Outsourcing Evaluation in RL Network

Kingshuk Jubaer Islam

University of Windsor

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Outsourcing Evaluation in RL Network

by

Kingshuk Jubaer Islam

A Thesis
Submitted to the Faculty of Graduate Studies
through Industrial and Manufacturing Systems Engineering
in Partial Fulfillment of the Requirements for
the Degree of Master of Applied Science at the
University of Windsor

Windsor, Ontario, Canada

2012

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January 20, 2010
DECLARATION OF ORIGINALITY

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ABSTRACT

This thesis addresses the qualitative investigation of the reverse logistics and outsourcing and a quantitative analysis of reverse logistic networks that covenant with the option of outsourcing or in-house remanufacturing. Two models are proposed with an objective of contributing to decision making process for reverse logistics outsourcing. The purpose is to find a set of decisions throughout the product life cycle that maximizes both outsourcing and in-house remanufacturing. These models will also verify two hypotheses: outsourcing is more likely to be an optimal solution when variance in return rate is high, and also when the product life cycle is short in length. Then, a solution approach is designed for solving this problem which follows MDP that considers the firm following a dynamic capacity model and also a stationary capacity model. Finally, computational analyses are performed to demonstrate the applicability of the model. Numerical results justify the two hypotheses.
DEDICATION

I dedicate this thesis to my family; my parents and my two sisters.

Who always prayed for me, inspired me and believed in me even when I lost belief in myself.

Thank you.
ACKNOWLEDGEMENTS

I take this opportunity to thank my supervisor; Dr. Walid Abdul-Kader, to take me in as a student and guide me through this rigorous process of becoming a Master’s graduate and on the way showing me to be a better person than I was.

I also thank my committee members; Dr. Fazle Baki and Dr. Ben Chaouch, for their constant support and insightful contributions into the thesis.

My acknowledgement will not be completed without a very special thanks to all my friends and family member who helped me throughout this time period. I cannot mention everybody here, but we all know how important you have been in my life. You have enriched my life in every possible way.
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CHAPTER 1

INTRODUCTION

1.1 Introduction

The traditional perception of product-producer relation has undergone a great change in the modern supply chain concept. Instead of there being only a one way flow from producer to user, many products (especially durable, consumables and machinery) witness a reverse flow from user to the producer. There is also another trend; subcontracting processes or logistics activities to another firm for maximizing profit and minimizing cost, which is commonly acknowledged as outsourcing. Outsourcing is gaining popularity really rapidly. In this research, the concepts of outsourcing are studied and mathematical models are developed to address the outsourcing strategy in the context of reverse supply chain.

1.2 Motivation

Factors like growing environmental concerns, rapid depletion of resources and legislation has led to the growing attention to Reverse Logistics (RL). RL is defined as the logistics activities consisting of products ranging all the way from used products no longer required by the customer to products not fulfilling the customer’s satisfaction in the market and are now being returned for further value adding activities to the Original Equipment Manufacturers (OEMs). The reasons why the products are returned can be seen as detrimental to a company and thus be avoided or can be considered as a
competitive advantage with potential for capturing further market share (Krumwiede & Sheu, 2002). Original Equipment Manufacturers (OEMs), instead of considering return of products a burden, now find it a source of immense opportunity in the business. Some value additive operations are completed on the returned products and they can thus again enter the market as reused/ repaired, refurbished or remanufactured products.

Once the reverse logistics is acknowledged by the Original Equipment Manufacturers as a potential aspect of the supply chain system, there comes, as a logical sequence, the option of outsourcing the RL operations to a third party, popularly known as 3rd party reverse logistics provider (3PRLP). Some manufacturers have inefficient, slow and expensive processes for handling the returned products and a considerable amount of value is lost when these returns cannot be processed quickly and completely (Rupnow, 2003) and it promotes the idea of outsourcing. There is also the fact that reverse logistics does not represent the core activity of a firm and the purpose of any company is not to manage the flow of products taken back from the sales point but rather to distribute such products to its customers. In short, comparative advantage of a firm plays an important role in popularizing outsourcing.

The concept of outsourcing, although has changed somewhat from the time it was first introduced, the question of “why” to outsource has remained the same from the very beginning. Possibility of acquiring lower costs due to economies of scale, achieving greater flexibility, opportunity of higher quality of service, better control of budget, faster set up of function or service, improved risk management, lower ongoing investment in
internal infrastructure are few of the reasons why the OEMs prefer to involve the 3rd party logistics provider.

Till now, there has not been a great amount of work done comprising of both the reverse logistics and outsourcing, and it is the motivation of this research to do a comprehensive study on outsourcing option in reverse logistics and help researchers gain a complete insight into the supply chain and also generate mathematical models that is going to help managers take the proper and accurate strategic decision as when to outsource.

1.3 Scope

Today the business scope of 3rd party reverse logistics provider (3PRLP) is much more than just managing warehouses or picking up and delivering customer orders as it was earlier. 3PRLP, although initially focused their strength on providing warehouses and transportation benefits for the OEMs; nowadays are more involved in the remanufacturing functions and they perform multiple tasks ranging from purchasing raw materials to remanufacturing the returned products for different firms.

Earlier, services or manufacturing opportunities were outsourced as a last resort but now it is relatively a strategic decision which is considered such that ensures profit. Before deciding on outsourcing decision, one needs to evaluate the outsourcing opportunities i.e., technological or process. Of course, during the process of deciding the strategy of outsourcing, one should consider all or part of the supply chain and whether this strategy would bring faster, more efficient and inexpensive options.
There have been many studies conducted on how to select the 3PRLP, but in contrast, very few researches have been explored on “when” to outsource. There has been some research conducted on the mathematical modeling of the problem of when it is beneficial to outsource rather than in-house remanufacturing. Miao (2009) employed Markov decision process to correlate between disposition and outsourcing. Hui-yun & Min-li, (2007) applied simulation tool to settle on the value of production outsourcing. Aras et al. (2004) categorized the returned products in their effort to study the outsourcing in reverse logistics network. Serrato et al. (2007) took the help of Markov decision process to generate a model that deals with outsourcing decision. Wang & Fan (2007) discussed outsourcing in terms of collecting the used products. In this research, various research work comprising of RL and outsourcing will be studied to gain insight into the problem formulation and solution techniques.

The research includes a thorough study of the reverse logistics and the outsourcing opportunities to the OEMs.

1.4 Research guideline

In this research, the concept of outsourcing is analyzed based on the simple fact that sometimes it will be profitable to outsource rather than continue with in-house remanufacturing. Since no company is expected to be continuously changing their policy about outsourcing, it becomes imperative for them to adopt a “take it or leave it” strategy; at least for a moderately long period of time. The company can take the option of managing the reverse logistics in house or may seek help of a 3PRLP by judging their
comparative advantages and disadvantages. Therefore, it is important to know exactly what is happening inside the supply chain.

Drawing upon the wide range of experience accumulated in the literature and with an objective of contributing to decision making process for reverse logistics outsourcing, the objective of the present research is to identify the suitability of outsourcing option for a particular firm dealing with reverse logistics by the help of an analytical model in which a threshold policy is introduced.

1.5 Thesis organization

The remainder of this document is presented as follows: Chapter 2 covers the related literature review, along with some critical analyses on reverse logistics and outsourcing. The research problem is defined and objectives are presented in chapter 3. A Markov model is formulated in chapter 4 considering a threshold that would make the decision of outsourcing easy. The solution of the model with examples and result analysis comes in chapter 5 followed by conclusions and recommendations for future work in chapter 6.
CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

Following extensive research, it is found that the literature on reverse supply chain or outsourcing is very rich and it is the goal of this research to create a link between outsourcing and reverse supply chain both qualitatively and quantitatively. The purpose of this chapter is to get a comprehensive idea for both in the historical perspective and also on some important policy issues now attracting academics, researchers and practitioners on the subject. A wide-ranging literature survey of both reverse supply chain and outsourcing is discussed in the following sections.

2.2 Reverse supply chain

Compared to other supply chain concepts, reverse supply chain is a relatively new term in the supply chain management. In the 1970’s, the study for the Club of Rome argues that there is a limit to the growth. The report announced that around 2050, Mankind is doomed to disintegrate (Meadows, 1974) and argues for collective effort to sustain the course of civilization. During the following decade, environmental disasters attracted the attention of the academics, politicians, the media and society in general addressed to such issues. Terms like recycling, reuse, resource reduction, environmental manufacturing responsibility and green products began to be familiar to all. Since the mid-nineties, especially in Europe, this was accompanied with legal enforcement of product and
material recovery or proper disposal (Brito & Dekker, 2002). It is safe to say that the waste reduction efforts promoted the idea of material recycles, which attains resources recycling, materials values addition and costs saved and gradually replacing a “one way” economy mode of traditional material operation (Zuqing & Baoyou, 2007).

The term “reverse logistics” is first proposed by Stock (1992), who submitted his report to the Council of Logistics Management (CLM) in 1992, in which he pointed out: “Reverse logistics is such logistics activity, which contains product returns, material substitution, goods reuse, waste abandons, re-handling, maintaining, remanufacturing and so on.” Reverse logistics is presently defined by CLM as “The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of origin to the point of consumption for the purpose of conforming to customer requirements” (Krumwiede & Sheu, 2002). The definition by CLM seems to widen the scope and penetrate the depth of the concept. It should be mentioned here that different researchers have used different terms, such as reverse flow logistics, reverse distribution, reverse logistics, closed loop supply chain systems and supply loops to represent the same activity or parts of it although all correspond to essentially the same logistics operations (Yellepeddi et al., 2006).

Besides all the environmental imperatives that cropped up during the 70’s and 80’s, changes or shifts in consumer behavior plays no lesser role in the development of reverse logistics. Ravi & Shankar, (2005) observes that there is an increase in the flow of returns of the products due to product recalls, warranty returns, service returns, end-of-use
returns, end-of-life returns and so on. It is found that the total value of product returns in the U.S. is estimated to be $100 billion annually. The Gartner group projects that merchants spent $10 billion per year processing returns, while web merchants alone spends $3.2 billion processing returns in 2001 (Tedechi, 2002).

It should be agreed upon at this point that reverse logistics is an untapped resource and can be argued that if effectively handled, may result in several benefits that include improved customer satisfaction, decreased investment level and reduced storage and distribution cost. It can be an area for improving profitability and customer satisfaction. Biehl et al. (2007) argues that the benefits to companies that use RL, along with recycling or remanufacturing, can be manifold. Such firms can save up to 40–60% of the cost of manufacturing a completely new product or cut down delivery lead times, if service parts or complex components are remanufactured rather than manufactured from scratch.

While the benefits of reverse logistics are real, a realistic mix of manufacturing and remanufacturing of a particular product still remains a gray area to policy planners. Aras et al. (2004), for example, suggests that product return rate is typically less than the demand rate, which suggests that it is impossible to satisfy the entire market demand via remanufacturing alone. Therefore, the coordination of manufacturing and remanufacturing facilities is crucial. A major difficulty arises, however, due to uncertainties with respect to quantity, quality and timing of the return.
Another important subject that needs to be discussed is the activities that reverse supply chain offers. Different researchers presented their version of activities. Blackburn et al. (2004) suggests that reverse supply chain are designed to carry out five processes:

1. Product acquisition: Obtaining the used products from the user.
2. Reverse logistics: Transporting the products to a facility for inspection and sorting.
3. Inspection and disposition: Assessing the condition of the returns and deciding on the form of reuse.
4. Remanufacturing or refurbishing: Returning the product to its original form.

Different researchers who addressed the issues, tried to exercise a policy that will help the OEMs take the appropriate decision regarding time, quality and quantity of returns. They experimented and attempted to develop models that would help understand the reverse logistics better.

Reverse supply chain is discussed here briefly with the concept, definition, benefits and activities. Outsourcing is discussed in the next section with the concept, benefit and definitions.
2.3 Outsourcing

Although outsourcing may be considered for both forward supply chain and reverse supply chain in any industry, in the context of the present research, outsourcing will be referred to as outsourcing in reverse supply chain.

Domberger (1998) states that outsourcing is transferring of goods and services that were carried out internally initially, to an external provider. Outsourcing is defined as the transferring of an internal function to an external organization (Ketler & Willems, 1999).

There are many reasons for outsourcing. One increasingly popular use of outsourcing is stimulated by the strategic shift in the ways in which the organizations are managing their business (Winkleman et al., 1993). In their effort to identify the motivating force for outsourcing, Lankford & Parsa, (1999) suggests that outsourcing can reduce cost, expand services and expertise, improve employee productivity and morale and create a more positive corporate image by focusing their resources on their core business, making organization’s plans more efficient and save both time and money. Fill & Visser, (2000) establishes that improvement of competitive pressures, improvement of quality and efficiency, rising of the potential for creating strategic business alliances and reducing internal administrative problems can be some sources of inspiration for outsourcing. Wu et al. (2005) mentions such factors as ability to purchase components, sub assemblies, finished products or services from outside suppliers when production capacity is limited as the main reasons behind outsourcing. It is also argued that competitive enterprises may satisfy their customers by improving service speed, flexibility and response capability.
This can be facilitated by suitable outsourcing design. On the other hand, Fisher et al. (2008) considers cost reduction and efficiency improvements, improvements in business performance, commercial exploitation of outsourcing provider capability, divesting the company of a problem and following the lead of others as the main reasons of outsourcing. Ordoobadi (2009) suggests 3 reasons behind outsourcing and they are: strategic, operational and financial reasons. Subcategories of these main reasons are given below:

Table 2.1: Reasons behind outsourcing (Ordoobadi, 2009)

<table>
<thead>
<tr>
<th>Main category</th>
<th>Sub category</th>
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<tr>
<td>Strategic reasons</td>
<td>Expansion to a new market</td>
</tr>
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<td></td>
<td>Ability to focus on core activities</td>
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<tr>
<td></td>
<td>Gaining access to world class technology improving customer service</td>
</tr>
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<td></td>
<td>Differentiation from competitors</td>
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<tr>
<td>Operational reasons</td>
<td>Lack of internal expertise</td>
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<tr>
<td></td>
<td>Labor issues</td>
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<td></td>
<td>Operational flexibility</td>
</tr>
<tr>
<td></td>
<td>Handling of non-value added activities</td>
</tr>
<tr>
<td>Financial reasons</td>
<td>Reduction in operating and transaction costs</td>
</tr>
<tr>
<td></td>
<td>Avoiding large investments</td>
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<td></td>
<td>Reduction in employee base</td>
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</table>
For a firm to adopt an outsourcing policy, two conditions are necessary and sufficient and they need to be satisfied simultaneously. The necessary condition, which may as well be termed as ability condition, comes into play when the firm can mobilize necessary resources to procure the service from outside to complement its own capacity. While the sufficient condition – again may be termed as profitability condition – is satisfied only when the firm finds it advantageous in terms of direct or indirect benefit. Behara et al. (1995) reasons, for example, that when the right tasks are contracted out, only then can the cost efficiency advantage be attained.

Some researchers specify more explicit benefits of outsourcing. Thus, Quinn & Hilmer, (1994) suggests that outsourcing contributes a significant part of an enterprise’s value chain activities. According to Wu et al. (2003), outsourcing enables an organization to better marshal its own resources and those of its external agents who have the required expertise and specific resources/technologies to accomplish all the tasks involved. According to Jiang et al. (2006), by outsourcing tasks to specialist organizations, firms may better focus on their most value-creating activities, thereby maximizing the potential effectiveness of those activities. Within the supply market, there is immense opportunity and outsourcing allows organizations to take advantage of it. It may be possible to state that the advantages of outsourcing not only benefit the customers but also suppliers who are able to make a profit from it. In a nutshell, outsourcing widens the business opportunities for small firms and higher profit for the larger organizations.

It may be pointed out at this stage that outsourcing has its risks too. Gavin & Matherly, (1997) classifies these risks and problems into three main aspects: people, process and
technology. “People” problem ranges from the reduction of loyalty to loss of internal expertise, emotional stress. “Process” problem arises from incompatibilities between service provider and organization, and the inability of organizations to sufficiently analyze their decision to outsource. Sweet (1994) argues that organizations are in danger of signing a blank check when dealing with outsourcing as it is very easy to be persuaded by the vendor to hand over the core technology details. Wu (2003) acknowledges this apprehension and stated that, there is a probability of some core technologies falling prey to potential technology predators because of inadequate protection and leading to a fall in market share. Though this issue is not going to be dealt with elaborately in the present work, it is worth mentioning here that proper care must be taken in order to avoid the problems with outsourcing that may cause damage to a business rather than fetching any advantage to it. As a combat strategy, Wee et al. (2010) in their paper proposes different outsourcing strategies to be deployed by different types of industries and stressed for a good working relationship with suppliers and customers as paramount to success.

