109E 00(M**6)+0.51476E-01(M**4)-0.56232E-03(M**2)+0.22398E 05 = 0
VLASOV         (M**8)+0.18730E 01(M**6)+0.79410E 00(M**4)-0.12699E 00(M**2)+0.22398E 05 = 0
PROPOSED(2)    (M**8)+0.86891E 00(M**6)-0.75515E-01(M**4)+0.47516E-01(M**2)+0.22398E 05 = 0
SCHORER        (M**8)+0.0          (M**6)+0.0          (M**4)+0.0          (M**2)+0.22398E 05 = 0
```

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.3121 -3.2935	3.1700 -1.3632	0.0	0.0
	PROPOSED(1)	1.3121 -3.2935	3.1700 -1.3633	0.27E-04	0.17E-02
3.0	DONNELL	1.3404 -3.2270	3.2357 -1.3368	0.14E 01	0.22E 01
	VLASOV	1.3121 -3.2936	3.1699 -1.3633	0.34E-02	0.43E-02
	PROPOSED(2)	1.3257 -3.2604	3.2031 -1.3496	0.72E 00	0.10E 01
	SCHORER	1.3385 -3.2314	3.2314 -1.3385	0.14E 01	0.20E 01

```
CHARACTERISTIC EQUATIONS FOR L/R= 4.0,R=120.0
FLUGGE         (M**8)+0.19263E 01(M**6) 0.86882E 00(M**4)-0.44986E-01(M**2)+0.70870E 04 = 0
PROPOSED(1)    (M**8)+0.19263E 01(M**6) 0.87342E 00(M**4)-0.41108E-01(M**2)+0.70870E 04 = 0
DONNELL        (M**8)-0.73735E-01(M**6) 0.16287E-01(M**4)-0.10008E-03(M**2)+0.70870E 04 = 0
VLASOV         (M**8)+0.19286E 01(M**6) 0.87235E 00(M**4)-0.71433E-01(M**2)+0.70870E 04 = 0
PROPOSED(2)    (M**8)+0.92626E 00(M**6)-0.55145E-01(M**4)+0.15112E-01(M**2)+0.70870E 04 = 0
SCHORER        (M**8)+0.0          (M**6) 0.0          (M**4)+0.0          (M**2)+0.70870E 04 = 0
```

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.1276 -2.8724	2.7255 -1.1884	0.0	0.0
	PROPOSED(1)	1.1276 -2.8724	2.7255 -1.1884	0.16E-04	0.17E-02
4.0	DONNELL	1.1604 -2.7956	2.8013 -1.1581	0.19E 01	0.29E 01
	VLASOV	1.1275 -2.8725	2.7254 -1.1884	0.25E-02	0.32E-02
	PROPOSED(2)	1.1432 -2.8343	2.7637 -1.1725	0.97E 00	0.14E 01
	SCHORER	1.1592 -2.7985	2.7985 -1.1592	0.19E 01	0.28E 01

DIMPLED CORRUGATED SHEET SHELL (CONT.)

CHARACTERISTIC EQUATIONS FOR L/R= 5.0,R=120.0

FLUGGE	(M**8)+0.19528E 01(M**6)+0.91229E 00(M**4)-0.35400E-01(M**2)+0.29028E 04 = 0
PROPOSED(1)	(M**8)+0.19528E 01(M**6)+0.91524E 00(M**4)-0.33281E-01(M**2)+0.29028E 04 = 0
DONNELL	(M**8)-0.47191E-01(M**6)+0.66713E-02(M**4)-0.26236E-04(M**2)+0.29028E 04 = 0
VLASOV	(M**8)+0.19543E 01(M**6)+0.91480E 00(M**4)-0.45717E-01(M**2)+0.29028E 04 = 0
PROPOSED(2)	(M**8)+0.95281E 00(M**6)-0.39046E-01(M**4)+0.62047E-02(M**2)+0.29028E 04 = 0
SCHORER	(M**8)+0.0 (M**6)+0.0 (M**4)+0.0 (M**2)+0.29028E 04 = 0

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.0007	-2.5870	2.4205	-1.0696	0.0	0.0
	PROPOSED(1)	1.0008	-2.5870	2.4204	-1.0696	0.0	0.17E-02
5.0	DONNELL	1.0377	-2.5010	2.5050	-1.0360	0.24E 01	0.37E 01
	VLASOV	1.0007	-2.5870	2.4204	-1.0696	0.20E-02	0.25E-02
	PROPOSED(2)	1.0180	-2.5444	2.4632	-1.0518	0.12E 01	0.18E 01
	SCHORER	1.0368	-2.5030	2.5030	-1.0368	0.24E 01	0.36E 01

CHARACTERISTIC EQUATIONS FOR L/R= 6.0,R=120.0

FLUGGE	(M**8)+0.19672E 01(M**6)+0.93767E 00(M**4)-0.27081E-01(M**2)+0.13999E 04 = 0
PROPOSED(1)	(M**8)+0.19672E 01(M**6)+0.93972E 00(M**4)-0.25747E-01(M**2)+0.13999E 04 = 0
DONNELL	(M**8)-0.32771E-01(M**6)+0.32173E-02(M**4)-0.87864E-05(M**2)+0.13999E 04 = 0
VLASOV	(M**8)+0.19683E 01(M**6)+0.93951E 00(M**4)-0.31748E-01(M**2)+0.13999E 04 = 0
PROPOSED(2)	(M**8)+0.96723E 00(M**6)-0.28531E-01(M**4)+0.29961E-02(M**2)+0.13999E 04 = 0
SCHORER	(M**8)+0.0 (M**6)+0.0 (M**4)+0.0 (M**2)+0.13999E 04 = 0

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	0.9064	-2.3777	2.1939	-0.9824	0.0	0.0
	PROPOSED(1)	0.9064	-2.3777	2.1939	-0.9824	0.19E-04	0.17E-02
6.0	DONNELL	0.9471	-2.2834	2.2865	-0.9458	0.29E 01	0.45E 01
	VLASOV	0.9064	-2.3778	2.1939	-0.9824	0.17E-02	0.20E-02
	PROPOSED(2)	0.9252	-2.3311	2.2408	-0.9628	0.15E 01	0.21E 01
	SCHORER	0.9465	-2.2850	2.2850	-0.9465	0.28E 01	0.44E 01

SPOT WELDED CORRUGATED SHELL

```
CHARACTERISTIC EQUATIONS FOR L/R= 1.0,R=120.0
FLUGGE        (M**8)-0.30380E 02(M**6)+0.23677E 03(M**4)-0.52563E 03(M**2)+0.21726E 08 = 0
PROPOSED(1)   (M**8)-0.30380E 02(M**6)+0.27587E 03(M**4)-0.88034E 03(M**2)+0.21726E 08 = 0
DONNELL       (M**8)-0.32380E 02(M**6)+0.30053E 03(M**4)-0.96492E 03(M**2)+0.21726E 08 = 0
VLASOV        (M**8)-0.10830E 02(M**6)+0.24044E 02(M**4)-0.12830E 02(M**2)+0.21726E 08 = 0
PROPOSED(2)   (M**8)-0.31380E 02(M**6)+0.28770E 03(M**4)-0.91621E 03(M**2)+0.21726E 08 = 0
SCHORER       (M**8)+0.0          (M**6)+0.0          (M**4)+0.0          (M**2)+0.21726E 08 = 0
```

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	3.3280 -7.2138	8.0624 -2.9748	0.0	0.0
	PROPOSED(1)	3.3357 -7.2104	8.0592 -2.9831	0.16E-02	0.28E 00
1.0	DONNELL	3.3446 -7.1837	8.0883 -2.9683	0.26E 00	0.50E 00
	VLASOV	3.2222 -7.4835	7.7862 -3.0966	0.25E 01	0.41E 01
	PROPOSED(2)	3.3401 -7.1971	8.0738 -2.9757	0.13E 00	0.36E 00
	SCHORER	3.1620 -7.6337	7.6337 -3.1620	0.39E 01	0.63E 01

