2013

Generalized Stability Conditions for an Ultra-Low Energy Electrostatic Charged Particle Storage Ring

Michael Sullivan
University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
Electronic Theses and Dissertations. 4953.
https://scholar.uwindsor.ca/etd/4953
Generalized Stability Conditions for an Ultra-Low Energy Electrostatic Charged Particle Storage Ring

by

Michael Sullivan

A Dissertation
Submitted to the Faculty of Graduate Studies
through the Department of Physics
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy at the
University of Windsor

Windsor, Ontario, Canada

© 2013 Michael Sullivan
Generalized Stability Conditions for an Ultra-Low Energy Electrostatic Charged Particle Storage Ring

by

Michael Sullivan

APPROVED BY:

___________________________
T. Leung, External Examiner
University of Waterloo

___________________________
R. Muscedere
Department of Electrical & Computer Engineering

___________________________
G. Drake
Department of Physics

___________________________
W. Kedzierski
Department of Physics

___________________________
T. Reddish, Advisor
Department of Physics

3 September 2013
Declaration of Co-Authorship / Previous Publication

I. Co-Authorship Declaration

I hereby declare that this thesis incorporates material that is result of joint research, as follows:

The theoretical study in Chapter 5 was the original idea of the author. The concepts and designs of the experimental software presented in Chapter 3, utilized to acquire data in Chapters 5, 6, and 7 was the work of the author. The development and implementation of the ion source presented in Chapter 2, Section 4.2 was the work of the author. All the data acquisition and subsequent analysis presented in Chapters 3, 5, 6 and 7 was performed by the author. The experimental software designed by the author was used by Theresa Spanjers to observe an isochronous mode of operation in her MSc research; this mode is briefly discussed at the end of Chapters 6 and 7.

The concept and design of the apparatus presented in Chapter 2 and the theoretical study of the symmetric case presented in Chapter 4 were completed by Drs. Peter Hammond and Timothy J. Reddish, along with their graduate students, prior to my start date of September 2008 and later published in 2009 [Hammond et al, New J. Phys. 11 (2009) 043033]. The apparatus was first implemented using electrons by Dr David Tessier, published in 2007 [Tessier et al, Phys. Rev. Lett. 99 (2007) 253201]. Section 4.2 of Chapter 2 is the work of the author; Section 4.1 of Chapter 2 is based on the research of Mike Nui and Dr. David Tessier.

I am aware of the University of Windsor Senate Policy on Authorship and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained written permission from each of the co-author(s) to include the above material(s) in my thesis.

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.

II. Declaration of Previous Publication

This thesis includes five original papers that have been previously published/submitted for publication in peer reviewed journals, as follows:

<table>
<thead>
<tr>
<th>Thesis Chapter</th>
<th>Publication title/full citation</th>
<th>Publication status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 2</td>
<td>The Design, Construction and Operation of an Electrostatic Storage Ring for Low Energy Charged Particles.</td>
<td>In Preparation</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>Concerning stability for an electrostatic charged particle storage device</td>
<td>In Preparation</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Spanjers et al, Isochronous Behaviour in Electron and Ion Storage for a Low Energy Electrostatic Storage Ring, Nuclear Instruments and Methods A</td>
<td>Submitted for Publication – May 2013</td>
</tr>
<tr>
<td>Chapter 7</td>
<td>A Compact, Low-energy, Mass Spectrometer based on an Electrostatic Ion Storage Ring</td>
<td>In Preparation</td>
</tr>
</tbody>
</table>

I certify that I have obtained a written permission from the copyright owner(s) to include the above published material(s) in my thesis. I certify that the above material describes work completed during my registration as graduate student at the University of Windsor.

I declare that, to the best of my knowledge, my thesis does not infringe upon anyone’s copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledged in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owner(s) to include such material(s) in my thesis.

I declare that this is a true copy of my thesis, including any final revisions, as approved by my thesis committee and the Graduate Studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.
Abstract

A low energy (~50 eV) electrostatic storage ring has been constructed that can store a recirculating bunch of either electrons or ions. The charged particle bunch ‘orbits’ within an apparatus consisting of four lenses and two hemispherical deflector analysers, arranged in a ‘race-track’ configuration of length 64.1 cm.

A theoretical study, using transfer matrices from charged particle optics for a ‘symmetric’ configuration of lens potentials, has been previously completed by Hammond et al. [New J. Phys. 11 (2009) 043033]. That approach was capable of predicting modes of storage which appeared as a resonant-like pattern. An ‘asymmetric’ configuration, new in this work and extending the previous study to apply to a more general case, has been completed and will be presented alongside experimental results. The level of agreement between the theoretical and experimental results is found to be excellent, and the robustness of the matrix formalism has eliminated the need to rely on computer simulation to achieve storage. This asymmetric arrangement of the lenses allows for greater flexibility in the operation of the ring, creating the potential for a more diverse range of applications and potentially aid in the design of future rings.

Several spectra for both electrons and positive ions are presented to provide an indication as to how the charged particle bunch evolves as more orbits are completed. The number of counts inevitably decreases as a function of orbit number due to loss mechanisms. Enhanced measurement techniques, as well as the matrix theory, have made storage of the bunch for over a hundred orbits routine, corresponding to over 65 m travelled, and this is observed directly from the spectra.

The application of the storage ring as a multi-pass time-of-flight mass spectrometer has been studied. The isotopes of krypton and xenon have been made to completely separate from one another out of a single pulse of ions. This is observed to occur after ~15 orbits of the ring, roughly 10 m of distance. Initial results have indicated that the mass resolution is approximately 5000. Limitations and potential improvements to the mass resolution are presented.
I dedicate this thesis to Heather and my family, for your unconditional support.
Acknowledgements

I would like to first thank Dr. Timothy Reddish, my supervisor, with whom I have had the privilege of studying during the course of my degree. Without your intervention during my undergrad years, along with the opportunity you gave me assisting in your laboratory during the summer of 2007, I would have never considered graduate school and for that, I am extremely grateful. To Dr. Peter Hammond, I have thoroughly enjoyed the synchrotron runs we had together. The assistance you were able to provide (from the other side of the world) has made the research towards this thesis possible. Thank you both for your guidance, patience, and support throughout the years.

To Dr. Penny Thorn and Dr. David Tessier thank you for the conversations and advice that you shared with me over coffee. To Dr. Arathi Padmanabhan, thank you for the encouragement you gave to me, I enjoyed the three years that we overlapped with each other. I would like to especially thank Theresa Spanjers, you have been a great colleague and a better friend. I have thoroughly enjoyed working alongside you and wish you all the best for your future.

I would also like to thank the entire Department of Physics; the exceptional faculty and staff that have worked exceptionally hard to allow for the completion of my studies. Finally, I wish to thank the rest of the undergraduate and graduate student communities for all your support. In particular, I wish to thank Dr. Lauren Moffatt, Dr. Anthony Karloff and Robert Petro, each of which have provided invaluable feedback about my work on many occasions with an open door policy.

Lastly, to my family who I can never thank enough: my parents, Stan and Cheryl Sullivan, my sister, Susan, and my brother, Steven. You have believed in me from the start and never cast doubt. To Heather Joyce, thanks for being there for me every time things were difficult; you kept me going. To all of the Sullivan, Lemmon and Joyce, Bering and Frei families, thank you for all of your enthusiasm and encouragement.
# Table of Contents

Declaration of Co-Authorship/Previous Publications iii  
Abstract v  
Dedication vi  
Acknowledgements vii  
List of Tables xii  
List of Figures xiii  
List of Abbreviations xvi

## Chapter 1: An Introduction to Electrostatic Storage Rings

1.1 Introduction 2  
1.2 Charged Particles in Electromagnetic Fields 3  
1.3 Storage Devices 4  
  1.3.1 Magnetic Elements 4  
  1.3.2 Electrostatic Elements 5  
1.4 Electrostatic Storage Systems 6  
  1.4.1 ELISA 6  
  1.4.2 DESIREE 8  
  1.4.3 Zaijman Trap 9  
  1.4.4 MULTUM 11  
  1.4.5 MiniRing 13  
  1.4.6 The Storage Ring at Windsor 14  
1.5 Summary 15  
1.6 References 17

## Chapter 2: Experimental Apparatus

2.1 Introduction 21
Chapter 3: Data Acquisition Methods

3.1 Introduction to Data Acquisition 39
3.2 Detection of Charged Particles 39
   3.2.1 Channel Electron Multipliers 40
   3.2.2 Signal Processing 41
3.3 LabVIEW Programs 44
   3.3.1 Variable Time of Flight 45
   3.3.2 Stability Testing 46
3.4 Data Interpretation 50
3.5 Summary 52
3.6 References 53

Chapter 4: Electrostatic Optics and the ‘Symmetric Condition’

4.1 Introduction 55
4.2 Electrostatic Optics 55
   4.2.1 Lenses 57
   4.2.2 Hemispherical Analysers 59
Chapter 5: Stability Conditions for an ‘Asymmetric’ Configuration

5.1 Introduction 73
5.2 Matrix Formalism 74
5.3 Basic Asymmetry 76
  5.3.1 Stability Plots 78
  5.3.2 36-18-18 Configuration 81
5.4 Type A 86
5.5 Type B 88
5.6 Fully Asymmetric 92
5.7 Summary 94
5.8 References 96

Chapter 6: Timing Properties of the Stored Particles

6.1 Introduction 98
6.2 Long-Term Storage 98
6.3 Background Scattering and Other Loss Mechanisms 101
6.4 Peak Width Analysis 102
6.5 Summary 106
6.6 References 107
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Summary of the Properties of current electrostatic devices</td>
<td>15</td>
</tr>
<tr>
<td>7.1</td>
<td>Naturally occurring isotopes of krypton and their relative abundances</td>
<td>118</td>
</tr>
<tr>
<td>7.2</td>
<td>Naturally occurring isotopes of xenon and their relative abundances</td>
<td>120</td>
</tr>
</tbody>
</table>
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Layout of ELISA</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>$\text{N}_2/\text{CO}$ mass doublet, acquired by ELISA</td>
<td>7</td>
</tr>
<tr>
<td>1.3</td>
<td>Layout of DESIREE</td>
<td>8</td>
</tr>
<tr>
<td>1.4</td>
<td>Layout of the Zajfmann linear trap</td>
<td>9</td>
</tr>
<tr>
<td>1.5</td>
<td>Operating modes within the Zajfmann trap</td>
<td>10</td>
</tr>
<tr>
<td>1.6</td>
<td>Layout of MULTUMS</td>
<td>11</td>
</tr>
<tr>
<td>1.7</td>
<td>$\text{N}_2/\text{CO}$ mass doublet, acquired by MULTUMS</td>
<td>12</td>
</tr>
<tr>
<td>1.8</td>
<td>Layout of MULTUMS II</td>
<td>12</td>
</tr>
<tr>
<td>1.9</td>
<td>Layout of the MiniRing</td>
<td>13</td>
</tr>
<tr>
<td>1.10</td>
<td>Layout of the storage ring at Windsor</td>
<td>14</td>
</tr>
<tr>
<td>2.1</td>
<td>Magnetic field from Helmholtz coils</td>
<td>22</td>
</tr>
<tr>
<td>2.2</td>
<td>Schematic and photo of the storage ring at Windsor</td>
<td>23</td>
</tr>
<tr>
<td>2.3</td>
<td>Schematic and photo of an electrostatic cylindrical lens</td>
<td>24</td>
</tr>
<tr>
<td>2.4</td>
<td>Schematic of a hemispherical deflector analyser</td>
<td>25</td>
</tr>
<tr>
<td>2.5</td>
<td>Photo of a hemispherical deflector analyser and correcting hoops</td>
<td>26</td>
</tr>
<tr>
<td>2.6</td>
<td>Configuration of the power supplies for the elements</td>
<td>27</td>
</tr>
<tr>
<td>2.7</td>
<td>Dimensions of the electron gun</td>
<td>29</td>
</tr>
<tr>
<td>2.8</td>
<td>Schematic and simulation of the filament and grid</td>
<td>30</td>
</tr>
<tr>
<td>2.9</td>
<td>Simulated acceleration of the electron gun</td>
<td>31</td>
</tr>
<tr>
<td>2.10</td>
<td>Schematic of the ion source</td>
<td>32</td>
</tr>
<tr>
<td>2.11</td>
<td>Photo of the ion source</td>
<td>32</td>
</tr>
<tr>
<td>2.12</td>
<td>Photo of the ion source with output for differential pumping</td>
<td>33</td>
</tr>
<tr>
<td>2.13</td>
<td>Pulsing of the hemispherical elements</td>
<td>35</td>
</tr>
<tr>
<td>3.1</td>
<td>Type and location of the ring’s particle detectors</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>Schematic of the signal processing electronics</td>
<td>42</td>
</tr>
<tr>
<td>3.3</td>
<td>Sample of an electron timing spectrum</td>
<td>43</td>
</tr>
<tr>
<td>3.4</td>
<td>Sample of an ion timing spectrum</td>
<td>43</td>
</tr>
<tr>
<td>3.5</td>
<td>Hemisphere detection pulse sequencing</td>
<td>45</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td>Raster scanning pattern</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>User interface for the stability testing software</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>Coding for the stability testing software</td>
<td></td>
</tr>
<tr>
<td>3.9</td>
<td>Relation of the single spectrum to the ‘linescan’</td>
<td></td>
</tr>
<tr>
<td>3.10</td>
<td>Relation of the ‘linescan’ to the ‘areascan’</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Simulation of a three-element lens with sample electron trajectories</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>Lens parameters defined for a single lens</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Schematic of lens with key lens parameters</td>
<td></td>
</tr>
<tr>
<td>4.4</td>
<td>Computer model of the storage ring for the symmetric configuration</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>Trace condition and $K_1K_2/f_1f_2$ ratio for the symmetric single orbit transfer matrix as a function of $V_2$</td>
<td></td>
</tr>
<tr>
<td>4.6</td>
<td>Linescan for $V_3/V_1 = 2$ with $V_1 = 18$ eV showing $(H, m)$ modes</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>Linescan for $V_3/V_1 = 3.5$ with $V_1 = 18$ eV showing $(H, m)$ modes</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>Lens parameters defined for a fully asymmetric storage ring</td>
<td></td>
</tr>
<tr>
<td>5.2</td>
<td>Stability logic diagram for satisfaction of trace condition for $V_3/V_1 = 3.5$</td>
<td></td>
</tr>
<tr>
<td>5.3</td>
<td>Trace value diagram for $V_3/V_1 = 3.5$ with $V_1 = 18$ eV</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td>Linear and angular magnification terms for single orbit transfer matrix</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>Stability logic diagram for satisfaction of all conditions for $V_3/V_1 = 3.5$</td>
<td></td>
</tr>
<tr>
<td>5.6</td>
<td>Stability logic diagram for $V_3/V_1 = 2$ with $V_1 = 18$ eV</td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td>Experimental result of $V_3/V_1 = 2$ with $V_1 = 18$ eV</td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td>Experimental results of Figure 5.7 with $(H, m)$ modes</td>
<td></td>
</tr>
<tr>
<td>5.9</td>
<td>Asymmetric linescan corresponding to the red line of Figure 5.7</td>
<td></td>
</tr>
<tr>
<td>5.10</td>
<td>Asymmetric linescan corresponding to the black line of Figure 5.7</td>
<td></td>
</tr>
<tr>
<td>5.11</td>
<td>Lens arrangement for ‘Type A’ configuration</td>
<td></td>
</tr>
<tr>
<td>5.12</td>
<td>Stability logic diagram and experimental results for ‘Type A’ set-up</td>
<td></td>
</tr>
<tr>
<td>5.13</td>
<td>Lens arrangement for ‘Type B’ configuration</td>
<td></td>
</tr>
<tr>
<td>5.14</td>
<td>Trace value diagram for ‘Type B’ configuration</td>
<td></td>
</tr>
<tr>
<td>5.15</td>
<td>Stability logic diagram and experimental results for ‘Type B’ set-up</td>
<td></td>
</tr>
<tr>
<td>5.16</td>
<td>Areascan and linescan experimental results for ‘Type B’ set-up</td>
<td></td>
</tr>
<tr>
<td>5.17</td>
<td>Trace condition diagram for a fully asymmetric configuration</td>
<td></td>
</tr>
<tr>
<td>5.18</td>
<td>Stability logic diagram and experimental results: fully asymmetric</td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6.1</td>
<td>Timing Spectrum for Ar⁺</td>
<td>99</td>
</tr>
<tr>
<td>6.2</td>
<td>Timing Spectrum for electrons</td>
<td>100</td>
</tr>
<tr>
<td>6.3</td>
<td>Peak amplitude and area as a function of orbit number</td>
<td>102</td>
</tr>
<tr>
<td>6.4</td>
<td>Peak width evolution for electrons</td>
<td>103</td>
</tr>
<tr>
<td>6.5</td>
<td>Peak width evolution for ions</td>
<td>104</td>
</tr>
<tr>
<td>6.6</td>
<td>Observation of an isochronous mode</td>
<td>105</td>
</tr>
<tr>
<td>7.1</td>
<td>Schematic of standard reflectron</td>
<td>111</td>
</tr>
<tr>
<td>7.2</td>
<td>Timing spectrum for Kr⁺</td>
<td>115</td>
</tr>
<tr>
<td>7.3</td>
<td>Peak areas for Kr⁺ isotopes</td>
<td>117</td>
</tr>
<tr>
<td>7.4</td>
<td>Single peak timing spectrum for Kr⁺</td>
<td>117</td>
</tr>
<tr>
<td>7.5</td>
<td>Timing spectrum for Xe⁺</td>
<td>119</td>
</tr>
<tr>
<td>7.6</td>
<td>Single peak timing spectrum for Xe⁺</td>
<td>120</td>
</tr>
<tr>
<td>7.7</td>
<td>Experimental and simulated data for Xe⁺</td>
<td>122</td>
</tr>
<tr>
<td>7.8</td>
<td>Ratio of peak separation to peak width as a function of orbit number</td>
<td>124</td>
</tr>
<tr>
<td>7.9</td>
<td>Mass resolution as a function of orbit number for a quadratic peak</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>width evolution</td>
<td></td>
</tr>
<tr>
<td>7.10</td>
<td>Sample quadratic and exponential curves as a function of orbit number</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>shown with Figure 6.6, which displayed similar trends</td>
<td></td>
</tr>
<tr>
<td>7.11</td>
<td>Mass resolution as a function of orbit number for an isochronous peak</td>
<td>127</td>
</tr>
<tr>
<td></td>
<td>width evolution</td>
<td></td>
</tr>
<tr>
<td>7.12</td>
<td>Simulation of heavy ions (m = 5000 amu) in a quadratic mode after</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>25, 50, 75 and 100 orbits completed</td>
<td></td>
</tr>
<tr>
<td>7.13</td>
<td>Simulation of heavy ions (m = 5000 amu) in an isochronous mode after</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>25, 50, 75 and 100 orbits completed</td>
<td></td>
</tr>
<tr>
<td>A.1</td>
<td>Hemisphere pulsing circuit</td>
<td>134</td>
</tr>
</tbody>
</table>
List of Abbreviations

AC – Alternating Current
BNC – Berkeley Nucleonics Corporation
CEM – Channel Electron Multiplier
DC – Direct Current
DESIREE – Double ElectroStatic Ion Ring ExpEriment
EI – Electron Ionization
ELISA – Electrostatic Ion Storage ring, Aarhus
ERS – Electron Recycling System
ESI – Electrospray Ionization
eV – Electron volts, unit of energy
FAB – Fast Atom Bombardment
FC – Faraday Cup
GB – Gun Base
GPIB – General Purpose Interface Bus
HDA – Hemispherical Deflector Analyser
ICP – Inductively Coupled Plasma
ISR – Ion Storage Ring
MALDI – Matrix-assisted Laser Desorption Ionization
MCP – Multi-Channel Plate
MCS – Multi-Channel Scalar
MP-TOF – Multi-Pass Time-Of-Flight
MULTUM – MULti-TUrn time-of-flight Mass spectrometer
NI – National Instruments
PCI – Peripheral Component Interconnect
TAC – Time to Amplitude Converter
TOF – Time-Of-Flight
TTL – Transistor – Transistor Logic
# Chapter 1: An Introduction to Electrostatic Storage Rings

1.1 Introduction 2
1.2 Charged Particles in Electromagnetic Fields 3
1.3 Storage Devices 4
  1.3.1 Magnetic Elements 4
  1.3.2 Electrostatic Elements 5
1.4 Electrostatic Storage Systems 6
  1.4.1 ELISA 6
  1.4.2 DESIREE 8
  1.4.3 Zaijman Trap 9
  1.4.4 MULTUM 11
  1.4.5 MiniRing 13
  1.4.6 The Storage Ring at Windsor 14
1.5 Summary 15
1.6 References 17
1.1 Introduction

Particle storage devices have existed for many decades and subsequently have been continually developed, resulting in a wide variety of traps and storage rings that have been created for studies involving all kinds of particles. Ions [1], electrons [2, 3], and neutral particles [4, 5] have all been trapped for the purpose of study within the field of atomic and molecular physics. This requires the use of radically different storage methods depending on each specific experiment, from a localized trap, where the kinetic energy of the particles and the volume of confinement are both extremely small, to a storage ring, where the kinetic energy of the particles can be varied, allowing the particles to traverse and be confined through a fixed, well-defined volume of space. These devices are used to study the properties of atomic and molecular particles, such as lifetime measurements [6, 7] and collision studies [8, 9], and they have several direct applications, such as mass spectrometry [10, 11]. These devices are interdisciplinary and are also used in chemistry, biology, and other fields outside of physics [12].

Electrostatic storage devices are of growing interest in the atomic and molecular community, particularly in the past two decades, giving rise to the development of a number of devices all over the world. One such device, the Electron Recycling Spectrometer, is an electrostatic storage ring, and was made operational by Tessier et al. for use as an electron spectrometer [13, 14]. In the present work, the storage of electrons using this spectrometer has been significantly improved as the stability of the storage ring is studied in detail. This ring has also successfully converted into an ion storage ring, which, while theoretically possible, has not been shown for ultra-low energies. The mean energy of the ion ring is \(~10\) eV, and is the only such device known to operate in this extremely low energy range. The potential applications include mass spectroscopy and crossed or merged beam experiments.

This chapter will briefly introduce the behaviour of charged particles in an electromagnetic field, followed by specific examples of electrostatic storage devices similar in nature to the ring in this thesis, in order to provide a basis of comparison of this device to the existing field. In Chapter 2, the layout of the storage ring will be presented, with details about the particle sources used and the method of injection of the particle beam into the
apparatus. Chapter 3 describes the detection and acquisition methods, along with the hardware and software required. Chapters 4 and 5 present a theoretical description of the ring using charged particle optics, with conditions for stable storage developed for this device to ‘predict’ successful storage and these conditions are compared to experimentally observed storage. In Chapter 6, the performance of the ring is studied in terms of the timing properties of the beam, along with the analysis of those results for the experiments completed with the device for both electron and ion storage. Chapter 7 presents studies of the device as a mass spectrometer, with estimates to its resolution.

1.2 Charged Particles in Electromagnetic Fields

Particle storage devices can use both electric and magnetic fields to manipulate the motion of a charged particle beam, such that the particles are confined. A particle moving nonrelativistically through an electromagnetic field experiences a force, giving by the equation

\[ \vec{F} = q(\vec{v} \times \vec{B} + \vec{E}) \]  

where \( q \) is the charge of the particle, \( \vec{v} \) is the particle velocity and \( \vec{E} \) and \( \vec{B} \) are the strength of the local electric and magnetic fields, respectively. Determining the local field that exists between two elements within the system, as well as defining the velocity of the particles passing through the field will allow this force to be calculated.

For the case of static electromagnetic fields, Maxwell’s equations are simplified with time independent fields, described by:

\[
\begin{align*}
\nabla \cdot \vec{E} &= \rho / \varepsilon_0 & \nabla \times \vec{E} &= 0 \\
\nabla \cdot \vec{B} &= 0 & \nabla \times \vec{B} &= \mu_0 \vec{j}
\end{align*}
\]

where \( \rho \) is the charge density, \( \varepsilon_0 \) is the permittivity of free space, \( \mu_0 \) is the permeability of free space and \( \vec{j} \) is the current density. If the beam intensity is low, the fields that exist in the presence of charged particles are approximated by the fields that exist in free space (i.e. with no charge or current) \[15\]. This further simplifies Maxwell’s equations by setting \( \rho = 0 \) and \( \vec{j} = 0 \) in equation 1.2. All the approximations made are appropriate for this thesis. As a
result, both the electric and magnetic field equations can be expressed using the Laplace equation for the electric scalar potential, \( V \), or the magnetic scalar potential, \( A \),

\[
\nabla^2 V = 0 \quad \text{or} \quad \nabla^2 A = 0
\]

respectively for each case.

1.3 Storage Devices

When the device uses electromagnetic fields in order to confine the particles to a limited volume of space, it can be classified as either a local trap or an accelerator. A local trap is capable of confining very low energy particles (<1 eV) to a well-defined region of space. Accelerators also confine charged particles to a fixed volume of space, which can be linear or circular. The energy of the particles that can be confined is significantly larger and the volume of space they are confined in can be large as well, although it is not a necessary condition. An example of a circular accelerator is a synchrotron, which is a large device shaped like a ring, with a typical circumference of a few hundred meters, used to produce a pulse of light through the acceleration of relativistic electrons, moving with an energy of up to 10 GeV.