The domain of outsourcing has moved from cleaning, catering and security to such critical areas as design, manufacturing, marketing, distribution and information systems (Jennings, 1997). According to a study by Canadian Logistics Skill Committee and Deloitte Consulting (July, 2005), outsourced supply chain activities in Canada demonstrate the following numbers as depicted in Figure 2.1.
Figure 2.1: Outsourced supply chain activities in Canada

According to recent estimates, annual sales of remanufactured products exceeded $50 billion in the United States alone (Guide & Wassenhove, 2003). Though there are no worldwide estimates of how many firms are now engaged in outsourcing, the number is increasing day by day. Dowlatshahi (2005), while accepting the claim adds further that the number of firms engaging in this sector is increasing day by day. Lieb & Randall,
(1996), Boyson et al. (1999), and Arroyo & Gaytán, (2007) argues that cost–benefit and operational efficiency are some of the criteria why outsourcing is being pursued.

Outsourcing, which is used to describe all subcontracting relationships between firms, and the hiring of workers in non-traditional jobs (Heshmati, 2003), is becoming more and more popular. One of the reasons is that the companies find they should work with their supply chain partners closely to improve the chain’s total performance. If they work closely, then they are able to retain competitive advantages in this fierce competitive environment, characterized by thin profit margins, high customer expectations for quality products and short lead times (Najla et al., 2007).

Since the early 1990s, outsourcing has been discussed under diverse aspects in both academic and business studies and operational practice (Kakabadse & Kakabadse, 2000; Lonsdale & Cox, 2000; Seuring, 2003; Espino- Rodriguez & Robaina, 2006; Holcomb & Hitt, 2007; Lacity et al., 2008;). For example, Grover et al. (1994) thinks that the success of outsourcing can be assessed in terms of attainment of benefits. Perry (1997) sees outsourcing as a means to obtain the competitive advantages of reliability, quality and cost from contracting out the production of goods and services. Boyson et al. (1999) emphasizes that outsourcing should be adopted as a means to gain strategic advantage rather than to remedy specific deficiencies. Krumwiede & Sheu, (2002) considers a model for market entry by a 3PRLP but the method to engage them into the business is still under work. Sohail & Sohal, (2003) suggests that functions performed by the third party can encompass the entire logistics process or selected activities within that process. Ravi & Shankar, (2005) proposes a combination of balanced scorecard and analytic network
process to provide a more realistic and accurate representation for conducting reverse logistics operations for end-of-life computers.

There has been little research work done on the mathematical implementation of outsourcing strategy as discussed earlier, and of course, there have been even fewer studies integrating both reverse logistics and outsourcing.

It is believed that this is a research gap that needs to be filled because integrating both reverse logistics and outsourcing is a subject matter that cannot be neglected much further. This research is intended to minimize this gap in the research area and come about with a suitable mathematical model that can be helpful to further researchers and also have a real life application.

The following studies illustrate some important research works and give a clear idea about the mathematical models that other researchers have already developed. At the end of this section possible improvement area will be addressed.

2.4 Some related papers

In the current research, the following parameters is used to portray the problem comprising of reverse logistics and outsourcing: product demand (forecasted), return rate, total return, units outstanding in the market after a specific time period, remanufacturing capacity, length of remanufacturing, managing the product after the last sales is made, transportation cost, inventory cost, inspection cost, shortage cost, outsourcing cost, fixed
internal cost, variable cost, investment cost, idle cost, salvage value, number of returned products disposed based on quality, products accepted for reuse, disposal cost.

In this section it will be depicted properly how other researchers have considered these parameters, how they differ from the concept used in this research and how the problems can be taken care off.

**Product demand**

One of the important parameters to be defined in the supply chain is the demand for product. With a clear idea of the forecasted demand, one can make the necessary changes in their policy.

Nembhard et al. (2003) describes the demand function as deterministic. This concept is acceptable only in the short term but when one is considering a longer time, this deterministic demand does not predict the future properly and there is a probability of error.

Aras et al. (2004) considers that the demand function follow a Poisson distribution.

Savaskan et al. (2004) on the other hand, assumes that the demand has a downward sloping linear relation with retail price. Both Aras and Savaskan’s proposal for demand prediction is suitable for special cases, but does not hold true much in general conditions.
Serrato et al. (2007) however uses a sales function comprising of length of product life cycle and maximum expected sales level as function parameters for the demand function, which is excellent in terms of portraying a real life scenario.

- In this current research, the demand function portrayed by Serrato et al. (2007) is closer to real life and is considered for the current model.

Return rate

Knowing exactly how the products are returning is a major advantage to the remanufacturer.

Savaskan et al. (2004) characterizes the return rate as a function of investment in collection activities and a scaling parameter. Here, the return rate can be thought of as the response of consumers who had kind of incentive for the product returns to the reverse channel. Similar forms of response function are used in advertising response model and product awareness.

Serrato et al. (2007) considers return rate as a constant number throughout the whole length of the analysis period. This concept is again true if one is considering a special case of reverse logistics.

Ordoobadi (2009) explains the return rate as a fraction, but did not elaborate on the parameter.
None of the researcher actually demonstrated any logical explanation for the return rate, but this is a very important parameter as explained earlier.

- In this research, the return rates are considered such that the expected number of returned products follows Poisson distribution (These are only assumed values to check if the program is running accordingly).

**Total return**

Total return volume is the consequence of the amount of units historically sold and a fraction of them returned through the reverse logistics system.

Aras et al. (2004) describes the product return following Poisson distribution.

Savaskan et al. (2004) explains the total return to consider the total cost of collection and characterized it by a function of return rate and the product demand.

Serrato et al. (2007) on the other hand explains the total return as a function of return rate and the number of products outstanding in the market. They considered the total return to follow binomial distribution.

Miao (2009) considers the total return such that the cost function is convex in nature.
• In this current research, the total return is assumed to follow a Poisson distribution. This has been confirmed by Teunter (2001), Brito & Dekker, (2002), Aras et al. (2004), Zhao et al. (2006).

Units outstanding in the market

This is one of the variables considered by Serrato et al. (2007) that is really important in defining their model. It gives the decision maker a better perception of the market and what is expected to return in the near future.

• As this current research is working with the strategy of in-house remanufacturing or outsourcing, the idea of number of products in the market gives the decision maker an extra advantage as he is aware of the sales, the number of products that have already returned and also the number that can be expected in the near future.

Remanufacturing capacity

Remanufacturing capacity is an important parameter in the field of supply chain. In the reverse supply chain study, one needs to be aware of the capacity of the remanufacturing process as it might be required to increase or decrease the capacity as well as knowing if this can be done properly.
Serrato et al. (2007) explains that the capacity of the firm in question can be increased or decreased as the problem requires and is a function of total returns from the market.

Ordoobadi (2009) suggests that the capacity can be estimated as a function of number of products returned from the market.

- In this research, the concept of remanufacturing capacity is aligned with the concept considered by both Serrato et al. (2007) and Ordoobadi (2009). The system capacity is a function of the products returning from the market.

**Length of remanufacturing**

To know for how long a firm will be continuing with the remanufacturing activities is really an important decision criterion for any firm. This gives the firm an indication on how to manage the resources available to them and also the cutoff point where they need to stop.

Nembhard (2003) correlates the length of remanufacturing to the number of cycles he wished to run the simulation model.

Serrato et al. (2007) describes the length of remanufacturing as the time when the last sale is made or the last epoch of their life cycle. This helped them understand the characteristics of the demand.
Ordoobadi (2009) aligns his length of remanufacturing with the product life cycle just as Serrato et al. (2007).

- In the current research, the length of remanufacturing has a relation with the product life cycle or the time unit when the last sale is made.

**Continue managing the returned products**

This is another parameter considered by Serrato et al. (2007). This parameter ensures that the remanufacturing operation goes on for a certain time limit where it would be profitable to run it and after that it might be a burden itself. It can also be the reason that no legal issues are faced as the firm in question takes back all the products and covers the basic ground.

**Transportation cost**

Transportation cost should be considered separately as they are an important part of supply chain system. Also not in all supply chain can the transportation cost be lumped in to total cost especially where it constitutes a large portion of the total cost.

Nembhard (2003) considers this cost as a delivery cost which is actually the transportation cost, but they considered it only during outsourcing. The transportation cost can be a part of the total cost, not only related with outsourcing.
Miao (2009) divides the transportation cost into two segments, one when the amount shipped is less than the transportation capacity and one for the amount more than the capacity of the transportation system.

- In this current research, the transportation cost is considered as an individual cost entity and considered for both the in-house remanufacturing and outsourcing which is not considered as we can see.

**Inventory holding cost**

Handling or inventory holding cost is important if one includes the study of the inventory in their research.

Nembhard (2003) considers handling cost as a part of the fixed cost, but only in terms of outsourcing. This handling cost actually arises from contracting and other related costs to make the contract.

Savaskan et al. (2004) also includes handling cost in the variable cost that comprises of collecting and handling returns.

- As it can be seen, two researchers included the handling cost, but considered them either as a part of fixed cost or variable cost. In this research, the handling or inventory cost is considered as a different cost entity and considered as a variable cost.
Inspection cost

This cost is considered in this current research and not been measured by other researchers described in the table. It is believed that the inspection entity is important in this current research as it defines which of the returned products go to remanufacturing and which go to disposal. Hence the inspection cost is included in this current research.

Unit shortage cost

Serrato et al. (2007) considers a shortage cost. Unit shortage cost is an important cost that has been involved in the current research just like Serrato et al. (2007), as it ensures that there is a motivation to undertake outsourcing. Otherwise, the firm would just pay the shortage cost rather than following an outsourcing option. It also ensures that the firm builds their own capacity rather than just paying the shortage cost.

Outsourcing cost

The current research is about deciding upon the in-house remanufacturing and outsourcing and the cost has been defined as the decision variable. Outsourcing cost is thus an integral part of the structure.
Nembhard et al. (2003) consider two costs for outsourcing: variable cost for outsourcing and fixed cost of outsourcing considering contracting, handling and other related costs coming from making the contract.

Serrato et al. (2007) on the other hand consider the outsourcing cost comprising of both the fixed cost and variable cost for producing one unit of the returned product.

Wang & Fan (2007) use the Savaskan et al. (2004) model and included this cost of outsourcing. They considers the outsourcing cost as a constant value involving negotiating, controlling and modifying the outsourcing contract as well as the miscellaneous cost.

Hui-yun & Min-li, (2007) consider the outsourcing cost as fixed cost in their model.

Miao (2009) divides the outsourcing cost into two segments and related them to the transportation capacity. One being when the amount outsourced is less than the transportation capacity and the other is for the extra amount that needed to be outsourced.

Ordoobadi (2009) considers the outsourcing cost that included information cost, subcontract cost and administrative cost and also divided them according to the number of units stored in the inventory.

- In the current research, the outsourcing cost is considered to include contracting, related costs coming from making the contract, fixed cost and variable cost. These costs are assumed to be deterministic in nature in any current scenario.
Fixed internal cost

Serrato et al. (2007) include fixed internal cost into their model but did not define what are the factors considered in this cost.

Hui-yun & Min-li, (2007) consider fixed cost for their model.

Ordoobadi (2009) considers fixed cost for remanufacturing and they included capitalized investment, overhead time and capacity for remanufacturing in their fixed cost.

- In the current research, set up cost, machine cost and overhead time are considered under the umbrella of fixed cost.

Unit variable cost

Aras et al. (2004) consider two remanufacturing costs based on the quality of the returned products. Remanufacturing costs depends on the quality of the returned products.

Savaskan et al. (2004) consider the cost of remanufacturing being less than manufacturing a new one. They considered that the cost of remanufacturing all products remain the same over time.

Serrato et al. (2007) consider the unit remanufacturing cost or the variable cost, but did not define the cost structure.
Ordoobadi (2009) considers variable cost and included labour, inventory carrying, supplies, energy and other costs.

- In the current research, the variable cost has been considered that included labour cost, overhead cost, supplies, energy and other related costs.

Unit investment cost

Serrato et al. (2007) consider the investment cost as unit cost required to increase the capacity by one unit.

- In the current research, the same concept has been used for investment cost. In the real life, it is also seen that there may occur some instances when increasing the capacity is required, and as it is also known, increasing the capacity means more machine, more worker, more overhead cost, more power. All these has been considered in the unit investment cost.

Unit salvage value

Aras et al. (2004) define salvage value as the negative of disposal cost.

Serrato et al. (2007) also consider salvage value in their model.
• This cost accounts for when the process is outsourced and the capacity is left unutilized; hence depreciation takes place and also if the decision is taken that the system capacity will be reduced to zero and consequently revenue will transpire for the OEM. Thus this value can be both cost and revenue depending on the decision taken.

**Number of products disposed based on quality**

Aras et al. (2004) define a quality level based on which they decided which products go to disposal section rather than remanufacturing.

• In this current research, the same concept has been considered. The products going through to disposal failed to cross the quality level that has been set for them.

**Products accepted for reuse**

Ordoobadi (2009) accepts products for reuse which are not sent to the scrap section as they are defective.

• The current research also considers the accepted products for reuse. It is nothing but the products that do pass the quality test, failing which it would have been sent to the disposal section.
Disposal cost

No other researcher considered in the table worked with disposal cost, but in the context of current research, it is felt that inclusion of disposal cost gives the model a better depiction of the real world scenario.

The formulation of the simulation problem described by Nembhard et al. (2003) can be stated as

\[ ds = \mu S dt + \sigma S dz \]

Where \( z \) is a Wiener process, \( \mu \) is the expected return in a risk-neutral world and \( \sigma \) is the volatility. S is the cost variable for unit production, unit outsourcing and unit delivery.

The formulation of the Markov problem depicted by Aras e al. (2004) can be stated as

\[ h_s = h_s^1 \left( \frac{R_1}{R_1 + R_2} \right) + h_s^2 \left( \frac{R_2}{R_1 + R_2} \right) \]

Where \( h \) is the serviceable inventory holding cost, \( h_s \) is average serviceable holding cost and \( R \) is the number of returned products remanufactured.

The final cost function is represented as

\[ C_i(s, Q_1, Q_2) = h\overline{I}_1 + h\overline{I}_2 + h_s \overline{I}_s + h_1^w \overline{W}_1 + h_2^w \overline{W}_2 + c_1 \overline{R}_1 + c_2 \overline{R}_2 + \delta_1 \overline{D}_1 + \delta_2 \overline{D}_2 + m\overline{M} \]
Where I is the remanufacturable inventory on-hand, I_s is the serviceable inventory on hand, W is the WIP inventory, D is the number of returned products disposed, Q is the disposable level, M is the number of products manufactured and B is number of outstanding remanufacturing orders at time t.

The Markov chain has five-dimensional state variable

\[ X(t) = (I_1(t), I_2(t), W_1(t), W_2(t), B(t): t \geq 0) \]

The formulation of the problem for centrally coordinated system considered by Savaskan et al. (2004) can be stated as

\[ \text{Max} \Pi^C = (\phi - \beta p)[p - c_m + \tau \Delta] - c_L \tau^2 - A \tau (\phi - \beta p) \]

The formulation of the problem for manufacture collecting system can be stated as

\[ \text{Max} \Pi^M = \left(\frac{\phi - \beta w}{2}\right)[w - c_m + \tau \Delta] - c_L \tau^2 - A \tau \left(\frac{\phi - \beta w}{2}\right) \]

The formulation of the problem for retailer collecting system can be stated as

\[ \text{Max} \Pi^R = \left(\frac{\phi + \beta c_M}{2}\right) - \frac{(\Delta - b)(b - A)(\phi - \beta c_M)}{2[4c_L - \beta(\Delta - A)(b - A)]} \]

The formulation of the problem for third party collecting system can be stated as

\[ \text{Max} \Pi^{3P} = (\phi - \beta p^{3P}) - [w - c_M + (\Delta - b)\tau^{3P}] \]
Where \( c \) is the cost of manufacturing and remanufacturing, \( p \) is the retail price, \( w \) is the unit wholesale price; \( b \) is the unit transfer price of a returned product from retailer/3rd party to manufacturer, \( \tau \) is the return rate, \( \Delta \) is the unit cost savings from reuse, \( A \) is the fixed payment given to consumer who returns a used product, \( \phi \) and \( \beta \) are positive integers.

