1995

Static and dynamic responses of simply supported and continuous skew composite bridges.

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STATIC AND DYNAMIC RESPONSES OF
SIMPLY SUPPORTED AND CONTINUOUS
SKEW COMPOSITE BRIDGES

by

TAREK IBRAHIM EBEIDO

A Dissertation
submitted to the Faculty of Graduate Studies and Research through
the Department of Civil and Environmental Engineering
in Partial Fulfilment of the Requirements for the
degree of Doctor of Philosophy at the
University of Windsor

Windsor, Ontario, Canada
1995
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ABSTRACT

The load distribution factor concept allows the design engineer to consider the longitudinal and transverse effects of wheel loads as two separate phenomena and thus simplifying the analysis and design of the bridge. North American codes of practice do not provide the design engineer with sufficient guidance regarding elastic load distribution factors for skew composite bridges. These codes of practice also require a check to be made on the strength of the sections of the bridge at the ultimate limit state, but they specify the use of elastic load distribution factors for the evaluation of the design forces at the ultimate limit state. Such specifications could lead to an extremely conservative design in some cases and to unsafe design in other cases, since these factors do not represent the actual behaviour of the bridge at that state. Under a moving truck, the bridge exhibits forces which are significantly greater than those under a static truck load. The Ontario Highway Bridge Design Code specifies the dynamic load allowance as a factor of the first flexural frequency of the bridge based on the beam theory. This latter theory is reliable in the case of right bridges without transverse diaphragms. However, in the case of skew bridges or bridges with transverse diaphragms the use of the beam theory leads to substantial errors in the determination of the first flexural frequency and hence the dynamic load allowance.

In this dissertation, the predictions of the elastic and the ultimate behaviours of single span simply supported and continuous two-span skew composite steel-concrete
bridges are presented. The dynamic response of such bridges is also investigated. A finite element analytical model based on the ABAQUS program, is used for the analyses. The analytical model was verified and substantiated by results from tests on six single span simply supported and three continuous skew composite steel-concrete bridge models. The finite element model is employed to conduct six extensive parametric studies on prototype skew composite bridges, including more than 2500 bridge cases. A statistical package for best fit is used to generate empirical formulas for the following: (i) elastic shear and moment distribution factors for simply supported and continuous skew composite bridges; (ii) ultimate span and support moment distribution factors for continuous skew composite bridges; and, (iii) first flexural frequency of such bridges. The following design parameters were considered when appropriate: angle of skew; bridge aspect ratio; girder spacing; number of lanes; number of girders; the effect of transverse intermediate diaphragms; longitudinal and transverse flexural rigidities; longitudinal and transverse ultimate moments of resistance of the bridge sections; eccentric and concentric truck loading cases; effect of dead load; and, the span ratio. From the results of this design oriented dissertation, the engineer would be able to design skew composite bridges more reliably and economically.

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TO MY FAMILY
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NOMENCLATURE

$A_i$  bridge aspect ratio (= span / width).

$[B]$  strain displacement matrix.

$b$  bridge width.

$D$  load distribution factor.

$D_i$  moment distribution factor at the $i^{th}$ loading increment.

$DLA$  dynamic load allowance.

$D_u$  moment distribution factor at the ultimate limit state.

$D_x$  flexural rigidity in the longitudinal direction.

$D_y$  flexural rigidity in the transverse direction.

$E$  modulus of elasticity of the concrete slab.

$[E]$  elasticity matrix.

$F$  maximum reaction in a continuous two-span composite girder.

$F_{max}$  maximum reaction from the finite element analysis.

$I$  moment of inertia of the bridge cross section.

$[K]$  structure stiffness matrix.

$[K_i]$  element $i$ stiffness matrix.

$L$  span length of the bridge.

$M$  moment of composite girder.

$m$  mass per unit length of the bridge.

$M_{max}$  maximum girder moment from the finite element analysis.

$M_{max_u}$  maximum girder moment at the ultimate limit state.
$m_{ni}$ ultimate transverse negative moment of resistance of composite section per unit length.

$m_{pl}$ ultimate longitudinal positive moment of resistance of composite section per unit width.

$m_{pi}$ ultimate transverse positive moment of resistance of composite section per unit length.

$m_a$ ultimate longitudinal negative moment of resistance of composite section per unit width.

$m_{tor}$ ultimate torsional moment of resistance of the concrete deck slab per unit width.

$N$ dimensionless parameter as a measure of girder spacing (= number of lanes / number of girders).

$[P]$ applied loads vector at the nodes.

$R$ rigidity ratio (= transverse rigidity / longitudinal rigidity).

$S$ spans ratio (= long span length / short span length).

$[U]$ displacement vector at the nodes.

$V$ reaction of composite girder.

$V_{max}$ maximum reaction at a bridge girder from the finite element analysis.

$\theta$ angle of skew in degrees.

$\rho$ reinforcement ratio.

$\omega_1$ first flexural frequency of the bridge using the flexural beam theory.

$\omega_s$ magnification factor.

$\omega_{1s}$ first flexural frequency of the bridge using the proposed method.
CHAPTER 1

INTRODUCTION

1.1 General

During the last few decades, composite steel-concrete structures have been used widely to form the basic superstructure for numerous bridges throughout the world. This was mainly due to the reduction in the structure's dead weight, better structural load-carrying capacity, and a considerable reduction in the bridge depth. Resulting from the demand of modern traffic routing, skew composite steel-concrete bridges have become wide spread. A survey, conducted by Kennedy and Tamberg in 1969, in the province of Ontario revealed that 35% of the total bridge deck area that had been built is on skew alignment.

Proper design of new bridges and evaluation of existing ones require accurate prediction of their structural response to truck loads. The load distribution factor concept allows the design engineer to consider the longitudinal and transverse effects of wheel loads as two separate phenomena, thus simplifying the analysis and design of the bridge. However, using inappropriate load distribution factors may lead to extremely conservative design forces or sometimes makes the design of the bridge unsafe. North American codes of practice do not provide the design engineer with sufficient guidance regarding load distribution factors of skew composite bridges. The American Association of State
Highway and Transportation Officials (AASHTO, 1994) has traditionally applied a load-distribution factor for both moment and shear, depending only on the center to center girder spacing. However, the AASHTO Code (1994) ignores the following significant parameters: (i) angle of skew; (ii) bridge aspect ratio; (iii) longitudinal and transverse rigidities; (iv) number of lanes; (v) number of girders; (vi) eccentric truck loading cases; and, (vii) the effect of dead load. The Ontario Highway Bridge Design Code (OHBDC, 1992) considers some parameters in addition to the girder spacing in establishing load-distribution factors for moments. However, the method is limited to bridges with skew parameters less than a certain value specified in the code. Both the AASHTO and the OHBDC methods are limited to simply supported bridges.

The above codes of practice requires the check of the strength of the sections of the bridge at the ultimate limit state. However, these codes of practice specify the use of elastic load distribution factors for the evaluation of the design forces which may lead to an extremely conservative design in some cases and to an unsafe design in other cases, since these factors do not represent the actual behaviour of the bridge at the ultimate limit state. A proper evaluation of the design forces at the ultimate limit state requires the consideration of the non-linear response of such bridges. The load redistribution phenomenon due to yielding should be also taken into account. For these reasons the distribution factors at the ultimate limit state are significantly different than those in the elastic range of loading.

Under a moving truck, the bridge exhibits forces which are significantly greater than those under a static truck load. Different methods are given by North American
codes of practice in order to take this effect into account in the design of bridges. The American Association of State Highway and Transportation Officials (AASHTO, 1994) has traditionally applied an impact factor depending only on the span length of the bridge. The Ontario Highway Bridge Design Code (OHBDC, 1992) specifies the dynamic load allowance as a factor of the first flexural frequency of the bridge. The formulation for the dynamic load allowance in the OHBD code (1992) was based on testing full-scale bridges and takes into account the dynamic amplifications due to the interaction between the vehicle and the bridge when their natural frequencies are close. The OHBD code (1992) recommends the use of the flexural beam theory to determine the first flexural frequency and hence the dynamic load allowance. The use of the flexural beam theory is reliable in the case of right bridges without diaphragms. However, in the case of skew bridges or in the presence of transverse intermediate diaphragms, the use of the beam theory leads to unacceptable errors in the determination of the first flexural frequency and hence the dynamic load allowance.

In the past, many researches had been carried out on right composite steel-concrete bridges. Comparatively little attention had been directed toward the behaviour of skew composite steel-concrete bridges. Moreover, a limited number of investigations had been conducted to predict the dynamic response of such bridges.

In this design-oriented dissertation, the candidate has added the following contributions to new knowledge in the field of structural engineering: (i) a method for the determination of the elastic shear and moment distribution factors for single span simply supported and continuous skew composite bridges; (ii) a method for the determination of
the span and support moment distribution factors for continuous skew composite bridges at the ultimate limit state; and, (iii) a method for reliably estimating the first flexural frequency and hence the dynamic load allowance of simply supported and continuous skew composite bridges.

1.2 Objectives and Scope

This study is an attempt to examine the static and dynamic responses of simply supported and continuous two-span skew composite steel-concrete bridges. The primary objectives of this investigation may be summarized as follows (see Figures 1.1 to 1.3):

1- To examine theoretically and experimentally the effect of angle of skew and type of transverse diaphragms connection on the elastic and ultimate behaviours of simply supported skew composite bridges from the standpoint of: (i) deflections of the longitudinal beams; (ii) strains in the longitudinal beams; (iii) reactions along the support lines; and, (iv) ultimate loads of the bridge.

2- To investigate theoretically and experimentally the effect of angle of skew and spans ratio (= long span length/short span length) on the elastic and ultimate behaviours of continuous two-span skew composite bridges.

3- To develop a method for the determination of the design forces for simply supported skew composite bridges in the elastic range of loading including both the reactions and the longitudinal girder moments.

4- To develop a method for the determination of the design forces for continuous two-unequal-span skew composite bridges in the elastic range of loading including the
following: (i) span girder moments; (ii) support girder moments; (iii) reactions at the intermediate interior support; and, (iv) longitudinal girder design shear forces.

5- To study the effect of the various parameters on the load distribution characteristics of two-unequal-span continuous skew composite bridges at the ultimate limit state.

6- To develop a reliable method for the determination of the load distribution factors for continuous two-unequal-span skew composite bridges at the ultimate limit state taking into account the non-linear response of such bridges and the load redistribution phenomenon due to yielding.

7- To investigate the effect of the various parameters on the natural frequencies and their corresponding mode shapes for simply supported and continuous skew composite steel-concrete bridges.

8- To develop a reliable method for the determination of the first flexural frequency of simply supported and continuous two-span skew composite steel-concrete bridges. The use of this method with the charts given in the OHBD code (1992) leads to a more reliable dynamic load allowance.

The above methods for the prediction of the static response of simply supported and continuous skew composite bridges were developed through six extensive parametric studies on prototype bridges subjected to both OHBDC and AASHTO truck loads as well as the dead load. The parametric studies included more than 2500 bridge cases. A statistical package for best fit was used to deduce the empirical formulas using the data generated in the parametric studies. The following design parameters were considered:
(i) angle of skew; (ii) bridge aspect ratio; (iii) girder spacing; (iv) number of lanes; (v) number of girders; (vi) the effect of transverse intermediate diaphragms; (vii) bridge longitudinal and transverse flexural rigidities; (viii) longitudinal and transverse moments of resistance of the bridge sections at the ultimate limit state; (ix) eccentric and concentric truck loading cases; and, (x) span ratio. The finite element model used for the parametric studies was verified and substantiated by means of an experimental study. The experimental study consisted of testing six single span simply supported and three continuous skew composite steel-concrete bridge models.

The method for the prediction of the first flexural frequency of skew composite bridges was developed using the data generated in a parametric study on prototype bridges. The parametric study included more than 150 bridges. The finite element model used for this parametric study was verified and substantiated by means of results from testing two continuous composite steel-concrete bridge models, presented by Grace and Kennedy (1984).

The contents of this dissertation are as follows:

CHAPTER II contains a review of the theoretical and the experimental study of the elastic and ultimate static response as well as the dynamic response of simply supported and continuous composite steel-concrete bridges.

CHAPTER III presents the finite element formulation and contains a full description for the parts of the 'ABAQUS' computer program (Hibbitt et al., 1989) used in the static and
dynamic analyses.

CHAPTER IV deals with the experimental program with a description of the bridge models tested in the laboratory, the instrumentation and materials used to accomplish the experimental program.

CHAPTER V presents the experimental and theoretical results for the numerous loading cases considered in the experimental study including deflections, strains, reactions, and collapse loads of the bridge models.

CHAPTER VI contains the discussion of results from the six extensive parametric studies carried out on prototype simply supported and continuous skew composite bridges and presents empirical formulas for the determination of the elastic and ultimate load distribution factors as well as the first flexural frequency of such bridges.

CHAPTER VII contains the summary and conclusions of the study as well as recommendations for future research.
CHAPTER II

LITERATURE REVIEW

2-1 General

Composite steel-concrete bridges have become one of the most widely used bridges today. Resulting from the demand of modern traffic routing, and for economical and architectural reasons, skew composite steel-concrete bridges have become a common and popular type of structure. A survey, conducted by Kennedy and Tamberg (1969) in the province of Ontario revealed that 35% of the total bridge deck area that had been built is on skew alignment. Only 1% of this skew bridge deck area has an angle of skew > 60°.

In the last forty years, considerable research effort was directed to study the static response of right composite steel-concrete bridges including load distribution characteristics in both the elastic and the ultimate ranges of loading. Several investigations were also directed to study the dynamic response of right composite steel-concrete bridges. Comparatively, little research effort was directed to study the static and dynamic responses of skew composite steel-concrete bridges.
The literature survey conducted is presented on the following: (i) elastic load distribution characteristics of composite steel-concrete bridges; (ii) codes of practice for load distribution in composite bridges; (iii) load distribution characteristics of composite bridges at the ultimate limit state; and, (iv) free vibration of composite steel-concrete bridges.

2-2 Elastic Load Distribution Characteristics of Composite Steel-Concrete Bridges

Newmark et al. (1946) conducted an experimental study on several simply-supported right composite steel-concrete bridge models. They showed that no significant effect on either the strains in the slab or the deflection of the longitudinal beams when interior diaphragms having a stiffness of 2.5% relative to those of the main beams were added. Newmark et al. (1948) tested five 1/4-scale simply-supported skew composite steel-concrete bridge models. Although their laboratory test results were too limited, they gave some design recommendations for skew composite bridges and proposed an approximate method to account for the effect of skew on the girder moments.

Chen et al. (1957) used the finite difference technique to analyze 18 simply-supported skew composite steel-concrete bridges. They used a digital computer to solve 39 simultaneous equations. All bridges considered had five longitudinal steel I-beam girders with different geometric properties and various angles of skew. The AASHTO H-type standard truck loading was applied on each bridge. Influence coefficients for moments and deflections were calculated and used to determine the maximum bending
moment in each longitudinal girder. Chen et al. proposed a set of empirical relations for the calculation of wheel load fractions in skew composite steel-concrete bridges. However, there were many sources of inaccuracy which affected their results and these were summarized by Marx et al. (1986) as follows:

"1- The applied concentrated load was converted to an equivalent uniformly distributed load occupying an area contained within four adjacent grid lines. Therefore, a wheel load was distributed over one eighth of the length and width of bridge. The length of the distributed load is unrealistically large.

2- The composite action between the longitudinal steel girders and the reinforced concrete deck slab was ignored which is not practical and leads to very conservative moments and deflections.

3- The AASHTO standard truck was simulated by using only one of the axle loads from each truck on a bridge.

4- Due to the computer limitations, a coarse 8 x 8 finite difference grid was used. The results for right bridges were compared with the exact solutions obtained by Newmark et al. (1948) and were in good agreement. They assumed that the finite difference grid was sufficient for skew bridges. There was no independent study of the influence of skew on the accuracy of the solution.

5- For large skew angles, where the reduction in girder moments is significant, very large scatter exists in the wheel load fraction data points. For example, for a bridge with a skew angle of 60°, the scatter is more than 55%. Therefore, any beneficial effect of skew was completely lost when the empirical relations were determined from their data."
Based on their study they concluded that: 

"(i) the effect of skew is always a reduction in girder moments for the rear wheel loads of standard H-trucks, and the greater is the skew angle, the larger is this reduction; (ii) the effects of front wheel loads vary from 0% to 17% of the corresponding effects of the rear wheel loads. This percentage is smaller for shorter bridges. The total reductions in girder moments for combined rear and front wheels are practically the same as the reduction in girder moments for rear wheels alone; (iii) for the truck loads considered, the critical load positions are generally one where the rear wheels are quite close to the skew center line of the bridge; and, (iv) for a uniformly distributed load over the entire bridge, the mid-span moment in a girder may be assumed to be roughly 1/5 of the sum of the girder moments across the skew center line of the bridge."

Lount (1957) reported the positive contribution of the transverse intermediate diaphragms in composite steel-concrete bridges. He concluded that such diaphragms may provide: better transverse wheel load distribution; increased bridge stiffness; reduced vibrational and deflection effect; decreased hazards from fatigue loading; enhanced bridge safety; and, allowed crossing of very heavy loads in emergencies. Hendry and Jaeger (1957) studied the effect of angle of skew on the load distribution characteristics of simply supported composite steel-concrete bridges. They considered bridges having three and four longitudinal girders with different angles of skew and geometric properties. They used the grid frame method where the elements of the bridge such as the longitudinal girders and the deck slab were replaced by an equivalent grillage of
interconnected beams. The equivalent stiffness properties for different elements of the grillage were determined to represent the actual bridge. The difficulty of the determination of these equivalent stiffness properties together with the failure of the results to give the actual distribution of loads indicate that the application of this method for the analysis of skew composite steel-concrete bridges is inappropriate. White et al. (1957) studied the effect of diaphragms on the load distribution characteristics of composite bridges. They found that about 80% of the test load was transferred through the deck slab and only about 20% was carried by the diaphragms.

Livesly (1964) used a simplified modelling of bridge superstructures as an open grid structure. The bridge was analyzed as an equivalent open grillage system with equivalent properties in the two directions. Carpenter and Magura (1965) concluded that the presence of intermediate transverse diaphragms has a significant influence on the load distribution over the bridge. They suggested that the effect of diaphragms should be taken into account in the design of the exterior girders and be ignored in the case of interior girders.

Mehrain (1967) studied the application of the finite element method on skew composite steel-concrete bridges. He developed several computer programs and tested them by studying the convergence of the results assuming different finite elements. He analyzed, elastically, skew composite plates under static loads by idealizing the bridge as an assemblage of plane stress, plate bending, and beam elements. Several solutions were
obtained for each element, then the individual elements were assembled to form the complete bridge. Thus, he obtained the solution for displacements and internal stresses. Results for deflections showed good agreement with experimental results obtained from tests on several bridge models. He showed that his computer programs approximated the true behaviours of skew composite bridges more closely than other methods, based on the effective flange width concept or equivalent orthotropic plate.

Gustafson and Wright (1968) analyzed two simply-supported skew composite steel-concrete bridges in order to account for the effect of the girder eccentricity, with respect to the slab, in the finite element analysis of skew composite steel-concrete bridges. They developed a finite element matrix method for eccentrically stiffened plates and then used the method to analyze skew composite bridges. They showed that the intermediate diaphragms enhance the distribution of wheel loads when applied to the interior girders. However, diaphragms do not affect the distribution of loads when applied to the exterior girders.

Powell and Bouwkamp (1969) and Powell and Buckle (1970) developed several computer programs for the analysis of simply-supported composite steel-concrete bridges. In these computer programs the bridge was idealized using different methods such as: equivalent grid; equivalent anisotropic plate; ribbed plate; and, equivalent isolated girder. Comparing the results from different programs they concluded that:

"...the grid idealization underestimates the transverse flexural deck stiffness in
the case of skew composite steel-concrete bridges.

2- The equivalent grid, equivalent plate, and ribbed plate idealizations give almost the same results.

3- The isolated girder idealization leads to results for design bending moments which are far from results from the other methods.

Descarto et al. (1979) analyzed a total of 120 simply-supported prestressed concrete beam-slab bridges. They used the finite element method where the deck slab was modelled using plate elements while the longitudinal girders were modelled using eccentric beam elements. Several parameters were varied as follows: bridge span varied from 10.4 m to 39.0 m; and, bridge width varied from 7.3 m to 21.9 m. Furthermore, they studied the effect of skew angle on the load distribution characteristics of prestressed concrete beam-slab bridges. They concluded that the distribution factor decreases with skew for the interior girders and increases for the exterior girders. In addition, they proposed wheel load distribution factors for skew bridges. The factors are based on the multiplication of the distribution factors of right bridges by corresponding skew correction factors. Fisher and Kostem (1979) concluded that the effectiveness of the intermediate transverse diaphragms and cross-framing on the distribution of live load ranges from 0% to 20% depending on the location of the load as well as the location of the measurements.

Bakht et al. (1979) presented a manual method for calculating the design live load longitudinal moments in superstructures for most of the types of right bridges. The bridge
types that can be analyzed by this method are: slab bridges, beam and slab bridges incorporating both steel and concrete beams, bridges incorporating wooden beams, and slabs on hollow trapezoidal or other such torsionally stiff beams. They used the orthotropic plate theory to develop the method and checked it by using the grillage analogy method. The effect of the various parameters on the transverse distribution of longitudinal moments was discussed. The method presented is useful to determine the design longitudinal moments and to investigate the effects of making changes in the design on load distribution characteristics. They indicated that the transverse distribution of longitudinal moments is dependent on the width of the design vehicle and independent of the number and spacing of axles in the design vehicle. They recommended that their method is applicable to design vehicles other than the Ontario highway bridge design truck provided that the center-to-center distance of the two lines of wheels is 1.8 m. They indicated that different modification factors, accounting for the multipresence of vehicles, can be easily accounted for in the determination of the governing longitudinal moments.

Kennedy (1983) conducted an experimental study on several skew bridge models having a scale of 1 : 6 in order to compare the effect of orthogonal diaphragms to that of oblique diaphragms on the load distribution characteristics of such bridges. He concluded that:

"1- Skew bridges with orthogonal diaphragms have a much better load distribution characteristics than bridges with oblique diaphragms.

2- The orthogonal arrangement of diaphragms enhances the stiffness of skew
bridges and increases their ultimate load-carrying capacity.

3- The orthogonal diaphragms in skew bridges lead to a less severe stress concentration at the obtuse corners.”

Bakht and Jaeger (1983) indicated that the transverse distance of a vehicle from the longitudinal free edge of a bridge has a significant effect on the governing longitudinal moments in the bridge. They showed that the fixed value of the vehicle edge distance, given in the OHBDC and AASHTO simplified methods, is smaller than encountered in most cases and therefore leads to conservative estimates of live-load longitudinal moments. They suggested a parameter to characterize the effect of vehicle edge distance and edge beams. They used this characterizing parameter to develop a graphical method to account for increasing the vehicle edge distance above the value assumed in the AASHTO and OHBDC methods. They confirmed the validity of the method by field test results. However, the method is strictly applicable only to those structures which can be realistically idealized as simple-span orthotropic plates of little or no skew. Furthermore, they indicated that taking account of actual vehicle edge distances is prudent especially for fatigue-related calculations for which the customary practice of assuming vehicles in the most eccentric positions is too far from reality.

Kennedy and Grace (1983) tested two 1/8-scale bridge models to investigate the influence of number of diaphragms, aspect ratio, skew, and cracking of the concrete deck on the transverse load distribution in right continuous composite steel-concrete bridges.
They used the orthotropic plate theory for the analytical study whose results were in good agreement with the experimental results. They proposed a method for the calculation of the longitudinal and transverse rigidities of the bridge for both cracked and uncracked concrete decks and compared results with those obtained using the rigidities suggested by the Ontario Highway Bridge Design Code (OHBDC, 1979). Some of the conclusions drawn were:

"1- The presence of steel I-beam diaphragms, when rigidly connected to the longitudinal girders, enhances significantly the transverse load distribution as well as the effectiveness of the orthotropic plate theory in predicting the elastic response of a continuous composite bridge. However, an increase in the number of diaphragms beyond a certain limit does not significantly improve the transverse load distribution.

2- Transverse cracking of the concrete deck at the intermediate support does not significantly influence the transverse distribution of the longitudinal moments there, nor of deflections and longitudinal and transverse moments at midspans of a continuous composite bridge.

3- A more economical design will result in using the load distribution method, suggested by the Ontario Highway Bridge Design Code (OHBDC, 1979), when the pertinent rigidities are calculated as recommended."

Marx et al. (1986) analyzed a total of 108 of two-lane simply supported skew composite bridges. They used the linear finite element method to determine the maximum bending moment in each girder for all the bridges considered. All bridges had five
longitudinal precast prestressed concrete I-girders and the AASHTO HS20-44 standard truck was applied on the two lanes representing a concentric-truck loading. The parameters considered were span length, girder spacing, and angle of skew. They developed a design method for simply supported skew composite bridges. The method consisted of skew reduction factors to be applied to distribution factors for right composite bridges. However, the method is limited since they did not consider the following significant factors: (i) bridge aspect ratio; (ii) bridge transverse rigidity; (iii) eccentric truck loading; and, (iv) the effect of dead load. Furthermore, the study was limited to bridges that have only end diaphragms and the minimum distance between the edge of the girder and the nearest truck wheels was taken as 0.6 m. Based on the study they concluded the following:

"1- The AASHTO wheel load distribution factor for the interior girders gives results which are between 12% on the unsafe side and 32% too large. For the exterior girders of composite steel-concrete bridges the AASHTO method yields results which are between 30% and 60% too large.

2- The presence of stiff end diaphragms can reduce the maximum bending moments in the interior girders of a skew bridge subjected to truck loads. The exterior girders are not significantly affected by the presence of end diaphragms.

3- The presence of skew results in significant reductions in the girder bending moment for both the exterior and the interior girders. For the interior girder, this reduction is always less than 5% for angles of skew up to 30° and is up to 38% when θ = 60°. The exterior girder is less affected by skew than the interior girder."
4- By changing the length of the slab overhang at the exterior girder from 0.48 m to 0.99 m, the resulting change in the exterior girder bending moment was only 3%, while the interior girders were not affected at all.

5- The maximum girder bending moments are insensitive to moderate changes in the girder torsional stiffness. The effect of an increase in the girder torsional stiffness on the maximum bending moments resulting from truck loads is similar to the effect of increasing the slab thickness.

6- In skew slab-and-girder bridges, the point of maximum bending moment in the exterior and first interior girders can shift with as much as 6% of the span away from midspan. The bending moment envelope diagrams for all girders are noticeably flat in the region of maximum bending moment."

Bakht and Moses (1988) examined the basic assumptions upon which the AASHTO method of lateral load distribution for right composite bridges is based. They concluded that while some assumptions may be defensible, others are not. They presented a semi-graphical method for the analysis of right slab-on-girder bridges. The method was developed by using the same principles on which the current AASHTO method is based and considered the longitudinal flexural rigidity, span length, vehicle edge distance, and edge stiffening. The method was verified by the finite element method and by field testing. However, the distribution factors calculated by using the method was close to the current AASHTO distribution factor. Furthermore, they presented a method to calculate the longitudinal moments and shear for fatigue design.
Nutt et al. (1988) conducted a number of sensitivity studies to determine the effect of varying some bridge geometric and stiffness as well as loading parameters on the wheel load distribution in right composite bridges. It was a very limited parametric study since an average reinforced concrete T-beam bridge was chosen and only one parameter was varied at a time. They used the finite element method and considered both the girder moment and shear for AASHTO standard truck HS20. The parameters considered in the study were: girder spacing, span length, flexural girder stiffness, slab thickness, skew, and continuity. They developed a set of formulas for the calculation of moment and shear distribution factors in right composite steel-concrete bridges. They also presented some correction factors to account for the effect of skew and continuity on the load distribution factors. However, the study was limited to bridges having one or two lanes and ignored several important parameters such as: bridge aspect ratio, bridge transverse rigidity, effect of dead load, and eccentric truck loading cases. Based on this study the following recommendations were drawn:

"1- The simple formula approach applicable to all bridge types and loading cases, given by different codes of practice, is not feasible. The specifications must provide formulas that consider more influencing factors and loading conditions for each type of bridge structures.

2- It is important to account for the effect of skew and continuity on the wheel load distribution of composite bridges. Analytical approach should be used to develop these correction factors and the limits of applicability of these formulas must be clearly defined.
3- Advanced methods of analysis should be used for bridges and loading cases which are not covered by the simple formulas. This will include cases where the geometric and stiffness properties fall outside the specified ranges used to develop the formulas or when unusual live load conditions are applied.

4- The exterior girders are more affected by slab overhangs, curbs, and parapets. Therefore, simple methods are more difficult to develop for exterior girders than interior girders. Curbs and parapets not only have an effect by limiting the exterior wheel position, but also because of their structural participation.

5- There are variations between field test results and finite element analysis results for live load distribution. It is recommended to adjust the stiffness of the secondary elements such as curbs, parapets, interior diaphragms, and bracing, or by assuming partial or full composite behaviour. Further basic research, including physical testing, is needed to understand all aspects of bridge behaviour."

Bakht and Jaeger (1989) showed that the load effects in the girders of a bridge could be related to one dimensionless parameter in terms of girder spacing, angle of skew, and span length. They indicated that using the same girder cross-section for bridges having a given span length but different girder spacings is contrary to practice. They referred that to the flexural rigidity of a girder and the associated portion of the deck slab which are proportional to the girder spacing. They also concluded that the longitudinal girders shears increase in magnitude with increase in the skew angle of the bridge.
Khaleel and Itani (1990) analyzed 112 pseudo continuous bridges using the finite element method. All bridges considered were two-lane bridges with five longitudinal I-girders. They assumed that the deck slab is simply supported at the abutments and continuous over the pier, while the supporting I-girders are discontinuous over the pier. They presented skew reduction factors for the girder moments for AASHTO truck loading, in terms of girder spacing, span length, and angle of skew. However, they did not consider other important parameters such as: bridge aspect ratio, longitudinal and transverse rigidities, eccentric truck loading cases, and the effect of dead load. They concluded that in some cases the AASHTO provisions for wheel load distribution factors for the interior girders in normal slab-and-girder bridges will result in an underestimation in the design moments by 60% while in other cases they will result in an overestimation by 40%. The AASHTO provisions will be more conservative in the case of skew bridges.

Tahini and Frederick (1992) presented a finite element analysis and modelling techniques for I-girder highway bridges. They studied the effect of various factors on the load distribution characteristics of right composite bridges such as: size and spacing of steel girders, presence of cross bracing, concrete slab thickness, span length, and composite and noncomposite behaviour. They concluded that the type of connection at the interface (composite and noncomposite construction) has a negligible effect on the wheel load distribution factors. However, the maximum deflections doubled when noncomposite construction was used. They showed that the most common types of channel diaphragm cross-bracing between longitudinal girders have a negligible effect on
the wheel load distribution factors. They also showed that changing the concrete slab thickness or doubling the size of the girders has a small effect on the wheel load distribution factors. Furthermore, they proposed a new formula to predict the wheel load distribution factors in right I-girder bridges as a function of girder spacing and span length. Results of using the formula showed good agreement when compared to the current AASHTO load distribution factors.

Bishara at al. (1993) used the finite element method and presented wheel load distribution factor expressions for both the interior and exterior girders of simply supported skew composite bridges of medium span length. The expressions were presented for two-lane, three-lane, and four-lane bridges under AASHTO HS20-44 standard truck. The expressions ignored several important factors such as: bridge aspect ratio, transverse and longitudinal rigidity, eccentric truck loading cases, and the effect of dead load. In addition, they concluded that the current AASHTO distribution factors are up to 85% conservative for the interior girders of right bridges, up to 70% conservative for the exterior girders of right bridges, up to 205% conservative for exterior girders of skew bridges, and up to 235% conservative for the exterior girders of skew bridges. Furthermore, they indicated that the interaction between cross-frames and longitudinal girders helps in the distribution of wheel loads to the stringers.

Tiedeman et al. (1993) tested one 0.4-scale right laboratory bridge model in order to investigate the validity of the current AASHTO method for the calculation of the shear
and the moment in the longitudinal girders of right composite steel-concrete bridges. They used the finite element method and compared the results to those obtained from experiments. They concluded that:

"1- The finite element analysis of the test bridge gave reactions and stresses that correlated very well with those from the experiment. The slight difference (0° to 11°) likely resulted from the rigid links that prevented the girder elements from distorting out of plane relative to the deck slab elements.

2- The AASHTO simple beam model for wheel-load distribution in conjunction with single girder analysis is a reasonably good method when the truck loads are applied directly over a pier. Therefore, the method can be used for reactions.

3- The AASHTO method is overly conservative when the load is away from the piers. The method overestimated the stresses in the girders by up to 308% and therefore it should not be used for the evaluation of the girder moments."

2-3 Codes of Practice for Load Distribution in Composite Bridges

The American Association of State Highway and Transportation Officials (AASHTO, 1994) provides the design engineer with a simplified method of calculating the girder moments and allows for using computer analysis for the design of bridges. The simplified method given by the AASHTO depends on the load distribution factor concept and have been used since 1931. The method is limited to right simply supported bridges and considers only the center to center girder spacing. The method is not applicable beyond a maximum specified girder spacing. The AASHTO specifies that the exterior
girder should be designed to sustain live loads at least equal to the load applied to the interior girders. Nutt et al. (1988) presented a historical background for the AASHTO method which can be summarized as follows: "(i) in the first edition of the AASHTO specifications (AASHTO, 1931) and in several following editions a value for the wheel load distribution factor was used depending on the type of decking in the bridge; (ii) in 1957 the values of the wheel load distribution factors were modified to account for the type of the supporting longitudinal girders; and, (iii) in the new editions of the AASHTO specifications the values of the factors were modified for some bridge types and new provisions were added for other bridge types." Furthermore, Nutt et al. (1988) summarized the weaknesses in the AASHTO method as follows:

"1- The provisions given by the AASHTO, which vary with bridge type, are in some cases verified by research while in other cases resulted by an engineering judgment. This caused some inconsistencies in the provisions for various bridge types.

2- The provisions consider several parameters for some bridge types in addition to girder spacing. However, these other parameters vary from one bridge type to the other.

3- The values of the load distribution factors have not been changed to account for the lane width which has been changed a number of times over the years.

4- The load distribution factors have not been changed to account for the provisions for the reduction in load intensity for multiple lane loading which were added in 1941."
The Ontario Highway Bridge Design Code (OHBDC, 1992) presents a method of determining the load distribution factors for composite I-girder bridges. Although the method is simple but it requires more effort than the method given by the American Association of State Highway and Transportation Officials (AASHTO, 1994). The method considers the flexural and torsional rigidities of the bridge in addition to the girder spacing. The method can be summarized in the calculation of a flexural parameter, \( \theta \), and a torsional parameter, \( \alpha \), and then by using design charts the engineer can determine the appropriate moment distribution factor and shear distribution factor for both the exterior and the interior girders. The OHBDC method is based on the idealization of the bridge to an equivalent orthotropic plate and then calculating the properties of the equivalent plate using prescribed methods. The OHBDC specifies reduction factors to account for the case of more than two traffic lanes being occupied at the same time. However, the OHBDC method is limited to the following bridge properties: (i) angle of skew < 20\(^\circ\); (ii) slight horizontal curvatures; (iii) certain maximum span length; (iv) certain maximum girder spacing; and, (v) no transverse diaphragms.

2-4 Load Distribution Characteristics of Composite Bridges at the Ultimate Limit State

Due to the rapid increase in the axle truck load in the last few decades, a very large number of overweight permits were issued in the United States of America. The problems that may result in overloading existing composite bridges were summarized by Soliman (1992) as follows: "(i) longitudinal and transverse cracking of the concrete deck
slab, (ii) spalling of the concrete cover; (iii) corrosion of the steel reinforcement; and, (iv) the cracked concrete deck may initiate the corrosion of the main steel girders under the slab due to the seepage of the deicing salt-laden water during the winter months." Hall and Kostem (1980) reported that in 1966 over 700,000 overweight permits were issued in forty states. This rose to 1,250,000 permits in 1975. As a result, numerous research efforts were directed to understand the overloading behaviour of composite bridges under unexpected heavy truck axle loads. These research efforts will be presented as follows:

Pillai and Lash (1967) modelled the actual bridge as an open grid. Each grid element was given flexural and torsional capacities. The results obtained were in fair agreement with the experimental results. In 1969, Reddy and Hendry used the virtual work approach to predict the collapse loads of simply supported right composite steel-concrete bridges. However, they did not consider the effect of transverse intermediate diaphragms.

Botzler and Colville (1979) used the finite difference method to predict the collapse load deformation response of continuous composite bridges. Their efforts were a continuation of the linear analysis of composite bridge models using the same technique. In addition, they used the finite element method to predict the ultimate load capacities of composite bridges. This was done using the first order theory; i.e. the small deflection theory and material non-linearities.

Kostem (1984) used the finite element method to analyze both composite steel-
concrete and concrete-concrete bridges. He considered the non-linear material behaviour and used an incremental iterative technique. The post-elastic response of the eccentrically stiffened plate was described by introducing a layered beam plate model. The concrete deck slab material was modelled as an elastic-perfectly plastic material. He developed several computer programs for the design of various types of bridges.

Cheung et al. (1987) investigated the behaviour and live load distribution characteristics of bridges beyond the working stress range. They tested a 1/4-scale right composite steel beam concrete deck bridge model for various loading conditions. They used the finite element method for the theoretical analysis and concluded that:

"1- The finite element method can be used efficiently and accurately in predicting the stress distribution of composite slab-on-girder bridges. Nonlinear behaviour and load redistribution phenomena are adequately modelled by the finite element method.

2- A considerable reduction of load distribution factors is observed between linear elastic stage and first yielding. This reduction is at least equal to the shape factor of the girder section. It is recommended that the plastic section modulus be used instead of elastic section modulus on proportioning steel girders at the ultimate limit state.

3- Load redistribution in bridge girders is insignificant before the formation of the first plastic hinge. Therefore, it can be ignored before that state.

4- Due to yielding of the bridge girders after the formation of the first plastic hinge, load redistribution and residual stress effects become important. A further study in this area is recommended to develop a method for considering the effects of various
parameters such as girder sections, bridge geometry, relative stiffness of the bridge deck
to girder, internal bracing, loading pattern, and loading steps. To achieve this, numerical
models are recommended rather than physical models."

Razaqpur and Nofal (1987, 1988, 1990) used the non-linear finite element
technique to analyze composite steel-concrete bridges. They conducted an extensive
parametric study in order to determine the trend of the load distribution characteristics of
composite steel-concrete bridges at the ultimate limit state. They considered several
parameters such as: bridge width, girder spacing, number of girders, loading positions, and
relative rigidity of the slab to that of the girder. However, they did not consider other
important parameters such as: bridge continuity, angle of skew, and bridges with
transverse intermediate diaphragms.

Soliman and Kennedy (1990) tested four right composite steel-concrete laboratory
bridge models in order to study the load distribution characteristics for composite bridges
with different diaphragm connections. They conducted a parametric study on prototype
simply supported and continuous bridges. They studied the effect of several factors on
the load distribution at both the service and at the ultimate limit states. They concluded
that moment-connected diaphragms in composite bridges have a better ability not only to
distribute effectively the service load but also any unforeseen overload. The distribution
factors at the ultimate load are larger in case of moment-connected diaphragms than those
in the case of shear-connected diaphragms. They found that the bridge aspect ratio has
a significant influence on the load distribution factors at the ultimate load. Furthermore, they recommended that there is an optimum number of moment-connected diaphragms for improving the load distribution characteristics of a bridge beyond which no improvement should be expected.

Kennedy and Soliman (1992) presented equations to predict the ultimate load capacity and the required ultimate moments of resistance for the design of a composite steel-concrete bridge. The equations were based on the yield line theory which can be used reliably to predict the ultimate loads of simple-span and continuous-span composite bridges subjected to AASHTO truck loading. Furthermore, they showed that the manner in which the transverse diaphragms are connected to the main longitudinal girders will have a significant influence on the ultimate load-carrying capacity of relatively wide bridges.

Helba and Kennedy (1994) tested five skew composite bridge models and used the finite element method to conduct a thorough study of the linear and non-linear behaviours of prototype skew composite bridges subjected to OHBDC truck loading. They presented results for the moment distribution factors and shear distribution factors at the ultimate limit state. They concluded that: "(i) a redistribution allowance of 15% and 22% could be safely assumed for longitudinal girder moments in simply-supported and continuous two-equal-span skew composite bridges, respectively; and, (ii) the distribution of shear at the supports of skew composite bridges is more critical than that of right bridges. The
maximum shears at the obtuse corner nearest to the load should be carefully checked and taken into account in the design of such bridges."

Holba and Kennedy (1994) conducted an investigation to predict the ultimate collapse loads of continuous skew composite bridges subjected to AASHTO truck loading. They showed that the yield-line method of analysis can be used to reliably predict the ultimate collapse loads of such bridges. They showed that the interaction between the skew angle and bridge aspect ratio can effect changes in the failure patterns. They presented equations for the ultimate load for the design of skew composite bridges and showed that the ultimate load is a function of the longitudinal and transverse moments of resistance for both hogging and sagging bending moments. Furthermore, they conducted a parametric study to investigate the influence of the various parameters on the failure pattern and the minimum collapse loads of skew composite bridges. The parameters considered were: skew angle, aspect ratio, span length, loading conditions, and moments of resistance. Some of the conclusions drawn were:

"1- For eccentric truck loading, the critical crack length is significantly affected by the bridge aspect ratio and to a much lesser extent by skew, while the critical location of the load is influenced significantly by both skew and aspect ratio.

2- For concentric truck loading, the inclination of the positive transverse failure line is a function of the number of loaded lanes as well as of skew."
2-5 Free Vibration of Composite Steel-Concrete Bridges

To study the dynamic response of composite steel-concrete bridges one requires to understand the free vibrational behaviour of such bridges, i.e. to determine the natural frequencies and the corresponding mode shapes. Grace (1986) reported that the lowest natural frequency of an average deck slab may vary between 2 Hz for long spans to 17 Hz for short spans. He showed that on the other hand, the frequency of vibration of a vehicle on its tires and suspension spring system usually falls in the range of 2-3.5 Hz. Therefore, an accurate estimate for the natural frequencies is required in order to avoid the state of resonance under loadings such as earthquake, wind, and truck. An example reported by Grace (1986) is the Tacoma Narrows Bridge which collapsed due to wind effects. The bridge failed in the second mode which was anti-symmetrical in the transverse direction due to weakness in the torsional rigidity of the bridge. The Ontario Highway Bridge Design Code (OHBDC, 1992) specifies the dynamic load allowance as a factor of the first flexural frequency of the bridge. The formulation for the dynamic load allowance in the OHBD code (1992) was based on testing a full-scale bridge. It takes into account the dynamic amplifications due to the interaction between the vehicle and the bridge when their natural frequencies are close. The OHBD code (1992) recommends the use of the flexural beam theory to determine the dynamic load allowance. The use of the flexural beam theory is reliable in the case of right bridges without diaphragms. However, in the case of skew bridges or in the presence of transverse intermediate diaphragms, the use of the beam theory leads to unacceptable errors in the determination of the first flexural frequency and hence in the dynamic load allowance.
Some research efforts have been directed to study the free vibrational behaviour of right composite steel-concrete bridges. However, very little effort has been directed towards the response of skew composite bridges. The research efforts already carried out are as follows:

Inglis (1934) presented some simplified assumptions in which the bridge was treated as a beam. However, he did not show if the results are comparable to the actual behaviour of the bridge or not. It is expected that these assumptions are applicable to narrow bridges with heavy girders and therefore they are useful in the case of railway plate girder bridges. Warburton (1954) studied the vibration of rectangular plates with any combination of boundary conditions. He used Rayleigh's method and presented an approximate technique for such vibrations. Also, he obtained the deflected shape by forming the product of the characteristic functions of two beams. In the same year, Biggs and Suer presented results of field measurements of dynamic deflections of simple span bridges due to the passage of heavy vehicles. They showed that the maximum observed amplitudes of vibration for the test bridges varied from 18 to 40% of the maximum static deflection. They indicated that these amplitudes are greater than that would be produced by a theoretical smoothly running load. They concluded that the most important factors that influence the vibration of the bridge are the dynamic characteristics of the vehicle itself and the condition of the roadway surface on the approaches.

Laura and Smith (1968) solved the problem of vibration of a fixed and simply supported plate in which stiffnesses were provided in two perpendicular directions. They
solved a set of a homogeneous equations and hence the frequencies of the plate were obtained. Although their solution satisfied the boundary conditions, it did not satisfy the governing differential equations of the system. Derby and Calcaterra (1971) compared the dynamic response of various bridge designs to individual heavy vehicle crossings and presented an evaluation of several techniques for reducing the resulting dynamic motions of long-span bridges. The types of bridges considered were concrete and orthotropic composite deck constructions using both low-alloy structural steel and high-strength steel. They concluded that the large dynamic motions associated with bridges employing higher strength steels can be reduced to the levels associated with bridges employing low-alloy steels by means of appropriate treatments. Furthermore, they presented a technique for the selection of vibration control treatments. They indicated that these treatments reduce the midspan dynamic motions from 20 to 40% when compared to the untreated bridges.

Eka (1971) developed a numerical method and a computer program for the analysis of skew composite steel-concrete bridges under static and dynamic loads. He used the finite element and the finite difference techniques in the solution of the problem. The bridge was idealized as an assembly of finite flat plate and beam elements. The vehicle was idealized as a single axle spring load having one wheel. Experimental results from testing three composite steel-concrete bridge models compared favourably with theoretical predictions. He concluded that the fundamental frequency of a skew composite bridge increases as the angle of skew increases, but the second and the third frequencies decrease with increasing skew up to about 70° beyond which they suddenly start to
increase. Ghobarah and Tso (1974) carried out an analysis for skew continuous composite bridge which suffered damage at the intermediate support as a result of the San Fernando earthquake. They showed that due to the torsional component of the bridge deck vibration, the outer columns were subjected to seismic forces three times larger than those acting on the inner columns of the center support. They concluded that: "(i) due to the skewness of the supports, coupled flexural and torsional vibration will be induced on skewed highway bridges. The natural frequencies of these vibrations are generally fairly high as a result of the typical dimensions of this type of bridges; (ii) the torsional response of the resulting coupled vibrations needs special attention in design; and, (iii) the dynamic characteristics of the bridge can be obtained by treating the supports as rigid. The flexibility of the interior supports appears to have only a minor influence on the frequencies and mode shapes of the bridge. Treating the supports as rigid will reduce the computational effort of the analysis."

Moss and Carr (1982) presented field measurements of the vibrational frequencies of three bridges with prestressed concrete beams and in situ concrete deck slabs. In each bridge, the beams were simply supported over each span while the composite deck is continuous over three or four spans. Also, they carried out a theoretical analysis and compared the results with results from field measurements. They presented values for the damping determined from the tests. They concluded that in the calculation of the vibrational frequencies for composite beam and slab bridges, more appropriate values for the moduli of elasticity for the concrete should be used instead of the values given in the
design codes.

Grace and Kennedy (1984, 1985) tested two continuous right composite steel-concrete bridge models in order to investigate the dynamic response of such bridges. They explored the analogy between a continuous prestressed composite bridge and a simply supported orthotropic plate. The effect of different parameters such as the bridge aspect ratio and the rigidity ratio on the natural frequencies were examined using the orthotropic plate theory and the results were compared to those from the beam theory. They also conducted fatigue tests to examine the behaviour of the longitudinal steel beams, shear connectors, and the prestressed wires in the concrete deck at the intermediate support. A computer program was developed for the formulation of the frequency equations from which the associated natural frequencies were found. They verified the results by using the finite element method. Furthermore, they proposed a series solution for the free vibration problem of skew waffle slabs using the orthotropic plate theory. They concluded that:

"1- The enhancement in the natural frequencies of continuous composite steel-concrete bridges can be achieved by reducing the cracks in the interior support moment region by prestressing the slab in that region of the bridge.

2- Prestressing the slab in the interior support moment area reduces the fatigue stress range and therefore both the bridge fatigue life and the ultimate load carrying capacity increase.

3- A fatigue-cracked continuous composite steel-concrete bridge can regain most
of its stiffness and load carrying capacity when properly repaired by welding plates, one to the exterior side of the web and one on the bottom flange of the longitudinal fatigue cracked steel beams.

4- The series solution of a skew orthotropic plate gives results which compared well to the experimental results verifying the assumptions and the mathematical derivation used.

5- Skew waffle slab bridges have higher natural frequencies than solid slab skew bridges."
CHAPTER III

THEORETICAL ANALYSES

3.1 General

With the rapid development and improvement of digital computers during the last few decades the use of the finite element method became common practice in almost all structural engineering problems. The finite element method is the most powerful and flexible method capable of providing a complete description of the structural response within the elastic and post-elastic loading stages as well as a detailed theoretical simulation of both steel and reinforced concrete structural components. The method is suitable for the analysis of skew composite steel-concrete bridges since one of its advantages is to deal with problems which have arbitrary arrangement of structural elements, material properties, and boundary conditions. In this chapter, a description of the finite element method is presented. The theoretical modelling of the different components of composite bridges in the linear and non-linear static analyses as well as in the dynamic analysis is explained. These components include the concrete deck slab, the steel reinforcement, the longitudinal steel girders, the transverse intermediate steel diaphragms, and the stud shear connectors. A general description of the finite element
program 'ABAQUS' (Hibbitt et al., 1989) which was used throughout this analysis is presented. The objectives of the theoretical analyses were:

1- To study the behaviour of simply supported and continuous skew composite steel-concrete bridges in both the elastic and post-elastic loading stages.

2- To predict the ultimate load carrying capacity of skew composite steel-concrete bridges.

3- To study the natural frequencies and the associated mode shapes of simply supported and continuous skew composite steel-concrete bridges.

In addition, the finite element analysis was employed to conduct seven extensive parametric studies on prototype composite bridges included more than 2650 bridge cases. The seven parametric studies were as follows:

1- Elastic shear distribution factors of simply supported skew composite steel-concrete bridges.

2- Elastic moment distribution factors of simply supported skew composite steel-concrete bridges.

3- Elastic span and support moment distribution factors for continuous skew composite steel-concrete bridges.

4- Elastic shear and reaction distribution factors for continuous skew composite steel-concrete bridges.

5- Span and support moment distribution factors for continuous skew composite steel-concrete bridges at the ultimate limit state.

6- Shear distribution factors for continuous skew composite steel-concrete bridges
at the ultimate limit state.

7- Dynamic response of simply supported and continuous skew composite steel-concrete bridges.

3.2 Finite Element Static Analysis

The first step in a finite element analysis is to discretize the structure into a set or different sets of structural components. Each set has a similar geometric pattern and physical assumption. Each finite element is interconnected with the adjacent elements by nodal points. Acting at each nodal point are nodal forces and the node is subjected to displacements. For each element a standard set of simultaneous equations can be obtained to relate these physical quantities. Assembling these elements to form the whole structure is equivalent physically to superimposing these element equations mathematically. The result is a large set of simultaneous equations which are suited for solution by computer. Applying the loading and boundary conditions for the structural problem the assembled set of equations can be solved and the unknown parameters found. Substituting these values back to each element formulation provides the displacements and stress distribution everywhere within each element.

The basic formulation of the finite element method relating the generalized displacement and the loads in a matrix form can be expressed as:

\[
[K] [U] = [P] \tag{3.1}
\]
where:

\[ [K] = \text{structure stiffness matrix}; \]
\[ [U] = \text{displacement vector at the nodes}; \text{ and} \]
\[ [P] = \text{applied loads vector at the nodes}. \]

The structure stiffness matrix \([K]\) is the assemblage of the element stiffness matrices \([K]_i\), given by:

\[
[K] = \Sigma [K]_i \tag{3.2}
\]

where

\[ [K]_i = \text{element } i \text{ stiffness matrix, defined as:} \]

\[
[K]_i = \int_{\nu} [B]^T [E] [B] \, d\nu \tag{3.3}
\]

In equ. (3.3) the integration is performed over the volume \(\nu\);

\([B] = \text{strain displacement matrix}; \text{ and } [E] = \text{elasticity matrix} \).

In a linear problem, loads are applied to a model and the response is obtained directly. In the non-linear finite element analysis several linear steps are taken to solve the problem. There are many solution procedures to solve non-linear problems. One of these solutions is the well known Newton's method which is a numerical technique for solving the non-linear equilibrium equations.

By solving a series of linear problems the non-linear solution of the problem is iteratively obtained. Let \([P_0]\) be the initial load, \([U_0]\) is the initial displacement, and \([K_0]\)
is the tangent stiffness at \([U_n]\) and \([P_n]\). The necessary load for the \(i^{th}\) cycle of the iteration process is determined by:

\[
[P_i] = [P] - [P_{i-1}]
\]  

(3.4)

where \([P]\) is the total load to be applied and \([P_{i-1}]\) is the load equilibrated after the previous step.

During the \(i^{th}\) step, an increment to the displacement is computed by the relation:

\[
[K_i] [\Delta U_i] = [P_i]
\]  

(3.5)

and, after the \(i^{th}\) iteration the total displacement is computed from

\[
[U] = [U_0] + \sum [\Delta U_i]
\]  

(3.6)

This procedure is repeated until the increment of displacements or the unbalanced forces become null or sufficiently close to null.

There are two methods used to compute the stiffness matrix. The first method is the tangent stiffness matrix at the end of the previous step which is the slope of the \([P]\)-\([U]\) curve at the point \([P_{i-1}], [U_{i-1}]\). The second method is a modified iterative technique which utilizes only the initial stiffness \([K_0]\).

Increments should be small to ensure correct modelling of history-dependent effects. The choice of increment size is a matter of computational efficiency. If the increments are too large, more iterations will be required. In Newton's method a large increment can prevent any solution from being obtained because the initial state is far away from the equilibrium state.
3.3 The ABAQUS Computer Program

ABAQUS is a multi-purpose finite element program developed and distributed by Electric Power Research Institute (EPRI) under the name ABAQUS-EPGEN. The program is used world-wide to estimate structural responses of power plant structures due to accident and operating stresses. However, it is used on a much wider scale to solve all types of structural problems.

Since ABAQUS is a batch program, the objective is to assemble a data deck which describes a problem so that ABAQUS can provide an analysis. Data decks for complex simulations can be large but can be managed without too much difficulty by using the convenience features built into the program's input structure.

A data deck for ABAQUS contains model data and history data. Model data define a finite element model: the elements, nodes, element properties, material definitions, nodal constraints, and any data that specify the model itself. History data define what happens to the model, the sequence of events or loadings for which the model's response is sought. In ABAQUS this history is divided by the user into a sequence of steps. Each step is a period of response of a particular type such as static loading or dynamic response. The definition of a step includes the procedure type, the control parameters for time integration for the non-linear solution procedures, loading, and output requests. What constitutes a step is a matter of choice on the part of the user. For example, a static load might be applied in one step or, if more detailed output is required
at higher load levels, the same analysis might be broken into two steps so that the output requests can be changed. A key aspect of the way ABAQUS provides simulations is that the steps define a sequence of events that follow one another in the sense that the state of the model at the end of the step provides the initial conditions for the start of the next step.