```
CHARACTERISTIC EQUATIONS FOR L/R= 2.0,R=120.0
FLUGGE        (M**8)-0.60950E 01(M**6)+0.35931E 01(M**4)+0.63072E 01(M**2)+0.13579E 07 = 0
PROPOSED(1)   (M**8)-0.60950E 01(M**6)+0.13368E 02(M**4)-0.12196E 02(M**2)+0.13579E 07 = 0
DONNELL       (M**8)-0.80950E 01(M**6)+0.18783E 02(M**4)-0.15077E 02(M**2)+0.13579E 07 = 0
VLASOV        (M**8)-0.12076E 01(M**6)-0.23712E 01(M**4)-0.32076E 01(M**2)+0.13579E 07 = 0
PROPOSED(2)   (M**8)-0.70950E 01(M**6)+0.15575E 02(M**4)-0.12033E 02(M**2)+0.13579E 07 = 0
SCHORER       (M**8)+0.0          (M**6)+0.0          (M**4)+0.0          (M**2)+0.13579E 07 = 0
```

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	2.2812 -5.2794	5.5202 -2.1812	0.0	0.0
	PROPOSED(1)	2.2868 -5.2770	5.5179 -2.1870	0.82E-03	0.26E 00
2.0	DONNELL	2.3012 -5.2383	5.5583 -2.1686	0.52E 00	0.88E 00
	VLASOV	2.2442 -5.3747	5.4224 -2.2243	0.13E 01	0.20E 01
	PROPOSED(2)	2.2939 -5.2577	5.5381 -2.1777	0.26E 00	0.56E 00
	SCHORER	2.2359 -5.3979	5.3979 -2.2359	0.16E 01	0.25E 01

SPOT WELDED CORRUGATED SHELL (CONT.)

```
CHARACTERISTIC EQUATIONS FOR L/R= 3.0,R=120.0
FLUGGE        (M**8)-0.15978E 01(M**6)-0.24853E 01(M**4)+0.90162E 00(M**2)+0.26823E 06 = 0
PROPOSED(1)   (M**8)-0.15978E 01(M**6)+0.18590E 01(M**4)-0.15466E 01(M**2)+0.26822E 06 = 0
DONNELL       (M**8)-0.35978E 01(M**6)+0.37102E 01(M**4)-0.13236E 01(M**2)+0.26822E 06 = 0
VLASOV        (M**8)+0.57439E 00(M**6)-0.12499E 01(M**4)-0.14256E 01(M**2)+0.26822E 06 = 0
PROPOSED(2)   (M**8)-0.25978E 01(M**6)+0.22846E 01(M**4)-0.72232E 00(M**2)+0.26822E 06 = 0
SCHORER       (M**8)+0.0       (M**6)+0.0       (M**4)+0.0       (M**2)+0.26822E 06 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.8382	-4.3701	4.4475	-1.8060	0.0	0.0
	PROPOSED(1)	1.8428	-4.3682	4.4455	-1.8107	0.55E-03	0.26E 00
3.0	DONNELL	1.8613	-4.3204	4.4946	-1.7891	0.78E 00	0.13E 01
	VLASOV	1.8184	-4.4219	4.3940	-1.8299	0.84E 00	0.13E 01
	PROPOSED(2)	1.8518	-4.3444	4.4702	-1.7997	0.39E 00	0.74E 00
	SCHORER	1.8256	-4.4073	4.4073	-1.8256	0.62E 00	0.11E 01

```
CHARACTERISTIC EQUATIONS FOR L/R= 4.0,R=120.0
FLUGGE        (M**8)-0.23754E-01(M**6)-0.18736E 01(M**4)-0.41689E 00(M**2)+0.84868E 05 = 0
PROPOSED(1)   (M**8)-0.23754E-01(M**6)+0.57014E 00(M**4)-0.65698E 00(M**2)+0.84868E 05 = 0
DONNELL       (M**8)-0.20238E 01(M**6)+0.11739E 01(M**4)-0.23558E 00(M**2)+0.84868E 05 = 0
VLASOV        (M**8)+0.11981E 01(M**6)-0.41356E 00(M**4)-0.80190E 00(M**2)+0.84868E 05 = 0
PROPOSED(2)   (M**8)-0.10238E 01(M**6)+0.37204E 00(M**4)-0.45323E-01(M**2)+0.84868E 05 = 0
SCHORER       (M**8)+0.0       (M**6)+0.0       (M**4)+0.0       (M**2)+0.84868E 05 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.5783	-3.8175	3.8188	-1.5776	0.0	0.0
	PROPOSED(1)	1.5823	-3.8159	3.8171	-1.5816	0.37E-03	0.25E 00
4.0	DONNELL	1.6043	-3.7604	3.8735	-1.5574	0.10E 01	0.16E 01
	VLASOV	1.5658	-3.8510	3.7840	-1.5934	0.63E 00	0.10E 01
	PROPOSED(2)	1.5929	-3.7882	3.8455	-1.5692	0.52E 00	0.93E 00
	SCHORER	1.5810	-3.8169	3.8169	-1.5810	0.12E-01	0.21E 00

SPOT WELDED CORRUGATED SHELL (CONT.)

