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THE BUCKLING OF BUILT-UP COMPRESSION MEMBERS
IN THE PLANE OF THE CONNECTORS

by

Ghada Elmahdy

A Thesis

Submitted to the Faculty of Graduate Studies and Research
through the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for the
Degree of Master of Applied Science at the
University of Windsor

Windsor, Ontario, Canada

1991

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ABSTRACT

THE BUCKLING OF BUILT-UP COMPRESSION MEMBERS
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Ghada Elmahdy

An examination of the different Standards and Specifications for the design of built-up compression members reveals a great variation in the value of the maximum slenderness ratio specified for the individual main member, and in the specified equivalent slenderness ratio. Also, the requirements of the Canadian Standard S16.1-M89 are confusing with regards to the requirements for the maximum slenderness ratio of the individual main member.

A series of tests was carried out on welded built-up model struts to verify the validity of the Canadian Standard S16.1-M89. Three parameters were studied: the separation between the main members, the number of connectors, and the slenderness ratio of the built-up strut acting as an integral unit. The buckling load and load-deflection curve, of each specimen, were determined from the tests.

The finite element method was used to predict the experimental results, but did not accurately reflect all of

the experimental results. Other analytical solutions were also attempted, and their validity with regards to the experimental results was examined.

The results of this investigation showed that there was a discrepancy in the Canadian Standard S16.1-M89, in that the requirement that the maximum slenderness ratio of an individual main member should be equal to or less than the integral slenderness ratio, does not agree with the experimental results, whereas the requirement that the maximum slenderness ratio of an individual main member should be less than 40 or 60% of the slenderness ratio of the built-up column about the axis perpendicular to the batten plates, does verify the experimental results. The experimental results also showed that built-up struts in which the main members are in contact had extra rigidity, and so did yield results which agree with those predicted using the Canadian Standard.

It is recommended for built-up columns with a space between the main members, that either the clause requiring the maximum slenderness ratio of an individual main member to be equal to or less than the integral slenderness ratio, be specifically stated that it does not apply to battened columns, or that the requirement for the equivalent slenderness ratio, be modified.

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NOMENCLATURE

a	the length of a panel from centre-to-centre of two consecutive batten plates.
a_1	the distance between the ends of two adjacent batten plates.
A	the total area of the cross-section.
A_b	the cross-sectional area of a batten plate.
A_m	the cross-sectional area of one main member.
$\bar{C}$	the modified carry-over stability function.
C, D	constants.
d	the centroidal distance between main members.
E	the modulus of elasticity.
f	the maximum deflection of the column at mid-height.
F, P	the axial force in the column.
F_r	the axial force in panel r of the main members.
F_y, σ_y	the yield stress.
g_a, g_b	the length of the end plates at the ends a and b of the column, respectively.
I, I_x	the moment of inertia of the entire cross-section about the axis perpendicular to the batten plates, Y , and the axis parallel to the batten plates, X , respectively.
I_1, I_{x1}	the moments of inertia of the cross-section of an individual main member about the y and x centroidal axes, respectively.

I_b	the moment of inertia of the batten plates about the horizontal centroidal axis perpendicular to the plane of the batten plates.
I_o	the moment of inertia of the entire cross-section about the axis perpendicular to the batten plates, neglecting the moment of inertia of the individual main members about their own centroidal axes.
k	the stiffness of the column.
k_1	the effective length factor of an individual main member between batten plates = 1.0.
K	the effective length factor of the integral column.
K_{eq}	the equivalent length factor.
$[K]$	the stiffness matrix.
$[K_E], [K_G]$	the elastic and geometric stiffness matrices, respectively.
l	the length of the column between end plates.
L	the total length of the column from knife edge to knife edge.
m	the number of main members.
M	the internal moment in the column due to bending.
M_{ab}, M_{ba}	the induced end moments in the column at ends a and b, respectively.
M_b	the bending moment in the batten plates.
M_{br}	the bending moment in batten plate r.
M_{fr}	the factored moment of resistance of the batten plates.
M_r	the bending moment in panel r of the main members.
n	the number of parallel planes of batten plates.
P, F	the axial force in the column.
P_1, P_2	the axial force in main member 1 and 2, respectively.
P_b	the actual buckling load of the built-up column.

P_0	the reciprocal of the shear factor.
P_c	the Euler buckling load.
P_f	the factored compressive load.
$\{P\}$	the load vector.
$\{\Delta P\}$	the load increment vector.
Q	the lateral shearing force acting on the column.
Q_b	the shearing force in the batten plates.
Q_{br}	the shearing force in batten plate r .
Q_{max}	the maximum shearing force at the ends of the column.
Q_r	the shearing force in panel r of the main members.
r, r_x	the radius of gyration of the entire cross-section about the centroidal axes perpendicular to the batten plates and parallel to the batten plates, respectively.
r_1	the radius of gyration of one main member about its centroidal axis perpendicular to the batten plates.
s	the length along the curved axis of a deflected column
$\bar{s}$	the modified stiffness stability function.
t	a variable.
$\{U\}$	the vector of nodal displacements.
$\{\Delta U\}$	the incremental displacement vector.
V	the strain energy due to deflection.
V_{tr}	the factored shear resistance of the batten plates.
W	the work done by the external buckling load.
x	the lateral deflection of the column.
x', z'	the linear degrees of freedom in the X-direction and the Z-direction, respectively.

x_0	the initial out-of-straightness at mid-height of the column.
z	the position along the length of the column.
z_b	the plastic section modulus of a batten plate.
z_l, z_r, z_m	the displacements in the Z-direction of the left, right, and middle nodes of the end plates, respectively.
α	the inverse of the critical load.
β	a constant equal to $\sqrt{P/EI}$.
γ	the additional slope of the deflection curve of the main members due to shear.
δ_1	the lateral displacement of the main members caused by the rotation of batten plates due to shear force.
δ_2	the lateral displacement of the main members due to the bending of the main members due to shear force.
Δ	the axial shortening of the column.
Δ_1	the axial displacement of the column due to axial shortening under an axial force.
Δ_2	the axial displacement of the column due to bending.
$\{\Delta\}$	the vector of relative displacements of the buckling mode.
θ	the angle of rotation of the batten plates.
θ'	the rotational degree of freedom.
θ_a, θ_b	the rotational disturbances of the column at ends a and b, respectively.
λ_1	the slenderness ratio of an individual main member, between ends of adjacent batten plates, about the axis perpendicular to the batten plates.
λ_{eq}	the equivalent slenderness ratio about the axis perpendicular to the batten plates.
λ_x, λ_y	the slenderness ratio of the integral column about the X and Y axes, respectively.

ρ the ratio of the buckling load to the Euler load.

σ_1, σ_b the maximum and buckling stresses, respectively, in the main member associated with maximum deflection at mid-height.

σ_y, F_y the yield stress.

ϕ the resistance factor.

Chapter 1

INTRODUCTION

1.1 General

Built-up compression members are used widely in structural engineering for bridge columns, columns in buildings, and truss members. Basically, built-up compression members are composed of two or more structural sections held together by transverse members, which could be battens, lacing bars or perforated plates. The function of these transverse members is to make the built-up column act as an integral unit, to hold the main members apart so that a large moment of inertia is achieved, and to form the shear connection between the main members. In this thesis built-up struts connected by welded batten plates are studied. Details of some common built-up sections are shown in Figure 1.1.

Built-up struts can be more economical than solid struts when the load is relatively small, or the length is quite long. Thus, if a solid section is used the slenderness ratio of the strut, which is the ratio of the effective length of the column to the radius of gyration, would be high. The stress at failure would tend to be well down the Eulerian part of the strut curve, and to be small compared with the yield

stress. A higher failing stress is obtained if the radius of gyration is increased by separating the two main members.

Built-up compression members can be considered as either simple or built-up struts depending on the plane of bending. If buckling occurs about the axis parallel to the battens, the batten plates would simply move with the main members, at the same time maintaining the separation between the main members, providing rotational restraint to the individual elements, but transferring little or no forces. Thus, this type of strut can be considered as a simple strut. On the other hand, if buckling occurs about the axis perpendicular to the battens, the batten plates would deform. So this type of strut can be considered as a built-up strut.

For built-up struts the effect of shearing forces on the behaviour of the column cannot be neglected, as it is for solid struts. For this reason, the number of batten plates used in a built-up column is an important aspect in its design. If the built-up strut is too sparsely connected, local buckling of the main members could reduce its load carrying capacity enough to prevent it from being a more economical solution to solid struts. Therefore, the most economical solution for battened struts is to make the slenderness ratios about both principal axes equal, and to ensure that the batten plates carry the required shear and moment, which can only be achieved by connecting the battens rigidly to the main members.

1.2 Objective

The equivalent slenderness ratio for columns that buckle in the plane of the connectors, differs from one Standard or Specification to another. Hence, the objective of this research is to develop rules for the design of built-up compression members when they buckle in this plane, from which a recommendation can be made for inclusion in the Canadian Standard S16.1-M89 [10].

In order to do this three parameters were studied; the slenderness ratio, the separation between the main members, and the number of connectors.

The purpose of this thesis is also to show that the familiar design rule of making the maximum slenderness ratio of the individual main members between connectors equal to or less than the slenderness ratio of the integral column will not lead to a buckling strength equal to that of the integral column [18].

1.3 Methodology

To acquire data that can be used to describe the behaviour of built-up compression members, tests were carried out on welded model struts. The motivation behind using model strut specimens rather than full-sized struts was to have better control over the fabrication and testing of each specimen. In this way, it was expected that more accurate results could be obtained, from which a theoretical study

could be performed.

Six model struts were tested, each being tested with four different numbers of batten plates. Two slenderness ratios were considered, 80 and 120, representing intermediate and slender columns, respectively. Each slenderness ratio was tested with three separations; zero, 4.02 and 7.85 mm. These separations were chosen so that the radius of gyration of the individual main members were $1/2$, $1/3$, and $1/4$ of the radius of gyration of the integral column about the axis perpendicular to the battens. Each model strut was tested with ends pinned about the axis perpendicular to the battens, and essentially fixed-end conditions about the axis parallel to the battens. In this way, buckling occurred in the plane of the battens.

As the purpose of this research was not to study the effect of torsional or torsional-flexural buckling on the behaviour of built-up columns, the main members of the model struts were taken as rectangular bars, rather than structural sections. As a result, the possibility of any local torsional buckling occurring was eliminated.

An iterative-incremental finite element program was used to predict the load-deflection behaviour and buckling load of the model struts.

Finally, an analytical solution for the equivalent slenderness ratio, and hence the buckling strength of built-up columns, was formulated.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

The earliest treatment of the problem of built-up compression members was by Engesser in 1888, and later in 1909. The problem was then investigated by Müller-Breslau (1910), Timoshenko and Gere (1936), Pippard (1948), Bleich (1952), Jones (1952), Koenigsberger and Mohsin (1956), Lin et al. (1970), Toosi (1972), and Ballio and Mazzolani (1983). These investigators contributed to the study of the buckling load and the frame behaviour of built-up columns as affected by the shear force.

The effect of the number of connectors used on built-up angle struts was studied by Temple, Schepers and Kennedy (1986), Temple, McCloskey and Calabrese (1987), Temple and Tan (1988), and Temple (1990).

The behaviour of sparsely connected built-up columns was discussed in a paper by Libove (1985). Finally, the design rules for built-up columns in the LRFD Specification [3], was discussed in a paper by Duan and Chen (1988).

In this chapter, a brief review of these references, and of the different design Standards and Specifications is given.

2.2 Previous Work

Timoshenko and Gere [31] derived an expression for the effect of shear on built-up columns with lacing bars, batten plates or perforated cover plates, as connectors. The analysis was based on determining the effect the shear deformation had on the buckling load, as a result of increasing the slope of the centre-line.

Pippard [22,24] suggested using the Perry-Robertson formula [23], to determine the load which would cause yielding at the outer-most fibre of the mid-height section. This formula, however, required the derivation of an expression for the buckling load of a battened column. This was derived using the assumption that the flanges, instead of being joined by a finite number of batten plates, were connected by a web which can apply flexural restraint to the flanges. Hence, the theoretical total moment applied by the web to the flanges was equated to the theoretical moment applied by all the batten plates to the flanges in order to find the equivalent effect. From this the buckling load was derived.

Bleich [7] derived an expression for the buckling load by means of the energy method. The resulting equation was similar to that of Timoshenko and Gere.

Jones [16] developed a theory for struts with lattice or batten bracing, with load eccentricities at the ends.

Koenigsberger and Mohsin [17] made a study of the design and load carrying capacity of welded battened struts composed

of two channels. Tests were carried out on a series of welded battened struts to determine: (1) the ratio between the collapse load and the permissible working load, (2) the stresses caused by an axial load as well as a load applied with a small eccentricity or shear, and (3) deflections and contractions of the struts under concentric loads up to the instance of collapse. The buckling load was determined analytically using Engesser's, Müller-Breslau's, Timoshenko's, Bleich's and Pippard's formulae. Timoshenko's formula was concluded to be the most appropriate.

Temple et al. [28,27,29] conducted a study of the interconnection requirements for starred, boxed, and back-to-back angles, respectively. Tests were carried out on struts and the test results were compared with the results of a finite element analysis. An examination of the requirements of the different Standards, for built-up compression members, with regard to the number of connectors and their design was also conducted.

Temple [26] discussed the behaviour of the connectors for double angle struts. It was proposed that connectors used to connect double angles be called interconnectors when buckling occurs in the plane perpendicular to the plane of the connectors, and battens when buckling occurs in the plane of the connectors. An example of torsional-flexural buckling was also given.

Libove [18] presented an elastic analysis for the initial

buckling and post-buckling behaviour, of simply supported built-up columns consisting of two slightly separated identical parallel elements. The cases of rigid connectors at the ends only, and rigid connectors at the ends and in the middle were discussed. The analysis concluded that the results of the initial buckling showed that the buckling load can be considerably below that corresponding to integral action, even if the slenderness ratio between connectors is less than that of the built-up column as a whole. The analysis also showed that the post-buckling behaviour of built-up columns was unstable, in that the load required to maintain a buckled configuration was less than that required to initiate buckling.

Duan and Chen [11] discussed the design rules for built-up columns when they buckle in the plane of the connectors, according to the AISC-LRFD Specification for Structural Steel Buildings [3]. It was shown that the proposed modified slenderness ratio formula is concise, simple, and physically clear, and that the calculated results were found to be in good agreement with available test results.

2.3 Standards and Specifications

The following sections discuss the requirements of several Standards and Specifications for the design of built-up compression members. Several steel Standards and Specifications, which include the Canadian, American, German,

and British Standard, were studied.

