Surface finish enhancement in a turning operation via control of the depth of cut.

Evangelos. Liasi

University of Windsor

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Surface Finish Enhancement in a Turning Operation
via Control of the Depth of Cut

by

Evangelos Liasi

A Dissertation
Submitted to the Faculty of Graduate Studies and Research
through the Department of Mechanical and Materials Engineering
in Partial Fulfilment of the Requirements for the Degree of
Doctor of Philosophy of Mechanical Engineering at the
University of Windsor
Windsor, Ontario, Canada

1996
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Abstract

During the past two decades there has been a steady increase in the number of computer numerically controlled (CNC) machine tools. Such problems as dimensional accuracy and surface finish are recently receiving widespread attention.

This theoretical investigation explores the possibility of decreasing the surface roughness by suppressing radial tool-workpiece relative displacements via control of the depth of cut. In order to achieve this goal various control strategies are developed and evaluated via simulation.

An integrated model of the turning process in the most flexible component which manifests itself in the surface roughness, i.e., the radial direction was generated from an extensive literature survey and is presented. Three major programs were developed in Matlab so as to simulate the turning process and to evaluate the effectiveness of controllers based on classical and modern techniques. These programs are general enough to be used for simulating other systems than the one used here as an example by just changing the "inputs" and "parameter" files which depict the input to the system and the system characteristics respectively.

First of all a proportional plus integral plus derivative (or simply PID) controller is designed and a typical machine turning centre is chosen from published literature in order to evaluate the effectiveness of this controller. Improvements of up to an order of magnitude in the tool-workpiece relative
displacement and a 16% reduction in surface roughness, \( R_a \) (i.e., form 3.22\( \mu m \) in the uncontrolled case to 2.69\( \mu m \) with control) values have been achieved. In the worst case scenario, before steady state is reached, the surface roughness, \( R_a \) was reduced by nearly six times.

Secondly, optimal control strategies were designed based on Linear Quadratic, LQ, and Linear Quadratic Gaussian (LQG) methods. Their effectiveness was also verified via simulation. Pole-placement and frequency shaped designs were carried out. Improvements superior to the PID controller were achieved especially with the frequency shaped LQG design. The frequency shaped design is, however, not justifiable because the improvement over the standard LQG design does not manifest itself in the final surface profile and it increases the controller complexity. Typical \( R_a \) values obtained are around 2.5\( \mu m \), which is the theoretical value determined from the tool nose radius and the feed rate.

Finally, a parameter adaptive controller was designed and its performance evaluated via simulations. A self tuning regulator (STR) based on LQG methods was developed. The tool-workpiece relative displacement was suppressed to very low levels (nanometers) which implies that the theoretical surface roughness value (i.e., the surface roughness is only determined by the tool nose radius and the feed) has been achieved. An added advantage with this type of controller is its ability to cope with process variations.

All controllers were stable and robust with high enough bandwidths to easily accommodate cutting speeds around 2000 rpm at 20 samples per revolution while satisfying the Nyquist sampling theorem. If a higher speed is required the number of samples per revolution should be reduced so as to be consistent with the Nyquist theorem. The same holds if a higher number of samples per revolution is desired, i.e., the cutting speed must be reduced.
Dedicated to
Christina,
my parents, Andreas and Andreani, my sister Vasilia,
my late grandparents Evangelos, Maria, Themistoklis and Christina,
and to all those who thirst for knowledge.
Acknowledgements

I would like to express my sincere gratitude to Dr. W.P.T. North for his guidance, patience and support throughout my graduate studies. Dr. North, you have been a constant source of strength and inspiration. It was your faith in me that made the completion of this study a reality.

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The music and lyrics of Axion Esti, and The Lines of the Horizons, along with WCSX kept me going during the late night and early morning hours I have spent in Essex Hall.

It is somehow saddening to write that Dr. J. Yuan who co-supervised this study for three years cannot share the joy of its completion.
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Nomenclature

The symbols used in this study are as shown below unless otherwise specified.

\( \sigma \)  standard deviation
\( \lambda \)  exponential forgetting factor
\( \alpha \)  rake angle
\( \phi \)  angle of approach
\( \Phi \)  matrix of \( \phi \)
\( \theta \)  estimate of \( \theta \)
\( \omega_b \)  bandwidth
\( \omega_c \)  crossover frequency
\( \theta_i \)  unknown parameters in regression model
\( \phi_i \)  regressors in regression model
\( \zeta_i \)  damping ratio
\( \phi_m \)  phase margin
\( \omega_s \)  sampling frequency
\( 2\omega_N \)  Nyquist frequency
\( A, A_n \)  area, chip cross-sectional area
AC  adaptive control
ACC  adaptive constrained control
\( A_i \)  Markov matrices
\( b \)  set-point weighting factor
\( B_s \)  damping constant of spindle
\( B_s \)  damping constant of cutting tool
\( B_w \) damping constant of workpiece
BUE built up edge
\( ch \) chamfer
CNC computer numerical control
\( C_o \) controllability matrix
\( d \) depth of cut
\( d_o \) initial (preset) depth of cut
\( E\{ \} \) expectation \{ \}
\( f \) feed rate
\( F \) resultant force
\( F \) system matrix
\( F_c \) control force
\( f_o \) initial (preset) feed rate
\( F_x \) cutting force in the radial direction
\( G \) system matrix
GAC geometric adaptive control
\( H \) system output matrix
\( H_N \) Hankel matrix
\( K \) gain vector
\( K_e \) feedback loop gain
\( K_e \) estimator gain vector
\( K_i \) empirical constants
\( K_s \) stiffness of spindle
\( K_t \) stiffness of cutting tool
\( K_w \) stiffness constant of workpiece
LQ linear quadratic
LQG linear quadratic gaussian
\( m_i \) empirically obtained exponents
\( M_s \) mass of spindle
\( M_s \) mass of cutting tool
\( M_w \) mass constant of workpiece
\( N(s) \) Laplace transform of input noise
\( NC \) numerical control
\( n_i \) empirically obtained exponents
\( O_b \) observability matrix
\( P \) solution to the matrix Riccati equation
\( P_e \) solution to the matrix Riccati equation for the Kalman-Bucy filter problem
\( PID \) Proportional-Integral-Derivative controller
\( P_s(s) \) Laplace transform of the setpoint variable
\( P_x(s) \) Laplace transform of the process variable
\( Q \) weight selection matrix
\( R \) weight selection matrix
\( R \) tool nose radius
\( R_s \) RMS surface roughness
\( RLS \) recursive least squares
\( RMS \) root mean square
\( STR \) self-tuning regulator
\( t \) time
\( T_d \) derivative time and the integral time.
\( T_i \) integral time
\( U_s(s) \) Laplace transform of the control variable
\( V \) cutting speed
\( v \) output noise
\( V \) performance index
\( w \) input noise
\( W_c \) crater wear
\( W_f \) flank wear
<table>
<thead>
<tr>
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<th>Description</th>
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<tr>
<td>$x$</td>
<td>state vector</td>
</tr>
<tr>
<td>$\hat{x}$</td>
<td>estimate of $x$</td>
</tr>
<tr>
<td>$x_\varepsilon$</td>
<td>state estimate vector</td>
</tr>
<tr>
<td>$x^f$</td>
<td>filtered state vector</td>
</tr>
<tr>
<td>$x_s$</td>
<td>spindle displacement (state)</td>
</tr>
<tr>
<td>$x_t$</td>
<td>tool displacement (state)</td>
</tr>
<tr>
<td>$x_w$</td>
<td>workpiece displacement (state)</td>
</tr>
<tr>
<td>$y$</td>
<td>system output (tool-workpiece relative displacement)</td>
</tr>
<tr>
<td>ZOH</td>
<td>zero order hold</td>
</tr>
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Chapter 1 Introduction

1.1 Introduction
Engineering may be described as the application of scientific laws to the design and implementation of systems. Laws allow the engineer to reason about models of a given system, whence making possible the transition from a specification to an implementation that satisfies its requirements. In this dissertation, an integrated model for a turning process is presented and control systems (in the radial direction) based on classical and modern control methods are designed so as to suppress the radial vibrations. The effectiveness of each control system is evaluated by means of computer simulations.

1.2 Metal Cutting
Metal cutting is one of the most important methods of removing unwanted material in the production of mechanical components. The importance of material removal operations can be realized by observing that nearly every device we use has at least one machined surface or hole, or alternatively by noting that the yearly cost associated with material removal operations in the United States is estimated to be equivalent to about ten percent of the gross national product [67].

Mechanical parts can be produced by changing the geometry of bulk material. This can be done via several methods:
- By putting material together.
- By moving material from one region to the other.
- By removing unnecessary material.
- By physical deformation.
Chapter 1 Introduction and Thesis Statement

The material removal operations, which incorporate these methods can be classified as follows:

- cutting
- grinding
- special techniques

Cutting operations involve the removal of macroscopic chips, grinding operations subdivide the material removed into smaller particles than in cutting, and special techniques such as electrochemical machining, electro-discharge machining and ultrasonic machining produce chips of atomic or subatomic size.

1.2.1 Principal Cutting Operations

The three most widely used cutting operations are:

- turning
- milling
- drilling

*Turning* is a process where a single point tool removes unwanted material to generate a surface of revolution. The variables adjusted by the operator (or automatically on a CNC-computer numerically controlled lathe) are the cutting speed, $V$, the feed, $f$ and the depth of cut, $d$. *Milling* is a process for producing flat and curved surfaces using multi-point cutting tools, and *drilling* is a process used to produce rough holes.

This work deals with the turning process and more specifically with the surface finish that results from a finish turning operation.

1.3 Historical Introduction

In order to fully appreciate the undertaking of this research study, a brief account of how material removal via turning has evolved is necessary.
Chapter 1  Introduction and Thesis Statement

The chisel was probably one of the first cutting tools used by a human being. The earliest stone implements were undoubtedly blunt but as experienced was gained the designs that were implemented were improved so as to require less effort.

Traditionally, a machinist does everything from machine set-up, to monitoring the process, to changing the tool, to sharpening the tool, etc. As demands and costs increase, machinists are being replaced by machines of increased levels of automation, including the application of numerically controlled (NC/CNC) machines.

The role of today's machinist, or rather CNC programmer, is to write the code that will tell the CNC how to machine the part, select the appropriate tooling, load and unload the machine, as well as monitor the machine.

Expert systems have in recent years made their appearance to assist in the monitoring of machines. Tool monitoring no longer has to rely on the vision, hearing, and touch skills of the machinist. Such examples include an expert system used to monitor drill condition [50] and intelligent tool monitoring using neural networks [63]. At this point it should also be noted that some expert systems are available commercially.

With the arrival of the numerical control technique, accurate positioning of the cutting tool has been achieved. In the last decade continuous attempts have been made, and are still considered, so as to control the metal-cutting process itself. In process measurements such as cutting temperature, tool vibration, and cutting force have been incorporated for the purpose of monitoring, with closed loop control of metal-cutting processes. More recently, the results of cutting operations - dimensional tolerance and surface finish - are given, deservedly, more attention. This can of course be attributed to advances in sensor technology.
1.4 The Thesis Objective

The Thesis Statement:

I. Successful control of the depth of cut, so as to suppress the tool-workpiece relative displacement, will improve the surface finish of the workpiece, as well as improve the dimensional tolerance of the workpiece.

II. Surface finish enhancement of workpieces that are used as parts of integrated machines can improve the machine performance and extend the life of the machine.

Based on the above statements, a theoretical investigation and simulation was carried out with the following objectives:

1. to control a turning operation for the purpose of decreasing the resulting surface roughness;

2. to propose and evaluate various control strategies;

3. to determine the required acceleration from an actuator which will control the resulting roughness;

4. to determine the required frequency response, and

5. to determine the required amplitude response.

Why the Thesis is Important:

The integrity of the machined surface is frequently one of the important outputs of the turning operation. The term surface integrity involves surface finish, freedom from cracks, chemical change, thermal change, and adverse residual stress. The most important one, and the main concern of this study, is surface finish.

In the control of machine tools almost all work reported in the last thirty years was of the ‘adaptive control’ nature [9, 12, 18, 21, 31, 45, 46, 51, 53, 54, 61, 70-73, 75, 76, 78, 80] with artificial intelligence (AI) also finding its way in recent years [12, 13]. Research and development efforts for the adaptive control of machine tools have been under way since the early 1960’s. [61, 79]. The most common
Chapter 1  Introduction and Thesis Statement

applications of adaptive control (AC) systems are for grinding, drilling, and milling (for example, see [70, 75, 76]). Turning has received some attention in the last ten to fifteen years [18, 21, 31, 45, 46, 51, 54, 71-73, 80].

The emphasis, has obviously been on adaptive constraint control (ACC) typically based upon force (and torque) measurement rather that geometric adaptive control (GAC) systems, based upon product dimensions and surface finish. This is primarily a result of the availability of less expensive and more reliable sensors for force and torque measurements. However, with recent advances in optical sensing techniques for part dimensions and surface finish, this trend can be expected to change in the coming years.

In addition, in the investigations based on force and torque measurements, the control variable was the feed rate [for example see 18, 46, 73]. This simply means that the control objective was achieved via feed regulation which is known to result in an irregular surface pattern and as a result, the implementation of some if not all control schemes (e.g. zero phase error adaptive control [71, 72]) for the purpose of enhancing the surface finish is further complicated if not made impossible.

Surface finish enhancement is the main goal of this study and control schemes will be proposed in order to achieve this objective, where the control variable is the depth of cut.

It is our belief that surface finish enhancement can be consummated by minimizing the tool-workpiece relative displacement (that is, radial vibration suppression) by proper regulation of the depth of cut, \( d \). Furthermore, incorporating the effects of surface micro-hardness on the cutting force, will give us further insight with respect to the modelling of the actual process, and in fact it will help us generate a more precise model which, as a consequence, will permit the design of better controllers to achieve the desired objective.
The surface finish of the individual components which constitute a machine plays an important role on various aspects of machine performance such as general functioning, lubrication, resistance to wear, load carrying capacity, fatigue life, control of dimensions, interference, or sliding fits, subsequent plating or painting, operations, etc. In general, an improvement in surface finish of the individual machine components often leads to improved performance, increased machine life, and increased cost.

The cost of finishing operations is significant. For example, [22, Fig 3.9 p.95] shows that finish turning results in an average surface roughness of $R \approx 63$ micro-inches RMS. In order to generate an average surface roughness of $R \approx 4$ micro-inches RMS, requires that the part be ground and then lapped. Each of these operations requires a different set up and different machines. As a result the cost of finishing is increased by a factor of two or three.

In a machining operation surface finish will depend on the type of chip formed, the cutting tool profile, the process geometry and operating variables. When a continuous chip with no built up edge is achieved, in theory, the tool profile will be reproduced on the work surface in the form of feed marks. In practical situations, however, phenomena such as tool wear, and tool-workpiece relative displacement cause the surface finish to deviate from the theoretical profile. Suppression of the tool-workpiece relative displacement will of course improve the surface finish.

Several relations that relate the surface roughness to the cutting speed, the feed rate, and the depth of cut are in existence. One such relation, perhaps the first, was obtained from an extensive experimental study by Hasegawa, Seireg and Lindberg [30] and later modified by Jang and Seireg [33-35]. Du et al. [20] took a slightly different approach in their study and the relation they presented as a result of their work accounts for tool wear. All models presented indicate that there is a general consensus with respect to the importance of the cutting parameters.
Chapter 1  Introduction and Thesis Statement

1.5 Specific Tasks Undertaken

Survey of Related Work
Due to the fact that this dissertation is of a multi-disciplinary nature a substantial
survey of literature from various domains in engineering (dynamics, vibrations,
controls, and materials) was undertaken with only the most relevant work cited. In
particular, the control of machine tools, machine tool monitoring -- for example, tool
wear, modelling of the cutting forces, metal cutting principles, and surface
roughness and surface topography generation models were surveyed.

Development of an Integrated Model for the Radial Component in Turning
An integrated model for the radial direction (the direction of interest) has been
developed from a collection of specific models put forth by various researchers
so as to enable us to design accurate controllers to meet our objective of
suppressing the tool workpiece relative displacement in finish turning.

Design of PID, Optimal (LQ/LQG), and Parameter Adaptive Control
Algorithms
Proportion plus Integral plus Derivative (PID), optimal (based on Linear
Quadratic and Linear Quadratic Gaussian - LQ/LQG methods) and parameter
adaptive controllers have been developed and evaluated via simulation. Many
techniques have been employed. For example, pole placement, pole-zero
cancellation, and gain selection were employed in the PID controller design.
Pole placement, frequency shaping and controller reduction have been
considered in the optimal controller design. In the design of the parameter
adaptive controller, a self-tuning regulator (STR) based on a linear quadratic
gaussian solution has been investigated. Stability, controllability and
observability were key issues considered. The assumptions made throughout
this investigation are listed in the following section.
Chapter 1 Introduction and Thesis Statement

Programs Written
Several programs were written (in Matlab) in order to carry out the design of the different controllers and observers, and for the purpose of evaluating the performance of each type of controller design investigated. The same programs were used to simulate the system performance, and to construct the corresponding surface topographies.

The program written for the design of the PID controller is very general and capable of assisting the designer with the control of any system as long as he/she defines the dynamics of the system. The designer is allowed to investigate the performance of a PID controller based on pole placement methods, pole-zero cancellation, and gain selection. In the design, bode plots, nyquist plots, power spectral density plots, and actual system performance plots are provided as a visual means of evaluating the controller that has been designed.

The program written for the design of the optimal controller based on (LQ/LQG) methods is also general enough to handle the design and evaluation of the controller as long as the designer provides the system dynamics, and the quadratic-weight matrices. The program can be used to carry out straight optimal (LQ) designs, designs based on pole placement, and frequency shaped designs. Furthermore, Kalman filter observers can be employed for complete LQG (linear-quadratic-gaussian) designs, where input and measurement noise are also assumed to be present.

In the design, bode plots, nyquist plots, power spectral density plots, and actual system performance plots are provided as a visual means of evaluating the controller that has been designed.

Finally, the program written for the design of the adaptive controller with a Kalman filter observer is also general enough to handle the design and evaluation of the control system. The designer is requested to provide the
matrices depicting the system dynamics, and the quadratic-weight matrices. Even though the program is intended for a self tuning regulator based on LQG methods with some modification it can be made to handle the design of self tuning PID controllers as well.

1.6 Assumptions
The course of this study was carried out under the following assumptions:
1. The external excitation of the tool comes from the variations in the micro-hardness of the part itself.
2. There is no tool breakage during cutting.
3. There is no tool wear during cutting; no built up edge (BUE).
4. Lathe and parts are properly aligned (that is, no run-out of the chuck).
5. Cutting is continuous.
6. No chatter is present.
7. Tool-workpiece relative displacement is available for feedback.
8. Tool nose radius is greater than both the feed and the depth of cut.

1.7 Organization of the Dissertation
In chapter 2, an integrated model of the turning process (in the radial-depth of cut direction) is presented, and some of the work carried out or currently being carried out by other researchers is highlighted. The model is then formulated in state space, and in transfer function representations. Finally, some fundamental control concepts are revisited. The three chapters that follow are dedicated to the controller design whose requirement is to suppress radial vibration.

In chapter 3, a Proportional plus Integral plus Derivative (PID) controller is designed to establish grounds for comparison. Such criteria as stability, noise rejection, pole-zero cancellation, and pole placement are considered.
Chapter 4 is dedicated to the design of an optimal controller using Linear Quadratic (LQ) methods. In the controller design, pole placement, and frequency shaping techniques are exploited. A Kalman filter observer is also incorporated in the Linear Quadratic Gaussian (LQG) design.

In chapter 5, an adaptive controller (Self Tuning Regulator (STR) based on LQG methods) with a Kalman filter observer is designed.

Finally, in chapter 6 the results that emerged from the investigation are summarized and some directions for future work are suggested.
Chapter 2  Modelling of the Turning Operation

2.1 Introduction
In this chapter an approximate dynamic model to the turning operation is derived and presented with a set of actual physical parameters as employed in the simulation work which is carried out for the evaluation of the different control strategies which are developed. In addition, cutting force and surface roughness models, as well as a surface micro-hardness distribution model are described. Finally, the possible directions for achieving the desired objective as considered are itemized and the chosen direction identified. All assumptions made in the modelling of the turning process are also highlighted at the appropriate section.

2.2 Problem Statement
An approximate model of the turning process is considered, subject to workpiece micro-hardness variations. The objective is to suppress radial vibrations, i.e., track the pre-set depth of cut. Measurements are the tool-workpiece relative vibration, and the scalar control is the depth of cut.

2.3 Convention
Before presenting the equations which describe the turning system, the convention that is followed must be established. Figure 2.1 depicts the turning system and the convention which is employed is defined. The convention is selected so as to be consistent with the convention commonly used on commercially available NC/CNC lathes.
Figure 2.1: Turning system and convention

The workpiece is usually mounted at the headstock (spindle) and whenever it is feasible and/or necessary it is also mounted at the tailstock, what is known as mounted between centres (as shown Figure 2.1). The cutting tool moves in the radial (depth of cut) direction depicted by the $X$-axis which has its origin along the centre of the workpiece, and the axial (feed) direction depicted by the $Z$-axis whose origin is defined by the operator. Motion in the tangential direction ($Y$-axis) is neither necessary nor commonly permitted. However, if someone requires the freedom for motion in the tangential direction for whatever reason, the machine can probably be modified so as to allow the extra degree of freedom.

2.4 Vibrations in Cutting

The analysis of vibration in machining is a rather complicated matter. This is because the machine tool, the cutting tool, and the part being machined constitute a system which has an infinite number of degrees of freedom. Even when the system
is reduced to a few major modes of vibration it is still very difficult to evaluate the compliance and damping properties of the system. In addition to the difficulties encountered in determining the vibration properties of the system, which are common to any vibrating machine, the metal cutting process itself also influences the vibration. For some cutting conditions, force fluctuations are inherent in the cutting process. These force fluctuations produce forced vibration. For other cutting conditions, the vibration may cause the cutting process to vary so that it supplies a positive input of energy to maintain the vibration. This is known as self excited vibration and is analogous to a positive feedback loop.

A practical situation which may occur is chatter which has the following undesirable effects: it may produce imperfections on the work surface, it may increase the rate of wear of the tool, and it may cause a high frequency noise which at best is unpleasant and can be harmful to nearby personnel. In this investigation it is assumed that chatter conditions are never reached, as it is usually the case in practical finishing operations in turning where only light cuts are involved.

The major part of this investigation deals with the effect of force fluctuations (forced vibration) on surface finish which results from a given turning operation. The relative motion between the cutting tool and the workpiece, which results from the force fluctuations, will be considered since this is one of the reasons why the surface finish profiles are not ideal. As the vibration is minimized, one expects the surface finish to be closer to the theoretical value which simply depends on the geometry of the tool, the depth of cut, and the feed rate. Once the process is made stable enough the only other contributing factor would be tool wear which is also neglected in this study since it is beyond the scope of this work.

Furthermore, tool wear in a finishing operation is expected to be much less severe than in a roughing operation. It is significant, however, when tool life is a factor to be considered. Tool wear will be discussed further in chapter 6, where directions for future work are detailed.
2.4.1 Nature of Vibrations in Cutting

The first and most obvious point to note is that the vibration will involve relative movement between the tool and the workpiece. The direction of this movement will be dictated by the elements of the system which are vibrating. Some possible modes of vibration for a turning operation are [8]:

- tool shank bending in (a) vertical plane; (b) horizontal plane
- workpiece bending between centres: (a) in a vertical plane; (b) in a horizontal plane headstock shaft in bending
- headstock or tailstock in bending: (a) in a vertical plane; (b) in a horizontal plane
- lathe bed bending (a) in a vertical plane; (b) in a horizontal plane
- tool post or compound slide bending in a vertical plane (a) parallel to the feed direction; (b) perpendicular to the feed direction.

The above list is far from complete but quite sufficient to demonstrate that the workpiece can move in almost any direction relative to the tool. Under vibrating conditions there may be fluctuations in the cutting speed, the feed, the radial depth of the layer removed and the inclination of the tool faces to the workpiece surface.

Surface finish, a major concern of every metal cutting operation is significantly affected by vibrations that are present during the cutting process. The most complete work (with respect to turning) to date was carried out by Jang and Seireg [35] who generated an empirical relationship between the peak-to-valley surface roughness and cutting parameters as functions of the tool's natural frequency. Their model was obviously a dynamic one and was based on the way that the cutting process is usually modelled [41, 59] i.e. a single degree of freedom system based on the most flexible component of a given machine tool, in this case the radial direction which seems to be the principal component that manifests itself in the measured surface roughness.

Since the deterministic component of surface roughness depends on the tool-workpiece relative motion, the dynamic model should at least simulate the
interaction between the two vibratory systems representing the tool and the workpiece structures as depicted in Figure 2.2.

