Theoretical and experimental analyses of wave motion in rockfill structures.

Mohamed Sameh Sami. Nasser

University of Windsor

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THEORETICAL AND EXPERIMENTAL ANALYSES
OF WAVE MOTION IN ROCKFILL STRUCTURES

A Dissertation
Submitted to the Faculty of Graduate Studies
through the Department of Civil Engineering
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
at the University of Windsor

by

Mohamed Sameh Sami Nasser

Windsor, Ontario
1974
Mohamed, Sameh Sami Nasser 1974

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TO,

Tamer and Sahar, and the children everywhere; may their life be safer, happier and more peaceful than ours.
ABSTRACT

This study treats the interaction of shallow water waves and rockfill embankments with impervious cores. The resultant unsteady flow through the rockfill is in the non-Darcy regimes. The Dupuit-Forchheimer assumption of parallel flow is invoked in deriving the hyperbolic system of partial differential equations, governing the flow in the rockfill.

The external wave drop rate at the rock-water interface is considered as either "fast" or "slow", depending on its value relative to the maximum fall velocity of the outcrop point; the fast drop case is resolved into different stages. The time dependent movement of the outcrop point is programmed as an entrance boundary condition in two mathematical models simulating the flow in the rockfill; one model for fast drop and the other for slow drop. The methods of characteristics and finite differences are utilized in developing the models which predict wave transmission and phreatic line profiles with time.

Using the characteristics-finite difference technique, the external and internal waves are coupled in a combined model, and the discontinuity of the solution at
the interface is treated by satisfying flow continuity according to the multi-stage movement of the outcrop point. In this model, the vertical displacement of the incident wave is used as an upstream boundary condition; a formula is suggested for asymmetric as well as symmetric waves. Also, an expression is proposed to account for centrifugal effects. The combined model yields wave transmission, reflection, run-up and impact wave height.

In the mathematical models, sloping embankments are transformed to equivalent rectangular sections with widths extending from the intersection of the slope and the mean water level to the impervious core. A formula is proposed to adjust the Chezy coefficient in order to allow for wave breaking losses. A new classification for waves running up slopes is introduced; a wave is fast or slow rising, depending on its uprush being faster or slower than the internal seepage rate.

An extensive experimental investigation, involving wave action in homogeneous rockfill embankments, was conducted. Crushed rock and quartz were utilized [size range 0.7 cm to 4.4 cm], as four different porous media, to build rectangular sections of various widths and also embankments with 2:1 sloping faces; the embankments were tested for several wave conditions. Wave steepness, embankment width and rock size were found to be major influencing factors. Empirical formulae were derived for transmission, reflection, run-up and rush-down.
The mathematical models were confirmed by typical experiments, representing most of the covered ranges. All the cases tested indicate good agreement between theory and experiment.
ACKNOWLEDGMENTS

I wish to express my deep appreciation and gratitude to my supervisor, Dr. J.A. McCorquodale, whose guidance and continuous encouragement made this endeavour possible. Of my association with Dr. McCorquodale, the educator and the scholar, I am indeed proud.

I am greatly indebted to Dr. J.K. Bemu for his useful comments, concern and encouragement, as well as his generous financial contribution to this project.

Thanks are due to the Civil Engineering technicians, Mr. G. Michalcuk and Mr. P. Feimer, for their help during the experimental investigation. Thanks are also due to the computer staff at the University of Windsor for their cooperation.

I am grateful to the Civil Engineering Department at the University of Windsor, and the National Research Council of Canada for the opportunity of conducting this study.

The suggestions of the Comprehensive Examination Committee are sincerely appreciated.

Last but not least, I am deeply grateful to my mother for her sacrifices, blessings, and moral support.
As for my wife, Elweya, I gratefully acknowledge that her tremendous assistance, patience and enthusiasm have put me through many a difficult day. To her I say "Thank you for sharing".

M.S. Nasser
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CHAPTER I
INTRODUCTION

Porous structures, such as rockfill dams and rubble-mound breakwaters, are frequently subjected to wave action. When water waves impinge on a rockfill embankment, the resultant unsteady, internal flow is often in one of the non-Darcy regimes. Thus, the governing equation is non-linear, and the solution becomes more difficult than that for Darcy flow. The complexity of the phenomenon, both physically and mathematically, is presumed to be the reason behind the semi-empirical nature of most previous research on permeable breakwaters.

The main theme of the present analysis is the study of the interaction of long shallow water waves and rockfill embankments with impervious cores. The importance of this problem in hydraulics and coastal engineering, and the lack of literature on the subject of unsteady non-Darcy flow were the prime motivating elements. To the writer's knowledge, only a few comprehensive, experimental studies of wave motion in rockfill embankments with impervious cores have been published. A limited number of analytical attempts have been made; the majority of which dealt with continuous breakwaters. Hence, many aspects of the problem
are still not well defined and warrant further exploration. A theoretical insight into the phenomenon considered is an essential undertaking in order to complement laboratory studies, and help in the interpretation and scaling-up of experimental results. On these bases, it was felt that a thorough analysis was needed to provide qualitative as well as quantitative information about wave transmission, reflection, run-up and rush-down for rockfill structures with impervious cores. The accomplishment of this goal requires a complete mathematical simulation of the phenomenon by coupling the external and internal wave motions in one model.

One of the applications of this study is the determination of the freeboards of the crest and impervious core of a rock protected dam. Freeboard allowances are precautions against overtopping of both the embankment and the core as a result of waves and wind set up. In countries like Canada, the upper few feet of the core may be subject to frost damage and wave run-up in this zone is undesirable; hence only the lower portion of the core is relied upon to prevent overtopping (Fig. 1.1). It is the usual practice, as a safety measure, to make the conservative assumption that the maximum water level on the core is the same as the maximum run-up on the rock face. Thus, the knowledge of the water level fluctuations within the rockfill should lead to a more efficient method of estimating rational freeboards, and
enable a designer to justify either lower-embankments or higher reservoir levels. This would result in definite economic benefits for existing and proposed projects. Another practical application of this study is to aid in establishing, more precisely, the height to which some porous, shore protection structures, e.g., gabion walls, could be erected.

In the light of the foregoing discussion, the objectives of the present work can be stated as follows:

1. The development of a mathematical model to trace internal flow profiles and determine wave transmission in the rockfill.

2. The coupling of the external and internal wave motions in order to model the interaction of both flows and illustrate their interdependence. This would yield wave transmission, reflection, run-up and impact wave height for a given incident wave.

3. An extensive experimental study, mainly to verify the mathematical models, but also to develop empirical relationships from the data. The latter would furnish a basis for comparison with other investigations.

In tackling the problem, the methods of characteristics and finite differences were adopted in formulating the mathematical models. For simulating the flow in the rockfill, the approach begins with considering the non-
linear equation of motion for one dimensional unsteady non-Darcy flow in a rectangular embankment. This equation and the continuity equation were discretized by finite differences and solved for the two dependent variables, the water elevation and the horizontal macroscopic velocity. The solution progresses from known initial conditions, and the ratio between the time and space increments was controlled by the characteristic directions in order to achieve stability and convergence. Two boundary conditions were satisfied; an entrance boundary condition which prescribes the time dependent movement of the outcrop point and a downstream condition indicating a zero normal velocity at the impervious core. The one unknown dependent variable along each boundary was obtained by manipulating a combination of the equations of motion and continuity. A challenging problem arises at the interface boundary since the inside and outside water levels may not coincide. In a rising phase the outcrop point coincides with the outside water level. In a falling phase, the outcrop point will only coincide with the outside water level if the latter drops at a rate equal to or slower than the maximum internal fall velocity; otherwise the drop is fast and a seepage face results. The fast drop case undergoes different stages, one of which is governed by a damped, non-linear ordinary differential equation.
In order to couple the external wave and internal flow, the shallow wave dynamic equation together with its continuity equation were discretized and utilized to advance the solution, in the outside region, in a similar manner to that used for the internal equations. The upstream boundary condition is originated in the mathematical model at one wave length away from the interface. This condition is formed by two sine functions that can describe the vertical displacement of both symmetric and asymmetric waves with respect to the mean water level. The discontinuity of the solution at the interface was dealt with by requiring flow continuity based on the phase of the outcrop point movement.

The modification of the mathematical models to solve sloping embankments necessitated the transformation of these embankments to equivalent rectangular sections. An equivalent rectangular section has a width equal to the horizontal distance from the core to the point of intersection of the still water level with the slope. Allowances were made to account for external wave breaking and internal high damping in the vicinity of the interfacial boundary.

In the experimental programme, crushed rock, in sizes ranging from 0.7 cm to 4.4 cm, was utilized, along with a 1.6 cm rounded quartz, to form four homogeneous porous media. The physical and hydraulic properties of
the media were predetermined experimentally. Rectangular embankments, ranging in width between 7 cm to 90.5 cm, and embankments with 2:1 sloping faces were subjected to waves of various characteristics. Empirical expressions were derived for transmission, reflection, run-up and rush-down.

Experimental confirmation for the mathematical models is presented for many cases covering most of the ranges tested in the laboratory.

The numerical solutions were executed on the IBM 360/65 computer at the University of Windsor. Listings of the programmes are attached.
CHAPTER II
REVIEW OF LITERATURE

2.1 Scope
As pointed out in the introduction, the state of the art does not adequately describe the phenomenon of wave absorption and attenuation in porous structures. However, there exist a few prominent contributions to this field, which are surveyed in this chapter. It is imperative for the present study to initiate the discussion by considering the natures of flow in porous media and their corresponding resistance equations. This would lead to a better understanding of the available literature pertinent to unsteady Darcy and non-Darcy flow, and the manifestation of the latter when rockfill embankments are subjected to water waves.

2.2 Flow Regimes in Porous Media
As early as 1901, Forchheimer (23) was the first to realize that Darcy's law was not universally valid for flow through porous media. He noted that for large velocities, the linear relationship between the hydraulic gradient and velocity breaks down. A number of publications followed, thereafter, on the hydrodynamics of this so-called non-Darcy flow.
There is some similarity between flow in porous media and flow past a single particle. In a porous medium, however, flow patterns interfere with subsequent influence on the drag affecting the grains of the medium. For a sphere moving through a fluid, Stokes (64) indicated that the exerted drag is proportional to the first power of the sphere's velocity, as long as the Reynolds number does not exceed 0.1. With increasing Reynolds number, the exponent on the velocity increases, approaching a value of 2 for very turbulent flow (56). The upper and lower limits, bounding the velocity exponent, also hold for flow in porous media, with Darcy's law being analogous to Stoke's. Hence, the regimes for saturated, Newtonian flow in a porous medium may be stated, based on an adaptation of the classifications introduced by Ward (68) and Wright (72), as follows:

1. "The Darcy or Laminar Regime", in which viscous forces predominate and inertial effects are negligible. In this regime the streamlines in the pores are stable and the head loss is directly proportional to the first power of the velocity.

2. "The Non-Linear Laminar or Steady Inertial Regime", i.e., viscous and inertial terms influence the flow which still follows stable streamlines in the pores. The head loss ceases to vary linearly with the velocity.
3. "The Turbulent Transitional Regime", where some of the streamlines in the pores become unstable; inertial actions predominate with viscous effects faintly existing. The head loss approaches dependence on the square of the velocity.

4. "The Fully Turbulent Regime", in which viscous effects practically disappear and only the inertial actions govern the flow. Almost no stable streamlines are present in the pores any more; the head loss may be assumed to vary with the square of the velocity.

Apart from the above classification, an additional non-Darcy flow regime exists for extremely small velocities at which the flow may become non-Newtonian. This regime is referred to by Kovacs (32,33) as microseepage, and is influenced, to a great extent, by fluid impurities and forces of molecular attraction.

2.3 The Resistance Equations

Numerous investigations have been made in order to establish friction equations for the description of non-linear flow behaviour through porous media (see, for example, references 4, 5, 13, 18, 21, 34, 45, 52, 53, 54, 68, 69). Several representations have emerged; each can be categorized under any one of the three following forms:
1. The exponential form,

\[ i = Nq^M \]  \hspace{1cm} (2.1)

2. The Forchheimer form,

\[ i = aq + bq^2 \]  \hspace{1cm} (2.2)

3. The general form,

\[ i = aq + bq^{1.5} + eq^2 \]  \hspace{1cm} (2.3)

where \( i \) = magnitude of the hydraulic gradient;
\( q \) = bulk or macroscopic velocity;
\( a, b \) and \( e \) are constants for a particular medium, fluid and flow regime;
\( N \) and \( M \) are variables depending on velocity;
\( M \) ranges from 1 in laminar flow to 2 in the fully turbulent case.

The first formula expresses both laminar and turbulent flows in one term. Though simple in form, the significance of this equation is not as illustrative as that of either Eq. 2.2 or Eq. 2.3. In the latter two equations laminar and turbulent flows are demonstrated separately, with the term \( bq^{1.5} \) being further included in Eq. 2.3 to account for transitional regimes. Despite the fact that the third equation may appear to be a more representative formula, Ng (47) found that this equation was only slightly better than the two term, Forchheimer
equation. He added that in the range of Reynolds number (600 - 4000), Forchheimer's equation gave as a good correlation with his experimental findings as the three term formula. Theoretical supports for the validity of Eq. 2.2 were presented by Ahmed and Sunada (2) and McCorquodale (38).

The constants a and b of Eq. 2.2 have been investigated, and a few expressions were proposed. Ward (68), using dimensional analysis, recommends that a and b be taken respectively as

\[ a = \frac{\nu}{k'} \]  \hspace{1cm} (2.4)

\[ b = \frac{f}{g k'} \]  \hspace{1cm} (2.5)

where \( f = \text{constant} \);

\( g = \text{acceleration of gravity} \);

\( \nu = \text{kinematic viscosity} \);

\( k' = \text{permeability} \).

Ward adapted the Kozeny-Carman equation (59) in the form

\[ k' = \frac{m^3}{J T' S_s (1-m)^2} \]  \hspace{1cm} (2.6)

to obtain

\[ k' = \frac{m^3 \lambda^2 M^2}{36 J T' (1-m)^2 g \ln \sigma g} \]  \hspace{1cm} (2.7)
where \( m \) = dry porosity;

\( J \) = a dimensionless constant depending on

the shape of the cross-section of flow;

\( M_g \) = geometric mean particle size;

\( T' \) = tortuosity;

\( \sigma_g \) = geometric standard deviation;

\( S_s \) = specific surface;

\( \lambda \) is a particle shape factor given by

\[
\lambda = \frac{1}{m} \left( 0.198 \frac{1}{\sigma_g} + 0.294 \right) - 0.330
\]  

(2.8)

Engelund (20) suggested the following expressions

for \( a \) and \( b \)

\[
a = \lambda_1 \frac{(1-m)^3}{m^2} \frac{\hat{u}}{D^2}
\]  

(2.9)

\[
b = \lambda_2 \frac{(1-m)}{m^3} \frac{\rho}{D}
\]  

(2.10)

in which \( \lambda_1 \) and \( \lambda_2 \) are shape factors,

\( \rho \) = density;

\( \hat{u} \) = dynamic viscosity;

\( D \) = grain size.

The studies of Ng (47), with angular materials,

indicated that Ward's porosity function may not be a

sufficient description of the turbulent term in Eq. 2.2.
McCrorquodale (38) then proceeded to modify Ward's work in order to obtain a general non-dimensional Forchheimer type equation for crushed rock. He gives this equation as

\[ Y = C_1 + C_2 X \]  \hspace{1cm} (2.11)

where \[ Y = \frac{gk'_i}{vq} \]

\[ X = \frac{\sqrt{k'iq}}{v} \]

\( C_1 \) and \( C_2 \) are dimensionless constants for a particular flow regime; \( k' \) is given by Eq. 2.7.

Nasser (45) utilized a least squares technique and pooled his own data with the data of Lane (35), McCrorquodale (38), Dudgeon (18) and Ng (47) to establish the constants \( C_1 \) and \( C_2 \). He obtained correlations with and without consideration of permeameters' wall effects.

The wall effect is a factor that is responsible for the variation in velocity near the wall of a permeameter due to porosity changes. The importance of this effect was recognized by Mott (44), Rose (55), Dudgeon (19) and McCrorquodale (38). The side wall of a permeameter, filled with a non-cohesive granular material, tends to create a higher porosity zone, of the order of half a particle diameter, to occur against the wall. Being dependent on
porosity, the velocity in this zone will be higher than the average velocity in the core, thereby causing the mean velocity of the whole cross-section to differ from the true velocity in an infinite medium. Dudgeon proposed a formula to account for this effect, based on the exponential resistance equation. McCorquodale, utilizing the Forchheimer equation, derived a wall correction expression.

The Forchheimer equation can be extended to describe the resistance for two and three dimensional non-Darcy flow (20, 31, 48). The equation then takes the form

\[ \nabla \phi = -F(q) \hat{q} \]  
\[ (2.12) \]

where \( \phi \) = piezometric head;

\[ q = |\hat{q}| \]

Combining Eq. 2.12 with the continuity equation,

\[ \nabla \cdot \hat{q} = 0 \]  
\[ (2.13) \]

yields

\[ \nabla \left( \frac{\nabla \phi}{F(q)} \right) = 0 \]  
\[ (2.14) \]

or

\[ \nabla \left( K(q) \nabla \phi \right) = 0 \]  
\[ (2.15) \]

where \( K(q) = \) hydraulic conductivity

\[ = 1/F(q) \]
The applicability of the Forchheimer equation, originally derived for one-dimensional flow, to two and three-dimensional cases was questioned by Wright (72), McCrorquodale (38) and Nasser (45). Their experimental studies with parallel and converging flow permeameters indicated that the resistance for converging flow was less than that for parallel flow by 13% to 30%. McCrorquodale's diverging flow tests showed an insignificant increase in the resistance. These deviations in resistance, noted in radial flow, are not rigorously justified by theoretical reasoning. It is surmised, however, that they could be attributed to the possibility that convergence inhibits turbulence and results in reducing the resistance; divergence, on the other hand, may induce separation so as to increase turbulence and consequently the resistance.

2.4 Studies on Unsteady Darcy and Non-Darcy Flow

In this section the literature concerning unsteady flow in porous media is reviewed, with special focus on the research most closely related to the present study. As a general remark, the hydraulic properties of the porous media were found to have been ignored in the majority of previous investigations. Nevertheless, intuitively, it is thought that these properties are important prerequisites if analytical or numerical solutions are sought.

The equation for unsteady Darcy flow is given by Hunt (26) as
\[-v \phi = a \dot{q} + \frac{1}{g m} \frac{\partial q}{\partial t}\]  

(2.16)

The term \( \frac{1}{g m} \frac{\partial q}{\partial t} \) is usually neglected in Darcy flow. Eq. 2.16 in the x and y dimensions can be combined with the continuity equation to yield

\[v^2 \phi (x,y,t) = 0\]  

(2.17)

Deweist (15) treated the damping of unsteady Darcy flow through a dam or levee with horizontal underdrain, when the head behind was raised considerably. Slow and rapid rises were dealt with, and experiments were conducted in a Hele-Shaw model. The analytical solution considered the unsteady flow as a time dependent perturbation of the final steady flow and the unsteady potential. The unsteady potential, a function of \(x, y\) and \(t\), was expanded in a power series which was truncated after the second term.

Wigle (70) applied a finite difference technique to solve unsteady free surface Darcy flow. He dealt, specifically, with the problem of rapid drawdown in a two dimensional rectangular domain. Essentially, Wigle solved the Laplace equation

\[v^2 \phi = 0\]  

(2.18)

where \(\phi\) represents the piezometric head as a function of \(x, y\) and \(t\).
His solution was obtained in small time steps, assuming steady boundary conditions. The free surface level, which is equal to the piezometric level, was adjusted with the supposition that a particle on the free surface would always remain there.

The two dimensional unsteady Darcy flow equations were utilized by Liggett (37), in a lagrangian form, to study initial motion during rapid drawdown in earth embankments. The solution was obtained in the form of a time series.

Karadi et al (28) developed a matrix method to solve a one dimensional unsteady seepage flow. In their method, the equations were solved in a non-dimensionalized form.

A variational approach was utilized by Witherspoon et al (71) to set up a finite element model for radial unsteady Darcy flow in an aquifer system.

Verma and Brutsaert (67) introduced finite difference solutions for the one and two dimensional unsteady Darcy flow in an unconfined aquifer. In the one dimensional case the Dupuit assumptions are used. Their two dimensional flow is governed by the Laplace equation, which was solved by successive over-relaxation. They also verified the numerical solutions by experiments conducted in a Hele-Shaw model.
Recently, Bruch (7) applied a finite element technique to solve unsteady Darcy flow from a reservoir into an unconfined aquifer or vice-versa. The Dupuit-Forchheimer approximation was utilized, and triangular as well as rectangular finite elements, with space and time coordinates, were used in discretizing the flow field.

Desai and Abel (14) discussed, concisely, finite difference and finite element solutions for transient unconfined seepage. The finite difference method was applied to two space dimensions, and the changing location of the free surface was obtained, at each time step, by satisfying the condition that the head at a point on the free surface must be equal to the elevation of the point. In applying the finite element method, a model was developed for one dimensional flow involving small variations in head. The one dimensional finite element model was based on a linearized form of the governing equation. Both numerical solutions were compared to experimental results.

Dracos (16, 17) applied the classical method of characteristics, based on the graphical intersections of the characteristic directions, to solve the Darcy flow equations

\[ \frac{d\xi}{dt} + u_\xi \frac{d\xi}{dx} + g \frac{d\eta}{dx} + \frac{\gamma'm}{\rho} \frac{u_\xi}{K_\xi} = 0 \]  \hspace{1cm} (2.19)
\[
\frac{\partial \eta}{\partial t} + u_\star \frac{\partial \eta}{\partial x} + (h_0 + \eta) \frac{\partial u_\star}{\partial x} = 0
\]  
(2.20)

where \( u_\star \) = horizontal pore velocity,
\( \eta \) = height of perturbation with respect to the mean water level;
\( h_0 \) = mean water level;
\( K_\star \) = conductivity for Darcy flow.

The solution was obtained for an infinite rectangular sand medium; comparisons with experimental results from a Hele-Shaw model were made. It would appear that Dracos' technique could only be efficient if the solution is restricted to very limited time advancements. Any attempt to expand such a method to a large scale problem would be too cumbersome and time consuming. In his work, however, Dracos (17) introduces an interesting analysis for the movement of the outcrop point, i.e., the point at which the phreatic line joins the seepage surface. He uses the movement of the outcrop point as a boundary condition in his solution of Eqs. 2.19 and 2.20. He assumes harmonic fluctuation, represented by a cosine function, for the free water level. Dracos indicates that the movement of the outcrop point may undergo different phases depending on the drop rate of the free water level, being faster or slower than the maximum fall velocity of the outcrop point. The writer adapted Dracos' analysis of the
movement of the outcrop point to the more-complex non-Darcy flow, and assumed a sine function for the free water level fluctuation. The writer's detailed derivation is given in the next chapter.

For one dimensional flow, the equation for unsteady non-Darcy flow may be written as (49)

$$i = aq + bq^2 + n \frac{\partial q}{\partial t} \quad \text{(2.21)}$$

where \(a, b\) and \(n\) are constants.

With the limitation that convergence, divergence and curvature of the macroscopic streamlines have negligible effects on the conductivity, McCorquodale (38) generalized the above equation to describe two and three dimensional flow. He examined the Navier-Stokes equations, and with the aid of a number of assumptions, derived the following equation.

$$\dot{\mathbf{q}} = - \left( \frac{1}{a + bq} \right) \left( \mathbf{v} \cdot \nabla \phi + \frac{1}{q_m} \frac{\partial q}{\partial t} \right) \quad \text{(2.22)}$$

Combining Eq. 2.22 with the continuity equation,

$$\mathbf{v} \cdot \dot{\mathbf{q}} = 0$$

yields the governing equation for two and three dimensional unsteady non-Darcy flow as

$$\nabla \cdot \left\{ \frac{1}{(a + bq)} \left( \nabla \phi + \frac{1}{q_m} \frac{\partial q}{\partial t} \right) \right\} = 0 \quad \text{(2.23)}$$
McCorquodale (39) solved the above equation, for rapid drawdown in rockfill, using a finite element technique. He utilized a lagrangian method to compute the free surface position at the end of each time increment. His solution is based on the assumption that the inertia term is small as compared to friction; this assumption, was supported by experiments with rock sizes up to 4 cm. The numerical solution was confirmed by rapid drawdown laboratory tests.

A similar theoretical approach to McCorquodale's was employed by Sollitt (61) in the derivation of the equation of motion for damped small amplitude water waves in a coarse porous medium. He arrived at an analogous form of Eq. 2.22. More emphasis will be given to Sollitt's work later in this chapter.

Lean (36) uses the following continuity and motion equations to describe non-Darcy flow through wave absorbers with simple geometries.

\[ n_t = - (h_0 u)_x \]  \hspace{1cm} (2.24)

and

\[ u_t = - g n_x - \frac{K'u|u|}{h_0} \] \hspace{1cm} (2.25)

where \( u \) = horizontal velocity;

\( K' \) = a resistance factor.

Lean linearizes the quadratic friction term in Eq. 2.25 through the transformation
\[ f' = \frac{8}{3\pi^2} \frac{K'}{h_0} U \]  

(2.26)

in which \( f' \) = constant,

\[ U = \text{amplitude of the local wave particle velocity.} \]

The factor \( f' \) is chosen such that the above term will yield the same energy loss per unit plan per cycle as the non-linear friction term. Hence, Eq. 2.25 reduces to the linear form

\[ u_t = -g\eta_x - f'u \]  

(2.27)

Lean's solution is strictly applicable to low waves which do not break at entry to the absorber.