All data definitions in ABAQUS are accomplished with option blocks which are sets of data describing a part of the problem definition. The user chooses these options that are relevant for a particular application. Each option is introduced by a keyword card. If the option requires data cards they follow the keyword.

One of the most useful features of the ABAQUS data definition method is the availability of sets. A set can be a set of nodes or a set of elements. The user provides a name for each set. That name then provides a means of referencing all of the members of the set. Sets are the basic reference throughout ABAQUS and the use of sets is recommended. Choosing meaningful set names makes it simple to identify which data belong to which part of the model.

The non-linear procedures in ABAQUS offer two approaches to obtain a convergent solution at minimum cost. Direct user control of increment size is one choice, whereby the user specifies the incrementation scheme. This is particularly useful in repetitive analyses where the user has a very good feel for the problem. Automatic
control is an alternate choice: the user defines a period of history (a step in the
terminology of the program) and at the same time specifies certain tolerances or error
measures. ABAQUS then automatically selects the increments to model the step. This
approach is usually more efficient because the user cannot predict the response ahead of
time. Automatic control in some cases increases the cost of the analysis over the cost
when the response is essentially predictable and direct user specification of increments is
adopted, but automatic control can save enormously over repeated user controlled running
of problem to obtain satisfactory incrementation scheme. In addition, automatic control
is extremely valuable in cases where the time or load increment varies widely through the
step. Ultimately, automatic control allows nonlinear problems to be run with confidence
without extensive experience with the problem.

3.4 Finite Element Modelling of Skew Composite Steel-Concrete Bridges

A convergence study was conducted to choose the finite element mesh. The finite
element mesh is usually chosen based on pilot runs and is a compromise between
economy and accuracy. A three-dimensional finite element analysis was used to model
skew composite steel-concrete bridges. To achieve this it is important first to divide the
bridge into several components. In the case of composite bridge there are two main
components which are the steel grid and the reinforced concrete deck slab. The steel grid
consists of: (i) the longitudinal girders which are supported on two abutments in case of
simply supported bridges and are supported on two abutments and a pier in the case of
continuous two-span bridges; (ii) end steel diaphragms; and, (iii) a number of transverse
intermediate steel diaphragms are used to enhance the load distribution characteristics of these bridges. These diaphragms may be moment-connected or shear-connected to the longitudinal girders. The reinforced concrete deck slab is supported by the steel grid. Stud shear connectors welded to the top flanges of the longitudinal girders are used to achieve the composite action between the longitudinal girders and the reinforced concrete deck slab. The ABAQUS has many types of elements in its element library. Several element types were tested in the pilot runs. In this section, element types used for different bridge components as well as material modelling in both the elastic and post-elastic loading stages are presented. It should be noted that the model presented herein was verified and substantiated by numerous loading cases on nine skew composite steel-concrete laboratory bridge models. Six single span bridge models were simply supported whereas three were continuous with two-unequal-spans. The details of the experimental study are presented in chapter IV.

3.4.1 Modelling of Deck Slab

A mesh of shell elements were used to model the reinforced concrete deck slab. The ABAQUS has many types of shell elements in its library. The one chosen was the four-node element named S4R in the ABAQUS library. The element was rectangular for right bridges whereas it was parallelogram in shape for skew bridges. The element had six degrees of freedom at each node, namely three displacements (U1, U2, U3) and three rotations (Φ1, Φ2, Φ3). Detailed description of the shell element S4R is shown in Figure 3.1.
3.4.2 Modelling of Longitudinal Girders

A three dimensional two-node beam element, named B31H in the ABAQUS library, was used to model the longitudinal steel girders. The element had two nodes with linear displacement function and six degrees of freedom at each node, these being three displacements (U1, U2, U3) and three rotations (Φ1, Φ2, Φ3). Detailed description of the beam element B31H is shown in Figure 3.2.

3.4.3 Modelling of Transverse Diaphragms and Their Connection to the Longitudinal Girders

The transverse intermediate steel diaphragms were modelled using the same beam element B31H used for the longitudinal girders. The multi-point constraints option (MPC) given by the ABAQUS was used to model the connection between the transverse diaphragms and the longitudinal girders. The experimental study, which will be presented in detail in chapter IV, included testing simply supported bridge models having different diaphragms-longitudinal beams types of connection. One of the bridge models had diaphragms shear-connected to the longitudinal beams by means of bolts between the web of the diaphragms and the web of the longitudinal beams. It was found that the diaphragm in this case is best modeled as a simply supported member transferring shear only. This was done by using multi-point constraint type 9 in the ABAQUS library. In another bridge model diaphragms were moment-connected to the longitudinal beams by welding both the flanges and the web of the diaphragms to the flanges and the web of the longitudinal beams. In this case the diaphragm was modeled as a fixed member
transferring both shear and moment. This was done by using multi-point constraint type 7 in the ABAQUS library.

3.4.4 Boundary Conditions

Two different nodal constraints were used in the analysis:

1- Boundary constraints: in which the vertical displacements were restricted for all nodes located along the support lines to model the two simple supports at the ends of the model as well as the intermediate pier support in the case of continuous bridges models.

2- Multi-point constraints (MPC): this option allows constraints between different degrees of freedom. It was used between the shell element nodes of the reinforced concrete deck slab and the beam element nodes of the longitudinal steel girders. The multi-point constraint type 7 in the ABAQUS library ensures full interaction between the reinforced concrete deck slab and the longitudinal steel girders, so it was used to model the presence of stud shear connectors. The definition of the MPC option is shown in Figure 3.3.

3.4.5 Reinforced Concrete Modelling

This section illustrates reinforced concrete modelling in the ABAQUS including the modelling of the plain concrete (CONCRETE option) and the modelling of the steel reinforcement (REBAR option). The bond and dowel interaction between the reinforcing steel and the concrete is modelled in the ABAQUS by adding some tension stiffening to
the plain concrete model. This is done by simulating the load transfer across cracks through the rebar.

3.4.5.1 Plain Concrete Modelling

The plain concrete model in ABAQUS program can be accomplished by using the CONCRETE option which is used to define the properties of plain concrete in the post-elastic range. Cracking is assumed to be the most important aspect of the behaviour of plain concrete and it dominates the modelling. Cracking is assumed to occur when the stresses reach a failure surface. This failure surface is taken to be a simple Coulomb line.

The plain concrete model adopted by ABAQUS program includes the material anisotropy resulting from cracking. The occurrence of cracks affects the state of stress and the material stiffness within the cracked elements. The material constitutive relations are calculated independently at each integration point of the finite element model.

When the principal stress components are dominantly compressive the response of the concrete is modelled by an elastic-plastic theory using a simple form of yield surface. When concrete is loaded in compression it exhibits initially elastic response. As the stress is increased some non-recoverable straining occurs and the response of the material softens. An ultimate stress is reached after which the material softens until it can no longer carry any stress. If the load is removed at some point after inelastic straining has occurred the unloading response is softer than the initial elastic response; this effect
is ignored in the model. When a uniaxial specimen is loaded in tension it responds elastically until at a stress range between 7\(^\circ\) and 10\(^\circ\) of the ultimate compressive stress, beyond which cracks form quickly. The cracking and compression responses of concrete that are incorporated in the model are illustrated by the uniaxial response of a specimen shown in Figure 3.4 (Ahmad, 1981). The surfaces used are shown in Figure 3.5 (Ahmad, 1981).

3.4.5.2 Reinforcement Modelling

The reinforcement model in ABAQUS can be accomplished by using the REBAR option which is used to define the properties and locations of reinforcement bars in the concrete elements. Rebars are defined singly or embedded in oriented surfaces that use a one-dimensional strain theory. These elements are superposed on the mesh of plain concrete elements and are used with standard metal plasticity models that describe the behaviour of the reinforcement material. Effects associated with the rebar/concrete interface, such as bond slip and dowel action, are modelled by introducing some tension stiffening to simulate load transfer across cracks through the rebar. The ABAQUS has the "TENSION STIFFENING" option which is an indirect technique for modelling the energy release with the propagation of cracks across the volume occupied by the material and the concrete/rebar interaction. The modelling of the tension stiffening is based on the work of Hegemier (1979). Figure 3.6 shows the response of the reinforced concrete in tension where the influence of tension stiffening is illustrated.
3.4.6 Modelling of Steel

The Von Mises yield criterion for isotropic metals behaviour is used in the metal plasticity models in the ABAQUS. The simple plasticity model, perfect plasticity, was used in this analysis where the yield surface acts as failure surface with no hardening parameters. The elastic perfectly plastic stress-strain relationship used in this analysis is shown in Figure 3.7.

3.5 Non-Linearity Control

In a linear problem, loads are applied to a model and the response is obtained directly. This may be not possible in a non-linear problem where the load must be applied gradually. In the non-linear analysis it is required to solve the non-linear equilibrium equations. The ABAQUS uses the Newton's method for this purpose. The solution is obtained as a series of increments with iterations within each increment to obtain equilibrium. Increments should be small to ensure correct modelling of history-dependent effects. The choice of increment size is a matter of computational efficiency. If the increments are too large, more iterations will be required. In the ABAQUS an automatic incrementation scheme is used to select the increment sizes based on these considerations.

In the ABAQUS, a force tolerance (PTOL) and a moment tolerance (MTOL) are provided to control the non-linear solution accuracy at each increment. The moment tolerance (MTOL) is equal to the force tolerance (PTOL) multiplied by the element
length. The moment tolerance (MTOL) is needed with elements having rotational degrees of freedom such as shell and beam elements. The accuracy and efficiency of the non-linear solution are significantly affected by the choice of the values of these tolerances at the beginning of the non-linear analysis. Very tight tolerances will require excessive number of iterations whereas large tolerances will give inaccurate results because the equilibrium equations will not be satisfied. The ABAQUS recommends values for the force tolerance (PTOL) and the moment tolerance (MTOL) equal to 0.1% to 1% of the actual loads. If the number of iterations exceeds the maximum allowed, the increment size is reduced. If this results in a smaller increment than was specified as a minimum in the input, the run is terminated. The user may define a maximum value of the load or specify a certain amount of displacement beyond which the solution is not of interest. If neither of the finishing conditions described above is specified, the analysis will continue for the number of increments defined at the beginning of the step.

For a non-linear analysis, the user assigns a time scale during the loading step to determine the variation of loads and other prescribed parameters through the step. The ABAQUS prints out a load proportionality factor, λ, at each increment during each step. The current magnitude for the load component can be defined as:

$$P_{\text{total}} = P_o + \lambda (P_{\text{ref}} - P_o)$$

(3.7)

where: $P_o =$ the magnitude of the load component at the start of the step; $\lambda =$ the load proportionality factor; and, $P_{\text{ref}} =$ the magnitude of the load component as defined by the user for the step.
3.6 Verification of the Finite Element Model

In order to verify and substantiate both the linear and the non-linear finite element models presented above, an extensive experimental study was conducted. The experimental study, which will be presented in detail in chapter IV, included testing nine skew composite steel-concrete laboratory bridge models. Six bridge models were simply supported whereas three were continuous with two unequal spans. The simply supported bridge models of single span had different skew angles and different types of diaphragm connections. The continuous bridge models had different skew angles and different spans ratio (long span length/short span length). The finite element model presented above was employed to analyze the nine bridge models tested. All loading cases applied to the bridge models in the laboratory were modelled theoretically using the finite element method. This theoretical effort included the following:

1- Elastic analysis of the steel grids alone before casting the concrete deck slab. The steel grids of the six simply supported bridge models were subjected to a total of 36 loading cases whereas the steel grids of the three continuous bridge models were subjected to a total of 30 loading cases. The loading patterns included either a single concentrated load applied at the mid span of an exterior or an interior beam or simulated truck loads applied eccentrically or concentrically to the steel grid.

2- Elastic analysis of the bridge models including 42 loading cases for the simply supported bridge models and 36 loading cases for the continuous bridge models. The same loading patterns described above for the steel grids were applied to the bridge models.
3- Ultimate load tests for the bridge models using an eccentric simulated truck load for the six simply supported bridge models and one of the three continuous bridge models and using a concentric simulated truck load for the other two continuous bridge models. The incremental loading approach was used up to failure. The last loading increment is the one at which the equilibrium cannot be achieved and the model is considered to be unable to carry any more load, and thus reaching its loading capacity. Figures 3.8 to 3.16 show the finite element meshes used to analyze the nine bridge models tested. In order to ensure that the development of plasticity and failure through the thickness of the concrete are adequately modelled, nine integration points were used through the thickness of the concrete deck slab.

3.7 Parametric Studies

The parametric studies aimed to cover the lack of information provided by North American codes of practice and included the following:

1- Shear distribution factors and moment distribution factors of right and skew simply supported composite steel-concrete bridges in the elastic range of loading. The OHBDC truck was used for this parametric study which included more than 700 bridge cases. The effect of the bridge dead load was also studied. The truck loadings included both eccentric and concentric loading cases. Several parameters were considered such as: angle of skew, bridge aspect ratio, girder spacing, number of lanes, number of girders, the effect of end diaphragms, the effect of transverse intermediate diaphragms, and the flexural rigidity ratio.
2- Shear distribution factors, reaction distribution factors, span moment distribution factors, and support moment distribution factors for right and skew continuous composite steel-concrete bridges in the elastic range of loading. The AASHTO truck was used for this parametric study and was applied both eccentrically and concentrically. The effect of the bridge dead load was also considered. This study included more than 1200 bridge cases. The parameters considered were the same as those considered in the first parametric study. In addition, the spans ratio was considered which can be defined as the long span length over the short span length.

3- Shear distribution factors, reaction distribution factors, span moment distribution factors, and support moment distribution factors for right and skew continuous composite steel-concrete bridges at the ultimate limit state. The AASHTO truck was used for this parametric study. The changes in the load distribution factors as the number of truck loads on the bridge is gradually increased up to the ultimate capacity of the bridge are investigated. The parameters considered were: angle of skew, bridge aspect ratio, girder spacing, spans ratio, number of lanes, and number of girders. In addition, the longitudinal and transverse ultimate moments of resistance for different sections in the bridge as well as the ultimate torsional moment of resistance of the concrete deck slab were considered. The use of the two trucks, i.e. OHBDC and AASHTO, in the parametric studies was necessary in order to provide the design engineer with the tools to design composite bridges subjected to either of the two loading trucks. It is important to note that the difference in the D factors based on the two truck systems is no more than 3%. This was also confirmed earlier by Bakht et al. (1979).
The main objectives of the parametric studies were:

1- To investigate the influence of all major variables affecting the shear, reaction, and moment distribution between the girders of a composite bridge.

2- To generate a database for shear, reaction, and moment distribution factors for composite bridges including more than 2500 bridge cases.

3- To develop empirical formulas for the design of the different girders of simply supported and continuous composite bridges. By using the proposed formulas the design engineer is able to estimate accurately the reactions, shear forces, and bending moments in the elastic stage of loading. The designer will be also able to design composite bridges at the ultimate limit state.

The complete details of the parametric studies are presented in chapter VI.

5.8 Dynamic Analysis of Skew Composite Steel-Concrete Bridges

The natural frequencies and the corresponding mode shapes of the structure are extracted in ABAQUS by eigenvalue techniques. The stiffness determined at the end of the previous step is used as the basis for the extraction. In this manner the vibrations of a structure can be modelled. The FREQUENCY option in the ABAQUS results in including the initial stress and displacements effects. The number of eigenvalues to be extracted has to be defined by the user.

Grace and Kennedy (1985) presented experimental results for the natural frequencies and the corresponding mode shapes of two continuous right composite steel-
concrete bridge models. In the present study, the ABAQUS was used to model the two bridge models. The finite element analysis results showed good agreement with the experimental results, thereby verifying the finite element model.

The verified finite element model was employed to conduct an extensive parametric study on prototype simply supported and continuous composite steel-concrete bridges including more than 150 bridge cases. The main objectives of the parametric study were: (i) to investigate the effect of all major parameters affecting the natural frequencies and the corresponding mode shapes of composite bridges. These parameters are: angle of skew, bridge aspect ratio, the presence of transverse diaphragms, longitudinal and transverse flexural rigidities, and continuity; (ii) to generate a database for the first three natural frequencies and the corresponding mode shapes including more than 100 cases of simply supported and continuous right and skew composite steel-concrete bridges; and, (iii) to develop an empirical formula for the estimation of the first flexural frequency of skew composite steel-concrete bridges. The formula is useful for any simply supported or continuous composite bridge with any skew angle and any arrangement of transverse intermediate diaphragms. Once the design engineer uses the formula to estimate the first flexural frequency the charts given by the Ontario Highway Bridge Design Code (OHBDC, 1992) can be used to determine the dynamic load allowance. The complete details of the parametric study is presented in chapter VI.
CHAPTER IV

EXPERIMENTAL STUDY

4.1 General

An extensive experimental program was undertaken to study the behaviour of simply supported and continuous skew composite steel-concrete bridges. The main objectives of this study was: (i) to verify and substantiate the linear and the non-linear finite element modelling used in the theoretical analysis; (ii) to study the effect of skew on the behaviour of composite bridges at both the elastic and the ultimate limit states; (iii) to study the behaviour of skew composite bridges with different types of diaphragm connections; and, (iv) to study the effect of the span ratio (= long span length / short span length) on the behaviour of two-span continuous skew composite bridges.

Tests were carried out on nine skew composite steel-concrete bridge models. Six single span bridge models were simply supported whereas three bridge models were continuous with two unequal spans. The simply supported bridge models were divided into two groups according to the skew angle and the type of connection between transverse intermediate diaphragms and the longitudinal steel beams. The first group (group I), including bridge models # 1, 2, and 3, was directed to study the effect of angle of skew on the following: (i) deflections of the longitudinal steel beams; (ii) strains of the
longitudinal steel beams; (iii) reactions of the steel beams at the two ends of each bridge model; and, (iv) the ultimate load-carrying capacity of the bridge models. This was done by changing the angle of skew while fixing all other parameters. The second group (group II), comprising of bridge models # 2, 4, 5, and 6, was intended to study the effect of the type of connection between intermediate transverse diaphragms and the longitudinal steel beams on the same measurements mentioned above. This was done by using the same geometry and arrangements of diaphragms and using different types of diaphragm connections.

The three continuous bridge models were divided into two groups according to the angle of skew and the ratio of the long span length to the short span length. The first group (group III), including bridge models # 7 and 8 was directed to study the effect of angle of skew on the behaviour of continuous composite steel-concrete bridges with two unequal spans. This was done by fixing all bridge details for the two models and changing the angle of skew from 0° in case of bridge model # 7 to 45° for bridge model # 8. The second group (group IV), comprising of bridge models # 8 and 9, was intended to study the behaviour of continuous composite steel-concrete bridges with different spans ratio (= long span length / short span length). All parameters were fixed for the two bridge models except that the long span length was increased for bridge model # 9 and therefore the spans ratio was increased from 1.4 in bridge model # 8 to 1.8 in bridge model # 9.
4.2 Description of Test Bridge Models

4.2.1 Simply Supported Bridge Models

Six 1 × 6 simply supported skew composite steel-concrete bridge models were tested. For all the bridge models the skew span was 1778 mm and the width, perpendicular to the traffic direction, was 1067 mm representing two traffic lanes. The longitudinal steel beams, the end diaphragms, and the transverse intermediate diaphragms were S75x8 of grade G40.21-M300W. The details of the simply supported bridge models will be described below.

Bridge model #1 was a simply supported skew composite steel-concrete bridge model with a skew angle of 30°. Four longitudinal steel beams were used so that the lateral spacing of beams (perpendicular to traffic) was 340 mm. The thickness of the deck was 38 mm and with isotropic reinforcement of 0.3% in each face. Stud shear connectors were welded to the top flange of the longitudinal steel beams, with a length of 31.8 mm, to provide interaction between the longitudinal steel beams and the reinforced concrete deck slab. Transverse intermediate diaphragms were used in this bridge model. The diaphragms were moment connected to the longitudinal beams. This was achieved by welding both the web and the flange of the diaphragms to the longitudinal beams. Detailed geometry of bridge model #1 is shown in Figure 4.1.

Bridge model #2 was the same as bridge model #1 except that the angle of skew was increased to 45°. Transverse intermediate diaphragms, moment connected to the longitudinal steel beams, were also used in this bridge model. Detailed geometry of bridge model #2 is shown in Figure 4.2. Bridge model #3 was the same as bridge
models # 1 and 2 except that the angle of skew was increased to 60°. Detailed geometry of bridge model # 3 is shown in Figure 4.3.

Bridge model # 4 was a simply supported skew composite steel-concrete bridge model with a skew angle of 45°. Four longitudinal steel beams were used so that the lateral spacing of beams was 340 mm. The thickness of the deck was 38 mm and with isotropic reinforcement of 0.3% in each face. The composite action between the longitudinal steel beams and the reinforced concrete deck slab was achieved by means of stud shear connectors of 31.8 mm length welded to the top flange of the longitudinal steel beams. In this bridge model, no intermediate transverse diaphragms were used. Only end steel diaphragms were provided at the two ends of the bridge model with no connection with the reinforced concrete deck slab (non-composite). Detailed geometry of bridge model # 4 is shown in Figure 4.4. Bridge model # 5 was the same as bridge model # 4 except that transverse intermediate diaphragms were used in this case shear connected to the longitudinal steel beams. This was done by using bolts between the web of the diaphragms and the web of the longitudinal steel beams. Detailed geometry of bridge model # 5 is shown in Figure 4.5. Bridge model # 6 was the same as bridge model # 5 except that the transverse intermediate diaphragms were moment connected to the longitudinal steel beams and were made composite with the reinforced concrete deck slab. This was done by welding both the web and the flange of the diaphragms to the longitudinal steel beams. In addition, stud shear connectors were welded on the top flange of the steel diaphragms, providing interaction between the diaphragms and the reinforced concrete deck slab. Detailed geometry of bridge model # 6 is shown in Figure 4.6.
4.6 A summary of the dimensions of the six single span simply supported bridge models is shown in Table 4.1.

4.2.2 Continuous Bridge Models

Three 1:5 continuous composite steel-concrete bridge models with two unequal spans were tested. For all the three models the width, perpendicular to traffic direction, was 1219 mm representing two traffic lanes. The longitudinal steel beams were S100x11 whereas the intermediate transverse diaphragms were S75x8. Both the longitudinal steel beams and the transverse intermediate diaphragms were of grade G40.21-M300W. In all the three models both the longitudinal steel beams and the reinforced concrete deck slab were simply supported at the two ends, on two roller supports representing the abutments. Both the beams and the deck slab were continuous over the intermediate pier support. The details of the three continuous bridge models will be described below.

Bridge model # 7 was a continuous right composite steel-concrete bridge model with two unequal spans. The long span length was 2134 mm, and the short span length was 1524 mm, with a span ratio of 1.4. Four longitudinal beams were used so that the spacing of beams was 381 mm. The thickness of the deck was 50 mm with isotropic reinforcement of 0.3% in each face. Stud shear connectors were welded to the top flange of the longitudinal steel beams with a length of 38.1 mm to provide interaction between the longitudinal steel beams and the reinforced concrete deck slab. It should be noted that stud shear connectors were supplied by Nelson Stud Welding Division of TRW Canada Limited, with the required length and diameter for all the bridge models tested. The
length and diameter of connectors were measured for some of the studs used and the variation in the measured dimensions were too small to be of significance. Transverse intermediate diaphragms were used in this bridge model. The diaphragms were moment connected to the longitudinal beams. This was done by welding both the web and the flange of the diaphragms to the web of the longitudinal beams. Detailed geometry of bridge model # 7 is shown in Figure 4.7. Bridge model # 8 was a continuous skew composite steel-concrete bridge model with two unequal spans. The angle of skew was \( \theta = 45^\circ \), with a long skew span length of 2134 mm, and a short skew span length of 1524 mm. The spans ratio was kept at 1.4 which is the same as in bridge model # 7. All other details for bridge model # 8 were the same as those for bridge model # 7. Detailed geometry of bridge model # 8 is shown in Figure 4.8. Bridge model # 9 was the same as bridge model # 8 except that the long span length was increased to 2743 mm and therefore the spans ratio was 1.8. Detailed geometry of bridge model # 9 is shown in Figure 4.9. A summary of the dimensions of the three continuous bridge models is shown in Table 4.2.

4.3 Materials

Local materials were used to prepare the bridge models.

4.3.1 Concrete

4.3.1.1 Cement

High Early Strength Portland cement CSA type 30, manufactured by Canada
Cement Company, was used for the concrete deck slabs of all bridge models. This type of Portland cement accelerates the hydration process resulting in rapid hardening and development of strength providing high strength within a week. Thus, it was possible to test the bridge models shortly after casting.

4.3.1.2 Fine Aggregate

Coarse sand from lake Erie was used as the fine aggregate in the concrete mix. This type of sand was available in the structural laboratory. It was assumed to be free of chemicals, coating of clay, or any other fine materials that may affect the hydration process and bond of cement paste.

4.3.1.3 Coarse Aggregate

Hard clean, crushed durable stone was used as the coarse aggregate in the concrete mix. The narrowest dimension between the sides of the formwork was equal to 32 mm and the concrete cover to the reinforcing wires was 8 mm. Therefore, the maximum size of stone was restricted to 6 mm.

4.3.1.4 Mixing Water

Natural tap water was used in the concrete mix. Water cement ratio of about 0.4 was selected to achieve the required concrete strength of 41 MPa. Mixing was done in an Eerich Counter Current Mixer model EA2(2W). The mixer had a maximum charging capacity of five cu.ft. and was operated electrically. Two batches of concrete mix were
required for each single span simply supported bridge model whereas four batches were required for each continuous bridge model. Three standard cylinders were cast for each bridge model. These cylinders were tested on the same day bridge model was tested to determine the concrete compressive strength. Two methods were used to determine the strain on the concrete. In the first method a compressometer was used to measure the compression under uniaxial load as shown in Figure 4.10. In the second method two strain gauges, of the same type used on the surface of the concrete deck of the bridge models, were installed on the middle of each side of the cylinder as shown in Figure 4.11. The design of the concrete mix is presented in appendix A.1.

4.3.2 Reinforcing Steel

The concrete deck slab for all the bridge models was reinforced by using a mesh of smooth welded wires. The sectional properties of the mesh were as follows: the diameter of the wires was 3.4 mm; the spacing was 152.4 mm center to center; and, the cross-sectional area was 9.07 mm². The load-elongation relationship for the reinforcing wires is shown in Figure 4.12. The yield strength was found to be 228 MPa with a modulus of elasticity of 207 GPa.

4.3.3 Steel Beams

The longitudinal steel beams, end diaphragms, and transverse intermediate diaphragms for the single span simply supported bridge models were S75x8. For the continuous bridge models the longitudinal steel beams were S100x11 whereas the
transverse intermediate diaphragms were S75x8. All the steel sections used were of grade G40.21-M300W. The sectional properties of the steel beams are shown in Table 4.3. Mechanical properties of the steel beams such as the yield strength and the tensile strength were obtained from a test program conducted using a 540-kN Tinus Olsen Universal Testing Machine. Eight tensile coupons were tested, four cut from the web of the supplied steel and four from the flange. The load-strain relationship for the steel beam material are shown in Figure 4.13. Figure 4.14 shows one of the tensile coupons being tested.

4.3.4 Stud Shear Connectors

Stud shear connectors of 9.5 mm diameter supplied by Nelson Stud Welding Division of TRW Canada Ltd. were used to provide interaction between the longitudinal steel beams and the reinforced concrete deck slab for all the bridge models tested. The length of the connectors was 31.8 mm for the single span simply supported bridge models #1 to #6 and it was 38.1 mm for the continuous bridge models #7 to #9. The spacing of the shear connectors was 100 mm for all the bridge models. The shear connectors were welded to the top flange of the longitudinal steel beams by using the Nelson stud equipment. Using the Nelson stud equipment rather than hand welding provided the following advantages, as reported by Grace (1986): (i) the concrete around the connectors is more satisfactorily compacted; (ii) equal shear is provided in all directions; and, (iii) any distortion that might result from hand welding is eliminated.

Nelson Stud Company supplied a complete line of stud welding system and power
source to meet the requirement of this research program. The line consisted of

1- The Nelson NS-20A HD, which is a heavy duty stud welding unit designed specially to weld small diameter studs.

2- Welding gun which is a semi-automatic lightweight pistol-shaped tool

3- Ceramic ferrule which served to shield the arc.

4- Stud shear connectors.

Figure 4.15 shows the Nelson stud welding system. Figure 4.16 shows the stud shear connectors welded to one of the beams.

In order to estimate the load carrying capacity of the stud shear connectors, push-out tests were conducted on six specimens. The push-out specimen consisted of a steel beam of the same steel section used for the bridge models, S100x11. The length of the steel beam was 610 mm with one stud shear connector welded to its flange on both sides. A rectangular concrete slab was attached to the flange of the steel beam on both sides. The slab was 305 mm x 460 mm with a thickness of 50 mm, which is the same thickness as the concrete slab in the continuous bridge models. The slab was reinforced by means of a reinforcing mesh with the same diameter and spacing used in the bridge models. Figure 4.17 shows the formwork and the reinforcement mesh of one of the push-out specimens. The specimen was subjected to compression and the slip between the beam and the slabs was measured by means of dial gauges until failure. Figure 4.18 shows one of the push-out specimens under compression. A typical experimental load-slip curve is presented in appendix A.5. It should be noted that the main objective of the push-out tests was to test the capacity of the shear studs. Earlier studies (Grace and Kennedy,
1985) have shown that shear studs designed according to OHBD (Code, 1992) have ample capacity in resisting shear even at high applied loads inducing fatigue in the main steel beams. The studies have shown that at failure loads including repetitive fatigue inducing loads there was no measurable slip between the deck slab and the main steel beams in composite steel-concrete bridges.

4.4 Formwork

The forms were made from plywood of 19-mm thickness. Two sizes of wood joists were used as stiffeners. The first size was 51 x 51 mm and the second size was 51 x 102 mm. Styrofoam sheets of 75-mm thickness was used for the single span simply supported bridge models #1 to #6 whereas 100-mm thick sheets were used for the continuous bridge models #7 to #9. Since the dimensions and geometry were not the same for all bridge models, nine forms were made one for each bridge model. Figures 4.19 to 4.22 show the formwork for different bridge models.

4.5 The Construction of Bridge Models

1- The longitudinal steel beams, end diaphragms, and transverse intermediate diaphragms were cut from a nominal length of 6.1 m rolled sections using an electrical saw. For the single span simply supported bridge models, #1 to #6, a part of the flange of the diaphragm section was cut so that the diaphragms web becomes flush with the web of the longitudinal beam.

2- The longitudinal steel beams for each bridge model, with stud shear connectors
welded on top of its flanges, together with the steel end diaphragms and the intermediate transverse diaphragms were assembled into a single frame using 6.35 mm fillet welding to ensure full interaction between the different parts of the steel grid. In the case of bridge model #5 the longitudinal steel beams and the transverse intermediate diaphragms were assembled together by means of bolted web connections.

3- The steel frame for each bridge model was then transferred to their supports at the loading frame. The supports consisted of steel rollers. Steel shims were used between the supports and the longitudinal steel beams to ensure full contact between the bridge and its supports. Furthermore, the ends of the steel grid were tied down firmly to the laboratory floor in order to prevent any uplift of the grid during loading. Each grid was loaded elastically using single concentrated loads and simulated truck loads applied at various positions. Deflections of the longitudinal steel beams were measured along longitudinal and transverse sections.

4- The steel grid for each bridge model was then put on a special steel bed with an attached vibrator. The required form was prepared and then painted with grease material, Vitrea oil 150, for easy form release after the setting of the concrete. The welded wire mesh was then placed in the form. The mesh was originally spaced at 150 mm center to center. Two layers were tied together and used in each face so that the spacing was 75 mm center to center. Figures 4.23 and 4.24 show the reinforcement meshes for bridge models # 6 and 9.

5- For each batch the required weights of concrete ingredients were prepared according to the concrete mix design presented in appendix A.1. The gravel, sand, and
cement were mixed first in the concrete mixer for a few minutes and then water was added in stages. During the casting operation, three standard cylinders 152 x 305 mm were prepared. All concrete in the bridge models was compacted by vibrating the bed and by steel rods with special care to ensure that no segregation occurred. The top of the concrete slab was then worked with a wooden screed to obtain a level surface. The top surface of the concrete slab was then given a smooth final finish by hand trowelling.

6- Wet burlap sheets were placed on the concrete surface few hours after casting. Curing continued for three or four days until the concrete surface seemed sufficiently wet. One day after casting, the three concrete cylinders were taken out of the aluminum moulds and placed in a sink of water in the curing room.

7- The bridge model was left to be air-cured for about one day, then the form was released. The bridge model was sprayed by water and left for another three days to gain its full strength. Finally, the bridge model was moved to its final place using an electrical crane.

4.6 Instrumentation

4.6.1. Strain Gauges on the Concrete

In order to measure the strain on the top surface of the reinforced concrete deck slab of the bridge models electrical strain gauges, type N11-FA-30-120-11, were used. The strain gauges had the following properties: (i) the length of the gauge was 30 mm; (ii) the resistance was 119.7 ohms; and, (iii) the gauge factor was 2.12. The concrete surfaces at the locations of the gauges were smoothed using sandpaper. All dust was
removed and the surfaces were then cleaned with acetone. A ratio of 65% of RTC epoxy resin A and 35% RTC epoxy activator B were mixed together and used for installing the strain gauges. After drying of surfaces, they were smoothed again with fine silicon carbide paper and then the gauges were mounted using the epoxy mix. The gauges were left for one day to make sure that the epoxy was fully hardened and then the wires were soldered to the strain gauges using lead wires. The gauges were then covered by gagekote #3 and cured for one day under room temperature. The wires were then connected to the automatic strain indicator. Figure 4.25 to 4.33 show the locations of the strain gauges on the tested bridge models.

4.6.2 Strain Gauges on Steel Beams

In order to measure the strains of the longitudinal steel beams foil strain gauges, of type N11-FA-10-120-11, were used. The gauges had the following properties: (i) the length of the gauge was 10 mm; (ii) the average resistance was 119.8 ohms; and, (iii) the gauge factor was 2.12. For the installation of the strain gauges the following steps were applied: (i) the surface of the steel was smoothed using sandpaper and then was cleaned with acetone; (ii) a conditioner was applied to the surface followed by a neutralizer; (iii) the gauges were mounted using an M-Bond AE10 adhesive with a 200 catalyst as bonding agent; (iv) pressure was applied on the gauge for one minute and then the wires were soldered to the strain gauges using lead wires; and, (v) the wires were then connected to the automatic strain indicator. Figures 4.34 to 4.36 show positions of the strain gauges on the steel beams for bridge models # 4, 7, and 9.
4.6.3 Mechanical Dial Gauges

Mechanical dial gauges having 0.025 mm travel sensitivity were used to measure the deflections of the longitudinal steel beams during the application of the load. The locations of the dial gauges are shown in Figures 4.37 to 4.45 and are seen in position in Figures 4.46 and 4.47.

4.7 Test Equipment

4.7.1 Hydraulic Jack

A hydraulic jack was used for the application of the load as shown in Figure 4.48. The jack had a capacity of 890 kN and was supported by a rigid portal frame.

4.7.2 Load Cells

Two types of load cells were used. A universal flat load cell was used to determine the value of the applied load whereas 24 cylindrical load cells were used to measure the reactions of the longitudinal steel beams and to monitor the uplift of the beams if any. The two types of load cells are described as follows:

4.7.2.1 Universal Flat Load Cell

A universal flat load cell model FL 500 (c) - 25 GKT. The capacity of the load cell was 890 kN. It was used to determine the value of the applied load. Figure 4.49 shows the Universal load cell attached to the hydraulic jack.
4.7.2.2 Cylindrical Load Cells

In order to measure the reactions at the supports of each bridge model a system was designed, consisting of a steel roller under the steel beams and a cylindrical load cell of 89 kN capacity was placed under each steel beam forming a line of load cells placed under the roller. Another line of load cells was placed above the support line to monitor the uplift if any. A total of 16 cylindrical load cells were used for the single span simply supported bridge models #1 to #6 whereas 24 load cells were used in case of continuous bridge models #7 to #9. Figures 4.50 to 4.58 show the arrangement of the cylindrical load cells for all the bridge models.

4.7.3 Automatic Strain Indicator

The automatic strain indicator manufactured by Vishay Intertechnology was used to record the strains during the application of the vertical loading. The strain indicator consisted of four main devices: the V1E-21 switch balance; ten digital strain indicators V1E-20; the scan controller V1E-25; and, the automatic printer V1E-22. Each strain indicator V1E-20 was able to record ten readings with a total capacity of one hundred strain gauges.

4.7.4 Loading Plate

The load was applied to the bridge deck by the hydraulic jack through a steel plate of thickness 25 mm. The steel plate was cut in the shape of a circle. The OHBDC specifies the contact area of a truck tire as 0.15 m². Scaling this area using a length scale
factor of 5 a circular loading plate with a diameter of 100 mm was used.

4.8 Experimental Setup and Test Procedure

After the forms were released each bridge model was transferred to its supports, at the loading frame, with extreme care and adequate precautions. Each single span simply supported bridge model was supported at both ends on steel rollers. In the case of continuous bridge models, they were supported at both end on steel rollers, representing the abutments, and they were supported also on an intermediate steel roller representing the pier. Steel shims were used to fill any gap between the supports and the longitudinal steel beams in order to ensure full contact between the bridge and its supports. The supporting system for each bridge model was firmly tied down to the laboratory floor to prevent the model from uplift during loading. Figure 4.59 shows the testing setup. Each bridge model was tested in three stages as follows:

4.8.1 Stage I, Elastic Loading of the Steel Grids

In this stage of loading, the steel grid of each bridge model was tested elastically before casting the concrete. For the steel grids of the simply supported bridge models #1 to #6 the following loading patterns were applied: (i) a single concentrated load was applied at the midspan of an external beam; (ii) a single concentrated load was applied at the midspan of the interior girder as shown in Figures 4.60 and 4.61; and, (iii) two beams were loaded simultaneously by two concentrated loads at their midspans as shown in Figures 4.62 and 4.63. The total number of loading cases applied on the steel grids
of the six simply supported bridge models was 36 cases. For the steel grids of the continuous bridge models #7 to #9 the following loading patterns were applied: (i) a single concentrated load was applied at the midspan of an exterior or an interior beam on either the long span or the short span as shown in Figures 4.64 and 4.65; (ii) two beams were loaded simultaneously by two concentrated loads at their midspans on either the long span or the short span as shown in Figure 4.66; and, (iii) two beams were loaded simultaneously by two concentrated loads at their midspans on both the long span and the short span as shown in Figures 4.67 and 4.68. The total number of loading cases applied on the steel grids of the three continuous bridge models was 30. Throughout this stage of loading only the deflections of the steel beams were measured.

4.8.2 Stage II, Elastic Loading of the Bridge Models

Before starting the test, each bridge model was subjected to a small load and then unloaded; this was repeated several times to ensure good seating of the bridge on its supports. Then, each bridge model was tested elastically using a single concentrated load or a simulated truck load applied at various locations. The six simply supported bridge models were subjected to a total of 42 loading cases. The following loading patterns were applied: (i) a single concentrated load was applied at the midspan of an exterior or an interior beam as shown in Figures 4.69 and 4.70; (ii) a simulated truck load applied on one of the two lanes of the bridge model representing an eccentric truck loading; and, (iii) two simulated truck loads applied on the two lanes representing a fully loaded bridge as shown in Figures 4.71 to 4.74. The three continuous bridge models were subjected to
a total of 36 loading cases. The following loading patterns were applied: (i) a single concentrated load was applied at the mid span of an exterior or an interior beam on the long span or the short span as shown in Figures 4.75 and 4.76; (ii) a simulated truck load occupying one of the two lanes of the bridge model on the long span or the short span representing an eccentric truck load on one of the spans as shown in Figures 4.77 to 4.79; (iii) a simulated truck load occupying one of the two lanes of the bridge model on both the long and the short span representing an eccentric truck load on both spans as shown in Figures 4.80 and 4.81; (iv) two simulated truck loads occupying the two lanes of the bridge model on the long span or the short span representing a concentric truck load on one of the spans as shown in Figures 4.82 and 4.83; and, (v) two simulated truck loads occupying the two lanes of the bridge model on both the long and the short span representing a fully loaded bridge model as shown in Figure 4.84. Throughout the 78 loading cases included in this stage the experimental measurements were directed to determine: (i) deflections of the longitudinal steel beams along longitudinal and transverse sections; (ii) strains in the longitudinal beams and in the reinforced concrete deck slab along longitudinal and transverse sections; (iii) reactions of the longitudinal steel beams along the support lines; and, (iv) any uplift of the bridge model.

4.8.3 Stage III, The Ultimate Load Test

Finally, each bridge model was tested to failure using simulated truck loads. For the six simply supported bridge models an eccentric simulated truck load was used for the ultimate load test. The simulated truck load was applied at the same location for the six
bridge models for comparison as shown in Figures 4.85 to 4.88. The continuous bridge models #7 and #9 were tested to failure using a concentric simulated truck load applied on both the long and the short spans representing a fully loaded bridge as shown in Figures 4.89 and 4.90. The continuous bridge model #8 was tested using an eccentric simulated truck load applied on both spans as shown in Figure 4.91. For all the bridge models, the load was applied at a constant rate in increments of 10 kN. After each increment the load was maintained constant during recording of the deflections and the strains. In addition to the measurements mentioned in stage II, the following were also recorded for each bridge model: (i) cracking load of the deck slab, (ii) load-carrying capacity of the bridge model; and, (iii) crack pattern of the deck slab after failure of the bridge model. After failure of the bridge model, the load was released slowly.
CHAPTER V

VERIFICATION OF THEORETICAL MODEL

5.1 General

The experimental study was undertaken to study the elastic and ultimate behaviour of simply supported and continuous skew composite steel-concrete bridges under single concentrated loads and simulated truck loads. The main objective of the experimental study was to verify and substantiate the linear and the non-linear finite element modelling used in the theoretical analysis. Other objectives of the experimental study were: (i) to study the effect of skew on the behaviour of simply supported and continuous skew composite bridges at both the elastic and the ultimate limit states; (ii) to study the effect of the type of diaphragm connection on the behaviour of skew composite bridges; and, (iii) to investigate the effect of the span ratio (= long span length/short span length) on the behaviour of two-unequal-span continuous skew composite bridges. In order to achieve the above objectives, tests were carried out on the nine skew composite steel-concrete bridge models.

The finite element method was used to model the nine bridge models tested. In this chapter, the experimental and theoretical results for the nine bridge models are presented and compared. The results presented include: deflections and strains of the
steel beams, reactions of the longitudinal beams along the support lines, and failure loads of the bridge models. It should be noted that the deflection behaviour was studied as one part of the serviceability limit state. Other parts of the serviceability limit state are crack widths and dynamic response. However, these parts are not discussed herein since they are outside the scope of this research. It should be noted that the dynamic response of composite bridges has been studied by Grace and Kennedy (1985). Because of the large number of loading cases considered, numerous amount of data was generated. For brevity, results for the most important loading cases will be presented. Five elastic loading cases were chosen for each simply supported bridge model in addition to the ultimate load test. The load positions for the five loading cases and for the ultimate load test are shown in Figures 5.1 to 5.6. For each continuous bridge model, eight elastic loading cases were chosen in addition to the ultimate load test. The load positions for the nine loading cases are presented in Figures 5.7 to 5.14.

5.2 Simply Supported Bridge Models

5.2.1 Bridge Model # 1

Bridge model # 1 was a simply supported skew composite steel-concrete bridge model with an angle of skew \( \theta = 30^\circ \) and with four longitudinal beams. Transverse intermediate diaphragms, moment connected to the longitudinal beams, were used for this bridge model. First, the model was loaded elastically using either a single concentrated load or a simulated truck load applied at various critical locations. Figures 5.15 to 5.19 show the experimental and the theoretical load-deflection relationships for the different loading cases. In all cases, the good agreement between the experimental and the
Theoretical results can be observed. The theoretical deflections are generally higher than the experimental deflections by less than 8% in most cases. In loading case #1, a single concentrated load was applied at the center of the bridge model. In this case, the interior beams were the ones that undergo maximum deflections with a difference of 14% greater than the deflections of the exterior beams. An eccentric simulated truck load, applied at the center of the bridge model, was used in loading case #2. In this case, the exterior beam at the loaded side of the bridge model was the one that exhibited maximum deflection. The difference between the deflections of the exterior and the interior beams was 31%. The eccentric simulated truck load was moved towards the obtuse corner of the bridge model in loading case #3. The exterior beam in this case also was the one that had the maximum deflection. However, the difference between the deflections of the exterior beam and the interior beams decreased to 22%. Comparing the results from loading cases #2 and #3 reveals that loading case #2, where an eccentric simulated truck load was applied at the center of the bridge model, is more critical than loading case #3, as expected. Concentric simulated truck loading, applied at the center of the bridge model, was used for loading case #4. The exterior beam was the one that exhibited the maximum deflection with a difference of 12% greater than that of the interior beam. The simulated truck load was moved towards the obtuse corners of the bridge model in loading case #5. The deflections of the exterior beams were greater than those of the interior beams by about 7%. Comparing the results for concentric and eccentric loading cases reveals that the concentric loading cases are more critical than the eccentric loading cases. The increase in the deflection in the case of concentric loading was about 6% for the exterior beams and about 27% for the interior beams.
The experimental and the theoretical load-strain relationships for the different loading cases are shown in Figures 5.20 to 5.24. Good correspondence is observed between the experimental and the theoretical results. The theoretical strains are generally higher than the experimental strains by about 6% in most cases. However, the theoretical strain for one of the interior beams in loading case #2 was higher than the experimental strain by about 12%. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the strain of the interior beam was higher than that of the exterior beam by 48%. Under an eccentric simulated truck load, applied at the center of the bridge model, the strain of the exterior beam was higher than that of the interior beam by 11%. However, when the eccentric simulated truck load was moved towards the obtuse corner of the bridge model, in loading case #3, the strain of the interior beam became higher than that of the exterior beam by 4%. Comparing the results for loading cases #2 and #3 revealed that when the eccentric simulated truck load was applied at the center of the bridge model, loading case #2, higher strains were produced in both the interior and the exterior beams, as expected. In the concentric simulated truck loading cases #4 and #5, the strains for the interior beams were higher than those for the exterior beams. Comparing results for concentric and eccentric loading cases showed that the concentric loading cases produces higher strains in the beams than in the eccentric loading cases.

Figures 5.25 to 5.27 show the experimental and the theoretical results for the support reactions for different loading cases. There is a good agreement between the experimental and the theoretical results. Under single concentrated load, applied at the
center of the bridge model in loading case #1, the distribution of the reactions at one of
the bridge supports was as follows: the reaction of the exterior beam close to the obtuse
corner represented 14.5% of the total applied load; 23.5% for the interior beam close to
the obtuse corner; 8.5% for the interior beam close to the acute corner; and, 3.5% for the
exterior beam close to the acute corner. The large differences in the reactions of various
beams in the bridge model can be readily observed. The reactions for the beams close
to the obtuse corner are much greater than those for the beams close to the acute corner.
The difference between the maximum reaction and the minimum reaction is about 85%.
In the case of an eccentric simulated truck loading, Figure 5.26, about 30% of the total
applied load is concentrated at the exterior beam close to the obtuse corner whereas only
11% of the total applied load was carried by the beam close to the acute corner. In the
case of concentric simulated truck load applied at the center of the bridge model, Figure
5.27, the load was distributed as follows: 14.6% of the total applied load was carried by
the exterior beam close to the obtuse corner; 18.8% was carried by the interior beam close
to the obtuse corner; 12.4% was carried by the interior beam close to the acute corner;
and, 4.7% was carried by the exterior beam near the acute corner. It should be noted that
the results for the reactions presented above for the three different loading cases confirms
the recommendations given by Helba and Kennedy (1994) that the reaction distribution
in skew bridges is more critical than that for normal bridges and therefore shear should
be considered in the design of skew bridges. Comparing the results for the eccentric and
the concentric loading cases revealed the following: (i) the eccentric loading case is the
one that produces the maximum reaction for the exterior beams; and, (ii) the concentric
loading case is the one that produces the maximum reaction for the interior beams.

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After the elastic loading of the bridge model it was finally tested to failure using an eccentric simulated truck loading applied at the center of the bridge model. The experimental and the theoretical load-deflection relationship for the ultimate load test of bridge model #1 is shown in Figure 5.28. It is observed that in the elastic range of loading and before the cracking of the concrete of the deck slab, the experimental results are in good agreement with the finite element results. The difference between the two sets of results did not exceed 6%. In the post-elastic range of loading up to the collapse of the bridge model, the experimental results are in reasonable agreement with the non-linear finite element results with a difference less than 12%. Figure 5.29 shows the experimental and theoretical load-strain relationships for the ultimate load test of bridge model #1. The same observations mentioned above for the deflections were found in this case. Table 5.1 shows the experimental and theoretical cracking loads for all the bridge models tested. Invariably the experimental cracking loads were higher than the theoretical ones. The reason for this is that first cracks in the experiments were observed only by eye. It has been shown (Park and Gamble, 1980) that such experimental cracking loads are usually between 65% to 75% those that are determined based on eye observation. This explains the differences noted between the experimental and the theoretical cracking loads in the study. Table 5.1 shows the experimental and theoretical ultimate loads for all the bridge models tested. The experimental ultimate load was 160 kN whereas the theoretical ultimate load was 152 kN, representing a difference of 5%. It should be noted that since the size of the concrete deck slab in the model was only 38 mm, its compressive strength would be higher than that exhibited by the tested cylinders (Sabnis et al., 1983). Using the compressive strength of concrete from the cylinder tests in the
ABAQUS will therefore result in a slightly smaller failure loads as demonstrated in Table 5.1. However, since the failure of the models was first initiated by yielding of the steel beams, the difference in the compressive strength of the concrete noted above due to the size effect will have very little effect on the magnitude of the failure loads of the models. Figures 5.30 and 5.31 show bridge model #1 after failure.

5.2.2 Bridge Model #2

Bridge model #2 was the same as bridge model #1 except that the angle of skew was increased to 45° instead of 30°. Figures 5.32 to 5.36 show the experimental and the theoretical load-deflection relationships for different loading cases. Good agreement is observed between the two sets of results. The difference between the two sets of results was less than 8% in most cases. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the deflection of the interior beam was greater than that of the exterior beam by about 30%. As a result of increasing the angle of skew from 30° to 45°, the deflections for both the exterior and the interior beams were significantly reduced in comparison to those of the corresponding loading case for bridge model #1. The reductions were about 19% for the exterior beam and about 23% for the interior beam. The above results show that the exterior beams were less affected by skew than the interior beams. An eccentric simulated truck load, applied at the center of the bridge model, was used in loading case #2. In this case, the exterior beam exhibited the maximum deflection, that being 33% greater than the deflection of the interior beam. The deflection of the exterior beam decreased by about 25% compared to that of bridge model #1, θ = 30°, whereas the deflection of the interior beam decreased by about 27%. It
should be noted that the deflections of the beams located at the unloaded side of the bridge model were also significantly reduced as a result of increasing the skew. The reductions were 40% for the interior beam and 29% for the exterior beam. Similar observations were found in loading case #3 where the eccentric simulated truck load was moved towards the obtuse corner of the bridge model. In loading case #4, a concentric simulated truck load was applied at the center of the bridge model. In this case also the deflections of the exterior beams were greater than those of the interior beams. It should be noted that the difference between the deflections of the exterior and the interior beams was increased to 20% in comparison to a difference of 12% in the case of bridge model #1 as a result of increasing the angle of skew to 45°. The reductions in the deflections of the beams were 27% for the exterior beam and 33% for the interior beam. Similar observations were found in loading case #5 where the concentric simulated truck load was moved towards the obtuse corners of the bridge model. Comparing the eccentric and the concentric loading cases revealed that the concentric loading cases are more critical than the eccentric loading cases. The concentric loading cases produced greater deflections for both the exterior and the interior beams.

The experimental and the theoretical load-strain relationships for different loading cases are presented in Figures 5.37 to 5.41. The difference between the experimental results and the theoretical results was less than 5% in most cases. In some cases, this difference increased to about 8%. However, in all cases it did not exceed 10%. It is observed that the strain of the interior beam was greater than that of the exterior beam in all the loading cases presented except in loading case #2 where an eccentric simulated
truck load was applied at the center of the bridge model. In this case, the strain of the exterior beam was higher than that of the interior beam by about 9%. It is also observed that the concentric loading cases produced higher strains, in both the exterior and the interior beams, than the eccentric loading cases. Comparing results for strains for bridge models #1 and #2, the following reductions in strains were observed in bridge model #2 as a result of increasing the skew angle to 45°: (i) under a single concentrated load, applied at the center of the bridge model, the strain was reduced by 18% for the exterior beam and by 27% for the interior beam; (ii) under an eccentric simulated truck load, applied at the center of the bridge model, the strain was reduced by 18% for the exterior beam and by 20% for the interior beam; and, (iii) under a concentric simulated truck load the reductions in the strains were 23% for the exterior beam and 28% for the interior beam. From the above results it is observed that the reductions in the strains, for both the exterior and the interior beams, due to increase in the angle of skew were higher in the case of concentric loading than that in the case of eccentric loading.

Figures 5.42 to 5.44 show the experimental and the theoretical results for the support reactions for three different loading cases. The good agreement between the two sets of results can be noted. In most of the cases, the difference between the experimental and the theoretical results was less than 10%. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the distribution of the reactions at one of the bridge supports was as follows: the reaction of the exterior beam close to the obtuse corner represented 10% of the total applied load; 35% for the interior beam close to the obtuse corner; 4% for the interior beam close to the acute corner; and,
1.5% for the exterior beam close to the acute corner. This shows that the reaction distribution in bridge model #2 is more critical than that of bridge model #1 as a result of increasing the angle of skew. The reaction of the interior beam close to the obtuse corner increased by 47% in comparison to bridge model #1. The reactions of the other beams decreased by different percentages. As an example, the reaction of the exterior beam close to the acute corner decreased from 1.4 kN in bridge model #1 to only 0.4 kN in bridge model #2, representing a decrease of about 71%. The difference between the maximum reaction and the minimum reaction is about 97% compared to 85% in the case of bridge model #1. In the case of an eccentric simulated truck loading applied at the center of the bridge model, Figure 5.43, about 35% of the total applied load was carried by the exterior beam close to the obtuse corner whereas only 1% of the total applied load was carried by the exterior beam close to the acute corner. The maximum reaction, at the exterior beam close to the obtuse corner, increased from 26.5 kN in the case of bridge model #1 to 31.0 kN in the case of bridge model #2, representing an increase of about 17%. In the case of concentric simulated truck load, applied at the center of the bridge model, Figure 5.44, the total applied load was distributed as follows: 16.7% of the total applied load was carried by the exterior beam close to the obtuse corner; 16.1% by the interior beam close to the obtuse corner; 12.3% by the interior beam close to the acute corner; and, 4% was carried by the exterior beam close to the acute corner. It should be noted that the distribution of the reactions presented above is more critical than that in the case of bridge model #1. However, the reaction distribution in the case of concentric loading was less affected by increasing the skew angle than that in the case of eccentric loading. It should be also noted that the maximum reactions in the different beams of the
bridge model were produced by the following loading cases: (i) the eccentric loading case produced the maximum reaction for the exterior beam close to the obtuse corner; and, (ii) the concentric loading case produced the maximum reactions for all other beams in the bridge model.