```
CHARACTERISTIC EQUATIONS FOR L/R= 5.0,P=120.0
FLUGGE        (M**8)+0.70480E 00(M**6)-0.11096E 01(M**4)-0.60229E 00(M**2)+0.34762E 05 = 0
PROPOSED(1)   (M**8)+0.70480E 00(M**6)+0.45441E 00(M**4)-0.41912E 00(M**2)+0.34762E 05 = 0
DONNELL       (M**8)-0.12952E 01(M**6)+0.48085E 00(M**4)-0.61755E-01(M**2)+0.34762E 05 = 0
VLASOV        (M**8)+0.14868E 01(M**6)+0.51490E-01(M**4)-0.51322E 00(M**2)+0.34762E 05 = 0
PROPOSED(2)   (M**8)-0.29520E 00(M**6)+0.32372E-01(M**4)+0.16173E-01(M**2)+0.34762E 05 = 0
SCHORER       (M**8)+0.0        (M**6)+0.0          (M**4)+0.0          (M**2)+0.34762E 05 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.4021	-3.4372	3.3931	-1.4203	0.0	0.0
	PROPOSED(1)	1.4057	-3.4357	3.3916	-1.4238	0.31E-03	0.26E 00
5.0	DONNELL	1.4308	-3.3735	3.4544	-1.3972	0.13E 01	0.20E 01
	VLASOV	1.3934	-3.4610	3.3680	-1.4318	0.51E 00	0.81E 00
	PROPOSED(2)	1.4178	-3.4047	3.4232	-1.4101	0.65E 00	0.11E 01
	SCHORER	1.4141	-3.4139	3.4139	-1.4141	0.46E 00	0.85E 00

```
CHARACTERISTIC EQUATIONS FOR L/R= 6.0,R=120.0
FLUGGE        (M**8)+0.11006E 01(M**6)-0.56700E 00(M**4)-0.55619E 00(M**2)+0.16764E 05 = 0
PROPOSED(1)   (M**8)+0.11006E 01(M**6)+0.51909E 00(M**4)-0.30192E 00(M**2)+0.16764E 05 = 0
DONNELL       (M**8)-0.89945E 00(M**6)+0.23189E 00(M**4)-0.20681E-01(M**2)+0.16764E 05 = 0
VLASOV        (M**8)+0.16436E 01(M**6)+0.32478E 00(M**4)-0.35640E 00(M**2)+0.16764E 05 = 0
PROPOSED(2)   (M**8)+0.10055E 00(M**6)-0.12451E 00(M**4)+0.16899E-01(M**2)+0.16764E 05 = 0
SCHORER       (M**8)+0.0        (M**6)+0.0          (M**4)+0.0          (M**2)+0.16764E 05 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.2725	-3.1553	3.0799	-1.3036	0.0	0.0
	PROPOSED(1)	1.2758	-3.1540	3.0786	-1.3068	0.27E-03	0.26E 00
6.0	DONNELL	1.3036	-3.0857	3.1473	-1.2781	0.16E 01	0.24E 01
	VLASOV	1.2661	-3.1734	3.0608	-1.3126	0.42E 00	0.69E 00
	PROPOSED(2)	1.2891	-3.1201	3.1132	-1.2919	0.78E 00	0.13E 01
	SCHORER	1.2909	-3.1165	3.1165	-1.2909	0.86E 00	0.14E 01

ISOTROPIC STEEL SHELL 2 IN. THICK

CHARACTERISTIC EQUATIONS FOR L/R= 1.0,R=120.0

FLUGGE	(M**8)-0.37478E 02(M**6)+0.50650E 03(M**4)-0.33006E 04(M**2)+0.42180E 07 = 0
PROPOSED(1)	(M**8)-0.37478E 02(M**6)+0.54598E 03(M**4)-0.36705E 04(M**2)+0.42177E 07 = 0
DONNELL	(M**8)-0.39478E 02(M**6)+0.58445E 03(M**4)-0.38456E 04(M**2)+0.42176E 07 = 0
VLASOV	(M**8)-0.17739E 02(M**6)+0.58931E 02(M**4)-0.19739E 02(M**2)+0.42081E 07 = 0
PROPOSED(2)	(M**8)-0.38478E 02(M**6)+0.56472E 03(M**4)-0.37481E 04(M**2)+0.42176E 07 = 0
SCHORER	(M**8)+0.0 (M**6)+0.0 (M**4)+0.0 (M**2)+0.42081E 07 = 0

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	2.8736 -5.5672	6.8483 -2.3290	0.0	0.0
	PROPOSED(1)	2.8869 -5.5602	6.8427 -2.3455	0.86E-03	0.71E 00
1.0	DONNELL	2.8971 -5.5272	6.8786 -2.3280	0.39E 00	0.82E 00
	VLASOV	2.6875 -5.9198	6.5278 -2.4335	0.37E 01	0.63E 01
	PROPOSED(2)	2.8919 -5.5437	6.8607 -2.3567	0.20E 00	0.64E 00
	SCHORER	2.5754 -6.2176	6.2176 -2.5754	0.72E 01	0.12E 02

CHARACTERISTIC EQUATIONS FOR L/R= 2.0,R=120.0

FLUGGE	(M**8)-0.78696E 01(M**6)+0.17789E 02(M**4)-0.33428E 02(M**2)+0.26307E 06 = 0
PROPOSED(1)	(M**8)-0.78696E 01(M**6)+0.27659E 02(M**4)-0.52845E 02(M**2)+0.26305E 06 = 0
DONNELL	(M**8)-0.98696E 01(M**6)+0.36528E 02(M**4)-0.60087E 02(M**2)+0.26304E 06 = 0
VLASOV	(M**8)-0.29348E 01(M**6)-0.27815E 01(M**4)-0.49348E 01(M**2)+0.26300E 06 = 0
PROPOSED(2)	(M**8)-0.88696E 01(M**6)+0.31594E 02(M**4)-0.53999E 02(M**2)+0.26304E 06 = 0
SCHORER	(M**8)+0.0 (M**6)+0.0 (M**4)+0.0 (M**2)+0.26300E 06 = 0

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.8996 -4.2068	4.5878 -1.7393	0.0	0.0
	PROPOSED(1)	1.9097 -4.2022	4.5836 -1.7503	0.90E-03	0.63E 00
2.0	DONNELL	1.9272 -4.1545	4.6332 -1.7281	0.78E 00	0.15E 01
	VLASOV	1.8452 -4.3279	4.4700 -1.7853	0.19E 01	0.29E 01
	PROPOSED(2)	1.9182 -4.1784	4.6085 -1.7390	0.39E 00	0.98E 00
	SCHORER	1.8211 -4.3965	4.3965 -1.8211	0.31E 01	0.47E 01

ISOTROPIC STEEL SHELL 2 IN. THICK (CONT.)

CHARACTERISTIC EQUATIONS FOR L/R= 3.0,R=120.0

FLUGGE	(M**8)-0.23865E 01	(M**6)-0.55750E 00	(M**4)-0.24461E 01	(M**2)+0.51958E 05	= 0
PROPOSED(1)	(M**8)-0.23865E 01	(M**6)+0.38290E 01	(M**4)-0.50632E 01	(M**2)+0.51954E 05	= 0
DONNELL	(M**8)-0.43865E 01	(M**6)+0.72155E 01	(M**4)-0.52751E 01	(M**2)+0.51953E 05	= 0
VLASOV	(M**8)-0.19324E 00	(M**6)-0.21839E 01	(M**4)-0.21932E 01	(M**2)+0.51951E 05	= 0
PROPOSED(2)	(M**8)-0.33865E 01	(M**6)+0.50222E 01	(M**4)-0.40725E 01	(M**2)+0.51953E 05	= 0
SCHORER	(M**8)+0.0	(M**6)+0.0	(M**4)+0.0	(M**2)+0.51951E 05	= 0

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.5123 -3.5210	3.6624 -1.4524	0.0	0.0
	PROPOSED(1)	1.5207 -3.5173	3.6589 -1.4612	0.87E-03	0.61E 00
3.0	DONNELL	1.5434 -3.4584	3.7191 -1.4352	0.12E 01	0.21E 01
	VLASOV	1.4853 -3.5859	3.5972 -1.4799	0.13E 01	0.19E 01
	PROPOSED(2)	1.5316 -3.4880	3.6892 -1.4479	0.59E 00	0.13E 01
	SCHORER	1.4869 -3.5898	3.5898 -1.4869	0.14E 01	0.24E 01

CHARACTERISTIC EQUATIONS FOR L/R= 4.0,R=120.0

FLUGGE	(M**8)-0.46740E 00	(M**6)-0.16518E 01	(M**4)-0.11232E 01	(M**2)+0.16439E 05	= 0
PROPOSED(1)	(M**8)-0.46740E 00	(M**6)+0.81562F 00	(M**4)-0.14115E 01	(M**2)+0.16438E 05	= 0
DONNELL	(M**8)-0.24674E 01	(M**6)+0.22830E 01	(M**4)-0.93886E 00	(M**2)+0.16438E 05	= 0
VLASOV	(M**8) 0.76630E 00	(M**6)-0.10869E 01	(M**4)-0.12337E 01	(M**2)+0.16438E 05	= 0
PROPOSED(2)	(M**8)-0.14674E 01	(M**6)+0.10493E 01	(M**4)-0.55835E 00	(M**2)+0.16438E 05	= 0
SCHORER	(M**8)+0.0	(M**6)+0.0	(M**4)+0.0	(M**2)+0.16438E 05	= 0

L/R		M1	M2	AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.2895 -3.0951	3.1269 -1.2755	0.0	0.0
	PROPOSED(1)	1.2970 -3.0920	3.1238 -1.2830	0.88E-03	0.59E 00
4.0	DONNELL	1.3240 -3.0237	3.1930 -1.2538	0.16E 01	0.27E 01
	VLASOV	1.2732 -3.1368	3.0840 -1.2945	0.96E 00	0.15E 01
	PROPOSED(2)	1.3098 -3.0580	3.1587 -1.2679	0.79E 00	0.16E 01
	SCHORER	1.2877 -3.1088	3.1088 -1.2877	0.36E 00	0.96E 00

ISOTROPIC STEEL SHELL 2 IN. THICK (CONT.)

CHARACTERISTIC EQUATIONS FOR L/R= 5.0,R=120.0