![Dynamic model of the system](image)

**Figure 2.2:** Dynamic model of the system

The equations of motion for the dynamic system, as depicted in Figure 2.2 are:

\[ M_s \ddot{x}_s + (B_t + B_w) \dot{x}_s + (K_s + K_w)x_s + B_w \dot{x}_w + K_w x_w = 0 \]

\[ M_w \ddot{x}_w + B_w \dot{x}_w + K_w x_w - B_s \dot{x}_s - K_s x_s = F_x \quad (2.1) \]

\[ M_t \ddot{x}_t + B_t \dot{x}_t + K_t x_t = F_c - F_x \]

where, the subscripts \(w, s,\) and \(t\) correspond to the workpiece, spindle, and tool respectively and \(M, K, B,\) and \(F\) are the mass, stiffness constant, damping constant and external force respectively. The block diagram for the above system is shown below in Figure 2.3 in state space representation.

![Block diagram of dynamic system](image)

**Figure 2.3:** Block diagram of dynamic system
The equations describing the above system in state space form are:

\[ x = Fx + Gu + G_w w \]

\[ y = H^T x + v \]  \hspace{1cm} (2.2)

where superscript \( T \) denotes the transpose of a matrix, \( w \) and \( v \) are input and output disturbances respectively, \( y \) is the tool workpiece relative displacement in the radial direction, and \( F, G, G_w, \) and \( H \) are the system matrices given by:

\[
F = \begin{bmatrix}
\frac{B_w + B_t}{M_t} & -\frac{K_w + K_t}{M_t} & \frac{B_w}{M_w} & \frac{K_w}{M_t} & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\frac{B_w}{M_w} & \frac{K_w}{M_w} & -\frac{B_w}{M_w} & -\frac{K_w}{M_w} & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{B_t}{M_t} & -\frac{K_t}{M_t} \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
0 & 0 & \frac{1}{M_w} & 0 & -\frac{1}{M_t} & 0
\end{bmatrix}^T \hspace{1cm} G_w = \begin{bmatrix}
0 & 0 & \frac{1}{M_w} & 0 & -\frac{1}{M_t} & 0
\end{bmatrix}^T
\]

\[
H = \begin{bmatrix}
0 & 0 & 0 & -1 & 0 & 1
\end{bmatrix}^T
\]

\[
x = [\dot{x}_t \ x_x \ x_w \ \dot{x}_w \ \dot{x}_i \ x_i]^T
\]

Please note that in the design of control strategies based on linear methods, additional terms may appear in the system matrix \( F \) which result from the linearization of non-linear models based on a first order Taylor expansion. For instance, the model used to compute the cutting force (i.e., equation 2.5 presented below) is dependent on the depth of cut which is equivalent to the sum of the pre-set depth of cut, \( d_o \), and the tool workpiece relative displacement in the radial direction, \( y \), obtained from the difference of state variables \( x_t \) and \( x_w \). This contribution is directly reflected in system matrix \( F \).
The dynamic model employed by Jang and Seireg will also be used to describe the system in the present investigation for the purpose of regulating the depth of cut. This is so as to compensate for the relative motion between the workpiece and the cutting tool. The result will be the minimization of the tool workpiece relative displacement which will result in a surface finish improvement.

The figure below (Figure 2.4) depicts the tool workpiece relative displacement as a part of a possible implementation arrangement.

![Diagram of tool workpiece relative displacement](image)

**Figure 2.4 - Physical interpretation of tool workpiece relative displacement and possible implementation arrangement.**

Based on the model described above Jang and Seireg [33-35] derived the empirical relation relating the peak-to-valley surface roughness to the cutting parameters as well as the machine structure's natural frequency. The resulting empirical relationship is presented in section 2.6. The tool bed is assumed relatively rigid, and the spindle structure, the workpiece and the cutting tool (with its support) are considered to have major influence on system vibration.

Existing empirical models which relate the cutting force to the cutting parameters $V$, $f$, and $d$ will be utilized in the present study. The surface finish is evaluated through consideration of the tool workpiece relative displacement. In
addition, tool profiles of a given section as well as surface topographies over a part or the whole finishing cycle are generated. They are generated by considering the tool geometry and the fact that the tool workpiece relative displacement is exactly how much the tool deviates from the desired location. In the generation of each cross-section the tool lead angle is also accounted for. The surface finish improvement becomes apparent from the examination of the surface topographies as well as the calculation of the $R_s$ surface roughness index.

Classical and modern control methods are employed for the purpose of designing control strategies so as to achieve the desired goal. Specifically, PID, LQG optimal, and parameter adaptive control strategies are developed, the limitations of such control schemes are highlighted, and their effectiveness evaluated via simulation.

The remaining sections of this chapter are dedicated to formulating an integrated model of the turning process, as used in this investigation.

2.5 Modelling the cutting force
There are a number of practical reasons why it is important to make accurate estimates of the magnitudes of forces necessary to cut materials. Some of these reasons are to estimate the motor size of new machine tools, to determine the limiting metal removal rates with existing machine tools, and to assist in the design of clamping and workholding devices.

Variations in cutting forces may also be used to evaluate the effectiveness of cutting fluids, in machinability evaluations of different materials, and as one sensing source in the area of adaptive control.

There is no theoretical model for the prediction of cutting forces, and furthermore, there is no known method to accurately predict the magnitude of the force components without conducting experiments. This is not surprising when one realizes the variables which have an effect on the cutting force. They are
feed, depth of cut, cutting speed, combination of tool angles, condition of lubrication, work material and/or type of microstructure, and sharpness (or dullness) of the cutting edge.

Over the years many empirical models have been developed after many experiments have been conducted. These empirical models are of various forms and account for different phenomena such as tool wear, surface micro hardness, etc. Metal cutting, however, is an extremely complex process and cannot be rationalized by a single simple mechanism. While a single mechanism may be predominant over a given range of operating conditions, it may be inadequate for another. This explains the wide range of views that have appeared in the literature and the sound experimental data provided in their support. Some of these models will be briefly discussed below:

The most prominent model, and perhaps the simplest, was the one presented by Kronenberg [47] in which the cutting forces are given as functions of only the chip cross-sectional area and the specific cutting force:

\[ F = K_c \cdot A_n \]  \hspace{1cm} (2.3)

where \( K_c \) is the specific or unit cutting force (N/mm\(^2\)) which is related to the hardness of the material being cut, and \( A_n \) the chip cross-sectional area (mm\(^2\)). The coefficient \( K_c \) is usually treated as a constant for evaluating the average cutting force. The chip cross-sectional area is typically the product between the feed and the depth of cut.

Many variations of equation 2.3 have been developed and used to achieve a variety of objectives. Perhaps the most widely used variation takes the form:

\[ F = K_c f^\alpha d^\beta \]  \hspace{1cm} (2.4)

where \( K_c \) is a cutting constant, and \( \alpha \) and \( \beta \) are constants. In the design of adaptive control strategies these constants are typically set equal to 1 [53, 73] which generally does not cause any serious difference between predicted and experimental results.
Chapter 2  Modeling the Turning Operation

The force model which is employed in this investigation is one proposed by Zhang and Kapoor [82, 83]:

\[ F = K_c \sigma d \left( \frac{h}{h_i} \right)^m \]  (2.5)

This model incorporates the effects of the workpiece micro-hardness by introducing the factor \( \left( \frac{h}{h} \right)^m \) where, \( h_i \) is the instantaneous surface micro-hardness, \( h \) is the mean micro-hardness, and \( m \) is the Meyer\(^1\) exponent.

This model is of particular interest since the surface micro-hardness plays a vital role in the resulting force profile, more specifically, force fluctuations. These force fluctuations in turn cause the system to vibrate and hence cause the surface finish to deteriorate.

Elbestawi et al [21] derived some rather comprehensive empirical relations for the three components of the resultant force acting on a single point tool in hard turning in the form:

\[ F_i = c^{k_{11}} f^{k_{12}} d^{k_{13}} V^{k_{14}} R^{k_{15}} \phi^{k_{16}} \alpha^{k_{17}} \psi c^{k_{18}} \]  (2.6)

where \( i = x, y, \) and \( z, k_{ij} \) are constants, \( e \) is the natural exponent, and \( R, \phi, \alpha, \psi, c \) are the tool nose radius, the angle of approach, the rake angle, and the chamfer respectively.

Perhaps the most comprehensive model has been put forth by Koren and Lenz [44]. It is an empirical model which considers the cutting parameters as well as both crater and flank wear of the cutting tool.

\[ F = [K_9 f^m (1 - K_{10} \alpha_i - K_{11} - K_{12} V)]d + K_{13} d W_f - K_{14} W_c \]  (2.7)

where, \( V, f, \) and \( d \) are the cutting parameters, \( K_i \) and \( n_i \) are constants, \( W_f \) and \( W_c \) are the flank and crater wear respectively and \( \alpha_i \) is the rake angle. Ulsoy and Park [60],

---

\(^1\) The Meyer exponent is an exponent related to the strain hardening of the material being cut.
obtained values for all the constants used above in their design of an on-line tool wear estimator.

2.5.1 Workpiece and the cutting force
The dimensions of the workpiece have a substantial effect on the cutting force that may be applied in practice. It may not be larger than the stability that the work permits in order to avoid out of roundness, bending, and other adverse effects. As a 'rule of thumb', the length of the workpiece should not exceed twelve times its diameter. In this investigation it is assumed that this condition is met.

2.6 Surface Finish
The integrity of the machined surface is frequently one of the important outputs of the turning operation. Surface integrity as a term involves surface finish, freedom from cracks, chemical change, thermal change, and adverse residual stress. The most important one, and the main concern of this study, is surface finish. Where surface finish is important a finishing operation is usually involved.

Surface finish is affected by the machining process. Regardless of the process (milling, turning or grinding) cutting tools leave a wake pattern of disrupted micro-structure on the surface of workpieces [47]. Even a finish process like grinding leaves its mark on the micro-structure of the workpiece. On moving parts, like shafts, surface finish is very important to the life and functioning of the component.

The finish produced by a tool is dependent on a number of conditions. There is general agreement that the cutting parameters feed, cutting speed, and depth of cut, as well as the tool nose radius influence the surface finish of a workpiece. Other studies indicate that tool wear [20, 68], material being machined [82, 83], and cutting tool material also have a profound effect on surface finish. Another factor
which influences the surface finish is the rigidity of the set-up. It is unlikely that a
good finish will be obtained if there are radial or end play in the machine spindle,
looseness in slides and gibbs, or if the workpiece flexed or vibrated under the cutting
forces [35]. Hasegawa, Seireg and Lindberg [30] derived an empirical relation
relating the first four parameters of the above list to the peak-to-valley roughness
viz.:

\[ R_{\text{max}} = k_o V^{m_1} f^{m_2} d^{m_3} r^{m_4} \]  
(2.8)

where, \( V \) is the cutting speed in fpm, \( f \) the feed rate in ipr, \( d \) the depth of cut in
inches \( r \) the tool tip radius in inches, and \( k_o, m_1, m_2, m_3, \) and \( m_4 \) are constants. Jang
and Seireg [33-35] later obtained another relation which accounts for the dynamics
of the cutting process (i.e. vibration) based on the dynamic model described above.
In this case the exponents \( m_5, m_6, m_7, \) and \( m_8 \) are functions of the systems natural
frequency:

\[ R_{\text{max}} = e^{c} V^{m_1} f^{m_2} d^{m_3} r^{m_4} \]  
(2.9)

where, \( c = 7.02 - 2.2(1 - \exp(-\omega/18.3)) \)

\[ m_5 = -0.3712 + 0.3712(1 - \exp(-\omega/21)) \]
\[ m_6 = 0.6302 + 1.3626(1 - \exp(-\omega/31.3)) \]
\[ m_7 = -0.5425 - 0.5425(1 - \exp(-\omega/10.3)) \]
\[ m_8 = -0.3419 - 0.6523(1 - \exp(-\omega/51.5)) \]

where \( \omega \) is the natural frequency of the tool assembly in Hz.

In their study they have also specified the range of applicability of their equation.
First of all the above relations are valid for structures which are rigid enough to be
practical (i.e. it has stiffness \( K_r \geq 10^8 \text{ N/m} \)). In addition, the above equations are
applicable for the following conditions:

- \( 61 \text{ m/min} < V < 305 \text{ m/min} \)
- \( 0.127 \text{ mm/rev} < f < 0.88 \text{ mm/rev} \)
- \( 0.31 \text{ mm} < d < 0.71 \text{ mm} \)
0.79 < r < 2.38 mm

Du, et al. [5] took a different approach in their investigation of surface finish. The model they proposed accounts for tool wear:

\[ R_{ma} = e^c V^{k5} f^{k2} d^{k3} (|w_f - k_5| + k_5)^{k_6} \]  

(2.10)

where, \( R_{ma} \) is the surface finish in RMS, \( w_f \) the flank wear, and, \( c, k_1, k_2, k_3, k_4 \) and \( k_5 \) are constants.

The model proposed by Du et al [5] for surface finish is more representative of the turning process as considered in the current investigation due to the fact that it incorporates tool wear which causes the cutting force to fluctuate which in turn causes the tool-workpiece relative displacement to fluctuate by exciting the system (forced vibration) under investigation.

The above list is far from complete. It was merely an introduction to the type of research which still goes on.

In the present work no such relation is utilized, nor is a new one presented. The tool workpiece relative displacement is obtained and then coupled with the cutting tool geometry to generate a complete surface topography. Roughness values can be generated if needed by simply employing well established mathematical/statistical relations along with a section of the surface topography. For instance, if the centre line average, \( CLA \), or \( R_a \), of the surface roughness (the average value of the departures both above and below its centre line) was to be calculated, this could be done by merely employing the equation presented below along with a section of the surface topography to obtain the necessary information.

\[ CLA = \frac{h_1 + h_2 + h_3 + \ldots + h_n}{L} = \frac{1}{L} \int_L^0 h_a dL \]  

(2.11)

where, \( h \) is the height of the profile above or below the centre line at points at unit distances apart (\( L \) units = sampling length). The centre line is a line drawn such that the sum of the areas embraced by the surface profile above the line is equal to the sum of those below the line (see Figures 2.3(a) and 2.3(b)).
Areas $A + C + E + G + I = Areas \ B + D + F + H + J + K \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2.12)$

Figure 2.5(a): Definition of centre line

Figure 2.5(b): Definition of CLA

In order to verify the results of this investigation, however, it is sufficient to compare tool workpiece relative displacements and this is what is actually done. The $R_s$ index is also calculated so as to establish grounds for comparison with published literature [for example, 83].

At this point it is also appropriate to recognize that in a turning operation having no disturbances (e.g., tool-workpiece relative displacements) the macro roughness is totally determined by the tool nose geometry and the feed rate.

Model Summary

The dynamic model for the tool-workpiece relative displacement in the radial direction is given by:

$M_s \ddot{x}_s + (B_s + B_w) \dot{x}_s + (K_s + K_w)x_s + B_w \dot{x}_w + K_w x_w = 0$

$M_w \ddot{x}_w + B_w \dot{x}_w + K_w x_w - B_s \dot{x}_s - K_s x_s = F_x \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2.1)$

$M_s \ddot{\hat{x}} + B_s \dot{\hat{x}} + K_s \hat{x} = F_c - F_x$

where, the cutting force in the same direction is given by:

$F_x = K_c f d \left( \frac{h}{h} \right)^m$ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2.5)$
Chapter 2  Modeling the Turning Operation

Typical values for the machine parameters are shown in Table 2.1 below. These values were taken from published literature [33-35] so as to make the simulations as realistic as possible. They are suitable when the following units are used: \( M_t \) [Kg], \( K_{aw} \) [N/m], \( B_{aw} \) [N*s/m] and \( K_c \) [N/m^2].

Table 2.1 - Parameter values used in the model evaluation

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<th>( B_t )</th>
<th>( K_t )</th>
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<td>27</td>
<td>128.5</td>
<td>1.2322E6</td>
<td>3.4782E9</td>
<td>0.454</td>
<td></td>
</tr>
</tbody>
</table>

2.7 Methods of Solution

Three different control strategies have been designed: proportional plus integral plus derivative (PID) control, which is a classical control approach, and linear optimal control based on linear quadratic (LQ) and linear quadratic gaussian (LQG) methods, and adaptive control (AC) which belong to what is often referred to as modern control theory.

First of all a PID controller was designed in order to establish ground for comparison. The fact that more than five decades after its adoption, the classical PID controller remains the most popular and most widely used theoretical as well as industrial controller and the fact that its general properties with regard to effectiveness, simplicity, and conditions of applicability are well acknowledged make the PID controller a cherished choice of control.

Regardless of these positive traits, however, the tuning of the gains of a PID controller still often appears to involve excessive costs, long start-up times, and less than optimal operating conditions.
Chapter 2  Modeling the Turning Operation

A linear optimal control strategy was also developed because the design criteria which were to be met are of an optimization nature i.e. to minimize the tool-workpiece relative displacement. Furthermore, the system which results from an optimal design will not only be stable, have a certain bandwidth, or satisfy any of the desirable constraints associated with classical control, but it is also supposed to be the best possible system.

The drawback with this method of control is the fact that the turning process which is to be controlled is assumed to be linear and the device which generates the control, is constrained to be linear. In addition, the linear constant coefficient approximation which results from the first order Taylor expansion of the available empirical models may be unsatisfactory since the operating conditions will be changing.

Finally, a parameter adaptive control strategy was developed. In this case the parameter vector i.e. the vector which incarnates the coefficients of the transfer function relating the tool workpiece relative displacement to the surface micro-hardness (and ultimately the cutting force) was updated at every sampling instant in order to accurately depict the cutting process. As a result the system was accurately represented at all sampling instants. On the other hand, this method of control was more involved due to the fact that adaptive controllers are inherently non-linear. Furthermore, the model of the cutting process is also non-linear which further complicates matters.

Adaptive control is definitely an excellent choice of control nonetheless because it compensates for any inaccuracies that may be present in the proposed empirical models which may lend themselves to scepticism and the fact that the turning process is a lot more complicated than the way it is generally modelled.

In the design of all three types of controllers observers (estimators) were also designed and the effect of including observers was investigated. Observers were
used so as to determine the states since as in most cases no on-line feedback of the states will be available.

In the case of adaptive control non-linear observers may also be used for estimating the surface roughness and cutting tool flank wear as in the work done by Ulsoy and Park [60]. In the case of linear optimal control the observers will of course have to be linear. The observers designed in the present study were Kaman filters with input and output disturbances.

The cutting force signal would be used to compute the relative motion between the workpiece and the cutting tool. These relative displacements coupled with the tool geometry will then be used to generate the surface topography for the machined surface in order to generate results which are as close to reality as possible. Sections of the surface topographies (as would be generated by a stylus trace for instance) as well as full 3D surface topographies before and after control actions are introduced are generated and compared.

2.8 Simulation of the Workpiece Micro-hardness and Radial Cutting Force

In order to make the simulations more realistic the workpiece surface will be "created" by using the documented hardness for a given workpiece material (the mean) as well as the covariance in order to randomly generate a surface (for a given sampling time) whose micro-hardness follows a normal distribution as demonstrated by Zhang and Kapoor [82, 83]. The covariance will vary with cutting conditions which reflects the proposed direction of the work. The equation predicting the covariance for a set of given conditions is:

$$\sigma_s^2 = 4\pi \sigma_x^2 \frac{n_t}{f_{dV}} \int \rho(r) W(r) r^2 dr$$  \hspace{1cm} (2.13)

where, $\sigma_x^2$ is the population mean variance, $\sigma_s^2$ is the sample mean variance, $\rho(r)$ is the correlation coefficient function, $W(r)$ is the geometric sample shape function, $r$ is the radius of the workpiece, and $n_t$ is the number of samples.
A typical surface micro-hardness distribution for a given workpiece having mean surface hardness of 120 BHN is shown in Figure 2.5. Actually, both a cross section and the entire surface micro-hardness distribution, over the duration of the cut, are shown.

![Hardness Variation (BHN)](image)

![material hardness (BHN)](image)

**Figure 2.6:** (a) Surface micro-hardness distribution (b) 3D surface micro-hardness distribution

Please note that j-th location corresponds to the location along the workpiece circumference which is sampled. This will be independent of the workpiece diameter since it only changes with the number of samples. The i-th location
corresponds to the cutting tool feed location. Again this only changes with feed. Specific units were purposely omitted so as to allow the simulation programs to handle any situation.

The predicted variations in the micro-hardness will be used to correct the expected cutting force signal. The cutting force signal would in turn be used to compute the relative motion between the workpiece and the cutting tool. These relative displacements coupled with the tool geometry will then be used to generate the surface topography for the machined surface in order to generate results which are as close to reality as possible. A typical force fluctuation profile and cutting force profile are shown in Figure 2.6.

Cross sections of the surface, as well as, three dimensional surface topographies, before and after control actions are introduced, are generated from the output signal (tool-workpiece relative displacement) coupled with the cutting geometry and tool nose radius, and compared. The considerations that have been dealt with in the design of the controllers are enumerated and defined in the following section.

2.9 Evaluation of the Controllers Designed
The effectiveness of each of the controllers designed is evaluated via simulation. The input vector(s) is/are generated so as to resonate the effects of micro-hardness variations and used to examine the output. In addition, stability, a basic requirement in any control system and the concepts of controllability, and Observability, (as defined in the following section) are also embracing in the design and evaluation of each controller design. The final controller, design, as mentioned above, is then employed to generate the system output subject to the resulting control law. The results are finally coupled with the cutting geometry and tool nose radius to generate surface cross sections as well as three dimensional surface topographies.
2.9.1 Stability

The design of linear control systems may be treated as a problem of arranging the closed loop poles and zeros in a way such that the corresponding system performs according to the prescribed requirements. Among the many forms of performance specifications in the design of control systems, the most significant requirement is that the system must be stable.

The following are some ways of determining the stability of a system:

- Direct computation of the eigenvalues of $F$
Chapter 2  Modeling the Tuning Operation

- Methods based on properties of the characteristic polynomial: Routh-Hurwitz criterion, calculation of the characteristic polynomial and investigating the characteristic equation.
- The root locus method.
- The Nyquist criterion.
- Lyapunov's method.

It follows from the theorem on asymptotic stability of linear systems [please see any of 3, 7, 36, 48, 49, 58, 78] that a straightforward way to test for the stability of a given system is to calculate the eigenvalues of the matrix $F$.

2.9.2 Controllability and observability

The concepts of controllability and Observability were introduced by Kalman. They play an important role in the design of control systems in state space. In fact, the conditions of controllability and Observability may govern the existence of a complete solution to the control system design problem. The solution to such a problem may be non-existent if the system considered is not controllable.

2.9.2.1 State controllability and output controllability

When designing a process, it is important to ensure that all important process variables can be changed conveniently. Controllability, is the word often used in this context. The system described by equations 2.2 is said to be state controllable at time $t_0$ if it is possible by means of an unconstrained control vector to transfer the system from any initial state $x(t_0)$ to any other state in a finite time interval. Controllability of a system in state space form can be tested by forming the so call controllability matrix

$$C_o = \begin{bmatrix} G & FG & \ldots & F^{s-1}G \end{bmatrix}$$  

(2.14)
Chapter 2  Modeling the Turning Operation

The system is said to be state controllable if the above matrix (r is the dimension of u and n is the order of the system) matrix has rank n, or contains n linearly independent column vectors. For a rigorous proof of the above statement the reader is referred to (Anderson and Moore [7]). The reader is also referred to Ogata [58], and Kailath[36] for different points of view.

In the practical design of a control system the designer may want to control the output rather than the states of the system. Complete state controllability is neither necessary nor sufficient for controlling the output of the system. For this reason it is desirable to define separately complete output controllability. Once again, consider the system defined by equations 2.2 which is said to be completely output controllable if it is possible to construct an unconstrained control vector u(t) that will transfer any given initial output y(t₀) to any final y(f) in a finite time interval t₀ ≤ t ≤ t. It can be proven that the condition for complete output controllability is as follows: The system described by:

\[ \dot{x} = Fx + Gu \]

\[ y = H^T x + Du \]

is completely output controllable if and only if the \( mx(n+1)r \) matrix

\[ C_o = \begin{bmatrix} HF & HFG & \ldots & HF^{n-1}G & D \end{bmatrix} \] (2.15)

is of rank m (m is the dimension of the output vector y). Note that the presence of the \( Du \) term always helps to establish output controllability.

It frequently happens that stability and controllability have antagonistic requirements. Traditionally, stability has been accentuated. It is however interesting to see that if a control system is used the basic system can instead be designed for controllability. The required stability can then be provided by the control system.
2.9.3 Observability

A system is said to be observable if every state \( x(t_o) \) can be determined from the observation of \( y(t) \) over a finite time interval, \( t_o \leq t \leq t_f \). The concept of Observability is useful in solving the problem of reconstructing state variables which are not accessible for direct measurement. State reconstruction is of course necessary in order to construct the control signals.

