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STARRED ANGLES SUPPORTING SECONDARY TRUSSES

by

Kenneth Hon-Wa Mok

A Thesis
submitted to the
Faculty of Graduate Studies and Research
through the Department of
Civil and Environmental Engineering
in Partial Fulfillment of the requirements for
the Degree of Master of Applied Science at
the University of Windsor

Windsor, Ontario, Canada

1988

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ABSTRACT

In some large industrial buildings it is common to use primary trusses in one direction and secondary trusses in the other. Due to its symmetrical configuration, a starred angle is frequently used as a web member in the primary truss to support the secondary truss, and that seems to be gaining in popularity. For design purposes, it is customary to neglect the effect of eccentricity and design such members as axially loaded columns.

An experimental research project has been completed in order to investigate the behaviour and strength of the starred angles supporting trusses. The main areas of concern were: (a) critical loads; (b) mode of buckling; (c) force and stress distributions. Various theories or standards were also reviewed to help in analyzing the test results.

This thesis illustrates the test procedures and provides the results, together with some preliminary conclusions. A simplified design method for such members is also proposed.

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TABLE OF CONTENTS

DEDICATION.	vi
ABSTRACT.	vii
ACKNOWLEDGEMENTS.	viii
LIST OF FIGURES	xii
LIST OF TABLES.	xv
NOMENCLATURE.	xvi

CHAPTER

I.	INTRODUCTION	1
	1.1 STARRED ANGLES.	1
	1.2 CONNECTION DETAIL	2
	1.3 JUSTIFICATION OF THE NEED	3
	1.4 OBJECTIVE	4
II.	LITERATURE REVIEW.	5
	2.1 GENERAL	5
	2.2 PREVIOUS WORK	5
	2.3 STANDARDS AND SPECIFICATIONS.	7
	2.3.1 Canadian Standards CAN3-S16.1-M84.	8
	2.3.2 ASCE Manual 52 , " Guide for Design of Steel Transmission Towers "	8
	2.3.3 ECCS Recommendations (1978).	10
	2.3.4 Other Standards.	10
	2.4 SUMMARY	11
III.	THEORETICAL BACKGROUND	12
	3.1 GENERAL	12
	3.2 CLASSICAL THEORETICAL ANALYSIS.	12
	3.2.1 Elastic Critical Load Under Concentric Loading.	12
	3.2.2 Elastic Critical Load Under Eccentric Loading.	13
	3.3 BEAM-COLUMN THEORETICAL MODELLING	15
	3.4 STRESS-STRAIN RELATIONSHIP.	15
	3.4.1 Hooke's Law.	15
	3.4.2 The Principle of Superposition	16
	3.4.3 Measurement of Surface Strains	17
	3.5 SAINT-VENANT'S PRINCIPLE.	18
IV.	EXPERIMENTAL PROCEDURE	20
	4.1 GENERAL	20
	4.2 TEST PROGRAM.	20
	4.3 TEST SET-UP	22
	4.3.1 End Conditions and Fixtures.	23

4.3.2	Loading Equipment.	24
4.3.3	Testing Frame for Type C Columns	24
4.3.4	Testing Frame for Type B Columns	25
4.3.5	Testing Frame for Type W Columns	26
4.3.6	Instrumentation.	26
4.4	TEST PROCEDURE.	28
4.4.1	Testing of Type C Columns.	28
4.4.2	Testing of Type B and W Columns.	28
4.5	DATA REDUCTION.	29
4.5.1	Load	29
4.5.2	Displacement.	29
4.5.3	Stress	30
V.	RESULTS AND ANALYSIS	32
5.1	GENERAL	32
5.2	PROPERTIES.	32
5.2.1	Geometric Properties	32
5.2.2	Mechanical Properties.	34
5.3	THEORETICAL PREDICTIONS	35
5.3.1	Type C columns	35
5.3.1.1	Classical Approach	35
5.3.1.2	North American Standards	36
5.3.1.3	Comparison	36
5.3.2	Type B and W Columns	37
5.3.2.1	Beam-Column Approach	37
5.3.2.2	Approach Using a Modified Slenderness Ratio.	38
5.4	EXPERIMENTAL RESULTS AND ANALYSIS	38
5.4.1	Failure Load	39
5.4.1.1	Test Summaries and Comparison with Theoretical Predictions	39
5.4.1.2	Combined Approach.	40
5.4.2	Load Deflection Curve.	41
5.4.2.1	Type C Columns	41
5.4.2.2	Type B Columns	42
5.4.2.3	Type W Columns	43
5.4.3	Stress and Force Distributions	44
5.4.3.1	Slender Type B Columns	44
5.4.3.2	Intermediate Type B Columns.	45
5.4.3.3	Type W Columns	47
5.4.3.4	Summary of Force Distribution.	47
VI.	DISCUSSION	49
6.1	GENERAL	49
6.2	ULTIMATE STRENGTH	49
6.3	LOAD TRANSFER MECHANISM	50
6.4	CONNECTION DESIGN	51
6.4.1	Joint 1.	51
6.4.2	Joint 2.	52
6.4.3	Joint 3.	53
6.5	SUMMARY	54

VII.	CONCLUSIONS AND RECOMMENDATIONS.	55
7.1	CONCLUSIONS	55
7.2	SCOPE FOR FURTHER RESEARCH.	56
APPENDIX A:	FIGURES	58
APPENDIX B:	TABLES.	115
APPENDIX C:	NONDIMENSIONAL BUCKLING CURVE (FROM ECCS RECOMMENDATIONS).	128
APPENDIX D:	BEAM-COLUMN ANALYSIS.	130
APPENDIX E:	DESIGN BASED ON ASCE MANUAL 52 AND ECCS RECOMMENDATIONS	135
APPENDIX F:	CURVE FITTING METHOD.	140
BIBLIOGRAPHY.		146
VITA AUCTORIS		149

LIST OF FIGURES

1.1	a) Physical Layout of Starred Angles in Roof Trusses	59
	b) Close-Up View of Starred Angles	59
1.2	Connection Detail.	60
1.3	The Coordinate Axes.	61
3.1	Theoretical Beam-Column Modelling	62
4.1	Detail of Type W Column.	63
4.2	a) Typical Dimensions of C2100 Specimens	64
	b) Typical Dimensions of C1270 Specimens	65
	c) Typical Dimensions of B2100 Specimens	66
	d) Typical Dimensions of B1270 Specimens	67
	e) Typical Dimensions of W2100 Specimens	68
4.3	Assembly of Double Knife Edges	69
4.4	a) Complete Set-Up for C2100 Specimens	70
	b) Complete Set-Up for C1270 Specimens	70
4.5	a) End Fixture with 222 kN Capacity Jack at the Top	71
	b) End Fixture with 222 kN Capacity Load Cell at the Bottom.	71
4.6	a) Close-Up View at the Connection End of the Truss	72
	b) Close-Up View at Other End of the Truss	72
4.7	The 1335 kN Capacity Jack at the Centre of the Truss.	73
4.8	Complete Set-Up for Type B Columns	74
4.9	a) Front View of the Bolted Connection	75
	b) Side View of the Bolted Connection.	75

4.10	The 45 kN Capacity Load Cell Under the Jack.	76
4.11	a) Side View of Type W Columns	77
	b) Close-Up View Showing Connection Joint.	77
4.12	a) Locations of Strain Gauges for B2100.1 and B1270.1	78
	b) Locations of Strain Gauges for Other Slender Specimens	79
	c) Pattern and Specifications of Strain Gauges	79
4.13	a) The Arrangement of Dial Gauges at Mid-Height.	80
	b) Numbering of Dial Gauges at Mid-Height.	80
4.14	The Arrangement of Dial Gauges for Specimen B1270.1	81
4.15	a) The Arrangement of Dial Gauges at Location of Strain Gauges	82
	b) Numbering of Dial Gauges at Location of Strain Gauges.	82
4.16	Calibration Curve for 222 kN Capacity Load Cell.	83
5.1	Idealized Rectangular Section.	84
5.2	a) Load-Deflection Curve for C2100.1	85
	b) Load-Deflection Curve for C2100.2	86
	c) Load-Deflection Curve for C2100.3	87
	d) Load-Deflection Curve for C1270.1	88
	e) Load-Deflection Curve for C1270.2	89
	f) Load-Deflection Curve for B2100.1	90
	g) Load-Deflection Curve for B2100.2	91
	h) Load-Deflection Curve for B1270.1	92
	i) Load-Deflection Curve for B1270.2	93
	j) Load-Deflection Curve for W2100.1	94
	k) Load-Deflection Curve for W2100.2	95

5.3	a) Buckling of Slender Type B Column	
	i) Front View.	96
	ii) Close-up View at the Top.	96
	b) Buckling of Intermediate Type B Column	
	i) Front View.	97
	ii) Side View	97
	c) Inelastic Buckling of B1270 Specimens	98
5.4	a) Stress Distribution in Angle A (B2100.1).	99
	b) Stress Distribution in Angle B (B2100.1).	100
	c) Force Distribution in B2100.1	101
5.5	a) Stress Distribution in Angle A (B2100.2).	102
	b) Force Distribution in B2100.2	103
5.6	a) Stress Distribution in Angle A (B1270.1).	104
	b) Stress Distribution in Angle B (B1270.1).	105
	c) Force Distribution in B1270.1	106
5.7	a) Stress Distribution in Angle A (B1270.2).	107
	b) Force Distribution in B1270.2	108
5.8	a) Stress Distribution in Angle A (W2100.1).	109
	b) Force Distribution in W2100.1	110
5.9	a) Stress Distribution in Angle A (W2100.2).	111
	b) Force Distribution in W2100.2	112
6.1	Load Transfer Mechanism.	113
6.2	Analysis of Joint 1.	114

LIST OF TABLES

1.	Actual vs. Nominal Cross-Sectional Properties	.116
2.	Mechanical Properties of Steel.	.117
3.	Theoretical Predictions of Critical Loads	.118
4.	Experimental Buckling Loads	.119
5.	Starred Angles Designed as Axially Loaded Members . . .	.120
6.	Force Distribution in Specimen B2100.1	.121
7.	Force Distribution in Specimen B2100.2	.122
8.	Force Distribution in Specimen B1270.1	.123
9.	Force Distribution in Specimen B1270.2	.124
10.	Force Distribution in Specimen W2100.1	.125
11.	Force Distribution in Specimen W2100.2	.126
12.	Maximum Force in Each Leg/Angle Expressed as a Percentage of the Buckling Load	.127

NOMENCLATURE

A	= area of the starred angle section
A_b	= area of the bolt
A_o	= area of an individual angle
B_r	= bearing resistance of the bolt
C_r	= compressive axial load in CSA S16.1
C_w	= warping constant
E	= modulus of elasticity
F_u	= ultimate strength of the gusset plate (or lug angle)
F_u'	= the specified tensile strength of the bolt metal
G	= modulus of rigidity
I_p	= polar moment of inertia
I_{ps}	= polar moment of inertia about the shear centre
I_u	= moment of inertia about the major principal axis of the starred angle section
I_v	= moment of inertia about the minor principal axis of the starred angle section
I_u', I_v'	= moment of inertia about the principal axes of an individual angle
J	= Saint-Venant's torsional constant
K	= effective length factor
L	= nominal length of column
M_u, M_v	= moments produced by the eccentric force in beam-column modelling
P	= total loads transmitted from the secondary truss to the primary truss

P_{ex} = experimental buckling loads
 P_n = nominal strength of axially loaded members in AISC Specification
 P_t = critical load for torsional buckling
 P_v = Euler load about the weak axis
 U, V, Z = coordinate axes for the starred angle section
 V_r = shear resistance of the bolt
 a = separation of individual angles in starred angle strut
 b = width of an individual angle
 d = diameter of the bolt
 e = eccentricity of the applied load in beam-column modelling
 e_d = end distance measured from the centre of the bolt to the end of gusset plate (or lug angle)
 e_u, e_v = u and v-coordinates of point of application of the eccentric axial load with reference to the centroidal principal axes
 e_x, e_y = coordinates of point of application of the external load with reference to the x and y axes
 n = number of bolts required in the joint
 r_t = radius of gyration for torsional buckling
 r_u, r_v = radii of gyration for flexural buckling in starred angle
 r_u', r_v' = radii of gyration for flexural buckling in individual angle
 t = thickness of angles
 t_p = thickness of the thinnest part in bearing connection
 u, v, z = coordinate axes for an individual angle

v_R = Poisson's ratio
 $\bar{x}, \bar{y}$ = coordinates of the centroid of an angle
 x_s, y_s = coordinates of shear centre with respect to the centroidal principal axes
 ϵ = strain
 λ = slenderness ratio defined in ECCS Recommendations
 ϕ = resistance factor in CSA S16.1
 ϕ_c = resistance factor in AISC Specification
 ϕ_z = rotation of the shear centre about the longitudinal axis (z-z axis)
 σ = stress

Chapter I

INTRODUCTION

1.1 STARRED ANGLES

Starred angles are frequently used as web members in trusses. They are easy to fabricate and erect due to the symmetry of their cross-section. Two angles are joined at their ends to the chord members, usually the webs of tee sections, to form a starred shape. The end connections are either welded or bolted. This arrangement provides accessibility to surfaces for maintenance and is usually used in the food and pharmaceutical industry to maintain a clean and hygienic environment. Starred angles are also used in the chemical industry where it is desirable to have access to the angle surfaces for corrosion protection.

Materials are used effectively in the starred angle arrangement. For example, the minimum radius of gyration is larger than that in the back-to-back arrangement when using the same equal leg angles. Lighter gusset plates are permitted in the starred angle arrangement if welded end connections are used. This is because a lower concentration of shear stresses is acting on the gusset plate.

The disadvantages of this arrangement should also be mentioned. The starred angle provides the smallest truss window size when comparing to the back-to-back arrangement or boxed-section. Bolted end connections are not economical since the bolts are used in single shear.

1.2 CONNECTION DETAIL

Roof trusses are an economical way to cover large spans with a minimum of material. They are often used in low-rise industrial buildings.

One arrangement often used to span large areas consists of two mutually perpendicular sets of trusses installed at different elevations. Secondary trusses frame into the vertical web members of the primary trusses. The secondary trusses are set at a lower elevation than the primary trusses to allow for roof beams which are supported by the secondary trusses. The tops of the roof beams and primary trusses are at the same elevation. A very convenient way of doing this is to use a starred angle web member in the primary truss to support the secondary truss (see Figures 1.1(a) and (b)). In this case, the use of back-to-back angles would not be acceptable due to the unsymmetrical connection and the boxed configuration would be completely impractical.

