2016

In-plane dynamic behaviour of conventional and hybrid cable network systems on cable-stayed bridges

Javaid Ahmad
University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
Ahmad, Javaid, "In-plane dynamic behaviour of conventional and hybrid cable network systems on cable-stayed bridges" (2016). Electronic Theses and Dissertations. 5795.
https://scholar.uwindsor.ca/etd/5795

This online database contains the full-text of PhD dissertations and Masters' theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000ext. 3208.
In-plane dynamic behaviour of conventional and hybrid cable network systems on cable-stayed bridges

By

Javaid Ahmad

A Dissertation
Submitted to the Faculty of Graduate Studies
through the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy
at the University of Windsor

Windsor, Ontario, Canada

2016

© 2016 Javaid Ahmad
In-plane dynamic behaviour of conventional and hybrid cable network systems on cable-stayed bridges

by

Javaid Ahmad

APPROVED BY:

______________________________
L. Caracoglia, External Examiner
Department of Civil and Environmental Engineering, Northeastern University, Boston

______________________________
V. Stoilov, Outside Reader
Mechanical, Automotive & Materials Engineering

______________________________
S. Das, Department Reader
Civil and Environmental Engineering

______________________________
A. El Ragaby, Department Reader
Civil and Environmental Engineering

______________________________
S. Cheng, Advisor
Civil and Environmental Engineering

______________________________
F. Ghrib, Co-Advisor
Civil and Environmental Engineering

Aug. 19, 2016
DECLARATION OF CO-AUTHORSHIP/PREVIOUS PUBLICATION

I. Co-Authorship Declaration

I hereby declare that this dissertation incorporates material that is the result of joint research undertaken with my Advisors, Dr. Shaohong Cheng and Dr. Faouzi Ghrib of the University of Windsor. In all cases, the key ideas, the primary contributions, and data analysis and interpretation were performed by the author of this dissertation. The contributions of the co-authors were primarily focused on the provision of the study and suggesting possible directions. Results related to this research are reported in Chapters 3 through 9, inclusive.

I am aware of the University of Windsor's Senate Policy on Authorship and I certify that I have properly acknowledged the contributions of the other researchers to my dissertation, and I have obtained written permission from my co-authors to include the above materials in my dissertation.

I certify that, with the above qualification, this dissertation, and the research to which it refers to, is the product of my own work.

II. Declaration of Previous Publication

There are 11 original papers based on the contents of the dissertation that have been previously published/submitted for publication in peer reviewed journals, as follows:

<table>
<thead>
<tr>
<th>Dissertation Chapter</th>
<th>Publication Title</th>
<th>Publication Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapters 3</td>
<td>Analytical study on in-plane free vibration of a cable network with straight alignment rigid cross-ties</td>
<td>Journal of Vibration and Control: 2015, Vol. 21(7), 1299–1320</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>Effect of cross-tie solution on the modal frequency and modal damping of cable networks</td>
<td>Proc: 7th IABMAS, Shanghai, China, 2014, ID: 05714P</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>Analytical study on in-plane modal properties of a general cable network</td>
<td>Proc: 9th Int. Symp. on Cable Dynamics, Shanghai, China, 2011, pp.39–46.</td>
</tr>
<tr>
<td>Chapter 7</td>
<td>Analytical study on the in-plane dynamic behaviour of hybrid cable networks and design optimization</td>
<td>Structural Control and Health Monitoring (Submitted)</td>
</tr>
<tr>
<td>Chapter 7</td>
<td>Formulating analytical model of conventional and hybrid cable system using a generalized approach</td>
<td>Journal of Sound and Vibration (Submitted)</td>
</tr>
</tbody>
</table>

I declare that, to the best of my knowledge, my dissertation does not infringe upon anyone’s copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my dissertation, published or otherwise, are fully acknowledged in accordance with the
standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owners to include such materials in my dissertation.