If the system is completely observable then given the output \( y(t) \) over a time interval \( t_o \leq t \leq t_f \), \( x(t) \) is uniquely determined. This requires that the rank of the \( m \times n \) matrix

\[
O_o = \begin{bmatrix}
H & HF & HF^2 & \ldots & HF^{n-1}
\end{bmatrix}
\]

is \( n \).

The properties of controllability and Observability will be exploited mostly in the design of the optimal controller based on LQ/LQG methods in chapter 4.
Chapter 3 Properties and Application of PID Control

3.1 Introduction
In this chapter the properties of classical control methods, particularly those of proportional-integral-derivative (PID) control, are highlighted. A PID controller is then designed and its effectiveness when employed for controlling the tool-workpiece relative displacement is evaluated through computer simulation.

3.2 PID Control
The PID controller is the most commonly used standard form of dynamic compensation in industrial instrumentation. Electronic, pneumatic, and digital PID controllers are available off the shelf in a wide variety of makes and types. Its basic proportional, integral, and derivative actions are also fundamental to other dynamic compensators, such as the phase-lag and phase-lead compensators. In addition, process engineers and operating personnel are familiar with it and most industrial single loop processes are robustly and stably controlled by it.

Conceptually the PID controller is a linear parameter-optimized controller with fixed structure given by the following transfer function:

\[ G_c(s) = K_p \left(1 + \frac{T_i}{s} + T_ds\right) \]  

(3.1)

where, \(K_p\) is the proportional gain, \(T_i\) is the integral time, and \(T_d\) is the derivative time. Constants \(K_p\), \(T_i\), and \(T_d\) are the controller parameters. The block diagram for the basic configuration of a PID controlled system is shown in Figure 3.1.
If \( e(t) \) is the input to the PID controller, the output \( u(t) \) from the controller is given by

\[
u(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_{-\infty}^{t} e(t)\,dt + T_d \frac{de(t)}{dt} \right]
\]  

(3.2)

Equation 3.1 can also be written as

\[
G_c(s) = K_p + \frac{K_i}{s} + K_ds
\]  

(3.3)

In this case, \( K_p \), \( K_i \), and \( K_d \) become controller parameters. Equivalently, the discrete transfer function (using the \( z \) operator) for a PID controller is given by equation 3.4.

\[
G_c(z) = \frac{q_o + q_1z^{-1} + q_2z^{-2}}{1 - z^{-1}}
\]  

(3.4)

for small sampling times \( q_o \), \( q_1 \), and \( q_2 \) can be obtained by discretization of the continuous PID controller.

### 3.2.1 PID Control of Systems

If a mathematical model of the system can be derived, then it is possible to apply various design techniques for determining the parameters of the controller that will meet the transient and steady state specifications of the closed-loop system. However, if the system is so complicated that its mathematical model cannot be easily obtained, then analytical approach to the design of a PID controller is not possible.
The designer must then resort to experimental approaches to the design of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Zeigler and Nichols suggested rules for tuning PID controllers (i.e., to set values of $K_p$, $K_i$, and $K_d$) based on experimental step responses or based on the value of $K_p$, that results in marginal stability with only the proportional control action used. Zeigler-Nichol rules (see [58]) are very convenient when mathematical models of systems are not known - these rules can of course also be applied to the design of systems with known mathematical models.

### 3.2.2 Tuning Rules for PID Controllers

More than five decades after its adoption, the classical PID controller remains the most popular and most widely used theoretical as well as industrial controller. Its general properties with regard to effectiveness, simplicity, and conditions of applicability are well acknowledged. Subsequent to extensive industrial experience, several attractive configurations have been proposed and many tuning methods have been proposed. Many methods rely on dominant poles, frequency domain, and pole placement techniques [17, 48].

Despite these positive traits, however, the tuning of the gains of a PID controller still often appears to involve excessive costs, long start-up times, and less than optimal operating conditions. These reasons continue to inspire research intended at refining and broadening the available PID state of knowledge [19, 26, 28].

Recent techniques have been proposed for the improvement of the tuning methods of PID controllers These include: formalizing experimental knowledge into expert rules [5], developing more effective quadratic performance minimization algorithms, and further improving the Ziegler-Nichols and Cohen-Coon methods [26, 28].
Chapter 3  Theory and Application of PID Control

PID controllers, as stated above, are very frequently used in industrial control systems. While there are neither industrial nor scientific standards for PID controller configurations, the action of the classical PID controller is usually described by equation 3.6 (Aside: Equation 3.6 appears different from equation 3.2 because in the work with analog controllers it was found advantageous to not let the derivative act on the command signal, and suitable to let only a fraction of the command signal, $\beta$, act on the proportional part):

$$U(s) = K_p \left\{ \beta P_s(s) - sT_d P_v(s) + \frac{P_v(s) - P_r(s)}{sT_i} \right\}$$  \hfill (3.6)

where, $U(s)$, $P_s(s)$, $P_v(s)$, denote the Laplace transforms of control, setpoint, and process variable respectively. Adjustable gains $K_p$, $\beta$, $T_i$, and $T_d$ are the feedback loop gain, the set-point weighting factor, the integral and derivative time. A typical tuning procedure is described below:

1. The feedback system is submitted to a set point variation and the gains $K_p$, $\beta$, and $T_d$ are modified to improve the input/output response.
2. The feedback system is subjected to an external perturbation and $T_i$ is modified to reduce the sensitivity of the process variable.
3. Steps 1 and 2 are iterated until a satisfactory overall input/output and sensitivity behaviour is obtained.

The above procedure is carried out while considering $T_d$ improves that transient response, $T_i$ improves sensitivity to perturbations, $\beta$ is used to adjust the DC gain, and $K_p$ is used for fine adjustment of both input/output and sensitivity response.

The implementation of these steps is greatly simplified by the application of the methods mentioned in section 3.2.1, i.e. controller tuning. However, several iterations are usually required before a satisfactory setting is attained. One of the difficulties is the unpredictable influence of unmodelled system behaviour such as saturation, coulomb friction, and higher order dynamics.
3.3 Controller Evaluation

The PID controller designed has been evaluated from the following perspectives: overall performance (verified via computer simulation), stability, relative stability (phase and gain margins), and robustness analysis, and noise rejection ability that the closed loop system possesses.

3.3.1 Results of the computer simulations

First of all, the open loop (no control) response of the system was obtained via computer simulation. The open loop system response, that is, the actual depth of cut and the tool-workpiece relative displacement are shown in Figure 3.2

![Figure 3.2 - (a) Actual depth of cut and (b) tool-workpiece relative displacement open loop response](image)

Figure 3.2 - (a) Actual depth of cut and (b) tool-workpiece relative displacement open loop response
For the sake of better understanding the results that have been obtained a tool profile over a 7mm distance, and a three dimensional surface topography have been generated. Please note that in the generation of these plots, the tool nose radius and the tool lead angle have also been incorporated. In addition, the tool profile has been 'normalized' in the sense that, what is shown is what one would obtain from a stylus measurement using a roughness measuring machine such as Taylor Hobson Talysurf. These plots are shown in Figure 3.3.

Figure 3.3 - (a) Section of the surface profile and (b) 3D surface topography of the uncompensated system-open loop
Please note that no specific units were assigned to the feed and cutting speed axes in the simulations. This was done purposely so as to allow the simulation programs to handle various feed rates, various tool nose diameters, and various workpiece diameters. The feed axis corresponds to thirty feed advances (regardless of the feed rate) with twenty points defining the tool nose radius. Twenty points were sampled along the workpiece circumference every revolution (regardless of the workpiece diameter).

The open loop poles (i.e., the roots of the characteristic polynomial) were also calculated so as to shed some light on how we should expect our system to behave. The open loop poles are located at:

-622+10777i, -622-10777i, -32+5334i, -32-5334i, -2+279i, -2-279i. Even though our system is open loop stable we expect its performance to be rather poor, which is also obvious from Figure 3.2, since the last two complex conjugate pairs are sufficiently close to the imaginary axis of the s-plane indicating that they cause the system to be lightly damped. A quick look at the power spectral density plot corresponding to the open loop system, shown in Figure 3.4, identifies the problem immediately. The resonances near 279 rad/s, and 5344 rad/s quickly identify the effect of the poles at -2±279i, and -32±5344i. The most profound effect on the power spectral density is of course due to the dominant pair of poles at -2±279i.

This fact promotes the design of the PID controller using the pole-zero cancellation technique. Other approaches as gain selection, and the selection of the proportional gain with pole placement were also investigated with results only nearly as good as pole-zero cancellation. The PID controller that results from applying the above pole-zero cancellation technique is given by equation 3.7 which is the transfer function of the controller.

\[ G_c(s) = 4.75946 + \frac{77719.5}{s} + s \]  (3.7)
The poles of the closed loop (compensated system) are located at: 
-143130, -650.5-10682i, -650.5+10682i, -76.18-2553.2i, -76.18+2553.2i.

At this point a practical difficulty which is associated with the pole-zero cancellation design deserves a mention to prevent the method from being used indiscriminately. The problem is that in practice exact cancellation of poles and zeros of transfer functions is seldom possible. This is because the process transfer function is usually determined through testing and physical modeling, and the fact that linearization of a nonlinear process and approximations of a complex process are inevitable. As a result the true poles and zeros of a process may not
be precisely modeled. In fact, even the order of the system may be higher than that of the system representation. Another difficulty is that the dynamic properties of the process may modulate due to the aging of the system components or due to alterations in the operating environment in such a way so as to cause the poles and zeros to move during operation. Due to these and other reasons, exact pole-zero cancellation is seldom possible in practical situations. In most cases, however, exact pole-zero cancellation is not really necessary to effectively improve the performance of the system (please see [48] for the proof). This is one of those cases. It was verified by selecting the controller gains so as to reflect poles and zeros very close to those that we wished to cancel. No significant change to the system performance was observed. The closed loop (compensated) system, with PID control, yields the results (computer simulation) shown in Figure 3.5. The tool-workpiece relative displacement has been reduced by nearly fifteen times as compared to Figure 3.2.
Figure 3.5 - (a) Actual depth of cut and (b) tool workpiece relative displacement after PID controller implementation-closed loop performance

For the sake of better understanding the results that have been obtained a tool profile over a 7mm distance and a three-dimensional surface topography have been generated. Once again, in the generation of these plots, the tool nose radius and the tool lead angle have also been incorporated. In addition, the tool profile has been 'normalized' in the sense that what is shown is what one would obtain from a stylus measurement using a roughness measuring machine such as a Taylor Hobson Talysurf. These plots are shown in Figure 3.6.
Figure 3.6 -- (a) Section of the surface profile and (b) 3D surface topography of the compensated system-closed loop.

The power spectral density plot corresponding to the closed loop (compensated) system is shown in Figure 3.7 where the effects of the dominant poles have diminished greatly. This outcome leads us to conclude that the poor open loop response is definitely associated (as expected) with the dominant complex conjugate pole pair identified earlier.
Figure 3.7 - (a) Power spectral density plot of closed loop system (closed loop performance, and (b) open and closed loop performance comparison.

This design is in a sense analogous to frequency shaped designs in optimal control, a technique that was exploited in the design of the optimal controller. More detail will be provided in chapter 4, where the optimal controller based on LQ methods is evaluated.

3.3.2 Stability, Relative Stability (Phase and Gain Margins), Robustness

The stability of the closed loop system was determined by both direct computation of the closed loop poles and construction of the Nyquist plot, shown in Figure 3.8.
Suffice to say that the closed loop system is quite stable.

The next question that needs to be answered is how stable is it? This question of relative stability can be answered by looking at the phase and gain margins. There are of course a number of ways to obtain these values (e.g., from the Nyquist plot, from the Bode plot, etc.). In this case the Bode plot was constructed and it is shown in Figure 3.9.
The phase margin $\phi_m$ is the distance of the phase angle curve above $-180^\circ$ at the crossover frequency $\omega_c$, where the magnitude plot crosses the 0dB axis. This is readily available from the Bode plot and it is $170^\circ$. The gain margin, on the other hand, is the distance of the magnitude below 0dB at the frequency where the phase margin $\phi_m=-180^\circ$. This value is also readily obtainable from the Bode plot and it is infinite.

![Bode plot](image)

**Figure 3.9: Bode magnitude and phase plots**

Of course, no real system has infinite gain margin. Such parasitic effects as stray capacitance, time delay, and the like will always prevent infinite gain
margin from being a physical reality. Some mathematical models of systems may, however, have infinite gain margins. Pending on accurate modeling, it can be concluded that these systems do have very high gain margins. This, in fact, seems to be the case with our system in conjunction with the fact that the parasitic effects have not been considered.

Finally, another piece of useful information that can be obtained from the Bode magnitude plot is the bandwidth, \( \omega_b \), a measure of the speed of response, of the system. The bandwidth of our system is approximately 1590 Hz.
Chapter 4  Properties and Application of Linear Quadratic Optimal Control

4.1 Introduction
In this chapter the properties of linear optimal control methods are discussed and LQ designs are implemented so as to minimize the tool-workpiece relative displacement in a finish turning operation. As a first step a standard LQ controller is designed. A LQ controller is then designed based on pole-placement methods. Furthermore, a LQG design is investigated, where a Kalman-Bucy filter observer is incorporated in the design of the control system. Finally, frequency shaping and frequency weighted balanced truncation (to reduce the order of the controller/observer) are considered.

4.2 Linear Optimal Control
The primary aim of a designer using classical design methods is to stabilize the system, while secondary aims may entail obtaining a certain transient response, bandwidth, disturbance rejection steady state error, and robustness to system variations or uncertainties. The designer's methods are a combination of analytical (e.g., Routh test), graphical (e.g., Nyquist plots), and empirically based knowledge.

Two of the main objectives of modern control, as opposed to classical control are to formalize control system design and to present solutions to a much wider class of control problems.
Chapter 4  Properties and Application of Linear Optimal Control

Optimal control is one particular branch of modern control that sets out to provide analytical designs which are of a very appealing type for the following reasons: The system which results from an optimal design will not only be stable, have a certain bandwidth, or satisfy any of the desirable constraints associated with classical control, but it is also supposed to be the best possible system.

Linear optimal control is a particular type of optimal control. The system which is to be controlled is assumed to be linear and the device which generates the control, is constrained to be linear. Linear controllers are consummated by employing quadratic performance indices. Methods which achieve linear optimal control are termed linear quadratic (LQ) methods.

4.3 Formulation of the LQ Problem
Consider a stabilizable system described by:

\[
\begin{align*}
    \dot{x} &= Fx + Gu \\
    y &= H^T x
\end{align*}
\]

with \( x(t_o) \) given, and entries \( F, G \) assumed continuous and design parameters \( Q \) (positive semi-definite) and \( R \) (positive definite) matrices with continuous entries. Define the performance index

\[
V = \int_{t_o}^{t} (u^T R u + x^T Q x^T) dt
\]

and the minimization problem as the task of finding an optimal control \( u^*, t \geq t_o \), minimizing \( V \) and the associated optimal performance index \( V^* (x(t_o), t_o) \).

For the case of state estimate feedback using LQG designs where the system also includes input and/or output additive noise which is white, gaussian with zero mean and the performance index described by equation 4.2 would actually be random, equation 4.3 is used where the expectation is over \( x(t_o) \).
\[ V = E\left\{ \int_{t_0}^{t} (u^T Ru + x Q x^T) dt \right\} \quad (4.3) \]

The optimal control law is given by \( u = K^T x \), where \( K = -R^{-1} G P \), with \( P \) obtained from the solution of the matrix Riccati equation

\[ \dot{P} = PF + F^T P - PG R^{-1} G^T P + Q \quad (4.4) \]

The following section is dedicated to the selection of the design parameters \( Q \) and \( R \).

4.3.1 Quadratic Weight Selection

It is very enlightening to study situations where the ratio of state to control weighting is very large or small so these two situations will be the focus of the following discussion.

4.3.1 Low State Weighting

For a problem parametrized by \( F, G, \rho Q, \) and \( R \) with \( \text{Re} \lambda_i < 0 \), when \( \rho \to 0 \) the closed-loop eigenvalues approach the stable open-loop eigenvalues and the reflections through the imaginary axis of the unstable eigenvalues (for the proof see [7]).

4.3.2 High State Weighting

High state weighting provides a means for placing closed-loop poles [29, 69]. First of all, the designer can control up to \((n-1)\) poles of the closed loop system, by choosing \( Q = \alpha d^T \) (for some vector \( d \)) with the zeros of \( d^T (sI - F)^{-1} G \) coinciding with these \((n-1)\) poles - \( I \) is the identity matrix. Then the designer can choose \( \rho \) (a variable which is used to multiply the matrix \( Q \)) to move the remaining pole towards infinity. The smaller \( \rho \) is (lower state weighting) the less accurate will be the pole placement. Another approach could be to set \( \rho \) by invoking a specification regarding the cross-over frequency [see 29].
\[ \omega_c = \sqrt{\rho|d^*G|} \]  

(4.5)

In this particular design, the pole placement method has been used and \( Q \) and \( R \) have been taken to be diagonal since the choice of \( Q, R \) entries in such a case is especially more readily reflective of physical insights.

4.4 Pole Placement

It is possible to define a modified regulator problem which achieves a closed loop system with a prescribed degree of stability \( \alpha \) [55]. That is for some prescribed \( \alpha > 0 \), the states \( x(t) \) approach zero as fast as \( e^{-\alpha t} \) in the continuous time case. It should be noted that a high degree of closed-loop stability may only be achieved at excessive control energy cost or controller complexity cost. As a result, the selection of \( \alpha \) must be a considered one.

The strategy adopted in solving this modified problem is to introduce transformations that will shift the closed-loop eigenvalues to the left (the s-plane is considered). The modified problem becomes:

\[ \dot{x} = (F + \alpha I)x + Gu \]  

(4.6)

The optimal gain vector \( K \), is calculated by solving equation 4.4-matrix Riccati equation with \( F \) replaced by \( F + \alpha I \).

4.5 Kalman-Bucy Filter Observer (LQG) Design

The implementation of the optimal control laws depends on the states of the controlled system being available for measurement. In practical situations, however, this is not often the case. The way to get around this problem is by designing observers (also called estimators) to estimate the unavailable states. A state observer must possess the following two properties:

1. It should be of the form of Figure 4.1, with inputs consisting of the system input and output, \( x_r(t) \) at time \( t \), an on-line estimate of \( x(t) \).
2. It should function in the presence of noise. Preferably it should be possible to optimize the action of the estimator in a noisy environment.

![Diagram of estimator structure](image)

**Figure 4.1: Desired estimator structure**

### 4.5.1 Full Order Kalman-Bucy Filter Observer Design

The particular material mentioned in this section is presented and discussed in the momentous paper by Kalman and Bucy [38] of which only some highlights are revisited.

We consider the design of observers for systems, like ours (equation 2.2 restated here), which are of the form:

\[
\begin{align*}
\dot{x} &= Fx + Gu + G_\omega w \\
y &= H'x + \nu
\end{align*}
\]  

(4.7)

with initial time \( t_0 \), finite, and \( \nu(.) \), \( w(.) \), \( x(t_o) \) independent and gaussian with

\[
\begin{align*}
E[\nu \nu^T] &= Q \\
E[\nu] &= 0 \\
E[ww^T] &= R \\
E[w] &= 0 \\
E \{ [x(t_o) - m][x(t_o) - m]\} &= P_{x_0} \\
E[x(t_o)] &= m
\end{align*}
\]

The observer equation is given by equation 4.8 which defines the optimal estimate \( x_\ast(t) \) of \( x(t) \):

\[
\dot{x}_\ast = Fx_\ast + Gu + K_\ast H' [x_\ast - x]
\]  

(4.8)

where,

\[
K_\ast = -P_\ast HR^{-1}
\]  

(4.9)
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\( P_\varepsilon \) is the solution to the matrix Riccati equation:

\[
-\dot{P}_\varepsilon = P_\varepsilon F + F^T P_\varepsilon - P_\varepsilon G R^{-1} G^T P_\varepsilon + Q \tag{4.10}
\]

As a result an on-line unbiased estimate \( x_\varepsilon(t) \) of \( x(t) \) is provided at time \( t \) by the arrangement shown in Figure 4.2 with equation 4.8 where the filter gain is given by equation 4.9.

![Diagram](image)

**Figure 4.2:** Full system observer structure

Equation 4.8 has the property that if \( x \) and \( x_\varepsilon \) are the same at some point \( t_0 \), then they will be the same for all \( t \geq t_0 \), since the third term on the right hand side will be zero for all \( t \).

The error equation, which can be obtained from equations 4.7 and 4.8 then becomes

\[
\frac{d}{dt}(x - x_\varepsilon) = (F + K_\varepsilon H^T)(x - x_\varepsilon) \tag{4.11}
\]

If the eigenvalues of \((F + K_\varepsilon H^T)\) have negative real parts, \( x - x_\varepsilon \) approaches zero at a certain exponential rate, and \( x_\varepsilon(t) \) will effectively track \( x(t) \) after a time interval...
determined by the eigenvalues of $F + K_s H^T$. The eigenvalues of $F + K_s H^T$ will have all negative real parts when the pair $[F, H]$ is completely detectable; see [7].

4.6 System Design Using a Kalman-Bucy Filter Observer (LQG design)

Section 4.4 dealt with the design of an optimal controller whereas section 4.5 dealt with the design of a Kalman-Bucy filter observer. The next step, which naturally follows, is to obtain a control system which is based on an LQG design (i.e., formulate a controller/observer augmented system). The equations which describe such a system, as shown in Figure 4.3 are:

$$\frac{d}{dt} \begin{bmatrix} x \\ x - x_e \end{bmatrix} = \begin{bmatrix} F + GK^T & -GK^T \\ 0 & F + KeH^T \end{bmatrix} \begin{bmatrix} x \\ x - x_e \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u_{ext}$$

$$y = \begin{bmatrix} H & 0 \end{bmatrix} \begin{bmatrix} x \\ x - x_e \end{bmatrix}$$

Figure 4.3: Augmented system representation

Clearly, with $x_e = x$ the control used is precisely the same as that obtained with true state feedback.

4.7 Frequency-Shaped Design

The standard linear quadratic based designs can be further improved by examining the frequency response of a given system and if necessary improve
the frequency characteristics of the system. Such designs are termed *frequency shaped designs*. Basic results for these designs can be found in [65, 66].

In frequency shaped designs, the system is augmented with frequency shaped filters so as to penalize their outputs in addition to other cost terms in the performance index. For instance if the performance response spectrum shows too much control (or output) energy in a certain frequency band, then the system can be augmented with a filter which only has a response in the same frequency band. Penalizing the output of such a filter in the performance index associated with the augmented system may improve the performance response, or at least permit trade-offs. It is obvious then that in frequency shaped designs, power spectral density plots in performance index selection are a essential ingredient to achieve a good LQG design. They can be used to quickly identify the frequencies which need to be further penalized. In addition, the Bode plots can also be used to obtain such information. The frequencies that need to be further penalized expose themselves as spikes in these plots. This will become apparent below as these methods are identified and used in our design.

In such a design approach, the augmented system then becomes:

\[
\begin{bmatrix}
\dot{x} \\
\dot{x}'
\end{bmatrix} = \begin{bmatrix}
F & 0 \\
G'H^T & F'
\end{bmatrix}\begin{bmatrix}
x \\
x'
\end{bmatrix} + \begin{bmatrix}
G' \\
0
\end{bmatrix}u
\]

(4.13)

where the original system is described as in equation (4.1), and the filter is described by \( \dot{x}' = F'x' + G'u' \), \( y' = H'r'x' \). Of course, \( u' = y_a \) and \( H'r'(sI - F')G' = W'(s) \). Without going into further details (the reader is referred to [7] suffice to say, the improvement is gained at the cost of controller complexity. In this particular case the order of the controller increased from six to eight.

The controller which results from such a design, is a frequency dependent one and takes the form:
\[ u(s) = K^T(s)x(s), \quad K^T(s) = K^T + K_f^T(sI - F^f)G^fH^T \] \hspace{1cm} (4.14)

A performance index of the form, \( V = E[x^TQx + u^TRu + x^TQ'u'x'] \) can be considered.

For our particular design, based on the information obtained from the power spectral density and bode plots, the frequency weighting transfer function \( W(s) \) was chosen to be a frequency weighted mountain. The corresponding transfer function takes the form:

\[ W(s) = \frac{s^2 + 2\zeta_1\omega_n s + \omega_n^2}{s^2 + 2\zeta_2\omega_n s + \omega_n^2} \] \hspace{1cm} (4.15)

with \( \zeta_1 > \zeta_2 \) and \( \omega_n \) is the resonant frequency of the system.

### 4.8 Controller/Observer Reduction

The techniques employed so far lead to a controller/observer with order equal to that of the system, i.e., sixth order. In this section, the existence of a simpler controller/observer (i.e., lower order) with performance characteristics nearly as good as those of the full order controller/observer will be investigated. Low order controllers are normally favoured over high order controllers, provided their performance is comparable because this way there are fewer things to go wrong in the hardware, or bugs to fix in the software. Basically, one is more likely to be able to identify parts of the controller as achieving certain subgoals of control, such as pole cancellation, or injection of a phase compensation. Finally, in discrete time implementation, the computational requirements will be less.
4.8.1 Frequency Weighted Balanced Truncation (Controller Order Reduction)

Keeping in mind the concepts brought forth in the previous section and the fact that the order of our system is high (sixth order), we considered whether there might be a simpler controller (i.e. a lower order controller) that would perform nearly as well as the full order controller that resulted from the linear quadratic design.