```
FLUGGE        (M**8)+0.42086E 00(M**6)-0.12231E 01(M**4)-0.89012E 00(M**2)+0.67336E 04 = 0
PROPOSED(1)   (M**8)+0.42086E 00(M**6)+0.35599E 00(M**4)-0.72397E 00(M**2)+0.67331E 04 = 0
DONNELL       (M**8)-0.15791E 01(M**6)+0.93513E 00(M**4)-0.24612E 00(M**2)+0.67329E 04 = 0
VLASOV        (M**8)+0.12104E 01(M**6)-0.42328E 00(M**4)-0.78957E 00(M**2)+0.67329E 04 = 0
PROPOSED(2)   (M**8)-0.57914E 00(M**6)+0.14556E 00(M**4)-0.90261E-01(M**2)+0.67329E 04 = 0
SCHORER       (M**8)+0.0         (M**6)+0.0         (M**4)+0.0          (M**2)+0.67329E 04 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.1400	-2.7992	2.7666	-1.1527	0.0	0.0
	PROPOSED(1)	1.1467	-2.7964	2.7638	-1.1594	0.87E-03	0.59E 00
5.0	DONNELL	1.1775	-2.7198	2.8409	-1.1273	0.20E 01	0.33E 01
	VLASOV	1.1290	-2.8287	2.7356	-1.1671	0.77E 00	0.12E 01
	PROPOSED(2)	1.1612	-2.7583	2.8028	-1.1427	0.98E 00	0.19E 01
	SCHORER	1.1518	-2.7806	2.7806	-1.1518	0.42E 00	0.10E 01

CHARACTERISTIC EQUATIONS FOR L/R= 6.0,R=120.0

```
FLUGGE        (M**8)+0.90338E 00(M**6)-0.74228E 00(M**4)-0.72808E 00(M**2)+0.32473E 04 = 0
PROPOSED(1)   (M**8)+0.90338E 00(M**6)+0.35435E 00(M**4)-0.48041E 00(M**2)+0.32470E 04 = 0
DONNELL       (M**8)-0.10966E 01(M**6)+0.45097E 00(M**4)-0.82424E-01(M**2)+0.32470E 04 = 0
VLASOV        (M**8)+0.14517E 01(M**6)-0.21461E-01(M**4)-0.54831E 00(M**2)+0.32470E 04 = 0
PROPOSED(2)   (M**8)-0.96622E-01(M**6)-0.97344E-01(M**4)-0.72622E-02(M**2)+0.32470E 04 = 0
SCHORER       (M**8)+0.0         (M**6)+0.0         (M**4)+0.0          (M**2)+0.32470E 04 = 0
```

L/R		M1		M2		AVERAGE ERROR%	MAXIMUM ERROR%
	FLUGGE	1.0305	-2.5788	2.5026	-1.0613	0.0	0.0
	PROPOSED(1)	1.0366	-2.5763	2.5000	-1.0673	0.85E-03	0.60E 00
6.0	DONNELL	1.0709	-2.4921	2.5843	-1.0327	0.24E 01	0.39E 01
	VLASOV	1.0226	-2.6010	2.4788	-1.0727	0.64E 00	0.11E 01
	PROPOSED(2)	1.0525	-2.5345	2.5426	-1.0492	0.12E 01	0.21E 01
	SCHORER	1.0514	-2.5383	2.5383	-1.0514	0.11E 01	0.20E 01

APPENDIX (III)

BOUNDARY CONDITIONS.

i- Shells Simply Supported On Four Edges.

The boundary conditions reduce to:

$$\left[M \right] \left\{ U \right\} = \left\{ N \right\} \qquad (50)$$

in which

$$\left\{ U \right\} = \left\{ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix} \right\}$$

$m_{11} = 1.0$; $m_{12} = 0.0$; $m_{13} = 1.0$; $m_{14} = 0.0$;
$m_{15} = 1.0$; $m_{16} = 0.0$; $m_{17} = 1.0$; $m_{18} = 0.0$;

$m_{21} = \underline{1} - \beta_1^2 + \alpha_1^2$; $m_{22} = 2\alpha_1\beta_1$; $m_{23} = \underline{1} - \beta_1^2 + \alpha_1^2$;

$m_{24} = -2\alpha_1\beta_1$; $m_{25} = \underline{1} - \beta_2^2 + \alpha_2^2$; $m_{26} = 2\alpha_2\beta_2$;

$m_{27} = \underline{1} - \beta_2^2 + \alpha_2^2$; $m_{28} = -2\alpha_2\beta_2$;

$$m_{31} = \underline{- \frac{B_\phi}{R^5k^3} (\beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4)} - \frac{B_\phi}{R^5k^3} (-\beta_1^6 + 15\alpha_1^2\beta_1^4 - 15\alpha_1^4\beta_1^2$$
$$+ \alpha_1^6) + \frac{2B_{x\phi}}{kR^3} (\beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4) - \frac{B_x k}{R^2} (-\beta_1^2 + \alpha_1^2) \quad ;$$

$$m_{32} = \underline{\frac{B_\phi}{R^5k^3} (4\alpha_1\beta_1^3 - 4\alpha_1^3\beta_1)} + \frac{B_\phi}{R^5k^3} (-6\alpha_1\beta_1^5 + 20\alpha_1^3\beta_1^3 - 6\alpha_1^5\beta_1)$$
$$- \frac{2B_{x\phi}}{kR^3} (4\alpha_1\beta_1^3 - 4\alpha_1^3\beta_1) - \frac{B_x k}{R^2} (2\alpha_1\beta_1) \quad ;$$

m_{33} = same expression of m_{31}, with $-\alpha_1$ instead of α_1 ;

m_{34} = same expression of m_{32}, with $-\alpha_1$ instead of α_1 ;