With the Markov decision process one needs to find the reward function and compare it with the other option of reward function and decide on the best policy. The reward function for the original equipment manufacturer described by Serrato et al. (2007) is:

\[
R_{t+1}(k_t, w_t, 0) = -c_1[n_tr - k_t]^+ - c_2[k_t - n_tr]^+ - c_3[n_tr] \\
- c_4 \sum_{j=0}^{n_t} (\min(j, n_tr) \triangleright p_{t+1} ((n_tr, w_{t+1} = w_t + j) | (k_t, w_t), 0)) \\
- c_5 \sum_{j=0}^{n_t} (\max(j - n_tr, 0) \triangleright p_{t+1} ((n_tr, w_{t+1} = w_t + j) | (k_t, w_t), 0))
\]

The reward function for the outsourcing option is:

\[
R_{t+1}(k_t, w_t, 1) = c_6(k_t) - c_7 \left( n_t + \sum_{t=1}^{T} S_t \right)
\]
Where \( c_1 \) to \( c_7 \) are the cost of investment cost, disinvestment cost, fixed internal cost, internal labor cost, shortage cost, salvage value and outsourcing cost respectively. \( k \) is the capacity of the firm, \( n \) is the units outstanding in the market, \( r \) is the return rate, \( p \) is the transition probability and \( w \) is the total units that have already returned.

The following equations is formulated by Hui-yun & Min-li, (2007) to get the best result out of the simulation:

\[
p(t + \Delta t) = p(t) \exp \left[ \left( \alpha_p - \frac{\sigma_p^2}{2} \right) \Delta t + \sigma_p \varepsilon_1 \sqrt{\Delta t} \right]
\]

\[
D(t + \Delta t) = D(t) \exp \left[ \left( \alpha_D - \frac{\sigma_D^2}{2} \right) \Delta t + \sigma_D \varepsilon_2 \sqrt{\Delta t} \right]
\]

\[
I_1(t + \Delta t) = I_1(t) \exp \left[ \left( \alpha_{I_1} - \frac{\sigma_{I_1}^2}{2} \right) \Delta t + \sigma_{I_1} \varepsilon_3 \sqrt{\Delta t} \right]
\]

\[
I_2(t + \Delta t) = I_2(t) \exp \left[ \left( \alpha_{I_2} - \frac{\sigma_{I_2}^2}{2} \right) \Delta t + \sigma_{I_2} \varepsilon_3 \sqrt{\Delta t} \right]
\]

Where \( p, D, I_1, I_2 \) are product price, market demand, cost of manufacturing in-house and cost of outsourcing respectively.

The optimization equation preferred by Miao (2009) is as follows:
\[ V_t(x) = \min_{0 \leq y_1 \leq x} \min_{0 \leq y_2 \leq y_1} \{ r(y_1 - y_2) + b(x - y_1) + b'y_2 + G(y_2) \]
\[ + E[V_{t+1}(y_1 - y_2 + D)] \]
Table 2.2: Parameters considered by different researchers relating outsourcing and reverse logistics

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Papers</th>
<th>Current research</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Game theory</td>
</tr>
<tr>
<td>Product demand</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Return rate</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Total return</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Units outstanding in the market</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Remanufacturing capacity</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Length of manufacturing/ remanufacturing</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Continue managing the returns for the product analyzed</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Inventory cost</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Inspection cost</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Unit shortage cost</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Outsourcing cost</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Fixed internal cost</td>
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<td>X</td>
</tr>
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<td>Unit variable cost</td>
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<td>Unit investment cost</td>
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<td>Unit idle cost</td>
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<td>Unit salvage value</td>
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<tr>
<td>Number of returned products disposed based on quality</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Products accepted for reuse</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

34
After going through a lot of researches that have been conducted in the field of reverse logistics and outsourcing, it is clear that the researchers have focused their research on some specific area, but there is a lot of grey area that can still be covered. This research work tries to focus on the most common areas covered by the researchers and also include some new ideas that would shed some light on the grey areas mentioned earlier.

Dynamic and stationary capacity is not a new concept and they both have been introduced in this research. Both concepts have some positive and negative effects. The examples discussed later in the thesis demonstrate which method is helpful in terms of remanufacturing and outsourcing. The dynamic capacity helps us understand at which point the capacity should be fixed and not be increased. The stationary capacity helps check against different fixed capacity and decides at what capacity level the model works best.

Some cost parameters are introduced in the model that represents a more realistic system. Idle cost of unused capacity is introduced in the model. Idle cost is an important parameter as there might be some cases when the system remains unutilized either completely when the whole lot is either outsourced or disposed or partially when the returns are less than capacity and the firm decides to do the remanufacturing in-house. In both of the cases, there is some capacity that remains unused and this cost portrays these real life situations.

Disposal cost is also introduced in the model as it is an integral part of the reverse supply chain and depicts the real life picture better if considered in any model.
The concept of transportation cost and inventory cost are two of the important cost parameters that are also included in the model. The transportation cost is important as it represents a large portion of the total cost. The inventory cost is also important as it tells us about the warehouse cost related to it.

Non stationary return rates for different period of time have been considered in this model, which is considered stationary by many of the earlier researchers. The return rate can remain stationary for a very short period of time or for a very exclusive product. But when one is considering a general model, the return rate should be dynamic in nature and this has been considered in this research.

In this section, some theoretical background study has been conducted on reverse logistics and outsourcing. Some related papers have been studied thoroughly against the parameters and concepts of this current research which is portrayed through the Table 2.2 for better understanding. In the end, in a nutshell it has been discussed what will be done later in the coming sections.
CHAPTER 3

PROBLEM DEFINITION

3.1 Introduction

In order to determine how and where the outsourcing option can be considered that would fetch the maximum benefit in reverse logistics, one must have a clear idea about the reverse supply chain itself.

3.2 Relating RL and outsourcing

Lebreton (2007) depicted a very lucid model comprising of both the forward and the reverse supply chain and pointed out the model evidently. The model is presented below (see Figure 3.1) with some explanation of the processes.

According to Lebreton (2007), the course for any product starts first from extraction of the raw material from the source. The process then moves forward through parts manufacturing and final assembly. The final stage of this chain is the customers who are served through sales. The reverse logistics starts from the point when customers commence returning the products. Various reasons for product returns have been discussed in detail earlier and hence it would not be discussed further.

As the products return, they undergo five generic activities to make sure that the returned products are properly handled for the value addition activities. The first step of this value adding process is acquisition. Acquisition consists of returning the products from market
to the point of recovery. Collection and procurement are the two core sub activities in this process. Once the products have returned, the selection process takes place in which the valuable products are identified, categorized and guided to one of the three recovery processes: disassembly, cannibalization and mechanical processing. Direct reuse or repair happens in this selection step. Refurbishing, use of spare parts and remanufacturing takes place in disassembly, cannibalization and mechanical processing respectively. At last, when all the other alternatives for value addition to the returned products are completed, disposal of the products are considered.

Figure 3.1: Overview of recovery process, adapted and modified from Lebreton, 2007
The question now is if any of these processes can be outsourced which may increase profit for the OEMs. In response to this subject, one needs to know the reasons behind outsourcing. Several reasons for outsourcing activities to a 3rd party in reverse supply chain are cited by different researchers and after careful analysis and eliminating duplications; Ordoobadi et al. (2009) classified three main reasons:

- Strategic reasons,
- Operational reasons and
- Financial reasons.

The authors emphasized a lot on these three different factors to disclose the complexity of reverse supply chain and outsourcing that appeared in normal business conditions. It is implicit that, to be acquainted with the complex situation arising from the outsourcing opportunity, one needs to comprehend these three reasons behind any outsourcing decision.

The key consideration of outsourcing decision is whether enough products are returning through the reverse supply chain system that establishing a reverse logistics system is profitable. For this, an elimination criterion should first be set up. A logical process flow chart is thus considered and it is shown below:
When the products are returned, the numbers are considered and a logical decision is reached whether the number of returned products ensures that creating a reverse logistics network is worth the cost. If the returned products are low in number, the RL is not considered and the project is terminated. If, however, the returned products are sufficiently high in number, the creation of a RL chain is considered. This criteria is set by the OEMs as this number may vary from company to company. In this research, however, it is assumed that either under legislative law or enough products do return that to continue with reverse supply chain is necessary and profitable.

Activities in reverse logistics can be considered as either operational or strategic. Operational activity is considered as warehousing and strategic is measured as value adding activities such as remanufacturing. Any one of these can be considered for outsourcing strategy. Operational activities are outsourced as long as the outsourcing
option is financially justified. In this research, only the remanufacturing option will be considered for outsourcing. The warehousing decision is not considered for outsourcing as it widens the scope out of this research extent.

Remanufacturing option is a decision making point; whether remanufacturing should be done in-house or outsourced needs to be decided. Now it is important to decide exactly at what stage in the reverse supply chain one can consider this option. A process chart is depicted below to give a clear idea about the flow of products.
According to the flowchart, the product outsourcing option can be considered just after the product acquisition and they are stored in the warehouse. One of the options of outsourcing can start from this point. At the warehouse the threshold is checked and based on that the decision is taken to either in-house remanufacturing or outsourcing. Threshold is defined as the benchmark cost when the outsourcing decision is taken and

*Figure 3.3: Outsourcing strategy*
this benchmark is set by the cost of remanufacturing in-house in each of the time epochs. Now a mathematical model is developed that ensures that the outsourcing decision is not taken just out of some qualitative measurements.

For this problem, the following approach can be undertaken.

- Economic analysis

This economic analysis is conducted to compare the cost of performing the remanufacturing process in-house against the cost of contracting it out to a third party recovery firm. The firm has the option of taking the outsourcing decision based on the economic analysis before taking the final decision. The purpose of this mathematical model is to provide the firm with suitable decision making tool.

3.3 Objectives

After identifying the problem, four objectives have been set up to answer the question posed earlier

- Categorize reverse logistics qualitatively according to two critical factors.
- Formulate a mathematical model and establish conditions for which the outsourcing decision will be optimal.
- Do an economic analysis that establishes conditions for which outsourcing will be optimal.
- Measure the efficiency of the model using regression analysis.
3.4 Hypothesis

After considering the characteristics for the reverse logistics network and also the elements to be considered in this research, the hypotheses (Serrato et al., 2007) to be verified in this research are as follows:

- Outsourcing is more likely to be an optimal decision when the variance in return rate is high.
- Outsourcing is more likely to be an optimal decision when the life cycle of the product in consideration is short in length.

3.5 Model formulation

After the problem definition, the next step is to decide on the methodology that can be used to study the outlined problem. In this research, the dilemma is identified as the outsourcing decision strategy which is actually a “take it or leave it” tactic because no firm is interested in changing their policy on a regular basis.

There are three types of methodologies from which one can choose from: exact, approximate and simulation methodology. From Table 2.2, it is comprehensible that most of the researchers favored the exact methodology over simulation and approximate methodology. Game theory and Markov decision model are two of the methodologies that most researchers are inclined to exercise in their research work. It would be wise to
decide on the actuality that exact methodology will be more acceptable to both researchers and in real life as it gives the exact answer of the stated problem.

In this research, the exact methodology will be used and a Markov decision model is considered to study the problem. The selection is justified through the course of this section. Later the model is also validated using the simulation technique.

In this section, the investigation of outsourcing will be conducted from two different points of view: quantitative and qualitative.

In the qualitative analysis, the reverse logistics network will be described and a brief emphasizing of its general characteristics will be shared. It is implicit that the two most important elements in characterizing outsourcing in reverse logistics network are Product Life Cycle (PLC) and variance in return rate. These two elements and how they affect the outsourcing decision are described in section 4.2 and 4.3.

Any problem should comply with some common characteristics before they can be modeled using a Markov decision process framework. The problem should be stochastic in nature, should have Markovian property and be dynamic in nature. After these conditions are fulfilled, a Markov decision process can be considered.

A stochastic process is defined to be simply an indexed set of of random variables \( \{X_t\} \), where the index \( t \) runs through a given set \( T \). \( T \) is taken to be the set of nonnegative integers and \( X_t \) represents a measurable characteristic of interest at time \( t \). In a stochastic process, there are some uncertainties in the evolution of the future portrayed by
probability distribution. The return rate considered in this research is known only after the current time has passed and the expected return rate is considered with a probability distribution. This criterion matches the characteristics of a stochastic process which states that the states are not known with certainty and can be expressed by a random variable. Return rate is believed to follow Poisson distribution which is also stochastic in nature (Fleischmann et al., 1997, Teunter 2001, Brito & Dekker, 2002, Zhao et al., 2006).

Uncertainties about the future lies at the heart of many decision problems but to say that the future is uncertain does not mean that nothing is known about it. There are some probabilities associated with each work and when these probabilities can be assessed, rational decision making becomes possible.

For a problem to be considered as a Markov chain there should be countable state space and the future system states should be independent of the past states. In the current research, the problem is thus formulated that they have a finite number of states comprising of RL system capacity and cumulative amount of returns. They also have the memoryless property which is indicated by the fact that to jump to the next state, one needs to know only the current state and the transition probability and is not dependent on anything else. Any model that is a blend of both Markov chains and decision making is called a Markov decision model. A Markov decision process is described by 4 characteristics:

- System state
- Decision set
• Transition probability
• Reward function

In this research, two scenarios will be considered, one with dynamic capacity and one with stationary capacity. The 4 characteristics stated above will differentiate from one another in these two scenarios and they are stated briefly in the following section.

System state:

The system state at decision epoch (beginning of a period) \( t \) for scenario 1 is defined by

\[(k_t, w_t) \text{for } t = 1, 2, \ldots, T\]

Where \( k_t \) is the reverse logistics capacity during period \( t \) and \( w_t \) is the cumulative number of units that have returned through the reverse logistics channel up to the end of period \( t \).

The system states are partially ordered according to \( w_t \). A partially ordered set consists of a set together with a binary relation which indicates that, for certain pair of elements in the set, one of the elements precedes the other. Such a relation is called a partial order to reflect the fact that not every pair of elements need be related: for some pairs, it may be that neither element precedes the other in the partially ordered set (Puterman, 1994).

The system state at decision epoch (beginning of a period) \( t \) for scenario 2 is defined by

\[(w_t) \text{for } t = 1, 2, \ldots, T\]
Where \( w_t \) is the number of units that have returned through the reverse logistics channel at the end of period \( t \).

**Decision set:**

For scenario 1, the purpose is to determine the alternative between outsourcing and in-house remanufacturing and hence it is assumed that at the end of any period \( t \), either of the two actions (outsource or in-house remanufacturing) will be taken.

- \( a = 0 \) Continuing with the reverse logistics activity internally by updating the firm’s capacity to the amount expected to return.

- \( a = 1 \) Follow an outsourcing strategy for the next period by engaging a third party reverse logistics provider by keeping the current capacity idle.

It is however assumed that at any time an outsourcing decision is taken; it will stay until the end of time epoch.

For scenario 2, the purpose is to determine whether the returned amounts should be split between in-house remanufacturing and outsourcing when the amount returned is more than the capacity and when the returned amount is less than capacity, to determine between in-house remanufacturing and outsourcing. It is assumed that at the end of any period \( t \), any one of the three actions will be taken.

- \( b = 0 \) Continuing with the reverse logistics in-house while the number of returns is less than the capacity
b = 1 Follow an outsourcing strategy for the next period by engaging a third party reverse logistics provider if the number of returned products less than the current capacity.

b = 2 Split the returned amounts into two sections. In-house remanufacturing up to the limit of capacity and outsource the rest.

Transition probability:

At a specified point in time, a decision maker observes the state of a system. Based on this state, the decision maker chooses an action. The action choice produces two results; the decision maker receives an immediate reward and the system evolves to a new state at a subsequent point in time according to a probability distribution determined by the action choice (transition probability). At this subsequent point in time, the decision maker faces a similar problem but now the system is in a different state and there may be different set of actions to choose from (Puterman, 1994).

Figure 3.4: Symbolic representation of a sequential decision problem (Puterman, 1994)
It is assumed that the returns follow a Poisson distribution, and given that the sales function is also known, the transition probability for both scenario 1 and scenario 2 between each state is as follows:

\[
F_{t+1}( (k_t + y/k_t, w_t + y) | (k_t, w_t), a/b ) = \begin{cases} 
\sum_{y=0}^{j} \frac{e^{-n_tr} (n_tr)^y}{y!} & \text{for } y = 0, 1, \ldots \\
0 & \text{otherwise}
\end{cases}
\]

where \( n_tr \) is the mean for Poisson distribution. The value of \( j \) is considered as the break-even point that the cost for in-house remanufacturing and outsourcing are equal. Any value over or below this point will influence the decision towards any of the two choices.

The concept of \( j \) can be better explained with the help of Figure 5.1. The capacity has been considered 20 for the given example. In scenario 1, the capacity will be updated at each time epoch according to the taken decision (either to outsource or remanufacture in-house) and the value of \( j \) will also change accordingly.

**Reward function:**

To define the cost function, cost of different entities in the reverse supply chain needs to be identified and thus cost for the different alternatives are calculated. In this thesis, the reward is actually cost, hence, it will be termed as cost function from now on.