Figures 5.45 and 5.46 show the experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of the bridge model over the complete range of loading up to the collapse load. In the load-deflection relationship, good agreement (less than 5% difference) is observed between the experimental and the theoretical results in the elastic range of loading whereas the agreement between the two sets of results was reasonable (less than 12% difference) in the post-elastic range of loading up to the collapse of the bridge model. For the load-strain relationship, the agreement between the two sets of results was good in both the elastic and the post-elastic ranges of loading. It should be noted that in the post-elastic range of loading, the theoretical results for strains compared better to the experimental results better than the results for deflections. This may be due to experimental errors since readings the deflections were taken manually whereas strains were taken automatically. Both the deflections and strains at the collapse of the bridge model were significantly reduced when compared to those for bridge model #1 as a result of increasing the skew. These reductions were in the range of 10%. However, the collapse load of the bridge model was greater than that of bridge model #1 by 15%. Figures 5.47 and 5.48 show bridge model #2 after collapse.
5.2.3 Bridge Model #3

Bridge model #3 was the same as bridge models #1 and #2 except that the angle of skew was increased to 60°. Figures 5.49 to 5.53 show the experimental and the theoretical load-deflection relationships for different loading cases. The experimental and the theoretical results were in good agreement with the difference not exceeding 10% in all cases. Significant reductions in the deflections of both the exterior and the interior beams are observed in comparison to those of bridge models #1 and 2. These reductions can be summarized as follows: (i) in the case of single concentrated load the deflection of the exterior beam decreased by 45% and 26% in comparison to that of bridge models #1 and 2, respectively; (ii) for the same loading case, the corresponding reductions for the interior beam were 49% and 31%; (iii) in an eccentric simulated truck loading case, applied at the center of the bridge model, the deflection of the exterior beam decreased by 44% and 25% in comparison to that of bridge models #1 and 2, respectively; (iv) for the same loading case, the corresponding reductions for the interior beam were 50% and 31%; (v) in a concentric simulated truck loading case, applied at the center of the bridge model, the deflection of the exterior beam decreased by 39% and 21% in comparison to that of bridge models #1 and 2, respectively; and, (vi) for the same loading case, the corresponding reductions for the interior beam were 45% and 24%. From these comparisons, the following observations can be made: (i) the exterior beam was less affected by an increase in the angle of skew than the interior beam; (ii) the reductions in the deflections for both the exterior and the interior beams are more significant in the case of eccentric loading than in the case of concentric loading; and, (iii) the reductions in the deflections of both the exterior and the interior beams are more significant when the skew
angle is increased from 45° to 60° than that when the skew angle is increased from 30° to 45°. It should be noted that concentric loading cases produced greater deflections for both the exterior and the interior beams than eccentric loading cases. In the case of single concentrated load, the interior beam was the one that exhibited the maximum deflection. However, in all cases of eccentric and concentric simulated truck loads the deflection of the exterior beam was greater than that of the interior beam.

Figures 5.54 to 5.58 show the experimental and the theoretical load-strain relationships for different loading cases. The maximum difference between the experimental and the theoretical strains was less than 10% in most cases. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the finite element method overestimated the strain for both the exterior and the interior beams by about 15%. This is probably due to an experimental error since the results in all other cases were in good agreement. Significant reductions in strains were observed in comparison to those in bridge models #1 and 2, as a result of increasing the angle of skew. These reductions were as follows: (i) in the case of a single concentrated load the reductions in the strain of the exterior beam were 42% and 26% in comparison to that of bridge models #1 and 2, respectively; (ii) for the same loading case, the corresponding reductions in the strain of the interior beam were 48% and 31%; (iii) for the case of an eccentric simulated truck load, applied at the center of the bridge model, the reductions in the strain of the exterior beam were 35% and 20% in comparison to that of bridge models #1 and 2, respectively; (iv) for the same loading case the corresponding reductions in the strain of the interior beam were 40% and 25%; (v) for the case of concentric
simulated truck load, applied at the center of the bridge model, the reductions in the strain of the exterior beam were 31\% and 17\% in comparison to that of bridge models #1 and 2, respectively; and, (vi) for the same loading case the corresponding reductions in the strain of the interior beam were 36\% and 21\%. It can be observed that the effect of skew is more significant in the case of eccentric loading than in the case of concentric loading. The interior beam is more affected by skew than the exterior beam. It should be noted that in the case of strains in the longitudinal steel beams also the concentric loading cases are more critical than the eccentric loading cases. In the case of eccentric simulated truck loading, applied at the center of the bridge model, the exterior beam exhibited the maximum strain. However, in all other cases the strain of the interior beam was higher than that of the exterior beam.

Figures 5.59 to 5.61 show the experimental and the theoretical results for the support reactions for different loading cases. The results from the finite element analysis compared well to those from tests. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the distribution of the reactions at one of the bridge supports was as follows: the reaction of the exterior beam close to the obtuse corner represented 7.5\% of the total applied load; 41\% for the interior beam close to the obtuse corner; 2\% for the interior beam close to the acute corner; and, -3\% for the exterior beam close to the acute corner. As a result of increasing the angle of skew, the reaction distribution in bridge model #3 was more critical than those of bridge models #1 and 2. The reaction of the interior beam close to the obtuse corner increased by 85\% and 24\% in comparison to bridge models #1 and 2, respectively. The reactions of all other
beams in the bridge model decreased by different percentages. For example, the reaction of the interior beam close to the acute corner decreased from 3.1 kN in the case of bridge model #1 to only 0.7 kN in bridge model #3. It is observed that the reaction of the exterior beam close to the acute corner had a negative value of -1.0 kN which indicates an uplift of the acute corner of the bridge model. The difference between the maximum and the minimum reactions was about 107% compared to 85% and 97% in the case of bridge models #1 and 2, respectively. In the case of an eccentric simulated truck load, applied at the center of the bridge model, Figure 5.60, about 41% of the total applied load was concentrated at the exterior beam close to the obtuse corner whereas uplift forces existed at the exterior beam close to the acute corner. The maximum reaction, at the exterior beam close to the obtuse corner, increased from 26.5 kN and 31.0 kN in the case of bridge models #1 and 2, respectively, to 36.4 kN in the case of bridge model #3. In the case of concentric simulated truck load, applied at the center of the bridge model, Figure 5.61, the total applied load was distributed as follows: 20% of the total applied load was carried by the exterior beam close to the obtuse corner; 18.7% was carried by the interior beam close to the obtuse corner; 10.9% was carried by the interior beam close to the acute corner; and, 1% was carried by the exterior beam close to the acute corner. It can be observed that the distribution of the reactions presented above is more critical than that of the corresponding loading case in bridge models #1 and 2, as a result of increasing the angle of skew.

Figures 5.62 and 5.63 show the experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of the bridge model. Good
agreement is observed between the two sets of results for both the deflections and the strains in both the elastic and the post-elastic range of loading. The maximum difference between the experimental and the theoretical results was less than 1.3\(^\circ\) in the post-elastic range of loading. The deflections and the strains at collapse were significantly reduced in comparison to those of bridge models #1 and 2, as a result of increasing the skew angle. The collapse load of the bridge model was greater than that of bridge models #1 and 2 by 36\% and 18.5\% respectively. Figure 5.64 shows bridge model #3 after collapse.

5.2.4 Bridge Model # 4

Bridge model # 4 was a simply supported skew composite steel-concrete bridge model, with angle of skew \( \theta = 45^\circ \) and four longitudinal beams. The bridge model was the same as bridge model # 2 except that no transverse intermediate diaphragms were used. Only end diaphragms were used for bridge model # 4. Figures 5.65 to 5.69 show the experimental and the theoretical load-deflection relationships for the different loading cases. Good agreement is observed between the two sets of results. The finite element method overestimated the deflections by less than 5\% in all cases except in loading case # 1. In this case, where a single concentrated load was applied at the center of the bridge model, the finite element overestimated the deflections by about 10\%. In this case, the deflection of the interior beam was greater than that of the exterior beam by 55\% compared to 30\% in the case of bridge model # 2 with moment connected intermediate transverse diaphragms. The deflection of the interior beam increased by about 38\% whereas that of the exterior beam decreased by about 20\%. The above results indicate that bridge model # 2 with moment connected transverse intermediate diaphragms had
better load distribution characteristics than bridge model # 4 with end diaphragms only. In loading case # 2, where an eccentric simulated truck load was applied at the center of the bridge model, the following changes were observed for the deflection of different beams of the bridge model in comparison to bridge model # 2: (i) the deflection of the exterior beam on the loaded side of the bridge model increased by 26%; (ii) the deflection of the interior beam on the loaded side of the bridge model increased by 31%; (iii) the deflection of the interior beam on the unloaded side of the bridge model increased by 58%; and (iv) the deflection of the exterior beam on the unloaded side of the bridge model decreased by 45%. Similar observations were found in loading case # 3 where the eccentric simulated truck load was moved towards the obtuse corner of the bridge model. In loading case # 4, a concentric simulated truck load was applied at the center of the bridge model. The changes in the deflections of the different beams in the bridge model in comparison to bridge model # 2, with moment connected transverse intermediate diaphragms, were as follows: (i) the deflection of the exterior beam increased by 14%; and, (ii) the deflection of the interior beam increased by 22%. Similar observations were found in loading case # 5 where the concentric simulated truck load was moved towards the obtuse corners of the bridge model. From the results for deflections presented above for the five loading cases, the following can be concluded: (i) the presence of moment connected transverse intermediate diaphragms results in significant reductions in the deflections of the longitudinal beams and therefore enhances the load distribution characteristics of the bridge; (ii) the interior beam is more significantly affected by the presence of transverse intermediate diaphragms than the exterior beam; and, (iii) the effect of the presence of transverse intermediate diaphragms is more significant in the case of
eccentric loading than in the case of concentric loading.

Figures 5.70 to 5.74 show the experimental and the theoretical load-strain relationships for different loading cases. Good agreement is observed between the two sets of results. The maximum difference between the experimental and the theoretical strains was less than 10% in most cases. Significant changes were observed in the strains of the beams in comparison to those in the case of bridge model # 2. In the case of a single concentrated load, the strain of the interior beam increased by 39% whereas that of the exterior beam decreased by 25%. In the case of eccentric simulated truck loading the following changes in the strain were observed in comparison to those of bridge model # 2 with moment connected transverse intermediate diaphragms: (i) the strain of the exterior beam on the loaded side of the bridge model increased by 15%; (ii) the strain of the interior beam on the loaded side of the bridge model increased by 17%; (iii) the strain of the interior beam on the unloaded side of the bridge model increased by 28%; and, (iv) the strain of the exterior beam on the unloaded side of the bridge model decreased by 33%. In the case of concentric simulated truck loading the strain in the exterior beam increased by 8% whereas that of the interior beam increased by 27%. The above results for strains confirm that the presence of transverse intermediate diaphragms, moment-connected to the longitudinal beams, enhances the load distribution characteristics of the bridge model.

Figures 5.75 to 5.77 show the experimental and the theoretical results for the support reactions for three different loading cases. Good agreement between the two sets
of results is observed. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the distribution of the reactions at one end of the bridge model was as follows: the reaction of the exterior beam close to the obtuse corner represented 13% of the total applied load; 26% for the interior beam close to the obtuse corner; 9% for the interior beam close to the acute corner; and, 2% for the exterior beam close to the acute corner. In the case of an eccentric simulated truck load applied at the center of the bridge model, Figure 5.76, about 43% of the total applied load concentrated at the exterior beam close to the obtuse corner whereas an uplift force of about 2% of the total applied load existed at the exterior beam close to the acute corner. The maximum reaction, at the exterior beam close to the obtuse corner, increased from 31.0 kN in the case of bridge model #2 to 38.7 kN in the case of bridge model #4, representing an increase of about 21%. In the case of concentric simulated truck load, applied at the center of the bridge model, Figure 5.77, the total applied load was distributed as follows: 22% of the total applied load was carried by the exterior beam close to the obtuse corner; 14% was carried by the interior beam close to the obtuse corner; 12% was carried by the interior beam close to the acute corner; and, 3% was carried by the exterior beam close to the acute corner. It should be noted that the distribution of the reactions presented above is more critical than that in the case of bridge model #2 with moment connected transverse intermediate diaphragms. However, the reaction distribution in the case of concentric loading was less critical than that in the case of eccentric loading. The effect of the presence of transverse intermediate diaphragms was more significant in the eccentric than the concentric loading cases.
Figures 5.78 and 5.79 show the experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of the bridge model over the complete range of loading up to the ultimate load. Good correspondence is noted between the experimental and the theoretical results for both the deflections and the strains. However, in the post-elastic range of loading, the theoretical results for strains compared to the experimental results better than the results for deflections. This is due to the same reasons mentioned earlier. The deflections and the strains at the ultimate load of the bridge model were significantly greater than those in the case of bridge model # 2 with moment connected intermediate transverse diaphragms. However, the collapse load of the bridge model was less than that of bridge model # 2 by 18%. Figures 5.80 and 5.81 show bridge model # 4 after failure.

5.2.5 Bridge Model # 5

Bridge model # 5 was the same as bridge models # 2 and 4, except that the transverse intermediate diaphragms were shear-connected to the longitudinal beams by means of bolts. Figures 5.82 to 5.86 show the experimental and the theoretical load-deflection relationships for different loading cases. Good agreement is observed between the two sets of results in all cases. The maximum difference between the experimental and the theoretical deflections was less than 7% in most cases. In loading case # 1, where a single concentrated load was applied at the center of the bridge model, the deflection of the interior beam was greater than that of the exterior beam by 46% compared to 55% in the case of bridge model # 4 with end diaphragms only and 30% in the case of bridge model # 2 with moment-connected transverse intermediate diaphragms.
The deflection of the interior beam increased by about 13% in comparison to that of bridge model #2 whereas the deflection of the exterior beam was almost the same as in bridge model #2. The above results indicate that bridge model #2 with moment-connected transverse diaphragms had a better load distribution characteristics than bridge model #5 with shear-connected transverse diaphragms. In loading case #2, where an eccentric simulated truck load was applied at the center of the bridge model, the following changes were observed in the deflection of the different beams of the bridge model in comparison to those in the case of bridge model #2: (i) the deflection of the exterior beam on the loaded side of the bridge model increased by 17%; (ii) the deflection of the interior beam on the loaded side of the bridge model increased by 12%; (iii) the deflection of the interior beam on the unloaded side of the bridge model increased by 33%; and, (iv) the deflection of the exterior beam on the unloaded side of the bridge model decreased by 18%. The above results confirm that the moment-connected diaphragms enhance the distribution of wheel loads more significantly than shear-connected diaphragms. However, a bridge with shear-connected transverse diaphragms has a better load distribution characteristics than a bridge with end diaphragms only. This was concluded when comparing bridge model #5 to bridge model #4. The following changes were observed for the deflection of the different beams of bridge model #5 in comparison to those of bridge model #4: (i) the deflection of the exterior beam on the loaded side of the bridge model decreased by 8%; (ii) the deflection of the interior beam on the loaded side of the bridge model decreased by 15%; (iii) the deflection of the interior beam on the unloaded side of the bridge model decreased by 16%; and, (iv) the deflection of the exterior beam on the unloaded side of the bridge model increased by 43%. In loading
case # 4, where a concentric simulated truck load was applied at the center of the bridge model, the deflection of the interior beam increased by 15° in comparison to that in the case of bridge model # 2 whereas the deflection of the exterior beam increased by 8°. It should be noted that the interior beam was more significantly affected by the type of transverse diaphragms connection than the exterior beam. It should be also noted that the type of transverse diaphragms connection influences the load distribution characteristics of skew composite bridges more significantly in eccentric loading cases than in concentric loading cases.

Figures 5.87 to 5.91 show the experimental and the theoretical load-strain relationships for different loading cases. Good correspondence is noted between the two sets of results in all cases. The difference between the experimental and theoretical strains was less than 6% in most cases. The results indicated that the type of transverse diaphragms connection had a significant influence on the values of the strains of the longitudinal beams. Under a single concentrated load, applied at the center of the bridge model, the strain in the interior beam increased by 16% in comparison to that in the case of bridge model # 2 with moment-connected diaphragms. However, the strain in the exterior beam did not change. In the case of eccentric simulated truck loading, the strain in the interior beam increased by 8% while that in the exterior beam increased by 7%. In the case of concentric simulated truck loading, the strain in the interior beam increased by 6% whereas that in the exterior beam increased by only 4%. Comparing results for deflections and strains revealed that the type of transverse diaphragms connection influences the values of deflections more significantly than the values of strains.
Figures 5.92 to 5.94 show the experimental and the theoretical results for the support reactions for three different loading cases. The good agreement between the two sets of results can be observed. In loading case #1, where a single concentrated load was applied at the center of the bridge model, the distribution of the reactions at one end of the bridge model was as follows: the reaction of the exterior beam close to the obtuse corner represented 14% of the total applied load; 32% for the interior beam close to the obtuse corner; 6% for the interior beam close to the acute corner; and, 1% for the exterior beam close to the acute corner. In the case of an eccentric simulated truck load, applied at the center of the bridge model, Figure 5.93, about 38% of the total applied load concentrated at the exterior beam close to the obtuse corner whereas only 0.3% was carried by the exterior beam close to the acute corner. The maximum reaction, at the exterior beam close to the obtuse corner, increased from 31.0 kN in the case of bridge model #2 with moment connected transverse diaphragms to 33.8 kN in the case of bridge model #5 with shear connected transverse diaphragms, representing an increase of 9%. In the case of concentric simulated truck load applied at the center of the bridge model, Figure 5.94, the total applied load was distributed as follows: 18% of the total applied load was carried by the exterior beam close to the obtuse corner; 15% was carried by the interior beam close to the obtuse corner; 12% was carried by the interior beam close to the acute corner; and, 5% was carried by the exterior beam close to the acute corner. It should be noted that the above reaction distribution is more critical than that in the case of bridge model #2 with moment-connected transverse diaphragms. However, the distribution is less critical than that in the case of bridge model #4 with end diaphragms only. It should be also noted that the type of transverse diaphragms connection influences
the reaction distribution more significantly in the case of eccentric loading than in concentric loading cases.

Figures 5.95 and 5.96 show the experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of the bridge model over the complete range of loading up to the collapse load. Good agreement is observed between the two sets of results in both the elastic and the post-elastic loading stages. Both the strains and the deflections at collapse were significantly increased in comparison to bridge model # 2 with moment-connected transverse diaphragms. However, these values were much lesser than those in the case of bridge model # 4 with end diaphragms only. The collapse load was also affected by transverse diaphragms. The load was less than that of bridge model # 2, with moment-connected transverse diaphragms, by 11.5%. However, this load was greater than that in the case of bridge model # 4, with end diaphragms only, by 9%. Figures 5.97 and 5.98 show bridge model # 5 after collapse.

5.2.6 Bridge Model # 6

Bridge model # 6 was the same as bridge model # 2 except that the transverse intermediate diaphragms were moment-connected to the longitudinal beams and were also made composite with the reinforced concrete deck slab. Figures 5.99 to 5.103 show the experimental and the theoretical load-deflection relationships for different loading cases. Although the modelling of the transverse diaphragms was very complex in this case, including using multipoint constraints at the diaphragms-longitudinal beam connection and between the diaphragms and the reinforced concrete deck slab, the finite element method
was capable of providing an accurate prediction of the deflections of the longitudinal beams. The difference between the experimental and the theoretical deflections was less than 10% in all cases. In loading case # 1, where a single concentrated load was applied at the center of the bridge model, the deflection of the interior beam was greater than that of the exterior beam by about 17% only compared to a difference of 30% in the case of bridge model # 2. The deflection of the interior beam decreased by 18% in comparison to that in the case of bridge model # 2 whereas the deflection of the exterior beam decreased by 6%. In loading case # 2, where an eccentric simulated truck load was applied at the center of the bridge model, the following changes in the deflections were observed in comparison to those in the case of bridge model # 2: (i) the deflection of the exterior beam at the loaded side of the bridge model decreased by 3%; (ii) the deflection of the interior beam at the loaded side of the bridge model decreased by 9%; (iii) the deflection of the interior beam at the unloaded side of the bridge model decreased by 8%; and, (iv) the deflection of the exterior beam at the unloaded side of the bridge model decreased by 9%. Similar observations were found in loading case # 3 where the eccentric simulated truck load was moved towards the obtuse corner of the bridge model. In loading case # 4, where a concentric simulated truck load was applied at the center of the bridge model, the deflection of the exterior beam decreased by 8% whereas that of the interior beam decreased by 10%. The results presented above show that moment-connected transverse diaphragms when made composite also with the reinforced concrete deck slab increase the stiffness of the bridge resulting in significant reductions in the deflections of the beams and a superior load distribution characteristics.
Figures 5.104 to 5.108 show the experimental and the theoretical load-strain relationships for different loading cases. Good correspondence is observed between the two sets of results. Significant changes were observed in the strains in the beams in comparison to those in the case of bridge model # 2. In the case of a single concentrated load, the strain of the interior beam decreased by 13% whereas that of the exterior beam decreased by 15%. In the case of eccentric simulated truck load the following changes in the strains were observed in comparison to those in the case of bridge model # 2, where the moment-connected transverse diaphragms were not made composite with the deck slab: (i) the strain of the exterior beam at the loaded side of the bridge model decreased by 4%; (ii) the strain of the interior beam at the loaded side of the bridge model decreased by 7%; (iii) the strain of the interior beam at the unloaded side of the bridge model decreased by 6%; and, (iv) the strain of the exterior beam at the unloaded side of the bridge model increased by 9%. In the case of concentric simulated truck loading, the strain of the exterior beam decreased by 4% whereas that of the interior beam decreased by 6%. The results presented above for strains confirm that moment-connected transverse diaphragms when made also composite with the reinforced concrete deck slab lead to superior load distribution characteristics.

Figures 5.109 to 5.111 show the experimental and the theoretical results for the support reactions for three different loading cases. Good agreement is noted between the two sets of results. In loading case # 1, where a single concentrated load was applied at the center of the bridge model, the distribution of the reactions at one end of the bridge model was as follows: the reaction of the exterior beam close to the obtuse corner
represented 11% of the total applied load, 34% for the interior beam close to the obtuse corner, 5% for the interior beam close to the acute corner; and, 1% for the exterior beam close to the acute corner. This reaction distribution is less critical than that of the corresponding loading case in bridge model # 2. In the case of an eccentric simulated truck load applied at the center of the bridge model, Figure 5.110, about 34% of the total applied load concentrated at the exterior beam close to the obtuse corner whereas the reaction of the exterior beam close to the acute corner represented only 1% of the total applied load. The maximum reaction, at the exterior beam close to the obtuse corner, decreased from 31.0 kN in the case of bridge model # 2 to 30.5 kN in the case of bridge model # 6, representing a decrease of 2%. In the case of concentric simulated truck load, applied at the center of the bridge model, the total applied load was distributed as follows: 16% of the total applied load was carried by the exterior beam close to the obtuse corner; 17% was carried by the interior beam close to the obtuse corner; 13% was carried by the interior beam close to the acute corner; and, 4% was carried by the exterior beam close to the acute corner. It should be noted that the reaction distributions presented above are less critical than those of the corresponding loading cases in bridge model # 2. It is concluded that moment-connected transverse intermediate diaphragms when made composite with the deck slab enhance the reaction distribution in skew composite bridges. This enhancement is more significant than that produced by using moment-connected transverse diaphragms having no connection with the deck slab.

Figures 5.112 and 5.113 show the experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of the bridge model over the
complete range of loading up to the ultimate load. Good agreement is observed between the experimental and the theoretical results in both the elastic and the post-elastic loading stages. The difference between the two sets of results was less than 11% in the post-elastic range of loading. The deflections and the strains at collapse of the bridge model were significantly less than those in the case of bridge model #2 where the moment-connected diaphragms were made non-composite with the deck slab. However, the ultimate load of the bridge model was greater than that in the case of bridge model #2 by only 2.2%. Figures 5.114 and 5.115 show bridge model #6 after collapse.

5.2.7 Mode of Failure of the Simply Supported Bridge Models

For all the six simply supported bridge models tested the final mode of failure can be summarized as follows: (i) cracking as well as crushing of the concrete on the top surface of the deck slab; (ii) cracking of the concrete on the bottom surface of the deck slab; and, (iii) yielding of the steel of the longitudinal steel beams. The failure modes of bridge models #1 to 6 with top and bottom views are shown in Figures 5.30 and 5.31, 5.47 and 5.48, 5.64 (top view for bridge model #3), 5.80 and 5.81, 5.97 and 5.98, 5.114 and 5.115, respectively.

5.2.8 Effect of Angle of Skew

Figure 5.116 shows the effect of angle of skew on the load-deflection relationship of different beams in which a concentric simulated truck load was applied at the center of the bridge model. The following can be observed: (i) the exterior beam exhibits the maximum deflection for all skew angles; (ii) the deflection of the exterior beam decreased
by about 23% when the angle of skew increased from 30° to 45° and by another 24% when the skew angle increased to 60°; and, (iii) the deflection of the interior beam decreased by about 26% when the skew angle increased from 30° to 45° and by another 33% when the skew angle increased to 60°. Figure 5.117 shows the effect of angle of skew on the load-strain relationship of different beams in which a concentric simulated truck load was applied at the center of the bridge model. Similar observations as those for the deflections were found for the strains except that the strain in the interior beam was higher than that in the exterior beam. Comparisons between support reactions for different skew angles in a concentric simulated truck loading case are presented in Figure 5.118. The following can be observed: (i) for all skew angles studied, there is a concentration of reaction at the beams close to the obtuse corner; (ii) this reaction concentration increases with increase in the skew angle; and, (iii) the reactions of the beams close to the acute corner decrease with increase in the angle of skew.

Based on the experimental results presented, the following recommendations can be given in the case of skew composite bridges:

1- The presence of skew results in significant reductions in the deflections and the strains in both the exterior and the interior beams. It is expected that similar reductions will be found in the design bending moments. It is suggested that design methods for normal bridges when used to design skew bridges will lead to an extremely conservative design.

2- Concentric loading cases control the design of skew composite bridges. Such loading cases result in higher deflections and strains and hence higher design bending moments than eccentric loading cases.
3- The exterior beam undergoes higher deflection and strain than the interior beam in most of the loading cases. Therefore, the exterior beam will control the design in skew composite bridges.

4- The presence of skew enhances the ultimate load carrying capacity of composite bridges.

5- The presence of skew results in a concentration of reaction at the beam close to the obtuse corner and in a decrease in the reactions of all other beams in the bridge. It is recommended that shear should be considered in the design of skew composite bridges.

5.2.9 Effect of Transverse Intermediate Diaphragms

The bridge models consisted of different cases of diaphragms, namely: (i) bridge model # 4 with end diaphragms only; (ii) bridge model # 5 with shear-connected transverse intermediate diaphragms; (iii) bridge model # 2 with moment-connected transverse intermediate diaphragms; and, (iv) bridge model # 6 with transverse intermediate diaphragms moment-connected to the longitudinal beams and made also composite with the reinforced concrete deck slab. Figures 5.119 to 5.121 show comparisons for deflections, strains, and reactions of the bridge models mentioned above. From these comparisons the following can be observed:

1- The longitudinal beams in bridge model # 4, with end diaphragms only, exhibit higher values of deflections and strains than all other bridge models with different types of transverse intermediate diaphragms. In addition, the reaction distribution in this case is more critical than that in all other bridge models. The reaction at the beam close to
the obtuse corner had a greater value whereas that at the beam close to the acute corner had a smaller value.

2- The deflections and the strains of the longitudinal beams in bridge model # 5, with shear connected transverse intermediate diaphragms, were less than those in the case of bridge model # 4. The presence of shear-connected transverse intermediate diaphragms enhanced the reaction distribution. The reaction at the beam close to the obtuse corner was less than that in the case of bridge model # 4.

3- Moment-connected transverse intermediate diaphragms used in bridge model # 2 resulted in a superior load distribution characteristics in comparison to bridge models # 4 and # 5. The deflections and the strains in the longitudinal beams were significantly reduced and the reaction distribution was much less critical.

4- No much difference was observed in the load distribution characteristics of bridge model # 6, where the moment connected transverse diaphragms were made composite with the deck slab, in comparison to bridge model # 2.

5- The presence of transverse intermediate diaphragms, moment connected to the longitudinal beams resulted in a significant increase in the ultimate load-carrying capacity of bridge model # 2 in comparison to that in the case of bridge models # 4 and # 5.

5.3 Continuous Bridge Models

5.3.1 Bridge Model # 7

Bridge model # 7 was a continuous right composite steel-concrete bridge model with four longitudinal beams. The span ratio (= long span length/short span length) was 1.4. Transverse intermediate diaphragms, moment-connected to the longitudinal beams,
were used for this bridge model. First, the model was loaded elastically using simulated truck loads applied at several critical locations. Figures 5.122 to 5.126 show the experimental and the theoretical load-deflection relationships for different loading cases. In all cases, the good agreement between the two sets of results can be observed. The difference between the two sets of results was less than 8% in most cases. In loading case # 1, an eccentric truck load was applied on the long span of the bridge model. In this case, the exterior beam was the one that exhibited the maximum deflection with a difference of 18% greater than the deflection of the interior beam. It should be noted that all beams in the unloaded short span exhibited upward deflections which represented about 20% of the downward deflections of the beams in the loaded long span. A concentric simulated truck load was applied on the long span of the bridge model in loading case # 2. In this case, the deflections of all the beams were almost the same in the loaded long span. In the unloaded short span both the exterior and the interior beams exhibited an equal upward deflection. In loading case # 3, the bridge was fully loaded by concentric simulated truck load applied on both the long and the short spans. In this case, the beams in both the long and the short spans exhibited downward deflections. It is observed that the deflection of the interior beam was a slightly greater than that of the exterior beam. It should be noted that the deflection of the beams in the long span was much greater than that of the beams in the short span. An eccentric simulated truck load, applied on both the long and the short spans, was used in loading case # 4. In this case, the deflection of the exterior beam was greater than that of the interior beam in both the long and the short spans. The difference between the deflections of the exterior and the interior beams in the long span was about 22%. In loading case # 5, an eccentric
simulated truck load was applied on the long and the short spans opposite to each other. In this case, the deflection of the exterior beam was greater than that of the interior beam by about 26% in the long span and by about 15% in the short span. Comparing the results for eccentric and concentric loading cases revealed that the latter produced the maximum deflection for both the exterior and the interior beams.

The experimental and the theoretical load-strain relationships for different loading cases are shown in Figures 5.127 to 5.131. Good correspondence is observed between the experimental and the theoretical results. The difference between the two sets of results was less than 6% in most cases. In loading case # 1, where an eccentric simulated truck load was applied on the long span of the bridge model, the strain in the exterior beam in the long span was greater than that in the interior beam by about 13%. It should be noted that the strain in the exterior beam in the unloaded short span was also greater than that in the interior beam by about 23%. However, the beams in the loaded long span exhibited tensile strains whereas those in the unloaded short span exhibited compressive strains. Under a concentric simulated truck load applied on the long span of the bridge model in loading case # 2, the following can be observed: (i) in the loaded long span the strain in the interior beam is greater than that in the exterior beam by about 13%; (ii) in the unloaded short span the strains in the exterior and in the interior beams are almost the same; and, (iii) the tensile strain in the beams in the loaded long span are greater than the compressive strain in the beams in the unloaded short span by about 73%. The results presented for other loading cases revealed that the exterior beam was the one that undergoes the maximum strain in the eccentric loading cases whereas in the concentric
loading cases the interior beam exhibited the maximum strain. However, comparing the results for strains in the eccentric and the concentric loading cases showed that the latter produced higher strains both in the exterior and in the interior beams.

Figures 5.132 to 5.134 show the experimental and the theoretical results for the support reactions for different loading cases. The good agreement between the two sets of results can be noted. In loading case # 1, where a concentric simulated truck load was applied on the long span of the bridge model, the following were observed: (i) at the simply supported end of the loaded long span, the reaction of the interior beam was greater than that of the exterior beam by 34%; (ii) the reactions at the interior support are higher than those at the simply supported end of the loaded long span by 84% on the average; and, (iii) uplift forces existed at the simply supported end of the unloaded short span. These forces represented about 38% of the reactions at the simply supported end of the loaded long span. When a concentric simulated truck load was applied on both the long and the short spans, Figure 5.133, the total applied load was distributed as follows: 17% of the total applied load was carried by the simply supported end of the long span; 71% was carried by the intermediate support; and, 12% was carried by the simply supported end of the short span. In the case of an eccentric simulated truck load applied on both the long and the short spans, the reaction of the interior beam was greater than that of the exterior beam at both the simply supported ends and also at the intermediate support. It is observed that uplift forces existed at the exterior beam located on the unloaded side of the bridge model. These uplift forces existed at both the simply supported ends and also at the intermediate support. Comparing results for different
loading cases the following observations were found: (i) the maximum reactions at the simply supported end of the long span are produced by loading the long span only; (ii) the maximum reactions at the simply supported end of the short span are produced by loading the short span only; and, (iii) the maximum reactions at the intermediate support are produced by loading both the long and the short spans.

The bridge model was finally tested to failure using a concentric simulated truck load on both the long and the short spans of the bridge model. The experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of bridge model # 7, over the complete range of loading up to the collapse load, are shown in Figures 5.135 and 5.136. Good agreement between the two sets of results is observed. The deflections and the strains in the beams in the long span are much higher than those in the beams in the short span. It should be noted that the beams in the short span behaved elastically up to the ultimate load of the bridge model. After the collapse of the bridge model, the load was removed and it was observed that the beams in the short span recovered all the deflections and the strains during the application of the load. Figures 5.137 and 5.138 show bridge model # 7 after failure.

5.3.2 Bridge Model # 8

Bridge model # 8 was the same as bridge model # 7 except that the angle of skew was increased to 45° instead of 0° in the case of bridge model # 7. Transverse intermediate diaphragms, moment-connected to the longitudinal beams, were used for this bridge model also. Figures 5.139 to 5.143 show the experimental and the theoretical load-
deflection relationships for different loading cases. Good correspondence is observed between the two sets of results in all loading cases. The difference between the two sets of results was less than 9% in most cases. In loading case # 1, where an eccentric simulated truck load was applied on the long span of the bridge model, the deflection of the exterior beam was greater than that of the interior beam by about 20% compared to 18% in the case of bridge model # 7. Significant reductions in the deflections of the beams were observed in comparison to those in bridge model # 7. These reductions can be summarized as follows: the deflection of the exterior beam in the loaded long span decreased by 14%; the deflection of the interior beam in the loaded long span decreased by 24%; the upward deflection of the exterior beam in the unloaded short span decreased by 49%; and, the upward deflection of the interior beam in the unloaded short span decreased by 23%. A concentric simulated truck load was applied on the long span of the bridge model in loading case # 2. In this case the following observations were found in comparison to bridge model # 7: (i) the exterior beam was the one that undergoes the maximum deflection; (ii) the transverse distribution of the deflection in the loaded long span was not uniform. The side of the bridge model where the acute corner was simply supported and the obtuse corner was continuous exhibited greater deflection than the other side; (iii) the deflection of the exterior beam decreased by 21% whereas that of the interior beam decreased by 25%; and, (iv) the transverse distribution of the upward deflection in the unloaded short span was also not uniform. In loading case # 3, a concentric simulated truck load was applied on both the long and the short spans. In this case the deflection of the exterior beam was greater than that of the interior beam. Compared to bridge model # 7, the deflection of the exterior beam decreased by 14%
whereas that of the interior beam decreased by 22%. The deflection of the beams in the short span represented 27% of that of the beams in the long span. Similar observations can be found for the other loading cases presented. Based on the results presented above, the following observations were found: (i) in all loading cases, the exterior beam was the one that undergoes the maximum deflection; and, (ii) concentric loading cases produced greater deflections, for both the exterior and the interior beams, than eccentric loading cases.

Figures 5.144 to 5.148 show the experimental and the theoretical load-strain relationships for different loading cases. Good agreement is found between the two sets of results. In loading case #1, where an eccentric simulated truck load was applied on the long span of the bridge model, the strain in the interior beam in the loaded long span was higher than that in the exterior beam. The strain in the exterior beam in the loaded long span decreased by 46% in comparison to that in the case of bridge model #7 whereas the strain in the interior beam decreased by 20%. In the unloaded short span, all the beams exhibited a compressive strain which was smaller than that in the case of bridge model #7 by 60% for the exterior beam and 39% for the interior beam. A concentric simulated truck load was applied on the long span in loading case #2. In this case, the transverse distribution of strain was not uniform in both the long and the short spans. In the long span the side of the bridge model where the acute corner was simply supported and the obtuse corner was continuous exhibited higher strain than that on the other side. Compared to bridge model #7, the strain in the exterior beam decreased by 42% whereas that in the interior beam decreased by 23%. It should be noted that all the
beams in the unloaded short span exhibited a compressive strain. In loading case \# 3, the bridge model was fully loaded by concentric simulated truck load on both the long and the short spans. In this case also, the strain in the interior beam was higher than that of the exterior beam. The strains in the beams in the short span were smaller than those in the long span by 56%. Similar observations were found for the other loading cases presented. Comparing the results for the eccentric and the concentric loading cases revealed that the latter was the one that produced the maximum strain in both the exterior and the interior beams. However, in all loading cases the strain in the interior beam was higher than that in the exterior beam.

Figures 5.149 to 5.151 show the experimental and the theoretical results for the support reactions for different loading cases. The good agreement between the two sets of results can be observed. Under a concentric simulated truck load applied on the long span of the bridge model, Figure 5.149, the distribution of the reactions at the simply supported end of the loaded long span was as follows: the reaction of the exterior beam close to the obtuse corner represented 16.5% of the total applied load; 12.8% was carried by the interior beam close to the obtuse corner; 10.5% was carried by the interior beam close to the acute corner; and, 2.4% was carried by the exterior beam close to the acute corner. At the intermediate support, the distribution of reactions was as follows: 28% of the total applied load was carried by the exterior beam close to the obtuse corner of the loaded long span; 21% was carried by the interior beam close to the obtuse corner; 19% was carried by the interior beam close to the acute corner; and, 5.5% was carried by the exterior beam close to the acute corner. All the beams at the simply supported end of the
unloaded short span exhibited uplift forces. The maximum reaction at the simply supported end of the loaded long span increased from 10.1 kN in the case of the rectangular bridge model # 7 to 14.4 kN in the case of bridge model # 8 with skew angle $\theta = 45^\circ$. This represents an increase of about 30%. At the intermediate support, the maximum reaction increased from 19.2 kN in the case of bridge model # 7 to 24.3 kN in the case of bridge model # 8 representing an increase of about 27%. In the case of fully loaded bridge model where a concentric simulated truck load was applied on both the long and the short spans of the bridge model, Figure 5.150, different distribution of reactions were found. The reactions at the simply supported end of the long span were smaller than those found in the previous loading case where the long span was only loaded. As a result of loading both spans the distribution of reactions at the intermediate support was less critical than that in the previous loading case. However, the values of the reactions were higher than those found in the previous loading case. Based on the results of reactions from the several loading cases studied, not shown herein for brevity, the following recommendations can by given:

1- The reactions at the simply supported end of either the long or the short span can be maximized by loading the long or the short span, respectively. This loading pattern will result in a critical reaction distribution with the reactions at the beams close to the obtuse corner being much greater than those at the beams close to the acute corner. In this case either the long or the short span can be treated as a simple-span bridge and the reactions can be distributed based on load-distribution factors of simply supported bridges with reasonable accuracy.

2- The reactions at the intermediate support can be maximized by loading both
spans. Therefore as a result of symmetry in the case of two-equal-span skew bridges, the
distribution of the reactions at the intermediate support is not critical and is very close to
that for right bridges.

3- For two-unequal-span skew bridges the distribution of reactions at the
intermediate support becomes more critical than that in the case of two-equal-span skew
bridges. The reactions of the beams close to the obtuse corner of the long span are
greater than those of the beams close to the acute corner.

4- The recommendations given above for the reactions at the intermediate support
do not apply to the design shear forces at this support. The distribution of shear forces
are more critical than that of the reactions. This point will be discussed in detail for
prototype bridges in chapter VI.

The bridge model was finally tested to failure using an eccentric simulated truck
load on both the long and the short spans of the bridge model. The experimental and the
theoretical load-deflection and load-strain relationships for the ultimate load test of bridge
model # 8, over the complete range of loading up to the collapse load, are shown in
Figures 5.152 and 5.153. Good agreement between the two sets of results is observed.
It should be noted that in this bridge model also, the beams in the short span behaved
elastically up to the failure of the bridge model. After the failure of the bridge model,
and upon the removal of the load it was observed that the beams in the short span
recovered all the deflections and the strains exhibited during the application of the load.
Figures 5.154 and 5.155 show bridge model # 8 after collapse.
5.3.3 Bridge Model # 9

Bridge model # 9 was the same as bridge model # 8 except that the long span length was increased to 2743 mm instead of 2134 mm in the case of bridge model # 8. Therefore the span ratio (= long span length / short span length) increased to 1.8. Figures 5.156 to 5.160 show the experimental and the theoretical load-deflection relationships for the different loading cases. Good agreement is observed between the two sets of results in all the loading cases. The difference between the two sets of results was less than 7% in most cases. In loading case # 1, when an eccentric simulated truck load was applied on the long span of the bridge model, the exterior beam was the one that exhibited the maximum deflection. As a result of increasing the long span length, the following changes in the deflections of the beams were observed in comparison to bridge model # 8: (i) the deflection of the exterior beam in the loaded long span increased by 52%; (ii) the deflection of the interior beam in the loaded long span increased by 76%; (iii) the upward deflection of the exterior beam in the unloaded short span increased by 37%; and, (iv) the upward deflection of the interior beam in the unloaded short span increased by 46%. Similar observations were found in loading case # 2 where a concentric simulated truck load was applied on the long span of the bridge model. However, different observations were found in loading case # 3 where a concentric simulated truck load was applied on both the long and the short spans. In this case the deflections of the beams in the long span increased significantly in comparison to those in the case of bridge model # 8, as a result of increasing the long span length. The deflection of one of the exterior beams in the short span also increased. However, the deflections of all other beams in the short span decreased by different percentages. In loading case # 4, where an eccentric
simulated truck load was applied on both the long and the short spans, the deflection of all the beams in the long span increased significantly whereas those of the beams in the short span of the bridge model decreased.

Figures 5.161 to 5.165 show the experimental and the theoretical load-strain relationships for different loading cases. Good correspondence is observed between the two sets of results. In loading case # 1, where an eccentric simulated truck load was applied on the long span of the bridge model, the tensile strain of the exterior beam in the loaded long span increased by 27% in comparison to that in bridge model # 8 as a result of increasing the long span length whereas that of the interior beam increased by 8% only. In the unloaded short span, the compressive strains of all the beams increased by different percentages in comparison to those in the case of bridge model # 8. In the case of a concentric simulated truck load applied on the long span of the bridge model, all the beams in the loaded long span exhibited tensile strains whereas those in the unloaded short span exhibited compressive strains. The values of both the tensile and the compressive strains were much higher in comparison to those in bridge model # 8, as a result of increasing the long span length. In a fully loaded bridge case, where a concentric simulated truck load was applied on both the long and the short spans, the beams in both spans exhibited tensile strains which were much higher than those in the case of bridge model # 8. These increases in the strains were as follows: the strain in the exterior beam in the loaded long span increased by 54%; the strain in the interior beam in the loaded long span increased by 19%; the strain in the exterior beam in the unloaded short span increased by 30%; the strain in the interior beam in the unloaded short span
increased by 19%. From the results for strains presented for different loading cases, the following can be observed: (i) the exterior beam was more significantly affected by increasing the long span length than the interior beam; (ii) when only the long span is loaded the strains in both the exterior and the interior beams in the unloaded short span increased significantly by increasing the long span length; and, (iii) the strains in both the exterior and the interior beams were more significantly affected by changing the long span length in concentric than eccentric loading cases.

Figures 5.166 to 5.168 show the experimental and the theoretical results for the support reactions for different loading cases. Good agreement is observed between the two sets of results. Significant changes in the reactions were observed in comparison to those in bridge model # 8, as a result of increasing the long span length. In the case of a concentric simulated truck load applied on the long span of the bridge model, Figure 5.166, the following observations were found in the reactions compared to those in the case of bridge model # 8: (i) the reactions at the simply supported end of the loaded long span decreased significantly; (ii) the reactions at the intermediate support increased; and, (iii) the uplift forces at the simply supported end of the unloaded short span increased. Different observations were found in the case of a concentric simulated truck load applied on both the long and the short spans, Figure 5.167. At the simply supported end of the long span all the reactions decreased significantly in comparison to those in bridge model # 8 except the reaction of the exterior beam close to the acute corner. At the intermediate support, the reactions of the beams at the side of the bridge model where the obtuse corner of the long span was simply supported and the acute corner was continuous.
decreased whereas the reactions of the beams located on the other side of the bridge model increased. At the simply supported end of the short span, the reactions of all the beams decreased except the reaction of the exterior beam close to the obtuse corner.

After the elastic loadings were completed, the bridge model was tested to failure using a concentric simulated truck load applied on both the long and the short spans. Figures 5.169 and 5.170 shows the experimental and the theoretical load-deflection and load-strain relationships for the ultimate load test of bridge model # 9. Good correspondence is observed between the two sets of results. The deflections and the strains in the beams in the long span were much higher than those in the beams in the short span. Figure 5.171 shows bridge model # 9 after failure.

5.3.4 Mode of Failure of the Continuous Bridge Models

For the continuous bridge models tested the final mode of failure can be summarized as follows: (i) cracking as well as crushing of the concrete on the top surface of the deck slab; (ii) cracking of the concrete on the bottom surface of the deck slab; (iii) yielding of the steel of the longitudinal steel beams; and, (iv) cracking of the concrete on the top surface of the deck slab as well as yielding of the steel beams at the pier support. It is expected also that the top reinforcement at the pier support will yield. The size of the steel reinforcement in the slab around the pier was too small to be instrumented with strain gauges. However, there were some strain gauges on the concrete surface around the pier support. The first cracking loads of the concrete deck around the pier support are shown in Table 5.1. On further loading beyond the cracking load, the readings of the gauges on the top surface of the concrete deck around the pier support became unreliable.
due to the presence of wide cracks. The failure modes of bridge models # 7, 8, 9 with top and bottom views are shown in Figures 5.137 and 5.138, 5.154 and 5.155, 5.171 (top view for bridge model # 9), respectively.

5.3.5 Effect of Angle of Skew

Figures 5.172 to 5.174 show the effects of angle of skew on the deflections, strains, and reactions by comparing the results of bridge model # 7 to those of bridge model # 8. From the comparison the effects of skew on the response of two-unequal-span composite bridges can be summarized as follows:

1- The presence of skew results in significant reductions in the deflections and the strains in both the exterior and in the interior beams. These reductions are more significant for the interior beams than those for the exterior beams. Similar reductions are expected in the design girder bending moments. It should be noted that concentric loading cases produce greater deflections and strains than eccentric loading cases.

2- The distribution of reactions at the simply supported ends of a skew composite bridge is more critical than that in a normal right composite bridge. The reactions of the beams close to the obtuse corner increase with the increase in the skew whereas those of the beams close to the acute corner decrease.

3- The distribution of reactions at the intermediate support of a two-equal-span continuous composite bridge is not significantly affected by skew. However, in a two-unequal-span continuous composite bridge the distribution of reactions at the intermediate support becomes more critical with skew. The reactions of the beams located at the side of the bridge where the acute corner of the long span is simply supported and the obtuse
corner is continuous increase with an increase in the angle of skew. However, in this case the reactions of the beams located on the other side of the bridge model decrease.

5.3.6 Effect of Spans Ratio

Figures 5.175 to 5.177 show the effect of the span ratio (= long span length/short span length) on the deflections, strains, and reactions. It should be noted that studying the effect of the spans ratio on the response can be done by either changing the long span length, while fixing the short span length or by changing the short span length while fixing the long span length. In the experimental study, the effect of the spans ratio was studied by changing the long span length while fixing the short span length. The long span length was increased from 2134 in bridge model # 8 to 2743 in bridge model # 9. Therefore, the spans ratio was increased from 1.4 to 1.8. Based on the experimental results the effects of the spans ratio can be summarized as follows:

1- Increasing the spans ratio results in significant increases in the deflections and the strains of the beams in the long span. The deflections and the strains of the beams in the short span increase also when only the long span is loaded. However, the deflections and the strains in the beams in the short span decrease when both the long and the short spans are loaded. The deflections and the strains in the exterior beams in both the long and the short spans are more significantly affected by changing the spans ratio than those in the interior beams.

2- Increasing the span ratio results in significant changes in the distribution of reactions at the simply supported ends of the bridge model and also at the intermediate support. These changes depend on the loading conditions.
CHAPTER VI

RESULTS FROM THE PARAMETRIC STUDIES

PART I: ELASTIC LOAD DISTRIBUTION IN SIMPLY SUPPORTED COMPOSITE BRIDGES

6.1.1 General

North American codes of practice do not provide the design engineer with sufficient guidance regarding load-distribution factors of simply supported skew composite bridges. The American Association of State Highway and Transportation Officials (AASHTO, 1994) has traditionally applied a load-distribution factor for both moment and shear, depending on the center to center girder spacing only. However, the AASHTO ignores the following significant parameters: (i) angle of skew; (ii) bridge aspect ratio; (iii) longitudinal and transverse rigidities; (iv) number of lanes; (v) number of girders; (vi) eccentric truck loading cases; and, (vii) the effect of dead load. The Ontario Highway Bridge Design Code (OHBDC, 1992) considers some parameters in addition to the girder spacing in establishing load-distribution factors for moment and suggests a skew parameter. The presence of skew reduces the longitudinal moments in the girders. However, it also causes high concentration of shear in the girder closest to the obtuse
corner and it reduces it in the girder closest to the acute corner as well as in the interior girders. Therefore there is a need for the generation of a database for both shear distribution factors and moment distribution factors leading to the formation of empirical formulas for the design of skew composite bridges.

In this part, the influence of several parameters on the shear-distribution and moment-distribution factors in simply supported skew composite steel-concrete bridges are studied. These parameters are: angle of skew, girder spacing, bridge aspect ratio, number of lanes, number of girders, end diaphragms, number of intermediate cross-beams, and loading conditions. A detailed parametric study is conducted on prototype composite steel-concrete bridges subjected to OHBDC truck loading and to dead load as well. The parametric study included more than 700 cases. The finite element modelling of the prototype bridges was verified and substantiated by means of experimental results from tests on six simply-supported skew composite steel-concrete bridge models described in chapter V. Based on this study empirical formulas for shear-distribution and moment-distribution factors, for both OHBDC truck loading and dead load, are generated.

6.1.2 Parametric Study

The objectives of the parametric study were: (i) to investigate the influence of all major variables affecting shear-distribution and moment-distribution between girders in the elastic range of loading; (ii) to generate a database for shear-distribution and moment-distribution factors including more than 700 bridge cases; and, (iii) to develop empirical formulas for elastic shear-distribution and moment-distribution factors corresponding to
OHBDC truck loading as well as to dead load. The parameters chosen for this study were: angle of skew, bridge aspect ratio, girder spacing, number of lanes, number of girders, end diaphragms, and intermediate cross-beams. The parametric study was based on the following assumptions: (i) the reinforced concrete deck slab and the supporting steel I-girders are in full composite action; (ii) all transverse diaphragms are moment-connected to the longitudinal girders; (iii) all materials are elastic and homogeneous; and, (iv) the effects of the curbs are ignored. The details of the parametric study will be described in the following sections.

6.1.2.1 Bridge Geometry

The number of lanes considered was 2, 3, and 4 lanes with bridge width of 8.4 m for two-lane bridges, 12.6 m for three-lane bridges, and 16.2 m for four-lane bridges. The span lengths were 16.2 m, 19.8 m, and 23.4 m. Based on the bridge widths and span lengths, the bridge aspect ratio (= span length / total width) varied from 1.0 to 2.9. The number of girders was 3, 4, and 5 for two-lane bridges, 4, 5, and 6 for three-lane bridges, and 5, 6, and 7 for four-lane bridges. Based on the bridge widths and number of girders, the girder spacing ranged from 1.5 m to 3.8 m. The angles of skew were 0°, 30°, 45°, and 60°.

6.1.2.2 Bridge Structural Properties

The steel sections used for the longitudinal girders were W610x155 for span length 16.2 m, W760x185 for span length 19.8 m, and W760x196 for span length 23.4 m. The
bridges were analyzed with the following steel grid conditions: bridges with no diaphragms; bridges with end diaphragms only; and, bridges with 1, 2, 3, 4, 5, and 6 lines of intermediate transverse diaphragms moment-connected to the longitudinal girders. The steel sections used for the transverse diaphragms were W360x33 for span length 16.2 m, W360x91 for span length 19.8 m, and W360x101 for span length 23.4 m. The longitudinal girders and the transverse diaphragms were of grade G40.21-M300W. For all bridges, 225-mm thick concrete deck slab of 30 MPa compressive strength was used and reinforced top and bottom by a steel mesh having an area of 400 mm²/m. All bridges were designed according to OHBD Code (1992).

6.1.2.3 Loading Conditions

Many loading cases were considered in the parametric study, using an OHBDC standard truck. The more than 700 bridge cases considered can be classified into five main categories, each comprising of more than 120 cases, according to the loading patterns. The categories were as follows: (i) one or two trucks applied on one side of the bridge representing an eccentric truck loading. The trucks were applied close to one of the supports to maximize the reactions; (ii) a concentric truck loading in which the bridge was fully loaded by two trucks for two-lane bridges, three trucks for three-lane bridges, and four trucks for four-lane bridges. The trucks were applied close to one of the supports to maximize the reactions; (iii) an eccentric truck loading similar to (i) except that the trucks were applied close to the mid span to maximize the longitudinal girder moments; (iv) a concentric truck loading similar to (ii) except that the trucks were applied
close to the mid span to maximize the longitudinal girder moments; and, (v) the dead load of the bridge in which the total weights of the reinforced concrete deck slab, the longitudinal steel girders, and the transverse steel diaphragms were smeared and applied on the bridge as a uniformly distributed load.