2.3.1 Canadian Standard, CAN/CSA-S16.1-M89, "Limit States Design of Steel Structures", [10]

Clause 19.1 covers the general requirements for built-up members in compression. Clause 19.1.3(c) requires that the slenderness ratio of any shape, based on the least radius of gyration of the component parts, between points of interconnection, shall not exceed the slenderness ratio of the built-up member. Clause 19.1.4 gives the equivalent slenderness ratio for compression members composed of two or more rolled shapes, separated from one another by intermittent fillers. Clause 19.1.8 states that the "open sides of compression members built up from plates or shapes shall be connected to each other by lacing, batten plates or perforated cover plates".

For battens, Clause 19.1.16 requires that they are placed such that:

"(a) when the slenderness ratio of the built-up member with respect to the axis perpendicular to the battens is equal to or less than 80% of the slenderness ratio with respect to the axis parallel to the battens, the spacing between battens shall be such that the slenderness ratio of a main component between ends of adjacent batten plates shall not exceed 50, and shall not exceed 70% of the slenderness

ratio of the built-up member with respect to the axis parallel to the battens;

- (b) when the slenderness ratio of the built-up member with respect to the axis perpendicular to the battens is more than 80% of the slenderness ratio with respect to the axis parallel to the battens, the spacing between battens shall be such that the slenderness ratio of a main component between ends of adjacent batten plates shall not exceed 40, and shall not exceed 60% of the slenderness ratio of the built-up member with respect to the axis perpendicular to the battens."

Clause 19.1.17 requires that "battens shall have a length of not less than the distance between lines of bolts or welds connecting them to the main components of the member, and shall have a thickness of not less than $1/60$ of this distance." This clause also gives the longitudinal shear force and moment that battens and their connections must resist, induced in them by a transverse shear acting on the main members equal to 2.5% of the factored axial compressive load on the column.

2.3.2 American Specifications, AISC

a) Specification for Structural Steel Buildings, "Allowable Stress Design (ASD)", 1989, [2]

Clause E4 requires that the slenderness ratio between

points of interconnection "does not exceed $3/4$ times of the governing slenderness ratio of the built-up member", and that at least two intermediate connectors be used along the length of the built-up member. Battened columns are not covered by the AISC Specifications.

b) Specification for Structural Steel Buildings, "Load and Resistance Factor Design (LRFD)", 1986, [3]

Contrary to the AISC-ASD Specification, Clause E4 requires that the slenderness ratio between the fasteners does not exceed the governing slenderness ratio of the built-up member. This clause also gives the modified slenderness ratio of built-up members when the buckling mode involves relative deformation that produce shear forces in the connectors.

2.3.3 German Standard, DIN 4114-1952, "German Buckling Specifications", [14]

Clause 8.213 requires that the slenderness ratio between batten plates must be equal to or less than $1/2$ the slenderness ratio about the axis parallel to the battens, or equal to 50 if the requirements of Clause 8.34 are met, which requires battens to be provided, at least, at the third points of the column. Clause 8.211 requires that built-up columns be designed against buckling at right angles to the axis parallel to the battens, as single member columns. Clause 8.212 gives the modified slenderness ratio required for the design against buckling at right angles to the axis perpendicular to the

battens.

Clause 8.31 requires the battens to be designed to resist the forces induced in them by an assumed transverse shear of 1.25% of the area times the allowable tensile stress acting on the main members.

2.3.4 British Standard, BS 5950 1985, "Code of Practice for Design in Simple and Continuous Construction: Hot Rolled Sections", [8]

Clause 4.7.9(c) requires that the maximum slenderness ratio of a main component, based on its minimum radius of gyration, between end welds of adjacent battens should not exceed 50. Also, the maximum slenderness ratio of a battened strut about the axis parallel to the battens should not be taken less than 1.4 times the maximum slenderness ratio of a main component. This clause also gives the equivalent slenderness ratio of a battened strut about the axis perpendicular to the battens.

Clause 4.7.9(d) requires that the thickness of the battens should not be less than $1/50$ of the minimum distance between welds or fasteners. Clause 4.7.9(e) requires that the battens, and the connections between them and the main components, be designed to carry the forces and moments induced by a transverse shear equal to or greater than 1% of the maximum factored axial load in the member.

2.3.5 Summary

Examining these Standards and Specifications leads to the conclusion that there is considerable variation among the different steel Standards, with regards to the equivalent slenderness ratio of the battened strut, and to the number and design requirements of the connectors. Therefore, research is required to develop consistent design rules for built-up compression members.

Chapter 3

FINITE ELEMENT ANALYSIS

3.1 Introduction

To predict the theoretical load carrying capacities of the model struts, the finite element method was used. This was done using a computer program, first as an eigenvalue program to predict the critical load, and second as an iterative-incremental program to predict the nonlinear load-deflection behaviour of the model strut.

A commercial finite element package, ABAQUS [1], was used. Initial stresses due to the manufacturing processes of the material used, or due to the welding of the model strut, were not included in the input owing to the difficulty of determining them. Geometric imperfections, such as the initial out-of-straightness of the model strut, were included in the input of the analysis, as the coordinates of the nodes.

A linear elastic, perfectly plastic type analysis was used to model the material properties, and as this is a large deflection problem, deformed geometry was used.

An analysis of the effect of the length of the end plate on the buckling load of the model strut was also performed, using stability functions.

3.2 The Finite Element Program

The commercial finite element package used, ABAQUS [1], is a batch program, where the input file indicates the options which are required. It can model one-, two-, or three-dimensional elements, for example a beam, plate, or solid continuum element, respectively. It can perform two dimensional or three-dimensional analyses. The input file required by ABAQUS consists of the model data and the history data.

The model data defines the finite element model, and includes the elements, nodes, nodal constraints, and material definition. Nodal constraints are used to restrict certain degrees of freedom at the nodes. Three types of nodal constraints were used: (1) multi-point constraints (MPC), which are predefined sets of constraints provided by ABAQUS, (2) equations, which are linear relationships written to define a certain relationship between degrees of freedom, and (3) boundary constraints, which define the boundary conditions. The material definition is the data given in the input describing the elastic properties of the material used.

The history data defines the sequence of events or loadings for which the model's response is sought. This history is described by a sequence of steps. In the models analyzed, a static load, applied in one step, using nonlinear geometry, was specified. The load applied was assigned up to 50 increments to reach the buckling load, and each increment

assigned 15 iterations in which to converge, or the increment was automatically reduced. The load tolerance for convergence was specified in the input.

3.3 Finite Element Method

The finite element method involves solving the equilibrium equations of the nodal degrees of freedom of the structure, in the form

$$[K]\{U\} = \{P\} \quad (3.1)$$

where $[K]$ is the stiffness matrix; $\{U\}$ is the vector of nodal displacements; and $\{P\}$ is the load vector.

In the case of the axially loaded column, the presence of lateral bending displacement causes the equilibrium equations to become nonlinear. To account for this, a geometric stiffness matrix, as well as the traditional elastic stiffness matrix, must be included in the equilibrium equations, such that

$$([K_E] - F[K_G])\{U\} = \{P\} \quad (3.2)$$

where $[K_E]$ is the elastic stiffness matrix; $[K_G]$ is the geometric stiffness matrix; and F is the axial force in the column. The derivation of these matrices can be found in

Reference [20].

3.3.1 Eigenvalue Procedure

This method was illustrated by Temple et al. [28]. Buckling is defined as the state at which the stiffness of a compression member goes to zero, and the lateral displacements, at least theoretically, become infinitely large. Thus, to determine the critical load, the determinant of the total stiffness matrix is equated to zero, as follows

$$|[K_E] - F[K_G]| = 0 \quad (3.3)$$

As the geometric stiffness matrix is singular, the only way to solve Equation 3.3 is to rewrite it in the form

$$|\alpha[I] - [K_E]^{-1}[K_G]| = 0 \quad (3.4)$$

Equation 3.4 is a typical eigenvalue problem where α , the eigenvalue, is the inverse of the critical load; and $[I]$ is the identity matrix. Therefore, to find the first critical load, the highest value of α must be found.

To find the corresponding buckling mode, the critical load is substituted into Equation 3.2, which is equated to zero, such that

$$([K_E] - \frac{1}{\alpha} [K_G])\{\Delta\} = 0 \quad (3.5)$$

in which $\{\Delta\}$ is the eigenvector, which represents the vector of relative displacements of the buckling mode.

3.3.2 Iterative-Incremental Procedure

As pointed out by Martin [20], it is necessary to keep the deformed geometry in mind when writing the equilibrium equations, Equation 3.2. This causes these equations to become nonlinear. In the finite element method, this is taken into account by an incremental step procedure, where the deformed geometry is corrected at the start of each iteration. This procedure was used by Temple et al. [28]. Therefore, Equation 3.2 should be written as

$$([K_E]_j^i - F_j [K_G]_j^i) \{\Delta U\}_j^i = \{\Delta P\}_j^i \quad (3.6)$$

where $\{\Delta U\}$ is the incremental displacement vector; $\{\Delta P\}$ is the load increment vector; i denotes the i th incremental loading step; and j denotes the j th iteration cycle.

To determine the load-deflection curve, the process described in Equation 3.6 is repeated for successive load increments. For the first load increment, the initial out-of-straightness is used as the geometry of the model to calculate the elastic and geometric stiffness matrices, and the change

in displacement vector is determined by an iterative procedure, such that

$$\{ [K_E]_j^i - F_j [K_G]_j^i \} \{\Delta U\}_j^i = \{\Delta P\}_j^i \quad (3.7)$$

where

$$\{\Delta P\}_j^i = \{P\}_{\text{ext}}^{i,j-1} - \{P\}_{\text{int}}^{i,j-1} \quad (3.8)$$

Here, $\{P\}_{\text{ext}}$ is the external load vector; and $\{P\}_{\text{int}}$ is the internal load vector calculated from the internal deformation. The solution of Equation 3.7, gives a corrected displacement vector from

$$\{U\}_j^i = \{U\}_{j-1}^i + \{\Delta U\}_j^i \quad (3.9)$$

and hence, the next iteration can be executed. Convergence was measured by limiting the value of all the components of $\{\Delta P\}$, in Equation 3.8, within a certain tolerance.

Using the estimated critical load obtained from the eigenvalue program, as an upper load limit, automatic load increments were applied. The automatic scheme for this procedure was based on the convergence of the iteration process of each increment, until the specified load tolerance was achieved. If the number of iterations exceeded the

maximum allowed, the increment size was reduced by a factor of four. If this resulted in a smaller increment than was specified as a minimum in the input, the run was terminated. In this way, the complete load-deflection curve was obtained. The load tolerance varied from run to run according to the estimated critical load, and was in the range of 1.0 to 10.0 newtons.

3.4 The Finite Element Model

The model strut was restrained so that buckling occurred only in the plane of the connectors, consequently, a two-dimensional finite element model was used. The main members, connectors, and end plates were all modelled by two-dimensional Euler-Bernoulli beam elements, with rectangular cross-sections. In these elements, it was assumed that the cross-section did not deform in its plane, nor warp out of its plane, so that the cross-sectional plane remained normal to the axis of the beam element. The coordinates of the nodes were defined, such that each main member was parabolic, having a maximum out-of-straightness at its mid-height.

The axes were taken such that the cross-section of the main members were in the X-Y plane, as shown in Figure 3.1. Buckling was modelled to take place about the Y axis, and so the analysis was carried out in the X-Z plane.

The connectors were modelled as one beam element, connected at its nodes to the two corresponding nodes on each

main member. They were assigned the dimensions of the two parallel connectors together, and as the distance between the centroids of the two main members was very small, were modelled as rigid beams. This was done by putting multi-point constraints on the nodes of the connectors. The rotations and displacements, in the X and Z directions, of these two nodes, were restrained.

The end plates were assigned constraints to model the rigid, pin-ended boundary condition. The top boundary was free to displace in the Z-direction, in order to allow for shortening, while the bottom boundary was prevented from displacing in the Z-direction. Each end plate was modelled by two beams with large moments of inertia, connected to each other by a common middle node. For both the top and bottom end plates, the rotation of the three nodes defining each plate, was made equal to ensure that the end plate remained rigid. The displacement, in the X-direction, of the middle node was prevented, whereas the displacements, in the Z-direction, of the two outer nodes were made equal in value, but opposite in direction. For the top boundary, a relation was specified for the displacements in the Z-direction, which was

$$z_1 + z_r - 2z_m = 0 \quad (3.10)$$

where z_1 is the displacement of the left node in the Z-

direction; z_r is the displacement of the right node in the Z-direction; and z_m is the displacement of the middle node in the Z-direction. For the bottom boundary, the displacement in the Z-direction of the middle node was prevented, consequently, the displacements of the two outer nodes, in the Z-direction, were made equal in value, but opposite in direction.

All other nodes were free to displace in any direction within the X-Z plane.

The material of the models was input as a linear elastic, perfectly plastic material, by defining the modulus of elasticity and yield stress of the material.

Five finite element models were used with zero, one, three, five, and seven connectors. Details of the nodal degrees of freedom for these models are shown in Figures 3.2 (a) to (e).

In the finite element model the load was applied on the middle node of the top end plate, in the negative Z-direction, in increments defined in one step.

3.5 Effect of Length of End Plates on Buckling Load

The stability functions derived by Livesley and Chandler [15], were used to investigate the effect of the length of the end plates, on the buckling load of the model strut.

These stability functions were derived on the assumption that the end plates are perfectly rigid, which is not true as

the actual end plates have a little flexibility. Therefore, the solution was on the safe side, as it exaggerated the effect of the length of the end plate on the buckling load.

It was found that the buckling load was increased by 0.53%, when the end plates were taken as rigid members. This is a negligible increase in the buckling load. The details of this investigation are given in Appendix C.

Chapter 4

ANALYTIC CRITERIA

4.1 Introduction

The overall behaviour of built-up compression members is dependent on the flexural deformability of the main members and batten plates. It is influenced by the deformations, resulting from bending moments and shear forces, on which lateral deflection of the main members, due to the presence of initial geometric imperfections, depends. This causes the effect of shear forces on deflections to be much greater for a built-up column than for a solid column, and thus decreases the buckling load.

To account for these effects on the buckling load of a built-up column, the concept of an equivalent slenderness ratio is adopted. In order to formulate the equivalent slenderness ratio, it is necessary to analyze the effect of local bending of the main members and batten plates.

The analysis of a battened column is conducted according to the classical assumption that there are imaginary hinges at the mid-points of the main members between batten plates, and at the mid-points of the batten plates, and also on the assumption that the deflected shape is sinusoidal.

As a result, the bending moment of the integral column is sinusoidal, and the bending moment of the system due to shear is triangular as described in Section 4.5. Therefore, if the two bending moments are superimposed on each other, the bending moment diagram for a battened column is as illustrated in Figure 4.1.

In this chapter, two methods of formulating the equivalent slenderness ratio are discussed, namely, Timoshenko's method [31] and Bleich's method [7]. The local failure of built-up columns is also discussed.