The earlier studies suggests that Markov decision model would be appropriate for the problem identified in this research goal and it is equipped to deal with these problems. Also Aras et al. (2004), Serrato et al. (2007) and Miao (2009) used Markov Decision
Model to illustrate the optimal policy for outsourcing. Serrato et al. (2007) identified a monotone deterministic optimal policy in light of their proposed MDM. It would be safe into considering the use of such policy that would give the opportunity to have the desired strategy of either to outsource or not during the horizon in analysis.

![Diagram of RL chain and costs considered in MDM](image)

*Figure 3.5: Relation between RL chain and costs considered in MDM*

The efficiency of the model will be checked with the help of regression analysis.
In this chapter, the area where the outsourcing option can be considered is depicted properly with the help of a flow diagram of the return process. The problem is stated as a Markov Decision Process. In the next chapter the problem is depicted qualitatively followed by a formulation of a mathematical model to solve this problem.
CHAPTER 4

METHODOLOGY

4.1 Methodology

The qualitative and the quantitative analyses are conducted in this chapter. The two factors important in describing the RL network; product life cycle and variability in return rate are discussed in section 4.2 and 4.3. The Markov Decision Model is depicted in section 4.4.

4.2 Product life cycle

Product Life Cycle (PLC) breaks down the cycle of the product and determines the necessary steps that need to be addressed once an item is returned. Product life cycle is essentially used to determine the actual “life” of a product (Smith, 2005).

There are two definitions about product life cycle. The first definition refers to the progress of a product from raw material through production and use to its final disposal.

The second definition, however, describes the evolution of a product measured by its sales over time. Smith (2005) described that every product passes through a series of phases in the course of life referred to as the product life cycle. These phases are development, introduction, growth, maturity, decline and cancellation.

The life cycle of a product is classified into 3 basic forms: product model, product form and product class. The product model indicates that the changes from one model to the
next are minor and that there may be small or few challenges along the way due to the shift in sales volume from switching from one model to another. The product form changes from one form to another causing major product changes or changes that could imply a partially new product. As for the product class, this would be all forms of a particular product, from the time when the product is initially in the market to when the last item is sold, which may lead to an eventual end of the product.

The characteristic of the life cycle provides a theoretical foundation regarding the possibilities of acquiring used products suitable for remanufacturing. Different companies in different industries will apply altered relations with the suppliers of the cores to get a sufficient number of cores for their remanufacturing operations (Smith, 2005). (Cores are used or broken down products or components). Factors such as the mean product lifetime, rate of technological innovation, and failure rate of components all influence the return rate of products from end-of-use and end-of-life. End-of-use returns (lease returns) refer to those situations where the user has a return opportunity at a certain stage of the life cycle of the product. Although end-of-use products are not new, they are often in a good or reasonable state. Johan (2009) referred to end-of-life returns to those returns where the products are at the end of their economic or physical lives.

As mentioned earlier, the product life cycle has six stages that the product passes through.

4.2.1 Development stage: During the development stage, it is considered to be an excellent time to begin thinking about the design for reverse logistics implications for a product. When a new model of an existing product is being developed, the new model is
considered to be a new product with minor changes to the old version; therefore few challenges should be expected (Smith, 2005). If the product is new then there should be some calculation done on the demand and expected forecast. But all in all, the state of reverse supply chain work is minimal.

![Figure 4.1: Product life cycle](image)

**Figure 4.1: Product life cycle**

**4.2.2 Introduction stage:** The introduction stage of a product class may have more defects than in any other stage due to the introduction of a completely new class of product. Product form introduces the familiarity of the product to consumers. If consumers are familiar with the product class from having previously purchased other forms of the product, they may be able to understand the new product easier than the completely new version. The firm starts to decide on the supply chain structure and especially reverse supply chain as products start to come back. This is normally the case with new products. When the products are similar to the product they replace, the sales
starts to pick up from the beginning as the return grows. In this case the products might skip the introduction phase and go straight to growth phase. In this stage the return rate might pick up at the start because of the different initial problems faced by the customers but it quickly stabilizes and stays low for the rest of the period.

4.2.3 Growth stage: The growth phase is the phase when CRCs (Central Recovery Centers) or individual recovery systems gain experience in determining what is wrong with each item and in learning the best process suited for the specific returned items. This phase will also help determine which brokers are best suited to handle the product in different conditions because with the production function scaling production volume, different types of defects will arise as the process is refined; therefore, slightly increasing customer returns from the introduction stage. Growth of the product class reports that the return volume will increase as the customer base grows beyond the early adopters and that many customers will not be as willing to put up with problems with the product and are more likely to return the products. The growth of the product model indicates that as sales increase, the volume of returns will also increase, but the rate of return may not change. This is the phase when remanufacturing volumes emerge and increase over time. Here, the core returns from end-of-use are limited. In this phase, the possibilities for generating good profit margins are high, mainly due to the high demand for remanufactured products with respect to the lower supply of products suitable for remanufacturing. As the end-of-use and end-of-life disposal rates are limited to failure rates and average usage periods, the possibility to manage the returns is low. In this phase, the greatest potential for acquiring cores is from other sources such as commercial
returns and other Secondary Channel Goods, as for example warranty claims and transportation damages.

**4.2.4 Maturity stage:** The maturity stage of a product initiates its main focus; cost reduction. Increasing revenues or taking advantage of every possible opportunity for reducing costs will keep retail prices low and because competition with other products is based on price, developing a technique to process returns quickly will be one of the ways to reduce costs (Smith, 2005). In this stage, the return rates from end-of-use increase to a point where the return rate exceeds the demand rate. There is a breakpoint between supply and demand. This breakpoint also has a significant impact on the competitive advantage for remanufacturing companies. Before the breakpoint, competitive advantage is based on, e.g., identifying potential products and the ability to acquire cores. After the breakpoint, this becomes less important, while on the other hand efficiency in the remanufacturing process increases in importance. As the supply of end-of-use and end-of-life products increases, an important issue is to limit and acquire only the cores that are most suitable for remanufacturing. Another characteristic in the latter stages of this phase, as well as in the decline phase, is that the quality of the cores can become lower; this in turn can cause a demand for new types of reprocessing operations (Johan, 2009).

**4.2.5 Decline stage:** Eventually the Product Life Cycle will reach the decline phase where keeping costs as low as possible is still pertinent. The decline phase of a product interprets the falling of the overall market, in which retailers may be less interested in selling the product that will cause consumers to be less interested in the product; leading to decreasing sales (Smith, 2005). The main danger in this phase is having excessively
high inventory levels of cores and remanufactured products when demand for remanufactured products decreases. The return rate starts to increase as people are more interested into newer products and replace the old ones.

**4.2.6 Cancellation stage:** Smith (2005) defines that the end of the Product Life Cycle is the cancellation phase, but the volume of product returns will continue to decrease before stopping overall. Challenges become more profound due to the terminated product(s). If a company cannot sell the product, it also may not even be able to give it away, regardless of promotions.

As indicated earlier, there should be different policies for different stages of life cycle of a product. There are normally two types of return. One for the end-of-life return and another one is end-of-use return (Hanafi 2004). According to Morana & Seuring, (2007) though, there are four types of product return, namely: end-of-life, end-of-use, commercial return and reusable items. For example, in the development stage there need not be any decision taken because no product is ready to return yet. In the introduction and growth stage, the firms need to prepare the customer for the new product. The products that do return in the growth stage and maturity stage tend to go through refurbishing before they are resold to the market. The return is high again in decline and cancellation phase and the products go through remanufacturing when returned in the decline and cancellation stage. In the cancellation stage, the extra steps that are taken can be landfill or cannibalization.
There are of course few factors that contribute to product return according to Hanafi (2004). They are social, technological and historical sales factor. These factors influence when and how many products do return. According to technological factors, the products returned can be divided into three categories according to their life cycle length.

Short product life cycle: Products with life of 1 to 3 years are considered having short product life cycles. Typical examples would be mobile phones, printer cartridges and other small appliances.

Medium product life cycle: Products with life of 3 to 8 years are thought of having medium life cycle. Typical example would be microwave oven, oven toaster, television, steam iron, tire, and many other products.

Long product life cycle: Products having life cycle of 8 to 15 years fall in this category. Refrigerator, washing machine, dishwasher, dryer are considered having long life cycle.

As discussed earlier, quality of products returned are also factors included in this discussion. Aras, et al., (2004) defined two types of product quality that are returned: high quality products and low quality products. Categorizing returned products and implementing the appropriate remanufacturing and disposal policy can lead to considerable cost savings. Giving priority to quality returns in remanufacturing is a better strategy under a wide range of cost and process parameters and as a consequence, the quality difference between high quality and low quality return increases and the quality of both return types decreases when the quality difference remains constant.
Although managers may not know when (or if) products will be coming back, they must be prepared to quickly process and handle the products on demand. Thus, prompt and accurate exchange and access to information should be considered a top priority.

As we see from this literature, there is a direct relationship between product life cycle and product returns. Different stages in the life cycle dictate the amount of product return and give managers an indication about future returns. The main problem regarding product life cycle is the concept and understanding exactly where the specific product stands in respect to life stages.

As it is explained in this section, the product life cycle determines the expected amount of returns for a particular product over time. The characteristics of return depend on the length of the life cycle as well as the characteristics of that product.

Also there is another type of return that was spoken about earlier which should be accounted for and that is end-of-use return. Warranty return suggests that return happens due to the fact that sometimes products do not meet consumer demand, wrong products are delivered, relaxed return policies. The cause and timing of the return is important as these two types of return (end-of-life and end-of-use or warranty return) portray different timing and also different state of returns.

The following table shows the different return drivers and also the timing of the return.
Table 4.1: Drivers and timing of product return

<table>
<thead>
<tr>
<th>Type of return</th>
<th>Drivers of return</th>
<th>Timing of return</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-of-life</td>
<td>Legislation.</td>
<td>Increasing trend starting from the end of maturity stage.</td>
</tr>
<tr>
<td>return</td>
<td>Short product life cycles.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Different types of defects arise as the product goes through refinement.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Introduction of new products.</td>
<td></td>
</tr>
<tr>
<td>End-of-use</td>
<td>Consumer does not know how to install or use the product.</td>
<td>Increasing with time.</td>
</tr>
<tr>
<td>return</td>
<td>Products do not meet consumer expectation.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wrong product delivered.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Relaxed return policies.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Usage.</td>
<td></td>
</tr>
</tbody>
</table>

4.3 Variance in return rate

The difference between forward and reverse supply chain is in uncertainty. In both cases there is supply of goods from the source. In reverse supply chain, however, the supply of raw material (products returned by customer) has a higher uncertainty than forward supply chain. One need to consider the amount of variation for different products and particularly return volume must be considered during the complete life cycle.
Return volume might be equal for different products in different stages of life cycle, but what really affects the reverse logistics process is the variability in the return volume. Higher variability complicates the management process. The return volume is important when one wants to work with reverse logistics network; the variance on the other hand dictates the strategic decision about outsourcing.

Let us first think about establishing a reverse logistics network. The return volume must be high enough so that the establishment of reverse logistics network is cost efficient. Now, if there is a high variation in the return volume, it may not be economically practical for a firm to develop the reverse logistics network to handle the return in-house. With the return varying heavily (more than some predetermined level), it will be too costly to continuously change its capacity in-house. This problem may be effectively handled if the help of a 3PRLP is taken, who actually specializes in these kinds of activities and can take advantage of the economies of scale to convert reverse logistics functions in a profit-creating activity into the closed-loop chain.

On the other hand, if the variance in return volume is relatively low, the firm may not need the help of any third party as it may be able to implement its own reverse logistics network.
4.4 Markov Decision Model (MDM)

4.4.1 Model development

Markov Decision Model proposed in this research examines the uncertainty of return in which outsourcing can be used as an option to remanufacture some of the returns. It is important to recognize the return volume and as it is known, return volume is nothing but a consequence of the amount of units historically sold by the firm, given that a fraction of them is returned through the RL system. Sales function is hypothesized which is related to the scenario under analysis. Sales function is a function of maximum expected sales and Product Life Cycle and can be characterized according to the historical data related to the scenario. The variance in the returns for each period during the entire planning horizon can now be considered. The following notations are defined for the proposed MDM:

\[ t = \text{Decision epoch, } t \in \{1,2,\ldots,T\} \]

\[ L = \text{Length of the Product Life Cycle depending on the particular RL scenario considered.} \]

\[ W = \text{Time length defined by the firm in continuously managing the returned products after the last sale is made. The choice for value of } W \text{ is subject to change and considered according to the need of the OEM.} \]

\[ T = \text{Length of the horizon under analysis, } T = L + W. \]
$M =$ Maximum expected sales level.

$r =$ Return rate. This is the fraction of sold units expected to be returned in the next period. $0 \leq r \leq 1$

$k_t =$ Reverse logistics capacity defined by the firm at the beginning of period $t$.

$x_t =$ Amount of units returned in period $t$, which is characterized by Poisson distribution.

$s_t =$ Amount of units sold by the firm during period $t$.

$S_t =$ Cumulative sales experienced by the firm from period 1 to the end of period $t$.

$w_t =$ Cumulative amount of units returned from period 1 to the end of period $t$.

$n_t =$ Amount of units outstanding in the market at the end of period $t$.

$c_i =$ Costs in reverse logistics system.

$y_t =$ Possible number of returns from the market at each time epoch = \{0\}, \{1\}, \ldots, \{n_t\}; x_t \in y_t$

$y$ actually defines the number of products that can come back from the market which has a maximum value of $n_t$ and a minimum value of 0. We use the values of $y$ to find out the total expected cost of outsourcing and in-house remanufacturing; based on which a decision is taken.
**Assumption 1**

A particular sales function is assumed with a maximum expected sales level, $M$, which is employed to establish the value of $s_t$ in each period. It is also given that $s_t$ determines the value of $S_t$

$$S_t = \sum_{t=1}^{t} s_t$$

With the availability of a sales function and maximum sales level expected, it is easier to estimate the values of $s_t$ and $S_t$, and these elements can be defined for a scenario in question and be estimated according to historical data.

The sale function comprises of maximum expected sales level and Product Life Cycle and follows the equation (Serrato et al., 2007):

$$s_t = \begin{cases} \frac{2M}{L} t, & t = 1, 2, \ldots, \frac{L}{2} \\ M - \frac{2M}{L}(t - \frac{L}{2} - 1), & t = \frac{L}{2} + 1, \ldots, L \end{cases}$$

**Assumption 2**

It is assumed that the number of returns in period $t + 1$ follows Poisson distribution with parameter $(n_t r)$ such that expected amount of returns in the next period is obtained by
\[ E(x_{t+1}) = n_t r \]  

(3)

Where \( n_t \) is the number of products outstanding at the end of period \( t \).

\[ n_t = S_t - w_t \]  

(4)

**Assumption 3**

Products that are declared to be of acceptable quality for remanufacturing are either outsourced or remanufactured in-house.

**4.4.2 Scenario 1 (Dynamic capacity)**

In scenario 1, all the returned products will be either completely outsourced or completely remanufactured in-house. Capacity is not considered fixed in this scenario; rather it can be increased to accommodate the number of returned products if necessary. So, the capacity is thought to be dynamic in nature for this scenario. The assumption in this case is that the capacity, although can be increased to accommodate the returned products, cannot be decreased as it would be unrealistic in technical, organizational and administrative perspective and non profitable in most cases. This implies that the capacity can be considered given in terms of units for the period in consideration. This assumption holds true for the scenario.
4.4.2.1 Model definitions

4.4.2.1.1 Decision epoch

Decision epoch \( t \) represents the end of period \( t \) and \( T \) represents the end of the problem horizon.

\[ t \in \{1, 2, \ldots, T\} \]  

(5)

It is also known,

\[ T = L + W \]  

(6)

Where, \( L \) is the length of the Product Life Cycle and \( W \) is the time length for which the firm continues managing the returned products after the last sale is made. This length ensures that a warranty or accomplishment of the legal requirements for managing returned products after period \( L \).

4.4.2.1.2 System state

The system state at the end of period \( t \) is defined by

\[ (k_t, w_t) \text{ for } t = 1, 2, \ldots, T \]  

(7)

Where \( k_t \) is the reverse logistics capacity during period \( t \) and \( w_t \) is the cumulative number of units that have returned through the RL system at the end of period \( t \).
\[ w_t = \sum_{i=1}^{t} x_i \]  \hspace{1cm} (8)

where \( x_i \) is the number of returns at time \( i \).

In reverse logistics, the option of outsourcing exists as a form of “Take it or leave it”. No firm will be interested in changing back and forth between capacity increasing and decreasing.

### 4.4.2.1.3 Decision

The following decision set is considered for scenario 1. The purpose of this model is to determine the decision of outsourcing and hence it is assumed that at the end of any period \( t \), either of the two following decisions can be taken.