There are numerous types of apparatus that use both electric and magnetic fields simultaneously to manipulate charged particle motion, such as the cyclotron, and many that operate using only electric or magnetic fields. This thesis will focus on electrostatic devices only. In all cases, the storage devices are made of elements that are capable of creating the electromagnetic fields that are used to manipulate and confine the particles. The general principles behind magnetic and electric elements are presented in the next two sections.

1.3.1 Magnetic Elements

Magnetic elements create a magnetic field in order to alter the path of a charged particle. As can be seen from the first term in equation 1.1, the Lorentz term, force exerted on the particle is perpendicular to the particle velocity. This implies that the magnetic force can do no work, and hence cannot change the kinetic energy of the charged particle [16]. Magnetic fields can affect a particle’s trajectory, which makes them ideal for focusing
(i.e. for use as a lens). This force is dependent on the velocity and thus the particle momentum; the force experienced by charged particles of identical fixed energy is determined in part by the mass. This complicates the ability to manipulate massive particles, as heavy particles will require a strong magnetic field in order to follow a similar path as a light particle. This will eventually require a relativistic treatment of the particle. Large magnetic fields require large current supplies, which are expensive, and the large currents create a significant amount of heat. This further requires cooling in order to protect the element from heat damage. As a way of avoiding this last issue, superconducting magnets can be used, as are used in synchrotron devices. This, however, adds its own elements of expense and complexity.

1.3.2 Electrostatic Elements

Similar to magnetic elements, electrostatic devices use electric fields to manipulate a charged particle beam, although they are capable of changing the energy of the particles. Examples of these elements include parallel plate deflectors, quadrupole lenses and spherical or cylindrical deflectors. Devices consisting of these electrostatic elements can be mechanically simpler to construct than their magnetic counterparts, as they require the use of extremely low current power supplies, reducing the cost and complexity as cooling is not required for the electrodes as in magnetic systems. Since the force depends only on the charge state of the particle and the electric field, particles with different masses will act identically, given the same energy and trajectories, with only the time frames differing. This makes it extremely simple for the user to operate the same electric elements with various types of particles, since the mass limitation is virtually non-existent given proper conditions, such as ultra high vacuum [12].

The main limitation for electric elements is the electrical breakdown, which affects the size of the electric fields that can be used. This occurs when the ions that exist in the region containing an electric field accelerate toward one of the electrodes. This ion, given sufficient energy and colliding with a residual neutral gas, will ionize the neutral gas target, which begins to accelerate as well and this continues until the gas inside the element is ionized. This was a main problem encountered during the development of early high-energy
particle accelerators, such as the Van de Graaff accelerators [17], and limited the total energy of an ion beam to less than 8 MeV [18].

1.4 Electrostatic Storage Systems

There are many examples of electrostatic storage devices, consisting of various electrostatic elements that together form several different geometries. These devices are termed electrostatic, despite the use of pulsing techniques commonly employed in these systems, as will be apparent later in the chapter. This is due to the fact that the electric fields are only pulsed (i.e. time dependent fields) on one small section of the device and only for the purpose of injection or detection of the particles. This pulsing has no effect on the particle energies or storage, since by the time the particles return to the pulsed section, the field has returned to its static value to allow containment within the device. During this entire process, the particle only experiences electric fields that are constant in time, as the pulsing has ideally occurred when the particles are a large distance away. This is unlike some devices, such as a cyclotron, where an AC field is responsible for energy manipulation as the particles travel through the gap region.

Electrostatic storage devices can differ not only by the geometric shape, but also by the overall size of the device, usually indicated by the orbital distance travelled by a charged particle. Another difference is the mean energy of the charged particle beam. Both of these factors can depend critically on the intended application of the storage ring. In the rest of the chapter, several electrostatic storage rings will be presented and compared regarding their construction, operating energy and base pressures, as well as the applications in order to gain an appreciation for the existing field.

1.4.1 ELISA

The ELectrostatic Ion Storage ring, Aarhus, otherwise known as ELISA, is a storage device that was built and tested in 1998. This device initially consisted of two
Figure 1.1 – The schematic for ELISA. The device consists of quadrupole lenses (QEV, QEH), parallel plate deflectors (DEV, DEH), as well as spherical deflectors (SDEH), pickup electrodes (UEH/V) and a radio frequency tube (RF). [19]

160° spherical deflectors, four 10° parallel plate deflectors and four electrostatic quadrupole doublets, giving focusing in both the vertical and horizontal directions [1]. The spherical deflectors have since been replaced with cylindrical deflectors. The general layout is horizontal and can be seen in Figure 1.1, along with other features specific to experimentation. The total flight distance is 7.62 m and the system was designed to run at 25 keV taking H+ 3.5 μs to complete an orbit [20]. The design of ELISA was used as the template for the KEK ring and the TMU E-Ring, both located in Japan [21-23].

Experimentation has been conducted with ELISA, which includes lifetime measurements of atoms and molecules; the metastable state Be+ (2s2p2 4P, J=3/2) was found to be 43.40 ± 0.10 μs [24]. The possibility of using ELISA as a mass spectrometer

Figure 1.2 – The signal from a mixed source of N2+ and CO+ at 10 ms and 20 ms after injection, indicating the potential to use ELISA in the field of mass spectrometry. [20]
has also been explored, with an initial test involving a mixed beam of \( \text{N}_2^+ \) and \( \text{CO}^+ \). As can be seen in the Figure 1.2, the two peaks correspond to each of the species within the beam and are clearly distinguishable, showing the successful separation according to mass.

1.4.2 DESIREE

The “Double ElectroStatic Ion Ring ExpEriment”, known as DESIREE, is a system formed from two storage rings that share a common straight section, as is shown in Figure 1.3. The rings themselves are based on the ELISA design with an orbital distance of 8.6 m each, consisting of 160° cylindrical analysers each surrounded by two 10° parallel plate deflectors [25, 26]. Separating the deflection arrangement are quadrupole lenses and field free drift regions.

The two rings store both positive and negative ions simultaneously, with a positive and a negative ring. The ions orbit in opposite directions so both travel the same direction through the common straight section. This apparatus allows for the study of the interaction of cations and anions and is also equipped for cryogenic cooling of the beam.
and has operated at internal temperatures of 13 K [26]. The cooling of the system helps improve the vacuum, which is critical to long term storage, and recreates the environment in which ion interactions occur in space [25-27].

1.4.3 Zajfman Trap

The Zajfman trap is a linear storage system that consists of two electrostatic mirrors that are faced towards one another, as shown in Figure 1.4. An ion beam with an initial fixed energy distribution is injected into the system when one of the mirrors is ‘turned off’. This occurs when all the elements of this mirror have a potential that is less than the kinetic energy of the incoming ions, and thus will not repel the beam. This mirror is turned back ‘on’ while the beam is being slowed down and reflected by the second non-pulsed mirror, allowing the particles to now be trapped in the space between the two. Each mirror consists of five non-grounded elements that characterize the mirror, and the two mirrors are separated by 2.27 m. The system is operated with energy of a few keV and an initial energy spread is ~1 eV [28].

![Diagram of Zajfman trap](image)

Figure 1.4 – The Zajfman trap consists of two electrostatic mirrors, shown by the two sets of parallel lines (labelled by the voltages, \( V_i \)) in the diagram above, that serve to reflect the beam between them. [28]
Initial studies involving the system include mass spectrometry, lifetime measurements as well as the investigation of beam dynamics. A “self-bunching” mode of operation has been observed in which the width of the ion bunch stored within the trap evolves slowly in time, or as a function of number of revolutions completed. This effect, seen in Figure 1.5, is due primarily to the ion-ion interactions within the beam [28, 29]. As a result, the use of the Zajfman trap as a mass spectrometer has been studied [28] with an initial mass resolution of the order of $10^6$. Furthermore, as a result of its simplicity in its design, layout and operation, similar linear configurations have been successfully employed by other groups [10, 11].

Figure 1.5 – The evolution of a stored ion beam for three distinct intervals of time in both the dispersive mode (left) and the “self-bunching” mode (right). As is evident, the two cases are initially similar after 150 $\mu$s, but the peak width evolves much differently for the 650 and 1650 $\mu$s, resulting in the loss of distinguishable peaks for the dispersive mode (left), while the “self-bunching” peaks not only remain separated, but are comparable to the initial peaks in terms of width. [28]
1.4.4 MULTUM

MULTUM (MULti-TUrn time-of-flight Mass spectrometer) is a system designed specifically for the purpose of being an ultrahigh resolution mass spectrometer for use in outer space. The system consists of four 157.1° cylindrical electric sectors separated by 16 quadrupole lenses and these elements are organized in a ‘figure eight’ shaped layout, seen in Figure 1.6. The original device had an orbital path length of just over 1.7 m and the ions have an energy typically around ~1.5 keV [30].

Toyoda and his collaborators studied in great detail the timing aspects of the particles as they travel through the device. They developed a ‘perfect focussing’ condition, capable of retaining both the spatial and temporal properties of the beam as it travels through each individual orbit [31]. This is a property critical to mass spectroscopy as the beam width in time does not vary as more orbits are completed, and initial testing of the device provides an estimated mass resolution of ~350000 [30], shown in Figure 1.7.

Figure 1.6 – The layout of the MULTUM storage ring. The ring portion contains four cylindrical electric sectors (Sector) and 16 quadrupole lenses (Q), with more lenses external to the ring (CQ, SQ) to focus from the source (SS) and focus into the detectors (CS), making it a fairly complex system. [30]
MULTUM was specifically designed for the purposes of a miniaturized, ultra-high resolution mass spectroscopy for portable use. In order to achieve this, the device needed to be as simple as possible. To meet this criterion, the ring was redesigned, shown in Figure 1.8, to remove the quadrapole lens elements and the cylindrical electric sectors, replacing them with toroidal electric sectors. This reduced the mass resolution that it obtained to an estimated resolution of 24000 [30]. This was later combined with a quadratic-field ion mirror to create a tandem time-of-flight mass spectrometer for the study of the fragments of collision induced dissociation [32].

Figure 1.7 – The CO, N2 doublet after more than 500 orbits completed. The doublet is clearly separated, and the estimated resolution is 350 000. [30]

Figure 1.8 – The simplified version of the device, center, with the quadrapole lenses and cylindrical electric sectors replaced by four toroidal electric sectors. It is shown with an external ion source, as well as an ion mirror (not a quadratic field mirror) as part of the detector. [30]
1.4.5 MiniRing

The MiniRing is the smallest electrostatic storage device mentioned in this work, having an orbit distance of only ~30 cm. The small size is reported by the group as an advantage when cooling the setup to low temperature [33]. It is based on two conical mirrors with four horizontal deflectors. The mirrors will contain the beam based on the condition given by:

\[
\frac{L}{4} \leq f < \infty
\]  (1.4)

where \( L \) is the separation between the mirrors and \( f \) is the focal length of the mirrors. This is the same condition as for storage of photons in optics [34, 35]. The purpose of the four deflectors is to bend the particles in an ovular shape, so the particles travel in different directions and detection can occur by switching off one deflector and allowing the beam to exit the orbital pathway. The MiniRing has shown successful operation at energies of 2-4 keV for \( \text{Ar}^+ \) and \( \text{He}^+ \), storing the ions for over 1 ms [33].

Figure 1.9 – Left – cross-sectional view of the MiniRing showing the orbital pathway. Right – The model of the ring indicating the four deflectors \( D_i \), the two conical mirrors \( C_i \), and three faraday cups \( F_i \), located in positions to collect the beam once the appropriate deflectors are turned off. [33]
1.4.6 The Storage Ring at Windsor

The storage ring at Windsor, referred to as the Electron Recycling Spectrometer or the Ion Storage Ring (ERS or ISR), used for the work presented in this thesis, consists of two 180° hemispheres separated by two lens stacks, each consisting of two lenses and a short field free drift region, Figure 1.10. The lenses serve to extract the particle beam leaving the field free interaction regions and transport it into the hemispheres and vice versa.

Originally designed as an ultrahigh resolution electron spectrometer, the device is well suited for collision studies involving low flux sources. This is because particles travelling through an interaction region have a low probability of undergoing a collision event on a single pass. Typically, the particles that do not interact are then discarded. Using the storage ring, however, these particles are ‘recollected’ and sent into another orbit, and are given another chance to interact.

Experimentation occurs within the field free regions; one of which contains a hypodermic needle and allows for the introduction of a target gas. Each is surrounded by channel electron multipliers that are oriented 90° with respect to the stored particle beam. These detectors are made to detect either positive or negative particles and both types can

Figure 1.10 – The layout of the storage ring at the University of Windsor, showing the particle source, the detectors, which are channel electron multipliers, the four lenses and two hemispheres.
detect neutral particles with sufficient energy, such as metastable atoms and photons, that could potentially arise from interactions. The entire length of the storage ring is approximately 64.1 cm and the mean energy of a particle beam is <30 eV, but the ring has operated primarily with a mean particle energy of ~10 eV. The ring has successfully stored both positive ions and electrons, though not simultaneously, proving that not only can this configuration store both positive and negative charges, but that the injection pulse and detection timing is fast enough to handle the much faster electrons.

1.5 Summary

While the devices mentioned are all electrostatic, they are different in several aspects. The different configurations of each of the devices is the most obvious, but the overall size of each device and the mean energy of their stored beams are of interest as these factors determine the usefulness of the devices for different applications. These properties have been summarized in Table 1.1.

<table>
<thead>
<tr>
<th>Device</th>
<th>Orbital Path Length (m)</th>
<th>Energy Range of Stored Beam (eV)</th>
<th>Operational Vacuum Base Pressure (torr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elisa</td>
<td>7.62</td>
<td>1000-100000</td>
<td>&lt;10^{-11}</td>
</tr>
<tr>
<td>Desiree*</td>
<td>8.6</td>
<td>1000-100000 each</td>
<td>10^{-14}</td>
</tr>
<tr>
<td>Zaijmann</td>
<td>4.54</td>
<td>1000-10000</td>
<td>~3 x 10^{-10}</td>
</tr>
<tr>
<td>Multums</td>
<td>1.28</td>
<td>1000-10000</td>
<td>~5 x 10^{-7}</td>
</tr>
<tr>
<td>MiniRing</td>
<td>~0.30</td>
<td>1000-10000</td>
<td>~7.5 x 10^{-7}</td>
</tr>
<tr>
<td>ERS/ISR</td>
<td>0.641</td>
<td>1-100</td>
<td>~2 x 10^{-6}</td>
</tr>
</tbody>
</table>

Table 1.1 – Comparison of the electrostatic storage devices in terms of size, energy and operating vacuum background pressure [20, 25, 26, 28, 30, 33].

As is evident in Table 1.1, there is a distinct difference, with no overlap, in the energies of the charged particles between the storage ring at Windsor and the others presented in this chapter. The storage ring at Windsor is the only ring known that has proven

* Not measured directly, the base pressure is an estimate from observed storage lifetimes. [26]
to successfully store an ion pulse at \(~10\) eV. Another key distinction is the operational vacuum base pressure, which has an effect on the storage capability of the devices, as a higher pressure means more background particles to scatter the stored beam. In this case the ring is not ideal, but this can be easily remedied with an upgrade to the vacuum pumps.

This chapter has presented several electrostatic storage devices along with their applications to demonstrate the capabilities of these apparatuses. While differing in several ways, as shown in Table 1.1, the devices fundamentally operate in the same manner, using static electric fields to confine charged particles. In the next chapter, the device briefly presented in Section 1.4.6 will be presented in detail.
1.6 References


Chapter 2: Experimental Apparatus

2.1 Introduction 21
2.2 Experimental Chamber 21
2.3 Layout of the Storage Ring 22
  2.3.1 Cylindrical Lenses 24
  2.3.2 Hemispherical Analysers 25
  2.3.3 Voltage Configuration 27
2.4 Particle Sources 28
  2.4.1 Electron Gun 28
  2.4.2 Ion Source 31
2.5 Hemisphere Pulsing 34
2.6 Summary 36
2.7 References 37
2.1 Introduction

The Windsor storage ring was briefly mentioned in Chapter 1 in order to give an overview of the system itself and provide a basis for comparison with other existing devices. This chapter will focus on giving a clear picture of the entire experimental system, with each major component discussed in detail. The design of the system, including experimental consideration, is presented in a systematic manner to clarify how the individual components of the apparatus function together as a storage ring. New in this work was the adaptation of the storage ring for use with positive ions. This required the development of an ion source, which will be discussed along with the electron source and the injection process. The next section will describe the vacuum chamber, in which the entire apparatus is placed.

2.2 Experimental Chamber

The actual storage ring is placed within a cylindrical vacuum chamber made of 304 stainless steel. It has many ports, allowing electrical and gas feed-throughs to enter the interior of the chamber, enabling external control of the internal components while still maintaining vacuum.

Vacuum pressure is obtained by the use of two pumps: a mechanical and a turbo molecular pump. The mechanical pump, Varian model DS602, is used to reach a low pressure ($10^{-3}$ to $10^{-4}$ Torr) suitable for the operation of the turbo molecular pump. At this point, the turbo molecular pump, Leybold model TW700, can be operated without significant risk of damage in order to reach the base pressure ($\sim 10^{-8}$ Torr).

The chamber is also wrapped in heating tape which shortens the time it takes to reach the base pressure, $\sim 5 \times 10^{-8}$ Torr, when the system is initially being placed under vacuum. This tape, along with projector bulbs inside the vacuum chamber, heats the metal surfaces of the chamber and the storage ring for outgassing purposes (i.e. water vapour) to keep the surfaces clean.

The chamber is surrounded by three sets of mutually perpendicular Helmholtz coils with the chamber resting at the center of all the square coils, as in Figure 2.1. Each pair of coils, with side length of $\sim 1.8$ m and separated by a distance also equal to 0.9 m, is large...
enough to fully contain the chamber at the center of the pair. The coils have a current flowing that creates a nearly uniform magnetic field in the space between each pair; see Figure 2.1 (left). This current is adjusted to cancel the magnetic field of the Earth as well as any constant direct current (DC) fields present in the lab.

The internal wall of the chamber is lined with two layers of mu-metal shielding. This shielding serves to reduce any residual alternating and direct current magnetic fields within the chamber, as they are not desired. This reduces the magnetic field inside the experimental chamber to <2 mG.

Figure 2.1 – Left – The magnetic field lines for an individual pair of helmholtz coils. The field lines are approximately uniform in the area between them. Right – A pair of coils is placed on the x, y, and z axis surrounding the experimental apparatus. This enables the cancellation of any external magnetic fields. [1]

2.3 Layout of the Storage Ring

The system contains two hemispherical deflector analysers* separated by two lens stacks, in a ‘racetrack’ configuration, as in Figure 2.2. Each lens stack contains two lenses: a lens to collect charged particles from the hemisphere and transport them towards a target region, and a lens to collect particles from the target/interaction region and transport them into the hemisphere. The hemispherical analysers bend the particles from the entrance to the exit of the hemisphere, which sends the beam from the exit of one lens stack to the entrance of the second lens stack, which is mechanically the same as the first. As seen in Figure 2.2,

* Also referred to as ‘HDAs’ or ‘hemispheres’ in the remainder of this thesis
the length of each of the lens stacks is 16.35 cm and the mean radius of the hemisphere is 5 cm, leading to a mean orbital distance of 64.1 cm. All of the elements within the lenses and hemispheres are made of oxygen-free copper and all the surfaces exposed to the particle beam have been gold-plated.

The target/interaction region, as its name implies, is the region of the ring where the stored particle beam is introduced to a neutral gas target. The target region is field-free in order to keep the energy of the beam well defined, and is surrounded by channel electron multipliers capable of measuring either excited neutrals, positive or negative particles, which are discussed in the following chapter. As the lenses and hemispheres are the only two types of components used to form the ring, they will now be discussed in the following sections.

Figure 2.2 – Left - The layout of the storage ring at Windsor, showing the positioning of the hemispheres and lenses, as well as the target (experimentation) regions and the overall scale of the device. [2] Right – Photo of the apparatus. The ring itself is made from oxygen-free copper; the outer component of the upper hemispherical analyser has been removed, revealing the gold-plated surface of the inner hemisphere. [2]
2.3.1 Cylindrical Lenses

The electrostatic lenses within the stacks have cylindrical symmetry and for charged particles they function in a similar manner as optical thick lenses. The main difference is that the electric potential determines particle motion rather than the index of refraction for the medium of the optical lens; this is described further in Chapter 4. Each of the lenses contain four elements, labelled by $V_1$, $V_2$, $V_3$, and $V_4$ in Figure 2.2, and two molybdenum apertures to restrict the beam size and range of particle trajectories. In practice, however, the four-element lens is used as a three-element lens by limiting the voltage difference between the two middle elements $V_2$ and $V_3$. In this three-element lens arrangement, the first element is used to determine the initial energy of the particle, the second is the focusing element and the third element determines the final energy of the particle as it leaves the lens. Separation of the middle elements allows for some compensation to any physical

Figure 2.3- Top – Schematic of one of the cylindrical lenses used within the ring, indicating the dimensions of each of the components [2]. Bottom – Photo of the one of the lenses, showing the elements (gold) and the source/target region. The insulating ceramics used to mechanically and electrically separate the lens elements are also shown (white).
misalignment within the length of the system, namely in the direction of the optical axis [2].

### 2.3.2 Hemispherical Analysers

The hemispherical analyser [3, 4] is a device that consists of a half sphere and a spherical shell with a larger inner radius. The two components fit nested and concentrically with each other so the distance separating them is constant over the *inner* curved surfaces. Ideally, a voltage is applied which creates a radial electric field ($\propto 1/r^2$) between the two elements. Given the correct voltages, this field allows a particle with a specific energy to undergo uniform circular motion when it enters the analyser on a paraxial trajectory, midway between the hemispherical surfaces. This is a simplified view of the hemisphere’s function within the storage ring. There is a range of positions and trajectories possible at the entrance of the hemisphere and, consequently, most of the particles undergo elliptical motion, just as for planetary motion in a gravitational field, as indicated in Figure 2.4.

![Hemisphere diagram](image)

**Figure 2.4** – Hemisphere containing an inner and outer electrode, which possess different voltages, $U_1$ and $U_2$, that bend and focuses the particle beam from the entrance to the exit. While the dashed line represents the central-most circular trajectory, most of the trajectories are elliptical and fall within the blue region shown.

One of the difficulties in operating hemispheres is the ability to create a perfectly radial electric field near its entrance and exit. The energy of the particles prior to entering the hemisphere should be fairly constant; if not, there may be some field penetration within the hemisphere, which adds a nonradial component to the electric field inside the hemisphere. The region outside the hemisphere therefore needs to be essentially field free at the mid-hemisphere potential, and requires the use of an aperture. This aperture will also
introduce a nonradial component of an electric field with the hemisphere, since the aperture is ‘visible’ to the inner region of the hemisphere. In order to minimize this effect, end correctors are used. These end correctors are a series of rings of various radii, referred to as ‘hoops’, which are similar geometrically to those used by Jost [4] and can be seen in Figure 2.5. They are thin in the radial dimension and carry the voltage potential that would exist naturally at that radial distance inside a spherical capacitor of the same dimensions as the hemisphere. Combining many of these elements with various radii, the radial field can be approximated at the entrance and exits of the hemisphere.

The voltage potential that is applied to the elements of the hemisphere determines the energy of the particles that pass through the hemisphere. This potential varies with the radius of the metallic surfaces facing the beam and is represented by:

$$V(r) = V_0 \left( \frac{2R_0}{r} - 1 \right)$$

(2.1)

where $eV_0$ is the pass energy (i.e. the mean energy of the exit beam) and $R_0$ as the mean radius of the hemisphere. This formula applies to both the spherical elements and the end correctors.

Figure 2.5 – Picture of the inner hemisphere and the hoops used to correct the fields near the entrance and exit, which are the two holes seen in the central ring. [2]
2.3.3 Voltage Configuration

The voltage set-up is a major component of the overall apparatus. The voltage difference between the particle source and any of the elements in which the particle will travel is the energy that the particle will possess while travelling within that element. Hence this voltage difference defines the energy of the charged particles. In practice, this is accomplished by referencing each of the power supplies relative to one another; ultimately each supply is referenced to one energy defining power supply. In this case the power supply to the target region, called ERS in Figure 2.6, defines the energy. While the target region itself is held at ground, the target region power supply shifts all the other voltage supplies for the ring elements relative to the ungrounded output of that supply, which defines the voltage where the kinetic energy of the particles is zero. This is shown in Figure 2.6, which presents the voltage configuration for storing electrons.

The voltage configuration of Figure 2.6 is used to make shifting the target energy simpler, as a change in the target energy is not a change to any one element, but rather a change in the ‘zero’ energy of the particles. The energy of the particles in the other

![Figure 2.6 – The typical voltage configuration for the elements of the storage ring when it is operating to store negative particles, indicating the voltages are defined with respect to one another. [2]](image-url)
regions remains the same, since the power supplies for those elements still represent the same voltage change from the ‘zero’ energy.