A low order design can be achieved via three methods:

1. Seek to obtain a low order controller directly (the direct approach)
2. Use LQG methods to design a full order controller, as a first step, and then as a second step, approximate the full order controller by a lower order controller (the indirect method)
3. Begin the whole design by approximating the system with a low order model and design a controller, using LQG methods, for the low-order model.

The second approach has been utilized and will be demonstrated below.

Before proceeding with the design of a lower-order controller the issue of performance will be addressed. First and foremost, controller reduction must preserve closed-loop stability. Other properties which should be preserved (as far as possible) are: closed-loop transfer function, phase margins, and robustness properties. These goals define what is known as a frequency weighted approximation problem.

Without going through all the theory here [see 46] we will simply state that it is sufficient to examine the Hankel singular values in order to obtain insight with respect to the value by which the order of the controller can be reduced.

The Hankel singular values are nothing more than the singular values of the nxn Hankel matrix which is an arrangement of the Markov matrices $A_i$. The
Markov matrices are obtained by expansion of the $p \times m$ matrix $W(s)$ which is assumed to be strictly proper:

$$W(s) = \frac{A_0}{s} + \frac{A_1}{s^2} + \frac{A_2}{s^3} + \ldots \ldots$$ \hspace{1cm} (4.14)

The $A_i$ are arranged to form truncated Hankel matrices $H_N$ as follows:

$$H_N = \begin{bmatrix}
A_0 & A_1 & \ldots & A_{N-1} \\
A_1 & A_2 & \ldots & A_N \\
\vdots & \vdots & \ddots & \vdots \\
A_{N-1} & A_N & \ldots & A_{2N-1}
\end{bmatrix}$$ \hspace{1cm} (4.15)

The next step is to find the first integer $r$ for which $\text{rank } H_r = \text{rank } H_{r+1} = \text{rank } H_{r+2} \ldots \ldots$. In such a case $r$ would be the dimension of minimal realization of $W(s)$.

### 4.9 Simulations and Results

The design steps carried out in the optimal design based on linear quadratic gaussian methods were as follows:

1. Standard linear quadratic design.
2. Pole-placement design.
3. Pole-placement design including a Kalman-Bucy filter observer.
4. Frequency shaped design.
5. Reduced order controller-frequency weighted balanced truncation.

All design steps were investigated through computer simulation using Matlab. The Matlab programs used to carry out these simulations are found in Appendix B. The conditions are as in the previous simulations (i.e., with PID controller). For the sake of consistency and fair evaluation the same input vector is used. The open loop response is omitted here since it is no different than before. A side note here: the Matlab programs are general enough, to permit anyone interested, to go through all of the above design steps for any
given system by making some minor alterations so as to be consisted with the theory.

4.9.1 Straight LQ Design
First of all, the closed-loop response of the system with an optimal controller (straight LQ design, obtained from the solution of the steady state Riccati equation) was obtained via computer simulation. The closed-loop system response—i.e., the actual depth of cut and the tool-workpiece relative displacement are shown in Figure 4.4.

![Graphs of depth of cut and relative displacement vs. time](image)

**Figure 4.4:** Actual depth of cut and tool-workpiece relative displacement plots closed loop response
The weight matrices used are as follows:

\[
Q = \begin{bmatrix}
q_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & q_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & q_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & q_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & q_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & q_{66}
\end{bmatrix}
\]

\[
R = [r]
\]

where, \( q_{11}=0, q_{22}=0, q_{33}=0.1, q_{44}=1.0, q_{55}=0.1, q_{66}=1.0, \) and \( r=0.0001. \)

Figure 4.5: Bode (a) magnitude and (b) phase plots—closed loop response
The resulting optimal gain matrix is given by:

\[
K = \begin{bmatrix}
-1.9694 & -883.4304 & 238291 & 1.1119 \times 10^5 & -31.4743 & 506.1683 \\
-3.5481 \times 10^{-6} & -0.0016 & 4.293 \times 10^{-5} & 0.2003 & -5.6704 \times 10^{-5} & 9.1191 \times 10^{-4}
\end{bmatrix}
\]

The corresponding closed-loop poles are located at:

-3.6569 \times 10^8, -1693.7 + 2278.4i, -1693.7 - 2278.4i, -637.44 + 10664i,

-637.44 - 10664i, -3.1632.

The Bode magnitude and phase plots, as well as the Nyquist plot were also constructed and are shown (for the first input) in Figure 4.5 and Figure 4.6 respectively. They provide us with insight with respect to closed loop stability and information on noise rejection ability.

![Nyquist plot](image)

**Figure 4.6:** Nyquist plot-closed loop system for input=$d_o$.
It is obvious from the above plots that the system has a very high gain margin and excellent noise rejection ability.

4.9.2 Straight LQ Design With Pole Placement

The dominant closed-loop poles were located closer to the imaginary axis than desired. For this reason, a pole-placement design was carried out as described in section 4.4. The prescribed degree of stability was chosen to be $\alpha=500$ (i.e., closed loop poles moved left of the -500 line). The closed-loop (pole-placement) system response—i.e., the actual depth of cut and the tool-workpiece relative displacement are shown in Figure 4.7.

![Graph](image)

Figure 4.7: (a) Actual depth of cut and (b) tool-workpiece relative displacement plots LQ controller with pole placement
Comparison of Figure 4.7 and Figure 4.4 quickly illustrates the improvement in the response of the system. The tool-workpiece relative displacement has been reduced by nearly 100 times and the resulting depth of cut tracks the preset depth of cut almost perfectly. This of course, implies better dimensional and surface roughness results. The closed-loop poles after pole-placement were placed at:

-3.6569e+006, -1739.4+2258.9i, -1739.4-2258.9i, -500.0101, -138.90+10664i, -138.90-10664i.

Figure 4.8: Nyquist plots-LQ design (pole placement) (a) input=$d_o$ (b) input=$h_i$. 
The resulting optimal gain matrix is given by:

\[ K = \begin{bmatrix}
-3.9158 & -6.5055 \times 10^3 & 16.4394 & 1.6853 \times 10^5 & -57.0396 & -3.8788 \times 10^4 \\
-7.0546 \times 10^{-6} & -0.0117 & 2.9617 \times 10^{-3} & 0.3036 & -1.0276 \times 10^{-4} & -0.0699 
\end{bmatrix} \]

For the sake of obtaining insight with respect to closed loop stability, the Nyquist plot has been generated and is shown in Figure 4.8.

It is readily apparent, by inspecting the Nyquist plots, that the closed-loop system is quite stable.

4.9.3 Pole-Placement Including a Kalman-Bucy Observer (LQG) Design

The next step is a LQG design where a statistical estimator (Kalman-Bucy filter observer) as described in sections 4.5 and 4.6 was also incorporated in the control system.

The poles of the Kalman-Bucy filter observer are located at:

-2997.6+278.77i, -2997.6-278.77i, -2968.0+5334.0i, -2968.0-5334.0i,
-2378.1+10777i, -2378.1-10777i.

Please note that the observer poles are located farther left than the control system poles, i.e., the observer is faster than the controller. This is a desirable effect since we want state estimates to be available before the control action is initiated. The associated optimal observer gain obtained form solving equations 4.9 and 4.10 is given by:

\[ K_o = \begin{bmatrix}
3.7868 \times 10^8 & -3.3837 \times 10^4 & -3.5615 \times 10^4 & 2.5465 \times 10^4 & 1.0067 \times 10^4 & 5.884 \times 10^4 
\end{bmatrix} \]

The system response obtained via simulation of the LQG design is depicted in Figure 4.9. From the examination of the estimation error, Figure 4.10, one can readily see that the error is so small that one can assume that \( x_o(t) \) effectively tracks \( x(t) \) almost instantaneously. For the sake of better understanding the results that have been obtained, a tool profile over a 7mm distance, and a three dimensional surface topography have been generated.
Figure 4.9: Actual depth of cut and tool workpiece relative displacement controller/estimator (LQG design)
Figure 4.10: Estimation Error

Please note that in the generation of these plots, the tool nose radius and the tool lead angle have been taken into account and are reflected in the plots. Also, the tool profile has been normalized so as to show what one may measure from an actual surface roughness measurement. These results are presented in Figure 4.11.
Figure 4.11: (a) section of the surface profile and (b) 3D surface topography

LQG design

The Bode magnitude and phase plots are shown in Figure 4.12 and the power spectral density plots of all the designs presented so far, are depicted in Figure 4.13 (note that, the LQ design and the LQG design power spectral density curves overlap). These two figures provide us with insight which leads to an effective frequency shaped design. For example, the spike on the Bode plots, as well as the higher energy exhibited below 2560 rad/s aliment the design of a frequency shaped controller.
Figure 4.12: Bode (a) magnitude and (b) phase plots-controller/observer (LQG design)
4.9.4 Frequency Shaping

A frequency shaped design was carried out as detailed in section 4.7. The frequency weighting transfer function was chosen to be a frequency weighted mountain. It has the form of equation 4.15 with \( \zeta_l=0.8, \ \zeta_r=0.0008, \) and \( \omega_n=2560 \) rad/s. The resulting frequency weighting transfer function takes the form:

\[
W(s) = \frac{s^2 + 4096s + 2560^2}{s^2 + 4.096s + 2560^2}
\]

The Bode magnitude and phase plots corresponding to the frequency weighting transfer function are shown in Figure 4.14.
Figure 4.14: Bode (a) magnitude and (b) phase plots-frequency shaping filter

The resulting augmented system performance is depicted in Figure 4.15. The system performance has been improved by nearly three times, however, this improvement is not expected to contribute significantly to the improvement of the surface roughness, or the final dimension for that matter, of the given workpiece. As a result, a frequency shaped design is not preferred since it increases the system order by two - a big price to pay in controller complexity for an insignificant improvement.
Figure 4.15: (a) Actual depth of cut (b) Tool workpiece relative displacement frequency shaped LQG design

The frequency weighted mountain, when augmented with the original system, is expected to reduce the power spectral density of the system, a result which is readily visible from Figure 4.16 and Figure 4.17. Please note the dramatic reduction in power spectral density (Figure 4.16), as well as the smoothing out of the 'spike' on the Bode magnitude (Figure 4.17).
Figure 4.16: Frequency shaped LQG design performance plot
Figure 4.17: Bode (a) magnitude and (b) phase plots-frequency shaped LQG design

Finally, the Nyquist plots for the LQG frequency shaped design is presented in Figure 4.18.
Figure 4.18: Nyquist plot-frequency shaped LQG design

4.9.5 Frequency-Weighted Balanced Truncation

The frequency-weighted balanced truncation (controller reduction) was initiated by computing the Hankel singular values of our system. These values were found to be, in descending order,

1228, 1035.2, 737.8230, 534.9255, 1.8385, 0.7815.
Since, the last two values are rather small compared to the first four, we can reduce the order of the controller by two (i.e., design a fourth order controller which should have a performance very close to that of the full order controller).

![Magnitude and Phase Comparison](image)

**Figure 4.19:** Bode (a) magnitude and (b) phase comparison with full and reduced order controllers

The reduced order controller is given, in transfer function form, by:

\[
C_r(s) = \frac{\left[ 2.1139s^4 - 1.0897 \times 10^{10}s^3 - 4.5996 \times 10^{13}s^2 - 8.5030 \times 10^{17}s - 4.4610 \times 10^{20} \right]}{\left[ 3.8083 \times 10^{-6}s^4 - 1.9632 \times 10^{4}s^3 - 8.2866 \times 10^{7}s^2 - 8.0369 \times 10^{14}s - 8.03969 \times 10^{14} \right]}
\]

\[
s^4 + 7.3956 \times 10^6s^3 + 2.4173 \times 10^{11}s^2 + 9.7705 \times 10^{14}s + 2.4644 \times 10^{19}
\]
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The Bode magnitude and phase plots of the closed loop transfer function (for input 1) that are shown in Figure 4.14 can be used as a measure of how the reduced order controller compares with the full-order controller. The two closed-loop transfer function magnitudes and phase plots obtained from $C$ and $C_r$ are virtually indistinguishable. Two other measures of comparison come from the phase and gain margins which based on the definition given in section 3.3.2 are the same when either $C$ or $C_r$ are used.

4.9.5 Stability, Relative Stability (Phase and Gain Margins), Robustness

The stability of the closed loop system was determined by both direct computation of the closed loop poles and construction of the Nyquist plots, shown in Figure 4.8 and Figure 4.18. Suffice to say that the closed loop system is quite stable.

The next question that needs to be answered is how stable is it? This question of relative stability can be answered by looking at the phase and gain margins as defined in section 3.3.2. The phase and gain margins of the system are very high, as it is readily available from the Bode magnitude and phase plots.

To ensure a specified attenuation (reduction) of noise components in the input above a certain frequency-noise rejection-the plot above that frequency should be below -20dB. In our case this is true for all frequencies in all the designs.
Chapter 5 Properties and Application of Adaptive Control

5.1 Introduction

In this chapter the properties of adaptive control and more specifically self-tuning regulators (STR) are discussed. A self-tuning regulator is then designed such as to minimize the tool-workpiece relative displacement. The linear quadratic guassian (LQG) design method is employed.

In most feedback control systems, small deviations in parameter values from their design values will not cause any problem in the normal operation of the system, as long as these parameters are inside the loop. If system parameters vary widely according to environmental changes, however, then the control system may demonstrate satisfactory response for one environmental condition but not another. In some cases, large variations of system parameters may even cause instability.

In the simplest analysis one may consider different sets of values of the system parameters. It is then desirable to design a control system that works well for all sets. As soon as this demand is formulated, the strict optimal control problem loses its importance. By asking for the best performance we have to abandon the best performance for one parameter set.

If the system transfer function or system state equation can be identified continuously, then we can compensate for variations in the transfer function or state equation of the system simply by varying adjustable parameters of the controller and thereby obtain satisfactory system performance continuously under various environmental conditions. Such an adaptive approach is very useful to cope with a
problem where the system is normally exposed to varying environments so that system parameters change from time to time.

5. 2 Adaptive Control

The term *adaptive* system has a variety of specific meanings, but it usually implies that the system is capable of accommodating unpredictable environmental changes, whether these changes arise within the system or external to it.

This concept is quite appealing to the designer since a highly adaptive system can accommodate abstemious engineering design errors or uncertainties and would compensate for the malfunction of minor system components in addition to accommodating for environmental changes, thereby increasing system reliability.

*Definition:* An adaptive control system is one that continuously and automatically measures the dynamic characteristics (such as the transfer function or state equation) of the system, compares them with the desired dynamic characteristics and uses the difference to vary adjustable system parameters (usually controller characteristics) or to generate an actuating signal so that the optimal performance can be maintained regardless of the environmental changes.¹

Adaptive control is based on the premise that there is some condition of operation or performance for the system that is better than any other. It is therefore, necessary to define what constitutes optimal performance. In adaptive control systems, the performance index, which is decided upon after stating the objectives, is used to depict such performance. The objectives are as diverse as the system to which they are applied but it is usually feasible to generalize the object of optimization to that of minimizing the cost of operation or

¹ *Modern Control Engineering, K. Ogata*
maximizing the profit. Some general characteristics deemed desirable are: reliability, selectivity, and applicability.

5.3 Adaptive Controllers
Most control systems can be divided into two main groups - feedforward adaptive controllers and feedback adaptive controllers.

Feedforward adaptive control systems are based on the fact that the changing properties of the system can be grasped by measurement of signals acting on the process. A feedforward adaptation mechanism can be designed if how the controller must be changed, dependent on these measurable signals, is known. Feedforward adaptive controllers have fast reaction to process changes because the process behaviour is known in advance and need not be identified with measurable input and output signals. All the effects based on unmeasured signals and disturbances, however, are neglected.

If the process behaviour changes cannot be determined directly by measurement of external process signals, feedback adaptive controllers have to be designed. These control schemes are characterized by the following: First, the changing properties or its signals can be observed by the measurement of different internal control loop signals. Second, in addition to the 'basic' control loop feedback the adaptation mechanism results in a additional feedback level. Finally, the closed loop yields a second feedback level which is non-linear.

An adaptive controller may consist of the following three functions:
1. *identification* of dynamic characteristics of the system
2. *decision* making based on the identification of the system
3. *modification* or *actuation* based on the decision model
5.4 Adaptive Control in Turning

A typical approach employed in industry for the selection of optimum machining parameters is to use a data bank. Past experience of the company is stored in a computer and then used to predict the optimum operating conditions for a given job. The problem with this approach is that the number of parameters required to characterize a job fully is too large to be used in practice. When only the most important variables are used the result is too approximate. In addition, the inputs to the process such as workpiece hardness, workshop temperature, or even operator fatigue are not static but vary with time. To do better, some form of adaptive control must be used in which performance is monitored and the machine adjusted according to the results obtained.

Adaptive control (with reference to turning) may be classified as:

1. Primary adaptive control - the actual optimization variable is measured or calculated and the machine adjusted stepwise to approach optimization.
2. Secondary adaptive control - a secondary variable such as rate of tool wear, cutting force, or cutting temperature are monitored and the machine parameters adjusted in the direction of the optimum based on an assumed relationship between primary and secondary variables.

Almost all applications of adaptive control in turning are of the secondary type, for example, adjusting the feed to control the cutting force. In this
Chapter 5 Properties and Application of Adaptive Control

particular case the adaptive control algorithm design is also of the secondary type. Since, there are no workshop proven in-process sensors for surface roughness one must be content to measure a secondary variable (vibrational amplitude) that is related in a complex way to the main variable of interest (surface roughness).

5.5 Why Adaptive Control?

One of the goals of adaptive control is to compensate for parameter variations. Parameters may vary due to non-linear actuators, changes in operating conditions of the process and non-stationary disturbances acting on the process.

In addition, adaptive control is suitable for systems with inexact models of the system dynamics. In this particular case two "conditions" are met. First of all, the model of the system dynamics is inexact since as stated earlier the turning system actually has infinite degrees of freedom but is approximated as a three degree of freedom system. Secondly, the variation in the surface hardness is treated as a non-stationary disturbance: The surface hardness of the workpiece is normally distributed about the mean surface hardness with standard deviation as high as twenty percent of the mean value. By adaptation the regulator can be adjusted to the changing characteristics of the disturbances.

5.6 Approaches to Adaptive Control

Three schemes for parameter adaptive control are presented here in a common framework. They are gain scheduling, model reference control, and self-tuning regulators. The starting point is an ordinary feedback control loop with a process and a regulator with adjustable parameters. The key problem is to find a convenient way of changing regulator parameters in response to changes in process and disturbance dynamics. The schemes differ only in the way the
parameters of the regulator are adjusted. Here, we would only detail the self tuning regulator. For the other two schemes the reader is referred to [2, 4, 15, 37, 39, 49, 56].

The self-tuning regulator is one method for adjusting the parameters. Such a system is shown in Figure 5.2. The regulator can be thought of as being comprised by two loops. The inner loop consists of the process and an ordinary linear feedback regulator. The parameters of the regulator are adjusted by the outer loop, which is composed of a recursive parameter estimator and a design calculation.

![Figure 5.2: Block diagram of a self-tuning regulator (STR)](image)

The self-tuning regulator was originally proposed by Kalman [39]. Recently, the self-tuning regulator has received a lot of attention because it is flexible, easy to understand, and easy to implement with microprocessors [4, 49].

The self-tuner also contains a recursive parameter estimator. Many different estimation schemes are available, for example, stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering, and the maximum likelihood method. The recursive least squares method will be detailed below since this is also the method used in the present design. For description of the other methods the reader is referred to Åström and Wittenmark [6], and Iserman, Lachmann, and Matko [32].
5.7 Real Time Parameter Estimation

On-line determination of process parameters is a key element in adaptive control. Either non parametric or parametric process and signal models can be used for controller design. Parametric models involve a finite number of parameters and permit the application of advanced controller design procedures with relatively little computational effort for a wide class of processes.

For real time estimation, in particular, recursive parameter estimation methods have been developed for linear time-invariant and time variant processes, for some classes of nonlinear processes, and for stationary and some non stationary signals. Non-parametric estimation methods include Estimation of Correlation Functions (ACF), and convolution. Parametric techniques on the other hand include Least squares (LS) method (non-recursive), Recursive Least Squares (RLS) method, and Extended Least Squares (ELS) method. The above list is by no means complete and only the method employed in the adaptive controller design will be detailed. The remaining can be found in literature (for example, Ljung and Söderström [52]).

5.7.2 Process and Signal Models

In the case of data processing and for identification using a computer, the process signals are sampled and digitized via an analogue to digital converter (ADC). This results in discrete signals which are quantized in amplitude and time - it is assumed that the quantization unit is small, so that the amplitudes can be considered as continuous. If the sampling occurs periodically with sampling time $T_s$, amplitude-modulated pulse trains are generated after the ADC. The digitized signals are processed by the digital computer using programmed algorithms. The output signal then calculated. If the digital computer is used for controlling a process and an analogue signal is required for the actuator the
calculated output signal is generated by a digital to analogue converter (DAC) followed by a hold device.

The input and output samplers do not operate synchronously. They are displaced by some time interval which results from the digital to analogue conversion and the data processing within the central processing unit. This interval is in general much smaller than the time constants of the actuators, processes, and sensors. As a result it is often neglected and the sampling is assumed synchronous.

5.7.3 Least Squares and Regression Models

In regression analysis, an observation (output) is hypothesized to be a linear combination of explanatory variables (inputs), and a set of observations is used to estimate the weighting on each variable such that some fitting criterion is optimized. The common criterion chooses the model parameters such that the sum of squares of the errors between the model outputs and the observations is minimized (least squares).

Gauss formulated the principle of least squares near the end of the eighteenth century and used it to determine the orbits of planets. According to the same principle the unknown parameters of a mathematical model can be chosen in such a way that

the sum of the squares of the differences between the actually observed and computed values, multiplied by numbers which measure the degree of precision, is a minimum.

It is particularly simple for a mathematical model and is usually written in the form

\[ y(t) = \varphi_1(t)\theta_1 + \varphi_2(t)\theta_2 + \ldots + \varphi_n(t)\theta_n = \varphi^T(t)\theta \]  (5.1)

where, \( y \) is the observed variable, \( \theta_1, \theta_2, \ldots, \theta_n \) are unknown parameters, and \( \varphi_1, \varphi_2, \ldots, \varphi_n \) are known functions that may depend on other known variables. The
variables \( \phi \), are called the \textit{regression variables} or the \textit{regressors}, and the model in equation 5.1 is also called the \textit{regression model}. The least-squares error can be written as

\[
V(\theta, t) = \frac{1}{2} \sum_{i=t}^{t'} e(i)^2 = \frac{1}{2} \sum (y(i) - \phi^T(i) \hat{\theta})^2 = \frac{1}{2} E^T E = \frac{1}{2} \|E\|^2
\]  

(5.2)

**Theorem: Least-squares estimation**

The function of equation 5.2 is minimal for parameters \( \hat{\theta} \) such that

\[
\Phi^T \Phi \hat{\theta} = \Phi^T Y
\]  

(5.3)

If the matrix \( \Phi^T \Phi \) is nonsingular, the minimum is unique and given by

\[
\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y
\]  

(5.4)

For the proof please see Åström and Wittenmark [6] p.61.

**Theorem: Recursive least-squares estimation (RLS)**

Assume that the matrix \( \Phi(i) \) has full rank for all \( i \geq t_0 \). The least squares estimate \( \hat{\theta} \) then satisfies the recursive equations 5.5, 5.6, and 5.7.

The recursive parameter algorithm is given by:

\[
\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - \phi^T(k) \hat{\theta}(k-1)]
\]  

(5.5)

The correcting vector is given by:

\[
K(k) = P(k-1) \varphi(k)(I + \phi^T(k) P(k-1) \varphi(k))^{-1}
\]  

(5.6)

and

\[
P(k) = [I - K(k) \phi^T(k)] P(k-1) \frac{1}{\lambda}
\]  

(5.7)

To start the recursive algorithm one sets

\[
\theta(0) = 0
\]

\[
P(0) = \alpha I
\]

with \( \alpha \) large. The expectation of the covariance matrix \( P \) is proportional to the covariance matrix of the parameter estimates.
\( E(P(k)) = \frac{1}{\sigma^2_* (k)} \text{cov}[\Delta \theta(k-1)] \)  \hspace{1cm} (5.8)

with

\( \sigma^2_* = E(e^T e) \)

### 5.7.4 Exponential Weighting With Variable Forgetting Factor

If the error is small, the estimation is correct or there is no excitation. In both cases it is required that \( \lambda(k) \approx 1 \). On the other hand, if the error is large, a small \( \lambda(k) \) is required so as to allow fast adaptation of the parameters. For precisely these reasons \( \lambda(k) \) is varied so as to maintain the information content constant. This is done by using the weighted sum of the a posteriori error (as proposed by Fortescue et. al., [23]). In recursive form this is given by:

\[
\lambda(k) = 1 - \frac{1}{\Sigma_0} [1 - \phi^T(k) y(k-1)] e^2(k) \]

\[ \Sigma_0 = \sigma^2_* N_0 \]

where \( \sigma^2_* \) is the variance of the noise and \( N_0 = 1/(1-\lambda_0) \). An alternative to exponential weighting with variable forgetting factor, especially for situations which require fast parameter changes is to change the covariance matrix. For a detailed description of this method the reader is referred to Iserman et. al.[32].