The worst situation of truss-to-truss connections occurs at the exterior members of the roof trusses. In those members, the secondary trusses are framed into the primary trusses from one side only. This provides unsymmetrical connections. Hence, this research will concentrate on the exterior members of the roof trusses. The truss-to-truss connections mentioned later are all of this specific type.

A typical connection detail between primary and secondary trusses is shown in Figure 1.2 . It will be noted that the work point of the connection coincides with the longitudinal centroidal axis of the starred angle. Thus in general design practice, the starred angle is assumed to be axially loaded. The open nature of the cross section of the starred angle, as shown in Figure 1.3 , raises some doubt about the distribution of load between the two angles in the starred angle.

1.3 JUSTIFICATION OF THE NEED

An analysis of a typical roof truss reveals that as much as 95 percent of the force acting on the vertical web members of the primary truss comes from the secondary truss. Thus the connection between the trusses is important. Research is required to

determine the behaviour of such connections as well as to determine the load carrying capacity of these starred angles.

1.4 OBJECTIVE

The object of this research is to determine the behaviour of the connections between secondary and primary trusses when starred angles are used as the vertical web members in the primary trusses.

To achieve this objective, the following were examined:

- (a) the load carrying capacities of the starred angle compression members;
- (b) the buckling behaviour of the starred angles;
- (c) the distribution of the applied load to the legs of the starred angles; and
- (d) the bolting requirements of truss - to - truss connection.

CHAPTER II

LITERATURE REVIEW

2.1 GENERAL

Literature on starred angle compression members is very limited. In the first part of this chapter , a brief survey of the contributions made by some of the researchers of starred angles is presented. Then, in the second part , steel standards and specifications as they relate to this research are discussed.

2.2 PREVIOUS WORK

Requirements with regard to the interconnection of starred angle compression members vary from code to code. The North American standards, CAN3-S16.1-M78 and AISC Specification, are the most liberal and require only one interconnector at mid-height . On the other hand, the British Standard⁽⁹⁾ is the most conservative and several pairs of interconnectors are required.

In 1978 , tests were conducted by Pauls and Stringer⁽¹⁸⁾ to investigate the interconnection of starred angle compression members. It was concluded that the number and type of interconnectors has a significant effect on the compressive strength of the

built-up section. However, no actual value for the number of interconnectors required or theoretical analysis of starred angled struts was made.

Two years later, Temple and Schepers ⁽²⁰⁾ undertook a series of tests to study the problem of interconnection of starred angles in detail. A total of twenty-nine starred angle compression members with the number of interconnectors varying from zero to five were loaded axially. It was found that starred angles with zero or one interconnector failed by buckling about the major principal axis of the component angles, i.e., the two angles did not act as an integral unit, whereas members having two or more interconnectors buckled about the minor principal axis as an integral unit. A corresponding change in the failure load was observed with the change in failure mode. To ensure buckling occurs about the weak axis of the entire cross-section, two interconnectors, one at each of the third points, should be used for any starred angle compression members made from equal leg angles. Test results also showed that the length of the end connector has a negligible effect on the load-carrying capacity of a starred angle when buckling occurs about the minor principal axis.

In 1986 , Tan (19) tested forty-four widely spaced back-to-back angles to explore the behaviour of such double angle compression members. The study indicated that only one interconnector was required at mid-height of any back-to-back double angle made from equal leg angles. The project also examined the magnitudes of the forces and moments to be resisted by the interconnectors and it was found that they were very small. Test results were verified by a theoretical study based on the finite element method.

2.3 STANDARDS AND SPECIFICATIONS

As was shown in Section 1.2 , the work point of the framing of a secondary truss into a primary truss coincides with the centroidal axis of the starred angle strut. It seems that the starred angle web member is axially loaded. However, the open nature of the cross-section might mean that the starred angle is eccentrically loaded. Hence, there is a question as to whether or not the starred angles supporting secondary trusses should be designed as concentrically or eccentrically loaded members.

The following sections discuss the applicable clauses in several steel standards and specifications related to the design of concentrically and eccentrically loaded compression angle members.

2.3.1 Canadian Standards CAN3-S16.1-M84 (12)

Clause 9.3.4 states that unless analysis shows that a smaller value is applicable, the effective length factor (K) shall be taken as 1.0 for compression members in trusses.

In normal practice, design engineers adopt this criterion to design the starred angle struts supporting secondary trusses.

2.3.2 ASCE Manual No.52, "Guide for Design of Steel Transmission Towers" (3)

The manual was developed to serve as a basis for the design of self-supporting steel transmission towers. It provides an accepted reference for various angle shapes used as pin-connected members in trusses.

Clause 4.7.4 of the Manual specifies K factors for angle compression members which depend upon the type of connection. It indicates that, for axially loaded leg members bolted in both faces at connections, the effective length factor K is taken as one. Modified slenderness ratios, KL/r (where L = whole length of the compression member and r = radius of gyration), used for other compression members are listed as follows :

(a) For members with concentric load at both ends of

the unsupported panel, and $L/r \leq 120$

$$KL/r = L/r \quad [2.1]$$

- (b) For members with concentric load at one end and normal framing eccentricity at the other end of the unsupported panel, and $L/r \leq 120$

$$KL/r = 30 + 0.75 * (L/r) \quad [2.2]$$

- (c) For members with normal framing eccentricities at both ends of the unsupported panel, and $L/r \leq 120$

$$KL/r = 60 + 0.50 * (L/r) \quad [2.3]$$

- (d) For members unrestrained against rotation at both ends of the unsupported panel, and $120 \leq L/r \leq 200$

$$KL/r = L/r \quad [2.4]$$

- (e) For members partially restrained against rotation at one end of the unsupported panel, and $120 \leq L/r \leq 225$

$$KL/r = 28.6 + 0.762 * (L/r) \quad [2.5]$$

- (f) For members partially restrained against rotation at both ends of the unsupported panel, and $120 \leq L/r \leq 250$

$$KL/r = 46.2 + 0.615 * (L/r) \quad [2.6]$$

The slenderness ratios recommended above are based on a review of formulas that have been used by the tower industry for many years and are supported by the results of laboratory and full-scale tower tests.

2.3.3 ECCS (European Convention for Constructional Steelwork) Recommendations (15)

For angles in compression loaded through one leg , the effect of eccentricity is taken into account by modifying the slenderness ratio as given below (R4.1.5.3) :

(a) For $(\lambda / \lambda_y) \leq 1.41$,

$$(\lambda / \lambda_y)_{\text{eff}} = 0.60 + 0.57 * (\lambda / \lambda_y) \quad [2.7]$$

where λ =slenderness ratio = KL/r ; $\lambda_y = \pi \sqrt{E/\sigma_y}$;

'E = modulus of elasticity; and σ_y = yield stress.

(b) For $(\lambda / \lambda_y) \geq 1.41$,

$$(\lambda / \lambda_y)_{\text{eff}} = (\lambda / \lambda_y) \quad [2.8]$$

(c) If the chords provide good end restraints to the web members and if at least two bolts or rivets in line are present at the end connections of the web members, for $1.41 \leq (\lambda / \lambda_y) \leq 3.5$,

$$(\lambda / \lambda_y)_{\text{eff}} = 0.35 + 0.75 * (\lambda / \lambda_y) \quad [2.9]$$

Values of buckling stress are then determined from the non-dimensional buckling curve 'b' (15) as shown in Appendix C .

2.3.4 Other Standards

A study of the British (9) and German Specifications (14) reveals that in the design of eccentrically loaded single-angle members in compression, the eccentricity is ignored. To compensate

for this, the strength of such angles is limited to a certain percentage of the strength of corresponding concentric axially loaded angles.

The Australian Standard (5) and AISC Specification⁽¹⁾, however, specifically mention that eccentricity cannot be ignored. Such members should be designed as members under biaxial bending.

2.4 SUMMARY

Starred angles are used extensively as structural members. Yet research on starred-angle compression members is very limited. An examination of the standards and specifications reveals nothing directly related to the design of starred-angle compression members in primary trusses.

To simplify the design of eccentrically loaded single-angle compression members, ASCE Manual 52 and ECCS Recommendations modify the slenderness ratio of such members to make use of the formulas applicable to concentrically loaded angles. In contrast, other standards and specifications either ignore the eccentricity or design the angle members as beam-columns.

CHAPTER III

THEORETICAL BACKGROUND

3.1 GENERAL

In this chapter, classical theoretical analysis of starred angles under concentric and eccentric loading is presented. A beam-column modelling of starred angles is discussed. Finally, a review of stress-strain relationship and the Saint-Venant's Principle are also included.

3.2 CLASSICAL THEORETICAL ANALYSIS

3.2.1 Elastic Critical Load Under Concentric Loading

For doubly-symmetric compound angles, in which the shear centre coincides with the centroid, failure will occur either by :

- (a) Euler flexural buckling about the weak axis (V-V axis in starred angles)

$$P_v = \frac{\pi^2 EI_v}{(K_v L)^2} \quad [3.1]$$

- (b) Torsional buckling about the shear centre

$$P_t = \frac{A}{I_{ps}} * \left[GJ + \frac{\pi^2 EC_w}{(K_t L)^2} \right] \quad [3.2]$$

Equation [3.2] can be expressed in terms of an equivalent radius of gyration, r_t , as follows :

$$P_t = \frac{\pi^2 E A r_t^2}{(K_t L)^2} \quad [3.3]$$

in which

$$r_t = \left\{ \frac{1}{I_{ps}} * \left[\frac{J (K_t L)^2}{2\pi^2 (1+\nu_R)} + C_w \right] \right\}^{\frac{1}{2}} \quad [3.4]$$

where K_v, K_t = effective length factors for flexural buckling about the V-V axis and torsional buckling, respectively; A = area of the cross-section of the column; G = modulus of rigidity; I_{ps} = polar moment of inertia about the shear centre = $I_u + I_v$; J = Saint-Venant's torsional constant; ν_R = Poisson's ratio and C_w = warping constant, generally taken as zero for starred angles (8).

Since Equations [3.1] and [3.2] are independent, the flexural buckling mode (about the weak axis) and the torsional buckling mode are uncoupled. The critical buckling load is then the smaller value of either P_v or P_t .

3.2.2 Elastic Critical Load Under Eccentric Loading

The theory of elastic buckling of a column, loaded with axial loads and biaxial eccentricities, was formulated by Timoshenko and Gere (1961). The basic differential equations (22) are :

$$EI_y u^{iv} + Pu'' + P(y_s - e_y) \phi_z'' = 0 \quad [3.5]$$

$$EI_x v^{iv} + Pv'' - P(x_s - e_x) \phi_z'' = 0 \quad [3.6]$$

$$EC_w \phi_z^{iv} - (GJ - Pe_y \beta_1 - Pe_x \beta_2 - PI_{ps}/A) \phi_z'' + P(y_s - e_y) u'' - P(x_s - e_x) v'' = 0 \quad [3.7]$$

$$\text{in which } \beta_1 = (1/I_x) (\int_A y^3 dA + \int_A x^2 y dA) - 2y_s$$

$$\beta_2 = (1/I_y) (\int_A x^3 dA + \int_A xy^2 dA) - 2x_s$$

where P = applied load; ϕ_z = rotation of shear centre about longitudinal z - z axis; I_x, I_y = moments of inertia about the centroidal principal major and minor axes, respectively; x_s, y_s = coordinates of shear centre with respect to centroidal principal axes; e_x, e_y = coordinates of point of application of the external load; and u, v = deflection components of the shear centre along the major and minor principal axes, respectively.

For members with cross-section having two axes of symmetry (e.g. starred angles), the critical load is computed from the cubic equation obtained by expanding the following determinant:

$$\begin{bmatrix} (P_y - P_{cr}) & 0 & P_{cr} e_y \\ 0 & (P_x - P_{cr}) & -P_{cr} e_x \\ P_{cr} e_y & -P_{cr} e_x & I_{ps} (P_t - P_{cr}) / A \end{bmatrix} \quad [3.8]$$

The above determinant is derived from solving the basic differential equations (Equations [3.5] to [3.7]). It should also be mentioned that P_x and P_y correspond to P_u and P_v in the case of starred angles.

3.3 BEAM-COLUMN THEORETICAL MODELLING

The framing of a secondary truss into a primary truss is achieved by bolted connections through one leg of the angle members. It is tacitly assumed that the line of action of the force acting on a bolt group passes through the centroid of the areas of the bolts. Providing the two angle components function as an integral unit at the place of truss-to-truss connection, it could then be argued that the external load P was applied at an eccentricity of e from the centroid of the cross-section (see Figure 3.1). As is demonstrated in books on statics, this eccentric force can be replaced by the centroidal force P and a couple of moment, $M_u = M_v = (P)(e)\cos 45^\circ$, along the principal axes. Such columns, subjected to both bending and axial compression, are termed beam-columns.

In Appendix D, starred angles supporting secondary trusses are analyzed as beam-columns following the design procedures suggested by the CSA Standard. Results are then compared with test data in Chapter V to check whether or not these members can be designed as beam-columns.

3.4 STRESS-STRAIN RELATIONSHIP

3.4.1 Hooke's Law

In the elastic region of the stress-strain diagram of steel, the stress σ is proportional to the strain ϵ . This relation is generally known as Hooke's Law. Imagine a thin rod subjected to a tensile force which results in a normal

stress, σ_x , uniformly distributed over the cross-section. The unit elongation of the element up to the proportional limit can be written as :

$$\epsilon_x = \sigma_x / E \quad [3.9]$$

where E is the modulus of elasticity of steel .

This extension gives rise to lateral contractions in the y and z directions, the unit contractions are given by

$$\epsilon_y = -v_R \epsilon_x = -v_R (\sigma_x/E) \quad [3.10]$$

$$\epsilon_z = -v_R \epsilon_x = -v_R (\sigma_x/E) \quad [3.11]$$

where v_R is Poisson's ratio and usually taken equal to 0.3 for structural steel.

3.4.2 The Principle of Superposition

This principle states that deformations and stresses, produced by several different systems of forces, can be superimposed, providing that the following conditions are satisfied:

- (a) the stress-strain relationship is linear, and
- (b) deformations are small and do not significantly affect the action of the external force system.