Storing both positive and negative particles in the ring is possible, but not at the same time. This configuration of the voltage supplies makes converting from positive to negative particles (or vice versa) relatively straightforward. One only has to reverse the polarities of all voltage supplies, and change the particle source and detectors, both of which are discussed later in this thesis. The next section will introduce the particle sources used with the storage ring.

2.4 Particle Sources

The storage ring is capable of storing both positively and negatively charged particles, and has demonstrated successful storage of both electrons and positive ions, although not simultaneously. These particles are first generated and injected into the ring before they are stored. The next two sections will describe the electron and ion sources that have been used and how they operate with the storage ring.

2.4.1 Electron Gun

The electron gun was designed by Nui [1] and modified by Tessier [2] to create and focus an electron beam so that the electrons are travelling nearly parallel with respect to the optical axis of the source lens stack. It consists of the filament (the ‘cathode’), filament holder (the ‘grid’*), an extraction plate (the ‘anode’) and followed by five aperture elements. A schematic of the gun is shown in Figure 2.7, which indicates the separation and dimensions of the elements.

Electrons are generated by passing a current through the filament. Electrons flowing through the hot wire are able by thermionic emission to escape the surface if the energy of the electron is greater than the work function of the filament material. The

* While referred to as a grid for historical reasons, it is in fact an electrode.
The filament in this application is made of tungsten, which has a work function of ~4.55 eV [6]. The shape of the filament itself is important, as electrons are emitted from the entire length of the filament wire. In order to create a ‘spot’ source, a hairpin shaped wire is used. Only the tip of the hairpin filament extends into the grid allowing only this portion of the wire to be the source of the electron beam; this is depicted in Figure 2.8. The electrons created elsewhere on the wire are repelled back. This functionality is also dependant on the shape of the grid, which is based on a Pierce geometry [7]. This serves to maximize the current density by ‘pushing’ the newly emitted electrons near the tip of the filament, which have a broad range of thermal energies (typically less than 1 eV and in a Maxwell-Boltzman distribution), towards the optical axis. This is accomplished by negatively biasing the Pierce electrode with respect to the local potential of the filament such that it repels the negative electrons. The anode, which is positively biased with respect to the cathode, accelerates the desired electrons towards the lens portion of the electron gun.

Figure 2.7 – Schematic of the electron gun, showing the aperture diameters (left) and the separation between elements (right) of the electron gun [1,2]
Following the anode there are two pairs of parallel deflectors, with the two sets being perpendicular to each other. These serve to help steer the beam and compensate for any minor misalignment between the filament or gun elements and the optical axis of the storage ring. The first two aperture elements are held at the anode potential in order to create a region that does not have an electric field in the direction of the optical axis. Between these two aperture elements are parallel plate deflectors, which create electric fields perpendicular to the particles motion. The acceleration due to the deflectors acts perpendicular to the beam and will not significantly affect the energy of the particles. The remaining three aperture elements are adjusted to create a beam with nearly parallel trajectories, shown in Figure 2.9, that enter the storage ring through the hemisphere. During this process the hemisphere operating potential are pulsed ‘off’, which is discussed later in this chapter, in Section 2.5.
2.4.2 Ion Source

The design and use of an ion source for the storage ring in Windsor is new in this work, allowing for the storage of positive ions in the ring. The source is based on electron impact ionization of a neutral gas target, creating a pulse of positive ions. This source is designed to fit the existing aperture elements of the electron gun, requiring that only the filament and grid are replaced by the ion source region. The ions can be injected and stored within the ring, provided that the elements within the ring have had their applied potentials reversed.

The electron gun can be converted to an ion gun by replacing the filament and the grid by an ion source region, leaving the anode and the other aperture elements from the electron gun in place. This ion source region is made from 316 stainless steel and has its own grid and filament, now mounted perpendicular to the optical axis of the gun, together with a Faraday cup and a hypodermic needle. The electron beam created by the filament, a gas target from the hypodermic needle and the extraction aperture, which is part of the optical axis of the storage ring, are all mutually perpendicular to one another, shown in Figures 2.10, 2.11.
The electron beam is pulsed for a fixed amount of time in a method similar to the electron gun pulsing scheme, discussed later in the chapter, allowing for control over the time in which the electrons can interact with the neutral gas. This creates a finite window

Figure 2.10 – Schematic of the ion source gun, showing all the components used to generate, extract and focus the newly formed ion pulse into the storage ring. The vertical solid line is the pathway of the newly created ions, the dotted line is the electron travel and coming out of the page is the direction of the needle that injects the target gas.

Figure 2.11 – A photo of the ion source region, along with the extraction and focusing electrodes. From this viewpoint, the hypodermic needle is seen on the right, the large conical portion is an adaptor for differential pumping of the source region. Mounted on the side of the source is the filament housing.
of time where ions are formed. This electron beam has an adjustable energy, allowing the electrons to interact with the neutral gas with energy between 0 eV and 150 eV. The impact energy is adjusted to allow for the maximization of ionization yield for various atoms or molecules, with a typical impact energy being \( \sim 70 \text{ eV} \) for \( \text{N}_2 \). The electrons are focused by a single lens element which controls the size of the beam and therefore minimizes the distance along the optical axis where the ionization of the target gas occurs. A Faraday cup is located opposite the filament to capture any electrons that do not ionize the gas.

The electron filament, its mount and the lens element are gas tight, leaving the source extraction aperture and the connection to the differential vacuum pump as the only points where the target gas can escape the ion source region, shown in Figure 2.12. By making the extraction aperture much smaller than the pump connection by a factor of \( \sim 10 \), the amount of gas entering the storage ring is reduced and thus contribution to loss of storage due to background scattering is reduced as well.

![Figure 2.12- Photograph of the actual source region, complete with the extraction lenses and the outlet for differential pumping, which is the focus of the rightmost region of the photo.](image)

The weak electric field from the extraction aperture penetrates the source region, such that any ion formed is subject to this weak potential and is gently accelerated and extracted by the “field penetration technique” [9]. These ions then travel into the same deflecting and focusing elements used in the electron gun. Since these are electrostatic elements, the ions will travel in the same manner as the electrons, given the electrostatic fields possess the same magnitude and the charged particle energy and trajectories remain the same as well.
2.5 Hemisphere Pulsing

As mentioned earlier in the discussion of the electron gun, pulsing of the filament and the hemisphere is required to both limit the temporal size of the particle bunch and to inject the particle bunch into the storage ring. Additional pulses are also used in order to turn ‘off’ the hemisphere a second time to eject the particle beam from the ring and to trigger the start signal of the measurement process, discussed in the next chapter. These pulses must be controlled very accurately with respect to one another; to ensure there are no timing errors, a unit operating with a single master clock, a delay-pulse generator (BNC model 555) [10] was used to control all the pulses required.

The system was designed for spectroscopy, and the first particles tested and stored were electrons [11]. This posed one challenge, however, as the low mass of the electrons, coupled with the short orbit distance, meant that the orbiting period is small (~300 ns) and so the pulsing used for injection must be very fast (<25 ns rise/fall time). Ultimately, this is beneficial, as the pulsing for particles such as ions, or more exotic particles, such as positrons or spin polarized electrons would only require the existing circuitry and have no need for upgrading.

In order to create a pulsed electron bunch, required for both electron and ion applications, one or more of the elements of the electron source, the grid, cathode or anode, must be pulsed. In this work, the cathode is the pulsed element, and this is accomplished by pulsing the floating voltage of the filament ~15 V positive for the generation of the electrons. During this pulse, the grid is more negative than the filament and thus accelerates the electrons away from the filament towards the electron anode. This, in essence, turns the electron gun ‘on’, allowing the electrons to travel towards the ring. The rest of the time the filament is more negative than the grid, and the close proximity of the two element forces the electrons towards the grid itself, collecting all the electrons before they can be accelerated into the system by the anode, turning the gun ‘off’. The electron gun operates in the ‘on’ state with a typical width of ~50 ns, and this repeats every ~30 μs.

Hemisphere pulsing requires that all elements are capable of switching from having a common voltage - the pass energy resulting in a field free region - to each element having a specific voltage, creating a radial electric field. This field is one in which charged particles
of ideal conditions (the pass energy on the optical axis) undergo uniform circular motion within the hemisphere. This is depicted in Figure 2.13, along with the electron source pulse.

In order to accomplish this, a TTL 0-5 V pulse with variable width and delay relative to the master clock is sent to an opto-isolator (7721), which isolates this 0-5 V pulse from ground. A complete pulsing diagram [2] can be found in Appendix 1. This allows for the pulse to float with respect to ground; specifically the float potential was the potential of the lower HDA pass energy. This isolated signal was then fed into four distinct summing amplifiers (AD811); one amplifier for each of the inner and outer hemispheres and the inner and outer hoops. These amplifiers float the potential of the hemispherical element of the lower hemisphere with respect to the pass energy when the TTL pulse is at 5 V triggering the storage mode and 0 V with respect to the pass energy potential when the TTL output is at 0 V, triggering the field-free state of the hemisphere [2].

Detection without scattering is possible by dumping the stored particle bunch directly into a particle detector, or through an analyser prior to entering a particle detector. This was accomplished by sending an additional pulse, whose timing properties are independently controlled. This pulse also triggers the hemisphere to return to its field free status, which allows the particles to leave the storage ring.

![Figure 2.13 – Pulsing of the hemisphere, along with the electron source, showing the voltage of the inner and outer elements, as well as the current of the electron pulse during injection. Injection occurs when the hemisphere elements are all at 12 eV. Afterwards, the hemisphere elements return to their potentials used for storage.](image-url)
2.6 Summary

The layout of the apparatus has been presented along with its constituent components: the lenses and the hemispherical deflector analyser. The generation of the charged particle pulse, for both electrons and positive ions, has been described, along with the procedure used to ‘inject’ and ‘extract’ this pulse into and out of the storage ring. Detection of the charged particles was not discussed in this chapter, but will be in Chapter 3, along with the data acquisition method and experimental control software.
2.7 References


    (Accessed September 19, 2013)

### Chapter 3: Data Acquisition Methods

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Introduction to Data Acquisition</td>
<td>39</td>
</tr>
<tr>
<td>3.2 Detection of Charged Particles</td>
<td>39</td>
</tr>
<tr>
<td>3.2.1 Channel Electron Multipliers</td>
<td>40</td>
</tr>
<tr>
<td>3.2.2 Signal Processing</td>
<td>41</td>
</tr>
<tr>
<td>3.3 LabVIEW Programs</td>
<td>44</td>
</tr>
<tr>
<td>3.3.1 Variable Time-of-Flight</td>
<td>45</td>
</tr>
<tr>
<td>3.3.2 Stability Testing</td>
<td>46</td>
</tr>
<tr>
<td>3.4 Data Interpretation</td>
<td>50</td>
</tr>
<tr>
<td>3.5 Summary</td>
<td>52</td>
</tr>
<tr>
<td>3.6 References</td>
<td>53</td>
</tr>
</tbody>
</table>
3.1 Introduction to Data Acquisition

The previous chapter introduced the components of the storage ring and its layout. The generation of the particles used in the device was described, along with the method by which they are injected into the ring for storage. In this brief chapter the method of detection of the charged particle beam is presented, along with the data acquisition process and the software used to perform experimentation. The bulk of the data presented in this thesis was acquired using custom-built software, designed and implemented by the author, written using the LabVIEW [1] platform. Each program was designed to perform specific experiments autonomously, once started, and these programs are described in terms of their functionality. Examples and explanations of the different presentations of the data are given, which is essential to the complete understanding of the remainder of the thesis.

3.2 Detection of Charged Particles

There are numerous detectors that surround the storage ring, placed to measure particles that result from the storage of the charged particle pulse. These may include measuring scattered particles, which result from collisions between the pulse and a neutral gas target, or measuring the beam as it is purposely ejected. The latter is most useful for time-of-flight measurements. The detectors can be configured to measure either positive or negative particles and both types can be used to measure neutral metastable atoms. The positions of the detectors are shown in Figure 3.1.

The detectors used are Channel Electron Multipliers (CEM) and a Faraday cup. The Faraday cup is located opposite the particle source and is only used for the optimization of the injection particle flux. This is done while both hemispheres are ‘off’ (i.e. both field-free regions). The CEMs are the main detectors used to measure the stored particles and can be fitted with various adaptors designed to accept specific particles (i.e. positive ions, electron, etc.) or to measure energetically resolved scattered particles through the use of an additional hemispherical analyser. This type of single particle detector is well suited for this
Figure 3.1 – The positions of the various detectors relative to the storage ring is shown above. The detectors used consist of channel electron multipliers (CEM) and a Faraday Cup (FC).

ring, as the low number of particles in the beam coupled with the low probability of a scattering event makes other methods (i.e. a pickup electrode) unfeasible. Since the CEM is the main detector used, their method of operation is described in more detail, in the next section.

3.2.1 Channel Electron Multipliers

A CEM is a detector that creates a voltage pulse from a single particle, and the continuous dynode type used in this apparatus, Philips model X919BL, was developed in the 1960’s [2]. This type of CEM generally consists of a glass tube coated internally with a semi-conducting, secondary emissive material. A large potential difference (~ 2000 V) is placed across the ends of the tube. In our application, the entrance end has a large collecting cone (~10 mm in diameter) with an opening into the narrow glass tube of ~1 mm in diameter, and the opposite (collection) end has a metal plate that collects the signal resulting from a particle entering the detector. The resistive surface allows for a continuously more positive potential from the entrance cone, where the particle enters the detector, to the
collection end. These detectors allow an incoming particle (of energy \( \sim 50-100 \) eV) to strike the entrance cone, which liberates an electron. This electron then is accelerated into the glass tube by the positive potential between the entrance cone and the tube. The electron will accelerate and strike the walls of the tube creating a few secondary electrons, each of which is accelerated by the more positive voltage farther into the tube. These newly created electrons strike the emissive walls deeper in the tube and each electron creates several more electrons. This process repeats until there are enough electrons to create a detectable charge pulse at the collection end of the tube. For the detection of positive ions, the entrance cone is biased to \(-2000\) V and the collection end is at 0 V, while for electrons the entrance cone is \(-100\) V and the collection end held at +2000 V. Once a detectable pulse has been generated for a particle incident on the detector, the signal is separated from the applied voltage through the use of a decoupling capacitor and it is sent through processing electronics, described in the following section.

### 3.2.2 Signal Processing

Each charge pulse created by the secondary electrons within the CEM represents an individual particle entering the detector. These pulses are sent through a series of electronics that amplify and filter the pulses that result from the CEM and the BNC unit, shown in Figure 3.2. First, the pulse from the CEM is sent to an amplifier (Phillips Scientific model 777), which helps to further increase the strength of the measured signal. This is necessary because the voltage pulse leaving the CEM is still weak, despite the build up of a large number of electrons.

After amplification, the signal is sent into a fast discriminator (Ortec model 934), which only accepts pulses that fall above a minimum threshold voltage, any pulses lower than this minimum are rejected and no output pulse is generated. The discriminator is necessary to remove the ‘dark counts’ that originate from within the CEM. These dark counts are caused by the thermionic emission of electrons at various positions along the glass tube due to the applied voltage. These electrons do not travel the full length of the tube, as their point of origin inside the tube is random. As a result, the cascade of secondary electrons
Figure 3.2 – The electronic circuit of the storage ring. The BNC unit is the main component, generating the pulses used to control the filament, the bottom hemisphere and the stop input of the Time to Amplitude Converter (TAC). Electrons (or ions) from the ring enter a CEM, either by scattering or a beam dump, create a signal that is processed through an amplifier, discriminator and a TAC (which is built into the PCI card used for ions), before entering a PCI card and interpreted through software.

is smaller and these types of pulses are weaker, which makes them easily eliminated by a minimum threshold.

For the detection of electrons, the output pulse from the discriminator is sent to a Time-to-Amplitude Converter (Tennelec TC826), or TAC. The TAC requires two input signals, a start and a stop pulse, and is capable of outputting a signal with amplitude that is proportional to this time difference of the start and stop signals. The signal from the discriminator triggers the start input of the TAC, and an additional signal is sent from the BNC unit to trigger the stop, creating an output pulse.

This output pulse is fed into the computer via a PCI card (Ortec Trump Pci-2k Multi-Channel Analyser) which analyses the amplitude of the pulse. This is recorded by software (Maestro-32 v5.3, [3]) that acts as a large histogram with 2048 channels (or bins). When a signal is received the software will add 1 to the appropriate bin, which is determined by the amplitude of the signal from the TAC. This process is repeated a large number of times and the result is a timing spectrum for the stored electron bunch. Figure 3.3 shows a timing spectrum as a result of electrons scattering from a neutral He target. There is a series of peaks, with each representing another pass of the stored electron pulse through the target region. Thus, each successive peak indicates another completed orbit of the pulse within the
Figure 3.3 – Timing spectrum of scattered electrons from a neutral gas target given in a logarithmic plot. Each of the peaks represents an orbit of the bunch through the storage ring, which has successfully contained the electrons for over 30 µs after the injection of the bunch.

There are several properties that can be obtained from this particular type of data display, such as orbit times, the exponential rate of decay and the peak width of the pulse. Analysis of these spectra is discussed in Chapter 6.

The detection of ions follows a similar process; the main difference is that the TAC is built into the data acquisition card (Ortec MCS-PCI) which also has its own software (MCS-32 v.2.11, [3]). The discriminator pulse is sent directly to the computer interface, along with a pulse from the BNC unit. The ion software functions in the same manner as the
electron software, though the number of channels is capable of scaling to 65536. This produces data which looks similar to the electron data shown in Figure 3.4; the most notable distinction being the time scale used in each. As for electrons, each peak represents the bunch after a number of orbits have been completed.

While the data acquisition electronics for detecting electrons and ions essentially function in the same manner, a single electronic circuit could not be used for both, due to the differing time scales. The electron orbit is typically 250-350 ns, while the ion orbit, depending not only on the energy but also the mass, has been observed to range from 30 to 200 µs. This is the only difference in the data acquisition process for electrons and ions, the other aspects of the experiment, such as the LabVIEW programs, do not change, and are presented in the next section.

3.3 LabVIEW Programs

In order to facilitate various types of experiments, LabVIEW was used to control voltages, pulse sequencing and the measurement process to obtain a higher degree of efficiency. Utilizing this platform, custom, purpose-built programs were developed and implemented by the author. The automation of the storage ring is new in this work, and has allowed for large amounts of data (with ~5000 data points) to be acquired in a short time (i.e. a few hours). Longer measurement sets that take the entire night or weekend are now possible, and have been successfully completed. This has allowed for extensive testing of the ability of the storage ring to maintain the particle bunch for a large number of orbits (i.e. its stability). Phenomena, such as surface insulation and changes in the source flux, will eventually place a limit on the overall length of a measurement. The storage ring is a passive device with no active feedback mechanisms, so it will not correct for any of these issues once an experiment is started. The next sections describe the stability software, which has been used to obtain much of the data in chapters 4 and 5, as well as the variable time-of-flight software, which has been used in Chapter 6 to obtain the ion data and Chapter 7 to operate the storage ring as a mass spectrometer.
3.3.1 Variable Time-of-Flight

Detection of the pulse without a target is possible if the lower hemisphere is pulsed a second time and the pulse is dumped directly into a CEM located beneath the hemisphere, on the optical axis of one of the lens stacks. This method of detection is based on a time-of-flight technique, where the detector is placed a specific distance away from a source. The particles are able to drift towards the detector and can separate based on their energy and mass. In this device, the detector can be moved to various different distances by allowing the particles to complete more, or less, orbits of the storage ring before detection. This type of operation is used to generate timing spectra, such as that seen in Figure 3.5, without scattering from a target, and is the method in which much of the data presented in chapters 6 and 7 have been obtained.

Figure 3.5 – Pulse sequencing shown for the two hemisphere pulses. The first pulse allows the particle beam to ‘inject’ into the storage ring. The second pulse ejects the particle beam out of the storage ring, into a detector. This second pulse is shifted relative to the injection pulse, shown above for three distinct times. The software is only able to detect particles that correspond to the second pulse.
A second pulse from the BNC unit is used to turn the hemisphere ‘off’. This pulse occurs at a time that is delayed from the initial injection pulse, discussed in Chapter 2. During this time, the hemisphere is field free, allowing the particle bunch to leave the storage ring. In order to acquire a complete timing spectrum, the second hemisphere ‘off’ pulse must be shifted computationally through the time window of interest. This is accomplished by altering timing between the two pulses, namely by delaying the second pulse with respect to the first, as shown in Figure 3.5. The delay commands are sent from the software, over the General Purpose Interface Bus, GPIB (NI PCI-GPIB), and to the BNC pulsing unit. This shift of the second hemisphere is easily expressed as a function of the current loop iteration, shown by:

\[ T_{\text{current}} = T_{\text{initial}} + n\Delta T \]  

(3.3)

This expression is contained within a ‘timing loop’, which completes each of the iterations of the loop in a fixed, user-defined amount of time, sending the result of this calculation once per iteration.

### 3.3.2 Stability Testing

The stability testing software was designed to provide experimental verification of the long-term storage of charged particles. As will be discussed in chapters 4 and 5, stable storage, or stability, depends critically on the voltages placed on the middle lens elements, \( V_2 \). These voltages will change the properties of the electrostatic lenses in the two lens stacks, such as magnification and the focal lengths, which will change the operation of the storage ring. Stability regions, such as Figure 3.10 later in this chapter and in chapters 4 and 5, were obtained by varying the voltages in the lenses and measuring the pulse after it has completed multiple orbits.

For this measurement of particle storage over a range of lens potentials, the pulses are routed from the discriminator into a National Instruments PCI counter/timer (NI PCI-6601). Pulses are counted for a user-defined period of time, typically around 1-5 seconds. Once this interval is finished, the number of counts is recorded as an element within an array and saved in a text file. The value saved at each element represents the number of particles measured.
by the detectors after a specific amount of time following injection. To test for long-term stability, it is advantageous to remove the counts received during the first several orbits. This is accomplished through the hardware by sending a TTL pulse from the BNC unit to the discriminator, which ‘gates’, or ignores, counts that arrive in the interval of time given by the width of the TTL pulse, which typically corresponds to ~10 orbits. The voltage combination is then changed and the measurement process repeated.

The software is used to define a voltage on the middle lenses for one or more lenses. The voltages are scanned in a raster pattern, shown in Figure 3.6, and the stored pulse is measured at all possible voltage combinations, as defined by the user. Computationally, this is accomplished through the use of nested ‘for’ loops and the definition of

Figure 3.6 – Raster scanning pattern used to test for long-term stability within the storage ring. The number of elements is dependent on the range of potentials and the step size; both are user defined.

three key values for each of the two lens potentials: the initial voltage $V_{\text{initial}}$, final voltage $V_{\text{final}}$, and the voltage step $\Delta V$. Each loop is tied to one of the lenses, and the lens values relate to their respective loop iteration number, $n$, by the simple formula:

$$V_{\text{current}} = V_{\text{initial}} + n\Delta V$$  \hspace{1cm} (3.1)

with a constraint applied that limits the number of voltages. In the case for a positive voltage step,

$$V_{\text{current}} \leq V_{\text{final}}.$$  \hspace{1cm} (3.2)

The position within the array of the result of this software represents a specific combination of the lens voltages, with one of the lenses representing the horizontal and the other the vertical axis of the array.

The voltages are generated using a National Instruments multifunction data acquisition card (NI PCI-6221), which has two 16-bit analog outputs. These outputs are capable of producing a voltage from 0 to 10 V with respect to true ground. In order to
increase this range, the voltages are sent through an amplifier, which linearly scales the voltages up by a factor of ~15, giving a usable range from 0 to ~150 V. The software has a user interface to simplify the start of testing, shown in Figure 3.7, where the input data is sent to the program, Figure 3.8, which functions as described previously.

The previous two software programs can be combined to save a timing spectrum, rather than a count number, for each point within an area. This will retain the specific information given in the spectral data. The result is a very long scan however, unless the number of voltage combinations is small, or only a single loop variable is used creating a line scan. The next section will present the different types of figures used in this thesis, along with their relation to one another.
Figure 3.8 – Stability testing software, coded in LabVIEW. LabVIEW has many pre-built routines that are represented by unique icons and require specific data to input and output for each one. The icons all are tied together using coloured wires, which represents the type of data that is sent from one icon to the next. Looping is indicated by each of the grey boxes above; one for the control of each of the voltages and a third that enables the entire experiment to repeat.
3.4 Data Interpretation

As will be seen in the rest of this thesis, there are several different sets of measurements that can be taken to characterize the storage within the ring. In this section, the data sets, along with the way that each set relates to one another, will be presented so that the meaning of each type will be clear. The figures presented from the various measurements have been created using Mathcad [4].