6.1.3 Shear Distribution Factor

In order to determine the shear distribution factor $D$, the maximum reaction $(V)$ of a simply supported composite girder under the effect of a line of wheel loads of an OHBDC truck was first calculated for all the prototype bridges studied in the parametric study. Based on the experimentally-calibrated finite element model the maximum reaction in each girder $(V_{\text{max}})$ was obtained for each prototype bridge. The shear distribution factor $D$ was then calculated from the following relationship:

$$D = \frac{V}{V_{\text{max}}} \quad (6.1)$$

The effect of the various design parameters on the shear distribution factor, $D$, is presented now:

6.1.3.1 Effect of Angle of Skew

The effect of skew on the shear distribution factor $D$ for the prototype bridges was examined for three types of loading: eccentric and concentric OHBDC truck loadings as well as for the dead load. The results show that the angle of skew is the most critical parameter that influences the shear distribution factor. The results confirmed the
recommendations of Helba and Kennedy (1994) that shear should be considered in the design of skew composite bridges. Figure 6.1 shows the effect of skew on the shear distribution factor for the girder close to the obtuse corner for a four-lane bridge with different aspect ratios and under concentric loading. It can be observed that the shear distribution factor decreases with increase in the angle of skew, $\theta$. The rate of this decrease for $\theta$ between $30^\circ$ and $60^\circ$ is greater than that for $\theta$ between $0^\circ$ and $30^\circ$. It is observed also that the effect of aspect ratio on the factor D increases with increase in $\theta$. Figure 6.2 shows the effect of skew on the shear distribution factor for the girder close to the acute corner for a four-lane bridge with different aspect ratios and under concentric truck loading. The shear distribution factor increases with increase in the angle of skew. This increase is higher for $\theta$ between $30^\circ$ and $60^\circ$ than that for $\theta$ between $0^\circ$ and $30^\circ$. It is also noted that an increase in the aspect ratio augments this increase. Similar observations were found for the effect of skew on the shear distribution factor in the case of an interior girder of a four-lane bridge under concentric loading (Fig. 6.3). However, the values of the shear distribution factors are higher for the girder close to the acute corner than those for the interior girders which indicates that the reactions for the interior girders are greater than that for the girder close to the acute corner. Figures 6.4 to 6.6 show the effect of skew on the shear distribution factor in the case of eccentric truck loading for a girder close to the obtuse corner, a girder close to the acute corner, and for an interior girder, respectively. The effects of skew in this case were similar to those in the case of concentric truck loading. It should be noted that the eccentric truck loading produces the most critical conditions for shear design of the girder close to the obtuse
corner and the girder close to the acute corner. However, for the interior girders the concentric loading cases are more critical than the eccentric loading cases. Figures 6.7 to 6.9 show the effect of skew on the shear distribution factor for a girder close to the obtuse corner, a girder close to the acute corner, and for an interior girder of a two-lane bridge under dead load. The results reveal the following: (i) the factor D decreases with increase in the angle of skew for the girder close to the obtuse corner; (ii) it increases with increase in skew for the girder close to the acute corner; and, (iii) it increases with increase in skew for an interior girder. Therefore there is a concentration of reaction at the girder close to the obtuse corner in the case of dead load also. It is suggested that the reactions due to the dead load be estimated carefully in the case of skew composite steel-concrete bridges.

6.1.3.2 Effect of Bridge Aspect Ratio

The results presented in Figure 6.10 reveal the influence of aspect ratio upon the shear distribution factor. These results are for the reactions at the end of the girder close to the obtuse corner of a two-lane bridge under concentric truck loading. It can be observed that the shear distribution factor decreases with increase in the aspect ratio. While this decrease is small for skew angles between 0° and 30°, it becomes significant for skew angles between 45° and 60°. Figure 6.11 shows the effect of aspect ratio on the shear distribution factor for the girder close to the acute corner of a three-lane bridge under eccentric truck loading. It is observed that the shear distribution factor decreases with an increase in the aspect ratio. This was also the case for the girder close to the
obtuse girder of a bridge under eccentric truck loading. However, for the interior girders the shear distribution factor increases with an increase in the aspect ratio.

Figure 6.12 shows the effect of aspect ratio on the shear distribution factor for an interior girder of a four-lane bridge under dead load. The following can be observed: (i) the factor increases with an increase in the aspect ratio for angle of skew $0 = 0^\circ$; (ii) the factor decreases with an increase in the aspect ratio for angles of skew $30^\circ$, $45^\circ$, and $60^\circ$; and (iii) the effect of aspect ratio on the shear distribution factor is almost the same for skew angles $30^\circ$, $45^\circ$, and $60^\circ$. For the girder close to the obtuse corner as well as the girder close to the acute corner, the shear distribution factor decreases with an increase in the bridge aspect ratio.

6.1.3.3 Effect of Girder Spacing

The spacing of longitudinal girders, $S$, is one of the most important factors affecting the shear distribution factor. This is reflected in the code of the American Association of State Highway and Transportation Officials (AASHTO, 1994) where the girder moments and shear may be calculated using the wheel load fraction of $S/5.5$. The results in this study reveal that the girder spacing has a significant influence on the shear distribution factor, $D$. The girder spacing is a factor of the bridge width and the number of longitudinal girders. The bridge width can be taken as the number of lanes multiplied by the lane width which is constant in most cases. Therefore the girder spacing could be related to the number of girders and the number of lanes without reference to the lane
width. Thus, it is suggested that a measure for the girder spacing be expressed by a
dimensionless factor, \( N \), defined by:

\[
N = \frac{\text{number of lanes}}{\text{number of girders}}
\]  
(6.2)

The relations between the shear distribution factor and the ratio \( N \) for the girder
close to the obtuse corner of a two-lane bridge with different angles of skew are presented
in Figure 6.13 for a concentric truck loading. It can be observed that the shear
distribution factor decreases with increase in the ratio \( N \). The effect is almost the same
for all skew angles. Similar effects were found for the girder close to the acute corner
and for an interior girder. The same observations were found for eccentric truck loading
cases. Figure 6.14 shows the effect of the ratio \( N \) on the shear distribution factor for the
girder close to the acute corner of a three-lane bridge under eccentric loading. The shear
distribution factor decreases with an increase in the ratio \( N \) for all skew angles. Different
variations were found under dead load. Figure 6.15 shows the effect of the ratio \( N \) on
the shear distribution factor for an interior girder of a four-lane bridge under dead load.
Under such load the shear distribution factor increases with an increase in the ratio \( N \) for
all angles of skew. Similar effects were found for the girder close to the obtuse corner
and for the girder close to the acute corner. It should be noted that an increase in the
shear distribution factor with increase in the ratio \( N \) in the case of dead load does not
necessarily mean a decrease in the value of the shear, since the shear in a simple girder
under dead load also changes with variation in the girder spacing.
6.1.3.4 Effect of Transverse Diaphragms

The effect of transverse diaphragms on the shear distribution was investigated for numerous prototype bridges in the following categories: (i) bridges with neither end diaphragms nor intermediate transverse diaphragms; (ii) bridges with only end diaphragms torsionally as well as flexurally stiff; and, (iii) bridges with only intermediate diaphragms torsionally as well as flexurally stiff and moment-connected to the longitudinal girders. The number of such intermediate diaphragms were varied from one to six. The longitudinal and transverse flexural rigidities were calculated for all the prototype bridges using the method presented by Kennedy and Grace (1983). Thus the flexural rigidity ratio, $R$, were calculated as:

$$ R = \frac{D_y}{D_x} \times 100 $$  \hspace{1cm} (6.3)

where: $D_x = \text{flexural rigidity in the longitudinal direction per unit width}$; and, $D_y = \text{flexural rigidity in the transverse direction per unit length}$.

The relation between the flexural rigidity ratio, $R$, and the shear distribution factor is presented in Figure 6.16 for the girder close to the obtuse corner of a four-lane bridge with different skew angles and under concentric truck loading. It is observed that the shear distribution factor increases with increase in the flexural rigidity ratio which means that the presence of moment-connected transverse intermediate diaphragms decreases the concentration of the reaction of the girder close to the obtuse corner. Skew appears to augment this effect. However, this increase levels out at a flexural rigidity ratio of about 18%. Figure 6.17 shows the effect of the flexural rigidity ratio on the shear distribution.
factor for the girder close to the acute corner of a three-lane bridge under eccentric loading. In this case the shear distribution factor decreases with an increase in the flexural rigidity ratio. The effect of the flexural rigidity ratio, R, on the shear distribution factor for an interior girder of a two-lane bridge under eccentric loading is presented in Figure 6.18. In this case also the shear distribution factor decreases with an increase in the flexural rigidity ratio. Furthermore the effect of the flexural rigidity ratio on the shear distribution factor is more significant in the case of eccentric truck loading than in the case of concentric truck loading as well as in the case of dead load. Comparing results for the three categories of bridges mentioned earlier shows that the presence of only end diaphragms has no significant effect on the distribution of shear, Figure 6.19.

6.1.3.5 Empirical Formulas for the Shear Distribution Factor, D

From the results of the parametric study carried out on some 400 cases of prototype bridge cases it became evident that the shear distribution factor, D, is governed by the following parameters: (i) angle of skew θ, in degrees; (ii) bridge aspect ratio, A_r; (iii) girder spacing expressed as the dimensionless parameter N, equal to the number of lanes/number of girders; and, (iv) the flexural rigidity ratio, R, in percent. Using a statistical package for best fit empirical formulas were generated for the factor D in terms of the above parameters. The shear distribution factors were determined for: (i) a girder close to the obtuse corner; (ii) a girder close to the acute corner; and, (iii) an interior girder. The loading conditions included concentric, eccentric OHBDC truck loadings as well as dead load. Based on preliminary results, it was revealed that more accurate
predictions for the shear distribution factor, D, are obtained if the best fit formulas are presented for two ranges of skew angles namely: \(\theta < 30^\circ\), and \(\theta > 30^\circ\). For a bridge with \(\theta=30^\circ\), the value of D is calculated from the equations listed below for these two ranges, with \(\theta=30^\circ\), and the smaller value of D is used for conservative design. Thus:

For girder close to the obtuse corner of a bridge under concentric OHBDC truck loading

For \(\theta < 30^\circ\):-

\[
D = \frac{0.51 \times R^{0.05} \times A_r^{0.05}}{(1 + \theta)^{0.04} \times N^{0.77}}
\]  
(6.4)

For \(\theta > 30^\circ\):-

\[
D = \frac{1.75 \times R^{0.15} \times A_r^{0.23}}{\theta^{0.5} \times N^{0.99}}
\]  
(6.5)

For girder close to the obtuse corner of a bridge under eccentric OHBDC truck loading

For \(\theta < 30^\circ\):-

\[
D = \frac{0.5 \times R^{0.07} \times A_r^{0.05}}{(1 + \theta)^{0.06} \times N^{0.64}}
\]  
(6.6)

For \(\theta > 30^\circ\):-

\[
D = \frac{2.6 \times R^{0.15} \times A_r^{0.22}}{\theta^{0.6} \times N^{0.72}}
\]  
(6.7)
For girder close to the obtuse corner of a bridge under dead load

For $\theta < 30^\circ$:

$$D = \frac{1.25 \times R^{1.19} \times N^{0.57}}{(1 + \theta)^{0.08} \times A_r^{0.11}}$$  \hspace{1cm} (6.8)

For $\theta \geq 30^\circ$:

$$D = \frac{3.3 \times R^{0.81} \times A_r^{0.51} \times N^{0.32}}{\theta^{0.84}}$$  \hspace{1cm} (6.9)

For girder close to the acute corner of a bridge under concentric OHBDC truck loading

For $\theta < 30^\circ$:

$$D = \frac{0.59 \times (1 + \theta)^{0.08} \times R^{0.012}}{A_r^{0.02} \times N^{0.76}}$$  \hspace{1cm} (6.10)

For $\theta \geq 30^\circ$:

$$D = \frac{0.06 \times \theta^{0.77}}{R^{0.012} \times A_r^{0.13} \times N^{0.63}}$$  \hspace{1cm} (6.11)

For girder close to the acute corner of a bridge under eccentric OHBDC truck loading

For $\theta < 30^\circ$:

$$D = \frac{0.7 \times (1 + \theta)^{0.08} \times R^{0.03}}{A_r^{0.05} \times N^{0.07}}$$  \hspace{1cm} (6.12)
For $\theta > 30^\circ$: 

$$D = \frac{0.07 \times \theta^{0.77} \times R^{0.02}}{A_r^{0.13} \times N^{0.06}}$$  \hspace{1cm} (6.13)$$

For girder close to the acute corner of a bridge under dead load 

For $\theta < 30^\circ$: 

$$D = \frac{1.58 \times (1 + \theta)^{0.018} \times R^{0.08} \times N^{0.61}}{A_r^{0.22}}$$  \hspace{1cm} (6.14)$$

For $\theta > 30^\circ$: 

$$D = \frac{0.62 \times \theta^{0.29} \times R^{0.06} \times N^{0.54}}{A_r^{0.31}}$$  \hspace{1cm} (6.15)$$

For interior girder of a bridge under concentric OHBDC truck loading 

For $\theta < 30^\circ$: 

$$D = \frac{0.43 \times (1 + \theta)^{0.05} \times A_r^{0.05}}{R^{0.01} \times N^{1.26}}$$  \hspace{1cm} (6.16)$$

For $\theta > 30^\circ$: 

$$D = \frac{0.11 \times \theta^{0.45} \times R^{0.0004}}{A_r^{0.02} \times N^{1.23}}$$  \hspace{1cm} (6.17)$$
For interior girder of a bridge under eccentric OHBDC truck loading

For \( \theta < 30^\circ \):

\[
D = \frac{0.72 \times (1 + \theta)^{0.1} \times A_r^{0.14}}{R^{0.06} \times N^{0.6}}
\]  

(6.18)

For \( \theta > 30^\circ \):

\[
D = \frac{0.07 \times 6^{0.76} \times A_r^{0.45}}{R^{0.07} \times N^{0.33}}
\]  

(6.19)

For interior girder of a bridge under dead load

For \( \theta < 30^\circ \):

\[
D = 0.83 \times (1 + \theta)^{0.02} \times R^{0.12} \times A_r^{0.12} \times N^{0.13}
\]  

(6.20)

For \( \theta > 30^\circ \):

\[
D = \frac{0.23 \times 6^{0.17} \times R^{0.3} \times A_r^{0.21}}{N^{0.06}}
\]  

(6.21)

6.1.3.6 Illustrative Example

Consider a three-lane bridge with five girders composite with a concrete deck slab designed according to Ontario Highway Bridge Design Code (OHBDC, 1992). The bridge details are as follows: Span of bridge = 20 m; total bridge width = 13.2 m; angle
of skew = 60°, girder spacing = 3 m, deck slab thickness = 0.225 m, five longitudinal
girders W760x185; and, three lines of moment-connected transverse diaphragms
W360x33. It is required to calculate the maximum design shear forces for the different
girders of the bridge. The aspect ratio, \( A_y = 20/13.2 = 1.52 \), and from (6.2) the ratio \( N = 3/5 = 0.6 \). Applying the method presented by Kennedy and Grace (1983), the flexural
rigidity in the longitudinal direction \( D_x = (226.5)(10^4)(E) \), where \( E = \) modulus of elasticity
of the concrete slab, and the flexural rigidity in the transverse direction considering the
effect of the transverse diaphragms, \( D_y = (25.7)(10^4)(E) \). From (6.3) the rigidity ratio \( R = \frac{(25.7)(10^4)(E)}{(226.5)(10^4)(E)} \times 100 = 11.3\% \). Applying a line of wheel loads of an
OHBDC truck on a simple composite girder with a slab of width 3 m and with span 20
m, the maximum end shear \( V = 244 \) kN, and due to its dead load \( V_{dl} = 189 \) kN. The
shear distribution factor, \( D \), for the girder close to the obtuse corner of the bridge under:
concentric truck loading and using (6.5), \( D = 0.59 \); therefore \( V_{max} = V/D = 244/0.59 = 414 \)
kN; under eccentric truck loading using (6.7), \( D = 0.51 \); thus \( V_{max} = 244/0.51 = 478 \); and,
under dead load, using (6.9), \( D = 0.79 \); or \( V_{max} = 189/0.79 = 239 \) kN. For the girder
close to the acute corner of the bridge under: concentric truck loading, using (6.11), \( D = 1.84 \); or, \( V_{max} = 244/1.84 = 133 \) kN; under eccentric truck loading using (6.13), \( D = 1.62 \);
or, \( V_{max} = 244/1.62 = 151 \) kN; and for the dead load using (6.15), \( D = 1.57 \); thus \( V_{max} = 189/1.57 = 120 \) kN. For an interior girder of the bridge under concentric truck loading,
using (6.17), \( D = 1.25 \); or, \( V_{max} = 244/1.25 = 195 \) kN; under eccentric truck loading, using
(6.19), \( D = 1.95 \); or, \( V_{max} = 244/1.95 = 125 \) kN; and, under the bridge dead load, using
(6.21), \( D = 1.1 \); and hence \( V_{max} = 189/1.1 = 172 \) kN.
In order to estimate the dynamic load allowance the first flexural frequency of the bridge was calculated as $\omega = 9.9$ Hz. Then, from the OHBD Code (1992) the corresponding dynamic load allowance = 25%. Thus the maximum design shear forces may be calculated as follows: For the girder close to the obtuse corner $V_{\text{design}} = 239 + (478)(1.25) = 837$ kN; for the girder close to the acute corner $V_{\text{design}} = 120 + (151)(1.25) = 309$ kN; and, for an interior girder $V_{\text{design}} = 172 + (195)(1.25) = 416$ kN. These values are used to design the shear connectors, the abutments and bearing plates for the bridge. It is important to observe the large differences in shear carried by the various girders in this bridge. Such large differences can be quite critical in the design of skew bridges in regions of high seismicity.

6.1.3.7 Summary of Findings

The conclusions found based on the above parametric study can be summarized as follows: (i) skew is the most critical parameter that influences the shear distribution in composite bridges. Increase in the skew angle reduces the shear distribution factor for the girder close to the obtuse corner, and it increases it for the girder close to the acute corner and for interior girders. This influence becomes more significant for skew angles greater than 30°; (ii) the shear distribution factor is very sensitive to a change in the girder spacing expressed in terms of the dimensionless factor, $N$ (=number of lanes/number of girders). An increase in the ratio, $N$, significantly reduces the shear distribution factor for all girders in the bridge; (iii) the presence of intermediate transverse diaphragms moment-connected to the longitudinal girders enhances the distribution of shear forces between
girders. An increase in the rigidity ratio, R (=transverse rigidity:longitudinal rigidity), increases the shear distribution factor for the girder close to the obtuse corner and reduces it for the girder close to the acute corner and for an interior girder. There appears to be a limiting value for R of about 18% beyond which no significant influence on the shear distribution factor is observed; and, (iv) an increase in the bridge aspect ratio reduces the shear distribution factor for all bridge girders supported by the abutment closest to the truck loading. The effect of the bridge aspect ratio increases with increase in the skew angle.

6.1.4 Moment Distribution Factor

In order to derive the moment distribution factor D, the maximum moment (M) of a simply supported composite girder under the effect of a line of wheel loads of an OHBDC truck or under the effect of dead load was first calculated for each prototype bridge. Then, the linear finite element model was used to obtain the maximum moment (M_{max}) in all the girders for each prototype bridge in the parametric study. The moment distribution factor D was then calculated from the following relationship:

$$D = \frac{M}{M_{max}} \quad (6.22)$$

The effect of different parameters on the moment distribution factor, D, will be presented in the following sections.
6.1.4.1 Effect of Angle of Skew

The results show that the angle of skew, \( \theta \), is the most important factor that influences the distribution of moments in composite bridges. The moment distribution factor \( D \) for the prototype bridges was determined for three types of loading: eccentric OHBDC truck loading, concentric OHBDC truck loading, and for the dead load. The girders of each prototype bridge were classified into two categories which are exterior girder and interior girder. Figure 6.20 shows the effect of skew on the moment distribution factor for an exterior girder of a two-lane bridge under concentric truck loading with different aspect ratios. It is observed that skew has no significant effect on the moment distribution factor for skews between 0\(^\circ\) and 30\(^\circ\). However, the moment distribution factor increases significantly with increase in skew for \( \theta \) between 30\(^\circ\) and 60\(^\circ\). It is also observed that the effect of aspect ratio on the moment distribution factor increases with increase in the skew angle.

Figure 6.21 shows the effect of skew on the moment distribution factor for an interior girder of a two-lane bridge with different aspect ratios and under concentric truck loading. It can be noted that the increase in the moment distribution factor with skew is greater than that in the case of an exterior girder. However, the value of the moment distribution factor for a right bridge, \( \theta = 0^\circ \), is almost the same for both the exterior and the interior girders. Thus, for the design of skew bridges the exterior girder is the controlling girder. Figures 6.22 and 6.23 show the effect of skew on the moment distribution factor of an exterior and an interior girders of a three-lane bridge under
eccentric truck loading. The effect of skew on the moment distribution factor under eccentric loading is similar to that in the case of concentric truck loading. However, the values of the moment distribution factors in the case of concentric truck loading are lower than those in the case of eccentric truck loading. This implies that the critical loading case for the design of skew bridges is the concentric truck loading case and thus for one-lane or two-lane bridges the concentric loading case will control the design since for such bridges the load modification factor equals to one. However, because of the reduction in the load intensity for multilane loading (OHBDC, 1992), in the case of bridges with three or more lanes both eccentric and concentric loading cases should be considered.

The effect of skew on the moment distribution factor for an exterior girder of a four-lane bridge under dead load is presented in Figure 6.24. It can be noted that this factor increases with increase in the angle of skew, the rate of this increase being much greater for $\theta$ between 30° and 60°. Similar observations were found for the effect of skew on the moment distribution factor of an interior girder of a four-lane bridge under dead load as shown in Figure 6.25.

6.1.4.2 Effect of Bridge Aspect Ratio

The results presented in Figure 6.26 reveal the effect of aspect ratio on the moment distribution factor for an exterior girder of a two-lane bridge under concentric loading. The moment distribution factor decreases with an increase in the aspect ratio for all skew angles. It is observed that in this case there is no interaction between the skew
angle and the aspect ratio. In the case of an interior girder of a bridge under concentric truck loading, the effect of aspect ratio on the moment distribution factor can be summarized as follows: (i) the aspect ratio has no effect on the moment distribution factor for a right bridge, $\theta = 0^\circ$; and, (ii) the effect of an increase in the aspect ratio on the moment distribution factor becomes significant for skew angles between $45^\circ$ and $60^\circ$. Figure 6.27 presents the results for the effect of aspect ratio on the moment distribution factor for an interior girder of a three-lane bridge under eccentric loading. The factor decreases with increase in the aspect ratio. While this decrease is small for skew angles $0^\circ$ and $30^\circ$, it becomes significant for skew angles $45^\circ$ and $60^\circ$. Different results were found for the effect of aspect ratio on the moment distribution factor for the exterior girder of a bridge under eccentric truck loading. In this case, the factor increases significantly with increase in the aspect ratio for all skew angles. Figure 6.28 shows the effect of aspect ratio on the moment distribution factor for the exterior girder of a four-lane bridge under dead load. In this case the factor decreases with increase in the aspect ratio. Similar observations were found for the effect of aspect ratio on the moment distribution factor for the interior girder of a bridge under dead load.

### 6.1.4.3 Effect of Girder Spacing

A dimensionless factor $N (= \text{number of lanes}/ \text{number of girders})$ was suggested as a measure for the girder spacing. The factor was explained in detail in section 6.1.3.3. Figure 6.29 shows the effect of the ratio $N$ on the moment distribution factor for an exterior girder of a two-lane bridge under concentric truck loading. It is observed that the
factor decreases significantly with increase in the ratio N. The effect is almost the same for all the skew angles considered. For an interior girder the effect of the ratio N on the factor is almost the same as in the case of an exterior girder. Figure 6.30 shows the effect of the ratio N on the moment distribution factor for an interior girder of a three-lane bridge under eccentric truck loading. It is noted that the factor decreases significantly with increase in the ratio N for all skew angles. Similar observations were found in the case of an exterior girder of a bridge under eccentric truck loading.

Under dead load, different effects of the ratio N were found for both the interior and the exterior girders of the bridge. In this case the moment distribution factor increases with increase in the ratio N for all the skew angles considered. Figure 6.31 shows the effect of the ratio N on the moment distribution factor for an exterior girder of a four-lane bridge under dead load. It should be noted that an increase in the moment distribution factor with increase in the ratio N in this case does not necessarily imply a decrease in the value of the girder moment, since the moment, M, of the simply supported composite girder also changes with changes in the girder spacing.

6.1.4.4 Effect of Transverse Diaphragms

The prototype bridges considered in the parametric study were examined first without transverse diaphragms. They were then analyzed with different number of intermediate transverse diaphragms moment-connected to the longitudinal girders. The number of such diaphragms were varied from one to seven. The longitudinal and
transverse flexural rigidities were calculated for all prototype bridges using the method presented by Kennedy and Grace (1983). The flexural rigidity ratio, \( P \), was then calculated using eqn. 6.3. The effect of the flexural rigidity ratio on the moment distribution factor for an exterior girder of a two-lane bridge under concentric truck loading is presented in Figure 6.32. The following can be observed: (i) the factor increases with increase in the flexural rigidity ratio and more so with an increase in the angle of skew; and, (ii) the factor is not affected significantly when the rigidity ratio exceeds 21%. Figures 6.33 and 6.34 show the effect of the flexural rigidity ratio on the moment distribution factor for an interior girder of a two-lane bridge under eccentric truck loading and under dead load. The effects were similar to that in the case of a bridge under concentric loading. However, the effect of the flexural rigidity ratio in the case of a bridge under eccentric truck loading is more significant than that in the case of a bridge under concentric truck loading or a bridge under dead load.

6.1.4.5 Empirical Formulas for the Moment Distribution Factor, \( D \)

Based on the data generated in the parametric study, analyzing more than 300 cases of prototype bridges, empirical formulas were generated for the moment distribution factor \( D \) using a statistical package for best fit. The moment distribution factors were determined for both exterior and interior girders. The empirical formulas are in terms of the following significant parameters: (i) angle of skew, \( \theta \) in degrees; (ii) bridge aspect ratio, \( A_c \); (iii) girder spacing expressed as the dimensionless parameter, \( N = \text{number of lanes/number of girders} \); and, (iv) the flexural rigidities ratio, \( R \) in percent. The three
loading conditions were: concentric, eccentric OHBDC truck loadings and dead load.

Based on preliminary results it was revealed that more accurate predictions for the moment distribution factor, $D$, are obtained if the best fit formulas are presented for two ranges of skew angles namely: $\theta < 30^\circ$ and $\theta > 30^\circ$. For a bridge with $\theta > 30^\circ$, the value of the factor $D$ is calculated from the equations listed below for these two ranges, with $\theta = 30^\circ$, and the smaller value of $D$ is used for conservative design. It should be noted that the effect of the bridge aspect ratio sometimes appear in the equations in a different way than explained earlier. This is due to the variation in the rigidity ratio with the aspect ratio because the section of the longitudinal girder changes. Thus the deduced formulas are:

**Exterior Girder of a Bridge under Concentric OHBDC Truck Loading**

For $\theta < 30^\circ$:

\[
(6.23) \quad D = \frac{0.52 \times (1 + \theta)^{0.008} \times R^{0.05} \times A_r^{0.05}}{N^{0.71}}
\]

For $\theta > 30^\circ$:

\[
(6.24) \quad D = \frac{0.15 \times \theta^{0.305} \times R^{0.1} \times A_r^{0.1}}{N^{0.75}}
\]

**Exterior Girder of a Bridge under Eccentric OHBDC Truck Loading**

For $\theta < 30^\circ$:

\[
(6.25) \quad D = \frac{0.53 \times (1 + \theta)^{0.02} \times R^{0.05} \times A_r^{0.24}}{N^{0.65}}
\]
For $\theta > 30^\circ$:

(6.26) \[ R = \frac{0.17 \times \theta^{0.3} \times K^{0.057} \times A_r^{0.38}}{N^{0.7}} \]

**Exterior Girder of a Bridge under Dead Load**

For $\theta < 30^\circ$:

(6.27) \[ D = \frac{1.26 \times (1 + \theta)^{0.01} \times N^{0.503}}{A_r^{0.132}} \]

For $\theta > 30^\circ$:

(6.28) \[ D = \frac{0.47 \times \theta^{0.27} \times K^{0.04} \times N^{0.44}}{A_r^{0.19}} \]

**Interior Girder of a Bridge under Concentric OHBDC Truck Loading**

For $\theta < 30^\circ$:

(6.29) \[ D = \frac{0.41 \times (1 + \theta)^{0.033} \times K^{0.05} \times A_r^{0.05}}{N^{0.82}} \]

For $\theta > 30^\circ$:

(6.30) \[ D = \frac{0.01 \times \theta^{1.02} \times K^{0.172} \times A_r^{0.065}}{N^{0.833}} \]
Interior Girder of a Bridge under Eccentric OHBDC Truck Loading

For $\theta \leq 30^\circ$:

\[
D = \frac{0.94 \times (1 + \theta)^{0.01} \times R^{0.04} \times A_r^{0.006}}{N^{0.1}}
\]

(6.31)

For $\theta > 30^\circ$:

\[
D = \frac{0.41 \times \theta^{0.198} \times R^{0.095} \times A_r^{0.01}}{N^{0.22}}
\]

(6.32)

Interior Girder of a Bridge under Dead Load

For $\theta < 30^\circ$:

\[
D = 1.02 \times (1 + \theta)^{0.02} \times R^{0.025} \times N^{0.3}
\]

(6.33)

For $\theta > 30^\circ$:

\[
D = \frac{0.04 \times \theta^{0.92} \times R^{0.11} \times A_r^{0.3}}{N^{0.17}}
\]

(6.34)

6.1.4.6 Illustrative Example

Consider a four-lane bridge with five longitudinal girders composite with a concrete deck slab. The bridge was designed according to Ontario Highway Bridge Design Code (OHBDC, 1992). The bridge details are as follows: Span of bridge = 24 m; total bridge width = 17.4 m; angle of skew = 45$^\circ$; girder spacing = 3.75 m; deck slab thickness = 0.225 m; five longitudinal girders W760x257; and, three lines of moment-
connected transverse diaphragms W360x33. It is required to evaluate the longitudinal
timber for exterior and interior girders of the bridge for eccentric and concentric truck
loading and also for the dead load. The aspect ratio, \( A = 24/17.4 \approx 1.38 \), and from [2] the ratio \( N = 4/5 = 0.8 \). Applying the method presented by Kennedy and Grace (1983),
the flexural rigidity in the longitudinal direction \( D_x = (257)(10^6)(E) \), where \( E \) - modulus
of elasticity of the concrete slab, and the flexural rigidity in the transverse direction
considering the effect of the transverse diaphragms, \( D_y = (23)(10^6)(E) \). From [3] the
rigidity ratio \( R = (23)(10^6)(E)/(257)(10^6)(E) = 8.9\% \). Applying a line of wheel loads of
an OHBDC truck on a simple composite girder having a 0.225 m deck slab and 3.75 m
wide with span 24 m, the maximum moment \( M = 1299 \, \text{kN.m} \). Due to dead load, for the
exterior girders \( M_{DL} = 1405 \, \text{kN.m} \), and for the interior girders \( M_{DL} = 1671 \, \text{kN.m} \). The
moment distribution factor, \( D \), for the exterior girders of the bridge under: concentric
truck loading and using [5], \( D = 0.73 \); therefore \( M_{max} = M/D = 1299/0.73 = 1779 \, \text{kN.m} \);
under eccentric truck loading using [7], \( D = 0.82 \); thus \( M_{max} = 1299/0.82 = 1584 \, \text{kN.m} \);
and, under dead load, using [9], \( D = 1.22 \); or \( M_{max} = 1405/1.22 = 1152 \, \text{kN.m} \). For the
interior girders of the bridge under: concentric truck loading, using [11], \( D = 0.87 \); or,
\( M_{max} = 1299/0.87 = 1493 \, \text{kN.m} \); under eccentric truck loading using [13], \( D = 1.13 \); or,
\( M_{max} = 1299/1.13 = 1150 \, \text{kN.m} \); and for the dead load using [15], \( D = 1.49 \); thus \( M_{max}
= 1671/1.49 = 1121 \, \text{kN.m} \).

In order to estimate the dynamic load allowance the first flexural frequency of the
bridge was calculated as \( \omega = 6.33 \, \text{Hz} \). Based on the OHBD Code (1992) the
corresponding dynamic load allowance is approximately 25\%. Thus the maximum design

150
moments may be calculated as follows: For the exterior girders \( M_{\text{design}} = 1152 + (1779)(1.25) \times 3376 \text{ kN.m} \), and for the interior girders \( M_{\text{design}} = 1121 + (1493)(1.25) = 2987 \text{ kN.m} \).

6.1.4.7 Summary of Findings

The conclusions drawn from the above parametric study can be summarized as follows: (i) in the evaluation of girder moments in skew composite bridges the exterior girder is the controlling girder for design and the concentric truck loading case is the critical case of loading, yielding maximum moments in both exterior and interior girders. The girder moment due to the bridge dead load is also significant and has to be evaluated carefully; (ii) skew is the most important parameter affecting girder moments in composite bridges. An increase in the skew angle significantly reduces the girder moments. The effect of skew becomes more significant for skew angles greater than 30°. The interior girders are more sensitive to a change in the skew angle than the exterior girders; (iii) the girder spacing is a very important factor influencing the moment distribution factor in composite bridges. The moment distribution factor decreases significantly with increase in the ratio, \( N \) (number of lanes/number of girders) which is a measure of the girder spacing; (iv) the presence of intermediate transverse diaphragms moment-connected to the longitudinal girders enhances the load distribution characteristics of the bridge. An increase in the rigidity ratio, \( R \) (= transverse rigidity/longitudinal rigidity), increases the moment distribution factor. However, using a rigidity ratio greater than approximately 21% is not economical since beyond this value it does not have much effect on the moment distribution factor; and, (v) the bridge aspect ratio has an influence on the moment distribution factor and it increases with increase in the skew angle.
RESULTS FROM THE PARAMETRIC STUDIES

PART II: ELASTIC LOAD DISTRIBUTION IN CONTINUOUS COMPOSITE BRIDGES

6.2.1 General

Simply supported structures are efficiently used for bridges up to a certain span limit, beyond which using continuous structures becomes more economical. As a result of continuity, support moments are developed and therefore span moments are significantly reduced. The design of continuous skew composite bridges and the evaluation of existing ones require an accurate prediction of the following: (i) span girder moments; (ii) support girder moments; (iii) reactions at the simply supported ends; (iv) reactions at the intermediate support; and, (v) girder shear forces at the intermediate support. The use of load-distribution factors given by codes of practice does simplify the analysis and design of bridges. However, using inappropriate load-distribution factors may lead to extremely conservative design moments especially in the case of continuous skew composite bridges. This also may lead to unsafe reactions and design shear forces in skew composite bridges. The load distribution factor given by the American Association of State Highway and Transportation Officials (AASHTO, 1994) leads to extremely conservative design girder moments for skew bridges since the method does
not consider the reduction in girder moments due to skew. Furthermore, the method given by the AASHTO leads to unsafe reactions and design shear forces since it does not consider the redistribution in such forces due to skew. In addition, the method does not account for the effect of bridge continuity, bridge aspect ratio, nor the presence of intermediate transverse diaphragms. The Ontario Highway Bridge Design Code (OHBDC, 1992) accounts for the bridge longitudinal and transverse rigidities in addition to the girder spacing. However, the method is limited to simply supported bridges with skew parameters less than certain value specified in the code. Therefore, there is lack of adequate information given by North American codes of practice regarding moment-distribution factors and shear-distribution factors for continuous skew bridges. There is a need for the generation of a database for both moment-distribution factors and shear-distribution factors leading to the formation of empirical formulas for the design of continuous skew composite bridges.

It was found that the reactions at the simply supported ends of continuous skew composite bridges can be estimated accurately by using the shear distribution factors previously developed in part I. However, different distribution was found for the reactions and the design shear forces at the intermediate support.

In this section, the influence of several parameters on the moment-distribution factors and shear distribution factors in continuous skew composite steel-concrete bridges are studied. Results from a parametric study on prototype two-span continuous composite
bridges subjected to AASHTO truck loading and to dead load are presented. The parametric study included more than 1200 bridge cases. The finite element model used in the study was verified and substantiated by results from tests on three continuous composite bridge models with two unequal spans subjected to simulated truck loading, chapter V. Empirical formulas were deduced for: (i) span moment distribution factors, (ii) support moment distribution factors; (iii) mid-support reaction distribution factors, and, (iv) mid-support shear distribution factors. The formulas were deduced for eccentric and concentric AASHTO truck loading as well as for dead load. The derived formulas are based on the bridge elastic response and are in terms of the following factors: angle of skew; girder spacing; bridge aspect ratio; spans ratio; number of lanes; number of girders; and, number of intermediate transverse diaphragms.

6.2.2 Parametric Study

The objectives of the parametric study were: (i) to investigate the influence of all major parameters affecting both the span and support moment-distribution factors in the elastic range of loading; (ii) to investigate the influence of all major parameters affecting the mid-support reaction distribution factors and the mid-support shear distribution factors in the elastic range of loading; and, (iii) to develop empirical formulas for such factors corresponding to AASHTO truck loading as well as to dead load. The parametric study was based on the following assumptions: (i) both the reinforced concrete deck slab and the longitudinal steel girders are simply supported at the abutments and continuous over the intermediate pier; (ii) The reinforced concrete deck slab and the supporting steel I-
girders are in full composite action; (iii) all transverse intermediate diaphragms are moment-connected to the longitudinal girders; (iv) all materials are elastic and homogeneous; and, (v) the effects of the curbs are ignored. The details of the parametric study are described in the following sections.

6.2.2.1 Bridge Geometry

The number of lanes considered was 2, 3, and 4 lanes with bridge width of 8.4 m for two-lane bridges, 12.6 m for three-lane bridges, and 16.2 m for four-lane bridges. The span length was 16.8 m, 20.4 m, and 24 m, with a total bridge length of 33.6 m, 40.8 m, and 48 m. Based on the bridge widths and the span lengths, the bridge aspect ratio (= long span length/width) varied from 1.0 to 2.85. The number of girders considered was 3, 4, and 5 for two-lane bridges, 4, 5, and 6 for three-lane bridges, and 5, 6, and 7 for four-lane bridges. For the above bridge widths and number of girders, the girder spacing ranged from 1.5 m to 3.8 m. The angles of skew were taken as 0°, 30°, 45°, and 60°. Some bridges were studied with different spans ratios i.e. long span length/short span length. For bridges with a long span of 16.8 m the short span was varied from 16.8 m to 8.4 m; for bridges with a long span of 20.4 m the short span was varied from 20.4 m to 10.2 m; and, for bridges with a long span of 24 m the short span was varied from 24 m to 12 m. Thus the spans ratio were 1.0, 1.33, and 2.0.
6.2.2.2 Bridge Structural Properties

The steel sections used for the longitudinal girders were W610x155 for span length of 16.8 m, W760x185 for span length of 20.4 m, and W760x190 for span length of 24 m. Bridges were analyzed with no diaphragms, and, with 1 to 6 lines of intermediate transverse diaphragms, for each span. All diaphragms were moment-connected to the longitudinal girders. The diaphragms were W360x33 for span length of 16.8 m, W360x91 for span length of 20.4 m, and W360x101 for span length of 24 m. The steel girders and the transverse diaphragms were of grade G40.21-M300W. For all bridges, 225-mm thick concrete deck slab of 30 MPa compressive strength was used and reinforced top and bottom by a steel mesh having an area of 400 mm²/m, therefore the reinforcement ratio was 0.3%.

6.2.2.3 Loading Conditions

Many loading cases were considered in the parametric study, using an AASHTO standard truck HS20-44. The more than 1200 bridge cases considered can be classified into nine main categories according to the loading patterns. The categories were as follows: (i) one or two trucks applied on one side of the bridge representing an eccentric truck loading. The trucks occupied one span only to maximize the span moments; (ii) an eccentric truck loading similar to (i) except that the trucks were applied on both spans to maximize the support moments; (iii) a concentric truck loading in which the bridge was fully loaded by two trucks for two-lane bridges, three trucks for three-lane bridges, and four trucks for four-lane bridges. The trucks were applied on one span only to maximize
the span moments, (iv) a concentric truck loading similar to (iii) except that the trucks were applied on both spans to maximize the support moments; (v) one or two trucks applied on one side of the bridge representing an eccentric truck loading. The trucks were applied on both spans and were arranged in a way to maximize the mid-support reactions; (vi) an eccentric truck loading similar to (v) except that the trucks were arranged in a way to maximize the shear forces at the intermediate support; (vi) a concentric truck loading in which the bridge was fully loaded by two trucks for two-lane bridges, three trucks for three-lane bridges, and four trucks for four-lane bridges. The trucks were applied on both spans and were arranged in a way to maximize the mid-support reactions; (vii) a concentric truck loading similar to (vi) except that the trucks were arranged in a way to maximize the shear forces at the intermediate support; and, (viii) the dead load of the bridge in which the total weights of the reinforced concrete deck slab, the longitudinal steel girders, and transverse diaphragms were applied on the bridge as a uniformly distributed load on both spans.

6.2.3 Span and Support Moment Distribution Factors

In order to derive the moment distribution factor D, influence line diagrams, for both span and support moments, were constructed for all prototype bridges considered in the parametric study. The maximum span and support moments (M) in a continuous two-span composite girder under a line of wheel loads of an AASHTO truck or under the effect of dead load were first calculated for each prototype bridge. To maximize the span moment due to the wheel loads only one span was loaded whereas two spans were loaded
to maximize the support moment. However, two spans were loaded in case of dead load for both span and support moments. The finite element model was then used to obtain the maximum span and support moments ($M_{\text{max}}$) in all the girders for each prototype bridge in the parametric study. Thus, the moment distribution factor $D$ was calculated using eqn. (6.22). The effects of the different factors on the moment distribution factor, $D$, will be presented in the following sections.

### 6.2.3.1 Effect of Angle of Skew

The results reveal that the presence of skew causes significant reductions in both span and support girder moments. The span and support moment distribution factors, $D$, for prototype bridges were determined for three types of loading: eccentric AASHTO truck loading, concentric AASHTO truck loading, and for dead load. Figure 6.35 shows the effect of skew on the span moment distribution factor for an exterior girder of a four-lane bridge under concentric truck loading with different aspect ratios. It is observed that skew has only a small effect on the factor $D$ for skews less than 30°. However, the factor $D$ increases significantly with increase in skew greater than 30°. It is also noted that there is no significant interaction between aspect ratio and skew on the moment distribution factor $D$ for skew angles between 0° and 30°. However, this interaction becomes more significant when skew angles are greater than 30°. Figure 6.36 shows the effect of skew on the span moment distribution factor for an interior girder of a four-lane bridge under concentric truck loading with different aspect ratios. It is observed that skew has a significant influence on the factor $D$ especially for skew angles greater than 30°. It is
observed also that the aspect ratio has no effect on the factor $D$ for skew angle $\theta = 0^\circ$.

However, the aspect ratio has a significant influence on the span moment distribution factor for all other skew angles considered. Figure 6.37 shows the effect of skew on the span moment distribution factor for an exterior girder of a two-lane bridge under eccentric truck loading. In this case the following can be observed: (i) the span moment distribution factor increases significantly with an increase in the skew angle; (ii) the effect of skew is more significant for skew angles greater than $30^\circ$; and, (iii) there is no interaction between skew and aspect ratio for all skew angles considered. Figure 6.38 shows the effect of skew on the span moment distribution factor for an interior girder of a two-lane bridge under eccentric loading. The effect of skew in this case is more significant than that in the case of an exterior girder. Significant interaction between the skew and the aspect ratio is observed in this case. The effect of skew on the span moment distribution factor in the case of a bridge with aspect ratio $= 2.0$ is more significant than that in the case of a bridge with aspect ratio $= 2.86$. Figure 6.39 shows the effect of skew on the span moment distribution factor for an exterior girder of a three-lane bridge under dead load. In this case the span moment distribution factor $D$ increases with an increase in the skew angle. No interaction between skew and bridge aspect ratio is observed in this case. Figure 6.40 shows the effect of skew on the span moment distribution factor for an interior girder of a three lane bridge under dead load. The effect of skew in this case is more significant than that in the case of an exterior girder. Significant interaction between skew angle and aspect ratio is observed in this case. It should be noted that the effect of skew angle on the span moment distribution factor in
the case of dead load is less significant in the case of skew angles less than 30° than that in the case of skew angles greater than 30°.

Figure 6.41 shows the effect of skew on the support moment distribution factor D for an exterior girder of a four-lane bridge under concentric truck loading. The following can be observed: (i) skew has no effect on the factor D for skew angles less than 30°; (ii) the factor D increases significantly with an increase in the skew angle for skew angles greater than 30°; and, (iii) for all skew angles there is no interaction between skew and aspect ratio. Similar observations were found for the effect of skew on the support moment distribution factor for an interior girder of a four-lane bridge under concentric loading as shown in Figure 6.42. However, the effect of skew on the moment distribution factor is more significant in the case of an interior girder than that in the case of an exterior girder. Figure 6.43 shows the effect of skew on the support moment distribution factor for an exterior girder of a two-lane bridge under eccentric loading. The same observations mentioned above in the case of concentric truck loading were found in this case and were found also in the case of an interior girder of a two-lane bridge under eccentric loading as shown in Figure 6.44. Significant reductions in the support girder moment with increasing the skew angle was found for both the exterior and the interior girders of a bridge under dead load. Figures 6.45 and 6.46 show the effect of skew on the support moment distribution factor for both the exterior and the interior girders of a three-lane bridge under dead load. It is observed that the factor D increases significantly with an increase in the skew angle. It is also noted that there is no
interaction between skew and aspect ratio in this case.

It should be noted that for both span and support moments the factor $D$ for an interior girder increases more significantly with an increase in the skew angle than in the case of an exterior girder. Thus, for the design of skew bridges the exterior girder becomes the more controlling girder for design as skew increases. However, the value of the factor $D$ for a right bridge, $\theta = 0^\circ$, is almost the same for both the exterior and the interior girders, as expected. While the effect of skew on the factor $D$ under eccentric truck loading is similar to that for the case of concentric truck loading, the values of the factor $D$ in the latter case are lower than the former. This implies that the critical loading case for the design of skew bridges is the concentric truck loading case and thus for one-lane or two-lane bridges the concentric loading case will control the design since for such bridges the load modification factor equals to one. However, because of the reduction in the load intensity for multilane loading (AASHTO, 1994), in the case of bridges with three or more lanes both eccentric and concentric loading cases should be considered.

6.2.3.2 Effect of Bridge Aspect Ratio

The results reveal that the bridge aspect ratio does have an influence on both the span and the support moment distribution factors. Figure 6.47 shows the effect of aspect ratio on the span moment distribution factor for an exterior girder of a four-lane bridge under concentric loading. It is observed that the factor $D$ increases with an increase in the aspect ratio. The rate of this increase is more significant for skew angles $45^\circ$ and $60^\circ$.
than that in the case of skew angles 0° and 30°. Similar observations were found for the
effect of aspect ratio on the span moment distribution factor in the case of an interior
girder of a bridge under concentric truck loading. However, different observations were
found in the case of an exterior or an interior girder of a bridge under eccentric truck
loading. Figure 6.48 shows the effect of aspect ratio on the span moment distribution
factor for an interior girder of a two-lane bridge under eccentric truck loading. In this
case, the span moment distribution factor D decreases with an increase in the bridge
aspect ratio. The effect of aspect ratio in this case is not significant in the case of skew
angles 0°, 30°, and 45°. However, this effect becomes significant for skew angle 60°. In
the case of a bridge under dead load the moment distribution factor decreases with an
increase in the aspect ratio for all the skew angles considered as shown in Figure 6.49.

Figure 6.50 shows the effect of the bridge aspect ratio on the support moment
distribution factor D for an exterior girder of a four-lane bridge under concentric truck
loading. It is observed that the support moment distribution factor decreases significantly
with an increase in the bridge aspect ratio. The same effect was found for an interior
girder of a bridge under concentric truck loading. Figure 6.51 shows the effect of aspect
ratio on the support moment distribution factor for an interior girder of a two-lane bridge
under eccentric truck loading. The effect of aspect ratio in this case is less significant
than that in the case of a bridge under concentric truck loading. Under dead load, the
support moment distribution factor decreases significantly with an increase in the bridge
aspect ratio for all the skew angles considered as shown in Figure 6.52.
6.2.3.3 Effect of Girder Spacing

The spacing of longitudinal girders, S, is one of the most critical factors affecting both the span and the support moment distribution factors. This is reflected in the AASHTO specifications (AASHTO, 1994) where the moment distribution factor is a function of the center to center girder spacing only. A dimensionless factor N (= number of lanes/number of girders) was suggested in part I of this chapter in equ. (6.2) as a measure for the girder spacing. The ratio N was calculated for all prototype bridges considered in the parametric study, and the relationships between the ratio N and the moment distribution factor D were determined for both span and support moments. Figure 6.53 shows the effect of the ratio N on the span moment distribution factor D for an exterior girder of a two-lane bridge under concentric truck loading. It is observed that the factor D decreases significantly with increase in the ratio N. This effect is almost the same for all the skew angles considered. A similar effect was found for an interior girder as well as in the case of eccentric truck loading as shown in Figure 6.54. Different effect for the ratio N was found in the case of dead load. Figure 6.55 shows the effect of the ratio N on the span moment distribution factor for an exterior girder of a three-lane bridge under dead load. In this case, the span moment distribution factor increases significantly with an increase in the ratio N.

Figure 6.56 shows the effect of the ratio N on the support moment distribution factor for an exterior girder of a two-lane bridge under concentric loading. In this case, the support moment distribution factor D decreases significantly with an increase in the
ratio N for all the skew angles considered. Similar observations were found for an interior girder of a bridge under concentric loading. Figure 6.57 shows the effect of the ratio N on the support moment distribution factor for an interior girder of a two-lane bridge under eccentric truck loading. Although the support moment distribution factor decreases with an increase in the ratio N, the effect of the ratio N in this case is less significant than that in the case of concentric truck loading. Different observations were found in the case of a bridge under dead load. Figure 6.58 shows the effect of the ratio N on the support moment distribution factor for an exterior girder of a three-lane bridge under dead load. In this case, the support moment distribution factor increases significantly with an increase in the ratio N for all the skew angles considered. It should be noted that for both the span and support moments in the case of a bridge under dead load, an increase in the moment distribution factor with increase in the ratio N does not necessarily imply a decrease in the value of the bridge girder design moment, since the moment, M, in a continuous two-span composite girder also changes in the girder spacing.

6.2.3.4 Effect of Transverse Intermediate Diaphragms

The experimental results presented in chapter V for the effect of transverse diaphragms on the load distribution characteristics of simply supported skew composite bridges revealed the importance of the presence of orthogonal intermediate transverse diaphragms moment-connected to the longitudinal girders. Tests on actual bridges (Boyce, 1977) and on bridge models (Kennedy et al., 1989) have shown that such connections lead to improved bridge stiffness, better load distribution characteristics, and
increased ultimate load capacity.

The prototype bridges considered in the parametric study herein were examined first without transverse diaphragms. They were then analyzed with different number of orthogonal intermediate transverse diaphragm moments-connected to the longitudinal girders. The number of such diaphragms were varied from one to six in each span. The longitudinal and transverse rigidities were calculated for all the prototype bridges using the method presented by Kennedy and Grace (1983). The flexural rigidity ratio, R, was then calculated using equ. (6.3). The relationships between the flexural rigidity ratio and the moment distribution factor were then determined for both the span and the support moments.

The results reveal that the span moment distribution factor D increases with increase in the flexural rigidity ratio, R, with the exterior girder being more affected than the interior girder. A change in R is more significant in the case of eccentric truck loading than that in the case of concentric truck loading or dead load, as expected. However, when a value of $R > 20\%$ its effect on the span and the support moment distribution factors D diminishes.

6.2.3.5 Effect of Spans Ratio

The spans ratio, $S$, is defined as the ratio between the long span length to the short span length. Bridges considered in the parametric study were studied with different short span lengths and a fixed length for the long span. The results reveal that the span
moment is reduced significantly with increase in the spans ratio, $S$, when the long span only is loaded. It was found also that the support moment diminishes significantly with an increase in the span ratio when both the long and the short spans are loaded. In the case of dead load, the span moment increases while the support moment decreases with an increase in the spans ratio. To study the effect of the spans ratio on the moment distribution factors, $D$, the moments in a single continuous composite girder were also determined for the different spans ratios considered. It should be noted that an increase or a decrease in the moment distribution factors does not necessarily indicate an increase or a decrease in the girder moment. Bearing this in mind, the following were observed:

(i) Both the span and support moment distribution factors $D$ increase with increase in the spans ratio $S$ for both interior and exterior girders for all cases of eccentric and concentric truck loadings as well as for dead load. However, the rate of the increase in the span moment distribution factor $D$ was higher in the case of skew angles $45^\circ$ and $60^\circ$ than in the case of skew angles $\leq 30^\circ$. For skew angles $\leq 30^\circ$ the support moment distribution factor $D$ decreases with increase in the spans ratio $S$ for both exterior and interior girders in all cases of eccentric and concentric truck loading and in the case of dead load; and,

(ii) the support moment distribution factor $D$ is more sensitive to a change in the spans ratio $S$ than the factor for the span moment.
6.2.3.6 Empirical Formulas for the Span and the Support Moment Distribution Factors, D

Based on the data generated from the parametric study, analyzing more than 600 cases of prototype bridges, empirical formulas were developed for both span and support moment distribution factor D using a statistical package for best fit. The moment distribution factors were determined for both exterior and interior girders. The empirical formulas are in terms of the following significant factors: (i) angle of skew, \( \theta \) in degrees; (ii) bridge aspect ratio, \( A_r = \text{long span length}/\text{bridge width} \); (iii) girder spacing expressed as the dimensionless parameter, \( N = \text{number of lanes}/\text{number of girders} \); (iv) spans ratio, \( S = \text{long span length}/\text{short span length} \); and, (v) the flexural rigidities ratio, \( R \) in percent.