I declare that this is a true copy of my dissertation, including any final revisions, as approved by my dissertation committee and the Graduate Studies office, and that this dissertation has not been submitted for a higher degree to any other University or Institution.
ABSTRACT

Stay cables on cable-stayed bridges are vulnerable to dynamic excitations due to their long flexible feature and low intrinsic damping. Connecting a vulnerable cable with the neighbouring ones through cross-ties to form a cable network is one of the commonly used field solutions. The current dissertation is dedicated to explore the in-plane dynamic behaviour of the conventional (cross-tie only) and hybrid (combined use of cross-ties and external dampers) cable networks used for controlling undesirable bridge stay cable vibrations. Their performances are evaluated based on the system in-plane stiffness, damping and the severity of local mode formation.

A number of analytical models have been developed to analyze the in-plane modal response of conventional cable networks by gradually extending the model of a basic undamped two-cable network with a rigid cross-tie to include the cross-tie stiffness, the damping property of main cables and cross-tie, and more number of main cables and cross-tie lines into the formulation. A damping transfer phenomenon between cable network elements having different damping properties was observed. Two criteria, the degree of mode localization (DML) coefficient and the local mode cluster (LMC), were proposed to quantify the severity of local mode formation. Based on the proposed analytical models, key system parameters which dictate the dynamic behaviour of conventional cable networks were identified. A parametric study was conducted to explore their respective role in influencing the in-plane stiffness, the damping ratio and the local mode formation of cable networks.

Analytical models of two-cable hybrid networks with different configurations have been developed to assess the system in-plane modal behaviour. A concept of
“isoquant curve” was proposed to optimize the performance of a selected hybrid system mode. A state-of-the-art generalized approach was developed to derive analytical models of a more complex conventional or hybrid cable network from a relatively simple parent system. Results indicated that the existing universal damping estimation curve for a single isolated damped cable was no longer applicable once the cable became part of a hybrid system. Thus, approximate relation equations were developed to predict the optimum damper size and the maximum attainable fundamental modal damping ratio of a basic two-cable hybrid system.

All the proposed analytical models were validated through independent numerical simulations using the commercial finite element software Abaqus 6.10. Besides, an experimental study was conducted for two-cable conventional and hybrid networks to not only verify the validity of the corresponding analytical and numerical models, but also evaluate the impact of different assumptions made in the formulation of these models on the system modal response.

The outcomes yielded from this study are expected to add valuable knowledge to comprehend the current understanding of the mechanics associated with the conventional and hybrid cable networks. The developed tools will greatly contribute to the bridge industry by assisting optimum design of conventional and hybrid cable networks, especially in the preliminary design stage. Besides, it is worthy pointing out that the current findings will also contribute to the knowledge of structural health monitoring, assessment and management of bridges, and the development of more sustainable civil infrastructures.
DEDICATION

I would like to dedicate my work to my mother and the memory of my father who made this work easy for me, and to my wife, Riffat.
ACKNOWLEDGEMENTS

I would like to express my deepest appreciation and gratitude to my research advisors, Dr. Shaohong Cheng and Dr. Faouzi Ghrib, for their patient guidance, fruitful discussions, and kind support during the course of this study. I believe such a technical research would not have been possible without their kind supervision.

Also I would like to thank Dr. Vesselin Stoilov for his technical assistance to solve some of the mathematical problems. The advice and comments made by Dr. Sreekanta Das and Dr. Amr El Ragaby are also assets for me to complete this project.

I must thank Dr. Luca Caracoglia at the Northeastern University, Boston, USA for taking the responsibility as the external reviewer of my dissertation.

I would like to thank laboratory technologist Mr. Lucian Pop for his help and suggestions with the experimental setup, and computer systems administrator Mr. Mark Gryn for his help for all the computer related issues. Lastly, I would also like to thank my colleagues, Ali, Kashif, and Gnanasekaran for their incredible help and support.

Finally, I would like to thank my family, Riffat, Usama, Sara and Talha, for their love, endless support and encouragement.
# TABLE OF CONTENTS