Figure 3.9 – Breakdown of the ‘line scan’ figures. The bottom is a logarithmic plot of the timing spectrum, which is converted into a coloured bar (middle) based on the amplitudes of the channels. Finally, the bar is combined with several others to form the complete picture seen at the top, where the boxed region is the dataset’s position in the figure.
The main type of data acquired is the standard spectrum dataset, seen in Figures 3.3 and 3.4, and have already been described. Another type of frequently used set of measurements is the result of a ‘line scan’, which moves the lens potentials, $V_2$, by a fixed amount, based on equation 3.1. At each pair of $V_2$ lens potentials, a timing spectrum is obtained. Combining several individual timing spectra to form an array creates this type of data. This is easy to do since the spectral data is a histogram whose values are numerically represented as a column, and these columns can be appended into a matrix. The first column represents the first scan, appended to that is the second scan, and the process is repeated for all the individual timing datasets. Each number in the array is then given a colour, based on its percentage relative to the maximum value in the array; red represents a high amount of storage, blue represents a low amount of storage and following the colour spectrum in between. The graphs, after a transposition, are displayed in colour, with the time after injection on the horizontal axis and the lens potential(s) on the vertical. An example of this data is given, as well as a breakdown in Figure 3.9.

Another set of measurements presented in this thesis is the ‘area scan’. In this type of measurement, all timing information is removed from the visual presentation. Like the line scan, the area scan is an array of numbers, however, in the line scan, each row represents an entire spectrum for single combination of lens voltages. In the area scan, each number in the array is representative of the integrated spectrum received for a unique combination of lens potentials. The axis labels on this type of figure are typically lens 1 & 2 vs. lens 3 & 4 $V_2$ potentials, or lens 1 & 4 vs. lens 2 & 3 $V_2$ lens potentials. The data is not necessarily the total counts received for a specific combination of potentials, but conditions can be applied in order to reduce the bias from the first few peaks, such as only allowing counts that arrive after a specific time to contribute towards the dataset. This allows for the study of long term recycling, after which other types of data can be acquired to observe peak width and lifetimes of the strongest regions of recycling. An example and its relation to the line scan can be found in Figure 3.10.
Figure 3.10 – The connection between the ‘area scans’ (right) and the ‘line scans’ (left) is made by setting a minimum time after injection, shown above left by the black line, and integrating all counts that occur after this time. This will turn the array into a vector, shown by the coloured bar in the center. For this example, the line scan falls along the white line indicated on the area scan.

3.5 Summary

The detection and data acquisition process has been discussed in detail. A method to test for the presence of a stored pulse within the ring and a time of flight measurement process has also been given, which has allowed for the detection of the stored particles without a scattering medium. These software programs, developed and implemented by the author, are used in the acquisition of the data that is presented in the rest of the thesis. In Chapter 4, a matrix formalism for the theoretical stability regions the storage ring for a ‘symmetric’ configuration based on the use of transfer matrices will be presented along with the experimental result of stability, acquired using a ‘line scan’. Chapter 5 will present the extension of the matrix formalism to a configuration that is ‘asymmetric’, with experimental data acquired using the ‘area scan’.
3.6 References

   (Accessed September 19, 2013)
   (Accessed September 19, 2013)
   (Accessed September 19, 2013)
Chapter 4: Electrostatic Optics and the ‘Symmetric Condition’

4.1 Introduction 55
4.2 Electrostatic Optics 55
  4.2.1 Lenses 57
  4.2.2 Hemispherical Analysers 59
4.3 General Storage Criteria 60
4.4 Symmetric Case 62
4.5 General Transfer Matrix 63
4.6 Experimental Results 67
4.7 Summary 69
4.8 References 71
4.1 Introduction

In Chapter 2, the storage ring was physically described in terms of the size and overall layout of the device. While the ‘racetrack’ configuration was presented, there was no mention as to how the potentials of each of the elements, mainly the lenses, were determined. This chapter develops an approach to determine the stability of the storage ring based on transfer matrices from charged particle optics for the case of a ‘symmetric’ potential configuration (i.e. in which each of the four lenses operate with the same potentials). As in standard optics, a transfer matrix for every region, in this case for each region involving an electric field, can be used to describe the result of the beam travelling through that particular region. This chapter will describe the derivation of an overall transfer matrix, which is a mathematical representation of the combined elements that form the ring, as described in Chapter 2. The overall transfer matrix determines how the position and trajectory of a particle change after a single orbit of the storage ring. Constraints are applied to this matrix in order to allow for storage and compared with experimental results, which have been obtained using the software discussed in Chapter 3. The next section introduces the topic of charged particle optics, with a focus on cylindrical lenses and hemispherical deflector analysers.

4.2 Electrostatic Optics

Electrostatic elements can be described through the use of transfer matrices. These matrices can be used to predict the change in position and trajectory of a charged particle moving in an electromagnetic field, and in particular we are interested in their movement within electric fields since the focus of this thesis is on electrostatic devices.

A particle moving along the optical axis (in the $z$ direction) within the beam can be expressed by its distance away from the optical axis in both the $x$ and $y$ directions, as well as by the angle with respect to the propagation axis in both the $x$ and $y$ directions, given by $x'$ and $y'$. An electrostatic element will manipulate the beam and potentially change all of the variables that describe the particle’s motion. Just as in light optics, this element can be described by using a matrix, which when it acts on the initial vector of parameters of the
particles, gives the final parameters, and ultimately, the final representation of the transferred beam. This can be expressed as

\[
\begin{bmatrix}
x'_2 \\
x'_1 \\
y'_2 \\
y'_1 \\
\end{bmatrix} =
\begin{bmatrix}
a_{00} & a_{10} & a_{20} & a_{30} \\
a_{01} & a_{11} & a_{21} & a_{31} \\
a_{02} & a_{12} & a_{22} & a_{32} \\
a_{03} & a_{13} & a_{23} & a_{33} \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
y_1 \\
y_2 \\
\end{bmatrix}
\] (4.1)

for an initial state 1 and final state 2. For example, quadrapole lenses are described in this manner, as the \( x \) and \( y \) directions are independent from one another. These lenses focus in one direction while defocusing in the other. In order to contain the beam in both directions, quadrapole lenses appear in pairs where the focusing and defocusing axis alternate, such that the overall combination is a converging lens. These lens combinations are used in some of the storage devices mentioned in the introduction, such as ELISA and MULTUM [1,2].

In the case of a cylindrically symmetric lens, the transfer matrix can be simplified, requiring only two parameters to describe the particle’s motion: the radial distance \( r \), and the angle \( r' \), with respect to the optical axis. This leads to a 2 x 2 matrix of the form

\[
\begin{bmatrix}
r'_2 \\
r'_1 \\
\end{bmatrix} =
\begin{bmatrix}
b_{00} & b_{10} \\
b_{01} & b_{11} \\
\end{bmatrix}
\begin{bmatrix}
r_1 \\
r_2 \\
\end{bmatrix}
\] (4.2)

This type of matrix describes both aperture and cylindrical lenses which are used in the present storage ring.

When a particle travels through multiple elements, each individual transfer matrix is multiplied in sequence to give an overall transfer matrix that describes the effect of all the elements on the particle. Consider a particle in initial state \( A \) that travels through \( N \) elements, each with a transfer matrix given by \( M_i \), for \( i = 1,2,..,N \) and the index value is also the order that the particle travelled through the elements. Then the new state of the particle given by \( B \) is expressed as:

\[
B = M_N \times M_{N-1} \times \ldots \times M_2 \times M_1 \times A
\] (4.3)

A single matrix, determined through the use of equation 4.3 and the use of the individual transfer matrices for cylindrical lenses and hemispherical deflector analysers, can represent the storage ring. The next sections will discuss lens and hemispherical deflector analysers, along with their respective transfer matrices.
4.2.1 Lenses

Electrostatic lenses consist of two or more voltage elements that serve to manipulate the trajectories of a particle beam in a similar manner to lenses in light optics. The main difference between the two is that in traditional light optics, the interface between two media with different indices of refraction is well defined. The angle between this interface and the trajectory of the incoming ray within the first medium determines the trajectory of the ray within the second medium. The bending of the trajectories only occurs at these boundaries. In charged particle optics, there is no clear boundary between the two media; in this case the media are the voltage potentials. Here there is a continuous change in potentials over a given distance, and the trajectories are bent gradually, rather than discontinuously as in light optics. The angle between the incoming trajectory and the equipotential curve at that point in space is what determines the new trajectory, and this process is repeated until the potential differences go to zero. This is more clearly shown in Figure 4.1.

![Figure 4.1](image)

The lenses used in the present storage ring are treated as three element cylindrical lenses with the same inner radius, and behave like optical thick lenses, like the one shown in Figure 4.1. Each lens contains two principle planes, indicated by $PP_1$ and $PP_2$, as well as two focal lengths, one corresponding to each of the principal planes and indicated by $f_1$ and $f_2$. Unlike light optics however, the principal planes are always reversed for electrostatic optics in the case of an accelerating lens [4]. The object and image distances are indicated by $P$ and $Q$, respectively. $K_1$ and $K_2$ represent the distance from the focal point to the object.
or image and finally $F_1$ and $F_2$ are the midfocal lengths, which can also be expressed as $F_1 = P - K_1$ and $F_2 = Q - K_2$, as seen in Figure 4.2, with a schematic of the physical lens shown in Figure 4.3. For a given object distance, there are two voltage potentials that will allow the lens to place the image in the same position along the optical axis. These two voltages, one high and one low, function by compressing (or expanding) the beam towards (or away) the optical axis.

![Figure 4.2](image1.png)

Figure 4.2 – The schematic diagram for an optical thick lens showing the principle planes, $PP_1$ and $PP_2$, principle foci, $PF_1$ and $PF_2$, the focal lengths, $f_1$ and $f_2$, the midfocal lengths, $F_1$ and $F_2$ and the object and image distances, $P$ and $Q$. [5]

![Figure 4.3](image2.png)

Figure 4.3 - Schematic of one of the cylindrical lenses used within the ring, indicating the object and image positions relative to the elements within the lens. [5]
In the storage ring, $P$ and $Q$ do not correspond to the positions of an object and image, but rather to key positions along the track. The four lenses separate the source from HDA 1, HDA 1 from the target, the target from HDA 2 and finally HDA 2 from the source. Thus $P$ and $Q$ represent the physical distances from the center of the source (or target) regions to the entrance of HDA 1 (or HDA 2) or vice versa.

Similar to optical systems, electrostatic lenses can be written as a matrix, which can be used to determine the final position of a particle from the optical axis and angle of the trajectory with respect to the optical axis, given an initial position and trajectory. This transfer matrix can be expressed as [6]:

$$m_{\text{lens}} = -\frac{1}{f_2} \begin{bmatrix} K_2 & K_1K_2 - f_1f_2 \\ 1 & K_1 \end{bmatrix}$$

(4.4)

Each lens in the storage ring can be expressed in this manner, and just as in optics, an overall transfer matrix can be found via matrix multiplication if the particle travels through multiple lenses.

### 4.2.2 Hemispherical Analysers

Hemispherical analysers have the property of focusing in the two dimensions perpendicular to the particle motion. This allows for an image of the particles to be formed at the exit of the hemisphere based on how the particles arrived at the entrance. Hemispherical analysers do not just reflect the beam at the exit, but also filters the particles according to energy. This is easily explained as the particles moving too slowly follow a radius of curvature too small and do not pass through the exit aperture. Particles travelling too quickly follow a radius too large and again do not pass through the exit aperture. This leaves a finite range of energies that will successfully travel through the hemisphere. This range is determined not only by the energy, but also by the mean radius of the hemisphere and the initial trajectory of the particle, as they travel along elliptical orbits for the most general case. The formula to determine the energy resolution of an analyser is approximated by [7]:

$$\frac{\Delta E_{\text{FWHM}}}{E_0} \approx \frac{r_s}{R_0} + 0.25\alpha^2$$

(4.5)
where \( r_s \) is the size of the apertures at the entrance and exit of the hemisphere, \( \alpha \) is the beam half angle, \( R_0 \) is the mean radius of the hemisphere, \( E_0 = eV_0 \) is the nominal pass energy and \( \Delta E \) is the full width at half maximum of the range of energies that will successfully travel through the hemisphere.

The transfer matrix for an idealized hemisphere is given by:

\[
\begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\]  

which clearly would produce an inverted image of the entrance image. This is not a true image, as aberrations, such as chromatic, have been ignored in this first-order approximation; higher order aberrations have been considered by Hammond et al. [8] This still, however, indicates that the device has ‘mirror-like’ properties, allowing the beam to be “reflected” onto a parallel optic axis that is separated from the original axis by a distance equal to the mean diameter of the hemisphere.

### 4.3 General Storage Criteria

When a transfer matrix has been developed for an electrostatic system, the present position and trajectory of a particle can be determined after travelling through the region represented by this matrix. In the case of the charged particle storage ring presented in Chapter 2, the entire ring can be represented by an overall transfer matrix, which consists of the individual lens and hemisphere matrices. Travelling multiple complete ‘orbits’ through the ring means the transfer matrix for a single orbit is applied multiple times, one for each orbit of the particle. This places a constraint onto the transfer matrix \( M \) for a complete revolution of a particle which is given [9] by

\[
\frac{1}{2} |Tr(M)| \leq 1.
\]  

This condition prevents the absolute value of the combined linear and angular magnifications from being greater than 2, represented by the matrix terms \( b_{00} \) and \( b_{11} \), respectively, from equation 4.2. Within the ring, the position and trajectory of a particle is physically limited through the use of apertures. Each of the four lenses contains two apertures - one to limit the size of the beam and another to limit the angular range of the particles that can pass through
each lens. If this condition is not met, the position and angle of trajectory of the particles would increase with each completed orbit and eventually their storage would be terminated by one of the lens apertures.

The factors that affect the angular spread of the beam are the acceleration ratio of each of the lenses and the position from the optical axis of the particle. This is expressed by the product of the beam parameters: position, trajectory and energy, which is a constant. This is given as:

\[ r \sqrt{eV} \sin \phi = a, \]  

(4.8)

where \( r \) is the distance of the object from the optical axis, \( eV \) is the energy and \( \phi \) is the pencil angle, half of the total angular range of trajectories possible from a point in the object, and \( a \) is the resulting constant. Since this product is a constant, it can be used to relate the object side of a lens to the image side, and an electron optical analog of Abbe’s sine condition from optics, with \( \sqrt{eV} \) replacing the index of refraction \( n \), can be expressed [10]:

\[ r_1 \sqrt{eV_1} \sin \phi_1 = r_2 \sqrt{eV_2} \sin \phi_2, \]  

(4.9)

where the indices 1 and 2 indicate the object side and the image side, respectively. For paraxial trajectories, which as we will see are appropriate for this work, this is called the Helmholtz-Lagrange equation for an electron lens [11] and is given by:

\[ r_1 \sqrt{eV_1} \tan \phi_1 = r_2 \sqrt{eV_2} \tan \phi_2. \]  

(4.10)

This relationship between the acceleration ratio and the angular spread can be seen in Figure 4.1, in which the slower particles (18 eV) have a larger angular range than the faster particles (54 eV) and the object/image sizes are the same.

The Helmholtz-Lagrange relationship is built into the matrix formalism through the determinant, and for a matrix \( M \), it is expressed as

\[ |M| = \frac{f_1}{f_2} = -\frac{eV_1}{\sqrt{eV_2}}. \]  

(4.11)

The matrix form for a single lens, equation 4.4, is in agreement with this expression. One of the constraints that can be applied to the matrix formalism is due to the periodic nature of the storage ring. The overall transfer matrix is the representation of a complete orbit within the ring, and while the position, trajectory and energy of a particle will change as it travels around the ring, when it crosses any given reference plane, it must possess the same energy
as it had on the previous orbit, which simplifies the right hand side of equation 4.11 to unity, placing a condition on the combinations of focal lengths used for all the lenses.

4.4 Symmetric Case

The purpose of Section 4.2 was to give background knowledge of charged particle optics, specifically, the components that form the storage ring at the University of Windsor. This will allow for the formulation of the theoretical limits of stable storage, based on the transfer matrices of the individual components, which arise from the overall transfer matrix when the constraints of Section 4.3 are applied.

As mentioned in Chapter 2, the storage ring consists of two hemispheres that are separated by two sets of lenses, each set containing two lenses. The simplest operation of the ring occurs when both hemispheres possess the same pass energy and all four lenses operate in the same manner. This makes the ring a fully symmetric setup, as can be seen in Figure 4.4, and will allow for the study of the conditions for the long-term storage of the particle beam. It is simpler to consider the ring consisting of two cells, each cell containing one hemisphere and the two adjacent lenses. A transfer matrix $M_{st}$ is used to represent the source*-to-target transformation and a second matrix $M_{ts}$ for the target-to-source

* The term source is maintained for historical reasons, referring to an electrode and not the actual particle source.
transformation. This allows the overall transfer matrix $M_{ss}$ for source-to-source to be expressed as:

$$M_{ss} = M_{ts}M_{st} \quad (4.12)$$

In order for full symmetry to occur, the two lenses that are used to transport the particles into the hemisphere must be the same as each other, and the two lenses that are used to collect the particles upon leaving the hemispheres must also be the same. Furthermore, the lenses collecting the particles at the exit of the hemisphere are the time reversal of the entrance lenses. This time reversed lens has the same form as equation 4.4, but the object and image points of interest have reversed, thus the indices on the transfer matrix are reversed, giving:

$$m_{\text{lens-time reversal}} = -\frac{1}{f_1} \begin{bmatrix} K_1 & K_1K_2 - f_1f_2 \\ 1 & K_2 \end{bmatrix} \quad (4.13)$$

For a charged particle to travel from the source region to the target region, it must go through an entrance lenses, followed by one of the hemispheres and finally travel through the collection lens. The half orbit transfer matrix is found after multiplying the transfer matrices in the same manner as in optics:

$$M_{st} = m_{\text{lens-time reversal}}m_{\text{hemisphere}}m_{\text{lens}}. \quad (4.14)$$

Completing the multiplication using equations 4.4, 4.6, and 4.13 leads to

$$M_{st} = \frac{1}{f_1f_2} \begin{bmatrix} f_1f_2 - 2K_1K_2 & 2K_1(f_1f_2 - K_1K_2) \\ -2K_2 & f_1f_2 - 2K_1K_2 \end{bmatrix}. \quad (4.15)$$

The target-to-source cell has its electrostatic components in exactly the same manner as the source-to-target cell, and with the lens and hemisphere symmetries imposed earlier, their transfer matrices will be identical, $M_{ts} = M_{st}$.

### 4.5 General Transfer Matrix

It is well known from circular accelerator theory [13, 14] that the overall transfer matrix $M_{ss}$ for a single orbit can be written in a generalized form that is similar to a
rotation matrix:

\[
M_{ss} = \begin{bmatrix}
\cos \theta & -L \sin \theta/L \\
\sin \theta & \cos \theta
\end{bmatrix}.
\] (4.16)

$L$ has units of length and $\theta$ physically corresponds to the rotation of the phase space ellipse, which is limited to an angle between 0 and $2\pi$. This matrix clearly has a determinant of 1, satisfying equation 4.11, which is expected since there is to be no overall acceleration of the particles when they return to the same position, and electrostatic potential, in the ring. It also satisfies the trace condition, equation 4.7, as the trace is simply $2 \cos \theta$.

As the particle travels through $N$ orbits, several of the overall transfer matrices are multiplied together to give the current position and trajectory. This matrix is given by

\[
M_{ss}^N = \begin{bmatrix}
\cos \theta & -L \sin \theta/L \\
\sin \theta & \cos \theta
\end{bmatrix}^N = \begin{bmatrix}
\cos N\theta & -L \sin N\theta/L \\
\sin N\theta & \cos N\theta
\end{bmatrix}.
\] (4.17)

This can be proved by induction (in Appendix 2). The ring consists of two identical cells, and similar expressions can also be used to represent the half orbit transfer matrix in a general case, which is given by:

\[
M_{st} = \begin{bmatrix}
\cos(\theta/2) & L \sin(\theta/2) \\
\sin(\theta/2) & \cos(\theta/2)
\end{bmatrix}.
\] (4.18)

The matrices given by equations 4.15 and 4.18 represent the same change in the motion of the particle, and by equating the corresponding the matrix elements, the following expressions can be found for $\cos(\theta/2)$ and $L^2$:

\[
L^2 = \frac{K_1}{K_2} (f_2 f_1 - K_1 K_2)
\] (4.19)

\[
\cos(\theta/2) = 1 - \frac{2K_1 K_2}{f_1 f_2}.
\] (4.20)

As can be seen, for $K_i \geq 0$, the minimum value of the product $K_1 K_2$ is 0, but this leads to an undefined $L^2$ term when $K_1 = 0$ or $K_2 = 0$. The maximum value for the product $K_1 K_2$ is $f_1 f_2$ in order to keep $L$ real. These extreme values lead to a half orbit transfer matrix equal to
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\quad \text{or} \quad 
\begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\] respectively; however, Hammond et al. (2009) have shown that these endpoints lead to an overall instability, such that the beam size and angle will expand as \( N \) increases [8]. This leads to a range of stable values for the product \( K_1K_2 \) that is given [8] by

\[0 < K_1K_2 < f_1f_2, \quad (4.21)\]

which can be seen in Figure 4.5.

While the single half orbit transfer matrix is unstable if it is equal to \( \pm I \), more than one half orbit transfer matrices multiplied together can equal \( \pm I \). Physically, the particles will eventually return to their starting conditions, or the inverted starting conditions, only after travelling through multiple half orbits. The product \( K_1K_2 \) falls within the range given by equation 4.21 and the resulting matrix

\[
M^H_{st} = \begin{bmatrix}
\cos \frac{\theta}{2} & L \sin \frac{\theta}{2} \\
\sin \frac{\theta}{2} & \cos \frac{\theta}{2}
\end{bmatrix}^H = \begin{bmatrix}
\cos \frac{H\theta}{2} & L \sin \frac{H\theta}{2} \\
\sin \frac{H\theta}{2} & \cos \frac{H\theta}{2}
\end{bmatrix}, \quad (4.22)
\]

allows for a stable beam. After travelling \('H'\) half orbits and returning to the original or

\[V_2\]

Figure 4.5– Shows the range for the value \( K_1K_2/ f_1f_2 \) as a function the lens potential as well as the trace condition for the overall transfer matrix for a system that has an energy of 36 eV in the source and target regions, and 18 eV in each of the hemispheres. Both conditions are simultaneously satisfied for the range of \( V_2 \) values from ~3 V to ~7 V and 85 V to 160 V, with the range in between omitted. [8]
inverted conditions, the matrix, given in equation 4.21, can be equated to ±I, resulting in

$$\cos\left(\frac{H\theta}{2}\right) = \pm 1.$$  

The angle to acquire the limits can be found by setting $$\frac{H\theta}{2} = m\pi$$, where $$m$$ is any integer in the range $$0 < m < H$$. The rotation of the phase space ellipse for each half orbit is $$\frac{\theta}{2} = \frac{m\pi}{H}$$, and substituting this angle back into equation 4.20 and applying the half angle formula for $$\cos\theta$$ results in

$$\cos\left(\frac{m\pi}{H}\right) = 1 - 2\sin^2\left(\frac{m\pi}{2H}\right) = 1 - \frac{2K_1K_2}{f_1f_2}.$$  

(4.23)

This yields the final condition [8] for stable orbits:

$$\sin^2\left(\frac{m\pi}{2H}\right) = \frac{K_1K_2}{f_1f_2}.$$  

(4.24)

which is ultimately determined by the $$(H, m)$$ modes, provided that the fraction $$\frac{m}{H}$$ is irreducible, so as to give a unique value for the right hand side.

For the case of a particle completing $$H$$ half-orbits, the appropriate values for $$m$$ are 1, 2,.., $$H - 1$$. The transfer matrix after completing $$H$$ orbits is given by equation 4.22. For $$m = \text{odd}$$, the condition for the rotation of phase space is $$\frac{\theta}{2} = \frac{m_{\text{odd}}\pi}{H}$$.

Substituting this in equation 4.22 gives

$$M_{st}^H = \begin{bmatrix} \cos\left(\frac{Hm_{\text{odd}}\pi}{H}\right) & L\sin\left(\frac{Hm_{\text{odd}}\pi}{H}\right) \\ -\sin\left(\frac{Hm_{\text{odd}}\pi}{H}\right) & \cos\left(\frac{Hm_{\text{odd}}\pi}{H}\right) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$  

(4.25)

for any odd integer of $$m$$. This implies that for odd values of $$m$$, the particle will return to an inverted position and trajectory after travelling $$H$$ half orbits in the system. Similarly, for $$m = \text{even}$$, $$\frac{\theta}{2} = \frac{m_{\text{even}}\pi}{H}$$ which leads to

$$M_{st}^H = \begin{bmatrix} \cos\left(\frac{Hm_{\text{even}}\pi}{H}\right) & L\sin\left(\frac{Hm_{\text{even}}\pi}{H}\right) \\ -\sin\left(\frac{Hm_{\text{even}}\pi}{H}\right) & \cos\left(\frac{Hm_{\text{even}}\pi}{H}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$  

(4.26)
Therefore, for even values of \( m \), the particle returns to its initial position and trajectory after completing \( H \) half orbits in the system.

