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Shear Strengthening of Single Web Prestressed Hollow Core Slabs Using Externally Bonded FRP Sheets

Yuanli Wu
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Shear Strengthening of Single Web Prestressed Hollow Core Slabs

Using Externally Bonded FRP Sheets

By

Yuanli Wu

A Thesis
Submitted to the Faculty of Graduate Studies
through the Department of Civil & Environmental Engineering
in Partial Fulfillment of the Requirements for
the Degree of Master of Applied Science
at the University of Windsor

Windsor, Ontario, Canada

2015

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DECLARATION OF ORIGINALITY

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ABSTRACT

Precast, prestressed hollow core (PHC) slabs are among the most common load bearing concrete elements in the world. They are widely used in floors and roofs of office, residential, commercial and industrial buildings. As no shear reinforcement can be provided due to fabrication restraint, the concrete shear strength is the only source of PHC slab shear capacity. However, in some design situations, typically when large concentrated or line loads are present, shear capacity of PHC slabs could be exceeded. Therefore, developing an efficient technique to strengthen shear capacity of PHC slabs is essential. The objective of this research project is to explore an innovative application of Fiber-Reinforced Polymer (FRP) composite sheets to strengthening web-shear capacity of PHC slabs by installing the sheets along the internal perimeter of the slab voids. As the first phase of exploring the feasibility and effectiveness of this new technique, experimental and finite element study has been carried out on full scale single web hollow core slabs with carbon FRP sheets bonded on the full perimeter of voids on both sides of the specimen. Several parameters including the width and number of FRP sheets have been investigated. Test results showed the effectiveness of this novel shear strengthening technique.
DEDICATION

To My Grandfather and Grandmother:

Derong Li & Huihua Chen

I appreciate everything you have done for me.
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CHAPTER 1 INTRODUCTION

1.1 Background

A prestressed hollow core (PHC) slab (Figure 1.1) is a precast prestressed concrete slab typically used in the construction of floors or roofs in multi-story apartment buildings. They are also popular to be used for bridge decks (especially the long span ones) and wall panels. A PHC slab is produced on casting beds. First, arrange the prestressing strands along the bed according to the predetermined tension value. Second, move the slab extruder to the bed, which will complete six steps at one time. They include extruding concrete for bottom, slab ribs and panels; compacting concrete; moulding hollow core; moulding slab edge for locking key and exiting. The PHC slabs are then cured under appropriate temperature and moisture condition for a period of time (usually 28 days). Finally, it can be cut and moved out of the production line.

Figure 1.1 Prestressed hollow core slabs

Figure 1.2 shows a typical cross-section of PHC slabs. Compared to an ordinary
reinforced concrete slab, a PHC slab with the same resistance capacity can save 40% to 50% of steel and 20% to 40% of concrete. Its reduced thickness would help to conserve headroom, which is particularly important in multi-story buildings. The prestressed strands in slabs not only increase its strength but also reduce the deformation under normal conditions. Moreover, it is an economical choice in civil construction because of fast building assembly, light self-weight and less quantity of material that will save more cost and time.

![PHC Slab](image1.png) ![Web Beam](image2.png)

**Figure 1.2 Typical cross-section of PHC slab and web beam**

Although PHC slabs are generally designed to resist flexural stress induced by assumed uniformly distributed loads, in practice, they could also be subjected to concentrated load or line load which would lead to shear failure, in particular, over regions close to the end support (Figure 1.3). However, it is very difficult to arrange stirrups in PHC slabs during the manufacturing process because of the production method. Therefore, PHC slabs mainly rely on the tensile strength of concrete to resist shear force. When higher shear capacity is needed, the current practice is to either fill the voids with additional concrete or use thicker slabs to satisfy the shear strength requirement. However, both solutions would add cost, increase self-weight and thus reduce the economy of PHC slabs. Therefore, there is an
urgent need to develop an efficient technique to strengthen the shear capacity of PHC slabs while still retain its advantages.

![Figure 1.3 Shear failure of slabs](image)

### 1.2 Motivations

The Prestressed Systems Incorporation (PSI) is a proven national leader in the manufacture and installation of prestressed/precast concrete solutions and has served for civil projects ranging from Ontario to Kentucky, and Indiana to New York. In recent years, externally bonded fiber reinforced polymer (FRP) composite sheets have been successfully used in the repair and rehabilitation of reinforced concrete members. Enlightened by such experience, PSI is keen to extend this technique to shear strengthening of PHC slabs. To evaluate the effectiveness of this approach and find out the impact of FRP sheet characteristics, including width, length and number of layers, on the shear capacity of PHC slabs with different prestressing levels, experimental studies, complemented by numerical simulations, is necessary. Physical tests would be very useful to validate numerical model,
results and proposed strengthening technology. Numerical simulation would help to understand the shear behaviour of PHC slabs with externally bonded FRP sheets and failure mechanism, predict possible failure mode and conduct parametric studies.

As shown in Figure 1.4, this shear strengthening approach requires to attach FRP sheets to the surface of inner hollow core along circumferential direction to improve shear capacity of PHC slabs. Compare with the traditional approaches, this is very creative because FRP sheets are usually installed on the external surface of concrete members for strengthening. Also, FRP sheets are mostly added to structure members after construction. Attaching FRP sheets prior to installation of PHC slabs as proposed in this novel shear strengthening technique would make it easy and convenient to operate, quicker and cheaper in terms of construction time and cost.

![Figure 1.4 PHC slabs with FRP](image)

Figure 1.4 PHC slabs with FRP
1.3 Objectives

The objectives of the current study are proposed as follows:

1. Explore the feasibility and effectiveness of an innovative application of Fiber-Reinforced Polymers (FRP) composite sheets to strengthening web-shear capacity of PHC slabs by installing the sheets along the internal perimeter of the slab voids.

2. Develop a finite element model of I-shaped single web beam and full width PHC slab to understand the shear behaviour of web beams and PHC slabs when equipped with externally bonded FRP sheets.
CHAPTER 2 LITERATURE REVIEW

Wide range of research programs on shear capacity of PHC slabs have been conducted in different institutes, universities and countries. This chapter reviews the existing studies on failure mode of PHC slabs, FRP materials and both experimental and numerical studies on concrete beams and slabs.

2.1 Failure Mode of PHC Slabs and Traditional Strengthening Methods

2.1.1 Web shear failure and flexure-shear failure

In general, two typical types of shear failure would occur in PHC slabs, i.e. the web shear failure and the flexural shear failure (Celal, 2011).

Web shear failure occurs when the maximum principle stress resulted from the combined effect of shear and bending exceeds the tensile strength of concrete. As a consequence, a crack would initiate from a point on the critical section. Subsequently, more and more cracks would form and propagate upwards and downwards in a straight inclined form joining the edge of the slab support with the top of the slab and eventually lead to total collapse of the slab. The failure is mainly induced by diagonal cracks formed in the PHC slab web at the critical section near the support.

Flexural shear failure is a very typical failure mode of slender and lightly prestressed members. When normal stress in the slab due to flexure exceeds the tensile strength of concrete, vertical tensile crack would initiate at slab bottom face, generally at the critical section where the maximum moment occurs. The crack would propagate through concrete cover till it hits the steel reinforcing bars. As load increases, the crack would not only widen
but also continuous to propagate vertically beyond the level of reinforcements, and reach the top surface of the slab. At the same time, concrete in the tension zone would exit work and those in the compression zone would exceed the ultimate compressive strength of concrete and finally lead to failure.

### 2.1.2 Key parameters affecting shear capacity of PHC slabs

Existing studies (Anderson, 1976, Palmer and Schultz, 2010) show that there are a number of factors that could influence the shear capacity of PHC slabs. Among these, the most important ones are identified to be:

- **a)** Size of the cross section: The depth of slab can directly affect its ultimate shearing strength, just as beams (Hawkins and Ghosh, 2006);
- **b)** Level of prestressing: Reinforcement prestressed at different levels would alter the bearing capacity of PHC slabs (ACI 318-08);
- **c)** Concrete strength: This is an important parameter since it will determine $f'_c$ and $f'_t$ (Anderson 1987, ACI 318-08);
- **d)** Shear span ratio: This parameter would also affect the ultimate shear capacity of PHC slabs (MacGregor and Bartlett 2000, EN 1168-08)

### 2.1.3 Traditional Strengthening Methods for PHC Slabs

There are a number of approaches conventionally used to increase strength of concrete slabs. External prestressing can be applied at certain part of slabs, which will not only help to increase the moment resistance capacity, but also to close cracks and reduce deflection.
Steel sheets can be externally bonded to the surface of concrete slabs to improve their stiffness and ultimate capacity. This, due to its simple technology and reliable outcome, is very popular in practice. Besides, improvement of strength, stiffness and stability of reinforced concrete slabs can also be achieved by increasing the height and width of a slab section. However, this would result in larger effective slab cross-section and thus increase its self-weight and operational space.

2.2 Carbon Fiber Reinforced Plastic Sheets (CFRP)

In recent years, fiber reinforced concrete became a more matured technology. It attracted much attention of the engineering community because of its simple and quick construction, great repairing effect and durable performance.

Based on different raw materials, carbon fiber can be divided into many types and each one has a different feature. The carbon fiber products which are used for civil engineering structures mainly consist of PAN-based carbon fiber precursor. Normally, the products are Carbon Fiber Sheet (Figure 2.1), Carbon Fiber Plate and Carbon Fiber Prestressed Bars. It has high tensile strength along fiber direction which is about a dozen times of ordinary carbon steel bars. Also, it is light and thin with low density that hardly increase the weight and volume of the structure after strengthening. Furthermore, CFPR has good capacity for waterproof, durability, fatigue and corrosion resistance. In addition, it is easy for operation and construction.
Carbon fiber material is first developed and taken into application in Japan. It was found that this kind of material could be applied easily and quickly to structural members with complex geometric shape. Therefore, the technique of using FRP material for strengthening is widely used in civil engineering. According to statistics, this type of material has already been used in construction, repair and rehabilitation of various reinforced concrete structure. A series of design construction standards and guidelines for carbon fiber reinforcement have been developed in the past two decades, in particular, in USA and Japan (SIKA, 1994, Kobayashi et al, 1995, ACI Report, 1995; 2000, Thomas et al, 1996, Tonen General Sekiyu K.K 1996; 1997).

The technology of carbon fiber reinforced concrete structure is widely used and has good performance for strengthening concrete structure. The research and application for high performance FRP repairing technology began in the early 1980s. Meier (1987) first tried to use FRP on bridge structure reinforcement in 1982, the high strength-mass density ratio of CFRP made it possible to design and construct cable supported bridges of greatly...
increased spans (Meier, 1987, Kim and Meier, 1991, Saul, 1991). Results showed that CFRP could resist endure at least three times higher in the amplitude of stress and a higher mean stress than the steel material for $2 \times 10^6$ cycles without damage (Meier, 1991).

Following this, the technology was developed rapidly in Japan, USA and Europe. In 1991, American Concrete Institute (ACI) established ACI 440 committee which was responsible for studying the applications of fiber reinforced composite materials to strengthening concrete and masonry structures. Canada also set up a national research and development committee to study FRP repairing technology. ACI held the first international conference on FRP technology in Canada in 1993, which was subsequently held every two years. A FRP reinforcement committee was developed by Japan civil society in March 1999. Other counties such as Germany and Switzerland actively involved in research in this field.

Compared to the traditional reinforcement techniques, because of the excellent physical and mechanical properties of FRP, its high strength and high elastic modulus can be fully used to improve the strength and ductility of reinforced concrete structure components and to achieve efficient reinforcement repairing. FRP reinforced concrete members have good durability and corrosion resistance to a variety corrosive substances that structures like buildings could be exposed to, such as acid, alkali and salt. Besides saving cost on rust proof, as is always required for steel reinforcing bars, FRP sheets could also protect concrete member internally. These would save an enormous amount of maintenance cost.
FRP reinforced technology can be widely used in various structural types (such as buildings, tunnels, culverts, chimneys), various structural shapes (such as rectangles, circles, surface structure) and various structural members (such as beams, columns, plates, arches, shells, piers). The fact that the application of FRP would not change the appearance of structures makes it particularly unique that no other available reinforcing methods could compete with. Most importantly, for those large-scale civil structures, such as large bridge piers and decks, tunnels, large cylinder and shell structure, it is impossible to use conventional reinforcing method while it can be successfully solved by FRP technology.