4.2 Timoshenko's Formula

To derive the equivalent slenderness ratio, Timoshenko [31] determined the effect the shear force would have on the lateral deflection of a built-up column. The formula taken for the buckling load is expressed by

$$P_b = \frac{P_o}{1 + \frac{P_o}{P_d}} \quad (4.1)$$

where P_b is the actual buckling load of the built-up column; P_o is the Euler buckling load; and $1/P_d$ is the quantity by which the shearing force Q is multiplied in order to obtain the additional slope γ of the deflection curve due to shear, such that

$$\gamma = \frac{Q}{P_d} \quad (4.2)$$

To obtain the lateral displacement produced by the shearing force Q , the deformation of an element of the strut, from centre-to-centre of two consecutive panels, as shown in Figure 4.2, is studied. Points of inflection are assumed at the ends of the element, and at the mid-point of the batten plate. The lateral deflection consists of the sum of displacements δ_1 , due to the bending of the batten plate, and δ_2 , due to the bending of the main members.

The couples $Qa/2$, acting at the ends of the batten plate, rotate it at its ends by an angle of rotation of

$$\theta = \frac{Qad}{12EI_b} \quad (4.3)$$

where a is the length of a panel from centre-to-centre of two consecutive batten plates; d is the centroidal separation between the main members; I_b is the moment of inertia of the batten plates about the horizontal centroidal axis perpendicular to the plane of the battens, and so EI_b is the flexural rigidity of the batten plates. As the battens are rigidly fixed to the main members, an equal rotation θ will occur in the main members causing a lateral displacement δ_1 at the ends of the element under discussion, such that

$$\delta_1 = \frac{\theta a}{2} = \frac{Qa^2 d}{24EI_b} \quad (4.4)$$

The displacement, δ_2 , due to the bending of the main members, can be determined from the expression for the deflection of a cantilever beam, so that

$$\delta_2 = \frac{Q}{2} \left(\frac{a}{2} \right)^3 \frac{1}{3EI_1} = \frac{Qa^3}{48EI_1} \quad (4.5)$$

where I_1 is the moment of inertia of a main member about the centroidal axis perpendicular to the battens, and so EI_1 is the flexural rigidity of one main member.

Therefore, the total angular displacement produced by the shearing force Q , is

$$\gamma = \frac{\delta_1 + \delta_2}{\frac{1}{2}a} = \frac{Qad}{12EI_b} + \frac{Qa^2}{24EI_1} \quad (4.6)$$

Hence, Equation 4.2 gives

$$\frac{1}{P_d} = \frac{ad}{12EI_b} + \frac{a^2}{24EI_1} \quad (4.7)$$

Substituting Equation 4.7 into Equation 4.1 gives

$$P_b = \frac{\pi^2 EI}{L^2} \cdot \frac{1}{1 + \frac{\pi^2 EI}{L^2} \left(\frac{ad}{12EI_b} + \frac{a^2}{24EI_1} \right)} \quad (4.8)$$

where I is the moment of inertia of the integral column about the axis perpendicular to the battens; and L is the length of the column.

The contribution to the angular displacement γ by the shear in the batten plates has been neglected, as it is very small in comparison with the other terms.

Since the expression for the buckling load is

$$P_b = \frac{\pi^2 EI}{(K_{eq}L)^2} \quad (4.9)$$

where K_{eq} is the equivalent length factor, then the equivalent slenderness ratio can be written as

$$\lambda_{eq} = \frac{K_{eq}L}{r} = \sqrt{\left(\frac{L}{r}\right)^2 + \frac{\pi^2 adI}{12r^2 I_b} + \frac{\pi^2 a^2 I}{24r^2 I_1}} \quad (4.10)$$

where r is the radius of gyration of the integral column about the axis perpendicular to the battens.

By substituting the geometric properties of the model strut specimens, it was found that the second term under the

radical sign on the right-hand side of Equation 4.10 has an insignificant effect on the value of the equivalent slenderness ratio, and so Equation 4.10 can be rewritten as

$$\lambda_{eq} = \sqrt{\left(\frac{L}{r}\right)^2 + \frac{\pi^2 I}{24 I_1} \left(\frac{a}{r}\right)^2} \quad (4.11)$$

Replacing $1/r^2$ in the second term by $2A_0/I$, where A_0 is the cross-sectional area of one of the main members, and introducing the radius of gyration of the main member, $r_1 = \sqrt{I_1/A_0}$, into Equation 4.11, gives

$$\lambda_{eq} = \sqrt{\left(\frac{L}{r}\right)^2 + \frac{\pi^2}{12} \left(\frac{a}{r_1}\right)^2} \quad (4.12)$$

Equation 4.12 shows that the equivalent slenderness ratio is composed of two parts, the first is the square of the slenderness ratio of the integral column, and the second is the square of the slenderness ratio of a single main member between consecutive battens, multiplied by a factor of $\pi^2/12$.

4.3 Bleich's Formula

Bleich [7] developed a formula to calculate the equivalent slenderness ratio of a pin-ended battened column. The derivation is based on the energy condition that the

transition from stable to unstable equilibrium of any elastic system, is characterized by

$$V - W = 0 \quad (4.13)$$

where V is the strain energy due to deflection; and W is the work done by the external buckling load P_b due to the vertical displacement of its point of application, which is caused by the lateral deflection and axial shortening.

The elastic strain energy of the distorted column, Figure 4.3, consists of three energy terms, which are due to:

- a) the axial shortening under the axial force F_c in the two main members,
- b) the local bending of the two main members, and
- c) the local bending of the batten plates.

The bending energy of the main member, associated with the sinusoidal deformation of the column axis, is at first neglected but is taken into account in the final formula for the buckling load. The effect of the distortion of the main members and the batten plates, due to the shear stresses which occur with bending, are disregarded. The total energy, the sum of the energy from each panel, is used to determine the buckling load.

Based on the above approach, Bleich derived the following equation for the equivalent slenderness ratio of a battened column,

$$\lambda_{oq} = \sqrt{\left(\frac{L}{r}\right)^2 + \frac{\pi^2 I_o}{24 I_1} \left(\frac{a}{r}\right)^2 + \frac{\pi^2 I_o a d}{12 I_b r^2}} \quad (4.14)$$

where I_o is the moment of inertia of the cross-section about the axis perpendicular to the battens, neglecting the moment of inertia of the individual components about their own centroidal axes, and is equal to, $2A_o(d/2)^2 = A_o d^2/2$. Again, the third term under the radical sign on the right-hand side of Equation 4.14 has been numerically proven to have a negligible effect on the equivalent slenderness ratio. So neglecting the third term, and replacing $1/r^2$ in the second term by $2A_o/I$, and introducing the radius of gyration of the main member, $r_1 = \sqrt{I_1/A_o}$, into Equation 4.14, gives

$$\lambda_{oq} = \sqrt{\left(\frac{L}{r}\right)^2 + \frac{\pi^2 I_o}{12 I} \left(\frac{a}{r_1}\right)^2} \quad (4.15)$$

4.4 Comparison Between Timoshenko's and Bleich's Formulae

Studying Equations 4.12 and 4.15, it can be seen that they differ only in the second term. In Timoshenko's formula, the moment of inertia in the numerator, I , is the moment of inertia of the total cross-section, and so has cancelled out with the moment of inertia I in the denominator. Whereas, in Bleich's formula, the moment of inertia in the numerator, I_o ,

is the moment of inertia of the total cross-section neglecting the moment of inertia of the individual main members about their own centroidal axes, $2I_1$.

Ballio and Mazzolani [6] suggested that the effect of neglecting the moment of inertia of the individual main members, $2I_1$, in comparison with their moment of inertia about the overall centroidal axis, would be negligible, and so recommended Bleich's formula, whereas Timoshenko's formula is on the safe side. Discussion of the results given by Equations 4.12 and 4.15 is given in Chapter 6.

Ballio and Mazzolani, however, also suggested that for built-up columns with closely spaced main members, as in the case of no separation, it is logical to reduce the effect of the strut's local slenderness ratio, a/r_1 , in the second term, by 80%, compared to that in struts having spaced elements. Thus, Equation 4.12 becomes

$$\lambda_{eq} = \sqrt{\left(\frac{L}{r}\right)^2 + 0.8 \frac{\pi^2}{12} \left(\frac{a}{r_1}\right)^2} \quad (4.16)$$

Discussion of this equation is also given in Chapter 6.

4.5 Local Failure of Built-Up Columns

In order to prevent a built-up column from failing at a load less than the buckling load obtained by using the equivalent slenderness ratio derived above, it is necessary to

prevent local failure of the battens and the main members, prior to primary failure of the column [7]. In this section, rules are established for the design of built-up columns to prevent premature failure of the column.

Each main member of the built-up column must be able to carry a load of $P_b/2$, safely, over the length between two adjacent batten plates, which is designated a . It seems logical to conclude that if $K_{eq}L/r$ is the equivalent slenderness ratio of the integral column, and k_1a/r_1 , the slenderness ratio of one main member, where r_1 refers to the radius of gyration about the centroidal minimum principal axis, the condition

$$\frac{k_1 a}{r_1} \leq \frac{K_{eq} L}{r} \quad (4.17)$$

must be satisfied, where k_1 indicates the effect of the restraint of the battens. Here k_1 , can be anything between 0.7 and unity, but for design purposes, it is on the safe side to use $k_1=1.0$. Therefore, the theoretical condition for the elimination of local buckling failure of the main member is

$$a \leq \frac{K_{eq} L}{r} r_1 \quad (4.18)$$

It is to be noted, however, that K_{eq} cannot be calculated until

the value of a is known, so it is more convenient to simply use the slenderness ratio of the integral column, so that

$$a \leq \frac{KL}{r} r_1 \quad (4.19)$$

where K is the effective length factor of the integral column.

The function of the batten plates is to carry the shear forces and the moments they produce when the column deflects under the buckling load. The shear forces in the batten plates can be calculated using the usual assumption of hinges at the mid-points of the panels and batten plates, as shown in Figure 4.4. The shear acting on the column Q , can be divided equally between the two main members, so that each member carries $Q/2$. From the equilibrium of the moments about the point of intersection of the batten plate with the main member, the shear at the mid-point hinge of the batten is Qa/d . From this, the maximum moment in the batten plate is found to be $Qa/2$. Details of the statical system and bending moment, due to shear, are shown in Figure 4.4.

The value of the shear force varies along the deflected column. It is equal to the component of the vertical buckling load in the direction perpendicular to the deflected centre-line of the column, and is a function of the deflection as shown in Figure 4.5. Therefore, assuming a sinusoidal deflection, the shear is greatest at the ends of the column,

where the slope of the line of deflection is greatest, and is denoted by Q_{max} , and is equal to zero at the centre of the column.

Bleich [7] proposed that since in the unstable state of equilibrium f , the maximum deflection at mid-height, is indeterminate, there is no defined value of Q indicating the magnitude of the shear force. Nevertheless, it seems logical to design the batten plates, so that they are just strong enough to carry the shearing forces which arise when the deflection reaches the magnitude at which the main member on the concave side of the bent column begins to yield. Then the highest stressed parts of the column will yield or collapse simultaneously.

The maximum stress in the main members σ_1 , associated with the maximum deflection at mid-height f , neglecting the moment of inertia of the main members about their centroidal axes, is

$$\sigma_1 = \frac{P_b}{2A_o} + \frac{P_b f}{A_o d} = \sigma_b + \frac{P_b f}{A_o d} \quad (4.20)$$

where P_b is the buckling load; σ_b the buckling stress; and A_o is the cross-sectional area of one main member. From this equation, the value of f can be defined at which σ_1 will be equal to the yield stress, σ_y ,

$$f = \frac{\sigma_y - \sigma_b}{P_b} A_o d \quad (4.21)$$

Assuming the sinusoidal deformation has a defining equation of $x = f \sin(\pi z/L)$, where x is the lateral deflection and z is the distance along the length of the column, the slope of the centre-line of the column at the ends is $x'|_{z=0} = f\pi/L$. Therefore, the maximum shearing force at the ends of the column is

$$Q_{\max} = P_b \frac{\pi}{L} f \quad (4.22)$$

Substituting f from Equation 4.21, and approximating d by $2r$, where r is the radius of gyration of the column in the plane of the battens, that is to say, $r = \sqrt{I_o/2A_o} = \sqrt{A_o d^2/4A_o}$, gives

$$Q_{\max} = 2\pi (\sigma_y - \sigma_b) A_o \frac{r}{L} \quad (4.23)$$

This formula shows that the shear varies considerably with the slenderness ratio.

Generally, in Standards, the shear is taken to be a percentage of the axial buckling load.

Chapter 5

EXPERIMENTAL PROCEDURE

5.1 Introduction

An experimental program was devised to accumulate enough data to determine the validity of the Canadian Standard S16.1-M89 [10], with regard to the design of built-up columns that buckle in the plane of the connectors. The specimens used were model struts, consisting of two bars of rectangular cross-section connected together by connectors. The material used to manufacture the model struts was AISI-SAE Designation 1010 as specified by the Metals Handbook [4]. It is a hot-rolled carbon steel with a carbon content of about 0.08-0.13%, so it models the behaviour of structural steel. Four 6.10 m long bars were used for the experimental program. Model struts with three different separations, each having an integral slenderness ratio of 80 and 120, were tested. Each strut was tested with four different numbers of connectors.

Tension tests were performed to determine the modulus of elasticity and yield stress of the material. Calibration tests were performed to determine the conversion factor for the load cells in order to change the strain to a load. Measurements of the geometric properties of each specimen were

also carried out.

5.2 Ancillary Tests

5.2.1 Tension Test

5.2.1.1 Specimens

Four tension test specimens were prepared in the Technical Support Centre, one from each of the four bars used to fabricate the model struts. The tension test specimens were designed according to the ASTM Standards [5], Designation: E8-89, but their dimensions were adapted to satisfy the dimensions of the bar, and the grips of the testing machine. The width of the specimens was taken as the width of the bar, and the thickness of the specimen was taken as the thickness of the bar. The dimensions of one of these specimens are shown in Figure 5.1.

5.2.1.2 Electric Resistance Strain Gauges

To measure the strain in the tension test specimen, electric resistance strain gauges were used. Two foil strain gauges were attached to each specimen, one on either side of the centre of the reduced section. In this way, any eccentricities of loading, causing bending strains in the specimens, were eliminated by taking the strain as the average strain of the two gauges. The strain gauges used had a gauge length of 5 mm, and a gauge factor of $2.09 \pm 1\%$. Each gauge was composed of a minute foil coil, 5 mm in length, with an

electric resistance of 199.9Ω , which ended at each end with a terminal. The terminals, in turn, were connected to two wires 30 mm in length. The foil coil and terminals were enclosed with a protective, plastic cover.

This type of strain gauge measures the strain, by the change of resistance of its coil, due to the deformation of the specimen.

5.2.1.3 Test Set-Up

The tension tests were carried out in the universal testing machine, in the University of Windsor's Strength of Materials Laboratory. The strain was measured by a strain indicator, and a switch and balance unit.