Hence if σ_x , σ_y and σ_z are uniform stresses acting in the x, y and z directions, respectively, the principle of superposition can be used to obtain the following equations:

$$\begin{aligned} \epsilon_x &= \sigma_x/E - v_R \sigma_y/E - v_R \sigma_z/E \\ &= [\sigma_x - v_R (\sigma_y + \sigma_z)] / E \end{aligned} \quad [3.12]$$

$$\epsilon_Y = [\sigma_Y - \nu_R (\sigma_X + \sigma_Z)] / E \quad [3.13]$$

$$\epsilon_Z = [\sigma_Z - \nu_R (\sigma_X + \sigma_Y)] / E \quad [3.14]$$

3.4.3 Measurement of Surface Strains

A convenient and accurate method for the measurement of strains on a surface is provided by electrical resistance strain gauges. In order to measure the strain of a material in the given direction, the gauge is glued to the surface of the material, with the wire folds running parallel to the direction in which the strain is to be measured. As the material elongates, the wire increases in length and decreases in diameter, causing the electrical resistance of the gauge to increase. The strain can thus be measured electrically.

When the principal directions are known, one gauge is placed along each of the principal directions to measure the strains, ϵ_1 and ϵ_2 (ϵ_X and ϵ_Y). Using Hooke's Law, Equations [3.12] to [3.14], the principal stresses σ_1 (σ_X) and σ_2 (σ_Y) are then calculated as :

$$\sigma_1 = E (\epsilon_1 + \nu_R \epsilon_2) / (1 - \nu_R^2) \quad [3.15]$$

$$\sigma_2 = E (\epsilon_2 + \nu_R \epsilon_1) / (1 - \nu_R^2) \quad [3.16]$$

The above equations are based on the assumption of plane stress, i.e. there is no stress normal to the surface to which the gauges are attached. Thus $\sigma_Z = 0$.

The arrangement of strain gauges used to measure strains in different directions at the same point is known as a strain rosette. The rosettes used to measure strains in transverse and longitudinal directions of the test specimens are called 90° rosettes. Sizes of gauges are carefully chosen to suit the limited space available in the test specimens. Dimensions and locations of strain rosettes will be discussed in Chapter IV .

3.5 SAINT-VENANT'S PRINCIPLE

Consider a member loaded concentrically at both ends. The elements in the immediate vicinity of the points of application of the loads are subjected to very large stresses. However, at a distance equal to, or greater than, the width of the member, stress distribution is nearly uniform across the cross-section. More precisely, the stress distribution may be assumed independent of the actual mode of application of the loads except in the immediate vicinity of the point of the application of the load. This theory is known as Saint-Venant's Principle, which applies not only to axial loadings, but to any type of load.

Hence for the purpose of computing stresses in a structural member, the given loading can be replaced by a simpler one provided both loadings are statically equivalent. It should also be noted that stresses cannot be computed in this manner in the immediate vicinity of the

points of application of the loads. Instead, advanced theoretical or experimental methods must be used in these areas.

CHAPTER IV

EXPERIMENTAL PROCEDURE

4.1 GENERAL

The experimental program involved the testing of eleven starred angle compression members . In order to reduce the number of variables the same size angles , $1\frac{3}{4} \times 1\frac{3}{4} \times \frac{3}{16}$ in. (i.e. $45 \times 45 \times 5$ mm) , were used for all tests. Consulting engineers and fabricators have indicated that most of the starred angles in practical applications have a slenderness ratio that falls in the " slender " range. Therefore, emphasis of the research was placed on slender columns. However , starred angles with slenderness ratios which fell in the "intermediate" range were also tested.

4.2 TEST PROGRAM

There were two different lengths of columns, 2100 and 1270 mm . These specimens had an L/r_y ratio of 124 and 75, which could be classified as "slender" and as of "intermediate" length.

Due to the different loading conditions and connection detail, the specimens were further classified into three categories :

(a) Type C columns -- The letter "C" indicates that the

specimens are loaded concentrically. To confirm that the present test set-up gave test results the same as those obtained in previous research, and to obtain "base" data that could be used as a basis for comparison with the load carrying capacities of starred angles loaded through secondary trusses, five specimens (three 2100 mm and two 1270 mm in length) were loaded axially.

(b) Type B columns -- The letter "B" stands for the bolted connections. These columns represent the most common type of starred angles supporting secondary trusses. As shown in Figure 1.2, Angle A and Angle B of the starred angle struts are bolted together at the place of truss-to-truss connection by using a gusset plate. With the help of a lug angle, the starred angle struts in a primary truss are then connected to a secondary truss. Four specimens, with bolted connections, were tested as web members of a truss and loads were applied through the secondary truss.

(c) Type W columns -- The letter "W" stands for the welded connections. During the fabrication process, it is sometimes inconvenient to bolt the two legs, one on each angle of the starred angle struts at the place of truss-to-truss connection. Instead, fillet welds are used to join them together (see Figure 4.1). However, the primary truss and the secondary truss are still connected together by using high strength bolts. As more rotational restraint is offered by the welding, a

question may arise as to whether or not there is any change in strength or distribution of forces . To investigate the difference between these and the type B columns, two specimens were built and tested . They were called type W columns.

All the specimens, having two interconnectors, were tested with pinned end conditions. The separation of angles, as shown in Figure 3.1, was chosen as 9.5 mm. Typical dimensions for all the specimens are shown in Figures 4.2 (a) to (e). As illustrated in the figures, each test specimen was designated by a test series number, the first letter of which indicated the type of column and the number following it represented the length in mm. The last digit following the period distinguished the various specimens of the same length and type. For example, specimen B2100.1 indicated that the column was a type B column, with a bolted connection, had a length of 2100 mm, and the last digit indicated that this was the first specimen of this type.

4.3 TEST SET-UP

All the tests were carried out in the Civil Engineering Structural Laboratory at University of Windsor . A testing frame and a steel truss were built to accommodate test specimens of variable length. This section describes the test set-up and pieces of equipment used to carry out the experiments.

4.3.1 End Conditions and Fixtures

All specimens were tested as columns with pinned ends about both principal axes using double knife edges (see Figure 4.3). These specially constructed end fixtures were designed to provide no rotational restraint at the ends of the strut. The principal axes of the specimens, U-U and V-V axes, were oriented at 45 degrees to the axes of the knife edges.

Each set of double knife edges consisted of three steel blocks separated by two hardened steel square bars. Dimensions of the two outer blocks and the inner one were 150 x 150 x 25 and 150 x 150 x 38 mm , respectively , while both squares bars were 25 x 25 x 150 mm .

To allow rotation of the bars at one edge only, one 90 degree groove was cut on one side of the upper and lower plates. The center plate had two perpendicular grooves, one on each side, with an included angle of 135 degrees to allow the entire specimen to rotate freely about any axis. To ensure a minimum of resistance to rotation, the assembled steel blocks and bars were only held in place by compressive load applied to the specimen . The plates closest to the specimen, however, were bolted to the tee sections at each end of the strut .

The knife edges were positioned such that the groove nearest the specimen in the top knife edge assembly was perpendicular to the groove nearest the specimen at the

bottom. Thus the nominal effective length of the specimen about each of the perpendicular axes were the same and were taken as the length from knife edge to knife edge about which rotation took place.

4.3.2 Loading Equipment

In order to obtain good data from the experiment, the loading equipment should be properly chosen. Hydraulic jacks and load cells were used to apply and record the loads. In addition, a steel bracket and a steel truss were specially constructed to transfer the load from the loading jacks to the specimens. Installation and operation of the equipment will be discussed in detail in the following sections.

4.3.3 Testing Frame for Type C Columns

The specimens were loaded axially in a loading frame as shown in Figure 4.4(a). In order to prevent any lateral movement at the ends of the strut, the frame was welded to the floor at the bottom. At the top, a steel bracket was fabricated and used as a top support for the specimens. This bracket was fastened to an existing column at a height that would accommodate the length of the slender specimens. For testing the shorter specimens, the bottom part of the specimens was raised to accommodate the length of the columns (see Figure 4.4(b)).

A 222 kN capacity hydraulic jack, attached to the upper bracket, was used to apply axial load through the top of the specimen (see Figure 4.5(a)). The jack was operated manually with a hydraulic pump. Directly under the knife edges, at the lower end of the specimen, a 222 kN load cell was used to measure the total applied load (see Figure 4.5(b)).

4.3.4 Testing Frame for Type B columns

These specimens were tested with a truss attached to the starred angles to determine the behaviour of starred angles when a load was applied from a secondary truss. A steel truss was fabricated from 90 x 90 x 10 mm angles, arranged in a back-to-back configuration. The truss was about 3.6 m in length, 1.2 m in height. The starred angle strut being tested was bolted to one end of the truss through a gusset plate with bolt holes (see Figure 4.6(a)). The other end of the truss was fastened to one of the existing columns of a structural testing frame in the Structural Laboratory at the University of Windsor (see Figure 4.6(b)).

A 1335 kN capacity hydraulic jack was located at the centre of the truss . The jack was operated through an electric hydraulic pump. To provide a point loading, a ball and socket joint was mounted to the truss and placed under the jack (Figure 4.7) . The complete testing set-up for the specimens is illustrated in Figure 4.8 .

Loads were transferred from the truss to the specimen through the bolted connection (Figures 4.9(a) and (b)). In this type of connection, the truss was fastened to one leg of one of the angles of the specimen using two M16 bolts. The two angle components of the starred angle specimen were bolted together to redistribute the force components amongst themselves. Moreover, at the top of the specimen, a 45 kN load cell was installed to record the axial loads applied from the jack at the top (Figure 4.10).

4.3.5 Testing Frame for Type W Columns

The testing frame was the same as that used for Type B columns. The only difference occurred at the place of truss-to-truss connection. Figure 4.11 shows the different views of Type W columns at the connection joint.

4.3.6 Instrumentation

Strain gauges were attached to the type B and W specimens in order to determine the force distribution along each leg of the specimen . Twenty-four 90 degree strain rosettes were installed along the four legs of specimens B2100.1 and B1270.1 at a distance of about 150 mm below the connection (see Figure 4.12(a)). This distance was chosen based on the Saint-Venant's Principle to avoid measurement of high stresses near the connection point, where stress concentration occurred.

The analysis of the results indicated that the force distribution could also be determined by installing strain gauges on only one angle of the specimen. Hence, in subsequent tests, only six 90 degree strain rosettes were used on angle A of each slender specimen (see Figure 4.12(b)). For the remaining intermediate length specimen B1270.2, since the stress at buckling exceeded the proportional limit of the material, twelve strain rosettes were required for angle A.

In this study, all the strain rosettes used were foil strain gauges (Type N22-FA-2-120-11) manufactured by Showa Measuring Instruments Company and obtained from Durham Instruments (see Figure 4.12(c)). Moreover, all the strain readings were recorded using the Vishay-Ellis digital strain indicator.

Since the buckling mode of the specimens was one of single curvature, maximum deflection occurred at or near mid-height. To measure the lateral displacements, six dial gauges were arranged along the two principal axes at mid-height of the specimens (see Figures 4.13 (a) and (b)). In the case of type B and W columns, there was some uncertainty about the buckling mode. Thus in one of the tests of a B1270 specimen, twelve dial gauges were set at six different locations along the specimen (see Figure 4.14). The recorded lateral displacements showed that the overall buckling mode could not be simply defined by this method. Hence in other tests, dial gauges were used only at

the mid-height and at the location of strain gauges. Due to the limited space, only four dial gauges could be used at the location of strain gauges, as shown in Figures 4.15(a) and (b).

4.4 TEST PROCEDURE

4.4.1 Testing of Type C Columns

The load was applied slowly in increments of 8.9 kN. After the system had reached equilibrium, the dial gauges were read. At a load of about 80 percent of the estimated failure load, the load increment was reduced to 2.2 kN. This load increment was maintained until the specimen failed, which was taken as the load at which a large increase in lateral deflections resulted from a small increase in load.

4.4.2 Testing of Type B and W Columns

The specimen was first loaded to about 4.45 kN by the 222 kN hydraulic jack at the top of the specimen. This load (4.45 kN) from the jack was kept constant throughout the experiment.

Then a load was applied to the specimen by the 1335 kN hydraulic jack at the centre of the truss. Total loads acting on the specimen were obtained from the 222 kN load cell at the bottom of the specimen. A constant load increment of 4.45 kN was used.

During the experiment, the load cell at the top of the specimen was frequently checked and, if necessary, the load was adjusted to maintain a constant force of 4.45 kN.

All the readings from the dial gauges and strain gauges were carefully read and recorded at each load interval.

4.5 DATA REDUCTION

The data obtained from the experiments should be reduced to a usable form for analysis. The following sections describe the methods used to reduce the load, displacement, and stress data.

4.5.1 Load

The 45 and 222 kN capacity flat load cells were calibrated prior to the testing. Using the calibration curve, the microstrain readings were converted to units of force in kN. As an example, Figure 4.16 shows the calibration curve for the 222 kN capacity load cell.

To ensure that the load cells functioned properly, the calibration was checked periodically during the experimental phase of this work. Only a small variation, less than one percent, was found in the calibration factor.

4.5.2 Displacement

Deflections could be determined for the individual angles of the specimen from the instrumentation used. For example, deflections of the two angles along V-axis at

mid-height were calculated as the average of the two dial gauge readings 1 and 2, or 3 and 4, respectively (see Figure 4.13(b)). Since the interconnectors made the two angles of the starred angle struts act as an integral members at the point of interconnection, the deflections could be measured for the specimen as a whole. In addition, data obtained from the experiments confirmed that deflections of the two individual angles along the principal axes were similar in magnitude and direction. Thus for simplification, the following criterion was adopted in measuring the deflections.

Deflection of the starred angle struts along the V-axis were calculated, for example, as the average of indicator readings on both angles at mid-height dial gauges 1,2,3 and 4 (see Figure 4.13), and at the location of strain gauges dial indicators 1 and 2 (see Figure 4.15(b)). Deflection along U-axis was recorded as the average of indicator readings 5 and 6 at mid-height, and 3 and 4 at the location of strain gauges.

4.5.3 Stress

The term stress denotes internal force per unit area. The resultant of internal forces acting on a cross-section can be computed by integrating stresses over the area.