Construction using FRP is very efficient. It involves no wet work, no large-scale construction equipment, no fixed-site facilities and occupancy for operation is less. Finished product of FRP is a piece of fabric with a width of 20cm, 30cm, 50cm or even 100cm. It can be rolled up easily and cut as needed. The flexibility of FRP ensures a nearly 100% bondage with the applied surface. Even if the surface is not smooth which could cause some bubbles after the FRP sheet is installed, they can be eliminated by injecting resin into air bubbles using syringe.

FRP material has very light weight, less than 1.0kg/m\(^2\) after installation (including the self-weight of resin). The thickness of a typical FRP sheet is only about 1.0mm. Therefore, using FRP as a strength reinforcing tool would not considerably add weight and size to the original member.
2.3 Experimental Study on Shear Strengthening of Reinforced Concrete Member using FRP

FRP composites are widely used to reinforce flexural and shear capacity of structural members, especially for concrete beams and slabs. The existing experimental studies on shear strengthening of reinforced concrete beams and slabs using FRP will be reviewed in this section.

2.3.1 Shear strengthening for reinforced concrete beams

The feasibility of using FRP composites to strengthen shear capacity of reinforced concrete beams was first explored by Berset (1992) at MIT. Glass fiber sheets were used to strengthen the lateral shear zone of reinforced concrete beams. By comparing the behaviour and capacity with the control beam, a shear capacity analysis model for the FRP strengthened beams was proposed, which the contribution of FRP sheets to the shear resistance capacity of the beam was found to be based on the maximum allowable strain.

The effectiveness of this shear strengthening technique was confirmed by Chajes et al. (1995) and Triantafilou (1998), of which the former series of tests showed a 50% to 60% increase in shear capacity, while the latter indicated a FRP orientation of 45° and 135° with beam axis would be more effective than the 90° arrangement.

Khalifa et al. (1999) investigated the shear behavior and failure mode of continuous reinforced concrete beams strengthened by CFRP sheets. The experimental results were used to validate a proposed shear design approach. A total of nine full-scale two-span
continuous reinforced concrete beams, as shown in Figure 2.2, were tested. All of them had a rectangular cross section of 150mm wide by 305 mm deep. Beams CW1, CO1 and CF1 were the control beams without CFRP. Beam CW2 was strengthened by two layers of CFRP sheet with the fiber oriented respectively at 90° and 0° with respect to beam axis and perpendicular to each other. Beam CO2 was strengthened by only one layer of CFRP sheet in the form of U-wrap strip. Each sheet strip had a width of 50mm and was oriented at 90° to the beam axis. The center-to-center spacing between the sheet strips was 125 mm. Beam CF2 and CO3 were strengthened using one continuous layer of CFRP sheet in U-wrap form. Beam CF4 was wrapped completely by one layer of CFRP sheet. Testing results showed that the externally bonded CFRP sheets could be used to enhance shear capacity of beams in both positive and negative moment regions. The shear strength was found to be increased by 22% to 135%. For beams without stirrups, the shear strengthening effect provided by CFRP was more considerable than those having adequate steel shear reinforcement.
Sundarraja and Rajamohan (2008) conducted a shear strengthening test on reinforced concrete beams using GFRP vertical side strips and U-wrap strips, as shown in Figure 2.3. The width of the GFRP strips in both cases were decided based on the provisions of ACI.
The load carrying capacity of the retrofitted beams was found to be greater than that of their respective control beams, which implied that the externally bonded FRP sheets were able to improve the shear capacity of reinforced concrete beams. While the GFRP vertical side strips inhibited the growth of diagonal cracks, U-wrap strips manifested better shear strengthening effect in terms of the load-deflection relation, although the load carrying capacity of the two were comparable. The amount of shear resistance provided by GFRP strips was found to depend on the strip width, i.e. greater strip width would offer higher shear capacity.

![GFRP vertical side strips](image)

Figure 2.3 GFRP vertical side strips of various widths (Sundarraja and Rajamohan, 2008)

Subsequently, Sundarraja and Rajamohan (2009) provided another set of experimental data from a shear strengthening test on reinforced concrete beams externally bonded by bi-directional GFRP strips, in the form of either inclined strips or U-wrap strips shown in Figure 2.4. Results indicated that GFRP strips were effective in shear strengthening of structures since the initial cracks appeared at higher loads than their respective control
beams. Also, the ultimate strength of beams could be increased when inclined or U-wrap GFRP strips were bonded, with the latter being more effective. The width of the CFRP strips was also found to be an important parameter determining the shear strengthening effect.

Figure 2.4 GFRP inclined side strips and U-strips of various widths (Sundarraja and Rajamohan, 2009)

Panigrahi et al. (2014) studied shear strengthening of reinforced concrete T-beams using epoxy bonded bi-directional GFRP sheets. The investigated parameters included the amount and distribution of GFRP, the bonding surface, the number of layers and the orientation of GFRP fibre. Results confirmed that the externally bonded GFRP technique could be used to enhance the shear capacity of reinforced concrete T-beams. The initial cracks in the strengthened beams were formed at higher loads compared to that of their respective control beams, and the ultimate load carrying capacity of the strengthened beams were increased. In addition, compared to a vertical (strips oriented at 90° to the beam axis) or “X-shape” (two strips cross each other, oriented at +45° and +135° to the beam axis) strip arrangement, strips oriented at 45° could offer more increment in shear capacity.
Among all tested configurations, the most effective one was found to be the U-wrap strips with end anchorage.

2.3.2 Shear strengthening for reinforced concrete slabs

Liu et al. (2008) tested seven full-scale FRP strengthened prestressed hollow core slabs under uniform load. It was found that the ultimate loading capacity of PHC slabs improved 48% -109% after strengthened by FRP sheets. Moreover, the ductility of the strengthened slabs was also considerably enhanced. The slab mid-span deflection was observed to increase significantly after the yielding of prestressed steel bars.

An innovative hybrid composite system, which combined the usage of high performance concrete and CFRP to enhance the bearing capacity of concrete slabs, was proposed and tested by Mosallamand et al. (2012). The proposed retrofitting scheme was applied to one-way slabs with two continuous spans. The ultimate loading capacity showed an increment up to 164% and the ductility, in terms of the maximum displacement, increased by up to 122%. Figure 2.5 gives the setup of the tests and the typical failure mode of the strengthened slab, which was dictated by shear and debonding of the concrete cover close to the intermediate support.
Li et al. (2014) conducted an experimental study on ten precast PHC slabs externally bonded with FRP sheets having different width, thickness and number of layers. Results confirmed again that externally bonded FRP sheets could be used for shear strengthening of PHC slabs. Observation on the failure mode revealed that while flexural failure occurred in PHC slabs with appropriate amount of FRP, excessive amount of FRP would lead to shear failure. An average increment of 17% for cracking load and 67% for ultimate load were achieved by strengthening PHC slabs with FRP. When failure occurred, the maximum displacement was found to be doubled. Furthermore, the initial bending stiffness of the concrete member could be enhanced with the increase of FRP amount.

2.3.3 Summary

Based on the above review, three main factors that would affect the shear strengthening effect of FRP on reinforced concrete members can be identified. They are

1. Configuration of FRP strips:

   This include a) I-shape strip, of which the FRP strip is only bonded to one side of the
beam surface; b) U-wrap strip, of which the FRP strip is bonded continuously to "wrap" the two side and the bottom surface of the beam; c) Full-wrap strip, of which the FRP strip wrapped around the beam surface;

2. Orientation of FRP fiber

In most applications, FRP strips are arranged to be perpendicular to the member axis (90° orientation). However, results from some experimental tests showed that the shear stiffness and shear capacity of concrete beams and slabs could be further enhanced if the FRP strips were oriented at 45° with respect to the member axis. The drawback of the 45° orientation was that the chance of having brittle failure would be higher than the 90° orientation arrangement.

3. Quantity of FRP

The shear strengthening effect of FRP mainly depends on the stress level of FRP before failure. The more the quantity of FRP is used, the smaller the strain in FRP at shear capacity would be. But, the effective stress in FRP would be really small and resulted in debonding failure when FRP amount is too much.

2.4 Numerical Study on Shear Strengthening of Reinforced Concrete Members Using FRP

The rapid development of finite element simulation technique and computer capacity make it possible to numerically simulate and study the behaviour of reinforced concrete members with FRP. Compare to the experimental and analytical approaches, it is cost
effective and more flexible to conduct parametric study. At present, the most popular commercial finite element softwares used in structure engineering are ANSYS and ABAQUS.

2.4.1 Numerical simulation using ANASYS

Fathelbab et al. (2011), Bennegadi et al. (2013) as well as Tara and Jagannatha (2014) all used ANSYS to numerically study the effectiveness of FRP in retrofitting reinforced concrete beams. The details of these finite element models, including the element types selected for concrete, steel reinforcement and FRP composite, the constitutive relation of different materials, the bond-slip relations between concrete and steel reinforcing bars, as well as that between concrete and FRP are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Structure Member</th>
<th>FRP Type</th>
<th>Constitutive Relation</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Concrete</td>
<td>Strand</td>
<td>FRP</td>
<td></td>
</tr>
<tr>
<td>Fathelbab et al. (2011)</td>
<td>RC beam</td>
<td>CFRP</td>
<td>smeared crack</td>
<td>elastic-perfectly plastic</td>
<td>linear elastic</td>
</tr>
<tr>
<td>Bennegadi et al. (2013)</td>
<td></td>
<td>HFRP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tara and Jagannatha (2014)</td>
<td></td>
<td>CFRP/GFRP</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reference</th>
<th>Element Type</th>
<th>Bond-slip Relation</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete</td>
<td>Strand</td>
<td>FRP</td>
<td>Concrete-Strand</td>
</tr>
<tr>
<td>Fathelbab et al. (2011)</td>
<td>Solid65</td>
<td>Link8</td>
<td>Solid46</td>
<td>perfect bonding</td>
</tr>
<tr>
<td>Bennegadi et al. (2013)</td>
<td>Solid45</td>
<td></td>
<td>Solid46</td>
<td>embedded</td>
</tr>
<tr>
<td>Tara and Jagannatha (2014)</td>
<td>Solid45</td>
<td></td>
<td>Solid45</td>
<td>N/A</td>
</tr>
</tbody>
</table>

(Solid45, Solid46 and Solid65: eight nodes solid element with three degrees-of-freedom at each node; Link8: 3D spar uniaxial tension-compression element with three degrees-of-freedom at each nodal end.)
Fathelbab et al. (2011) numerically analyzed the behaviour of CFRP strengthened simple reinforced concrete beams subjected to uniform loads in flexure, shear and a combination of flexure and shear. The CFRP sheets were externally bonded to the beam surface. The simulation was focused on the impact of different configuration of CFRP sheets on the flexure, shear and combination flexure and shear behaviour of the retrofitted beams. Results obtained from parametric study suggested that compared to one- or two-layer scheme, using three to five layers of CFRP sheets would yield a noticeable increase in beam loading capacity and a dramatic improvement of beam ductility. In order to delay the formation of cracks and control the failure mode, it was recommended to extend the installation of CFRP sheets to the sides of beams.

Bennegadi et al. (2013) developed a three-dimensional nonlinear finite element model to seek optimization solution of using Hybrid Fiber Reinforced Polymer (HFRP) plate to externally strengthen reinforced concrete beams. The model used embedded bonding relation between concrete and steel reinforcement, while the friction coefficient between concrete and HFRP was assumed to be 0.1. Overall, the results obtained from numerical simulation were found to agree well with the experimental data, except a stiffer behaviour was predicted by the finite element model in the linear range. A parametric study was conducted to evaluate the effects of HFRP plate height, width and thickness on the response of retrofitted beams. The thickness of the HFRP plate was found to be the most important optimization factor. Further, the numerical analysis results indicated that to stabilize stress
concentration at the edges of HFRP plate and to avoid debonding near the interface, the reduction of plate length should not be significant.

The numerical study by Tara and Jagannatha (2014) aimed at clarifying the effectiveness of natural bio-based woven jute FRP in shear strengthening of reinforced concrete beams, as well as to understand the effect of various FRP configurations (full and strip wrapping techniques), different types of FRP composites (CFRP and GFRP) on the retrofitting efficiency. A perfectly bonded relation between concrete and FRP was assumed in the simulation. Results showed that by applying the bio-based woven jute FRP along the entire length of reinforced concrete beams, both shear strength and stiffness of the beams were enhanced. The formation of shear cracks was significantly reduced and the initial cracking load was considerably increased.