The three loading conditions considered were: concentric, eccentric AASHTO truck loadings as well as dead load. Based on preliminary results it was revealed that more accurate predictions for the moment distribution factors, D, are obtained if the best fit formulas are presented for two ranges of skew angles namely: \( \theta < 30^\circ \) and \( \theta > 30^\circ \). For a bridge with \( \theta = 30^\circ \), the value of the factor D is calculated from the equations listed below for these two ranges, with \( \theta = 30^\circ \), and the smaller value of D is used for conservative design. Thus the deduced formulas are as follows:

(a) Span Moments

exterior girder of a bridge under concentric AASHTO truck loading

for \( \theta < 30^\circ \):

\[
D = \frac{0.52 \times (1 + \theta)^{0.01} \times R^{0.01} \times A_r^{0.06} \times S^{0.1}}{N^{0.6}}
\]  

(6.35)
for $\theta > 30^\circ$: -

\[
D = \frac{0.12 \times 6^{0.4} \times R^{0.05} \times A_r^{0.2} \times S^{0.3}}{N^{0.45}} \quad (6.36)
\]

exterior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$: -

\[
D = \frac{0.49 \times (1 + \theta)^{0.02} \times R^{0.04} \times A_r^{0.2} \times S^{0.1}}{N^{0.58}} \quad (6.37)
\]

for $\theta > 30^\circ$: -

\[
D = \frac{0.23 \times 6^{0.19} \times R^{0.06} \times A_r^{0.3} \times S^{0.4}}{N^{0.63}} \quad (6.38)
\]

exterior girder of a bridge under dead load

for $\theta < 30^\circ$: -

\[
D = \frac{1.34 \times (1 + \theta)^{0.01} \times R^{0.01} \times N^{0.4} \times S^{0.01}}{A_r^{0.3}} \quad (6.39)
\]

for $\theta > 30^\circ$: -

\[
D = \frac{0.74 \times 6^{0.18} \times R^{0.02} \times N^{0.6} \times S^{0.2}}{A_r^{0.4}} \quad (6.40)
\]
interior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$:

$$D = \frac{0.44 \times (1 + \theta)^{0.04} \times R^{0.05} \times S^{0.1}}{A_r^{0.01} \times N^{0.68}} \quad (6.41)$$

for $\theta > 30^\circ$:

$$D = \frac{0.0072 \times \theta^{1.1} \times R^{0.16} \times S^{0.5}}{A_r^{0.02} \times N^{0.6}} \quad (6.42)$$

interior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$:

$$D = \frac{0.85 \times (1 + \theta)^{0.01} \times R^{0.03} \times S^{0.1}}{A_r^{0.01} \times N^{0.08}} \quad (6.43)$$

for $\theta > 30^\circ$:

$$D = \frac{0.28 \times \theta^{0.34} \times R^{0.05} \times S^{0.4}}{A_r^{0.2} \times N^{0.06}} \quad (6.44)$$

interior girder of a bridge under dead load

for $\theta < 30^\circ$:

$$D = \frac{1.16 \times (1 + \theta)^{0.02} \times R^{0.03} \times N^{0.5} \times S^{0.06}}{A_r^{0.02}} \quad (6.45)$$
for $\theta > 30^\circ$ :-

$$D = \frac{0.04 \times \theta^{0.9} \times R^{0.06} \times N^{0.4} \times S^{0.4}}{A_r^{0.15}}$$  \hspace{1cm} (6.46)

(b) Support Moments

exterior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.41 \times (1 + \theta)^{0.002} \times R^{0.1}}{A_r^{0.06} \times N^{1.0} \times S^{0.1}}$$  \hspace{1cm} (6.47)

for $\theta > 30^\circ$ :-

$$D = \frac{0.098 \times \theta^{0.33} \times R^{0.15} \times S^{0.5}}{A_r^{0.06} \times N^{0.9}}$$  \hspace{1cm} (6.48)

exterior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.55 \times (1 + \theta)^{0.003} \times R^{0.07}}{A_r^{0.01} \times N^{0.65} \times S^{0.06}}$$  \hspace{1cm} (6.49)

for $\theta > 30^\circ$ :-

$$D = \frac{0.30 \times \theta^{0.15} \times R^{0.1} \times S^{0.5}}{A_r^{0.02} \times N^{0.7}}$$  \hspace{1cm} (6.50)
exterior girder of a bridge under dead load

for $\theta < 30^\circ$:

$$D = \frac{1.86 \times (1 + \theta)^{0.003} \times N^{0.64}}{A_r^{0.44} \times S^{0.3}} \quad (6.51)$$

for $\theta > 30^\circ$:

$$D = \frac{0.94 \times \theta^{0.2} \times N^{0.7}}{A_r^{0.4} \times S^{0.3}} \quad (6.52)$$

interior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$:

$$D = \frac{0.35 \times (1 + \theta)^{0.03} \times R^{0.08}}{A_r^{0.08} \times N^{0.15} \times S^{0.1}} \quad (6.53)$$

for $\theta > 30^\circ$:

$$D = \frac{0.011 \times \theta^{1.0} \times R^{0.13} \times S^{0.5}}{A_r^{0.1} \times N^{0.9}} \quad (6.54)$$

interior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$:

$$D = \frac{0.94 \times (1 + \theta)^{0.02} \times R^{0.04}}{A_r^{0.04} \times N^{0.13} \times S^{0.2}} \quad (6.55)$$
for $\theta > 30^\circ$ :-

$$D = \frac{0.12 \times 6^{0.61} \times R^{0.08} \times S^{0.4}}{A_r^{-0.06} \times N^{0.15}}$$  \hspace{1cm} (6.56)$$

interior girder of a bridge under dead load

for $\theta < 30^\circ$ :-

$$D = \frac{1.33 \times (1 + \theta)^{0.02} \times R^{0.02} \times N^{0.46}}{A_r^{0.05} \times S^{0.3}}$$  \hspace{1cm} (6.57)$$

for $\theta > 30^\circ$ :-

$$D = \frac{0.087 \times 6^{0.74} \times R^{0.06} \times N^{0.42} \times S^{0.3}}{A_r^{0.1}}$$  \hspace{1cm} (6.58)$$

6.2.3.7 Illustrative Design Example

Consider a three-lane two-unequal-span continuous skew bridge with five girders composite with a concrete deck slab. Both the longitudinal steel girders and the reinforced concrete deck slab are simply supported at the abutments and continuous over the pier. The bridge details are as follows: bridge long span length = 20 m; bridge short span length = 14 m; total bridge length = 34 m; total bridge width = 13.2 m; angle of skew = 45°; girder spacing = 3 m; deck slab thickness = 0.225 m; five longitudinal girders W610x241; three lines and two lines of moment connected transverse diaphragms W310x67 in the long span and in the short span, respectively. It is required to: calculate the maximum design span and interior support moments for both the exterior and the interior girders of the bridge; check the strength of the assumed sections; and, check the
maximum deflection. The aspect ratio, \( A_c = 20/13.2 = 1.52 \), the span ratio \( S = 20/14 = 1.43 \), and from (6.2) the ratio \( N = 3/5 = 0.6 \). Applying the method presented by Kennedy and Grace (1983), the flexural rigidity in the longitudinal direction \( D_x = (82.3)(10^5)(E) \), where \( E \) = modulus of elasticity of the concrete slab, and the flexural rigidity in the transverse direction considering the effect of transverse diaphragms, \( D_y = (14.6)(10^5)(E) \). From (6.3) the rigidity ratio \( R = (14.6)(10^5)(E)/(82.3)(10^5)(E) = 17.7\% \). For a continuous girder with a composite concrete slab of width 3 m, having a long span of 20 m and a short span of 14 m, subjected to a line of wheel loads of an AASHTO truck one can find: the maximum span moment, when only the long span is loaded, \( M_{\text{span}} = 516 \text{ kN.m} \); the maximum support moment, when both spans are loaded, \( M_{\text{support}} = 630 \text{ kN.m} \); due to dead load, for the exterior girders \( M_{\text{span, D.L.}} = 442 \text{ kN.m} \) and \( M_{\text{support, D.L.}} = 541 \text{ kN.m} \), and for the interior girders \( M_{\text{span, D.L.}} = 629 \text{ kN.m} \) and \( M_{\text{support, D.L.}} = 771 \text{ kN.m} \). The span moment distribution factor, \( D \), for the exterior girders of the bridge under: concentric truck loading and using (6.36), \( D = 0.93 \); therefore \( M_{\text{max, span}} = (0.9)(M_{\text{span}})/D = (0.9)(516)/(0.93) = 499 \text{ kN.m} \), using the reduction factor of 0.9 for multilane loading; under eccentric truck loading using (6.38), \( D = 1.02 \); thus \( M_{\text{max, span}} = 516/1.02 = 506 \text{ kN.m} \); and, under dead load, using (6.40), \( D = 1.03 \); or \( M_{\text{max, span}} = 442/1.03 = 429 \text{ kN.m} \). For the interior girders of the bridge under: concentric truck loading, using (6.42), \( D = 1.22 \); or \( M_{\text{max, span}} = (0.9)(516)/(1.22) = 381 \text{ kN.m} \); under eccentric truck loading using (6.44), \( D = 1.29 \); or \( M_{\text{max, span}} = 516/1.29 = 400 \text{ kN.m} \); and, for the dead load using (6.46), \( D = 1.36 \); thus \( M_{\text{max, span}} = 629/1.36 = 463 \text{ kN.m} \). The support moment distribution factors, \( D \), for the exterior girders of the bridge under: concentric truck loading and using (6.48), \( D = 1.17 \);
therefore $M_{\text{max, support}} = (0.9)(M_{\text{support}})D = (0.9)(630)/(1.17) = 485$ kN.m, under eccentric truck loading using (6.50), $D = 1.19$; thus $M_{\text{max, support}} = 630/1.19 = 529$ kN.m; and, under dead load, using (6.52), $D = 1.07$; or $M_{\text{max, support}} = 541/1.07 = 506$ kN.m. For the interior girders of the bridge under: concentric truck loading, using (6.54), $D = 1.29$; or $M_{\text{max, support}} = (0.9)(630)/(1.29) = 440$ kN.m; under eccentric truck loading using (6.56), $D = 1.8$; or, $M_{\text{max, support}} = 630/1.8 = 350$ kN.m; and, for the dead load using (6.58), $D = 1.49$, thus $M_{\text{max, support}} = 771/1.49 = 517$ kN.m. It should be noted that the design moments based on the AASHTO distribution factor are $M_{\text{max, span}} = 901$ kN.m. and $M_{\text{max, support}} = 1103$ which are significantly greater than the design moments found above. This indicates that the AASHTO distribution factor is extremely conservative for skew continuous composite bridges.

To check the sections at the ultimate limit state, the AASHTO live load factor $= 1.3 (1.67 [1 + \text{impact factor}]) = 2.17 (1 + 50/[125 + 66]) = 2.74$, and the dead load factor $= 1.3$. Applying these factors to the above calculated maximum moments for both the exterior and the interior girders, shows that the exterior girder controls with a maximum factored span moment of 1944 kN.m, and a maximum factored support moment of 2107 kN.m. Strength calculations of the composite section showed that the assumed section is adequate in the span moment region with a resisting moment of 3439 kN.m. > 1944 kN.m. required, and also in the support moment region with a resisting moment of 2552 kN.m. > 2107 kN.m. required.

For the maximum deflection, the exterior girder controls and therefore it will be considered. The maximum deflection of a girder composite with the deck slab due to:
dead load, including the effect of creep, = 18.5 mm; a line of wheel loads of an AASHTO truck = 15.3 mm. The deflection of the exterior girder in the bridge, using the span moment distribution factors (which are not significantly different from those for deflections), under dead load = 18.5/1.03 = 18.0 mm; under concentric truck loading = (15.3)(0.9)/(0.93) = 14.8 mm; and, under eccentric truck loading = 15.3/1.02 = 15.0 mm. Therefore the eccentric truck loading case controls. Using the AASHTO formula for the impact factor I = 26%, therefore the deflection due to live load including impact = (15.0)(1.26) = 18.9 mm. This deflection represents a ratio of span/1058 which is less than the maximum allowable deflection for live load given by AASHTO of span/800.

6.2.3.8 Summary of Findings

The conclusions found in the above parametric study can be summarized as follows: (i) in the design of continuous skew composite bridges the exterior girder is the controlling girder in terms of both span and support moments. For one-lane or two-lane bridges the concentric truck loading case is the critical case, yielding maximum moments in both exterior and interior girders. However, for bridges having three or more lanes, both eccentric and concentric truck loading cases should be compared to determine the maximum moments for design; (ii) both the span and the support girder moments decrease significantly with an increase in the skew angle. Skew has a greater influence on the design of interior girders than exterior girders. The effect of skew becomes more significant for skews greater than 30°; (iii) moment distribution factors are very sensitive to changes in the girder spacing. These factors decrease significantly with increase in the
ratio, \( N \) (= number of lanes/number of girders) which is a measure of the girders spacing. (iv) for bridges with skew angles greater than 30°, both span and support girder moments decrease significantly with an increase in the span ratio, \( S \) (= long span length/short span length); and (v) the presence of intermediate transverse diaphragms moment-connected to the longitudinal girders enhances the load distribution characteristics of the bridge. An increase in the rigidities ratio, \( R \) (= transverse rigidity/longitudinal rigidity), increases the moment distribution factor to a limiting value of \( R = 20\% \), beyond which no increase in the factor is incurred.

### 6.2.4 Reaction and Shear Distribution Factors at the Intermediate Support

In order to design a skew continuous composite steel-concrete bridge properly the following design forces and moments should be first estimated accurately: (i) the span girder moment; (ii) the support girder moment; (iii) the reactions at the simply supported ends of the bridge; (iv) the reactions at the intermediate interior support; and (v) the shear forces at the intermediate interior support. The span and the support design girder moments can be estimated accurately by using the moment distribution factors given in section 6.2.3 of this chapter. It was found that the reactions at the simply supported ends can be estimated accurately by using the shear distribution factors for simply supported bridges presented in section 6.1.3 of this chapter. However, a different distribution of the reactions and the shear forces was found at the intermediate interior support.

In order to derive the reaction and the shear distribution factors at the intermediate
interior support, influence line diagrams for both reactions and shear forces were constructed for all prototype bridges considered in the parametric study. The maximum reaction (F) and the maximum shear force (V) in a continuous two-span composite girder under a line of wheel loads of an AASHTO truck or under the effect of dead load were first calculated for each prototype bridge. To maximize the reaction at the intermediate support due to wheel loads two spans were loaded and the wheel loads were arranged in a way to produce the maximum reaction. Two spans were loaded also in order to maximize the shear force near the intermediate support. However, the wheel loads in this case were arranged in a way to produce the maximum shear force. In the case of dead load, the total weight of the steel girder and the concrete deck slab was applied on the two spans for both the reaction and the shear force.

The finite element model was then used to obtain the maximum reactions ($F_{\text{max}}$) and shear forces ($V_{\text{max}}$) at the intermediate support in all the girders for each prototype bridge in the parametric study. Thus the reaction distribution factor and the shear distribution factor at the intermediate support were calculated from the following relationships:

$$D_{\text{reaction}} = \frac{F}{F_{\text{max}}} \tag{6.59}$$

$$D_{\text{shear}} = \frac{V}{V_{\text{max}}} \tag{6.60}$$

The effects of the different influencing factors on the reaction and shear distribution
factors at the intermediate support will be presented in the following sections

6.2.4.1 Effect of Angle of Skew

Results presented in part I of this chapter revealed that there is a concentration of reaction of the girder close to the obtuse corner of simply supported bridges and that this concentration increases with an increase in the angle of skew. It was also concluded that the reactions of the girder close to the acute corner and of the interior girders decrease with an increase in the skew angle. It was found herein that this is also the case for the reactions at the simply supported ends of a continuous two-span skew composite bridge. However, different distribution of reactions was found at the intermediate support. Since a skew continuous bridge with two equal spans consists of two parts symmetrical about the intermediate support and since the maximization of the reactions at such support requires the simultaneous application of the truck loads on the two spans producing a symmetrical type of loading about the intermediate support. Therefore, the distribution of the reactions at the intermediate support is almost uniform for the two exterior girders and also for the interior girders. This distribution of the reactions at the intermediate support was found to be not significantly affected by increasing the skew angle. Figure 6.59 shows the effect of skew on the reaction distribution factor for an exterior girder of a three-lane two-equal-span continuous bridge under concentric truck loading. It is observed that the reaction distribution factor is constant for all the skew angles considered. It is also noted that the factor decreases with an increase in the bridge aspect ratio. Figure 6.60 shows the effect of skew on the reaction distribution factor for an
interior girder of a three-lane two-equal-span continuous bridge under concentric loading. In this case the reaction distribution factor increases with an increase in the skew angle. However, this increase is not significant. It is observed that the factor decreases with an increase in the bridge aspect ratio but without any interaction with the angle of skew. The same observations were found for an exterior and interior girders of a two-lane two-equal-span continuous bridge in the case of eccentric truck loading as shown in Figures 6.61 and 6.62. Figure 6.63 shows the effect of skew on the reaction distribution factor for an exterior girder of a three-lane two-equal-span continuous bridge under dead load. In this case the skew was found to have no effect on the reaction distribution factor. The factor decreases with an increase in the bridge aspect ratio without any interaction with the skew angle. Figure 6.64 shows the effect of skew on the reaction distribution factor for an interior girder of a three-lane bridge under dead load. The following were observed: (i) skew has almost no effect on the reaction distribution factor in the range of $\theta = 0^\circ$ to $30^\circ$; (ii) skew has a small effect on the factor in the range of $\theta = 30^\circ$ to $60^\circ$; (iii) the reaction distribution factor decreases with an increase in the bridge aspect ratio; and, (iv) there is no interaction between the aspect ratio and the skew angle.

From the above discussion, it is clear that the angle of skew has no significant on the reactions distribution at the intermediate support of a two-equal-span continuous composite steel-concrete bridge. However, different reactions distribution was found in the case of two-unequal-span continuous composite bridges. In this case the problem becomes unsymmetrical in terms of geometry and loading conditions. As a result, the
following were observed: (i) the reaction of the exterior girder, located on the side of the bridge where the acute corner of the long span is simply supported and the obtuse corner of such span is continuous, increases with an increase in the angle of skew; (ii) the reaction of the exterior girder, located on the side of the bridge where the obtuse corner of the long span is simply supported and the acute corner of such span is continuous, decreases with an increase in the angle of skew; and, (iii) the reaction of the interior girders decrease with an increase in the angle of skew. Therefore, the interaction between the skew and the unequal spans produces more critical reactions distribution at the intermediate interior support than that in the case of two-equal-span skew bridge.

Comparing the results of reactions at the intermediate interior support for the exterior and the interior girders and for concentric and eccentric loading cases revealed the following: (i) the reaction of the interior girder is greater than that of the exterior girder for right bridges, $\theta = 0^\circ$, and for all the skew angles considered; (ii) the reactions due to dead load is significant and should be carefully estimated for design; (iii) eccentric loading cases are the ones that produce the maximum reactions for the exterior girders; and (iv) concentric loading cases are the ones that produce the maximum reactions for the interior girders.

The distribution of the reactions at the intermediate support discussed above does not apply to the distribution of the design shear forces at such support. The distribution of shear forces at the intermediate support is more critical than that in the case of
reactions. Figure 6.65 shows the effect of skew on the shear distribution factor for an exterior girder of a three-lane bridge under concentric truck loading. It is observed that the shear distribution factor decreases significantly with an increase in the angle of skew reflecting a concentration of shear forces at the exterior girder. The effect of skew was more significant in the range of $\theta = 30^\circ$ to $60^\circ$ than that in the range of $\theta = 0^\circ$ to $30^\circ$. It is also observed that the effect of aspect ratio increases with increasing the skew angle. Figure 6.66 shows the effect of skew on the shear distribution factor for an interior girder of a three-lane bridge under concentric truck loading. In this case the shear distribution factor increases with an increase in the skew angle reflecting a decrease in the design shear forces for the interior girders. This increase was also more significant for skew angles greater than $30^\circ$. The significant interaction between the aspect ratio and skew angle is also noted.

Similar observations were found in the case of eccentric truck loading cases for both the exterior and the interior girders as shown in Figures 6.67 and 6.68. Figure 6.69 shows the effect of skew on the shear distribution factor for an exterior girder of a three lane bridge under dead load. It is observed that the shear distribution factor decreases significantly with an increase in the skew angle. In this case, the interaction between the aspect ratio and the angle of skew is not significant. Figure 6.70 shows the effect of skew on the shear distribution factor for an interior girder of a three-lane bridge under dead load. It is noted that the shear distribution factor increases significantly with an increase in the skew angle. It is also observed that the rate of this increase is more
significant for skew angles greater than 30°.

Comparing the results of the shear forces at the intermediate interior support for
the exterior and the interior girders and for concentric and concentric loading cases
revealed the following: (i) in concentric truck loading cases the shear force for the interior
girder was greater than that for the exterior girder in the case of skew angle θ = 0°.
However, for all other skew angles considered, the shear force at the exterior girder was
greater than that at the interior girder; (ii) in eccentric truck loading cases, the shear force
at the exterior girder was greater than that at the interior girder for angle of skew θ = 0°
and for all other skew angles considered; (iii) concentric truck loading cases produce
higher shear forces in the interior girders than eccentric loading cases; (iv) eccentric truck
loading cases produce higher shear force in the exterior girder than concentric loading
cases for skew angles 0° and 30°; and, (v) concentric truck loading cases produce higher
shear force in the exterior girder than eccentric truck loading cases for skew angles 45°
and 60°.

6.2.4.2 Effect of Bridge Aspect Ratio

The results reveal that the bridge aspect ratio does have an influence on the
distribution of both the reactions and the shear forces at the intermediate interior support.
Figure 6.71 shows the effect of the bridge aspect ratio on the reaction distribution factor
for an exterior girder of a two-lane bridge under concentric truck loading. It is observed
that the reaction distribution factor decreases with an increase in the bridge aspect ratio.
for all the skew angles considered. It is also noted that skew has no effect on the factor. Similar observations were found for the interior girder of a bridge under concentric truck loading. Figure 6.72 shows the effect of aspect ratio on the reaction distribution factor for an interior girder of a two-lane bridge under eccentric truck loading. In this case the reaction distribution factor increases with an increase in the aspect ratio for all the skew angles considered. However, in the case of an exterior girder of a bridge under eccentric truck loading, the reaction distribution factor decreases with an increase in the aspect ratio. Figure 6.73 shows the effect of aspect ratio on the reaction distribution factor for an exterior girder of a three-lane bridge under dead load. It is observed that the reaction distribution factor decreases with an increase in the bridge aspect ratio for all the skew angles considered. Similar observations were found in the case of an interior girder of a bridge under dead load.

Figure 6.74 shows the effect of the bridge aspect ratio on the shear distribution factor for an exterior girder of a two-lane bridge under concentric truck-loading. It is observed that the factor decreases with an increase in the bridge aspect ratio. It is also noted that skew bridges were more significantly affected by changing the bridge aspect ratio than a right bridge. Similar observations were found in the case of an interior girder of a bridge under concentric truck loading. However, different observations were found in the case of eccentric truck loading. Figure 6.75 shows the effect of the bridge aspect ratio on the shear distribution factor for an interior girder of a two-lane bridge under eccentric truck loading. In this case the shear distribution factor increases slightly with
an increase in the bridge aspect ratio. This increase was more significant in the case of skew angles 45° and 60° than that in the case of skew angles 0° and 30°. In the case of an exterior girder of a bridge under eccentric truck loading, the shear distribution factor decreases slightly with an increase in the bridge aspect ratio. Figure 6.76 shows the effect of the bridge aspect ratio on the shear distribution factor for an exterior girder of a three-lane bridge under dead load. It is observed that the factor decreases with an increase in the bridge aspect ratio for all the skew angles considered. Similar observations were found in the case of an interior girder of a bridge under dead load.

6.2.4.3 Effect of Girder Spacing

The results reveal that the girder spacing is the most important factor affecting the distribution of both the reactions and the shear forces at the intermediate interior support. The ratio $N$ (= number of lanes/number of girders), which was suggested as a measure to the girder spacing equ. (6.2), was calculated for all the prototype bridges in the parametric study. The relationships between the ratio $N$ and the reaction and shear distribution factors were then found. Figure 6.77 shows the effect of the ratio $N$ on the reaction distribution factor for an exterior girder of a three-lane bridge under concentric truck loading. It is observed that the reaction distribution factor decreases significantly with an increase in the ratio $N$, reflecting an increase in the value of the reaction. Similar observations were found in the case of an interior girder of a three-lane bridge under eccentric truck loading as shown in Figure 6.78. However, different effect of the ratio $N$ on the reaction distribution factor was found in the case of dead load. Figure 6.79
shows the effect of the ratio $N$ on the reaction distribution factor for an exterior girder of a two-lane bridge under dead load. In this case the reaction distribution factor increases significantly with an increase in the ratio $N$ for all the skew angles considered. Similar observations were found in the case of an interior girder of a bridge under dead load.

Figure 6.80 shows the effect of the ratio $N$ on the shear distribution factor for an exterior girder of a three-lane bridge under concentric truck loading. It is observed that the shear distribution factor decreases with an increase in the ratio $N$, reflecting an increase in the design shear force. The effect was almost the same for all the skew angles considered. Similar observations were found in the case of an interior girder of a bridge under concentric truck loading and for an exterior girder of a bridge under eccentric truck loading. Figure 6.81 shows the effect of the ratio $N$ on the shear distribution factor for an interior girder of a bridge under eccentric truck loading. It is observed that the effect of the ratio $N$ on the shear distribution factor is less significant than that in the case of an interior girder of a bridge under concentric truck loading. Different effects were found in the case of a bridge under dead load. Figure 6.82 shows the effect of the ratio $N$ on the shear distribution factor for an exterior girder of a two-lane bridge under dead load. In this case the shear distribution factor increases significantly with an increase in the ratio $N$ for all the skew angles considered. It should be noted that an increase in the distribution factor with an increase in the ratio $N$, for both the reactions and the shear forces, in the case of a bridge under dead load does not necessarily imply a decrease in the value of the reaction or the bridge girder design shear force, since the reaction (F) and
the shear force \( V \) in a continuous two-span composite girder also changes with change in the girder spacing.

### 6.2.4.4 Effect of Transverse Intermediate Diaphragms

The prototype bridges considered in the parametric study herein were examined first without transverse diaphragms. They were then analyzed with different number of orthogonal intermediate transverse diaphragms moment-connected to the longitudinal girders. The number of such diaphragms were varied from one to six in each span. The longitudinal and transverse rigidities were calculated for all the prototype bridges using the method presented by Kennedy and Grace (1983). The flexural rigidities ratio, \( R \), was then calculated using eqn. (6.3). The relationships between the rigidity ratio, \( R \), and the reaction and shear distribution factors were then determined.

The results reveal that the intermediate transverse diaphragms have only a small influence on the distribution of the reactions at the intermediate interior support. The reaction distribution factor for an exterior girder increases with an increase in the flexural rigidity ratio, reflecting a decrease in the value of the reaction. For an interior girder, the reaction distribution factor decreases with an increase in the flexural rigidity ratio, reflecting an increase in the value of the reaction. The effect of the presence of the intermediate diaphragms was more significant in the case of an eccentric truck loading than that in the case of a concentric truck loading or in the case of dead load.
However, the presence of intermediate transverse diaphragms, moment-connected to the longitudinal girders, plays an important role in the distribution of shear forces at the intermediate interior support. It was mentioned before that the presence of skew causes a critical distribution of shear forces at such support in which the shear forces concentrate at the exterior beams and decrease at the interior beams. The presence of intermediate diaphragms decreases the shear force at the exterior girder and increases it at the interior girders and therefore makes the distribution of the shear forces at the intermediate support more uniform. The results revealed that the shear distribution factor for an exterior girder increases significantly with an increase in the flexural rigidity ratio in all cases of eccentric and concentric truck loading as well as in the case of dead load. It was also found that the shear distribution factor decreases significantly with an increase in the flexural rigidity ratio for all cases of loading.

For both the exterior and the interior girders, the effect of the flexural rigidity ratio on the shear distribution factor was more significant for bridges with skew angles 45° and 60° than that for bridges with skew angles 0° and 30°. The effect of the flexural rigidity ratio on the shear distribution factor for both the exterior and the interior girders was more significant in the case of eccentric truck loading than that in the case of concentric truck loading or dead load.
6.2.4.5 Effect of Span Ratio

The span ratio, S, is defined as the ratio between the long span length to the short span length. Bridges considered in the parametric study were studied with different short span lengths and a fixed length for the long span. The results reveal that the reactions at the intermediate support decreased significantly with increase in the spans ratio, S. It was also found that the shear at the intermediate support decreases with increase in the spans ratio. However, this decrease is much less significant than that in the case of the reactions at the intermediate support. The above observations were found in all cases of eccentric and concentric truck loading as well as in the case of dead load.

To study the effect of the spans ratio on the reaction and shear distribution factors, D, the reactions and shear forces in a single continuous composite girder were also determined for the different spans ratios considered. It should be noted that an increase or a decrease in the reaction and the shear distribution factors does not necessarily indicate an increase or a decrease in the value of the reaction or in the girder design shear force. Bearing this in mind, the following were observed: (i) in the cases of eccentric and concentric truck loading and in the case of dead load, the reaction distribution factor for an exterior girder increases with an increase in the spans ratio for skew angle $\theta = 0^\circ$; (ii) for all other skew angles considered, $30^\circ$, $45^\circ$, and $60^\circ$, the reaction distribution factor decreases with an increase in the spans ratio; (iii) in the cases of eccentric and concentric truck loading and in the case of dead load, the reaction distribution factor for an interior girder decreases with an increase in the spans ratio for skew angle $\theta = 0^\circ$; (iv) for all
other skew angles considered, 30°, 45°, and 60°, the reaction distribution factor increases with an increase in the spans ratio; (v) in the cases of eccentric and concentric truck loading as well as in the case of dead load, the shear distribution factor for an exterior girder increases with an increase in the spans ratio; and, (vi) for an interior girder of a bridge under eccentric or eccentric truck loading or under dead load the shear distribution factor increases with an increase in the spans ratio.

6.2.4.6 Empirical Formulas for the Reaction and Shear Distribution Factors at the Intermediate Support

Based on the data generated from the parametric study, analyzing more than 600 cases of prototype bridges, empirical formulas were developed for both reaction and shear distribution factors at the intermediate support using a statistical package for the best fit. The factors were determined for both exterior and interior girders. The empirical formulas are in terms of the following significant factors: (i) angle of skew, \( \theta \) in degrees; (ii) bridge aspect ratio, \( A_l = \text{long span length/bridge width} \); (iii) girder spacing expressed as the dimensionless parameter, \( N = \text{number of lanes/number of girders} \); (iv) spans ratio, \( S = \text{long span length/short span length} \); and, (v) the flexural rigidities ratio, \( R \) in percent. The three loading conditions considered were: concentric, eccentric AASHTO truck loadings as well as dead load. Based on preliminary results it was revealed that more accurate prediction for the reaction and shear distribution factors, \( D \), are obtained if the best fit formulas are presented for two ranges of skew angles namely: \( \theta < 30^\circ \) and \( \theta > 30^\circ \). For a bridge with \( \theta = 30^\circ \), the value of the factor \( D \) is calculated from the equations listed below for these two ranges, with \( \theta = 30^\circ \), and the smaller value of \( D \) is used for conservative design. Thus the deduced formulas are as follows:
(a) Reactions at the Intermediate Support

exterior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.41 \times R^{0.1} \times A_r^{0.13} \times S^{0.1}}{(1 + \theta)^{0.002} \times N^{0.93}}$$  \hspace{1cm} (6.61)

for $\theta > 30^\circ$ :-

$$D = \frac{0.387 \times R^{0.16} \times A_r^{0.17}}{\theta^{0.02} \times N^{0.96} \times S^{0.25}}$$  \hspace{1cm} (6.62)

exterior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.47 \times R^{0.96} \times A_r^{0.07}}{(1 + \theta)^{0.002} \times N^{0.72} \times S^{0.06}}$$  \hspace{1cm} (6.63)

for $\theta > 30^\circ$ :-

$$D = \frac{0.496 \times R^{0.1} \times A_r^{0.07}}{\theta^{0.02} \times N^{0.72} \times S^{0.34}}$$  \hspace{1cm} (6.64)

exterior girder of a bridge under dead load

for $\theta < 30^\circ$ :-

$$D = \frac{1.11 \times N^{0.09}}{(1 + \theta)^{0.001} \times A_r^{0.09} \times S^{0.07}}$$  \hspace{1cm} (6.65)
for $\theta > 30^\circ$ :

$$D = \frac{1.17 \times R^{0.003} \times N^{0.09}}{\theta^{0.015} \times A_r^{0.09} \times S^{0.26}}$$  \hspace{1cm} (6.66)$$

interior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$ :

$$D = \frac{0.41 \times (1 + \theta)^{0.004} \times R^{0.01}}{A_r^{0.03} \times N^{0.27} \times S^{0.01}}$$  \hspace{1cm} (6.67)$$

for $\theta > 30^\circ$ :

$$D = \frac{0.287 \times \theta^{0.056} \times R^{0.06} \times A_r^{0.07} \times S^{0.13}}{N^{0.34}}$$  \hspace{1cm} (6.68)$$

interior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$ :

$$D = \frac{1.6 \times (1 + \theta)^{0.01} \times S^{0.22}}{R^{0.11} \times A_r^{0.09} \times N^{0.17}}$$  \hspace{1cm} (6.69)$$

for $\theta > 30^\circ$ :

$$D = \frac{1.08 \times \theta^{0.12} \times S^{0.8}}{R^{0.12} \times A_r^{0.12} \times N^{0.17}}$$  \hspace{1cm} (6.70)$$
interior girder of a bridge under dead load

for $\theta < 30^\circ$ :-

$$D = \frac{0.75 \times (1 + \theta)^{0.002} \times R^{0.107} \times A_r^{0.11} \times S^{0.03}}{N^{0.11}}$$  \hspace{1cm} (6.71)$$

for $\theta > 30^\circ$ :-

$$D = \frac{0.647 \times \theta^{0.062} \times R^{0.064} \times A_r^{0.07} \times S^{0.14}}{N^{0.03}}$$  \hspace{1cm} (6.72)$$

(a) Shear Forces at the Intermediate Support

exterior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.514 \times R^{0.04} \times A_r^{0.09} \times S^{0.22}}{(1 + \theta)^{0.02} \times N^{0.72}}$$  \hspace{1cm} (6.73)$$

for $\theta > 30^\circ$ :-

$$D = \frac{0.678 \times R^{0.097} \times A_r^{0.11} \times S^{0.17}}{\theta^{0.149} \times N^{0.85}}$$  \hspace{1cm} (6.74)$$

exterior girder of a bridge under eccentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.535 \times R^{0.04} \times A_r^{0.04} \times S^{0.29}}{(1 + \theta)^{0.01} \times N^{0.63}}$$  \hspace{1cm} (6.75)$$
for $\theta > 30^\circ$ :-

$$D = \frac{0.66 \times R^{0.09} \times A_r^{0.06} \times S^{0.3}}{\theta^{0.103} \times N^{0.65}}$$

(6.76)

exterior girder of a bridge under dead load

for $\theta < 30^\circ$ :-

$$D = \frac{1.07 \times R^{0.01} \times N^{0.1} \times S^{0.2}}{(1 + \theta)^{0.03} \times A_r^{0.07}}$$

(6.77)

for $\theta > 30^\circ$ :-

$$D = \frac{1.78 \times R^{0.08} \times A_r^{0.025} \times N^{0.14} \times S^{0.22}}{\theta^{0.23}}$$

(6.78)

interior girder of a bridge under concentric AASHTO truck loading

for $\theta < 30^\circ$ :-

$$D = \frac{0.52 \times (1 + \theta)^{0.02} \times S^{0.18}}{R^{0.03} \times A_r^{0.1} \times N^{1.1}}$$

(6.79)

for $\theta > 30^\circ$ :-

$$D = \frac{0.273 \times \theta^{0.194} \times S^{0.3}}{R^{0.05} \times A_r^{0.04} \times N^{1.16}}$$

(6.80)
interior girder of a bridge under eccentric AASHTO truck loading

for \( \theta < 30^\circ \):

\[
D = \frac{1.41 \times (1 + \theta)^{0.066} \times S^{0.19}}{R^{0.02} \times A_r^{0.02} \times N^{0.17}} \tag{6.81}
\]

for \( \theta > 30^\circ \):

\[
D = \frac{0.35 \times 6^{0.575} \times S^{0.1}}{R^{0.09} \times A_r^{0.01} \times N^{0.001}} \tag{6.82}
\]

interior girder of a bridge under dead load

for \( \theta < 30^\circ \):

\[
D = \frac{1.06 \times (1 + \theta)^{0.02} \times A_r^{0.12} \times N^{0.25} \times S^{0.3}}{R^{0.02}} \tag{6.83}
\]

for \( \theta > 30^\circ \):

\[
D = \frac{0.95 \times 6^{0.047} \times A_r^{0.24} \times N^{0.22} \times S^{0.39}}{R^{0.03}} \tag{6.84}
\]

6.2.4.7 Illustrative Example

Consider a three-lane two-unequal-span continuous skew bridge with five girders composite with a concrete deck slab. Both the longitudinal steel girders and the reinforced concrete deck slab are simply supported at the abutments and continuous over the pier. The bridge details are as follows: bridge long span length = 20 m; bridge short span length = 14 m; total bridge length = 34 m; total bridge width = 18.0 m; angle of
skew = 60\(^\circ\); girder spacing = 3 m; deck slab thickness = 0.225 m; five longitudinal girders W610x82; three lines and two lines of moment connected transverse diaphragms W310x21 in the long span and in the short span, respectively. It is required to estimate the maximum reactions and design shear forces at the intermediate support for both the exterior and the interior girders of the bridge. The aspect ratio, \( A_r = 20/18 = 1.11 \), the spans ratio \( S = 20/14 = 1.43 \), and from (6.2) the ratio \( N = 3/5 = 0.6 \). Applying the method presented by Kennedy and Grace (1983), the flexural rigidity in the longitudinal direction \( D_x = (82.3)(10^5)(E) \), where \( E \) = modulus of elasticity of the concrete slab, and the flexural rigidity in the transverse direction considering the effect of transverse diaphragms, \( D_y = (14.6)(10^5)(E) \). From (6.3) the rigidity ratio \( R = (14.6)(10^5)(E)/(82.3)(10^5)(E) = 17.7\% \). For a continuous girder with a composite concrete slab of width 3 m, having a long span of 20 m and a short span of 14 m, subjected to a line of wheel loads of an AASHTO truck one can find that the maximum reaction at the intermediate support \( F = 303 \) kN. and the maximum shear force \( V = 190 \) kN. Due to dead load, for both the exterior and the interior girders \( F_{DL} = 393 \) kN. and \( V_{DL} = 216 \) kN. The intermediate support reaction distribution factor, \( D \), for the exterior girders of the bridge under: concentric truck loading and using (6.62), \( D = 0.905 \); therefore \( F_{max} = (0.9)(F)/D = (0.9)(303)/(0.905) = 301 \) kN., using the reduction factor of 0.9 for multilane loading; under eccentric truck loading using (6.64), \( D = 0.802 \); thus \( F_{max} = 303/0.802 = 378 \) kN.; and, under dead load, using (6.66), \( D = 0.927 \); or \( F_{max} = 393/0.927 = 424 \) kN. For the interior girders of the bridge under: concentric truck loading, using (6.68), \( D = 0.931 \); or \( F_{max} = (0.9)(303)/(0.931) = 293 \) kN; under eccentric truck loading using (6.70),
\[ D = 1.727; \text{ or } F_{\text{max}} = \frac{303}{1.727} = 175 \text{ kN}; \text{ and, for the dead load using (6.72), } D = 1.197, \]

thus \( F_{\text{max}} = \frac{393}{1.197} = 328 \text{ kN}. \) The intermediate shear distribution factors, D, for the exterior girders of the bridge under: concentric truck loading and using (6.74), \( D = 0.836; \)

therefore \( V_{\text{max}} = (0.9)(V)/D = (0.9)(190)/(0.836) = 205 \text{ kN}; \) under eccentric truck loading using (6.76), \( D = 0.892; \) thus \( V_{\text{max}} = 190/0.892 = 213 \text{ kN}; \) and, under dead load, using (6.78), \( D = 0.902; \) or \( V_{\text{max}} = 216/0.902 = 239 \text{ kN}. \) For the interior girders of the bridge under: concentric truck loading, using (6.80), \( D = 1.036; \) or \( V_{\text{max}} = (0.9)(190)/(1.036) = 165 \text{ kN}; \) under eccentric truck loading using (6.82), \( D = 2.395; \) or, \( V_{\text{max}} = 190/2.395 = 79 \text{ kN}; \) and, for the dead load using (6.84), \( D = 1.065; \) thus \( V_{\text{max}} = 216/1.065 = 203 \text{ kN}. \)

Using the AASHTO formula for the impact factor \( I = 26\%. \) Thus, the maximum reactions and design shear forces at the intermediate support may be calculated as follows:

for the exterior girder the maximum reaction for design \( F_{\text{design}} = 424 + (378)(1.26) = 900 \text{ kN}; \) the maximum design shear force \( V_{\text{design}} = 239 + (213)(1.26) = 507 \text{ kN}; \) for the interior girder the maximum reaction for design \( F_{\text{design}} = 328 + (293)(1.26) = 697 \text{ kN}; \) and, the maximum design shear force \( V_{\text{design}} = 203 + (165)(1.26) = 411 \text{ kN}. \)

6.2.4.8 Summary of Findings

The conclusions drawn from the above parametric study can be summarized as follows: (i) the reactions and shear forces at the simply supported ends of a two-span continuous skew composite bridge can be estimated accurately using the shear distribution factors for simply supported skew bridges presented in part I of this chapter; (ii) the
distribution of the reactions at the intermediate support of a two-equal-span continuous composite bridge is uniform and is not affected by increasing the skew angle; (iii) the distribution of the reactions at the intermediate support of a two-unequal-span continuous composite bridge is significantly affected by skew. Increasing the angle of skew for two-unequal-span bridge increases the reaction of the exterior girder and decreases it for the interior girder; (iii) the distribution of shear forces at the intermediate support is critical in both cases of two-equal-span and two-unequal-span skew bridges. The shear forces increase at the exterior girders and decrease at the interior girders with increasing the skew angle; (iv) both the reaction and the shear distribution factors at the intermediate support are very sensitive to changes in the girder spacing. These factors decrease significantly with an increase in the ratio, $N$ (= number of lanes/number of girders) which is a measure of the girder spacing; (v) an increase in the spans ratio (= long span length/short span length) reduces the shear forces at the intermediate support and more significantly reduces the reactions at such support.
RESULTS FROM THE PARAMETRIC STUDIES

PART III: LOAD DISTRIBUTION

CHARACTERISTICS OF CONTINUOUS COMPOSITE BRIDGES AT THE ULTIMATE LIMIT STATE

6.3.1 General

The elastic responses of simply supported composite bridges can be determined by using load distribution factors given by different codes of practice. The American Association of State Highway and Transportation Officials (AASHTO, 1994) considers the center to center girder spacing only in establishing elastic load distribution factors for simply supported right composite bridges. The Ontario Highway Bridge Design Code (OHBDC, 1992) considers several parameters in addition to the girder spacing such as the bridge longitudinal and transverse rigidities. However, the method is limited to simply supported bridges with skew parameters less than a certain value specified in the code. The load distribution factors given by these codes of practice allows the design engineer to treat these complex indeterminate bridges as a simple beam by using influence lines.
Due to the rapid increase in the truck axle load in the last few decades, a very large number of overweight permits were issued in the United States of America. Overloading a composite steel-concrete bridge may result in several problems such as: (i) longitudinal and transverse cracking of the concrete deck slab; (ii) spalling of the concrete cover; (iii) corrosion of the steel reinforcement; and, (iv) corrosion of the main steel girders under the slab due to the seepage of the deicing salt-laden through the cracked concrete deck during the winter months, (Soliman, 1992). For this reason together with the introduction of the ultimate limit state design philosophy, it is clear that a study of the behaviour of composite bridges beyond the working stress range is urgent. A proper ultimate design of a continuous composite bridges requires a reliable evaluation of the design forces at the ultimate limit state.

North American Codes of practice requires strength check of the sections of the bridge at the ultimate limit state. However, these codes of practice specify the use of elastic load distribution factors for the evaluation of the design forces at the ultimate limit state which leads to an extremely conservative design in some cases and to an unsafe design in other cases. It should be noted that these factors do not represent the actual behaviour of the bridge at the ultimate limit state. A proper evaluation of the design forces of a continuous composite bridge at the ultimate limit state requires the consideration of the non-linear response of such bridges. The load redistribution phenomena due to yielding should be also taken into account. For these reasons the distribution factors at the ultimate limit state are significantly different than those in the
elastic range of loading.

In this section, the influence of several parameters on the span and support moment-distribution factors at the ultimate limit state in continuous skew composite steel-concrete bridges are studied. These factors are: angle of skew, girder spacing, bridge aspect ratio, spans ratio, number of lanes, number of girders, intermediate transverse diaphragms, ultimate longitudinal and transverse moments of resistance of the composite sections, and ultimate torsional moment of resistance of the concrete deck slab. The reaction and shear distribution factors of continuous skew composite bridges at the ultimate limit state are also investigated. A detailed parametric study is conducted on prototype two-span continuous skew composite steel-concrete bridges subjected to AASHTO truck loading. The parametric study included more than 600 bridge cases. The response of each bridge in each loading case was traced up to failure. The finite element model used in the study was verified and substantiated by results from tests on three continuous composite steel-concrete bridge models and six single span bridge models subjected to simulated truck loading, and discussed in chapter V. Based on this study empirical formulas for span and support moment-distribution factors, for AASHTO truck loading, at the ultimate limit state are presented.

6.3.2 Parametric Study

The objectives of the parametric study were: (i) to study the behaviour of continuous two-span skew composite steel-concrete bridges over the complete range of
loading up to the collapse load. In this study the non-linear response and the load 
redistribution phenomena due to yielding are taken into account; (ii) to investigate the 
influence of all major parameters affecting the span and support moment-distribution 
factors at the ultimate limit state; (iii) to generate a data base for the span and support 
moment-distribution factors at the ultimate limit state which represent the actual response 
of the bridges at such loading stage; (iv) to develop empirical formulas for such factors 
corresponding to AASHTO truck loading; and, (v) to study the reaction and shear 
distribution factors for continuous two-span skew composite steel-concrete bridges at the 
ultimate limit state. The parametric study was based on the following assumptions: (i) 
both the reinforced concrete deck slab and the longitudinal steel girders are simply 
supported at the abutments and continuous over the intermediate pier; (ii) the reinforced 
concrete deck slab and the supporting steel I-girders are in full composite action; and, (iii) 
all transverse intermediate diaphragms are moment-connected to the longitudinal girders. 
The details of the parametric study are described below:

6.3.2.1 Bridge Geometry

See page 155, section 6.2.2.1.

6.3.2.2 Bridge Structural Properties

See page 156, section 6.2.2.2.
6.3.2.3 Material Properties

(i) The Concrete

Concrete compressive strength \( \approx 30 \text{ MPa}, \)
Modulus of elasticity \( \approx 28,000 \text{ MPa}, \)
Poisson's ratio \( = 0.15 \)

(ii) The Steel of the Longitudinal Girders and the Transverse Diaphragms

Yield strength \( = 300 \text{ MPa}; \)
Modulus of elasticity \( = 200,000 \text{ MPa}; \)
Poison's ratio \( = 0.3 \)

(iii) The Reinforcing Steel

Yield strength \( = 400 \text{ MPa}; \)
Modulus of elasticity \( = 200,000 \text{ MPa} \)

6.3.2.4 Loading Conditions

Numerous loading cases were considered in the parametric study, using an AASHTO standard truck HS20-44. The more than 600 bridge cases considered can be classified into eight main categories, each comprising of more than 75 cases, according to the loading patterns. The categories were as follows: (i) one or two trucks applied on one side of the bridge representing an eccentric truck loading. The trucks occupied one span only to maximize the span moments; (ii) an eccentric truck loading similar to (i)
except that the trucks were applied on both spans to maximize the support moments; (iii) a concentric truck loading in which the bridge was fully loaded by two trucks for two-lane bridges, three trucks for three-lane bridges, and four trucks for four-lane bridges. The trucks were applied on one span only to maximize the span moments; (iv) a concentric truck loading similar to (iii), except that the trucks were applied on both spans to maximize the support moments; (v) one or two trucks applied on one side of the bridge representing an eccentric truck loading. The trucks were applied on both spans and were arranged in a way to maximize the mid-support reactions; (vi) an eccentric truck loading similar to (v) except that the trucks were arranged in a way to maximize the shear forces at the intermediate support; (vi) a concentric truck loading in which the bridge was fully loaded by two trucks for two-lane bridges, three trucks for three-lane bridges, and four trucks for four-lane bridges. The trucks were applied on both spans and were arranged in a way to maximize the mid-support reactions; and, (vii) a concentric truck loading similar to (vi) except that the trucks were arranged in a way to maximize the shear forces at the intermediate support.

6.3.2.5 Incremental Loading Scheme

The wheel loads of AASHTO standard trucks were arranged on each bridge in the way explained above. The number of trucks applied on the bridge lanes was varied from one to four to represent eccentric and concentric truck loading cases for bridges which have number of lanes 2, 3, and 4. The values of the wheel loads were increased in increments up to the collapse of each bridge. At the first loading increment the actual
values of wheel loads of the AASHTO standard truck were used and the number of vertically stacked trucks was considered to be one truck. Following that the values of the wheel loads were increased and the number of the vertically stacked trucks was determined at each loading increment up to the collapse of the bridge. The actual values of the wheel loads of an AASHTO truck are 17.8 kN, and 71.2 kN. Therefore, when the values of the wheel loads applied on the bridge are for example 35.6 kN, and 142.4 kN, the number of vertically stacked trucks applied on the bridge is equal to two trucks. Therefore the number of vertically stacked trucks applied on the bridge can be defined as:

\[
\text{No. of Vertically Stacked Trucks} = \frac{\text{Load at an increment}}{\text{Wheel load of an AASHTO truck}}
\]

(6.85)

It should be noted that there is no relation between the number of the trucks applied on the bridge lanes and the number of the vertically stacked trucks. For example, when a concentric truck loading is applied on a four-lane bridge and it is mentioned that the number of trucks is two, this means that four trucks are applied on the bridge lanes and that the wheel loads of these four trucks have values which are double the actual values of the wheel loads of a standard AASHTO truck. The number of vertically stacked trucks was increased from one up to the collapse of the bridge and the behaviour of the bridge was traced over the complete range of loading.
6.3.3 Moment Distribution Factor at the Ultimate Limit State, $D_u$

Influence line diagrams, for both span and support moments were constructed for all prototype bridges considered in the parametric study. The maximum span and support moments ($M$) in a continuous two-span composite girder under a line of wheel loads of an AASHTO standard truck was first calculated for each prototype bridge. To maximize the span moment due to wheel loads only one span was loaded whereas two spans were loaded to maximize the support moment. The non-linear finite element model was then used to obtain the maximum span and support moments ($M_{\text{max}}$) in all the girders for each prototype bridge at each loading increment. The number of vertically stacked trucks was determined also at each loading increment. The moment distribution factor at the $i^{th}$ loading increment was calculated from the following relationship:

$$D_i = \frac{M \times \text{(number of trucks)}_i}{(M_{\text{max}})_i} \quad (6.86)$$

The span and the support moment distribution factors was calculated at all the loading increments up to the ultimate load of the bridge. At the ultimate load the number of vertically stacked trucks was determined and the maximum span and support moments ($M_{\text{max}})_u$ for all the girders for each prototype bridge in the parametric study were obtained from the finite element analysis. Then, the span and the support moment-distribution factors at the ultimate limit state were calculated using the following relationship:

$$D_u = \frac{M \times \text{(number of trucks)}_u}{(M_{\text{max}})_u} \quad (6.87)$$

The effects of the different influencing factors on the span and the support moment-
distribution factors at the ultimate limit state, $D_{ue}$ will be presented in the following sections.

6.3.3.1 Effect of Angle of Skew

The effect of skew was studied for two cases of eccentric and concentric AASHTO truck loadings. The results reveal that the presence of skew does have an influence on both the span and the support moment distribution factors not only at the elastic range of loading but also over the complete range of loading up to the collapse of the bridge. Figure 6.83 shows the effect of skew on the span moment distribution factor for an exterior girder of a four-lane bridge under eccentric truck loading. This effect is presented for continuous bridges with skew angles 0°, 30°, and 60° over the complete range of loading up to the collapse load of each bridge. The following can be observed: (i) in the elastic range of loading, (number of vertically stacked trucks = one), the span moment distribution factor increases with an increase in the skew angle which indicates a decrease in the girder bending moment. While this increase is not significant when the skew angle increases from 0° to 30°, it becomes significant when the skew angles increases to 60°; (ii) before the formation of the first plastic hinge, the span moment distribution factor is almost not affected by increasing the number of trucks for all the skew angles considered; (iii) after the formation of the first plastic hinge, as expected the redistribution phenomenon starts to take place and the span moment distribution factor increases with an increase in the number of trucks; (iv) the degree of redistribution is almost the same for skew angles 0° and 30°; (v) the degree of redistribution for a bridge
with skew angle $\theta = 60^\circ$ is up to 20% whereas for a bridge with skew angle of $30^\circ$ it is less than 10%; (vi) the formation of the first plastic hinge for a bridge with skew angle $\theta = 60^\circ$ occurs at a number of vertically stacked trucks greater than that in the case of skew angles $0^\circ$ and $30^\circ$; (vii) the span moment distribution factor at the ultimate limit state, $D_u$, increases with an increase in the skew angle. However, this increase is not significant for skew angles less than $30^\circ$; and, (viii) a bridge with skew angle $\theta = 60^\circ$ collapses at a greater number of vertically stacked trucks than bridges with skew angles $0^\circ$ and $30^\circ$.

Figure 6.84 shows the effect of skew on the span moment distribution factor for an interior girder of a two-lane bridge under eccentric truck loading. The following can be observed: (i) in the elastic range of loading, (number of vertically stacked trucks = one), the span moment distribution factor increases with an increase in the skew angle. This increase is more significant when the skew angle is greater than $30^\circ$; (ii) before the formation of the first plastic hinge, for skew angles $0^\circ$ and $30^\circ$, the span moment distribution factor is almost not affected by increasing the number of trucks; (iii) before the formation of the first plastic hinge, for skew angle $\theta = 45^\circ$, the span moment distribution factor decreases with an increase in the number of trucks; (iv) after the formation of the first plastic hinge, for all the skew angles considered, the span moment distribution factor increases significantly with an increase in the number of trucks; (v) the degree of redistribution increases with an increase in the angle of skew; (vi) the span moment distribution factor at the ultimate limit state, $D_u$, increases significantly with an increase in the skew angle; and, (vii) the collapse load of the bridge increases.
significantly with an increase in the skew angle.

Comparing the results for the effect of skew on the span moment distribution factors for both the exterior and the interior girders revealed the following observations: (i) in the case of a rectangular bridge the behaviour of both the exterior and the interior girders are the same over the complete range of loading and both having the same span moment distribution factor at the ultimate limit state; (ii) the interior girder is more significantly affected by increasing the angle of skew than the exterior girder over the complete range of loading; and, (iii) for bridges with skew angles 30°, 45°, and 60° the exterior girder becomes the controlling girder for design over the complete range of loading. Furthermore, at the ultimate limit state the exterior girder has a smaller span moment distribution factor, $D_u$, than the interior girder.