DECLARATION OF CO-AUTHORSHIP/PREVIOUS PUBLICATION iii

ABSTRACT vi

DEDICATION viii

ACKNOWLEDGEMENTS ix

LIST OF FIGURES xiv

LIST OF TABLES xxiii

CHAPTER 1 Introduction 1

1.1 Background 1

1.2 Types of wind-induced cable vibrations 3

1.3 Countermeasures 6

1.4 Motivations 12

1.5 Objectives 14

CHAPTER 2 Literature Review 16

2.1 Free vibration of a single cable 16

2.2 Modal behaviour of cable networks 19

2.2.1 In-plane frequency 19

2.2.2 Modal damping 23

2.2.3 Energy Distribution 26

2.2.4 Local mode formation 27

2.2.5 Role of different system parameters 29

2.3 Hybrid system 31

2.4 Summary 33

CHAPTER 3 Analytical Study on Modal Behaviour of Cable Networks 35

3.1 Undamped Two-Cable Networks 35

3.1.1 System characteristic equation 36

3.1.2 Twin-cable network with a flexible cross-tie at arbitrary location 40

3.1.3 Symmetric two-cable network with a flexible cross-tie at mid-span 47

3.1.4 Asymmetric two-cable network with a flexible cross-tie at one-third span 50

3.2 Damped Two-Cable Network 54
3.2.1 System characteristic equation 56
3.2.2 Application examples 62
  3.2.2.1 Twin-cable rigid cross-tie network 63
  3.2.2.2 Symmetric DMT two-cable rigid cross-tie network 70
  3.2.2.3 Asymmetric DMT two-cable rigid cross-tie network 80
  3.2.2.4 Twin-cable damped flexible cross-tie network 87
  3.2.2.5 Symmetric DMT damped flexible cross-tie Cable Network 96
  3.2.2.6 Asymmetric DMT damped flexible cross-tie Cable Network 102

3.3 Summary 108

CHAPTER 4 Modal Analysis of General Cable Network 110
  4.1 Formulation of analytical model 110
  4.2 Application to cable networks with real configurations 114
    4.2.1 Five Cable Network 115
    4.2.2 Cable Network on the Fred Hartman Bridge 118
  4.3 Summary 124

CHAPTER 5 Formation of Local Modes 126
  5.1 Mode localization in cable-stayed bridges 127
  5.2 Degree of mode localization and system parameters 128
    5.2.1 Degree of mode localization 135
    5.2.2 Role of system parameters on degree of mode localization 136
    5.2.2.1 Frequency ratio 138
    5.2.2.2 Mass-tension ratio 145
    5.2.2.3 Cross-tie stiffness 149
  5.3 Local mode clusters 154
    5.3.1 Cross-tie position effect 157
    5.3.2 Cross-tie stiffness effect 167
    5.3.3 Effect of number of cross-tie lines 174
  5.4 Summary 176

CHAPTER 6 Effect of System Parameters on Modal Behaviour of Cable Networks 179
  6.1 Length ratio 181
  6.2 Frequency ratio 189
  6.3 Mass-tension ratio 194
  6.4 Cross-tie position 199
6.5 Cross-tie stiffness
6.6 Number of cross-tie lines
6.7 Summary

CHAPTER 7 Hybrid System
7.1 Introduction
7.2 Formulation of the Analytical Model
7.3 Model validation
7.4 Generalized approach
   a) A single taut cable attached with a transverse linear viscous damper
   b) A single taut cable with a transverse linear viscous damper and a transverse linear spring
   c) A single taut cable with two transverse linear viscous dampers
   d) A two-cable network with connection to ground
   e) A two-cable network with transverse linear viscous dampers installed on both cables
7.5 Parametric Study
   7.5.1 Cross-tie stiffness
   7.5.2 Spacing between damper and cross-tie
   7.6 Design Optimization
7.7 Damping estimation curves for hybrid systems
   7.7.1 Effect of cross-tie stiffness on the damping estimation curve
   7.7.2 Effect of cross-tie position on the damping estimation curve
   7.7.3 Approximate relation equation for estimating damping in a two-cable hybrid system
7.8 Summary

CHAPTER 8 Experimental Study on the In-plane Modal Behaviour of Pure Cable Networks and Hybrid Systems
8.1 Experimental setup
   8.1.1 Main cables
   8.1.2 Cross-ties
   8.1.3 Passive linear viscous damper
   8.1.4 Load cells
   8.1.5 Hydraulic pumps
   8.1.6 Electronic dynamic smart shaker
   8.1.7 Signal generator
8.1.8 Accelerometer 307
8.1.9 Data acquisition (DAQ) system 307