m_{35}, m_{36}, m_{37} & m_{38} = similar expressions as m_{31}, m_{32}, m_{33} & m_{34}, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$m_{41} = \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{2B_{x\phi}R^2}{D_{x\phi}} - \frac{B_xR^2}{D_x}\right)(\alpha_1^3 - 3\alpha_1\beta_1^2) + \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{B_\phi}{D_xR^2k^4} + \frac{2B_{x\phi}}{D_xk^2}\right)(5\alpha_1\beta_1^4 - 10\alpha_1^3\beta_1^2 + \alpha_1^5) + \left(\frac{B_xR^4k^2}{D_{x\phi}}\right)(\alpha_1) - \left(\frac{B_\phi}{D_xR^2k^4}\right)(-7\alpha_1\beta_1^6 + 35\alpha_1^3\beta_1^4 - 21\alpha_1^5\beta_1^2 + \alpha_1^7) \quad ;$$

$$m_{42} = -\left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{2B_{x\phi}R^2}{D_{x\phi}} - \frac{B_xR^2}{D_x}\right)(\beta_1^3 - 3\alpha_1^2\beta_1) + \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{B_\phi}{D_xR^2k^4} + \frac{2B_{x\phi}}{D_xk^2}\right)(\beta_1^5 - 10\alpha_1^2\beta_1^3 + 5\alpha_1^4\beta_1) + \left(\frac{B_xR^4k^2}{D_{x\phi}}\right)(\beta_1) - \left(\frac{B_\phi}{D_xR^2k^4}\right)(-\beta_1^7 + 21\alpha_1^2\beta_1^5 - 35\alpha_1^4\beta_1^3 + 7\alpha_1^6\beta_1) \quad ;$$

m_{43} = same expression of m_{41}, with $-\alpha_1$ instead of α_1 ;

m_{44} = same expression of m_{42}, with $-\alpha_1$ instead of α_1 ;

m_{45}, m_{46}, m_{47} & m_{48} = similar expressions as m_{41}, m_{42}, m_{43} & m_{44}, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$m_{51} = e^{2\alpha_1\phi_e} \cdot \cos 2\beta_1\phi_e \;;\quad m_{52} = e^{2\alpha_1\phi_e} \cdot \sin 2\beta_1\phi_e \;;$$

$$m_{53} = e^{-2\alpha_1\phi_e} \cdot \cos 2\beta_1\phi_e \;;\quad m_{54} = e^{-2\alpha_1\phi_e} \cdot \sin 2\beta_1\phi_e \;;$$

$$m_{55} = e^{2\alpha_2\phi_e} \cdot \cos 2\beta_2\phi_e \;;\quad m_{56} = e^{2\alpha_2\phi_e} \cdot \sin 2\beta_2\phi_e \;;$$

$$m_{57} = e^{-2\alpha_2\phi_e} \cdot \cos 2\beta_2\phi_e \;;\quad m_{58} = e^{-2\alpha_2\phi_e} \cdot \sin 2\beta_2\phi_e \;;$$

$$m_{61} = (e^{2\alpha_1\phi_e} \cos 2\beta_1\phi_e)\,(\underline{1} - \beta_1^2 + \alpha_1^2) - 2\alpha_1\beta_1 e^{2\alpha_1\phi_e} \sin 2\beta_1\phi_e;$$

$$m_{62} = (e^{2\alpha_1\phi_e} \sin 2\beta_1\phi_e)\,(\underline{1} - \beta_1^2 + \alpha_1^2) + 2\alpha_1\beta_1 e^{2\alpha_1\phi_e} \cos 2\beta_1\phi_e;$$

$$m_{63} = (e^{-2\alpha_1\phi_e} \cos 2\beta_1\phi_e)\,(\underline{1} - \beta_1^2 + \alpha_1^2) + 2\alpha_1\beta_1 e^{-2\alpha_1\phi_e} \sin 2\beta_1\phi_e;$$

$$m_{64} = (e^{-2\alpha_1\phi_e} \sin 2\beta_1\phi_e)\,(\underline{1} - \beta_1^2 + \alpha_1^2) - 2\alpha_1\beta_1 e^{-2\alpha_1\phi_e} \cos 2\beta_1\phi_e;$$

m_{65}, m_{66}, m_{67} & m_{68} = similar expressions as m_{61}, m_{62}, m_{63} & m_{64}, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$m_{71} = \left(\frac{2B_{x\phi}}{R^3 k} - \underline{\frac{B_\phi}{R^5 k^3}}\right) \Big\{ (e^{2\alpha_1\phi_e} \cos 2\beta_1\phi_e)(\beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4) + (e^{2\alpha_1\phi_e} \sin 2\beta_1\phi_e)(4\alpha_1\beta_1^3 - 4\alpha_1^3\beta_1) \Big\} - \left(\frac{B_\phi}{R^5 k^3}\right) .$$

$$\Big\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,(-\beta_1^6 + 15\alpha_1^2\beta_1^4 - 15\alpha_1^4\beta_1^2 + \alpha_1^6) +$$

$$(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,(-6\alpha_1\beta_1^5 + 20\alpha_1^3\beta_1^3 - 6\alpha_1^5\beta_1)\Big\} - (\frac{B_x k}{R}) .$$

$$\Big\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,(-\beta_1^2 + \alpha_1^2) + (e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,.$$

$$(-2\alpha_1\beta_1)\Big\} \quad ;$$

$$m_{72} = (\frac{2B_{x\phi}}{R^3\,k} - \underline{\frac{B_\phi}{R^5k^3}})\Big\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,(\beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4) +$$

$$(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,(-4\alpha_1\beta_1^3 + 4\alpha_1^3\beta_1)\Big\} - (\frac{B_\phi}{R^5k^3}) .$$

$$\Big\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,(-\beta_1^6 + 15\alpha_1^2\beta_1^4 - 15\alpha_1^4\beta_1^2 + \alpha_1^6) +$$

$$(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,(6\alpha_1\beta_1^5 - 20\alpha_1^3\beta_1^3 + 6\alpha_1^5\beta_1)\Big\} - (\frac{B_x k}{R}) .$$

$$\Big\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,(-\beta_1^2 + \alpha_1^2) + (e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,.$$

$$(2\alpha_1\beta_1)\Big\} \quad ;$$

m_{73} = similar expression to m_{71}, with $-\alpha_1$ instead of α_1 ;

m_{74} = similar expression to m_{72}, with $-\alpha_1$ instead of α_1 ;

m_{75}, m_{76}, m_{77} & m_{78} = similar expressions to m_{71}, m_{72}, m_{73} & m_{74}, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$m_{81} = \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{2B_{x\phi}R^2}{D_{x\phi}} - \frac{B_xR^2}{D_x}\right)\Big\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)(\beta_1^3 - 3\alpha_1^2\beta_1)$$

$$+ (e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(-3\alpha_1\beta_1^2 + \alpha_1^3)\Big\} + \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{B_\phi}{D_xR^2k^4}\right.$$

$$\left. + \frac{2B_{x\phi}}{D_xk^2}\right)\Big\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)(-\beta_1^5 + 10\alpha_1^2\beta_1^3 - 5\alpha_1^4\beta_1) +$$

$$(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(5\alpha_1\beta_1^4 - 10\alpha_1^3\beta_1^2 + \alpha_1^5)\Big\} + \left(\frac{B_xR^4k^2}{D_{x\phi}}\right) \cdot$$

$$\Big\{(-\beta_1\, e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e) + (\alpha_1\, e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\Big\} -$$

$$\left(\frac{B_\phi}{D_xR^2k^4}\right)\Big\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)(\beta_1^7 - 21\alpha_1^2\beta_1^5 + 35\alpha_1^4\beta_1^3 -$$

$$7\alpha_1^6\beta_1) + (e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(-7\alpha_1\beta_1^6 + 35\alpha_1^3\beta_1^4 -$$