Figure 6.85 shows the effect of skew on the span moment distribution factor at the ultimate limit state, $D_u$, for an exterior girder of a three-lane bridge under concentric truck loading. This effect is presented for bridges with different aspect ratios. It is observed that the factor $D_u$ increases with an increase in the skew angle which indicates a decrease in the girder bending moment. This increase in the factor $D_u$ is not significant when the skew angle $\theta$ is less than 30°. However, for skew angles greater than 30° the factor $D_u$ increases significantly with an increase in the skew angle. It is also observed that the effect of the bridge aspect ratio on the span moment distribution factor at the ultimate limit state, $D_u$, increases with an increase in the skew angle. Similar observations were found for the effect of skew on the span moment distribution factor at the ultimate limit state, $D_u$, for an interior girder of a bridge under concentric truck loading as shown in
Figure 6.86. Comparing the results for the span moment distribution factors at the ultimate limit state, $D_o$, for both the exterior and the interior girders of a bridge under concentric truck loading revealed that the interior girder is much more significantly affected by an increase in the skew angle than the exterior girder and that the exterior girder becomes the controlling girder for design for skew angles 30°, 45°, and 60°.

Different results were found for the support moment distribution factor. Figure 6.87 shows the effect of skew on the support moment distribution factor for an exterior girder of a two-lane bridge under eccentric truck loading. The effect is presented over the complete range of loading up to the collapse of the bridge. It is observed that the support moment distribution factor in the elastic range of loading, (number of vertically stacked trucks = one), is not affected by an increase in the skew angle from 0° to 30°. However, this factor increases significantly when the skew angle is greater than 30°. The support moment distribution factor begins to change significantly when the number of trucks exceeds one and the following can be observed: (i) before the formation of the first plastic hinge, for all the skew angles considered, the support moment distribution factor decreases significantly with increasing the number of vertically stacked trucks; (ii) the first plastic hinge forms in bridges with angles of skew $\theta = 45°$ and 60° at greater number of vertically stacked trucks than in bridges with skew angles 0° and 30°; (iii) after the formation of the first plastic hinge, for all the skew angles considered, the support moment distribution factor start to increase significantly with increasing the number of vertically stacked trucks up to the collapse of the bridge; (iv) for all the skew angles considered, the support moment distribution factor at the ultimate limit state is much
smaller than that in the elastic range of loading. (v) the support moment distribution factor at the ultimate limit state, \( D_u \), is almost the same for skew angles 0° and 30°. (vi) for skew angles greater than 30° the support moment distribution factor at the ultimate limit state, \( D_u \), increases significantly with an increase in the skew angle; and, (vii) the collapse load of the bridge increases with an increase in the skew angle.

Similar observations were found in the case of the support moment distribution factor for an interior girder of a bridge under eccentric truck loading as shown in Figure 6.88. However, the effect of skew on the support moment distribution factor is more significant in the elastic range of loading than that at the ultimate limit state. Comparing the results of the support moment distribution factor at the ultimate limit state for both the exterior and the interior girders revealed that the interior girder was much more significantly affected by increasing the skew angle. The exterior girder is the controlling girder for skew angles 30°, 45°, and 60°.

Figure 6.89 shows the effect of skew on the support moment distribution factor at the ultimate limit state for an exterior girder of a four-lane bridge under concentric truck loading. The effect is presented for bridges with different aspect ratios. It is observed that the factor \( D_u \) is not significantly affected when the skew angle is increased from 0° to 30°. However, for bridges with skew angles greater than 30° the factor increases significantly with an increase in the angle of skew, \( \theta \). It is noted that there is almost no interaction between the aspect ratio and the skew angle in this case. Figure 6.90 shows the effect of skew on the support moment distribution factor at the ultimate limit state, \( D_u \), for an interior girder of a four-lane bridge under concentric truck loading.
Similar observations are found as in the case of an exterior girder of a bridge under concentric truck loading. However, the effect of skew on the support moment distribution factor at the ultimate limit state is more significant in the case of an interior girder than that in the case of an exterior girder.

For both the span and the support moments the effect of skew on the moment distribution factor at the ultimate limit state, $D_u$, under eccentric truck loading is similar to that for the case of concentric truck loading. However, the values of the factor $D_u$ in the latter case are lower than the former. This implies that the critical loading case for the design of skew bridges is the concentric truck loading case and thus for one-lane or two-lane bridges the concentric loading case will control the design since for such bridges the load modification factor equals to one. However, because of the reduction in the load intensity for multilane loading (AASHTO, 1994), in the case of bridges with three or more lanes both eccentric and concentric truck loading cases should be considered.

### 6.3.3.2 Effect of Bridge Aspect Ratio

The results reveal that the aspect ratio does have an influence on both the span and the support moment distribution factors. Figure 6.91 shows the effect of the bridge aspect ratio on the span moment distribution factor for an exterior girder of a right four-lane bridge under concentric truck loading. This effect is presented over the complete range of loading up to the collapse of the bridge. The following can be observed: (i) in the elastic range of loading, (number of vertically stacked trucks = one), the span moment distribution factor increases with an increase in the bridge aspect ratio; (ii) the number of
vertically stacked trucks needed for the formation of the first plastic hinge decreases significantly with an increase in the bridge aspect ratio, as expected; (iii) the span moment distribution factor at the ultimate limit state, \( D_u \), decreases significantly with an increase in the bridge aspect ratio; and, (iv) the number of vertically stacked trucks needed for the collapse of the bridge decreases significantly with an increase in the aspect ratio, as expected. Similar observations were found for the effect of the bridge aspect ratio on the span moment distribution factor for an interior girder of a right bridge under concentric truck loading (Fig. 6.92).

Figure 6.93 shows the effect of the bridge aspect ratio on the span moment distribution factor for an exterior girder of a four-lane bridge under eccentric loading. The bridge has a skew angle \( \theta = 30^\circ \) and the results are presented over the complete range of loading up to the collapse of the bridge. In this case, the span moment distribution factor at the ultimate limit state, \( D_u \), decreases significantly with an increase in the bridge aspect ratio. Furthermore, the number of trucks needed for the collapse of the bridge decreases significantly with an increase in the aspect ratio. Similar observations were found in the case of an interior girder of a 30° skew bridge under eccentric truck loading as shown in Figure 6.94. Similar observations were found also in the case of an exterior or an interior girder of a 60° skew bridge under concentric truck loading as shown in Figures 6.95 and 6.96.

Figure 6.97 shows the effect of bridge aspect ratio on the support moment distribution factor for an exterior girder of a right two-lane bridge under eccentric truck loading. The following can be observed: (i) in the elastic range of loading, number of
trucks = one, the support moment distribution factor decreases significantly with an increase in the bridge aspect ratio; (ii) this is also the case before the formation of the first plastic hinge; (iii) the number of trucks needed for the formation of the first plastic hinge decreases significantly with an increase in the bridge aspect ratio; and, (iv) the support moment distribution factor at the ultimate limit state, $D_u$, decreases with an increase in the bridge aspect ratio. It is also noted that the effect of the bridge aspect ratio is more significant in the elastic range of loading than that at the ultimate limit state. Similar observations were found for the interior girder of a right bridge under eccentric truck loading as shown in Figure 6.98. Similar observations were found also for the exterior and the interior girders of a $30^\circ$ and $45^\circ$ bridge under concentric truck loading as shown in Figures 6.99 to 6.102.

### 6.3.3.3 Effect of Girder Spacing

A dimensionless factor $N (= \text{number of lanes/number of girders})$ was suggested in part I of this chapter in equ. (6.2) as a measure for the girder spacing. The factor $N$ was calculated for all prototype bridges considered in the parametric study and the relationships between the ratio $N$ and the moment distribution factor at the ultimate limit state, $D_u$, were determined for both the span and the support moments. The results revealed that the girder spacing has a significant influence on the span and the support moment distribution factors at the ultimate limit state. Figure 6.103 shows the effect of the ratio $N$ on the span moment distribution factor at the ultimate limit state, $D_u$, for an exterior girder of a two-lane bridge under eccentric truck loading. It is observed that the
factor $D_u$ decreases significantly with an increase in the ratio $N$. This effect is almost the same for all the skew angles considered. Similar observations were found in the case of an interior girder of a bridge under eccentric truck loading as shown in Figure 6.104. Similar observations were found also in the case of an exterior or an interior girder of a bridge under concentric truck loading as shown in Figures 6.105 and 6.106.

Figures 6.107 to 6.110 shows the effect of the ratio $N$ on the support moment distribution factor at the ultimate limit state, $D_u$, for both the exterior and the interior girders of a bridge under eccentric or concentric truck loading. In all these cases the factor $D_u$ decreases significantly with an increase in the ratio $N$. This effect is almost the same for all the skew angles considered.

6.3.3.4 Effect of Transverse Intermediate Diaphragms

The results from the tests on six simple span and three continuous skew composite bridge models, presented in chapter V, revealed the importance of the presence of orthogonal intermediate transverse diaphragms moment-connected to the longitudinal girders. Tests on actual bridges (Boyce, 1977) and on bridge models (Kennedy et al., 1989) have shown that such connections lead to improved bridge stiffness, better load distribution, and increased ultimate load capacity. One of the objectives of the present study was to investigate the effect of the presence of intermediate transverse diaphragms moment-connected to the longitudinal girders on the load distribution characteristics of skew continuous composite steel-concrete bridges at the ultimate limit state. The prototype bridges considered in the parametric study herein were examined first without

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transverse intermediate diaphragms. They were then analyzed with different number of orthogonal intermediate transverse diaphragms moment-connected to the longitudinal girders. The number of such diaphragms were varied from one to six in each span. The results revealed that the effect of the presence of such diaphragms is more significant at the ultimate limit state than in the elastic range of loading. Their beneficial effects can be summarized as follows: (i) for a bridge with transverse intermediate diaphragms moment-connected to the longitudinal girders the number of vertically stacked trucks needed for the formation of the first plastic hinge is greater than that required in the case of a bridge without diaphragms. The number of such trucks increases with the increase in the number of transverse diaphragms; (ii) both the span and the support moment-distribution factors at the ultimate limit state, $D_u$, increase with increase in the number of transverse intermediate diaphragms; and, (iii) the collapse load of the bridge increases with increase in the number of transverse diaphragms.

A method was developed by Kennedy and Soliman (1992) for the calculation of the moments of resistance of the composite sections considering the effect of the transverse intermediate diaphragms. Furthermore the method takes into account the degree of fixity between the transverse diaphragms and the longitudinal girders. This method was employed herein to calculate the moments of resistance of the composite sections for all the prototype bridges considered in the parametric study. For each prototype bridge the following were determined: (i) ultimate longitudinal positive moment of resistance of composite section per unit width, $m_{p_l}$; (ii) ultimate transverse positive moment of resistance per unit length taking into account the effect of transverse
intermediate diaphragms if any, $m_{pl}$, (iii) ultimate longitudinal negative moment of composite section per unit width, $m_\ell$, (iv) ultimate transverse negative moment of resistance per unit length taking into account the effect of transverse intermediate diaphragms if any, $m_{tr}$, and, (v) ultimate torsional moment of resistance of the concrete deck slab per unit width, $m_{tor}$.

To obtain a relationship between the ultimate moments of resistance and the span moment distribution factor at the ultimate limit state, $D_u$, the following dimensionless parameters were calculated for all the prototype bridges in the parametric study:

$$M_p = \frac{m_{pl}}{m_{pl}} \times 100$$  \hspace{1cm} (6.88)

$$M_{tor} = \frac{m_{tor}}{m_{pl}} \times 100$$  \hspace{1cm} (6.89)

The above parameters were found to have a significant influence on the span moment distribution factor at the ultimate limit state, $D_u$. For both the exterior and the interior girders the factor $D_u$ increases significantly with an increase in the parameters $M_p$ and $M_{tor}$. However, an increase in the parameter $M_p$ affects the factor $D_u$ more significantly than an increase in the parameter $M_{tor}$. The effect of both parameters on the factor $D_u$ is more significant in eccentric truck loading cases than in concentric truck loading cases. It was also found that an increase in these parameters results in an increase in the collapse load of the bridge. It should be noted that the effect of the above parameters is more significant in the case of bridges with skew angles 45° and 60° than that in the case of bridges with skew angles 0° and 30°.
In the case of the support moment-distribution factor at the ultimate limit state $D_u$, the following dimensionless parameters were calculated for all the prototype bridges considered in the parametric study:

\[ M_n = \frac{m_{nt}}{m_s} \times 100 \quad (6.90) \]

\[ M_{n_{\text{var}}} = \frac{m_{n_{\text{var}}}}{m_s} \times 100 \quad (6.91) \]

The results revealed that the above parameters do have an influence on the support moment distribution factor at the ultimate limit state $D_u$. For both the exterior and the interior girders, the support moment distribution factor $D_u$ increases significantly with an increase in the parameters $M_n$ and $M_{n_{\text{var}}}$. The effect of these parameters is more significant in the case of eccentric than concentric truck loading cases. Furthermore, the effects of the parameters $M_n$ and $M_{n_{\text{var}}}$ on the factor $D_u$ are more significant in the case of bridges with skew angles 45° and 60° than those in the case of bridges with skew angles 0° and 30°.

### 6.3.3.5 Effect of Span Ratio

The span ratio, $S$, is defined as the ratio between the long span length to the short span length. Bridges considered in the parametric study were studied with different short span lengths and a fixed length for the long span. The results reveal that the span moment is reduced significantly with increase in the span ratio when only the long span is loaded. It was found also that the support moment decreases significantly with increase
in the span ratio when both the long and the short spans are loaded. To study the effect of the span ratio on the moment distribution factors at the ultimate limit state, $D_u$, the moments is a single continuous composite girder was also determined for the different span ratios considered. It should be noted that an increase or a decrease in the moment distribution factors $D_u$ does not necessarily indicate an increase or a decrease in the girder moment. Bearing this in mind, the following were observed: (i) both the span and the support moment distribution factors at the ultimate limit state $D_u$ increase with increase in the spans ratio $S$ for both interior and exterior girders for all cases of eccentric and concentric truck loadings. However, the rate of this increase is higher in the case of skew angles $45^\circ$ and $60^\circ$ than in the case of skew angles $\leq 30^\circ$; and, (ii) the support moment distribution factor at the ultimate limit state is more sensitive to a change in the spans ratio $S$ than the distribution factor for the span moment.

6.3.3.6 Empirical Formulas for the Moment Distribution Factor at the Ultimate Limit State, $D_u$

Based on the data generated from the parametric study, analyzing more than 300 cases of prototype bridges, empirical formulas were developed using a statistical package for best fit for both the preliminary design and the final design of continuous composite steel-concrete bridges at the ultimate limit state. In fact, the formulas deduced for the preliminary design and the final design represent one design method with two steps. In the first step, the design engineer knows the geometrical properties of the bridge based on the intersection ($\theta$, $A_r$, $S$, and $N$). Therefore, it is possible for him to select
preliminary steel sections. In the second step, the formulas are more accurate since they recognize the ultimate moments of resistance of the sections of the bridge as an additional factor. With the assumed steel sections from the first step, the design engineer calculates the moments of resistance of the different sections of the bridge. He then proceeds to use the formulas to calculate the final design moments at the ultimate limit state. Finally he checks the strength of the selected sections.

6.3.3.6.1 Preliminary Design

In this case, simple approximate formulas were generated for both the span and the support moment distribution factors at the ultimate limit state, $D_w$. It is recommended that these formulas be used only in the preliminary design of continuous composite bridges at the ultimate limit state, for a preliminary selection of steel sections. The moment distribution factors were determined for both exterior and interior girders. The effect of the moments of resistance of the different sections of the bridge is neglected in this case for simplicity. The moment distribution factors were determined in both cases of eccentric and concentric truck loading cases. Load modification factors given in the AASHTO code (AASHTO, 1994) were applied to the moment distribution factors in the cases of concentric loading and the results were compared to those from eccentric loading cases. The smaller factors were used in the generation of the formulas. The empirical formulas are in terms of the following significant geometrical factors. (i) angle of skew, $\theta$ in degrees; (ii) bridge aspect ratio, $A_r = \text{long span length}/\text{bridge width}$; (iii) girder spacing expressed as the dimensionless parameter, $N = \text{number of lanes}/\text{number of}$
girders, and, (iv) span ratio, \( S = \text{long span length}/\text{short span length} \). Based on preliminary results it was revealed that more accurate predictions for the moment distribution factors, \( D_u \), are obtained if the best fit formulas are presented for two ranges of skew angles namely: \( \theta \leq 30^\circ \) and \( 30^\circ < \theta \leq 60^\circ \). Thus the deduced formulas for preliminary design are as follows:

(a) Span Moments

For an Exterior Girder

for \( \theta \leq 30^\circ \):

\[
D_u = \frac{0.721 \times (1 + \theta)^{0.013} \times S^{0.06}}{A_r^{0.006} \times N^{0.58}}
\]  \hspace{1cm} (6.92)

for \( 30^\circ < \theta \leq 60^\circ \):

\[
D_u = \frac{0.174 \times 6^{0.45} \times S^{0.36}}{A_r^{0.07} \times N^{0.37}}
\]  \hspace{1cm} (6.93)

For an Interior Girder

for \( \theta \leq 30^\circ \):

\[
D_u = \frac{0.76 \times (1 + \theta)^{0.04} \times S^{0.04}}{A_r^{0.01} \times N^{0.59}}
\]  \hspace{1cm} (6.94)
for $30^\circ < \theta \leq 60^\circ$:

$$D_u = \frac{0.047 \times \theta^{0.87} \times S^{0.19}}{A_r^{0.01} \times N^{0.28}}$$  \hspace{1cm} (6.95)

(b) Support Moments

For an Exterior Girder

for $\theta \leq 30^\circ$:

$$D_u = \frac{0.39 \times (1 + \theta)^{0.01}}{A_r^{0.06} \times N^{0.84} \times S^{0.21}}$$  \hspace{1cm} (6.96)

for $30^\circ < \theta \leq 60^\circ$:

$$D_u = \frac{0.08 \times \theta^{0.48} \times S^{0.25}}{A_r^{0.06} \times N^{0.64}}$$  \hspace{1cm} (6.97)

For an Interior Girder

for $\theta \leq 30^\circ$:

$$D_u = \frac{0.36 \times (1 + \theta)^{0.04}}{A_r^{0.01} \times N^{0.97} \times S^{0.23}}$$  \hspace{1cm} (6.98)

for $30^\circ < \theta \leq 60^\circ$:

$$D_u = \frac{0.013 \times \theta^{1.02} \times S^{0.3}}{A_r^{0.01} \times N^{0.61}}$$  \hspace{1cm} (6.99)
6.3.3.6.2 Final Design

In this case more accurate formulas were generated for both the span and the support moment distribution factors at the ultimate limit state, $D_u$. It is recommended that these formulas be used in the final design of continuous composite steel-concrete bridges at the ultimate limit state. The moment distribution factors were determined for the exterior and the interior girders in both cases of eccentric and concentric AASHTO truck loading. The empirical formulas are in terms of the following significant factors: (i) angle of skew, $\theta$ in degrees; (ii) bridge aspect ratio, $A_r = \text{long span length}/\text{bridge width}$; (iii) girder spacing expressed as the dimensionless parameter, $N = \text{number of lanes}/\text{number of girders}$; and, (iv) span ratio, $S = \text{long span length}/\text{short span length}$. In addition, the formulas recognize the moments of resistance of different sections of the bridge in terms of the two dimensionless parameters $M_p$ and $M_{pnr}$ in the case of the span moment distribution factors and the two dimensionless parameters $M_n$ and $M_{nur}$ in the case of the support moment distribution factors. For accurate predictions of the moment distribution factors, $D_u$, the best fit formulas are presented for two ranges of skew angles namely: $0 \leq 30^\circ$ and $30^\circ < \theta \leq 60^\circ$. Thus the deduced formulas are as follows:

(a) Span Moments

**Exterior girder of a bridge under concentric AASHTO truck loading**

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.55 \times (1 + \theta)^{0.013} \times S^{0.13} \times M_p^{0.05} \times M_{pnr}^{0.13}}{A_r^{0.006} \times N^{0.62}}$$

(6.100)
for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.12 \times \theta^{0.453} \times S^{0.5} \times M_p^{0.135} \times M_{por}^{0.06}}{A_r^{0.07} \times N^{0.49}}$$ (6.101)

Exterior girder of a bridge under eccentric AASHTO truck loading

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.67 \times (1 + \theta)^{0.015} \times S^{0.41} \times M_p^{0.11} \times M_{por}^{0.03}}{A_r^{0.007} \times N^{0.54}}$$ (6.102)

for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.11 \times \theta^{0.45} \times S^{0.68} \times M_p^{0.15} \times M_{por}^{0.19}}{A_r^{0.008} \times N^{0.61}}$$ (6.103)

Interior girder of a bridge under concentric AASHTO truck loading

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.34 \times (1 + \theta)^{0.044} \times S^{0.14} \times M_p^{0.06} \times M_{por}^{0.51}}{A_r^{0.01} \times N^{0.59}}$$ (6.104)

for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.013 \times \theta^{0.87} \times S^{0.36} \times M_p^{0.1} \times M_{por}^{0.76}}{A_r^{0.01} \times N^{0.3}}$$ (6.105)
Interior girder of a bridge under eccentric AASHTO truck loading

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.30 \times (1 + \theta)^{0.04} \times S^{0.34} \times M_P^{0.07} \times M_{pior}^{0.63}}{A_r^{0.015} \times N^{0.7}}$$  \hspace{1cm} (6.106)

for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.012 \times \theta^{0.87} \times S^{0.55} \times M_P^{0.11} \times M_{pior}^{0.89}}{A_r^{0.014} \times N^{0.41}}$$  \hspace{1cm} (6.107)

(b) Support Moments

Exterior girder of a bridge under concentric AASHTO truck loading

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.199 \times (1 + \theta)^{0.01} \times M_n^{0.004} \times M_{nor}^{0.35}}{A_r^{0.03} \times N^{0.77} \times S^{0.2}}$$  \hspace{1cm} (6.108)

for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.042 \times \theta^{0.48} \times S^{0.4} \times M_n^{0.064} \times M_{nor}^{0.26}}{A_r^{0.08} \times N^{0.66}}$$  \hspace{1cm} (6.109)

Exterior girder of a bridge under eccentric AASHTO truck loading

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.16 \times (1 + \theta)^{0.01} \times M_n^{0.011} \times M_{nor}^{0.45}}{A_r^{0.02} \times N^{0.88} \times S^{0.002}}$$  \hspace{1cm} (6.110)

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for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.036 \times \theta^{0.48} \times S^{0.56} \times M_n^{0.07} \times M_{n_{tor}}^{0.35}}{A_r^{0.08} \times N^{0.77}}$$  \hspace{1cm} (6.111)$$

**Interior girder of a bridge under concentric AASHTO truck loading**

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.21 \times (1 + \theta)^{0.04} \times M_n^{0.027} \times M_{n_{tor}}^{0.27}}{A_r^{0.03} \times N^{0.94} \times S^{0.2}}$$  \hspace{1cm} (6.112)$$

for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.0087 \times \theta^{1.02} \times S^{0.45} \times M_n^{0.1} \times M_{n_{tor}}^{0.1}}{A_r^{0.05} \times N^{0.68}}$$  \hspace{1cm} (6.113)$$

**Interior girder of a bridge under eccentric AASHTO truck loading**

for $\theta \leq 30^\circ$ :-

$$D_u = \frac{0.22 \times (1 + \theta)^{0.03} \times M_n^{0.03} \times M_{n_{tor}}^{0.43}}{A_r^{0.04} \times N^{1.08} \times S^{0.33}}$$  \hspace{1cm} (6.114)$$

for $30^\circ < \theta \leq 60^\circ$ :-

$$D_u = \frac{0.0091 \times \theta^{1.0} \times S^{0.72} \times M_n^{0.11} \times M_{n_{tor}}^{0.25}}{A_r^{0.03} \times N^{0.85}}$$  \hspace{1cm} (6.115)$$

**6.3.4 Reaction and Shear Distribution Factors at the Ultimate Limit State, $D_u$**

An extensive parametric study was conducted on prototype continuous composite bridges under AASHTO standard truck loading in order to study the shear and reaction
distribution factors at the ultimate limit state, $D_u$. Both eccentric and concentric truck loading cases were considered. Several parameters were varied such as: angle of skew, bridge aspect ratio; girder spacing; number of lanes, number of girders; spans ratio, and, number of transverse intermediate diaphragms. The reaction and the shear distribution factors were determined over the complete range of loading up to the collapse load of the bridge. At such load the reaction and the shear distribution factors at the ultimate limit state were determined. The results reveal that the degree of redistribution in the reactions and shear does not exceed 3% and therefore can be neglected. Both the reaction and the shear distribution factors are almost constant over the complete range of loading up to the ultimate load. The distribution factors at the ultimate limit state are almost equal to those found in the elastic range of loading. It is recommended that the elastic reaction and shear distribution factors presented in part II be used for the determination of the reactions and the shear at the ultimate limit state in the case of continuous composite bridges. It should be noted that the above recommendation agrees with the provision given in the AASHTO code (AASHTO, 1994) which permits the use of the elastic load distribution factors for the ultimate design of composite bridges.

6.3.5 Summary of Findings

The conclusions found in the above study can be summarized as follows: (i) the provisions given by North American codes of practice which permit the use of the elastic load distribution factors for the ultimate design of composite bridges are not valid in the case of the span and the support moments in rectangular and skew continuous composite
bridges. These provisions lead to an extremely conservative span moments and to an unsafe support moments. The span and the support distribution factors at the ultimate limit state should reflect the actual behaviour of the bridge at such state. Both the non-linear behaviour and the load redistribution phenomena should be taken into account in the determination of the distribution factors at the ultimate limit state; (ii) the span and the support moment distribution factors at the ultimate limit state are influenced by the following factors: angle of skew, bridge aspect ratio, girder spacing, number of lanes, number of girders, spans ratio, and the presence of transverse intermediate diaphragms; and, (iii) the provisions given by North American codes of practice which permits the use of the elastic load distribution factors for the ultimate design of composite bridges are valid in the case of the reaction and the shear in continuous composite bridges. The degree of redistribution in this case does not exceed 3% and therefore can be neglected. The reaction and shear distribution factors at the ultimate limit state are almost equal to those found in the elastic range of loading.
6.4.1 General

To study the dynamic response of composite steel-concrete bridges it is important to understand the free vibrational behaviour of such bridges, i.e. to determine the natural frequencies and the corresponding mode shapes. An accurate estimate for the natural frequencies is required in order to avoid the state of resonance under earthquake, wind, and under truck loading. An example reported by Grace (1986) is the Tacoma Narrows Suspension Bridge which collapsed due to wind effects. The bridge failed in the second mode which was anti-symmetrical in the transverse direction due to weakness in the torsional rigidity of the bridge.

Under a moving truck, the bridge exhibits forces which are significantly greater than those resulting from a static truck load. Different methods are given by North American codes of practice to account for this effect in the design of bridges. The American Association of State Highway and Transportation Officials (AASHTO, 1994)
has traditionally applied an impact factor depending only on the span length of the bridge. The Ontario Highway Bridge Design Code (OHBDC, 1992) specifies the dynamic load allowance as a factor of the first flexural frequency of the bridge. The formulation for the dynamic load allowance in the OHBD code (1992) was based on testing full-scale bridges and it takes into account the dynamic amplifications due to the interaction between the vehicle and the bridge when their natural frequencies are the same. The OHBD code (1992) recommends the use of the flexural beam theory to determine the first flexural frequency and hence the dynamic load allowance. The use of the flexural beam theory is reliable in the case of right bridges without diaphragms. However, in the case of skew bridges or with the presence of transverse intermediate diaphragms, the use of the beam theory leads to unacceptable errors in the determination of the first flexural frequency and hence the dynamic load allowance.

In this section, the influence of several parameters on the natural frequencies and the corresponding mode shapes of composite steel-concrete bridges is studied. These parameters are: angle of skew, bridge aspect ratio, number of lanes, number of intermediate transverse diaphragms, and continuity. A detailed parametric study was conducted on prototype simply supported and continuous two-span skew composite steel-concrete bridges. The parametric study included more than 150 bridges. The finite element model used in the study was verified and substantiated by results from tests on two continuous composite steel-concrete bridge models presented by Grace and Kennedy (1985). The validity of the use of the flexural beam theory in the determination of the
first natural frequency, as recommended by the OHBDC (1992), is investigated in the case of skew composite bridges and in the case of bridges with transverse intermediate diaphragms. Based on this study a method is suggested for the determination of the first flexural frequency of skew composite bridges. The method takes into account the presence of transverse intermediate diaphragms. The use of this method with the charts given by the OHBDC (1992) leads to a more accurate estimate for the dynamic load allowance of skew composite bridges.

6.4.2 Finite Element Model Verification

Grace and Kennedy (1985) tested two continuous composite steel-concrete bridge models. Reinforced concrete was used throughout the deck slab of the first bridge model whereas the deck slab of the second model in the region of the pier support was prestressed. The geometrical and structural properties for the two models were identical. The span length of the bridge models was 3.7 m with a total length of 7.4 m and a width of 0.91 m. The steel grid of the bridge models consisted of three longitudinal beams and five lines of cross beams. The steel sections used for the longitudinal beams were S130x22 whereas those used for the cross beams were S75x8. The steel used for both the longitudinal beams and the cross beams was of grade G40.21-M300W. The thickness of the deck slab was 50 mm for the two bridge models. Stud shear connectors welded to the top flanges of the longitudinal steel beams were used to provide the composite action between the longitudinal steel beams and the deck slab.
The finite element model, described in chapter III, was used to analyze the first bridge model where reinforced concrete was used throughout the deck slab. The first three natural frequencies and their corresponding mode shapes were determined. Table 6.1 shows a comparison between the results of the current analysis and the experimental results presented by Grace and Kennedy (1985). In both sets of results the first mode was a symmetrical mode (flexural mode) and the results for the first natural frequency showed excellent agreement with a difference of only 2.5%. The second mode was an antisymmetrical mode (torsional) and the difference between the two sets of results for the second natural frequency was 3%. The third mode was a symmetrical mode (flexural mode) and the difference between the two sets of results for the third natural frequency was 3.5%.

6.4.3 Parametric Study

The objectives of the parametric study were: (i) to investigate the influence of all major variables affecting the natural frequencies and the corresponding mode shapes of composite bridges; (ii) to generate a database for the first three natural frequencies and the corresponding mode shapes by analyzing more than 150 bridge cases; (iii) to test the validity of the use of the flexural beam theory in the determination of the first flexural frequency in the case of skew bridges; and, (iv) to develop a reliable method for the determination of the first flexural frequency for skew composite bridges taking into account the effect of transverse intermediate diaphragms. The parameters chosen for this study were: angle of skew, bridge aspect ratio, number of lanes, number of intermediate
transverse diaphragms and continuity. The parametric study was based on the assumptions mentioned earlier in page 126. The details of the parametric study were presented in page 126.

6.4.4 Effect of Angle of Skew

Numerous prototype bridges were analyzed in the parametric study with different angles of skew, aspect ratios, number of lanes, and different number of transverse intermediate diaphragms. For all the right bridges considered, the mode shapes were found to be as follows: (i) the first mode was symmetrical in the transverse direction which means that it is a flexural mode, Figure 6.111; (ii) the second mode was anti-symmetrical in the transverse direction which means that it is a torsional mode, Figure 6.112; and, (iii) the third mode was symmetrical in the transverse direction (Flexural mode), Figure 6.113. Similar mode shapes were found also for all the skew bridges considered in the parametric study as shown in Figures 6.114 to 6.116.

Figure 6.117 shows the effect of skew on the first natural frequency of two-lane bridges with different aspect ratios. It is observed that there is an increase in the first natural frequency with an increase in the skew angle. This increase reflects the increase in the stiffness of the bridge as the angle of skew increases. It is also noted that the rate of such increase is more significant for skew angles greater than 30°. The first natural frequency for a bridge with an aspect ratio = 2.0, for example, were as follows: the first natural frequency increased from 4.2 Hz to 4.7 Hz when the skew angle increased from
0° to 30° representing an increase of about 11%; the first natural frequency increased from 4.2 Hz to 5.1 Hz when the skew angle increased from 0° to 45° representing an increase of about 21%; and, the first natural frequency increased from 4.2 to 5.7 when the skew angle increased from 0° to 60° representing an increase of about 35%. It is observed that there is no interaction between the bridge aspect ratio and the angle of skew. Figure 6.118 shows the effect of skew on the first natural frequency of three-lane bridges having different aspect ratios. The increases in the first natural frequency for a bridge with an aspect ratio = 1.3, for example, were 15%, 29%, and 54% when the skew angle increased from 0° to 30°, 45°, and 60°, respectively. It should be noted that the effect of skew in this case was more significant in the case of bridges having small aspect ratios i.e. for relatively wide bridges. Similar observations were found in the case of four-lane bridges as shown in Figure 6.119.

For all the prototype bridges considered in the parametric study the second mode shape was anti-symmetrical in the transverse direction representing a torsional mode. The effect of skew in this case was also significant. Figure 6.120 shows the effect of skew on the second natural frequency for two-lane bridges with different aspect ratios. It is observed that the second natural frequency increases with an increase in the skew angle with no interaction with the aspect ratio. The increases in the second natural frequency due to skew for a bridge with an aspect ratio = 2.0, for example, were as follows: the second natural frequency increased from 5.4 Hz to 6.2 Hz when the skew angle increased from 0° to 30° representing an increase of about 14%; the second natural frequency
increased from 5.4 Hz to 6.6 Hz when the skew angle increased from 0° to 45° representing an increase of about 22%; and, the second natural frequency increased from 5.4 Hz to 7.1 Hz when the skew angle increased from 0° to 60° representing an increase of about 31%. Figure 6.121 shows the effect of skew on the second natural frequency of three-lane bridges with different aspect ratios. The increases in the natural frequency for a bridge with an aspect ratio = 1.3 i.e. relatively wide bridge, for example, were 12%, 24%, and 41% when the skew angle increased from 0° to 30°, 45°, and 60°, respectively. Similar observations were found for the second natural frequency in the case of four-lane bridges as shown in Figure 6.122.

The skew also affects the third natural frequency. However the effect of skew in this case was less significant than that in the case of the first and the second natural frequencies. In this case the corresponding mode shape was symmetrical in the transverse direction representing a flexural mode. Figure 6.123 shows the effect of skew on the third natural frequency of two-lane bridges with different aspect ratios. It is observed that the third natural frequency increases with increase in the angle of skew. The increases in the third natural frequency due to skew for a bridge with an aspect ratio = 2.0, for example, were as follows: the third natural frequency increased from 10.8 Hz to 11.3 Hz when the skew angle increased from 0° to 30° representing an increase of about 5%; the third natural frequency increased from 10.8 Hz to 11.7 Hz when the skew angle increased from 0° to 45° representing an increase of about 8%; and, the third natural frequency increased from 10.8 Hz to 12.2 Hz when the skew angle increased from 0° to 60°.
representing an increase of about 13%. Figure 6.124 shows the effect of skew on the third natural frequency of three-lane bridges with different aspect ratios. The increases in the third natural frequency due to skew for a bridge with an aspect ratio = 1.3, for example were 4%, 7%, and 13% when the skew angle increased from 0° to 30°, 45°, and 60°, respectively. Similar observations were found for the third natural frequency in the case of four-lane bridges as shown in Figure 6.125.

6.4.5 Effect of Bridge Aspect Ratio

The prototype bridges considered in the parametric study were analyzed with different aspect ratios. In all cases of two-lane, three-lane, and four-lane bridges, the bridge width was kept constant whereas the span length was varied according to the aspect ratios. The steel sections for the longitudinal girders were also varied according to the span length. Heavier steel sections were used for larger span lengths. The results reveal that the bridge aspect ratio has a significant influence on the first three natural frequencies considered. However, the corresponding mode shapes were not affected by changing the bridge aspect ratio. For all the aspect ratios considered the first mode shape was symmetrical in the transverse direction representing a flexural mode. Figure 6.126 shows the effect of the bridge aspect ratio on the first natural frequency of two-lane bridges having different skew angles. It is observed that the first natural frequency decreases significantly with an increase in the bridge aspect ratio for all the skew angles considered. This is expected since the increase in the span length results in a decrease in the stiffness of the bridge and therefore a corresponding decrease in the natural frequency.
frequency. The decreases in the first natural frequency due to an increase in the aspect ratio for a bridge with a skew angle $\Theta = 45^\circ$ were as follows: the first natural frequency decreased from 5.1 Hz to 3.9 Hz when the aspect ratio increased from 2 to 2.4 representing a decrease of about 24%; and, the first natural frequency decreased from 5.1 Hz to 2.9 Hz when the aspect ratio increased from 2.0 to 2.9 representing a decrease of about 43%. Similar observations were found for the first natural frequency in the cases of three-lane and four-lane bridges as shown in Figures 6.127 and 6.128.

The second mode shape was always anti-symmetrical in the transverse direction representing a torsional mode. The second mode shape was not affected by changing the bridge aspect ratio. However, the second natural frequency was significantly affected by changing the bridge aspect ratio. Figure 6.129 shows the effect of the bridge aspect ratio on the second natural frequency of two-lane bridges with different skew angles. It is observed that the second natural frequency decreases significantly with an increase in the bridge aspect ratio for all the skew angles considered. The decreases in the second natural frequency due to an increase in the aspect ratio for a bridge with a skew angle $\Theta = 45^\circ$ were as follows: the second natural frequency decreased from 5.7 Hz to 4.4 Hz when the bridge aspect ratio increased from 2.0 to 2.4 representing a decrease of about 22%; and, the second natural frequency decreased from 5.7 Hz to 3.6 Hz when the bridge aspect ratio increased from 2.0 to 2.9 representing a decrease of about 37%. Similar observations were found for the second natural frequency in the case of three-lane and four-lane bridges as shown in Figures 6.130 and 6.131.
For all the aspect ratios considered, the third mode shape was always symmetrical in the transverse direction representing a flexural mode. The third natural frequency was significantly affected by varying the aspect ratio. Figure 6.132 shows the effect of the bridge aspect ratio on the third natural frequency of two-lane bridges with different skew angles. It is observed that the third natural frequency decreases significantly with an increase in the bridge aspect ratio for all the skew angles considered. The decreases in the third natural frequency due to an increase in the bridge aspect ratio for a bridge with an angle of skew $\theta = 45^\circ$, for example, were as follows: the third natural frequency decreased from 11.7 Hz to 8.3 Hz when the aspect ratio increased from 2.0 to 2.4 representing a decrease of about 29%; and, the third natural frequency decreased from 11.7 Hz to 6.3 Hz when the aspect ratio increased from 2.0 to 2.9 representing a decrease of about 46%. Similar observations were found for the third natural frequency in the case of three-lane and four-lane bridges as shown in Figures 6.133 and 6.134.

6.4.6 Effect of Transverse Intermediate Diaphragms

The effects of transverse intermediate diaphragms on the natural frequencies and their corresponding mode shapes were studied for all the prototype bridges in the following categories: (i) bridges with no transverse diaphragms; and, (ii) bridges with one to six transverse intermediate diaphragms moment-connected to the longitudinal girders. The longitudinal and transverse rigidities were calculated for all the prototype bridges using the method presented by Kennedy and Grace (1983). Thus the rigidity ratio, $R$, was calculated using equation 6.3.
The presence of transverse intermediate diaphragms did not affect the mode shapes of all the prototype bridges considered in the parametric study. However, the diaphragms had a significant influence on the natural frequencies. This influence increases with increase in the number of such diaphragms. Figure 6.135 shows the effect of the rigidity ratio on the first natural frequency of two-lane bridges with different angles of skew. The following can be observed: (i) the first natural frequency increases with an increase in the rigidity ratio; and, (ii) the effect of the rigidity ratio is more significant in the case of skew angles 45° and 60° than that in the case of skew angles 0° and 30°. The above observations reveal the importance of the presence of transverse intermediate diaphragms moment-connected to the longitudinal girders. The presence of such diaphragms increases the first flexural frequency of the bridge leading to a smaller dynamic load allowance. Similar observations were found for the effect diaphragms on the second and the third natural frequencies as shown in Figures 6.136 and 6.137. However, in these cases the effect of diaphragms was more significant than that in the case of the first natural frequency.

6.4.7 The Validity of the Flexural Beam Theory in the Estimation of the First Flexural Frequency of Skew Composite Bridges

The Ontario Highway Bridge Design Code (OHBDC, 1992) specifies the dynamic load allowance (DLA) as a function of the first flexural frequency of the bridge and recommends the use of the flexural beam theory for the determination of such frequency. The first flexural frequency \( \omega_1 \) was calculated for all the prototype bridges considered in
the parametric study using the well known flexural beam theory equation. This equation is given by Humar (1990) in the form of:

\[ \omega_1 = \pi^2 \sqrt{\frac{E I}{m L^4}} \]  \hspace{1cm} (6.116)

where: \( \omega_1 \) = the first flexural frequency of the bridge; \( E \) = modulus of elasticity of the longitudinal steel beams; \( I \) = moment of inertia of the bridge cross section; \( m \) = mass per unit length of the bridge; and, \( L \) = span length of the bridge.

The first flexural frequency for all the prototype bridges considered in the parametric study was determined also using the finite element method. The results were compared to those from the flexural beam theory. Table 6.2 shows a comparison between the first flexural frequency calculated by the two methods for three two-lane right bridges, \( \theta = 0^\circ \), and three two-lane skew bridges, \( \theta = 60^\circ \). It can be observed that the flexural beam theory is capable of predicting the first flexural frequency for right bridges with different aspect ratios. The flexural beam theory underestimates very slightly the first flexural frequency in comparison to the finite element method by 4\%, 4.8\%, and 2.6\% for bridges with aspect ratios 2.0, 2.4, and 2.9, respectively. These differences in the first flexural frequency are not significant and therefore the recommendation of the OHBD code (1992) that the flexural beam theory be used in the calculation of the first flexural frequency is valid in the case of right bridges. However, it can be also observed that in the case of skew bridges the flexural beam theory failed to predict reliable values for the first flexural frequency. In the case of bridges with skew angle, \( \theta = 60^\circ \) for example, the
flexural beam theory underestimated the first flexural frequency in comparison to the finite element method by 27\%, 27\%, and 32\% for bridges with aspect ratios 2.0, 2.4, and 2.9, respectively. It should be noted that these differences are much greater for bridges having small aspect ratios. Therefore the recommendation of the OHBD code (1992) that the flexural beam theory be used in the calculation of the first flexural frequency is not valid in the case of skew bridges. Results, not shown herein for brevity, reveal that the use of the flexural beam theory in the calculation of the first flexural frequency is unacceptable also in the case of right bridges with transverse intermediate diaphragms. The presence of such diaphragms moment-connected to the longitudinal girders leads to an increase in the first flexural frequency of the bridge. However, the flexural beam theory equation has no response to the presence of these diaphragms which leads to an unacceptable underestimation of the first flexural frequency of the bridge and therefore to incorrect estimate for the dynamic load allowance.

6.4.8 The Proposed Method for the Estimation of the First Flexural Frequency

Figure 6.138 shows the response of the flexural beam theory equation and the finite element method to a change in the angle of skew. Both methods yielded almost the same first flexural frequency for a rectangular bridge, i.e. $\theta = 0^\circ$. As the skew angle increases the first flexural frequency predicted using the finite element method increases. However, the first flexural frequency predicted using the flexural beam theory is the same for all the skew angles considered. The flexural beam theory has also no response to the presence of intermediate transverse diaphragms. It is suggested that in the case of skew
bridges or in the case of right bridges having intermediate transverse diaphragms, a simple beam idealization is inadequate and it becomes necessary to consider the bridge as a three-dimensional structure. A method is suggested herein for the estimation of the first flexural frequency of any composite bridge with or without transverse diaphragms. The method can be summarized in the calculation of the first flexural frequency, \( \omega_1 \), using the flexural beam theory equation by treating any bridge as a right bridge without transverse diaphragms and then a magnification factor, \( \omega_s \), is applied to \( \omega_1 \). The magnification factor, \( \omega_s \), takes into account the effect of skew and the effect of the presence of transverse intermediate diaphragms. Finally the first flexural frequency, \( \omega_{1s} \), can be calculated from the following relationship:

\[
\omega_{1s} = \omega_1 \times \omega_s \quad (6.117)
\]

The magnification factor, \( \omega_s \), was determined for two categories of bridges, namely bridges with no transverse intermediate diaphragms and bridges with transverse intermediate diaphragms. The derivation of the magnification factor for the two categories will be presented in the following sections.

6.4.8.1 Bridges with no Intermediate Transverse Diaphragms

From the results of the parametric study carried out on some 100 prototype bridges having no transverse diaphragms it became evident that there is a need for a magnification factor to be applied to the results of using the flexural beam theory equation. The magnification factor in this case is a function of: (i) angle of skew \( \theta \), in degrees; and, (ii) bridge aspect ratio, \( A_r \), (= span length/total bridge width). The magnification factor, \( \omega_s \),
was calculated for all the prototype bridges considered in the parametric study using the following relationship:

\[
\omega_\theta = \frac{\omega_{\text{finite element analysis}}}{\omega_{\text{flexural beam theory}}}
\]  \hspace{1cm} (6.118)

Using a statistical package for best fit, empirical formulas were generated for the factor \(\omega_\theta\) in terms of the above parameters. Based on preliminary results, it was revealed that more accurate predictions for the magnification factor are obtained if the best fit formulas are presented for two ranges of skew angles namely: \(\theta < 30^\circ\), and \(\theta > 30^\circ\). For a bridge with a skew angle \(\theta = 30^\circ\), the value of the factor \(\omega_\theta\) is calculated from the equations listed below for these two ranges, with \(\theta = 30^\circ\), and the smaller value of \(\omega_\theta\) is used for conservative design. Thus:

for \(\theta < 30^\circ\):

\[
\omega_\theta = \frac{1.01 \times (1 + \theta)^{0.023}}{A_f^{0.01}}
\]  \hspace{1cm} (6.119)

for \(\theta > 30^\circ\):

\[
\omega_\theta = \frac{0.154 \times \theta^{0.551}}{A_f^{0.09}}
\]  \hspace{1cm} (6.120)

6.4.8.2 Bridges with Intermediate Transverse Diaphragms

In this case the presence of intermediate transverse diaphragms further increases the first flexural frequency of the bridge and therefore the problem becomes more complicated. The magnification factor in this case is governed by the following
significant parameters: (i) angle of skew, $\theta$ in degrees, (ii) bridge aspect ratio, $A_r (= \text{span length/total bridge width})$; and, (iii) the flexural rigidity ratio, in percent ($= \text{flexural rigidity in the transverse direction per unit length/flexural rigidity in the longitudinal direction per unit width}$). The magnification factor $\omega_s$ was calculated for all the 150 bridges considered in the parametric study using equation (6.118). Using a statistical package for best fit, empirical formulas were generated for the factor $\omega_s$ in terms of the above parameters. The best fit formulas are presented for two ranges of skew angles as explained earlier. Thus:

for $\theta < 30^\circ$ :-

$$\omega_s = \frac{0.98 \times (1 + \theta)^{0.023} \times R^{0.016}}{A_r^{0.008}}$$  \hspace{1cm} (6.121)

for $\theta > 30^\circ$ :-

$$\omega_s = \frac{0.176 \times \theta^{0.507} \times R^{0.025}}{A_r^{0.11}}$$  \hspace{1cm} (6.122)

6.4.9 Dynamic Response of Continuous Skew Composite Bridges

Figures 6.139 to 6.141 shows the mode shapes of a skew continuous composite bridge. It is observed that the first mode shape is symmetrical in the transverse direction representing a flexural mode. The second mode shape is anti-symmetrical in the transverse direction representing a torsional mode. The third mode shape is also a flexural mode. The above mode shapes were found for all the prototype continuous bridges considered in the parametric study. The above mode shapes were the same as
those found in the case of simply supported bridges discussed earlier. Therefore, continuity has no effect on the mode shapes. Table 6.3 shows a comparison between the natural frequencies of single-span and two-span continuous bridges. It is observed that the results for the first and the second natural frequencies are almost the same for single-span and two-span continuous bridges. It is observed also that there is only small difference, not exceeding 5%, in the third natural frequency for single-span and two-span continuous bridges. It is suggested therefore that the method presented above for the determination of the first flexural frequency of simply supported bridges be utilized for two-span continuous bridges provided that the length of long span be used in the calculation of the bridge aspect ratio.

6.4.10 Illustrative Example

Consider a three-lane single span bridge with five longitudinal girders composite with a concrete deck slab designed according to the Ontario Highway Bridge Design Code (OHBDC, 1992). The bridge details are as follows: span of the bridge = 16.8 m; total bridge width = 14 m; angle of skew = 60°; girder spacing = 3 m; deck slab thickness = 0.225 m; five longitudinal girders W610x174; and, three lines of moment connected transverse intermediate diaphragms W360x33. It is required to calculate the first flexural frequency of the bridge and hence estimate the dynamic load allowance using the OHBD code (1992).
Step (1) calculation of the first flexural frequency using the flexural beam theory equation:

Equation (6.116) is used. The modulus of elasticity of steel \( E = 200 \times 10^9 \text{ kN/m}^2 \), the moment of inertia of the bridge cross section \( I = 0.00471 \text{ m}^4 \), the mass per unit length of the bridge cross section \( m = 18.2 \text{ kN/m} \); and, the span of the bridge \( l = 18 \text{ m} \). Therefore first flexural frequency of the bridge, using the flexural beam theory is \( \omega_1 = 4.2 \text{ Hz} \).

Step (2) calculation of the magnification factor \( \omega_m \) using the proposed method

To account for the effect of skew and the presence of intermediate transverse diaphragms the magnification factor will be calculated using equation (6.122). Applying the method presented by Kennedy and Grace (1983), the flexural rigidity in the longitudinal direction \( D_x = 157 \times 10^5 \times E_c \), where \( E_c \) = modulus of elasticity of the concrete deck slab, and the flexural rigidity in the transverse direction considering the effect of the transverse diaphragms, \( D_y = 23.7 \times 10^5 \times E_c \). From equation (6.3) the rigidity ratio \( R = (23.7 \times 10^5 \times E_c) / (157 \times 10^5 \times E_c) = 15\% \). The aspect ratio \( A_r = 16.8/14 = 1.2 \). The magnification factor for the bridge using equation (6.122) with the angle of skew \( \theta = 60^\circ \), becomes \( \omega_1 = 1.47 \).

Step (3) calculation of the correct first flexural frequency of the bridge

Using equation (6.117), and with \( \omega_1 = 4.2 \text{ Hz} \) and the magnification factor \( \omega_m = 1.47 \), the estimated first flexural frequency of the skew bridge is, \( \omega_{1,1} = 4.2 \times 1.47 = 6.17 \text{ Hz} \).
Step (4) determination of the dynamic load allowance

From the Ontario Highway Bridge Design Code (OHBDC, 1992), the dynamic load allowance (DLA), corresponding to a first flexural frequency of 6.17 Hz, is 25%. It should be noted that the DLA for this bridge is 40% if the flexural beam theory equation is used without correction.

6.4.11 Summary of Findings

The conclusions drawn from the above study can be summarized as follows: (i) there is a definite improvement in the dynamic characteristics of composite bridges as a result of skew, since the presence of skew results in an increase in the stiffness of the bridge. An increase in the skew angle increases the natural frequencies of the bridge. However, this increase does not affect the corresponding mode shapes. In all cases the first and the third mode shapes were symmetrical in the transverse direction representing a flexural mode whereas the second mode shape was anti-symmetrical in the transverse direction representing a torsional mode; (ii) the bridge aspect ratio does have a significant influence on the natural frequencies of the bridge. An increase in the bridge aspect ratio results in a decrease in the first three natural frequencies; (iii) the presence of intermediate transverse diaphragms, moment-connected to the longitudinal girders improves the dynamic characteristics of composite bridges. The natural frequencies increase with an increase in the rigidity ratio. This increase is more significant for bridges with skew angles > 30°; (iv) the natural frequencies and their corresponding mode shapes are almost the same for single-span and two-span continuous bridges; and, (v) the use of the flexural
beam theory in the determination of the first flexural frequency, as specified in the OIHBDC
code (1992) is reliable only in the case of right bridges without diaphragms. In the case
of skew bridges or with the presence of intermediate transverse diaphragms, a simple
beam idealization is inadequate. In these cases it is necessary to consider the bridge as
a three-dimensional structure.
CHAPTER VII

SUMMARY AND CONCLUSIONS

7.1 Summary

Detailed experimental and theoretical studies were carried out to investigate the elastic and ultimate static responses as well as the dynamic response of simply supported and continuous skew composite steel-concrete bridges. Several design parameters were considered such as: angle of skew, bridge aspect ratio, girder spacing, number of lanes, number of girders, longitudinal and transverse rigidities, effect of transverse intermediate diaphragms, moments of resistance of the bridge sections at the ultimate limit state, spans ratio, and loading pattern. A detailed literature review was conducted in order to establish the foundation for the study. It was observed that the previously cited literature had concentrated on right simply supported composite steel-concrete bridges.

The experimental study was carried out by testing six single span simply supported and three continuous skew composite steel-concrete bridge models under single concentrated loads as well as eccentric and concentric simulated truck loads. It should be noted that the effects of temperature, shrinkage, and creep for deck slabs were not
considered herein because they are outside the scope of this study. The finite element method was used in the analyses. The analytical effort covered the nine tested bridge models and was extended to conduct six extensive parametric studies on prototype bridges subjected to both AASHTO and OHBDC standard truck loads as well as to dead load. The parametric studies included more than 2500 bridge cases. The data generated in these parametric studies was used to deduce empirical formulas for both the elastic and ultimate moment and shear distribution factors for the design of both simply supported and continuous skew composite steel-concrete bridges. The analytical effort also included another extensive parametric study on the dynamic response of simply supported and continuous skew composite steel-concrete bridges. The parametric study included more than 150 bridge cases. Based on this parametric study a method was developed for the estimation of the first flexural frequency of simply supported and continuous skew composite bridges. The use of this method with the charts given in the OHBD code (1992) leads to a reliable dynamic load allowance for such bridges.

7.2 Conclusions

Based on the experimental and the theoretical results the following conclusions are drawn:

1- The good agreement between the experimental and the theoretical results supports the reliability of using the finite element modelling to predict the elastic and ultimate responses as well as the dynamic response of skew composite steel-concrete bridges.
**For the static study:**

2- The presence of skew results in significant reductions in the deflections, strains, and bending moments for both the exterior and the interior girders. It also causes a concentration of reaction at the girder close to the obtuse corner and a decrease in the reactions of all other girders in the bridge. It is suggested that using the simplified methods for right bridges given by different codes of practice for the design of skew bridges will lead to an extremely conservative design girder bending moments and to an unsafe reactions and design shear forces.