### 5.8 Self-Tuning Regulator (STR) Design

The automatic tuning of controllers for industrial processes has stimulated both theoretical and practical interest for many years. Although, it is generally recognized that the conventional PID controller is especially effective in practice, its original tuning and the maintenance of good tuning is a time consuming activity, particularly if the process dynamics are slow.

Self-tuning control is one approach to the automatic tuning problem. It can be perceived either as a tuning aid for control laws that are more complex
than PID but which have fixed parameters or as a method for controlling time-varying processes in a consistent way. Usually, it is not an optimal control law but if properly used it is capable of achieving good tuning and control performance. Furthermore, self-tuning controllers require modest and predictable computing power. When one considers that the PID algorithm only takes a small fraction of the computational resources of a typical industrial microprocessor based regulator, it is seen that there is scope with current technology to implement a self-tuning regulator in a microprocessor at a cost not greatly exceeding that of the PID based system. The combination of cost and performance constitute ground for the acceptance of the self-tuner by industry on grounds of convenience even for loops where there is no need for adaptive capability.

The self-tuning regulator has three main components. There is a standard feedback law in the form of a difference equation which acts upon a set of values such as the output and feed-forward signals, and which generates the new control action, a recursive parameter observer monitors the process’s input and output and generates an estimate of the system dynamics, and finally the parameter estimates are fed into a control design algorithm which in turn generates the new set of coefficients for the feedback law.

In an adaptive system it is understood that the regulator parameters are modified continuously. This suggests that the regulator parameters follow revisions in the process. It is, however, difficult to investigate the convergence and stability properties of such systems. To simplify the problem it can be assumed that the process has constant but unknown parameters. The adaptive controller should converge to these values even when the process is unknown. Such an adaptive controller is termed self-tuning, since it automatically tunes the controller to the desired performance. The self tuning regulator is based on the concept of separating the estimation of the unknown parameters from the design
of the controller. The estimated parameters are treated as if they are true and the uncertainties are ignored (certainty equivalence principle).

Any of several estimation techniques can be used. The techniques are: stochastic approximation, generalized least squares, extended least squares, and maximum likelihood.

The self-tuning regulator is very compliant with respect to the design method. Essentially any design technique can be accommodated. Self-tuning regulators based on minimum variance control, LQG control, pole placement, phase and amplitude margins, and model following, have been evaluated by other researchers [2, 14, 15, 25].

Self-tuning algorithms can be divided into two major classes: direct and indirect algorithms. In an indirect algorithm, there is an estimation of an indirect process model. The parameters of the controller are obtained via the design procedure. Direct self-tuning regulators, on the other hand, are based on direct estimation of the controller parameters.

5.8.1 Linear Quadratic STRs
The linear quadratic design procedure is, as mentioned above, one of the design methods that can be employed in the design of a self-tuning regulator. The process model can be in polynomial or state space form. The state space form is given below:

\[ x(k+1) = Fx(k) + Gu(k) + \Gamma v(k) \] (5.10)

and the measurement equation

\[ y(k) = Hx(k) + m(k) \] (5.11)

where \( \Gamma \) is the input noise matrix, \( v(k) \) is a statistically independent vector input noise with mean

\[ E(v(i)) = \overline{v(k)} \]

and covariance matrix
\[ E(\nu(i)\nu^T(j)) = V\delta_{ij} \]

\( \delta_{ij} \) is the Kronecker delta functions.

There are two ways to construct indirect linear quadratic self-tuning regulators. The first is based on spectral factorization, and the second is based on the Ricatti equation. Only the second will be detailed here (section 5.9), since it is the method used in the present design. For the reader who is interested in the first the reader is referred to Åstrom and Wittenmark [6].

### 5.9 Parameter Adaptive Control

Parameter adaptive control algorithms based on recursive parameter estimation methods together with suitable control algorithms will be discussed below. The foundations of this section were laid in previous sections: recursive parameter estimation, and design of control algorithms. In this section attention is turned to the design of appropriate combinations of recursive parameter estimators and control algorithms. Figure 5.3 depicts a parameter adaptive system.

![Diagram of parameter adaptive state controller with parameter adaptive state estimation](image)

**Figure 5.3:** Parameter adaptive state controller with parameter adaptive state estimation
Chapter 5 Properties and Application of Adaptive Control

Parameter adaptive controllers can be classified in terms of process model, parameter estimation (and possibly state estimation), the information about the process, the criterion for controller design, the control algorithm, and other functions.

To build up a deterministic state controller the following methods can be used:

1. Recursive parameter estimation:
   (a) Dynamic behaviour-RLS or RELS or modified versions
   (b) Static behaviour:
       (i) differencing;
       (ii) implicit or explicit d.c. value estimation.

2. State controller:
   (a) State controller calculation:
       (i) recursive matrix Ricatti equation;
       (ii) pole assignment;
   (b) state variable calculation:
       (i) state observer;
       (ii) state reconstruction.

3. Compensation of offsets:
   (a) integrator in the observer;
   (b) integrator in parallel to the state controller;
   (c) integrator before or after the process model;
   (d) setting $Y_0=W(k)$ and implicit or explicit d.c. value stimulation (1(b)(ii)).

Of all the possible combinations which present themselves the following was chosen and designed:

1. Recursive parameter estimation:
   (a) Dynamic behaviour-RLS
   (b) Static behaviour-implicit or explicit d.c. value estimation (ii)
Chapter 5 Properties and Application of Adaptive Control

2. State controller:
   (a) State controller calculation - recursive matrix Ricatti equation (i)
   (b) state variable calculation: state observer - Kalman-Bucy filter (i)

3. Compensation of offsets-setting $y_{oo}=\bar{W}(k)$ and implicit or explicit d.c. value stimulation (1(b)(ii)).

The performance criterion for the controller is given by:

$$ V = x^T(N)Qx(N) + \sum_{k=0}^{N-1}[x^T(k)Qx(k) + rK_p^2u^2(k)] $$  \hspace{1cm} (5.12)

The parameters of the gain matrix are obtained via the recursive equation:

$$ K_{N-j} = (R + G^TP_{N-j}G)^{-1}G^TP_{N-j+1}F $$  \hspace{1cm} (5.13)

The matrix Ricatti difference equation is given by:

$$ P_{N-j} = Q + F^TP_{N-j+1}[I - G(R + G^TP_{N-j+1}B)^{-1}G^TP_{N-j+1}]F $$  \hspace{1cm} (5.14)

$Q$ is chosen as a diagonal matrix. Feasible values are:

$1 \leq q_{ii} \leq 3$

$0.05 \leq r \leq 0.5$

$P_N = Q$ is the initial matrix.

5.10 State Observers

In practice the state variables $x(k)$ are not directly measurable for many processes. They have to be reconstructed using measurable quantities.

The state vector and the process output are assumed to be contaminated by noise. The disturbed process is represented by the following vector difference equation:

$$ x(k+1) = Fx(k) + Gu(k) + \Gamma v(k) $$  \hspace{1cm} (5.15)

and the measurement equation

$$ y(k) = Hx(k) + n(k) $$  \hspace{1cm} (5.16)
where $\Gamma$ is the input noise matrix, $\nu(k)$ is a statistically independent vector input noise with mean

$$E(\nu(i)) = \nu(k)$$

and covariance matrix

$$E(\nu(i)\nu^T(j)) = V \delta_{ij}$$

$\delta_{ij}$ is the Kronecker delta functions.

A Kalman-Bucy filter observer was designed for the purpose of calculating the state variables based on the following procedure:

1. Predict the process state $\hat{x}^*(k+1)$, based on the last estimate $\hat{x}(k)$ determined by:

$$\hat{x}^*(k+1) = F\hat{x}(k) + Gu(k)$$

2. Predict the estimated state vector error covariance matrix $P^*(k+1)$, based on the last covariance of the estimated state vector error $\hat{P}(k)$:

$$P^*(k+1) = F\hat{P}(k)F + \Gamma V T^T$$

3. Determine the correction matrix $K(k)$ using:

$$K(k) = P^*(k)[H^T[HP^*(k)H^T + N]^{-1}}$$

4. Determine the state vector estimate from its prediction, from the correction matrix and from the measured output of the processes at time $k$:

$$\hat{x}(k) = \hat{x}^*(k) + K(k)[y(k) - Hx^*(k)]$$

5. Calculate the covariance of the estimated state vector error $\hat{P}(k)$ from its predicted value:

$$\hat{P}(k) = P^*(k) - K(k)HP^*(k)$$
Figure 5.4 illustrates the corresponding block diagram of the Kalman-Bucy filter.

The augmented system as illustrated in Figure 5.5 is represented by:

$$
\begin{bmatrix}
    x(k+1) \\
    \tilde{x}(k+1)
\end{bmatrix} =
\begin{bmatrix}
    F - GK_s & GK_s \\
    0 & F - KHF
\end{bmatrix}
\begin{bmatrix}
    x(k) \\
    \tilde{x}(k)
\end{bmatrix}
$$

(5.22)

where the error for the current state estimator is given by equation 5.23

$$
\tilde{x}(k) = x(k) - \hat{x}(k)
$$

(5.23)

The Principle of Duality

The optimal state controller and the Kalman-Bucy filter are very similar from a mathematical point of view. This is readily obvious by inspecting table 5.1. Please note that, the manipulated vector weighting matrix R for the optimal state controller corresponds to the covariance matrix of the output noise N of the
Kalman-Bucy filter and the state weighting matrix $Q$ corresponds to the covariance matrix of the state noise $I^r \lor I^t$ respectively. The other parameters are self explanatory.

**Table 5.1: Correspondence between optimal control and optimal filtering**

<table>
<thead>
<tr>
<th>Optimal State controller</th>
<th>Kalman-Bucy filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-k$</td>
<td>$k$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F^r$</td>
</tr>
<tr>
<td>$G$</td>
<td>$H^r$</td>
</tr>
<tr>
<td>$K$</td>
<td>$K_u$</td>
</tr>
<tr>
<td>$R$</td>
<td>$N$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$I^r \lor I^t$</td>
</tr>
</tbody>
</table>

**The Separation Theorem**

The control action for a process with unmeasurable state variables consists of the state observer and the state controller using the estimated state variables:

$$u(k) = -K_u \hat{x}(k)$$  \hspace{1cm} (5.24)

The complete closed-loop control system as illustrated in Figure 5.5 is described by equation 5.26, where the process is described by the state equation 5.25, the error for the current state estimator is given by equation 5.23, and the control action is described by equation 5.21:

$$x(k+1) = Fx(k) + Gu(k)$$ \hspace{1cm} (5.25)

$$x(k+1) = Fx(k) - GK_u[x(k) - \hat{x}(k)]$$ \hspace{1cm} (5.26)

The augmented system is given by equation 5.22.

In closing, it should be stated that the poles of the state controller (without the observer) are exactly the poles of the closed loop system. The poles of the observer can be designed independently without affecting the poles of the
controller which are, as mentioned above, the same as the closed loop system poles.

![Diagram](image)

**Figure 5.5: State controller with observer (Kalman-Bucy filter)**

5.11 Robustness in Parameter Adaptive Control

This section deals with the sensitivity and robustness properties of digital adaptive state controllers and state observers. For optimal state feedback designed on the solution of equation 5.12 with positive definite $Q$ and $r > 0$ to be specified for the dynamic system represented by equation 5.17 the solution to the Ricatti equation 5.14 if the system is stable and controllable (as in our case). The optimal control feedback law, equation 5.23 always gives a closed-loop stable system, that is the eigenvalues of equation 5.27 all lie inside the unit circle.

$$x(k+1) = (F - GK)x(k) \quad (5.27)$$

5.11.1 Stability Robustness of State Controllers

The merits of optimal state controller design more than ensure stable closed loops for linear processes. Beyond these properties state feedback a closed
loop system also if a nondynamic non-linearity, $nL$, is inserted in the path between $K$ and $G$ (see Figure 5.6).

![Diagram](image)

**Figure 5.6:** Linear state space model $(F,G,H)$ with non-linearity $nL$ in the feedback loop

### 5.11.2 Stability and Optimality Robustness of Discrete Optimal Observers

The duality that exists between optimal controllers and optimal observers can be used to derive conclusions that pertain to the robustness of the optimal observer.

### 5.12 Simulations and Results

The applicability of the self-tuning regulator based on LQG methods was verified via simulation. The Matlab programs used to carry out these simulations are found in Appendix C. The conditions are as in the previous two simulations (i.e., with the PID controller and LQG design). For the sake of consistency and fair evaluation, the same input vector is used. The open loop response will therefore be omitted here since it is no different than before. To construct the dynamic state model, the input, output, and state variables have to be selected. The inputs to a turning process are the feed, $f$, the cutting speed, $v$, and the depth of cut, $d$. In this model the cutting speed and feed are assumed constant. The
input variable is the depth of cut so as to permit compensation for tool-workpiece relative motion in the radial direction. The output is selected to be the tool workpiece relative displacement which can be measured during the process. The state variables are position and velocity components of each vibrating component of the system i.e., tool post, workpiece, and the chuck.

As a second step in the construction of the model, the state and output equations have to be determined. For this, relationships reported in the literature are reviewed and those which define the simplest and clearest relationship between the model variables (input, output, and state variables) are selected.

Simulation Results of the Adaptive Control Strategy

For simulation purposes the model parameters were selected from typical values reported in the literature (see Table II.1). Many sets of simulations were carried out with a typical run depicted in Figure 5.7. Please note that the results shown in Figure 5.7 are generated by using the identical input values as with the simulation results presented for the PID control and linear optimal (LQG) control strategies.

For the sake of better understanding the results that have been obtained a tool profile over a 7mm distance and a three-dimensional surface topography have been generated.
Figure 5.7: (a) Actual depth of cut; (b) Tool-workpiece relative displacement

adaptative control strategy (self-tuning regulator)

Once again, in the generation of these plots, the tool nose radius and the tool lead angle have also been incorporated. In addition, the tool profile has been 'normalized' in the sense that what is shown is what one would obtain from a stylus measurement using a roughness measuring machine such as Taylor Hobson Talysurf. These plots are shown in Figure 5.8.
Figure 5.8: (a) Section of the surface profile and (b) 3D surface topography of the closed loop system - LQG self-tuning regulator.
Chapter 6 Discussion, Conclusions, and Recommendations

6.1 Introduction
In this chapter the results obtained from the simulations carried out will be summarized and commented on. The different control strategies proposed and tested via simulation will be evaluated. In addition, controller implementation will be discussed. Finally, directions for future work will be suggested with some relevant information and references cited. A different control strategy that may be useful in surface roughness reduction will also be briefly mentioned.

6.2 Discussion
The goal of this study was to investigate control strategies to control the depth of cut in a turning operation so as to minimize the tool-workpiece relative displacements. This will of course reduce the surface roughness and improve the dimensional accuracy of a given workpiece.

Recognizing that the macro-roughness in a turning operation is totally determined by the tool nose radius and the feed rate, minimization of the tool-workpiece relative displacement would allow us to approach this theoretical limit.

Under the conditions investigated here, it is assumed that the external excitation of the cutting tool comes from the variations of the workpiece's surface
micro-hardness. A model for this input disturbance due to spatial variations in micro-hardness, was selected from published literature.

Also, from published literature, typical machine turning centre stiffness, mass, and damping were selected so as to evaluate the effectiveness of the depth of cut control strategies.

Control strategies were proposed to control the relative movement of the tool with respect to the workpiece in an attempt to eliminate this relative displacement. In such an event, the surface roughness would be determined by the tool nose radius, and the feed rate for the finishing cut operation as no noise would be superposed on the tool displacement.

A summary of the control strategies, bandwidth, and tool-workpiece relative displacement, and acceleration follows.

6.2.1 Simulation Results (Summarized)

PID Control
The open loop response as it is apparent from Figure 3.2 and Figure 3.3 yields results that are unsatisfactory for jobs that require some sort of precision. PID control can be used to significantly improve both the surface finish and the dimensional accuracy of a given workpiece (see Figure 3.5 and Figure 3.6). In addition the power spectral density is reduced significantly as it is obvious from Figure 3.7. The resulting control system is both stable and robust. A reduction of approximately 16% in the RMS surface roughness has been achieved.

6.2.1.2 LQ and LQG Optimal Control
Optimal control based on linear quadratic methods was a further improvement over PID control. The tool-workpiece relative displacement was further reduced especially when the design included pole placement (see Figure 4.7). The results obtained from the LQG design (state controller and Kalman-Bucy filter
observer) were better yet. Figure 4.9 and Figure 4.11 testify. Finally, the LQG design with frequency shaping yields the best results as one can gather from Figure 4.15. This design, however, is not justifiable since the improvement will not manifest itself on the workpiece and the cost associated with this design - system order increased by two - is a price too high to pay.

6.2.1.3 Adaptive Control (STR)
Parameter adaptive LQG self-tuning regulator based on an LQG design and recursive least square identification was also extremely successful as is quite evident from Figure 5.7. The tool-workpiece relative displacement has been reduced to insignificant levels with quite low state weighting.

6.3 Conclusions
The conclusions which emerged from this study are as follows:
1. Proportional-Integral-Derivative (PID) control can be employed in order to suppress the tool workpiece relative displacement in turning. Even though it is not capable of suppressing these displacements to levels which will not appear in the final evaluation of a given workpiece (i.e., surface roughness and final dimensional accuracy) it is capable of reducing them by a substantial amount to make it attractive for a wide class of machining where ultra high precision is not required. Its off the shelf availability makes it an even more attractive choice.
2. Linear optimal control using linear quadratic methods based on pole placement and especially LQG designs can be employed to successfully suppress tool workpiece relative displacements to levels (nanometers) where the pre-set depth of cut is tracked almost perfectly and as a consequence the resulting surface roughness will approach the theoretical, i.e., only the tool signature will be portrayed on the surface - the resulting profile is determined
by the tool nose radius and the feed rate. Furthermore, the final workpiece geometrical accuracy will be met at the same levels of precision. The fact that the system which results from an optimal design will not only be stable, have a certain bandwidth, or satisfy any of the desirable constraints associated with classical control (PID), but it is also supposed to be the best possible system. The results obtained testify to this. The ability to control to nanometers is important in that it means control can be perfect. Measurement technology, however, is only possible to $1\mu m$ and even unnecessary at the nanometer level.

3. Parameter adaptive control via a self-tuning regulator based on LQG methods can also be employed successfully (as demonstrated via simulation) to suppress tool-workpiece relative displacements to levels analogous to those of linear optimal control based on LQG designs. The added advantage of such a system is its capability to accommodate process variations.

Table 6.1 gives a brief summary of the simulation results for the system used in the evaluation of the different control methods. This should give the reader an idea of what is expected.

**Table 6.1: Summary of Simulation Results**

<table>
<thead>
<tr>
<th>control strategy</th>
<th>system bandwidth (Hz)</th>
<th>maximum peak-to-peak relative motion ($\mu m$)</th>
<th>$\sim R_s$ ($\mu m$)</th>
<th>tool acceleration (maximum peak-to-peak in $m/s^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>no control</td>
<td>N/A</td>
<td>300</td>
<td>3.22</td>
<td>257.8</td>
</tr>
<tr>
<td>PID</td>
<td>1590</td>
<td>40</td>
<td>2.69</td>
<td>16.806</td>
</tr>
<tr>
<td>Optimal LQ</td>
<td>1634</td>
<td>1.5</td>
<td>2.545</td>
<td>16.403</td>
</tr>
<tr>
<td>Optimal LQG</td>
<td>1634</td>
<td>0.05</td>
<td>2.545</td>
<td>16.214</td>
</tr>
<tr>
<td>Adaptive (STR)</td>
<td>N/A</td>
<td>0.0025</td>
<td>2.545</td>
<td>15.809</td>
</tr>
</tbody>
</table>
The $R_e$ value shown in Table 6.1 for the no control case is consistent with the experimental results of [83] for the same machining conditions.

6.4 Prospects for Future Work

In this section, possible directions for future work are suggested. In addition, some of our ideas on how the suggested directions may be pursued are also presented.

6.4.1 Controller Implementation

In the case of PID control and the case of optimal control based on LQG design methods, the focus was exclusively on the design of an analog controller for an analog system. The focus will now shift to the actual implementation of these controllers. The initial design was analog so as not to lose physical insight, concerning our analog system, which is often the case when a sampled-data representation is introduced.

In addition, insight related to frequency domain notions is also more straightforward to achieve in continuous time, due to the absence of aliasing effects and possible distortions of frequencies in the mapping from continuous to sampled data representations. Even though, dead beat responses are not obtainable by taking an analog controller and implementing it in digital form, while direct digital designs can secure such responses, and multirate digital designs offer freedoms not achievable with analog designs, it is a well known fact that, if it is not possible to obtain a good design in continuous time it is not possible to obtain a good digital design. In practice, both transformation to digital form of an analog design and direct digital design can be found.

The structure that replaces an analog compensator is shown in Figure 6.1.
Figure 6.1: Digital implementation of a controller
(adopted from [7])

The key issues that need to be addressed are:

1. Choice of sampling rate.
2. Role and design of the analog prefilter.
3. The determination of the discrete-time transfer function matrix implemented in the computer.
4. The choice of the state variable realization for the transfer function matrix refer to in 3.

No digital compensator can mimic exactly an analog controller and all trade-offs are related to cost and performance. One may observe that if the sampling frequency associated with the digital controller becomes infinitely small (impossible) the digital controller will duplicate the analog one. This of course will require the hardware to be infinitely fast, which is out of the question. Any increase in sampling frequency must always be weighted against hardware cost. Sampling frequency selection is discussed below.

6.4.1.1 Sampling Frequency Selection
Sample frequency selection is based on the Nyquist sampling theorem: Let \( s(t) \) for \( t \in (-\infty, \infty) \) be an analog symbol strictly bandlimited to frequency \( \omega_N \) and suppose samples of \( s(t) \) are obtained at a frequency of \( \omega_s \). Then, one can reconstruct \( s(t) \) for all \( t \) from these samples if and only if \( \omega_s > 2\omega_N \), where \( 2\omega_N \) is
the Nyquist frequency. For the reader who is interested in a proof and discussion, nearly any book on digital filters can be referred to; for instance [81].

Practical utilization of the sampling theorem requires that the sampling frequency be significantly higher than twice the maximum frequency of interest (i.e., the closed-loop frequency).

As a first step, account must also be taken of controller effectiveness, sensitivity to system parameter variation, responsiveness to command changes, smoothness of response to command changes, and closed-loop stability retention in the presence of additional delay. These issues will now be addressed.

Regulator Effectiveness
One task of the controller is to suppress the effects of random disturbances. Due to the sampling interval, the digital controller will not be as effective as its analog counterpart, since in effect, the controller acts in open-loop between one sample and the next.

Sensitivity to System Parameter Variation
The point that is at issue in this case is the ability of the controller to suppress the effects of system parameter variations away from the nominal value for which the controller was designed. From [32], the lower the sampling frequency, the greater the destructive effect on performance of a particular parameter variation in the system.

Responsiveness to Command Changes
Frequently, the interconnection of controller and system will provide a unity feedback system in which the system output should follow externally applied reference signals. One effect of a digital controller is to introduce a delay, which
may be unacceptable, between the application of the externally applied reference and the initiation of the system response.

Smoothness of Response to Command Change
The D/A converter and hold replace a discrete time signal to a piecewise constant continuous time signal equal to the most recent discrete value. This may cause the response to be fluctuating to unacceptable levels, and/or unwanted resonances may be excited.

Implications for Closed Loop Stability
The hold device will introduce time delay equal to one-half the sampling interval. One way to dealing with the delay is to augment further dynamics approximating the delay before the design of the analog controller. Another way is by increasing the sampling frequency.

6.4.1 Development of a Program Package for Parameter Adaptive Control in turning
The development of a real time program package for parameter adaptive control may be considered. Some of the features that may be offered are:
- modules for real time application
- identification methods in open and closed-loop
- controller design methods for PID and optimal control
- adaptive control - STR based on LQG methods and PID STR.
The user will only have to enter information describing the system to be controlled (mass, stiffness, and damping properties of the system) as well as information regarding the tool geometry and the workpiece material. This will have to be done keeping in mind that the user will not necessarily have knowledge of adaptive control.
6.4.2 Including Cutting-Tool Wear in the Modelling of the Turning Process

Cutting-tool wear may be incorporated in the modelling in order to get a more comprehensive model of the turning process. The following section is dedicated to the analysis of cutting-tool wear and a summary of some of the existing empirical models developed by various researchers.