8.2 Test procedures 308
8.2.1 Free vibration test 308
8.2.2 Forced vibration test 312

8.3 Experimental results and discussion 315
8.3.1 Cable network 316
8.3.2 Hybrid systems 325

8.4 Summary 339

CHAPTER 9 Conclusions and Recommendation 341

9.1 Conclusions 341
9.1.1 In-plane stiffness 343
9.1.2 Damping increment 345
9.1.3 Formation of local modes 346
9.1.4 Hybrid system 347

9.2 Recommendations 348

Appendix A: Copyright permission 351
Appendix B: Copyright permission 352

REFERENCES 353

VITA AUCTORIS 366
LIST OF FIGURES

Figure 2.1 Components of displacement for a single suspended cable vibration 17
Figure 2.2 Schematic diagram of experimental model used by Yamaguchi and Nagahawatta (1995) 19
Figure 2.3 Modeling cable network on the Fred Hartman Bridge. (a) Three-dimensional network on real bridge; (b) Equivalent two-dimensional model used by Caracoglia and Jones (2005b) 23
Figure 2.4 Schematic diagram of experimental model used by Yamaguchi et al. (2001) 25
Figure 2.5 Schematic diagram of experimental model used by Sun et al. (2007) 26
Figure 2.6 Local mode plateaus for different configuration of cross-ties observed by Caracoglia and Jones (2005b) 28
Figure 2.7 Model developed by Zhou et al. (2011) where single taut cable is connected by multiple springs 30
Figure 3.1 Schematic diagram of the mathematical model for an undamped two-cable network 36
Figure 3.2 Schematic diagram of a symmetric twin-cable network with flexible cross-tie at arbitrary point 40
Figure 3.3 Transformation of first ten modes of a symmetric twin-cable network as flexibility parameter $\Psi$ varies from 0 to 1.0 and cross-tie locates at quarter span 44
Figure 3.4 Evolution of LS and RS modes of a symmetric twin-cable network with cross-tie located at quarter span and flexibility parameter $\psi$ varies from 0 to 1.0 46
Figure 3.5 Symmetric SMT two-cable network with unequal length main cables and flexible cross-tie 47
Figure 3.6 A few typical modes of a symmetric SMT cable network with system parameters as frequency ratio $\eta_2=0.883$, segment ratio $\epsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi=1.0$ 50
Figure 3.7 First ten modes of an asymmetric DMT two-cable network with a flexible 53
cross-tie ($\psi=1.0$) at $\varepsilon=1/3$

Figure 3.8 Schematic diagram of the mathematical model for a damped two-cable network

Figure 3.9 Non-dimensional modal damping ratio as a function of the non-dimensional cross-tie position, $\varepsilon$, for a twin-cable network

Figure 3.10 Mode shapes (real part) of global and local modes having the same modal frequency and modal damping: a) $\varepsilon=1/3$, b) $\varepsilon=1/2$, c) $\varepsilon=3/4$

Figure 3.11 Selected modes (real part) of a twin-cable network with a rigid cross-tie at 1/3 span

Figure 3.12 First four modes (real part) of a symmetric DMT two-cable system with a rigid cross-tie at mid-span

Figure 3.13 Non-dimensional damping ratio of the fundamental mode as a function of non-dimensional damping relation parameter for a symmetric DMT two-cable network

Figure 3.14 Non-dimensional damping ratio of the fundamental mode as a function of cross-tie position for a symmetric DMT two-cable network

Figure 3.15 Optimum cross-tie position range for a symmetric DMT two-cable network

Figure 3.16 First ten modes of an asymmetric two-cable system with a rigid cross-tie at three-quarter span

Figure 3.17 Optimum cross-tie position range for an asymmetric two-cable network

Figure 3.18 First ten modes of a twin-cable network with a damped flexible cross-tie ($K_c=30.54$ kN/m, $C_c=1.0$ kN·s/m) at $\varepsilon=1/3$

Figure 3.19 First ten modes of a twin-cable network with a damped flexible cross-tie ($K_c=30.54$ kN/m, $C_c=1.0$ kN·s/m) at $\varepsilon=2/5$

Figure 3.20 First ten modes of a symmetric DMT two-cable network with a damped flexible cross-tie ($K_c=30.54$ kN/m, $C_c=1.0$ kN·s/m) at $\varepsilon=1/2$