5.2.1.4 Test Procedure

Prior to the attachment of the strain gauges onto the tension specimen, the dimensions of the specimen were measured and recorded. In particular, the dimensions of the reduced section at its smallest cross-section was carefully measured so that the stress could be calculated at this section. The measurements were taken using vernier callipers with a minimum reading of 0.02 mm.

The test began by gradually loading the specimen, holding it tightly in place to prevent slippage by means of two external handles for the first 1000 N. Strain readings were taken every 4 kN, and the readings were recorded. As the estimated yield load was approached, the load interval was gradually decreased to 0.5 kN. At the yield load, the load

dropped suddenly, and then remained approximately constant, while the strain increased rapidly. When the load began to increase again, more readings were taken at intervals of 1 kN, until failure or until the strain gauges ceased to function. A tension test specimen, fixed in the universal testing machine, with its strain gauges attached to it, can be seen in Figure 5.2.

In this way, a stress-strain curve was obtained, from which the required properties of the material were acquired.

5.2.2 Calibration Test

Two calibration tests were carried out for the two load cells used in the compression tests. The calibration tests were carried out in the universal testing machine.

5.3 Compression Tests

5.3.1 Test Program

Six sets of compression tests, each consisting of four tests, were carried out on model struts. Each model strut specimen was composed of two rectangular bars, as main members, with dimensions of approximately 6.35 x 25.4 mm, connected together by batten plates. The objective of each test was to obtain an experimental load-deflection curve, and to determine the load carrying capacity of the model strut.

The tests were carried out on specimens with integral slenderness ratios of 80 and 120. Since the specimens with a

slenderness ratio of 120 buckled elastically, only one specimen was required for all four tests. The specimens with a slenderness ratio of 80, however, buckled inelastically, and so a separate specimen was required for each test. Therefore, for the 24 tests carried out, only 15 different specimens were required, which were made in the Technical Support Centre.

Specimens with three different separations were made for both the slenderness ratios. The separations used were zero, 4.02, and 7.85 mm. For the slenderness ratio of 80 these separations corresponded to a length of 293.0, 441.3, and 588.0 mm, respectively. Whereas, for the slenderness ratio of 120 these separations corresponded to a length of 440.0, 660.5, and 881.0 mm, respectively, measured from knife edge to knife edge.

Each specimen was tested with four different numbers of connectors in order to vary and reduce the slenderness ratio of the individual main members. All the specimens were tested with zero, one, three, and seven connectors, except the 293.0 mm specimen which, due to its short length, was tested with zero, one, three, and five connectors. End connectors were also used at each end of the specimen to make sure that the two main members stayed together at the ends. For the specimens with a slenderness ratio of 120, the number of connectors was increased after each test on the same specimen.

Two parallel planes of connectors were welded to the narrow width of the main members, using Tungsten Inert Gas

(T.I.G.) welding. This type of welding was chosen as it caused the least amount of residual stresses. The connectors all had the same cross-sectional dimensions, which were 4.76 x 12.7 mm. The length of the connectors depended on the separation of the specimens, but in general, they had a length equal to the separation of the main members plus an overlap of 4.76 mm on both main members. Details of the compression specimens can be seen in Figures 5.3 (a) to (m).

The moment of inertia of the cross-section about the axis perpendicular to the connectors, was the smallest for the specimens with zero and 4.02 mm separation, and so buckling occurred in the plane of the connectors. For the specimens with 7.85 mm separation, the smallest moment of inertia was about the axis parallel to the connectors, but as this axis was essentially fixed at the ends, and the perpendicular axis was pinned at the ends, buckling still occurred in the plane of the connectors.

The tests were all designated by a test number, such as 441-4-3. The first number indicates the nominal length of the specimen, the second number indicates the nominal separation between the main members of the specimen, and the third number specifies the number of connectors.

5.3.2 Test Set-Up

Two testing frames were used to conduct the compression tests, a universal testing machine, and a shop-fabricated testing frame. In this section, a description of the pieces

of equipment used to apply and measure the load, and the test procedure, is given.

5.3.2.1 End Conditions

The model strut specimens were all tested with pin-ended conditions about the axis perpendicular to the connectors, and essentially fixed-ended conditions about the axis parallel to the connectors. These end conditions were achieved by means of knife edges, parallel to each other, at each end of the specimen.

Two types of knife edges were used for the specimens, one for the specimens with no separation, and one for the specimens with a separation of 4.02 and 7.85 mm. Each pair of knife edge assemblies were composed of two parts, a part fixed to the testing machine or frame, and a part fixed to the specimen. The part of each pair of knife edges fixed to the testing machine or frame, was the same for both types of knife edges. It consisted of a 50.8 mm square plate, with a thickness of 12.7 mm. On one side of the plate, there was a central groove with an included angle of 135° , and a depth of 3.18 mm, running across the whole width of the plate. These plates were fixed to the top and bottom of the testing machine or frame by means of two socket head cap screws, screwed through counter bores located in opposite diagonal corners of the plate. These two plates were carefully adjusted each time the loading frame was assembled to make sure that they were aligned directly one above the other, such that the grooves

were parallel to each other. Details of these plates are shown in Figure 5.4. The second part of each pair of knife edges, for the specimen with no separation, consisted of a metal piece with an inclined part of depth 12.7 mm, and a straight part of depth 6.35 mm. The width of this piece was 25.4 mm, which narrowed to an apex of 90° . The length of this piece was 38.1 mm. Inside the straight part, there was a 12.7 x 25.4 mm pocket with a depth of 6.35 mm into which the two main members of the specimen fitted. Details of this knife edge are shown in Figure 5.5 (a).

The second part of each pair of knife edges for the specimens with a separation of 4.02 and 7.85 mm consisted of two parts, a square base plate and a steel square bar. Two sizes of knife edges were used. The first size used consisted of a 50.8 mm square base plate with a thickness of 25.4 mm, welded onto the ends of the two main members. On the side away from the main members, there was a groove with an included angle of 90° , and a depth of 12.7 mm. This groove was centred between the main members, and ran along the whole length of the base plate. Along with this plate, there was a 19.05 mm steel square bar, with a length of 47.63 mm. This size of knife edge assembly was later considered to be unnecessarily large, and so the size of the assembly was reduced for subsequent specimens. The smaller sized knife edge had a 50.8 mm base plate with a thickness of 12.7 mm. The groove had an included angle of 90° and a depth of 6.35

mm. A 12.7 mm square bar, with a length of 47.63 mm, was used with this base plate. In this type of knife edge, the square bar fitted snugly into the groove of the base plate, so that no rotation between these two pieces occurred. Details of this knife edge is shown in Figure 5.5 (b).

The edge of the specimens rotated about the vertex of each of the grooves in the plates attached to the testing machine or frame. This insured a minimum of resistance to rotation. The whole knife edge assembly, was held together by the compressive load applied to the specimen.

5.3.2.2 Universal Testing Machine

The universal testing machine was used to test the shorter three model strut specimens, that is the 293.0, 440.0, and 441.3 mm long specimens.

The lower plate of the knife edge assembly was attached to a 76.2 x 355.6 x 12.7 mm bottom plate as shown in Figure 5.6 (a). This plate was attached to the table of the testing machine by means of four steel pins inserted through four holes in the plate, and four holes existing in the testing machine. In this way, the bottom plate was prevented from moving. This also ensured that proper alignment was maintained.

The upper plate of the knife edge was attached with socket head cap screws to a 152.4 x 152.4 x 9.5 mm plate, which had a notch which fitted into the opening of the lower head, as shown in Figure 5.6 (b). This plate was fixed to the

lower head by means of a second plate on top of the lower head, connected to the first by a screw. A nut was tightened against the second plate, so that the first plate was held tightly to the lower head.

The alignment of the knife edge assembly was carefully checked using a plumb line. Details of the whole set-up in the universal testing machine, can be seen in Figure 5.7.

The load was applied to the specimen by means of a piston below the table. The load was read directly from the digital display.

5.3.2.3 Testing Frame

The model strut specimens with lengths of 588.0, 660.5, and 881.0 mm, were too long to fit into the universal testing machine. Thus a shop fabricated welded frame in the University of Windsor's Structures Laboratory was used to test these specimens.

This frame was composed of two steel columns, held together at the top and bottom, by steel beams welded to the columns. Each column had two triangular brackets, welded onto its lower end, forming a sturdy base for the frame. The frame also had a third, adjustable arm, which could be positioned at any height along its length.

A jack was attached to the lower beam by means of a framing system composed of a lower plate with four rods and two upper bars. The upper bars were tightly held down over the jack by the four threaded rods. The lower plate of the

knife edge was fastened to the piston of the jack. Details of this arrangement are shown in Figure 5.8 (a).

The load cell was attached to a plate, which was attached to the adjustable arm. The upper plate of the knife edge was fastened to the load cell. Details of this arrangement are shown in Figure 5.8 (b).

The adjustable arm was used in two positions, one for the 881.0 mm specimens, and one for the 588.0 and 660.5 mm specimens. For the 588.0 mm specimens, a 50.8 mm high cylindrical seat was used to make up the difference in height. Details of this seat, and of the whole set-up, can be seen in Figure 5.9.

5.3.2.4 Loading Equipment

The hydraulic jack used in the shop fabricated frame was short and stocky with a capacity of 224 kN. The piston of the jack had a stroke of 50.8 mm. The jack was operated manually, by means of a hydraulic pump with a capacity of 69.4 MPa, attached to the jack by a flexible tube. Details of this assembly can be seen in Figure 5.10.

5.3.2.5 Load Cell

Two load cells were used. A load cell with a capacity of 44.8 kN was used for all the specimens tested in the testing frame except for the 588.0 mm specimens, with three and seven connectors. For these specimens a load cell with a capacity of 112.0 kN was used.

The loads measured by the load cell were preset by

setting the strain reading on the strain indicator to the required value as determined by the calibration test. Then the specimen was loaded until the dial on the strain indicator was balanced and steady.

5.3.2.6 Instrumentation

Due to the simple buckled shape of the model strut specimens, one of buckling in the plane of the connectors, obtaining the data for lateral displacements was not complicated. A dial gauge was placed at the mid-height of one of the main members, perpendicular to the long side of the rectangular cross-section. For the specimens with zero connectors, two dial gauges were used, one at mid-height of each main member, to measure the lateral deflection of each main member.

The shortening of the specimens was also measured. For the specimens tested in the universal testing machine, this was done by measuring the relative displacement between the top plate and the bottom plate adjacent to the specimen. For the specimens tested in the testing frame, this was done by measuring the displacement of the piston. However, the results obtained were much larger than those theoretically predicted. This was probably due to the wearing down of the knife edges, and the compressing of any gaps between the different pieces of the loading assembly, under the effect of the compressive load. Although these readings of axial shortening were recorded, no further attempt was made to

compare them to the theoretical results. The theoretical analysis of the shortening of a column is given in Appendix D.

The piston of the hydraulic jack in the testing frame was also checked for lateral displacement by placing a dial gauge horizontally against it.

Two types of dial gauges were used. The first type was a mechanical dial gauge with a minimum reading of 0.01 mm. The second type was a digital dial gauge, also with a minimum reading of 0.01 mm. Each dial gauge was supported by a flexible arm attached to a magnetic base. The magnetic bases were attached to the frame.

Details of the assembly of dial gauges for the tests conducted in the universal testing machine and the testing frame are shown in Figures 5.11 (a) and (b), respectively.

5.3.3 Test Procedure

The model strut specimens were first measured to determine the initial out-of-straightness. As buckling occurred in the plane of the connectors, only the out-of-straightness in this plane was measured. This can be done in the laboratory by taking displacement measurements, measuring from a taut wire or a straight edge to each main member of the model strut. The latter approach was preferred as the model struts were quite short and by using a straight edge the vernier callipers could be held against both the strut and the straight edge to obtain the out-of-straightness reading. This method would not be suitable for longer specimens, as the

straight edge could sag if it was too long. The set-up for the out-of-straightness measurements is shown in Figure 5.12.

A metal straight edge was placed over two standard blocks positioned at the two ends of one of the main members. At mid-height, quarter points, and adjacent to the blocks, the distance was measured from the top of the straight edge to the main member of the specimen using vernier callipers, making sure that the callipers were perpendicular to the specimen and the straight edge. This procedure was followed for both main members of each specimen. The length of each specimen was also measured from the vertices of the knife edges.

The compression test was conducted by placing the specimen in the testing arrangement being used. The dial gauges were positioned, and the strain indicator was connected to the load cell, and zeroed. A small load was applied to the specimen, ensuring that the top and bottom ends of the specimen were aligned with and parallel to the upper and lower knife edge plates. The specimen was then unloaded until the load just reached zero and the dial gauges were again zeroed.

The specimens were loaded, slowly, in increments ranging from 2 to 8 kN, according to the predicted failure load. As the load approached its predicted failure load, or the deflections increased too rapidly, the increments were reduced from 2 to 1, and eventually to 0.5 kN. At each increment, the dial gauge readings were recorded when equilibrium was reached.

The specimens with a slenderness ratio of 80, and no separation, were unloaded when they buckled inelastically and collapsed. The specimens with a slenderness ratio of 80, and a separation of 4.02 or 7.85 mm, were unloaded when the load started to drop. Whereas the specimens with a slenderness ratio of 120 were unloaded when a precalculated deflection, within the elastic limit of the model strut, was reached.

5.3.4 Data Reduction

5.3.4.1 Load

The results of the calibration tests were used to convert the microstrain readings of the strain indicator, into kN. Calibration curves for the 44.8 and 112.0 kN capacity load cells were plotted, as shown in Figures 5.13 (a) and (b), respectively.

5.3.4.2 Out-Of-Straightness

From the data obtained as described in the previous section, the initial out-of-straightness was calculated. This was done by assuming a straight line relationship, between the points at the two blocks. The out-of-straightness of these two points was assumed to be zero. The relative out-of-straightness of each intermediate point was calculated by subtracting the reading at this point from the value of the reading of the imaginary straight line between the end points, at this point.

The out-of-straightness varied from approximately zero,

to almost $L/250$, where L is the length of the member from knife edge to knife edge. In these tests, no limitations were put on the out-of-straightness.

The out-of-straightness results were used in the iterative-incremental procedure to predict the theoretical load-deflection curves.

Chapter 6

RESULTS AND DISCUSSION

6.1 Introduction

This chapter presents and discusses the results of the experimental data and the theoretical calculations made for the model strut specimens tested.

The experimental data presented includes the geometric and mechanical properties of the compression test specimens determined from data obtained during the experimental program.

The requirements of different design Standards, presented in Chapter 2, for the design of built-up compression members and their connectors, are discussed and compared with the experimental results obtained.

The experimental and theoretical load-deflection curves are plotted. The experimental curves are plotted from the data obtained from the compression tests. The theoretical curves are plotted from data obtained from the finite element program, described in Chapter 3. A comparison is made of the experimental and theoretical load-deflection curves.