As an example, consider a particle that returns to an inverted trajectory and position after three half orbits, travelling 1.5 times around the ring. This is the \((3, 1)\) mode of operation. In order to apply this mode to the storage ring, equation 4.24 is used to provide constraints on the parameters of the lens,

\[
\sin^2\left(\frac{1\cdot\pi}{2\cdot3}\right) = \frac{K_1K_2}{f_1f_2} \Rightarrow \frac{f_1f_2}{4} = K_1K_2. \tag{4.27}
\]

The two cases, when \( M_{\mu''} = \pm I \), do not both lead to stable orbits when aberrations are considered, such as those due to the hemispherical analysers. The cases where the particle returns to its original position after travelling \( H \) orbits, represented by the \( m = \) even cases, are likely unstable due to the accumulation of aberrations. On the contrary, the cases where the particle returns to an inverted position and angle, given by \( m = \) odd, and remain stable [8]. This leads to a ‘resonance’-like pattern of unstable and stable regions of storage, and which has been observed, and is shown in the section to follow.

### 4.6 Experimental Results

The theory presented in the previous sections yields the \((H, m)\) conditions for stability, which does provide information about the stability of the storage for a given set of operating potentials. As previously discussed, combinations of the \( H \) and \( m \) that provide the longest storage are the \( m = \) odd conditions, while the \( m = \) even conditions are unstable. In order to use the conditions derived in the previous sections to find combinations of lens potentials that are stable/unstable, parameterization coefficients given by Harting and Read [15] are used to find \( K_1, K_2, f_1, \) and \( f_2 \) for a given \( V_2 \) value. The paraxial approximation used by Harting and Read is valid for the storage ring, as defining apertures located within the lenses restrict the angular spread of the trajectories of the particles. This method yields the voltage regions for which both equations 4.7 and 4.21 are satisfied. Figure 4.5, shown earlier in the chapter, shows that both conditions are satisfied for two regions, from \( \sim 3 \) V to \( \sim 7 \) V and \( \sim 85 \) V to \( \sim 160 \) V. It is within these regions that (4.24) can be used, leading to the \((H, m)\) values which give stable and unstable predictions for the corresponding lens potentials.
Both of the stable/unstable \((H, m)\) modes can be seen in the ‘line scan’ shown in Figure 4.6, with blue (stable) and red (unstable) labels indicating the type. There are clearly many more combinations which are not labelled because they will be incorporated into the regions labelled by the dominant mode. The level of agreement is excellent between the predicted stability modes corresponding to \((H, m)\) combinations \((2, 1), (3, 1), (4, 1)\) and the unstable combinations \((3, 2), (5, 2), (7, 2)\), though no other modes for stable conditions can be observed.

\[
\frac{V_2}{V_1} = 2
\]

Figure 4.6 – Line scan for the symmetric case where \(V_3/V_1 = 2\) and the ratio \(V_2/V_1\) converted to a voltage based on \(V_1 = 18\) eV. The modes solved for have been indicated on the right hand side at the corresponding voltage given on the left hand side, with stable modes in blue and unstable voltages in red. [5]

This approach has been applied to a voltage configuration of source/target energy of 63 eV and HDA 1, HDA 2 pass energy of 18 eV, which is a ratio of 3.5, shown in Figure 4.7.
The \((H, m)\) approach used for this voltage configuration includes an additional stability region that is not present in Figure 4.6. This could be due to the increased voltage ratio, which increases the change in the angular distribution, via the Helmholtz-Lagrange equation. This would increase the angular aberrations, as well as the significance of the \((9, 4)\) mode, making the region near 148 V unstable.

\[
V_3/V_1 = 3.5
\]

Figure 4.7 – Line scan for the symmetric case where \(V_3/V_1 = 3.5\) and the ratio \(V_2/V_1\) converted to a voltage based on \(V_1 = 63\) eV. The modes are indicated on the right with stable modes in labelled in blue and unstable modes in red. The vertical band of counts at \(\sim10-15\) µs is due to metastable states of He, a result of the interaction within the target region.

### 4.7 Summary

In this chapter, the electrostatic elements that form the storage ring were introduced, along with the concept of transfer matrices. An overall transfer matrix representing a half orbit unit cell, was found for a symmetric arrangement of potentials and criteria for stability.
within a particle accelerator was applied to find operating potentials for the lens elements, given that the other potentials are defined. This theoretical model was compared with experimental results and demonstrated the success of the matrix formalism, even while considering a first-order treatment of the hemispheres and the lack of physical symmetry in the diameters of the beam defining apertures currently present in the ring. This is an indication that the existence of aberrations does not fundamentally inhibit stability, but rather makes only certain modes unstable.

A symmetric voltage configuration is common in most rings, and while it is useful for storage in our system, experimental results indicate storage is improved by breaking this symmetric arrangement. In the next chapter, conditions for an asymmetric configuration are presented, along with experimental considerations for the motivation behind applying such an arrangement to the storage ring.
4.8 References

Chapter 5: Stability Conditions for an ‘Asymmetric’ Configuration

5.1 Introduction 73
5.2 Matrix Formalism 74
5.3 Basic Asymmetry 76
  5.3.1 Stability Plots 78
  5.3.2 36-18-18 Configuration 81
5.4 Type A 86
5.5 Type B 88
5.6 Fully Asymmetric 92
5.7 Summary 94
5.8 References 96
5.1 Introduction

In the previous chapter, a theoretical description of the storage ring, using transfer matrices, was developed to represent the system when it operated with a symmetric potential configuration. Breaking the symmetric setup presented in Chapter 4 leads to a diverse range of practical applications and that are in excellent agreement with experimental results. This chapter presents the theoretical stability conditions for an asymmetric configuration of lens potentials, using an approach based on transfer matrices similar to Chapter 4, and experimental data to verify these conditions.

Each element of the ring is externally controlled by its own unique power supply and the charged particle beam is always contained physically within these elements. This is a significant experimental advantage, as it allows the energy of the particles to be varied for each of the uniquely defined elements within the ring, without manipulation of any of the other elements or the particle source. The two hemispheres, for example, can be operated at different pass energies from one another, allowing one of the hemispheres to possess a lower pass energy and operate as an energetic prism, defining and maintaining the energy resolution after every orbit. The other hemisphere, meanwhile, acts as a mirror and serves only to maintain the flux of the beam, since the energy resolution will be poorer, as indicated by equation 4.5. Furthermore, the target and source lens stacks can be operated differently from one another. While appearing trivial at first, it allows for the energy of the particles within the target lens stack to be scanned (i.e. during a crossed beam experiment) while keeping the rest of the ring and the particle source itself constant. For this type of scan, it is only the target region and the middle lens element in lenses 2 and 3 that need to be altered. By not changing the pass energy of the hemispheres, the energy resolution of the beam remains the same, since the hemispheres will operate with the same pass energy during the entire experiment.

Regions of storage can be found theoretically by using the matrix description for the transfer matrix for a complete orbit and applying the trace condition, equation 4.7. In order to find the lens voltages satisfying this condition, the parameterization values found in Harting and Read [1] are used, and ultimately, regions of potential storage obtained. This approach provides large regions where stability is possible, even though experimentally these
region are found to be smaller than predicted, as the matrices have been treated in an idealized manner. Analogous \((H, m)\) substructure to that of the symmetric operation, evident in the experimental data presented in this chapter, will not be expanded theoretically at this point. The purpose of the matrix formulation is to find a set of voltages that serve as a starting point for experimentation without the need to rely on electron optical simulation \([3, 4]\). Once symmetry is broken, the computer can no longer exploit the symmetries during simulation leading to many more calculations that require significant computational power and any numerical inaccuracies (i.e. rounding) are going to have a cumulative effect on the results, which may be misleading \([2]\). The matrix approach relies only on the single orbit transfer matrix, however, and does not require this significant amount of computation. It can be applied easily to any number of systems as described in the following sections.

5.2 Matrix Formalism

The fully asymmetric case can also be expressed in matrix formulation just as the symmetric condition. The unit cells representing the half orbits will not be the same; they can even have an overall acceleration (i.e. the determinant \(\neq 1\)) as long as the returning half orbit decelerates the beam by the same amount. In assuming the most general case, when the two pass energies are different from each other, as well as the source and target potentials having different energies, all four of the lenses will operate with different voltage ratios, requiring each to be uniquely treated. Unlike the symmetric case where time reversed lenses can be considered, in this most general case, there is no simplification in the matrix that can be applied.

Figure 5.1 indicates the labelling used in the forming of the transfer matrices. As before, the orbits will start in the source region and the lenses are numbered in the order in which they are encountered until the beam returns to the source. The first half orbit consists of lenses 1 and 2, and the second half of lenses 3 and 4.

Reflection symmetries are evident along two planes A and B. ‘Type A’ asymmetric lens configuration occurs when symmetry is broken along reflection A. This occurs when the two hemispheres operate with different pass energies and the source energy equals the
target energy. This produces different voltage ratios for the lenses next to each hemisphere. The two lenses on each of the hemispheres are still time reversals of one another (i.e. lens 1 is a time-reversal of lens 2, and lens 3 is a time-reversal of lens 4). ‘Type B’ asymmetry occurs when the two hemispheres operate with the same pass energy, but the source and target energies are not equal. In this case, lens 1 is a time reversal of lens 4, and lens 2 a time reversal of lens 3. When the source, target and the two hemispheres have different energies, the system is fully asymmetric.

Using the transfer matrices discussed in Section 4.2, along with the labelling convention shown in Figure 5.1, expressions for both the $M_{st}$ and $M_{ts}$ matrices are expressed as:

\[
M_{st} = -\frac{1}{f_2 f_4} \begin{bmatrix}
K_2 K_3 + K_3 K_4 - f_3 f_4 & K_4 (K_1 K_2 - f_1 f_2) + K_1 (K_3 K_4 - f_3 f_4) \\
K_2 + K_3 & K_1 K_2 + K_1 K_3 - f_1 f_2
\end{bmatrix}
\]  

(5.1a)

\[
M_{ts} = -\frac{1}{f_6 f_8} \begin{bmatrix}
K_6 K_8 + K_7 K_8 - f_7 f_8 & K_8 (K_5 K_6 - f_5 f_6) + K_5 (K_7 K_8 - f_7 f_8) \\
K_6 + K_7 & K_5 K_6 + K_5 K_7 - f_5 f_6
\end{bmatrix}
\]  

(5.1b)
These matrices are fully generalized and work with the fully asymmetric case; nevertheless, it still is advantageous to introduce symmetry when it applies. To this end, the next section will start from the fully symmetric case, but allow the $M_{st}$ and $M_{ts}$ matrices to differ from one another, creating the most basic ‘asymmetric’ configuration.

5.3 Basic Asymmetry

In Chapter 4, the symmetric source-to-target transfer matrix, and the target-to-source matrix were the same, both given by equation 4.15. Now, the two matrices are given the freedom to be different from one another. Each section will be representative of a lens-hemisphere-lens combination, with the two lenses being time reversals of one another. The two transfer matrices can be written in terms of the lens 1 and lens 3 variables as:

\[
M_{st} = \frac{1}{f_1 f_2} \begin{bmatrix} f_1 f_2 - 2K_1 K_2 & 2K_1 (f_1 f_2 - K_1 K_2) \\ -2K_1 & f_1 f_2 - 2K_1 K_2 \end{bmatrix}
\]

(5.2a)

\[
M_{ts} = \frac{1}{f_5 f_6} \begin{bmatrix} f_5 f_6 - 2K_5 K_6 & 2K_5 (f_5 f_6 - K_5 K_6) \\ -2K_5 & f_5 f_6 - 2K_5 K_6 \end{bmatrix}
\]

(5.2b)

The half orbit transfer matrices can also be described in a manner similar to equation 4.16, but using two different phase shifts, $\alpha$ and $\beta$, for each of the two unique cells, along with $L_1$ and $L_2$, which are characteristic constants with units of length. This is appropriate since the cells are mirror-symmetric [5] and are as follows:

\[
M_{st} = \begin{bmatrix} \cos(\alpha) & L_1 \sin(\alpha) \\ -\frac{\sin(\alpha)}{L_1} & \cos(\alpha) \end{bmatrix}
\]

(5.3a)

\[
M_{ts} = \begin{bmatrix} \cos(\beta) & L_2 \sin(\beta) \\ -\frac{\sin(\beta)}{L_2} & \cos(\beta) \end{bmatrix}
\]

(5.3b)

As before, by comparing the corresponding matrix elements, the phase shifts and the characteristic constants are found for the source-to-target matrix to be

\[
L_1^2 = \frac{K_1}{K_2} (f_1 f_2 - K_1 K_2)
\]

(5.4)
\[
\cos(\alpha) = 1 - \frac{2K_1K_2}{f_1f_2}, \quad (5.5)
\]

and similar equations can be found corresponding to the return matrix \( M_{ts} \) by equating equations 5.2b and 5.3b.

An expression for storage can be obtained for the two matrices given in equation 5.3, then applying the trace condition, equation 4.7. The overall transfer matrix is found by performing the multiplication given in equation 4.3 to give

\[
M_{ss} = \begin{bmatrix}
\cos(\alpha) & L_1 \sin(\alpha) \\
-\frac{\sin(\alpha)}{L_1} & \cos(\alpha)
\end{bmatrix}
\begin{bmatrix}
\cos(\beta) & L_2 \sin(\beta) \\
-\frac{\sin(\beta)}{L_2} & \cos(\beta)
\end{bmatrix}
\quad (5.6)
\]

\[
M_{ss} = \begin{bmatrix}
\cos(\alpha) \cos(\beta) - \frac{L_1}{L_2} \sin(\beta) \sin(\alpha) & L_2 \sin(\beta) \cos(\alpha) + L_1 \sin(\alpha) \cos(\beta) \\
-\frac{1}{L_2} \sin(\beta) \cos(\alpha) + \left(\frac{1}{L_1}\right) \sin(\alpha) \cos(\beta) & \cos(\alpha) \cos(\beta) - \frac{L_2}{L_1} \sin(\beta) \sin(\alpha)
\end{bmatrix}.
\quad (5.7)
\]

Applying the trace condition to this matrix gives

\[
\frac{1}{2} \left| 2 \cos(\alpha) \cos(\beta) - \left(\frac{L_1}{L_2} + \frac{L_2}{L_1}\right) \sin(\beta) \sin(\alpha) \right| \leq 1. \quad (5.8)
\]

The simplification of equation 5.8 is presented in full in Appendix 3, but the result leads to the expression for stability given by:

\[
0 \leq \frac{L_1}{K_1} + \frac{L_2}{K_2} \leq \frac{f_1, f_2, f_5, f_6}{K_2 K_6(K_1 + K_5)}. \quad (5.9)
\]

This condition is analogous\(^*\) to equation 4.21 from the previous section for the symmetric case. In the sections that follow, this condition will be applied to different configurations to provide a prediction for the ranges of stability. Results that have been obtained experimentally will then be compared to these predictions.

\(^*\) While the origin of this condition is equivalent, it does not reduce to equation 4.21, as the unit cell has changed from half of one orbit to one full orbit.
5.3.1 Stability Plots

The theory, as described in the previous section, can be used to make stability plots, which indicate where beam storage is possible. These plots are based on equation 5.9 with the lens potentials for $M_{ts}$ and $M_{st}$ on each of the x and y axis, respectively. Figure 5.2 is one such plot, for source and target energy of 63 eV and the hemispheres with HDA pass energy of 18 eV, giving a ratio for $V_3/V_1$ of 3.5. The ratio $V_2/V_1$ is varied for the first half of the orbit, which alters $K_1$, $K_2$, $f_1$, and $f_2$ for lenses 1 and 2 within this half. Similarly, the $V_2/V_1$ ratio is varied for the second half of the storage ring, altering the corresponding lens parameters for lenses 3 and 4. The lens parameters are then tested using equation 5.9 to produce a logic plot, which indicates whether the lens combination is stable or not.

Figure 5.2 shows that there are four main areas of storage possible, which correspond to the different combinations of the high and low focus potentials for the two pairs of lenses. The main region of stability, corresponding to each pair of lenses operating with a ‘high focus’ potential, is located in the range of ~105 V to 185 V. This is the simplest region to use because the storage is stable over the range of tens of volts, unlike the other three regions where stability is affected by changes of <1 V. This upper region is also less

Figure 5.2 - A logic diagram indicating the stable regions (green) where storage can occur for a $V_3/V_1$ ratio of 3.5. The same features are found in the $V_3/V_1$ ratio of 2; however the storage regions will be shifted and the areas of stability slightly smaller.
sensitive to aberrations than the other three regions, as the high $V_2$ potential of the lens compresses the beam closer to the optic axis than a low $V_2$ potential [1]. The resonant substructure apparent in the experimental data of the symmetric case shown in Figure 4.7 is not given; however, the voltages of the observed storage in this figure, ~115 V to ~160 V, fall entirely within the stable region shown in Figure 5.2, which indicates storage is possible over the range 105 V to 185 V for the symmetric case.

One feature that is apparent within the stability plot is that each of the four distinct regions is further divided into two sections. This can be explained by looking at the actual values of the trace condition, equation 4.7. Recall, the trace condition places a limit on the linear and angular magnification; such that the absolute value of both of these combined is less than 2. Figure 5.3 shows the value of the trace as a function of the $V_2$ lens potentials. As can be seen, the value of the trace is -2 where the two sections meet, and drops below this value in the adjacent green regions, which divides the two sections. This is shown more clearly in Figure 5.4, which shows the linear and angular magnification terms individually; i.e. $(M_{ss})_{00}$ and $(M_{ss})_{11}$, respectively. Each of the magnifications is asymmetric

![Figure 5.3](image)

Figure 5.3 – The value of the trace when the trace condition is satisfied as the lens potentials are varied for a $V_3/V_1$ ratio of 3.5. The large green region represents non-valid combinations of lenses, which has been arbitrarily set to 0. The borders where the two regions meet have a trace of -2, and are separated due to the trace dropping below -2.
about the leading diagonal, which quickly falls to -3 towards one of the corner regions. These significantly large values indicate the mechanism for the trace violation. For the upper left corner region, it is the linear magnification that is responsible and for the lower right corner, it is the angular magnification.

Figure 5.4 – The numerical values for the linear and angular magnification for the complete transfer matrix, $M_{ss}$, as a function of the lens potentials. The values that lie between -3 and 3 have been plotted, while values outside of this range have been set to either 3 or -3 in order to preserve the contrast in the graphs. Each show an asymmetry about the leading diagonal. Note the scale change from the previous figure.

Another feature of Figures 5.2 and 5.3 that has yet to be addressed is the presence of the ‘tails’ that stretch beyond the edges of the figures. These will also yield results that are not conducive to long-term storage of the particle bunch. Inspection of these regions for a lens combination of $(\text{Lens } 1 \& 2 \ V_{2}, \text{ Lens } 3 \ & 4 \ V_{2}) = (225, ~160) \ V$ will give a linear magnification term that is ~1.5 and an angular magnification term that is ~ -1.5. Equation 4.7 is satisfied; however, the physical implication of these values is that they will both allow the beam to expand as more orbits are completed and are inherently unstable. These tails are then eliminated from the predictive plot by imposing the additional symmetry in the half matrix, namely, that both magnifications remain in the range -1 to 1 for each of the half matrices. This leads to the stability plot shown in Figure 5.5, predicting the regions where storage will likely occur. There are two plots given in this figure; the first shows the trace value when the full array of constraints are satisfied, and the second is a simpler logic plot.
indicating whether the combination of lens potentials is stable or not. This second plot will be used most often in the sections that follow.

Figure 5.5 – Left - the value of the trace condition is given only when (5.9) is satisfied. Right – A logic diagram indicating where (5.9) is satisfied (shown in green) for a \( V_3/V_1 \) ratio of 3.5.

5.3.2 36-18-18 Configuration

Figure 5.6 is the result of the theory for the system when the source and target are both at 36 eV and the HDA pass energies are both at 18 eV, plotted with the lens potentials for \( M_{si} \) and \( M_{st} \) on each of the \( x \) and \( y \) axis, respectively. This leads to a fixed acceleration ratio for each of the lenses, \( V_3/V_1 = 2 \).

As in Figure 5.2, there are four distinct regions of stability, each consisting of two areas due to the magnification values. The potentials of these regions agree with Figure 4.5, which indicated that storage for the completely symmetric configuration of potentials was possible for a lens potential of 3-7 V and 85-160 V, as can be seen on the leading diagonal.
Figure 5.6 – This figure indicates the stable regions (green) where storage can occur or a $V_3/V_1$ ratio of 2.0, based on theoretical framework involving matrices. This figure is very similar to Figure 5.5 right, except the stable regions appear over a lower range of $V_2$ lens potentials.

Experimentally, the result of the matrix formalism can be tested using the technique and software described in Chapter 3. The result is a region of long-term storage that is shown in Figure 5.7. This figure represents the experimental result for a $V_3/V_1$ ratio of 2 and is in agreement with the theoretical figure, Figure 5.6. It is of note, however, that the storage regions achieved experimentally are smaller than the theoretical plot. This can be easily explained as the matrix formalism is based on the most ideal case and does not consider several factors, namely aberrations within the lenses and the hemispheres which will be detrimental to the beam.

The general shape of the stability region is different between the experimental and theoretical plots. There is an asymmetry of the experimental storage when compared with the white diagonal line in Figure 5.7, which corresponds to the fully symmetric condition. This is not predicted in Figure 5.6, which indicates that storage would be symmetric about the diagonal. Based on Figure 5.4, one area shows the theoretical angular magnification of
the charged particle bunch is relatively constant, and it is the linear magnification that is changing as the lens potentials are varied. In the other area, this role is reversed, such that the linear magnification is relatively constant and the angular magnification is changing as the lens potentials are varied. This asymmetry can be further attributed to the mechanical layout of the storage ring. The two lenses that make up each half orbit were treated symmetrically within the derivation of the condition for stability, equation 5.9. Throughout the storage ring, however, the diameters of the defining apertures for the four lenses are not all identical, and thus a true time reversal of the lenses is not strictly valid. The lenses transport the particles while apertures maintain a bound on the positions and trajectories, and the matrix approach used does not account for the use of apertures within the ring.

Figure 5.7 – Experimental result of the stability program for a $V_3/V_1$ ratio of 2 showing the main storage region, which is divided into the two distinct subsections. Three distinct lines are shown, the white line corresponding to the fully symmetric configuration, and the black and red lines, which are nearly reflections of one another about the white diagonal, are discussed later in the text.
Figure 5.8 - The diagonal line represents the potentials for which the setup is fully symmetric. The $(H, m)$ modes found in Chapter 4 are shown with stable modes in white and unstable modes in black, and correspond to the same conditions observed in Figure 4.6.

Another feature that appears in the experimental data is a diagonal boundary present in the upper right region, observed in Figure 5.7, which contradicts the ‘box-like’ boundaries predicted in Figure 5.6. This boundary follows the same general shape as the magnification contours seen in Figure 5.5, which provides a possible explanation for this feature. Experimentally, the charged particle bunch shows a propensity for stability when the trace of the single orbit transfer matrix is negative, with the best storage found when this value is close to -2. For this value, both of the position and trajectory would (nearly) invert after one complete orbit, similar to the $(2, 1)$ mode in the symmetric case.

Figure 5.8 shows a zoomed-in version of the two distinct areas of storage, along with the symmetric condition given by the white line. Along this line the $(H, m)$ storage modes are labelled and correspond directly to Figure 4.6. However, these are not the only modes of storages that are apparent, as can be seen in Figure 5.8 (right). The labelling of these other modes would require an alternate method of identification that is outside of the scope of this thesis.

The difference in storage above and below the diagonal especially becomes apparent when comparing two “line scans” performed for asymmetric combinations of lenses. Figures 5.9 and 5.10 show the storage time of an electron beam for specific combinations of lens potentials falling along either the red or black line shown on Figure 5.7. The red line
Figure 5.9 – Contour plot representing storage as a function of time for many combinations of the lens 1 and 2 and lens 3 and 4 potentials. These potentials fall along the red line shown on Figure 5.7, and shows the uppermost region is capable of storing an electron beam for over 30 \( \mu s \) (~100 orbits).

Figure 5.10 – Contour plot representing storage as a function of time for many combinations of the lens 1 and 2 and lens 3 and 4 potentials. These potentials fall along the black line shown on Figure 5.7. The strong upper region seen in Figure 5.9 is not observed, which highlights the difference in the storage seen above and below the white line in Figure 5.7.

passes through the center of each of the storage subsections, while the black line passes along the edge of each region of stability. This shows that the storage ring does not respond in a symmetric manner, as the difference between the red and black lines is merely that the voltages of lenses 1 and 2 \((M_{st})\) and the voltages of lenses 3 and 4 \((M_{ts})\) have been reversed. Furthermore, the strong storage found only in the upper subsection suggests that the asymmetric voltage arrangement within the lenses is optimal, given the arrangement of the
apertures and the magnifications of the lenses, for long term-storage of a charged particle bunch.

The previous sections have shown that an asymmetric configuration is not only predictable through an approach involving matrices, but experimentally ideal for the apparatus. These sections, however, have only considered the simplest asymmetries. The next section will build on this set-up by considering the use of different HDAs.

5.4 Type A

In the previous section, the HDA pass energies were equal to one another, while the lenses surrounding one hemisphere were ‘allowed’ to operate differently from the lenses of the other hemisphere, i.e. away from the leading diagonal in the stability plots. This section will describe the operation when the HDAs are operated differently from one another. Unlike the previous section, this requires that the lenses adjacent to one hemisphere operate differently from the lenses of the other hemisphere. As previously mentioned, there is an advantage to breaking the symmetry in the two hemispheres and this asymmetry will be referred to as ‘type A’.