2.4.2 Numerical simulation using ABAQUS

The numerical studies on shear strengthening of reinforced concrete beams using FRP by Ebead et al. (2002), Hu et al. (2004) and Chen et al. (2012) were all conducted in ABAQUS. The details of the finite element models are summarized in Table 2.2.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Structural Member</th>
<th>FRP Type</th>
<th>Constitutive Relation</th>
<th>Element Type</th>
<th>Bond-slip Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ebead et al. (2002)</td>
<td>RC slab</td>
<td>CFRP/</td>
<td>damaged plasticity</td>
<td>concrete</td>
<td>perfect bonding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GFRP</td>
<td></td>
<td>strand</td>
<td></td>
</tr>
<tr>
<td>Hu et al. (2004)</td>
<td>RC beam</td>
<td>FRP</td>
<td>damaged plasticity</td>
<td>solid</td>
<td>perfect bonding</td>
</tr>
<tr>
<td>Chen et al. (2012)</td>
<td>RC beam</td>
<td>FRP</td>
<td>smeared crack</td>
<td>CPS4</td>
<td>interface element</td>
</tr>
</tbody>
</table>

Both experimental study and numerical simulation were performed by Ebead et al. (2002) to investigate the effectiveness of using CFRP sheet strips and GFRP laminates to strengthen two-way slabs. The interface between concrete and FRP, and that between concrete and steel reinforcing bars were all assumed to be perfect bonding in the developed finite element model. The retrofitted specimens with CFRP strips showed a roughly 40% average improvement of the ultimate loading capacity, whereas those strengthened by GFRP laminates manifested an average increment of 31%.

Hu et al. (2004) conducted a nonlinear finite element simulation to predict the ultimate loading capacity of reinforced rectangular concrete beams strengthened by FRP sheets. Two configurations, i.e. externally bond FRP sheets on the bottom surface or on both sides.
of a beam, were analyzed. A constitutive model of FRP was proposed and used in the simulation. This, along with those for concrete and reinforcing steel, were validated individually by the existing experimental data. It was found that for beams having high reinforcement ratio and strengthened by FRP at the bottom surface, their behaviour was not affected by beam length. However, the beam length became an important factor for those with low reinforcement ratio. In addition, a comparison between the two FRP configurations indicated that if the same number of FRP sheet layers were used, externally bonding FRP sheets on both sides of the beam would yield lower ultimate strength than bonding FRP sheets on the bottom surface of the beam.

Chen et al. (2012) developed an advanced finite element model to investigate the impact of interface modelling, i.e. the interface between concrete and steel stirrups, that between concrete and steel tension bars, and the one between concrete and FRP, on the predicted FRP shear strengthening effect on reinforced concrete beams. Results showed that it was crucial to choose an appropriate bonding model between FRP and concrete. The assumption of a perfect bond between them might greatly over-estimate the shear capacity of the beam. The bonding condition between steel tension bars and concrete would affect the distribution and the direction of diagonal cracks and thus have a significant effect on the shear capacity of beams.

It is worth mentioning that extensive research on strengthening reinforced concrete members using FRP were carried out at Hong Kong Polytechnic University. Most of the
numerical studies by this research group used ABAQUS, with the supplement of many user subroutines. Teng et al. (2009) refined a new design-oriented stress-strain model for FRP-confined concrete, which could be directly used for practical design and included in design codes and specifications. Zhang et al (2014) developed a bond strength model for CFRP strips mounted near concrete surface and demonstrated high accuracy of the proposed model by comparing the predicted results with the testing data of fifty-one specimens from seven existing studies. Zhang and Teng (2014) proposed a novel numerical approach to predict end cover debonding failure of reinforced concrete beams strengthened in flexure with either externally bonded or near-surface mounted FRP. The radial stress exerted by the steel tension bars onto the surrounding concrete was for the first time identified to be an important factor for FE approach. Results obtained from this new finite element approach, in terms of the load-deflection curve, the ultimate loading capacity and the crack pattern, were found to agree well with the experimental data.

2.4.3 Numerical simulation using other tools

Besides, other commercial computer-aided engineering (CAE) software and self-developed program were used by different researchers to simulate the behaviour of reinforced concrete members strengthened by FRP. These numerical work are summarized in Table 2.3.
Table 2.3 Simulation details

<table>
<thead>
<tr>
<th>Reference</th>
<th>Subject with FRP</th>
<th>Constitutive Model</th>
<th>Program</th>
<th>Reference</th>
<th>Bond-slip Model</th>
<th>Element Type</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Concrete-Strand</td>
<td>Concrete-FRP</td>
<td>Concrete-FRP</td>
</tr>
<tr>
<td>Teng (2009)</td>
<td>strengthened RC slabs</td>
<td>elastic–plastic</td>
<td>composite</td>
<td>Ahmed</td>
<td>adhesive layer</td>
<td>adhesive layer</td>
<td>shell layer</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2011)</td>
<td></td>
<td></td>
<td>self-developed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>N/A</td>
<td>adhesive joints element</td>
<td>solid</td>
</tr>
<tr>
<td>Yousef</td>
<td>shear-strengthened RC</td>
<td>cscm-concrete</td>
<td>composite-damage</td>
<td>Denise</td>
<td>interface element</td>
<td>interface element</td>
<td>self-developed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>solid</td>
</tr>
<tr>
<td>Ahmed</td>
<td>shear-strengthened RC</td>
<td>smeared crack</td>
<td>elastic tensile</td>
<td>Denise</td>
<td>interface element</td>
<td>interface element</td>
<td>self-developed</td>
</tr>
<tr>
<td>Ahmed</td>
<td>shear-strengthened RC</td>
<td>smeared crack</td>
<td>elastic tensile</td>
<td>Ahmed</td>
<td>discrete truss elements</td>
<td>plane stress</td>
<td>truss</td>
</tr>
</tbody>
</table>

### 2.4.4 Summary

Based on the above review of numerical studies on shear strengthening of reinforced concrete members using FRP, it can be concluded that to accurately predict the behaviour of the retrofitted member, it is essential to properly simulate the following key elements in the finite element model. They include:

1. The constitutive models of concrete, reinforcing steel bars and FRP;
2. The models of interface between concrete and FRP, and that between concrete and reinforcing steel bars;

3. The element type selected for modelling concrete, reinforcing steel bars and FRP;

4. The material properties of concrete, reinforcing steel bars and FRP.
CHAPTER 3 EXPERIMENTAL STUDY

Shear behaviour of concrete members is a complex problem that has yet been fully understood. Since experimental study is one of the most important, reliable and effective methods to understand behaviour of structures, an experiment is conducted in the Structures lab at the University of Windsor to study the performance of PHC slabs externally bonded by CFRP sheets. The experimental program is mainly to verify the proposed shear strengthening method in this paper and the numerical simulation results in Chapter 4.

3.1 Test Specimens

According to the existing experience, the most common type of shear failure in PHC slabs is web shear failure. It occurs when the maximum principle stress resulted from the combined effect of shear and bending exceeds the tensile strength of concrete. As a consequence, a crack would initiate from a point on the critical section. Subsequently, more and more cracks would form and propagate upwards and downwards in a straight inclined form joining the edge of the slab support with the top of the slab and eventually lead to total collapse of the slab. The failure is mainly induced by diagonal cracks formed in the PHC slab web at the critical section near the support. Therefore, if the shear capacity of slab webs can be enhanced, the shear capacity of the entire PHC slab can be increased as well.

To explore the feasibility and effectiveness of this new technique in terms of the enhancement of ultimate shear capacity and to optimize the effect of different test
parameters, the technique was applied first to I-shaped single web beams cut longitudinally from the full width PHC slabs, which can be considered as single web hollow core slabs. The I-shaped single web beams are externally strengthened by Carbon FRP (CFRP) sheets bonded along the full perimeter on both sides of the specimen, as shown in Figure 3.1. The studied parameters include the length of the strengthened zone and the thickness of CFRP sheets.

Figure 3.1 Proposed shear strengthening techniques for single web hollow core slabs

In this research, a total of sixteen I-shaped single web beams, cut from full PHC slabs, were used in the experimental study. There are eight W1-Series specimens (1-prestressing strand web beams) and eight W2-Series specimens (2-prestressing strand web beams) which corresponding to the low prestressing and the medium prestressing PHC slabs, respectively. The cross-section of these two types of specimens are shown in Figure 3.2 and Figure 3.3.
Using W1-Series specimens as an example, two of them (W1-C1 and W1-C2) are used as control beams, to which no FRP sheets were applied, where “W” represents web beam specimen, “1” represents one prestressed steel strand, and “C” represents control beam. Three different lengths of the strengthened zone, i.e. 300 mm, 450 mm and 600 mm, were investigated. In addition, two different thicknesses of FRP sheets, i.e. two layers (one layer per side) and four layers (two layers per side), were investigated. The ID of the rest six specimens is defined to include these characteristics of FRP sheets with a pattern of W1-number of FRP sheets-length of FRP sheets. For example, in specimen W1-4-300, four FRP sheets (two on each side) of 300 mm length are externally bonded to the web beam specimen that has 1 prestressing strands. For W2-Series group, the only difference is that “2” represent two prestressed steel strands, the other ID rules are the same as those in the
W1-Series specimens. Table 3.1 lists the details of each specimen.

### Table 3.1 Parameters of PHC web beams used in the experimental study

<table>
<thead>
<tr>
<th>Specimen-ID</th>
<th>No. of FRP Sheets</th>
<th>Strengthened zone width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1-C1, W2-C1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W1-C2, W2-C2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W1-2-300, W2-2-300</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>W1-2-450, W2-2-450</td>
<td>2</td>
<td>450</td>
</tr>
<tr>
<td>W1-2-600, W2-2-600</td>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>W1-4-300, W2-4-300</td>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>W1-4-450, W2-4-450</td>
<td>4</td>
<td>450</td>
</tr>
<tr>
<td>W1-4-600, W2-4-600</td>
<td>4</td>
<td>600</td>
</tr>
</tbody>
</table>

All specimens were fabricated, cured and cut at the supplier site, then shipped to the Structures lab at the University of Windsor for testing. The specimen surface where the FRP sheets would be applied was cleaned and polished using a steel brush drum driven by a drill, before attaching the FRP sheets. The uni-directional FRP sheets were applied in the vertical direction to the web surface using a wet layup process. Both the CFRP sheets and the concrete surface were first saturated by epoxy resin, then the sheets were directly bonded to the specimen surface, as shown in Figure 3.4. The direction of FRP fibres were oriented along the web depth and perpendicular to the longitudinal axis of beams. They were left indoor 7 days for curing before testing.
3.2 Experimental Test Setup

All sixteen specimens had a length of 4575 mm, a depth of 300 mm and a width of 284 mm, which represents the centre to centre distance between two adjacent webs in a full PHC slab. Each W1-Series web beam specimen had one longitudinal prestressed steel strands, with 13 mm diameter, at the bottom. While in W2-Series group, they had two longitudinal prestressed steel strands with the same diameter. All specimens were tested over a simply supported clear span of 4499 mm while the bearing width at each support is 76 mm. The support condition is a pin at the loading end and a roller on the other end, so that no axial force would be generated by rotation or the constraint of the web beams at the support. For the W1-Series specimens, one concentrated load was applied monotonically till failure at a distance of 600 mm from the pin support via a very stiff steel plate of 190 mm width. This results in a shear span to depth ratio \((a/d)\) of 2.0. While the shear span to depth ratio of 2.5 was used for the W2-Series web beams, which leads to the concentrated load located at a distance of 750mm from the pin support. The details of the experimental
test setup is shown in Figure 3.5.

![Diagram](image)

Figure 3.5 Schematics of experimental setup for W1-Series specimens (unit: mm)

(Numbers in “(”) were used for W2-Series specimens)

### 3.3 Instrumentations and Testing Procedures

Both W1-Series and W2-Series web specimen were instrumented at the loaded end. As shown in Figure 3.6(a), two linear variable displacement transducers (LVDT) were installed, one at the loaded end on one prestressing strand to measure its end slippage during tests and another at the top flange of the beam specimen adjacent to the loading point to measure the vertical displacement. In addition, four displacement transducers, Pi-gauges, were used to measure strain at different locations. One is placed on the surface of the specimen top flange in the longitudinal direction near the loading position to measure the top concrete compressive strain while the other three transducers were arranged in the longitudinal...
direction along the center line of the web in the shear-tension region, as shown in details in Figure 3.6(b). These three critical points are calculated according to EN 1168 (2008) code, FE model predication and CAN/CSA-A23.3-04 (2004) code. In addition, three electrical foil strain gauges were glued on the surface of the FRP sheets, in the vertical direction at the center point of each Pi-gauge to monitor the FRP tensile strain.