The equivalent slenderness ratios of the built-up struts are calculated using Timoshenko's formula [31], Bleich's formula [7], and the Canadian Standard [10], Clause 19.1.4.

The buckling loads are then calculated using the equations in Clause 13.3.1 of the Canadian Standard S16.1-M89. These buckling loads are studied, and their validity established by comparing them to the experimental results. The buckling loads from the finite element program are also presented. The forces in the connectors are calculated according to the Canadian Standard S16.1-M89, and also using the method described in Chapter 4 and the buckling load obtained from the experimental results.

The effect of the separation between the main members and the number of connectors on the experimental and theoretical results are discussed. Finally, the implications of the present design Standards and Specifications are discussed with respect to the buckling load of the column, and the forces in the connectors.

6.2 Properties

6.2.1 Geometric Properties

The specimens used were all made from rectangular steel bars of nominal dimensions $1 \times 1/4$ in. in US Customary Units, that is, 25.4×6.35 mm in SI units. The actual cross-section of the bars was curved along the width of the rectangle. The actual cross-sectional area was calculated neglecting the curvature of the ends. The actual dimensions of the bars, which were measured for each bar, varied from specimen to specimen from the nominal dimensions by -1.34% to $+0.16\%$.

The properties of the cross-section used for the theoretical calculations, can be determined as follows:

$$A = 2A_o \quad (6.1 \text{ a})$$

$$I_x = 2 I_{x1} \quad (6.1 \text{ b})$$

$$I_o = A_o \frac{d^2}{2} \quad (6.1 \text{ c})$$

$$I = 2 \left[I_1 + A_o \left(\frac{d}{2} \right)^2 \right] = 2 I_1 + I_o \quad (6.1 \text{ d})$$

$$r = \sqrt{\frac{I}{A}} \quad (6.1 \text{ e})$$

$$r_1 = \sqrt{\frac{I_1}{A_o}} \quad (6.1 \text{ f})$$

where A is the total area of the cross section; A_o is the cross-sectional area of one main member; I_x and I are the moments of inertia of the entire cross-section about the X and Y axes, respectively; I_{x1} and I_1 are the moments of inertia of one main member about the x and y axes, respectively; I_o is the moment of inertia of the entire cross-section about the axis perpendicular to the battens neglecting the moment of inertia of the individual components about their own axes; r and r_1 are the radii of gyration of the entire cross-section and the individual cross-section about the axes perpendicular

to the battens, respectively; and d is the centroid to centroid distance between the main members parallel to the plane of the battens.

The mid-height initial out-of-straightness of the specimens, as measured by the procedure described in Chapter 5, varied from approximately zero to $1/250$ of the length of the specimen. The mid-height initial out-of-straightness of each specimen is shown in Table 4.

6.2.2 Mechanical Properties

The mechanical properties of the compression specimens were determined from the tension tests described in Chapter 5. Four tension test specimens were tested, one from each bar used to make the compression specimens. For each test, the modulus of elasticity E and the yield stress σ_y were determined. A stress-strain curve was also drawn for each tension test specimen. Figure 6.1 shows the stress-strain curve for tension test specimen 3. The values of the modulus of elasticity and the yield stress, varied between 198 600 to 211 100 MPa and 321.4 to 338.5 MPa, respectively. The results of each tension test are listed in Table 1. The calculations for each compression specimen were made using the values of the modulus of elasticity and the yield stress corresponding to the bar from which they were made.

6.3 Requirements of the Standards and Specifications

6.3.1 Canadian Standard, CAN/CSA-S16.1-M89, [10]

The Canadian Standard, in Clause 19.1, gives several requirements for the spacing of connectors for built-up compression members. The main requirement given in this clause is Clause 19.1.3(c), which requires that

$$\lambda_1 \leq \lambda_y \quad (6.2)$$

where λ_1 is the maximum slenderness ratio of an individual main member between ends of adjacent batten plates; and λ_y is the slenderness ratio of the integral column with respect to the axis perpendicular to the battens using $K=1.0$ for pinned end conditions. Clause 19.1.16 also requires that

$$\begin{array}{ll} \lambda_1 < 50 & \text{If } \lambda_y \leq 0.8\lambda_x \\ \lambda_1 < 0.7\lambda_x & \end{array} \quad (6.3 \text{ a})$$

$$\begin{array}{ll} \lambda_1 < 40 & \text{If } \lambda_y > 0.8\lambda_x \\ \lambda_1 < 0.6\lambda_y & \end{array} \quad (6.3 \text{ b})$$

where λ_x is the slenderness ratio of the integral column with respect to the axis parallel to the battens using $K=0.5$ for fixed end conditions.

Calculating λ_y and λ_x from the geometric properties given in Table 2, it can be seen that λ_y is much greater than λ_x for

all the model struts tested, so the second condition given in Equation 6.3, is valid. Therefore, Equations 6.2 and 6.3 b, contradict each other. That is, the maximum slenderness ratio of one main member must not be greater than 40 or 60% of λ_y , and not, as stated in Clause 19.1.3(c), less than λ_y . The results of the analysis of this concept is discussed in Section 6.9.

Clause 19.1.4 requires that the equivalent slenderness ratio for compression members composed of two or more shapes in contact or separated from one another by welded connectors shall be

$$\lambda_{eq} = \sqrt{\left(\frac{KL}{r}\right)^2 + \left(0.65 \frac{a}{r_1}\right)^2} \quad (6.4)$$

where a is the centre-to-centre distance between connectors; r_1 is the minimum radius of gyration for one main member, which is about the axis perpendicular to the battens; $K=1.0$ for the case of buckling occurring in the plane of the connectors, which has pinned end conditions; L is the overall length of the column, which is from knife edge to knife edge; and r is the radius of gyration of the integral column with respect to the axis about which buckling occurs, which is the axis perpendicular to the battens. The results of this equivalent slenderness ratio for each model strut are listed in Table 3. The buckling load based on this equivalent

slenderness ratio is calculated according to Clause 13.3.1 of the Canadian Standard and is listed in Table 5.

The Canadian Standard, in Clause 19.1.17, specifies for connectors that their length shall not be less than the distance between lines of welds connecting them, and that their thickness shall not be less than 1/60 of this distance. The first condition was approximately satisfied by the struts with a separation of zero and 4.02 mm, but for the struts with a separation of 7.85 mm, the length was slightly short. The second condition was sparingly satisfied for all the struts.

Clause 19.1.17 also requires that the battens be proportioned to resist, simultaneously, a longitudinal shear force and a bending moment given by

$$Q_b = \frac{0.025 P_f a}{n d} \quad (6.5 \text{ a})$$

$$M_b = \frac{0.025 P_f a}{2 n} \quad (6.5 \text{ b})$$

where Q_b is the longitudinal shear in the batten (kN); M_b is the moment which must be resisted by the batten (kN·mm); P_f is the factored compressive load (kN); a is the longitudinal centre to centre distance between battens (mm); d is the distance between lines of bolts or welds connecting the batten to each main component (mm); and n is the number of parallel planes of battens. The factored resistances of the batten

plates, with respect to bending and shear, were calculated according to Clauses 13.5 and 13.4.2, respectively. The results of the factored forces and factored resistances for the model struts are discussed in Section 6.6.

6.3.2 American Specifications, AISC, [2,3]

In the ASD Manual [2], it is required that

$$\lambda_1 \leq 0.75 \lambda_y \quad (6.6)$$

whereas, in the LRFD Manual [3], the same requirement is specified as given in Equation 6.2. As Equation 6.6 is more critical it will be assumed as valid. Therefore, the slenderness ratio of an individual main member can be up to 0.75 times the integral slenderness ratio of the built-up strut, in comparison to the 0.6 suggested in the Canadian Standard.

The modified slenderness ratio specified in Clause E4 of the LRFD Manual is

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_1} - 50\right)^2} \quad \text{for } \frac{a}{r_1} > 50 \quad (6.7 \text{ a})$$

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_o \quad \text{for } \frac{a}{r_1} \leq 50 \quad (6.7 \text{ b})$$

where $(KL/r)_o$ is the slenderness ratio of the integral column.

This equation gives results for the model struts that are slightly less than those obtained using Bleich's formula, but better than those obtained by the Canadian Standard. However, as the comparison being made is mainly between the results given by the Canadian Standard and the results given by Timoshenko's and Bleich's formulae, the equivalent slenderness ratio as given in the LRFD Specification are not listed.

6.3.3 German Standard, DIN 4114-1952, [13]

The German Standard gives two criteria for the maximum spacing of the connectors. These are that

$$\frac{a}{r_1} \leq \frac{1}{2} \lambda_x \quad (6.8 \text{ a})$$

$$\frac{a}{r_1} \leq 50 \quad (6.8 \text{ b})$$

providing that at least two connectors are used at the third points of the built-up column. Substituting the geometric properties of the model struts into the right-hand side of Equation 6.8 a, the maximum slenderness ratio a/r_1 , of an individual main member ranges from 10.07 to 30.28 for the specimens with a length of 293.0 to 881.0 mm, respectively. As the lower limit of this inequality is unreasonably small, the first condition of Equation 6.8 will be neglected, and the second will be taken as valid.

Clause 8.212 requires that the equivalent slenderness

ratio for built-up columns that buckle at right angles to the axis perpendicular to the battens be

$$\lambda_{eq} = \sqrt{\left(\frac{KL}{r}\right)^2 + \frac{m}{2}\left(\frac{a}{r_1}\right)^2} \quad (6.9)$$

where m is the number of main members. In our case $m=2$ and so the equivalent slenderness ratio is reduced to that of the Canadian Standard without any factor being applied to the slenderness ratio of the individual main member. However, the results given by this equation are too conservative, and so are not listed.

The German Standard requires the battens to be designed to resist an assumed lateral shear on the column equal to 1.25% of the area times the allowable stress, half the value required by the Canadian Standard.

6.3.4 British Standard, BS 5950 1985, [8]

The requirements of the British Standard for the maximum spacing of connectors given in Clause 4.7.9(c) are

$$\lambda_1 \leq 50 \quad (6.10 \text{ a})$$

$$1.4 \lambda_1 \leq \lambda_x \quad (6.10 \text{ b})$$

As was shown in the requirements of the German Standard, if

the geometric properties are substituted into the right hand side of Equation 6.10 b, the maximum slenderness ratio of the individual main member between ends of adjacent battens would be 14.38 for the case of the specimen with a length : 293.0 mm, which is too small. Therefore, the first condition will be taken as valid.

Clause 4.7.9(c) also specifies the equivalent slenderness ratio of built-up columns, about the axis perpendicular to the battens as

$$\lambda_{eq} = \sqrt{\left(\frac{KL}{r}\right)^2 + \left(\frac{a_1}{r_1}\right)^2} \quad (6.11)$$

where a_1 is the distance between ends of adjacent battens. This equation is the same as specified by the German Standard, except for the second term where the maximum slenderness ratio of the main member is measured from centre-to-centre of adjacent battens in the German Standard, and from ends of adjacent battens in the British Standard, and is still too conservative.

Clause 4.7.9(e) requires that the battens resist a lateral shear force on the column of 1% of the maximum factored axial load, as opposed to the 2.5% required by the Canadian Standard. This is discussed in Section 6.9.

6.4 Load-Deflection Curves

6.4.1 Theoretical Load-Deflection Curves

The theoretical load-deflection curve for each specimen was plotted from the results of the finite element iterative-incremental procedure described in Chapter 3.

Figures 6.2 to 6.7 show the theoretical and experimental curves superimposed, for the specimens with zero, 4.02, and 7.85 mm separation, for each of the integral slenderness ratios of 120 and 80, respectively. Each of the above mentioned figures are divided into four parts, showing the theoretical and experimental curves of the specimens with zero, one, three, and five or seven connectors. Figures 6.7 (a) and (c) show only the experimental curves, and not the theoretical curves, because of the poor agreement between them. This was caused by the fact that the strut first deflected in one direction and then at higher loads deflected in the other direction, which was probably caused by an irregular initial out-of-straightness.

All the curves neglect the initial out-of-straightness of the specimen. Thus all the curves start at zero deflection. As this is done for both the experimental and theoretical curves the comparison is the same.

For the specimens with zero connectors, the load-deflection curves are plotted for each main member, except for the case of no separation where the main members were modelled to move together. For the specimens with one, three, five or

seven connectors, the load-deflection curve is plotted for only one main member, as the two members move together.

Figure 6.8 shows the theoretical load-deflection curves of the specimen with an integral slenderness ratio of 80 and no separation, for the cases of zero, one, three, and five connectors, with an initial out-of-straightness of 0.1 mm. It can be seen that for the cases of zero and one connector, the curves coincide, and give the same buckling load. Whereas, adding three and five connectors increases the load.

The theoretical load-deflection curves were checked using the following equation

$$x = \frac{x_0}{\left(1 - \frac{P}{P_0}\right)} \quad (6.12)$$

where x is the deflection; x_0 is the initial out-of-straightness; P is the axial load; and P_0 is the buckling load. The term $1/(1-P/P_0)$ is known as the magnification factor. The theoretical load-deflection curves show good agreement with this equation.

6.4.2 Experimental Load-Deflection Curves

The experimental load-deflection curves are shown in Figures 6.2 to 6.7 for all the specimens tested. It can be seen that the experimental load-deflection curves are in reasonable agreement with the theoretical curves at small

loads, for most of the specimens, with the exception of the specimens with an integral slenderness ratio of 120 and no separation, with zero, one, and three connectors. This is due to the initial rapid increase in the deflection of the model strut at small loads. This rapid increase in deflection does not occur in the theoretical analysis and so the two curves are not in good agreement, as shown in Figures 6.2 (a) to (c).

The experimental results also include the failure load, the initial out-of-straightness at mid-height measured for each specimen and, as described in Section 6.5.3, the predicted experimental failure load for a specimen with an initial out-of-straightness of $L/1000$, where L is the length of the integral column from knife edge to knife edge. This data is listed in Table 4.

6.4.3 Comparison Between Experimental and Theoretical Load-Deflection Curves

For the specimens with no separation, where the two main members are in contact, the experimental load-deflection curves give a buckling load that is about the same or higher than the theoretical curves. This is probably due to the extra rigidity of the model strut gained by the support of one main member by the other. That is to say, the compressive stress in the main member on the convex side of the deflected column is theoretically less than the compressive stress in the main member on the concave side, and thus assists the member on the concave side to resist additional displacements.

This would increase the strength of the strut. In the finite element program, pinned rigid beams were added between the two main members to keep them apart at a constant separation, but the added support could not be modelled, and so resulted in a lower buckling load.

For the specimens with a separation of 4.02 and 7.85 mm, the theoretical and experimental load-deflection curves show good agreement for the cases of zero and one connector. Whereas, for the cases of three and seven connectors, the theoretical and experimental load-deflection curves are not in good agreement. The theoretical curve gives a considerably higher buckling load than the experimental curve. This discrepancy is probably caused by the extra initial residual stress induced into the main members by the welding of the greater number of connectors onto the main members. These residual stresses were not included in the finite element program as their value was not known.