- \( a = 0 \): Continuing with the reverse logistics activities internally by updating the firm’s in-house remanufacturing capacity to the amount expected to return in the next period:

\[ k_{t+1} = E(x_{t+1}) = n_t r \]  \hspace{1cm} (9)

- \( a = 1 \): Follow an outsourcing strategy for the next period by engaging a third party reverse logistics provider by keeping the current capacity idle.

It is however assumed that any time an outsourcing decision is taken; it will stay until the end of the time epoch.
Figure 4.2: Decision criteria for scenario 1

4.4.2.1.4 Transition probabilities

It is assumed earlier that the number of returned products follows a Poisson distribution, and given that the sales function is also known, the transition probability between each state is as follows:
For $a = 0$

\[ F_{t+1}(k_t + y, w_t + y) | (k_t, w_t), 0) = \begin{cases} \sum_{y=j}^{n_t} \frac{e^{-n_t r}(n_t r)^y}{y!} & \text{for } y = 0, 1, \ldots \ldots \quad (10, a) \\ 0 & \text{otherwise} \end{cases} \]

from the Poisson distribution defined for the number of returned products when $y$ is larger than $k_t$.

\[ F_{t+1}(k_t, w_t + y) | (k_t, w_t), 0) = \begin{cases} \sum_{y=j}^{n_t} \frac{e^{-n_t r}(n_t r)^y}{y!} & \text{for } y = 0, 1, \ldots \ldots \quad (10, b) \\ 0 & \text{otherwise} \end{cases} \]

from the Poisson distribution defined for the number of returned products when $y$ is smaller than $k_t$.

For $a = 1$

\[ F_{t+1}(k_t, w_t + y) | (k_t, w_t), 1) = \begin{cases} \sum_{y=0}^{j} \frac{e^{-n_t r}(n_t r)^y}{y!} & \text{for } y = 0, 1, \ldots \ldots \quad (11) \\ 0 & \text{otherwise} \end{cases} \]

from the Poisson distribution defined for the number of returned products.

Where $j$ is the break-even point at which point the expected cost of outsourcing and in-house remanufacturing is equal. Any return, either side of the break-even point will incline the decision to either of the actions. The reader can refer to Table 5.3, 5.4 and 5.5 and Figure 5.1 in pages 94, 95, 98 and 96 respectively for better explanation of the value
of j. The following figure gives an example of the transition probability, states and actions.

\[ \text{Figure 4.3: Example of transition probability} \]

Action, \( a = 0 \); Remanufacture in-house

\( a = 1 \); Outsource

This problem is similar to Puterman (1994) explained earlier and here it is also seen that at each time epoch, there are two states that one can jump to and based on the cost function, only one decision is taken and the system jumps to that state only. The crossed states indicate that these states are not chosen and this decision is based on the cost function.

When one is in state \((0, 0)\) (taken from the example illustrated later) at time epoch 0, there are two new states one can go to. The new states are dependent upon the in-house
remanufacturing capacity and the number of returned products and according to the number of returned products (20) for time epoch 1, the new states can be (0, 20) or (20, 20). In the first instance, the returned products are outsourced and hence the in-house remanufacturing capacity remains 0. In the second instance, the decision is to remanufacture in-house and the capacity has been updated accordingly. The cost function is now calculated (outsourcing cost: 416.55 and in-house remanufacturing cost: 408.03) and it has been found that it is more profitable to remanufacture in-house in the first time epoch. So the state at time epoch 1 is (20, 20).

According to the definition provided in Introduction to Operations Research (Hillier and Lieberman), stationary transition probability implies that the transition probability do not change over time. The non-stationary transition probability is defined as: A Markov chain with non-stationary transition probabilities is allowed to have a different transition matrix for each time \( t \) (Sigman, 2007). So, from the two definitions explained above, it is clear that the transition probability here is stationary as these values will not change for each time.

In this model, the mean time between arrivals is considered to be exponential. The arrival of minimal batch is on an average 1 month. For this model, the minimal batch is considered to be 0 (minimal and minimum are same, minimum is a constant value and minimal refers to a range and for this model, the range is from 0 to \( n_{pr} \)). When the inter-arrival time is considered exponential, the arrival occurrence is considered to follow Poisson distribution and thus the use of Poisson in Markov decision process can be justified.
4.4.2.1.5 Cost function

It is important to classify the related costs of the RL chain in order to define the cost function. Here capacity unit represents the firm’s ability to process one returned item.

$c_1$: Unit investment cost: Cost of increasing the capacity of the RL by one unit to meet the demand for the returned products.

$c_2$: Unit idle cost: This represents the cost when the capacity remains unutilized in case the returned products are outsourced and also when the decision of in-house remanufacturing is considered but the number of returned products is less than the capacity available for remanufacturing.

$c_3$: Fixed cost: The cost of setup, machine cost, electricity, and order processing cost are considered as fixed cost.

$c_4$: Unit variable cost: Variable cost of labor and overhead cost are considered as unit variable cost of the firm which in turn can be thought of as unit remanufacturing cost.

$c_5$: Unit shortage cost: This cost is paid by the firm if it refuses to take responsibility of the returned products and thus get away by paying only a shortage cost.

$c_6$: Unit salvage value: This cost accounts for when the process is outsourced and the capacity is left unutilized; hence depreciation takes place and also if the decision is taken that the system capacity will be reduced to zero and consequently revenue will transpire
for the OEM. Thus, this value can be both cost and revenue depending on the decision taken.

$c_7$: Unit outsourcing cost: This is the cost of outsourcing one unit which consists of both the variable cost and fixed cost on part of the 3rd party involved.

$c_8$: Unit transportation cost: This cost represents transporting one unit of returned products back to the warehouse.

$c_9$: Unit inventory cost: This cost accounts for handling the returned products in the warehouse.

$c_{10}$: Unit inspection cost: This is the cost of inspecting for quality of the returned products to decide on the disposal or remanufacturing option.

$c_1, c_2, c_3, c_4, c_5, c_7, c_8, c_9, c_{10} > 0$ as they represent costs for the firm, $c_6$ is unrestricted in sign as there is no reason to take for granted that it will become cost or not. Given that reverse logistics does not represent a core activity for the firm, profit from remanufacturing is not considered.

Some relationships among these cost variables exist. They are as follows:

Inequality (12) implies that the cost of investment while increasing the capacity is more than when the capacity is left unutilized.

\[ c_1 \geq c_2 \] (12)
Cost of unutilized capacity is less than maintaining for an additional period is given by.

\[ c_3 + c_9 \geq c_2 \]  
(13)

Cost of outsourcing is more than the variable cost as outsourcing comprises both variable and fixed costs while in-house is considering only variable cost.

\[ c_7 > c_4 \]  
(14)

Total internal cost should be less than shortage cost as it gives the motivation to develop internal capacity, given that the internal cost of having the capacity for one additional period and then processing one additional unit.

\[ c_1 + c_3 + c_4 + c_9 + c_{10} \leq c_5 \]  
(15)

These cost parameters define the following cost structures:

Investment cost: \( c_1 (y - k_t)^+ \)  
(16)

Idle cost: \( c_2 (k_t - y)^+ \)  
(17)

Fixed cost: \( c_3 \)  
(18)

Variable cost: \( c_4 (y) \)  
(19)

Shortage cost: \( c_5 (E[(X - k_t)^+]) \)  
(20)

where \( X \) is Poisson distributed with mean \( (n_{t,r}) \)

Transportation cost: \( c_8 (y) \)  
(21)

Inventory cost: \( c_9 (E[\min(X, k_t)]) \)  
(22)
where X is Poisson distributed with mean (n₀r)

Inspection cost:

\[ c_{10}(E[\min(X, k_t)]) \]  where X is Poisson distributed with mean (n₀r)  \hspace{1cm} (23)

These are for the case when the remanufacturing option is taken to be in-house.

The expected total cost for in-house remanufacturing is:

\[
R_{t+1}(k_t, w_t, 0) = \sum_{y=0}^{n_t} (c_1[y - k_t]^+ + c_2[k_t - y]^+ + c_3 + c_4[y]) + c_5(E[(X - k_t)^+]) + c_8[y] + c_9(E[\min(X, k_t)]) + c_{10}(E[\min(X, k_t)]) \times P(y) \hspace{1cm} (24)
\]

Salvage value: \( c_6[k_t] \)  \hspace{1cm} (25)

Outsourcing cost: \( c_7[y] \)  \hspace{1cm} (26)

The expected total cost for outsourcing is:

\[
R_{t+1}(k_t, w_t, 1) = \sum_{y=0}^{n_t} (c_6[k_t] + c_7[y] + c_8[k_t]) \times P(y) \hspace{1cm} (27)
\]

Where, \( P(y) = \frac{e^{-(n_0r)(n_0r)^y}}{y!} \) for \( y = 0, 1, 2, \ldots \ldots \)

Based on the two costs (equation 24 and 27), the final decision is taken. As explained earlier in Figure 3.3, the concept of disposal is also introduced in the system, and the
decision maker has that flexibility to decide if the products can be disposed of too if they still crossed the acceptable quality level.

**4.4.2.1.6 System dynamics**

Recalling the MDM defined above, we can identify that during each period \( t \), the system:

1. Has a facility with in-house capacity of size \( k_{t-1} \) at the beginning of each period, \( w_{t-1} \) returned units and \( n_{t-1} \) units outstanding in the market (sold but not yet returned).
2. Computes the sales using sales function.
3. Computes the expected returns for the period, \( x_t = n_{t-1} r \).
4. Computes the transportation cost, fixed cost, variable cost, inspection cost, inventory cost, shortage cost, investment cost, idle cost, shortage cost and outsourcing cost based on the equations 16 to 23, 25 and 26.
5. Computes the expected total in-house remanufacturing cost \( R_t((k_{t-1}, w_{t-1}), 0) \) and expected total outsourcing cost \( R_t((k_{t-1}, w_{t-1}), 1) \) based on equations 24 and 27.
6. Transition probability is calculated based on equations 10 and 11.
7. In-house remanufacturing or outsourcing decision is taken and the in-house capacity is adjusted accordingly.
8. The expected amount of return and the in-house remanufacturing capacity determines the new state.
9. The process again starts from step 1.
The system dynamics can be depicted properly with the help of a flow chart:

![Flow chart for system dynamics for Dynamic capacity model](image)

- **Start**
- **Calculate sales through sales function**: Equation 2
- **Compute expected return**: Equation 3
- **Compute individual cost in the system**: Equations 16 to 23; 25, 26
- **Compute expected total cost**: Equations 24 and 27
- **Calculate transition probability**: Equations 10 and 11
- **Take a decision**
- **Adjust capacity according to taken decision**: Equation 9
- **Update outstanding products in the market**: Equation 4
- **End of time horizon?**
- **Yes**: Terminate
- **No**

*Figure 4.4: Flow chart for system dynamics for Dynamic capacity model*
4.4.3 Scenario 2 (Stationary capacity)

In scenario 2, there will be fixed capacity for in-house remanufacturing. Hence, there will be three decisions to choose from. If the number of returned products is more than the capacity, then the returned products can be split between in-house remanufacturing and outsourcing or be completely outsourced. The amount remanufactured in-house will be equal to its capacity and the rest of the returned products will be outsourced. In other options, when the number of returned products is less than the in-house capacity, the total amount can be completely remanufactured in-house or outsourced.

For scenario 2, the capacity is considered to be stationary, hence it cannot be increased.

4.4.3.1 Model definitions

4.4.3.1.1 Decision epoch

Decision epoch $t$ represents the end of period $t$ and $T$ represents the end of problem horizon.

$t \in \{1, 2, \ldots, T\}$  \hspace{1cm} (28)

It is also known,

$T = L + W$  \hspace{1cm} (29)
Where $L$ is the length of the Product Life Cycle and $W$ is the time length for which the firm continues managing the returned products. This length ensures that a warranty or accomplishment of the legal requirements for managing returned products after period $L$, when the last sale is made.

### 4.4.3.1.2 System state

The system state at decision epoch $t$ is defined by

$$(w_t) \text{ for } t = 1, 2, \ldots, T$$

(30)

Where $w_t$ is the cumulative number of units that have returned through the RL channel at the end of period $t$.

$$w_t = \sum_{i=1}^{t} x_i$$

(31)

where $x_i$ is the return at time $i$.

### 4.4.3.1.3 Decision

The following decision set is considered for scenario 2. The purpose of this model is to determine the decision of outsourcing and hence it is assumed that at the end of any period $t$, any one of the three decisions can be taken.

$b = 0$: Continuing with the reverse logistics activities in-house as the number of returns is less than the available capacity for remanufacturing.
\[ y \leq k_t \]

b = 1: Follow an outsourcing strategy for the next period by engaging a third party reverse logistics provider for each of the cases when the number of returned products is more than the current available capacity and also less than the available remanufacturing capacity.

b = 2: Split the returned amounts into two sections. In-house remanufacturing up to the limit of available remanufacturing capacity and outsource the rest.

*In-house remanufacturing*, \([y]\)

*Outsourcing*, \([ (k_t - y) ]\)
Figure 4.5: Decision criteria for scenario 2

- Returned products
  - Is the quality level acceptable?
    - NO: Disposal
    - YES
      - Is it worthy to split the products?
        - YES: Split
        - NO
          - Is it worthy to remanufacture it in-house?
            - NO: Outsource
            - YES
              - b = 0: In-house remanufacturing
              - b = 1
              - b = 2

End
4.4.3.1.4 Transition probabilities

It is earlier assumed that the number of returned products follows a Poisson distribution, and given that the sales function is also known, the transition probability between each state is as follows:

For $b = 0$

$$F_{t+1}((w_t + y)|(w_t), 0) = \sum_{y=0}^{n_t} \frac{e^{-nt}(nt)^y}{y!} \quad \text{for } y = 0, 1, \ldots \ldots \quad (32)$$

from the Poisson distribution defined for the number of returned products.

For $b = 1$

$$F_{t+1}((w_t + y)|(w_t), 1) = \sum_{y=0}^{l} \frac{e^{-nt}(nt)^y}{y!} \quad \text{for } y = 0, 1, \ldots \ldots \quad (33)$$

from the Poisson distribution defined for the number of returned products.

For $b = 2$

$$F_{t+1}((w_t + y)|(w_t), 2) = \left\{ \begin{array}{ll} \sum_{y=j}^{n_t} \frac{e^{-nt}(nt)^y}{y!} & \text{for } y = 0, 1, \ldots \ldots \quad (34) \\ 0 & \text{otherwise} \end{array} \right.$$ 

from the Poisson distribution defined for the number of returned products.

Again in the scenario, the disposal option can be included based on the cost factor and also if the returned products fail to pass the acceptability test.
**4.4.3.1.5 Cost function**

It is important to classify the related costs of the RL chain in order to define the cost function. Here capacity unit represents the firm’s ability to process one returned item.

$c_1$: Unit idle cost: This represents the cost when the capacity remains unutilized in case the returned products are outsourced and also when the decision of in-house remanufacturing is considered but the returned products are less than the capacity available for remanufacturing.

$c_2$: Fixed cost: The cost of set up, machine cost, electricity, order processing is considered as fixed internal cost.

$c_3$: Unit variable cost: Variable cost of labor and overhead cost are considered as unit internal labor cost of the firm which in turn can be thought of as unit remanufacturing cost.

$c_4$: Unit shortage cost: This cost is paid by the firm if it refuses to take responsibility of the returned products and thus get away with paying a shortage cost.

$c_5$: Unit salvage value: This cost accounts for when the process is outsourced and the capacity is left unutilized; hence depreciation takes place and also if the decision is taken that the system capacity will be reduced to zero and consequently revenue will transpire for the OEM. Thus this value can be both cost and revenue depending on the decision taken.
$c_6$: Unit outsourcing cost: This is the cost of outsourcing one unit which consists of both the internal labor and fixed cost on part of the 3$^{rd}$ party involved.

$c_7$: Unit transportation cost: This cost represents transporting one unit of returned products back to the warehouse.

$c_8$: Unit inventory cost: This cost accounts for handling the returned products in the warehouse.

$c_9$: Unit inspection cost: This is the cost of inspecting for quality of the returned products to decide on the disposal or remanufacturing option.