The voltage potential schematic can be seen in Figure 5.11. The applied symmetries used in the derivation of equation 5.9 are still valid in this particular voltage arrangement. Therefore, equation 5.9 is used to find the theoretical regions of stability for ‘type A’ configurations.

![Figure 5.11 – Setup for type A, each distinct pattern indicates the elements that share the same potential voltage. Clearly the hemispheres are different, but the two lenses that surround each hemisphere are the same, indicating that one acts as a time-reversal of the other. The source and target regions are also equal.](image)
Figure 5.12 displays the theoretical result of equation 5.9 as compared to experimental result. This figure represents the configuration in which the source and the target regions are both at 36 eV, while HDA 1 is at 9 eV and HDA 2 at 18 eV. As before, there are four main areas of recycling, with two of these areas, corresponding to a high-high and a high-low combination of these lens potentials are shown in this figure. As can be seen, however, the voltage range of storage for lenses 3 and 4 has shrunk from ~60 V, which relates to a ‘high focus’ lens, to ~3 V for the ‘low focus’ lens. This difference, along with the increase in aberrations for a low focus lens, makes the upper region preferable for study. While similar in shape to Figure 5.2, the theoretical plot in Figure 5.12 has a scale change on the x-axis. The change in the voltage ratio of lenses 1 and 2, due to the differing hemisphere, is the origin of this difference.

Figure 5.12 – Left – the theoretically stable regions given by the shaded regions for the type A setup, showing all four of the high/low focussing combinations. Right – Two of the stability areas that were experimentally found, corresponding to the high/high and the low/high lens focussing combinations. Strong long term recycling is indicated in red and weak or no recycling is in blue.
The result shown in Figure 5.12 is in remarkable agreement with the result of the matrix condition. Again, the theoretical plot shows regions of storage that are larger in size than the experimental figure, as well as the presence of the upper boundary; however, the reasons for these differences have been previously discussed. One feature that is different from Figure 5.7 is the size and amount of storage of the two regions have changed, the lower region now has more stability than the upper. However, the injection process would have changed between the two data sets, changing the initial range of trajectories.

This is not the only asymmetry that can exist within the ring. Another is to make the source and target operate at different energies, while leaving the hemispheres equal to one another. This is now presented in the following section.

### 5.5 Type B

The source and target regions are not required to possess the same voltage within the apparatus, just as the HDAs in the previously. In this section, the hemispheres are kept the same, while the source and target are allowed to differ from one another. This asymmetry is called a ‘type B’ and can be seen schematically in Figure 5.13. In this lens configuration, the lenses surrounding the source, lenses 1 and 4, are time-reversed representations of one another, as are the lenses 2 and 3, which surround the target region.

![Figure 5.13 – Setup for a type B setup, each pattern represents a voltage potential. In this case, the hemispheres are the same, but the source and target regions are different. The two lenses in each stack are the same, but different from the other one.](image)
Although this set-up appears entirely different from those already presented, equation 5.9 will still be valid. The two half matrices are redefined for convenience, and represent one hemisphere and one entire lens stack. Both lenses in each of the lens stack are time reversals of one another and the hemisphere is still considered to a first order approximation, which is similar to when the two lenses adjacent to each hemisphere were time reversals of one another. This has the effect of making the entire orbit a mirror symmetric setup, for which the diagonal elements, \((M_{ss})_{00}\) and \((M_{ss})_{11}\), are be equal to one another [5]. Unlike the case for type A, this implies that \(L_1 = L_2\) and equation 5.7 collapses down to a form similar to equation 4.16. Equation 5.9 can still be used however, along with this simplification.

Using equation 5.9, theoretical storage regions can be obtained, as seen in Figure 5.14, which shows only the ‘upper right’ region of storage. Comparison with Figure 5.6 shows that the shape of the storage region has changed significantly. Both plots are representative of the predicted storage for a setup of source = target = 36 eV and HDA 1 = HDA 2 = 18 eV, but the coupling of the lenses has changed. The major distinction between the two regions is the predicted tails for the type B arrangement are not removed entirely with the additional constraints on the magnification. Instead, the box-like shape of the regions is replaced with a ‘bowtie’, as there is a change in the size and shape of both regions.

![Figure 5.14 – The theoretical plot as a result of equation 5.9, showing the value of the trace condition for valid solutions. The shape has changed significantly as a result of the different combination of lenses.](image-url)
Figure 5.15 – Left – the theoretically stable regions given by the shaded regions for the type B setup, showing the uppermost region of stability. Right – The main stability region that was observed experimentally.

Figure 5.15 shows the theoretical plot for a configuration of source = 54 eV, target = 36 eV, HDA 1 = HDA 2 = 18 eV. This plot is compared with experimental results, which together, show the predictability of the storage regions for a type B asymmetry. The general shape of the two is very similar; the experimental region was focused on the upper right storage region, as indicated on the graph. The strongest regions of storage found in the experimental data relate to the trace condition when it is near −2, shown in Figure 5.14.

The timing profile of the long-term storage can be seen in the ‘line scan’, Figure 5.16. This scan indicates the storage that occurs for combinations of the lens potentials that can be found along the black line and it shows that a resonant-like structure, similar to that seen in the fully symmetric case. The clear pattern distinguishes between stable and unstable regions of storage and the cause of this pattern is due to aberration attributed to the angular spread within the hemispheres. This figure indicates that long-term storage is possible for multiple combinations of lens potentials, easily maintaining the electron beam for over 30 μs, which is approximately 100 orbits, or a path of 65 m for a mean energy of ~25 eV.
Figure 5.16 – Top – Experimental result of the stability region as a function of source and target lenses. Plotted on this graph is a black line, which indicates the specific lens combinations used to the line scan, seen on the bottom. Bottom – Line scan showing the strength of the time structure of the storage area seen above. A resonant-like pattern can be seen, similar to the symmetric case.
5.6 Fully Asymmetric Set-up

The last potential configuration that has to be discussed is the fully asymmetric condition. In this case, the source energy does not equal the target energy, and HDA 1 ≠ HDA 2. One of the key differences between this setup and the others up until this point is the presence of an overall acceleration within the half matrix representations. This acceleration, which is because of the potential differences in both the source/target and HDA 1/HDA 2, make the general matrix representation, equation 5.9, no longer a valid representation for an indication of storage. The generalized form, given by equations 5.3a and 5.3b no longer applies, since the determinant of these matrices do not account for this acceleration. This is not so large an issue as the trace condition can be applied directly to the transfer matrix for an entire orbit, which is found using equations 5.1a and 5.1b, to provide the storage regions. Furthermore, these matrices combine to form an overall transfer matrix which must have a determinant of exactly 1, since this matrix represents the return of the particles to the same point within the storage ring and thus the particles must possess the same energy.

A fully asymmetric set-up is possible when each of the lenses operate completely independent of one another (i.e. there are no time reversals or identical lenses). This, as mentioned in the introduction, will allow all of the target, source, HDA 1 and HDA 2 regions to possess unique potentials, which thereby allows the particles to traverse these elements with unique energies. The hemispheres will act in a prism/mirror arrangement while the source and target region will also differ from one another, allowing for incident energy scans of the target region, leaving the rest of the ring constant.

A theoretical plot for the predicted storage regions can be produced for a fully asymmetric potential arrangement, seen as Figure 5.17. In this case, the source is 28 eV, the target is 20 eV, the pass energy for HDA 1 is 6 eV and for HDA 2 is 18 eV. As before, the two source lenses are fixed and different, with middle lens elements of 47.5 V for lens 1 and 129 V for lens 4. Four regions of storage are predicted; and as before, the largest one is the high focus lens potentials of the two lenses. The absence of the second area seen in the previous plots can be attributed to the fixing of the source two lenses. In all the other cases both of the lens stacks were being manipulated, and each of the lens stacks
Figure 5.17 – Theoretical plot predicting the regions of storage for the fully asymmetric configuration of target = 20 eV, Source = 28 eV, HDA 1 = 6 eV, HDA 2 = 18 eV potential arrangement. Here, the $V_2$ lens potentials on the source side are fixed at 47.5 V (Lens 1) and 129 V (Lens 4) while the target lenses (2 and 3) are varied to search to stability.

Figure 5.18 – A portion of Figure 5.17 (Left) shown with the storage found experimentally (Right) for the same configuration. The experimental region is much smaller than the predicted region, but this is to be expected due to aberrations.
predetermines how the charged particle bunch is moved from one hemisphere into the other.

Figure 5.19 shows the storage found experimentally alongside a zoomed in version of Figure 5.18, which corresponds to the same voltage ranges. As is evident, there is storage appearing in one region and it is located in position that was predicted. The voltage range over which this was seen is smaller than predicted, but previously mentioned, the predicted plot uses ideal hemispheres and this approximation is sufficient to explain the deviation. The experimental data are located near the ‘elbow’ of the theoretical plot; however, the location of the two does not exactly coincide with one another. This is due to the use of the parameterization method from Harting and Read, in which the cardinal lengths of the lenses are accurate to ~10% [1]. Despite this, the level of agreement is excellent and allows for the voltage potentials to be positioned at values most likely to produce storage for this fully asymmetric configuration.

5.7 Summary

Stability within the storage ring is a significant factor in determining the usefulness of the device as well as the flexibility in its operation. This chapter demonstrated that the apparatus is capable of operating in a nonsymmetric arrangement, with the elements of the ring possessing different potentials throughout, while still maintaining a stable containment of the particles within the device. Both theoretical, based on the use of charged particle transfer matrices, and experimental data have been studied for various potential arrangements. The matrix formalism has been shown to provide regions where storage will occur, and also observed experimentally. Asymmetry leads to an increase in operational flexibility as one can simultaneously maintain energy resolution of the particle beam via the hemisphere pass energy, while experimentally varying the incident energy of the particles within a specific region of the ring and still maintain storage. This asymmetric setup can have interesting applications for other existing devices. For example, in the DESIREE setup, Section 1.4.2, where two storage rings share a common section, this can provide the possibility to alter the properties of the straight section while keeping the remainder of the ring fixed or vice versa. The approach developed in this chapter does not require the use of charged particle simulation, and while these programs provide results that give a great deal
of insight as to the particle trajectories within a system, in this application they may provide results which are misleading when considering multiple orbits, as numerical inaccuracies will be cumulative. The removal of symmetries also leads to more calculations within simulation, which require more time or increased computational power, which is minimized using the matrix approach since it only relies on the single orbit transfer matrix. The next chapter will focus on the time evolution of the stored pulse that is successfully stored using this matrix approach.
5.8 References


Chapter 6: Timing Properties of the Stored Particles

6.1 Introduction 98
6.2 Long-Term Storage 98
6.3 Background Scattering and Other Loss Mechanisms 101
6.4 Peak Width Analysis 102
6.5 Summary 106
6.6 References 107
6.1 Introduction

The previous two chapters focused on a theoretical approach to finding long-term storage conditions within the ring for a variety of voltage configurations. The stability plots found in these chapters, particularly Chapter 5, can be used to obtain voltage potentials that will lead to long-term storage of the charged particle bunch. Once the storage is achieved, however, the evolution of the bunch itself can be studied as a function of orbit time. For example, the existence of an exponential decay, representing a significant loss mechanism, is present in the time spectra that are obtained for both ions and electrons. This chapter studies the evolution of the charged particle beam in terms of the time profile of the stored particle bunch as a function of orbit time.

6.2 Long-Term Storage

Long term storage was achieved through the use of the stability plots to find a suitable set of operating voltages for the system. This was further enhanced by optimizing the voltages of the gun and the current in the Helmholtz coils to allow for longer storage. Figure 6.1 shows a data set for Ar⁺, operating asymmetrically – target/source is 36 eV, HDA 2 pass energy is 15 eV and for the HDA 1 it is 6 eV. The data set shows the excellent storage of the low-energy ions with an orbit time of ~98.2 µs. The bunch is observed for over 7.5 ms, which represents ~75 revolutions, or nearly 50 m of travel around the ring at an operating pressure of 2×10⁻⁶ mbar. As can be seen, there is a decrease in the amplitude and an increase in the width of each of the peaks as more orbits are completed.

Figure 6.2 shows a data set for an electron bunch confined within the storage ring, which is operating asymmetrically in a type A configuration – target/source is 18 eV, for HDA 2 the pass energy is 18 eV and for HDA 1 it is 6 eV. The bunch is recycled for over 55 µs, with a statistically significant number of counts (>100 peak amplitude) in each peak for the entire window. The total storage time, along with the mean orbit time of ~340 ns, represents ~160 revolutions, or over 100 m of travel around the ring. As can be seen in the figure, there is a change in both amplitude and the width of each of the peaks as more orbits are completed, the same trends as observed for ions. These trends indicate that the while the
system may be operating with a stable configuration of voltage potentials, other factors are still affecting the timing properties of the bunch – namely the amplitude and the time width of the peak. While the broadening of the peak widths is partially responsible for the decrease in the amplitude of the peak, the area of the peak, which is a superior indicator of the actual number of counts in the bunch, is also shown to be decreasing as more orbits are completed; this is discussed in the following section.

Figure 6.1 – Storage of Ar+ ions for an asymmetric ‘type A’ configuration. The pulse can be observed completing over 75 revolutions; representing a mean travel distance of ~50 m. Notable features of the pulse are the decreasing amplitude and increasing peak width as function of time. Top – A linear plot of the storage showing the exponential decay of the ion bunch. The data have been scaled by 20x and 100x to display the data taken at a time further from the injection. Bottom – A logarithmic plot showing the entire spectrum.
Figure 6.2 – Electron storage for a type A configuration – Target = Source = 18 eV, HDA 1 = 6 eV, HDA 2 = 18 eV.  Top – Entire timing spectrum indicating the successful storage of the electron beam for over 55 μs.  The orbit time is ~340 ns, which indicates that the electron pulse has completed over 160 revolutions, travelling for more than 100 m.  The bottom left figure shows storage of the bunch from 10-15 μs and in the bottom right, storage from 45-50 μs.  These highlight the distinct difference in amplitude and peak width for two different time intervals, as well as the lack of any background noise.
6.3 Background Scattering and Other Loss Mechanisms

The number of particles in the beam reduces as more orbits are completed, as has been observed in the previous section. This is potentially due to several factors; the main two being due to collisions with background gases and optical aberrations. Other factors have been minimized during the design stage of the system. These include mechanical misalignment, “patch” fields*, voltage instabilities, noise and stray magnetic fields.

Aberrations are unavoidable and can only be minimized. There are two main types of aberrations, spherical and chromatic. As the particle bunch travels though the lenses, the different particle energies within the bunch create various focal points along the optical axis, otherwise known as chromatic aberration. Spherical aberrations are due to the “over filling” of the lenses. This occurs when the particle trajectories occupy a larger percent of the lens element, and are subject to the extreme regions of the potential curves within the lens. As a result, the particles are over focussed and this leads to different focal points along the optical axis based on the trajectory and angle.

Collisions occur with background gas leading to charged particle scattering throughout each orbit. This is an energy dependent loss mechanism linked to the total scattering cross section. This is an indicator of the quality of the system’s vacuum, as increased pressure creates more collisions and reduces the mean free path of the particles.

These major loss mechanisms create an exponential decay in the number of counts within each peak. This exponential decay can be given by

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$  \hspace{1cm} (6.1)

where $N_0$ is the initial number of particles in the beam, $t$ is the time after the first detected peak and $\tau$ is the lifetime of the beam. Figure 6.3 shows the decreasing amplitude of the bunch as more orbits are completed. This is deceptive, however, as the distribution of the counts within the peak is also changing as will be the further discussed

---

* A “patch” field is due to voltage differences along the surfaces of the elements generating the electric fields. These can be due to any build-up of impurities on the element.
in the next section. A more accurate indicator is the peak area, also shown in Figure 6.3, which displays the natural logarithm of the area versus time. The linear fit shows that while both show the decay given by equation 6.1 is appropriate, the two different slopes, which can be used to find the lifetime of the beam, is an indication that the count distribution of the peaks is changing. Neither fit holds for the first ~10 orbits, due to the particles with inherently unstable energies and trajectories being removed from the system. The single decay indicates a dominant loss process and Tessier et al. have shown that the residual background gas pressure is the key factor in the decay [1].

### 6.4 Peak Width Analysis

The peaks do not retain the same timing profile as more orbits are completed. As mentioned in the previous subsection, there is an interval in time where the pulse is “cleaned up” of unstable orbits after injection. This has the effect of narrowing the peaks for approximately 5 to 10 complete orbits. Here the range of energies and trajectories not suited for long-term storage are removed from the apparatus through collisions with one of the eight apertures. Afterwards, the widths slowly become wider each successive completion. This widening is potentially due to two factors: energy and trajectories. The beam in not mono-energetic, so the particles within the bunch will be travelling at slightly different velocities in the same region of space. During the straight portions of the ring this will result
in particles with higher energies requiring a shorter time to travel the length of the stacks. In the hemispheres, the trajectories of the particles will lead to a spread in transit time [2].

The particles in the bunch are subject to space charge, where the charged particles within the bunch interact with each other due to the mutual Coulomb repulsion, which is largest at the narrow waists of the beam. This can have the effect of altering the energy of each particle, since particles at the “front” and “rear” of the bunch are accelerated away from the center, which increases the energy range. Space charge also alters the trajectories of the particles, as particles next to one another will repel and accelerate away from each other and away from the optical axis. Both Coulomb processes will alter the energy and/or the distance travelled for an individual particle, which will have an effect on the orbit time for the particle, ultimately changing the width of the peaks, as the range of orbit times changes.

Another factor is due to the various trajectories possible within the storage ring. By travelling through the ring on various pathways, the particles will travel distances different from one another. This clearly will affect the time for each individual particle to complete orbits. Both factors are responsible for altering the beam width simultaneously and result in the gradual increase in the timing profile of the beam.

The beam width of the \( n^{th} \) orbit, \( \delta t_n \), can be modelled be a quadratic fit [3] given by

\[
\delta t_n = \sqrt{W_0^2 + n^2 \Delta T^2},
\]

where \( W_0 \) is the initial temporal width of an injection pulse that is ideal and requires no further clean up and \( \Delta T \) is the time spread per orbit. Figures 6.4 and 6.5 show this fit as it
Figure 6.5 - The experimental width of the Argon peaks shown in Figure 6.1 given as a function of the orbit number (blue). The red line indicates the fit as given by equation 6.2, with \( W_0 = 5.1 \pm 0.1 \, \mu s \) and \( \Delta T = 0.57 \pm 0.04 \, \mu s \).

has been applied to the electron and ion spectra displayed earlier in the chapter. As can been seen in both, this fit is accurate in determining the width of the peaks that occur after the first few orbits.

The \( \Delta T \) consists of both the time spread due to the source, \( \Delta T_{\text{source}} \), and the time spread due to the various conditions within the ring, \( \Delta T_{\text{ISR}} \). Since these processes are independent, this can be expressed [3] as

\[
\Delta T^2 = \Delta T_{\text{source}}^2 + \Delta T_{\text{ISR}}^2 .
\] (6.3)

The time spread due to the source is caused by the initial energy spread of the ions within the ion source. This will create an energy spread of the beam within the ring, and as mentioned above, causes an increased range of peak widths as a function of orbit number. The time spread due to the ring conditions is based on the varying paths available to the ion pulse, leading to a range of path lengths and therefore a range in orbit times within each pulse.

The ions do not leave the source region at the same time, as indicated by the \( W_0 \) term in equation 6.2. This is due primarily to two additional factors: the width of the electron beam used in the impact ionization, and the initial direction of travel of the newly created ion. The latter of these is far more dominant and is controlled partially by the strength of the extraction field. A strong extraction potential is capable of reversing a low energy ion headed in the opposite direction of the ring. Thus if two ions created with the same energy at the same location, one headed directly into the ring and one headed directly away from the ring, the time spread is the time it takes for the opposite ion to stop, reverse direction and return to the point of origin.
The peak widths have been observed to evolve in a manner other than described by the quadratic fit, equation 6.2. In this type of operation, the peak width of the charged particle bunch essentially stops increasing as more orbits are completed. This type of behaviour is referred to as ‘isochronous behaviour’, and has been observed in the storage ring for both electrons and ions by Theresa Spanjers [4, 5].

Figure 6.6 shows the peak widths for two electron data sets, both seen to store for over 35 μs. In the first data set, indicated by the red points, the widths evolve as before and can be fitted by equation 6.2, shown by the black line. The peak widths can be seen increasing as more orbits are completed, and near the end of storage the width of the peak is ~55 ns. The orbit time for these spectra is ~300 ns, which means that the width of the peak is roughly 1/6th of this time. The blue data set is seen to stop increasing after ~10 orbits and the peak widths remain ~35 ns for the rest of storage. For this case, the width to orbit ratio is 1/9.

![Figure 6.6](image)

Figure 6.6 – The peak width evolution for two distinct electron spectra, indicated by the red and blue points. The red points increase as predicted by the quadratic fit, equation 6.2, which is shown as a solid line. The blue set remains relatively constant once the peak width reaches ~33 ns and cannot be fit according to equation 6.2. This behaviour is called isochronous behaviour, and has been observed both electrons and positive ions. [4]

The matching of energies and trajectories likely causes isochronous behaviour. Particles with higher energies take slightly longer paths through the apparatus than those with lower energies such that the orbit times for all are similar [3]. This behaviour is useful
for applications that rely on the width of the beam to remain narrow, such as mass spectrometry. In the next section, mass spectrometry will be considered as a potential application of the storage ring, and operating with this behaviour remains a way to improve the initial mass resolution.

6.5 Summary

This chapter presented timing spectra for both electrons and ions, highlighting the evolution of the pulse as a function of the number of orbits completed. An exponential decay, which is due to scattering off background gas, as well as a function to describe the peak width spread, was applied to spectra for both electrons and ions. The mechanisms behind these phenomena were also described. A new type of behaviour, isochronous, was mentioned for a case where the electron width ceased to increase. The next chapter will focus on the use on ions within the ring, specifically in the function of a mass spectrometer. For this particular application, the timing properties of the beam as discussed in this chapter become extremely important to the ability to separate unequal masses. These factors and the initial performance of the ring are analysed in Chapter 7.
6.6 References


Chapter 7: Ion Storage and the Application to Mass Spectrometry

7.1 Introduction 109
7.2 Mass Spectrometry 109
  7.2.1 Time-Of-Flight 109
  7.2.2 Reflectron 111
  7.2.3 Electrostatic MultiPass Time-Of-Flight 112
7.3 Krypton 114
  7.3.1 Data Analysis 116
7.4 Xenon 118
7.5 Data Simulation 121
7.6 Resolution of the System 123
7.7 Application to Heavy Ions and Biomolecules 127
7.8 Summary 129
7.9 References 130
7.1 Introduction

In the earlier chapters, the experimental apparatus was discussed in detail, along with theoretical study of the stability of storage for a completely asymmetric set-up and the experimental confirmation of the long-term storage of the apparatus under these operating conditions. The previous chapter revealed some insight to the overall time structure of the pulse as a function of the number of orbits completed. Up until this point in the thesis, however, there has been no direct application of the device. This chapter will now focus on the storage of ions, specifically, the potential application of this device to the field of mass spectrometry.

7.2 Mass Spectrometry

Mass spectrometry is the technique of separation of one atomic species from another based on the charge to mass ratio of the particles that form this substance. There are several different methods to accomplish this based on the use of various combinations of electromagnetic fields, with different advantages to each type. Common examples include time-of-flight mass spectrometer, the reflectron, and the magnetic mass spectrometer. The first two examples are briefly introduced in the next sections.

7.2.1 Time-Of-Flight

Time-Of-Flight (TOF) mass spectroscopy is a common method to separate particles based on the charge-to-mass ratio. It most generally consists of an ion source, an extraction plate, a long field-free drift region and finally a detector. An electron beam ionizes a target gas and once this occurs the ions are subjected to an electric potential due to the extraction plate. This accelerates the ions out of the source region; the energy of the ions determined by the charge state, $q_i$, on each ion. Upon ionization, the potential energy of the particles is determined by $q_i \Delta V$, where $\Delta V$ is the difference between the local potential at which the ionization occurred for each particle and the potential of the extraction plate. This is
converted into kinetic energy by the time the particles pass into the region beyond the extraction plate, which is at a fixed voltage across the entire length in order to maintain a field free region. Once in this region, particles with the same charge state, and thus the same energy, will separate based solely on the individual mass of each particle, given that the resolution of the device is sufficient. This separation is clear when considering a rough calculation* for two different masses with equal kinetic energies:

\[
\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2
\] (7.5)

which, using the fact that they are traveling in a field free region of length \(d\), can be written

\[
\frac{1}{2} m_1 \left( \frac{d}{t_1} \right)^2 = \frac{1}{2} m_2 \left( \frac{d}{t_2} \right)^2.
\] (7.6)

The particles travel the same distance until reaching the detector and so it is evident from the above equation that the time it takes a particle of one mass will relate to the time it takes a particle of a different mass by the equation:

\[
t_1 = \sqrt{\frac{m_1}{m_2}} \times t_2
\] (7.7)

In order to improve resolution of the device, it is necessary to lengthen the drift tube, giving the faster and slower particle bunches enough time to separate from one another.