Since the individual angles of the test specimens were subjected to combined axial forces and bending moments, the stress distribution was very complicated. Stresses varied

along the length as well as the thickness of the cross-section. To simplify the computations, average stress on the centreline of the section was used. The aim was to eliminate the variation of stress across the thickness of the angle due to local bending.

Strain gauges were attached at the same locations on both sides of each leg of the angles. Data obtained from the tests, namely ϵ_1 and ϵ_2 , were input into Equations [3.15] and [3.16] to calculate the longitudinal stresses at the gauge locations. Stress on the mid-section was then taken as the average of the two adjacent stress readings, e.g. 3 and 4 in Fig.4.12(a), and, 5 and 6 in Fig. 4.12(b). Using an appropriate curve fitting method, stress distribution along the corresponding angle was plotted to compute the internal forces.

Chapter V

RESULTS AND ANALYSIS

5.1 GENERAL

This chapter focuses on the following aspects : (a) theoretical predictions of critical loads, and (b) analysis of experimental results. The geometric and mechanical properties of the test specimens are first determined. Values obtained are then used to calculate the theoretical critical loads and study the experimental data.

5.2 PROPERTIES

5.2.1 Geometric Properties

The computations of the geometric properties of angles are based on the idealized rectangular section in which the toe and fillet radii have been omitted (see Figure 5.1). The geometric properties of starred angles are determined as follows:

(a) For a Single Angle :

$$A_o = (2b - t)t \quad [5.1a]$$

$$\bar{x} = \bar{y} = \frac{b^2 + bt - t^2}{2(2b - t)} \quad [5.1b]$$

$$I_x = \frac{1}{12}tb^3 + bt\left(\frac{b}{2} - \bar{y}\right)^2 + \frac{1}{12}(b-t)t^3 + (b-t)t\left(\bar{y} - \frac{t}{2}\right)^2 \quad [5.1c]$$

$$I_y = I_x \quad [5.1d]$$

$$r_x = r_y = (I_x / A_o)^{0.5} \quad [5.1e]$$

$$I_{xy} = -bt \left(\frac{b}{2} - \bar{y} \right) \left(\bar{x} - \frac{t}{2} \right) - t(b-t) \left(\bar{y} - \frac{t}{2} \right) \left(\frac{b+t}{2} - \bar{x} \right) \quad [5.1f]$$

$$I_u' = I_{MIN} = \frac{I_x + I_y}{2} - \left[\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2 \right]^{\frac{1}{2}} \quad [5.1g]$$

$$I_v' = I_{MAX} = \frac{I_x + I_y}{2} + \left[\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2 \right]^{\frac{1}{2}} \quad [5.1h]$$

$$r_u' = (I_u' / A_o)^{0.5} \quad [5.1i]$$

$$r_v' = (I_v' / A_o)^{0.5} \quad [5.1j]$$

$$J = \frac{1}{3} t^3 (2b-t) \quad [5.1k]$$

(b) For a Starred Angle :

$$A = 2 A_o \quad [5.1l]$$

$$I_u = A [(r_u')^2 + 2 (\bar{y} + a/2)^2] \quad [5.1m]$$

$$r_u = (I_u / A)^{0.5} \quad [5.1n]$$

$$I_v = 2 I_v' \quad [5.1o]$$

$$r_v = (I_v / A)^{0.5} \quad [5.1p]$$

where b,t = width and thickness of angle leg, both are measured experimentally; a = separation of individual angles; A_o = the area of an individual angle; A = the total area of starred angle section; I', I = the moment of inertia of the individual and starred angle, respectively; r', r = the radii of gyration of the individual and starred angle,

respectively; x , y = the coordinates of the centroid of an individual angle; J = Saint-Venant's torsional constant; the subscripts u' , v' , x and y = the coordinate axes which are defined in Fig. 5.1; the subscripts u and v = the major and minor principal axes of the starred angle sections.

Table 1 lists all the cross-sectional properties computed. The nominal geometric properties are evaluated from those values listed in CISC Handbook⁽¹¹⁾ for a 1-3/4 x 1-3/4 x 3/16 in. angle. A conversion from Imperial to SI units is made for such angles. The actual cross-sectional properties are used in calculating the theoretical critical loads and analyzing experimental results.

5.2.2 Mechanical Properties

Tension tests were used to obtain the mechanical properties of the angles. Five test specimens were prepared according to ASTM A370-77⁽²⁾. These specimens were selected randomly from the batch of steel used to construct starred angles. Two strain gauges were then attached to each specimen. Each specimen was tested to rupture using the Universal Testing Machine in the Strength of Materials Laboratory.

The average values of the yield stress and the modulus of elasticity were found to be 390 and 205400 MPa, respectively (see Table 2). These values were used in all the computations.

5.3 THEORETICAL PREDICTIONS

Since the behaviour of the starred angles supporting trusses is unknown, various approaches are used to predict the critical loads. In all the classical theories critical loads are bifurcation-type loads, the smallest load at which neutral equilibrium occurs, and are determined for an "ideal" member. Table 3 summarizes the theoretical critical loads as determined by different theories or standards. The significances and accuracies of each approach will be discussed in the following sections.

5.3.1 Type C Columns

5.3.1.1 Classical Approach

The classical theories of elastic buckling of concentrically loaded columns (see Section 3.2.1) are used to predict the critical loads for C2100 specimens. The Euler's load for flexural buckling about the minor axis (P_v) is obtained by solving equation [3.1] and is equal to 103 kN. From equation [3.2], the torsional-flexural buckling load (P_t) is computed as 477 kN. The ultimate concentric axial load is the smaller of P_v or P_t , i.e. 103 kN. Therefore, all the slender Type C columns will fail by flexural buckling about the minor axis.

For the intermediate length columns, i.e. C1270 specimens, the stress at buckling exceeds the proportional limit of the material. In the inelastic range, Equations [3.1] and [3.2] can still be applied provided that the

elastic modulus is replaced by the tangent modulus. Since the tangent modulus is a variable which depends on the stress level, it will not be considered here.

5.3.1.2 North American Standards

Table 3 also lists the nominal compressive strength calculated in accordance with the AISC Specification and CSA S16.1. In all these calculations a resistance factor of 1.0 has been used. There is no great difference in strength according to the two standards for slender specimens. However, there is about a 10% difference in strength for the intermediate length columns. In both cases, the Canadian Standard is more conservative than the AISC Specification.

5.3.1.3 Comparison

For slender columns, Euler's flexural buckling load has a value much higher than the nominal strengths suggested by the two American standards. This can be explained by the fact that Euler's solution is based on the assumption in which the column is an idealized member, homogeneous, perfectly straight and contains no residual stresses from rolling or fabrication. However, in the North American Standards, both the effects of residual stress and those caused by initial out-of-straightness were considered in formulating the relationship for the column strength.

5.3.2 Type B and W Columns

5.3.2.1 Beam-Column Approach

In Section 3.3, starred angles supporting secondary trusses were modelled as beam-columns (see also Figure 3.1). The classical approach based on the work of Timoshenko and Gere is now applied. Solving Equation [3.8], the elastic buckling load of slender column under eccentric axial load is found to be 94 kN. Failure of such columns occurs by combined bending and torsion. However, computation of normal stress at failure indicates that stress in the extreme fibres has a value which is much higher than the yield stress of 390 MPa. Since Timoshenko's theory was formulated on the assumption that the material was linearly elastic, Equation [3.8] cannot be applied in this case, and the solution obtained above is no longer valid.

In Appendix D, starred angles supporting secondary trusses are analyzed as beam-columns following the design procedure given in Clause 13.8 of CSA S16.1. A failure load of 41 kN was obtained. Since starred angle sections are classified as non-compact sections according to Clause 11 of CSA S16.1, this ultimate load will cause the extreme fibre of the cross-section to yield. However, experimental results (which are discussed in Section 5.4) indicated that all the Type B and W specimens had a failure load over 90 kN. In addition, stress across the cross-section was still well below the yield stress. Once again the procedure in

Clause 13.8 produced an invalid solution. Hence it confirms that the beam-column approach cannot be applied to solve the problem of starred angles supporting secondary trusses.

5.3.2.2 Approach Using a Modified Slenderness Ratio

In ASCE Manual 52 and ECCS Recommendations, the design of eccentrically loaded single-angle members are simplified by adopting a modified slenderness ratio. The ultimate strength of Type B and W columns are computed using the two standards. Detailed calculations are shown in Appendix E and the final results are listed in Table 3. The effective length factors, K , of B1270 specimens are found to be 1.30 and 1.15 according to ASCE Manual 52 and ECCS Recommendations, respectively. However, 'K' values stay the same and are equal to one for B2100 and W2100 specimens.

5.4 EXPERIMENTAL RESULTS AND ANALYSIS

Results collected from the test program are used to study the behaviour of starred angle struts, with specific emphasis on the following areas: (a) failure load; (b) load-deflection curves; and (c) stress and force distributions. All the experimental curves are plotted using the reduced data as described in Chapter IV.

5.4.1 Failure Load

5.4.1.1 Test Summaries and Comparison with Theoretical Predictions

The experimental failure loads and the buckling modes for all the test specimens are tabulated in Table 4. In all cases, the tested specimens behaved consistently and buckled about the V-axis, the minor principal axis of the entire cross-section.

Type C specimens were columns tested individually. The experimental failure loads agree quite well with the theoretical predictions. Hence it can be concluded that Euler's formula and the design equations given in CSA S16.1 or AISC Specification can all be used in predicting the critical loads of axially loaded starred angles.

Type B and W specimens were columns tested with the truss. As shown in Table 4, the failure loads of B2100 specimens were very close to those of the C2100 specimens. There was about 6-8% increase in the critical loads for W2100 specimens. The difference may be due to experimental errors, imperfections of the set-up, etc. Another possible explanation is that welded connections provide more stiffness to the starred angles than the bolted connection. Since the calculated percentage increase is small, for safety purposes, it should not be included in the design. Therefore, Type B and W columns are assumed to have the same capacity.

For B1270 specimens, the failure loads are 133.8 kN and 123.8 kN. There is an unexpected drop in capacity of about 40% when compared with the C1270 specimens. This abrupt change may be caused by the excessive stress induced in Leg 1 and Leg 3 (see Figure 1.3) due to the uneven stress distribution, which will be discussed in later sections. At this point, it can be concluded that starred angles, of intermediate length, supporting secondary trusses cannot be simply designed as axially loaded columns.

In comparison with the results obtained by ASCE Manual 52 and ECCS Recommendations as listed in Table 3, the experimental results are quite consistent with the theoretical predictions for B2100 and W2100 specimens. In contrast, for B1270 specimens, both specifications give a higher values (16-25% more) than the experimental results.

5.4.1.2 Combined Approach

In order to achieve a better correlation between the experimental results and theoretical predictions, a combined approach is used. In this approach, starred angles supporting trusses are designed as axially loaded members based on the nominal strength given in AISC and CSA Standards. However, the effective length factors are modified first in accordance with the formulas listed in ASCE Manual 52. The reason for using ASCE Manual to

calculate the modified slenderness ratio is that it gives a more conservative effective length factor than the ECCS Recommendations.

Table 5 lists a comparison of the results obtained by this approach with the experimental data. It is observed that by modifying the slenderness ratio, both AISC Specification and CSA Standard can give better results than all the previous theoretical predictions mentioned before. Details of the new approach will be discussed in next chapter.

5.4.2 Load Deflection Curve

5.4.2.1 Type C Columns

Figures 5.2(a), 5.2(b) and 5.2(c) show the theoretical and experimental curves for specimens C2100.1, C2100.2 and C2100.3, respectively. The theoretical solutions are obtained from the results of a linear mixed incremental-iterative finite element program written by Tan⁽¹⁹⁾. Only one curve is plotted for each of the axes as it is assumed that the interconnectors force the individual angles to take the same displaced shape. This assumption is confirmed experimentally. All these curves indicate that the specimens failed principally about the V-V axis. The shapes of the experimental curves tend to agree well with the theoretical ones. Discrepancies between the results may be due to the improper modelling of the columns or errors in measuring the out-of-straightness.

Figures 5.2(d) and 5.2(e) show the experimental load-deflection curves for specimens C1270.1 and C1270.2. Again, failure occurred about the V-axis. No theoretical analysis is performed on these specimens as the program mentioned above is only valid for elastic columns.

In general, for all the concentrically loaded starred angles, deflections in the U-direction govern, hence buckling occurs about the V-axis. This confirms the results of previous research conducted by Temple and Schepers⁽²⁰⁾.

5.4.2.2 Type B Columns

The experimental load-deflection curves for specimens B2100.1 and B2100.2 are plotted in Figures 5.2(f) and (g), respectively. Both specimens at failure had the U-deflection about four times that observed in the V-direction. In comparison with Figures 5.2(a) to (c), the experimental mid-height deflections of B2100 specimens are much greater than that of C2100 specimens.

Figures 5.2(h) and (i) give the experimental load-deflection curves for B1270.1 and B1270.2 specimens. Both specimens deflected in a combination of the U and V directions prior to failure. Deflection was predominantly in the U direction where the U displacement was about twice that in the V direction.

Overall Type B columns behaved in a similar manner and had the failure loads which were quite consistent. The shapes of specimens after buckling are illustrated in Figures 5.3(a) to (c) .

5.4.2.3 Type W Columns

Figures 5.2(j) and (k) show the experimental load-deflection curves for specimens W2100.1 and W2100.2, which indicate the same tendency to bend about the V-V axis as in the case of Type B columns. Comparing with Figures 5.2(f) and (g), it can be seen that the mid-height deflection is smaller for W2100 specimens than that of B2100 specimens.

Intuitively the reduction in deflection can be explained by the fact that more flexural restraint is offered by the welded connection. This increased flexural stability should be accompanied by an increase in load carrying capacity as shown in Table 4. However, the difference may also due to the effect of initial out-of-straightness. More experiments are required before a firm conclusion can be drawn.

Simple theory would suggest that bending without twisting occurs only when the line of action of the applied load passes through the locus of the shear centres, i.e. axis of twist. Hence there is a possibility of torsional-flexural buckling instead of the pure flexural buckling for Type B and W columns. The angles of rotation

about the longitudinal axis are calculated from the dial gauge readings. A maximum value of about one degree is found. The angles of rotation are so small that they can be neglected. This phenomenon is considered to be due to the increased torsional restraint provided by the secondary truss. Therefore, it can be concluded that Type B and W columns failed by pure flexural buckling about the minor principal axis of inertia.