Kondo (29) treated the problem of wave transmission and reflection for permeable structures in a manner that resembles Lean's method. In particular, he develops an analytical approach to long waves interacting with homogeneous, vertical-faced breakwaters. He solves the one dimensional equation of motion for periodic, linearly damped free surface flow; his linearized equation has the form

\[ \frac{1}{mg} \frac{\partial u}{\partial t} + \frac{u}{k^*} + \frac{\partial \eta}{\partial x} = 0 \]  

(2.28)

subject to the continuity equation,
\[ m \frac{\partial h}{\partial t} + (h_0 + n) \frac{\partial h}{\partial x} = 0 \]  \(2.29\)

where \( \tau \) = square root of tortuosity,

\( k^* \) is a constant coefficient of permeability that constitutes a velocity, \( U^* \), determined in advance and represents \( U(x) \) of the whole range considered; \( U(x) \) was evaluated at the middle of the structure.

The solutions of Lean and Kondo are similar in that they both yield exponential decay of wave amplitude in the direction of wave propagation. Lean mainly computes reflection coefficients for high porosity, wire mesh absorbers. Kondo, on the other hand, assumes linear wave theory to apply outside the breakwaters, couples the external and internal solutions via the continuity of horizontal velocity and pressure at the sea-breakwater interfaces and obtains reflected and transmitted wave amplitudes.

Kondo and Toma (30) carried out an experimental study on the effects of incident wave characteristics and thickness of structure on reflection and transmission. They used an idealized porous medium, comprising a lattice of circular cylinders. Their plots indicate that while the transmission decreases with increasing wave steepness, the reflection was not significantly affected by this parameter. They report that an increase in the ratio of
the structure's width to the wave length causes a nearly exponential decrease in transmission, and an increase in reflection which hits a maximum when this ratio is between 0.2 and 0.6: thereafter reflection starts to decrease and remains almost uniform when the ratio is greater than 0.6. The authors point out that a standing wave pattern was observed within the porous medium when the structure's width reached about 0.25 or more of the wave length.

Johnson et al (27), and Cross et al (12) conducted experimental studies on scale effects in the Froude modelling of wave transmission. The second study also included reflection. Cross et al employed gravel with diameters of 1.37 in., 0.762 in. and 0.324 in. in testing three different models. Utilizing the largest model as the prototype, prototype to model ratios were 1/1.80 and 1/4.23. In the work of Johnson and his colleagues particle sizes, ranging from 0.9 cm. to 3.6 cm., were used to construct three models of lengths varying between 15 cm. and 60 cm.

A brief laboratory investigation on wave reflection was made by Straub et al (65) in 1957. Experimental reflection coefficients were determined for highly permeable, as well as impermeable, wave absorbers.

Earlier in 1949, Caldwell (8) conducted systematic studies on the reflection of solitary waves. Caldwell utilized permeable and impermeable breakwaters with
vertical and sloping faces. He also examined the case of a vertical rock breakwater with impermeable shoreward face. A dimensional analysis was made and empirical expressions for the ratio of energy absorbed by the structure were presented.

Thornton and Calhoun (66) carried out one of the very few field studies on wave reflection and transmission for a rubblemound breakwater. The prototype in their research was the Monterey breakwater in California. They installed two wave sensors seaward from the structure and a third one in the harbour. The three instruments were placed in a line normal to the breakwater, and the measurements were taken only when the incoming waves arrived with their crests approximately parallel to the breakwater in order to ensure normal wave incidence. Despite the fact that the measurements were recorded at fixed locations, Thornton and Calhoun managed to devise a method which enabled them to separate incident and reflected wave components through the application of linear wave theory. They plotted reflection and transmission coefficients versus wave frequency, and reported that these coefficients displayed a dependence on tidal stage and incident wave amplitude. They came to the conclusion that the treatment of wave reflection and transmission as a linear, stationary, random process could be considered a reliable and useful technique for at least moderate wave conditions.
Fallon (22) conducted an experimental study on discontinuous composite wave absorbers. Wave energy absorption was determined, based on the characteristics of the input wave and of the absorber. The experimental model consisted of an impervious, lower slope that levelled off at a berm supporting a stone filled, upper slope. Reflection was measured for different conditions attained by varying wave length and height, water depth, average stone diameter, both pervious and impervious slopes and depth and width of the berm. Fallon concludes that wave absorption increases with wave steepness and berm width, and with decreasing angles of the upper and lower slopes. He postulates that enlarging the berm width beyond five stone diameters does not cause significant change in wave absorption nor does varying the stone size or water depth. He indicates that minimum reflection occurred when the berm depth was between 0.25 and 0.5 of the water depth, and that additional berm depths increased reflection.

Brian and Herbich (6) investigated the relationship between the coefficients of reflection and transmission and several cylindrical pile-group configurations. They utilized circular piles in different rectangular arrangements and one staggered pattern. They presented plots of the reflection coefficient against the transverse spacing between the piles. It was concluded that transmission and reflection decrease with a decrease in
the steepness of the waves. The reflection coefficient decreased with an increase in pile spacings but did not exhibit significant change by staggering the piles; however, staggering reduced transmission. The conclusion of Brian and Herbich regarding the effect of wave steepness on transmission is contrary to the findings of all other investigators; yet no explanation for the apparent contradiction was offered.

Sollitt (61,62), like Lean and Kondo, resorts to a linearizing technique to solve small amplitude wave motion in three breakwater configurations, namely crib style, pile array and trapezoidal sections. The linearization of the equation of motion is achieved by requiring that both the linear and non-linear friction laws account for the same amount of energy dissipation during one wave cycle. His method yields a potential flow problem, satisfied by an eigen series solution. Linear wave theory is assumed to apply outside the structure. Sollitt determines the transmitted, reflected and internal wave amplitudes by matching the general solutions at the sea-breakwater interfaces, based on continuity of pressure and horizontal mass flux. His analytical solutions were verified by laboratory tests. From his study, Sollitt concludes that the transmission decreases with decreasing wave length and breakwater porosity, and increasing wave height and breakwater width. Reflection was found to decrease with increasing wave length and
breakwater porosity, and decreasing breakwater width. In order to apply his analytical solution to the trapezoidal breakwaters, Sollitt (61) introduces what he terms an equivalent rectangular section to replace the trapezoidal one, provided that both sections possess the same submerged volume. The reason for the equivalent embankment to have the same submerged volume as the trapezoidal one was not clearly explained. It would appear, though not explicitly stated, that Sollitt must have arrived at this approximation based on a correlation with his experimental results. Sollitt then obtains an estimate of the energy loss, due to the breaking of waves on the windward slope, by modifying a wave breaking criterion proposed by Miche for impermeable slopes. The criterion consists in an adjustment of the reflection coefficients. Sollitt then proceeds to combine breaking losses and internal damping to obtain the solution which is reported to reproduce the essential features of his trapezoidal breakwaters.

The writer adopted the equivalent rectangular section hypothesis to solve the sloping faced embankments of this study. Yet, the writer's approach in applying the concept digresses from Sollitt's in two main issues. Firstly, replacing the sloping embankment by a rectangular section having the same submerged volume was found to exaggerate internal damping and consequently underestimate transmission. Secondly, it is the writer's conviction
that losses due to external wave breaking and those resulting from internal damping ought to be separable. The present study, therefore, takes a diversified line from Sollitt's; this will be elaborated on by the analyses in the coming chapters.

The most relevant theoretical attempt, found in the literature, for treating wave motion in a porous structure with an impervious core was reported by McCorquodale (40). He modified his rapid drawdown finite element model and applied it to the problem of wave propagation through a rectangular rockfill embankment with an impervious core. He utilizes Eq. 2.23 assuming that the term \( \frac{1}{2} \frac{\partial \tilde{q}}{\partial t} \) is small compared to \( |\nabla \phi| \) and has nearly the same line of action as \( \nabla \phi \) and \( \tilde{q} \). On this basis, he introduces the transformation

\[
\nabla \xi = \nabla \phi + \frac{1}{g m} \frac{\partial \tilde{q}}{\partial t}
\]

(2.30)

to reduce the governing equation, Eq. 2.23, to

\[
\nabla \cdot \left\{ K(|\nabla \xi|) \nabla \xi \right\} = 0
\]

(2.31)
in which

\[
K(|\nabla \xi|) = \frac{a}{2b} \left\{ \frac{1 + 4b |\nabla \xi|/a^2 - 1}{|\nabla \xi|} \right\}
\]
He expresses Eq. 2.31 in the variational form

$$\delta x = \int_0^t \int_{A(T)} \delta \left\{ K(|\nabla \xi|) (|\nabla \xi|)^2 + G(|\nabla \xi|) \right\} \, dA \, dT = 0$$

where \( t_0 \) = initial time;

\( A(T) \) = solution domain;

\( G(|\nabla \xi|) \) is a function introduced to ensure that continuity requirements are satisfied.

McCordquodale represents the tailwater piezometric boundary condition by a periodic time function and uses triangular elements in discretizing his solution domain. At the end of each time increment, the position of the free surface is calculated from the known previous value and the surface particle velocity during that increment.

Finally, the method of characteristics, utilized frequently in solving such flow problems as flood waves and water hammers (1, 24), is not known to have ever been applied to unsteady non-Darcy flow. Moreover, the use of finite difference techniques to solve this problem is missing from the literature. The present analysis introduces the application of both methods, combined, for the determination of wave action in rockfill.
CHAPTER III
THEORETICAL DEVELOPMENTS

The developments in this chapter are presented first for the case of rectangular rockfill embankments with impervious cores (Fig. 3.1). At a later stage, the modifications required for the inclusion of sloping sections are discussed.

3.1 Internal Governing Equations

If the Dupuit-Forchheimer assumption of parallel flow is applied to the unsteady non-Darcy regimes, the equations of motion and continuity, in a coarse granular medium, can be derived from the unit thickness flow element shown in Fig. 3.2.

Considering the pore velocity, \( u_n/m \), the acceleration term, \( \frac{d}{dt} \frac{u_n}{m} \), is expanded to

\[
\frac{\partial u_n}{\partial t} + \frac{u_n}{m} \frac{\partial u_n}{\partial x}
\]  

(3.1)

giving an effective force of

\[
\rho (\eta_n + h_0) m dx \left\{ \frac{\partial u_n}{\partial t} + \frac{u_n}{m} \frac{\partial u_n}{\partial x} \right\}
\]  

(3.2)
where \( \eta_n \) = perturbation height with respect to the mean water level, \( h_0 \);
\[ u_n = \text{horizontal component of the macroscopic velocity}; \]
\[ m = \text{porosity}; \]
the subscript \( n \) refers to the internal (rockfill) region.

An inertia correction coefficient, based on the principle of virtual mass (51), can be imposed on the first term of expression 3.1, and a Coriolis type correction factor (11) may be used with the second term. The effects of these two corrections could be significant if the particles of the porous body are widely separated. However, in a compacted medium the flow pattern around each grain interferes with the adjacent ones and this tends to drastically reduce the effects. Therefore, for the purpose of this study, both correction factors are taken equal to unity.

Neglecting vertical acceleration, the difference in the pressure, \( P \), obtaining across the element will be

\[ -\rho(h_0 + \eta_n) mg \frac{\partial(h_0 + \eta_n)}{\partial x} \, dx \quad (3.3) \]

The friction force is given by

\[ -\rho(h_0 + \eta_n) g \mid \text{dx} \quad (3.4) \]
where \( i \) is the hydraulic gradient expressed in the Forchheimer form as

\[
i = (a + b|u_n|) u_n
\]

Equating 3.2 to 3.3 and 3.4 yields the following equation of motion:

\[
\frac{\partial u_n / m}{\partial t} + \frac{u_n}{m} \frac{\partial u_n / m}{\partial x} + g \frac{\partial (h_0 + \eta_n)}{\partial x} = -gi
\]

In order to partially account for the vertical component, \( v_n \), of the macroscopic velocity, its effect may be incorporated in the friction term. Thus, Eq. 3.5 is written as

\[
i = (a + b|q_*|) u_n
\]

where \( q_* \) is the effective macroscopic velocity containing both vertical and horizontal components on the basis of an energy weighted average such that

\[
q_*^2 = \frac{v_n^2}{2} + u_n^2
\]

Putting \( F = (a + b|q_*|) \), and multiplying by \( m \), Eq. 3.6 becomes
\[ \frac{\partial u_n}{\partial t} + \frac{u_n}{m} \frac{\partial u_n}{\partial x} + gm \frac{\partial \eta_n}{\partial x} = -gmF u_n \]  

(3.9)

The equation of continuity is deduced by equating the net inflow into the control interval with the rate of increase of fluid over the interval.

The net inflow is

\[ -\rho \frac{\partial}{\partial x} \left[ u_n \left( h_0 + \eta_n \right) \right] \, dx = -\rho \left\{ u_n \frac{\partial}{\partial x} \left( h_0 + \eta_n \right) + \left( h_0 + \eta_n \right) \frac{\partial u_n}{\partial x} \right\} \, dx \]  

(3.10)

The rate of increase of mass is

\[ \rho m \, dx \frac{\partial}{\partial t} \left( h_0 + \eta_n \right) \]  

(3.11)

Equating 3.10 and 3.11 gives

\[ \frac{\partial \eta_n}{\partial t} + \frac{u_n}{m} \frac{\partial \eta_n}{\partial x} + \frac{(h_0 + \eta_n)}{m} \frac{\partial u_n}{\partial x} = 0 \]  

(3.12)

Eqs. 3.9 and 3.12 describe the one-dimensional unsteady non-Darcy flow in the x-direction.

3.2 The Characteristics Developments

The total differentials of the horizontal macroscopic velocity, \( u_n \), and the perturbation height, \( \eta_n \), are
respectively

\[
    d\eta_n = \frac{\partial \eta_n}{\partial x} \, dx + \frac{\partial \eta_n}{\partial t} \, dt
\]

and

\[
    d\eta_n = \frac{\partial \eta_n}{\partial x} \, dx + \frac{\partial \eta_n}{\partial t} \, dt
\]

Writing Eqs. 3.12, 3.9, 3.13 and 3.14 in a matrix form yields

\[
    \begin{bmatrix}
        \frac{(h_o + \eta_n)}{m} & 0 & \frac{u_n}{m} & 1 \\
        -\frac{u_n}{m} & 1 & g_m & 0 \\
        dx & dt & 0 & 0 \\
        0 & 0 & dx & dt
    \end{bmatrix}
    \begin{bmatrix}
        \frac{\partial u_n}{\partial x} \\
        \frac{\partial u_n}{\partial t} \\
        \frac{\partial \eta_n}{\partial x} \\
        \frac{\partial \eta_n}{\partial t}
    \end{bmatrix}
    =
    \begin{bmatrix}
        0 \\
        -g_m F \eta_n \\
        d\eta_n
    \end{bmatrix}
\]

(3.15)

In order for more than one solution to propagate from a point (a non-unique solution), the determinant of the matrix of coefficients, \( A \), must be zero.

Setting the 4 x 4 determinant to zero leads to the following positive and negative characteristic directions:

\[
    \alpha_n = \frac{dx}{dt} = \frac{u_n}{m} + \sqrt{g(h_o + \eta_n)}
\]

(3.16)

\[
    \beta_n = \frac{dx}{dt} = \frac{u_n}{m} - \sqrt{g(h_o + \eta_n)}
\]

(3.17)

Since \( g(h_o + \eta_n) \) is non-zero and positive, the system (Eq. 3.15) is hyperbolic, and the problem lends
itself to treatment by the method of characteristics.

A solution by a pure method of characteristics requires two more equations besides 3.16 and 3.17. These can be derived by first replacing any one of the columns in \( A \) by the vector on the right hand side of Eq. 3.15. Choosing the first column, for instance, then according to Cramer's rule:

\[
\frac{\partial u_n}{\partial x} = \text{Det.} \begin{bmatrix}
0 & 0 & \frac{u_n}{m} & 1 \\
-gmF_u & 1 & gm & 0 \\
du_n & dt. & 0 & 0 \\
d\eta_n & 0 & dx & dt
\end{bmatrix}
\]

Now for the system to be consistent, i.e. a solution exists for \( \frac{\partial u_n}{\partial x} \), the rank of the augmented matrix must equal the rank of \( A \). Therefore, the numerator of the right hand side of Eq. 3.18 should also be zero. This leads to the equations of motion along the characteristic directions, \( \alpha_n \) and \( \beta_n \), as follows:

\[
\frac{d}{dt} \left\{ \frac{u_n}{m} + 2\sqrt{g(h_o + \eta_n)} \right\} = -gF_u \text{ on } \alpha_n \quad (3.19)
\]

and

\[
\frac{d}{dt} \left\{ \frac{u_n}{m} - 2\sqrt{g(h_o + \eta_n)} \right\} = -gF_u \text{ on } \beta_n \quad (3.20)
\]
An alternative way of arriving at the four characteristic equations, with some physical rationalization (11), is to solve Eqs. 3.9, 3.12, 3.13 and 3.14 simultaneously for $\frac{\partial \eta_n}{\partial x}$. This gives

$$
\frac{\partial \eta_n}{\partial x} = \frac{\frac{du_n}{dt} + \frac{m}{(h_o + \eta n)} \frac{d^2n}{dt^2} \left( \frac{dx}{dt} - \frac{u_n}{m} \right) + gmF_n}{(\frac{dx}{dt})^2 - 2\frac{u_n}{m} \frac{dx}{dt} + \left( \frac{u_n}{m} \right)^2 - (h_o + \eta_n)g}
$$

(3.21)

It can be assumed that a wave comprises a huge number of infinitesimal surges, each of which displays a discontinuous surface profile. Thus, the water surface breaks at the point of discontinuity, and the slope has two values. This indicates that $\frac{\partial \eta_n}{\partial x}$ must be indeterminate, or $\frac{\partial \eta_n}{\partial x} = 0/0$. Setting the denominator of Eq. 3.21 to zero leads to Eqs. 3.16 and 3.17. When the numerator is equated to zero, Eqs. 3.19 and 3.20 are obtained after substituting the values of $a_n$ and $b_n$, respectively, for $dx/dt$.

Eqs. 3.16 and 3.17 may be solved to determine the characteristic directions on which $u_n$ and $\eta_n$ propagate. Eqs. 3.19 and 3.20 can then be used to find the values of $u_n$ and $\eta_n$ at the points of intersection of the $a_n$ and $b_n$ curves in the $x-t$ plane. The implementation of such a set up on a digital computer is rather difficult for a large scale problem, and the execution would be very time consuming. However, a combination of the method of characteristics and finite differences would greatly suppress these problems.
3.3 Finite Difference Formulations

In using a finite difference scheme for the problem at hand, it is convenient to make the two dependent variables, \( u_n \) and \( \eta_n \), dimensionally identical. Therefore, the following transformation is introduced:

\[
c_n = \sqrt{g(h_o + \eta_n)}
\]  

(3.22)

where \( c_n \) represents internal local wave celerity, and is utilized as a measure of \( \eta_n \).

Based on this transformation, the equations of motion and continuity can be altered, and expressed in terms of \( u_n \) and \( c_n \) through the substitution

\[
d\left\{ g(h_o + \eta_n) \right\} = d(c_n^2) = 2c_n dc_n
\]

(3.23)

Thus, Eqs. 3.9 and 3.12 become respectively

\[
\frac{\partial u_n}{\partial t} + \frac{u_n}{m} \frac{\partial u_n}{\partial x} + 2c_n \frac{\partial c_n}{\partial x} = -gF \frac{u_n}{m}
\]

(3.24)

\[
2 \frac{\partial c_n}{\partial t} + \frac{c_n}{m} \frac{\partial u_n}{\partial x} + 2 \frac{u_n}{m} \frac{\partial c_n}{\partial x} = 0
\]

(3.25)

The characteristic directions, as given by Eqs. 3.16 and 3.17, transform to

\[
\sigma_n = \frac{dx}{dt} = \frac{u_n}{m} + c_n
\]

(3.26)
\[
\beta_n = \frac{dx}{dt} \bigg|_n = \frac{u_n}{m} - c_n
\]

(3.27)

The selection of a finite difference scheme to discretize the equations of motion and continuity should be based on the fact that the initial data for a hyperbolic equation determine the solution only within the region bounded by the initial line and its terminal characteristics (1, 60). An explicit scheme with central differences in space and forward differences in time (see Fig. 3.3) would fulfill this requirement. Thus, the derivatives in Eqs. 3.24 and 3.25 can be approximated as follows:

\[
\frac{\partial u_n}{\partial t} = \frac{u_n(i,j+1) - u_n(i,j)}{\Delta t} ; \quad \frac{\partial u_n}{\partial x} = \frac{u_n(i+1,j) - u_n(i-1,j)}{\Delta x_n}
\]

\[
\frac{\partial c_n}{\partial t} = \frac{c_n(i,j+1) - c_n(i,j)}{\Delta t} ; \quad \frac{\partial c_n}{\partial x} = \frac{c_n(i+1,j) - c_n(i-1,j)}{\Delta x_n}
\]

It is noted that \( \Delta x \) is referred to, in this study, as the space increment; the actual grid spacing is only \( \Delta x/2 \).

By means of these relations, Eqs. 3.24 and 3.25 are written, in finite difference notation, respectively as:

\[
u_n(i,j+1) = u_n(i,j) - \frac{\Delta t}{\Delta x_n} \left\{ \frac{u_n(i,j)}{m} \left[ u_n(i+1,j) - u_n(i-1,j) \right] + 2mc_n(i,j) \left[ c_n(i+1,j) - c_n(i-1,j) \right] + q_m \Delta x_n F(i,j) u_n(i,j) \right\}
\]

(3.28)
\[ c_n(i,j+1) = c_n(i,j) - \frac{\Delta t}{m \Delta x_n} \left\{ u_n(i,j) \left[ c_n(i+1,j) - c_n(i-1,j) \right] + \frac{c_n(i,j)}{2} \left[ u_n(i+1,j) - u_n(i-1,j) \right] \right\} \]  
(3.29)

As a general rule, for convergence and stability considerations, the characteristic directions serve as a guide to control the discretization of the x-t plane; a sample discretization scheme for the present problem is depicted in Fig. 3.4. The ratio between the time and space increments is then to be chosen according to the condition

\[ \frac{\Delta t}{\Delta x_n} < \frac{1}{2} \left( \frac{u_n}{m} + c_n \right)^{-1} \]  
(3.30)

which follows from Eqs. 3.26 and 3.27.

Hypothetically, the above condition is supposed to be evaluated at each time increment. However, the critical \( \frac{\Delta t}{\Delta x_n} \) ratio, that would ensure convergence and stability at all times, can be calculated and generalized over the whole solution domain, based on the maximum possible values of \( u_n \) and \( c_n \).

Eqs. 3.28 and 3.29 can then be solved to advance the values of \( u_n \) and \( c_n \) at all the grid points, exclusive of the boundaries, from given initial conditions.
3.4 Initial Conditions

In the present analysis it is assumed that the water level is originally stagnant. Accordingly, the solution must be initiated by the two conditions

\[ u_n = 0 \text{ and } c_n = \sqrt{gh_0} \text{ at } t = 0. \]

These initial conditions are illustrated in Fig. 3.4.

3.5 Boundary Conditions

3.5.1 Core Boundary Condition

At the downstream end of the embankment there is no flow across the impervious core; thus, the physical boundary condition, at this end, specifies a zero normal velocity at all times. This condition is referred to in this study as the core boundary condition, and is given by (see Fig. 3.4)

\[ u_n = 0 \text{ for all the values of } t. \]

3.5.2 Entrance Boundary Condition

A time dependent function is needed to represent the movement of the outcrop point (point C in Fig. 3.1). Such a function may be utilized as an entry boundary condition at the water-rockfill interface.

The treatment, presented here, is a modification of Dracos' analysis (17) for Darcy flow to describe the more complex non-linear regimes.

As long as water is flowing into a porous body, the free water level and the phreatic line may reasonably
be assumed to rise together and meet at the same point on
the interface. However, when water is exiting from the
medium, the outcrop point will only coincide with the
outside level if the latter drops at a rate equal to or
less than the maximum internal fall velocity, $V_\star$, under a
unit piezometric gradient. Should the external drop rate
exceed $V_\star$, a seepage face will be interposed between the
free water level and the phreatic line.

In order to proceed with the analysis, the free
water level is assumed to fluctuate sinusoidally as
given by (see Fig. 3.5)

$$V_F = h_0 + A_\theta \sin \frac{2\pi}{T} t \quad (3.31)$$

where $V_F$ = elevation of the free water level at
the interface with respect to channel
bottom;

$A_\theta$ = amplitude of the impact wave;

$T$ = period of incident wave.

Differentiating Eq. 3.31 with respect to $t$ gives
the vertical velocity of fluctuation, $V_F$, as

$$V_F = \frac{dV_F}{dt} = \frac{2\pi}{T} A_\theta \cos \frac{2\pi}{T} t, \quad (3.32)$$

with the maximum rate, $V_{F\text{max}}$, being

$$V_{F\text{max}} = \frac{2\pi}{T} A_\theta \quad (3.33)$$
The maximum fall velocity, \( V_* \), of the outcrop point, under a unit piezometric gradient, can be deduced from the Forchheimer friction equation as follows:

\[
V_* = -\frac{k_*}{m} \sin^2 \theta
\]  
(3.34)

in which \( \theta \) is the angle of inclination of the rockface with the horizontal;

\( k_* \) is the non-Darcy, hydraulic conductivity corresponding to \( V_* \) and is given by

\[
k_* = \frac{1}{a + bm |V_*|}
\]  
(3.35a)

The value of \( V_* \) can directly be calculated from the Forchheimer equation, viz.

\[
V_* = \frac{-a/bm + \sqrt{(a/bm)^2 + 4 \sin^2 \theta / m^2 b}}{2}
\]  
(3.35b)

From Eqs. 3.32 and 3.34 the drop rate of the free water level will be termed:

'slow' if \( V_F \leq -\frac{k_*}{m} \sin^2 \theta \)  
(3.36a)

or

'fast' if \( V_F > -\frac{k_*}{m} \sin^2 \theta \)  
(3.36b)

Substituting the right hand side of Eq. 3.32 for \( V_F \) and introducing the dimensionless parameter \( W = \frac{k_* T}{2\pi m A_o} \), Eqs. 3.36 can be rearranged to
\[ \frac{1}{W} \cos \frac{2\pi}{T} t < -\sin^2 \theta \quad \text{for slow drop} \quad (3.37a) \]

and

\[ \frac{1}{W} \cos \frac{2\pi}{T} t > -\sin^2 \theta \quad \text{for fast drop} \quad (3.37b) \]

Equating the terms in Eq. 3.37 yields

\[ t^* = \frac{T}{2\pi} \cos^{-1}(-W \sin^2 \theta) \quad (3.38) \]

where \( t^* \) is the time during which the external and internal movements are identical.