Figure 3.6 Instrumentation setup of W1-Series specimens (unit: mm)

(Numbers in “()” were used for W2-Series specimens)
Also, the FEM was implemented to predict an ultimate shear capacity of the control test specimen. Following the Deutsche Norm. (2008), the load was applied in two steps. The specimens were first loaded to 70% of the estimated failure load at a rate of 10 kN per minute then unloaded. In the second load cycle, the load was applied monotonically at the same rate until failure by displacement control. Figure 3.7 shows a typical web beam specimen during loading stage.

![Figure 3.7 A typical web beam specimen during loading stage](image)

**3.3 Results of W1-Series**

**3.3.1 Experimental observation**

Originally, a shear span to depth ratio \((a/d)\) of 2.5, the same as the W2-Series specimens, was designed for the W1-Series specimens in the test. However, the test result of the first control beam (W1-C1) showed a typical bending failure instead of shear failure. It is believed that compared with the 2-strand web beam, bending capacity of 1-strand web beam is lower, which leads to bending failure. Therefore, the shear span ratio was adjusted
to 2.0 for the rest seven W1-Series specimens to ensure the unstrengthened web beams would fail in shear.

During the test of the second control beam W1-C2, two vertical flexural cracks were observed when the load reached respectively 63 kN and 75 kN before failure. Close to the failure load, an inclined crack initiated at the support and propagated rapidly through the web up to the top flange close to the loading plate. It covered the entire shear-tension region with a longitudinal distance of 595 mm from the support to the top end of the flange. The maximum width of this critical crack was 3.2 mm. When the load reached 77.8 kN, the specimen failed suddenly in diagonal shear. Therefore, the ultimate shear capacity of W1-C2 was 77.8 kN. Clearly, by adjusting the shear span ratio from 2.5 to 2.0, the second control beam failed in the shear failure mode. Figure 3.8(a) shows the failure mode and crack pattern of W1-C2.

Cracking patterns and failure mode of the three W1-2-Series specimens are given in Figure 3.8 (b), (c), (d) and Figure 3.9. The first vertical crack of all three web beams appeared at the bottom flange at loading end when the load was 24 kN, 13 kN and 22 kN, respectively (Figure 3.9(a), (b), (c)). For W1-2-450 and W1-2-600 specimens, when load increased to around 70-80 kN, the formation of three vertical flexural cracks was observed, which was believed to result from the more ductile behaviour of the strengthened specimens. A critical inclined crack initiated at the inner edge of the support bearing plate at the loading end and grew upwards till reached top flange of the specimen at the inner edge near the
loading plate when the load reached respectively 81.4 kN and 89.3 kN (Figure 3.8(c), (d)).

The horizontal distance between the loading end and crack top end was measured to be 377 mm and 300 mm, and the maximum crack width was 4.2 mm and 4.8 mm, respectively. The failure mode of these two specimens belonged to shear failure. However, in the case of the W1-2-300 web beam, as the load reached around 67 kN, a vertical flexural crack appeared at the bottom flange with a horizontal distance of 645 mm from the loading end. With the further increase of the load, the crack became wider and grew upward until eventually failed in flexure at 88.7 kN. It is worth noting that there was a pre-existing crack in W1-2-300 formed during the transportation (at 1265 mm from loading end). It extended over the entire cross-section and consumed extra energy during the loading stage which led to a ductile type failure mode as observed. Theoretically, this specimen should also fail in a brittle failure mode with one critical shear failure crack like the other two W1-2-Series specimens.

The W1-4-Series specimens manifested different cracking pattern and failure mode. For all three strengthened beams, the first vertical crack appeared at the bottom flange of the loading end before the load reached 30 kN (Figure 3.10). As shown in Figure 3.8 (e), a critical flexural crack was initiated at the bottom flange of W1-4-300 at 74 kN, which propagated vertically through the beam web and reach the top flange. The horizontal distance between the crack top end and the loading end was 580 mm. While this crack expanded and continuously propagated upwards with the increase of load, three more
flexural cracks appeared close to the mid-span and measurable vertical deflection was observed before the beam failed in a ductile bending failure mode at 89.6 kN. Such critical flexural cracks also formed in W1-4-450 and W1-4-600 specimens, with respectively horizontal distance of 670 mm and 710 mm between the crack top end and the loading end. Also, 2-3 more flexural cracks were formed prior to failure. The recorded ultimate failure load was respectively 98.1 kN (W1-4-450) and 89.1 kN (W1-4-600). No formation of diagonal shear crack appeared before beam failure. Both specimens failed in ductile mode.

In all the strengthened specimens, no de-bonding or delamination of the FRP sheets were observed before failure. The FRP sheets were de-attached manually from the concrete surface to study the cracking pattern. Also, formation of vertical and radial cracks were observed around the prestressing strand at the loading end before the load reached 30 kN during the first loading stage. The occurrence of these early vertical cracks were mainly due to the high prestressing force in the steel strands. The surrounding concrete, which was much weaker, was thus easier to damage at the loading end. This would reduce the specimen stiffness at the very beginning of the first loading stage. In addition, for W1-2-450 and W1-2-600 specimens, the critical shear failure cracks were found to propagate through the top flange of the specimens in the compression zone, as shown in Figure 3.9 (d) and (e).
(a) W1-C2

(b) W1-2-300

(c) W1-2-450

(d) W1-2-600
Figure 3.8 W1-Series web beam failure results
3.3.2 Ultimate load

The testing results of the W1-Series web beam specimens are summarized in Table 3.2. The shear failure of the control web beam W1-C2 occurs at 77.8 kN. The failure load of W1-2-300, W1-2-450 and W1-2-600 are 88.7 kN, 81.4 kN and 89.3 kN, respectively. Besides, W1-2-300 specimen, which has a pre-existing crack, fails in a ductile mode, whereas the other two strengthened specimens had brittle failure mode. This set of results
show that the web beam with the longest FRP strengthening zone (600 mm) is more effective and can improving the ultimate shear failure load by 14.8%. More tests are needed to confirm this finding. However, the ultimate load capacity of the W1-4-Series beams was found to be greatly increased with the application of the proposed shear strengthening technique. In addition, all three strengthened specimens failed in ductile bending failure mode. They are 89.6 kN, 87.5 kN and 98.1 kN, respectively for W1-4-300, W1-4-450 and W1-4-600. These indicate that the failure load was increased by 15.2%, 12.5% and 26.1% when 4 layers of CFRP sheets were bonded to the web surface for a strengthening length of 300, 450 and 600 mm, respectively. And the specimen strengthened with 600 mm sheets achieves the highest ultimate load capacity as expected. Most importantly, the shear tension region of the W1-4-Series specimens was reduced and the flexural region expanded at the same time when 4 layers of FRP sheets were bonded. It implies that by increasing the thickness of FRP sheets and the width of the strengthened zone, ductility of the studied web beams can be considerably enhanced, improving the shear capacity to a certain extent.

Table 3.2 Test results of W1-Series web beam specimens

<table>
<thead>
<tr>
<th>Specimen-ID</th>
<th>No. of FRP Sheets</th>
<th>Failure load (kN)</th>
<th>Percentage of improvement</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1-C2</td>
<td>0</td>
<td>77.8</td>
<td>-</td>
<td>brittle</td>
</tr>
<tr>
<td>W1-2-300</td>
<td>2</td>
<td>88.7</td>
<td>14.0%</td>
<td>ductile</td>
</tr>
<tr>
<td>W1-2-450</td>
<td>2</td>
<td>81.4</td>
<td>4.6%</td>
<td>brittle</td>
</tr>
<tr>
<td>W1-2-600</td>
<td>2</td>
<td>89.3</td>
<td>14.8%</td>
<td>brittle</td>
</tr>
<tr>
<td>W1-4-300</td>
<td>4</td>
<td>89.6</td>
<td>15.2%</td>
<td>ductile</td>
</tr>
<tr>
<td>W1-4-450</td>
<td>4</td>
<td>87.5</td>
<td>12.5%</td>
<td>ductile</td>
</tr>
<tr>
<td>W1-4-600</td>
<td>4</td>
<td>98.1</td>
<td>26.1%</td>
<td>ductile</td>
</tr>
</tbody>
</table>
3.3.3 Load-deflection behaviour

Figure 3.11 (a) gives the load-displacement relationships of the control beam and the three W1-2-Series specimens. The trend and pattern of the W1-C2 and the W1-2-Series (except W1-2-300 with a pre-existing crack) curves are found to be very similar, whereas, the failure load of W1-2-300 and W1-2-600 are apparently higher than that of the control beam. All three W1-2-Series ((except W1-2-300) curves have a drop at the load of 60-70 kN due to the occurrence of the first vertical flexural crack. Besides, the maximum displacements at loading position of the three strengthened specimens are much larger than that of the control beam, with an average value of 12-13 mm as compared to only 3-4 mm of W1-C2. This implies the improvement in the ductility of the W1-2-Series specimens due to the proposed strengthening technique. Further the sudden drop in the load-deflection curves of W1-C2, W1-2-450 and W1-2-600 specimens after failure corresponds to brittle shear failure occurred in these three specimens.

In the case of the W1-4-Series specimens, the first drop in the load-deflection curve of W1-4-300 and W1-4-600 occurs around 70 kN due to formation of the first vertical flexural crack. As can be seen in Figure 3.11(b), the average maximum displacement of the thress strengthened specimens exceeds 20 mm, which is more than five times of that of the control beam. This clearly shows the improvement in the ductility of the W1-4-Series beams. In addition, it is observed that the load-displacement curve of each strengthened specimen has a higher maximum flat region before failure, suggesting higher load carrying capacity and
ductile failure mode. Although the W1-4-600 specimens exhibit the best shear strengthening effects in terms of the enhancement in shear capacity and ductility, considerable improvement has also been achieved in W1-4-300 and W1-4-450 specimens. Compared to 600 mm, a CFRP strengthened zones with length of 300 mm would be easier for the installation of the FRP sheets and allow the proposed shear strengthening technique more feasible in real application.

As shown in Figure 3.12, the specimen W1-4-450 suffered a significant strand slippage at the loading lend, which leads to the low initial stiffness shown in Figure 3.11(b).
(b) W1-4-Series results

Figure 3.11 Load-deflection behaviour

Figure 3.12 Load - end slip relationship
3.3.4 Crack pattern

Figure 3.13 shows the crack profile of each W1-Series specimen at failure. As can be observed, W1-C2, W1-2-450 and W1-2-600 beams all had a diagonal shear crack formed at the instant of failure, indicating a typical shear failure. All W1-Series specimens have 2-4 vertical flexural cracks formed before failure. For W1-2-450 and W1-2-600 beams, the effective shear-tension region is reduced and shifted towards the support. This lead to an increase of the flexural region so the two specimens manifest a more ductile behaviour. As explained earlier, the vertical crack resulted from ductile bending failure of W1-2-300 could be mainly due to the presence of pre-cracking in the specimen.

On the other hand, all W2-4-Series specimens show a vertical flexural failure crack near the loading position and 2-3 more flexural cracks formed close to the mid-span before bending failure. Clearly, the cracking pattern and distribution indicate the typical bending failure. Compared to the control beam, the ductility of specimens, strengthened by 2 layers of FRP sheets, has improved significantly due to broadening of the flexural region. Based on the testing results, applying 2 layers of FRP sheets showed much stronger enhancement of shear capacity as compared to two layer configuration. It eventually leads to bending failure due to the relatively lower flexural capacity of the one strand specimens. Further, it is interesting to note that the critical failure cracks in all three strengthened specimens appeared out of the FRP strengthening zone. This could be mainly due to the presence of four FRP sheets close to the specimen support, which would considerably enhance the shear
resistance capacity in this local region.

Figure 3.13 Critical crack profiles (Cont.)
Figure 3.13 Critical crack profiles (unit: mm)

3.3.5 Strain behaviour

Figure 3.14 portrays the relation between the load and the longitudinal compressive strain in concrete at the top surface of the flange at 465 mm from the loading end. It can be noticed that for W1-2-Series specimens, the measured values of the top concrete compressive strain are very close to each other. For W1-2-450 and W1-2-600, the recorded concrete compressive strains were much smaller than that of the control beam before failure, which were well below the ultimate concrete crushing strain limit of 3500 microstrain.
Similarly, the concrete compressive strain of W1-4-450 and W1-4-600 specimens at failure were much lower than that of the control beam, too, and far below the ultimate crushing strain limit of concrete. Although all W1-4-Series web beams manifested a bending failure, there were no concrete crushing on the top compressive region because the prestressing steel strand has not yielded at failure.