This setup was further enhanced by applying spatial focusing conditions in order to minimize time spread caused by the different initial positions and velocities [1,2]. Pulsing a portion of the source region allows for the ions within the bunch to gain different amounts of energy, which is dependent on the position of the ions. Ions that are in the front of the bunch gain less energy than ions in the back, making them slightly slower. After entering the drift region, the faster particles in the back will gradually catch the slower particles at the front, which initially minimizes the time spread of the pulse. Altering the amplitude and frequency of the pulsed extraction plate will change the energy difference across the pulse, which determines how long it takes the pulse to become a minimum with respect to time. Using this type of source, the mass resolution of a drift tube was increased [1].

* This calculation assumes there is no angular spread in velocities and every particle is moving along the length of the drift tube.
While simple in its design, the resolution of the drift tube is ultimately limited by its size, as it relies on the difference in velocity between masses to separate them. This however, is not the only electrostatic mass spectrometer to be developed; another is the mass reflectron which is based on an electrostatic mirror, as discussed in the next section.

### 7.2.2 Reflectron

First developed in 1973 by Boris Mamyrin [3], its concept and design were successful and is now commercially available [4, 5]. The mass reflectron consists of an ion source, a set of parallel plate deflectors, an electrostatic mirror and a detector. While more complex than the drift tube with the addition of the electrostatic mirror, the reflectron is superior in its ability to separate masses.

![Figure 7.1- Schematic of standard reflectron setup, showing the source (1,2), the focussed beam leaving the source, containing all the masses (3), the deceleration and reflecting regions (4,5), the detector (6) and electronics (7,8,9). [6]](image)

The ions are extracted from the source region, which is based on the design by Wiley and McLaren[1], and enter the deflecting field that gives the ions a small deflection towards the electrostatic mirror, located a distance $L$ away from the deflecting field. The ions enter the electrostatic mirror at an angle; this does not reflect the ions back towards the source, but to the detector located a distance $L$ away from the mirror, adjacent to the ion source. This is shown in Figure 7.1.
The mass reflecton is designed to have a second-order focussing that allows the ions to reach the detector with timing information that is nearly independent of energy [6]. This is achieved by the use of the electrostatic mirror, which compensates for the energy spread (and hence the time spread caused from the drift regions) since the ions of higher energy require more time to undergo the reflection process.

The initial resolution of this type of device was reported at 3500; however improvements, such as the development of a delayed extraction MALDI technique [7] and the implementation of second order focussing techniques have increased this value to 9000+ [4].

7.2.3 Electrostatic MultiPass Time-Of-Flight

The charged particle storage ring is not a standard TOF system; this is obvious, as the particles need to be constantly accelerated in order to maintain them in the space desired. The portion of the ring that contains a field free region (i.e. a drift region) is relatively small (~20%), mainly the source and target regions. Within these regions, the difference in time between two distinct masses is the same as the drift region of a typical TOF system. In the remainder of the ring, particles experience acceleration perpendicular to their motion within the hemispheres and acceleration that can have components both parallel and perpendicular to the direction of their travel within the four lenses.

In order to see how effective the storage ring will be as a mass spectrometer, it is important to see if and how the flight times of particles with different masses will change the period of storage for each. These will be the first-order approximation, as no angular or energetic deviations will be considered.

The case of the hemisphere of mean radius \( r \) is to be considered first. A particle at the pass energy, \( E \), will undergo uniform circular motion, thus at all points within the hemisphere, the particle has an instantaneous perpendicular acceleration due to a radial electric field. This perpendicular field will not change the energy, only the direction. The time it takes to travel through 180° is given by:

\[
\frac{T}{2} = \frac{\pi v}{\nu} = \pi \sqrt{\frac{m}{2E}}
\]  

(7.8)
where $T$ is the period due to uniform circular motion, $m$ is the mass of the particle. This can be written in terms of the period to mass ratio and the physical aspects as

$$\frac{T_i}{\sqrt{m_i}} = \pi \sqrt{\frac{2}{E}} . \tag{7.9}$$

A second mass traveling with the same energy along the same radius of motion can also be expressed with equation 7.8. This mass will clearly have a different period as long as it is different from the first. This is because the pass energy is fixed as well as the radius of the hemisphere; the relationship between two masses can be given as:

$$\frac{T_1}{\sqrt{m_1}} = \frac{T_2}{\sqrt{m_2}} \Rightarrow T_1 = \sqrt{\frac{m_1}{m_2}} \times T_2 \tag{7.10}$$

This relationship is identical to that of a field free region, which is expected since the energy of the particles is held constant in both cases.

As particles move in the four lenses, it is possible for them to experience an electric field with components both perpendicular and parallel to the instantaneous direction of travel. The component of the electric field that is parallel to the instantaneous direction of travel alters the energy of the particle whereas the component perpendicular to the motion alters its trajectory. The perpendicular component has no effect on the velocity, only changing the instantaneous trajectory of motion, therefore has a similar effect as the hemisphere on the time structure of the masses. The component parallel to the motion alters the velocity of the ions due to the change in potential energy, $U$, given by

$$U = q\Delta V . \tag{7.11}$$

This is independent of the mass of the particle, and so the change in energy will be the same for all masses. Therefore, ions have the same energy regardless of their mass as they pass the same point of the ring. If one considers a small enough distance $d$ such that the electric field is fairly constant, then the final energy of the ion is given by:

$$\frac{1}{2} mv_i^2 = \frac{1}{2} mv_i^2 + q\Delta V . \tag{7.12}$$

This can be expressed using Newton’s second law as

$$\frac{1}{2} mv_i^2 = \frac{1}{2} mv_i^2 + mad \tag{7.13}$$
where \( a \) is the magnitude of the particle acceleration. Since the change in potential energy is the same for all ions within the ring, then
\[
m_1 a_1 d = m_2 a_2 d .
\] (7.14)

The equation of motion for two different masses over the same distance \( d \) can be expressed as:
\[
v_{01} t_1 + \frac{1}{2} a_1 t_1^2 = v_{02} t_2 + \frac{1}{2} a_2 t_2^2 .
\] (7.15)

Using equations 7.5 and 7.14, this leads to
\[
\left[ \sqrt{\frac{m_2}{m_1}} v_{02} \right] t_1 + \frac{1}{2} \frac{m_2}{m_1} a_2 t_1^2 = v_{02} t_2 + \frac{1}{2} a_2 t_2^2 .
\] (7.16)

Equality in equation 7.16 clearly occurs when
\[
t_1 = \sqrt{\frac{m_1}{m_2}} \times t_2
\] (7.17)

which is the same condition reached in equation 7.7 for the drift region, and equation 7.10 for the hemispherical analyser. The storage ring therefore separates masses the same as a classic TOF system, even though ions undergo accelerations within the lenses and the hemisphere. This is very advantageous since the storage ring does not need to physically change to increase the flight time and hence the resolution; the particles just need to complete more orbits. Such a system is referred to as a MultiPass Time-Of-Flight system (MP-TOF). In the follow section, the first multi-isotope substance that was used, krypton, will be presented and the isotopes shown to follow equation 7.17.

### 7.3 Krypton

The first gas with multiple isotopes of significant abundance tested in the storage ring was krypton. \( \text{Kr}^+ \) was injected into the storage ring and was measured after it completed successive orbits, using the variable time-of-flight software described in Chapter 3. Krypton has six naturally occurring isotopes, given in Table 7.1, that fall in the mass range of 78-86 amu. Figure 7.2 shows a spectrum of \( \text{Kr}^+ \), obtained with an asymmetric configuration of the ring. As is evident, storage occurred for well over 7 ms, at which point data ceased to
be acquired. The mean orbit time for the bulk krypton bunch is ~131 µs, obtained by measuring the distance between the peaks of the first several orbits, prior to clear isotope separation.

Since $^{84}$Kr is the isotope of the largest abundance, the orbit time found for the bunch corresponds to the orbit time of this isotope. The orbit times of the other isotopes can then be found using equation 7.17, as shown in the previous section. As the ions revolved in the ring, isotopes began to separate from one another as early as the 5th orbit, occurring about 700 µs after injection or after having travelled ~3.25 m. The isotopes continue to separate after each orbit and after around 2 ms of storage all the isotopes are clearly distinguishable from one another, shown in Figure 7.2.

Figure 7.2 – (top) Time spectrum of a pulse of krypton, showing storage for 7 ms. Initially a single pulse, the isotopes separate out as more orbits are completed, which are more clearly visible in the figures below. (Bottom Left) The isotopes for three orbits between 1.6 and 2.05 ms after injection, 5 of the 6 isotopes are clearly visible. (Bottom Right) Three orbits 3.1 to 3.5 ms after injection. At this point the light isotopes are catching the heavier isotopes from the previous orbit, seen at 3.2 ms.
The separation of the isotopes will continue until the faster ions from one orbit catch the slower ions from the previous orbit. This first is observed at 3.3 ms, when $^{80}\text{Kr}$ merges into the $^{86}\text{Kr}$ peak. At this point it shows the usefulness of recording over the course of many orbits; if the time 3.3 ms had been picked and the spectra record for only a single orbit, then the detection of the $^{80}\text{Kr}$ peak would have not been possible. For times beyond this, the pulses of the different isotopes can be observed passing through one another. No detectable effects, however, can be seen altering the different isotopes when this occurs, and this will be addressed later in the chapter.

In addition to the number of orbits that the ions complete, another key factor in the resolution is the width of the peaks. As is clear, the widths gradually increase as the particles complete more orbits. The trend can be fitted with the same formula used to describe the electron peak width, given by equation 6.2.

### 7.3.1 Data Analysis

The peak areas can by found be integrating the peaks of the isotopes that have clearly separated from the other ones. Each of the isotopic species is subject to an exponential decay, due mainly to scattering off background gas. The natural logarithm of the intensity, or the total area, of the peaks then yields a linear trend. The data, along with the fits, are plotted as a function of time in Figure 7.3 for each of the constituent isotopes. As can be seen in the graph, the exponential decay for each is approximately the same, indicated by the slopes of the linear fits being the same. This suggests that there is no discernable difference in the loss mechanisms based on mass. The $y$-intercepts of these fits then can be used to find the percent abundance of each, representing the initial amounts of each isotope. These values can be found in the Table 7.1.

An alternative way to find the abundance of each isotope is to find a distance (or time) at which each of the isotopes is separated from the others. Figure 7.4 shows such a time, where all six isotopes are nearly distinct. A Gaussian fit is applied to each of the peaks and thus the peak areas are found. These areas can then be used to find the percentage of each isotope, which is shown in Table 7.1. In this example, however, the isotopes do not correspond to the same orbit, $^{78}\text{Kr}$ and $^{80}\text{Kr}$ have travelled an extra orbit and are subject to
further losses. This is not apparent in the calculation, which has both isotopes significantly higher than their naturally occurring abundance [8].

![Figure 7.3](image1)

Figure 7.3 - The logarithm of the peak area as a function of time. Only points that represent clearly separated peaks are included in the fit of the decays of the individual isotopes. The points that appear above the red line at ~6 ms are not included in any fit, as they correspond to when the $^{82}$Kr peak merges with the $^{86}$Kr peak. The lightest isotope, $^{78}$Kr, was not distinguishable above the noise in this type of analysis.

![Figure 7.4](image2)

Figure 7.4 – A ‘single orbit’ showing the six isotopes of krypton - from left to right are $^{82}$Kr, $^{83}$Kr, $^{84}$Kr, $^{78}$Kr, $^{86}$Kr and $^{80}$Kr. The lightest two, $^{78}$Kr and $^{80}$Kr have travelled an extra orbit, and therefore appear later in the spectrum. While a Gaussian distribution is expected, the pulsing of the hemisphere during the injection and detection procedure can interfere with the edges of the ion pulse, producing some asymmetry in the count distribution.
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Accepted Natural Abundance</th>
<th>Experimental Abundance - Linear Fit Method</th>
<th>Experimental Abundance – Single Peak Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{78}$Kr</td>
<td>0.35</td>
<td>-</td>
<td>2.2 ± 0.2</td>
</tr>
<tr>
<td>$^{80}$Kr</td>
<td>2.28</td>
<td>2.60 ± 0.02</td>
<td>3.7 ± 0.2</td>
</tr>
<tr>
<td>$^{82}$Kr</td>
<td>11.58</td>
<td>11.59 ± 0.07</td>
<td>14.4 ± 0.5</td>
</tr>
<tr>
<td>$^{83}$Kr</td>
<td>11.49</td>
<td>11.91 ± 0.04</td>
<td>10.5 ± 0.5</td>
</tr>
<tr>
<td>$^{84}$Kr</td>
<td>57.00</td>
<td>52.13 ± 0.08</td>
<td>54.4 ± 0.7</td>
</tr>
<tr>
<td>$^{86}$Kr</td>
<td>17.30</td>
<td>23.41 ± 0.04</td>
<td>14.9 ± 0.5</td>
</tr>
</tbody>
</table>

Table 7.1 – Calculated abundance for the data given in Figure 7.2. $^{78}$Kr was not clearly distinguishable against the background for the linear fitting method.

A clear discrepancy between the two methods and the accepted abundances can be seen in Table 7.1. While the linear fit method is not able to be performed on $^{78}$Kr due to the small abundance, this is likely due to the shortened data acquisition method used which does not allow sufficient time for the peak to build to a statistically significant peak. On the contrary, the single peak method allows for this time, but not all the isotopes correspond to the same orbit. This could be addressed by allowing for a shorter flight time for the isotopes, compressing the spectrum.

### 7.4 Xenon

Xenon is another gas that contains naturally occurring isotopes, having 9 in total. This makes it another good candidate to test the feasibility of the storage ring as a mass spectrometer. Xenon recycled in the storage ring for well over 10 ms with excellent signal to noise ratio. The entire spectrum is shown in Figure 7.5, with the orbit time for the largest isotope, $^{132}$Xe being 172 µs. From this value, the orbit times for the other isotopes can be found, again using equation 7.17 with their respective masses.

The natural logarithm of the peak areas is plotted as a function of time and the linear fits completed, as before. The same process is taken in order to determine the abundance of each of the isotopes that were detected with significant statistics. The results are shown in Table 7.2. Once again however, the quickness of the scan has resulted in some irregularities.
in the intensity of consecutive peaks of the same isotope, leading to a linear fit with greater error.

Figure 7.5 – (top) Time spectrum of a pulse of xenon, showing successful storage for over 10ms. The isotopes separate from one another, as more orbits are completed, which are more clearly visible in the figures below. (Bottom Left) The isotopes are shown for three orbits between 2.0 and 2.5 ms after injection. Seven of the nine isotopes are identifiable. (Bottom Right) Three orbits 4.0 to 4.5 ms after injection, showing the isotopes have clearly separated at this point.

In an attempt to reduce the error, the spectrum was not scanned through multiple orbits, but kept fixed recording over one singular orbit. The orbit was selected so that the entire spectrum was recorded simultaneously, with the inclusion of the two peaks of small intensity. The single peak spectrum, Figure 7.6, corresponds to data recorded for several minutes, contrasting the seconds of acquisition during the scan mode. While unfortunately, the mass peaks as not entirely separated at this point in time, the peaks, however, are clearly detectable. Finding the abundance is done once again using a Gaussian fit, with the results given in Table 7.2 compared with the peak integration done on the fully separated peaks.
Figure 7.6 – A single orbit spectrum for xenon, showing all 9 isotopes. These peaks are fitted with a Gaussian distribution in order to find the abundance of each within the sample. While the peaks are asymmetric, the last two peaks in particular, it is likely a result of the injection and detection procedure, which may interfere with the edges of the ion pulse.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Accepted Natural Abundance</th>
<th>Experimental Abundance - Single Peak Method</th>
<th>Experimental Abundance – Linear Fit Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{124}$Xe</td>
<td>0.10</td>
<td>0.042 ± 0.006</td>
<td>-</td>
</tr>
<tr>
<td>$^{126}$Xe</td>
<td>0.09</td>
<td>0.13 ± 0.02</td>
<td>-</td>
</tr>
<tr>
<td>$^{128}$Xe</td>
<td>1.92</td>
<td>1.9 ± 0.5</td>
<td>2.01 ± 0.04</td>
</tr>
<tr>
<td>$^{129}$Xe</td>
<td>26.44</td>
<td>29.0 ± 0.7</td>
<td>25.4 ± 0.2</td>
</tr>
<tr>
<td>$^{130}$Xe</td>
<td>4.08</td>
<td>4.6 ± 0.6</td>
<td>4.2 ± 0.1</td>
</tr>
<tr>
<td>$^{131}$Xe</td>
<td>21.18</td>
<td>21.2 ± 0.6</td>
<td>21.4 ± 0.2</td>
</tr>
<tr>
<td>$^{132}$Xe</td>
<td>26.89</td>
<td>26.8 ± 0.7</td>
<td>27.0 ± 0.2</td>
</tr>
<tr>
<td>$^{134}$Xe</td>
<td>10.44</td>
<td>9.4 ± 0.6</td>
<td>10.8 ± 0.1</td>
</tr>
<tr>
<td>$^{136}$Xe</td>
<td>8.87</td>
<td>6.9 ± 0.4</td>
<td>9.2 ± 0.1</td>
</tr>
</tbody>
</table>

Table 7.2 – The isotopes of xenon, along with the accepted abundances [8] and the calculated abundances using a single orbit method and a linear fitting method. There is discrepancy between the two values, likely caused by the fast scanning method used to acquire data.
### 7.5 Data Simulation

Data can be simulated by the representation of each peak of the spectrum with a Gaussian distribution. The distribution for each peak is combined with all the others to form a similar spectrum of the data. This is useful when determining the charge to mass ratio of unknown particles that are being stored in the storage ring, using a calibration gas prior to the experiment. The Gaussian peak evolves due to factors that alter the particle beam, such as an exponential decay and the peak width evolution, covered in the previous chapter/section. Each peak is described by the following formulae

\[
y(t, ot(n), dt_n, A(n)) = A(n)e^{\left(-\frac{(t-ot(n))^2}{2(s(dt_n))^2}\right)}
\]  

(7.1)

\[
A(n) = A_0e^{-\frac{ot(n)}{\tau}}
\]  

(7.2)

\[
s(dt_n) = \frac{dt_n}{2\sqrt{\ln(2)}}
\]  

(7.3)

where \(A(n)\) represents the amplitude, \(s(dt_n)\) the standard deviation based on peak width, \(dt_n\), \(ot(n)\) is the time after injection of the centroid of the \(n^{th}\) peak, \(A_0\) is the area of the initial Gaussian peak, \(\tau\) is the lifetime of the exponential decay. The peak width \(dt_n\) is the same as that given by equation 6.2. A calculated peak is found for each peak of a measured spectrum, and the calculated peaks are combined to recreate the data.

For a spectrum with several masses, a few assumptions can be made for the data under specific circumstances. First, the peak width after a given orbit is the same for all species in the sample. While this is not valid over the entire mass range due to the pulsing of the hemisphere, for similar masses this is acceptable, since the time of the different particles to travel through the hemisphere will be similar. Next, the exponential decay for each mass is the same, but not the lifetime. It is therefore more appropriate to use a decay formula to give the area of the peak after the \(n^{th}\) orbit, \(N(n)\), based on the mean-free path, \(\lambda\), the distance travelled after the \(n^{th}\) orbit, \(x(n)\), and the initial area, \(N_0\), given below.

\[
N(n) = N_0e^{-\frac{x(n)}{\lambda}}
\]  

(7.4)
This reduces the area of each mass by the same percentage after they have travelled the same distance. Finally the ratios between the orbit times are based on the square root of the ratio of the masses.

The data were also recreated by using the formulae for peak width evolution given by equation 6.2, allowing for an exponential decay, using either equation 6.1 or 7.4, and using the natural abundances given in Table 7.2. These equations are applied to a single peak, which has an initial width and height based on the first peak of the experimental data. The simulated data show the entire mass spectrum along with the evolution of this spectrum as it completes more orbits, shown in Figure 7.7.

Figure 7.7 – Experimental data (left) showing the evolution of a Xe+ pulse as it is stored within the ring. The naturally occurring isotopes of the xenon pulse can be seen separating from one another, as the different isotopes travel with different velocities. The simulated data (right) is the result of using a peak width, which varies as the orbit, an exponential decay, and setting the isotope abundance to the current accepted values. [8] The agreement between the two is excellent.
The simulation and the data are clearly in excellent agreement for the positions of the mass peaks as well as the width of the mass peaks. The largest discrepancy is the intensity of the peaks, as the counts in a single peak are only accumulated for a few seconds and this statistically will result in some variation. As for the experimental data shown earlier in the chapter for xenon and krypton, the simulated data also shows the peaks of the different isotopes passes through one another. These occur in both the experimental and simulated data after the same number of orbits. As the amplitudes of the two peaks remain in the same proportion, and the peak width does not change, there is no detectable effect as the peaks merge and demerge. This can be stated with confidence, as the simulated data are purely mathematical, and each of the peaks is calculated individually. In the next two sections, the mass resolution of the apparatus will be estimated, and data, corresponding to heavy molecules, will be simulated in order to provide an indication of the expected performance.

7.6 Resolution of the System

The resolution of the apparatus can be estimated in order to qualify how the apparatus functions as a mass spectrometer. Using the assumption that the timing variables, such as the orbit time and the peak widths, of heavy ions evolve in a manner similar to lighter ions and are related by the square root of the mass ratio, this resolution can be determined. This will determine whether a specific mass range from heavy ions can be run successfully within the ring.

The time required for a mass $M$ to complete $n$ orbits is related to the distance travelled and the average energy of the particle, $\bar{K}$. For $n$ orbits, orbital path length $L$, this is expressed as:

$$t_n = \frac{nL}{v} = \frac{nL\sqrt{M}}{\sqrt{2\bar{K}}}.$$  \hspace{1cm} (7.18)

The difference in time for particles with two adjacent mass numbers to complete the same number of orbits, given that the average energy is the same, can therefore be expressed as

$$\Delta t_n = \frac{nL}{\sqrt{2\bar{K}}} \left(\sqrt{M} + 1 - \sqrt{M}\right).$$  \hspace{1cm} (7.19)
which will be a positive quantity, since the heavier mass will require longer time. This can be approximated using a binomial expansion to give

$$\Delta t_n \approx \frac{nL}{\sqrt{2K}} \frac{1}{2\sqrt{M}} = \frac{n_{\text{orbit}}}{2M}$$

(7.20)

and simplified by using the fact that the denominator is twice the average momentum. This is not the only value that needs to be considered, as the peak widths are also increasing as a function of orbit, as seen in the previous chapter, and this will affect the ability to detect the mass peaks distinctly. By assuming the peak width evolution can also be expressed using equation 6.2, the ratio of the separation between peak centers and the peak width can be found:

$$\left(\frac{\Delta t}{\Delta T}\right)_n \approx \frac{n_{\text{orbit}}}{2M} \frac{1}{\sqrt{W_0^2 + n^2 \Delta T^2}}$$

(7.21)

In order for these masses to be distinguishable, this ratio must at least be 1 at the time of detection. While this will clearly not give completely separated peaks, for the purposes of identification of the mass number it will suffice.

A plot of this ratio as a function of orbit number can be seen in Figure 7.8, showing the minimum number of orbits required to make this ratio larger than 1, for two cases. The masses are 4000 and 4001, both with the same average energy of 10 eV and a time spread per orbit of 0.1 µs. In the first case, represented by the red line, the initial temporal width is 2 µs, while the second case, given by the blue line, the initial width is only 1 µs. As can be seen in the figure, the case with the larger initial width will need twice as many orbits in order to be able to detect the two masses separate from one another. This shows the importance of the timing parameters of the beam on the resolution of the system.

Figure 7.8 - The ratio of the time between peaks and the peak widths is plotted as a function of orbit number for $M = 4000$ amu, $\Delta T = 0.1 \mu s$, $K = 10$ eV, $t_{\text{orbit}} = 0.1$ ms is plotted for $W_0 = 2 \mu s$ and $W_0 = 1 \mu s$. While in both cases the peaks can be resolved, it requires only about half of the orbits for the $1 \mu s$ case.
Solving for $M$ in equation 7.21 can provide an estimate of the mass resolution. In doing so, the orbit time for a single orbit, at a fixed, average energy must be accounted for using equation 7.18, which adds another factor of $\sqrt{M}$ and is given by
\[
\frac{\Delta t}{\delta t} \approx \frac{n}{2M} d \frac{\sqrt{M} \frac{1}{\sqrt{W_0^2 + n^2 \Delta T^2}}}{2K} \tag{7.22}
\]
and rearranging this gives
\[
M \approx \left(\frac{\delta t}{\Delta t} \right) \frac{nd}{2} \frac{1}{\sqrt{2K \sqrt{W_0^2 + n^2 \Delta T^2}}} \tag{7.23}
\]
Assuming a ratio of the time between peaks and the peak widths 1 and plotting the result as a function of orbit number shows the largest mass that can be separated from an adjacent mass number within that many orbits. This is shown in Figure 7.9, and the approximate mass resolution seen after $\sim$50 orbits, which has been easily obtained within the storage ring, is $\sim$5000. As can be seen, the mass resolution will be limited after this point, and no further revolutions of the bunch will aid in the improvement of the resolution. This saturation is caused by the peak width evolution of the ion pulse, specifically $n\Delta T$, which is the most critical factor in determining the resolution of the apparatus.