The above equation along with the conditions indicated by Eqs. 3.37 provide a criterion for evaluating the movement of the outcrop point as follows:

1) For \( W \sin^2 \theta > 1 \), \( t^* \) (from Eq. 3.38) either does not exist or it is equal to \( T/2 \). This represents a slow drop case for which the elevation, \( Y_c \), of the outcrop point can be calculated from

\[ Y_c = Y_F = h_0 + A_0 \sin \frac{2\pi}{T} t \quad (3.39) \]

2) For \( W \sin^2 \theta < 1 \), the movement of the outcrop point undergoes various phases (see Fig. 3.5):

a. \( 0 < t < t^* ; \frac{1}{W} \cos \frac{2\pi}{T} t < -\sin^2 \theta \)

From the time 0 to \( T/4 \) there is a rising phase, followed by a slow drop phase
during the interval of time indicated by 
\[ t' = t^* - \frac{T}{4} \]. For this stage, the 
movement of the outcrop point is governed 
by Eq. 3.39.

b. \[ t^* < t \leq \left( \frac{3T}{4} - t' \right) ; \] \[ \frac{1}{W} \cos \frac{2\pi}{T} t > -\sin^2 \theta \]

This indicates a fast drop stage; therefore the outcrop point falls at its 
maximum rate, \( V_* \). Thus,

\[ y_c = (h_o + A_o \sin \frac{2\pi}{T} \left( \frac{T}{4} + t' \right) ) \]

\[ - V_* (t - (\frac{T}{4} + t')) \]  \hfill (3.40)

c. \[ \left( \frac{3T}{4} - t' \right) < t < t_c \]

At \( t = \frac{3T}{4} - t' \), the outcrop point curve 
will have reached its peak deviation, \( \Delta Y_p' \)
from the free water level curve; there-
after the two curves start to converge, 
i.e. \( \Delta Y_p \) diminishes indicating a decel-
erating motion for the outcrop point.
Assuming that the deceleration varies 
linearly with \( \Delta Y \) and referring to Fig. 
3.5, the driving force, per unit mass, 
can be expressed as

\[ g \sin^2 \theta \frac{\Delta Y}{\Delta Y_p} = \]

\[ g \sin^2 \theta \frac{\Delta Y}{\Delta Y_p} (y_c - (h_o + A_o \sin \frac{2\pi}{T} t)) \]  \hfill (3.41)
Now, it is possible to formulate an equation with non-linear viscous damping for this phase as follows:

\[ \ddot{Y}_C + g m (a + mbY_C) \dot{Y}_C + \frac{g \sin^2 \theta}{\Delta Y_p} Y_C \]

\[ = \frac{g \sin^2 \theta}{\Delta Y_p} \left( h_o + A_o \sin \frac{2\pi}{T} t \right) \quad (3.42) \]

in which \( \Delta Y_p = 2A_o \sin \frac{2\pi}{T} t^* - V_*(\frac{T}{2} - 2t') \) \( (3.43) \)

Eq. 3.42 is subject to the initial conditions

\[ \dot{Y}_{ci} = -\frac{k^*}{m} \sin^2 \theta \]

and

\[ Y_{ci} = (h_o + A_o \sin \frac{2\pi}{T} t^*) - V_*(\frac{T}{2} - 2t') \quad t = \frac{3T}{4} - t' \]

There is no exact analytical solution for Eq. 3.42; however, numerical methods may be used (42).

Eq. 3.42 may be written in a reduced form as

\[ \ddot{Y}_C + (a^* + b^* |Y_C|) \dot{Y}_C + \delta \dot{Y}_C = \delta f(t) \quad (3.44) \]

where \( a^* = gma; b^* = gm^2b; \delta = g \sin^2 \theta/\Delta Y_p; \)

\[ f(t) = h_o + A_o \sin \frac{2\pi}{T} t \]

Referring to Fig. 3.6, the derivatives in this equation can be replaced by finite differences as follows:

\[ Y_C = \frac{Y_C(t + \Delta t) + Y_C(t - \Delta t) - 2Y_C(t)}{\Delta t^2} \]
\[ Y_c(t) = \frac{Y_c(t) - Y_c(t - \Delta t)}{\Delta t} \]

Also, in order to improve stability, \( Y_c \) can be approximated, based on a time average, by

\[ Y_c = \frac{Y_c(t + \Delta t) + Y_c(t - \Delta t)}{2} \]

Substituting these expressions into Eq. 3.44 gives

\[ Y_c(t + \Delta t) = \left( \frac{2\Delta t^2}{2 + \Delta t^2} \right) \left\{ \delta \left[ f(t) - \frac{Y_c(t - \Delta t)}{2} \right] \right. \]

\[ - \left[ a^* + b^*(Y_c(t) - Y_c(t - \Delta t))/\Delta t \right] \]

\[ \frac{\partial}{\partial t} \left[ Y_c(t) - Y_c(t - \Delta t) \right]/\Delta t \]

\[ \left. - \left[ \frac{Y_c(t - \Delta t) - 2Y_c(t)}{\Delta t^2} \right] \right\} (3.45) \]

Eq. 3.45 can be solved for \( Y_c(t + \Delta t) \) in terms of the values at the two previous time steps.

The time \( t_c \) (Fig. 3.5) is the time at which the inside and outside water surfaces rejoin, and marks the upper limit for the damping phase.

d. \( t_c < t < T \)

This stage represents another rising phase during which the movement of the outcrop point is again described by Eq. 3.39.
3.6 The Unknowns At The Boundaries

From the foregoing analysis it is obvious that only one of the two dependent variables is known at each boundary. The other dependent variable remains to be found. The discretized equation of motion, Eq. 3.28, may be used to get the unknown velocity at the interfacial boundary, whereas the difference form of the continuity equation, Eq. 3.29, may be applied to obtain the unknown celerity at the impervious core. However, this approach does not utilize the full information available at each boundary. A better approximation of the unknown dependent variable at the one boundary could be achieved if the value of the other dependent variable, specified by the boundary condition at time \((j+1)\), is included. This can be done through manipulation of the equations of motion and continuity. Thus, Eqs. 3.24 and 3.25 are added to yield

\[
\frac{3u_n}{3t} + 2m \frac{3c_n}{3t} + \left( \frac{u_n}{m} + c_n \right) \frac{3u_n}{3x} + 2m \left( \frac{u_n}{m} + c_n \right) \frac{3c_n}{3x} = -gmF u_n \tag{3.46}
\]

Discretizing this equation, using forward differences both in space and time, and substituting a value of zero for \(u_n\) at the \(i^{th}\) column, the following reduced form is obtained:
\[ c_n(i,j+1) = c_n(i,j) + 2 \frac{\Delta t}{\Delta x_n} \left\{ c_n(i,j)u_n(i-1,j)/2m \right. \]

\[ \left. - c_n(i,j) \left[ c_n(i,j) - c_n(i-1,j) \right] \right\} \] (3.47)

This expression is an \( a_n \)-characteristic difference equation which can be used to calculate the celerity at the core.

Similarly, subtracting Eq. 3.25 from Eq. 3.24 leads to

\[ \frac{\partial u_n}{\partial t} - 2m \frac{\partial c_n}{\partial t} + \left( \frac{u_n}{m} - c_n \right) \frac{\partial u_n}{\partial x} = 2m \left( \frac{u_n}{m} + c_n \right) \frac{\partial c_n}{\partial x} \]

\[ = -\gamma_m F u_n \] (3.48)

Upon discretization as before, the following \( \beta_n \)-characteristic difference equation is obtained:

\[ u_n(i,j+1) = u_n(i,j) + 2m \left[ c_n(i,j+1) - c_n(i,j) \right] \]

\[ + \frac{2\Delta t}{\Delta x_n} \left\{ 2m \left[ u_n(i,j)/m - c_n(i,j) \right] \right. \]

\[ \left. \left[ c_n(i+1,j) - c_n(i,j) \right] - \left[ u_n(i,j)/m - c_n(i,j) \right] \right\} \]

\[ \left. \left[ u_n(i+1,j) - u_n(i,j) \right] - \Delta x_n \gamma_m F(i,j) u_n(i,j)/2 \right\} \] (3.49)

The unknown velocity at the interfacial boundary can be computed by means of Eq. 3.49.
3.7 External Governing Equations

In the present study long shallow water waves are assumed outside the embankments. The assumption implies a near hydrostatic pressure distribution; the equation of motion for a long shallow water wave is (24,63)

\[ \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + g \frac{\partial e}{\partial x} = -g S_f \]  

(3.50)

and the equation of continuity is

\[ \frac{\partial \eta_e}{\partial t} + u_e \frac{\partial \eta_e}{\partial x} + (h_o + \eta_e) \frac{\partial u_e}{\partial x} = 0 \]  

(3.51)

where \( u_e \) = external horizontal velocity;
\( \eta_e \) = external perturbation height;
\( S_f \) = friction slope = \( \frac{u_e |u_e|}{C_f^2 (h_o + \eta_e)} \);
\( C_f \) = Chezy friction factor; the bed is assumed horizontal.

In order to partially compensate for the assumption of hydrostatic pressure, allowances for centrifugal effects in the system can be introduced. The derivation to follow is directed towards deducing a Chezy friction coefficient to account for possible additional energy dissipation due to hydrodynamic pressure, based on a given wave steepness, \( S \), and the Chezy coefficient, \( C_f \), representing the prevailing friction conditions in the waterway.
Assuming a sinusoidal wave and referring to Fig. 3.7a, the total head loss, $h_T$, between sections 1 and 2 may be expressed as

$$h_T = h_f + \epsilon h_c$$  \hspace{1cm} (3.52)

where $h_f =$ frictional head loss between section 1 and 2;

$h_c =$ difference in centrifugal pressure head between the two sections;

$\epsilon =$ a proportionality factor.

The approximate centrifugal pressure, $p$, can be computed by Newton's second law for a mass of water having a unit cross-section and height, $d$, as follows (11):

$$p = \frac{\gamma}{g} d \frac{v^2}{r}$$  \hspace{1cm} (3.53)

where $\gamma =$ specific weight of water;

$\frac{v^2}{r} =$ centrifugal acceleration;

$v =$ mean velocity of flow;

$r =$ radius of curvature of water surface.

Thus, the upward, centrifugal pressure head at section 1 is

$$h_{c1} = -\frac{(h_o + Hi/2)}{gr} v^2$$  \hspace{1cm} (3.54)

and the downward pressure head at section 2 is
\[ h_{c2} = \frac{(h_o - H_i/2)}{gr} v^2 \] (3.55)

where \( H_i \) is the incident wave height.

Subtracting Eq. 3.54 from Eq. 3.55 yields

\[ h_c = \frac{2 h_o}{gr} v^2 \] (3.56)

The frictional head loss, \( h_f \), can be represented by

\[ h_f = \frac{v^2 L/2}{C_f h_o} \] (3.57)

in which \( L \) is the incident wave length.

Also, the total head loss, \( h_T \), may be assumed as follows:

\[ h_T = \frac{v^2 L/2}{C_t h_o} \] (3.58)

where \( C_t \) is a Chezy factor that accounts for both frictional and centrifugal effects.

Substituting Eqs. 3.56 through 3.58 into Eq. 3.52 leads to

\[ \frac{1}{C_t^2} = \frac{1}{C_f^2} + \epsilon \frac{4 h_o^2}{gLr} \] (3.59)

Since the radius of curvature, \( r \), is inversely proportional to the wave steepness, \( S \), therefore,
\( r = I/S \) \hspace{1cm} (3.60)

where I is a constant.

Thus, Eq. 3.59 can be rearranged to

\[
c_t^2 = \frac{g/L \frac{C_f^2}{C^2}}{gL + c' h^2 O_f C^2} \hspace{1cm} (3.61)
\]

in which \( c' = 4\varepsilon/I \).

Eq. 3.61 indicates that the steeper the wave the more the energy dissipation due to centrifugal effects.

The value of \( C_t \) can directly be found from Eq. 3.59 in the form

\[
c_t = \sqrt{\frac{g/L r C_f^2}{gLr + c' h^2 O_f C^2}} \hspace{1cm} (3.62)
\]

From Fig. 3.7b the value of \( r \) may be approximated as follows:

\[
cos \theta' = \frac{Hi/2}{\sqrt{(Hi/2)^2 + (L/4)^2}} \cdot \frac{\sqrt{(Hi/2)^2 + (L/4)^2}/2}{r} \]

or

\[
r \approx \left[ \frac{(Hi/2)^2 + (L/4)^2}{Hi} \right] /H \hspace{1cm} (3.63)
\]

The value of the factor \( c' \) is to be determined by correlation with laboratory experiments.

Now, writing Eqs. 3.51 and 3.50, along with the total differentials of \( u_e \) and \( \eta_e \) in a matrix form yields
\[
\begin{bmatrix}
(h_0 + \eta_e) & 0 & u_e & 1 \\
u_e & 1 & g & 0 \\
dx & dt & 0 & 0 \\
0 & 0 & dx & dt
\end{bmatrix}
\begin{bmatrix}
\frac{\partial u_e}{\partial x} \\
\frac{\partial u_e}{\partial t} \\
\frac{\partial \eta_e}{\partial x} \\
\frac{\partial \eta_e}{\partial t}
\end{bmatrix}
= \begin{bmatrix}
0 \\
-gS_f \\
du_e \\
d\eta_e
\end{bmatrix}
\]

(3.64)

which is a hyperbolic system of differential equations, similar to the one obtained for the internal flow.

The characteristic directions can be derived as before and are given by

\[
a_e = \frac{dx}{dt} \bigg|_+ = u_e + \sqrt{g(h_0 + \eta_e)}
\]

(3.65)

\[
b_e = \frac{dx}{dt} \bigg|_- = u_e - \sqrt{g(h_0 + \eta_e)}
\]

(3.66)

Introducing the transformation

\[
c_e = \sqrt{g(h_0 + \eta_e)} = \text{external wave local celerity},
\]

Eqs. 3.65 and 3.66 become

\[
a_e = \frac{dx}{dt} \bigg|_+ = u_e + c_e
\]

(3.67)

\[
b_e = \frac{dx}{dt} \bigg|_- = u_e - c_e
\]

(3.68)
Also, Eqs. 3.50 and 3.51, in terms of $c_e$, are respectively

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + 2c_e \frac{\partial c_e}{\partial x} = -gS_f$$  \hspace{1cm} (3.59)

$$2 \frac{\partial c_e}{\partial t} + c_e \frac{\partial u_e}{\partial x} + 2u_e \frac{\partial c_e}{\partial x} = 0$$  \hspace{1cm} (3.70)

Applying the same discretization technique, used for the internal flow equations, the above two equations become

$$u_e(i,j+1) = u_e(i,j) - \frac{\Delta t}{\Delta x_e} \left\{ u_e(i,j) \left[ u_e(i+1,j) - u_e(i-1,j) \right] \right\}$$

$$- u_e(i,j) \left[ c_e(i+1,j) - c_e(i-1,j) \right] + 2c_e(i,j) \left[ c_e(i+1,j) - c_e(i-1,j) \right] + g\Delta x_e S_f(i,j)$$  \hspace{1cm} (3.71)

$$c_e(i,j+1) = c_e(i,j) - \frac{\Delta t}{\Delta x_e} \left\{ u_e(i,j) \left[ c_e(i+1,j) - c_e(i-1,j) \right] + \frac{c_e(i,j)}{2} \left[ u_e(i+1,j) - u_e(i-1,j) \right] \right\}$$  \hspace{1cm} (3.72)

The $\alpha_e$ and $\beta_e$ characteristics provide the stability and convergence rule for the finite difference solution of Eqs. 3.71 and 3.72, viz.
\[
\frac{\Delta t}{\Delta x_e} \leq \frac{1}{\sqrt{\beta (u_e + c_e)}}
\] (3.73)

Eqs. 3.71 and 3.72 can be used to compute the values of \(u_e\) and \(c_e\) at all mesh points other than those lying on the boundaries.

3.8 The Coupling of the External and Internal Waves

3.8.1 Initial Conditions and Discretization of the \(x-t\) Plane

The initial conditions, used for the internal region, also apply outside the embankment. Thus, at \(t = 0\):

\[c_n = c_e = \sqrt{gh_0}\]  and \[u_n = u_e = 0\].

In discretizing the \(x-t\) plane, both the internal and external convergence and stability conditions (Eqs. 3.30 and 3.73) must be satisfied. To expedite the computations, a larger space increment can be assigned to the outer region. Yet, it is advantageous to utilize identical space increments for the inside and outside regions adjacent to the interface in order to simplify the treatment of the discontinuity in this vicinity. Furthermore, the use of only one time step is preferable for programming purposes. On these bases, the ratio between the time and space increments has to be set according to the smaller space increment \(\Delta x_n\) and the maximum possible values of the velocity and celerity attainable over the entire solution domain.

A sample discretization scheme for the combined solution is illustrated in Fig. 3.8.
3.8.2 End Boundary Conditions

In order to develop a coupled external and internal model, a function, simulating the vertical displacement of the incident wave is required as an upstream boundary condition in the external region. Unless the wave amplitude is small compared to the water depth, the wave is apt to be asymmetric about the mean water level (24, 63). An input function is proposed here to represent the vertical displacement of an asymmetric, or trochoidal shaped, wave. This function consists of two discontinuous sine waves for the phases above and below the mean water level as shown in Fig. 3.9. The amplitudes, \( A_1 \) and \( A_2 \), and the periods, \( T_1 \) and \( T_2 \), of the two waves are proportioned so that volume continuity is satisfied, i.e.

\[
\int_0^{T_1/2} A_1 \sin \frac{2\pi}{T_1} t \, dt = \int_0^{T_2/2} A_2 \sin \frac{2\pi}{T_2} t \, dt \quad (3.74)
\]

From which

\[
\frac{A_1}{A_2} = \left[ \frac{\cos \frac{2\pi}{T_2} (T_2/2) - 1}{\cos \frac{2\pi}{T_1} (T_1/2) - 1} \right] \cdot \frac{2\pi/T_1}{2\pi/T_2}
\]

or

\[
\frac{A_1}{A_2} = \frac{T_2}{T_1} \quad (3.75)
\]
The ratio \( \lambda_1 / \lambda_2 \) can be assigned any value, depending on the nature of the desired incident wave, and \( T_2 / T_1 \) can then be calculated.

In case of a symmetrical wave, \( \lambda_1 \) is equal to \( \lambda_2 \) and \( T_1 \) equals \( T_2 \); hence, the two functions reduce to one continuous sine wave.

Once the vertical displacement of the incident wave is specified by a boundary condition, the wave celerity at the upstream section can be calculated. In order to determine the unknown velocity, \( u_e \), at this section, a \( \beta_e \)-characteristics difference equation is required. Thus, subtracting Eq. 3.70 from Eq. 3.69 gives

\[
\frac{\partial u_e}{\partial t} - \frac{\partial c_e}{\partial t} + (u_e - c_e) \frac{\partial u_e}{\partial x} - 2(u_e - c_e) \frac{\partial c_e}{\partial x} = -gS_f \tag{3.76}
\]

Using forward differencing in space and time, Eq. 3.76 takes the form

\[
u_e(i,j+1) = u_e(i,j) + 2 \left[ c_e(i,j+1) - c_e(i,j) \right] + 2 \frac{\Delta t}{\Delta x_e} \left\{ 2 \left[ u_e(i,j) - c_e(i,j) \right] - \left[ c_e(i+1,j) - c_e(i,j) \right] \right. \\
\left. \left[ c_e(i+1,j) - c_e(i,j) \right] - \left[ u_e(i+1,j) - u_e(i,j) \right] \right. \\
\left. - c_e(i,j) + u_e(i+1,j) - u_e(i,j) \right. \\
\left. - \Delta x_e g S_f(i,j)/2 \right\} \tag{3.77}
\]
which can be used to compute \( u_e \) at the upstream boundary.

The downstream core boundary condition is the same zero normal velocity, specified earlier. Thus, Eq. 3.47 is again in order for the calculation of the unknown celerity at the impervious core.

### 3.8.3 Interfacial Boundary Condition

The main difficulty, encountered at the rock-water interface, arises in the discontinuity of the solution. This problem can be dealt with by satisfying flow continuity requirements, according to the analysis of the multi-phased movement of the outcrop point. It is noted that the external waves are assumed to reach the rockfill embankments at normal incidence.

In a rising phase, the outside and inside water levels at the interface coincide. In this case, an equation for the common, interfacial water elevations, as represented by the wave celerity, may be derived as follows:

Adding the external equations of motion and continuity, Eqs. 3.69 and 3.70, yields

\[
\frac{\partial u}{\partial t} + \frac{\partial c}{\partial t} + (u_e + c_e) \frac{\partial u}{\partial x} + 2(u_e + c_e) \frac{\partial c}{\partial x} = -gS_f
\]  

(3.78)

Discretizing this equation, using forward differences, leads to the following \( a_e \)-characteristic difference equation (see Fig. 3.8):
\[
\frac{c_e(i,j+1) = c_e(i,j) - [u_e(i,j+1) - u_e(i,j)]}{2} - \frac{\Delta t}{\Delta x_n} \left\{ 2 \left[ u_e(i,j) + c_e(i,j) \right] \\
\left[ c_e(i,j) - c_e(i-1,j) \right] + \left[ u_e(i,j) + c_e(i,j) \right] \left[ u_e(i,j) - u_e(i-1,j) \right] \\
+ \Delta x_n g S_f(i,j)/2 \right\}
\] (3.79)

The $\beta_n$-characteristic equation, Eq. 3.49, can be rearranged and written as

\[
c_n(i,j+1) = c_n(i,j) + \frac{1}{2m} \left[ u_n(i,j+1) - u_n(i,j) \right] - \frac{\Delta t}{m \Delta x_n} \left\{ 2m \left[ u_n(i,j)/m - c_n(i,j) \right] \\
\left[ c_n(i+1,j) - c_n(i,j) \right] - \left[ u_n(i,j)/m \right] - \Phi \\
- c_n(i,j) \left[ u_n(i+1,j) - u_n(i,j) \right] \\
- \Delta x_n g m F(i,j) u_n(i,j)/2 \right\}
\] (3.80)

In Eqs. 3.79 and 3.80 it is assumed that the interface boundary lies at the i\textsuperscript{th} column.

Since for a rising phase $c_e = c_n = c_{en}$ and $u_e = u_n = u_{en}$ at both $(i,j)$ and $(i,j+1)$, Eqs. 3.79 and 3.80 can be combined, after eliminating the unknown velocity at $(i,j+1)$, to give
\[
c_{en}(i,j+1) = c_{en}(i,j) - \frac{\Delta t}{(1+m)\Delta x_n} \left\{ 2 \left[ u_e(i,j) + c_e(i,j) \right] \right. \\
\left. \left[ c_{e(i,j)}^{(i+1,j)} - c_e(i-1,j) \right] + \left[ u_e(i,j) + c_e(i,j) \right] \right. \\
\left. \left[ u_e(i,j) - u_e(i-1,j) \right] + \Delta x_n g S_f(i,j)/2 \right. \\
+ 2m \left[ u_n(i,j)/m - c_n(i,j) \right] \left[ c_n(i+1,j) \\
- c_n(i,j) \right] - \left[ u_n(i,j)/m - c_n(i,j) \right] \right. \\
\left. \left[ u_n(i+1,j) - u_n(i,j) \right] \right. \\
- \Delta x_n g m F(i,j) u_n(i,j)/2 \right\} (3.81)
\]

Having computed \( c_{en}(i,j+1) \), the value of \( u_{en}(i,j+1) \) can be obtained directly from either Eq. 3.79 or Eq. 3.80.

The same procedure can also be applied to determine the common interfacial velocity and celerity for a slow drop case.

In a fast drop case, the phases, obtained for the movement of the outcrop point, serve to provide a boundary condition that allows the calculation of the inside celerity, \( c_n(i,j+1) \) at the interface. Thus, the determination of the inside velocity, \( u_n(i,j+1) \), is then possible by means of Eq. 3.49. From flow continuity at the water-rock interface,

\[
c_n^2(i,j+1) u_n(i,j+1) = c_e^2(i,j+1) u_e(i,j+1) (3.82)
\]
The left hand side of Eq. 3.82 is now known, and can be put equal to, say, \( K_1 \). Hence,

\[
u_e(i,j+1) = K_1/c_e^2(i,j+1)
\]  

(3.83)

Substituting Eq. 3.83 into Eq. 3.79 leads to the following cubic equation in \( c_e(i,j+1) \):

\[
c_e^3(i,j+1) + K_2 c_e^2(i,j+1) + K_1/2 = 0
\]  

(3.84)

in which

\[
K_2 = \Delta t/\Delta x_n \left\{ 2 \left[ u_e(i,j) + c_e(i,j) \right] \left[ c_e(i,j) - c_e(i-1,j) \right] + \left[ u_e(i,j) + c_e(i,j) \right] \left[ u_e(i,j) - u_e(i-1,j) \right] + \right\}
\]

\[
\Delta x_n \left[ g S_f(i,j)/2 - c_e(i,j) - u(i,j)/2 \right]
\]

Eq. 3.84 is to be solved for \( c_e(i,j+1) \); thereafter \( u_e(i,j+1) \) can be found from Eq. 3.79.