(a) W1-2-Series
(b) W1-4-Series (the data of W1-4-300 was lost due to reading errors)

Figure 3.14 Load vs longitudinal concrete compressive strain at the surface of the top flange for W1-Series specimens

3.4 Results of W2-Series

3.4.1 Experimental observation

The two control specimens showed almost identical behaviour. At 30 kN, the first crack appeared at the bottom surface near the loading end at the face of the bearing plate. Close to failure load, an inclined crack initiated at the support and propagated quickly through the beam web up to the top flange close to the loading plate, over the entire shear-tension region with a distance of 780 mm from the support. The maximum width of this critical shear crack was 3.2 mm. There was no formation of vertical flexural crack prior to beam failure. Both W2-1C and W2-2C failed suddenly in diagonal shear at an ultimate load of 81.5 kN.
and 76.3 kN, respectively. An average value of 78.9 kN for the ultimate shear capacity of the control specimens will be considered in the comparison. Figure 3.15(a) shows the typical cracking and failure shape of the control specimens.

Figure 3.15(b) shows a typical cracking pattern and failure shape of W2-2-Series specimens. The first vertical crack of W2-2-450 web beam appeared at the bottom of the flange at loading end when the load was 13 kN. When the load reached 79.9 kN, a critical inclined crack initiated at the inner edge of the support bearing plate at the loading end and grew upwards till reached top flange of the specimen at the inner edge of the loading plate. The horizontal distance between the crack top end and the beam loading end was measured to be 715 mm, and the maximum crack width was 2.0 mm. No formation of vertical flexural crack was observed before beam failure. The failure mode of this specimen belonged to shear failure, too.

The W2-4-series specimens showed different cracking behaviour before failure. The cracking pattern and failure mode of W2-4-300 web beam is shown in Figure 3.15(c). The first vertical crack appeared at the bottom of the loading end when the load reached only 10 kN. Further, in all three cases, vertical flexural cracks formed at the bottom of the beam close to mid-span prior to beam failure. In the case of W2-4-300, one such crack appeared at around 90 kN, whereas two vertical flexural cracks were observed in W2-4-450 and W2-4-600, when the load reached respectively around 80 kN and 100 kN. This resulted in more ductile behaviour before failure. The critical inclined crack initiated at the specimen bottom
at 174 mm from the pin support at the loading end and extended to the top flange close to the middle of the loading plate. The recorded ultimate shear failure load was 102.1 kN. Further, the critical shear cracks in both W2-2-450 and W2-4-300 web beams were observed to grow through the beam web. In all the strengthened specimens, no de-bonding or delamination of the FRP sheets was observed before failure. After the test, the FRP sheets were de-attached manually from the concrete surface to study the main inclined shear crack.

In addition, for all eight tested web beam specimens, the critical shear failure cracks were found to break through the top flange of the specimens in the compression zone, as can be seen in Figures 4.16(a), (b) and (c). Also, during the tests, formation of vertical and radial cracks were observed around the prestressing strand at the loading end before the load reached 35 kN. Figures 4.16(d), (e) and (f) show some sample cases. The same as the W1-Series specimens, these early vertical cracks occurred because of the high prestressing force in the steel strands, which would also reduce the specimen stiffness at the very beginning of the first loading stage.

(a) W2-C1
Figure 3.15 W2-Series web beam failure results

Figure 3.16 Cracking patterns of W2-Series at top flange and around the strands at loading end
3.4.2 Ultimate load

The testing results of the 8 W2-Series web beam specimens are summarized in Table 3.4. The average shear failure load of W2-C control web beams is 78.9 kN, which is in very good agreement with the FEM predicted ultimate capacity of 82.7 kN and has a difference of only 4%. The failure load of W2-2-300, W2-2-450 and W2-2-600 are 76.8 kN, 77.9 kN and 83.5 kN, respectively, which implies that using two layers of FRP sheets to strengthen the shear capacity of the studied web beam is not adequate. More tests are needed to confirm this finding. The failure mode of the W2-2-Series specimens remains the same as that of the control beam, i.e. failed in brittle shear failure mode. However, shear capacity of the W2-4-Series webs were found to be greatly enhanced with the application of the proposed shear strengthening technique. The ultimate shear failure load of W2-4-300, W2-4-450 and W2-4-600 are 102.1 kN, 99.6 kN and 114.2 kN, respectively. In other words, compared to the shear failure load of the control beam, the shear capacity of PHC webs increased by 29.4%, 26.2% and 44.7% when 4 layers of CFRP sheets were bonded to the web surface for a length of strengthening zone being 300, 450 and 600 mm, respectively. These testing results imply that by increasing the thickness of FRP sheets and the length of the strengthened zone, shear capacity of the studied web beams can be considerably increased.
<table>
<thead>
<tr>
<th>Specimen-ID</th>
<th>No. of FRP Sheets</th>
<th>Failure load (kN)</th>
<th>Percentage of improvement</th>
<th>No. of Flexural cracks before failure</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>W2-C1</td>
<td>0</td>
<td>81.5</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>W2-C2</td>
<td>0</td>
<td>76.3</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>W2-2-300</td>
<td>2</td>
<td>76.8</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>W2-2-450</td>
<td>2</td>
<td>79.9</td>
<td>8.7%</td>
<td>0</td>
<td>brittle</td>
</tr>
<tr>
<td>W2-2-600</td>
<td>2</td>
<td>83.5</td>
<td>8.7%</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>W2-4-300</td>
<td>4</td>
<td>102.1</td>
<td>29.4%</td>
<td>1 (at 90 kN)</td>
<td></td>
</tr>
<tr>
<td>W2-4-450</td>
<td>4</td>
<td>99.6</td>
<td>26.2%</td>
<td>2 (at 80 kN)</td>
<td></td>
</tr>
<tr>
<td>W2-4-600</td>
<td>4</td>
<td>114.2</td>
<td>44.7%</td>
<td>2 (at 100 kN)</td>
<td></td>
</tr>
</tbody>
</table>

### 3.4.3 Load-deflection behaviour

Figure 3.17(a) shows the load-displacement relationships of the control beam and the three W2-2-Series specimens. The trend and pattern of the four curves are found to be very similar. Besides, the maximum displacements of the four beams at the loading position are almost the same, which are about 4-5 mm, indicating a sudden and brittle failure occurred in these four specimens. On the other hand, in the case of the W2-4-series specimens, the maximum displacement at the loading point are much larger. In particular, as can be seen from Figure 3.17(b), it exceeds 10 mm in W2-4-450 and W2-4-600. This is more than two times of that of the control beam and clearly shows the improvement in the ductility of the W2-4 series specimen of which 2 layers of FRP sheets are bonded at each side of the web. Further, although W2-4-600 specimens exhibit the best shear strengthening effects in terms of enhancing shear capacity, considerable improvement was also achieved in W2-4-300
and W2-4-450 specimens. Again, the shorter strengthening zone (300 mm and 450 mm) would be easier for practical application, comparing with the 600 mm length.

Figure 3.18 shows the relationship between the load and the end slip of the prestressing strands at the loaded end. It can be noticed that specimen W2-4-450 suffered from significant strand slippage at the loading end mainly due to the formation of early cracks on the loaded face around the strands. This also explains the low stiffness of W2-4-450 specimen shown in Figure 3.17(b).
Figure 3.17 Load-deflection behaviour of W2-Series specimens

Figure 3.18 Load-end slip relationship
3.4.4 Crack pattern

Figure 3.19 shows the crack profile of each specimen at failure. As can be observed, in all cases, a diagonal shear crack formed at the instant of failure, indicating a typical shear failure. The orientation of the cracks is between 17°-46° with respect to the specimen longitudinal axis. A closer look at this set of results reveals that in the case of the control beam, the critical shear failure point is very close to the intersection of the specimen longitudinal axis and the interior line connecting the inner edges of support and loading bearing plates, which is consistent with the existing studies by Celal (2011) and Yang (1994). For W2-2-Series specimens, the effective shear-tension region is reduced and shifted towards the support. This suggests an increase of the flexural region so the specimen would manifest a more ductile behaviour. However, it is interesting to note that although a reduction of shear-tension region also occurs in all three W2-4 series specimens, it is found to be shifted away from the support. This could be mainly due to the presence of the four FRP sheets which are close to the specimen support. They would considerably enhance the shear resistance capacity in this local region. The fact that the application of four FRP sheets showed much stronger enhancement of shear capacity than the two FRP sheets arrangement seems to suggest that there could exist a minimum requirement of FRP sheets to obtain satisfactory shear strengthening effect. More tests are needed to clarify this.
Figure 3.19 Critical shear crack profiles (Cont.)
Figure 3.19 Critical shear crack profiles (unit: mm)
3.4.5 Strain behaviour

Figure 3.20 shows the relation between the load and the top longitudinal concrete compressive strain measured on the top flange at 565 mm from the loading end. It can be seen from the Figure 3.20(a) that for W2-2-series, the measured top concrete compressive strain are very close to that measured in the control beam as there was no sizable change in the ultimate capacity. However, for W2-4-450 and W2-4-600 as shown in Figure 3.20(b), less compressive strains were recorded up to the 80 kN capacity of the control beam. Meanwhile, the W2-4-450 and W2-4-600 recorded the same concrete compressive strain at failure as W2-C but at much higher load. All the concrete compressive strain readings at failure were well below the ultimate crushing strain limit in concrete, i.e. 3500 microstrain, which also ensure that the failure was controlled by the shear strength.

(a) W2-2-Series
Figure 3.20 Load vs top longitudinal concrete compressive strain relationships of W2-Series specimens

Figure 3.21 shows the load vs longitudinal tensile strain (vertical direction of the webs) relationship measured on FRP sheets at the longitudinal centre line of the web, 225 (dv) mm away from the support. It can be noticed that the relation is linear for W2-2-300, W2-2-600 and bilinear for W2-4-300, W2-4-600 strengthened specimens with a strengthened zone length of 300 mm and 600 mm. Up to about 80 kN load, the measured strains of the FRP sheets are very small and less than the cracking strain of the concrete. Beyond the 80 kN load limit, the strain increases rapidly in the FRP sheets in W2-4-300 and W2-4-600 and exceeds the cracking strain of the concrete. This indicates that after the first shear crack developed in the concrete, the 4 layers of FRP sheets acts as stirrups and contributes to
resisting the tensile strains in the vertical direction, thus provide the enhancement in the overall shear capacity.

Figure 3.21 Load vs longitudinal tensile strain in FRP sheets relationships

3.5 Interface between FRP and Concrete

The bonding condition between the FRP sheets and the concrete surface directly governs the effect of shear strengthening. Figure 3.22 and Figure 3.23 show some typical interfaces of both W1-Series and W2-Series specimens before and after the FRP sheets were manually peeled off after specimen failure. Inspections of the failure regions of the specimens showed that the FRP sheets demonstrated an excellent bonding performance. As can be seen from Figure 3.22 that there is actually no damage of the FRP sheet itself, nor
to their interface to concrete. It still bonds perfectly with the concrete surface even after shear failure occurred. This could be mainly attributed to the limited displacement under shear failure and the high bonding capacity of the epoxy resin. In addition, it was very hard to take off the FRP sheets after the tests. When peeled off manually after failure, it can be noticed that part of the concrete material still attached to the sheet, as shown in Figure 3.23(a) and (b).

![Figure 3.22 Interface between FRP and concrete of W1-Series specimens](image)

(a) At failure  (b) After manually peel off

(c) At failure  (d) After manually peel off
Figure 3.23 Interface between FRP and concrete of W2-Series specimens
CHAPTER 4 FINITE ELEMENT SIMULATION

Finite element modeling (FEM) is a very effective and economic tool to study the complex behaviour of shear strengthening for PHC slabs using externally bonded FRP sheets. ABAQUS is one of the most powerful commercial finite element softwares for various application and it is widely used for simulating in structural problems. ABAQUS 6.14 will be used in the current study for numerical analysis.

In ABAQUS, models can be created through graphical user interface or typing in command using input files. Though it is convenient and easy to use graphical user interface, for certain modeling functions, type in command is the only choice and it is more efficient. The numerical models in the current study are developed by combining both approaches. This chapter presents details of the developed FEM models and simulation. This includes types of element selected for simulating of different components in a PHC slab and FRP sheets, the constitutive model for each element, mesh size, boundary conditions, and how the convergence in solution is controlled. Finally, the model is validated using the experimental results.
4.1 Constitutive Relation

4.1.1 Concrete

Concrete is a typical type of inhomogeneous material that mainly consists of cement, sand and gravel, which leads to complicated mechanical properties. Thus, choose a proper constitutive model to describe its stress-strain relationship is very important. Also, this is one of the key elements in a successful numerical simulation.