3- In the evaluation of girder moments in skew composite bridges the exterior girder is the controlling girder for design. For one-lane or two-lane bridges the concentric truck loading case is the critical case yielding maximum moments in both the exterior and the interior girders. However, for bridges having three or more lanes, both eccentric and concentric truck loading cases should be compared to determine the maximum moments for design. The girder moment due to the bridge dead load is also significant and has to be evaluated carefully.

4- It is important to use intermediate transverse diaphragms moment-connected to the longitudinal girders. The presence of such diaphragms has the following beneficial effects: (i) reduces the strains, deflections, and longitudinal girder bending moments; (ii) reduces the reaction at the girder close to the obtuse corner and increases it for both the girder close to the acute corner and the interior girders which means a more uniform
reaction distribution for skew bridges; and, (iii) increases the ultimate load carrying capacity of skew composite bridges.

5- In the case of two-span continuous skew composite bridges, the reactions at the simply supported end of either the long or the short span can be maximized by loading the long span or the short span, respectively. This loading pattern will result in a critical reaction distribution with the reactions at the girder close to the obtuse corner being much greater than those at the girder close to the acute corner and the interior girders. In this case, it is suggested that either the long or the short spans can be treated as a simple-span bridge and the reactions can be distributed with reasonable accuracy based on the deduced load-distribution factors of simply supported bridges.

6- For two-span continuous skew composite bridges the reactions at the intermediate support can be maximized by loading both spans. Therefore, as a result of symmetry in the case of two-equal-span skew bridges, the distribution of the reactions at the intermediate support is not critical and is very close to that for right bridges. However, in the case of two-unequal-span skew bridges the distribution of reactions at the intermediate support becomes more critical. The reactions of the girders close to the obtuse corner of the long span are greater than those of the girders close to the acute corner.

7- The distribution of shear forces at the intermediate support is critical in both
cases of two-equal-span and two-unequal-span continuous skew bridges. The shear forces increase at the exterior girders and decrease at the interior girders as the skew angle increases.

8- The moment and shear distribution factors for both simply supported and continuous skew bridges are very sensitive to changes in the girder spacing. These factors decrease significantly with increase in the ratio, N (= number of lanes/number of girders) which is a measure of the girder spacing.

9- In the case of two-span continuous bridges, both span and support girder moments decrease significantly with an increase in the span ratio, S (=long span length/short span length).

10- The span and the support moment distribution factors at the ultimate limit state are influenced by the following factors: angle of skew, bridge aspect ratio, girder spacing, number of lanes, number of girders, spans ratio, loading pattern, the presence of intermediate transverse diaphragms, and the moments of resistance of the bridge sections at the ultimate limit state.

11- The provisions of the North American codes of practice which permit the use of the elastic load distribution factors for the ultimate design of composite bridges are not valid in estimating the span and support moments in continuous composite bridges. These
provisions lead to an extremely conservative span moments and to unsafe support moments. The span and the support moment distribution factors at the ultimate limit state should reflect the actual behaviour of the bridge at such state. Both the non-linear behaviour and the load redistribution phenomena should be taken into account in the determination of the distribution factors at the ultimate limit state.

12- The provisions of the North American codes of practice which permit the use of the elastic load distribution factors for the ultimate design of composite bridges are valid in the case of the reaction and the shear in continuous composite bridges. The degree of redistribution in this case is very small and can be neglected. The reaction and shear distribution factors at the ultimate limit state are almost equal to those found in the elastic range of loading.

For the dynamic study:

13- There is a definite improvement in the dynamic characteristics of composite bridges as a result of skew. An increase in the skew angle increases the natural frequencies of the bridge. However, this increase does not affect the corresponding mode shapes.

14- The presence of intermediate transverse diaphragms, moment-connected to the longitudinal girders improves the dynamic characteristics of composite bridges. The natural frequencies increase with an increase in the rigidity ratio. This increase is more
significant for bridges with skew angles $\geq 30^\circ$.

15- The natural frequencies and their corresponding mode shapes are almost the same for single-span and two-span continuous bridges.

16- The use of the flexural beam theory in the determination of the first flexural frequency as specified in the OHBD code (1992) is reliable only in the case of right bridges without transverse diaphragms. In the case of skew bridges or in the presence of intermediate transverse diaphragms, a simple beam idealization is inadequate. In such cases, it is necessary to consider the bridge as a three-dimensional structure.

7.3 Recommendations for Future Research

It is recommended that future research efforts be directed towards the following:

1- The study of the load distribution characteristics of skew composite bridges supported by non-parallel support lines.

2- The study of the load distribution characteristics of simply supported and continuous curved composite bridges at the ultimate limit state.

3- The study of the dynamic characteristics of curved composite bridges.

4- The study of the effects of temperature, shrinkage, and creep for deck slabs in skew composite bridges.
REFERENCES

ACI Committee 318 (1989), "Building Code Requirements for Reinforced Concrete (ACI 318-89) and Commentary - ACI 318 R-89", American Concrete Institute, Detroit, 360 pp.


American Concrete Institute (1989), "ACI Manual of Concrete Practice", Detroit, Michigan, 48219.


Derby, T.F., and Calcaterra, P.C. (1971), "Control of Single Span Highway Bridge
Vibrations". Highway Research Record, No. 354, Highway Research, pp. 27-44

Descartos, E.S. (1979), "Live-Load Distribution in Skewed Prestressed Concrete I-Beam and Spread Box-Beam Bridges", Fritz Engineering Laboratory, Report No. 3873, Lehigh University, Bethlehem, Pa., Aug.


Hall, J.C., and Kostem, C.N. (1980), "Overloading Behaviour of Steel Highway
Bridges", Fritz Engineering Laboratory, Report No. 435.1, Lehigh University, Bethlehem, PA, August


Humar, J.L. (1990), "Dynamics of Structures", Prentice-Hall Inc.


Kennedy, J.B. (1983), "Orientation of Ribs in Waffle-Slab Skew Bridges", Journal of


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TABLE 4.1 Description of the Simply Supported Single Span Bridge Models

<table>
<thead>
<tr>
<th>Bridge model number</th>
<th>Angle of skew, $\theta$ (degrees)</th>
<th>Skew span (mm)</th>
<th>Skew width (mm)</th>
<th>Deck thickness (mm)</th>
<th>Type of transverse diaphragms</th>
</tr>
</thead>
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<tr>
<td>#1</td>
<td>30</td>
<td>1778</td>
<td>1232</td>
<td>38</td>
<td>Diaphragms moment-connected to the longitudinal beams</td>
</tr>
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<td>#2</td>
<td>45</td>
<td>1778</td>
<td>1509</td>
<td>38</td>
<td>Diaphragms moment-connected to the longitudinal beams</td>
</tr>
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<td>60</td>
<td>1778</td>
<td>2134</td>
<td>38</td>
<td>Diaphragms moment-connected to the longitudinal beams</td>
</tr>
<tr>
<td>#4</td>
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<td>1778</td>
<td>1509</td>
<td>38</td>
<td>End diaphragms only</td>
</tr>
<tr>
<td>#5</td>
<td>45</td>
<td>1778</td>
<td>1509</td>
<td>38</td>
<td>Diaphragms shear-connected to the longitudinal beams</td>
</tr>
<tr>
<td>#6</td>
<td>45</td>
<td>1778</td>
<td>1509</td>
<td>38</td>
<td>Diaphragms moment-connected to the longitudinal beams and made composite with the deck slab</td>
</tr>
<tr>
<td>Bridge model number</td>
<td>Angle of skew, θ (degrees)</td>
<td>Skew width (mm)</td>
<td>Deck thickness (mm)</td>
<td>Long span length (mm)</td>
<td>Short span length (mm)</td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------------------</td>
<td>----------------</td>
<td>-------------------</td>
<td>----------------------</td>
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</tr>
<tr>
<td># 7</td>
<td>0</td>
<td>1219</td>
<td>51</td>
<td>2134</td>
<td></td>
</tr>
<tr>
<td># 8</td>
<td>45</td>
<td>1724</td>
<td>51</td>
<td>2134</td>
<td></td>
</tr>
<tr>
<td># 9</td>
<td>45</td>
<td>1724</td>
<td>51</td>
<td>2743</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 4.3 SECTIONAL PROPERTIES OF STEEL BEAMS

<table>
<thead>
<tr>
<th></th>
<th>AREA (mm²)</th>
<th>I_x (mm⁴)</th>
<th>I_y (mm⁴)</th>
<th>DEPTH (mm)</th>
<th>WIDTH (mm)</th>
<th>FLANGE THICKNESS (mm)</th>
<th>WEB THICKNESS (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S100x11</td>
<td>1450</td>
<td>2.55 x 10⁶</td>
<td>0.324 x 10⁶</td>
<td>102</td>
<td>68.0</td>
<td>7.4</td>
<td>4.8</td>
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<td>S75x8</td>
<td>1070</td>
<td>1.04 x 10⁶</td>
<td>0.190 x 10⁶</td>
<td>76.0</td>
<td>59.0</td>
<td>6.6</td>
<td>4.3</td>
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Table 5.1 Cracking Loads and Collapse Loads for the Tested Bridge Models

<table>
<thead>
<tr>
<th>Bridge model number</th>
<th>Cracking Load (kN)</th>
<th>Collapse Load (kN)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Finite element analysis</td>
</tr>
<tr>
<td># 1</td>
<td>104</td>
<td>81</td>
</tr>
<tr>
<td># 2</td>
<td>131</td>
<td>99</td>
</tr>
<tr>
<td># 3</td>
<td>164</td>
<td>131</td>
</tr>
<tr>
<td># 4</td>
<td>89</td>
<td>79</td>
</tr>
<tr>
<td># 5</td>
<td>96</td>
<td>82</td>
</tr>
<tr>
<td># 6</td>
<td>137</td>
<td>102</td>
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<td># 7</td>
<td>184</td>
<td>123</td>
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<td># 8</td>
<td>153</td>
<td>101</td>
</tr>
<tr>
<td># 9</td>
<td>120</td>
<td>99</td>
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</table>
Table 6.1 Comparison of Results of the Present Finite Element Analysis to Experimental Results (Grace and Kennedy, 1985)

<table>
<thead>
<tr>
<th></th>
<th>Experimental Results (Grace, 1986) Hz</th>
<th>Present Finite Element Analysis Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Natural Frequency (Flexural)</td>
<td>18.0</td>
<td>17.6</td>
</tr>
<tr>
<td>Second Natural Frequency (Torsional)</td>
<td>21.0</td>
<td>20.4</td>
</tr>
<tr>
<td>Third Natural Frequency (Flexural)</td>
<td>27.0</td>
<td>26.1</td>
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</tbody>
</table>
Table 6.2 The First Flexural Frequency Calculated by the Finite Element Method and the Flexural Beam Theory

<table>
<thead>
<tr>
<th>Number of lanes</th>
<th>Aspect ratio</th>
<th>Angle of skew (degrees)</th>
<th>First flexural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Finite element analysis</td>
</tr>
<tr>
<td>Two</td>
<td>2.0</td>
<td>0</td>
<td>4.36</td>
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<tr>
<td>Two</td>
<td>2.4</td>
<td>0</td>
<td>3.36</td>
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<tr>
<td>Two</td>
<td>2.9</td>
<td>0</td>
<td>2.31</td>
</tr>
<tr>
<td>Two</td>
<td>2.0</td>
<td>60</td>
<td>5.69</td>
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<tr>
<td>Two</td>
<td>2.4</td>
<td>60</td>
<td>4.38</td>
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<tr>
<td>Two</td>
<td>2.9</td>
<td>60</td>
<td>3.30</td>
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Table 6.3 Comparison of the Natural Frequencies of Single-Span Bridges to Two-Span Continuous Bridges

<table>
<thead>
<tr>
<th>Number of lanes</th>
<th>Aspect ratio</th>
<th>Angle of Skew (degrees)</th>
<th>Natural Frequency (Hz)</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Single-span bridge</td>
<td>Two-span continuous bridge</td>
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<tr>
<td>Two</td>
<td>2.0</td>
<td>0</td>
<td>4.36&lt;sup&gt;A&lt;/sup&gt;</td>
<td>4.39&lt;sup&gt;A&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.69&lt;sup&gt;B&lt;/sup&gt;</td>
<td>5.69&lt;sup&gt;B&lt;/sup&gt;</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>10.94&lt;sup&gt;C&lt;/sup&gt;</td>
<td>10.6&lt;sup&gt;C&lt;/sup&gt;</td>
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<tr>
<td>Two</td>
<td>2.86</td>
<td>45</td>
<td>2.93</td>
<td>3.00</td>
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<td>6.33</td>
<td>6.50</td>
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<td></td>
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<td>9.87</td>
<td>9.23</td>
</tr>
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<td>Three</td>
<td>1.82</td>
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<tr>
<td></td>
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<td></td>
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<td>Four</td>
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<td>4.43</td>
<td>4.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.10</td>
<td>5.09</td>
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<td></td>
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<td>8.35</td>
<td>7.97</td>
</tr>
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<td>Four</td>
<td>1.48</td>
<td>45</td>
<td>3.08</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.92</td>
<td>3.94</td>
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<tr>
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<td></td>
<td></td>
<td>6.00</td>
<td>5.75</td>
</tr>
</tbody>
</table>

Note:  
<sup>A</sup> first natural frequency (flexural mode).  
<sup>B</sup> second natural frequency (torsional mode)  
<sup>C</sup> third natural frequency (flexural mode)
Fig. 1.1 Static Analysis Layout (Theoretical Analysis)
Fig. 1.2 Static Analysis Layout (Experimental Study)
Fig. 1.3 Dynamic Analysis Layout
- Four Nodes Element
- Displacement Components: U1, U2, U3, \( \phi_1, \phi_2, \phi_3 \) (U is the displacement and \( \phi \) is the rotation)
- Stress Components: S1, S2, S12
- Force Components: N1, N2, Q1, Q2, Q12
- Moment Components: M1, M2, M12
- REBAR Option Available in Two Directions

FIG. 3.1 SHELL ELEMENT S4R
- Two Nodes Element
- Displacement Components: $u_1, u_2, u_3, \phi_1, \phi_2, \phi_3$
- Force Components: Total Torque $T$
  Warping Torque $T_w$
  Bending Moment $M_1, M_2$
  Shearing Force $V_1, V_2$

FIG. 3.2 BEAM ELEMENT B31H
FIG. 3.4 Uniaxial Behaviour of Plain Concrete
FIG. 3.5 Concrete Failure Surfaces In Plane Stress
FIG. 3.6 Tension Stiffening Model for Concrete DUE TO REINFORCEMENT
FIG. 3.7 Idealized Stress Strain Relation for Steel
FIG. 3.8 Typical Finite Element Mesh for Bridge Model #1
FIG. 3.9 Typical Finite Element Mesh for Bridge Model #2

Shell Elements (S4R)
Beam Elements (B31H)
Beam Elements (B31H)
FIG. 3.10 Typical Finite Element Mesh for Bridge Model #3
FIG. 3.11 Typical Finite Element Mesh for Bridge Model #4
FIG. 3.12 Typical Finite Element Mesh for Bridge Model #5

Shell Elements (S4R)
Beam Elements (B31H)
Beam Elements (B31H)
FIG. 3.13 Typical Finite Element Mesh for Bridge Model #6
FIG. 3.14 Typical Finite Element Mesh for Bridge Model #7
FIG. 3.16 Typical Finite Element Mesh for Bridge Model #9
(a) Transverse Section

(b) Plan

Note: All dimensions are in mm

FIG. 4.1 Detailed Geometry of Bridge Model #1
(a) Transverse Section

(b) Plan

Note: All Dimension are in mm

FIG. 4.2 Detailed Geometry of Bridge Model #2
FIG. 4.3 Detailed Geometry of Bridge Model z3
(a) Transverse Section

Simple Support

Exterior Beam (D)

S 75x8

b=1067

45°

Interior Beam (C)

Interior Beam (B)

Exterior Beam (A)

End Diaphragm

S 75x8

(b) Plan

Note: All dimensions are in mm

FIG. 4.4 Detailed Geometry of Bridge Model #4
FIG. 4.5 Detailed Geometry of Bridge Model #5
(a) Transverse Section

(b) Plan

Note: All Dimension are in mm

FIG. 4.6 Detailed Geometry of Bridge Model #6
FIG. 4.8 Detailed Geometry of Bridge Model #8

Note: All Dimensions are in mm
FIG. 4.9 Detailed Geometry of Bridge Model #9

Note: All Dimension are in mm
FIG. 4.11 Compression Test On Concrete Cylinders (Using Strain Gauges On Concrete)
FIG. 4.12 Load-Elongation Relationship for Reinforcing Wires
FIG. 4.13 Load-Strain Relationship for Steel Beams
FIG. 4.15 Nelson Stud Welding System
FIG. 4.16  Stud Shear Connectors Welded To One Of The Beams
FIG. 4.20  Formwork For Bridge Model # 5
FIG. 4.23 Reinforcement Meshes For Bridge Model # 6
FIG. 1.25 Strain Gauge Arrangement for Bridge Model #1

- Strain Gauges (Top on Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top on Concrete and Bottom on Steel Cross-Beams)
- Strain Gauges (Top on Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top on Concrete and Bottom on Steel Cross-Beams)

FIG. 4.26 Strain Gauge Arrangement for Bridge Model #2
- Strain Gauges (Top on Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top on Concrete and Bottom on Steel Cross-Beams)

FIG. 4.27 Strain Gauge Arrangement for Bridge Model #3
- Strain Gauges (Top on Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top on Concrete)

FIG. 4.28 Strain Gauge Arrangement for Bridge Model #1
- Strain Gauges (Top on Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top on Concrete and Bottom on Steel Cross-Beams)

FIG. 4.29 Strain Gauge Arrangement for Bridge Model #5
FIG. 4.30 Strain Gauge Arrangement for Bridge Model #6

- Strain Gauges (Top on Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top on Concrete and Bottom on Steel Cross-Beams)
FIG. 4.31 Strain Gauge Arrangement for Bridge Model #7

- Strain Gauges (Top Concrete and Bottom on Longitudinal Steel Beams)
• Strain Gauges (Top Concrete and Bottom on Longitudinal Steel Beams)
- Strain Gauges (Top Concrete and Bottom on Longitudinal Steel Beams)

FIG. 4.33 Strain Gauge Arrangement for Bridge Model #9
FIG. 4.34 Positions Of Strain Gauges On The Steel Beams Of Bridge Model # 4
FIG. 4.35 Positions Of Strain Gauges On The Steel Beams Of Bridge Model #7
FIG. 4.39 Dial Gauge Arrangement for Bridge Model #3
FIG. 4.43 Dial Gauge Arrangement for Bridge Model #7
FIG. 4.44 Dial Gauge Arrangement for Bridge Model #8
FIG. 4.46 Dial Gauges In Position (Bridge Model # 2)
FIG. 4.48 The Hydraulic Jack (890 kN Capacity)
FIG. 4.49  The Universal Flat Load Cell (890 kN Capacity)
FIG. 4.50 Load Cell Arrangement for Bridge Model #1

- Load Cells (under the Steel Beams and Above the Concrete Deck Slab)
Load Cells (Under the Steel Beams and Above the Concrete Deck Slab)
• Load Cells (Under the Steel Beams and Above the Concrete Deck Slab)

FIG. 4.53  Load Cell Arrangement for Bridge Model #1
- Load Cells (Under the Steel Beams and Above the Concrete Deck Slab)

FIG. 4.54 Load Cell Arrangement for Bridge Model #5
Load Cells (Under the Steel Beams and Above the Concrete Deck Slab)

FIG. 4.55 Load Cell Arrangement for Bridge Model #6
Load Cells (Under the Steel Beams and Above the Concrete Deck Slab)

FIG. 4.56 Load Cell Arrangement for Bridge Model #7
• Load Cells (Under the Steel Beams and Above the Concrete Deck Slab)

FIG. 4.57 Load Cell Arrangement for Bridge Model #8
- Load cells (under the steel beams and above the concrete Deck Slab)

FIG. 4.58  Load Cell Arrangement for Bridge Model #9
FIG. 4.60 Steel Grid Of Bridge Model #3 Under Single Concentrated Load
FIG. 4.61 Steel Grid Of Bridge Model # 5 Under Single Concentrated Load
FIG. 4.62 Steel Grid Of Bridge Model # 3 Under Eccentric Simulated Truck Load
FIG. 4.64 Steel Grid Of Bridge Model # 7 Under Single Concentrated Load
(Long Span Only Is Loaded)
FIG. 4.65 Steel Grid Of Bridge Model # 9 Under Single Concentrated Load
(Long Span Only Is Loaded)
FIG. 4.66 Steel Grid Of Bridge Model #9 Under Eccentric Simulated Truck Load (Long Span Only Is Loaded)
FIG. 4.67 Steel Grid Of Bridge Model # 7 Under Eccentric Simulated Truck Load
(Both Spans Are Loaded)
FIG. 4.68 Steel Grid Of Bridge Model # 8 Under Eccentric Simulated Truck Load
(Both Spans Are Loaded)
FIG. 4.71 Bridge Model #1 Under Concentric Simulated Truck Load
FIG. 4.74 Bridge Model # 5 Under Concentric Simulated Truck Load
FIG. 4.75 Bridge Model # 7 Under Single Concentrated Load (Long Span Only Is Loaded)
FIG. 4.77 Bridge Model # 9 Under Eccentric Simulated Truck Load
(Short Span Only Is Loaded)
FIG. 4.78 Bridge Model # 8 Under Eccentric Simulated Truck Load
(Long Span Only Is Loaded)
FIG. 4.80 Bridge Model # 7 Under Eccentric Simulated Truck Load
(Both Spans AreLoaded)
FIG. 4.81 Bridge Model # 9 Under Eccentric Simulated Truck Load (Both Spans Are Loaded)
FIG. 4.82 Bridge Model # 9 Under Concentric Simulated Truck Load
(Short Span Only Is Loaded)
FIG. 4.83 Bridge Model # 9 Under Concentric Simulated Truck Load
(Long Span Only Is Loaded)
FIG. 4.84 Bridge Model # 8 Under Concentric Simulated Truck Load
(Both Spans Are Loaded)
FIG. 4.86 Test Setup For The Ultimate Load Test For Bridge Model # 2
FIG. 4.88  Test Setup For The Ultimate Load Test For Bridge Model # 5
FIG. 4.89  Test Setup For The Ultimate Load Test For Bridge Model # 7
FIG. 4.91 Test Setup For The Ultimate Load Test For Bridge Model # 9
FIG. 5.1 Loading Case #1 for Simply Supported Bridge Models
(Single Concentrated Load)
FIG. 5.3 Loading Case #2 for Simply Supported Bridge Models
(Eccentric Simulated Truck Loading)
FIG. 5.4 Loading Case #4 for Simply Supported Bridge Models
(Concentric Simulated Truck Loading)
FIG. 5.5 Loading Case #5 for Simply Supported Bridge Models (concentric Simulated Truck Loading)
FIG. 5.6 The Ultimate Load Test for Simply Supported Bridge Models (Eccentric Simulated Truck Loading)
FIG. 5.7 Loading Case #1 for Continuous Bridge Models
(Eccentric Simulated Truck Loading on the Long Span)
FIG. 5.8 Loading Case #2 for Continuous Bridge Models (Eccentric Simulated Truck Loading on the Long Span)
FIG. 5.9 Loading Case #3 for Continuous Bridge Models
(Fully Loaded Bridge)
FIG. 5.11 Loading Case #5 for Continuous Bridge Models
(Opposite Simulated Truck Loading on the Two Spans)
FIG. 5.13 Loading Case #7 for Continuous Bridge Models
(Eccentric Simulated Truck Loading on the Short Span)
FIG. 5.14 Loading Case #6 for Continuous Bridge Models (Concentric Simulated Truck Loading on the Short Span)
FIG. 5.15 Load-Deflection Relationship
Bridge Model #1, Loading Case #1

Load (kN) vs. Deflection (mm)

- Experimental
- Theoretical

Legend:
- mid-B1
- mid-B2
- mid-B3
- mid-B4
- mid-B1
- mid-B2
- mid-B3
- mid-B4
FIG. 5.16 Load-Deflection Relationship
Bridge Model #1, Loading Case #2
FIG. 5.17 Load-Deflection Relationship
Bridge Model #1, Loading Case #3
FIG. 5.18 Load-Deflection Relationship
Bridge Model #1, Loading Case #4
FIG. 5.20 Load-Strain Relationship  
Bridge Model #1, Loading Case #1
FIG. 5.21 Load-Strain Relationship
Bridge Model #1, Loading Case #2

![Graph showing load-strain relationship for Bridge Model #1, Loading Case #2.](image-url)
FIG. 5.22 Load-Strain Relationship
Bridge Model #1, Loading Case #3

Load (kN)

Strain (microstrain)

Experimental
Theoretical

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.23 Load-Strain Relationship
Bridge Model #1, Loading Case #4

Load (kN)

Strain (microstrain)

Experimental
Theoretical

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.25 Support Reactions for Bridge Model #1
Loading Case #1, Single Concentrated Load

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
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<td>1.4</td>
<td>4.8</td>
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<td>Finite Element Method</td>
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<td>3.0</td>
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<td>3.0</td>
<td>1.5</td>
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<td>(B)</td>
<td>(C)</td>
<td>(D)</td>
<td>(E)</td>
<td>(F)</td>
<td>(G)</td>
<td>(H)</td>
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<td>1.4</td>
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<td>1.6</td>
<td>13.8</td>
<td>18.2</td>
<td>9.8</td>
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</table>

**FIG. 5.26 Support Reactions for Bridge Model #1 Loading Case #2, Eccentric Simulated Truck Loading**
<table>
<thead>
<tr>
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<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
</tr>
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<tr>
<td><strong>Experimental</strong></td>
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<td>11.0</td>
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<td>11.3</td>
<td>3.9</td>
<td>13.2</td>
<td>16.1</td>
<td>11.3</td>
<td>4.2</td>
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**FIG. 5.27 Support Reactions for Bridge Model #1**
**Loading Case #4, Concentric Simulated Truck Loading**
FIG. 5.28 Load Deflection Relationship
Bridge Model #1, The Final Test

Load (kN) vs. Deflection (mm)

- Experimental
- Theoretical

mid-B1 ▼ mid-B2 ○ mid-B3 → mid-B4 ■ mid-B1 ▼ mid-B2 ○ mid-B3 × mid-B4
FIG. 5.29 Load Strain Relationship
Bridge Model #1, The Final Test

Experimental

Theoretical

Load (kN)

Strain (microstrain)
FIG. 5.30  Crack Pattern of Deck Slab For Bridge Model # 1 At Failure (Top Surface)
FIG. 5.31 Crack Pattern Of Deck Slab For Bridge Model #1 At Failure (Bottom Surface)
FIG. 5.32 Load-Deflection Relationship
Bridge Model #2, Loading Case #1
FIG. 5.33 Load-Deflection Relationship
Bridge Model #2, Loading Case #2
FIG. 5.34 Load-Deflection Relationship
Bridge Model #2, Loading Case #3
FIG. 5.35 Load-Deflection Relationship
Bridge Model #2, Loading Case #4
FIG. 5.36 Load-Deflection Relationship
Bridge Model #2, Loading Case #5
FIG. 5.37 Load-Strain Relationship
Bridge Model #2, Loading Case #1

![Graph showing load-strain relationship for Bridge Model #2, Loading Case #1. The graph displays experimental and theoretical load-strain data points for various strain values.](image-url)
FIG. 5.38 Load-Strain Relationship
Bridge Model #2, Loading Case #2

Load (kN)

[Graph showing experimental and theoretical curves for load-strain relationship with strain values ranging from 0 to 1200 microstrains and load values ranging from 0 to 100 kN. Different markers and line styles represent various mid-B points (B1, B2, B3, B4).]
FIG. 5.39 Load-Strain Relationship
Bridge Model #2, Loading Case #3

Load (kN) vs. Strain (microstrain)

Experimental vs. Theoretical

mid-B1 — — mid-B2 — — mid-B3 — — mid-B4
FIG. 5.40 Load-Strain Relationship
Bridge Model #2, Loading Case #4
FIG. 5.41 Load-Strain Relationship
Bridge Model #2, Loading Case #5
<table>
<thead>
<tr>
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<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
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<tbody>
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**FIG. 5.42** Support Reactions for Bridge Model #2
Loading Case #1, Single Concentrated Load
### Support Reaction (kN) at

<table>
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**FIG. 5.43 Support Reactions for Bridge Model #2**

**Loading Case #2, Eccentric Simulated Truck Loading**
### Support Reactions (kN) at

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**FIG. 5.44 Support Reactions for Bridge Model #2 Loading Case #4, Concentric Simulated Truck Loading**
FIG. 5.45 Load Deflection Relationship
Bridge Model #2, The Final Test

- Experimental
- Theoretical
FIG. 5.46 Load Strain Relationship
Bridge Model #2, The Final Test
FIG. 5.47 Crack Pattern Of Deck Slab For Bridge Model # 2 At Failure (Top Surface)
FIG. 5.49 Load-Deflection Relationship
Bridge Model #3, Loading Case #1

Load (kN)

Deflection (mm)

Experimental
Theoretical

mid-B1 —— mid-B2 —— mid-B3 —— mid-B4

mid-B1 ▽ mid-B2 ■ mid-B3 ○ mid-B4
FIG. 5.51 Load-Deflection Relationship
Bridge Model #3, Loading Case #3

Load (kN) vs. Deflection (mm)

- Experimental
- Theoretical

Legend:
- mid-B1
- mid-B2
- mid-B3
- mid-B4
- mid-B1
- mid-B2
- mid-B3
- mid-B4
FIG. 5.53 Load-Deflection Relationship
Bridge Model #3, Loading Case #5

- Experimental
- Theoretical

Load (kN) vs. Deflection (mm)

Legend:
- mid-B1
- mid-B2
- mid-B3
- mid-B4
- mid-B1 (triangle)
- mid-B2 (square)
- mid-B3 (circle)
- mid-B4 (diamond)
FIG. 5.54 Load-Strain Relationship
Bridge Model #3, Loading Case #1

Load (kN)

Strain (microstrain)

Experimental
Theoretical

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.55 Load-Strain Relationship
Bridge Model #3, Loading Case #2

Experimental
Theoretical

Strain (microstrain)
0 100 200 300 400 500 600 700 800 900

Load (kN)
0 5 10 15 20
FIG. 5.56 Load-Strain Relationship
Bridge Model #3, Loading Case #3
FIG. 5.57 Load-Strain Relationship
Bridge Model #3, Loading Case #4
FIG. 5.58 Load-Strain Relationship
Bridge Model #3, Loading Case #5
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<td>15.0</td>
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**FIG. 5.59 Support Reactions for Bridge Model #3**

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**FIG. 5.60** Support Reactions for Bridge Model #3
Loading Case #2, Eccentric Simulated Truck Loading
### Support Reaction (kN)

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<td>16.2</td>
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**FIG. 5.61** Support Reactions for Bridge Model #3
Loading Case #4, Concentric Simulated Truck Loading
FIG. 5.62 Load Deflection Relationship
Bridge Model #3, The Final Test
FIG. 5.63 Load Strain Relationship
Bridge Model #3, The Final Test

![Graph showing load strain relationship with experimental and theoretical lines.](image-url)
FIG. 5.65 Load-Deflection Relationship
Bridge Model #4, Loading Case #1
FIG. 5.66 Load-Deflection Relationship
Bridge Model #4, Loading Case #2
FIG. 5.67 Load-Deflection Relationship
Bridge Model #4, Loading Case #3
FIG. 5.69 Load-Deflection Relationship
Bridge Model #4, Loading Case #5
FIG. 5.70 Load-Strain Relationship
Bridge Model #4, Loading Case #1

Load (kN)

Experimental
Theoretical

Strain (microstrains)

mid-B1 ➔ mid-B2 ➔ mid-B3 ➔ mid-B4 ➔ mid-B1 ▼ mid-B2 ■ mid-B3 ○ mid-B4
FIG. 5.71 Load-Strain Relationship
Bridge Model #4, Loading Case #2
FIG. 5.73 Load-Strain Relationship
Bridge Model #4, Loading Case #4
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**FIG. 5.75** Support Reactions for Bridge Model #4 Loading Case #1, Single Concentrated Load
### Support Reaction (kN) at

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**FIG. 5.76 Support Reactions for Bridge Model #4**

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<td>11.7</td>
<td>10.6</td>
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**FIG. 5.77 Support Reactions for Bridge Model #4**  
Loading Case #4, Concentric Simulated Truck Loading
FIG. 5.78 Load Deflection Relationship
Bridge Model #4, The Final Test

Experimental
Theoretical

Load (kN)

Deflection (mm)

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.79 Load Strain Relationship
Bridge Model #4, The Final Test
FIG. 5.80 Crack Pattern Of Deck Slab For Bridge Model # 4 At Failure (Top Surface)
FIG. 5.81 Crack Pattern Of Deck Slab For Bridge Model # 4 At Failure (Bottom Surface)
FIG. 5.82 Load-Deflection Relationship
Bridge Model #5, Loading Case #1
FIG. 5.83 Load-Deflection Relationship
Bridge Model #5, Loading Case #2

Experimental
Theoretical

mid-B1 — mid-B2 — mid-B3 — mid-B4

Load (kN)
100  20  30  40  50  60

Deflection (mm)
0   1   2   3   4   5   6
FIG. 5.85 Load-Deflection Relationship
Bridge Model #5, Loading Case #4

Load (kN)

Deflection (mm)

Experimental
Theoretical

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.86 Load-Deflection Relationship
Bridge Model #5, Loading Case #5
FIG. 5.87 Load-Strain Relationship
Bridge Model #5, Loading Case #1

- Experimental
- Theoretical

Legend:
- mid-B1
- mid-B2
- mid-B3
- mid-B4
FIG. 5.89 Load-Strain Relationship
Bridge Model #5, Loading Case #3
FIG. 5.91 Load-Strain Relationship
Bridge Model #5, Loading Case #5

![Graph showing Load-Strain Relationship for Bridge Model #5 and Loading Case #5. The graph includes both experimental and theoretical data points, with different markers for mid-B1, mid-B2, mid-B3, and mid-B4 sections, plotted against Strain (microstrain) and Load (kN).]
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<td>10.9</td>
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**FIG. 5.92 Support Reactions for Bridge Model #5**  
Loading Case #1, Single Concentrated Load
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**FIG. 5.93** Support Reactions for Bridge Model #5
Loading Case #2, Eccentric Simulated Truck Loading
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<td>13.4</td>
<td>10.1</td>
<td>4.4</td>
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**FIG. 5.94 Support Reactions for Bridge Model #5**
Loading Case #4, Concentric Simulated Truck Loading
FIG. 5.95 Load Deflection Relationship
Bridge Model #5, The Final Test
FIG. 5.96 Load Strain Relationship
Bridge Model #5, The Final Test

Load (kN) vs. Strain (microstrain)

- Experimental
- Theoretical

Legend:
- mid-B1
- mid-B2
- mid-B3
- mid-B4
FIG. 5.99 Load-Deflection Relationship
Bridge Model #6, Loading Case #1

Experimental
Theoretical

mid-B1
mid-B2
mid-B3
mid-B4

478
FIG. 5.101 Load-Deflection Relationship
Bridge Model #6, Loading Case #3
FIG. 5.102 Load-Deflection Relationship  
Bridge Model #6, Loading Case #4
FIG. 5.103 Load-Deflection Relationship
Bridge Model #6, Loading Case #5
FIG. 5.104 Load-Strain Relationship
Bridge Model #6, Loading Case #1
FIG. 5.105 Load-Strain Relationship
Bridge Model #6, Loading Case #2

Load (kN) vs. Strain (microstrain)

- Experimental
- Theoretical

Legend:
- mid-B1
- mid-B2
- mid-B3
- mid-B4
- mid-B1
- mid-B2
- mid-B3
- mid-B4
FIG. 5.106 Load-Strain Relationship
Bridge Model #6, Loading Case #3

---

Experimental
Theoretical

\[ \text{Load (kN)} \]

\[ \text{Strain (microstrain)} \]

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.108 Load-Strain Relationship
Bridge Model #6, Loading Case #5

Load (kN)

0  100  80  60  40  20  0

Strain (microstrain)

0  100  200  300  400  500  600

Experimental

Theoretical

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
<table>
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<tr>
<th></th>
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<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
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FIG. 5.109 Support Reactions for Bridge Model #6
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**FIG. 5.110 Support Reactions for Bridge Model #6 Loading Case #2, Eccentric Simulated Truck Loading**
### Support Reaction (kN) at

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**FIG. 5.111** Support Reactions for Bridge Model #6 Loading Case #4, Concentric Simulated Truck Loading
FIG. 5.113 Load Strain Relationship
Bridge Model #6, The Final Test

Load (kN)

Strain (microstrain)

Experimental
Theoretical

mid-B1  mid-B2  mid-B3  mid-B4  mid-B1  mid-B2  mid-B3  mid-B4
FIG. 5.115 Crack Pattern Of Deck Slab For Bridge Model # 6 At Failure (Bottom Surface)
FIG. 5.116 Effect of Skew on Load-Deflection Relation
Concentric Simulated Truck Loading

Load (kN) vs. Deflection (mm)
FIG. 5.117 Effect of Skew on Load-Strain Relation
Concentric Simulated Truck Loading
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<td>-0.7</td>
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</table>

**FIG. 5.118 Effect of Skew on Reaction Distribution**

Concentric Simulated-Truck Loading
FIG. 5.119 Effect of Diaphragms on Load-Deflection Relation
Concentric Simulated Truck Loading

Load (kN) vs. Deflection (mm)

- mid-B1, model #2
- mid-B2, model #2
- mid-B3, model #2
- mid-B4, model #2
- mid-B1, model #4
- mid-B2, model #4
- mid-B3, model #4
- mid-B4, model #4
- mid-B1, model #5
- mid-B2, model #5
- mid-B3, model #5
- mid-B4, model #5
- mid-B1, model #6
- mid-B2, model #6
- mid-B3, model #6
- mid-B4, model #6
FIG. 5.120 Effect of Diaphragms on Load-Strain Relation
Concentric Simulated Truck Loading

[Graph showing the relationship between load and strain for different model configurations]
<table>
<thead>
<tr>
<th>Support Reaction (kN) at</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
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<td>12.5</td>
<td>6.3</td>
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<td>18.6</td>
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<td>2.7</td>
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<td>17.6</td>
<td>6.8</td>
<td>2.5</td>
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**FIG. 5.121** Effect of Diaphragms on Reaction Distribution
Concentric Simulated-Truck Loading

500
FIG. 5.122 Load-Deflection Relationship
Bridge Model #7, Loading Case #1
FIG. 5.124 Load-Deflection Relationship
Bridge Model #7, Loading Case #3

Experimental
--- Theoretical

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.125 Load-Deflection Relationship
Bridge Model #7, Loading Case #4

- Experimental
- Theoretical

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.126 Load-Deflection Relationship
Bridge Model #7, Loading Case #5

- Experimental
- Theoretical

- at A  - at B  - at C  - at D  - at E  - at F  - at G  - at H
FIG. 5.127 Load-Strain Relationship
Bridge Model #7, Loading Case #1
FIG. 5.128 Load-Strain Relationship
Bridge Model #7, Loading Case #2

Experimental
Theoretical

Strain (microstrain)

Load (kN)

at A
at B
at C
at D
at E
at F
at G
at H
FIG. 5.129 Load-Strain Relationship
Bridge Model #7, Loading Case #3

Load (kN)

Strain (microstrain)

- Experimental
- Theoretical

at A  at B  at C  at D  at E  at F  at G  at H
FIG. 5.131 Load-Strain Relationship
Bridge Model #7, Loading Case #5

Strain (microstrain)

Load (kN)

--- Experimental
--- Theoretical

at A --- at B ↔ at C ← at D --- at E ← at F --- at G → at H
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<tr>
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<th></th>
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<td>(C)</td>
<td>(D)</td>
<td>(E)</td>
<td>(F)</td>
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<td></td>
<td></td>
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<td></td>
<td>(G)</td>
<td>(H)</td>
<td>(I)</td>
<td>(J)</td>
<td>(K)</td>
<td>(L)</td>
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<td>-3.6</td>
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<td>-2.9</td>
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FIG. 5.132 Support Reactions for Bridge Model #7
Long Span Loaded, Concentric Simulated Truck Loading
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<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
<th>(I)</th>
<th>(J)</th>
<th>(K)</th>
<th>(L)</th>
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<td>8.1</td>
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<td>34.9</td>
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<td>7.4</td>
<td>7.4</td>
<td>7.3</td>
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**Support Reaction (kN)**

**FIG. 5.133 Support Reactions for Bridge Model #7**
Both Spans Loaded, Concentric Simulated Truck Loading

512
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<td>11.2</td>
<td>5.9</td>
<td>-2.1</td>
<td>-1.9</td>
<td>23.1</td>
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<tr>
<td>Finite Element Method</td>
<td>8.0</td>
<td>10.8</td>
<td>6.9</td>
<td>-1.9</td>
<td>-1.5</td>
<td>22.2</td>
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<td>37.8</td>
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**FIG. 5.134 Support Reactions for Bridge Model #7**
Both Spans Loaded, Eccentric Simulated Truck Loading
FIG. 5.135 Load Deflection Relationship
Bridge Model #7, The Final Test

- Experimental
- Theoretical

Load (kN) vs. Deflection (mm)

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.136 Load Strain Relationship
Bridge Model #7, The Final Test

Load (kN)

Strain (microstrain)

Experimental

Theoretical

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.138 Deflected Shape Of Bridge Model # 7 After Failure
FIG 5.139 Load-Deflection Relationship
Bridge Model #8, Loading Case #1

- Experimental
- Theoretical

Load (kN) vs. Deflection (mm)

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.140 Load-Deflection Relationship
Bridge Model #8, Loading Case #2

Experimental
Theoretical

Load (kN)

Deflection (mm)

- at A  at B  at C  at D  at E  at F  at G  at H
FIG. 5.141 Load-Deflection Relationship
Bridge Model #8, Loading Case #3

Load (kN) vs. Deflection (mm) plot showing experimental and theoretical data points at various locations labeled A through H.
FIG.5.143 Load-Deflection Relationship
Bridge Model #8, Loading Case #5

---

Experimental
Theoretical

\[ \text{Load (kN)} \]

\[ \text{Deflection (mm)} \]

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H

---
FIG. 5.144 Load-Strain Relationship
Bridge Model #8, Loading Case #1

Load (kN)

Strain (microstrain)

Simple Support
G Skew
F Skew
H Skew
Simple Support

Experimental
Theoretical

at A —— at B —— at C —— at D —— at E —— at F —— at G —— at H
FIG. 5.145 Load-Strain Relationship
Bridge Model #8, Loading Case #2
FIG. 5.146 Load-Strain Relationship
Bridge Model #8, Loading Case #3

Load (kN)

Strain (microstrain)

--- Experimental

--- Theoretical

--- at A --- at B --- at C --- at D --- at E --- at F --- at G --- at H
FIG. 5.147 Load-Strain Relationship
Bridge Model #8, Loading Case #4

---

- Experimental
- Theoretical

---

at A → at B → at C → at D → at E → at F → at G → at H
FIG. 5.148 Load-Strain Relationship
Bridge Model #8, Loading Case #5

Load (kN) vs. Strain (microstrain)

Experimental
Theoretical

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
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<td>9.1</td>
<td>2.1</td>
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<td>10.2</td>
<td>8.7</td>
<td>2.3</td>
<td>23.2</td>
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<td><strong>Support Reaction (kN)</strong></td>
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<td></td>
<td></td>
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<tr>
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<td>-6.4</td>
<td>-3.3</td>
<td>N.A.</td>
<td>-0.6</td>
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<tr>
<td><strong>Experimental</strong></td>
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<td><strong>Finite Element Method</strong></td>
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<td>-2.7</td>
<td>-2.3</td>
<td>-0.7</td>
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FIG. 5.149 Support Reactions for Bridge Model #8
Long Span Loaded, Concentric Simulated Truck Loading
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<tr>
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<td></td>
<td>(G)</td>
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<td>Finite Element Method</td>
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</tbody>
</table>

**FIG. 5.150 Support Reactions for Bridge Model #8**
Both Spans Loaded, Concentric Simulated Truck Loading
<table>
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<th>Support Reaction (kN)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>at</strong></td>
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<td></td>
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<tr>
<td>(A)</td>
<td>17.5</td>
<td>9.4</td>
</tr>
<tr>
<td>(B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>2.1</td>
<td>0.0</td>
</tr>
<tr>
<td>(D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E)</td>
<td>-2.0</td>
<td>23.8</td>
</tr>
<tr>
<td>(F)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
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</tr>
<tr>
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<tr>
<td><strong>Finite Element Method</strong></td>
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<td></td>
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<tr>
<td>(G)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(I)</td>
<td>1.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>(J)</td>
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<tr>
<td>(K)</td>
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<td>23.3</td>
</tr>
<tr>
<td>(L)</td>
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<td></td>
</tr>
</tbody>
</table>

**Support Reaction (kN) at**

|                |                       |                  |
| (G)            | 38.1                  | 37.6             |
| (H)            |                       | -4.2             |
| (I)            |                       | 7.9              |
| (J)            |                       | N.A.             |
| (K)            |                       | 2.3              |
| (L)            |                       |                  |
| **Experimental** |                       |                  |
|                |                       |                  |
| **Finite Element Method** | 40.6   | 39.3             |
|                |                       | -4.5             |
| (J)            | 7.1                   | 10.9             |
| (K)            |                       | 1.8              |
| (L)            |                       |                  |

**FIG. 5.151 Support Reactions for Bridge Model #8**
Both Spans Loaded, Eccentric Simulated Truck Loading
FIG.5.152 Load Deflection Relationship
Bridge Model #8, The Final Test

![Graph showing experimental and theoretical load deflection relationship for bridge model #8. The graph displays multiple curves at different points A, B, C, D, E, F, G, and H, along the x-axis (Deflection in mm) and y-axis (Load in kN). The graph compares experimental and theoretical values, with annotations for different support conditions: Simple Support, Pier Support, and Skew Support.]
FIG. 5.153 Load Strain Relationship
Bridge Model #8, The Final Test

---

- Experimental
- Theoretical

- at A
- at B
- at E
- at F
- at G
- at H
FIG. 5.154 Crack Pattern Of Deck Slab For Bridge Model # 8 At Failure (Top Surface)
FIG. 5.155 Crack Pattern Of Deck Slab For Bridge Model # 8 At Failure (Bottom Surface)
FIG. 5.156 Load-Deflection Relationship
Bridge Model #9, Loading Case #1

Load (kN) vs. Deflection (mm)

- Experimental
- Theoretical

Points at:
- A
- B
- C
- D
- E
- F
- G
- H
FIG. 5.157 Load-Deflection Relationship
Bridge Model #9, Loading Case #2
FIG. 5.158 Load-Deflection Relationship
Bridge Model #9, Loading Case #3

Experimental
Theoretical

Load (kN)

0
100
150
200

Deflection (mm)

0
1
2
3
4
5
6
7

--- at A --- at B --- at C --- at D --- at E --- at F --- at G --- at H
FIG. 5.159 Load-Deflection Relationship
Bridge Model #9, Loading Case #4

- Experimental
- Theoretical

---

at A — at B — at C — at D — at E — at F — at G — at H
FIG. 5.160 Load-Deflection Relationship
Bridge Model #9, Loading Case #5

- Experimental
- Theoretical

Load (kN) vs. Deflection (mm)

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.161 Load-Strain Relationship
Bridge Model #9, Loading Case #1

- Experimental
- Theoretical

- at A → at B → at C ▲ at D → at E ← at F → at G → at H
FIG. 5.162 Load-Strain Relationship
Bridge Model #9, Loading Case #2

- Experimental
- Theoretical

---

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.163 Load-Strain Relationship
Bridge Model #9, Loading Case #3

---

Experimental
Theoretical

---

Load (kN)

Strain (microstrain)

---

at A → at B → at C → at D → at E → at F → at G → at H
FIG. 5.164 Load-Strain Relationship
Bridge Model #9, Loading Case #4

Experimental
Theoretical

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H

Load (kN)

Strain (microstrain)
FIG. 5.165 Load-Strain Relationship
Bridge Model #9, Loading Case #5

[Graph showing load-strain relationship with various points and lines indicating experimental and theoretical results.]

- at A — at B — at C — at D — at E — at F — at G — at H
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FIG. 5.166 Support Reactions for Bridge Model #9
Long Span Loaded, Concentric Simulated Truck Loading
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**FIG. 5.167 Support Reactions for Bridge Model #9**  
Both Spans Loaded, Concentric Simulated Truck Loading
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<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
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</tbody>
</table>

FIG. 5.168 Support Reactions for Bridge Model #9
Both Spans Loaded, Eccentric Simulated Truck Loading
FIG. 5.169 Load Deflection Relationship
Bridge Model #9, The Final Test
FIG. 5.170 Load Strain Relationship
Bridge Model #9, The Final Test

- Experimental
- Theoretical

Load (kN) vs Strain (microstrain)

- at A
- at B
- at C
- at D
- at E
- at F
- at G
- at H
FIG. 5.172 Effect of Skew on Load-Deflection Relation
Both Spans Loaded, Concentric Simulated Truck Loading
FIG. 5.173 Effect of Skew on Load-Strain Relation
Both Spans Loaded, Concentric Simulated Truck Loading
<table>
<thead>
<tr>
<th>Support Reaction (kN) at</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge Model #7 (\theta=0^\circ)</td>
<td>6.4</td>
<td>8.1</td>
<td>7.4</td>
<td>6.8</td>
<td>25.9</td>
<td>34.9</td>
<td>35.2</td>
<td>27.9</td>
</tr>
<tr>
<td>Bridge Model #8 (\theta=45^\circ)</td>
<td>12.5</td>
<td>8.3</td>
<td>7.8</td>
<td>1.8</td>
<td>27.1</td>
<td>40.9</td>
<td>36.9</td>
<td>21.0</td>
</tr>
</tbody>
</table>

**FIG. 5.174 Effect of Skew on Reaction Distribution**
Both Spans Loaded, Concentric Simulated Truck Loading
FIG. 5.175 Effect of Spans Ratio on Load-Deflection Relation
Both Spans Loaded, Concentric Simulated Truck Loading

- Simple Support
- Plate Support

Load (kN)

Deflection (mm)

- at A, model #8 — at B, model #8 — at C, model #8 — at D, model #8
- at A, model #9 — at B, model #9 — at C, model #9 — at D, model #9
FIG. 5.176 Effect of Spans Ratio on Load-Strain Relation
Both Spans Loaded, Concentric Simulated Truck Loading

![Graph showing load-strain relation with different markers for different conditions and models.](image-url)
<table>
<thead>
<tr>
<th>Bridge Model #8</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_2/L_1 = 1.4 )</td>
<td>12.5</td>
<td>8.3</td>
<td>7.8</td>
<td>1.8</td>
<td>27.1</td>
<td>40.9</td>
<td>36.9</td>
<td>21.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bridge Model #9</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(F)</th>
<th>(G)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_2/L_1 = 1.8 )</td>
<td>11.6</td>
<td>7.4</td>
<td>6.9</td>
<td>3.4</td>
<td>28.0</td>
<td>42.1</td>
<td>32.6</td>
<td>19.9</td>
</tr>
</tbody>
</table>

**FIG. 5.177** Effect of Spans Ratio on Reaction Distribution
Both Spans Loaded, Concentric Simulated Truck Loading
FIG. 6.1 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges Girder Close to the Obtuse Corner of a Four-Lane Bridge Under Concentric Loading
FIG. 6.2 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges Girder Close to the Acute Corner of a Four-Lane Bridge Under Concentric Loading
FIG. 6.3 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges
An Interior Girder of a Four-Lane Bridge Under Concentric Loading

Shear Distribution Factor (D)

Angle of Skew (degrees)

Aspect Ratio = 1.0
Aspect Ratio = 1.26
Aspect Ratio = 1.48
FIG. 6.4 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges Girder Close to the Obtuse Corner of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.5 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges Girder Close to the Acute Corner of a Three-Lane Bridge Under Eccentric Loading.
FIG. 6.6 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.7 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges Girder Close to the Obtuse Corner of a Two-Lane Bridge Under Dead Load
FIG. 6.8 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges Girder Close to the Acute Corner of a Two-Lane Bridge Under Dead Load
FIG. 6.9 Effect of Skew on the Shear Distribution Factor of Simply Supported Bridges
An Interior Girder of a Two-Lane Bridge Under Dead Load
FIG. 6.10 Effect of Aspect Ratio on the Shear Distribution Factor
Girder Close to the Obtuse Corner of a Two-Lane Bridge Under Concentric Loading
FIG. 6.11 Effect of Aspect Ratio on the Shear Distribution Factor
Girder Close to the Acute Corner of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.12 Effect of Aspect Ratio on the Shear Distribution Factor
An Interior Girder of a Four-Lane Bridge Under Dead Load
FIG. 6.13 Effect of Girder Spacing on the Shear Distribution Factor
Girder Close to the Obtuse Corner of a Two-Lane Bridge Under Concentric Loading
FIG. 6.14 Effect of Girder Spacing on the Shear Distribution Factor
Girder Close to the Acute Corner of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.15 Effect of Girder Spacing on the Shear Distribution Factor
An Interior Girder of a Four-Lane Bridge Under Dead Load
FIG. 6.16 Effect of Flexural Rigidity Ratio, $R$, on the Shear Distribution Factor
Girder Close to the Obtuse Corner of a Four-Lane Bridge Under Concentric Loading
FIG. 6.17 Effect of Flexural Rigidity Ratio, R, on the Shear Distribution Factor
Girder Close to the Acute Corner of a Three-Lane Bridge Under Eccentric Loading.

- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees
FIG. 6.18 Effect of Flexural Rigidity Ratio, R, on the Shear Distribution Factor An Interior Girder of a Two-Lane Bridge Under Eccentric Loading

- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees
FIG. 6.19 Shear Distribution for Different Diaphragm Cases
Three-Lane Bridge Under Eccentric Truck Loading
FIG. 6.20 Effect of Skew on the Moment Distribution Factor of Simply Supported Bridges
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading

[Graph showing the effect of skew on moment distribution factor (D) for different aspect ratios]
FIG. 6.21 Effect of Skew on the Moment Distribution Factor of Simply Supported Bridges
An Interior Girder of a Two-Lane Bridge Under Concentric Loading

![Graph showing the effect of skew on the moment distribution factor (D) for different aspect ratios. The graph plots the moment distribution factor (D) against the angle of skew (degrees). The aspect ratios are 2.0, 2.4, and 2.9, indicated by different line styles.](image-url)
FIG. 6.22 Effect of Skew on the Moment Distribution Factor of Simply Supported Bridges
An Exterior Girder of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.23 Effect of Skew on the Moment Distribution Factor of Simply Supported Bridges
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.24 Effect of Skew on the Moment Distribution Factor of Simply Supported Bridges
An Exterior Girder of a Four-Lane Bridge Under Dead Load

Moment Distribution Factor (D)

Angle of Skew (degrees)

Aspect Ratio = 1.0
Aspect Ratio = 1.26
Aspect Ratio = 1.48
FIG. 6.25 Effect of Skew on the Moment Distribution Factor of Simply Supported Bridges

An Interior Girder of a Four-Lane Bridge Under Dead Load

- Aspect Ratio = 1.0
- Aspect Ratio = 1.26
- Aspect Ratio = 1.48

Moment Distribution Factor (D) — Angle of Skew (degrees)
FIG. 6.26 Effect of Aspect Ratio on the Moment Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading
FIG. 6.27 Effect of Aspect Ratio on the Moment Distribution Factor
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading

Moment Distribution Factor (D)

Aspect Ratio

Skew = 0 degrees
Skew = 30 degrees
Skew = 45 degrees
Skew = 60 degrees
FIG. 6.28 Effect of Aspect Ratio on the Moment Distribution Factor
An Exterior Girder of a Four-Lane Bridge Under Dead Load
FIG. 6.29 Effect of Girder Spacing on the Moment Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading
FIG. 6.30 Effect of Girder Spacing on the Moment Distribution Factor
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading

Graph showing the moment distribution factor (D) against the ratio (N) for different skew angles:

- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees
FIG. 6.31 Effect of Girder Spacing on the Moment Distribution Factor
An Exterior Girder of a Four-Lane Bridge Under Dead Load
FIG. 6.32 Effect of Flexural Rigidity Ratio, $R$, on the Moment Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading
FIG. 6.33 Effect of Flexural Rigidity Ratio, R, on the Moment Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.34 Effect of Flexural Rigidity Ratio, R, on the Moment Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Dead Load
FIG. 6.35 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Four-Lane Bridge Under Concentric Loading
FIG. 6.36 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Four-Lane Bridge Under Concentric Loading

![Graph showing the effect of skew on the moment distribution factor of a bridge with different aspect ratios. The graph plots the moment distribution factor (D) against the angle of skew (degrees).]

- Aspect Ratio = 1.05
- Aspect Ratio = 1.26
- Aspect Ratio = 1.48
FIG. 6.37 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.38 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.39 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges

An Exterior Girder of a Three-Lane Bridge Under Dead Load
FIG. 6.40 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges

An Interior Girder of a Three-Lane Bridge Under Dead Load

Moment Distribution Factor (D)

Angle of Skew (degrees)
FIG. 6.41 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Four-Lane Bridge Under Concentric Loading
FIG. 6.42 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Four-Lane Bridge Under Concentric Loading

![Graph showing the effect of skew on support moment distribution factor with different aspect ratios.](image-url)

- Aspect Ratio = 1.05
- Aspect Ratio = 1.26
- Aspect Ratio = 1.48
FIG. 6.43 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Eccentric Loading

![Diagram of moment distribution factor](image)

- **Aspect Ratio = 2.0**
- **Aspect Ratio = 2.43**
- **Aspect Ratio = 2.86**

- **Moment Distribution Factor (D)**
- **Angle of Skew (degrees)**
FIG. 6.44 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.45 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Three-Lane Bridge Under Dead Load

![Graph showing the effect of skew on the support moment distribution factor for a continuous bridge.](image)

- Aspect Ratio = 1.28
- Aspect Ratio = 1.55
- Aspect Ratio = 1.82

Moment Distribution Factor (D) vs. Angle of Skew (degrees)
FIG. 6.46 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Three-Lane Bridge Under Dead Load
FIG. 6.47 Effect of Aspect Ratio on the Span Moment Distribution Factor
An Exterior Girder of a Four-Lane Bridge Under Concentric Loading
FIG. 6.48 Effect of Aspect Ratio on the Span Moment Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading

- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees

Moment Distribution Factor (D)

Aspect Ratio
FIG. 6.49 Effect of Aspect Ratio on the Span Moment Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Dead Load
FIG. 6.50 Effect of Aspect Ratio on the Support Moment Distribution Factor
An Exterior Girdor of a Four-Lane Bridge Under Concentric Loading
FIG. 6.51 Effect of Aspect Ratio on the Support Moment Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.52 Effect of Aspect Ratio on the Support Moment Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Dead Load
FIG. 6.53 Effect of Girder Spacing on the Span Moment Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading
FIG. 6.54 Effect of Girder Spacing on the Span Moment Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.55 Effect of Girder Spacing on the Span Moment Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Dead Load
FIG. 6.56 Effect of Girder Spacing on the Support Moment Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading

The ratio (N) is plotted on the x-axis, ranging from 0.35 to 0.75. The y-axis represents the Moment Distribution Factor (D), ranging from 1.3 to 0.7. Three lines are shown, each representing different skew angles:
- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees
FIG. 6.57 Effect of Girder Spacing on the Support Moment Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.58 Effect of Girder Spacing on the Support Moment Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Dead Load

[Graph showing the effect of skew on moment distribution factor vs. ratio (N)]

- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees
FIG. 6.59 Effect of Skew on the Reaction Distribution Factor of Continuous Bridges

An Exterior Girder of a Three-Lane Bridge Under Concentric Loading

![Graph showing the effect of skew on the reaction distribution factor for a three-lane bridge with different aspect ratios.](image)
FIG. 6.60 Effect of Skew on the Reaction Distribution Factor of Continuous Bridges
An Interior Girder of a Three-Lane Bridge Under Concentric Loading
FIG. 6.61 Effect of Skew on the Reaction Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.62 Effect of Skew on the Reaction Distribution Factor of Continuous Bridges
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.63 Effect of Skew on the Reaction Distribution Factor of Continuous Bridges

An Exterior Girder of a Three-Lane Bridge Under Dead Load

![Graph showing the effect of skew on the reaction distribution factor for a continuous bridge with aspect ratios of 1.28, 1.55, and 1.82. The graph plots the reaction distribution factor (D) against the angle of skew (degrees).]
FIG. 6.64 Effect of Skew on the Reaction Distribution Factor of Continuous Bridges
An Interior Girder of a Three-Lane Bridge Under Dead Load

![Graph showing reaction distribution factor (D) against angle of skew (degrees) for different aspect ratios.]

- Aspect Ratio = 1.28
- Aspect Ratio = 1.55
- Aspect Ratio = 1.82
FIG. 6.65 Effect of Skew on the Shear Distribution Factor of Continuous Bridges
An Exterior Girder of a Three-Lane Bridge Under Concentric Loading
FIG. 6.66 Effect of Skew on the Shear Distribution Factor of Continuous Bridges
An Interior Girder of a Three-Lane Bridge Under Concentric Loading

Shear Distribution Factor (D)

Angle of Skew (degrees)

Aspect Ratio = 1.28
Aspect Ratio = 1.55
Aspect Ratio = 1.82
FIG. 6.67 Effect of Skew on the Shear Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Eccentric Loading

Shear Distribution Factor (D)

Angle of Skew (degrees)

Aspect Ratio = 2.0
Aspect Ratio = 2.43
Aspect Ratio = 2.86
FIG. 6.68 Effect of Skew on the Shear Distribution Factor of Continuous Bridges
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading

Shear Distribution Factor (D)

Angle of Skew (degrees)

Aspect Ratio = 2.0
Aspect Ratio = 2.43
Aspect Ratio = 2.86
FIG. 6.69 Effect of Skew on the Shear Distribution Factor of Continuous Bridges
An Exterior Girder of a Three-Lane Bridge Under Dead Load

Shear Distribution Factor (D)

Angle of Skew (degrees)

- Aspect Ratio = 1.28
- Aspect Ratio = 1.55
- Aspect Ratio = 1.82
FIG. 6.70 Effect of Skew on the Shear Distribution Factor of Continuous Bridges
An Interior Girder of a Three-Lane Bridge Under Dead Load

Shear Distribution Factor (D)

Angle of Skew (degrees)

Aspect Ratio = 1.28
Aspect Ratio = 1.55
Aspect Ratio = 1.82
FIG. 6.71 Effect of Aspect Ratio on the Reaction Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading
FIG. 6.72 Effect of Aspect Ratio on the Reaction Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.73 Effect of Aspect Ratio on the Reaction Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Dead Load

- Skew = 0 degrees
- Skew = 30 degrees
- Skew = 45 degrees
- Skew = 60 degrees
FIG. 6.74 Effect of Aspect Ratio on the Shear Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading
FIG. 6.75 Effect of Aspect Ratio on the Shear Distribution Factor
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.76 Effect of Aspect Ratio on the Shear Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Dead Load

Shear Distribution Factor (D)

Aspect Ratio

Skew = 0 degrees
Skew = 30 degrees
Skew = 45 degrees
Skew = 60 degrees
FIG. 6.77 Effect of Girder Spacing on the Reaction Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Concentric Loading

[Graph depicting the effect of girder spacing on the reaction distribution factor with a diagram of a three-lane bridge under concentric loading.]
FIG. 6.78 Effect of Girder Spacing on the Reaction Distribution Factor
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading

![Graph showing the effect of girder spacing on the reaction distribution factor. The graph plots the reaction distribution factor (D) against the ratio (N), with different lines representing different skew angles: Skew = 0 degrees, Skew = 30 degrees, Skew = 45 degrees, and Skew = 60 degrees.]
FIG. 6.79 Effect of Girder Spacing on the Reaction Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Dead Load

![Graph showing the effect of girder spacing on reaction distribution factor. The graph plots Reaction Distribution Factor (D) against the ratio (N), with lines representing different skew angles: Skew = 0 degrees, Skew = 30 degrees, Skew = 45 degrees, Skew = 60 degrees.]
FIG. 6.80 Effect of Girder Spacing on the Shear Distribution Factor
An Exterior Girder of a Three-Lane Bridge Under Concentric Loading
FIG. 6.81 Effect of Girder Spacing on the Shear Distribution Factor
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading

Shear Distribution Factor (D)

Skew = 0 degrees
Skew = 30 degrees
Skew = 45 degrees
Skew = 60 degrees

The Ratio (N)

0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8
FIG. 6.82 Effect of Girder Spacing on the Shear Distribution Factor
An Exterior Girder of a Two-Lane Bridge Under Dead Load

Shear Distribution Factor (d)

The Ratio (\%)

Skew = 0 degrees
Skew = 30 degrees
Skew = 45 degrees
Skew = 60 degrees
FIG. 6.83 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Four-Lane Bridge Under Eccentric Loading
FIG. 6.84 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges

An Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.85 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Exterior Girder of a Three-Lane Bridge Under Concentric Loading
FIG. 6.86 Effect of Skew on the Span Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Interior Girder of a Three-Lane Bridge Under Concentric Loading
FIG. 6.87 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Eccentric Loading

![Graph showing the effect of skew on the support moment distribution factor for a continuous bridge under eccentric loading. The graph plots the moment distribution factor (D) against the load (number of trucks) for different skew angles: 0 degrees, 30 degrees, and 45 degrees.]
FIG. 6.88 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Three-Lane Bridge Under Eccentric Loading
FIG. 6.89 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Exterior Girder of a Four-Lane Bridge Under Concentric Loading
FIG. 6.90 Effect of Skew on the Support Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Interior Girder of a Four-Lane Bridge Under Concentric Loading
FIG. 6.91 Effect of Aspect Ratio on the Span Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Four-Lane Bridge Under Concentric Loading, Skew = 0 degrees
FIG. 6.92 Effect of Aspect Ratio on the Span Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Four-Lane Bridge Under Concentric Loading, Skew = 0 degrees
FIG. 6.93 Effect of Aspect Ratio on the Span Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Four-Lane Bridge Under Eccentric Loading, Skew = 30 degrees

[Graph showing the effect of aspect ratio on moment distribution factor]
FIG. 6.94 Effect of Aspect Ratio on the Span Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Four-Lane Bridge Under Eccentric Loading, Skew = 30 degrees
FIG. 6.95 Effect of Aspect Ratio on the Span Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Four-Lane Bridge Under Concentric Loading, Skew = 60 degrees
FIG. 6.96 Effect of Aspect Ratio on the Span Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Four-Lane Bridge Under Concentric Loading, Skew = 60 degrees
FIG. 6.97 Effect of Aspect Ratio on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Eccentric Loading, Skew = 0 degrees
FIG. 6.98 Effect of Aspect Ratio on the Support Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Two-Lane Bridge Under Eccentric Loading, Skew = 0 degrees

![Graph showing the effect of aspect ratio on the support moment distribution factor for continuous bridges. The graph plots the moment distribution factor against the number of trucks, with different lines representing various aspect ratios.]
FIG. 6.99 Effect of Aspect Ratio on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Three-Lane Bridge Under Concentric Loading, Skew = 30 degrees
FIG. 6.100 Effect of Aspect Ratio on the Support Moment Distribution Factor of Continuous Bridges

An Interior Girder of a Three-Lane Bridge Under Concentric Loading, Skew = 30 degrees

Aspect Ratio = 1.28
Aspect Ratio = 1.55
Aspect Ratio = 1.82

Load (Number of Trucks)

Moment Distribution Factor (D)
FIG. 6.101 Effect of Aspect Ratio on the Support Moment Distribution Factor of Continuous Bridges
An Exterior Girder of a Two-Lane Bridge Under Concentric Loading, Skew = 45 degrees
FIG. 6.102 Effect of Aspect Ratio on the Support Moment Distribution Factor of Continuous Bridges
An Interior Girder of a Two-Lane Bridge Under Concentric Loading, Skew = 45 degrees
FIG. 6.103 Effect of Girder Spacing on the Span Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Exterior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.104 Effect of Girder Spacing on the Span Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Interior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.105 Effect of Girder Spacing on the Span Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Exterior Girder of a Three-Lane Bridge Under Concentric Loading

![Graph showing the effect of girder spacing on the span moment distribution factor of continuous bridges at the ultimate limit state, with different skew angles (0, 30, 45, and 60 degrees).]
FIG. 6.107 Effect of Girder Spacing on the Support Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Exterior Girder of a Two-Lane Bridge Under Eccentric Loading
FIG. 6.109 Effect of Girder Spacing on the Support Moment Distribution Factor of Continuous Bridges at the Ultimate Limit State, Exterior Girder of a Three-Lane Bridge Under Concentric Loading
FIG. 6.111 First Mode Shape Of A Simply Supported Right Four-Lane Bridge
FIG. 6.112 Second Mode Shape Of A Simply Supported Right Four-Lane Bridge
FIG. 6.113  Third Mode Shape Of A Simply Supported Right Four-Lane Bridge
FIG. 6.114 First Mode Shape Of A Simply Supported Skew Three-Lane Bridge, $\theta = 45^\circ$
FIG. 6.116 Third Mode Shape Of A Simply Supported Skew Three-Lane Bridge, $\theta = 45^\circ$
FIG. 6.117 Effect of Skew on the First Natural Frequency of Two-Lane Bridges

![Graph showing the effect of skew on the first natural frequency of two-lane bridges with different aspect ratios.]
FIG. 6.118 Effect of Skew on the First Natural Frequency of Three-Lane Bridges

---

Aspect Ratio = 1.3
Aspect Ratio = 1.55
Aspect Ratio = 1.82
FIG. 6.119 Effect of Skew on the First Natural Frequency of Four-Lane Bridges

![Graph showing the effect of skew on the first natural frequency of four-lane bridges.](image)

- Aspect Ratio = 1.0
- Aspect Ratio = 1.26
- Aspect Ratio = 1.48
FIG. 6.120 Effect of Skew on the Second Natural Frequency of Two-Lane Bridges
FIG. 6.121 Effect of Skew on the Second Natural Frequency of Three-Lane Bridge
FIG. 6.122 Effect of Skew on the Second Natural Frequency of Four-Lane Bridges
FIG. 6.123 Effect of Skew on the Third Natural Frequency of Two-Lane Bridges

![Graph showing the effect of skew on the third natural frequency of two-lane bridges. The graph plots the angle of skew (degrees) on the x-axis and the third natural frequency (Hz) on the y-axis. There are three lines representing different aspect ratios: Aspect Ratio = 2.0, Aspect Ratio = 2.4, and Aspect Ratio = 2.9. The lines show an increasing trend as the angle of skew increases.]
FIG. 6.124 Effect of Skew on the Third Natural Frequency of Three-Lane Bridges

Aspect Ratio = 1.3
Aspect Ratio = 1.55
Aspect Ratio = 1.82

Third Natural Frequency (Hz)

Angle of Skew (degrees)
FIG. 6.125 Effect of Skew on the Third Natural Frequency of Four-Lane Bridges

- Aspect Ratio = 1.0
- Aspect Ratio = 1.26
- Aspect Ratio = 1.48
FIG. 6.126 Effect of Aspect Ratio on the First Natural Frequency of Two-Lane Bridges
FIG. 6.127 Effect of Aspect Ratio on the First Natural Frequency of Three-Lane Bridges
FIG. 6.128 Effect of Aspect Ratio on the First Natural Frequency of Four-Lane Bridges
FIG. 6.129 Effect of Aspect Ratio on the Second Natural Frequency of Two-Lane Bridges
FIG. 6.130 Effect of Aspect Ratio on the Second Natural Frequency of Three-Lane Bridges
FIG. 6.131 Effect of Aspect Ratio on the Second Natural Frequency of Four-Lane Bridges
FIG. 6.132 Effect of Aspect Ratio on the Third Natural Frequency of Two-Lane Bridges

- Skew=0 degrees
- Skew=30 degrees
- Skew=45 degrees
- Skew=60 degrees

Third Natural Frequency (Hz) vs. Aspect Ratio
FIG. 6.133 Effect of Aspect Ratio on the Third Natural Frequency of Three-Lane Bridges
FIG. 6.134 Effect of Aspect Ratio on the Third Natural Frequency of Four-Lane Bridges
FIG. 6.135 Effect of Flexural Rigidity Ratio, R, on the First Natural Frequency of Two-Lane Bridges

Skew = 0 degrees
Skew = 30 degrees
Skew = 45 degrees
Skew = 60 degrees

Flexural Rigidity Ratio, R (percent)

First Natural Frequency (Hz)
FIG. 6.136 Effect of Flexural Rigidity Ratio, R, on the Second Natural Frequency of Two-Lane Bridges
FIG. 6.137 Effect of Flexural Rigidity Ratio, $R$, on the Third Natural Frequency of Two-Lane Bridges
FIG. 6.138 Effect of Skew on the First Flexural Frequency of Four-Lane Bridges
Finite Element Analysis versus Flexural Beam Theory
FIG. 6.139  First Mode Shape Of A Continuous Skew Four-Lane Bridge, $\theta = 45^\circ$
FIG. 5.140 Second Mode Shape Of A Continuous Skew Four-Lane Bridge, $\theta = 45^\circ$
FIG. 5.41 Third Mode Shape Of A Continuous Skew Four-Lane Bridge, $\theta = 45^\circ$
APPENDIX A.1

CONCRETE MIX DESIGN
A.1.1 General

The design of the concrete mix used for the deck slabs of the laboratory bridge models was based on the guidelines presented in the ACI manual of concrete practice (ACI, 1989).

A.1.2 Materials

Dry mass of coarse aggregate = 1600 kg/m³

Moisture content of coarse aggregate = 2%

Moisture content of fine aggregate = 4%

Maximum size of coarse aggregate = 9.5 mm

Fineness modulus of fine aggregate (FM) = 2.56

High early strength cement (type 30)

Natural tap water

A.1.3 Required Characteristics

Compressive strength $f'_c = 41.3$ MPa

Slump = 80 mm

Air content = 0%

A.1.4 Design

Step (1) - Maximum Size of Aggregate:

Take maximum size = 10 mm to produce the maximum strength at a certain water
cement ratio.

Step (2) - Estimation of Mixing Water Content:

Using Table 5.3.3, for slump = 3-4 inch (76.2-101.6 mm), and nominal maximum size of aggregate = 3/8 inch (10 mm);

\[ W = 228 \text{ kg/m}^3 \]

Step (3) - Selection of Water-Cement ratio:

Using Table 5.3.4 (a), W/C = 0.4

Step (4) - Calculation of Cement content

Cement = \(\frac{228}{0.4} = 570\) kg/m\(^3\)

Step (5) - Estimation of Coarse Aggregate Content:

Using Table 5.3.6, for fineness modulus of 2.56 and maximum size of 10 mm, the coarse aggregate content = 0.484. As the dry mass of coarse aggregate = 1600 kg/m\(^3\), the dry weight of coarse aggregate = 0.484 x 1600 = 775 kg/m\(^3\).

Step (6) - Estimation of Fine Aggregate Content:

Using Table 5.37.1, the weight of fresh concrete = 2278 kg/m\(^3\), therefore:

fine aggregate = 2278 - (228 + 570 + 775) = 705 kg/m\(^3\)

Step (7) - The Final Estimated Batch:

(a) Water = 228 kg/m\(^3\)
(b) Cement = 570 kg/m\(^3\)
(c) Coarse Aggregate = 775 kg/m\(^3\)
(d) Fine Aggregate = 705 kg/m\(^3\)

700
Step (8) - Calculation of the Total Concrete Volume:

The total volume of concrete needed for a concrete deck slab and three 6 inch (152.4 mm) by 12 inch (304.8 mm) test cylinders:

Volume of a deck slab of the simply supported bridge model = 0.07 m³
Volume of a deck slab of the continuous bridge model = 0.27 m³
Volume of cylinders = 0.02 m³
Total volume for a simply supported bridge model = 0.07 + 0.02 = 0.09 m³
Total volume for a continuous bridge model = 0.27 + 0.02 = 0.29 m³

Step (9) - Final Batch For the Needed Volume of Concrete

For a Simply Supported Bridge Model:

(a) Water = 0.09 x 228 = 20.5 kg

(b) Cement = 0.09 x 570 = 51.3 kg

(c) Coarse Aggregate = 0.09 x 775 = 69.75 kg

(d) Fine Aggregate = 0.09 x 705 = 63.45 kg

For a Continuous Bridge Model:

(a) Water = 0.29 x 228 = 66.1 kg

(b) Cement = 0.29 x 570 = 165.3 kg

(c) Coarse Aggregate = 0.29 x 775 = 224.8 kg

(d) Fine Aggregate = 0.29 x 705 = 204.5 kg
Final Ratios:

Water/Cement ratio = 0.4

Aggregate/Cement ratio = 2.6

Fine Aggregate/Coarse Aggregate ratio = 0.91

This leads to the following mix proportions:

(W : C : CA: FA) = (1 : 2.5 : 3.4 : 3.1)
APPENDIX A.2

"ABAQUS' INPUT DATA"
A.2.1 Elastic Analysis of a Simply Supported Single Span Skew Composite Bridge

* HEADING

******************************************************************************

Elastic Analysis of a 3 lane skew composite bridge with five longitudinal girders
Simply supported bridge
Angle of skew = 60 degrees
OHBDC Truck Loading

******************************************************************************

** DATA CHECK
*PREPRINT, ECHO=YES, MODEL=NO, HISTORY=NO

********** COORDINATES OF NODES

*NODE
101,0,0,0,0,0,0
135,20.4,0,0,0,0
201,4.8,3,0,0,0
235,25.2,3,0,0,0
301,9.6,6,0,0,0
335,30.0,6,0,0,0
340,14.4,9,0,0,0
401,14.4,9,9,0,0
435,34.8,9,0,0,0
501,19.2,12,0,0,0
535,39.6,12,0,0,0
1801,-0.96,-0.6,0.496
1835,19.44,-0.6,0.496
6201,20.16,12.6,0.496
6235,40.56,12.6,0.496

********** END COORDINATES OF NODES

********** NODES GENERATION

*NGEN,NSET=LEFT
1801,6201,100
*NGEN,NSET=BRIGHT
1835,6235,100
*NFILL,NSET=DECK
BLEFT,BRIGHT,34,1
*NGEN,NSET=D1
2001,2035
*NGEN,NSET=D2
3001,3035
*NGEN,NSET=D3
4001,4035
*NGEN,NSET=D4
5001,5035
*NGEN,NSET=D5
6001,6035
*NGEN,NSET=G1
101,135
*NGEN,NSET=G2
201,235
*NGEN,NSET=G3
301,335
*NGEN,NSET=G4
401,435
*NGEN,NSET=G5
501,535
*NSET,NSET=B
G1,G2,G3,G4,G5
*NSET,NSET=SUPL
135,235,335,435,535
*NSET,NSET=SUPR
101,201,301,401,501
*NSET,NSET=OH1
2025,2620,3625,4220,5425,6020
*NSET,NSET=OH2
2017,2612,3617,4212,5417,6012,
2019,2614,3619,4214,5419,6014
*NSET,NSET=OH3
2007,2602,3607,4202,5407,6002
**************** END OF NODES GENERATION
**************** BEAM ELEMENT DEFINITION
*ELEMENT, TYPE=B31H
1,101,102
2,201,202
3,301,302
4,401,402
5,501,502
**************** END OF BEAM ELEMENT DEFINITION
**************** BEAM ELEMENTS GENERATION
*ELGEN,ELSET=GIRD1
1,34,1,40
*ELGEN,ELSET=GIRD2
2,34,1,40
*ELGEN,ELSET=GIRD3
3,34,1,40
*ELGEN,ELSET=GIRD4
4,34,1,40
*ELGEN,ELSET=GIRD5
5,34,1,40
*ELSET,ELSET=GIRD
GIRD1,GIRD2,GIRD3,GIRD4,GIRD5
7,D1,G1
7,D2,G2
7,D3,G3
7,D4,G4
7,D5,G5

********************** END OF MULTI POINT CONSTRAINTS

********************** BOUNDARY CONDITIONS

*BOUNDARY
SUPL,3
SUPR,3

********************** END OF BOUNDARY CONDITIONS

********************** MODEL PLOTTING

*DETAIL,ELSET=EDECK
*DRAWR
0
-16,05

********************** END OF MODEL PLOTTING

********************** LOADING AND OUTPUT

*STEP,LINEAR
STATIC
*LOAD
OH1,3,-30
OH2,3,-80
OH3,3,-100
*ELPRINT,ELSET=GIRD1
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD2
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD3
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD4
SM1, SF2, SF3
*ELPRINT,ELSET=GIRD5
SM1, SF2, SF3
*NODEPRINT,TOTALS=NO,NSET=G1
U3
*NODEPRINT,TOTALS=NO,NSET=G2
U3
*NODEPRINT,TOTALS=NO,NSET=G3
U3
*NODEPRINT,TOTALS=NO,NSET=G4
U3
*NODEPRINT,TOTALS=NO,NSET=G5
U3
*NODEPRINT,TOTALS=YES
RF3
CF3
*END STEP

************************ END OF LOADING AND OUTPUT
A.2.2 Elastic Analysis of a Continuous Two Span Skew Composite Bridge

* HEADING

Elastic Analysis of a 4 lane skew composite bridge with six longitudinal girders
Continuous bridge
Angle of skew = 45 degrees
AASHTO Truck Loading

** DATA CHECK
*PREPRINT, ECHO=YES, MODEL=NO, HISTORY=NO

*************** COORDINATES OF NODES

*NODE
101.0,0.,0.
141.48.0,0.,0.
201.3,0.3,0.
241.51.0,3.,0.
301.6,0.6,0.
341.54.0,6.,0.
401.9,0.9,0.
441.57.0,9.,0.
501.12.,12.,0.
541.60.0,12.,0.
601.15.,15.0,0,0
641.63.0,15.0,0,0
1801.-6.,-61.496
1841.47.4,-61.496
7201.15.6,15.61.496
7241.63.6,15.61.496

*************** END COORDINATES OF NODES

710
**************** NODES GENERATION

*NGEN,NSET=BLEFT
1801,7201,100

*NGEN,NSET=BRIGHT
1841,7241,100

*NFILL,NSET=DECK
BLEFT,BRIGHT,40,1

*NGEN,NSET=D1
2001,2041

*NGEN,NSET=D2
3001,3041

*NGEN,NSET=D3
4001,4041

*NGEN,NSET=D4
5001,5041

*NGEN,NSET=D5
6001,6041

*NGEN,NSET=D6
7001,7041

*NGEN,NSET=G1
101,141

*NGEN,NSET=G2
201,241

*NGEN,NSET=G3
301,341

*NGEN,NSET=G4
401,441

*NGEN,NSET=G5
501,541

*NGEN,NSET=G6

711
601,641
*NSET,NSET=B
G1,G2,G3,G4,G5,G6
*NSET,NSET=SUPL
141,241,341,441,541,641
*NSET,NSET=SUPR
101,201,301,401,501,601
*NSET,NSET=MIDDLE
121,221,321,421,521,621
*NSET,NSET=ASH1
2006,2604,3606,4204,5406,4808,7006,6408,
2016,2614,3616,4214,5416,4818,7016,6418,
2032,2630,3632,4230,5432,4834,7032,6434
*NSET,NSET=ASH2
2003,2601,3603,4201,5403,4805,7003,6405,
2009,2607,3609,4207,5409,4811,7009,6411,
2013,2611,3613,4211,5413,4815,7013,6415,
2025,2623,3625,4223,5425,4827,7025,6427,
2029,2627,3629,4227,5429,4831,7029,6431,
2035,2633,3635,4233,5435,4837,7035,6437,
2039,2637,3639,4237,5439,4841,7039,6441
*************** END OF NODES GENERATION
*************** BEAM ELEMENT DEFINITION
*ELEMENT,TYPE=P31H
1,101,102
2,201,202
3,301,302
4,401,402
5,501,502
6,601,602

712
*************** END OF BEAM ELEMENT DEFINITION
*************** BEAM ELEMENTS GENERATION

*ELGEN,ELSET=GIRD1
1,40,1,40
*ELGEN,ELSET=GIRD2
2,40,1,40
*ELGEN,ELSET=GIRD3
3,40,1,40
*ELGEN,ELSET=GIRD4
4,40,1,40
*ELGEN,ELSET=GIRD5
5,40,1,40
*ELGEN,ELSET=GIRD6
6,40,1,40
*ELSET,ELSET=GIRD
GIRD1,GIRD2,GIRD3,GIRD4,GIRD5,GIRD6

*************** END OF BEAM ELEMENTS GENERATION
*************** SHELL ELEMENT DEFINITION

*ELEMENT,TYPE=S4R
7,1801,1802,2002,2001

*************** END OF SHELL ELEMENT DEFINITION

*************** SHELL ELEMENTS GENERATION

*ELGEN,ELSET=EDECK
7,40,1,40,27,200,1

*************** END OF SHELL ELEMENTS GENERATION

*************** STEEL DEFINITION

*MATERIAL,NAMESPACE=STEEL

*ELASTIC
200.0E6,0.3

*************** END OF STEEL DEFINITION
CONCRETE DEFINITION
*MATERIAL,NAME=RC
*ELASTIC
28.0E6,0.15
END OF CONCRETE DEFINITION

STEEL I-BEAM DEFINITION
*BEAM SECTION,SECTION=I-BEAM,ELSET=GIRD,MATERIAL=STEEL
0.385,0.770,268,268,025,025,016
0,1,0
END OF STEEL I-BEAM DEFINITION

DECK SLAB DEFINITION
*SHELL SECTION,ELSET=EDECK,MATERIAL=RC
0.225,9
END OF DECK SLAB DEFINITION

MULTIPOINT CONSTRAINTS
*MPC
7,D1,G1
7,D2,G2
7,D3,G3
7,D4,G4
7,D5,G5
7,D6,G6
END OF MULTIPOINT CONSTRAINTS

BOUNDARY CONDITIONS
*BOUNDARY
SUPL,3
SUPR,3
MIDDLE,3
END OF BOUNDARY CONDITIONS

MODEL PLOTTING
*DETAIL, ELSET=EDECK

*DRAW
0

-16,...,05

************************** END OF MODEL PLOTTING

************************** LOADING AND OUTPUT

*STEP, LINEAR

*STATIC

*CLOAD
ASH1,3,-17.8.
ASH2,3,-71.2

*ELPRINT, ELSET=GIRD1
SM1, SF2, SF3

*ELPRINT, ELSET=GIRD2
SM1, SF2, SF3

*ELPRINT, ELSET=GIRD3
SM1, SF2, SF3

*ELPRINT, ELSET=GIRD4
SM1, SF2, SF3

*ELPRINT, ELSET=GIRD5
SM1, SF2, SF3

*ELPRINT, ELSET=GIRD6
SM1, SF2, SF3

*NODEPRINT, TOTALS=NO, NSET=G1
U3

*NODEPRINT, TOTALS=NO, NSET=G2
U3

*NODEPRINT, TOTALS=NO, NSET=G3
U3

*NODEPRINT, TOTALS=NO, NSET=G4

715
U3
*NODEPRINT,TOTALS=NO,NSET=G5
U3
*NODEPRINT,TOTALS=NO,NSET=G6
U3
*NODEPRINT,TOTALS=YES
RF3
CF3
*END STEP
A.2.3 Ultimate Analysis of a Continuous Two Span Skew Composite Bridge

* HEADING

** DATA CHECK
*PREPRINT, ECHO=YES, MODEL=NO, HISTORY=NO

****************************** COORDINATES OF NODES

*NODE
101,0.,0.,0.
141,48.0,0.,0.
201,1.8,3.,0.
241,49.8,3.,0.
301,3.6,6.,0.
341,51.6,6.,0.
401,5.4,9.,0.
441,53.4,9.,0.
501,7.2,12.,0.
541,55.2,12.,0.
601,9.0,15.0,0.0
641,57.0,15.0,0.0
1801,-36,-6.,496
1841,47.64,-6.,496
7201,9.36,15.6,496
7241,57.36,15.6,496

****************************** END COORDINATES OF NODES
********** NODES GENERATION **********

*NGEN,NSET=BLEFT
1801,7201,100

*NGEN,NSET=BRIGHT
1841,7241,100

*NFILL,NSET=DECK
BLEFT,BRIGHT,40,1

*NGEN,NSET=D1
2001,2041

*NGEN,NSET=D2
3001,3041

*NGEN,NSET=D3
4001,4041

*NGEN,NSET=D4
5001,5041

*NGEN,NSET=D5
6001,6041

*NGEN,NSET=D6
7001,7041

*NGEN,NSET=G1
101,141

*NGEN,NSET=G2
201,241

*NGEN,NSET=G3
301,341

*NGEN,NSET=G4
401,441

*NGEN,NSET=G5
501,541

*NGEN,NSET=G6
*NSET,NSET=B
G1,G2,G3,G4,G5,G6
*NSET,NSET=SUPL
141,241,341,441,541,641
*NSET,NSET=SUPR
101,201,301,401,501,601
*NSET,NSET=MIDDLE
121,221,321,421,521,621
*NSET,NSET=ASH1
2012,2611,3612,4211,5412,4813,7012,6413,
2022,2621,3622,4221,5422,4823,7022,6423
*NSET,NSET=ASH2
2005,2604,3605,4204,5405,4806,7005,6406,
2009,2608,3609,4208,5409,4810,7009,6410,
2015,2614,3615,4214,5415,4816,7015,6416,
2019,2618,3619,4218,5419,4820,7019,6420

*************** END OF NODES GENERATION

*************** BEAM ELEMENT DEFINITION

*ELEMENT,TYPE=B31H
1,101,102
2,201,202
3,301,302
4,401,402
5,501,502
6,601,602

*************** END OF BEAM ELEMENT DEFINITION

*************** BEAM ELEMENTS GENERATION

*ELGEN,ELSET=GIRD1
1,40,1,40
*ELGEN,ELSET=GIRD2
2,40,1,40
*ELGEN,ELSET=GIRD3
3,40,1,40
*ELGEN,ELSET=GIRD4
4,40,1,40
*ELGEN,ELSET=GIRD5
5,40,1,40
*ELGEN,ELSET=GIRD6
6,40,1,40
*ELSET,ELSET=GIRD
GIRD1,GIRD2,GIRD3,GIRD4,GIRD5,GIRD6
************************ END OF BEAM ELEMENTS GENERATION
************************ SHELL ELEMENT DEFINITION
*ELEMENT,TYPE=S4R
7,1801,1802,2002,2001
************************ END OF SHELL ELEMENT DEFINITION
************************ SHELL ELEMENTS GENERATION
*ELGEN,ELSET=EDECK
7,40,1,40,27,200,1
************************ END OF SHELL ELEMENTS GENERATION
************************ STEEL DEFINITION
*MATERIAL,NAME=STEEL
*ELASTIC
200.0E6,3
*PLASTIC
300.0E3,0.
************************ END OF STEEL DEFINITION
************************ CONCRETE DEFINITION
*MATERIAL,NAME=RC
*ELASTIC
28.E6,15
*CONCRETE
13.E3,0.
30.E3,9.E-4
*FAILURE RATIOS
1.16,0755
*TENSION STIFFENING
0.,15.E-4
*SHEAR RETENTION
1.,1000.

*************** END OF CONCRETE DEFINITION

*************** STEEL I-BEAM DEFINITION
*BEAM SECTION,SECTION=I-BEAM,ELSET=GIRD,MATERIAL=STEEL
.385,,770,,268,,268,,025,,025,,016
0,1,0

*************** END OF STEEL I-BEAM DEFINITION

*************** DECK SLAB DEFINITION
*SHELL SECTION,ELSET=EDECK,MATERIAL=RC
.225,9

*************** END OF DECK SLAB DEFINITION

*************** REINFORCEMENT DEFINITION
*REBAR,ELEMENT= SHELL,MATERIAL=ST2,GEOMETRY=ISOPARAMETRIC,
NAME=X1
EDECK,7.9E-5,2,-0.1,1
*REBAR,ELEMENT= SHELL,MATERIAL=ST2,GEOMETRY=ISOPARAMETRIC,
NAME=X1
EDECK,7.9E-5,2,0.1,1
*REBAR,ELEMENT= SHELL,MATERIAL=ST2,GEOMETRY=ISOPARAMETRIC,
NAME=Y1

721
EDECK,7.9E-5,2,-0.1,2
*REBAR,ELEMENT=SHALLOW,MATERIAL=ST2,GEOMETRY=ISOPARAMETRIC,
NAME=Y2
EDECK,7.9E-5,2,0,1,2
*MATERIAL,NAME=ST2
*ELASTIC
200.E6
*PLASTIC
400.E3
*********************** END OF REINFORCEMENT DEFINITION
*********************** MULTI POINT CONSTRAINTS
*MPC
7,D1,G1
7,D2,G2
7,D3,G3
7,D4,G4
7,D5,G5
7,D6,G6
*********************** END OF MULTIPOINT CONSTRAINTS
*********************** BOUNDARY CONDITIONS
*BOUNDARY
SUPL,3
SUPR,3
MIDDLE,3
*********************** END OF BOUNDARY CONDITIONS
*********************** MODEL PLOTTING
*DETAIL,ELSET=EDECK
*DRAW
0
-16,....,05
END OF MODEL PLOTTING

LOADING AND OUTPUT

*STEP,INC=25,CYC=11,SUBMAX
*STATIC,PTOL=27.,MTOL=23.,RIKS
.05,1.
*CLOAD
ASH1,3,-356.
ASH2,3,-1424.
*ELPRINT,ELSET=GIRD1
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD2
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD3
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD4
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD5
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD6
SM1,SF2,SF3
*NODEPRINT,TOTALS=NO,NSET=G1
U3
*NODEPRINT,TOTALS=NO,NSET=G2
U3
*NODEPRINT,TOTALS=NO,NSET=G3
U3
*NODEPRINT,TOTALS=NO,NSET=G4
U3
*NODEPRINT,TOTALS=NO,NSET=G5
U3
*NODEPRINT,TOTALS=NO,NSET=G6
U3
*NODEPRINT,TOTALS=YES
RF3
CF3
*END STEP

****************** END OF LOADING AND OUTPUT
A.2.4 Dynamic Analysis of a Simply Supported Single Span Skew Composite Bridge

* HEADING

---------------------------------------------------------------------
Dynamic Analysis of a 3 lane skew composite bridge with five longitudinal girders
Simply supported bridge
Angle of skew = 60 degrees

---------------------------------------------------------------------

** DATA CHECK
*PREPRINT, ECHO=YES, MODEL=NO, HISTORY=NO

*************** COORDINATES OF NODES

*NODE
101,0,0,0,0,0
135,20,4,0,0,0
201,4,8,3,0,0
235,25,2,3,0,0
301,9,6,6,0,0
335,30,0,6,0,0
401,14,4,9,9,0,0
435,34,8,9,0,0,0
501,19,2,12,0,0,0
535,39,6,12,0,0,0
1801,-0,96,-0,6,0,496
1835,19,44,-0,6,0,496
6201,20,16,12,6,0,496
6235,40,56,12,6,0,496

*************** END COORDINATES OF NODES

*************** NODES GENERATION

*NGEN,NSET=BLEFT
1801,6201,100

725
*NGEN,NSET=BRIGHT
1835,6235,100
*NFILL,NSET=DECK
BLEFT,BRIGHT,34,1
*NGEN,NSET=D1
2001,2035
*NGEN,NSET=D2
3001,3035
*NGEN,NSET=D3
4001,4035
*NGEN,NSET=D4
5001,5035
*NGEN,NSET=D5
6001,6035
*NGEN,NSET=G1
101,135
*NGEN,NSET=G2
201,235
*NGEN,NSET=G3
301,335
*NGEN,NSET=G4
401,435
*NGEN,NSET=G5
501,535
*NSET,NSET=B
G1,G2,G3,G4,G5
*NSET,NSET=SUPL
135,235,335,435,535
*NSET,NSET=SUPR
101,201,301,401,501
*************** END OF NODES GENERATION
*************** BEAM ELEMENT DEFINITION

*ELEMENT, TYPE=B31H
1,101,102
2,201,202
3,301,302
4,401,402
5,501,502

*************** END OF BEAM ELEMENT DEFINITION

*************** BEAM ELEMENTS GENERATION

*ELGEN,ELSET=GIRD1
1,34,1,40

*ELGEN,ELSET=GIRD2
2,34,1,40

*ELGEN,ELSET=GIRD3
3,34,1,40

*ELGEN,ELSET=GIRD4
4,34,1,40

*ELGEN,ELSET=GIRD5
5,34,1,40

*ELSET,ELSET=GIRD
GIRD1,GIRD2,GIRD3,GIRD4,GIRD5

*************** END OF BEAM ELEMENTS GENERATION

*************** SHELL ELEMENT DEFINITION

*ELEMENT,TYPE=S4R
7,1801,1802,2002,2001

*************** END OF SHELL ELEMENT DEFINITION

*************** SHELL ELEMENTS GENERATION

*ELGEN,ELSET=EDECK
7,34,1,40,22,200,1

727
END OF SHELL ELEMENTS GENERATION

STEEL DEFINITION

*MATERIAL,NAME=STEEL

*DENSITY
7.83

*ELASTIC
200.0E6,0.3

END OF STEEL DEFINITION

CONCRETE DEFINITION

*MATERIAL,NAME=RC

*DENSITY
2.49

*ELASTIC
28.0E6,0.15

END OF CONCRETE DEFINITION

STEEL I-BEAM DEFINITION

*BEAM SECTION,SECTION=I-BEAM,ELSET=GIRD,MATERIAL=STEEL
0.383,0.766,0.267,0.267,0.024,0.015
0,1,0

END OF STEEL I-BEAM DEFINITION

DECK SLAB DEFINITION

*SHELL SECTION,ELSET=EDECK,MATERIAL=RC
0.225,9

END OF DECK SLAB DEFINITION

MULTI POINT CONSTRAINTS

*MPC
7,D1,G1
7,D2,G2
7,D3,G3
7,D4,G4
7D5,G5

*************** END OF MULTI POINT CONSTRAINTS

*************** BOUNDARY CONDITIONS

*BOUNDARY
SUPL,3
SUPR,1,4
SUPR,6

*************** END OF BOUNDARY CONDITIONS

*************** MODEL PLOTTING

*DETAIL,ELSET=EDECK

*DRAW
0
-16.,...05

*************** END OF MODEL PLOTTING

*************** NATURAL FREQUENCY EXTRACTION

*STEP

*FREQUENCY
3

*************** END OF NATURAL FREQUENCY EXTRACTION

*************** PLOTTING OF MODE SHAPES

*PLOT

*VIEWPOINT
1,1,1

*DISPLACED
U

*************** END OF PLOTTING OF MODE SHAPES

*************** OUTPUT

*ELPRINT,ELSET=GIRD1
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD2
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD3
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD4
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD5
SM1,SF2,SF3
*NODEPRINT,TOTALS=NO,NSET=G1
U3
*NODEPRINT,TOTALS=NO,NSET=G2
U3
*NODEPRINT,TOTALS=NO,NSET=G3
U3
*NODEPRINT,TOTALS=NO,NSET=G4
U3
*NODEPRINT,TOTALS=NO,NSET=G5
U3
*NODEPRINT,TOTALS=YES
RF3
*END STEP

****************** END OF THE OUTPUT
A.2.5 Dynamic Analysis of a Continuous Two Span Skew Composite Bridge

* HEADING

******************************************************************************

Dynamic Analysis of a 4 lane skew composite bridge with six longitudinal girders
Continuous bridge
Angle of skew = 45 degrees

******************************************************************************

** DATA CHECK

*PREPRINT, ECHO=YES, MODEL=NO, HISTORY=NO

****************************** COORDINATES OF NODES

*NODE
101,0.0,0.0.
141,48.0,0.0.
201,3.0,3.0.
241,51.0,3.0.
301,6.0,6.0.
341,54.0,6.0.
401,9.0,9.0.
441,57.0,9.0.
501,12.0,12.0.
541,60.0,12.0.
601,15.0,15.0
641,63.0,15.0
1801,-6.0,-61.496
1841,47.4,-61.496
7201,15.6,15.6,1.496
7241,63.6,15.6,1.496

****************************** END COORDINATES OF NODES

****************************** NODES GENERATION
*NGEN,NSET=BLEFT
1801,7201,100
*NGEN,NSET=BRIGHT
1841,7241,100
*NFILL,NSET=DECK
BLEFT,BRIGHT,40,1
*NGEN,NSET=D1
2001,2041
*NGEN,NSET=D2
3001,3041
*NGEN,NSET=D3
4001,4041
*NGEN,NSET=D4
5001,5041
*NGEN,NSET=D5
6001,6041
*NGEN,NSET=D6
7001,7041
*NGEN,NSET=G1
101,141
*NGEN,NSET=G2
201,241
*NGEN,NSET=G3
301,341
*NGEN,NSET=G4
401,441
*NGEN,NSET=G5
501,541
*NGEN,NSET=G6
601,641

732
*NSET,NSET=B
G1,G2,G3,G4,G5,G6
*NSET,NSET=SUPL
141,241,341,441,541,641
*NSET,NSET=SUPR
101,201,301,401,501,601
*NSET,NSET=MIDDLE
121,221,321,421,521,621

*************** END OF NODES GENERATION
*************** BEAM ELEMENT DEFINITION

*ELEMENT,TYPE=B31H
1,101,102
2,201,202
3,301,302
4,401,402
5,501,502
6,601,602

*************** END OF BEAM ELEMENT DEFINITION
*************** BEAM ELEMENTS GENERATION

*ELGEN,ELSET=GIRD1
1,40,1,40
*ELGEN,ELSET=GIRD2
2,40,1,40
*ELGEN,ELSET=GIRD3
3,40,1,40
*ELGEN,ELSET=GIRD4
4,40,1,40
*ELGEN,ELSET=GIRD5
5,40,1,40
*ELGEN,ELSET=GIRD6
6,40,1,40
*ELSET,ELSET=GIRD
GIRD1,GIRD2,GIRD3,GIRD4,GIRD5,GIRD6
************************** END OF BEAM ELEMENTS GENERATION
************************** SHELL ELEMENT DEFINITION
*ELEMENT,TYPE=S4R
7,1801,1802,2002,2001
************************** END OF SHELL ELEMENT DEFINITION
************************** SHELL ELEMENTS GENERATION
*ELGEN,ELSET=EDECK
7,40,1,40,27,200,1
************************** END OF SHELL ELEMENTS GENERATION
************************** STEEL DEFINITION
*MATERIAL,NAME=STEEL
*DENSITY
7.83
*ELASTIC
200.0E6,0.3
************************** END OF STEEL DEFINITION
************************** CONCRETE DEFINITION
*MATERIAL,NAME=RC
*DENSITY
2.49
*ELASTIC
28.0E6,0.15
************************** END OF CONCRETE DEFINITION
************************** STEEL I-BEAM DEFINITION
*BEAM SECTION,SECTION=I-BEAM,ELSET=GIRD,MATERIAL=STEEL
0.385,0.770,.268,.268,.025,.025,.016
0,1,0
END OF STEEL I-BEAM DEFINITION

DECK SLAB DEFINITION
*SHELL SECTION,ELSET=EDECK,MATERIAL=RC
0.225,9

END OF DECK SLAB DEFINITION

MULTIPOINT CONSTRAINTS
*MPC
7,D1,G1
7,D2,G2
7,D3,G3
7,D4,G4
7,D5,G5
7,D6,G6

END OF MULTIPOINT CONSTRAINTS

BOUNDARY CONDITIONS
*BOUNDARY
SUPL,3
SUPR,3
MIDDLE,1,4
MIDDLE,6

END OF BOUNDARY CONDITIONS

MODEL PLOTTING
*DETAIL,ELSET=EDECK
*DRAW
0
-16,,,,,05

END OF MODEL PLOTTING

NATURAL FREQUENCY EXTRACTION
*STEP
*FREQUENCY
END OF NATURAL FREQUENCY EXTRACTION

PLOTTING OF MODE SHAPES

*PLOT
*VIEWPOINT
1,1,1
*DISPLACED
U

END OF PLOTTING OF MODE SHAPES

OUTPUT

*ELPRINT,ELSET=GIRD1
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD2
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD3
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD4
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD5
SM1,SF2,SF3
*ELPRINT,ELSET=GIRD6
SM1,SF2,SF3
*NODEPRINT,TOTALS=NO,NSET=G1
U3
*NODEPRINT,TOTALS=NO,NSET=G2
U3
*NODEPRINT,TOTALS=NO,NSET=G3
U3
*NODEPRINT,TOTALS=NO,NSET=G4
U3
\*NODEPRINT,TOTALS=NO,NSET=G5
U3
\*NODEPRINT,TOTALS=NO,NSET=G6
U3
\*NODEPRINT,TOTALS=YES
RF3
\*END STEP

******************** END OF THE OUTPUT
APPENDIX A.3

CALIBRATION OF LOAD CELL
FIG. A.3.1 Calibration of Load Cell 890 KN Capacity

Load (KN) vs Strain Reading (micro strain)
APPENDIX A.4

MULTIPLE REGRESSION ANALYSIS
When a dependent variable depends on more than one independent variable, the prediction of the mean of a dependent variable can be improved by multiple regression analysis. The multiple regression method establishes the effect of each independent variable with the other independent variables kept constant, (Kennedy and Neville, 1986).

**Multiple Regression Equation**

Consider the general case of a relation between the population mean value of the dependent variable \( y \) and independent variables \( x_1, x_2, \ldots, x_k \). This can be expressed as follows:

\[
E(y) = b_o + b_1x_1 + b_2x_2 + \ldots + b_kx_k \tag{A.4.1}
\]

where: \( b_o \) is a constant and \( b_1, b_2, \ldots, b_k \) are partial regression coefficients.

The multiple regression model for the distribution of \( y \) can be expressed as:

\[
y = b_o + b_1x_1 + b_2x_2 + \ldots + b_kx_k + e \tag{A.4.2}
\]

where \( e \) is the error or residual.

An estimate of equation (A.4.1) can be expressed as:

\[
y' = b'_o + b'_1x_1 + b'_2x_2 + \ldots + b'_kx_k \tag{A.4.3}
\]

where \( b'_o, b'_1, b'_2, \ldots, b'_k \) are point estimators of the population parameters \( b_o, b_1, b_2, \ldots, b_k \).

It should be noted that the function \( y \) should be a linear function of the unknown parameters \( b'_o, b'_1, b'_2, \ldots, b'_k \) since this condition makes it easy to apply the principle of least squares. It is to be noted also that equation (A.4.3) represents a plane in \((k + 1)\)
dimensions. This plane passes through the centroid, \(x_1, x_2, x_3, \ldots, x_k, y\) of all the observed values. Therefore:

\[
y = b'_o + b'_1x_1 + b'_2x_2 + \cdots + b'_kx_k \tag{A.4.4}
\]

Substituting in equation (A.4.3), yields:

\[
y' - y = b'_1(x_1 - x_i) + b'_2(x_2 - x_i) + \cdots + b'_k(x_k - x_i) \tag{A.4.5}
\]

The coefficients are obtained using the method of least squares by differentiating the square of the error function with respect to the unknown coefficients \(b_i\)s. If \(y_i\) is an observed value of the dependent variable \(y\) then the error in \(y_i\) relative to \(y'\), which is given by equation (A.4.3) is

\[
\varepsilon_i = y_i - y'_i \tag{A.4.6}
\]

Taking the partial derivatives of the above equation with respect to \(b_i\)s and equating each expression to zero we obtain the following normal equations:

\[
nb'_o + (\Sigma x_{1i})b'_1 + (\Sigma x_{2i})b'_2 + \cdots + (\Sigma x_{ki})b'_k = \Sigma y_i \tag{A.4.7}
\]

\[
(\Sigma x_{1i})b'_o + (\Sigma x_{2i}^2)b'_1 + (\Sigma (x_1x_{2i}))b'_2 + \cdots + (\Sigma (x_1x_{ki}))b'_k = \Sigma x_{1i}y_i
\]

\[
(\Sigma x_{2i})b'_o + (\Sigma x_{1i}x_{2i})b'_1 + \cdots + (\Sigma x_{k-1}x_{2i})b'_{k-1} + (\Sigma x_{2i}^2)b'_k = \Sigma x_{2i}y_i
\]

where summation signs indicates summation over all the data points, \(i = 1, 2, 3, \ldots, n\).
APPENDIX A.5

PUSH-OUT RESULTS

(LOAD-SLIP RELATIONSHIP)
APPENDIX A.6

AASHTO AND OHBDC TRUCKS
FIG. A.6.1 AASHTO Standard Truck Used in the Parametric Study (Collins and Mitchel, 1991)

746
FIG. A.6.2 OHBDC Standard Truck Used in the Parametric Study (Collins and Mitchel, 1991)
VITA AUCTORIS

TAREK IBRAHIM EBEIDO

The author was born on 23 April, 1967 in Cairo, Egypt. In 1984 he completed his high school education at "The English School", Alexandria, Egypt. In September of 1984 he joined the Faculty of Engineering at Alexandria University. In June of 1989 he graduated from Alexandria University with a degree of Bachelor of Civil Engineering (Distinction with degree of honour). In September 1989 he was employed by Alexandria University as a demonstrator in the Structural Engineering Department. At the same time he joined, as a part time design engineer, a civil engineering consulting office in Alexandria. In May 1992 he enrolled in M. A. Sc. program in the Department of Civil and Environmental Engineering at the University of Windsor, Windsor, Ontario, Canada, and worked as a teaching and research assistant. In July 1993 the author obtained a Degree of Master of Applied Science in Civil Engineering from the University of Windsor. The author prepared this dissertation in partial fulfilment of the requirements for the Degree of Doctor of Philosophy in Civil Engineering.