3.9 Treatment of Sloping Embankments

The problem of sloping faced embankments subjected to water waves gives rise to many difficulties such as the phenomenon of wave breaking and the complexity in dealing with the boundary condition on the slope.

The equivalent rectangular section hypothesis, introduced by Sollitt*(61), could be a convenient tool in overcoming the above mentioned difficulties. However,
recourse to such an approximation requires drastic modifications of Sollitt's approach in order to suit the technique of the present study.

Sollitt replaces his trapezoidal breakwaters with rectangular ones having the same submerged volume as shown in Fig. 3.10. As discussed in Chapter II, Sollitt obtains an estimate of the wave breaking losses and combines them with internal damping; this effectively increases his linearized friction coefficient.

The writer, on the other hand, believes that the external and internal losses must not be combined, and that estimating external breaking is but a means by which the remaining energy, available for reflection and transmission, can be determined. The change in internal damping, if any, should be confined to the immediate vicinity of the interface; for the rest of the embankment, the original frictional characteristics apply. The degree of change in internal damping depends primarily on the horizontal velocity component of the uprushing external wave. If this velocity is less or equal to the maximum internal velocity, the internal resistance stays unchanged. Should the external velocity exceed the maximum seepage rate of the rock-fill embankment, extra energy dissipation will be induced in the vicinity of the interface. The reason for this is the tendency for air entrainment, which would create a region of unsaturated flow (see Figs.
4.23 and 4.25 which will be discussed in the next chapter). The notion of separating external and internal losses was also confirmed by experimental evidence.

Based on the above discussion, the present study introduces a new classification for waves running up slopes as "fast" or "slow" rising waves, depending on their rising velocity being, respectively, faster or slower than the internal seepage rate.

Now, the external and internal losses still need to be incorporated in the method of solution. An attempt to accurately evaluate wave breaking losses is a very ambitious task that warrants lengthy studies. However, for practical purposes, these losses may be accounted for by adjusting the Chezy friction coefficient. Thus, the following form is proposed:

\[
C_b = C_t \left[ 1 - \frac{G}{(\cos \theta) \ell^n S^z} \right]
\]

(3.85)

in which \( C_b \) = a Chezy coefficient that allows for wave breaking losses;

\( \alpha \) = constant \( \leq 1 \) and is taken as unity in this analysis;

\( G, \ell, n, z \) are constants to be evaluated by trial and error from experimental findings.

The essence of this expression can be illustrated by considering the upper and lower extremes of the
variables $\theta$, $D$ and $S$, taking $G' = e^{\frac{G}{(\cos\theta)^{\frac{n}{2}} DS}}$, as follows:

if $D$ or $S \to \infty$, $G' \to 1$;

therefore $C_b \geq 0$ indicating very high resistance;

if $D$, $S$ or $\cos\theta \to 0$, $G' \to 0$;

therefore $C_b = C_t$ indicating no wave breaking;

when $\cos\theta \to 1$, angle $\theta$ is neutral, and

conditions are solely determined by the

values of $S$, and $D$.

As for the internal damping, if the horizontal component, $V_{Fh}$, of the rising velocity exceeds the maximum velocity, $V_*$, in the rockfill, the excess internal damping may be estimated based on the following proportionality:

$$F' = (\frac{V_{Fh}}{V_*})^{f''} \cdot F$$

(3.86)

where $F'$ is the new non-Darcy friction term to account for the added dissipation;

$F$ is the original friction term;

$f''$ is an exponent.

This relationship can be written in the form

$$F' = G^* (\frac{V_{Fh}}{V_*})^{f''} \cdot F$$

(for $V_{Fh} > V_*$)

(3.87)

where $G^*$ is a constant which, along with $f''$, can be found by experimental fitting.
The detailed steps of the solution for the developments presented in the preceding sections are illustrated in Chapter V.
CHAPTER IV

EXPERIMENTAL INVESTIGATION AND ANALYSIS

The principal purpose of the extensive experimental programme, conducted for the present study, was to provide adequate evaluation and verification of the mathematical models.

Since the direct application of these models is for rockfill structures, crushed rock was used in building the embankments.

It is noted again that most of the references cited in the literature review chapter ignored the determination of the hydraulic properties of the porous media. For this investigation, the physical and hydraulic properties of the materials used were predetermined, experimentally, by the writer (45).

Wave flume experiments were carried out for different rectangular configurations as well as embankments with 2:1 sloping faces for a range of wave conditions.

Multiple linear regression analysis yielded empirical formulae for transmission, reflection, run-up and rundown.
4.1 Materials Used and Their Properties (see reference 45).

A sieve analysis was made to separate the crushed rock into three different sizes. Only one rounded quartz size, provided by the supplier, was used.

The crushed rock and the rounded quartz were utilized to compose four homogeneous porous media. A sample of thirty particles was selected, at random, from each material and used to determine the following physical properties: specific weight, $\gamma_g$; specific surface, $S_s$; shape factor, $\alpha_s$; and average particle size, $D$. The specific surface is the ratio of the surface area of grains to their volume. The shape factor of a particle is defined as the product of its specific surface and nominal size or the equivalent diameter of a sphere having the same volume as the particle. The particle size, $D$, was based on the geometric mean diameter, i.e., the $n^{th}$ root of $n$ observations calculated from the nominal sizes of the particles of each sample. Fig. 4.1 shows a photograph of the four materials used.

Flow-head loss experiments were conducted, in a steady flow permeameter, to establish the hydraulic properties of porous media, i.e., the Darcy and non-Darcy friction coefficients, $a'$ and $b'$, in the Forchheimer equation. The details of this permeameter are drawn in Fig. 4.2. The top wide section was designed for the purpose of dissipating the inflow turbulence to avoid air...
bubble entrainment which, if present, could interfere with the flow and cause false piezometer readings. The data were utilized to obtain the friction relationships for the four media.

Writing the Forchheimer equation in the form

$$\frac{i}{u_n} = a' + b' \cdot u_n,$$

(4.1)
a least squares technique was employed to correlate \(i/u_n\) and \(u_n\) and deduce the intercept, \(a'\), on the \(i/u_n\) axis and the slope, \(b'\), of the line of best fit.

Table 4.1 lists the physical and hydraulic properties for the four media used. Shown also are the standard errors of the means for these properties.

### 4.2 Laboratory Facilities for Wave Experiments

The test flume (Fig. 4.3) is made of aluminum, with plexiglass walls supported by vertical beams spaced at 6 ft. intervals in the longitudinal direction. The entrance to the flume is regulated by a steel gate; this gate along with the tail gate were completely closed and sealed to prepare a wave tank for the present experimental study. The distance between both gates is 36 ft., and the flume is 1.5 ft. wide. The walls of the flume are 3 ft. high in the first 6 ft. from the upstream side, i.e. the test section, and the rest of the flume is 2 ft. deep. There are two side grooves, located at about 3 ft. from the upstream gate, in which a steel screen was inserted.
The wave machine consists of a slider-crank mechanism, and was set to generate shallow water waves. It has a variable speed motor linked by a chain to two 2 ft. diameter flywheels. The flywheels have holes drilled at different radii to allow variation in stroke. The desired stroke is achieved by connecting two steel rods to the appropriate holes; the rods drive a vertical steel plate, in a reciprocating motion, on two horizontal tracks along the top of the flume's side walls. The wave conditions can be varied through different combinations of strokes and speeds. The wave machine was mounted at a location that left a clear distance of 30 ft. between the vertical steel screen and the mid-position of the plate creating the disturbance. A steel guard was used to shield the entire wave equipment during operation. The guard could be lifted to provide access to the flywheels in order to change the stroke. Fig. 4.4 shows a photograph of the set up.

4.3 The Rockfill Embankments

The four porous media were utilized to construct different homogeneous embankments. A rectangular embankment of width, B, equal to 90.5 cm, extending from the vertical screen to the upstream control gate, was tested for each of the 4.4 cm rock, the 1.7 cm rock and the 1.6 cm quartz. The available quantity of the 0.7 cm rock was not sufficient for such a wide embankment, and only
permitted a maximum section thickness of 49 cm. Rectangular sections of widths equivalent to approximately 10 and 30 particle sizes were examined for the 1.7 cm rock, the 0.7 cm rock and the quartz. Space limitations, due to the fixed position of the retaining screen, prohibited the construction of a 30 particle size wide section for the 4.4 cm rock, but a 10 particle size section was possible. The dimensions of the different rectangular embankments used are given in Figs. 4.5 to 4.8. Overtopping of the embankments was not allowed during the experiments. For the 90.5 cm embankment the upstream gate was utilized as the impervious core. The back of the shorter sections was supported by a solid vertical steel plate to simulate the core.

The sloping embankments were built with 2:1 sloping faces. The choice of this particular slope emerges from the fact that most sloping porous structures have faces with inclinations ranging, approximately, between 1.25:1 and 3:1. Thus, a 2:1 slope is a reasonable average representation. For the sloping sections, a flexible steel screen was laid on the slope to prevent any scouring during wave action; the screen was held in place by means of a wooden support. The dimensions of the sloping embankments are illustrated in Figs. 4.9 to 4.12.

The embankments were built, and in situ porosity was measured as follows:
1. The solid steel plate, simulating the impervious core, was erected at the desired location. Modelling clay and waterproof, cloth tape were used to seal the sides and bottom of the plate.

2. In the cases of rectangular embankments, the rock was poured between the core and the screen and then levelled without compaction. For sloping sections, the rock was guided into the desired configuration, which was marked on the wall of the flume. Each amount of rock was weighed before pouring.

3. Having completed the construction of the embankment, the total weight of the rock used was deduced. Knowing the specific weight of the rock, its actual volume could be calculated. This together with the total volume occupied by the rock yielded the dry porosity, \( m \), as follows:

\[
m = \frac{\text{Tot. wt. of rock}/\gamma_g}{\text{Vol. of test section}}
\]

4.4 Experimental Procedure

A typical wave test was carried out as follows:

1. After building the embankment and determining in situ porosity, the flume was filled with water to the required mean water level, \( h_0 \), and the temperature was recorded. Amarnath
red dye was added and mixed with the water for photographic purposes.

2. The stroke was set and the wave machine was started; the motor was then adjusted to the speed that would minimize surface disorders and secondary waves. The wave period, \( T \), was measured, using a stop watch, by calibrating the rotation of a point, marked on the front flywheel over 10 revolutions; this step was repeated three times. The average period was then calculated, and checked by measuring the time of travel of the disturbing plate between its extreme forward and backward positions.

3. The wave celerity, \( c \), was determined by timing a wave peak over different distances between the vertical beams of the flume and recording the travel time by a stop watch. The average of all readings was considered. The wave length, \( L \), was then computed from \( L = c T \).

4. Nodes and anti-nodes (loops) were observed to form, alternately, about every quarter wave length. After the loops and nodes stabilized, their heights were recorded and used to compute incident and reflected wave heights as will be shown shortly. The locations of the loops and nodes were also recorded and utilized to check the calculated wave length.
5. The run-up, \( R_u \), and rush-down, \( R_d \), were measured at the face of the embankment, and water level fluctuations at the core were recorded. In addition, the maximum and minimum water levels in the rockfill were measured at different sections. Movies (16 mm black and white) were taken during the tests to trace experimental phreatic line profiles with time.

The observed nodes and loops are the result of the superposition of the incident and reflected waves, which yields a partial standing wave (clapotis) having the same period as the incident wave (24, 43, 57). The profile of the standing wave envelope oscillates in one place, giving rise to relative maxima (loops) and minima (nodes). In the majority of experiments, the nodes and loops did occur, alternately, at about every quarter wave length. At a loop location, the horizontal and vertical motions are minimum and maximum respectively, whereas a reversed phenomenon takes place at a node location. Hence, the loops result when the crests of the incident and reflected waves compound yielding a maximum surface elevation; the troughs superimpose at the same place, half a period later, to produce a maximum surface depression. Likewise, the nodes occur where the crests and troughs of the incident and reflected waves subtract.
In the present study, the calculations of the incident and reflected wave heights were approximated according to the linear wave theory (1; 24). Thus,

\[
\text{loop height } = h_p = H_i + H_r \tag{4.2}
\]

\[
\text{node height } = h_n = H_i - H_r \tag{4.3}
\]

therefore,

\[
H_i = \frac{h_p + h_r}{2} = \text{apparent incident wave height} \tag{4.4}
\]

and

\[
H_r = \frac{h_p - h_r}{2} = \text{apparent reflected wave height} \tag{4.5}
\]

The measurements of \( h_p \) and \( h_r \) were taken at different locations and their average values were used in determining \( H_i \) and \( H_r \). No measurements were made in the immediate vicinity of the embankments in order to avoid surface disorders. It should be pointed out that the loop and node measurements were recorded only when the profile of the standing wave became uniform and steady after a few traverses of the test section. This state was attained when the energy transmitted to the embankment balanced the input energy by the wave machine.

4.5 Results

The data of the rectangular embankments are documented in Tables 4.2 to 4.12; all length dimensions are given in centimeters and wave periods are in seconds. Each table presents the set of experiments carried out
for a certain rock size and embankment width. Within each
set the mean water level and wave characteristics were
varied in order to test different conditions; the results
of every individual test are listed in 4 rows. The first
row contains the information about the incident and reflected
waves along with the mean water level used. The second row
provides the horizontal distances, \( X \), at which the maximum
and minimum water elevations were recorded in the rockfill,
starting with \( X = 0 \) at the outcrop section. The last two
rows show the corresponding maximum and minimum water
elevations, \( Y_X \) and \( Y_M \) respectively, with the final readings
being at, or close to, the impervious core.

Tables 4.13 to 4.16 present the results of the
sloping embankments. These tables are patterned after those
for the rectangular sections, with \( X = 0 \) representing the
beginning of the sloping face at the flume's bottom along
which all the horizontal distances were measured. The
first reading appearing in the \( Y_X \) row refers to the ver-
tical upper limit of run-up, while the first one in the \( Y_M \)
row corresponds to the minimum vertical limit of rush-
down.

It is noted that the temperature in all the
experiments ranged between \( 18^\circ C \) and \( 22^\circ C \). This range is
similar to the one over which the resistance coefficients,
\( a' \) and \( b' \), given in Table 4.1, were determined.
4.6 Observations and Remarks

Figs. 4.13 to 4.20 show photographs of different rectangular embankments subjected to wave action. The waves, in most experiments, were not sinusoidal, but rather they had a trochoidal (or non-linear) type shape with their crests having higher curvature than their troughs, i.e., the waves were asymmetric with respect to the mean water level.

In some cases, waves impinging on the vertical face caused high splashes (Figs. 4.13 and 4.14). In such cases, extra care was taken in reading the maximum run-up in order to minimize the error due to the interference of splashing.

It was observed that most of the wave energy entering the rockfill was dissipated within a short distance from the face of the embankment (see for example Figs. 4.13 and 4.15). This was particularly noticeable for the steeper waves.

In the majority of the experiments for the 4.4 cm rock embankments, the seepage face has been found negligible, indicating a slow drop behaviour (Fig. 4.16). On the other hand, fast drop was experienced in almost all the tests of the 0.7 cm rock embankments (Fig. 4.17). The 1.7 cm rock and the quartz displayed both fast and slow drop cases, on a more balanced basis, depending on the external wave conditions (Figs. 4.19 and 4.20). It is noted that the waves in the vast majority of the tests on the rectangular embankments were non-breaking.
Some typical wave experiments on sloping embankments are illustrated by photographs in Figs. 4.21 to 4.24.

The change in the wave profile (Fig. 4.21) as the wave climbed the slope was clearly seen, especially for the steeper waves which further steepened until breaking occurred.

According to the fast and slow rising classification, introduced in Chapter III, breaking was considerably more pronounced for the fast rising waves. Slow rising waves were smoothly transmitted inside the rockfill, demonstrating a similar behaviour to that observed for rectangular embankments (Fig. 4.22). A fast rising wave experiment was characterized by a region of unsaturated flow encountered below the position of maximum run-up; this may be regarded, in a sense, as internal wave breaking (Fig. 4.23). Water was observed to infiltrate from that position, and the entrapped air induced blockage of the pores thereby causing a region of high damping. One of the movies, taken for the fast rising wave experiments, was studied using a movie analyzer to reveal this interesting phenomenon. The successive phreatic line profiles, starting from the maximum run-up down to the minimum limit of rush-down, are reproduced in Fig. 4.25.

A "pumping" type of phenomenon was observed to cause an upward shift of the mean water level in the embankments during wave action. Some examples are
presented in Figs. 4.26 to 4.32, with the corresponding maximum and minimum internal water elevations and the original mean water level. Each point on the new, shifted, mean water level is plotted as the average of the corresponding maximum and minimum water elevations. The upward shift, with respect to the original mean water level, reaches a maximum at the face of the embankment and gradually diminishes towards the impervious core. This shift appears to be, on the average, about 20% of the local wave height, \( H \), defined as the difference between the maximum and minimum local water elevations. However, the upward shift was pronounced in some cases to the extent that even the minimum water elevations were above the original mean water level (Figs. 4.30 and 4.31). It is noted that the pumping effect was also predicted by the mathematical models. Fig. 4.33 illustrates a typical upward shift as obtained by the combined solution for a quartz embankment.

4.7 The Role of Wave Steepness

The effect of varying the wave steepness, \( S = \frac{H}{L} \), on wave transmission and reflection was deduced by plotting \( S \) versus transmission and reflection coefficients. The transmission coefficient, \( C_T \), is defined in this study as an index of the ratio of the transmitted wave energy at the impervious core to the incident wave energy. Thus, \( C_T \) is expressed as
\[ C_T = \frac{m H_T}{H_i} \quad (4.6) \]

where \( H_T \) = transmitted wave height at the core.

The coefficient of reflection, \( C_R \), is given by

\[ C_R = \frac{H_F}{H_i} \quad (4.7) \]

The variations of \( C_T \) and \( C_R \) with \( S \) are shown for the rectangular embankments in Figs. 4.34 to 4.37. The data of all the rectangular embankments, tested for the one rock-size, are presented in a pair of figures; one for transmission and the second for reflection. The data, in each figure, are stratified according to the embankment width. The trend of the effect of \( S \) on \( C_T \) is indicated in the appropriate figures; a decrease in \( C_T \) with increasing \( S \) is noted. The observed reflection values do not show any definite trend with the variation of \( S \). It can be seen from all the transmission figures that the wider the embankment the lesser the transmission. Also, some increase in reflection is noted with the decrease of the embankment width.

The plots of \( S \) against \( C_T \) and \( C_R \) for the sloping embankments are illustrated in Figs. 4.38 to 4.41. Not only does the transmission coefficient show a decreasing trend with the increase in wave steepness, but also the reflection coefficient is displaying a similar behaviour, just slightly milder.
Generally, the rock size also affects the transmission and reflection. By comparing the figures of different rock sizes, it is apparent that the bigger the rock size the more the transmission and the lesser the reflection. This is conceivable on the basis that the smaller the rock the larger the surface area of the solid section and hence, the more the resistance offered by the medium to the flow.

4.8 Corrections for Wall Effect and Porosity

As pointed out before, the values of the resistance coefficients, \( a' \) and \( b' \), were found utilizing a steady flow permeameter whose cross-section is different from that of the flume in which the wave experiments were conducted. Consequently, the values of \( a' \) and \( b' \), to be used in the present analysis, should be corrected for differences in porosity and wall effect between the two apparatuses.

Using the expressions of \( a \) and \( b \) (Eqs. 2.4 and 2.5), with the value of \( k' \) given by Eq. 2.7, the new resistance coefficients corrected for porosity will be

\[
a_m = a' \cdot \frac{f(m')}{f(m)}
\]

and

\[
b_m = b' \cdot \sqrt{\frac{f(m')}{f(m)}}
\]

where \( a_m \) and \( b_m \) are the coefficients corresponding to the test flume;
\[ f(m') = m'^3 / (1-m')^2; \]  
\[ m' \]  
is the porosity in the steady flow permeameter;
\[ f(m) = m^3 / (1-m)^2; \]  
\[ m \]  
is the porosity in the flume.

It was shown in Chapter II that the wall zone of a permeameter is a zone of higher porosity, thus higher velocity, than the core of the permeameter. Taking the wall effect into account, the Forchheimer equation may be written in the form,

\[ i = a' \left( u_n \cdot C_w \right) + b' \left( u_n^2 \cdot C_w^2 \right) \]  
(4.10)

where \( C_w \) is the wall correction factor defined as \( (38) \)

\[ C_w = q_w / q_O \]  
(4.11)

in which \( q_w \) is the macroscopic or bulk velocity in an infinitely large permeameter;
\( q_O \) is the apparent macroscopic velocity in the finite permeameter.

Based on the Forchheimer type, Ward equation \( (68) \), McCorquodale \( (38) \) derived an expression for \( C_w \). The expression, for rectangular permeameters, is given by

\[ C_w = \left[ 1 + 2 \frac{L_e}{L_T} \frac{D}{\sqrt{A_r}} \right]^{-1} \]  
(4.12)
where \( A_T \) = total area of the permeameter cross-section;

\( L_e \) = effective length of the wall zone perimeter;

\( L_T \) = length of permeameter perimeter.

As discussed by Dudgeon (19) and illustrated by the above formula, the wall effect is independent of the flow rate. Therefore, \( C_w \) may be applied to adjust \( a' \) and \( b' \) instead of \( u_n \) in Eq. 4.10. Thus,

\[
a_w = a' \cdot \frac{C_{wf}}{C_w^*} \tag{4.13}
\]

\[
b_w = b' \cdot \left( \frac{C_{wf}}{C_w^*} \right)^2 \tag{4.14}
\]

where \( a_w \) and \( b_w \) are the resistance coefficients corrected for the wall effect;

\( C_{wf} \) and \( C_w^* \) are the wall correction values for the test flume and the steady flow permeameter, respectively.

Combining the porosity and wall corrections, the expressions for \( a \) and \( b \) to be used for the embankments become

\[
a = a' \cdot \frac{f(m')}{f(m)} \cdot \frac{C_{wf}}{C_w^*} \tag{4.15}
\]

and

\[
b = b' \cdot \sqrt{\frac{f(m')}{f(m)}} \cdot \left( \frac{C_{wf}}{C_w^*} \right)^2 \tag{4.16}
\]
Applying the above formulae, using an average cross-section for all the embankments of each material size yields the adjusted values for the resistance coefficients, a and b, given in Table 4.17. It is emphasized that the average porosity for the embankments of each rock size was very close to that obtained, for the same rock size, in the steady flow permeameter. Therefore, the value of the porosity correction was trivial.

Since (as mentioned in section 4.5) the temperatures under which the wave experiments were conducted were within the same range covered in the flow-head loss tests, no temperature correction was necessary.

4.9 Dimensional and Regression Analysis

4.9.1 Rectangular Embankments

For the rectangular embankments, the wave transmission, reflection, run-up and rush-down are assumed to be functions of the following variables

\[(H_i, L, B, D, m, C_r, R_n, E, h_0, c)\]

where

- \(C_r\) = approximate wave celerity = \(\sqrt{gh_0}\);
- \(E\) = incident wave energy = \(\rho g L H_i^2 / 8\);
- \(R_n\) = representative Reynolds number = \((V_r/m)D\);
- \(V_r\) = a representative macroscopic velocity in the rockfill calculated from

\[V_r = \frac{S}{a + b|V_r|} \]
In order to obtain dimensionless expressions for transmission, reflection, run-up and rush-down, each was expressed as a ratio of the incident wave height, $H_i$.

Thus, as defined earlier

$$C_T = \frac{m H_t}{H_i}$$

and

$$C_R = \frac{H_r}{H_i}$$

In addition,

$$U_r = \frac{R_u}{H_i} = \text{dimensionless run-up}$$

and

$$D_r = \frac{R_d}{H_i} = \text{dimensionless rush-down}$$

The influencing variables were grouped in dimensionless parameters and, after a logarithmic transformation, multiple linear regression (46) was used to fit the data and determine the exponents of the assumed functions. Different combinations of these functions were tried, and the following empirical formulae gave the best correlation coefficient, $R$.

$$C_T = \frac{(1 + 15 S)^{0.844}}{(1 + \frac{B}{D}) (1 + \frac{B}{D} S R_N^{1/5})^{1.04}} \quad (4.17)$$

$R = 0.9$

$$C_R = \frac{0.21}{R_N^{0.21}} \left( \frac{E}{\rho C_* H_i^2} \right)^{0.172 B} \left( \frac{B}{D} \right)^{0.05} \quad (4.18)$$

$R = 0.90$
\[ U_r = 5.18(S)^{0.899} \left( \frac{E}{\rho C_s^2 H_i^2} \right)^{0.935} \left( \frac{h_o}{H_i} \right) ; \quad (4.19) \]

\[ R = 0.90 \]

\[ D_r = 1.02(S)^{0.611} \left( \frac{E}{\rho C_s^2 H_i^2} \right)^{0.604} \left( \frac{h_o}{H_i} \right)^{0.092} \left( \frac{D}{Bm} \right) ; \quad (4.20) \]

\[ R = 0.72 \]

It is noted that Eq. 4.17 was formulated by trial, based on the observed range of variables and the fact that \( C_T \rightarrow 1 \) as \( S \) and B/D approach zero.

In the preceding analysis, the incident wave energy, \( E \approx \rho g L H_i^2 / 8 \) (24, 43), was introduced as a variable. The dimensionless term containing \( E \), in a reduced form, is \( (L/8h_o) \); this term brings the role of \( (L/h_o) \) into play and its inclusion in the formulation was found to produce the best correlation for reflection, run-up and rush-down.