Wear is usually undesirable and should be minimized. This is definitely the case with cutting tool wear.

Wear may be classified into several types:
- **Attritious** (small particle) wear associated with adhesion, prowl formation, and shear-plane ends.
- **Abrasive** wear (due to cutting action of hard particles)
- **Erosive** wear (cutting action of particles in a fluid)
- **Diffusion** wear at high temperatures
- **Corrosive** wear (due to chemical attack of a surface)
- **Fracture** wear chipping of brittle surfaces.

Practical wear situations almost never involve just one of these types of wear. Tool wear in a turning operation involves four types. Cutting tool wear, which can be defined as the total loss of mass between the sliding pairs, may be classified as follows: adhesive wear (attritious wear), abrasive wear, diffusion wear, and fatigue wear. All of the above mentioned types of wear are generally present in combination. The cutting conditions, however, dictate the predominant wear mechanism.

Under normal conditions of cutting, cutting tools fail by a process of wear which is due to the interaction between the chip and the tool or between the workpiece and the tool. After some time has elapsed in the cutting process, wear takes place at four regions of the cutting tool [16]:
- **wear land** - more or less a uniform wear zone on the flank of the cutting tool.

Under low speeds, wear land is dominant. It can be easily measured
optically in terms of its length $w$. In general, before a tool is replaced the wear land is in the range of $0.25 \leq w \leq 1.27$mm.

- crater wear - a dished out section on the tool face, which forms where one would normally expect most wear due to high contact stresses and high interface temperatures. At low cutting speeds crater wear is usually insignificant.

- nose radius wear - forms at the nose radius near the relief face. The wear in this zone is to some extent a continuation of the wear and around the nose radius, and partially a series of grooves. These grooves have been found to be spaced a distance equal to the feed.

- outer diameter groove - forms at the outer diameter of the workpiece and can become quite large compared to the other zones. It is not harmful, however, since it is not generally associated with the finished surface of the workpiece.

Figure 6.2: Geometry of tool wear

Figure 6.2, depicts the geometry of tool wear [15]. The conditions that cause the surface integrity to deteriorate, correlate to the growth of the wear land. At higher cutting speeds, both the rate of flank wear and the rate of crater wear increase, but crater wear dominates. However, the volume available to be worn away before total destruction is significantly greater for crater wear than for wear land wear, which
compensates for the greater rate of wear on the face. Analyses of cutting tool wear have traditionally emphasized flank wear and the reason is the more direct influence it has on the quality of the product. A summary of the models proposed by some researchers is presented below.

Tool Wear Models

On-line tool wear estimation has been investigated by many researchers (for example, [10, 40, 42, 43, 64, 74, 81]). Despite years of research in the area, no reliable on-line tool wear measurement technique was found [80]. For example, direct measurement of tool wear using optical methods can only be employed when the tool is not in contact with the workpiece. Radiometric techniques on the other hand, have not proven to be practical for production environments. Indirect methods that rely on the relationship between tool wear and a measurable signal, such as torque, force, temperature, or vibration, to estimate the wear have also been extensively investigated.

Flank Wear Models

Koren [43], in one model, assumed that wear occurs by two mechanisms: a thermally activated one $W_1$ and a mechanically activated one $W_2$. The total wear being equal to the sum of the two:

$$W = W_1 + W_2$$  \hspace{1cm} (6.1)

As the tool wear land grows, the cutting force will also increase due to the sliding between the wear land and the workpiece. The power component of the cutting force is taken as:

$$F = F_0 + C_w W$$  \hspace{1cm} (6.2)

where, $F_0$ is the initial value of power force, $F$ is the value of power force after wear is initiated, and $C_w$ is a constant.

The wear transfer function based on this analysis was derived as:
\[ W(s) = \frac{[C_1 + s(C_0 + \tau C_3)] F_0(s)}{\tau s^2 + (1 - \tau C_1 C_3 - \tau C_2 C_3)s - C_1 C_3 - C_2 C_3} \]  \hspace{1cm} (6.3)

where, \( C_0, C_1, C_2, C_3 \) are constants.

Koren and Lenz later derived another model for flank wear where they obtained relations for the rates of flank wear due to abrasion and diffusion:

\[ \frac{l_0}{V} \dot{w}_f + \dot{w}_d = \frac{K_1 \cos \alpha_r F}{fd} \]  \hspace{1cm} (6.4)

\[ \dot{w}_f = K_2 \sqrt{V} \exp\left[\frac{-K_3}{273 + \theta_f}\right] \]  \hspace{1cm} (6.5)

where, \( \dot{w}_f \) and \( \dot{w}_d \) are the flank wear caused by abrasion and diffusion respectively, \( \alpha_r \) is the effective rake angle, and \( K_1, K_2, K_3, \) and \( l_0 \) are constants.

The tool-work temperature \( \theta_f \) is as derived by Chao and Trigger [11] who are among the few who have investigated the effects of tool-work temperatures:

\[ \theta_f = K_8 V^n f^m + K_7 \omega_f^n \]  \hspace{1cm} (6.6)

The total flank wear and the associated constraint can be represented by:

\[ w_f = w_f + w_d \]  \hspace{1cm} (6.7)

\[ w_f \leq 0.3mm \]  \hspace{1cm} (6.8)

**Crater Tool Wear**

Usui et al.[77] derived a crater wear model:

\[ \dot{w}_c = K_4 V \exp\left[\frac{-K_5}{273 + \theta_c}\right] \]  \hspace{1cm} (6.9)

where, \( w_c \) is the crater wear, \( F \) is the cutting force, \( \theta_c \) is the chip temperature, and \( K_4 \) is a constant. The chip temperature \( \theta_c \) is given by:

\[ \theta_c = K_8 V^n f^m d^m \]  \hspace{1cm} (6.10)

where, \( K_8, n_4, n_5, n_6 \) are constants.
Koren and Lenz utilized the above crater wear model along with their model for flank wear and presented an empirical model for the cutting force [24]:

\[ F = [K_9 f^{"}(1 - K_{10}\alpha_r) - K_{11} - K_{12}V]d + K_{13}dW_f - K_{14}W_c \]  (6.11)

Ulsoy and Park [60], obtained values for the parameters used above in their design of a non-linear tool wear estimator. For the reader who is interested in the actual values, the cited reference contains the complete list.

### 6.4.4 Another Type of Control System

The information detailed above can be useful in the design of a yet another type of control system. The block diagram of the control system we are proposing is shown in Figure 6.3. The controller is basically a constant gain controller and relies on information from assist curves - information on tool wear, as well as constraint information - for the value of the resulting loop gain.

![Block diagram of control system](image)

Figure 6.3: (a) closed-loop system and (b) controller of the proposed control system
References


References


Appendix A: Matlab Files for Input Parameters and PID Control

Program Which Defines the Physical Parameters of the System.

% ****************************************************************************************************************
% * This m file defines all the system's physical parameters. It is called upon *
% * by turnlqr.m in the design of an optimal controller using LQG methods and *
% * turnpid.m in the design of a PID controller. *
% ****************************************************************************************************************

% defining the system parameters ...
% t=tool, w=workpiece, s=spindle

Mt=27; % tool post mass (Kg)
Kt=1.2322E6; % tool post stiffness (N/m)
Bt=128.5; % tool post damping (N-sec/m)
Mw=7.829; % workpiece mass (Kg)
Kw=2.4190E8; % workpiece stiffness (N/m)
Bw=261; % workpiece damping (N-sec/m)
Ms=34; % chuck mass (Kg)
Ks=3.6345E9; % structure stiffness (N-sec/m)
Bs=43071.91; % structure damping (N-s)
R=0.8E-3; % tool nose radius [m]
lead=pi/12; % tool lead angle [rad]
sample=20; % number of samples per revolution
rpm=2000; % spindle rotational speed
Ts=1/(rpm/60*sample); % sampling interval
ti=0; % initial time [s]
tf=15; % final time [s]

% defining the plant parameters ..................
% plant -> dx/dt = Fx + [G Gw] [u w]
% equations -> y = H'x + Du
% state vector -> x = [Xs dot Xs Xw dot Xw Xt dot Xt]'
%
Kc=3.4782E9;  % specific cutting force constant for steel (N/m^2)
dc=0.500E-3;  % initial depth of cut (m)-point of linearization
k=1;
for t=ti:Ts:tf
    if k<3300
        do(k)=dc;
    elseif k>6600
        do(k)=dc;
    else
        do(k)=dc/2;
    end
    k=k+1;
end
do=do';
of=0.25E-3;  % initial feedrate (m/rev)
hmean=126;  % workpiece mean hardness (BHN)
variance=176;  % variance (BHN^2)
S=sqrt(variance);  % standard deviation (BHN)
m=0.454;  % meyer exponent
Ko=Kc*fo;
f11=-(Bs+Bw)/Ms; f12=-(Kw+Ks)/Ms; f13=Bw/Ms; f14=Kw/Ms; f15=0;
f16=0; f21=1; f22=0; f23=0; f24=0; f25=0; f26=0; f31=Bw/Mw;
f32=Kw/Mw; f33=-Bw/Mw; f34=-Kw+Ko)/Mw; f35=0; f36=Ko/Mw;
f41=0; f42=0; f43=1; f44=0; f45=0; f46=0; f51=0; f52=0; f53=0;
f54=Ko/Mt; f55=-Bt/Mt; f56=-(Kt+Ko)/Mt; f61=0; f62=0; f63=0;
f64=0; f65=1; f66=0;
F=[f11 f12 f13 f14 f15 f16
    f21 f22 f23 f24 f25 f26
    f31 f32 f33 f34 f35 f36
    f41 f42 f43 f44 f45 f46
    f51 f52 f53 f54 f55 f56
    f61 f62 f63 f64 f65 f66];
G=[0 0 Ko/Mw 0 -Ko/Mt 0];
gw=Ko*dc*m/hmean; Gw=[0 0 gw/Mw 0 -gw/Mt 0];
H=[0 0 0 -1 0 1]; D=[0];
clear f11 f12 f13 f14 f15 f16 f21 f22 f23 f24 f25 f26;
clear f31 f32 f33 f34 f35 f36 f41 f42 f43 f44 f45 f46;
clear f51 f52 f53 f54 f55 f56 f61 f62 f63 f64 f65 f66;
clear Mt Mw Ms Bt Bw Bs Kt Kw Ks variance;
Program Which Generates the Input Vectors/Matrices.

```matlab
% This file is used to generate surface microhardness variation, force variations, and provide results in graphical form. The simulation start and finish times are also defined. This file is invoked by turnpid.m and turnlqr.m

eps=2e-22;
ti=0; tf=15; %% initial and final times of the simulation
i=1;
t1=ti:Ts:tf;
for t=ti:Tt:tf
    micro(i)=randn; % vector to generate the micro-
i=i+1; % hardness distribution
end
delta_hi=microl*S/max(abs(micro)); % microhardness vector
hi=hmean+delta_hi; % micro-hardness @ location i
w=logspace(-2,6,300); % frequency range for frequency response
set(0,'DefaultFigureColor','white');
set(0,'defaulttextcolor','black');
set(0,'defaultaxesxcolor','black');
set(0,'defaultaxesycolor','black');
set(0,'defaultaxeszcolor','black');
set(0,'defaultsurfacexcolor','black');
figure;
subplot(2,1,1); plot(t1,hi,'black');
title('Workpiece Hardness Distribution');
xlabel('time (s)'); ylabel('Hardness Variation (BHN)');
count=1;
for i=1:30,
    for j=1:sample,
        hard(i,j)=hi(count);
        count=count+1;
    end
end
map=[0 0 0]; colormap(map);
s=[30 60];
subplot(2,1,2); mesh(hard,s);
title('Typical 3D surface micro-hardness distribution');
xlabel('j-th location'); ylabel('i-th section');
```
% Zlabel('material hardness (BHN)');
vv=cov(hi);
ss=sqrt(vv);
Fcut=(Ko*do*m/hmean)*delta_hi; % force fluctuations corresponding
% to hardness variations.
figure;
subplot(2,1,1); plot(t1,Fcut,'black');
title('Profile of Force Fluctuations');
xlabel('time (s)'); ylabel('Force (N)'); grid;
CutF=Ko*do*(hi/hmean).^m;
subplot(2,1,2); % Cutting Force Profile.
plot(t1,CutF,'black'); title('Cutting Force Profile');
xlabel('time (s)'); ylabel('Force (N)');
grid; clear CutF Fcut;
ui1=do; % contribution of initial depth of cut
ui2=delta_hi; % contribution of microhardness variation

% ui is the input vector after linearization.
% NOTE: this is not the 'actual' input since part of it involves
% the output variable and is reflected in the transfer function.

% % transfer function matrix corresponding to state space representation
% % NOTE: (1) vectors are the polynomial coefficients in descending powers of s.
% % (2) the vectors were obtained from 'Mathematica' program transfnf.ma

% for do=0.5E-3m & fo=0.25E-3m
NUM=[143273.38 1.84204E8 1.7337E13 1.144209E15 1.0695E20];
DEN=[1 1.31259 1.45147E8 4.36396E10 3.32705E15 1.9110415E16
2.57684725E20];

% for do=0.305E-3m & fo=0.508E-3m
% NUM=[291132 3.74302E8 3.52289E13 2.93033E15 2.17322E20];
% DEN=[1 1.31259 1.452954E8 4.38297E10 3.34495E15 2.05987E16
3.68056E20];

% low pass filter .....

wn=255;
NUMfilter=[wn];
DENfilter=[1 wn];
pack;
Matlab Program Which Generates the Open Loop Response

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% * This program is used to generate the open loop time response, open %
% * the open loop frequency response, as well as generate root locus and %
% * Nyquist plot. It is invoked by files turnpid.m and turnlqr.m %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open loop response ......

ui=ui1+m/hmean*do.*ui2;
[NUMd,DENd]=bilinear(NUM,DEN,1/Ts);
do,do]=disim(NUMd,DENd,ui); yo=do-do;
figure; subplot(2,1,1); plot(t1,do,'black');
title('Actual Depth of Cut-Open Loop Response');
xlabel('time (s)'); ylabel('depth of cut (m)');
subplot(2,1,2); plot(t1,yo,'black');
title('Tool-Workpiece Relative Displacement-Open Loop Response');
xlabel('time (s)'); ylabel('relative displacement (m)');
[h,W]=freqs(NUM,DEN);
ho=abs(h(1:150)); wo=W(1:150);
figure; plot(wo,ho,'black');
title('Power Spectral Density Plot-Open Loop Response');
xlabel('frequency (rads/s)'); ylabel('power spectral density');
% bode phase/magnitude plots for open loop
[mago,phaseo]=bode(NUM,DEN,w);
[Gm,pm,Wcg,Wcp]=margin(mago,phaseo,w); % gain & phase margins
magodB=20*log10(mago);
figure; subplot(2,1,1); % begin with magnitude plot
semilogx(w,magodB,'black');
title('Bode Magnitude and Phase Plots-Open Loop');
xlabel('magnitude (dB)'); grid;
subplot(2,1,2); % now generate phase plot
semilogx(w,phaseo,'black');
xlabel('frequency (rad/s)'); ylabel('phase (deg)'); grid;
figure; % nyquist/root locus plots for open loop
subplot(2,1,1); rlocus(NUM,DEN); % root locus plot
title('Root Locus Plot for Open Loop System');
subplot(2,1,2); nyquist(NUM,DEN);
title('Nyquist Plot for the Uncompensated System');
pack;
Matlab Program for PID Controller Design.

```matlab
% * This program generates a PID controller in order to minimize tool *
% * workpiece relative displacements. The system matrices, initial and *
% * final simulation times, as well as the input vector ui are as defined *
% * in pmers.m. The transfer function is obtained through use of *
% * MATHEMATICA file trasfnf.ma *
% *
% *
% * NUM(s)  bo*s^m + b1*s^(m-1) + ... + bm *
% * G(s) = ----------- = ---------------------------------- *
% *
% * DEN(s)  ao*s^n + a1*s^(n-1) + ... + an *
% *
% *

pmers();                          % loading the system parameters

inputs % Generate input vectors, surface microhardness variation, force
% variations, and provide results in graphical form. File inputs.m
% is used.
%
%

% transfer function matrix corresponding to state space representation
% NOTE: (1) vectors are the polynomial coefficients in descending powers of s.
% ^^^^ (2) the vectors were obtained from 'Mathematica' program transfnf.ma
%

% for do=0.5E-3m & fo=0.25E-3m
NUM=[143273.38 1.84204E8 1.7337E13 1.144209E15 1.0695E20];
DEN=[1 1312.59 1.45147E8 4.36396E10 3.32705E15 1.9110415E16
     2.57684725E20];
%
% for do=0.305E-3m & fo=0.508E-3m
% NUM=[291132 3.74302E8 3.52289E13 2.93033E15 2.17322E20];
% DEN=[1 1312.59 1.452954E8 4.38297E10 3.44955E15 2.05987E16
     3.68056E20];
%

% Open loop response ......

olresp % compute open loop response using file olresp.m
%

% Given a PID compensator -> Gc=Kp+Ki/s+Kd*s=(Kd*s^2+Kp*s+Ki)/s, gains
ui2=m*do./hmean.*delta_hi;
discr0=[' ', ]; try=1; newtry=1;
zeros=roots(NUM);
while newtry == 1
    poles=roots(DEN);
discr1=['The open loop poles are located at: '];
disp(discr1); disp(discr0);
disp(poles); disp(discr0);
discr2=['The open loop zeros are located at: '];
disp(discr2); disp(discr0);
disp(zeros); disp(discr0);
discr3=['Choose one of the following options for the design of the PID ' 
'compensator:

1. pole-zero cancellation (minimize the effect of 2 residues)
2. Ki selection and desired closed loop pole location
3. select all gains (KP, KI, KD)
4. EXIT (enter a number >= 4)
'];
disp(discr3); disp(discr0);
ans1=input('please enter your choice now -> ');
if ans1 == 1
    poles=roots(DEN);
disp(discr0);
discr4=['
Please enter the open loop poles you wish to have cancelled. '];
disp(discr4); disp(discr0);
discr5=['Once more the open loop poles are ...... '];
disp(discr5); disp(poles); disp(discr0);
ans2=input('enter the number corresponding to the first pole ->');
while ans2 > length(DEN)
    discr_no=['This choice is not acceptable. '
'Make another choice ... ']; disp(discr_no);
    ans2=input('enter the number corresponding to the first pole ->');
end
s1=poles(ans2);
if ans2 == 1


Appendix A: Matlab Programs for Input Parameters and PID Control

```matlab
% opoles(1)=0; %
elseif ans2 == 2 %
    opoles(2)=0; %
elseif ans2 == 3 %
    opoles(3)=0; % cancelling selected pole #1
elseif ans2 == 4 %
    opoles(4)=0; %
elseif ans2 == 5 %
    opoles(5)=0; %
else %
    opoles(6)=0; %
end %
ans3=input('enter the number corresponding to the second pole ->');
while ans3 > length(DEN)
    disp(discr_no);
    ans3=input('enter the number corresponding to the second pole ->');
end
s2=opoles(ans3);
if ans3 == 1 %
    opoles(1)=0; %
elseif ans3 == 2 %
    opoles(2)=0; %
elseif ans3 == 3 %
    opoles(3)=0; % cancelling selected pole #2
elseif ans3 == 4 %
    opoles(4)=0; %
elseif ans3 == 5 %
    opoles(5)=0; %
else %
    opoles(6)=0; %
end %
DENC=[0 1]; one=[0 1];
for M=1:length(opoles)
    if opoles(M) == 0
        DENC=conv(DENC,one);
    else
        DENC=conv(DENC,[1 -opoles(M)]);
    end
end
DENC=conv(DENC,[1,0]); % adding extra pole @ origin due to compensator
num1=[1 -s1]; num2=[1 -s2]; GcNUM=conv(num1,num2); GcDEN=[0 1 0];
KD=[1 0 0]*GcNUM'; KP=[0 1 0]*GcNUM'; KI=[0 0 1]*GcNUM';
elseif ans1 == 2
```

Appendix A: Matlab Programs for Input Parameters and PID Control

Kl=input('Enter the integrator gain Kl -> '); 
s1=input('Enter the desired location of a desired closed loop pole -> '); 
beta=atan2(imag(s1),real(s1));

% This portion of the program evaluates the magnitude and phase angle in 
% radians of the open-loop transfer function G(s)H(s) for the specified 
% complex variable s = a + jw. Num and den are the row vectors containing 
% the polynomial coefficients. This will assist in the evaluation of the 
% rest of the necessary information.

sp=abs(s1); 
m=length(NUM); n=length(DEN); 
for j=1:m 
    sm(j)=s1^(m-j); 
end 
for i=1:n 
    sn(i)=s1^(n-i); 
end 
NUM1=sm*NUM'; 
DEN1=sn*DEN'; 
GH=NUM1/DEN1; 
mag=abs(GH); 
phase=atan2(imag(GH),real(GH)); 
[mag phase]; % magnitude and phase response vectors 
if imag(s1)==0 
    KP=input('Enter the gain Kp of the PID compensator -> '); 
    fprintf('

') 
    ghr=mag*cos(phase); 
    KD=-1/(s1*ghr)-KP/s1-Kl/(s1*s1); 
else 
    KP=-sin(beta+phase)/(mag*sin(beta))-(2*Kl*cos(beta))/sp; 
    KD=sin(phase)/(sp*mag*sin(beta))+Kl/(sp*sp); 
end 
Z0=KP/KD; 
else ans1 == 3 
    KP=input('Enter the proportional gain KP -> '); 
    Kl=input('Enter the integrator gain Kl -> '); 
    KD=input('Enter the differentiator gain KD -> '); 
else 
    discr6=['You DO NOT want any Compensation !!']; 
    disp(discr0); disp(discr6); disp(discr0); 
    GcNUM=[0 0 1]; GcDEN=[0 0 1]; 
end
% The characteristic Equation of the compensated system is now formed ...
% 
if ans1 >= 4
    discr7=[
        'Open Loop response has already been provided. The program will now'
        'terminate. Re-run the program if you wish to include compensation.'];
    disp(discr0); disp(discr7); disp(discr0);
else

% the characteristic equation of the compensated system is now formed.

    if ans1 == 1
        n=length(DENC);
    else
        n=length(DEN);
    end
    m=length(NUM);
    if n > m
        o=zeros(1,n-m); mk=[o,1];
        if ans1 == 1
            NUM2=conv(NUM,mk);
        end
    else
        NUM2=NUM;
    end
    if ans1 == 1
        NUMopen=NUM2;
        DENopen=abs(DENC);
    else
        if KI == 0
            GcNUM=[KD KP]; GcDEN=[0 1];
        else
            GcNUM=[KD KP KI]; GcDEN=[0 1 0];
        end
        DENopen=conv(GcDEN,DEN); NUMopen=conv(GcNUM,NUM);
    end
    m1=length(NUMopen); n1=length(DENopen);
    if n1>m1
        NUM3=zeros(1,n-m);
        NUMopen=[NUM3 NUMopen];
    end
Appendix A: Matlab Programs for Input Parameters and PID Control

```
% The compensator (PID controller) transfer function is:
discr11=[
    'Gc = %g',KP,fprintf(' + %g',KI),fprintf(' + %g',KD),fprintf(' in

DENcsls=DENopen+NUMopen;
NUMopen=abs(NUMopen); DENcsls=abs(DENCsls);
GH=NUMopen/DENCsls;

Row vectors of polynomial coefficients of the compensated system:

Open-loop numerator:'), disp(NUMopen)
Open-loop denominator:'), disp(DENopen)
Closed-loop denominator:'), disp(DENcsls)

Roots of the compensated characteristic equation:

Root on the RHP, system is unstable. Change s1 and/or s2'
and/or gain(s) KI and/or KP and/or KD and repeat.

Roots on the LHP, system is stable.