In addition to the above mentioned differences, the rest of the minor discrepancies between the theoretical and experimental load-deflection curves shown in Figures 6.2 to 6.7 may be a result of:

a) Experimental

- i) inaccuracies in the measurement of the initial out-of-straightness,
- ii) inaccuracies in the measurement of the physical and mechanical properties of the specimens,

- iii) imperfections in setting up the test specimens, and
- iv) the load is not applied concentrically.

b) Theoretical

- i) neglecting the effect of the end plates,
- ii) the assumption that the connectors are perfectly rigid and rigidly fixed to the main members, and
- iii) neglecting the effect of shear deformation in the main members and connectors in the finite element procedure.

In general, both the theoretical and experimental load-deflection curves show an increase in buckling load for each specimen as the number of connectors are increased. However, adding seven connectors did not result in a buckling load equal to the Euler load of the integral column.

6.5 Theoretical Investigation

6.5.1 Timoshenko's Method

Substituting the geometric properties of each specimen, from Table 2, into Equation 4.12, gives the equivalent slenderness ratio of the built-up model strut. These results are listed in Table 3.

Using the mechanical properties of each specimen, listed in Table 1, and substituting into the equations given in Clause 13.3.1 of the Canadian Standard S16.1-M89, the buckling load is obtained. The results of the buckling loads for all the specimens are listed in Table 5.

6.5.2 Bleich's Method

The same procedure as described above is repeated using Equation 4.15 for the equivalent slenderness ratio, and Clause 13.3.1 of the Canadian Standard S16.1-M89 for the buckling load. The results of the equivalent slenderness ratio and the buckling load for each specimen are given in Tables 3 and 5, respectively.

6.5.3 Finite Element Method

The buckling load of each model strut specimen was determined using the iterative-incremental procedure described in Chapter 3. The initial out-of-straightness measured for each specimen was used as the initial geometric out-of-straightness for the finite element model, and the mechanical properties measured for the bar from which each specimen was made were used as the material properties for the finite element model.

The equations for the buckling load in Clause 13.3.1 of the Canadian Standard S16.1-M89, were derived for an initial out-of-straightness of $L/1000$, where L is the total length of the specimen from knife edge to knife edge. So, in order to properly compare the results of the theoretical buckling loads obtained by these equations with the experimental results, the experimental buckling load of a specimen with an initial out-of-straightness of $L/1000$ is required. Therefore, the finite element method was also used to predict the experimental buckling load, for a specimen with an initial out-of-

straightness of $L/1000$. This was done by determining the buckling load of the finite element model with an initial out-of-straightness of $L/1000$. Then, by multiplying it by the ratio of the experimental buckling load of the specimen to the theoretical buckling load of the specimen derived by the finite element method with the exact out-of-straightness, the experimental buckling load for the specimen with an initial out-of-straightness of $L/1000$ can be predicted. This process was followed, rather than using the absolute value of the buckling load of the specimen with an initial out-of-straightness of $L/1000$ determined by the finite element method, because for the specimens with three and seven connectors the theoretical buckling load was greater than the experimental buckling load for the same out-of-straightness.

The buckling loads determined by the finite element iterative-incremental procedure for the actual out-of-straightness and for the predicted experimental buckling load with an out-of-straightness of $L/1000$ are given in Tables 5 and 4, respectively.

6.5.4 Comparison and Discussion of the Theoretical and Experimental Results

Table 3 shows the equivalent slenderness ratio for each model strut calculated using Timoshenko's formula, Equation 4.12, Bleich's formula, Equation 4.15, and the Canadian Standard, Equation 6.4. As was stated in Section 4.4, Equations 4.12 and 4.15 differ only in the second term, where

the moment of inertia is I in Timoshenko's formula and I_c in Bleich's formula. The maximum percent difference between the theoretical equivalent slenderness ratios as determined by these two formulae is 9.3%, which occurs in the case of no separation. For this case, the moment of inertia of the individual main members about their own centroidal axes I_1 , has a significant effect on the overall moment of inertia of the specimen about the axis perpendicular to the connectors. Although the minimum difference in the theoretical equivalent slenderness ratio as determined by these two equations can be quite small, as small as 0.5% in some cases, it was found that Timoshenko's formula gave results for the buckling load which are in better agreement with the experimental results.

The results of the equivalent slenderness ratio specified by the Canadian Standard are up to 25.6% less than those obtained from Timoshenko's method for the specimens that were sparsely connected, but are as low as 0.9% less for the specimens that had seven connectors.

Table 5 lists the buckling load of each specimen as determined by the finite element method, Timoshenko's method, Bleich's method, and the method specified by the Canadian Standard.

Comparing the finite element buckling load to the experimental failure load for struts with the same initial out-of-straightness shows that they tend to be in reasonable agreement for the cases of zero and one connector. For the

cases of three and seven connectors, the experimental failure load is greater than the finite element buckling load for the specimens with no separation, and less than it for the specimens with a separation of 4.02 and 7.85 mm. So, the finite element method did not correctly predict the experimental results in all cases.

Figures 6.9 (a) and (b) demonstrate the differences between the buckling loads obtained using Timoshenko's formula, Bleich's formula, the Canadian Standard, and the predicted experimental buckling load for specimens with an out-of-straightness of $L/1000$, for the case of one connector. Referring to Figure 6.9 (a), it can be seen that for the specimen with no separation, all three solutions predict a lower buckling load than the experimental failure load, so all are safe solutions, but the buckling load predicted by the Canadian Standard is the closest to the experimental. For the specimen with a separation of 4.02 mm, the buckling load given by the Canadian Standard is very high compared to the experimental failure load, and so is an unsafe solution, even if it is multiplied by the resistance factor of 0.9. Timoshenko's and Bleich's solutions are both very close to the experimental failure load, Timoshenko's is on the safe side, Bleich's is safe when multiplied by the resistance factor of 0.9. For the specimen with a separation of 7.85 mm, again the buckling load given by the Canadian Standard is too high, and so is an unsafe solution. Both Timoshenko's and Bleich's

solutions, are slightly too high, but by multiplying Timoshenko's solution by 0.9 it is approximately equal to the experimental failure load.

For the case of seven connectors, the differences in these solutions are very small, but to unify the solution, Timoshenko's formula should be used.

Figure 6.10 shows an example of the relationship between the equivalent slenderness ratio and the buckling load for the specimen with an integral slenderness ratio of 120 and a separation of 4.02 mm, for the cases of zero, one, three, and seven connectors. For the case of zero connectors, which is not allowed by any Standard, all the solutions—the Canadian Standard, Timoshenko's formula, and Bleich's formula—give a lower buckling load than the experimental failure load, and so are safe. For the cases of one and three connectors, the Canadian Standard gives a buckling load that is unsafe, while both Timoshenko's and Bleich's formulae are valid. For the case of seven connectors, all the solutions are approximately equal. This figure also shows that the difference in equivalent slenderness ratio derived by the different formulae is much greater for sparsely connected columns.

Thus, it can be concluded that for specimens with separated main members, Timoshenko's formula is the most suitable solution. Whereas for the specimens with no separation, it may be better to apply Ballio and Mazzolani's [6] suggestion, discussed in Section 4.4, to reduce the

constant in the second term in Equation 4.12. Ballio and Mazzolani suggested reducing the constant 0.82 by 80% giving a constant of approximately 0.65, as shown in Equation 4.16, but it would be more appropriate to reduce it by 50% giving a constant of 0.41. In this way, Timoshenko's formula for the equivalent slenderness ratio would be reduced to that of the Canadian Standard which gave the best results for the specimens with no separation.

Figures 6.11 (a) and (b) show the ratio of the experimental failure load to the Euler load of the integral column, for the cases of zero, one, three, five, and seven connectors, for the specimens with an integral slenderness ratio of 120 and 80, respectively. Comparing Figures 6.11 (a) and (b) shows that the experimental failure load is closer to the Euler buckling load for the case of the integral slenderness ratio of 120 than 80. Figures 6.11 also show a drop in the ratio of the experimental failure load to the Euler load, with the increase in separation. For example, for the specimens with an integral slenderness ratio of 120 and seven connectors, the ratio is 0.957 for no separation and 0.669 for a separation of 7.85 mm.

6.6 Forces in the Connectors

The experimental forces in the connectors were not measured as the connectors used for the model strut specimens were very small, and so connecting electrical strain gauges to

them was impossible. However, the factored resistance of the connectors was calculated according to the Canadian Standard S16.1-M89, Clause 13, and compared to the results of the forces in the connectors according to the Canadian Standard S16.1-M89, Clause 19.1.17. As all the model struts had 4.76 x 12.7 mm rectangular cross-section connectors, the factored resistance of the connectors is the same for all the struts, such that

$$V_{fr} = 0.66 \phi F_y A_b \quad (6.13 \text{ a})$$

$$M_{fr} = \phi z_b F_y \quad (6.13 \text{ b})$$

where ϕ , the resistance factor, is taken as 1.0; V_{fr} is the factored shear resistance of the connectors; M_{fr} is the factored moment of resistance of the connectors; F_y is the yield stress; A_b is the cross-sectional area of a connector; and z_b is the plastic section modulus of a connector. Substituting the values into these equations gives a shear resistance of 13.10 kN and a moment of resistance of 63.03 kN·mm. The maximum shear force and bending moment in the connectors, as determined by Clause 19.1.17 of the Canadian Standard, is 8.29 kN and 57.90 kN·mm, respectively, so that theoretically the connectors are safe. As nothing happened to the connectors during the tests, it can be assumed that the requirements of the Canadian Standard for the forces

in the connectors are adequate.

The maximum shear force in the column calculated from Equation 4.23 gave values for the shear force and bending moment in the connectors, which in most cases, were too large when compared to those calculated by the Canadian Standard. So this equation will be disregarded as unsuitable.

6.7 Effect of Separation

The experimental results show that the separation between the main members of the built-up struts played an important role in determining the magnitude of the buckling load. The specimens with no separation failed at loads that were much greater than those predicted by the Canadian Standard, no matter what the slenderness ratio of the individual main member was. This result was also verified by the finite element method. The most probable reason for this, as was mentioned in Section 6.4.3, is that the main member on the convex side of the deflection supports the other main member which tends to deflect more due to the extra compressive stress in it from bending, and so increases the rigidity of the strut. Whereas, for the specimens with separated main members, the results obtained were just safe if the slenderness ratio of the individual main member was according to the Standard, and not necessarily safe if not. Also, comparing the experimental results for specimens 440-0-3 and 881-7-7, both of which have the same equivalent slenderness

ratio, the former, with no separation, has a buckling load of 39.9 kN, and the latter, with a separation of 7.85 mm, a buckling load of 29.2 kN. Hence, for built-up columns with separated main members the load carrying capacity of the strut should be reduced by increasing its equivalent slenderness ratio by multiplying the term for the slenderness ratio of the individual main member by a greater factor.

Figures 6.9 (a) and (b) show the drop in the load carrying capacity of a specimen with one connector as the separation increases. They also show the increased discrepancy of the Canadian Standard as the separation increases.

Therefore, the effect of the separation between the main members is an important factor in the design of built-up struts.

6.8 Effect of Number of Connectors

As the question of the permissible slenderness ratio of the individual main members is a major point in this research, the number of connectors used is an important factor. Naturally, increasing the number of connectors decreases the equivalent slenderness ratio of the built-up strut and so increases the buckling load, as shown in Figure 6.10.

However, the number of connectors required depends mainly on two factors. First, the requirements of the Canadian Standard, and second, the question of the economy of the

design of the built-up strut. To satisfy the requirements of the Canadian Standard S16.1-M89, with respect to the slenderness ratio of an individual main member, more than one connector may be required and is determined by the spacing of the main members. Considering the aspect of economy of design, Figures 6.11 (a) and (b) show that the experimental failure load can be as low as 22% of the corresponding Euler buckling load when only one connector is used and the main members are separated. It is quite possible in this case that using a solid column would be more economical than a built-up column. Whereas, for the specimen with seven connectors and the same separation, the failure load reached 67% of the Euler load.

6.9 Analysis of the Requirements of the Standards

All the Standards examined have different requirements for the number and design of the connectors, and the equivalent slenderness ratio. The European Standards examined specify a maximum slenderness ratio for each main member, between batten plates, of not more than 50 with, according to the German Standard, at least two connectors. Whereas, as explained in Section 6.3.1, the Canadian Standard specifies two conditions for this slenderness ratio. First, that it is not greater than the slenderness ratio of the integral column, and second, that it is not greater than 40 or 60% of the slenderness ratio of the integral column about the axis

perpendicular to the battens. In our case 40 governs.

Comparing the results listed in Tables 4 and 5, all the specimens with seven connectors have a local slenderness ratio of less than 40, with the exception of specimen 881-7-7, and the results given by the Canadian Standard agree with the experimental values with an initial out-of-straightness of $L/1000$, when multiplied by the resistance factor of 0.9. Whereas, specimens 881-7-3 and 588-7-3 which have individual slenderness ratios close to the slenderness ratio of the integral column do not have theoretical failure loads that agree with the experimental results. So it appears that the design rule in the Canadian Standard, requiring that the maximum slenderness ratio of the individual main members between adjacent battens be equal to or less than the slenderness ratio of the integral column is not applicable to battened columns.

For the equivalent slenderness ratio, the European Standards examined specify requirements which are too conservative in view of the limitation of 50 given for the maximum slenderness ratio of the individual components. The Canadian Standard, however, specifies an equivalent slenderness ratio, that is valid for the clause that specifies the slenderness ratio of the individual components be less than 40, but is not valid for the clause that specifies it be equal to or less than the slenderness ratio of the integral column, when the main members are separated from each other.

The design Standards all specify the same equation for the bending moment and shear force to be resisted by the connectors, with the exception of the American Specifications which do not mention it. However, they differ with regards to the lateral shear force acting on the column. The Canadian Standard specifies a lateral shear force of 2.5% of the factored axial load, the German Standard specifies 1.25% of the area times the allowable stress, and the British Standard specifies 1% of the factored axial load. The experimental results did not provide the evidence required to justify the use of a shear and moment based on a lateral shear force of less than 2.5% of the factored axial load, and so it cannot be concluded that the Canadian Standard is incorrect.

The experimental and theoretical results of this research indicate that the requirements of the Canadian Standard S16.1-M89 for built-up compression members are inadequate and should be changed.

Chapter 7

OBSERVATIONS, CONCLUSIONS, AND RECOMMENDATIONS

7.1 Observations

The following observations can be noted from this research:

- a) There is significant variation in the North American and European Standards with regard to the number and design of connectors as well as with regard to the equivalent slenderness ratio for built-up compression members.
- b) The specification for the maximum slenderness ratio of an individual main member between battens is unclear in the Canadian Standard S16.1-M89.