Although the particulars of the parameter \( E \) can be formed by a combination of other variables considered, presenting \( E \) as an independent variable is believed to be more meaningful due to its physical significance. Also, it should be pointed out that the velocity \( V_r \) is not the actual macroscopic velocity because the wave steepness, \( S \), is not the true internal gradient. However, the use of \( S \) in the Forchheimer equation was intended to emphasize the relationship between wave steepness and the gradient at which the wave enters the embankment.
In order to illustrate the scatter in the data of transmission, reflection, run-up and rush-down, plots of the observed values versus the ones computed by the above equations are shown in Figs. 4.42 to 4.45. In these figures, the reflection, run-up and rush-down are related to the mean water level, $H_0$, instead of the incident wave height, $H_i$; this was found to provide a clearer representation of the scatter for the mentioned parameters.

4.9.2 Sloping Embankments

The analysis of the sloping sections was carried through by considering the same influencing variables assumed for the rectangular embankments. Since only one slope of 2:1 was tested, it is not feasible to enter the angle of inclination $\theta$, in the analysis. Yet, an independent analysis could be made to derive empirical expressions, based only on the data of the sloping embankments used. This, however, would not reveal the part played by the angle, $\theta$. Therefore, the data for the sloping embankments were analyzed by imposing the dimensionless functions, used in Eqs. 4.17 to 4.20, raised to the same exponents and deducing the effect of the angle on the constant in each equation. This was achieved by utilizing a least squares technique to obtain the new constants pertaining to the 2:1 slope. Thus, for the sloping embankments, the equivalents of Eqs. 4.17 to 4.20 become respectively
$$C_T = \frac{0.86(1 + 15 S)^{0.844}}{(1 + \frac{B}{D})^{0.142}(1 + \frac{B}{D} S R^{1/5})^{1.04}}$$ \hspace{1cm} (4.21)

$$R = 0.87$$

$$C_R = \frac{1.97}{R^{0.21}(\frac{E}{\rho C_s^2 H_i^2})^{0.172}(B)^{0.05}\rho C_s^2 H_i^2}$$ \hspace{1cm} (4.22)

$$R = 0.88$$

$$U_r = 4.96(S)^{0.899}(\frac{E}{\rho C_s^2 H_i^2})^{0.935}\frac{h_o}{H_i}$$ \hspace{1cm} (4.23)

$$R = 0.86$$

$$D_r = 0.98(S)^{0.6611}(\frac{E}{\rho C_s^2 H_i^2})^{0.604}\frac{h_o}{H_i}(\frac{D}{Bm})^{0.092}$$ \hspace{1cm} (4.24)

$$R = 0.7$$

The effective width, B, for a sloping embankment was taken equal to the horizontal distance, between the core and the sloping face, along the mean water level. The constant in each of the above equations is slightly lower than the corresponding one for rectangular embankments. The scatter plots for the sloping sections are presented in Figs. 4.46 to 4.49.
CHAPTER V.
EVALUATION OF THE THEORETICAL
AND EXPERIMENTAL ANALYSES

In this chapter the various mathematical models are presented, and the details of the numerical solutions are illustrated.

Transmission curves along with the values of reflection coefficient, run-up and impact wave height as predicted by the models are compared with the corresponding experimental results.

Typical phreatic line profiles, as obtained from the 16 mm movies taken for the experiments, are shown and compared to those determined by the computer programmes.

The sources of experimental errors and the limitations in the mathematical models are discussed.

Finally, some comparisons with a few previous investigations are introduced and evaluated.

5.1 The Mathematical Models

The models are developed first for rectangular embankments. The modifications required to include sloping embankments are given in section 5.1.3.
5.1.1 The Internal Models

In chapter III it is indicated that if a solution for the internal equations alone is sought, the movement of the outcrop point will be needed as an entrance boundary condition. The analysis of this movement introduced the "fast" and "slow" drop cases; each one of these two cases is treated in a separate computer model. According to the criterion

\[ W \sin^2 \theta \geq 1 \rightarrow \text{slow drop case} \]

or

\[ W \sin^2 \theta < 1 \rightarrow \text{fast drop case}, \]

the appropriate internal computer model is entered.

The discretization of the x-t plane for the internal models is shown in Fig. 3.4.

The "Slow Drop" Model

In the slow drop model, the experimentally determined impact wave height, \( H_I \), was programmed as the entrance boundary condition. As pointed out in chapter IV, the waves, observed in the laboratory investigation, were (in most experiments) trochoidal in shape. Hence, in order to simulate, in the model, the asymmetry of the impact wave with respect to the mean water level, the function derived in chapter III for the vertical.
displacement of the incident wave (Eq. 3.74) was utilized. The amplitudes $A_1$ and $A_2$ were proportioned based on the experimentally observed values, and then $T_1$ and $T_2$ were calculated.

The maximum internal velocity was computed from Eq. 3.35, and the maximum celerity was obtained from the highest possible water level at the outcrop section. These maximum values were substituted in the stability and convergence condition, Eq. 3.30, to determine the ratio $\Delta t/\Delta x_n$ which was used throughout the solution. The space increment, $\Delta x_n$, was selected according to the embankment width, and then $\Delta t$ was deduced.

The input data required for the slow drop model are the impact wave height $H_i$, wave period, $T$, medium properties $a$, $b$ and $m$; mean water level, $h_o$, $A_1/A_2$, $T_1/T_2$, $\Delta t$, $\Delta x_n$ and the number of nodal points covering the width of the section.

The discretized equations of motion and continuity, Eqs. 3.28 and 3.29, were used to advance the values of $u_n$ and $c_n$, at all the internal mesh points, from the initial conditions, $u_n = 0$ and $c_n = \sqrt{gh_o}$ ($\eta_n = 0$), as shown in Fig. 3.4. The unknown velocity at the left boundary and unknown celerity at the core boundary were found by means of Eqs. 3.49 and 3.47 respectively.

An iteration procedure was utilized to adjust the non-linear friction term, $F$, as well as the
velocities and celerities by averaging the values between 
two successive time advancements. The process was 
terminated after 3 to 5 iterations or a tolerance of 
0.001, which ever occurred first.

At the end of each time increment, the celerity 
at all the grid points were transformed to a water level 
by means of Eq. 3.22 in the form

\[(n_n + h_o) = c_n^2 / g\]  \hspace{1cm} (5.1)

Also, the values of the velocity and celerity were used 
as starting values for the next time advancement.

The "Fast Drop" Model

The formulation of this model is similar to that 
of the slow drop model. The only difference between the 
two models lies in the treatment of the entrance boundary 
condition. For a fast drop case, the movement of the 
outcrop point has been resolved into various phases. The 
phase governed by the non-linear, damped equation, Eq. 
3.44, introduces a difficulty into the fast drop model. 
The proposed finite difference form of this equation, 
Eq. 3.45, was solved; in a subroutine, in the computer 
model. It is necessary that the time increment, \(\Delta t\), 
used in the main model be the same for the solution of 
the damped equation. Otherwise, matching the corres-
ponding values, obtained from two different time steps, 
would lead to complicated interpolations. Nevertheless,
the use of the same time increment may cause a problem at
the initial time (known in advance) at which the damped
equation starts to apply. There is no guarantee that
this initial time will exactly coincide with the end of a
regular \( \Delta t \). Therefore, the calculation of \( Y_c \) for the
phase prior to the viscous damping phase was continued,
by means of Eq. 3.40, until the time, \( t \), equalled or
exceeded the initial time, \( t_i \), for the damped equation.
Another time step, \( \Delta t' \), smaller than \( \Delta t \) was then defined
as:

\[
\Delta t' = t - t_i \tag{5.2}
\]

The solution for the damped equation was initiated
by substituting the known initial values, \( Y_{c_i} \) and \( Y_{c_i}' \),
into the finite difference forms of \( Y_c \) and \( Y_c \) proposed in
section 3.5.2. Thus, (see Fig. 5.1)

\[
Y_c(t_i - \Delta t') = \frac{Y_{c_i} - Y_{c_i}' \cdot \Delta t'}{\Delta t'} \tag{5.3}
\]

Therefore,

\[
Y_c(t_i + \Delta t') = Y_c' \cdot \Delta t'^2 - Y_c(t_i - \Delta t') + 2Y_{c_i} \tag{5.4}
\]

in which \( Y_c \) was calculated from the damped equation
itself, Eq. 3.44, i.e.

\[
Y_c = \delta \cdot f(t) - (a^* + b^*|Y_{c_i}|) Y_{c_i} - \delta Y_{c_i} \tag{5.5}
\]
The second advancement with a full $\Delta t'$ would then form with the previous $\Delta t'$ an unbalanced arm, which was treated by approximating $\ddot{Y}_c$, $\dot{Y}_c$ and $Y_c$, based on a weighted average, as follows:

\[
\ddot{Y}_c = \frac{\dot{Y}_c(t+\Delta t) - \dot{Y}_c(t) - \dot{Y}_c(t) - \dot{Y}_c(t-\Delta t')}{\Delta t} - \frac{1}{2}(\Delta t + \Delta t')
\]

\[
\dot{Y}_c = \frac{Y_c(t) - Y_c(t-\Delta t')}{\Delta t'}
\]

\[
Y_c = \frac{\Delta t' [Y_c(t-\Delta t')] + Y_c(t+\Delta t)}{1 + \Delta t'/\Delta t'}
\]

These relationships were substituted in Eq. 3.44 which was then solved for $Y_c(t+\Delta t)$. After this step, the solution proceeded with a uniform $\Delta t$ using Eq. 3.45. The outside and inside water levels were continuously checked until they coincided and a rising phase started.

Experimentation with the fast drop model indicated that a drop rate of about one half of the maximum fall velocity, $V_*$, would give very close values to those obtained by the finite difference solution of the damped equation. Therefore, the elevation of the outcrop point, in the damping phase, could also be approximated by (see Fig. 3.5).

\[
Y_c = (h_o + A_o \sin \frac{2\pi}{\frac{T}{4} + t'}) - 0.5V_*(t - (\frac{T}{4} + t'))
\]
In addition to the input data of the slow drop model, the time, \( t^* \), as given by Eq. 3.38 is also needed for the fast drop model, while \( A_1/A_2 \) and \( T_1/T_2 \) are omitted. In the fast drop model, the value of the run-up was used as the amplitude \( A_0 \).

In both the fast and slow drop models, space increments, \( \Delta x_n \), ranging between 15 cm to 45 cm were utilized along with time steps of 0.02 sec. to 0.06 sec., depending on the individual case treated. The solution, in most cases, was stopped after 5 wave periods.

A flow chart illustrating the successive steps of the solution for either of the internal models is shown in Fig. 5.2.

5.1.2 The Combined Model

The finite difference equations pertinent to the internal and external waves were developed and presented in chapter III. A typical discretization of the x-t plane for coupling both waves is shown in Fig. 3.8.

The internal and external stability and convergence conditions, Eqs. 3.30 and 3.73, were satisfied by considering the space increment inside the rockfill, \( \Delta x_n \), and generalizing one \( \Delta t \) over the outer and inner regions based on estimates of the maximum values of \( u_e \) and \( c_e \). The maximum value of \( c_e \) occurs at the interface, where the maximum run-up represents the highest possible water elevation. Since the impact wave height is an output not an input in the combined
model, the estimate of the maximum $c_e$ was based on the amplitude of the incident wave height. As there is no means of predicting the maximum value of the velocity, $u_e$, in advance, it was taken equal to the maximum $c_e$. This estimate of the maximum $u_e$ exaggerates its value, but also compensates for the underestimated maximum $c_e$.

Thus, $\Delta x_n$ was chosen in the range of 15 cm to 45 cm as for the internal models; $\Delta t$ was then calculated from

$$\Delta t = \frac{\Delta x_n}{4(\text{maximum } c_e)} \quad (5.7)$$

Preliminary runs of the computer model, using one $\Delta t/\Delta x_n$ over the whole solution domain, indicated that the above condition produces very stable solutions in the internal region. However, some error growth was noticed to occur in the vicinity of the upstream boundary and radiate into the outside region; this dictated a further reduction in the ratio $\Delta t/\Delta x_n$, by factors ranging between 1/5 and 1/10, depending on the case being treated. Hence, in order to achieve consistent stability in both regions, the ratio between the time and space increments had to be adjusted by increasing the space increment in the external region and/or decreasing $\Delta t$. In order to accelerate the execution of the computations, a larger space increment, $\Delta x_e$, was utilized in the external region, and the
reduction of $\Delta t$ was avoided whenever possible. Values of $\Delta x_e$ between 46 cm and 200 cm were used according to the incident wave length; $\Delta t$ was in the range of 0.004 sec. to 0.02 sec.

The constant in Eq. 3.62, proposed to take account of possible additional energy loss due to centrifugal effects, was evaluated by experimentation with the combined model for a variety of observed cases. A value of 1.2 for the constant of proportionality, $c'$, was found to give the best results. Thus, the adjusted Chezy friction coefficient, $C_t$, was calculated from

$$C_t = \frac{\sqrt{g L r C_e^2}}{\sqrt{g L r + 1.2 h_o C_f^2}}$$  \hspace{1cm} (5.8)

Since the test flume has plexiglass walls and aluminum bottom, a value of 830 cm$^3$/sec (approximately 150 ft$^3$/sec) was chosen for $C_f$ to express the smooth conditions in the flume; $C_t$ was then computed for each individual case.

The discretized internal equations of motion and continuity, Eqs. 3.28 and 3.29, were used to advance the values of $u_n$ and $c_n$ from their initial conditions at all the grid points, within the rockfill embankment, exclusive of the boundaries. Likewise, the external finite difference equations, Eqs. 3.71 and 3.72, were
utilized, outside the embankment, to advance the values of $u_e$ and $c_e$.

The vertical displacement of the incident wave forms the left boundary condition which was originated in the model at one wave length away from the face of the rockfill section. The choice of a distance of one wave length is based on the fact that this is the minimum distance which would ensure the generation of at least one loop and one node. Furthermore, exceeding the distance of one wave length requires additional computational time and the stability of the solution would be more sensitive. The two discontinuous sine functions, suggested in chapter III (Eqs. 3.74 and 3.75), were used in modelling the upstream boundary condition; the amplitudes and periods were proportioned based on laboratory observations. The unknown velocity at this boundary was calculated by means of Eq. 3.77. Similarly, the unknown celerity at the impervious core, where a zero normal velocity condition applies, was found from Eq. 3.47.

In order to simplify the treatment of the interface, the half outside space increment, immediately before the interface, was taken equal to $\Delta x_n/2$. Therefore, the space derivatives at the outside i column, $\Delta x_n/2$ away from the interface, were approximated by the adjacent values according to a weighted average, based on
the inverse ratios of the external and internal space increments. Thus, at this location (see Fig. 3.8)

\[
\frac{\partial u_e}{\partial x} = \frac{1}{\Delta x_e/2} \left\{ \frac{u_e(i,j) - u_e(i-1,j)}{\Delta x_e/2} \right\} + \frac{1}{\Delta x_n/2} \left\{ \frac{u_e(i+1,j) - u_e(i,j)}{\Delta x_n/2} \right\}
\]

\[
\left\{ \frac{2}{\Delta x_e} + \frac{2}{\Delta x_n} \right\}
\]

(5.9)

\[
\frac{\partial c_e}{\partial x} = \frac{1}{\Delta x_e/2} \left\{ \frac{c_e(i,j) - c_e(i-1,j)}{\Delta x_e/2} \right\} + \frac{1}{\Delta x_n/2} \left\{ \frac{c_e(i+1,j) - c_e(i,j)}{\Delta x_n/2} \right\}
\]

\[
\left\{ \frac{2}{\Delta x_e} + \frac{2}{\Delta x_n} \right\}
\]

(5.10)

These relations were used to discretize the external equations of motion and continuity in order to obtain expressions for \( u_e \) and \( c_e \) at the particular \( i \) column in question.

In the combined model, the solution at the interfacial boundary is governed by a set of conditional control statements for testing the phase of the movement of the outcrop point. When the testing indicated a rising phase, Eqs. 3.81 and 3.80 were automatically entered to calculate the values of the common celerity and velocity, \( c_{en} \) and \( v_{en} \). For a falling phase, the
outside vertical velocity, \( V_F \), at the interface was calculated at each time step as follows:

\[
V_F(i, j+1) = \frac{c_{en}^2(i, j+1) - c_{en}^2(i, j)}{g \Delta t} \quad (5.11)
\]

This velocity was then compared to the maximum fall velocity, \( V_* \), of the outcrop point. As long as \( V_F \) remained less or equal to \( V_* \), Eqs. 3.81 and 3.80, used for the rising phase, were also applied to determine \( c_{en} \) and \( u_{en} \) for this slow drop phase. A fast drop phase was encountered when \( V_F \) exceeded \( V_* \); this phase was dealt with in the following manner:

1. Once \( V_F > V_* \), the outcrop point was allowed to drop at its maximum rate, \( V_* \). Thus, the elevation of the outcrop point was computed from

\[
\eta_n(i, j+1) = \eta_n(i, j) - V_* \Delta t \quad (5.12)
\]

from which \( c_n(i, j+1) \) was calculated and then \( u_n(i, j+1) \) was found by Eq. 3.49. These values of \( c_n(i, j+1) \) and \( u_n(i, j+1) \) were utilized, according to flow continuity requirements, to derive the cubic equation

\[
c_e^3(i, j+1) + K_2 c_e^2(i, j+1) + K_1/2 = 0 \quad (3.84)
\]

as shown in chapter III.
The exact solution of cubic equations may yield one or more real roots, depending on the coefficients in the equation. It was difficult, for the present problem, to program a general criterion by which the proper real root could be selected. Therefore, it was necessary to resort to another means of solution for the cubic equation; the Newton iterative method (10) was employed for this purpose. This method (see Fig. 5.3) states that an unknown root to satisfy a function, \( f(x) \), can be obtained by first attempting an approximate root, \( x_1 \). The tangent, corresponding to \( x_1 \), to the \( f(x) \) curve meets the \( x \)-axis at \( x_2 \) with an angle \( \psi \), say. Thus,

\[
\tan \psi = \frac{f(x_1)}{x_1 - x_2} = \frac{df}{dx} x_1 = f'(x_1)
\]  

(5.13)

from which

\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}
\]  

(5.14)

The process is then repeated until it converges to the required root.

The Newton method was applied to Eq. 3.84, assuming an initial root equal to the value of \( c_e \) at the previous time row, \( j \). The procedure was successful, and convergence was attained in about cycles.
Having determined $c_e(i,j+1)$, the value of $u_e(i,j+1)$ was calculated by the $\alpha_e$-characteristic equation, Eq. 3.79, in the form

$$u_e(i,j+1) = u_e(i,j) - 2 \left[ c_e(i,j+1) - c_e(i,j) \right]$$

$$- \frac{2\Delta t}{\Delta x_n} \left\{ 2 \left[ u_e(i,j) + c_e(i,j) \right] c_e(i,j) \right. - c_e(i-1,j) \left. \right\} + \left[ u_e(i,j) + c_e(i,j) \right]$$

$$\left\{ \left[ u_e(i,j) - u_e(i-1,j) \right] + \Delta x_n g S_f(i,j)/2 \right\}$$

(5.15)

2. The elevation of the outcrop point and the outside water level were continuously compared until the maximum divergence between them was reached; thereafter the outcrop point started to drop according to the damped equation. For convenience, this phase was approximated, as a result of the experimentation with the fast drop model, by invoking half the maximum drop rate, $V_\star$. Thus,

$$\eta_n(i,j+1) = \eta_n(i,j) - 0.5 V_\star \Delta t$$

(5.16)

The procedure illustrated in the previous step was performed once more to determine the unknowns at the interface.
3. When the inside and outside water elevations coincided, another rising phase began and Eqs. 3.81 and 3.80 were then applied.

Throughout the solution, the non-linear friction terms, $S_f$ and $F$, along with the external and internal velocities and celerities were adjusted within each time increment as described for the internal models.

The reflection coefficient was obtained from the combined model by scanning the successive free water level profiles to locate the loops and nodes, whose heights were used to determine the reflection coefficient following the procedure given in Chapter IV.

A master flow chart, showing the order of calculations for the combined model, is presented in Fig. 5.4.

5.1.3 The Models for the Sloping Embankments

The necessary modifications in the mathematical models to solve sloping embankments were made according to the treatment proposed in section 3.9.

Preliminary runs of the computer models indicated that the best correlation with the experimental results is achieved if transformation of the sloping embankment to a rectangular one is based on the intersection of the mean water level with the slope. This can be argued on the basis that the entire wave action takes place about the mean water level. Sollitt's identical submerged volume, rectangular section (Fig.
3.10) tended to considerably underestimate transmission. It would appear that this is because Sollett's approximation exaggerates the extension of the rockfill into the external wave region which consequently results in overestimating the overall internal resistance. This, however, may have not significantly affected Sollett's results due to his small amplitude, almost sinusoidal waves. For the large amplitude, trochoidal shaped waves of this study, exaggeration of internal damping would be more critical. Hence, the sloping embankments in the present study were replaced by the rectangular section shown in Fig. 5.5 in compliance with the results of the experimentations with the mathematical models.

The equation proposed to account for wave breaking losses, Eq. 3.85, was evaluated using different experimental conditions to find the values of the constant, $C$, and the exponents, $l$, $n$ and $z$. Based on experimentation with the combined model, this equation takes the final form

$$C_b = C_t \left[ 1 - e^{-\frac{0.135}{\cos \theta D S^{0.5}}} \right]$$  \hspace{1cm} (5.17)

Also, based on experimentation with the models, the internal friction term, $F'$, (Eq. 3.87), was approximated by
\[ F' = \left( \frac{V_{\text{FH}}}{V_*} \right)^{0.5} \cdot F \quad \text{(for} \quad V_{\text{FH}} > V_* \text{)} \quad \text{(5.18)} \]

Since the rising velocity ranges between a minimum of zero to a maximum of \( \frac{2\pi A_0}{T} \sec \theta \) (see section 3.5.2), an average value for \( V_{\text{FH}} \) was calculated, based on half the maximum rising velocity, and used to find \( F' \) for each case treated. It is noted that in the appropriate internal model, \( V_{\text{FH}} \) was computed utilizing the amplitude, \( A_0 \), of the observed impact wave. In the combined model, the incident wave height was substituted instead of \( A_0 \) for the calculation of \( V_{\text{FH}}' \).

The adjustments obtained by Eqs. 5.17 and 5.18 were applied only to the half space increment adjacent to the interface in the outside and inside regions, respectively. The resistance values at the first column, prior to and after the interface, had to be balanced by a weighted average. Thus, (see Fig. 5.6)

\[ F(i+1) = \frac{F' + F}{2} \quad \text{(5.19)} \]

and

\[ C_t(i-1) = \frac{C_b\left( \frac{\Delta x_e}{\Delta x_n} \right) + C_t}{\frac{\Delta x_e}{\Delta x_n} + 1} \quad \text{(5.20)} \]

After a sloping embankment had been transformed to an equivalent rectangular section, the vertical
projections of the run-up and rush-down, measured in the laboratory, were programmed in the internal models as the impact wave height at the face of the transformed section. In the coupled solution, the combined model predicts the vertical impact wave at the face of the transformed section.

Apart from the above mentioned modifications, the procedures for the solution of the sloping embankments are identical to those described for the rectangular sections.

Listings of the computer models are given in Appendix C. The modifications for the inclusion of sloping embankments are indicated.

5.2 Verification

Fifteen different cases, covering most of the experimental range of variables for the rectangular embankments, were used to verify the combined model and, also, the appropriate internal model. Dimensionless transmission curves were obtained by plotting the local transmission coefficient, \( mH/H_1 \), versus the horizontal distance, \( X \), in the rockfill over the incident wave height, \( H_1 \). The theoretical and experimental transmission curves are presented in Figs. 5.7 to 5.21; there is good agreement in all cases. Also shown in the figures are the values of the run-up, impact wave height and reflection coefficient, \( C_R \), as
predicted by the combined model, and they conform satisfactorily with the corresponding experimental results.

Experimental phreatic line profiles, traced by means of a movie analyzer from the motion pictures, are presented in Figs. 5.22 to 5.25 for some typical, rectangular embankment cases. The corresponding profiles, computed by the mathematical models, are also compared to the experimental ones in the same figures. Each figure gives four different profiles that correspond to the maximum and minimum water elevations, both at the outcrop section and the impervious core.

The confirmation of the proposed numerical solutions for the sloping embankments was obtained utilizing twelve representative cases. The comparisons between the predicted and experimental results are illustrated in Figs. 5.26 to 5.37. The part of the experimental transmission curve marked "transformed section" should be discarded from the comparison, as it represents the actual results of the region, adjacent to the interface, in which the models are treating the slope as a vertical line. Good agreement, between theory and experiment, is characteristic of the sloping embankment results.

The numerical values were taken from the models, based on the results of the third wave period in order
to ensure compliance with the experimental measurements, which were recorded after the standing wave envelope had stabilized.

The predicted transmission coefficients at the core, $C_T$, are plotted against the observed ones in Fig. 5.38 for all the cases used for verification. A similar plot for the reflection coefficients is shown in Fig. 5.39. The balance of the scattered points about the $45^\circ$ line, in both figures, does not offer a concrete evidence as to whether the models consistently overestimate or underestimate transmission or reflection. The deviations noted between the predicted and observed values may be explained on the basis of the probable experimental errors and the limitations of the mathematical models which are discussed in the following two sections.

5.3 Experimental Errors

The sources of experimental errors in this study may be summarized as follows:

1. The errors associated with the determination of the friction coefficients, $a'$ and $b'$, from the flow-head loss experiments. The standard errors of the means in the values of $a'$ and $b'$ are given in table 4.1. The discharge for the steady flow permeameter was measured by means of a flow meter connected to a venturi-meter.
The calibration error for the venturi-meter is estimated at approximately ±2%; the flow meter could be read within an accuracy of ±1.0 USGPM. The local pressure might have been affected by the orientation of material particles around the entrance of piezometers at the wall of the permeameter. This would be more critical for the 4.4 cm rock which is liable to a more pronounced bridging effect than the other rock sizes.