5.4.3 Stress and Force Distributions

5.4.3.1 Slender Type B Columns

At the beginning of the experiments, the stress and force distributions were unknown. Hence, strain gauges were installed along every leg of specimen B2100.1. Using the strain data and the procedure given in Section 4.5.3, stress distribution along each leg of the specimen was obtained. The curve fitting method used to interpolate the stress data will be discussed in Appendix F.

Figures 5.4(a) and (b) show the stress distribution curves of specimen B2100.1 for Angle A and Angle B, respectively. In these figures, the angle has been straightened out to lie flat on the sheet of paper. Both angles display a similar trend. The stress distribution is a straight-line function at small loads. Gradually, the relationship becomes nonlinear when the load is increased. When the specimen buckles, the extreme fibres on one end of

each angle are subjected to tensile stress while the other end has a compressive stress up to 300 MPa. The maximum stress is still below the yield stress.

Forces are then calculated from the stresses. It is observed that the area under the stress-distribution curve represents the integral $\int \sigma \, dA$, which is equal to the force. Figure 5.4(c) shows the force distribution in specimen B2100.1. Forces in Angle A are always higher than that in Angle B. Legs 1 and 3 of the specimen carry more loads than the other two legs.

In order to design the truss-to-truss connection, the forces acting at Leg 1, Leg 2 and the total force in Angle B are required (details will be discussed in Section 6.3). Therefore, in specimen B2100.2, strain gauges were installed along Angle A of the specimen only. Besides, as it was illustrated in Figures 5.4(a) and (b), the stress distribution was approximately a linear function. So strain gauges were installed at only three locations of Angle A. Figures 5.5(a) and (b) show the stress and force distributions for specimen B2100.2. It is apparent that the results obtained are very similar to that of specimen B2100.1.

5.4.3.2 Intermediate Type B Columns

Similar to the slender Type B columns, strain gauges were located at both angles of specimen B1270.1, whereas they were used only on Angle A of specimen B1270.2.

Figures 5.6(a), (b), (c) and Figures 5.7(a) and (b) show the stress and force distribution curves for specimen B1270.1 and B1270.2, respectively.

For the intermediate columns, stresses are induced beyond the proportional limit of the material after a certain loading. The analytical investigation is carried out by assuming that the columns are made of an elastic-perfectly plastic material. Therefore, if the stresses calculated in any part of the test specimens exceed the yield strength, they all will be taken as equal to σ_y . This explains why the assumed stress distribution curves flatten in Figures 5.6(a) and 5.7(a).

It is observed that Leg 1 of both specimens was in the plastic state when the columns failed. Moreover, about one-half of Legs 2 and 4 were subjected to tensile stress. Another interesting observation is that the slope of the stress distribution curve of Leg 3 of specimen B1270.1 (see Figure 5.6(b)) decreases when the load is close to buckling load. The change in behaviour might be due to the local buckling which happened in Leg 3 of the specimen (see Figure 5.3 (c)).

This phenomenon seems to contradict Clause 11.2 of CSA S16.1 which states that local buckling will not happen provided that the width to thickness ratio of the compression member (equal to 9.3 for the section used) is less than the limiting value of $200/\sqrt{\sigma_y}$ (equal to 10.1 in this case). The same clause also implies that at failure

the stresses in the extreme fibres of the non-compact section (starred angle section) have just reached the yield stress. However, in order to obtain the ultimate load carrying capacity of the test specimens, the load was increased even though most of the leg of the angle had yielded. Since the width to thickness ratio of the specimen was so close to the limiting value, it is not unusual that local buckling occurred in the test specimens.

5.4.3.3 Type W Columns

Strain gauges were used on Angle A of Type W specimens only. The stress and force distribution curves are plotted in Figures 5.8(a), (b) and Figures 5.9(a), (b) for specimens W2100.1 and W2100.2, respectively. No significant difference is observed when compared to the slender Type B specimens.

5.4.3.4 Summary of Force Distribution

Tables 6 to 11 show the force distributions for all Type B and W columns in tabulated forms. Overall the results are quite consistent. There is an uneven stress distribution across the cross-sections of the specimens. More than 50 percent of the forces from the truss is transmitted to Angle A. Legs 1 and 3 of the specimen carry a large percentage of the applied load.

Maximum forces occurring in each leg and angle are also highlighted in Tables 6 to 11. They are the critical values which should be used in the design of joints. Table 12 lists the maximum forces expressed as a percentage of the buckling load. A range of force distribution is also estimated for each leg of the starred angle section.

CHAPTER VI

DISCUSSION

6.1 GENERAL

Two design methods are proposed in this chapter. The first estimates the ultimate strength of starred angles supporting trusses. The second is concerned with the design of joints at the place of truss-to-truss connection. In addition, the load transfer mechanism is discussed.

6.2 ULTIMATE STRENGTH

Since the starred angles supporting secondary trusses are no longer axially loaded, then according to the experimental results and theoretical investigations, the slenderness ratios of such member should be computed based on the formulas given by ASCE Manual 52 as follows:--

(a) For $L/r \leq 120$,

$$\frac{KL}{r} = 60 + 0.50 * \frac{L}{r} \quad [6.1]$$

(b) For $L/r > 120$,

$$\frac{KL}{r} = \frac{L}{r} \quad [6.2]$$

Then the ultimate strength of the starred angles can be calculated using the design equations for axially loaded columns given in CSA S16.1 or AISC Specification. Table 5 lists a comparison of the load carrying capacities of the test specimens computed by this approach with the experimental results.

6.3 LOAD TRANSFER MECHANISM

A study of typical structural drawings and a discussion with a fabricator indicate that as much as 95 percent of the force in the vertical web member of the truss is transmitted from the secondary truss. This force is divided into two parts at Joint 1 (see Figure 6.1). A larger portion of the force will go to Angle A, while the smaller portion will pass through the lug angle to Joint 3. Through the gusset plate, some of the force will redistribute among Legs 2 and 3 using the bolts at Joints 2 and 3. Then from Leg 3, a small amount of the total force in Angle B will go to Leg 4.

In general, total load transmitted by the truss is acting at Joint 1. From experimental analysis, about one-half, but less than 50 percent of the total force will go to Angle B through the bolts at Joint 3 (see Table 12). Only a small amount of the load (10-15%) is acting on the bolts at Joint 2.

6.4 CONNECTION DESIGN

A simple procedure is suggested for the design of joints at the place of truss-to-truss connection. Each joint will be discussed separately under the following sections.

6.4.1 Joint 1

Connection at Joint 1 is made by using bolts in bearing or shear as shown in Figure 6.2(a) .

In a bearing type joint, the bolt is in close contact with the gusset plate (or lug angle) so that bearing stresses are produced at the cylindrical contact surface between the bolt and the plate (or lug angle). Since the bearing capacity of the bolt is greater than that of the plate (or lug angle), failure occurs in bearing in the gusset plate (or lug angle). The bearing resistance of one bolt according to Clause 13.10 of CSA S16.1 is given as: --

$$Br = \phi e_d t_p F_u \leq 3 \phi t_p d F_u \quad [6.3]$$

where ϕ = resistance factor = 0.67; e_d = distance measured from the centre of the bolt to the end of gusset plate (or lug angle); t_p = thickness of the thinnest part being connected ; F_u = ultimate strength of the gusset (or lug angle); and d = diameter of the bolt.

Another mode of joint failure is by shearing of the bolt itself across the two shear planes (see Figure 6.2(b)), which is termed as failure in double shear. The shear resistance of one bolt in accordance with Clause 13.11.2 of CSA S16.1 is taken as :

$$V_r = (2) (0.60) \phi A_b F_u' \quad [6.4]$$

where A_b = area of one bolt; and F_u' = the specified tensile strength of the bolt metal.

Hence the number of bolts required at Joint 1 is given by the larger value of the following relations:

$$n \geq P / B_r \quad [6.5]$$

$$n \geq P / V_r \quad [6.6]$$

where P = the total loads transmitted from the secondary truss to the primary truss.

It is common in practice to assume that all of the fasteners in a line share equally in carrying the load. In fact, the fasteners towards the end of the joint carry the largest portion of the load. However, this effect is not considered in this study.

6.4.2 Joint 2

At Joint 2, Leg 2 is connected to the gusset plate on one side only. There is only one faying surface and the bolts used here are in single shear.

A good design practice is to make the gusset plate section symmetrical, Joints 2 and 3 should contain the same number of bolts. As it was mentioned before, only a small

amount (10-15%) of the total load is acting on Leg 2. The design of the bolts in Joint 2 is less critical when compared to Joint 3. Hence the number of bolts used in Joint 2 should depend on the design of Joint 3 which will be discussed in the following section.

6.4.3 Joint 3

Angle B is connected to Angle A through the gusset plate at Joint 3. Hence total force acting in Angle B (40-50% of the applied load) is used for the design. As it is illustrated in Figure 6.1, a large amount of the force transmitted to Angle B comes from the lug angle. Although some force may come from other sources, say Leg 2, this is only a small amount and may be neglected. Hence it is assumed that the bolts in Joint 3 are subjected to shearing across a single surface as the lug angle and gusset plate bear up against the fasteners. Again, the factored resistance of one bolt in single shear is :

$$V_r = 0.60 \phi A_b F_u \quad [6.7]$$

In this case only one-half of the load transmitted from the secondary truss to the primary truss is used in determining the number of bolts required at Joints 2 or 3, i.e.

$$n \geq \frac{1}{2} P / B_r \quad [6.8]$$

$$\text{and } n \geq \frac{1}{2} P / V_r \quad [6.9]$$

where the value of B_r is defined in Equation [6.3].

6.5 SUMMARY

In general, all the three joints should contain the same number of bolts if failure of the joints occur by shearing of the bolts, i.e. Equations [6-6] and [6-9] govern. On the other hand, only one-half of the number of bolts used in Joint 1 are required for Joints 2 and 3 when failure of joints is caused by excessive bearing stress. In that case, Equations [6-5] and [6-8] are used.

Chapter VII

CONCLUSIONS AND RECOMMENDATIONS

In this research program the behaviour of starred angles supporting secondary trusses is studied. Special emphasis is placed on determining the load carrying capacities and the force distributions of such members.

7.1 CONCLUSIONS

The following conclusions are based on the preceding work:

- (a) There is no specific requirement in the Standards with regard to the design of the starred angle struts and the truss-to-truss connection when starred angles are used as vertical web members in the primary trusses.
- (b) The experimental study indicates that starred angles supporting trusses, especially those of intermediate length, cannot be simply designed as concentrically loaded columns. The failure mode of all the test specimens was one of flexural buckling about the minor principal axis.
- (c) Analysis of starred angles supporting trusses based

on beam-column analogy results in high stresses at the extreme fibres of the cross-section, hence proved to be unsuitable.

- (d) A simplified method to predict the critical load is suggested: the slenderness ratios of starred angles are modified according to ASCE Manual 52 . Then formulas for concentrically loaded columns given in CSA S16.1 can be applied.
- (e) Legs 1 and 3 (see Figure 1.3) carry much more axial load than Legs 2 and 4.
- (f) Total load transmitted from a secondary truss to a primary truss is used to determine the number of bolts required at Joint 1. Only one-half of the load is used to design the connections at Joints 2 and 3.

7.2 SCOPE FOR FURTHER RESEARCH

Although the suggested approach appears promising, further studies are needed before the method can be proposed for general use. Areas where further research would be valuable are:

- (a) The design method developed here , probably with some modifications, can be applied to starred angles composed of unequal leg angles. An experimental and analytical study of such members would be of value.
- (b) An exploratory study was conducted by Haaijer et al

to investigate the feasibility of using a finite element program to simulate the eccentric load tests of single-angle columns (16). Hence modelling of starred angles supporting trusses with a finite element program would be a useful way to predict the effective-length factors, K , for such members.

Appendix A

FIGURES

(a) PHYSICAL LAYOUT OF STARRED ANGLES IN
ROOF TRUSSES

(b) CLOSE-UP VIEW OF STARRED ANGLES

FIGURE 1.1

CONNECTION DETAIL
FIGURE 1.2

THE COORDINATE AXES
FIGURE 1.3

THEORETICAL BEAM-COLUMN MODELLING

FIGURE 3.1

(a) Top View

(b) Side View
(section 1-1)

FIG. 4.1 DETAIL OF TYPE W COLUMN

Note: All dimensions are in mm.

TYPICAL DIMENSIONS OF C2100 SPECIMENS
FIGURE 4.2(a)

Note: All dimensions are in mm.

TYPICAL DIMENSIONS OF C1270 SPECIMENS
FIGURE 4.2(b)

Note: All dimensions are in mm.

TYPICAL DIMENSIONS OF B2100 SPECIMENS
FIGURE 4.2(c)

Note: All dimensions are in mm.

TYPICAL DIMENSIONS OF B1270 SPECIMENS
FIGURE 4.2(d)

Note: All dimensions are in mm.

TYPICAL DIMENSIONS OF W2100 SPECIMENS
FIGURE 4.2(e)

ASSEMBLY OF DOUBLE KNIFE EDGES

FIGURE 4.3

(a) COMPLETE SET-UP FOR C2100 SPECIMENS (b) COMPLETE SET-UP FOR C1270 SPECIMENS

FIGURE 4.4

(b) END FIXTURE WITH 222 kN CAPACITY LOAD
CELL AT THE BOTTOM

(a) END FIXTURE WITH 222 kN CAPACITY JACK
AT THE TOP

FIGURE 4.5

(a) CLOSE-UP VIEW AT THE CONNECTION END OF THE TRUSS

(b) CLOSE-UP VIEW AT OTHER END OF THE TRUSS

FIGURE 4.6

THE 1335 kN CAPACITY JACK AT THE CENTRE OF THE TRUSS

FIGURE 4-7

COMPLETE SET-UP FOR TYPE B COLUMNS

FIGURE 4.8

(b) SIDE VIEW OF BOLTED CONNECTION

(a) FRONT VIEW OF BOLTED CONNECTION

FIGURE 4.9

THE 45 kN CAPACITY LOAD CELL UNDER THE JACK

FIGURE 4.10

(b) CLOSE-UP VIEW SHOWING CONNECTION JOINT

(a) SIDE VIEW OF TYPE W COLUMNS

FIGURE 4.11

LOCATIONS OF STRAIN GAUGES FOR B2100.1 & B1270.1

FIGURE 4.12 (a)

FIGURE 4.12 (b)
LOCATIONS OF STRAIN GAUGES
FOR OTHER SLENDER SPECIMENS

FIGURE 4.12 (c)
PATTERN AND SPECIFICATIONS
OF STRAIN GAUGES.