According to existing studies, there are five constitutive theories which can be used for concrete:

1. Linear elastic and non-linear elastic constitutive relation based on an elastic model (Chen and Saleeb, 1982).

   Concrete can be considered as a linear elastic homogeneous material before cracking and the linear elastic constitutive relation can be used in analysis. However, since the stress-strain relationship is curvilinear under multi-axial stresses, so non-linear constitutive, which is based on elastic theory, is only applicable to monotonic loading condition.

2. Elastic-plastic and hardening plasticity constitutive relation based on the classical plasticity theory (Chen and Saleeb, 1982).

   This is based on the plastic flow theory and considers concrete loading path and hardening, but it cannot reflect concrete softening behaviour.

3. Constitutive relation based on the combination of fracture theory and plasticity theory (Chen, 1982).
In fracture theory, the stiffness degradation in concrete is considered to be caused by micro cracks of concrete. This constitutive model is capable to reflect the concrete softening behaviour.

4. Theoretical description of concrete constitutive model developed from the viscous material constitutive relation (Jiang et al. 2005).

5. Constitutive relation developed based on the damage theory and the elastic-plastic fracture model (Chen, 1994).

Although, many studies have been conducted for developing a constitutive relation model for concrete, the result is yet satisfactory because of complicated forms of equations used in these models. In addition, the accuracy associated with existing models mostly depend on many parameters and functions obtained from experiments. So the existing models still need to be improve. At the same time, concrete constitutive model plays an important role in finite element simulation. Therefore, the first step is to choose a proper concrete constitutive model.

In ABAQUS/Standard, there are two concrete constitutive models: concrete smeared cracking model and concrete damage plasticity model. The latter will be adopted in the present work.

Concrete damage plasticity (CDP) model is based on the damage plasticity model proposed by Lubliner et al. (1989) and by Lee and Fenves (1998). The differences in tensile and compressive properties of concrete are taken into account and it can simulate
unrecoverable material degeneration caused by damage under low hydrostatic pressure. This degradation is mainly presented as the differences between tensile and compressive yield strength of concrete in macroscopic scale, the softening behaviour after yield in tension, the softening-hardening behaviour after yield in compression, different stiffness reduction factor, damage due to tension and compression, and partial stiffness recovery under cyclic loading, etc. The main theory of damage plasticity model (Tomasz, 2005, Birtel and Mark, 2006, ABAQUS Theory Manual 6.14) are summarized below:

In the CDP model, the stress-strain relation are governed by scalar damaged elasticity:

\[
\sigma = (1 - d)D_0^{el}: (\varepsilon - \varepsilon^{pl}) = D^{el}: (\varepsilon - \varepsilon^{pl})
\]

where \( \sigma \) is Cauchy stress tensor and \( d \) is the scalar stiffness degradation variable. \( \varepsilon \) is the strain tensor, \( D_0^{el} \) is the initial (undamaged) elastic stiffness of the material, while \( D^{el} = (1 - d)D_0^{el} \) is the degraded elastic stiffness tensor. The effective stress tensor is defined as:

\[
\bar{\sigma} \equiv D_0^{el}: (\varepsilon - \varepsilon^{pl})
\]

\[
d = d(\bar{\sigma}, \tilde{\varepsilon}^{pl})
\]
Damaged states in tension and compression are characterized independently by two hardening variables, $\bar{\varepsilon}_{cp}^\text{pl}$ and $\bar{\varepsilon}_{tc}^\text{pl}$, which are referred to equivalent plastic strain in tension and compression, respectively. The evolution of the hardening variables is given by the following expression:

$$\bar{\varepsilon}_{cp}^\text{pl} = \left[ \begin{array}{c} \bar{\varepsilon}_{tc}^\text{pl} \\ \bar{\varepsilon}_{cp}^\text{pl} \end{array} \right] \quad \text{and} \quad \dot{\bar{\varepsilon}}_{cp}^\text{pl} = h((\bar{\sigma}, \bar{\varepsilon}_{cp}^\text{pl}) \cdot \dot{\bar{\varepsilon}}_{cp}^\text{pl}$$

(4.5)

Cracking (tension) and crushing (compression) in concrete are represented by increasing values of the hardening (softening) variables. These variables control the evolution of the yield surface and the degradation of the elastic stiffness.

The yield function represents a surface in effective stress space which determines the states of failure or damage. For the inviscid plastic-damage model the yield function is:

$$F(\bar{\sigma}, \bar{\varepsilon}_{cp}^\text{pl}) \leq 0$$

(4.6)

Plastic flow is governed by a flow potential function $G(\bar{\sigma})$ according to nonassociative flow rule:

$$\dot{\bar{\varepsilon}}_{cp}^\text{pl} = \lambda \frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}}$$

(4.7)

The plastic potential function $G$ is also defined in the effective stress space.

In the CDP model, it is assumed that the failure mechanisms of concrete material are mainly due to tensile cracking and compressive crushing. Evolution of yield or failure is determined by two variables: $\bar{\varepsilon}_{tc}^\text{pl}$ (the equivalent tensile plastic strain) and $\bar{\varepsilon}_{cp}^\text{pl}$ (the equivalent compressive plastic strain). Also, it is assumed that the uniaxial tensile and compressive response of concrete is characterized by damage plasticity, as illustrated in
Figure 4.1 Response of concrete to uniaxial loading in tension and compression


It can be seen from Figure 4.1(a) that under uniaxial tension, the stress-strain response
of concrete follows a linear elastic relation until reaches the failure stress, \( \sigma_0 \). The failure stress corresponds to the onset of micro-cracking in the concrete material. Beyond the failure stress, the formation of micro-cracks is represented macroscopically by a softening stress-strain response, which induces strain accumulation in concrete. Under uniaxial compression the response is linear until reaches the yielding stress \( \sigma_{c0} \), as shown in Figure 4.1(b). In the plastic regime the response is typically characterized by stress hardening followed by strain softening beyond the ultimate stress, \( \sigma_{cu} \). This model, although still somewhat simple, captures the main features of concrete response.

When concrete specimen is unloaded from any point on the strain softening portion of the stress-strain curves in Figure 4.1, the unloading response is weakened and the elastic stiffness of the material appears to be degraded. This degradation is characterized by two variables in the CDP model, i.e., \( d_t \) and \( d_c \), if \( E_0 \) is the initial (undamaged) elastic stiffness of the material, the stress-strain relations under uniaxial tension and compression loading are, respectively:

\[
\sigma_t = (1 - d_t)E_0(\varepsilon_t - \varepsilon_{tp}) \tag{4.8}
\]

\[
\sigma_c = (1 - d_c)E_0(\varepsilon_c - \varepsilon_{cp}) \tag{4.9}
\]

It defines the effective tensile and compressive cohesion stresses as:

\[
\bar{\sigma}_t = \frac{\sigma_t}{(1 - d_t)} = E_0(\varepsilon_t - \varepsilon_{tp}) \tag{4.10}
\]

\[
\bar{\sigma}_c = \frac{\sigma_c}{(1 - d_c)} = E_0(\varepsilon_c - \varepsilon_{cp}) \tag{4.11}
\]

The effective cohesion stresses determine the size of the yield (or failure) surface.
In reinforced concrete the specification of postfailure behaviour generally means giving the postfailure stress as a function of cracking strain, \( \varepsilon^{ck} \). The cracking strain is defined as the total strain minus the elastic strain corresponding to the undamaged material,

\[
\varepsilon^{ck} = \varepsilon_t - e^{el}_{0t}, \quad \text{where} \quad e^{el}_{0t} = \sigma_t/E_0, \quad \text{as shown in Figure 4.2.}
\]

\[
\varepsilon^{pl} = \varepsilon^{ck} - \frac{d_t \sigma_t}{(1 - d_t) E_0}
\]

Figure 4.2 Definition of the cracking strain (ABAQUS User’s Manual 6.14, 2014)

Hardening data are given in terms of an inelastic strain, \( \varepsilon^{in}_c \), instead of plastic strain, \( \varepsilon^{pl}_c \). The compressive inelastic strain is defined as the total strain minus the elastic strain corresponding to the undamaged material, \( \varepsilon^{in}_c = \varepsilon_c - e^{el}_{0c} \), where \( e^{el}_{0c} = \sigma_c/E_0 \), as illustrated in Figure 4.3.

\[
\varepsilon^{pl}_c = \varepsilon^{ck}_c - \frac{d_c \sigma_c}{(1 - d_c) E_0}
\]

(4.13)
4.1.2 Reinforcement

In PHC slabs, steel strands mainly provide tensile resistance and determine the ductility of the member. The constitutive relationship of reinforcement is the basis for studying strength, stiffness and ductility of shear strengthened PHC slabs. It should include elastic stage, yielding stage and hardening stage. Also, the curvature of the constitutive relation curve should decrease in the transition zone from the elastic stage to the plastic stage. The prestressing strand in PHC slabs belongs to high strength steel which does not have an apparent yield stage. This kind of stress-strain relation can be described by the dual slash-curve model (Yu, 2002) shown in Figure 4.4. Segment $ab$ on the stress-strain curve represents the softening stage, which can be defined as

$$
\sigma_s = \frac{\sigma_b \varepsilon_b - \sigma_a \varepsilon_a}{\varepsilon_b - \varepsilon_a} + \frac{\varepsilon_a \varepsilon_b (\sigma_a - \sigma_b)}{\varepsilon (\varepsilon_b - \varepsilon_a)}
$$

(4.14)
where, $\sigma_a$, $\sigma_b$, $\varepsilon_a$ and $\varepsilon_b$ are the stress and strain at points $a$ and $b$, respectively.

![Stress-strain curve for steel strand](image)

**Figure 4.4 Stress-strain curve for steel strand**

### 4.1.3 FRP

FRP material used for shear strengthening of PHC slabs can be assumed as an elastic material (Yu, 2002), of which the typical constitutive model used in finite element simulation is defined by eq. (3.8) and portrayed in Figure 4.5.

$$\sigma_{FRP} = E_{FRP} \varepsilon_{FRP}$$  \hspace{1cm} (4.15)

![Stress-strain curve for FRP](image)

**Figure 4.5 Stress-strain curve for FRP**
4.2 Element Type and Material Properties

4.2.1 Concrete

The behaviour of concrete is simulated using element type C3D8R in ABAQUS. As shown in Figure 4.6, it is a three-dimensional, 8-node, reduced-integration solid element that is capable of taking into account the nonlinear elastic-plastic characteristics of concrete. In the meantime, it has the ability to simulate plastic deformations, crushing, cracking and even shear transfer at crack interface without shear locking problem. The geometry of the element is defined by eight corner nodes with three translational degrees-of-freedom (UX, UY, and UZ) at each node. But it is worth noticing that the hourglassing could be a problem in stress and displacement analyses when adopt C3D8R element. Since the elements have only one integration point, it is possible for them to distort in such a way that the strains calculated at the integration point are all zero, which, in turn, leads to uncontrollable distortion of the mesh. Though the first-order, reduced-integration elements in ABAQUS include hourglass control, it still should be used with reasonably fine meshes.

Figure 4.6 C3D8R element
A total of thirty 100 × 200 mm (4 × 8 in) and fifteen 150 × 300 mm (6 × 12 in) cylinders were prepared and cured under the same conditions as their reference slabs and web beams to evaluate the compressive and tensile strength of concrete at the test time. The average compression test results for the ultimate compressive strength of concrete $f'_c$ are 59.3 MPa for W1 specimens (web beams with 1 prestressing strand) and 52.2 MPa for W2 specimens (web beams with 2 prestressing strands).

Within the elastic zone of concrete stress-strain relation curve, the Young’s modulus $E_c$ should be taken as the average secant modulus based on stresses of 0.25 $f'_c$ and 0.5 $f'_c$. In the current study, it is 22167 MPa for W1 specimens and 23946 MPa for W2 specimens, respectively. In the FE model, the Poisson’s ratio of concrete is assumed to be 0.26.

The ultimate tension capacity of concrete is determined by performing standard split testing on fifteen concrete cylinders, which gives an average of results for approximately 3.35 MPa.

The other parameters associated with concrete material properties required in the CDP model in ABAQUS are listed in Table 4.1.