2. The errors associated with the determination of in situ porosities. These would result from measuring rock weights and volumes. The weight was read within an accuracy of ±0.2 lbs. The volume of the rock was deduced by means of the experimentally determined specific weight, which was subjected to an estimated error of 1%. The maximum error in the calculation of the total volume of the rockfill embankment is estimated to be about ±2%.

3. The errors in measuring wave periods and celerities. The human response error in operating the stop watch is roughly ±0.5 sec. An additional error in obtaining celerities could have resulted from the estimation of the distance travelled by the wave. This error was about ±8 cm.
4. The errors associated with the manual readings of the unsteady water elevations. These are attributed to the speed at which markings of the water elevations on the plexiglass were made. The error in an individual reading was estimated to be ± 0.5 cm in the external region, and about ± 0.2 cm in the internal region. Moreover, due to water fluctuation in the external region, the readings are estimated to be within an accuracy of ±4%. Inside the rockfill, surface tension effects, more pronounced the smaller the rock size, caused the readings to be within an estimated accuracy of ±1.5%.

5. Interpretation Errors. These could have affected the reading of the run-up due to the interference of splashing in case of rectangular embankments, and the region of unsaturated flow encountered in some cases of the sloping sections. The error is estimated to be about ± 2 cm. A small error, of about 0.2 cm, might have resulted from neglecting the refraction of light through the plexiglass. This would tend to slightly overestimate the upper water elevation and underestimate the lower ones.

6. Errors in determining the incident and reflected wave heights. These errors may have
resulted due to the assumption of the validity of the node-loop technique for the non-linear, trochoidal shaped waves of this study. The technique was originally derived from the linear wave theory, and would yield the most accurate values for small amplitude, sinusoidal waves. The use of the loop-node technique may have underestimated the reflection coefficients in this study.

5.4 Errors and Limitations in the Mathematical Models

The errors and limitations in the mathematical models may be classified as follows:

1. Idealization Limitations

The input data for the models were taken from the experimental values which were subjected to the already discussed errors. Furthermore, it is assumed in the models that the physical system is homogeneous; this is only true in a statistical sense. The use of constant friction coefficients throughout the models is open to question. As discussed in chapter II, previous studies (38, 45, 72) indicated that the value of the non-Darcy term, b, may depend on the convergence and divergence of the macroscopic streamlines. Also, the Dupuit-Forchheimer assumption of parallel flow leads to a possible error resulting from approximating the gradient by
using the horizontal distance, $dx$, rather than the true inclined one. This would be more significant when the water level is at the maximum position of run-up because the internal water surface slope would then be steepest. When the wave is receding, however, the effect would not be as serious since the slope of the phreatic line is quite mild. As a remedy for this approximation, a restriction in the models was imposed to prevent the value of the gradient from exceeding unity. Nevertheless, it was found that that condition was never reached in any of the cases evaluated.

2. Discretization Errors

The mesh size used in discretizing the $x$-$t$ plane was examined by assigning different values to the space increment; $\Delta t$ was calculated according to the convergence and stability rule. The internal models were run for typical fast drop and slow drop cases using a 90.5 cm embankment. Each model was run twice with $\Delta x_n$ equal to first 18 cm and then 45 cm. The finer grid gave a slightly better agreement with the experimental results than the coarser one, but required longer execution time.

It was indicated earlier that the ratio between the time and space increments, in the
combined model, had to be reduced to about 1/5 to 1/10 of its theoretical value in the outside region in order to achieve consistent stability in both regions. It was also pointed out that the ratio was adjusted mainly by increasing $\Delta x_e$ until stability was attained. Experimentation with the combined model showed that very large values of $\Delta x_e$ relative to the wave length, L, tended to flatten out the external wave in some cases.

3. Computational Errors

Single precision accuracy was used in performing the computations with the associated truncation and round-off errors.

5.5 Comparisons With Other Investigations

The finite element solution developed by McCorquodale (40) for a rectangular rockfill embankment with an impervious core furnishes the most relevant comparison with the present study. The transmission curves obtained by the internal models and McCorquodale's finite element method are shown in Figs. 5.40 and 5.41 for slow and fast drop cases respectively. The two solutions are also compared to the corresponding experimental curves. There is good agreement among the three transmission curves. The finite element solution provides a better approximation for the two dimensional velocity pattern, while the characteristic-finite
difference models are capable of dealing with larger macroscopic inertial effects. The proposed internal models require considerably less computer storage and take less than one minute of execution time while the finite element model consumes about five minutes on an IBM 360/65 computer. Although the finite element method has never been applied to solve sloping embankments, it is a very powerful numerical technique for treating irregular geometries.

Some comparisons with the experimental results of Sollitt (61), Kondo and Toma (30) and Johnson et al (27) are presented in Figs. 5.42 to 5.44. In these comparisons it is assumed that the multiplication of the transmitted wave height at the core by the porosity would compensate for exit losses and further damping in the leeward side in case of continuous breakwaters. It is emphasized, however, that these comparisons should be regarded as approximate because:

a) the nature of the embankments tested is different from that of the breakwaters used in the other investigations due to the existence of the impervious cores in the present study;

b) only ranges are compared not identical values of rock sizes and embankment widths.
Considering these limitations, the present experimental study is in a reasonable conformity with the other investigations.
CHAPTER VI
DISCUSSION

6.1 Advantages of the Proposed Numerical Technique

From the foregoing presentations, the merits of the characteristic-finite difference technique, used in this study, can be stated as follows:

1. The phenomenon considered is governed by a hyperbolic system of equations; thus the method of characteristics, being an established powerful means of treating hyperbolic partial differential equations, was employed in developing the mathematical models. The characteristic directions furnish a general rule to establish convergence and stability if a finite difference scheme is utilized to advance the solution. The combination of the methods of characteristics and finite differences makes the computer modelling simpler than with the method of characteristics alone which requires more operations and, thus, might be more time consuming.
2. The proposed approach is capable of treating larger macroscopic inertial effects than the finite element method, which also consumes significantly more computer storage and time. Furthermore, the present models can deal directly with the non-linear terms in the governing equations; this offers a more realistic approximation than linearizing techniques.

3. The computer programmes are formulated in a general fashion that requires minimal input data, storage and time. This should help to encourage engineers to apply the proposed models.

6.2 Justification of Assumptions

The basic assumptions used in the mathematical formulations are supported by the good agreement found for the wide variety of verification cases.

Shallow water waves, which form the external part of the combined model, have many applications in actual cases. Since the shallow wave equation is based on the assumption of hydrostatic pressure distribution, allowances were made for possible centrifugal effects.

Normal incidence of waves at the embankments is very nearly true in reality as waves tend to
refract in shallow water and attack structures perpendicularly (43).

The two discontinuous sine functions, assumed to simulate the vertical displacement of the incident waves, provide a relatively simple representation of the complex trochoidal shaped waves, observed in the laboratory investigation. Excellent correlation has been found between this representation and the experimental measurements.

It appears that the restriction of the gradient (i.e. not to exceed unity) in addition to the inclusion of the vertical component of the macroscopic velocity in the conductivity function do compensate for the assumption of parallel flow.

The results of this study indicate that the use of the loop-node technique in both the physical and mathematical (combined) models is consistent. Nevertheless, it is believed that the technique may have underestimated the incident wave height and, more severely, underestimated the reflected wave height, thus leading to an overall underestimation of the reflection coefficients. Non-linear waves do not obey the principle of superposition; at a loop location, the measured height would be less than the actual value due to energy dissipation. Trochoidal shaped waves would possess more energy than sinusoidal waves, but, being
steeper, they also give rise to additional energy dissipation and more breaking losses. The two contradictory phenomena are partially compensating. Since the loop-node technique is a direct derivation from the linear wave theory, it is believed that for the wave heights measured in this study, apparent incident wave height and apparent reflected wave height are more appropriate terminology.

The inclusion of a wall correction factor in the values of the resistance coefficients is believed to have contributed towards improving the agreement between theory and experiment. It is well established by previous investigations (19, 38, 45) that the wall effect criterion should be taken into account for the results of different apparatuses, to be compared on a consistent basis.

It is obvious from these arguments, and the fact that the mathematical models have not consistently overestimated or underestimated the experimental results that the slight deviation between the predicted and observed values can be attributed to the experimental errors and the limitations in the mathematical models.
6.3. **Convergence and Stability**

The discretization of the $x-t$ plane for the application of the explicit finite difference scheme, shown in chapter III, was controlled by the characteristic directions. This ensures that the finite difference pattern lies within the region of dependence of the governing differential equations, which automatically guarantees convergence. As for stability, the criterion provided by the characteristic directions has been found to produce very stable solutions in case of the internal models. The same criterion was first applied in the formulation of the combined model, but a reduction in the ratio between the time and space increments was further required in order to achieve stability in the external region. Thus, the upper limit of the criterion (the equality of the left and right sides) had to be precluded: this indicates that the stability of the numerical solution in the external region is more sensitive. It is believed that this sensitivity is due mainly to physical reasons rather than mathematical implications. The short period waves, the absence of significant damping in the external region and the amplifications resulting from continuous reflection are potential causes. In addition, the use of an irregular grid, i.e. the smaller space increment adjacent to the interface, and the discretization of the smooth wave
form by straight lines could be contributing factors.

From the above discussion, it can be concluded that for the type of waves, dealt with in this study, stability is more restrictive than convergence. It can also be stated that for low friction cases, the boundary conditions would affect the stability of the solution, whereas for high damping cases the boundary conditions do not have as much significance. This argument is supported by the fact that the criterion provided by the characteristic directions leads to very stable solutions for the rockfill region, as well as for flood wave problems when treated by a similar numerical technique (41). Proofs for stability, using for example a Fourier series analysis (9, 50, 60) are only concerned with the internal equations and do not involve the effect of boundary conditions.

6.4 The "Pumping" Phenomenon and Seepage Face

The "pumping" type of effect, observed in the experiments, was also detected in the mathematical models (Fig. 4.33). The phenomenon is demonstrated by an upward shift of the mean water level indicating that the internal water level, during wave action, fluctuates about a mean level higher than the original one. This behaviour is thought to result from the trochoidal shaped waves, the "clapotis" effect and the existence of a seepage face in the falling phase.
The combined model indicated that a small seepage face could develop, for non-Darcy regimes, even in a slow drop case; this was also confirmed by experimental evidence and McCorquodale's finite element solution. This phenomenon, which is different from Dracos' findings for Darcy flow, may be attributed to the fact that the phreatic line, in non-Darcy flow, is not tangential to the rock face but meets it at about 45°. This is due to the centrifugal, inertial features in non-Darcy flow and the more pronounced wall effect (for coarse granular media) at the interfacial boundary. Therefore, the value of the maximum fall velocity, \( V_\text{f} \), was reduced by about 30% in the models, to allow for the apparent phenomenon.

6.5 Comments on the Experimental Study

The results of the laboratory investigation have been shown to be reliable by the comparisons with the results of other investigations (Figs. 5.42 to 5.44). These comparisons indicate that the agreement for transmission is slightly better than that for reflection. The possibility that the use of the loop-node technique may have underestimated reflection in the present study could be an underlying reason behind the discrepancy in reflection coefficients. Nevertheless, it is stressed again that the nature of
the experiments conducted is uniquely different from that of the previous ones.

The test flume was short, relative to some of the wave lengths used, and no filter was utilized in front of the wave machine. Yet, it was possible to make use of resonance, successfully, and to obtain reliable results.

A total of about 350 laboratory tests were conducted in order to furnish a sufficient sample. Because of the random nature of wave experiments, the tests were sorted, carefully, and only those reported in tables 4.2 to 4.16 were considered and the rest were rejected. A certain test was rejected when its results demonstrated an unstable pattern, regarding loop and node heights.

The empirical formulae for transmission, reflection, run-up and rush-down all gave good correlations. However, their validity is only guaranteed under similar conditions to those for which they were derived. The correlations for the sloping sections were as good as those obtained for the rectangular embankments. An independent empirical analysis for the sloping sections may lead to better representation of the parameters considered. This would probably be the case for the reflection formula,
since wave steepness was found to have some effect on the reflection in case of the sloping sections, while the reflection formula for rectangular embankments does not contain the steepness as a variable. It is surmised that the non-breaking waves, the energy dissipation resulting from the roughness of the reflecting surface, and the inclusion of very long waves (5 sec. up) in the empirical analysis for rectangular embankments could be among the inherent reasons for the wave steepness not displaying a defined effect on reflection. On the other hand, the decrease of reflection with an increase in wave steepness for sloping sections is attributed to the breaking of waves on the slope which increases with wave steepness giving rise to more energy loss. It is emphasized that applying a separate analysis to the data of the sloping sections would have not indicated the trend of the effect of slope on the results. This was achieved by imposing the same parameters used for the rectangular embankments on the sloping section analysis and determining the effect of the slope on the constants in the empirical formulae. The effect of other slopes may qualitatively be determined by evaluating Eq. 5.17 to detect whether a further increase or decrease in the constants of the empirical formulae is required corresponding to a different slope. Examination of this equation, under identical
conditions, indicates that a slope milder than the 2:1 slope used would necessitate a further decrease in the constants of Eqs. 4.20 to 4.23 and vice-versa for a steeper slope. It is noted that Eq. 5.17 is empirical and should only be considered valid for slightly different slopes from the one tested. Its applicability to other slopes must be confirmed by experiments.

6.6 Remarks on the Flow Regimes in Porous Media

The different Newtonian, saturated flow regimes in porous media were classified in chapter II. Many researchers have tried to establish an upper limit for the applicability of Darcy's law, but no consistent answers have been found. Most of the efforts have been directed towards representing this upper limit by a "critical Reynolds number". Subsequently, various definitions were suggested for a characteristic length in the Reynolds number expression. The discrepancies in the proposed values of the "critical Reynolds number" are beyond coincidence, as they range between 0.1 and 75. Therefore, unless turbulence measurements are associated with experiments, no firm distinction between the different regimes can be considered reliable. Although no such measurements were made in the laboratory study, it is assured that the ranges covered lie in the non-Darcy regimes.
6.7 **Practical Applications**

The proposed mathematical models, as presented, are strictly applicable to rockfill dams and shore protection structures with impervious cores.

The internal models are formulated to trace phreatic line profiles with time and determine wave transmission at any horizontal distance within the rockfill. These models are intended for use in conjunction with experiments, since the experimentally determined impact wave height is required as an input boundary condition. However, if experimental equipment is not available, and it is desired to apply these models for a given incident wave, two alternatives are suggested:

a) the values of the run-up and rush-down may be calculated by appropriate empirical formulae and programmed as the entrance boundary condition;

b) the impact wave height may be taken approximately as 1.5 to 2 times the incident wave height, based on the findings of this study and the standing wave phenomenon (58).

Any arbitrary time dependent function, describing the movement of the outcrop point, may be used as an entrance boundary condition in the internal models. Therefore, these models are not necessarily restricted to external shallow water waves.
The combined model, which is only valid under shallow wave conditions, provides a broader picture of the wave action as it also simulates the external wave motion. Knowledge of the incident wave information permits the direct application of this model to a given embankment to predict wave transmission, reflection, run-up and impact wave height. Moreover, the model yields the free water surface profiles with time as well as the phreatic line profiles.

In general, the proposed models treat the impervious core as being vertical, while in rockfill dams cores are built with different slopes for construction purposes. This, however, is not a serious deviation since most of the wave energy is dissipated within a very short distance from the rock-water interface. Thus, by the time the wave reaches the core it will have lost most of its kinetic energy so that the use of either a vertical or sloping core would not significantly alter the results. Also, the transmission value obtained by the use of a vertical core would be on the safer side as it would tend to be higher than that determined by utilizing a sloping core. It is recommended, for all practical purposes, that the inclined core be replaced by a vertical one, located at the position of intersection of the mean water level with the upstream slope of the core.
The computer models can be simplified to solve Darcy flow. In addition, the proposed technique can conveniently be adapted to simulate wave action before, inside and past rubble mound breakwaters. The combined model can also be modified to treat linear external waves.

Finally, the external part of the combined model can be separated and applied to the case of impervious breakwaters in order to determine impact wave height, run-up and reflection. At the conclusion of the experimental programme, some tests were carried out for a vertical plexiglass barrier. The data are given in Table 6.1; the YX and YM columns refer, respectively, to the maximum and minimum limits of the impact wave. It was, therefore, possible to verify the last application for three cases. A zero normal velocity boundary condition was assumed at the vertical barrier and the solution was performed as described in chapter V. Good agreement was found between the predicted and experimental values as illustrated in Table 6.2.
CHAPTER VII
CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

From the theoretical and experimental analyses presented, the following conclusions are made:

1. The combination of the methods of characteristics and finite differences provides an efficient numerical technique for simulating unsteady non-Darcy flow on digital computers. The computer modelling can be formulated in a general fashion that requires minimal storage, input data and run time.

2. Waves impinging on a rockfill embankment are classified as "fast" or "slow" dropping, depending on their drop rate relative to the maximum fall velocity in the rockfill. In non-Darcy flow, a small seepage face may develop, even in a slow drop case because of inertial and wall effects.

3. For sloping rockfill embankments subjected to water waves, a region of unsaturated flow
occurs below the position of maximum run-up when wave up-rush is faster than the internal seepage rate. This induces higher damping in the vicinity of the interface than in the rest of the embankment. The relative values of the up-rush and seepage rates provide a criterion for classifying waves running-up slopes, as fast or slow rising.

4. A sloping rockfill embankment can be treated by the characteristics-finite difference technique by transforming the embankment to an equivalent rectangular section whose upstream vertical face passes through the point of intersection of the slope with the mean water level; allowances for wave breaking losses and high internal damping near the interface should be incorporated in the solution.

5. The "clapotis" effect, trochoidal shaped waves and the existence of a seepage face tend to cause an upward shift of the mean water level inside the rockfill during wave action; this shift is on the average about 20% of the local wave height. The upward shift reaches a maximum at the face of the embankment and gradually diminishes towards the impervious core.
6. The empirical expressions for transmission, reflection, run-up and rush-down, Eqs. 4.17 to 4.20 and Eqs. 4.21 to 4.24, all gave good correlation and may be used to predict these parameters for similar conditions to those covered in the present study. The 2:1 slope of the embankment face causes the values of transmission, reflection and impact wave height to drop below their corresponding values in case of a vertical faced embankment.

7. Wave transmission, run-up and rush-down display pronounced reliance on wave steepness. Reflection from the rectangular embankments does not show any definite trend although reflection from the sloping sections exhibits a decreasing trend with increasing wave steepness.

8. Wave transmission to the impervious core increases with increasing conductivity and decreasing wave steepness and embankment width. Reflection from rockfill embankments with impervious cores decreases with increasing conductivity and width of embankment.
9. When a water wave attacks a rockfill structure, most of its energy is attenuated within a short distance (of the order of 5 to 10 particle diameters) from the entrance to the structure; this is more pronounced for steeper waves. The findings of the present study indicate, clearly, that the conservative assumption of the run-up on the core being the same as that on the embankment face is a wasteful exaggeration. This conclusion can be utilized to effect significant economical gains in designing rockfill structures.

7.2 Recommendations for Future Research

The present study gives rise to some interesting topics that warrant further attention. The following are a few potential future extensions:

1. The proposed empirical formulae should be improved by the results of more experiments and field data. In particular, experiments with various sloping embankments are required to determine, more accurately, the degree of influence of the angle of inclination. In the experiments, a means of evaluating the values of the incident and reflected wave heights, obtained by the loop-node method would provide better estimates of these
parameters for non-linear waves. This may be achieved if a very long channel is used to allow the incident wave to propagate over a sufficient length before the interference of reflection; consequently, the direct measurement of the incident wave height alone would be possible. Prototype confirmation would also be useful for supporting the mathematical models.

2. The role of unsaturated flow in the phenomenon of internal breaking, for fast rising waves, should be evaluated by a careful theoretical and experimental study. A special permeameter can be designed to investigate the phenomenon for different rock sizes, and a gamma ray apparatus, for example, can be used to measure in situ water contents. Also, a rigorous and detailed study to evaluate wave breaking losses on permeable slopes is a logical progression.

3. The analysis of the virtual mass factor for flow in porous media has never attracted researchers. A study to evaluate this factor, which has always been assumed equal to unity, may prove useful. The phenomenon can be
investigated, experimentally, in a two dimensional simulated porous medium for which velocity and turbulence are to be monitored utilizing, for instance, hot film and/or laser equipment.

4. The necessary modifications to adapt the proposed numerical technique to rockfill structures without impervious cores should be considered. Experimental as well as prototype data are to be collected for verification.

5. The benefits of the methods of characteristics and finite elements could be coupled in order to analyze wave action in layered and non-homogeneous porous media. This approach would be a more effective tool to deal with sloping embankments. Also, deep water waves could be treated by this technique.

6. Flow visualization experiments, using a birefringent fluid, have successfully been conducted, at the University of Windsor, to study shear patterns over cast ripple beds and for flow around obstructions. A similar study can be made to examine the shear patterns in the different regimes of flow through an idealized two dimensional porous medium.
7. Recently, Henry (25) devised a simulation of long water waves on an analog computer. He dealt with equations similar to the external wave equations used in this study, i.e. Eqs. 3.50 and 3.51. It may be feasible to adopt his method for the solution of the governing equations for non-Darcy flow.

8. For the fast drop model, the extension of the analysis of the outcrop point movement, using an asymmetric external wave, would provide a better approximation of this movement and a more accurate estimate of the seepage face.
APPENDIX A

FIGURES
Fig. 1.1. A Typical Unsteady Non-Darcy Flow Problem.
Fig. 3.1. Definition of the Problem for a Rectangular Embankment.
Fig. 3.2. Unit Thickness Flow Element Used in Developing The Governing Equations.
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Interface
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Fig. 4.10. Dimensions of the 1.7 cm Rock, Sloping Embankment.
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Max. & Min. Elevs.

Shifted M. W. L.

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Fig. 5.1. Layout for the Finite Difference Solution of the Damped Equation.
Fig. 5.2. Flow Chart for Internal Models.

Input Data

$t \rightarrow$ Initial Conditions

Test Phase

Outcrop B.C.: Rising

Outcrop B.C.: Fast or Slow drop

Cal. outcrop vel. ($u_n$)

Adjust $u_n$ & F

No

No

Calculate $u_n$ & $c_n$ for all internal points

Yes

Core B.C.: $u_n \rightarrow$

Calculate Core Celerity, $c_n$

Adjust $c_n$

Yes

Print Elevation

Prepare for Advancement, $t \rightarrow t + t$

Yes

IF(t-S1)
Fig. 5.3. Newton's Method.
Fig. 5.4. Master Flow Chart for Combined Model.

Input Data

$t = 0$; Initial Conditions

U/S B.C. $\Rightarrow c_e$; Calculate $u_e$

Calculate $c_e$ & $u_e$ for all Internal Points

Calculate $c_e$ at interface

Test Condition

Rising

$c_e \cdot c_n \cdot c_en;
\quad u_e \cdot u_n \cdot u_en$

Falling

Slow

Test Case

Fast
Test phase of outcrop point
Calculate \( Y_c \)

Calculate \( c_n \) & \( u_n \)

Calculate \( c_e \) (Cubic Eq.)
Calculate \( u_e \)

Calculate \( c_n \) and \( u_n \) at all internal points

Core B.C.; \( u_n = 0 \),
Calculate \( c_{II} \)

Print Elevation at all points

Prepare for advancement; \( t = t + \Delta t \)

IF \( t < 51 \)

Stop

adjust \( u_n, c_n, F \)

Yes

No
Fig. 5.5. Equivalent Rectangular Section for the Present Study.
Fig. 5.6. Adjustment of Resistance for Sloping Embankments.
Fig. 5.7. Verification Case No. 1 for Rectangular Embankments.
Fig. 5.8. Verification Case No. 2 for Rectangular Embankments.

\[ \frac{mH}{H_i} \]

- \( D = 4.4 \text{ cm}, \ B = 90.5 \text{ cm} \)
- \( h_0 = 30 \text{ cm}, \ S = 0.026 \)

<table>
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<tr>
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<th>THEO.</th>
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<td>( H_i ) (cm)</td>
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<tr>
<td>( R_u ) (cm)</td>
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</tr>
<tr>
<td>( C_R )</td>
<td>0.28</td>
<td>0.33</td>
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</table>

- **Combined Model**
- **Slow Drop Model**
- **Experiment**
Fig. 5.9. Verification Case No. 3 for Rectangular Embankments.
Fig. 5.10. Verification Case No. 4 for Rectangular Embankments.
D = 1.7 cm, B = 90.5 cm
h = 30 cm, S = 0.022

\[ \frac{mH}{H_i} \]

<table>
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<td>( H_i ) (cm)</td>
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<td>( R_u ) (cm)</td>
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<td>( C_R )</td>
<td>0.47</td>
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Fig. 5.11. Verification Case No. 5 for Rectangular Embankments.
Fig. 5.12. Verification Case No. 6 for Rectangular Embankments.

D = 1.7 cm, B = 90.5 cm

\( h_0 = 30 \text{ cm}, S = 0.015 \)

\[
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{THEO} & \text{EXP} \\
\hline
H_i \text{ (cm)} & 6.5 & 8.0 \\
R_u \text{ (cm)} & 4 & 6 \\
C_R & 0.43 & 0.46 \\
\hline
\end{array}
\]
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<th>23.0</th>
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<th>0.350</th>
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<td>Theo.</td>
<td>22.0</td>
<td>15.4</td>
<td>0.365</td>
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</tbody>
</table>

- Combined Model
- Fast Drop Model
- Experiment

**Fig. 5.13. Verification Case No. 7 for Rectangular Embankments.**

- $D = 17 \text{ cm}, B = 90.5 \text{ cm}$
- $h_0 = 38 \text{ cm}, s = 0.033$
Fig. 5.14. Verification Case No. 8 for Rectangular Embankments.
Fig. 5.15. Verification Case No. 9 for Rectangular Embankments.
### Fig. 5.16. Verification Case No. 10 for Rectangular Embankments.