Figure 7.9 – The largest mass as a function of orbit number in which the ratio of the time between peaks and the peak widths remains $>1$. $\Delta T = 0.1 \mu$s, $K = 10$ eV, $\tau_{\text{orbit}} = 0.1$ ms is plotted for $W_0 = 2 \mu$s. Left - the mass resolution saturated at $\sim$5500 within 1000 orbits. At this point, even if more orbits were obtainable, the resolution would not increase. Right - the first 100 orbits in which the mass resolution increases over this range.
Operation when the pulse is displaying isochronous behaviour, discussed within the previous chapter, will minimize the spread of the peak widths and lead to an improved mass resolution. This can be represented by a generic exponential function of the form:

\[
dt_n = A \left( 1 - Be \frac{n}{C} \right)
\]  

(7.24)

where \(A, B, C\) are arbitrary constants and \(n\) is the number of orbits completed. The peak widths under this model will stop increasing as more orbits are completed. Figure 7.10 shows the difference between an exponential fit, given by equation 7.24 and a quadratic fit, given by equation 6.2. Here it is clear the benefit of the isochronous behaviour of the particle bunch, as the peak widths of the quadratic fit continuously increase.

This mathematical relation can be applied to provide an estimate of the resolution of the apparatus when the apparatus is operating in an isochronous mode. Starting from equation 7.23, the peak width term can be replaced with equation 7.24 to give:

\[
M \approx \left( \frac{\partial \phi}{\partial t} \right)_n \frac{nd}{2} \sqrt{\frac{1}{2K}} \frac{1}{A \left( 1 - Be \frac{n}{C} \right)}.
\]  

(7.25)

Figure 7.10 – Right - A quadratic fit (red) shown with an exponential fit (blue) as given by equation 7.24. Despite the difference in the initial peak widths, the quadratic fit allows the peak widths to continuously increase as more orbits are completed, while for the exponential fit it saturates after many orbits. Left – The isochronous behaviour shown previously in Figure 6.3 for comparison [9, 10].
Figure 7.11 shows the mass resolution after completing $n$ orbits, as determined by equation 7.25. Unlike Figure 7.9, the mass resolution continuously increases as more orbits are completed. This is easily understood, since the peak widths become constant, but the difference in orbit times between two unique masses is still present. In this operating mode, the limiting factor to the mass resolution will become the number of orbits the particle bunch can complete.

Figure 7.11 – The largest mass that can be separated after the $n^{th}$ orbit when the system is operating in an isochronous mode. Unlike the quadratic fit, the mass resolution continuously improves as more orbits are completed by the particle bunch.

### 7.7 Application to Heavy Ions and Biomolecules

The performance of the mass spectrometer for the application with heavy ions can be quantified using data simulation, as discussed earlier. As was shown in the previous section, the mass resolution will improve for the first ~50 orbits, afterwards limited by the peak width regardless of the number of orbits completed by the bunch. The main difference between the simulated data and the experimental data acquired for xenon and krypton will be in the orbit time and the time spread per orbit, which both depend on the mass. These will both increase, specifically the peak width, which is the limiting factor affecting the resolution.

Figure 7.12 is a simulated data set consisting of two masses, 5000 and 5001 amu, after they have travelled through 25, 50, 75 and 100 orbits. The ions are shown to separate for the first ~50 orbits, leading to a small valley between the peaks of the two species. After this point, while the ions will continue to separate, the peak widths have increased and the
resolution is no longer improving, as indicated by the valley between the two masses. Ideally, this valley would continue to increase until the masses were distinct. However, it stops ‘growing’, which is an indication that further orbits are no longer beneficial to the mass resolution.

Figure 7.12 – Simulation of an ion bunch after it has completed 25, 50, 75 and 100 orbits. The bunch consists of two masses, 5000 and 5001. The mass are shown to separate for the first ~50 orbits, afterwards the peak width becomes too large and more orbits do not improve the separation, as is evident with comparison of the masses between after 50, 75 and 100 orbits have been completed.

The isochronous behaviour can also be simulated for the same conditions. Figure 7.13 is a simulation of two masses, 5000 and 5001, and as before, they have travelled 25, 50, 75 and 100 orbits before the bunch is observed. The bunch has not yet separated after 25 orbits and the two masses remain indistinguishable. After 50 orbits however, the two masses are clearly separating and the separation is comparable to the same distance as seen in Figure 7.12. The final distances, corresponding to 75 and 100 orbits, are a significant improvement over the previous simulation, as the two masses continue to separate from one another. This demonstrates the advantage of operating in such a mode to the mass resolution.

Figure 7.13 - Simulation of an ion bunch after it has completed 25, 50, 75 and 100 orbits. The bunch consists of two masses, 5000 and 5001, which are shown to continuously separate as more orbits are completed, as the peak width has become a constant value.
One issue that will become more critical in operating in such a mode will be scattering off background gas. This can be partially corrected for by using improved vacuum technology, but will also depend on the ion source method used; currently only a simple source based on electron impact has been implemented with the storage ring.

7.8 Summary

The application of the storage ring as a mass spectrometer has been discussed and presented. Data from krypton and xenon was presented, showing the fully resolved isotopes after only ~10 orbits. The initial estimate of the mass resolution is ~5000, showing potential to be competitive, although it is currently limited by the width of the peaks. Although this value is reduced in order for fully separated peaks to be detected, an improvement to the ion source will also have a significant impact on the peak widths, and thus the resolution. The potential to be used in conjunction with the isochronous behaviour briefly introduced in Chapter 6 and this has been briefly discussed, ultimately showing that the apparatus can become a very competitive device for the field of mass spectrometry.
7.9 References

Summary and Conclusion

An ultra-low energy (~30 eV) electrostatic storage ring, consisting of two hemispherical deflector analysers and four cylindrically symmetric lenses, has been made to store either electrons or positive ions for over 100 orbits, corresponding to ~65 m. This device will be capable of storing more ‘exotic’ particles such as positrons and negative ions with ease. These particles can then be used in lifetime or collision studies, as the periodic nature of the device allows for multiple orbits to be completed, giving the particles longer storage time or more passes through an interaction region containing a neutral gas target. The details regarding the construction of the device, along with the data acquisition method and the controlling software are presented.

To aid in the operation of the device, matrix formalism has been applied to the fully symmetric case in order to predict stable storage regions. This study has shown that storage is not only found, but exists in a resonant-like pattern, with stable and unstable modes alternating with one another. The fully symmetric case has, in this work, been extended to the most general case where no symmetry with the lens potentials exists. This has allowed for regions of storage to be found theoretically without the need for computationally intensive simulations. Experimental data, found through the use of custom-built software, is shown to be in excellent agreement with the theoretical predictions, despite the absence of aberrations within the model and symmetry in the lens potentials. The asymmetric lens configuration is found to be optimal for specific combinations of lens potentials as compared with the symmetric case, which suggests that other storage rings currently operating in a fully symmetric configuration may benefit from removing symmetry.

A study of the apparatus as a potential mass spectrometer has been completed, with xenon and krypton being the first species that have had their constituent isotopes successfully separated. The data acquired from these data sets helped to develop a computer simulation that could be scaled to heavier masses, providing a prediction to the performance of the storage ring. Initial mass resolutions, for the purpose of identification, are approximately 5000, indicating that the ring shows potential to be a small, inexpensive alternative to existing devices and yet still remain competitive.
**Future Work**

The storage ring has proven to perform adequately for both positive ions and electrons; however, there are a few improvements that can be applied to enhance the apparatus. The ion source can be improved through the use of a pulsed laser which will minimize the time and the volume in which ionization occurs. This change should help to reduce the initial peak width from ~2-5 µs to the 10-100 ns range that is currently obtained by other devices presented in Chapter 1.

The isochronous mode, which has been observed for electrons, has the potential to improve the mass resolution of the storage ring. Operating in such a mode has been elusive, however, and the principle behind this mode is not yet understood. Further study on the energy resolution and timing properties are required for this mode. As this mode has only been observed while operating the storage ring asymmetrically, a computer simulation may offer some insight to this mode, specifically about the trajectories of the particles. A computer model could also be used to compare with the matrix approach and the experimental results.

The storage device has the potential to be used in the study of negative ions, as the storage ring has successfully been used for electrons and for the heavier positive ions. Furthermore, this has been achieved with mean particle energy from ~5 to 30 eV. The lower limit is currently limited by the particle flux and the upper limit is due to the range of the power supplies currently used.

A second generation of this ring is currently under construction at the University of Western Australia. This ring is slightly larger in size with additional lenses placed in each of the lens stacks for operational flexibility. As a result of the additional lenses, the overall transfer matrix is more complicated, as there are additional matrices for these lenses. A study involving the matrix approach presented in this thesis has yet to be applied to this ring to provide suitable conditions for stability.
Appendices

1  Hemisphere Pulsing Circuit  134
2  Overall Transfer Matrix after N Orbits  135
3  Derivation of Equation 5.9  136
Appendix 1 – Hemisphere Pulsing Circuit

![Circuit Diagram](image)

Figure A.1 – Circuit diagram for the hemisphere, showing both the Opto – Isolator, which isolates the incoming TTL pulse from ground, and the summing amplifier, which gives the hemispherical elements a voltage based on the incoming pulse. The TTL pulse triggers the output voltage $V_{out}$, to switch between two voltages: one representing a field-free state and one representing a storage state of the hemisphere. In the diagram, $C_1 = C_2 = 47.0 \ \mu F$, $C_3 = 1.0 \ \mu F$, $C_4 = 370 \ \text{pF}$, $R_1 = R_2 = 680 \ \Omega$, $R_3 = 220 \ \Omega$, $R_4 = R_G = 330 \ \Omega$.

---

Appendix 2 – Overall Transfer Matrix after N Orbits

The overall transfer matrix after traversing \( N \) cells is simply the single cell transfer matrix to the \( N \textsuperscript{th} \) power, which is given by equation 4.17, shown below.

\[
M_{ss}^N = \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix}^N = \begin{bmatrix}
\cos N\theta & L \sin N\theta \\
-\frac{\sin N\theta}{L} & \cos N\theta
\end{bmatrix}
\]

This can be proved by induction. First, for the case where \( N=1 \) this is clearly true, as

\[
M_{ss}^1 = \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix} = \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix}
\]

Assume this is true for all \( M \); that is the relation

\[
\begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix}^M = \begin{bmatrix}
\cos M\theta & L \sin M\theta \\
-\frac{\sin M\theta}{L} & \cos M\theta
\end{bmatrix}
\]

holds for all \( M \). For \( M+1 \) number of cells traversed

\[
M_{ss}^{M+1} = \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix}^{M+1} = \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix}^M \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix} = \begin{bmatrix}
\cos M\theta \cos \theta - \sin M\theta \sin \theta & L \sin \theta \cos M\theta + L \sin M\theta \cos \theta \\
-\frac{\sin M\theta \cos \theta - \cos M\theta \sin \theta}{L} & \cos M\theta \cos \theta - \sin M\theta \sin \theta
\end{bmatrix}
\]

by using the inductive step. Performing the multiplication leads to

\[
M_{ss}^{M+1} = \begin{bmatrix}
\cos M\theta \cos \theta - \sin M\theta \sin \theta & L \sin \theta \cos M\theta + L \sin M\theta \cos \theta \\
-\frac{\sin M\theta \cos \theta - \cos M\theta \sin \theta}{L} & \cos M\theta \cos \theta - \sin M\theta \sin \theta
\end{bmatrix}
\]

Using the Sum-Difference formulae for sine and cosine

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha
\]

gives:

\[
M_{ss}^{M+1} = \begin{bmatrix}
\cos \theta & L \sin \theta \\
-\frac{\sin \theta}{L} & \cos \theta
\end{bmatrix}^{M+1} = \begin{bmatrix}
\cos(M + 1)\theta & L \sin(M + 1)\theta \\
-\frac{\sin(M + 1)\theta}{L} & \cos(M + 1)\theta
\end{bmatrix}
\]

Since this holds for arbitrary \( M \), it is true by induction.
Appendix 3 – Derivation of Equation 5.9

The half matrices were expressed in two forms, one specific for lenses and one general form:

\[
M_{tt} = \frac{1}{f_1 f_2} \begin{bmatrix} f_1 f_2 - 2K_1 K_2 & 2K_1 (f_1 f_2 - K_1 K_2) \\ -2K_2 & f_1 f_2 - 2K_1 K_2 \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & L_1 \sin(\alpha) \\ -\frac{\sin(\alpha)}{L_1} & \cos(\alpha) \end{bmatrix}
\]

\[
M_{tt} = \frac{1}{f_5 f_6} \begin{bmatrix} f_5 f_6 - 2K_5 K_6 & 2K_5 (f_5 f_6 - K_5 K_6) \\ -2K_6 & f_5 f_6 - 2K_5 K_6 \end{bmatrix} = \begin{bmatrix} \cos(\beta) & L_2 \sin(\beta) \\ -\frac{\sin(\beta)}{L_2} & \cos(\beta) \end{bmatrix}
\]

Equating the corresponding matrix elements, the phase shifts and the characteristic constants are found to be

\[
L_1^2 = \frac{K_1}{K_2} (f_1 f_2 - K_1 K_2) \quad L_2^2 = \frac{K_5}{K_6} (f_5 f_6 - K_5 K_6)
\]

\[
\cos(\alpha) = 1 - \frac{2K_1 K_2}{f_1 f_2} \quad \cos(\beta) = 1 - \frac{2K_5 K_6}{f_5 f_6}.
\]

These can be used to express the overall transfer matrix as given in (5.7):

\[
M_{tt} = \begin{bmatrix} \cos(\alpha) \cos(\beta) - \frac{L_1}{L_2} \sin(\beta) \sin(\alpha) & L_2 \sin(\beta) \cos(\alpha) + L_1 \sin(\alpha) \cos(\beta) \\ \left(-\frac{1}{L_2}\right) \sin(\beta) \cos(\alpha) + \left(-\frac{1}{L_1}\right) \sin(\alpha) \cos(\beta) & \cos(\alpha) \cos(\beta) - \frac{L_2}{L_1} \sin(\beta) \sin(\alpha) \end{bmatrix}
\]

Applying the trace condition (4.7) gives

\[
\frac{1}{2} \left| \cos(\alpha) \cos(\beta) - \frac{L_1}{L_2} \sin(\beta) \sin(\alpha) + \cos(\alpha) \cos(\beta) - \frac{L_2}{L_1} \sin(\beta) \sin(\alpha) \right| \leq 1
\]

Removing the absolute value brackets, this becomes

\[
-2 \leq \cos(\alpha) \cos(\beta) - \frac{L_1}{L_2} \sin(\beta) \sin(\alpha) - \frac{L_2}{L_1} \sin(\beta) \sin(\alpha) \leq 2
\]

\[
-2 \leq \cos(\alpha) \cos(\beta) - \left(\frac{L_1}{L_2} + \frac{L_2}{L_1}\right) \sin(\beta) \sin(\alpha) \leq 2
\]

\[
-2 \leq \cos(\alpha) \cos(\beta) - \frac{L_1^2 + L_2^2}{L_1 L_2} \sin(\beta) \sin(\alpha) \leq 2 \quad (A3.1)
\]
Setting \( a = \frac{K_1K_2}{f_1f_2}, \ b = \frac{K_4K_6}{f_5f_6} \) will allow for the sine and cosine terms to be expressed as

\[
\cos(\alpha) = 1 - 2a \\
\sin(\alpha) = \sqrt{1 - (1 - 2a)^2} = 2\sqrt{a}\sqrt{1 - a},
\]

and the \( L \) term as

\[
L_a = K_1 \sqrt{\frac{1 - a}{a}}.
\]

Similar terms can be found for \( \cos(\beta), \sin(\beta) \) and \( L_2 \). Substituting these into \((A3.1)\) gives:

\[
-2 \leq 2(1 - 2a)(1 - 2b) - \left( \frac{K_1^2 \frac{1 - a}{a} + K_5^2 \frac{1 - b}{b}}{K_1 \left[1 - a\right] + K_5 \left[1 - b\right]} \right) \leq 2\sqrt{a}\sqrt{1 - a} 2\sqrt{b}\sqrt{1 - b} \leq 2
\]

\[
-2 \leq 2(1 - 2a)(1 - 2b) - \left( \frac{K_1^2 \frac{1 - a}{a} + K_5^2 \frac{1 - b}{b}}{K_1K_5} \right) 4ab \leq 2
\]

Divide by 2 and simplify the large term in parenthesis.

\[
-1 \leq -1 - 2a - 2b + 4ab - \frac{2K_1}{K_5} b(1 - a) - \frac{2K_5}{K_1} a(1 - b) \leq 1
\]

Subtract 2 from all sides and group terms.

\[
-2 \leq -a \left( 2 + \frac{2K_5}{K_1} \right) - b \left( 2 + \frac{2K_1}{K_5} \right) + ab \left( 4 + \frac{2K_1}{K_5} + \frac{2K_5}{K_1} \right) \leq 0
\]

Divide by 2 and express bracketed terms with a common denominator.

\[
-1 \leq -aK_5 \left( \frac{K_1 + K_5}{K_1K_5} \right) - bK_1 \left( \frac{K_5 + K_1}{K_1K_5} \right) + ab \left( \frac{2K_1K_5 + K_1^2 + K_5^2}{K_1K_5} \right) \leq 0
\]

\[
-1 \leq -aK_5 \left( \frac{K_1 + K_5}{K_1K_5} \right) - bK_1 \left( \frac{K_5 + K_1}{K_1K_5} \right) + ab \left( \frac{K_1 + K_5}{K_1K_5} \right)^2 \leq 0
\]
Divide through by \( \frac{K_1 + K_5}{K_1K_5} \), which is valid provided the overall term is positive.

\[
- \frac{K_1K_5}{K_1 + K_5} \leq -aK_1 - bK_1 + (K_1 + K_5)ab \leq 0
\]

\[
- \frac{K_1K_5}{K_1 + K_5} \leq bK_1(a - 1) + aK_5(b - 1) \leq 0
\]

Divide through by the product \( ab \), which again is valid provided the product is positive.

\[
- \frac{1}{ab} \frac{K_1K_5}{K_1 + K_5} \leq K_1 \frac{a - 1}{a} + K_5 \frac{b - 1}{b} \leq 0
\]

Divide through by -1 and multiply middle terms by 1

\[
0 \leq \frac{K_1^2}{K_1} \frac{1 - a}{a} + \frac{K_5^2}{K_5} \frac{1 - b}{b} \leq \frac{1}{ab} \frac{K_1K_5}{K_1 + K_5}
\]

Substitute to eliminate \( a, b \).

\[
0 \leq \frac{L_1^2}{K_1} + \frac{L_2^2}{K_5} \leq \left( \frac{f_1f_2}{K_1K_2} \right) \left( \frac{f_5f_6}{K_5K_6} \right) \frac{K_1K_5}{K_1 + K_5}
\]

Simplify to give equation 5.9

\[
0 \leq \frac{L_1^2}{K_1} + \frac{L_2^2}{K_5} \leq \frac{f_1f_2f_5f_6}{K_2K_6(K_1 + K_5)}
\]
Alternative Derivation for Equation 5.9

In the previous derivation, the use of the sin and $L$ terms involving square roots may lead to concerns involving the sign, as only the positive root was used. In this derivation, no such substitutions are used to provide the same result.

The half matrices were expressed in the form:

$$
M_{st} = \frac{1}{f_1 f_2} \begin{bmatrix}
    f_1 f_2 - 2 K_1 K_2 & 2 K_1 (f_1 f_2 - K_1 K_2) \\
    -2 K_2 & f_1 f_2 - 2 K_1 K_2
\end{bmatrix}
$$

$$
M_{ss} = \frac{1}{f_5 f_6} \begin{bmatrix}
    f_5 f_6 - 2 K_5 K_6 & 2 K_5 (f_5 f_6 - K_5 K_6) \\
    -2 K_6 & f_5 f_6 - 2 K_5 K_6
\end{bmatrix}
$$

This can be multiplied to give the transfer matrix after one complete orbit.

$$
M_{ss} = M_{ss} M_{st} = \begin{bmatrix}
    M_{00} & M_{10} \\
    M_{01} & M_{11}
\end{bmatrix}
$$

where

$$
M_{00} = \frac{1}{f_1 f_2 f_5 f_6} \left( [f_1 f_2 - 2 K_1 K_2] [f_5 f_6 - 2 K_5 K_6] - 2 K_2 (2 K_5 [f_5 f_6 - K_5 K_6]) \right)
$$

$$
M_{01} = \frac{1}{f_1 f_2 f_5 f_6} \left( -2 K_6 (f_1 f_2 - 2 K_1 K_2) - 2 K_2 (f_5 f_6 - 2 K_5 K_6) \right)
$$

$$
M_{10} = \frac{1}{f_1 f_2 f_5 f_6} \left[ 2 K_1 (f_1 f_2 - K_1 K_2) [f_5 f_6 - 2 K_5 K_6] + 2 K_5 (f_5 f_6 - K_5 K_6) (f_1 f_2 - 2 K_1 K_2) \right]
$$

$$
M_{11} = \frac{1}{f_1 f_2 f_5 f_6} \left[ -2 K_6 (2 K_1 (f_1 f_2 - K_1 K_2)) + (f_5 f_6 - 2 K_5 K_6) (f_1 f_2 - 2 K_1 K_2) \right]
$$

The trace is just the sum of the diagonal elements and using the trace condition (4.7), this gives

$$
-2 \leq \frac{1}{f_1 f_2 f_5 f_6} \left( [f_1 f_2 - 2 K_1 K_2] [f_5 f_6 - 2 K_5 K_6] - 2 K_2 (2 K_5 [f_5 f_6 - K_5 K_6]) \right) + \frac{1}{f_1 f_2 f_5 f_6} \left( -2 K_6 (2 K_1 (f_1 f_2 - K_1 K_2)) + (f_5 f_6 - 2 K_5 K_6) (f_1 f_2 - 2 K_1 K_2) \right) \leq 2
$$
Multiplying the terms together and all sides by \( f_1 f_2 f_3 f_6 \) leads to:

\[-2 f_1 f_2 f_3 f_6 \leq (f_1 f_2 f_3 f_6 - 2K_1 K_2 f_3 f_6 - 2K_5 K_6 f_1 f_2 + 4K_1 K_2 K_5 f_6) - 4K_2 K_5 (f_3 f_6 - K_5 K_6) - 4K_5 K_1 (f_3 f_2 - K_1 K_2)\]

Combining like terms together gives

\[-2 f_1 f_2 f_3 f_6 \leq 2 f_1 f_2 f_3 f_6 - 4K_3 K_5 f_1 f_2 f_3 f_6 + 8K_1 K_2 K_5 f_6 - 4K_2 K_5 (f_3 f_6 - K_5 K_6) - 4K_5 K_1 (f_3 f_2 - K_1 K_2)\] 

Subtracting \( 2 f_1 f_2 f_3 f_6 \) and then divide by -4 leads to

\[
0 \leq K_1 K_2 f_5 f_6 + K_2 K_6 f_1 f_2 - 2K_1 K_2 K_5 f_6 + K_2 K_5 (f_3 f_6 - K_5 K_6) + K_5 K_1 (f_3 f_2 - K_1 K_2) \leq f_1 f_2 f_3 f_6
\]

Combining terms and factoring

\[
0 \leq (K_1 f_5 f_6 - K_1 K_2 K_5 K_6) + (K_5 f_1 f_2 - K_1 K_2 K_5) + K_2 K_5 (f_3 f_6 - K_5 K_6) + K_5 K_1 (f_3 f_2 - K_1 K_2) \leq f_1 f_2 f_3 f_6
\]

Or written as

\[
0 \leq (K_1 + K_5)(f_5 f_6 - K_5 K_6) + (K_5 + K_1)(f_1 f_2 - K_1 K_2) \leq f_1 f_2 f_3 f_6
\]

Dividing through by \( K_2 K_6 (K_1 + K_5) \) gives

\[
0 \leq \frac{(f_5 f_6 - K_5 K_6)}{K_6} + \frac{(f_1 f_2 - K_1 K_2)}{K_2} \leq \frac{f_1 f_2 f_3 f_6}{K_2 K_6 (K_1 + K_5)}
\]

Finally, since \( L_1^2 = \frac{K_1}{K_2} (f_1 f_2 - K_1 K_2) \) and \( L_2^2 = \frac{K_5}{K_6} (f_5 f_6 - K_5 K_6) \), this leads to

\[
0 \leq \frac{L_1^2}{K_1} + \frac{L_2^2}{K_2} \leq \frac{f_1 f_2 f_3 f_6}{K_2 K_6 (K_1 + K_5)}
\]
Vita Auctoris

Michael Robert Sullivan was born in 1984 in Windsor, Ontario. He graduated from Kingsville District High School in 2003, where he went on to study at the University of Windsor, obtaining a B.Sc in Honours Physics and Mathematics in 2008. He is currently a candidate for the Doctoral degree in Physics at the University of Windsor and hopes to graduate in the Fall 2013.