N22-FA-2-120-11 (by SHOWA)
Resistance : 120 ohms
Approx. Gauge Factor : 2.0
Base Diameter : 8 mm
Grid Length : 2 mm
Grid Width : 1.6 mm

(a) THE ARRANGEMENT OF DIAL GAUGES AT MID-HEIGHT

(b) NUMBERING OF DIAL GAUGES AT MID-HEIGHT

FIGURE 4.13

THE ARRANGEMENT OF DIAL GAUGES FOR SPECIMEN B1270.1

FIGURE 4.14

(a) THE ARRANGEMENT OF DIAL GAUGES AT LOCATION OF STRAIN GAUGES

(b) NUMBERING OF DIAL GAUGES AT LOCATION OF STRAIN GAUGES

FIGURE 4.15

CALIBRATION CURVE FOR 222 kN CAPACITY LOAD CELL
FIGURE 4.16

Note: Shear Centre (S.C.) is located at the intersection of the centrelines of the legs of the angle

IDEALIZED RECTANGULAR SECTION

FIGURE 5.1

LOAD-DEFLECTION CURVE FOR C2100.1
FIGURE 5.2 (a)

LOAD-DEFLECTION CURVE FOR C2100.2
FIGURE 5.2 (b)

LOAD-DEFLECTION CURVE FOR C2100.3
FIGURE 5.2 (c)

LOAD-DEFLECTION CURVE FOR C1270.1
FIGURE 5.2 (d)

LOAD-DEFLECTION CURVE FOR C1270.2
FIGURE 5.2 (e)

LOAD-DEFLECTION CURVE FOR B2100.1
FIGURE 5.2 (f)

LOAD-DEFLECTION CURVE FOR B2100.2,
FIGURE 5.2 (g)

LOAD-DEFLECTION CURVE FOR B1270.1
FIGURE 5.2 (h)

LOAD-DEFLECTION CURVE FOR B1270.2
FIGURE 5.2 (i)

LOAD-DEFLECTION CURVE FOR W2100.1
FIGURE 5.2 (j)

LOAD-DEFLECTION CURVE FOR W2100.2
FIGURE 5.2 (k)

(ii) CLOSE-UP VIEW AT THE TOP

(i) FRONT VIEW

FIGURE 5.3(a) BUCKLING OF SLENDER TYPE B COLUMNS

(i) FRONT VIEW

(ii) SIDE VIEW

FIGURE 5.3(b) BUCKLING OF INTERMEDIATE TYPE B COLUMNS

INELASTIC BUCKLING OF B1270 SPECIMENS

FIGURE 5.3(c)

FORCE DISTRIBUTION IN B2100.1
FIGURE 5.4 (c)

FORCE DISTRIBUTION IN B2100.2
FIGURE 5.5 (b)

STRESS DISTRIBUTION IN ANGLE B (B1270.1)
FIGURE 5.6 (b)

FORCE DISTRIBUTION IN B1270.1
FIGURE 5.6 (c)

FORCE DISTRIBUTION IN B1270.2
FIGURE 5.7 (b)

FORCE DISTRIBUTION IN W2100.1
FIGURE 5.8 (b)

FORCE DISTRIBUTION IN W2100.2
FIGURE 5.9 (b)

LOAD TRANSFER MECHANISM

FIGURE 6.1

ANALYSIS OF JOINT 1
FIGURE 6.2

Appendix B

TABLES

TABLE 1. ACTUAL vs. NOMINAL CROSS-SECTIONAL PROPERTIES

	Properties	Units	Actual Values	Nominal Values	Percent Difference
Single Angle	b	mm	43.7	44.5	1.8
	t	mm	4.7	4.8	2.1
	x or y	mm	12.7	13.0	2.4
	A_o	mm ²	391	400	2.3
	I_x or I_y	mm ⁴	70400	74900	6.4
	r_x or r_y	mm	13.4	13.7	2.2
	I_u'	mm ⁴	28500	29900	4.9
	r_u'	mm	8.5	8.6	1.2
	I_v'	mm ⁴	112300	120000	6.9
	r_v'	mm	16.9	17.3	2.4
	J	mm ⁴	2920	3020	3.4
Starred Angle	A	mm ²	783	800	2.2
	I_u	mm ⁴	533100	561900	5.4
	r_u	mm	26.1	26.5	1.5
	I_v	mm ⁴	224500	240000	6.9
	r_v	mm	16.9	17.3	2.4

TABLE 2. MECHANICAL PROPERTIES OF STEEL

Properties	Specimens					Average
	1	2	3	4	5	
Modulus of Elasticity (MPa)	211100	205000	205900	206000	199200	205400
Modulus of Rigidity (MPa)	81200	78800	79200	79200	76600	79000
Yield Stress (MPa)	399	381	399	388	384	390
Ultimate Stress (MPa)	551	544	553	554	551	551
Percent Elongation (%)	18	19	19	18	19	19

TABLE 3. THEORETICAL PREDICTIONS OF CRITICAL LOADS

Type of Columns	Length (mm)	Theories or Standards Applied	Critical Loads (kN)	Comments
C	2100	Euler's Elastic Theory	103	Buckling about V-axis
		CAN3-S16.1-M84	88	$\phi = 1.0$
		AISC--LRFD	91	$\phi_c = 1.0$
C	1270	CAN3-S16.1-M84	177	$\phi = 1.0$
		AISC--LRFD	194	$\phi_c = 1.0$
B, W	2100	Timoshenko and Gere	94	Bending and Torsion
		CAN3-S16.1-M84 - Clause 13.8	41	Beam-Column $\phi = 1.0$
		ASCE Manual 52	100	$K = 1.0$
		ECCS Recommendations	83	$K = 1.0$
B	1270	ASCE Manual 52	166	$K = 1.30$
		ECCS Recommendations	148	$K = 1.15$

TABLE 4. EXPERIMENTAL BUCKLING LOADS

Specimen Number	Failure Load P_{ex} (kN)	Failure Mode
C2100.1	89.0	Buckle about V-axis
C2100.2	93.9	Buckle about V-axis
C2100.3	93.4	Buckle about V-axis
C1270.1	197.3	Buckle about V-axis
C1270.2	216.9	Buckle about V-axis
B2100.1	91.9	Buckle about V-axis
B2100.2	91.9	Buckle about V-axis
B1270.1	133.8	Buckle about V-axis
B1270.2	123.8	Buckle about V-axis
W2100.1	97.9	Buckle about V-axis
W2100.2	99.9	Buckle about V-axis

TABLE 5. STARRED ANGLES DESIGNED AS AXIALLY LOADED MEMBERS

Specimen Number	Nominal Strength *		$\frac{P_{ex}}{P_n}$ (%)	$\frac{P_{ex}}{C_r}$ (%)
	AISC - LRFD $P_n (\phi_c = 1.0)$ (kN)	CSA S16.1 $C_r (\phi = 1.0)$ (kN)		
C2100.1	90.6	88.0	98	101
C2100.2	90.6	88.0	104	107
C2100.3	90.6	88.0	103	106
C1270.1	194.3	177.5	102	111
C1270.2	194.3	177.5	112	122
B2100.1	90.6	88.0	101	104
B2100.2	90.6	88.0	101	104
B1270.1	142.1	124.3	94	108
B1270.2	142.1	124.3	87	100
W2100.1	90.6	88.0	108	111
W2100.2	90.6	88.0	110	113

Notes : # Values are based on the modified slenderness ratios suggested by ASCE Manual 52

* P_{ex} are experimental buckling loads as listed in Table 4.

TABLE 6. FORCE DISTRIBUTION IN SPECIMEN B2100.1

Applied Loads, kN	Angle A			Angle B		
	Leg 1 kN (%) *	Leg 2 kN (%)	Total kN (%)	Leg 3 kN (%)	Leg 4 kN (%)	Total kN (%)
4.5	0.8 (19.4)	1.0 (22.3)	1.8 (41.7)	1.3 (28.3)	1.4 (30.0)	2.7 (58.3)
8.9	3.2 (36.2)	1.5 (17.0)	4.7 (53.2)	2.9 (32.7)	1.3 (14.1)	4.2 (46.8)
13.3	4.3 (32.6)	2.5 (18.4)	6.8 (51.0)	4.2 (31.8)	2.3 (17.2)	6.5 (49.0)
17.8	5.6 (31.3)	3.4 (19.1)	9.0 (50.4)	5.6 (31.5)	3.2 (18.1)	8.8 (49.5)
22.2	7.0 (31.7)	4.3 (19.1)	11.3 (50.8)	7.0 (31.6)	3.9 (17.6)	10.9 (49.2)
26.7	9.1 (33.9)	4.9 (19.4)	14.0 (52.3)	8.6 (32.3)	4.1 (15.4)	12.7 (47.7)
31.1	11.2 (36.0)	5.5 (17.6)	16.7 (53.6)	10.3 (33.0)	4.2 (13.4)	14.4 (46.4)
35.6	13.1 (36.9)	6.2 (17.2)	19.3 (54.1)	11.8 (33.3)	4.5 (12.6)	16.3 (45.9)
40.0	17.4 (43.4)	6.2 (15.5)	23.6 (58.9)	14.2 (35.4)	2.2 (5.7)	16.4 (41.1)
44.5	19.2 (43.0)	6.8 (15.2)	25.9 (58.2)	15.9 (35.5)	2.8 (6.3)	18.7 (41.8)
48.9	22.8 (46.6)	7.0 (14.3)	29.8 (60.9)	17.8 (36.2)	1.4 (2.9)	19.2 (39.1)
53.4	25.1 (46.6)	7.5 (14.0)	32.6 (60.6)	19.6 (36.4)	1.6 (3.0)	21.2 (39.4)
57.8	28.4 (48.8)	7.7 (13.2)	36.1 (62.0)	21.7 (37.2)	0.5 (0.8)	22.1 (38.0)
62.3	30.1 (48.3)	8.0 (12.8)	38.1 (61.1)	23.3 (37.3)	1.0 (1.6)	24.3 (38.9)
66.7	31.9 (47.7)	8.3 (12.4)	40.2 (60.1)	25.1 (37.6)	1.5 (2.3)	26.7 (39.9)
71.2	33.9 (47.5)	9.5 (11.8)	42.4 (59.3)	27.1 (38.0)	1.9 (2.7)	29.0 (41.7)
75.6	35.5 (47.0)	9.6 (11.3)	44.1 (58.3)	29.1 (38.4)	2.5 (3.3)	31.6 (41.7)
80.1	37.4 (46.7)	8.4 (10.6)	45.9 (57.3)	31.3 (39.2)	2.8 (3.5)	34.2 (42.7)
84.5	38.9 (46.0)	8.2 (9.7)	47.1 (55.7)	33.9 (40.3)	3.4 (4.0)	37.3 (44.3)
89.0	40.1 (45.1)	7.5 (8.4)	47.6 (53.5)	36.9 (41.6)	4.3 (4.9)	41.3 (46.5)
91.9	44.1 (48.0)	6.2 (6.7)	50.3 (54.7)	40.5 (44.1)	1.1 (1.2)	41.7 (45.3)

Notes: # Values in parentheses are the forces in each leg / angle expressed as percentages of the applied loads.

* These are the maximum forces occurring in the corresponding components.

TABLE 7. FORCE DISTRIBUTION IN SPECIMEN B2100.2

Applied Loads, kN	Angle A			Angle B	
	Leg 1 kN (%) *	Leg 2 kN (%)	Total kN (%)	Total kN (%)	
4.5	1.7 (37.6)	0.6 (13.5)	2.3 (51.1)	2.2	(48.9)
8.9	2.5 (28.6)	1.8 (20.1)	4.3 (48.7)	4.6	(51.3)
13.3	3.9 (29.1)	2.7 (20.6)	6.6 (9.7)	6.7	(50.3)
17.8	5.4 (30.4)	3.6 (20.1)	9.0 (50.5)	8.8	(49.5)
22.4	7.1 (31.6)	4.2 (18.9)	11.3 (50.5)	11.1	(49.5)
26.7	8.5 (31.9)	5.2 (19.3)	13.7 (51.2)	13.0	(48.8)
31.1	10.0 (32.0)	6.1 (19.7)	16.1 (51.7)	15.0	(48.3)
35.6	11.6 (32.6)	6.9 (19.2)	18.5 (51.8)	17.1	(48.2)
40.0	13.7 (34.1)	7.5 (18.8)	21.2 (52.9)	18.8	(47.1)
44.5	15.6 (35.1)	8.0 (18.0)	23.6 (53.1)	20.9	(46.9)
48.9	17.5 (35.7)	8.8 (18.0)	26.3 (53.7)	22.6	(46.3)
53.4	19.4 (36.4)	9.2 (17.1)	28.6 (53.5)	24.8	(46.5)
57.8	22.8 (39.4)	10.1 (17.5)	32.9 (56.9)	24.9	(43.1)
62.3	25.0 (40.2)	10.2 (16.3)	35.2 (56.5)	27.1	(43.5)
66.7	28.1 (42.1)	10.5 (15.8)	38.6 (57.9)	28.1	(42.1)
71.2	30.4 (42.7)	10.7 (15.0)	41.1 (57.7)	30.1	(42.3)
75.6	32.3 (42.7)	10.6 (14.1)	42.9 (56.8)	32.7	(43.2)
80.1	34.3 (42.8)	10.4 (13.0)	44.7 (55.8)	35.4	(44.2)
84.5	35.9 (42.5)	10.1 (11.9)	46.0 (54.4)	38.5	(45.6)
89.0	38.5 (43.3)	8.7 (9.8)	47.2 (53.1)	41.8	(46.9)
91.6	40.4 (44.1)	6.8 (7.4)	47.2 (51.5)	44.4	(48.5)
91.9	41.1 (44.7)	5.4 (5.9)	46.5 (50.6)	45.4	(49.4)

Notes: # Values in parentheses are the forces in each leg / angle expressed as percentages of the applied loads.

* These are the maximum forces occurring in the corresponding components.