<table>
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<tr>
<th></th>
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<tr>
<td>$H_i$ (cm)</td>
<td>11</td>
<td>13</td>
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<tr>
<td>$R_u$ (cm)</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>$C_R$</td>
<td>0.45</td>
<td>0.48</td>
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</table>

$D = 0.7 \text{ cm}$, $B = 49 \text{ cm}$

$h_o = 22 \text{ cm}$, $S = 0.17$
Fig. 5.17. Verification Case No. 11 for Rectangular Embankments.
Fig. 5.18. Verification Case No. 12 for Rectangular Embankments.
**Fig. 5.10. Verification Case No. 13 for Rectangular Embankments.**

<table>
<thead>
<tr>
<th>Model Type</th>
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<tr>
<td>crack width (cm)</td>
<td>22</td>
<td>24</td>
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<tr>
<td>crack depth (cm)</td>
<td>15.5</td>
<td>14.0</td>
</tr>
<tr>
<td>crack ratio</td>
<td>0.476</td>
<td>0.412</td>
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</table>

**Description:**
- **THEO.** refers to theoretical calculations.
- **EXP.** refers to experimental observations.
- **H1** (cm): crack width.
- **R_u** (cm): crack depth.
- **CR** ratio: crack ratio.

Additional information:
- **D** = 1.6 cm, **B** = 90.5 cm
- **h** = 38 cm, **s** = 0.03

The graph compares the theoretical and experimental results, with the x-axis representing **X/H1** and the y-axis representing **mH/H1**.
Fig. 5.20. Verification Case No. 14 for Rectangular Embankments.
Fig. 5.21. Verification case No. 1.5 for Rectangular Embankments.

EXP.

THEO.

\[ \frac{mH}{H_1} \]

\begin{tabular}{cccc}
H \_1 (cm) & 155 & 12.5 & 0.50 \\
R \_u (cm) & 16.0 & 11.8 & 0.49 \\
C \_R & & & \\
\end{tabular}

Combined & Fast Drop Models

Experiment

\[ D = 16 \text{ cm}, B = 50.5 \text{ cm} \]

\[ h_0 = 30 \text{ cm}, s = 0.027 \]
Fig. 5.22. Experimental and Theoretical Phreatic Line Profiles for a 4.4 cm Rock Embankment.
Fig. 5.23. Experimental and Theoretical Phreatic Line Profiles for a 1.7 cm Rock Embankment.
Fig. 5.24. Experimental and Theoretical Phreatic Line Profiles for a 0.7 cm Rock Embankment.
**FAST DROP**

\[ S = 0.027, \quad B = 50.5 \text{ cm} \]

\[ h_0 = 30 \text{ cm} \]

**Fig. 5.25.** Experimental and Theoretical Phreatic Line Profiles for a 1.6 cm Quartz Embankment.
Fig. 5.26. Verification Case No. 1 for Sloping Embankments.
Fig. 5.27. Verification Case No. 2 for Sloping Embankments.
Fig. 5.28. Verification Case No. 3 for Sloping Embankments.

The diagram shows a comparison between theoretical (THEO) and experimental (EXP) data points for various parameters. The parameters include:

- \( H_1 \) (cm)
- \( R_\text{u} \) (cm)
- \( C_R \)

Theoretical values are:
- \( H_1 = 19.0 \) cm
- \( R_\text{u} = 16.0 \) cm
- \( C_R = 0.378 \)

Experimental values are:
- \( H_1 = 19.2 \) cm
- \( R_\text{u} = 14.2 \) cm
- \( C_R = 0.395 \)

The diagram includes lines for the combined model, fast drop model, and experiment, illustrating the comparison between them.
Fig. 5.29. Verification Case No. 4 for Sloping Embankments.

$\frac{mH}{H_i}$ vs. $\frac{X}{H_i}$

<table>
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<td>$R_u$ (cm)</td>
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<tr>
<td>$C_R$</td>
<td>0.36</td>
<td>0.34</td>
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</tbody>
</table>

$D = 1.7 \text{ cm}$, $h = 22 \text{ cm}$

$S = 0.017$

- Combined Model
- Fast Drop Model
- Experiment
Fig. 5.30. Verification Case No. 5 for Sloping Embankments.
D = 1.7 cm, \( h_0 = 30 \) cm
\[ S = 0.016 \]

<table>
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<th>Theo.</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_1 ) (cm)</td>
<td>12.0</td>
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</tr>
<tr>
<td>( R_u ) (cm)</td>
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<td>9.5</td>
</tr>
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<td>( C_R )</td>
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- Combined Model
- Slow Drop Model
- Experiment
Fig. 5.32. Verification Case No. 7 for Sloping Embankments.
Fig. 5.33. Verification Case No. 8 for Sloping Embankments.
Fig. 5.34. Verification Case No. 9 for Sloping Embankments.
Fig. 5.35. Verification Case No. 10 for Sloping Embankments.
D = 1.6 cm, h₀ = 38 cm
S = 0.024

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Fig. 5.36. Verification Case No. 11 for Sloping Embankments.
Fig. 5.37. Verification Case No. 12 for Sloping Embankments.
Fig. 5.38. Comparison of the Predicted and Observed Transmission at the Core.
Fig. 5.39. Comparison of the Predicted and Observed Reflection.
Fig. 5.40. Comparison Between the Finite Element Method and the Proposed Technique for a Slow Drop Case.

H/2A₀

X/2A₀

D = 4.4 cm
B = 122 cm
S = 0.02
Fig. 5.41. Comparison between the Finite Element Method and the Proposed Technique for a Fast Drop Case.

- -- Charac.- Fin. Diff. Solution
- - - Finite Element Solution
- - - Experiment

\[ D = 1.6 \text{ cm} \]
\[ B = 90.5 \text{ cm} \]
\[ S = 0.03 \]
Fig. 5.42a. Comparison with Sollitt's Results for Transmission.
Fig. 5.42b. Comparison With Sollitt's Results for Reflection.
Fig. 5.43a. Comparison With the Results of Kondo and Toma for Transmission.
Fig. 5.43b. Comparison With the Results of Kondo and Toma for Reflection.
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<td>16.5 &amp; 50.5</td>
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Fig. 5.44. Comparison of Transmission Coefficients With the Results of Johnson et al.
APPENDIX B

TABLES
TABLE 4.1

PHYSICAL AND HYDRAULIC PROPERTIES OF THE TEST MATERIALS

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<th>( a_s ) sec/cm</th>
<th>( a' ) (sec/cm)</th>
<th>( b' ) (sec/cm)(^2)</th>
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<td>2.59</td>
<td>9.4 ± 0.09</td>
<td>0.033 ± 0.0028</td>
<td>0.0034 ± 0.0004</td>
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<td>*1.7 ± 0.04</td>
<td>2.62</td>
<td>11.0 ± 0.13</td>
<td>0.0081 ± 0.0033</td>
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<tr>
<td>*0.7 ± 0.03</td>
<td>2.58</td>
<td>13.6 ± 0.33</td>
<td>0.0163 ± 0.0064</td>
<td>0.0316 ± 0.0014</td>
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<tr>
<td>*1.6 ± 0.04</td>
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<td>0.0092 ± 0.0037</td>
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* Crushed Rock

* Rounded Quartz
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| 38.00 | 1.30 | 2.07 | 12.50 | 5.50 |
| \(X\) | 7.0 | 9.5 | 20.5 | 40.5 | 84.5 |
| \(YX\) | 42.5 | 42.5 | 43.5 | 41.0 | 40.0 |
| \(YM\) | 35.0 | 33.0 | 34.0 | 37.5 | 37.5 |

| 39.00 | 1.30 | 3.29 | 6.25 | 2.75 |
| \(X\) | 7.0 | 9.5 | 20.5 | 40.5 | 84.5 |
| \(YX\) | 42.5 | 42.5 | 41.5 | 41.0 | 40.0 |
| \(YM\) | 35.0 | 33.0 | 34.5 | 37.5 | 37.5 |

| 38.00 | 1.29 | 1.83 | 11.00 | 3.50 |
| \(X\) | 7.0 | 9.5 | 20.5 | 41.5 | 24.5 |
| \(YX\) | 54.0 | 48.0 | 44.0 | 41.0 | 41.0 |
| \(YM\) | 31.0 | 31.0 | 33.0 | 36.0 | 36.0 |

| 34.00 | 1.40 | 6.19 | 23.00 | 5.50 |
| \(X\) | 5.0 | 9.5 | 20.5 | 40.5 | 84.5 |
| \(YX\) | 54.0 | 31.0 | 43.0 | 42.0 | 41.0 |
| \(YM\) | 31.0 | 31.0 | 33.0 | 34.0 | 34.0 |

| 38.00 | 5.00 | 9.15 | 16.00 | 5.00 |
| \(X\) | 7.0 | 9.5 | 20.5 | 40.5 | 84.5 |
| \(YX\) | 51.0 | 51.0 | 43.0 | 43.0 | 43.0 |
| \(YM\) | 31.0 | 31.0 | 32.0 | 32.0 | 32.0 |

<p>| 36.00 | 5.20 | 4.45 | 25.50 | 8.50 |
| (X) | 0.0 | 9.5 | 20.5 | 40.5 | 84.5 |
| (YX) | 70.0 | 50.0 | 48.0 | 46.0 | 45.0 |
| (YM) | 70.0 | 27.0 | 27.0 | 30.0 | 30.0 |</p>
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$x = 2.0$ | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |

$y = 22.00$ | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 | 22.00 |


$y = 3.0$ | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 |

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$y = 4.0$ | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |


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$y = 33.5$ | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 | 33.5 |

$y = 27.5$ | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 | 27.5 |
TABLE 4.4 (Continued)

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Table 4.5

Results of the 1.7 cm of CX, 52.0 cm rectangular embankment

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## RESULTS OF THE 1.76 & 14.5 Cm RECTANGULAR TANKMENT

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### Table 4.7

**RESULTS OF THE 2.7 CM PUCK ON L OBLONG EMBANKMENT**

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# RESULTS OF THE 0.7 CM CYCLOK 21.0 CM RECTANGULAR EMBANKMENT

(M = 0.487)

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### Table 4.9

**Results for the 3.7 m Nickel, 7.0 m Cylinder Embankment**

\( m = 0.480 \)

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**TABLE 4.10**

**RESULTS OF THE L.6 cm QUARTZ, 20.5 cm RECTANGULAR LUMINANCE (M = 0.376)**
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For each row, the values in the table represent the properties of a material or substance under different conditions, with \( n \) indicating the number, \( T \) the temperature, \( L/100 \) the length per 100 units, and \( H_0 \) and \( H_1 \) possibly indicating different material properties or measurements. The values are given in a structured format, likely for analysis or recording purposes in a scientific context.
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RESULTS OF THE 1.7 CM TRY, SLIDING EMBANKMENT
(m = 0.42)

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<th>hᵣ</th>
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| 22.00 | 7.46 | 5.28 | 12.00 | 4.00 |
| x = 7.0 | 3.0 | 90.0 | 140.0 | 164.0 |
| yₓ = - | - | 4.0 | 25.0 | 24.0 | 23.9 |
| yᵧ = - | 20.0 | 21.8 | 22.9 | 22.8 | 22.8 |

| 27.00 | 1.16 | 4.82 | 14.00 | 5.00 |
| x = 9.0 | 12.0 | 54.0 | 100.0 | 140.0 | 164.0 |
| yₓ = - | - | 24.2 | 27.0 | 24.8 | 24.2 | 24.3 |
| yᵧ = - | 17.0 | 21.9 | 22.0 | 22.7 | 23.2 | 23.3 |

| 27.00 | 2.16 | 4.54 | 10.50 | 3.50 |
| x = 9.0 | 35.0 | 52.0 | 100.0 | 140.0 | 164.0 |
| yₓ = - | - | 33.0 | 25.0 | 24.8 | 24.0 | 23.9 |
| yᵧ = - | 16.0 | 29.5 | 21.5 | 22.0 | 22.4 | 22.3 |

| 27.00 | 1.12 | 1.66 | 11.00 | 4.00 |
| x = 9.0 | 3.0 | 52.0 | 100.0 | 140.0 | 164.0 |
| yₓ = - | - | 33.0 | 25.0 | 24.8 | 24.0 | 23.9 |
| yᵧ = - | 17.0 | 27.0 | 23.5 | 23.0 | 23.8 | 23.7 |

| 22.00 | 2.62 | 4.03 | 3.10 | 1.50 |
| x = 9.0 | 4.0 | 41.0 | 100.0 | 140.0 | 164.0 |
| yₓ = - | - | 26.5 | 24.0 | 22.0 | 22.7 | 22.8 |
| yᵧ = - | 23.5 | 22.0 | 27.0 | 22.0 | 22.0 |

| 39.00 | 1.32 | 2.13 | 13.30 | 5.30 |
| x = 0.0 | 50.0 | 61.0 | 92.0 | 120.0 | 154.0 |
| yₓ = - | - | 44.2 | 36.0 | 34.7 | 34.7 |
| yᵧ = - | 25.0 | 31.0 | 22.0 | 22.5 | 33.0 |

<p>| 39.00 | 1.75 | 7.53 | 6.50 | 2.00 |
| x = 0.0 | 54.0 | 62.0 | 75.0 | 89.0 | 120.0 | 154.0 |
| yₓ = - | - | 37.0 | 32.7 | 31.6 | 31.6 |
| yᵧ = - | 20.7 | 29.0 | 21.7 | 20.7 | 30.7 | 30.5 |</p>
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<th>5.69</th>
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<th>6.00</th>
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<td>2.0</td>
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<td>yX =</td>
<td>-</td>
<td>-</td>
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</tr>
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<td>yM =</td>
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<td>23.0</td>
<td>31.0</td>
<td>31.4</td>
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<th>5.69</th>
<th>17.00</th>
<th>6.00</th>
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<td>7.00</td>
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<td>yX =</td>
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<td>-</td>
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<td>3.50</td>
</tr>
<tr>
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<td>12.0</td>
<td>23.0</td>
<td>31.0</td>
<td>31.4</td>
</tr>
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<tr>
<th>30.00</th>
<th>3.00</th>
<th>5.69</th>
<th>17.00</th>
<th>6.00</th>
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<td>2.0</td>
<td>3.00</td>
<td>7.00</td>
<td>10.00</td>
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<td>-</td>
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<td>23.0</td>
<td>31.0</td>
<td>31.4</td>
</tr>
<tr>
<td>$h_0$</td>
<td>$T$</td>
<td>$L/1000$</td>
<td>$H^r$</td>
<td></td>
</tr>
<tr>
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<td>-----</td>
<td>----------</td>
<td>-------</td>
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</tr>
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<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
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<td>19.5</td>
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<td>72.0</td>
<td>84.0</td>
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<td>25.7</td>
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<tr>
<td>22.00</td>
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<td>5.79</td>
<td>9.83</td>
<td>13.00</td>
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<td>25.0</td>
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<td>$Y_M$ = -</td>
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<td>29.5</td>
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<tr>
<td>22.00</td>
<td>3.42</td>
<td>5.79</td>
<td>9.83</td>
<td>13.00</td>
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<tr>
<td>$X$ = 9.0</td>
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<td>52.0</td>
<td>76.0</td>
<td>84.0</td>
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<td>-</td>
<td>-</td>
<td>33.0</td>
<td>24.5</td>
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<td>$Y_M$ = -</td>
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<td>22.9</td>
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<td>$Y_M$ = -</td>
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<td>19.2</td>
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<td>22.0</td>
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<tr>
<td>Material</td>
<td>a (sec/cm)</td>
<td>b (sec/cm)^2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>------------</td>
<td>---------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.4 cm Rock</td>
<td>0.004</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.7 cm Rock</td>
<td>0.009</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7 cm Rock</td>
<td>0.017</td>
<td>0.033</td>
<td></td>
<td></td>
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<tr>
<td>1.6 cm Quartz</td>
<td>0.010</td>
<td>0.021</td>
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<td>TEST NO</td>
<td>$h_0$</td>
<td>$T$</td>
<td>$L/100$</td>
<td>$w_1$</td>
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<tr>
<td>---------</td>
<td>-------</td>
<td>-----</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>1</td>
<td>38.0</td>
<td>5.74</td>
<td>11.6</td>
<td>13.0</td>
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<tr>
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<td>38.0</td>
<td>5.15</td>
<td>9.5</td>
<td>14.0</td>
</tr>
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<td>3</td>
<td>38.0</td>
<td>2.64</td>
<td>4.4</td>
<td>5.0</td>
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<tr>
<td>4</td>
<td>38.0</td>
<td>1.83</td>
<td>3.1</td>
<td>4.9</td>
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<tr>
<td>5</td>
<td>30.0</td>
<td>2.2</td>
<td>3.7</td>
<td>17.0</td>
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<tr>
<td>6</td>
<td>30.0</td>
<td>4.19</td>
<td>7.1</td>
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<td>30.0</td>
<td>5.50</td>
<td>7.7</td>
<td>24.0</td>
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<tr>
<td>8</td>
<td>22.0</td>
<td>4.45</td>
<td>7.0</td>
<td>15.0</td>
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<tr>
<td>9</td>
<td>22.0</td>
<td>2.52</td>
<td>4.0</td>
<td>3.8</td>
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</tbody>
</table>

All length dimensions are in centimeters.
Wave periods are in seconds.
### TABLE 6.2

**Verification for the Solid Vertical Barrier**

<table>
<thead>
<tr>
<th>Case No. (From Table 6.1)</th>
<th>Impact Wave Height (cm)</th>
<th>Run-Up (cm)</th>
<th>Reflection Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Observed: 21.5</td>
<td>16.5</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>Predicted: 19.5</td>
<td>14.0</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>Observed: 30.0</td>
<td>22.0</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>Predicted: 30.2</td>
<td>21.0</td>
<td>0.48</td>
</tr>
<tr>
<td>8</td>
<td>Observed: 17.0</td>
<td>12.0</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>Predicted: 18.5</td>
<td>12.4</td>
<td>0.64</td>
</tr>
</tbody>
</table>
APPENDIX C

COMPUTER PROGRAMMES
A, B = friction coefficients in the Ergun's equation
DP = DP
I = number of internal points in X-direction
H = mean water level
H1 = impact wave height
I = I = wave position
NT = time increment
UX = space increment
UX = maximum internal velocity
UX = horizontal macroscopic velocity
U = vertical macroscopic velocity
G = friction term
C = gravity
W = water elevation above bottom

DIMENSION U(11), X(11), U(11), C(11), H(11), Z(30), W(10)
REAL VC
VMAX = 0
DT = 0.05
A = 0.001
B = 0.016
AP = 0.413
CC = 1.0
FF = 1.0
H1 = 1.0
W1 = 3.0
UX = (SOM(1/2) + 2.0) - A/2.0
G = 9.81
J = IVMAX = 1
NX = 29.5
TP = 0.73
TDL = 0.5
PD = 1.44
KK = 0
F = 0.0
D = 1.0
T = 1.0

C INITIAL CONDITIONS
1 1 = 1, IVMAX
U(1) = 0.0
C(1) = SOM(TC)(G#HC)
H(1) = H1
1 CONTINUE
PRINT 20
7 FORMAT (15X, 7 (IV = 1), X, 6H, T = 
2 FORMAT (7X, 11E12.5)
24 T = T + 0.1
KJ = KJ + 1
G/S = C
U(1) = (IVMAX) = 7.4
JL = ?
G = 0.75 + (IVMAX) + 0.25 = (J)
4000 C(I^+X) = C(I^XX) - (2*67*M/N)*C(I*M) + (5.3X(I) - UI(J))/AP^2
X = CP(C(I^+X) - C(J))
W(IJ) = CP(I^+X)
F(IJ) = 0.5*U(IJ) + 0.05*(UI(JJ))
JJ = 1 + I
IF(UJ = 5) 400 C,J,CG,C5000
5000 CONTINUE

C INTERNAL POINT

10 11 I = 2 + J
20 *C = 0.5*C(I) + 0.5*(C(I+1) + C(I-1))
30 F = 0.5*U(I) + 0.5*(U(I+1) + U(I-1))
900 U(I) = C(I) - (U(I)/X*(A+P)) + (5.3X(C(I+1) - U(I-1)) + EII*(C(I+1) - C(I-1)))
900 V(I) = A11 + U(I)/2 - (C(I)*S) + S*N*T(2,0)
900 IF(V(I),2) GOTO 900
900 G = S*N*T(V(I)*2 + EII ** 2)
900 F = (A + N**2)*EII

6000 U(I) = U(I1) - (U(I)/X)*F*U(I1) + U(I-1))/AP +
X 2.3*AP + CCI=1 - C(I-1) + S = 0*X*AP +

50 = SQRT(V(I) + U(I))
20 = SQRT(V(I) + U(I))
70 = SQRT(V(I) + U(I))
F(I) = (F + T(K))/2.0
20 *C = 0.5*C(I) + 0.5*C(I)
20 EII = (EII + U(I))/2.3
1000 CONTINUE

11 CONTINUE

C VCR

30 I = (T - 0.5*T(I))/2.0
51 T(I) = T(I) + EII
50 T(I) = T(I) + EII
50 T(I) = T(I) + EII

30 CONTINUE

50 C(I) = SQRT(3**I)
X = 2.3*(C(I) + 2 - C(I))**2
0 = SQRT(V(N) + U(I))
F = 0.5*U(I)
50 = 0.15**2(I) + 0.25**2(I)

2000 U(I1) = U(I1) + 2.0*(C(I) - C(I1)) - 2.0*(U(I1)/X) +
X*(U(I2) - U(I1)) + C(I1)/2 - 2.0*(C(I) - C(I)) + EII)/AP-C(I1)*
X(I1) + 2.3*AP + 5.3X(I1)**2
900 IF(HS(UX) = AS(6,1),1) 60,70,70
60 IF(U1(I) = UC1) 80,90
80 U1(I) = UC
70 CONTINUE

90 U1(I) = U
70 CONTINUE
Q = SQR(T(V+2 + U*R(1)**2))
Z(K) = (A + 0.0*5)*R(1)
F = (F+2.0)/(2.0)
E = (EU + U*R(1))/2.0

J = K+1
IF(K=15)2000,2000,3000

3000 CONTINUE
DO 100 I=1,I'MAX
100 W(I) = (CR(I)**2)/G

C ~PRINT SOLUTION
IF((K1+1)/50)2010,K1) 600,601,601

601 PRINT 21
21 FORMAT (/I)
PRINT 22,7,17
22 FORMAT (15X,3HT,=,F8.3,5X,2HT,=,F8.3)
PRINT 13, (W(I), I=1,IMAX)
13 FORMAT (7X,11F9.2)