7.2 Conclusions

The following conclusions are based on the discussion of the theoretical and experimental results:

- a) The experimental results show that the design rule specified in the Canadian Standard S16.1-M89 in Clause 19.1.3(c), requiring the maximum slenderness ratio of an individual main member between ends of adjacent batten plates to be equal to or less than the integral slenderness ratio of the built-up column, is not valid

when used in combination with the requirement given by the Canadian Standard for the equivalent slenderness ratio of the column. In addition the use of Clause 19.1.3(c) does not give a failure load equal to that of the integral column.

- b) The requirement of the Canadian Standard S16.1-M89, for the maximum slenderness ratio of an individual main member between ends of adjacent batten plates, to be equal to or less than 40, was proven to be valid by the experimental results.
- c) Timoshenko's formula gives the best verification of the experimental results.
- d) There is an increase in failure load with the increase in the number of connectors.
- e) Due to the reduction in load carrying capacity, it is uneconomical to use only one connector, for built-up compression members with a separation between the main members.
- f) The effect of there being a separation between the main members, as opposed to no separation, is considerable.
- g) The initial out-of-straightness has a considerable effect on the failure load.
- h) The shear forces and bending moments in the connectors, specified by the Canadian Standard S16.1-M89 seem to be adequate.

7.3 Recommendations

As the objective of this research is to determine design rules for built-up columns, from which a recommendation can be made for inclusion in the Canadian Standard S16.1-M89, the following recommendations are made:

- a) Built-up columns composed of two main members with no separation, have extra rigidity, and so the present Standard adequately specifies their load carrying capacity.
- b) Based on the discussion given on pages 70-74 and 76-77, it is suggested for built-up columns composed of separated main members that either;
 - i) eliminate Clause 19.1.3(c) which allows the maximum slenderness ratio of an individual main member to be equal to or less than the integral slenderness ratio, and retain Clause 19.1.4 which specifies the equivalent slenderness ratio, or
 - ii) retain Clause 19.1.3(c) allowing the maximum slenderness ratio of an individual main member to be equal to or less than the integral slenderness ratio, and specify an equivalent slenderness ratio as derived by Timoshenko, that is

$$\lambda_{eq} = \sqrt{\left(\frac{L}{r}\right)^2 + 0.82 \left(\frac{a}{r_1}\right)^2} \quad (7.1)$$

Option (i), is probably more economical, as the load carrying capacity drops considerably when too few connectors are used.

7.4 Further Research

The topics that could be further looked into are:

- a) The effect of the number of connectors on the load carrying capacity of a strut, when buckling occurs in the plane perpendicular to the plane of the connectors.
- b) The effect of the number of connectors for short columns.
- c) The theoretical origin of Clause 19.1.16 of the Canadian Standard S16.1-M89, and more experimental verification of it for both cases of the slenderness ratio of the integral column about the axis perpendicular to the battens, less than or equal to, and greater than, 80% of the slenderness ratio of the integral column about the axis parallel to the battens.

Appendix A

FIGURES

BUILT-UP COLUMN SECTIONS

FIGURE 1.1

No Separation

4.02 nm Separation

7.85 nm Separation

THE CROSS-SECTIONS AND
COORDINATE AXES

FIGURE 3.1

FINITE ELEMENT MODEL FOR SPECIMENS
WITH THREE CONNECTORS

FIGURE 3.2 (c)

FINITE ELEMENT MODEL FOR SPECIMEN
WITH FIVE CONNECTORS

FIGURE 3.2 (d)

STATICAL SYSTEM FOR BUILT-UP BATTENED COLUMNS

FIGURE 4.1

STATICAL SYSTEM FOR TIMOSHENKO'S FORMULA

FIGURE 4.2

STATICAL SYSTEM FOR BLEICH'S FORMULA

FIGURE 4.3

(a) Forces Acting On A Panel

(b) Bending Moment Diagram

STATICAL SYSTEM AND BENDING MOMENT
CAUSED BY THE SHEARING FORCES

FIGURE 4.4

SINUSOIDAL LINE OF DEFLECTION WITH
PERPENDICULAR SHEARING FORCE

FIGURE 4.5

TENSION TEST SPECIMEN

FIGURE 5.1

TENSION TEST SPECIMEN FIXED IN THE
UNIVERSAL TESTING MACHINE

FIGURE 5.2

DETAILS OF SPECIMENS WITH NO SEPARATION AND ZERO CONNECTORS

FIGURE 5.3 (a)

DETAILS OF SPECIMENS WITH NO SEPARATION
AND ONE CONNECTOR

FIGURE 5.3 (b)

DETAILS OF SPECIMENS WITH NO SEPARATION
AND THREE CONNECTORS

FIGURE 5.3 (c)

DETAILS OF SPECIMEN WITH NO SEPARATION
AND SEVEN CONNECTORS

FIGURE 5.3 (c)

DETAILS OF SPECIMENS WITH A SEPARATION OF 4.02 mm
AND ZERO CONNECTORS

FIGURE 5.3 (f)

DETAILS OF SPECIMENS WITH A SEPARATION OF 4.02 mm
AND ONE CONNECTOR

FIGURE 5.3 (g)

DETAILS OF SPECIMENS WITH A SEPARATION OF 4.02 mm
AND THREE CONNECTORS

FIGURE 5.3 (h)

DETAILS OF SPECIMENS WITH A SEPARATION OF 7.85 mm
AND ZERO CONNECTORS

FIGURE 5.3 (j)

DETAILS OF SPECIMENS WITH A SEPARATION OF 7.85 mm
AND ONE CONNECTOR

FIGURE 5.3 (k)

DETAILS OF SPECIMENS WITH A SEPARATION OF 7.85 mm
AND THREE CONNECTORS

FIGURE 5.3 (1)

BASE PLATE OF KNIFE EDGE

FIGURE 5.4

END PLATE FOR SPECIMENS WITH NO SEPARATION

FIGURE 5.5 (a)

END PLATE AND SQUARE FOR SPECIMENS
WITH SEPARATED MAIN MEMBERS

FIGURE 5.5 (b)

KNIFE EDGE ASSEMBLY ATTACHED TO LOWER
TABLE OF UNIVERSAL TESTING MACHINE

FIGURE 5.6 (a)

KNIFE EDGE ASSEMBLY ATTACHED TO LOWER
HEAD OF UNIVERSAL TESTING MACHINE

FIGURE 5.6 (b)

SET-UP OF KNIFE EDGE ASSEMBLY IN
THE UNIVERSAL TESTING MACHINE

FIGURE 5.7

HYDRAULIC JACK ASSEMBLY WITH LOWER PLATE OF KNIFE EDGE
ATTACHED TO THE PISTON IN SHOP-FABRICATED FRAME

FIGURE 5.8 (a)

LOAD CELL ASSEMBLY WITH UPPER PLATE OF KNIFE EDGE
ATTACHED TO IT IN SHOP-FABRICATED FRAME

FIGURE 5.8 (b)

THE CYLINDRICAL SEAT ON THE PISTON OF THE JACK
DURING THE TESTING OF SPECIMEN 588-7-1

FIGURE 5.9

THE LOADING AND LOAD MEASURING ASSEMBLY
DURING THE TESTING OF SPECIMEN 660-4-7

FIGURE 5.10

ASSEMBLY OF DIAL GAUGES IN UNIVERSAL TESTING MACHINE
DURING THE TESTING OF SPECIMEN 441-4-1

FIGURE 5.11 (a)

ASSEMBLY OF DIAL GAUGES IN SHOP-FABRICATED FRAME
DURING THE TESTING OF SPECIMEN 588-7-0

FIGURE 5.11 (b)

SET-UP FOR MEASUREMENT OF OUT-OF-STRAIGHTNESS

FIGURE 5.12

44.8 kN CAPACITY LOAD CELL

CALIBRATION CURVE

FIGURE 5.13 (a)

112 kN CAPACITY LOAD CELL

CALIBRATION CURVE

FIGURE 5.13 (b)

TENSION TEST

FIGURE 6.1

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=120$, NO SEPARATION
AND ZERO CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.2 (a)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=120$, NO SEPARATION
AND ONE CONNECTOR

LOAD-DEFLECTION CURVES

FIGURE 6.2 (b)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=120$, NO SEPARATION
AND THREE CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.2 (c)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=120$, NO SEPARATION
AND SEVEN CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.2 (d)

EXPT. CURVE (2) EXPT. CURVE (1)
 THEO. CURVE (2) THEO. CURVE (1)

SPECIMEN WITH $L/r=120$, A SEPARATION OF 4.02 mm
 AND ZERO CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.3 (a)

EXPT. CURVE THEO. CURVE
SPECIMEN WITH $L/r=120$, A SEPARATION OF 4.02 mm
AND ONE CONNECTOR

LOAD-DEFLECTION CURVES

FIGURE 6.3 (b)

—●— EXPT. CURVE —▲— THEO. CURVE
 SPECIMEN WITH $L/r=120$, A SEPARATION OF 4.02 mm
 AND THREE CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.3 (c)

EXPT. CURVE THEO. CURVE
 SPECIMEN WITH $L/r=120$, A SEPARATION OF 4.02 mm
 AND SEVEN CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.3 (d)

EXPT. CURVE {1} EXPT. CURVE {2}
 THEO. CURVE {1} THEO. CURVE {2}

SPECIMEN WITH $L/r=120$, A SEPARATION OF 7.85 mm
 AND ZERO CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.4 (a)

—○— EXPT. CURVE —△— THEO. CURVE
SPECIMEN WITH $L/r=120$, A SEPARATION OF 7.85 mm
AND ONE CONNECTOR

LOAD-DEFLECTION CURVES

FIGURE 6.4 (b)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=120$, A SEPARATION OF 7.85 mm
AND THREE CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.4 (c)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=120$, A SEPARATION OF 7.85 mm
AND SEVEN CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.4 (d)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, NO SEPARATION
AND ZERO CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.5 (a)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, NO SEPARATION
AND ONE CONNECTOR

LOAD-DEFLECTION CURVES

FIGURE 6.5 (b)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, NO SEPARATION
AND THREE CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.5 (c)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, NO SEPARATION
AND FIVE CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.5 (d)

EXPT: CURVE {1} EXPT: CURVE {2}
 THEO: CURVE {1} THEO: CURVE {2}

SPECIMEN WITH $L/r=80$, A SEPARATION OF 4.02 mm
 AND ZERO CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.6 (a)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, A SEPARATION OF 4.02 mm
AND ONE CONNECTOR

LOAD-DEFLECTION CURVES

FIGURE 6.6 (b)

EXPT. CURVE THEO. CURVE
SPECIMEN WITH $L/r=80$, A SEPARATION OF 4.02 mm
AND THREE CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.6 (c)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, A SEPARATION OF 4.02 mm
AND SEVEN CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.6 (d)

EXPT. CURVE (1) EXPT. CURVE (2)

SPECIMEN WITH $L/r=80$, A SEPARATION OF 7.85 mm
AND ZERO CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.7 (a)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, A SEPARATION OF 7.85 mm
AND ONE CONNECTOR

LOAD-DEFLECTION CURVES

FIGURE 6.7 (b)

—○— EXPT. CURVE

SPECIMEN WITH $L/r=80$, A SEPARATION OF 7.85 mm
AND THREE CONNECTORS

LOAD-DEFLECTION CURVE

FIGURE 6.7 (c)

EXPT. CURVE THEO. CURVE

SPECIMEN WITH $L/r=80$, A SEPARATION OF 7.85 mm
AND SEVEN CONNECTORS

LOAD-DEFLECTION CURVES

FIGURE 6.7 (d)

—+—+—+— ZERO CONNECTORS —+—+—+— ONE CONNECTOR
 —+—+—+— THREE CONNECTORS —+—+—+— FIVE CONNECTORS

SPECIMENS WITH $L/r=80$, NO SEPARATION
 AND AN OUT-OF-STRAIGHTNESS OF 0.1 mm

THEORETICAL LOAD-DEFLECTION CURVES

FIGURE 6.8

— — —

EXPT. RESULTS
BLEICH'S EQN.

· · ·

CANADIAN STD.
TIMOSHENKOS EQN.

SPECIMENS WITH $L/r=120$
AND ONE CONNECTOR
SEPARATION VERSUS P_b

FIGURE 6.9 (a)

—
x x x

EXPT. RESULTS
BLEICH'S EQN.

• • •
o o o

CANADIAN STD.
TIMOSHENKOS EQN.

SPECIMENS WITH $L/r=80$

AND ONE CONNECTOR

SEPARATION VERSUS P_b

FIGURE 6.9 (b)

EXPT. RESULTS
BLEICH'S EQN.

CANADIAN STD.
TIMOSHENKO'S EQN.

SPECIMENS WITH $L/r=120$
WITH A SEPARATION OF 4.02 mm

$(L/R)_{eq}$ VERSUS P_b

FIGURE 6.10

: : : SEP.=0.0mm . . . SEP.=4.02mm
 : : : SEP.=7.85mm

SPECIMENS WITH $L/r=120$

RATIO OF EXPERIMENTAL BUCKLING LOAD
 TO EULER LOAD VERSUS NO. OF CONNECTORS

FIGURE 6.11 (a)

: : : SEP.=0.0mm • • • SEP.=4.02mm
 Δ Δ Δ SEP.=7.85mm

SPECIMENS WITH $L/r=80$

RATIO OF EXPERIMENTAL BUCKLING LOAD
 TO EULER LOAD VERSUS NO. OF CONNECTORS

FIGURE 6.11 (b)

Appendix B

TABLES

Table 1
RESULTS OF TENSION TESTS

Specimen	1	2	3	4	Average
Cross-Sectional Area of Reduced Section (mm) ²	79.89	78.00	79.76	78.06	78.93
Modulus of Elasticity (MPa)	208900	199200	211100	198600	204400
Yield Stress (MPa)	321.4	326.9	338.5	326.7	328.4

Table 2
GEOMETRIC PROPERTIES

Specimen	440-0	660-4	881-7	293-0	441-4	588-7
$A_o \text{ (mm)}^2$	160.3	159.3	160.3	160.3	159.3	158.6
$A \text{ (mm)}^2$	320.5	318.5	320.5	320.5	318.5	317.3
$I_{x1} \text{ (mm)}^4$	8482	8428	8482	8482	8428	8302
$I_1 \text{ (mm)}^4$	540.3	530.1	540.3	540.3	530.1	529.7
$I_o \text{ (mm)}^4$	3242	8511	16180	3242	8511	15950
$I_x \text{ (mm)}^4$	16960	16860	16960	16960	16860	16600
$I \text{ (mm)}^4$	4322	9571	17260	4322	9571	17010
$r \text{ (mm)}$	3.67	5.48	7.34	3.67	5.48	7.32
$r_1 \text{ (mm)}$	1.84	1.82	1.84	1.84	1.82	1.83

- NOTES:
- 1- I and I_1 are the moments of inertia of the integral member and one component main member, respectively, about the axes perpendicular to the battens.
 - 2- The last two rows, r and r_1 , are the radii of gyration of the integral member and one component main member, respectively, about the axes perpendicular to the battens.
 - 3- For all the model strut specimens, the moment of inertia of both battens about the horizontal axis perpendicular to the plane of the battens is $I_{bb}=1626 \text{ mm}^4$.