TABLE 8. FORCE DISTRIBUTION IN SPECIMEN B1270.1

Applied Loads, kN	Angle A			Angle B		
	Leg 1 kN (%) *	Leg 2 kN (%)	Total kN (%)	Leg 3 kN (%)	Leg 4 kN (%)	Total kN (%)
4.5	1.3 (28.8)	1.1 (24.8)	2.4 (53.6)	1.2 (26.4)	0.9 (20.0)	2.1 (46.4)
9.9	2.9 (32.0)	2.1 (23.3)	4.9 (55.3)	2.5 (27.8)	1.5 (16.9)	4.0 (44.7)
22.2	7.7 (34.8)	5.2 (23.4)	12.9 (58.2)	6.1 (27.3)	3.2 (14.5)	9.3 (41.8)
44.5	20.2 (45.4)	9.2 (20.7)	29.4 (66.1)	13.3 (29.9)	1.8 (4.0)	15.1 (33.9)
66.7	35.2 (52.7)	12.9 (19.3)	48.0 (72.0)	21.9 (32.8)	-3.2 (-4.8)	18.7 (28.0)
89.0	48.6 (54.6)	15.8 (17.6)	64.4 (72.4)	31.2 (35.1)	-6.7 (-7.5)	24.6 (27.6)
97.9	54.6 (55.8)	16.3 (16.7)	71.0 (72.5)	35.3 (36.1)	-8.4 (-8.6)	26.9 (27.5)
102.3	56.6 (55.3)	16.8 (16.4)	73.4 (71.7)	37.7 (36.8)	-8.7 (-8.5)	29.0 (28.3)
106.8	58.9 (55.2)	16.8 (15.7)	75.7 (70.9)	39.9 (37.4)	-8.9 (-8.3)	31.1 (29.1)
111.2	61.6 (55.4)	16.8 (15.1)	78.4 (70.5)	41.6 (37.4)	-8.8 (-7.9)	32.8 (29.5)
115.7	64.2 (55.5)	16.2 (14.0)	80.4 (69.5)	44.1 (38.1)	-8.8 (-7.6)	35.3 (30.5)
117.4	65.9 (56.1)	15.6 (13.3)	81.5 (69.4)	45.1 (38.4)	-9.2 (-7.8)	35.9 (30.6)
120.1	68.1 (56.7)	13.2 (11.0)	81.3 (67.7)	47.2 (39.3)	-8.4 (-7.0)	38.8 (32.3)
122.1	71.8 (58.8)	11.5 (9.5)	83.4 (68.3)	46.8 (38.3)	-9.1 (-6.6)	38.7 (31.7)
124.5	73.9 (59.3)	10.0 (8.0)	83.8 (67.3)	48.6 (39.0)	-7.9 (-6.3)	40.7 (32.7)
126.8	75.4 (59.5)	8.9 (7.0)	84.3 (66.5)	50.1 (39.5)	-7.6 (-6.0)	42.5 (33.5)
129.0	76.5 (59.3)	5.6 (4.3)	82.0 (63.6)	54.2 (42.0)	-7.2 (-5.6)	47.0 (36.4)
131.2	75.9 (58.6)	3.8 (2.9)	90.7 (61.5)	57.6 (43.9)	-7.1 (-5.4)	50.5 (38.5)
133.4	76.2 (57.1)	2.0 (1.5)	78.2 (58.6)	61.5 (46.1)	-6.3 (-4.7)	55.2 (41.4)
133.8	75.0 (56.8)	-1.3 (-1.0)	59.1 (55.8)	63.7 (47.6)	-4.3 (-3.2)	59.1 (44.2)

Notes: # Values in parentheses are the forces in each leg / angle expressed as percentages of the applied loads.

* These are the maximum forces occurring in the corresponding components.

TABLE 9. FORCE DISTRIBUTION IN SPECIMEN B1270.2

Applied Loads, kN	Angle A			Angle B	
	Leg 1 kN (%)*	Leg 2 kN (%)	Total kN (%)	Total kN (%)	
8.9	2.4 (27.3)	1.8 (20.6)	4.3 (47.9)	4.6 (52.1)	
22.2	10.6 (47.8)	4.3 (19.1)	14.9 (66.9)	7.4 (33.1)	
44.5	21.9 (49.2)	7.6 (17.0)	29.5 (66.2)	15.0 (33.8)	
66.7	33.2 (49.8)	9.8 (14.7)	43.0 (64.5)	23.7 (35.5)	
89.0	45.2 (50.8)	11.7 (13.1)	56.9 (63.9)	32.1 (36.1)	
97.9	50.7 (51.8)	14.4 (14.7)	65.1 (66.5)	32.8 (33.5)	
102.3	53.4 (52.2)	11.5 (11.2)	64.9 (63.4)	37.4 (36.6)	
106.8	56.3 (52.7)	11.0 (10.3)	67.3 (63.0)	39.5 (37.0)	
111.2	56.5 (50.8)	10.1 (9.1)	66.6 (59.9)	44.6 (40.1)	
115.7	59.9 (51.8)	8.4 (7.3)	68.4 (59.1)	47.3 (40.9)	
117.4	66.8 (56.9)	3.9 (3.3)	70.7 (60.2)	46.7 (39.8)	
120.1	69.9 (58.2)	1.9 (1.6)	71.8 (59.8)	48.3 (40.2)	
122.1	76.3 (62.5)	-0.7 (-0.6)	75.6 (61.9)	46.5 (38.1)	
123.8	76.4 (61.7)	-0.6 (-5.1)	70.1 (56.6)	53.7 (43.4)	

Notes: # Values in parentheses are the forces in each leg / angle expressed as percentages of the applied loads.

* These are the maximum forces occurring in the corresponding components.

TABLE 10. FORCE DISTRIBUTION IN SPECIMEN W2100.1

Applied Loads, kN	Angle A			Angle B	
	Leg 1 kN (%) *	Leg 2 kN (%)	Total kN (%)	Total kN (%)	
4.5	1.6 (35.9)	1.2 (25.6)	2.8 (61.5)	1.7 (38.5)	
8.9	3.0 (33.9)	2.2 (24.6)	5.2 (58.5)	3.7 (41.5)	
13.3	4.3 (32.1)	3.3 (24.7)	7.6 (56.8)	5.7 (43.2)	
17.8	5.9 (33.4)	4.2 (23.4)	10.1 (56.8)	7.7 (43.2)	
22.2	7.3 (33.2)	5.3 (23.7)	12.6 (56.9)	9.6 (43.1)	
26.7	8.7 (32.7)	6.0 (22.3)	14.7 (55.0)	12.0 (45.0)	
31.1	10.4 (33.3)	6.6 (21.2)	17.0 (54.5)	14.1 (45.5)	
35.6	11.8 (33.3)	7.5 (21.0)	19.3 (54.3)	16.3 (45.7)	
40.0	13.7 (34.2)	8.8 (21.9)	22.5 (56.1)	17.5 (43.9)	
44.5	16.1 (36.2)	9.4 (21.0)	25.5 (57.2)	19.0 (42.8)	
48.9	18.0 (36.8)	10.4 (21.2)	28.4 (58.0)	20.5 (42.0)	
53.4	19.9 (37.4)	11.3 (21.1)	31.2 (58.5)	22.2 (41.5)	
57.8	22.1 (38.3)	11.6 (20.1)	33.7 (58.4)	24.1 (41.6)	
62.3	23.9 (38.4)	12.4 (20.0)	36.3 (58.4)	26.0 (41.6)	
66.7	26.4 (39.5)	12.7 (19.0)	39.0 (58.5)	27.7 (41.5)	
71.2	28.7 (40.3)	13.5 (19.0)	42.2 (59.3)	29.0 (40.7)	
75.6	30.9 (40.9)	13.7 (18.1)	44.6 (59.0)	31.0 (41.0)	
80.1	32.7 (40.8)	14.1 (17.6)	46.8 (58.4)	33.3 (41.6)	
84.5	35.2 (41.6)	14.2 (16.9)	49.4 (58.5)	35.1 (41.5)	
89.0	36.9 (41.4)	14.9 (16.8)	51.8 (58.2)	37.2 (41.8)	
93.4	39.5 (42.3)	14.0 (15.0)	53.5 (57.3)	39.9 (42.7)	
97.9	40.9 (41.7)	13.6 (13.9)	54.5 (55.6)	43.4 (44.4)	

Notes: # Values in parentheses are the forces in each leg / angle expressed as percentages of the applied loads.

* These are the maximum forces occurring in the corresponding components.

TABLE 11. FORCE DISTRIBUTION IN SPECIMEN W2100.2

Applied Loads, kN	Angle A			Angle B	
	Leg 1 kN (%)*	Leg 2 kN (%)	Total kN (%)	Total kN (%)	
4.5	2.2 (49.8)	1.0 (20.9)	3.2 (70.7)	1.3 (29.3)	
13.3	6.3 (47.3)	4.0 (30.0)	10.3 (77.3)	3.0 (22.7)	
22.2	10.8 (48.6)	6.0 (27.1)	16.8 (75.7)	5.4 (24.3)	
26.7	13.6 (51.0)	6.2 (23.2)	19.8 (74.2)	6.9 (25.8)	
31.1	15.6 (50.1)	7.4 (23.6)	23.0 (73.7)	8.2 (26.3)	
35.6	18.3 (51.4)	8.2 (23.1)	26.5 (74.5)	9.1 (25.5)	
40.0	20.2 (50.4)	9.1 (22.6)	29.2 (73.0)	10.8 (27.0)	
44.5	22.2 (49.8)	9.9 (22.3)	32.1 (72.1)	12.4 (27.9)	
48.9	24.3 (49.6)	10.4 (21.3)	34.7 (70.9)	14.2 (29.1)	
53.4	26.9 (50.4)	11.2 (20.9)	38.1 (71.3)	15.3 (28.7)	
57.8	30.2 (52.2)	11.0 (19.1)	41.2 (71.3)	16.6 (28.7)	
62.3	31.3 (50.2)	13.5 (21.7)	44.8 (71.9)	17.5 (28.1)	
66.7	34.8 (52.2)	11.0 (16.5)	45.8 (68.7)	20.9 (31.3)	
71.2	35.8 (50.3)	12.1 (17.0)	47.9 (67.3)	23.3 (32.7)	
75.6	36.3 (48.0)	14.0 (18.5)	50.3 (66.5)	25.3 (33.5)	
80.1	41.8 (52.2)	9.8 (12.2)	51.6 (64.4)	28.5 (35.6)	
84.5	40.0 (47.3)	13.6 (16.1)	53.6 (63.4)	30.9 (36.6)	
89.0	42.3 (47.5)	12.5 (14.0)	54.7 (61.5)	34.3 (38.5)	
93.4	43.2 (46.2)	12.7 (13.6)	55.9 (59.8)	37.6 (40.2)	
97.9	45.7 (46.7)	11.6 (11.8)	57.3 (58.5)	40.6 (41.5)	
99.9	46.9 (46.9)	11.1 (11.1)	58.0 (58.0)	42.0 (42.0)	

Notes: # Values in parentheses are the forces in each leg / angle expressed as percentages of the applied loads.

* These are the maximum forces occurring in the corresponding components.

TABLE 12. MAXIMUM FORCE IN EACH LEG/ANGLE EXPRESSED AS A PERCENTAGE OF THE BUCKLING LOAD*

Test Specimen	Angle A			Angle B		
	Leg 1 (%)	Leg 2 (%)	Total (%)	Leg 3 (%)	Leg 4 (%)	Total (%)
B2100.1	48	9	55	44	5	45
B2100.2	45	12	51	—*	—*	49
B1270.1	58	13	63	48	-7 ~	44
B1270.2	61	12	61	—*	—*	43
W2100.1	42	15	56	—*	—*	44
W2100.2	47	14	58	—*	—*	42
Estimated Range	40-60~	10-15~	50-65	45-50	0-5	40-50~

Notes: # Maximum forces and buckling loads are obtained from Tables 6 to 11.

* Data are not available.

~ Negative value means the leg is subjected to tension.

^ They are the values used in designing the joints at the place of truss-to-truss connection.

Appendix C

NONDIMENSIONAL BUCKLING CURVE

(FROM ECCS RECOMMENDATIONS, 1978)

Appendix D

BEAM-COLUMN ANALYSIS

BEAM-COLUMN ANALYSIS

In the following example, the starred angles supporting secondary trusses are modelled as beam-columns (see Figure 3.1). Critical loads are estimated using the design procedures suggested by Clause 13.8 of CSA standard CAN3-S16.1-M84. In this case, the member size is first chosen and then checked against the two interaction equations to calculate the critical load, C_f .

Example : Critical Load for Specimens B2100

Data : The required cross-section properties for specimens B2100 are obtained from Tables 1 and 2 as follows:-

$$b = 43.7 \text{ mm} ; \quad t = 4.7 \text{ mm} ;$$

$$A = 783 \text{ mm}^2 ;$$

$$I_u = 533100 \text{ mm}^4 ; \quad I_v = 224500 \text{ mm}^4 ;$$

$$S_u = 14160 \text{ mm}^3 ; \quad S_v = 7260 \text{ mm}^3 ;$$

$$F_y = \sigma_y = 390 \text{ MPa}.$$

where S_u and S_v are the elastic section moduli of the starred angle section about the U and V axes, respectively.

Also, from Figure 3.1 :--

$$C_f = P ; \quad e = 27.8 \text{ mm} ;$$

$$M_{fu} = (P)(e)\cos 45^\circ = 19.6 P ;$$

$$M_{fv} = (P)(e)\sin 45^\circ = 19.6 P ;$$

where C_f , M_{fu} and M_{fv} = the factored axial compressive resistance and moment resistances which are defined in Clause 13.3, 13.6(b) and 13.5(b) (or

13.5(c)(i) and (c)(iii)), respectively.

ϕ (resistance factor) is taken as 1.0 because the load under consideration ~~is~~ the ultimate load.

Solution :

A satisfactorily designed beam-column must satisfy the following three requirements:

(a) Local Buckling Requirement

From Table 1 of CSA S16.1,

$$\frac{b}{t} = 9.25 < \frac{200}{\sqrt{F_y}} = 10.13$$

The starred angles can be classified as Class 3 sections. This insures that local buckling will not occur at stresses up to and including the yield stress.

(b) Strength Requirement

The strength of the beam-column can be checked by assuming the column to fail by yielding in the extreme fibre of the cross-section.