<table>
<thead>
<tr>
<th>Dilation Angle $\psi$</th>
<th>Eccentricity</th>
<th>$\sigma_{pl}/\sigma_{c,0}$</th>
<th>$K_c$</th>
<th>Viscosity Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.1</td>
<td>1.16</td>
<td>0.667</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

According to ABAQUS Theory Manual 6.14 (2014), Tomasz (2005) and Javier et al. (2013), the definition of the above parameters are based on the following theories:

The flow potential $G$ used in the concrete damage plasticity model is the Drucker-
Prager hyperbolic function. It neglects damage and is accepted in the form of

\[ G = q - p \tan \psi \]  
(4.16)

where \( q \) is the von Mises stress, \( p \) is the pressure invariant of the stress tensor, and \( \psi \) is the dilation angle. The dilation angle can be obtained from tri-axial tests, which is

\[ \tan \psi = \frac{3}{2} \frac{2 \nu_p - 1}{1 + \nu_p} \]  
(4.17)

where \( \nu_p \) is the Poisson’s ratio. For high confining pressure, the yield condition for undamaged concrete is defined as:

\[ F = q - 3\alpha p + \gamma \sigma_{\text{max}} \]  
(4.18)

where \( \alpha \) is related to the ratio of the bi-axial stress, \( \sigma_b \), to the uniaxial stress, \( \sigma_c \), through

\[ \frac{\sigma_b}{\sigma_c} = \frac{1 - \alpha}{1 - 2\alpha} \]  
(4.19)

\( \sigma_{\text{max}} \) is the maximum principal stress (compression being negative), and \( \gamma \) is defined by

\[ \gamma = \frac{3(1 - K_c)}{2K_c - 1} \]  
(4.20)

where \( K_c \) is a material parameter (the ratio between the second stress invariant on the tensile meridian \( q_{(TM)} \), and that on the compressive meridian \( q_{(CM)} \)) between 0.5 and 1.0. \( \alpha \) can be expressed as a function of the friction angle \( \phi \):

\[ \alpha = \frac{3K_c + K_c \sin \phi + \sin \phi - 3}{6K_c + \sin \phi - 2K_c \sin \phi - 3} \]  
(4.21)

Then, if the effects of the effective compressive cohesion stress \( \sigma_c \) is considered, the actual yield condition can be expressed as:

\[ F = q - 3\alpha p + \gamma \sigma_{\text{max}} + \sigma_c(\varepsilon_p) \]  
(4.22)

The viscosity parameter represents the relaxation time of the visco-plastic system. It is used
for the visco-plastic regularization of the concrete constitutive equations in ABAQUS (Standard) analysis. This parameter is ignored in ABAQUS (Explicit) with a default value of 0.0. However, to help achieve convergence during analysis, it is usually set as $\mu = 0.0005$.

4.2.2 Reinforcement

The prestressing strands of PHC slabs are modeled using element type T3D2. It is a three-dimensional and 2-node truss element. This truss element is capable of resisting axial compression and tension forces but not moment. It has characteristics of plasticity, large deflection, and large strain deformations. The geometry of the element is defined by node numbers at its two ends (1 and 2), as shown in Figure 4.7. Also, the cross-sectional area of the element which would represent area of the strands is the additional information required to define geometry of this element.

![Figure 4.7 T3D2 truss element](image)

According to supplier’s (PSI Company) data, the prestressing strands are 7-wire, low-relaxation strands which have a diameter of 13 mm and ultimate tensile strength of 1860 MPa. The jacking force is 128.63 kN and modulus of elasticity $E_p$ and Poisson’s ratio for
the strands are taken as 196,500MPa and 0.3, respectively. The prestressing strands do not have identifiable yielding point as in the case of regular mild steel and there exists no strain hardening before rupture. An appropriate stress-strain relation for low relaxation strands of Grade 270ksi (1860 MPa) according to PCI manual for the design of hollow core slabs (1998) and ASTM A416 is given in eq. (4.10) and portrayed in Figure 4.8.

\[
f_{ps} = 200 \times 10^3 \varepsilon_{pf} \left\{ 0.025 + \frac{0.975}{\left[ 1 + 121 \varepsilon_{pf}^6 \right]^{0.167}} \right\}
\]  

(4.23)

![Stress-strain curve for strand elements](image)

Figure 4.8 Stress-strain curve for strand elements

To estimate prestress loss for strand itself, elastic shortening (ES) is taken into consideration (PCI design handbook, 2004):

\[
ES = K_{es}E_{psf_{ct}}/E_{ci}
\]

(4.24)

where \( K_{es} = 1.0 \) for pretensioned members, \( E_{ps} \) is modulus of elastic of prestressing tendons,
$E_{ci}$ is modulus of elasticity of concrete at time prestress is applied, $f_{cir}$ is net compressive stress in concrete at center of gravity of prestress has been applied to the concrete.

$$f_{cir} = K_{cir} \left( \frac{P_i}{A_g} + \frac{P_i e^2}{I_g} \right) - \frac{M_g e}{I_g}$$  \hspace{1cm} (4.25)

where $K_{cir}$ is 0.9 for pretensioned members, $P_i$ is initial prestress force (after anchorage seating loss), $e$ is eccentricity if center of gravity if tendons with respect to center of gravity of concrete at the cross section considered, $A_g$ is area of gross concrete section at the cross section considered, $I_g$ is moment of inertia of gross concrete section at the cross section considered, $M_g$ is bending moment due to dead weight of prestressed member and any other permanent loads in place at time of prestressing.

The prestressing force in strand is simulated by defining a temperature load, as given in eq. (4.11), on the strands to make them shrink, which subsequently would introduce prestressing force to concrete.

$$T = \frac{-F}{E_p \times A \times e_c}$$  \hspace{1cm} (4.26)

where $T$ is the loading temperature, $F$ is the required prestressing force, $E_p$ is the elastic modulus of reinforcement, $A$ is the cross-sectional area of strand (the effective cross-sectional area of strand is 100 mm$^2$ in the current study), and $e_c$ is the thermal expansion coefficient of strand (value 0.00001 is used).

The strand prestress can also be added to concrete by setting it as an initial condition. However, through both approaches simulate prestress in strands, the initial condition approach would only give stress increment, not the actual stress level in the strands and
concrete.

### 4.2.3 CFRP

The CFRP sheets are simulated by element type SC8R (Figure 4.9). This is an 8-node quadrilateral in-plane general-purpose continuum shell element with reduced integration and finite membrane strain. It has hourglass control capability.

![8-node continuum shell (ABAQUS User’s Manual 6.14, 2014)](image)

Conventional shell elements have displacement and rotational degrees of freedom. In contrast, continuum shell elements discretize an entire three-dimensional body. The element thickness is determined from the element nodal geometry. Continuum shell elements have only displacement degrees of freedom. From a modeling point of view, continuum shell elements look like three-dimensional continuum solids, but their kinematic and constitutive behavior are similar to the conventional shell elements. Figure 4.10 illustrates the differences between a conventional shell and a continuum shell element (ABAQUS User’s Manual 6.14, 2014).
SikaWrap-900C CFRP sheets and Sikadur300 Epoxy were used in strengthening the PHC web specimens. The SikaWrap-900C is a uni-directional, fleece stabilized, stitched and heavy carbon fiber fabric for the wet application process of structural strengthening. The mechanical properties of the FRP sheets with epoxy resin, provided by the supplier and determined by tensile tests on representative samples in accordance with ASTM D3039, showed a linear elastic stress-strain relationship with average values of elastic modulus and tensile strength being 94000 MPa and 1012 MPa, respectively.

4.2.4 Steel bearing plates

The support plates of the loading beam are simulated by an eight-node solid element, C3D8R, which is the same as that used to simulate concrete. This is to avoid the convergence problems, which may occur at the loading or the reaction points between the
steel bearing plate element and the concrete element.

The material properties of these elements are assumed to be linear-elastic. Consequently, the modulus of elasticity and the Poisson’s ratio are the only essential properties need to be specified in the model. They are taken as 200 GPa and 0.3, respectively. Further, in the FE model, tie constraint is applied at the concrete-steel plate interface.

4.2.5 Reinforcement-concrete interface

In the current study, the embedded element is used to simulate the interface between reinforcement and concrete, i.e. an element or a group of elements is/are specified to be embedded in the “host” elements. This is achieved in ABAQUS by comparing the geometric relation between nodes of the embedded elements and the host elements. If a node of an embedded element is found to coincide with that in a host element, the translational degrees-of-freedom at the node of the embedded element are eliminated and constrained to the interpolated values of the corresponding degrees-of-freedom of the host element. So the node becomes an “embedded” node. The embedded elements are allowed to have rotational degrees-of-freedom, but these rotations are not constrained by the embedding defining. Multiple embedded elements is allowed in ABAQUS and it is appropriate to be used in nonlinear analysis (ABAQUS User’s Manual 6.14).

In addition, the interfacial effect (bond slip) between concrete and strands can be reflected by introducing “tension stiffing” in CDP model, which simulates the load transfer
function of strands in concrete cracking region. This command can define the strain softening of cracking concrete that is a simplify way to consider the bond slip effect (ABAQUS Theory Manual 6.14).

### 4.3 FE Model Geometry and Boundary Conditions

Figures 4.11 and 4.12 show respectively the cross-section of the low and medium prestressing PHC slabs and the corresponding web beam specimens used in this study. Figures 4.13(a) and (b) give a typical 3D view of the geometry of the FE model developed for the control web beams and the strengthened web beams. The X-axis is along the longitudinal direction of the beam, the Y- and the Z-axes are within the beam cross-section, and along respectively the vertical and the horizontal directions.

![Cross-section of low prestressing HPC slab and web beam](image1)

**Figure 4.11 Cross-section of low prestressing HPC slab and web beam**

![Cross-section of medium prestressing HPC slab and web beam](image2)

**Figure 4.12 Cross-section of medium prestressing HPC slab and web beam**
To simulate the pin support on the loading side of the model, the nodes on the bottom face of the support plate are constrained in the x, y and z directions. However, for a roller support condition on the other side, the nodes on the bottom face of the support plate are constrained only in the y and z directions, as shown in Figure 4.14. It is worth mentioning that constrain displacement of the above nodes in a specified directions is achieved by
setting the corresponding displacement to zero.

![Figure 4.14 Boundary conditions of support plates (Left-Pin, Right-Roller)](image)

### 4.4 Solution Control

The behaviour of a PHC web beam subjected to a vertical line load is simulated first. The nonlinear analysis is performed in two steps. In the first step, prestress in the strands is introduced to concrete by reducing temperature of strands and make them shrink. Obviously, this step will cause a slight camber along the slab length. More importantly, the “*Model Change” command is used to remove the tie constraints respectively between steel plates and concrete, CFRP sheets and concrete in the first step. This is to avoid convergence problem in the process of introducing prestressing force. The second step is to simulate the gradual application of a line load in the vertical direction as what is done during the physical tests to cause shear failure of the web beam. The increments in load is taken as 1.0 kN, the magnitude of the load increases until failure of the beam occurred.

The Full Newton method, which is a numerical technique for solving the nonlinear equilibrium equations, is adopted in this FE model. The convergence criterion is based on force. The tolerance limit of 0.05 is used in the first loading step to ease the solution, whereas a tolerance limit of 0.005 is specified for the second step. The simulation is
terminated when the solution becomes difficult to converge.