$$21\alpha_1^5\beta_1^2 + \alpha_1^7)\Big\} \quad ;$$

$$m_{82} = \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{2B_{x\phi}R^2}{D_{x\phi}} - \frac{B_xR^2}{D_x}\right)\Big\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(-\beta_1^3 + 3\alpha_1^2\beta_1)$$

$$+ (e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)(-3\alpha_1\beta_1^2 + \alpha_1^3)\Big\} + \left(\frac{B_\phi}{D_{x\phi}k^2} - \frac{B_\phi}{D_xR^2k^4}\right.$$

$$\left. + \frac{2B_{x\phi}}{D_xk^2}\right)\Big\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(\beta_1^5 - 10\alpha_1^2\beta_1^3 + 5\alpha_1^4\beta_1) +$$

$$(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)(5\alpha_1\beta_1^4 - 10\alpha_1^3\beta_1^2 + \alpha_1^5)\Big\} + \left(\frac{B_xR^4k^2}{D_{x\phi}}\right) \cdot$$

$$\left\{(\beta_1 e^{2\alpha_1\phi_e} \cos 2\beta_1\phi_e) + (\alpha_1 e^{2\alpha_1\phi_e} \sin 2\beta_1\phi_e)\right\} -$$

$$\left(\frac{B_\phi}{D_x R^2 k^4}\right)\left\{(e^{2\alpha_1\phi_e} \cos 2\beta_1\phi_e)(-\beta_1^7 + 21\alpha_1^2\beta_1^5 - 35\alpha_1^4\beta_1^3\right.$$

$$+ 7\alpha_1^6\beta_1) + (e^{2\alpha_1\phi_e} \sin 2\beta_1\phi_e)(-7\alpha_1\beta_1^6 + 35\alpha_1^3\beta_1^4 -$$

$$\left.21\alpha_1^5\beta_1^2 + \alpha_1^7)\right\} \quad ;$$

m_{83} = same expression of m_{81}, with $-\alpha_1$ instead of α_1 ;

m_{84} = same expression of m_{82}, with $-\alpha_1$ instead of α_1 ;

m_{85}, m_{86}, m_{87} & m_{88} = similar expressions to m_{81}, m_{82}, m_{83} & m_{84}, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$n_1 = \frac{12\,p}{\pi}\left(\frac{1}{D_{x\phi}k^2} + \frac{4}{R^2k^4D_x}\right) \quad ; \quad n_2 = 0.0 \quad ;$$

$$n_3 = -\frac{12\,p}{R\,k^3\,\pi} \quad ; \quad n_4 = 0.0$$

$$n_5 = \frac{12\,p}{\pi}\left(\frac{1}{D_{x\phi}k^2} + \frac{4}{R^2k^4D_x}\right) \quad ; \quad n_6 = 0.0 \quad ;$$

$$n_7 = -\frac{12\,p}{R\,k^3\,\pi} \quad ; \quad n_8 = 0.0 \quad .$$

ii- Shells With Longitudinal Stiffeners In Valleys.

The boundary conditions reduce to:

$$\left[\overline{M}\right]\left\{U\right\} = \left\{\overline{N}\right\} \qquad (51)$$

in which

$$\overline{m}_{11} = -\frac{B_\phi}{R^3}\left(\alpha_1 + \alpha_1^3 - 3\alpha_1\beta_1^2\right) + \frac{2B_{x\phi}}{R}k^2\alpha_1 \quad ;$$

$$\overline{m}_{12} = -\frac{B_\phi}{R^3}\left(\beta_1 - \beta_1^3 + 3\alpha_1^2\beta_1\right) + \frac{2B_{x\phi}}{R}k^2\beta_1 \quad ;$$

$$\overline{m}_{13} = -\overline{m}_{11} \quad ; \quad \overline{m}_{14} = \overline{m}_{12} \quad ;$$

$\overline{m}_{15}$, $\overline{m}_{16}$, $\overline{m}_{17}$ & $\overline{m}_{18}$ = similar expressions as $\overline{m}_{11}$, $\overline{m}_{12}$, $\overline{m}_{13}$ & $\overline{m}_{14}$, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$\overline{m}_{41} = \frac{B_\phi}{R^3}\left(-\beta_1^2 + \alpha_1^2 + \beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4\right) - \frac{2B_{x\phi}k^2}{R}\left(-\beta_1^2 + \alpha_1^2\right) + B_x R\, k^4 \quad ;$$

$$\overline{m}_{42} = \frac{B_\phi}{R^3}\left(2\alpha_1\beta_1 - 4\alpha_1\beta_1^3 + 4\alpha_1^3\beta_1\right) - \frac{2B_{x\phi}k^2}{R}\left(2\alpha_1\beta_1\right) + B_x R\, k^4 \quad ;$$

$\overline{m}_{43} = \overline{m}_{41}$; $\overline{m}_{44}$ = similar expression as $\overline{m}_{42}$, with $-\alpha_1$ instead of α_1 ;

$\overline{m}_{45}$, $\overline{m}_{46}$, $\overline{m}_{47}$ & $\overline{m}_{48}$ = similar expressions as $\overline{m}_{41}$, $\overline{m}_{42}$, $\overline{m}_{43}$ & $\overline{m}_{44}$, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$\bar{m}_{51} = -\frac{B_\phi}{R^3}\left\{(e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,(\underline{-\beta_1} + \beta_1^3 - 3\beta_1\alpha_1^2) + (e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,(\underline{\alpha_1} - 3\alpha_1\beta_1^2 + \alpha_1^3)\right\} + \frac{2B_{x\phi}k^2}{R}\cdot \left\{(-\beta_1\, e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e) + (\alpha_1\, e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\right\} \;;$$

$$\bar{m}_{52} = -\frac{B_\phi}{R^3}\left\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)\,(\underline{\beta_1} - \beta_1^3 + 3\alpha_1^2\beta_1) + (e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\,(\underline{\alpha_1} - 3\alpha_1\beta_1^2 + \alpha_1^3)\right\} + \frac{2B_{x\phi}k^2}{R}\cdot \left\{(\beta_1\, e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e) + (\alpha_1\, e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\right\} \;;$$

$\bar{m}_{53}$ = similar expression as $\bar{m}_{51}$, with $-\alpha_1$ instead of α_1 ;

$\bar{m}_{54}$ = similar expression as $\bar{m}_{52}$, with $-\alpha_1$ instead of α_1 ;

$\bar{m}_{55}$, $\bar{m}_{56}$, $\bar{m}_{57}$ & $\bar{m}_{58}$ = similar expressions as $\bar{m}_{51}$, $\bar{m}_{52}$, $\bar{m}_{53}$ & $\bar{m}_{54}$, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$$\bar{m}_{81} = \frac{B_\phi}{R^3}\left\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(\underline{-\beta_1^2 + \alpha_1^2} + \beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4) + (e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)(\underline{-2\alpha_1\beta_1} + 4\alpha_1\beta_1^3 - 4\alpha_1^3\beta_1)\right\} - \frac{2B_{x\phi}k^2}{R}\cdot \left\{(e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e)(-\beta_1^2 + \alpha_1^2) - (e^{2\alpha_1\phi_e}\sin 2\beta_1\phi_e)\cdot (2\alpha_1\beta_1)\right\} + B_x R\, k^4\, (e^{2\alpha_1\phi_e}\cos 2\beta_1\phi_e) \;;$$

$$\bar{m}_{82} = \frac{B_\phi}{R^3} \Big\{ (e^{2\alpha_1 \phi_e} \sin 2\beta_1 \phi_e)(\underline{-\beta_1^2 + \alpha_1^2} + \beta_1^4 - 6\alpha_1^2\beta_1^2 + \alpha_1^4) +$$

$$(e^{2\alpha_1 \phi_e} \cos 2\beta_1 \phi_e)(\underline{2\alpha_1\beta_1} - 4\alpha_1\beta_1^3 + 4\alpha_1^3\beta_1)\Big\} - \frac{2B_{x\phi}k^2}{R} \cdot$$

$$\Big\{(e^{2\alpha_1 \phi_e} \sin 2\beta_1 \phi_e)(-\beta_1^2 + \alpha_1^2) + (e^{2\alpha_1 \phi_e} \cos 2\beta_1 \phi_e) \cdot$$

$$(2\alpha_1\beta_1)\Big\} + B_x R\, k^4\, (e^{2\alpha_1 \phi_e} \sin 2\beta_1 \phi_e) \quad ;$$