Figure 3.21 First ten modes of an asymmetric DMT two-cable network with a damped flexible cross-tie ($K_c=30.54$ kN/m, $C_c=1.0$ kN·s/m) at $\varepsilon=1/3$

Figure 3.22 Effect of undamped cross-tie stiffness parameter $\psi_o$ on modal frequency and modal damping ratio of the lowest in-phase and out-of-phase global
modes of an asymmetric DMT cable network

Figure 3.23 Effect of cross-tie damping coefficient $C$ on modal frequency and modal damping ratio of the lowest in-phase and out-of-phase global modes of an asymmetric DMT cable network

Figure 4.1 Schematic diagram of general cable network with multiple lines of cross-ties

Figure 4.2 First ten modes of five-cable network with two lines of flexible cross-ties ($\psi_1=\psi_2=0.01$) at a position of $\varepsilon_1,\varepsilon_2=1/3$

Figure 4.3 The “AS-line” unit on the south tower of the Fred Hartman Bridge (Caracoglia and Zuo, 2009)

Figure 4.4 First ten modes of the cable network on the Fred Hartman Bridge with three lines of cross-ties ($\psi_1=\psi_2=\psi_3=0.01$) at a position of $\varepsilon_1,\varepsilon_2=\varepsilon_1,\varepsilon_2=1/3$

Figure 5.1 Schematic diagram of two-cable network with two transverse cross-ties

Figure 5.2 First ten modes of two-cable network with two lines of flexible cross-ties ($\psi_1=\psi_2=0.1$) at a position of $\varepsilon_1=0.35$ and $\varepsilon_2=0.25$

Figure 5.3 Schematic layout of Networks A to C used for the understanding of mode localization

Figure 5.4(a) Non-dimensional fundamental frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Networks A, B and C with non-dimensional cross-tie flexibility parameter $\Psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.4(b) Non-dimensional fundamental frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Networks A, B and C with non-dimensional cross-tie flexibility parameter $\Psi=1.0$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.5(a) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Modes 2, 3 and 4 in Network A with non-dimensional flexibility parameter $\Psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.5(b) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a
function of frequency ratio parameter for Modes 2, 3 and 4 in Network B with non-dimensional flexibility parameter $\psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.5(c) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Modes 2, 3 and 4 in Network C with non-dimensional flexibility parameter $\psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.6 Mode cross-over behaviour of Mode 2 and Mode 3 in Network A with non-dimensional flexibility parameter $\psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.7 Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Networks A, B and C with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility $\psi=0.01$.

Figure 5.8(a) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Modes 2, 3 and 4 in Network A with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility $\psi=0.01$.

Figure 5.8(b) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Modes 2, 3 and 4 in Network B with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility $\psi=0.01$.

Figure 5.8(c) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Modes 2, 3 and 4 in Network C with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility $\psi=0.01$.

Figure 5.9 Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Networks A, B and C with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$.

Figure 5.10(a) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Modes 2, 3
and 4 in Network A with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$.

Figure 5.10(b) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Modes 2, 3 and 4 in Network A with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$.

Figure 5.10(c) Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Modes 2, 3 and 4 in Network A with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$.

Figure 5.11 Mode “reshaping” of Mode 2 in Network B with frequency ratio parameter $\eta_2=0.92$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.12 Sample layout of cable networks with Configurations A, B and C: (a) Layout of Configuration A ($\varepsilon_{1,1}=0.1, 0.3, 0.5, 0.65$ for Network A1, A2, A3 and A4, respectively); (b) Layout of Network B2; (c) Layout of Network C3

Figure 5.13 Effect of cross-tie position on the modal frequency of Networks A1 to A4 (Single line of cross-ties, $\psi=0.01$): (a) Modes 1-10; (b) Modes 11-20

Figure 5.14 Effect of cross-tie position on the fundamental frequency and position of first LMC ($\psi=0.01$)

Figure 5.15 Effect of cross-ties position on the modal frequency of Network B1 to B5 (two lines of cross-ties, $\psi=0.01$)

Figure 5.16 Effect of cross-tie stiffness on the modal frequency of Network C1 to C3: (a) Network C1 (Single line of cross-tie evenly installed along main cable 3); (b) Network C2 (Two lines of cross-tie evenly installed along main cable 3); (c) Network C3 (Three lines of cross-ties evenly installed along main cable 3)