% closed loop response (PID compensation) ......
% 

[NUMopenD,DENCslsD]=bilinear(NUMopen,DENCsls,1/Ts);
[d_do,x_do]=dlsim(NUMopenD,DENCslsD,ui1);
[NUMd,DENod]=bilinear(NUM,DENCsls,1/Ts);
[d_hi,x_hi]=dlsim(NUMd,DENod,ui2);
dac=d_do+d_hi; clear x_hi x_do d_do d_hi;
figure; subplot(2,1,1); plot(t1,dac,'black');
title('Actual Depth of Cut-Closed Loop Response');
xlabel('time (s)'); ylabel('depth of cut (m)');
yc=dac-do;
subplot(2,1,2); plot(t1,yc,'black');
title('Tool-Workpiece Relative Displacement-Closed Loop Response');
xlabel('time (s)'); ylabel('relative displacement (m)');
```

% filtered results-filter as defined in lpfilt.m
% lpfilt;
[h, W]=freqs(NUMopen, DENclsd);
hc=abs(h(1:150)); figure; subplot(2,1,1); plot(wo, hc,'black');
title('Power Spectral Density Plot-Closed Loop Response');
ylabel('power spectral density');
subplot(2,1,2); plot(wo, ho, wo, hc,'black');
title('Power Spectral Density Plot-Closed and Open Loop Response');
xlabel('frequency (rads/s)');

if try~=1
    discrt=['would you like to try a different compensator?';
    newtry=input('please enter yes(1) or no(0)');
end
end

figure; % bode phase/magnitude plots for closed loop system
[magc, phasec, w]=bode(NUMopen, DENclsd, w);
magcdB=20*log10(magc);
subplot(2,1,1); semilogx(w, magcDB,'black');
title('Bode Magnitude and Phase Plots-Closed Loop');
ylabel('magnitude (dB)'); grid;
subplot(2,1,2); semilogx(w, phasec,'black');
xlabel('frequency (rad/s)'); ylabel('phase (deg)'); grid;
figure; % nyquist plot for closed loop system
nyquist(NUMopen, DENopen);
title('Nyquist Plot for the Compensated System');

% this section of the program program generates a cross-section of the
% surface profile as well as 3D surface topology. The tool-workpiece
% relative displacements are further processed (tool geometry is
% incorporated) to generate the 'actual' surface profile. Sections
% equivalent to the sampling per revolution are generated and meshed to
% generate a 3D surface topology.

% --------------------------------------------------
% Due to the presence of the tool lead angle the coordinate y*** is
% related to the projection of the system response. Hence....
% (lead=lead angle; feeds=no of feeds in each cross-section)
% --------------------------------------------------

pack
count=1;
feeds=30;
yo1=(yo)*sin(lead); yc1=(yc)*sin(lead);
for i=1:541,
    ytopo(i)=0; ytopc(i)=0;
end
topol1=[ytopo'; topol2=[ytopc'];
for counter=1:10,
    for i=1:feeds,
        yo2(i)=yo1(count); yc2(i)=yc1(count);
        count=count+sample;
    end
%
-----------------------------------------------------------------------------
% generating the centre points of the cutting tool as it moves along the
% given cross-section of the workpiece....
%-----------------------------------------------------------------------------

for i=1:feeds-1,
    Cx(i)=(2*i-1)/2*fo; Cyo(i)=abs(yo2(i)); Cyc(i)=abs(yc2(i));
end
%
-----------------------------------------------------------------------------
% generating the points of intersection between adjacent circles produced
% by the tool geometry....
% Equations used: (x^2-Cx(k))^2+(y^2-Cyo(k))^2=R^2
% (x^2-Cx(k+1))^2+(y^2-Cyo(k+1))^2=R^2
% NOTE: ALL OTHER VARIABLES ARE FOR CONVENIENCE DURING THE
% INTERMEDIATE STEPS
%
-----------------------------------------------------------------------------

for k=1:(feeds-2),
    a=Cx(k+1)-Cx(k); bo=(Cyo(k)-Cyo(k+1)); co=Cyo(k+1)^2-Cyo(k)^2-a^2;
    c2o=4*a^2+4*bo^2; c1o=4*bo*co-8*a^2*Cyo(k+1);
    c0o=co^2-4*a^2*R^2+4*a^2*Cyo(k+1)^2;
    yo_int(k)=(-c1o+sqrt((c1o^2-4*c2o*c0o)/(2*c2o)))/2*c2o;
    xo_int(k)=sqrt(R^2-(yo_int(k)-Cyo(k))^2)+Cx(k);
    bc=Cyc(k)-Cyc(k+1); cc=Cyc(k+1)^2-Cyc(k)^2-a^2;
    c2c=4*a^2+4*bc^2;
    c1c=4*bc*cc-8*a^2*Cyc(k+1); c0c=cc^2-4*a^2*R^2+4*a^2*Cyc(k+1)^2;
    yc_int(k)=(-c1c+sqrt((c1c^2-4*c2c*c0c))/(2*c2c));
    xc_int(k)=sqrt(R^2-(yc_int(k)-Cyc(k))^2)+Cx(k);
end
%
-----------------------------------------------------------------------------
% generating points in order to clearly define the circles which depict the
Appendix A: Matlab Programs for Input Parameters and PID Control

% surface profile....
% -> 20 points are used to generate the tool signature on the surface for each
%     feed increment.
%  
% count1=1; count2=20;
x topo(1)=0; xtopc(1)=0;
inco=xo_int(1)/20; incc=xc_int(1)/20;
for g=1:(feeds-3),
    for h=count1:count2;
        y topo(h)=1-sqrt(R^2-(x topo(h)-Cx(g))^2)+Cy(g);
x topo(h+1)=x topo(h)+inco;
    end
    count1=count1+1; count2=count2+20;
inco=(xo_int(g+1)-xo_int(g))/20;
incc=(xc_int(g+1)-xc_int(g))/20;
end
y topo(h+1)=1-sqrt(R^2-(x topo(h+1)-Cx(g))^2)+Cy(g);
y topo(h+1)=1-sqrt(R^2-(x topo(h+1)-Cx(g))^2)+Cy(g);
count=counter;
y topo=y topo-min(y topo); y topo=y topo-min(y topo);
topol1=[topol1 y topo']; topol2=[topol2 y topo'];
end
dummy10=[0 0 0 0 0 0 0 0 0 0]; surf=[dummy10' eye(10)];
topol1=surf*topol1'; topol2=surf*topol2';

% Plotting a particular crossection of the surface topology followed by the 3D
% surface topography ....
%  
s=[30 60];
figure; subplot(2,1,1);
plot(x topo,y topo,'black');
title('Cross Section of the Surface Topography-Open Loop');
xlabel('distance along workpiece (m)');
ylabel('tool signature (m)');
map=[0 0 0]; colormap(map);
subplot(2,1,2); mesh(topol1,s);
title('Surface Topography-Open Loop');
zlabel('Depth of Cut (m)');
ylabel('Cutting Speed (pi*D/20)');
xlabel('Feed (fo/20)');
figure; subplot(2,1,1);
plot(xtopc,ytopc,'black');
title('Cross Section of the Surface Topography-Closed Loop');
xlabel('distance along workpiece (m)');
ylabel('tool signature (m)');
map=[0 0 0]; colormap(map);
subplot(2,1,2); mesh(topol2,s);
title('Surface Topography-Closed Loop');
zlabel('depth of cut (m)');
ylabel('cutting speed (pi*D/20)');
xlabel('Feed (fo/20)');
Appendix B: Matlab Programs for LQ/LQG Optimal Controller Design

Matlab Program for LQ and LQG Optimal Controller Design.

% *---------------------------------------------------------------------%
% * This program generates an optimal control law for the purpose of     *
% * suppressing the tool-workpiece relative displacement during a turning *
% * operation. Variations in the workpiece hardness are also depicted and *
% * assumed to be the major factor causing these relative displacements.  *
% *---------------------------------------------------------------------%

%
% Loading the physical parameters which define the model of
% the turning process. They can be found and altered if
% desired in the file pmeters.m
%

pmetersl;

% use Rt for tool nose radius because R is a weight matrix
Rt=R;
inputs;

% open loop response (repeated) ......
% ...
oiresp % go through open loop response;

ui=[ui1' ui2']; clear ui1 ui2;
G1=[G Gw]; D1=[D D];
[NUMo,DENo]=ss2tf(F,G1,H,D1,1);

% check for controllability before designing a controller.....
% ...
olpoles=eig(F); % computing the open loop poles
n=size(F)*[1 0];
Appendix B: Matlab Programs for LQ and LQG Optimal Control

Co=ctrb(F,G1);
if rank(Co)==n
discr0=[' '];
discr1=['pair [F,G1] is controllable->controller can be designed. '];
disp(discr1); disp(discr0);

% _______________________________________________________
% quadratic weight matrices ..... 
% _______________________________________________________ 

q11=0; q22=0; q33=0.1; q44=1; q55=0.1; q66=1;
Q=diag([q11 q22 q33 q44 q55 q66]);
RowCol=size(G1); Col=RowCol(2); r=0.0001; R=r*eye(Col);

% _______________________________________________________
% obtaining the optimal feedback gain matrix .... 
% _______________________________________________________

[K,P,E]=lqr(F,G1,Q,R); % E=closed loop eigenvalues
discr3=['Optimal controller has been designed... '
' The gain matrix K is: ', disp(discr3); disp(discr0); K

% _______________________________________________________
% Closed loop response ..... 
% _______________________________________________________

Fc=F-G1*K;
[yc,xc]=lsim(Fc,G1,H,D1,ui,t1);
figure;
subplot(2,1,1); plot(t1,yc+do,'black');
title('Actual Depth of Cut-Closed Loop Response');
xlabel('time (s)'); ylabel('depth of cut(m)');
subplot(2,1,2); plot(t1,yc,'black');
title('Tool-Workpiece Relative Displacement-Closed Loop Response');
xlabel('time (s)'); ylabel('relative displacement (m)');
figure; % bode phase/magnitude plots for closed loop system
[magc,phasc]=bode(Fc,G1,H,D1,1,w);
magdB=20*log10(magc);
subplot(2,1,1); semilogx(w,magdB,'black');
title('Bode Magnitude and Phase Plots-Closed Loop');
ylabel('magnitude (dB)'); grid;
subplot(2,1,2); semilogx(w,phasc,'black');
xlabel('frequency (rad/s)'); ylabel('phase (deg)'); grid;
clear phassec magc magcDB;
figure; % nyquist plot for closed loop system
nyquist(Fc,G,H,D,1);
title('Nyquist Plot for Closed Loop System'); % pause

%------------------------------------------------------------------------
% Pole placement: left of line s=-alpha ....
%------------------------------------------------------------------------

l6=eye(size(F));
alpha=500; % time constant
Fp=F+alpha*l6;
[Kpp,Ppp,Epp]=lqr(Fp,G1,Q,R); % Epp=closed loop poles
Fpp=F-G1*Kpp; % after pole placement
[ycpp,xcpp]=lsim(Fpp,G1,H,D1,ui,t1);
figure;
 subplot(2,1,1); plot(t1,ycpp+do,'black');
title('Actual Depth of Cut-Pole Placement');
xlabel('time (s)'); ylabel('relative displacement (m)');
 subplot(2,1,2); plot(t1,ycpp,'black');
title('Tool-Workpiece Relative Displacement-Pole Placement');
xlabel('time (s)'); ylabel('relative displacement (m)');
 figure; % nyquist/root locus plots after pole placement
% subplot(2,1,1); riocus(Fpp,G,H,D);
% title('Root Locus Plot for Open Loop System-Pole Placement');
% subplot(2,1,2);
% subplot(2,1,1); nyquist(Fpp,G1,H,D1,1,w);
% title('Nyquist Plot for Closed Loop System-Pole Placement (input=do)');
% subplot(2,1,2); nyquist(Fpp,G1,H,D1,2,w);
% title('Nyquist Plot for Closed Loop System-Pole Placement (input=hi)');

%------------------------------------------------------------------------
% check for observability before designing an observer...
%------------------------------------------------------------------------

Ob=obsv(F,H);
if rank(Ob)==n
    discr4=['pair [F,H] is completely observable -> observer can be designed..'];
disp(discr4); disp(discr0);

%------------------------------------------------------------------------
% state estimator (observer) design ....
Appendix B: Matlab Programs for LQ and LQG Optimal Control

% note: estimator should be 5-10 times faster than regulator

OutputNoise=1e-7*(randn(size(t1))-0.1);
InputNoise=0.005*(randn(size(t1))-0.5);
QKalman=cov(InputNoise);
RKalman=cov(OutputNoise);
clear InputNoise OutputNoise;
beta=5*alpha; F0=Fp+beta*F6; % guaranteeing estimator performance
%q=Gw*h*Gw'; v=cov(h);
[Ke,Pe,Ee]=lqe(F0,Gw,H,QKalman,RKalman); % Ee=eigenvalues of F+Ke*H'
discr5=['Kalman filter has been designed...';
'The gain matrix Ke is:'];
disp(discr5); disp(discr0); Ke'

% Controller-Observer overall system and Response ....
% dx/dt=(F-GK)x+GKe -> state equation
% de/dt=(F-KeH)e -> observer error equation

Foc=[F-G1*Kpp G1*Kpp zeros(6) F-Ke*H];
zero2=[0 0 0 0 0 0 0 0 0 0];
Goc=[G1' zero2];
Hoc=[H 0 0 0 0 0 0 0 0 0];
Doc=[D1];
figure;
yoc,xoc=lsim(Foc,Goc,Hoc,Doc,ui,t1);
subplot(2,1,1); plot(t1,yoc+do,'black');
title('Actual Depth of Cut-Controller/Observer System');
xlabel('time (s)'); ylabel('depth of cut (m)');
subplot(2,1,2); plot(t1,yoc,'black');
title('Tool-Workpiece Relative Displacement-Controller/Observer System');
xlabel('time (s)'); ylabel('relative displacement (m)'); % pause
figure;
plot(t1,yoc-ycpp,'black');
title('Difference Between Observer/Controller System and Pole-Placement');
ylabel('Difference in Tool-Workpiece Realitive Displacement (m)');
xlabel('time (s)'); % pause
[NUMc1,DENC1]=ss2tf(Fc,G1,H,D1,1);
[NUMc2, DENc2] = ss2tf(Fc, G1, H, D1, 2);
[hc1, W] = freqs(NUMc1, DENc1); [hc2, W] = freqs(NUMc2, DENc2);
[NUMpp1, DENpp1] = ss2tf(Fpp, G1, H, D1, 1);
[NUMpp2, DENpp2] = ss2tf(Fpp, G1, H, D1, 2);
[hpp1, W] = freqs(NUMpp1, DENpp1); [hpp2, W] = freqs(NUMpp2, DENpp2);
[NUMoc1, DENoc1] = ss2tf(Foc, Goc, Hoc, Doc, 1);
[NUMoc2, DENoc2] = ss2tf(Foc, Goc, Hoc, Doc, 2);
[hoc1, W] = freqs(NUMoc1, DENoc1); [hoc2, W] = freqs(NUMoc2, DENoc2);
figure;

plot(W(1:80), abs(hc1(1:80)+hc2(1:80)), W(1:80), abs(hpp1(1:80)+hpp2(1:80)), W(1:80), abs(hoc1(1:80)+hoc2(1:80)), W(1:80), abs(hpp1(1:80)+hpp2(1:80)), W(1:80), abs(hoc1(1:80)+hoc2(1:80)), W(1:80)); % power spectral density plot
    title('Power Spectral Density Plot-Closed Loop (All Designs)');
    xlabel('frequency (rads/s)'); ylabel('power spectral density');
    clear W hpp1 hpp2 hoc1 hoc2 hc1 hc2; % pause
figure; % bode phase/magnitude plots for open loop
[magoc, phaseoc] = bode(Foc, Goc, Hoc, Doc, 1, w);
magocdB = 20*log10(magoc);
    subplot(2, 1, 1); semilogx(w, magocdB, 'black');
    title('Bode Magnitude and Phase Plots - Observer/Controller');
    ylabel('magnitude (dB)'); grid;
    subplot(2, 1, 2); semilogx(w, phaseoc, 'black');
    xlabel('frequency (rad/s)'); ylabel('phase (deg)'); grid;
    clear phaseoc magoc magocdb; % pause
    % pause
figure; % nyquist plot for closed loop
    subplot(2, 1, 1); nyquist(Foc, Goc, Hoc, Doc, 1, w);
    title('Nyquist Plot Closed Loop - Observer/Controller (input=do)');
    subplot(2, 1, 2); nyquist(Foc, Goc, Hoc, Doc, 2, w);
    title('Nyquist Plot Closed Loop - Observer/Controller (input=hi)');
else
    discr6 = ['one or more states are not observable->observer has not been designed'];
    disp(discr6); disp(discr0);
end

% this section of the program program generates a cross-section of the
% surface profile as well as 3D surface topology. The tool-workpiece
% relative displacements are further processed (tool geometry is
% incorporated) to generate the 'actual' surface profile. Sections
% equivalent to the sampling per revolution are generated and meshed to
% generate a 3D surface topology.
count=1;
feeds=30;
yoc1=(yoc)*sin(lead);
for i=1:541,
ytopoc(i)=0;
end
topol1=[ytopoc];
for counter=1:sample,
  for i=1:feeds,
yoc2(i)=yoc1(count);
  count=count+sample;
end

for l=1:feeds-1,
  Cx(l)=(2*l-1)/2*fo; Cyoc(l)=abs(yoc2(l));
end

for k=1:(feeds-2),
a=Cx(k+1)-Cx(k); bo=(Cyoc(k)-Cyoc(k+1)); co= Cyoc(k+1)^2-Cyoc(k)^2-a^2;
c2o=4*a^2+4*bo^2; c1o=4*bo*co-8*a^2*Cyoc(k+1);
c0o=co^2-4*a^2*R^2+4*a^2*Cyoc(k+1)^2;
yoc_int(k)=(-c1o+sqrt(c1o^2-4*c2o*c0o))/(2*c2o);
xoc_int(k)=sqrt(R^2-(yoc_int(k)-Cyoc(k))^2)+Cx(k);
Appendix B: Matlab Programs for LQ and LQG Optimal Control

end

% generating points in order to clearly define the circles which
% depict the surface profile....
% -> 20 points are used to generate the tool signature on
% the surface for each feed increment.
% count1=1; count2=20;
xtopoc(1)=0;
inoc=xoc_int(1)/count2;
for g=1:(feeds-3),
    for h=count1:count2;
        ytopoc(h)=1-sqrt(Rt^2-(xtopoc(h)-Cx(g))^2)+Cyoc(g);
        xtopoc(h+1)=xtopoc(h)+inoc;
    end
    count1=count2+1; count2=count2+20;
inoc=(xoc_int(g+1)-xoc_int(g))/20;
end
ytopoc(h+1)=1-sqrt(Rt^2-(xtopoc(h+1)-Cx(g))^2)+Cyoc(g);
count=counter;
ytopoc=ytopoc-min(ytopoc);
topol1=[topol1 ytopoc];
end
surf=zeros(sample,1) eye(sample);
topol1=surf*topol1;

% Plotting a particular crosssection of the surface topography
% followed by the 3D surface topography ....
% s=[30 60];
figure; subplot(2,1,1);
plot(xtopoc,ytopoc,'black');
title('Cross Section of the Surface Topography-Optimal (LQ) Design');
xlabel('distance along workpiece (m)');
ylabel('tool signature (m)');
map=[0 0 0]; colormap(map);
subplot(2,1,2); mesh(topol1,s);
title('Surface Topography-Optimal (LQ) Design');
zlabel('Depth of Cut (m)');
ylabel('Cutting Speed (pi*D/20)');
xlabel('Feed [*fo(m)]');
Appendix B: Matlab Programs for LQ and LQG Optimal Control

\[ \text{discr10=['would you like a frequency shaped design?']}; \]
\[ \text{disp(discr10); disp(discr0);} \]
\[ \text{ansfs=input('enter 0 for No or anything else for yes->');} \]
\[ \text{if ansfs ~= 0} \]
\[ \text{fsbr; \% call the frequency shaping routine} \]
\[ \text{end} \]

\% Evaluate the performance cost associated with each design considered ...
\% \text{cost}=E[\text{sum}[x'(t)Qx(t)+u'(t-1)Ru(t-1)]] \text{for } t=to->tf
\%

\[ \text{[yo, xo]=lsim(F,G1,H,D1,ui,t1);} \]
\[ \text{costo}=0; \text{costpp}=0; \text{costoc}=0; \text{costfs}=0; \text{costbr}=0; \]
\[ \text{for } k1=1:tf/Ts+1 \]
\[ \text{% performance cost associated with open loop plant} \]
\[ \text{pcq(k1)=xo(k1,::)*Q*xo(k1,::);} \]
\[ \text{pcr(k1)=ui(k1,::)*R*ui(k1,::);} \]
\[ \text{costo=costo+pcq(k1)+pcr(k1);} \]
\[ \text{% performance cost associated with pole placement-LQ design} \]
\[ \text{pcq(k1)=xcpp(k1,::)*Q*xcpp(k1,::);} \]
\[ \text{pcr(k1)=ui(k1,::)*R*ui(k1,::);} \]
\[ \text{costpp=costpp+pcq(k1)+pcr(k1);} \]
\[ \text{% performance cost associated with observer/controller-LQG design} \]
\[ \text{pcq(k1)=xcq(k1,1:6)*Q*xcq(k1,1:6);} \]
\[ \text{pcr(k1)=ui(k1,::)*Kpp*R*Kpp*ui(k1,::);} \]
\[ \text{costoc=costoc+cov(pcq(k1))+cov(pcr(k1));} \]
\[ \text{if ansfs ~= 0} \]
\[ \text{% performance cost associated with reduced order controller-LQG design} \]
\[ \text{pcq(k1)=xcfs(k1,::)*Q*xcfs(k1,::);} \]
\[ \text{pcr(k1)=ui(k1,::)*R*ui(k1,::);} \]
\[ \text{costfs=costfs+cov(pcq(k1))+cov(pcr(k1));} \]
\[ \text{% performance cost associated with observer/controller-LQG design} \]
\[ \text{pcq(k1)=xcbr(k1,::)*Q*xcbr(k1,::);} \]
\[ \text{pcr(k1)=ui(k1,::)*R*ui(k1,::);} \]
\[ \text{costbr=costbr+pcq(k1)+pcr(k1);} \]
\[ \text{end;} \]
\[ \text{end;} \]
\[ \text{discr11=['The performance cost associated with the open loop plant is:']}; \]
\[ \text{disp(discr11); disp(discr0); costo} \]
\[ \text{discr12=['The performance cost associated with the closed loop plant'} \]
\[ \text{'with pole placement is:'}; \]
\[ \text{disp(discr0); disp(discr12); disp(discr0); costpp} \]
\[ \text{discr13=['The performance cost associated with the closed loop plant'} \]
\[ \text{'with LQG regulator:'}; \]
\[ \text{disp(discr0); disp(discr13); disp(discr0); costpp} \]
\[ \text{if ansfs ~= 0} \]
Appendix B: Matlab Programs for LQ and LQG Optimal Control

discr14=['The performance cost associated with the closed loop ' 'plant with LQG regulator and frequency shaping is: ']);
disp(discr0); disp(discr14); disp(discr0); costfs
discr15=['The performance cost associated with the closed loop ' 'plant with LQG regulator (reduced order controller ' ]); 
disp(discr0); disp(discr15); disp(discr0); costbr
end
else
discr7=['one or more states are not controllable->the program will now terminate'];
disp(discr7); disp(discr0);
end

Matlab Program for Frequency Shaped Designs and Controller Reduction

%  _______________________________________
%  | Frequency Shaping -> Frequency Weighting Valley..... |
%  | _______________________________________
%  
%  s^2+2*zeta1*wn*s+wn^2
%  W(s)=---------------------------
%  | s^2+2*zeta2*wn*s+wn^2 |
%  
%  % NOTE: Frequency Shaping is based on the natural frequency
%  % —— that exhibits the highest contribution to the power spectral density.
%  %  _______________________________________

zeta1=0.8; zeta2=0.0008; Wn=2560;
numf=[1 2*zeta1*Wn Wn^2];
denf=[1 2*zeta2*Wn Wn^2];
[Ff,Gf,Hf,Df]=tf2ss(numf,denf);
[magf,phasef]=bode(Ff,Gf,Hf,Df,1,w);
magfdB=20*log10(magf);
figure;
subplot(2,1,1); semilogx(w, magfdB);
title('Bode Magnitude and Phase Plots-Frequency Weighting Valley');
ylabel('magnitude (dB)'); grid;
subplot(2,1,2); semilogx(w, phasef);
xlabel('frequency (rad/s)'); ylabel('phase (deg)'); grid;

%  _______________________________________
%  | Frequency Shaped Controller ...... |
%  %  _______________________________________