$c_1, c_2, c_3, c_4, c_6, c_7, c_8, c_9 > 0$ as they represent costs for the firm, $c_5$ is unrestricted in sign as there is no reason to take for granted that it will become cost or not. Given that reverse logistics does not represent a core activity for the firm, profit from remanufacturing is not considered. Some relationships among these cost variables are presumed and they are as follows:

Cost of unutilized capacity is less than maintaining for an additional period.

$$c_2 + c_8 \geq c_1$$  \hspace{1cm} (35)

Cost of outsourcing is more than the internal labor cost as outsourcing comprises both variable and fixed costs while in-house is considering only variable cost.

$$c_6 > c_3$$  \hspace{1cm} (36)
Total internal cost should be less than shortage cost as it gives the motivation to develop internal capacity, given that the internal cost of having the capacity for one additional period and then processing one additional unit.

\[ c_2 + c_3 + c_8 + c_9 \leq c_4 \]  \hspace{1cm} (37)

These cost parameters define the following cost structures:

Idle cost: \( c_1 (k_t - y)^+ \)  \hspace{1cm} (38)

Fixed internal cost: \( c_2 \)  \hspace{1cm} (39)

Variable cost: \( c_3 [y] \)  \hspace{1cm} (40)

Shortage cost: \( c_4 (E[(X - k_t)^+]) \)  \hspace{1cm} (41)

where \( X \) is Poisson distributed with mean \( (n_{t,r}) \)

Transportation cost: \( c_7 (y) \)  \hspace{1cm} (42)

Inventory cost: \( c_8 (E[\min(X, k_t)]) \)  \hspace{1cm} (43)

where \( X \) is Poisson distributed with mean \( (n_{t,r}) \)

Inspection cost:

\[ c_9(E[\min(X, k_t)]) \]  \hspace{1cm} (44)

where \( X \) is Poisson distributed with mean \( (n_{t,r}) \)

These are for the case when the remanufacturing option is taken to be in-house.

The total cost for in-house remanufacturing =
\begin{align*}
R_{t+1}(\omega_t, 0) &= \sum_{y=0}^{n_t} (c_1[y - k_t]^+ + c_2 + c_3[y] + c_4(E[(X - k_t)^+]) + c_7[y] \\
&\quad + c_8(E[\min(X, k_t)] + c_9(E[\min(X, k_t)]) \times P(y) \\
&= \text{(45)}
\end{align*}

Salvage value: \( c_5 k_t \)  \hspace{1cm} (46)

Outsourcing cost: \( c_6[y] \)  \hspace{1cm} (47)

The total cost for outsourcing = \( R_{t+1}(\omega_t, 1) = \sum_{y=0}^{n_t}(c_5k_t + c_6[y] + c_1k_t) \times P(y) \)  \hspace{1cm} (48)

The total cost for splitting the returned products =

\begin{align*}
R_{t+1}(\omega_t, 2) &= \sum_{y=0}^{n_t} (c_2 + c_3[y] + c_4(E[(X - k_t)^+]) + c_7[y] + c_8(E[\min(X, k_t)]) \\
&\quad + c_9(E[\min(X, k_t)]) \times P(y) \\
&= \text{(49)}
\end{align*}

Where, \( P(y) = \frac{e^{-(n_{tr})(n_{cr})y}}{y!} \text{ for } y = 0, 1, 2, \ldots\ldots \)

Based on the three costs (equation 45, 48 and 49), the final decision is taken. As explained earlier in Figure 3.3, the concept of disposal is also introduced in the system, and the decision maker has that flexibility to decide if the products can be disposed of too if they still crossed the acceptable level.
4.4.3.1.6 System dynamics

Recalling the MDM defined above, we can identify that during each period $t$, the system:

1. Has a facility with a fixed in-house capacity of size $k$, $w_{t-1}$ returned units and $n_{t-1}$ units outstanding in the market (sold but not yet returned).
2. Computes the sales using sales function.
3. Computes the expected returns for the period, $x_t = n_{t-1}r$.
4. Computes the transportation cost, fixed cost, variable cost, inspection cost, inventory cost, shortage cost, investment cost, idle cost shortage cost and outsourcing cost based on the equations 38 to 44, 46 and 47.
5. Computes the expected total in-house remanufacturing cost $R_t((w_{t-1}), 0)$ and expected total outsourcing cost $R_t((w_{t-1}), 1)$ and expected total split cost based on the equations 45, 48 and 49.
6. Transition probability is calculated based on the equations 32 to 34.
7. In-house remanufacturing, outsourcing or split decision is taken
8. The expected amount of return determines the new state.
9. The process again starts from step 1.

The system dynamics can be depicted properly with the help of a flow chart:

In the next section the theory will be tested with appropriate numerical examples.
Figure 4.6: Flow chart for system dynamics for Stationary capacity model
CHAPTER 5

NUMERICAL EXAMPLE

5.1 Numerical examples

In the numerical example, the two scenarios will be checked against two different return rates with low variance and high variance. Also the two models will be checked against different life cycles. For sales function (equation 2), an arbitrary value for $M$ is considered i.e., $M = 100$. Sales values are updated at each time epoch as can be seen from step 1 of Figure 4.4 and Figure 4.6.

At first let us take a look at the two models from different variance of return rates.

The return rates are randomly generated and shown in the following tables along with their variances. In real world situations, one would have the historical data to calculate the return rates at each time epoch but for this particular problem no data is available, so these return rates are generated randomly using Stat fit which are to be used in the current example. The return rates for these scenarios are considered such that the number of returned products ($n_r$) follows Poisson distribution. It has been found that the number of returned products follows Poisson distribution while the return rates ($r$) follow Lognormal distribution for these two scenarios (Dynamic and Stationary). See Appendix A for detailed information.

The return rates are randomly generated and shown in the following tables along with their variances.
Table 5.1: Return rate with lower variance

<table>
<thead>
<tr>
<th>0.96</th>
<th>0.35</th>
<th>0.33</th>
<th>0.23</th>
<th>0.15</th>
<th>0.22</th>
<th>0.28</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.4</td>
<td>0.41</td>
<td>0.42</td>
<td>0.43</td>
<td>0.44</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

Variance = 0.0314

Table 5.2: Return rate with higher variance

<table>
<thead>
<tr>
<th>0.96</th>
<th>0.33</th>
<th>0.32</th>
<th>0.21</th>
<th>0.15</th>
<th>0.19</th>
<th>0.33</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.39</td>
<td>0.4</td>
<td>0.41</td>
<td>0.42</td>
<td>0.43</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

Variance = 0.0319

These return values are considered for the life cycle (L) of both 10 and 18. The value of W is considered to be 4 for this problem. The choice of W value is subjective to change and can be chosen differently for different type of products and also for different facilities. The expected return is then calculated using the return rates considered in Table 5.1 and Table 5.6 taking into account equation 3 as can be seen from Figure 4.4 and Figure 4.6 (step 2).

5.2 Scenario 1 with different variance of return rates

Some relationships between costs have been assumed earlier in equations 12 to 15 for scenario 1. The following costs have been considered arbitrarily fulfilling the conditions stated earlier.
The cost considered for scenario 1 is as follows:

\[
\begin{align*}
&c_1 = 2 & c_2 = 2 & c_3 = 3 & c_4 = 4 & c_5 = 17 \\
&c_6 = 12 & c_7 = 40 & c_8 = 8 & c_9 = 5 & c_{10} = 3
\end{align*}
\]

According to the system dynamics, the total cost associated with each cost variable is calculated individually (step 3) followed by calculation of total cost (step 4). On the basis of the given information, we compute the transition probability (step 5) and take an action (step 6). The states are updated accordingly (step 7) and also the amount of units outstanding in the market \((n_i)\) (step 8) are computed for each time epoch. All the information are regarding Figure 4.4.

The following partial table shows the expected cost of outsourcing and in-house remanufacturing for each time epoch \((t)\) and for each value of \(y\) (expected number of returns) following the equations 24 and 27.
Table 5.3: Partial table for expected cost of outsourcing and in-house remanufacturing for different values of $y$ for $L = 10$. 

<table>
<thead>
<tr>
<th>$t$</th>
<th>Expected cost</th>
<th>$y=0$</th>
<th>$y=1$</th>
<th>$y=2$</th>
<th>$y=3$</th>
<th>$y=4$</th>
<th>$y=5$</th>
<th>$y=6$</th>
<th>$y=7$</th>
<th>$y=8$</th>
<th>$y=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Outsource</td>
<td>3.52e-06</td>
<td>6.76e-05</td>
<td>0.000649</td>
<td>0.004156</td>
<td>0.019948</td>
<td>0.076601</td>
<td>0.245122</td>
<td>0.672334</td>
<td>1.613602</td>
<td></td>
</tr>
<tr>
<td></td>
<td>In-house</td>
<td>3.70e-06</td>
<td>6.85e-05</td>
<td>0.000649</td>
<td>0.004130</td>
<td>0.019749</td>
<td>0.075643</td>
<td>0.241622</td>
<td>0.661829</td>
<td>1.587090</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Outsource</td>
<td>0.000233</td>
<td>0.003725</td>
<td>0.029336</td>
<td>0.152114</td>
<td>0.585640</td>
<td>1.788664</td>
<td>4.521852</td>
<td>13.739373</td>
<td>30.30027</td>
<td></td>
</tr>
<tr>
<td></td>
<td>In-house</td>
<td>0.000452</td>
<td>0.006246</td>
<td>0.043108</td>
<td>0.198509</td>
<td>0.685465</td>
<td>1.893215</td>
<td>4.356630</td>
<td>9.510518</td>
<td>22.7252</td>
<td></td>
</tr>
<tr>
<td>3</td>
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Let us take $t = 13$ for example and see how these values are generated for each values of $y$ following equation 24 and 27.
Table 5.4: A complete breakdown of the Poisson distribution, outsourcing cost and in-house remanufacturing cost for each values of y, time epoch 13

<table>
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<th>y</th>
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<th>In-house cost</th>
<th>y</th>
<th>P(y)</th>
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<th>In-house cost</th>
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The following table gives the expected cost of outsourcing and in-house remanufacturing based on different values of $y$ for time epoch $t = 13$ by multiplying the cost of outsourcing and in-house remanufacturing with the probability found in Table 5.4.

Table 5.5: Complete table for expected outsourcing and in-house remanufacturing cost for different values of $y$ for time epoch, $t = 13$

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<th>Expected Outsourcing cost</th>
<th>Expected In-house cost</th>
<th>$y$</th>
<th>Expected Outsourcing cost</th>
<th>Expected In-house cost</th>
<th>$y$</th>
<th>Expected Outsourcing cost</th>
<th>Expected In-house cost</th>
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The expected total outsourcing cost = $\sum_{y=0}^{n_t} P(y) \times (Outsourcing\ cost) = 1.75E+3$

The expected total in-house cost = $\sum_{y=0}^{n_t} P(y) \times (In\ -\ house\ cost) = 2.14E+3$
The following figure is the graphical representation of the Table 5.5:

![Graph](image-url)

**Figure 5.1: Determining the value of j**

It has been found that the intersection point of the two lines is at \( y = 23.02 \). So the value of \( j \) is 24 for time epoch 13. Similarly, the intersection point for different time epoch has been identified and the value of \( j \) has been used to find the transition probability. Based on the value of \( j \), the transition probability for outsourcing at time epoch 13 is 0.9916 and transition probability for in-house remanufacturing is 0.0084.

Let us take a look at a different time epoch, time epoch 1 for example. At step 1, the sale is calculated using equation 2 and the sale value is 20. Using equation 3 we know that the expected return is 20. The total cost comprises of individual costs (equations 16 to 23, 25, and 26) and the total cost of in-house remanufacturing is 408.03 and total cost of outsourcing is 416.55 calculated by equations 24 and 27 respectively. In step 5, the
transition probability is calculated and transition probability of outsourcing is 4.587E-9 and transition probability of in-house remanufacturing is 0.9999. The decision taken is to remanufacture the returned products in-house and the system state is thus updated to (20, 20) in step 7. Outstanding products in the market is now 0. We again start from step 1 and calculate the sale and at the next time epoch and the sale are 40 and we follow the Figure 4 again until the end of time horizon.

Based on the given inputs (length of the product life cycle, sales, return rate) and the criteria defined (cost structures), the program solves the MDM and shows the optimal action to be taken at each decision epoch. The decision can be seen in the figures below: (See Appendix B I for the Matlab program).

The decision can be seen in the figure below:

![Lower variance in return rate with L = 10](image)

*Figure 5.2: Scenario 1 with lower variance in return rate with life cycle considered as 10*
Figure 5.3: Scenario 1 with higher variance in return rate with life cycle considered as 10

The information is summed up in the following table and the decision taken at each time epoch is also included.

0 = decision to remanufacture in-house

1 = decision to outsource

Table 5.6: Outsourcing and in-house remanufacturing decisions for scenario 1 with different variance of return rates

<table>
<thead>
<tr>
<th>Time epoch</th>
<th>Lower variance</th>
<th>Higher variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
As it can be identified from Figures 5.2 and 5.3 and also from the Table 5.6, the higher variance in return rate has a higher number of outsourcing option as optimal decisions. A greater variance on the return rate increases the probability of crossing the threshold, which is defined as the breakeven cost for both decisions; the outsourcing and the remanufacturing in-house in each of the time epochs considered. This implies that greater variance in the return rate increases the uncertainty about the volume of units put into the corresponding RL system, which forces the firm to follow an outsourcing strategy and take advantage by involving a 3PRLP in managing the returned items.
5.3 Scenario 2 with different variance of return rates

Some relationships between costs have been assumed earlier from equations 36 to 38. The following costs have been considered arbitrarily fulfilling the conditions stated earlier.

The cost considered for the scenario is as follows:

\[
c_1 = 2 \quad c_2 = 3 \quad c_3 = 5 \quad c_4 = 16 \quad c_5 = 12 \\
c_6 = 40 \quad c_7 = 8 \quad c_8 = 5 \quad c_9 = 3
\]

Based on the given inputs (length of the product life cycle, sales, return rate) and the criteria defined (cost structures), the program solves the MDM and shows the optimal action to be taken at each decision epoch. The decision can be seen in the figures below: (See Appendix B II). The files can be found in Appendix D.

![Lower variance in return rate with L = 10](image)

*Figure 5.4: Scenario 2 with lower variance in return rate with life cycle considered as 10*
In both the cases, the gap in the middle offers the chance to split the returned products and consider both outsource and in-house. And in both cases, the decision is in favor of outsource and splitting.

The results are summarized in the following table.

0 = decision to remanufacture in-house

1 = decision to outsource

2 = decision to split the returned products
Table 5.7: Outsourcing, in-house remanufacturing and splitting decisions for scenario 2 with different variance in return rates

<table>
<thead>
<tr>
<th>Time epoch</th>
<th>Lower variance</th>
<th>Higher variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Decision to outsource | 4 | 5 |

This result also shows that the variance in return rate affects the decision criteria for outsourcing.

The first hypothesis is thus proved as the higher variance in return rate inclines the decision towards outsourcing. This hypothesis is true for scenario 2.
The effect of capacity on scenario 2 is justified but still a number of runs are made to test the sensitivity of the model. The results are shown below:

**Figure 5.6: Sensitivity analysis for scenario 2**

With the varying capacity, it can be seen that the hypothesis “outsourcing is more likely to be an optimal decision when the variance in return rate is high” holds true for all the cases. The regression analysis calculated later confirms that there are more variables that can be considered which might help us better explain this condition where all the outsourcing decisions for higher variance in return rate should show more outsourcing options than lower variance in return rate. The regression analysis has $R^2$ value as 0.77 and adjusted $R^2$ value as 0.76 which explains that there are more variables which can be considered that will make the model better.
Now the two models will be checked against different life cycle and see if the hypothesis is still valid in these cases.

5.4 Scenario 1 and 2 with different life cycle

The complexity of this situation increases when the life cycle for any type of products is extremely short which requires quick but adequate decisions for these RL systems in order to efficiently respond to such changing conditions. The following figure shows the model with $L = 18$.

![Figure 5.7: Scenario 2 with lower variance in return rate with life cycle considered as 18](image)

The following table depicts the predicament. (See Appendix C and D for details)
Table 5.8: Number of outsourcing decisions taken for different life cycle for scenario 1

<table>
<thead>
<tr>
<th>Life Cycle</th>
<th>Number of outsourcing decisions</th>
<th>Percentage of the life cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>20%</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>17%</td>
</tr>
</tbody>
</table>

The following figure shows the model with $L = 18$.

*Figure 5.8: Scenario 2 with higher variance in return rate with life cycle considered as 18*

The following table depicts the predicament.
Table 5.9: Number of outsourcing decisions taken for different life cycle for scenario 2

<table>
<thead>
<tr>
<th>Life Cycle</th>
<th>Number of outsourcing decisions</th>
<th>Percentage of the life cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>50%</td>
</tr>
<tr>
<td>18</td>
<td>8</td>
<td>45%</td>
</tr>
</tbody>
</table>

It is evident from the above tables that, as the length of the life cycle decreases, the decision to outsource increases in both the cases for scenario 1 and scenario 2. The outsourcing option is more logical when the life cycle is shorter and this can be effectively accomplished by involving a 3PRLP which specializes in these activities and can take advantage of the economies of scale to convert RL functions in a profit creating activity into the closed-loop chain.

5.5 Simulation

From the previous model, it is established that the outsourcing decision is related to the variance of return rate and this return rate is generated randomly which followed lognormal distribution. The higher variability in return rate inclined the decision towards outsourcing.