Figure 5.17 Effect of cross-tie stiffness on the fundamental frequency and position of the 1st LMC of Network C1 to C3: (a) Network C1 (Single line of cross-tie evenly installed along main cable 3); (b) Network C2 (Two lines of cross-
tie evenly installed along main cable 3); (c) Network C3 (Three lines of cross-ties evenly installed along main cable 3)

Figure 5.18 Effect of number of cross-ties on modal frequency of first 20 modes (Cross-ties evenly installed along cable 3): (a) $\psi=0.01$; (b) $\psi=0.10$

Figure 6.1 Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the length ratio parameter, $\lambda$, for three different cross-tie positions: (a) Network-1; (b) Network-2; (c) Network-3

Figure 6.2 Non-dimensional fundamental frequency, $\Omega/\pi$, and the DML coefficient as a function of the length ratio parameter, $\lambda$, for four different cross-tie flexibility parameters (SMT cable network, $\eta_2=0.667$, $\varepsilon=1/2$)

Figure 6.3(a) Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the frequency ratio parameter, $\eta_2$, for four SMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at the mid-span ($\varepsilon=1/2$)

Figure 6.3(b) Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the frequency ratio parameter, $\eta_2$, for four SMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at the one-third span ($\varepsilon=1/3$)

Figure 6.4 Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the frequency ratio parameter $\eta_2$ for a symmetric SMT two-cable network with a flexible cross-tie installed at mid-span ($\varepsilon=1/2, \lambda_2=1.2$)

Figure 6.5(a) Non-dimensional modified frequency, $\Omega/\pi$, and DML coefficient as a function of the mass-tension parameter, $\gamma$, for four symmetric DMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at mid-span ($\varepsilon=1/2$)

Figure 6.5(b) Non-dimensional modified frequency, $\Omega/\pi$, and DML coefficient as a function of the mass-tension parameter, $\gamma$, for four symmetric DMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at mid-span ($\varepsilon=1/3$)

Figure 6.6 Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of
the mass-tension ratio parameter $\gamma$ for a symmetric DMT two-cable network with a flexible cross-tie installed at mid-span ($\varepsilon=1/2$, $\eta_2=0.833$ and $\lambda_2=1.2$)

Figure 6.7 Sample layout of cable networks with Configurations A, B, C and D: (a) Layout of Network A1 in Configuration A; (b) Layout of Network B4 in Configuration B; (c) Layout of Network C6 in Configuration C; (d) Layout of Network D6 in Configuration D

Figure 6.8 Effect of cross-tie position on the fundamental frequency and position of the first local mode cluster of Network A ($\psi=0.01$)

Figure 6.9 Effect of cross-tie position on the fundamental frequency and position of the first local mode cluster of Network $B_i$ ($i=1$ to 6, $\psi=0.01$)

Figure 6.10 Mode-frequency evolution curves for Networks B1, B3 and B4 ($\psi=0.01$)

Figure 6.11 Effect of cross-tie position on the fundamental frequency and position of the first local mode cluster of Network $C_i$ ($i=1$ to 6)

Figure 6.12 Effect of cross-tie position on the fundamental frequency and position of the first local mode cluster of Network $D_i$ ($i=1$ to 6)

Figure 6.13 Mode-frequency evolution curves for selected cable networks (three lines of cross-ties, $\psi=0.01$)

Figure 6.14 Mode-frequency evolution curves for selected cable networks (four lines of cross-ties, $\psi=0.01$)

Figure 6.15 Mode-frequency evolution curves of the twelve-cable network in different layouts: (a) Network B4; (b) Network C6; (c) Network D6

Figure 6.16 Non-dimensional frequency, $\Omega/\pi$, damping ratio and DML coefficient as a function of cross-tie stiffness parameter $\psi$ for the fundamental mode of an asymmetric DMT two-cable network with a flexible cross-tie at one-third span ($\varepsilon=1/3$)

Figure 6.17 Effect of number of cross-tie lines on the modal frequency of the first 50 modes of cable networks A6, B4, C6 and D6: (a) $\psi=0.01$; (b) $\psi=0.10$

Figure 7.1 Schematic layout of a typical hybrid system with damper installed on main cable 1