zero1=zeros(2,6); 
zero2=zeros(2,2);
zero5=zeros(2,16);
Fdfs=[ Fp 
   zero1
   Gf'H   Ff ];
Fdfs1=[ F 
   zero1
   Gf'H   Ff ];
Gdfs=[G1' zero2];
Hdfs=[Df'H Hf];
Ddfs=[D D];
Q1=[ Q 
   zero1
   zero1' 
   zero2];
Qdfs=Q1+Hdfs'*Hdfs;
[Kdfs,Pdfs,Edfs]=lqr(Fdfs,Gdfs,Qdfs,R); % Edfs=closed loop poles
Fdfs=Fdfs1-Gdfs*Kdfs; % after frequency shaping
figure;
[ydfs,xdfs]=lsim(Fdfs,Gdfs,Hdfs,Ddfs,ui,t1);
plot(t1,ydfs);
title('Tool-Workpiece Relative Displacement-Frequency Shaping');
xlabel('time (s)'); ylabel('relative displacement (m)');
[NUMfs1,DENfs1]=ss2tf(Fdfs,Gdfs,Hdfs,Ddfs,1);
[NUMfs2,DENfs2]=ss2tf(Fdfs,Gdfs,Hdfs,Ddfs,2);
[hfs1,W]=frolems(NUMfs1,DENfs1); [hfs2,W]=frolems(NUMfs2,DENfs2);
figure;
plot(W(1:100),abs(hfs1(1:100)+hfs2(1:100))); % power spectral density plot
xlabel('frequency (rads/s)'); ylabel('power spectral density');
figure; % nyquist plot after frequency shaping
subplot(2,1,1); nyquist(Fdfs,Gdfs,Hdfs,Ddfs,1);
title('Nyquist Plot-Frequency Shaping (input=do)');
subplot(2,1,2); nyquist(Fdfs,Gdfs,Hdfs,Ddfs,2);
title('Nyquist Plot-Frequency Shaping (input=hi)');

% Controller-Observer overall system and Response ....
% dx/dt=(F-GK)x+GKe -> state equation
% de/dt=(F-KeH)e -> observer error equation
% 
Fdfs=[Fdfs-Gdfs*Kdfs  Gdfs*Kdfs 
   zeros(8,8)  Fdfs-Kdfs*Hdfs];
zero2=zeros(2,6);
Gdfs=[Gdfs' zeros(2,8)];
Hocfs=[Hcfs zeros(1,8)];
Docfs=[D1];
figure;
[yocfs,xocfs]=lsim(Focfs,Gocfs,Hocfs,Docfs,ui,t1);
subplot(2,1,1); plot(t1,yocfs+do,'black');
title('Actual Depth of Cut-Controller/Observer System');
xlabel('time (s)'); ylabel('depth of cut (m)');
subplot(2,1,2); plot(t1,yocfs,'black');
title('Tool-Workpiece Relative Displacement-Controller/Observer System');
xlabel('time (s)'); ylabel('relative displacement (m)'); % pause
[NUMocfs1,DENocfs1]=ss2tf(Focfs,Gocfs,Hocfs,Docfs,1);
[NUMocfs2,DENocfs2]=ss2tf(Focfs,Gocfs,Hocfs,Docfs,2);
[hocfs1,W]=freqs(NUMocfs1,DENocfs1);
[hocfs2,W]=freqs(NUMocfs2,DENocfs2);
figure;
plot(W(1:80),abs(hocfs1(1:80)+hocfs2(1:80)));
% power spectral density plot
xlabel('frequency (rads/s)'); ylabel('power spectral density');
clear W hpp1 hpp2 hoc1 hoc2 hc1 hc2; % pause
figure; % bode phase/magnitude plots for open loop
[magocfs,phaseocfs]=bode(Focfs,Gocfs,Hocfs,Docfs,1,w);
magocfsdB=20*log10(magocfs);
subplot(2,1,1); semilogx(w,magocfsdB,'black');
title('Bode Magnitude and Phase Plots - Observer/Controller');
ylabel('magnitude (dB)'); grid;
subplot(2,1,2); semilogx(w,phaseocfs,'black');
xlabel('frequency (rad/s)'); ylabel('phase (deg)'); grid;
clear phaseocfs magocfs magocfsdB; % pause
figure; % nyquist plot for closed loop
subplot(2,1,1); nyquist(Focfs,Gocfs,Hocfs,Docfs,1,w);
title('Nyquist Plot - LQG design with frequency shaping (input=do)');
subplot(2,1,2); nyquist(Focfs,Gocfs,Hocfs,Docfs,2,w);
title('Nyquist Plot - LQG design with frequency shaping (input=hi)');

% Controller and observer reduction ..............
% balanced realization and truncation....
pack;
Ir=eye(size(F));
s=[Ir zero1];
K1=s*Kcfs'; Ke1=s*Kefs;
Appendix B: Matlab Programs for LQ and LQG Optimal Control

\( \text{Irr} = \text{eye(size(F))}; \)
\( \text{ss} = [\text{zero1 Irr}]; \)
\( \text{K2} = \text{ss}^* \text{Kcf}'; \text{Ke2} = \text{ss}^* \text{Kefs}; \)

\% full order controller
\% frequency weighted balanced truncation

\[ \begin{align*}
\text{[Ac, Bc, Cc, Dc]} &= \text{reg(Fpp, G1, H, D1, Kpp, Ke)}; \\
\text{[Fbr, Gbr, Hbr, Hn, T]} &= \text{balreal(Ac, Bc, Cc)}; \quad \% \text{Hn are Hankel} \\
& \quad \% \text{singular values} \\
\text{elimc} &= [5, 6]; \quad \% \text{vector describing states to eliminate} \\
\text{[Fr, Gr, Hr, Dr]} &= \text{modred(Fbr, Gbr, Hbr, D1', elimc)}; \quad \% \text{reduced controller} \\
\text{[NumRedCont, DenRedCont]} &= \text{ss2tf(Fr, Gr, Hr, Dr)}; \quad \% \text{and transfer function} \\
\text{[Ae, Be, Ce, De]} &= \text{estim(Fpp, G1, H, D1, Ke)}; \quad \% \text{Hn are Hankel} \\
& \quad \% \text{singular values} \\
\text{elim}= [5, 6]; \quad \% \text{vector describing states to eliminate} \\
\text{[Fre, Gre, Hre, Dre]} &= \text{modred(Fbre, Gbre, H, D, elim)}; \quad \% \text{reduced controller} \\
\text{[NumRedEst, DenRedEst]} &= \text{ss2tf(Fre, Gre, Hre, Dre)}; \quad \% \text{and transfer function} \\
\text{discr1} &= \{'\text{the reduced order observer transfer function is:}'\} \\
\text{disp(discr1); NumRedEst} \quad \% \text{display numerator} \\
\text{DenRedEstt} \quad \% \text{display denominator} \\
\text{[magfoc, phasefoc]} &= \text{bode(Ac, Bc, Cc, Dc, 1, w)}; \\
\text{magfocdB} &= 20*\log10(\text{magfoc}); \\
\text{[magroc, phasero]} &= \text{bode(Fr, Gr, Hr, Dr, 1, w)}; \\
\text{magrocdB} &= 20*\log10(\text{magroc}); \\
\text{figure; plot(w, magfocdB, w, magrocdB)}; \quad \quad \% \text{bode magnitude plot-full and reduced order controllers}\) \\
\text{title('bode magnitude plot-full and reduced order controllers');} \\
\text{xlabel('frequency (rad/s)');} \\
\text{ylabel('magnitude (dB)');} \\
\text{Fro} &= [\text{Fpp} - \text{G1}^* \text{Hr} \quad \% \text{augmented system} \\
\text{Gr}^* \text{Ht} \quad \text{Fr}]; \quad \% \text{performance evaluation} \\
\text{dummy1} &= \text{min(size(G1))}; \text{dummy2} = \text{max(size(G1)-length(elimc));} \\
\text{Gro} &= [\text{G1}' \text{ zeros(dummy1, dummy2)}]; \quad \% \text{performance evaluation} \\
\text{Hro} &= [\text{H zeros(1, dummy2)}]; \quad \% \text{controller performance evaluation} \\
\text{[magrs, phars]} &= \text{bode(Fro, Gro, Hro, D1, 1, w)}; \quad \% \text{of reduced order} \\
\text{magrsdB} &= 20*\log10(\text{magrs}); \quad \% \text{controller} \\
\text{figure; semilogx(w, magrsdB, w, magrocdB)}; \quad \% \text{title('bode magnitude plot-full and reduced order controllers');} \\
\text{xlabel('frequency (rad/s)');} \\
\text{ylabel('magnitude (dB)');} \\
\text{nyquist(Fro, Gro, Hro, D1);} \)
% this section of the program generates a cross-section of the
% surface profile as well as 3D surface topology. The tool-workpiece
% relative displacements are further processed (tool geometry is
% incorporated) to generate the 'actual' surface profile. Sections
% equivalent to the sampling per revolution are generated and meshed to
% generate a 3D surface topology.

% %
% % Due to the presence of the tool lead angle the coordinate y*** is
% % related to the projection of the system response. Hence....
% % (lead=lead angle; feeds= no of feeds in each cross-section)
% %
% count=1;
feeds=30;
yoc1=(ycfs)*sin(lead);
for i=1:541,
    ytopoc(i)=0;
end
topol1=[ytopoc'];
for counter=1:sample,
    for i=1:feeds,
        yoc2(i)=yoc1(count);
        count=count+sample;
    end

% 
% generating the centre points of the cutting tool as it moves along the
% given cross-section of the workpiece....
%
for l=1:feeds-1,
    Cx(l)=(2*l-1)/2*fo; Cyoc(l)=abs(yoc2(l));
end

% 
% generating the points of intersection between adjacent circles produced
% by the tool geometry....
% Equations used: (x*_int-Cx*(k))^2+(y*_int-Cy*(k))^2=R^2
% (x*_int-Cx*(k+1))^2+(y*_int-Cy*(k+1))^2=R^2
% ALL OTHER VARIABLES ARE FOR CONVENIENCE DURING THE
% INTERMEDIATE STEPS
%
for k=1:(feeds-2),
    a=Cx(k+1)-Cx(k); bo=(Cyoc(k)-Cyoc(k+1)); co=Cyoc(k+1)^2-Cyoc(k)^2-a^2;
    c2o=4*a^2+4*bo^2; c1o=4*bo*co-8*a^2*Cyoc(k+1);
    c0o=co^2-4*a^2*Rt^2+4*a^2*Cyoc(k+1)^2;
    yoc_int(k)=(-c1o+sqrt(c1o^2-4*c2o*c0o))/(2*c2o);
    xoc_int(k)=sqrt(Rt^2-(yoc_int(k)-Cyoc(k))^2)+Cx(k);
end

% generating points in order to clearly define the circles which
% depict the surface profile....
% -> 20 points are used to generate the tool signature on
% the surface for each feed increment.

count1=1; count2=20;
xtopoc(1)=0;
incoc=xoc_int(1)/count2;
for g=1:(feeds-3),
    for h=count1:count2,
        ytopoc(h)=1-sqrt(Rt^2-(xtopoc(h)-Cx(g))^2)+Cyoc(g);
        xtopoc(h+1)=xtopoc(h)+incoc;
    end
    count1=count2+1; count2=count2+20;
    incoc=(xoc_int(g+1)-xoc_int(g))/20;
end
ytopoc(h+1)=1-sqrt(Rt^2-(xtopoc(h+1)-Cx(g))^2)+Cyoc(g);
counter=counter;
ytopoc=ytopoc-min(ytopoc);
topol1=[topol1 ytopoc];
end
surf=[zeros(sample,1) eye(sample)];
topol1=surf*topol1;

% Plotting a particular crosssection of the surface topography
% followed by the 3D surface topography ....

s=[30 60];
figure; subplot(2,1,1);
plot(xtopoc,ytopoc,'black');
title('Cross Section of the Surface Topography-Optimal (LQ) Design');
xlabel('distance along workpiece (m)');
ylabel('tool signature (m)'); pause
map=[0 0 0]; colormap(map);
subplot(2,1,2); mesh(topol1,s);
title('Surface Topography-Optimal (LQ) Design');
ylabel('Depth of Cut (m)');
xlabel('Cutting Speed (pi*D/20)');
xlabel('Feed [*f0(m)]'); pause
Appendix C: Matlab Programs for Adaptive Control (STR)

Matlab Program Defining the System Parameters (Non-Linear)

% This m file defines all the system's physical parameters. It is called upon
% by turnstr.m in the design of a parameter adaptive controller-self tuning
% regulator using LQG methods
% ..............................................................

% defining the system's physical parameters ...
% t=tool, w=workpiece, s=spindle
% ..............................................................

Mt=27;  % tool post mass (Kg)
Kt=1.2322E6;  % tool post stiffness (N/m)
Bt=128.5;  % tool post damping (N-sec/m)
Mw=7.829;  % workpiece mass (Kg)
Kw=2.4190E8;  % workpiece stiffness (N/m)
Bw=261;  % workpiece damping (N-sec/m)
Ms=34;  % chuck mass (Kg)
Ks=3.6345E9;  % structure stiffness (N-sec/m)
Bs=43071.91;  % structure damping (N-s)
rpm=3000;  % cutting speed [rpm]
s=10;  % number of samples along the
       % circumference
R=0.8;  % tool nose radius [mm]
lead=pi/12;  % tool lead angle [rad]
sample=20;  % number of samples per revolution
rpm=1200;  % spindle rotational speed
Ts=1/(rpm/60*sample);  % sampling interval

% defining the plant parameters ............
% plant -> dx/dt = Fx + Gu + Mv
% equations ->  y = H'x + w
% state vector ->  x = [Xs_dot Xs Xw_dot Xw Xt_dot Xt]'
Kc=3.4782E9; % specific cutting force constant for steel (N/m^2)
de=0.500E-3; % initial depth of cut (m)
for k=1:6001
  if k<2000
    do(k)=de;
  elseif k>4000
    do(k)=de;
  else
    do(k)=de/2;
  end
end
fo=.250E-3; % initial feedrate (m/rev)
Km=1000; % actuator gain (N/amp)
h=126; % workpiece mean hardness (BHN)
variance=176; % variance (BHN^2)
S=sqrt(variance); % standard deviation (BHN)
m=0.454; % meyer exponent
Ko=Kc*fo;
order=6; % order of the system
f11=(Bs+Bw)/Ms; f12=-(Kw+Ks)/Ms; f13=Bw/Ms; f14=Kw/Ms; f15=0;
f16=0; f21=1; f22=0; f23=0; f24=0; f25=0; f26=0; f31=Bw/Mw;
f32=Kw/Mw; f33=-Bw/Mw; f34=-Kw/Mw; f35=0; f36=0;
f41=0; f42=0; f43=1; f44=0; f45=0; f46=0; f51=0; f52=0; f53=0;
f54=0; f55=-Bt/Mt; f56=-Kt/Mt; f61=0; f62=0; f63=0;
f64=0; f65=1; f66=0;
F=[f11 f12 f13 f14 f15 f16
  f21 f22 f23 f24 f25 f26
  f31 f32 f33 f34 f35 f36
  f41 f42 f43 f44 f45 f46
  f51 f52 f53 f54 f55 f56
  f61 f62 f63 f64 f65 f66];
G=[0 0 1/Mw 0 1/Mt 0];
gw=Ko*de*m/h; Gw=[0 0 gw/Mw 0 -gw/Mt 0]; % v=Gw*hi
H=[0 0 0 1 0 1]; D=[0]; M=[0 0 0 1 0 1];
ti=0; tf=15; % initial and final times of the simulation
i=1;
t1=ti:Ts:tf;
for t=0:Ts:tf
  delta_hi(i)=3*(randn);
end
Appendix C: Matlab Programs for Adaptive Control (STR)

i=i+1;
end
hi=h+delta_hi;
w=logspace(-2,6,300); % frequency range for frequency response
set(0,'DefaultFigureColor','white');
set(0,'defaulttextcolor','black');
set(0,'defaultaxesxcolor','black');
set(0,'defaultaxesycolor','black');
set(0,'defaultaxeszcolor','black');
set(0,'defaultsurfaceedgecolor','black');

Matlab Program for the Adaptive Controller (STR) Design

global FD GD H D K flag u k StateError Goc Hoc Doc Foc Correction

% * This program simulates a tuning process with indirect adaptive
% * control implemented. A self tuning regulator (STR) is designed based
% * on a recursive least squares (RLS) estimation with exponential
% * forgetting. The system response is computed in order to compare the
% * results with those obtained from the designed optimal control law.
% * The systems physical parameters as well as the hardness distribution
% * are the same as those obtained from the execution of tmlqr.m such
% * that a basis for comparison is established. As a result this program
% * should be executed tmlqr6 is executed since it requires some results
% * generated or defined in tmlqr.m. The transfer function represen-
% * tation is used and it was obtained by using MATHEMATICA.

% Loading System Physical Parameters .....

parametersn;

set(0,'DefaultFigureColor','white');
set(0,'defaulttextcolor','black');
set(0,'defaultaxesxcolor','black');
set(0,'defaultaxesycolor','black');
set(0,'defaultaxeszcolor','black');
set(0,'defaultsurfaceedgecolor','black');

M=[0 0 0 1 0 1];

[NumC,DenC]=ss2tf(F,G,H,D); % continuous system transfer function
[NumD,DenD]=c2dm(NumC,DenC,Ts,'zoh'); % discrete transfer function
[FD,GD]=c2d(F,G,Ts);

% Augmented System: Adaptive Controller, Kalman Filter
% Foc is evaluated below as required parameters become available

Goc=[GD' 0 0 0 0 0];
Hoc=[H 0 0 0 0 0];
Doc=[D];

% Simulation of the workpiece surface micro-hardness .......

i=1; h=120;
for t=0:Ts:tf
    micro(i)=randn;
i=i+1;
end
delta_hi=micro*S/max(abs(micro));
hi=h+delta_hi;

% quadratic weight matrices .....
% Initial conditions/start up values for RLS parameter equation, % state estimation and recursive Riccati equation ........
% %

k=1;
y_estimate(k)=0; x=[0;0;0;0;0;0]; % initial conditions
StateError=[0;0;0;0;0;0]; xoc=[x;StateError];
ti=0; u(k)=Ko*do(k)*(hi(1)/h)^m;
flag=1; yoc(k)=strmto(ti,xoc);
lamda(k)=0.9; % forgetting factor
No=1/(1-lamda(k));
Sigma=1E-7;
alpha=1E3;
for index=2:order
   a_index(index)=DenD(index);
b_index(index)=NumD(index);
end
theta_estimate=[a_index b_index]'; % theta=parameter vector
P=alpha*eye(12); nu(1)=yoc(1); % covariance matrices
PRiccati=Q; Correction=[0;0;0;0;0;0];
xEstimate=[0 0 0 0 0 0];

% % Simulation for 6000 time steps. Each sampling step is equivalent % to the sampling period ie Ts=0.0025s ..............
%

for k=2:1:6001 % dt=0.0025s -> k=1=dt -> k=100=400*dt=1s
   u(k)=Ko*(do(k)+yoc(k-1))*(hi(k)/h)^m;
   %
   % NOTE: Eventhough indexes are not used the following
   % ^^^^^^ algorithm is recursive.
   %
   % STEP 1: PARAMETER ESTIMATION............
   % ^^^^^^ Recursive Least Squares (RLS) with time-varying
   % forgetting factor
   %
   tf=ti+Ts;
   Foc=[FD-GD*K   GD*K
        zeros(6) FD-Correction*H*FD];
   xoc=Foc*xoc;
   yoc(k)=Hoc*xoc;
x=xoc(1:6);
if k>order
    count1=1;
    for j=k-1:-1:k-order
        yPhi(count1)=yoc(j);
        uPhi(count1)=u(j);
        count1=count1+1;
    end
    phi=[-yPhi uPhi]';
else
    for count1=1:k
        yPhi(count1)=yoc(count1);
        uPhi(count1)=u(count1);
    end
    for count2=k+1:order
        yPhi(count2)=0;
        uPhi(count2)=0;
    end
    phi=[-yPhi uPhi]';
end
error(k)=yoc(k)-phi*theta_estimate;
gamma=inv(lamda(k-1)+phi'*phi)*phi';
P=(eye(12)-gamma*phi')*P/lamda(k-1);
theta_estimate=theta_estimate+gamma*error(k);
lamda(k)=1-1/Sigma*(1-phi'*gamma)*(error(k))^2;

% STEP 2: State controller (Recursive Ricatti Equation)......
% 
% K=inv(R+GD'*PRiccati*GD)*GD'*PRiccati*FD; % state controller calculation
PRiccati=Q+FD'*PRiccati*(eye(6)-
GD*inv(R+GD'*PRiccati*GD)*GD'*PRiccati)*FD;
%
%
% state variable calculation (Kalman Filter)
%
% From the Principle of Duality........
%
% Optimal State Controller    Kalman Filter
% --------------------------- --------------------------
% -k k
% F' F
% G H'
% R RKalman
% QKalman

% if k==2
%  PPredict=x'*x'; PEstimate=x'*x';
end;
xPred=pF*D*xEstimate+GD*u(k); % xPredict=x^A
PPredict=pF*PEstimate*D'+QKalman;
Correction=PPredict*H'*inv(H'*PPredict*H'+RKalman);
xEstimate=xPredict+Correction*(yoc(k)-H'*xPredict);
PEstimate=PPredict-Correction*H'*PPredict;
StateError=x-xEstimate; % StateError=x-
xo=x+[x;StateError];
ti=ti+Ts;
end

% System response after STR implementation ......

subplot(2,1,1); plot(t,yoc+do,'black');
title('Actual Depth of Cut-STR');
ylabel('depth of cut(m)');
subplot(2,1,2); plot(t,yoc,'black');
title('Tool Workpiece Relative Displacement - STR');
ylabel('tool-workpiece relative displacement(m)');
figure;
plot(t,error,'black');
xlabel('time (s)'); ylabel('error (m)');

% this section of the program program generates a cross-section of the
% surface profile as well as 3D surface topology. The tool-workpiece
% relative displacements are further processed (tool geometry is
% incorporated) to generate the 'actual' surface profile. Sections
% equivalent to the sampling per revolution are generated and meshed to
% generate a 3D surface topology.
%
% Note: Due to the presence of the tool lead angle the coordinate y*** is
% ^^^^^ related to the projection of the system response. Hence....
% (lead=lead angle; feeds=no of feeds in each cross-section)
%
count=1;
% generating the centre points of the cutting tool as it moves along the 
% given cross-section of the workpiece....

for i=1:feeds, 
    yoc2(i)=yoc1(count); 
    count=count+sample; 
end 

% generating the points of intersection between adjacent circles produced 
% by the tool geometry....
% Equations used: (x_int-Cx*(k))^2+(y_int-Cy*(k))^2=R^2
% (x_int-Cx*(k+1))^2+(y_int-Cy*(k+1))^2=R^2
% ALL OTHER VARIABLES ARE FOR CONVENIENCE DURING THE
% INTERMEDIATE STEPS

for k=1:(feeds-2), 
    a=Cx(k+1)-Cx(k); bo=(Cyoc(k)-Cyoc(k+1)); co=Cyoc(k+1)^2-Cyoc(k)^2-a^2; 
    c2o=4*a^2+4*bo^2; c1o=4*bo*co-8*a^2*Cyoc(k+1); 
    c0o=co^2-4*a^2*Rt^2+4*a^2*Cyoc(k+1)^2; 
    yoc_int(k)=(-c1o+sqrt(c1o^2-4*c2o*c0o))/(2*c2o); 
    xoc_int(k)=sqrt(Rt^2-(yoc_int(k)-Cyoc(k))^2)+Cx(k); 
end 

% generating points in order to clearly define the circles which 
% depict the surface profile....
% -> 20 points are used to generate the tool signature on 
% the surface for each feed increment.
count1=1; count2=20;
xtopoc(1)=0;
incoc=xoc_int(1)/count2;
for g=1:(feeds-3),
    for h=count1:count2;
        ytopoc(h)=1-sqrt(Rt^2-(xtopoc(h)-Cx(g))^2)+Cyoc(g);
        xtopoc(h+1)=xtopoc(h)+incoc;
    end
    count1=count2+1; count2=count2+20;
    incoc=(xoc_int(g+1)-xoc_int(g))/20;
end
ytopoc(h+1)=1-sqrt(Rt^2-(xtopoc(h+1)-Cx(g))^2)+Cyoc(g);
count=counter;
ytopoc=ytopoc-min(ytopoc);
topol1=[topol1 ytopoc'];
end
surf=[zeros(sample,1) eye(sample)];
topol1=surf\topol1';

%-------------------------------------------------------------------------------------------------
% Plotting a particular crosssection of the surface topography
% followed by the 3D surface topography ....
%-------------------------------------------------------------------------------------------------

s=[30 60];
figure; subplot(2,1,1);
plot(xtopoc,ytopoc,'black');
title('Cross Section of the Surface Topography-Adaptive Control');
xlabel('distance along workpiece (m)');
ylabel('tool signature (m)'); pause
map=[0 0 0]; colormap(map);
subplot(2,1,2); mesh(topol1,s);
title('Surface Topography-Adaptive Control Strategy');
xlabel('Depth of Cut (m)');
ylabel('Cutting Speed (pi*D/20)');
xlabel('Feed (fo(m))');
clear global
VITA AUCTORIS

Evangelos Liasi was born in Nicosia, Cyprus in 1968. In 1980, he and his family emigrated to Canada. He obtained the B.A.Sc. Degree in Mechanical Engineering from the University of Toronto in 1989 and the M.A.Sc. Degree in Mechanical Engineering from the University of Windsor in 1990. After being in the work force for a year, he returned to the University of Windsor to pursue the degree of Doctor of Philosophy in Mechanical Engineering. Currently he is a candidate for the same and a Post-Doctoral Fellow at the Machine Automation Laboratory at the University of Windsor.