The same scenario is now simulated in this section. In this section, the number of returned products is assumed to follow a Poisson distribution and are generated randomly (using default Poisson distribution function of Pro Model) rather than calculating from sales function and return rate. The condition will be checked if the return rate variance still affects the outsourcing decision. The variance in return rate is calculated from the number
of returned products and sales, which is not considered in the simulation; rather it is calculated at the end of the model output. As the scope of the problem is very generic and constraints that define and shape the simulation model are generic in nature; the model created here is generic in nature too. In other words, this model is a conceptual representation of problem under study. The objective of the model is to validate and verify the hypothesis presented in the paper and act as a guideline for future work on a specific industry type, where the constraint(s) and objective function parameters are known. To the fact that the Product Life Cycle of a product cannot be large (here it is a very large number if counted in months) puts a limitation on the model. For this generic model to work, the mean arrival rate has to be scaled down so that it is within the working range.

These simulation runs also give an idea of how the system works under different conditions; number of returns in this case.

5.6 Scenario 1

In this current simulation, to generate the random numbers, the mean value of Poisson distribution is considered 21 and 42 and the model is run for 5 replications. The reason for considering mean value of 21 and 42 is nothing but to show that how the variance in return rate affects the decision making criteria. These values are subjective and can be considered such that act in accordance with the limitation of the model explained earlier.
Figure 5.9: Simulation with mean 21 for Poisson distribution for scenario 1

Figure 5.10: Simulation with mean 42 for Poisson distribution for scenario 1
Now, to find the effect of return rate variability, five runs are considered and the variance in the return rate is shown in the following figures:

**Figure 5.11:** Variance in return rate for simulation with mean for Poisson distribution as 21 and 42 respectively for scenario 1

**Figure 5.12:** Number of outsourcing decisions for simulation with mean for Poisson distribution as 21 and 42 respectively for scenario 1
The result shows that the variance in the return rate influences the decision of outsourcing.

5.7 Scenario 2 with capacity 25

In the scenario 2, the mean value of Poisson distribution is considered 21 and 42 as the previous model and the model is run for 5 replications with capacity being stationary at 25 and 50 respectively. At the end the results are taken to an excel file and the result is as follows:

![Scenario 2 / Capacity 25 / P(21)](image)

*Figure 5.13: Scenario 2 with mean 21 for Poisson distribution and capacity 25*

When the model is run with capacity 5 and mean value of 10, the following result follows:
Figure 5.14: Scenario 2 with mean 42 for Poisson distribution and capacity 25

Now to consider the effect of return rate variance, let us consider the replications and the variance of the return rates are shown in the following figures:

Figure 5.15: Variance in return rate for scenario 2 with mean for Poisson distribution as 21 and 42 respectively for scenario 2
The result shows that the variance in the return rate influences the decision of outsourcing.

**5.8 Scenario 2 with capacity 50**

Now the model is run with capacity 50 and the mean for Poisson distribution as 21 and 42 respectively and the results are shown below:
Figure 5.17: Scenario 2 with mean 21 for Poisson distribution and capacity 50

Figure 5.18: Scenario 2 with mean 42 for Poisson distribution and capacity 50
Now to consider the effect of return rate variance, let us consider the three runs and the variance of the return rates are shown in the following figures:

**Figure 5.19: Variance in return rate for scenario 2 with mean for Poisson distribution as 21 and 42 respectively**

**Figure 5.20: Number of outsourcing decisions taken for scenario 2 with mean for Poisson distribution as 21 and 42 respectively**
The result shows that the variance in the return rate influences the decision of outsourcing.

5.9 Scenario analysis using regression analysis

The efficiency of the model is calculated by adjusted $R^2$. The calculation is done in an excel sheet. The result is as follows:

Table 5.10: Regression analysis for scenario 1

<table>
<thead>
<tr>
<th></th>
<th>Regression statistics for outsourcing</th>
<th>Regression statistics for in-house remanufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
<td>0.996382227</td>
<td>0.963444335</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.992777544</td>
<td>0.928224986</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.992623328</td>
<td>0.926692424</td>
</tr>
<tr>
<td>Standard Error</td>
<td>203.0914666</td>
<td>470.2357859</td>
</tr>
<tr>
<td>Observations</td>
<td>288</td>
<td>288</td>
</tr>
</tbody>
</table>

The $R^2$ value from the regression analysis indicates that the forecast of the future outcomes can be predicted with 99.27% and 92.82% accuracy. The adjusted $R^2$ value is the number of explanatory terms (product outstanding in the market; $n_t$, in-house capacity; $k_t$, number of returned products; $x_t$, cumulative returned products; $w_t$, product life cycle; $L$ and maximum expected sales level; $M$) in a model and the value of 0.9926 and 0.9267 indicates that this model is quite explanatory.
The ANOVA table is presented below:

**Table 5.11: ANOVA table for in-house remanufacturing for scenario 1**

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>6</td>
<td>803558765</td>
<td>133926460</td>
<td>605.66857</td>
</tr>
<tr>
<td>Residual</td>
<td>282</td>
<td>62135196</td>
<td>221121</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>288</td>
<td>865693961</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.12: ANOVA table for outsourcing for scenario 1**

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>6</td>
<td>1593150191</td>
<td>265525031</td>
<td>6437.57228</td>
</tr>
<tr>
<td>Residual</td>
<td>282</td>
<td>11590166</td>
<td>41246</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>288</td>
<td>1604740358</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The R^2 value from the regression analysis indicates that the prediction of the future outcomes can be predicted with 96.34% accuracy while deciding on the fact of outsourcing. On the other hand, when one is considering the factor of in-house remanufacturing, the accuracy is only of 69.27%. The adjusted R^2 value is the number of explanatory terms in a model and the value of 0.9593 and indicates that this model is quite explanatory when explaining the outsourcing scenario but in the case of in-house remanufacturing, this model is not as explanatory as the outsourcing scenario. Hence it can be concluded that the number of factors required explaining the model needs to be increased in case of in-house remanufacturing decision.

The ANOVA table is presented below:

<table>
<thead>
<tr>
<th></th>
<th>Regression statistics for outsourcing</th>
<th>Regression statistics for in-house remanufacturing</th>
<th>Regression statistics for split decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
<td>0.981554318</td>
<td>0.832337454</td>
<td>0.971489176</td>
</tr>
<tr>
<td>R^2</td>
<td>0.963448881</td>
<td>0.692785638</td>
<td>0.943791219</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.959254712</td>
<td>0.683792476</td>
<td>0.939248509</td>
</tr>
<tr>
<td>Standard Error</td>
<td>354.7518419</td>
<td>307.0099111</td>
<td>783.4496987</td>
</tr>
<tr>
<td>Observations</td>
<td>288</td>
<td>288</td>
<td>288</td>
</tr>
</tbody>
</table>
Table 5.14: ANOVA table for in-house remanufacturing for scenario 2

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>6</td>
<td>59939244</td>
<td>9989874</td>
<td>127.18516</td>
<td>4.394 E-77</td>
</tr>
<tr>
<td>Residual</td>
<td>282</td>
<td>26579934</td>
<td>94255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>288</td>
<td>86519178</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.15: ANOVA table for outsourcing for scenario 2

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>6</td>
<td>935462584</td>
<td>155910430</td>
<td>1486.6444</td>
<td>1.265E-209</td>
</tr>
<tr>
<td>Residual</td>
<td>282</td>
<td>35489381</td>
<td>125848</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>288</td>
<td>970951965</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.16: ANOVA table for split for scenario 2

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>6</td>
<td>2906317827</td>
<td>484386304</td>
<td>947.0019338</td>
<td>3.5664E-183</td>
</tr>
<tr>
<td>Residual</td>
<td>282</td>
<td>173089747</td>
<td>613793</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>288</td>
<td>3079407574</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.10 Comparison with Serrato et al. (2007) model

The basic difference between Serrato et al. (2007) model and the current model is the inclusion of new variables which better represents the real life situations and also improves the efficiency of the model, which is represented by the regression analysis below. There is also another improvement which is depicted by the fact that number of observations decreased significantly along with the time required to solve the model than the model Serrato et al. (2007) proposed. The explanation with example is presented after the regression analysis.

Serrato et al. (2007) studied the reverse supply chain and came up with a model to decide on the outsourcing decision. The model computes both the in-house cost and also the outsourcing cost and decides whether to outsource or not. The parameters considered for this model are $L = 4$ and $M = 3$. The values generated by the model are put into regression analysis and the result is as follows:
Table 5.17: Regression analysis for the model by Serrato et al. (2007)

<table>
<thead>
<tr>
<th>Regression statistics for outsourcing</th>
<th>Regression statistics for in-house remanufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
<td>0.590608055</td>
</tr>
<tr>
<td>R Square</td>
<td>0.348817875</td>
</tr>
<tr>
<td>Adjusted R Square</td>
<td>0.345687192</td>
</tr>
<tr>
<td>Standard Error</td>
<td>24.21795131</td>
</tr>
<tr>
<td>Observations</td>
<td>632</td>
</tr>
<tr>
<td></td>
<td>0.638572455</td>
</tr>
<tr>
<td></td>
<td>0.40777478</td>
</tr>
<tr>
<td></td>
<td>0.404927543</td>
</tr>
<tr>
<td></td>
<td>21.97541672</td>
</tr>
<tr>
<td></td>
<td>632</td>
</tr>
</tbody>
</table>

In the current model, the number of observations is directly related to number of times the returns take place. The return takes place at each time epoch and hence, if one is considering 14 time epoch, (for example, it can be days or months), there will be 14 observations in total. At every time epoch when the products are returned, the system is checked for the most profitable option and the states are updated accordingly. For the regression analysis of current model, number of considered observations regarding model parameter values for $L = 10$ and $18$ and also for different return rates (both with higher and lower variance). The value of $M$ is considered $50$, $100$, $150$ and $200$. That is why there are $288$ ($W=4$) observations in total. Whereas, Serrato et al. (2007) in their model used a different approach which considers every possible states taking into account the number of products outstanding in the market, $(0,1,2,\ldots, n_t)$ at each time epoch for each outstanding products $(0,1,2,\ldots, n_t)$, in the market in the previous time epoch regardless of
the return rate, hence the number of observations is extremely high as shown in the later table. The number of observations from the model by Serrato et al. (2007), are exponential in nature and increases very rapidly when the value of \( L \) or \( M \) is increased and thus making it infeasible to work with when these values are large, which will be in most real life cases.

This table is created by varying the value of \( L \) (length of the product life cycle) and the value of \( M \) (maximum expected sales level). This table is produced based on the parameters considered by Serrato et al. (2007) in their model and it gives the number of observations for various values of \( L \) and \( M \).

*Table 5.18: No. of observations varying \( L \) and \( M \) for model by Serrato et al. (2007)*

<table>
<thead>
<tr>
<th></th>
<th>( M = 3 )</th>
<th>( M = 4 )</th>
<th>( M = 5 )</th>
<th>( M = 6 )</th>
<th>( M = 7 )</th>
<th>( M = 8 )</th>
<th>( M = 9 )</th>
<th>( M = 10 )</th>
<th>( M = 11 )</th>
<th>( M = 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 3 )</td>
<td>123</td>
<td>256</td>
<td>360</td>
<td>617</td>
<td>973</td>
<td>1213</td>
<td>1759</td>
<td>2446</td>
<td>2878</td>
<td>3821</td>
</tr>
<tr>
<td>( L = 4 )</td>
<td>632</td>
<td>1042</td>
<td>2681</td>
<td>3799</td>
<td>7722</td>
<td>10087</td>
<td>17798</td>
<td>22101</td>
<td>35485</td>
<td>42569</td>
</tr>
<tr>
<td>( L = 5 )</td>
<td>1489</td>
<td>7353</td>
<td>14346</td>
<td>25392</td>
<td>67992</td>
<td>88354</td>
<td>192988</td>
<td>273503</td>
<td>376743</td>
<td>686625</td>
</tr>
</tbody>
</table>

The 3D plot shows that the number of observations rises rapidly with increment of both \( L \) and \( M \) values.
Figure 5.21: No. Of observations varying L and M for model by Serrato et al. (2007)

The following table gives a clear picture about the time taken to solve the model for Serrato et al., (2007).

Table 5.19: Time (sec) taken to solve for model by Serrato et al. (2007)

<table>
<thead>
<tr>
<th>M=3</th>
<th>M=4</th>
<th>M=5</th>
<th>M=6</th>
<th>M=7</th>
<th>M=8</th>
<th>M=9</th>
<th>M=10</th>
<th>M=11</th>
<th>M=12</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=3</td>
<td>0.079832</td>
<td>0.138292</td>
<td>0.098447</td>
<td>0.121204</td>
<td>0.151858</td>
<td>0.175018</td>
<td>0.23189</td>
<td>0.351913</td>
<td>0.473287</td>
</tr>
<tr>
<td>L=4</td>
<td>0.126256</td>
<td>0.172148</td>
<td>0.52174</td>
<td>1.294092</td>
<td>4.121378</td>
<td>4.90752</td>
<td>19.8255</td>
<td>32.86074</td>
<td>84.12181</td>
</tr>
<tr>
<td>L=5</td>
<td>0.302034</td>
<td>3.875137</td>
<td>13.77272</td>
<td>43.84063</td>
<td>280.5314</td>
<td>472.5438</td>
<td>2089.752</td>
<td>4076.094</td>
<td>9320.311</td>
</tr>
</tbody>
</table>
The 3D plot shows that time taken to solve the model rises rapidly with increment of both \( L \) and \( M \) values.

![3D plot showing time taken to solve the model](image)

**Figure 5.22: Time (sec) taken to solve Serrato et al. (2007) model**

### 5.11 Complexity

In the current model, (scenario 1, dynamic capacity model) there is a while loop and it continues \( n \) times 1 nested for loop and there are also 7 *if-else* statements which has a cost of \((1*6) = O(6)\). The overall costing is \( O(n^2+6) \). In this model \( n \) is larger than 6 is and negligible compared to \( n \). So the final complexity of the algorithm is \( O(n^2) \). In the second model (stationary capacity model), there is also a while loop 1 nested for loop and only 5 *if-else* statements. So the final complexity of the algorithm is also \( O(n^2) \).
On the other hand, complexity of the model given in Serrato et al. (2007) has 2 \textit{while-while} nested loops, 1 \textit{while-for} nested loop and 1 \textit{for-for} nested loop. It also has 4 \textit{for-if-else} nested loop and 2 \textit{if-else} loop. So the complexity of this model is $O(4n^2+6n)$. It can be written as $O(n^2)$. 
CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

Reverse logistics can be profitable for any company if applied correctly. The RL network is characterized using two critical factors; product life cycle and variance in return rate, which correspond to the first objective in this research. The convenience of using this categorization to analyze RL is covered in the research work.

The second and the third research objectives are addressed by developing a Markov decision model for RL systems, which modeled the RL outsourcing decision based upon the implied uncertainty of the return rate. Several critical elements are considered in the model which is important characteristics of a RL network, i.e., return rate, length of product life cycle, sales performance, costs incurred in reverse logistics. The time length of the existence of RL system is also considered in the problem. The uncertainty implied in the model is represented by the expected amount of returned units which is defined by the outstanding units in the market and the rate of return considered.

There is a hypothesis that outsourcing is a suitable option when the return rate had greater variance. This comment is supported by showing analytically that the probability of crossing the threshold is larger when there is greater variance in the return rate. The life cycle is also a factor and it is proven that the longer life cycle had a smaller probability of crossing the threshold than shorter life cycle.
Two different scenarios are created using a Matlab program. In the first instance the capacity is considered dynamic while in the second instance the capacity is considered static. These two scenarios are considered for the variance in return rate and also with product life cycle. Both the results proved the hypothesis: outsourcing is a more suitable option when the variance in return rate is high and also the outsourcing is a better option when the product life cycle is short in nature.

6.2 Future work

The conclusion stated above suggested that all the research objectives are achieved.

Only remanufacturing is considered for outsourcing in this research. There are other RL activities that can be considered for outsourcing too, for example, return acquisition, central recovery centers, transportation, and disassembly centers. Any one of these or all together can be considered for outsourcing.

Some relative costs are considered in this research but it is felt that there are also other costs inclusion of which can depict a more real life scenario.

The number of returned products is considered following Poisson distribution, what if they followed any other distribution? Whether it is possible to create a better sales function comprising of more than maximum sales expected and life cycle? What if the 3PRLP fail to perform adequately? What if profit is considered for the RL activities?

These questions represent an interesting basis for any further research.
APPENDIX A

In this research, return rates are generated using random number generator from the StatFit from the Pro Model software. Different distribution is considered for this random number generation. Different distributions considered for this experiment are: Erlang, Exponential, Gamma, Normal, Triangular, Uniform and Weibull distribution. The results are then considered as input for the two scenarios and the resulting number of returned products is put into the StatFit to see which distribution they follow.