Figure 7.2 Experimental setup of a hybrid system (Sandanam, 2015)
Figure 7.3  First ten modes of a two-cable hybrid network with damper position $\varepsilon_1=0.065$ and cross-tie position $\varepsilon=1/3$. ($c=19.1$ N·s/m, $K_c=1210$ kN/m)

Figure 7.4(a)  A taut cable with a transverse linear external viscous damper
Figure 7.4(b)  A taut cable attached with transverse linear spring and viscous damper
Figure 7.4(c)  A taut cable attached with two transverse linear viscous dampers
Figure 7.4(d)  Two-cable hybrid system with external viscous damper on neighboring cable
Figure 7.4(e)  Two-cable hybrid system with two external viscous dampers on main cable

Figure 7.5  Effect of cross-tie flexibility parameter $\psi$ on the fundamental modal damping ratio of a two-cable hybrid system ($\eta_2=0.80$)

Figure 7.6  Effect of cross-tie flexibility parameter $\psi$ on the DML coefficient of the fundamental mode of a two-cable hybrid system ($\eta_2=0.80$)

Figure 7.7  Effect of damper-cross-tie spacing parameter $\varepsilon_{1,2}$ on the fundamental modal damping ratio of a two-cable hybrid system ($\eta_2=0.80$)

Figure 7.8  Fundamental mode of a two-cable hybrid system for selected values of cross-tie position and stiffness

Figure 7.9  Effect of damper-cross-tie spacing parameter $\varepsilon_{1,2}$ on the DML coefficient of the fundamental mode of a two-cable hybrid system ($\eta_2=0.80$)

Figure 7.10 Isoquant curves representing in-plane frequency, modal damping ratio and DML of a two-cable hybrid system fundamental mode ($\eta_2=0.80$)

Figure 7.11 Isoquant curves representing in-plane frequency, modal damping and DML on $\psi$-$\varepsilon_{1,2}$ plane of a two-cable hybrid system in the numerical example

Figure 7.12 Damping estimation curves for a single damped cable and a symmetric two-cable hybrid system ($\varepsilon_{1,1}=0.0235$, $\varepsilon_c=0.50$, $\eta=0.82$, $\lambda=1.2$ and $\gamma=0.91$)

Figure 7.13 Damping estimation curves for a single damped cable and a symmetric two-cable hybrid system ($\varepsilon_{1,1}=0.0235$, $\psi=0.0$, $\eta=0.82$, $\lambda=1.2$ and $\gamma=0.91$)

Figure 8.1  Experimental setups of three different systems: (a) Cable network; (b) Hybrid system A; (c) Hybrid system B

Figure 8.2  Passive linear viscous damper
Figure 8.3  Universal Flat Load Cells mounted at the cable ends
Figure 8.4  Setup of Hydraulic pump
Figure 8.5  Electronic dynamic smart shaker installed on the target cable (bottom cable)

Figure 8.6  HP signal generator for dynamic smart shaker

Figure 8.7  PC-based AstroDAQ Xe data acquisition system

Figure 8.8  Cross-tie connected with the main cables

Figure 8.9  Half-power method used to calculate the damping ratio

Figure 8.10 A fraction of sample acceleration time history raw data of the cable network (rigid cross-tie located at 1/4L)

Figure 8.11 Power spectral density curve of the cable network (rigid cross-tie located at 1/4L)

Figure 8.12 Extracted fundamental modal displacement time history of the cable network (rigid cross-tie located at 1/4L)

Figure 8.13 In-plane fundamental frequency of a two-cable network as a function of cross-tie position \( \varepsilon_c \) (\( \eta=0.79 \))

Figure 8.14 Fundamental modal damping ratio of a two-cable network as a function of cross-tie position \( \varepsilon_c \) (\( \eta=0.79 \))

Figure 8.15 Frequency-response curve of hybrid system A (rigid cross-tie located at 1/4L)

Figure 8.16 Figure 8.16: In-plane fundamental frequency of a two-cable hybrid system A as a function of cross-tie position \( \varepsilon_c \) (\( \eta=0.79 \))

Figure 8.17 Fundamental modal damping ratio of a two-cable hybrid system A as a function of cross-tie position \( \varepsilon_c \) (\( \eta=0.79 \))