4.5 FEM Validation

Figure 4.15 presents the FEM results for load-deflection behaviour of W1-Series web beams. Compared to the experimental results in Figure 3.11, FEM was able to effectively predict the ultimate loading capacity, ductility behaviour and the FRP strengthening effect. Moreover, the increasing trend of the ultimate load and displacement obtained from W1-Series FEM is very consistent with the physical test results. As can be seen in Table 4.2, except W1-2-300 and W1-4-450 specimens, which gained unexpected testing data due to pre-existing discussed before, the maximum difference between the experimentally obtained ultimate failure load and the numerical predication is less than 5% in most FEM simulation results.
Figure 4.15 Load-deflection behaviour of W1-Series obtained from FEM
Table 4.2 Comparison of W1-Series results

<table>
<thead>
<tr>
<th>FE Model-ID</th>
<th>Failure load (kN)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>ABAQUS</td>
<td>Difference</td>
<td></td>
</tr>
<tr>
<td>W1-C</td>
<td>77.8</td>
<td>74.1</td>
<td>4.7%</td>
<td></td>
</tr>
<tr>
<td>W1-2-300</td>
<td>88.7</td>
<td>79.2</td>
<td>10.7%</td>
<td></td>
</tr>
<tr>
<td>W1-2-450</td>
<td>81.4</td>
<td>83.5</td>
<td>2.5%</td>
<td></td>
</tr>
<tr>
<td>W1-2-600</td>
<td>89.3</td>
<td>87.9</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
<td>W1-4-300</td>
<td>89.6</td>
<td>89.9</td>
<td>0.3%</td>
<td></td>
</tr>
<tr>
<td>W1-4-450</td>
<td>87.5</td>
<td>95.1</td>
<td>8.6%</td>
<td></td>
</tr>
<tr>
<td>W1-4-600</td>
<td>98.1</td>
<td>102.1</td>
<td>4.1%</td>
<td></td>
</tr>
</tbody>
</table>

The load-deflection behaviour simulation results for W2-Series web beams are illustrated in Figure 4.16. Clearly, compared to the experimental results (Figure 3.18), the developed finite element model well simulated the improved behaviour of the FRP strengthened single web PHC slab. Table 4.3 summarizes the experimental and numerical results of the failure load for the W2-Series specimens. The difference between tests and FEM results ranges from 2.6% to 5.6%, except for W2-4-450 specimen of which an imperfection problem existed prior to the test. The above FEM results of both W1-Series and W2-Series specimens clearly verifies the validity of the proposed numerical model.
Figure 4.16 Load-deflection behaviour of W2-Series obtained from FEM
Table 4.3 Comparison of W2-Series results

<table>
<thead>
<tr>
<th>FE Model-ID</th>
<th>Experiment</th>
<th>ABAQUS</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>W2-C</td>
<td>81.5</td>
<td>78.7</td>
<td>3.4%</td>
</tr>
<tr>
<td>W2-2-300</td>
<td>76.8</td>
<td>80.8</td>
<td>5.2%</td>
</tr>
<tr>
<td>W2-2-450</td>
<td>79.9</td>
<td>84.4</td>
<td>5.6%</td>
</tr>
<tr>
<td>W2-2-600</td>
<td>83.5</td>
<td>87.9</td>
<td>5.2%</td>
</tr>
<tr>
<td>W2-4-300</td>
<td>102.1</td>
<td>98.3</td>
<td>3.7%</td>
</tr>
<tr>
<td>W2-4-450</td>
<td>99.6</td>
<td>107.9</td>
<td>7.6%</td>
</tr>
<tr>
<td>W2-4-600</td>
<td>114.2</td>
<td>117.2</td>
<td>2.6%</td>
</tr>
</tbody>
</table>

The load-deflection relationship is the most important evaluation criterion for the performance of a finite element model since it represents the overall behaviour of specimens during the test. Figure 4.17 (a), (b), (c) and (d) give some sample load-deflection behaviour comparison results between experiment and FEM. It is found that the pattern of both curves is very similar that they have agreeable initial stiffness, ultimate failure load, deflection and the same developing trend.

Moreover, some sample relationships between the load and the top longitudinal concrete compressive strain as well as between the load and the longitudinal tensile strain in FRP sheets are portrayed repectively in Figures 4.18 and 5.19. As can be seen from these two figures, FEM can well predict the compressive strain in concrete and the tensile strain in FRP. The maximum error between the numerical results and the experimental data is 8.4% and 9.7%, respectively, which is acceptable.
(a) W1-C

(b) W2-C
Figure 4.17 Load-deflection behaviour
(a) W1-2-600 (max. difference: 2.8%)

(b) W1-4-600 (max. difference: 5.9%)
Figure 4.18 Load vs top longitudinal concrete compressive strain relationships

(c) W2-2-600 (max. difference: 8.4%)

(d) W2-4-600 (max. difference: 6.7%)
(a) W2-4-300 (max. difference: 9.7%)

(b) W2-4-600 (max. difference: 5.1%)
Figure 4.19 Load vs longitudinal tensile strain in FRP sheets relationships

Figure 4.20 gives four samples to compare the specimen cracking pattern observed in the experiments with the plastic strain (PEEQ) results obtained from numerical simulation. Although the two sets of results do not match exactly, the distribution of plastic strain and crack propagating trend in the shear-tension region obtained from FEM are consistent with the failure crack from the testing results.
(a) W1-C

(b) W2-C
Figure 4.20 Experimental cracking pattern and plastic strain (PEEQ) in FE model

4.6 Parametric Study

The developed FEM in this research can be used to conduct parametric study since the accuracy of predictions is good. According to the experimental study, there are three important parameters for shear strengthening of single web prestressed hollow core slabs
with FRP sheets, i.e. strengthening length, width and thickness.

The reasons that the strengthening length is not taken into consideration are: firstly, the strengthening efficiency of 300, 450, and 600 mm FRP sheets have already been investigated in the experiment and the FEM simulation; secondly, the production width of FRP sheets is usually 300 mm that it is hard to cut it as random length. Furthermore, FRP sheets more than 600 mm would be difficult for application in the real construction work. Therefore, this section only investigates the effect of two key parameters that are expected to influence the shear capacity of PHC web beams, which are strengthening thickness and effective strengthening width.

The numerical model used in the parametric study had the same geometric details, concrete and strand material properties of the model that was presented to simulate W2-Series specimen.

**4.6.1 Strengthening thickness**

The effect of the strengthening thickness was evaluated by applying one layer (1 sheets each side), two layers (2 sheets each side) and three layers (3 sheets each side) FRP sheets to the single web PHC slab in FEM. The increase rate of the ultimate shear failure load was used as the based for comparison. Figure 4.21 gives a sample set of W2-Series-600 FEM results, the ultimate load of 1-, 2- and 3-layer models are respectively 87.9 kN, 117.2 kN and 136.9 kN. So the average increasing rate of ultimate loading capacity is 27.9% per layer and the ductility increases 70.6% per layer.
As shown in Table 4.4, the average enhancing rate of ultimate load resulted from increasing strengthening thickness is much higher than that from increasing strengthening length. Therefore, it suggests that increase FRP strengthening thickness will be more effective than increase length for shear strengthening of prestressed hollow core web beams.

Figure 4.21 FEM parametric study results of strengthening thickness effect
<table>
<thead>
<tr>
<th>FE Model-ID</th>
<th>Strengthening length (mm)</th>
<th>Failure load (kN)</th>
<th>Average Enhancing rate</th>
<th>Failure load (kN)</th>
<th>Average Enhancing rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td></td>
<td></td>
<td>ABAQUS</td>
<td></td>
</tr>
<tr>
<td>W2-2-Series</td>
<td>300</td>
<td>76.8</td>
<td>4.3%</td>
<td>80.8</td>
<td>4.4%</td>
</tr>
<tr>
<td></td>
<td>450</td>
<td>79.9</td>
<td></td>
<td>84.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>83.5</td>
<td></td>
<td>87.9</td>
<td></td>
</tr>
<tr>
<td>W2-4-Series</td>
<td>300</td>
<td>102.1</td>
<td>5.9%</td>
<td>98.3</td>
<td>9.6%</td>
</tr>
<tr>
<td></td>
<td>450</td>
<td>99.6</td>
<td></td>
<td>107.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>114.2</td>
<td></td>
<td>117.2</td>
<td></td>
</tr>
</tbody>
</table>

### 4.6.2 Effective strengthening width

In this research, the strengthening width of FRP sheets has an arc shape due to the hollow core in the PHC slabs. It is easily to realize that only part of FRP sheets will work efficiently according to the tensile mechanical properties. Also, the strengthened FRP sheets are actually utilized its vertical tensile capacity to enhance shear force resistance. Therefore, it is necessary to study the effective strengthening area in the FRP sheets and three different scenarios (180°, 120° and 90°) are developed to solve this problem, as shown
in Figure 4.22(a), (b) and (c).

![Diagram](image1)

![Diagram](image2)

![Diagram](image3)

Figure 4.22 Three different scenarios of strengthening width

Table 4.5 gives the simulation results of the three different strengthening width based on the W2-4-600 FE model. Compared to the 180° case, the ultimate failure load of 115.6 kN corresponding to the 120° case, decreased only 1.4%. While, it drops 8.3% to 107.5 kN in the 90° case. Clearly, the most effective strengthening width would be around 120°. Moreover, the distribution of the maximum tensile principle stress results are shown in Figure 4.23. During the time-history analysis in FEM, it is visible that the tensile region expands from the middle to both sides along the FRP vertical fiber direction and the most efficient working area is in the middle region of FRP sheets.

Table 4.5 W2-4-600 FEM parametric study results

<table>
<thead>
<tr>
<th>Strengthening Width</th>
<th>Failure load (kN)</th>
<th>Percentage of reduction (compare with 180°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>117.2</td>
<td>-</td>
</tr>
<tr>
<td>120°</td>
<td>115.6</td>
<td>1.4%</td>
</tr>
<tr>
<td>90°</td>
<td>107.5</td>
<td>8.3%</td>
</tr>
</tbody>
</table>
Figure 4.23 The distribution of maximum principle stress in FRP sheets
CHAPTER 5 CONCLUSION

This study proposed a new method to strengthen shear capacity of PHC slabs by installing the FRP sheets along the internal perimeter of the slab voids. To explore the feasibility and effectiveness of this new technique, experimental study of full scale I-shaped single web beams, which are longitudinally cut out from PHC slabs and externally strengthened by Carbon FRP (CFRP) sheets bonded along the full perimeter on both sides of the specimen, was conducted. The studied parameters included the length and the width of the strengthened zone and the thickness of the CFRP sheets.

In the numerical study part of this research, a finite element simulation was conducted using ABAQUS software. The main objective of this phase is to develop a finite element model of an I-shaped single web beam to understand the shear behaviour of web beams when equipped with externally bonded FRP sheets. The model was validated by the experimental results and parametric study was performed.

5.1 Summary of Experimental Study

1. The experimental results show that the shear capacity of web beams strengthened by four FRP sheets have a significant increase of about 20% to 40%, which indicates the new technique is effective in shear strengthening for single web hollow core slabs when the amount of strengthening FRP sheets is adequate. Thus, it is expected that the proposed technique would also be effective when applied to the full width hollow core slabs.
2. W2-Series specimens strengthened with only two FRP sheets are not effective under the current test condition and materials.

3. Strengthening PHC web beams with 4 CFRP sheets, 2 on each side, can not only increase the ultimate shear capacity, but also enhance their ductility before failure. Also, the shear-tension region is reduced and the flexural region is increased.

4. The testing data indicates that minimum FRP thickness and strengthened zone length are required in order to obtain satisfactory strengthening effects.

5.2 Summary for Finite Element Modeling

1. The FEM developed using the ABAQUS software can simulate the shear behaviour of FRP strengthened I-shaped single web beam with a reasonable accuracy. The predicted load-deflection and load-strain relations agree well with the corresponding experimental data.

2. The damage-plasticity model can effectively simulate concrete material properties and mechanical behaviours. Also, according to the experimental data, the average bond-slip between the prestressing strand and the concrete beams is very limited (less than 0.8 mm) and an excellent bonding performance is demonstrated between FRP sheets and concrete surface. These results strongly proved that the embedded element and tie used respectively for simulating bonding between prestressing strand and concrete, as well as that between FRP sheets and concrete are reasonable and effective.
3. Results yielded from the parametric study suggest that increase of FRP strengthening thickness will be more effective for shear strengthening of prestressed hollow core web beams. Further, the effective strengthening width of FRP sheets covers a 120˚ arc region along the middle portion of the hollow core perimeter.

5.3 Future Work

Based on the findings and conclusions of the current study, the following recommendations are made for future research:

1. There are still six low prestressing and six medium prestressing full width PHC slabs for the second phase of the research. So it is necessary to make a detailed experiment plan for these twelve specimens to conduct further study.

2. Finite element model of full width PHC slabs needs to be developed and validated by the experimental results. More parameters such as the span ratio and the FRP fiber direction need to be investigated.

3. More experimental and numerical studies need to be performed for PHC slabs with different depth and reinforcement ratio.
APPENDIX A:

The strengthening effect comparisons of strengthened zone length and thickness
Effect of strengthened zone length on the behaviour of W1-Series specimens
Effect of strengthened zone length on the behaviour of W2-Series specimens
(a) W1-Series-300 specimens results

(b) W1-Series-450 specimens results
Effect of strengthened thickness on the behaviour of W1-Series specimens

(c) W1-Series-600 specimens results

(a) W2-Series-300 specimens results
Effect of strengthened thickness on the behaviour of W2-Series specimens
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