Lognormal distribution

Table A1: Random number generated following Lognormal distribution

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>0.96</td>
<td>0.35</td>
<td>0.33</td>
<td>0.23</td>
<td>0.15</td>
<td>0.22</td>
<td>0.28</td>
<td>0.3</td>
<td>0.35</td>
</tr>
<tr>
<td>0.4</td>
<td>0.41</td>
<td>0.42</td>
<td>0.43</td>
<td>0.44</td>
<td>0.44</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

The parameters considered to generate these values are

(minimum = 2.74e -002, \( \mu = -1.14 \), \( \sigma = 0.439 \))

The values have been put into the StatFit and the result is as follows:
Figure A1: Random number generated following Lognormal distribution

The result shows that the random number generated follows Lognormal distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products ($x_i$) is checked to see which distribution they follow.
Figure A2: Number of returned products for Lognormal distribution

The result shows that if the return rate is considered following Lognormal distribution then the number of returned products follows Poisson distribution.

Erlang distribution

Table A2: Random numbers generated following Erlang distribution

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>0.28</td>
<td>0.31</td>
<td>0.16</td>
<td>0.04</td>
<td>0.10</td>
<td>0.22</td>
<td>0.76</td>
<td>0.09</td>
</tr>
<tr>
<td>0.11</td>
<td>0.17</td>
<td>0.30</td>
<td>0.04</td>
<td>0.21</td>
<td>0.23</td>
<td>0.37</td>
<td>0.28</td>
<td>0.08</td>
</tr>
</tbody>
</table>
The parameters considered to generate these values are

\[(\text{minimum} = 4.\times 10^{-02}, \, m = 1. \, , \, \beta = 0.173)\]

The values have been put into the StatFit and the result is as follows:

\[\text{Figure A3: Random numbers generated following Erlang distribution}\]

The result shows that the random number generated follows Erlang distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.
These numbers are now put into the models created in Matlab and the number of returned products ($x_i$) is checked to see which distribution they follow.

Figure A4: Number of returned products for Erlang distribution

The result shows that if the return rate is considered Erlang then the number of returned products does not follow Poisson distribution.
Exponential distribution

Table A3: Random number generated following Exponential distribution

<table>
<thead>
<tr>
<th>0.59</th>
<th>0.06</th>
<th>0.49</th>
<th>0.53</th>
<th>0.25</th>
<th>0.20</th>
<th>0.14</th>
<th>0.05</th>
<th>0.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.04</td>
<td>0.02</td>
<td>0.44</td>
<td>0.36</td>
<td>0.30</td>
<td>0.37</td>
<td>0.49</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The parameters considered to generate these values are

(minimum = 1.e-002, $\beta = 0.258$)

The values have been put into the StatFit and the result is as follows:

![Image of StatFit output for Exponential distribution]

Figure A5: Random number generated following Exponential distribution
The result shows that the random number generated follows Exponential distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products \((x_0)\) is checked to see which distribution they follow.

*Figure A6: Number of returned products for Exponential distribution*

The result shows that if the return rate is considered following Exponential distribution then the number of returned products does not follow Poisson distribution.
Gamma distribution

Table A4: Random number generated following Gamma distribution

<table>
<thead>
<tr>
<th>0.42</th>
<th>0.60</th>
<th>0.14</th>
<th>0.19</th>
<th>0.25</th>
<th>0.15</th>
<th>0.53</th>
<th>0.46</th>
<th>0.18</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.78</td>
<td>0.57</td>
<td>0.71</td>
<td>0.85</td>
<td>0.18</td>
<td>0.57</td>
<td>0.93</td>
<td>0.91</td>
</tr>
</tbody>
</table>

The parameters considered to generate these values are

(minimum = -2.19, $\alpha = 102$, $\beta = 2.63\times10^{-2}$)

The values have been put into the StatFit and the result is as follows:

Figure A7: Random number generated following Gamma distribution
The result shows that the random number generated follows Gamma distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products \((x_0)\) is checked to see which distribution they follow.

\[\text{Figure A8: Number of returned products for Gamma distribution}\]

The result shows that if the return rate is considered following Gamma distribution then the number of returned products does not follow Poisson distribution.
Normal distribution

Table A5: Random number generated following Normal distribution

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<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.82</td>
<td>0.58</td>
<td>0.24</td>
<td>0.28</td>
<td>0.82</td>
<td>0.43</td>
<td>0.19</td>
<td>0.48</td>
</tr>
<tr>
<td>0.20</td>
<td>0.56</td>
<td>0.17</td>
<td>0.21</td>
<td>0.46</td>
<td>0.29</td>
<td>0.63</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.34</td>
</tr>
</tbody>
</table>

The parameters considered to generate these values are

\((\mu = 0.387, \sigma = 0.219)\)

The values have been put into the StatFit and the result is as follows:

Figure A9: Random number generated following Normal distribution
The result shows that the random number generated follows Normal distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products \( (x_0) \) is checked to see which distribution they follow.

![Figure A10: Number of returned products for Normal distribution](image)

The result shows that if the return rate is considered following Normal distribution then the number of returned products does not follow Poisson distribution.
Triangular distribution

Table A6: Random number generated following Triangular distribution

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.51</td>
<td>0.57</td>
<td>0.52</td>
<td>0.58</td>
<td>0.22</td>
<td>0.64</td>
<td>0.21</td>
<td>0.80</td>
</tr>
<tr>
<td>0.40</td>
<td>0.39</td>
<td>0.43</td>
<td>0.67</td>
<td>0.20</td>
<td>0.60</td>
<td>0.39</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.46</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.83</td>
<td></td>
</tr>
</tbody>
</table>

The parameters considered to generate these values are

(minimum = 7.72e-002, maximum = 0.901, mode = 0.57)

The values have been put into the StatFit and the result is as follows:

![Figure A11: Random number generated following Triangular distribution](image-url)
The result shows that the random number generated follows Triangular distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products ($x_o$) is checked to see which distribution they follow.

![Image of fitted distributions and data table]

*Figure A12: Number of returned products for Triangular distribution*

The result shows that if the return rate is considered following Triangular distribution then the number of returned products does not follow Poisson distribution.
**Uniform distribution**

**Table A7: Random number generated following Uniform distribution**

<table>
<thead>
<tr>
<th>0.50</th>
<th>0.51</th>
<th>0.40</th>
<th>0.25</th>
<th>0.08</th>
<th>0.23</th>
<th>0.26</th>
<th>0.52</th>
<th>0.34</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.64</td>
<td>0.88</td>
<td>0.08</td>
<td>0.53</td>
<td>0.89</td>
<td>0.69</td>
<td>0.87</td>
<td>0.21</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The parameters considered to generate these values are

(minimum = 8.e-002, maximum = 0.89)

The values have been put into the StatFit and the result is as follows:

![Fitted Density](chart.png)

**Figure A13: Random number generated following Uniform distribution**
The result shows that the random number generated follows Uniform distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products ($x_0$) is checked to see which distribution they follow.

Figure A14: Number of returned products for Uniform distribution

The result shows that if the return rate is considered following Uniform distribution then the number of returned products does not follow Poisson distribution.
The parameters considered to generate these values are

(minimum = 0, \( \alpha = 1.94 \), \( \beta = 0.524 \))

The values have been put into the StatFit and the result is as follows:
The result shows that the random number generated follows Weibull distribution. Although due to very small numbers generated, the numbers can be seen fit for a numerous number of distributions.

These numbers are now put into the models created in Matlab and the number of returned products \( (x_o) \) is checked to see which distribution they follow.

![Figure A16: Number of returned products for Weibull distribution](image)

The result shows that if the return rate is considered following Weibull distribution then the number of returned products does not follow Poisson distribution.
APPENDIX B I

Scenario 1

clear
clc

L = 18; %Length of the product life cycle
W = 4; %For how long the returns will be taken back
T = L+W;

sales = [11 22 33 44 56 67 78 89 100 100 89 78 67 56 44 33
22 11 0 0 0 0]; %Sales values calculated from the sales function
r = [0.96 0.52 0.34 0.37 0.32 0.18 0.26 0.15 0.29 0.35 0.24 0.2 0.33 0.12 0.15 0.2 0.19 0.21
0.11 0.08 0.06 0]; %Return rate
C = [2 2 3 4 17 12 40 8 5 3]; %Costs

sum_sales = 0; %Total sales initialization
t = 1; %Time initialization
wt = 0; %Total return initialized

RL = zeros(22,10);
ktemp = zeros(2,24);

while t < T+1

    if t <= 1

        ktemp(1,t) = 0; %deciding on the capacity

    else

        ktemp(1,t) = ktemp(2,t-1); %deciding on the capacity

    end

    RL(t,1) = t; %Time epoch
    sum_sales = sum_sales + sales(t); %Total sales
    nt = sum_sales - wt; %Products outstanding in the market
    x = nt*r(t); %Expected return
    RL(t,4) = ceil(x); %Rounded up because of the small amounts
    wt = wt + RL(t,4); %Total returns
    RL(t,2) = sum_sales - wt; %Products outstanding in the market at the end of time epoch
    RL(t,3) = ktemp(1,t); % Capacity
    RL(t,5) = wt; %Total returns
RLSum8 = 0;
RLSum9 = 0;

for y = 0:nt

    transportation_cost = c(8)*y; %Transportation cost

    if y >= RL(t,3) %If return is more than current capacity
        inspection_cost = c(10)*y; %Inspection cost
    else
        inspection_cost = c(10)*RL(t,3); %Inspection cost
    end

    fixed_cost = c(3); %Fixed cost
    variable_cost = c(4)*y; %Variable cost

    if y >= RL(t,3) %If return is more than current capacity
        inventory_cost = c(9)*y; %Inventory cost
        shortage_cost = c(5)*y; %Shortage cost
        investment_cost = c(1)*(y-RL(t,3)); %Investment cost
        idle_cost = 0; %Unutilized cost
    else
        inventory_cost = c(9)*RL(t,3); %Inventory cost
        shortage_cost = c(5)*(RL(t,3)-y); %Shortage cost
        idle_cost = c(2)*(RL(t,3)-y); %Unutilized cost
        investment_cost = 0;
    end

    RL(t,9) = transportation_cost+inspection_cost+fixed_cost+inventory_cost+variable_cost+shortage_cost+investment_cost+idle_cost; %In-house cost
    RL(t,8) = c(7)*y+c(6)*RL(t,3)+c(2)*RL(t,3); %Outsource cost

    poissonValue = poisspdf(y,x);

    RLSum8 = RLSum8 + RL(t,8)* poissonValue;
    RLSum9 = RLSum9 + RL(t,9)* poissonValue;

    RLSUM8Two(t,y+1) = RL(t,8);
    RLSUM9Two(t,y+1) = RL(t,9);
    RLSprob(t,y+1) = poissonValue;

```matlab
y = y + 1;

end

RL(t,8) = RLSum8;
RL(t,9) = RLSum9;

if RL(t,9) > RL(t,8)
    ktemp(2,t) = ktemp(1,t);
else
    if ceil(x) < ktemp(1,t)
        ktemp(2,t) = ktemp(1,t);
    else
        ktemp(2,t) = ceil(x);
    end
end

RL(t,3) = ktemp(2,t); %Capacity

syms b;

if x >= RL(t,3)
    z = solve(c(8)*b+c(10)*b+c(3)+c(4)*b+c(9)*b+c(5)*b+c(1)*(b-RL(t,3))-c(7)*b-c(6)*RL(t,3)-c(2)*RL(t,3));
    a = double(z);
    if a <= 0
        a = 0;
    else
        a = a;
    end
else
    z = solve(c(8)*b+c(10)*RL(t,3)+c(3)+c(4)*b+c(9)*RL(t,3)+c(5)*(RL(t,3)-b)+c(2)*(RL(t,3)-b)-c(7)*b-c(6)*RL(t,3)-c(2)*RL(t,3));
    a = double(z);
end

RL(t,6) = poisscdf(ceil(a),x); %Transition probability for outsourcing
```
RL(t,7) = 1-RL(t,6); %Transition probability for in-house remanufacturing

RL(t,10) = nt;
t=t+1;
end
APPENDIX B II

Scenario 2

clear
c1c

L = 18;
W = 4;
T = L+W;

sales = [11 22 33 44 56 67 78 89 100 100 89 78 67 56 44 33 22 11 0 0 0]; %Sales calculated from the sales function
r = [0.96 0.52 0.34 0.37 0.32 0.18 0.26 0.15 0.17 0.29 0.35 0.24 0.2 0.33 0.12 0.15 0.2 0.19 0.21 0.11 0.08 0.06 0 0 0]; %Return rate following Lognormal distribution
c = [2 3 5 16 12 40 8 5 3]; %Costs

sum_sales = 0; %Total sales initialization
t = 1; %Time initialization
wt = 0; %Total return initialized
kt = 50; %Fixed capacity

RL = zeros(22,15);

while t < T+1
    RL(t,1) = t; %Time epoch
    sum_sales = sum_sales + sales(t); %Total sales
    nt = sum_sales - wt; %Products outstanding in the market
    x = nt*r(t); %Expected return
    RL(t,4)= ceil(x);
    wt = wt + RL(t,4); %Total return
    RL(t,2) = sum_sales - wt; %Final number of products in the market
    RL(t,3) = wt; %Total return
    RLSum8 = 0;
    RLSum9 = 0;
    RLSum10 = 0;
    RLSum11 = 0;
    for y = 0:nt
        transportation_cost = c(7)*y; %Transportation cost
        if y >= kt %If return is more than current capacity
            inspection_cost = c(9)*y; %Inspection cost
        end
    end
end
else

    inspection_cost = c(9)*kt;  %Inspection cost

end

fixed_cost = c(2);  %Fixed cost
variable_cost = c(3)*y;  %Variable cost
poissonValue = poisspdf(y,x);

if y >= kt  %If return is more than current capacity

    inventory_cost = c(8)*y;  %Inventory cost
    shortage_cost = c(4)*y;  %Shortage cost
    RL(t,13) = c(6)*(y-kt) + inventory_cost + variable_cost + shortage_cost + transportation_cost + inspection_cost + fixed_cost;  %Split cost
    RL(t,12) = c(6)*y + c(5)*kt + c(1)*kt;  %Outsource cost
    RLSum8 = RLSum8 + RL(t,12) * poissonValue;
    RLSum9 = RLSum9 + RL(t,13) * poissonValue;

else

    inventory_cost = c(8)*kt;  %Inventory cost
    shortage_cost = c(4)*(kt-y);  %Shortage cost
    idle_cost = c(1)*(kt-y);  %Unutilized cost
    RL(t,15) = transportation_cost + inspection_cost + fixed_cost + inventory_cost + variable_cost + shortage_cost + idle_cost;  %In-house cost
    RL(t,14) = c(6)*y + c(5)*kt + c(1)*kt;  %Outsource cost
    RLSum10 = RLSum10 + RL(t,14) * poissonValue;
    RLSum11 = RLSum11 + RL(t,15) * poissonValue;

end

RLSUM8Two(t,y+1) = RL(t,12);
RLSUM9Two(t,y+1) = RL(t,13);
RLSUM10Two(t,y+1) = RL(t,14);
RLSUM11Two(t,y+1) = RL(t,15);
RLprob(t,y+1) = poissonValue;

y = y + 1;

dasis is the initial

if x > kt
    RL(t,8) = RLSum8;
    RL(t,9) = RLSum9;
else
    RL(t,10) = RLSum10;
    RL(t,11) = RLSum11;
end
RL(t,3) = kt;
syms b;

if x >= kt
    z = solve(c(7)*b+c(9)*b+c(2)+c(3)*b+c(8)*b+c(4)*b+c(6)*(b-kt) -
              c(6)*b-c(5)*kt-c(1)*kt);
    a = double(z);
    if a <= 0
        a = 0;
    else
        a = a;
    end
else
    z = solve(c(7)*b+c(9)*kt+c(2)+c(4)*b+c(8)*kt+c(4)*(kt-b) +
              c(1)*(kt-b)-c(6)*b-c(5)*kt-c(1)*kt);
    a = double(z);
end
RL(t,6) = poisscdf(ceil(a),x); %Transition probability
RL(t,7) = 1-RL(t,6);
t=t+1;
end
**APPENDIX C**

*Table C 1: Scenario 1 with lower variance (L = 10, W = 4)*

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>n</td>
<td>k</td>
<td>x</td>
<td>w</td>
<td>P (Outsourcing)</td>
<td>P (In-house)</td>
<td>Outsourcing cost</td>
<td>In-house cost</td>
<td></td>
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<td>14</td>
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## APPENDIX D

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