From Clause 13.8.3(a),

$$\frac{C_f}{C_r} + \frac{M_{fu}}{M_{ru}} + \frac{M_{fv}}{M_{rv}} \leq 1.0 \quad [D-1]$$

in which

$$C_r = \phi A F_y = 1.0 (783) (390) = 305 \text{ kN}$$

$$M_{ru} = \phi S_u F_y = 1.0 (14160) (390) = 5520 \text{ kN-mm}$$

$$M_{rv} = \phi S_v F_y = 1.0 (7260) (390) = 2830 \text{ kN-mm}$$

Then, C_f can be estimated from Equation [D-1] as:

$$\frac{C_f}{305} + \frac{19.6 C_f}{5520} + \frac{19.6 C_f}{2830} = 1.0$$

$$C_f = \underline{72.7 \text{ kN}} \quad [D-2]$$

(c) Stability Requirement

This section checks the overall member failure caused by excessive stresses due to the second-order effects from moment amplification.

From Clause 13.8.3(b),

$$\frac{C_f}{C_r} + \frac{w_u M_{fu}}{M_{ru} \left(1 - \frac{C_f}{C_{eu}}\right)} + \frac{w_v M_{fv}}{M_{rv} \left(1 - \frac{C_f}{C_{ev}}\right)} \leq 1.0 \quad [D-3]$$

in which

$w_u = w_v = 0.85$ (for conservative computation)

C_{eu} = Euler's load about U-axis = 245 kN

C_{ev} = Euler's load about V-axis = 103 kN

From Clause 13.3, $C_r = 88.0 \text{ kN}$

From Clause 13.6(b), $M_{ru} = 5520 \text{ kN-mm}$

From Clause 13.5(b), $M_{rv} = \phi S_v F_y = 2830 \text{ kN-mm}$

Hence, Equation [D-3] becomes

$$\frac{C_f}{88.04} + \frac{0.85 (19.6 C_f)}{5520 \left(\frac{245 - C_f}{245}\right)} + \frac{0.85 (19.6 C_f)}{2830 \left(\frac{103 - C_f}{103}\right)} = 1.0$$

After solving the above equation,

$$C_f = \underline{40.6 \text{ kN}}$$

[D-4]

Conclusion :

Comparing [D-2] and [D-4], [D-4] will govern. Thus, the ultimate load carrying capacity of starred angle supporting trusses based on the Beam-Column design procedure in Clause 13.8 is equal to 40.6 kN.

Appendix E
DESIGN BASED ON
ASCE MANUAL NO. 52
AND
ECCS RECOMMENDATIONS

SECTION E.1

ASCE MANUAL NO. 52E.1.1 ALLOWABLE COMPRESSIVE STRESS

The allowable compressive stress, F_a , specified in the Manual is actually the ultimate stress.

From Clause 4.7 of the Manual,

$$(b/t)_{\text{lim}} = 80 / \sqrt{F_y} \quad [E-1]$$

When $(b/t) \leq (b/t)_{\text{lim}}$,

$$F_{\text{cr}} = F_y \quad [E-2]$$

When $(b/t)_{\text{lim}} \leq (b/t) \leq 144 / \sqrt{F_y}$,

$$F_{\text{cr}} = \left[1.677 - 0.677 \frac{b/t}{(b/t)_{\text{lim}}} \right] F_y \quad [E-3]$$

When $(b/t) \geq 144 / \sqrt{F_y}$,

$$F_{\text{cr}} = \frac{9500}{(b/t)^2} \quad [E-4]$$

where b = width of the leg measured from the root of the fillet; t = thickness of leg; F_y = nominal yield stress in ksi; and F_{cr} = effective yield stress in ksi.

From Clause 4.6 of the Manual, when $KL/r \leq C_c$,

$$F_a = \left[1 - \frac{1}{2} \left(\frac{KL/r}{C_c} \right)^2 \right] F_{\text{cr}} \quad [E-5]$$

When $KL/r \geq C_c$,

$$F_a = \frac{286,000}{(KL/r)^2} \quad [E-6]$$

where $C_c = \pi \sqrt{2E/F_{cr}}$; E = modulus of elasticity in ksi;
 L = unbraced length ; r = radius of gyration; K = effective
length factor; and F_a = ultimate stress in ksi.

E.1.2 B/W 2100 SPECIMENS

From Equation [E-1], $(b/t)_{lim} = 10.51$

Since $(b/t) = 8.25 < (b/t)_{lim}$, then

$$F_{cr} = F_y = 390 \text{ MPa} = 56.6 \text{ ksi}$$

$$\text{Also } \frac{L}{r_v} = \frac{2100 \text{ mm}}{16.9 \text{ mm}} = 124 > 120$$

$$\text{From Equation [2-4], } \frac{KL}{r} = \frac{L}{r_v} = 124 \Rightarrow K = \underline{1.0}$$

$$C_c = \pi \left[\frac{2 (205400 \text{ MPa})}{390 \text{ MPa}} \right]^{\frac{1}{2}} = 102 < \frac{KL}{r_v}$$

From Equation [E-6], $F_a = 18.6 \text{ ksi} = 128 \text{ MPa}$

$$\Rightarrow P_{cr} = \text{AREA} * F_a = 783 \text{ mm}^2 * (128 \text{ MPa}) = \underline{100 \text{ kN}}$$

E.1.3 B1270 SPECIMENS

$$\frac{L}{r_v} = \frac{1270 \text{ mm}}{16.9 \text{ mm}} = 75 < 120$$

$$\text{From Equation [2-3], } \frac{KL}{r} = 60 + 0.5 \left(\frac{L}{r_v} \right) = 97.5$$

$$\Rightarrow K = 97.5 / 75.0 = \underline{1.30}$$

Since $C_c = 102 > KL/r$, from Equation [E-5],

$$F_a = 30.7 \text{ ksi} = 212 \text{ MPa}$$

$$\text{Hence, } P_{cr} = \text{AREA} * F_a = \underline{166 \text{ kN}}$$

SECTION E.2

ECCS RECOMMENDATIONSE.2.1 B/W 2100 SPECIMENS

$$\lambda = \text{slenderness ratio} = \frac{KL}{r_v} = \frac{2100 \text{ mm}}{16.9 \text{ mm}} = 124$$

$$\lambda_y = \pi \left(\frac{E}{\sigma_y} \right)^{\frac{1}{2}} = \pi \left(\frac{205400 \text{ MPa}}{390 \text{ MPa}} \right)^{\frac{1}{2}} = 72$$

$$\text{Since } \frac{\lambda}{\lambda_y} = \frac{124}{72} = 1.72 > 1.41, \text{ from Equation [2.8],}$$

$$\left(\frac{\lambda}{\lambda_y} \right)_{\text{eff}} = \frac{\lambda}{\lambda_y} = 1.72$$

$$\Rightarrow K = \text{effective length factor} = \frac{(\lambda / \lambda_y)_{\text{eff}}}{(\lambda / \lambda_y)} = \underline{1.0}$$

From the Nondimensional Column Curve in Appendix C,

$$\frac{\sigma_{cr}}{\sigma_y} = 0.2709$$

$$\begin{aligned} \Rightarrow \sigma_{cr} &= \text{critical stress} \\ &= 0.2709 * 390 \text{ MPa} \\ &= 106 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \Rightarrow P_{cr} &= \text{critical load} \\ &= \text{Area} * \sigma_{cr} \\ &= 783 \text{ mm}^2 * 106 \text{ MPa} \\ &= \underline{83 \text{ kN}} \end{aligned}$$

E.2.2 B1270 SPECIMENS

$$\lambda = \frac{KL}{r_v} = \frac{1270 \text{ mm}}{16.9 \text{ mm}} = 75$$

$$\frac{\lambda}{\lambda_y} = \frac{75}{72} = 1.04 < 1.41$$

From Equation [2.7],

$$\left(\frac{\lambda}{\lambda_y} \right)_{\text{eff}} = 0.60 + 0.57 * 1.04 = 1.19$$

$$\Rightarrow K = \frac{1.19}{1.04} = \underline{1.15}$$

From the Nondimensional Column Curve in Appendix C,

$$\frac{\sigma_{cr}}{\sigma_y} = 0.4848$$

$$\Rightarrow \sigma_{cr} = 0.4848 (390 \text{ MPa}) = 189 \text{ MPa}$$

$$\begin{aligned} \Rightarrow P_{cr} &= \text{Area} * \sigma_{cr} \\ &= 783 \text{ mm}^2 * 189 \text{ MPa} \\ &= \underline{148 \text{ kN}} \end{aligned}$$

Appendix F

CURVE FITTING METHOD

CURVE FITTING METHOD

Curve fitting involves the determination of a continuous function, $y = f(x)$, which results in the most reasonable fit of experimentally measured values. The following sections will discuss the techniques used to interpolate the stress distribution curve for the test specimens.

F.1 SPECIMENS B2100.1, B1270.1 AND B1270.2

Strain gauges were installed at three locations along each leg of specimens B2100.1, B1270.1, and Legs 1 and 2 of specimen B1270.2. The plots of stress (y values) and distance along the corresponding leg (x values) are shown in Figures 5.4(a) and (b), 5.6(a) and (b), and 5.7(a) for the specimens mentioned above. Due to the scattering and the limited data, the stress distribution can best be represented by either a linear form or a second order polynomial.

F.1.1 LINEAR FORM USING METHOD OF LEAST-SQUARES FIT

Examining the stress distribution curves carefully, it is found that at small loads, the measured data suggests a curve fitted by a straight line. That is,

$$y = f(x) = c_1 + c_2 x \quad [F-1]$$

The constants c_1 and c_2 can be determined from a "least-squares" criterion which will result in a "best-fit" straight line representation of the experimental data. From the three measured values of x and y , the following equations are obtained.

$$c_1 + c_2 x_1 = Y_1$$

$$c_1 + c_2 x_2 = Y_2$$

$$c_1 + c_2 x_3 = Y_3$$

[F-2]

Since c_1 and c_2 are the only unknowns, there are three equations with two unknowns. No solution is possible when solving the three equations simultaneously. In this case, the method of least-squares fit is applied. Equation [F-2] can be rewritten in the form

$$c_1 + c_2 x_1 - Y_1 = r_1$$

$$c_1 + c_2 x_2 - Y_2 = r_2$$

$$c_1 + c_2 x_3 - Y_3 = r_3$$

[F-3]

where r_1 , r_2 and r_3 are called the residuals which would all be zero in the case of a perfect fit. However, the residuals will generally not be zero due to experimental errors. A best fit is obtained by making them as small as possible. This is accomplished by finding values for c_1 and c_2 such that the sum of the squares of the residuals is a minimum. That is,

$$\sum_{i=1}^3 (r_i)^2 = \text{minimum}$$

[F-4]

Examining Equation [F-3], r_i is a function of the constants c_1 and c_2 . It follows from Equation [F-4] that

$$\frac{\delta}{\delta c_j} \sum_{i=1}^3 (r_i)^2 = 0$$

$$\Rightarrow \sum_{i=1}^3 r_i \frac{\delta r_i}{\delta c_j} = 0 \quad \text{for } j = 1, 2. \quad [\text{F-5}]$$

where $\frac{\delta}{\delta c_i}$ implies partial differentiation w. r. t. ' c_i '.

Expanding Equation [F-5] will give :

$$r_1 \frac{\delta r_1}{\delta c_1} + r_2 \frac{\delta r_2}{\delta c_1} + r_3 \frac{\delta r_3}{\delta c_1} = 0$$

$$r_1 \frac{\delta r_1}{\delta c_2} + r_2 \frac{\delta r_2}{\delta c_2} + r_3 \frac{\delta r_3}{\delta c_2} = 0$$

[F-6]

The following partial derivatives can be obtained from Equation [F-3] as :

$$\frac{\delta r_1}{\delta c_1} = \frac{\delta r_2}{\delta c_1} = \frac{\delta r_3}{\delta c_1} = 1.0$$

$$\frac{\delta r_1}{\delta c_2} = x_1, \quad \frac{\delta r_2}{\delta c_2} = x_2, \quad \frac{\delta r_3}{\delta c_2} = x_3$$

[F-7]

Substituting Equations [F-3] and [F-7] into Equation [F-6] yields

$$3 c_1 + c_2 \sum_{i=1}^3 x_i - \sum_{i=1}^3 y_i = 0$$

$$c_1 \sum_{i=1}^3 x_i + c_2 \sum_{i=1}^3 x_i^2 - \sum_{i=1}^3 y_i x_i = 0$$

[F-8]

Equation [F-8] can be solved simultaneously for the unknown constants c_1 and c_2 which will give the best fit, $y = c_1 + c_2 x$, to the three known data values.

F.1.2 POLYNOMIAL FUNCTIONS

As it was mentioned in the preceding sections, the stress distribution along any leg of the test specimens would change from a straight line to a curve after the load had reached certain values. In this case, a polynomial function is required in interpreting the stress data to compute the force distribution.

Polynomials are frequently used in curve fitting. In general, an $n-1$ degree polynomial has the form

$$y = c_1 + c_2x + c_3x^2 + \dots + c_nx^{n-1} \quad [F-9]$$

Using the least-squares approach, this polynomial function will give the closest fit for n given distinct data points. For example, in specimen B2100.1, three stress data are recorded along each leg. Hence, the polynomial used is in the form of

$$y = c_1 + c_2x + c_3x^2 \quad [F-10]$$

The constants c_1 , c_2 and c_3 can be solved simultaneously using the three measured data points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) .

F.2 SPECIMENS B2100.2, W2100.1 AND W2100.2

Strain gauges were installed at only three locations on Angle A of these specimens. The curve fitting techniques mentioned in Section F.1 can also be applied in this case. However, Angle A of specimens B2100.2, W2100.1 and W2100.2 is analyzed as a whole unit and only one curve is derived for the stress distribution.

In contrast, for the specimens in Section F.1, each angle was divided into two components (legs) which were analyzed separately. Hence, two functions were formulated for the stress distribution in each angle for that case.

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VITA AUCTORIS

KENNETH HON-WA MOK

The author was born on July 28th, 1963 in Hong Kong. In 1980 , he completed his secondary school education at Wah Yan College, Hong Kong .

He arrived in Canada in September 1981 for his Grade 13 program at Central Collegiate in Oshawa , Ontario. Then, he joined the University of Windsor , where he was granted undergraduate scholarship for four consecutive years . In May of 1986, he graduated with a degree of Bachelor of Applied Science (First Class Honours) in Civil Engineering and obtained the Board of Governors Medal .

In September of 1986 , he joined the Master's program in Civil Engineering Department at the same university .