Table 3
THEORETICAL EQUIVALENT SLENDERNESS RATIOS

Specimen	Timoshenko's Formula	Bleich's Formula	Canadian Standard
440-0-0	222.4	201.7	180.0
440-0-1	152.1	144.8	137.4
440-0-3	128.7	126.6	124.5
440-0-7	122.2	121.6	121.1
660-4-0	299.6	285.4	230.6
660-4-1	182.6	176.8	155.5
660-4-3	138.7	136.8	130.2
660-4-7	125.3	124.8	123.0
881-7-0	417.4	405.2	310.6
881-7-1	233.1	227.7	186.9
881-7-3	156.2	154.2	139.8
881-7-7	130.0	129.4	125.3
293-0-0	139.9	127.5	114.7
293-0-1	98.3	94.1	89.8
293-0-3	84.9	83.6	82.5
293-0-5	82.1	81.5	81.0
441-4-0	201.8	192.2	155.1
441-4-1	122.7	118.7	104.3
441-4-3	92.9	91.6	87.1
441-4-7	83.8	83.4	82.2
588-7-0	269.0	261.2	200.8
588-7-1	151.4	148.0	122.1
588-7-3	102.8	101.6	92.6
588-7-7	86.5	86.1	83.6

Table 4

EXPERIMENTAL RESULTS AND BUCKLING LOADS

Specimen	Slenderness Ratio Of Individual Components	Initial Out-Of- Straight. (mm)	Expt. Failure Load (kN)	Predicted Expt. Failure Load For Out-Of Str. L/1000 (kN)
440-0-0	199.6	0.43	33.0	28.7
440-0-1	96.4	0.15	33.7	31.5
440-0-3	44.7	0.15	42.8	39.9
440-0-7	18.9	0.25	46.1	44.1
660-4-0	295.5	1.61	15.4	12.8
660-4-1	144.3	1.15	16.9	17.9
660-4-3	68.6	1.80	25.8	29.1
660-4-7	30.8	1.80	29.7	33.5
881-7-0	433.9	3.53	4.0	7.1
881-7-1	213.5	2.43	9.2	9.6
881-7-3	103.3	2.45	17.7	19.2
881-7-7	48.2	3.10	24.8	29.2
293-0-0	119.7	0.30	64.3	65.6
293-0-1	56.4	0.03	76.0	62.0
293-0-3	24.8	0.50	68.0	74.8
293-0-5	14.2	0.40	68.7	73.9
441-4-0	197.0	1.00	27.0	31.8
441-4-1	95.0	0.32	39.4	38.1
441-4-3	44.0	0.70	51.0	54.9
441-4-7	18.5	0.43	59.2	59.0
588-7-0	276.1	0.30	20.5	18.1
588-7-1	134.6	0.88	22.2	23.1
588-7-3	63.8	0.62	45.0	45.5
588-7-7	28.4	1.03	57.0	59.8

Table 5

THEORETICAL BUCKLING LOADS

Specimen	Finite Element Method (kN)	Timoshenko's Method (kN)	Bleich's Method (kN)	Canadian Standard (kN)
440-0-0	34.4	12.6^	15.2^	18.8^
440-0-1	32.0	25.6^	27.6^	29.8^
440-0-3	40.2	32.8^	33.6^	34.4^
440-0-7	41.7	35.4^	35.6^	35.8^
660-4-0	16.1	7.0^	7.7^	11.8^
660-4-1	17.9	17.4^	18.5*	23.7x
660-4-3	28.2	28.3^	28.9^	31.1*
660-4-7	32.6	32.9^	33.1^	33.8*
881-7-0	4.0	3.6^	3.8^	6.5^
881-7-1	9.6	11.1~	11.6x	16.7x
881-7-3	21.9	23.5x	24.1x	28.1x
881-7-7	29.5	31.3*	31.5*	33.0~
293-0-0	61.7	29.0^	33.3^	38.7^
293-0-1	77.3	47.9^	50.8^	54.1^
293-0-3	65.3	58.4^	59.6^	60.6^
293-0-5	68.1	61.0^	61.6^	62.1^
441-4-0	27.3	14.4^	15.8^	23.8^
441-4-1	41.0	33.9^	35.6^	42.7~
441-4-3	58.5	50.0^	51.0^	54.6^
441-4-7	70.8	57.5^	57.9^	59.0^
588-7-0	22.2	9.0^	9.5^	15.4^
588-7-1	25.0	26.0~	26.9x	35.8x
588-7-3	54.5	45.7*	46.4*	52.7x
588-7-7	68.0	57.7^	58.1^	60.4*

Notes Relating To Table 5:

- 1- All the theoretical critical loads are multiplied by a resistance factor, ϕ , of 1.0.
- 2- $\sim$: The load is less than that of the predicted experimental failure load with an initial out-of-straightness of $L/1000$, without being reduced by the resistance factor 0.9.
- *: The load is less than that of the predicted experimental failure load with an initial out-of-straightness of $L/1000$, after being reduced by the resistance factor 0.9.
- $\sim$: The load is approximately equal to the predicted experimental failure load with an initial out-of-straightness of $L/1000$, after being reduced by the resistance factor 0.9.
- x: The load is greater than the predicted experimental failure load with an initial out-of straightness of $L/1000$, even after being reduced by the resistance factor 0.9.

Appendix C

EFFECT OF THE END PLATES ON THE BUCKLING LOAD

The stability function method, as described by Horne and Merchant [15], is used to determine the effect of the end plates on the buckling load of the model struts. In their derivation of the modified stability functions, Horne and Merchant have assumed the end plates to be perfectly rigid, which is not the true case, as the end plates have a little flexural flexibility. Therefore, the results obtained using these modified stability functions are exaggerated.

The model strut is modelled as one element, pinned at each end, with perfectly rigid end plates of length g_a and g_b at each end, respectively, as shown in Figure C.1. When the disturbances θ_a and θ_b (Figure C.1) are applied at the ends of the element, the following matrix is established

$$\begin{bmatrix} \bar{s}k & \bar{s}\bar{c}k \\ \bar{s}\bar{c}k & \bar{s}k \end{bmatrix} \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} = \begin{bmatrix} M_{ab} \\ M_{ba} \end{bmatrix} \quad (C.1)$$

where $\bar{s}$ is the modified stiffness stability function; $\bar{c}$ is the modified carry-over stability function; k is the stiffness of the member; θ_a and θ_b are disturbances applied at each end of

the member; and M_{1L} and M_{1R} are the moments induced by these disturbances.

At the point of instability, the determinant of the 2 x 2 matrix goes to zero, such that

$$\begin{vmatrix} \bar{s}k & \bar{s}\bar{c}k \\ \bar{s}\bar{c}k & \bar{s}k \end{vmatrix} = 0 \quad (C.2)$$

giving

$$(\bar{s}k)^2 - (\bar{s}\bar{c}k)^2 = 0 \quad (C.3)$$

Dividing Equation C.3 by k^2 , and factorizing it, gives two roots, namely

$$\bar{s} - \bar{s}\bar{c} = 0 \quad , \quad \bar{s} + \bar{s}\bar{c} = 0 \quad (C.4)$$

Substituting the expressions for the stability functions $\bar{s}$ and $\bar{s}\bar{c}$, from Horne and Merchant [15], the only unknown variable in Equations C.4 is ρ , which is the ratio of the buckling load to the Euler load of the column, calculated using the length l , of the centre part of the column. The most critical case of the specimens tested was specimen 660-4, in which $g_a = g_b = 45$ mm, and so $l = 570$ mm. Substituting the geometric properties of this strut into Equations C.4, the value of ρ which satisfies

them can be found by trial and error. It was found that the first equation of Equation C.4 was satisfied at a value of $p=0.75$, whereas the second equation was not satisfied within reasonable limits of p , and so was discarded. Therefore, the buckling load of the model strut, assuming perfectly rigid end plates, is 43.36 kN.

Analyzing this strut for the case of the end plates having the same flexibility as the rest of the column, that is $g_u=g_l=0$, gives $p=1$. Therefore, the buckling load is equal to the Euler load of the strut, calculated this time using the total length of the strut, and is equal to 43.13 kN. Hence, it can be seen that the effect of the rigid end plates on the buckling load is about 0.53%, and is negligible.

It can therefore be concluded, that the end plates have a negligible effect on the buckling load of a strut.

SIMPLE STRUT WITH RIGID END PLATES

FIGURE C.1

Appendix D

THE SHORTENING OF AN AXIALLY LOADED COMPRESSION MEMBER

The shortening of a deflected compression member Δ , is caused by two factors. The first, Δ_1 , is the axial displacement of the compression member due to its axial shortening under the axial force, and the second, Δ_2 , is the axial displacement of the compression member due to the lateral deflection of the column, caused by the bending of the column, due to the presence of initial imperfections. The latter of these factors is negligible at small loads, but has a more notable effect close to the buckling failure load of the column. The axial displacement due to the axial shortening of the column, Δ_1 , can be expressed by

$$\Delta_1 = \frac{PL}{AE} \quad (D.1)$$

where P is the axial force; L is the length of the column; A is the area of the cross-section; and E is the modulus of elasticity.

The axial displacement due to the lateral deflection of the column, Δ_2 , caused by the bending of the column, can be

derived by considering the element of length ds , measured along the axis of the deflected column [32], as shown in Figure D.1 (a) and (b). The projection of this element on the z -axis has a length of dz . Therefore, the difference between the length ds and its vertical projection can be expressed by

$$ds - dz = \sqrt{dz^2 + dx^2} - dz = dz \sqrt{1 + \left(\frac{dx}{dz}\right)^2} - dz \quad (D.2)$$

where dx is the difference in deflection in the x -direction along the length ds of the element, as shown in Figure D.1 (b). The term under the radical sign can be expanded, and thus simplified, by the series

$$(1 + t^2)^{\frac{1}{2}} = 1 + \frac{t}{2} - \frac{t^2}{8} + \frac{t^3}{16} - \dots \quad (D.3)$$

where t is any variable. If t is very small with respect to unity, the terms of a higher order than t can be disregarded. So, Equation D.2 can be rewritten as

$$ds - dz = dz \left(1 + \frac{1}{2} \left(\frac{dx}{dz} \right)^2 \right) - dz = \frac{1}{2} \left(\frac{dx}{dz} \right)^2 dz \quad (D.4)$$

Assuming that the curved length of the deflected column does

not change its length while bending, the difference between the total length of the beam and the chord length, $a'b$, which is the axial shortening due to bending, Δ_1 , can be calculated by integrating Equation D.4 along the length L ,

$$\Delta_2 = 2 \int_0^{\frac{L}{2}} \frac{1}{2} \left(\frac{dx}{dz} \right)^2 dz \quad (D.5)$$

It is, therefore, necessary to derive an expression for the deflection of a column due to bending under an axial load. This can be done by examining the equilibrium of the free body diagram shown in Figure D.1 (c), given

$$\begin{aligned} EI \frac{d^2x}{dz^2} &= -M \\ &= -Px \end{aligned} \quad (D.6)$$

where d^2x/dz^2 is the second derivative of the variable deflection x ; I is the moment of inertia of cross-section about the axis perpendicular to the plane of the paper; M is the internal moment in the column at the cut end; and x is the lateral deflection of the cut end. Simplifying this differential equation gives

$$\frac{d^2x}{dz^2} + \frac{P}{EI} x = 0 \quad (D.7)$$

where $\sqrt{P/EI}$ can be taken as the constant β . The solution to this homogeneous differential equation is

$$x = C \cos \beta z + D \sin \beta z \quad (D.8)$$

where C and D are constants. Substituting for the boundary conditions

$$\begin{aligned} &\text{at } z=0 \quad x=0 \\ &\text{and at } z=L/2 \quad x=f \end{aligned}$$

the constants are determined to be $C=0$ and $D=f/(\sin \beta L/2)$. By substituting these constants into Equation D.8, the complete expression for the deflection of the column is found to be

$$x = \frac{f}{\sin \frac{\beta L}{2}} \sin \beta z \quad (D.9)$$

It is now possible to determine the value of Δ_c from Equation D.5. Differentiating Equation D.9 with respect to z , then squaring it, gives

$$\left(\frac{dx}{dz}\right)^2 = \frac{\beta^2 f^2}{\sin^2 \frac{\beta L}{2}} \cos^2 \beta z \quad (D.10)$$

Substituting Equation D.10 into Equation D.5, gives

$$\Delta_2 = \int_0^{\frac{L}{2}} \left(\frac{\beta^2 f^2}{\sin^2 \frac{\beta L}{2}} \cos^2 \beta z \right) dz \quad (D.11)$$

Performing this integration and substituting the limits gives the expression

$$\Delta_2 = \frac{\beta^2 f^2}{\sin^2 \frac{\beta L}{2}} \left(\frac{L}{4} + \frac{\sin \beta L}{4 \beta} \right) \quad (D.12)$$

Therefore, the total shortening of a column is

$$\Delta = \Delta_1 + \Delta_2 = \frac{PL}{AE} + \frac{\beta^2 f^2}{\sin^2 \frac{\beta L}{2}} \left(\frac{L}{4} + \frac{\sin \beta L}{4 \beta} \right) \quad (D.13)$$

The shortening of the model struts was measured during the execution of the experiments. Equation D.13 was applied to specimen 440-0-0 for two cases, the first near the beginning of loading, and the second close to the failure load. The geometric and mechanical properties, $L=440.0$ mm, $A=320.5$ mm², $I=4322$ mm⁴, and $E=208\,900$ MPa, were substituted into Equation D.13. The load and lateral deflection at the beginning and the end of the increment, measured during the test, were substituted for each case, and the difference of theoretical values for Δ was compared to the shortening of the

column measured experimentally during the load increment. For the first case, the load was increased from 16 to 20 kN, and the lateral deflection increased from 0.355 to 0.432 mm, giving a value of $\Delta=0.027$ mm, whereas the shortening of the column measured during the test was 0.049 mm. For the second case, the load was increased from 30 to 32 kN, and the lateral deflection increased from 0.72 to 0.815 mm, giving a value of $\Delta=0.014$ mm, whereas the shortening of the column measured during the test was 0.02 mm. It can be seen that the experimental measurements are nearly twice as much as the theoretical predictions.

Hence, it can be concluded that the theoretical predictions of the shortening of a column are much smaller than experimentally measured. This is probably due to the wearing down of the knife edges, and also the tightening of any gaps between the various parts of the knife edges.

THE SHORTENING OF A COLUMN DUE TO BENDING

FIGURE D.1

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