600 CONTINUE
C PREPARE FOR ADVANCEMENT
DO 19 I=1,I'MAX
C(I) = F(I)
U(I) = UR(I)
19 CONTINUE

C CONTROL TIME FOR N/S & C.
IF(T-F*IP) 43,49,49
49 E = E+1.
EF = TE+1.
48 CONTINUE
IF(T=5*IP) 24,55,55
55 STOP

END
C INITIAL CONDITIONS
ND 1, I=1, IMAX
UI(I)=0.0
C(I)=SIN(T/GA)
HN(I)=H0
1 CONTINUE
PRINT 20
20 FORMAT (15X,7H0T=0.00,5X,6HT=0.00)
PRINT 20,(HN(I),I=1,1,IMAX)
2 FORMAT (7X,11F8.2)
KXX=1
TCQ=IP
F0=0.0
24 T=T+NT
C N/S 9.0
UP(I)=0.0
ER=C(I)
4000 C=IMAX = C(I*IMAX) - (2.*T/NX) * (5.*E*E*(-U(J))/AP +
X *AP*CG1*CG(I*IMAX) - C(I))
C INTERNAL POINTS
50 I=1, J
K=0
MC=C(I)
F=U(I)
900 C(I)= C(I) - (CT/10*N*AP) * (5*M*C*(U(I+1) - U(I-1)) + E*E*(C(I+1) -
X C(I-1)))/
V = AP*CG(I)*2 - C(I)*2)/(G*AP*T*SQRT(2.0))
IF(K,KT,2) GO TO 6000
IF(V,2*M/F + V)*2
F = (A + F)*F
6000 IF(I,LN,C,J, 1, T) 6001
UI(I) = UI(I) - (CT/10*X)*(E*E*(U(I+1) - U(I-1))/AP +
X *AP*CG1*MC*(C(I+1) - C(I-1)) + G*NX*AP*F)
GO TO 6002
6001 UI(I) = UI(I) - (CT/10*X)*(E*E*(U(I+1) - U(I-1))/AP +
X *AP*CG1*MC*(C(I+1) - C(I-1)) + G*NX*AP*F)
6002 OR = SP*T(W*W2 + UI(I)*W2)
Z = (AP*OR)/UI(I)
F = 0.5*(F+Z)
MC = (MC + CG(I))/2.0
FU = (FU + UI(I))/2.0
950 K = K + 1
IF(K, 900,9)0,1000
1000 CONTINUE
11 CNV IFUE
C N/S 9.0
T=1-(I+0.25)*P*T11)) S0, S0, S1
51 T=1-(E+0.75)*P*T11)) S2, S2, S3
53 T=T-TCQN) 54, 54, 57
50 W=MC*W+AIN(6.2832*T/TP)
GO TO 56
57 \( H = H + A \cdot \sin(6.2832 \cdot (1 + 0.25 \cdot TP + T1) / TP) - AK \cdot (T - ((E + 0.25) \cdot TP + T1)) \)
58 \( 764 \text{ to } 56 \)
59 \( H = H + A \cdot \sin(6.2832 \cdot (2.5 \cdot TP + T1) / TP) - AK \cdot (0.5 \cdot TP - 2.5 \cdot T1) \)
60 \( X = 0.5 \cdot AK \cdot (T - ((E + 0.75) \cdot TP - T1)) \)
61 \( \text{KKK = KKK + 1} \)
62 \( \text{PRINT 305, T} \)
63 \( \text{CALL DUMP} \)
64 \( \text{PRINT 305, T} \)
65 \( \text{305 FORMAT}(2X, FR. 9) \)
66 \( \text{HFO = H0 + A0 \cdot \sin(6.2832 \cdot T / TP)} \)
67 \( \text{PRINT 306, HELD, H} \)
68 \( \text{306 FORMAT}(2X, 4NH, 7E-10, 3X, 2NH, 7E-10) \)
69 \( \text{IF(H0 = H) 56, 150, 150} \)
70 \( \text{TCON = T} \)
71 \( \text{301 FORMAT}(2X, FR. 5) \)
72 \( \text{GO TO 56} \)
73 \( \text{57 IF(T - (T + 0.0) \cdot TP) 61, 61, 55} \)
74 \( \text{61 H = H + A0 \cdot \sin(0.2332 \cdot T / TP)} \)
75 \( \text{56 CR(1) = SQRT(G \cdot HI)} \)
76 \( K = 0 \)
77 \( \text{AC = 0.5 \cdot (C(1) \cdot CR(1))} \)
78 \( \text{V = AP \cdot (CR(1) \cdot SP2 - C(1) \cdot SP2) / (C \cdot \text{NT} \cdot \text{SQRT(2.0)})} \)
79 \( \text{Q = SQRT(V \cdot SP2 + U(1) \cdot SP2)} \)
80 \( \text{F = (V + R \cdot SP0) \cdot U(1)} \)
81 \( \text{EU = U(1)} \)
82 \( 2000 \text{ UP(1) = U(1) + 2.0 \cdot (C(1) - C(1)) + 2.0 \cdot (\text{FT} / \text{DT})} \)
83 \( \text{x1 = (U(1) - U(1)) \cdot \text{EU}} \)
84 \( \text{x2 = A \cdot \text{AP} \cdot U(1) \cdot SP2 + 0.5 \cdot P \cdot C(2) \cdot SP2 \cdot (0.5 \cdot max)} \)
85 \( \text{IF(x2 > x1) 60, 70, 70} \)
86 \( \text{60 IF(x1 < 100) 80, 80, 90} \)
87 \( \text{80 UP(1) = -Ux} \)
88 \( \text{GO TO 70} \)
89 \( \text{90 UF(1) = Ux} \)
90 \( \text{70 CONTINUE} \)
91 \( \text{QX = SQRT(V \cdot SP2 + UR(1) \cdot SP2)} \)
92 \( \text{Z = (A + R \cdot QF) \cdot U(1)} \)
93 \( \text{F = 0.5 \cdot (SP2 + Z)} \)
94 \( \text{EU = EU + UR(1) / 2.0} \)
95 \( 2500 \text{ K = K + 1} \)
96 \( \text{IF(K < 3) 2000, 3000, 3000} \)
97 \( \text{3000 CONTINUE} \)
98 \( \text{NO 100 I = L, I \times X} \)
99 \( \text{100 HN(T) = (CR(1) \times SP2) / G} \)
100 \( \text{C PRINT SOLUTION} \)
101 \( \text{PRINT 21} \)
102 \( \text{21 FORMAT}(//) \)
103 \( \text{PRINT 22, NT, T} \)
104 \( \text{22 FORMAT}(15X, 3NH, T = FR. 3, 5X, 2NH, T = FR. 3) \)
105 \( \text{PRINT 13, (H0(I), I = 1, I \times X)} \)
106 \( \text{13 FORMAT}(7X, 11FR. 2) \)
C PREPARE FOR ADVANCEMENT

Do 19 I=1,IMAX
C(I)=CR(I)
U(I)=UR(I)
19 CONTINUE
IF(T-((E+1.0)*TP)+0.0005) 160,161,161
161 E=E+1.0
TPE=(E+1.0)*TP
PRINT 700,T,TPE
700 FORMAT(2X,2HT=',F15.10,,F15.10')
KKK=0.0
TCON=T+TP
PRINT 162
162 FORMAT(5X,10HT,'L1CHF:D')
160 CONTINUE
IF(T-5.0*TP) 164,55,55
164 CONTINUE
GO TO 24
55 PRINT 600
600 FORMAT(12X,13HT,'EXCEPTED')
STOP
END
SUBROUTINE DAMP

C FINITE DIFFERENCE SOLUTION TO DAMPING EQUATION
COMMON AO, TP, TL, AK, T, GT RET, MM, NT, GD, TP, GT, HK, E, CC
CC=3.6*CC
D7=2.*AO*5IN(6.2832*(0.25*TP+T1)/TP)-AK*(0.5*TP-2.*T1)
PRINT 25, D7, T, KK
25 FORMAT(2X, 2F8.3, 1X)
GAMA = (G*RET)/U20
FT=GAMA*(CC*T)*5IN(6.2832*T/TP))
IF(KKK=2) 15, 15, 16
16 IF(KKK=3) 17, 17, 18
15 TIN=0.75+(1.*TP-T1)
D7T=T-T1
PRINT 30, D7T
30 FORMAT(2X, 4D8.3, 1X)
D7T=-AK.
ZT=HT+D7T*5IN(6.2832*(0.25*TP+T1)/TP)-AK*(0.5*TP-2.*T1)
ZTM=ZT-ZTM/T0
ZD7T=GAMA*(H)*L*5IN(6.2832*T/TP))=G*AP*(Z+3*AP+B)(ZD7T)*1.4
XCC =7D7T-GAMA*Z7T
ZTM=ZD7T*Z7T*2-ZTM*1+2*Z7
PRINT 26, Z7, ZTM, Z7T, ZD7T, ZTM1
26 FORMAT(2X, 4F10.5)
H=ZTM1
ZTM1=ZT
ZT=ZTM1
GO TO 19
17 D7T=1./DDT
X1=1.0/((1.0/(ZT+1))+(GAMA *DDT/DDT))
X2=1.0/(ZT+1)+(GAMA *DDT/DDT)
X4=1.0/((ZT-ZTM1)/DDT)*1.4/((ZT-ZTM1)/DDT)*1.4
7 TP1=X1*(X2*X3+X4)
PRINT 27, ZTP1
27 FORMAT(2X, 4F10.5)
H=ZTP1
ZTM1=ZT
ZT=ZTP1
GO TO 19
18 D7T=1./(ZT+1)
Y1=1.0/((1.0/(ZT+1))+(GAMA *ZT))
Y2=1.0/(ZT+1)+(GAMA *ZT)
Y4=1.0/((ZT-ZTM1)/DDT)*1.4/((ZT-ZTM1)/DDT)*1.4
Y4=-(GAMA *ZTM1/2.)*ET
ZTP1=Y1*(Y3+Y4)
PRINT 28, ZTP1
28 FORMAT(2X, 4F10.5)
H=ZTP1
ZTM1=ZT
ZT=ZTP1.
19 CONTINUE
RETURN
END
THE SIMPLIFIED MODEL

A, B = FRICTION COEFFICIENTS IN THE FURCHEIMER EQUATION
AP = POROSITY
I.MAX = NUMBER OF NODEAL POINTS IN X-DIRECTION
II = EXTERNAL NODEAL POINTS IN X-DIRECTION
H = MEAN WATER LEVEL
H.T = INCIDENT WAVE HEIGHT
U = HORIZONTAL VELOCITY
C = CELESTIAL
H.N = WATER ELEVATION ABOVE BOTTOM
P = WAVE PERIOD
U.X = MAXIMUM INTERNAL VELOCITY
V = VERTICAL MACROSCOPIC VELOCITY
F = FRICTION TERM
D.X = INTERNAL SPACE INCREMENT
D.X = EXTERNAL SPACE INCREMENT
C.Z = THE SHELL FRICTION COEFFICIENT
Z.C = SHELLY FRICTION IN REGION OF WAVE BREAKING
C.C = EXTERNAL CELESTIAL AT INTERFACE
C.I = INTERNAL CELESTIAL AT INTERFACE
U.D = EXTERNAL HORIZONTAL VELOCITY AT INTERFACE
U.I = INTERNAL HORIZONTAL VELOCITY AT INTERFACE
H.N.D = EXTERNAL WATER ELEVATION AT INTERFACE
H.N.I = INTERNAL WATER ELEVATION AT INTERFACE
V.D = VERTICAL EXTERNAL VELOCITY AT INTERFACE
V.I = VERTICAL INTERNAL VELOCITY AT INTERFACE
W = COEFFICIENT OF INCREASED INTERNAL DAMPING NEW INTERFACE

NOTE: SYMBOLS MARKED * REPRESENT MODIFICATIONS FOR SLOPING SECTIONS
FOR RECTANGULAR SECTIONS, W = 1.0 * Z.C = C.Z

DIMENSION C(45), U(45), U.P(45), U.B(45), H.N(45)
REAL W
I.MAX = 17
II = 11
H.T = 22.0
H.I = 10.5
A = 0.0019
P = 0.015
AP = 0.42
D.X = 40.0
D.X = 134.0
T.X = 0.5*(D.X*Y.X)
D.T = 0.02
T.P = 4.16
T.D = 4.40 T.P
T.D = 1.6* T.P
W = 1.12
D = 0.5*(1.0+W)
C.H = 1.0
C.C = 1.0
M.T = 0
C.Z = 600.0
Z.C = 300.0
FN=XX/N
Z7=(7*(C0+2)/(C0+1.0))
(UX=0.5*(1.0+T((A+I)*#2+4.0/n)) - A/R)
UX=0.75*UX
J=IMAX-1
IJJ=I1-1
I1J=I1+1
T=0.0
F=0.0
EF=1.0
G=981.0
C INITIAL CONDITIONS
K=1
K2=IJJ
2 DO 1 I=K1,I2
L(I)=0.0
C(I)=SQRT(T(s/I))
1 M(I)=HJ
TFK2=(D*IMAX) GJ T J 3
K=1+IJJ
K2=IMAX
GO TO 50
3 CONTINUE
C=SQRT(T(s/I))
M(I)=0.0
U(I)=0.0
G(I)=SQRT(T(s/I))
M(I)=HJ
M(I)=HJ
GO TO 50
T=T+DT
K1=KJJ+1
C TIME STEPS
1 IF(T-(C0.5+T(I+C0.5T(I))) 4,4,5
2 IF(T-(EF*TF)) 6,6,7
4 H=M+H0.8D+H*5*GR((G2.2532/TD1)*(T-E*TP))
GO TO 7
6 H=M+2.01H*GR((G2.2532/TD2)*(T+0.5*(TP2-TD1)-E*TP))
7 CONTINUE
F=1(I)=S*(T(G=1)
K=0
AC=0.5*(C(I)+C(I-1))
F(I)=J(1)
10 H0(I)=H+(1)(+2*GR(1)(-C(I))+2.0*TXX)*(2.0*CECH*AC)*(C2-C1)-X(CU-AC)*H(I(I)-H(I)(-3.5x(C+2)#X(I(I)+B15*Ein(I(I))))/CZ=AZ*AC*AC)
IF(H<(5-H((1)-+C0I)) 8,8,9
9 K=K+1
IF(K=-3) 10,10,13
8 CONTINUE
C OUTSIDE INTERFACE POINTS

K = 0
MC = C(I)
E = U(I)

12 IF(K = 0. or I = J) GO TO 205

U(I) = U(I) - (0.5*F)*((U(I-I) - U(I-I+1))/2 + 0.5*MC*(C(I+1) - C(I-1)) +
X(I)*X(I) + (F)*ABS(U(I))/VZ*MC*MC))

C(I) = C(I) - (0.5*MC*(U(I+1) - U(I-1)) + E)*C(I+1) - C(I-1))

GO TO 201

205 U(I+1) = U(I)

C(I) = C(I)

X = 0.5*X

U(I) = U(I) - (0.5*F)*((X*(U(I+1) - U(I-1)))/X + X*(U(I+1) - U(I))/
X)*X + X*F*(C(I+1) - C(I))/X)

C(I) = C(I) - (0.5*MC*(U(I+1) - U(I-1)))/X + X*(U(I+1) -
U(I+1))/X)

GO TO 201

MC = 0.5*(MC + CF(I))

F = F*(E(U(I+1) - U(I))/2) + 1

10 IF(K = 0) 12, 12, 13

3 CONTINUE

11 CONTINUE

INTFACE

IF(C(I) > GT(CO)) GO TO T; 14

I = 0

C = C(I)

U = U(I)

MC = C(I)

E = U(I)

F = (F - U(I))

C = 1.0

502 C = C + 0.5*F

X = (C(I+1) - C(I)) + E*U(I)

X = 0.5*(C(I) + C(I+1))

U = 0.5*(U(I) + U(I+1))

V = 0.5*(C(I+1) - C(I))

F = (F + 2) + (F - 2) + (F)*V

501 K = K + 1

IF(K = 1) 502, 502, 500

503 CONTINUE
C TEST WHETHER 1) FALLING OR 2) SINGING
   IF(XA*0.01) 690,690,601
   600: V-G(CA**2 - C**2)/(1T*G)
C TEST WHETHER 3) SLow DROp OR 4) FAST DROP
   IF(U-V) - (UX*dp)) 601,601,14
601 UX=USCFI = CI
   GO TO 15
14 YI = CI*2/G
   YC = YI = UX*0.5AP
   CI = SRT(C*YR1)
   K = 0
   MC = 0.5*(CI*CP1)
   EI = UJ
   F = (X/R)/(UX*UJ) + Y
20 IF(YI+2.0*(Y**1.021) - (2.0*MC*UX) + (UY)**1.0)**1.0 - (UW+U)**1.0
   X1(C1J-1J) + (C1J-1J) + 0.5*MC*AP*UX*MC
   V = 0.01*(C1J-UX*)/(C1J-UX*) + 0.5*MC*AP*UX*MC
   IF(UX**0.5(UX*1.01)) 17,17,16
16 O = SRT(V**2+UX**2)
   GO TO 100
17 O = SRT(V**2+UX**2)
100 F = 0.5*(F+2)
   Z = 0.5*(E+U11)
   IF(180*(F-2) - 0.001) 18,18,18
19 K = +1
   IF(K-5) 20,2,16
18 CONTINUE
C INFINITE INTERVAL SOLUTION OF CUBIC EQUATION BY THE NEWTON METHOD
   UI = 1.0*CFI*1**2
   A = 0.1/(X1**2 + (UY**1.021) + (UY)**1.0) + (UC**1.01)
   X + 0.5*(U**2) = X**1 + Y**1**1.0**1**1.0 + (UX**0.5)**1**1.0**1**1.0
   A3 = 0.5*UC
   X = C1
   K = 0
605 K = +1
   FX = X**3 + H2*(X**2) + A3
   PDX = 3*(X**2) + 2*X**2
   IF(130*(FX)-0.001) 610,615,610
610 IF(ARF(FPX)-1.0**50) 620,615,620
670 XP = FX/FPX
   IF(K-5) 605,605,615
615 CONTINUE
CEnd
   X = N - 2 - (C1)**1.021 - (2.0*UX)*
   X(2.0**1.021)*MC**1.021)
   X + 7.5*(C1**2) + X**1**1.0**1**1.0
   X/(2*C1**1.021)**1.021)
CHECK WHETHER DIVerging OR 21) CONVERGING
IF((CPI-CRD)-(C1-C0)+0.01) 21,15,15
21 Y1=0.5*(UX*U) /
C1=SQR(T(C2*Y1))
K=0
MC=0.5*(C1+C1)
F=0.1
F=(4.0*(UX)*)
26 URI=UI+2.0*(C1-C1) -2.0*(T/UX) *((U(I J)-U(I))*E1-MC)/AP-2.0*
X(C(I J J)-C(I))*(U(I J J)-U(I J J)) + 0.5*E1*AP*2.0*(X**2)
V=AP*(C1+2-C1*2)/(C1*1.14)
IF((UX-ABS(UI)) 22,22,23
23 URI=UX
22 0=SN*T(V**2+UI)**2)
Z=(n+n)*/C1
E=0.5*(F+F)
F=0.5*((F+1))
IF((ABS(F-2)>0.001) 24,24,25
25 K=K+1
IF(K-31) 24,25,26
24 CONTINUE
FINITE DIFFERENCE SOLUTION OF CUBIC EQUATION BY THE NEWTON METHOD
UR=URI**2
AP=AP*Y/(X**2+1+UH*MC) + (C1-C1) +(U(1)+U(1 J))
X+ 0.5*(G**2)*X*(I J)+APS(U(C)) / (7C*ZC+C1*MC) ) -0.5*U-UC
AP=C0
K=0
705 K=K+1
FX=X**3+2*X*(X**2)+A3
FPX = 3*X*(X**2) + 2*2+X
IF((ABS(FX)>0.001) 710,715,710
710 T=(ABS(FPX)-1.0-5) 720,715,720
720 X=X+X/FPX
IF(K=5) 705,705,715
715 CONTINUE
C=7*X
UP=U(I)=7.0*(C1-C1)+(0.0*2/UX)*
X(2*U(I J J))+C1) + (U(I)+U(1 J J))
X+ 0.5*(G**2)*X*(I J)+APS(U(C)) / (7C*ZC+C1*MC)
C CHECK WHETHER 21) CONTINUE OR 15) DID NOT MEET
IF(CPI-CRD) 27,27,15
27 CPI=C1
UR=UI
15 CONTINUE
C INSIDE INTERNAL POINTS
K=1
UT(I J J)=UI
C(I J J)=C1
32 CR(I) = C(I) - (CT/16XAP)*J*(5*W*W*(U(I+1) - U(I-1)) + UE*W(C(I+1) - X C(I-1)))
   V = DAP*(CR(I)**2 - C(I)**2)/(5*CT**1.414)
   IF(K,CT,1) GOTO 29
   O = SO*T(V**2 + UE**2)
   F = (A + W*O)**2
29 IF(I,EN,IJJ) GOTO 400
   UR(I) = U(I) - (D/T/X)*[UE*(U(I+1) - U(I-1))/AP +
   X 2.*AP*X(1/2 - C(I+1) - C(I-1))/AP +
   G**2/2)**2 + UR(I)**2]
   Z = (1 + D/2)*UR(I)
   F = 0.5*(F + Z)
   MC = 0.5*(MC + CR(I))
   EH = (EH + UE(I))/2.0
   IF(IS(F - Z) < 0.001) 3Q,30,31
31 K = K + 1
   IF(K = 3) 32,32,30
30 CONTINUE
29 CONTINUE
35 CR(I) = C(I)/MAX = 0.0
   EP = (C(I)**2)
   IF(I,MAX,I) 35,35,35
   CR(I) = C(I)/MAX
   K = K + 1
   IF(K = 3) 36,36,36
36 U(I) = C(I)**2/A
   IF(K = 2,GO TO 35)
   K1 = IJ
   K2 = IJ
   GO TO 38
37 CONTINUE
   HV = CR(I)**2/5
   HN = CR(I)**2/5
   PRINT SOLUTION, DEPENDENT VARIABLES: C, U, HN WITH TIME
   IF(KIJ/100)*100-KIJ) 1000,2000,2000
2000 PRINT 40,T,KIJ
40 FORMAT(2X,F5.2,8X,T,KIJ,F8.3,10X,4X,KIJ,,6)
   IF(TN 60,1000,1000)
   IF(TN 42,GO TO 41)
41 FORMAT(2X,F3.2,5X,F8.2,3X,F3.2,2X,F8.2,2X,10F8.2)
   IF(TN 42,GO TO 41)
60 CONTINUE
C PREPARE FOR NEXT TIME ADVANCEMENT
   K1 = 1
   K2 = IJ
47 DO 45 I=K1,K2
   C(I) = CR(I)
45 U(I) = UR(I).
   IF (K2.EQ.IMAX) GO TO 46
   K1 = IJJ
   K2 = IMAX
   GO TO 47
46 CONTINUE
   UI=UPI
   UN=URN
   CI=CRI
   CO=CRO
C CONTROL TIME FOR U/S N.C. AND STOP IF TIME EXCEEDS 5 PERIODS
   IF(T+1.*TP) 48,,47
48 E = E+1.
   EE = FF+1.
49 CONTINUE
   IE(T-5.*TP) 50,,51
51 STIP END


64. Stokes, G.G., Cambridge Transactions, 1851 (see Ref. 56).


70. Wigle, R.B., "A Numerical Analysis Solution of Water Table Drawdown Between Tile Drains", a Thesis Presented to the University of Windsor, Canada, in 1967, in Partial Fulfillment of the Requirements for the M.A.Sc. degree.


APPENDIX E

NOMENCLATURE
\[ A \]
Matrix of coefficients

\[ A_1, A_2 \]
Wave amplitudes

\[ A_0 \]
Amplitude of impact wave

\[ A_T \]
Total area of permeability cross section

\[ A(T) \]
Solution domain

\[ a, a' \]
Darcy term in Forchheimer's equation

\[ B \]
Width of embankment

\[ b, b' \]
Non-Darcy term in Forchheimer's equation

\[ C_b \]
A Chezy coefficient allowing for wave breaking

\[ C_f \]
Chezy friction coefficient

\[ C_R \]
Coefficient of reflection

\[ C_T \]
Coefficient of transmission

\[ C_t \]
A Chezy factor accounting for frictional and centrifugal effects

\[ C_{w1}, C_{w2}, C_{wf} \]
Wall correction factors

\[ C_1, C_2 \]
Dimensionless constants

\[ c_e \]
External local wave celerity

\[ c_n \]
Internal local wave celerity

\[ c_s \]
Approximate wave celerity

\[ D \]
Particle size

\[ D_r \]
Dimensionless rush-down

\[ d \]
Water depth

\[ E \]
Incident wave energy

\[ e \]
Constant

\[ F \]
Non-Darcy friction term

\[ F' \]
Non-Darcy friction term accounting for added internal dissipation
\( f, f' \) Constants
\( f'' \) Exponent
\( f(\ ) \) Function
\( G, G' \) Constants
\( G(\ ) \) A continuity function
\( g \) Acceleration of gravity
\( H \) Local wave height
\( H_i \) Impact wave height
\( H_i \) Incident wave height
\( H_r \) Reflected wave height
\( H_t \) Transmitted wave height at the core
\( h_c \) Difference in centrifugal pressure head
\( h_n \) Node height
\( h_p \) Loop height
\( h_o \) Mean water depth
\( h_T \) Total head loss
\( I \) Constant
\( i, j \) Magnitude of the hydraulic gradient
\( i, j \) Indices
\( J \) Dimensionless constant
\( K' \) A resistance factor
\( K_s \) Conductivity for Darcy flow
\( K(\ ) \) Conductivity function
\( k' \) Permeability
\( k' \) Constant coefficient of permeability
\( k_w \) Non-Darcy conductivity corresponding to \( V \).
L  Wave length
L_e  Effective length of wall zone perimeter
L_T  Length of permeameter perimeter
l  Constant
M_g  Geometric mean diameter
M,N  Variables depending on velocity
m,m'  Porosity
n  Constant
P  Pressure
p  Centrifugal pressure
q  Macroscopic velocity
q_e  Effective macroscopic velocity
q_o  Apparent macroscopic velocity in a finite permeameter
q_m  Bulk velocity in an infinitely large permeameter
R  Correlation coefficient
R_N  Representative Reynolds number
R_d  Rush-down
R_u  Run-up
r  Radius of curvature of water surface
S  Wave steepness
S_f  Friction slope
S_s  Specific surface
T'  Period of incident wave
T  Tortuosity
T_1,T_2  Wave periods
t  time
\( \Delta t \)  
Time increment  
\( \Delta t' \)  
A time step  
\( U \)  
Amplitude of local wave particle velocity  
\( U_r \)  
Dimensionless run-up  
\( u \)  
Horizontal macroscopic velocity  
\( u_e \)  
External horizontal velocity  
\( u_n \)  
Horizontal component of internal macroscopic velocity  
\( u_x \)  
Horizontal pore velocity  
\( V_F \)  
Rate of fluctuation of free water level  
\( V_{Fh} \)  
Horizontal component of rising velocity  
\( V_{m} \)  
Maximum internal fall velocity  
\( V_r \)  
Macroscopic velocity for \( R_N \)  
\( v \)  
Mean velocity of flow  
\( v_n \)  
Vertical component of internal macroscopic velocity  
\( W \)  
Dimensionless parameter  
\( x \)  
Horizontal distance in rockfill  
\( x_1, x_2, \ldots \)  
Roots  
\( \Delta x_e \)  
External space increment  
\( \Delta x_n \)  
Internal space increment  
\( Y_c \)  
Elevation of outcrop point  
\( Y_F \)  
Elevation of free water level at interface  
\( Y_M \)  
Minimum water elevation  
\( Y_X \)  
Maximum water elevation  
\( z \)  
Constant
\[ \alpha_e \] External positive characteristic direction
\[ \alpha_n \] Internal positive characteristic direction
\[ \alpha_s \] Shape factor
\[ \beta_e \] External negative characteristic direction
\[ \beta_n \] Internal negative characteristic direction
\[ \gamma \] Specific weight of water
\[ \gamma_g \] Specific weight of rock
\[ \epsilon, \epsilon' \] Proportionality factor
\[ \eta \] Perturbation height w.r.t. mean water level
\[ \eta_e \] External perturbation height
\[ \eta_n \] Internal perturbation height
\[ \theta \] Angle of inclination of rockface with horizontal
\[ \theta' \] Angle for determination of radius of curvature
\[ \lambda_1, \lambda_2 \] Particle shape factors
\[ \mu \] Dynamic viscosity
\[ \nu \] Kinematic viscosity
\[ \rho \] Density
\[ \sigma_g \] Geometric standard deviation
\[ \tau \] Square root of tortuosity
\[ \phi \] Piezometric head
\[ \psi \] An angle
\[ \omega \] Constant
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