$\bar{m}_{83}$ = similar expression as $\bar{m}_{81}$, with $-\alpha_1$ instead of α_1 ;

$\bar{m}_{84}$ = similar expression as $\bar{m}_{82}$, with $-\alpha_1$ instead of α_1 ;

m_{85}, m_{86}, m_{87} & m_{88} = similar expressions as $\bar{m}_{81}$, $\bar{m}_{82}$, $\bar{m}_{83}$ & $\bar{m}_{84}$, respectively, with α_2 & β_2 instead of α_1 & β_1 ;

$\bar{m}_{ij} = m_{ij}$ for $i = 2, 3, 6$ & 7 , $j = 1, 2, \ldots, 8$;

$\bar{n}_1 = 0.0$; $\bar{n}_4 = \dfrac{4\,p\,R}{\pi} \cos^2 \phi_e$; $\bar{n}_5 = 0.0$;

$\bar{n}_8 = \dfrac{4\,p\,R}{\pi} \cos^2 \phi_e$;

$\bar{n}_i = n_i$ for $i = 2, 3, 6$ & 7 .

<u>N.B.</u>

The underlined terms are those which drop when using Donnell's approximations.

APPENDIX (IV)

STRESS RESULTANTS FOR DIFFERENT CASES OF LOADING.

a- Uniform Snow Load on Horizontal Projection (p)

$$p_x = 0$$

$$p_\phi = -\frac{4p}{\pi} \cos(\phi_e - \phi) \sin(\phi_e - \phi) \sin \frac{\pi}{L} x$$

$$p_z = \frac{4p}{\pi} \cos^2(\phi_e - \phi) \sin \frac{\pi}{L} x$$

$$N_x = -\frac{12p}{\pi R k^2} \cos 2(\phi_e - \phi) \sin \frac{\pi}{L} x + (N_x)_b$$

$$N_\phi = -\frac{4pR}{\pi} \cos^2(\phi_e - \phi) \sin \frac{\pi}{L} x + (N_\phi)_b$$

$$N_{x\phi} = -\frac{6pL}{\pi^2} \sin 2(\phi_e - \phi) \cos \frac{\pi}{L} x + (N_{x\phi})_b$$

b- Radial one wave pressure (amplitude = p)

$$p_x = 0$$

$$p_\phi = 0$$

$$p_z = \frac{4p}{\pi} \sin \frac{\pi}{\phi_e} \phi \sin \frac{\pi}{L} x$$

$$N_x = -\frac{4pL^2}{R\phi_e^2 \pi} \sin \frac{\pi}{\phi_e} \phi \sin \frac{\pi}{L} x + (N_x)_b$$

$$N_\phi = -\frac{4pR}{\pi} \sin \frac{\pi}{\phi_e} \phi \sin \frac{\pi}{L} x + (N_\phi)_b$$

$$N_{x\phi} = -\frac{4pL}{\pi \phi_e} \cos \frac{\pi}{\phi_e} \phi \cos \frac{\pi}{L} x + (N_{x\phi})_b$$

c- Radial Half wave Pressure (amplitude = p)

$$p_x = 0$$

$$p_\phi = 0$$

$$p_z = \frac{4p}{\pi} \sin \frac{\pi}{2\phi_e} \phi \sin \frac{\pi}{L} x$$

$$N_x = - \frac{pL^2}{R\phi_e^2 \pi} \sin \frac{\pi}{2\phi_e} \phi \sin \frac{\pi}{L} x + (N_x)_b$$

$$N_\phi = - \frac{4pR}{\pi} \sin \frac{\pi}{2\phi_e} \phi \sin \frac{\pi}{L} x + (N_\phi)_b$$

$$N_{x\phi} = - \frac{2pL}{\phi_e \pi} \cos \frac{\pi}{2\phi_e} \phi \cos \frac{\pi}{L} x + (N_{x\phi})_b$$

d- Constant Tangential Load (-p/4)

$$p_x = 0$$

$$p_\phi = - \frac{p}{\pi} \sin \frac{\pi}{L} x$$

$$p_z = 0$$

$$N_x = (N_x)_b$$

$$N_\phi = (N_\phi)_b$$

$$N_{x\phi} = - pL \cos \frac{\pi}{L} x + (N_{x\phi})_b$$

In which, $(N_x)_b$, $(N_\phi)_b$ and $(N_{x\phi})_b$ are the bending solutions.

They have the same expression as that given by equation 40, in which the constants $A_{N_x} \ldots H_{N_x}, A_{N_\phi} \ldots H_{N_\phi}, A_{N_{x\phi}} \ldots H_{N_{x\phi}}$ for N_x, N_ϕ, and $N_{x\phi}$, respectively, are as follows:

$$A_{N_x} = \left\{ \frac{2B_{x\phi}}{R^3} - \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 \right\} (A\beta_1^4 - 4\alpha_1 B\beta_1^3 - 6\alpha_1^2 A\beta_1^2 + 4\alpha_1^3 B\beta_1 + \alpha_1^4 A)$$

$$- \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 (-A\beta_1^6 + 6\alpha_1 B\beta_1^5 + 15\alpha_1^2 A\beta_1^4 - 20\alpha_1^3 B\beta_1^3 - 15\alpha_1^4 A\beta_1^2$$

$$+ 6\alpha_1^5 B\beta_1 + \alpha_1^6 A) - \frac{B_x}{R} \left(\frac{\pi}{L}\right)^2 (-A\beta_1^2 + 2\alpha_1 B\beta_1 + \alpha_1^2 A)$$

$$B_{N_x} = \left\{ \frac{2B_{x\phi}}{R^3} - \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 \right\} (B\beta_1^4 + 4\alpha_1 A\beta_1^3 + 6\alpha_1^2 B\beta_1^2 - 4\alpha_1^3 A\beta_1 + \alpha_1^4 B)$$

$$- \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 (-B\beta_1^6 - 6\alpha_1 A\beta_1^5 + 15\alpha_1^2 B\beta_1^4 + 20\alpha_1^3 A\beta_1^3 - 15\alpha_1^4 B\beta_1^2$$

$$- 6\alpha_1^5 A\beta_1 + \alpha_1^6 B) - \frac{B_x}{R} \left(\frac{\pi}{L}\right)^2 (-B\beta_1^2 - 2\alpha_1 A\beta_1 + \alpha_1^2 B)$$

$$C_{N_x} = \left\{ \frac{2B_{x\phi}}{R^3} - \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 \right\} (C\beta_1^4 + 4\alpha_1 D\beta_1^3 - 6\alpha_1^2 C\beta_1^2 - 4\alpha_1^3 D\beta_1 + \alpha_1^4 C)$$

$$- \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 (-C\beta_1^6 - 6\alpha_1 D\beta_1^5 + 15\alpha_1^2 C\beta_1^4 + 20\alpha_1^3 D\beta_1^3 - 15\alpha_1^4 C\beta_1^2$$

$$- 6\alpha_1^5 D\beta_1 + \alpha_1^6 C) - \frac{B_x}{R} \left(\frac{\pi}{L}\right)^2 (-C\beta_1^2 - 2\alpha_1 D\beta_1 + \alpha_1^2 C)$$

$$D_{N_x} = \left\{ \frac{2B_{x\phi}}{R^3} - \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 \right\} (D\beta_1^4 - 4\alpha_1 C\beta_1^3 + 6\alpha_1^2 D\beta_1^2 + 4\alpha_1^3 C\beta_1 + \alpha_1^4 D)$$

$$- \frac{B_\phi}{R^5} \left(\frac{L}{\pi}\right)^2 (-D\beta_1^6 + 6\alpha_1 C\beta_1^5 + 15\alpha_1^2 D\beta_1^4 - 20\alpha_1^3 C\beta_1^3 - 15\alpha_1^4 D\beta_1^2$$

$$+ 6\alpha_1^5 C\beta_1 + \alpha_1^6 D) - \frac{B_x}{R} \left(\frac{\Pi}{L}\right)^2 (-D\beta_1^2 + 2\alpha_1 C\beta_1 + \alpha_1^2 D)$$

$$A_{N_\phi} = \frac{B_\phi}{R^3} (-A\beta_1^2 + 2\alpha_1 B\beta_1 + \alpha_1^2 A + B\beta_1^4 - 4\alpha_1 B\beta_1^3 - 6\alpha_1^2 A\beta_1^2$$

$$+ 4\alpha_1^3 B\beta_1 + \alpha_1^4 A) - \frac{2B_{x\phi}}{R} \left(\frac{\Pi}{L}\right)^2 (-A\ {}_1^2 + 2\ {}_1 B\ {}_1 + {}_1^2 A)$$

$$+ AB_x R \left(\frac{\Pi}{L}\right)^4$$

$$B_{N_\phi} = \frac{B_\phi}{R^3} (-B\beta_1^2 - 2\alpha_1 A\beta_1 + \alpha_1^2 B + B\beta_1^4 + 4\alpha_1 A\beta_1^3 + 6\alpha_1^2 B\beta_1^2$$

$$- 4\alpha_1^3 A\beta_1 + \alpha_1^4 B) - \frac{2B_{x\phi}}{R} \left(\frac{\Pi}{L}\right)^2 (-B\beta_1^2 - 2\alpha_1 A\beta_1 + \alpha_1^2 B)$$

$$+ BB_x R \left(\frac{\Pi}{L}\right)^4$$

$$C_{N_\phi} = \frac{B_\phi}{R^3} (-C\beta_1^2 - 2\alpha_1 D\beta_1 + \alpha_1^2 C + C\beta_1^4 + 4\alpha_1 D\beta_1^3 - 6\alpha_1^2 C\beta_1^2$$

$$- 4\alpha_1^3 D\beta_1 + \alpha_1^4 C) - \frac{2B_{x\phi}}{R} \left(\frac{\Pi}{L}\right)^2 (-C\beta_1^2 - 2\alpha_1 D\beta_1 + \alpha_1^2 C)$$

$$+ CB_x R \left(\frac{\Pi}{L}\right)^4$$

$$D_{N_\phi} = \frac{B_\phi}{R^3} (-D\beta_1^2 + 2\alpha_1 C\beta_1 + \alpha_1^2 D + D\beta_1^4 - 4\alpha_1 C\beta_1^3 + 6\alpha_1^2 D\beta_1^2$$

$$+ 4\alpha_1^3 C\beta_1 + \alpha_1^4 D) - \frac{2B_{x\phi}}{R} \left(\frac{\Pi}{L}\right)^2 (-D\beta_1^2 + 2\alpha_1 C\beta_1 + \alpha_1^2 D)$$

$$+ DB_x R \left(\frac{\Pi}{L}\right)^4$$

$$A_{N_{x\phi}} = \left\{ \frac{2B_{x\phi}}{R^2} \left(\frac{\pi}{L}\right) - \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) \right\} (-B\beta_1^3 - 3\alpha_1 A\beta_1^2 + 3\alpha_1^2 B\beta_1 + \alpha_1^3 A)$$

$$- \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) (B\beta_1^5 + 5\alpha_1 A\beta_1^4 - 10\alpha_1^2 B\beta_1^3 - 10\alpha_1^3 A\beta_1^2 + 5\alpha_1^4 B\beta_1$$

$$+ \alpha_1^5 A) - B_x \left(\frac{\pi}{L}\right)^3 (B\beta_1 + \alpha_1 A)$$

$$B_{N_{x\phi}} = \left\{ \frac{2B_{x\phi}}{R^2} \left(\frac{\pi}{L}\right) - \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) \right\} (A\beta_1^3 - 3\alpha_1 B\beta_1^2 - 3\alpha_1^2 A\beta_1 + \alpha_1^3 B)$$

$$- \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) (-A\beta_1^5 + 5\alpha_1 B\beta_1^4 + 10\alpha_1^2 A\beta_1^3 - 10\alpha_1^3 B\beta_1^2 - 5\alpha_1^4 A\beta_1$$

$$+ \alpha_1^5 B) - B_x \left(\frac{\pi}{L}\right)^3 (-A\beta_1 + \alpha_1 B)$$

$$C_{N_{x\phi}} = \left\{ \frac{2B_{x\phi}}{R^2} \left(\frac{\pi}{L}\right) - \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) \right\} (-D\beta_1^3 + 3\alpha_1 C\beta_1^2 + 3\alpha_1^2 D\beta_1 - \alpha_1^3 C)$$

$$- \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) (D\beta_1^5 - 5\alpha_1 C\beta_1^4 - 10\alpha_1^2 D\beta_1^3 + 10\alpha_1^3 C\beta_1^2 + 5\alpha_1^4 D\beta_1$$

$$- \alpha_1^5 C) - B_x \left(\frac{\pi}{L}\right)^3 (D\beta_1 - \alpha_1 C)$$

$$D_{N_{x\phi}} = \left\{ \frac{2B_{x\phi}}{R^2} \left(\frac{\pi}{L}\right) - \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) \right\} (C\beta_1^3 + 3\alpha_1 D\beta_1^2 - 3\alpha_1^2 C\beta_1 - \alpha_1^3 D)$$

$$- \frac{B_\phi}{R^4} \left(\frac{L}{\pi}\right) (-C\beta_1^5 - 5\alpha_1 D\beta_1^4 + 10\alpha_1^2 C\beta_1^3 + 10\alpha_1^3 D\beta_1^2 - 5\alpha_1^4 C\beta_1$$

$$- \alpha_1^5 D) - B_x \left(\frac{\pi}{L}\right)^3 (-C\beta_1 - \alpha_1 D)$$

The expressions of E, F, G & H corresponding to N_x, $N_{x\phi}$ & $N_{x\phi}$ are similar to those of A, B, C & D, respectively, with α_2 & β_2 instead of α_1 & β_1.

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