Figure 8.18 In-plane fundamental frequency of a two-cable hybrid system B as a function of cross-tie position \( \varepsilon_c \) (\( \eta=0.79 \))

Figure 8.19 Fundamental modal damping ratio of a two-cable hybrid system B as a function of cross-tie position \( \varepsilon_c \) (\( \eta=0.79 \))
LIST OF TABLES

Table 3.1  Comparison of in-plane modal properties of a symmetric twin-cable network with rigid ($\psi=0$) and flexible ($\psi=1.0$) cross-tie at quarter span 42

Table 3.2  In-plane modal properties of a symmetric unequal length two-cable network with system parameters as frequency ratio $\eta_2=0.88$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4), mass-tension ratio $\gamma_1=\gamma_2=1$ and flexibility parameter $\psi=1.0$ 48

Table 3.3  Comparison of modal properties of an asymmetric DMT two-cable network with flexible or rigid cross-tie at $\varepsilon=1/3$ 51

Table 3.4  In-plane modal frequency and modal damping of a twin-cable network with a rigid cross-tie at one-third span 68

Table 3.5  In-plane modal frequency and modal damping of a symmetric two-cable network with a rigid cross-tie at mid-span 71

Table 3.6  In-plane modal frequency and modal damping of an asymmetric two-cable network with a rigid cross-tie at three-quarter span 82

Table 3.7  Comparison of modal properties of a twin-cable network with damped flexible or rigid cross-tie at $\varepsilon=1/3$ 90

Table 3.8  Comparison of modal properties of a twin-cable network with damped flexible or rigid cross-tie at $\varepsilon=2/5$ 93

Table 3.9  Comparison of modal properties of a symmetric DMT two-cable network with damped flexible or rigid cross-tie at $\varepsilon=1/2$ 98

Table 3.10  Comparison of modal properties of an asymmetric DMT two-cable network with damped flexible or rigid cross-tie at $\varepsilon=1/3$ 102

Table 4.1  Modal properties of a general five-cable network with two lines of flexible cross-ties ($\psi_1=\psi_2=0.01$) evenly installed along the target cable ($\varepsilon_{1,1}=\varepsilon_{1,2}=1/3$) 117

Table 4.2  Physical properties of main cables in the “A-line’’ unit of main span of the Fred Hartman Bridge 118

Table 4.3  Modal properties of “A-line’’ cable network at Fred Hartman Bridge with two lines of flexible cross-ties ($\psi_1=\psi_2=0.01$) evenly installed along the 121
target cable ($\varepsilon_{1,1}=\varepsilon_{1,2}=1/3$)

Table 5.1 Modal properties of a general two-cable network with two lines of flexible cross-ties at a position of $\varepsilon_1=0.35$ and $\varepsilon_2=0.25$ ($\psi_1=\psi_2=0.1$) 133

Table 5.2 Non-dimensional modal frequencies of cable networks A1 to A4 (Single line of cross-ties, $\psi=0.01$) 159

Table 5.3 Summary of local mode cluster and pairs of local modes for cable networks B1 to B5 164

Table 5.4 Summary of local mode cluster and pairs of local modes for cable network C1 169

Table 5.5 Summary of local mode cluster and pairs of local modes for cable network C2 170

Table 5.6 Summary of local mode cluster and pairs of local modes for cable network C3 170

Table 6.1 Summary of fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks B1 to B6 206

Table 6.2 Summary of fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks C1 to C6 211

Table 6.3 Summary of fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks D1 to D6 217

Table 6.4 Summary of the fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks B4, C6 and D6 217

Table 6.5 Summary of non-dimensional system frequency and the first local mode cluster attributes for four optimized cable networks in term of positions of cross-ties within the first 50 modes 222

Table 7.1 Fundamental modal frequency and damping ratio of isolated cables and hybrid system with damper position $l_{1,1}=0.55m$ ($\varepsilon_1=0.065$) and cross-tie position $\varepsilon=1/3$ ($c=19.1$ N·s/m, $K_c=1210$ kN/m) 240

Table 7.2 Modal properties of the two-cable hybrid system in the numerical example 241

Table 7.3 The optimum damper size and the maximum attainable fundamental modal damping ratio of a single damped cable and a symmetric two-cable hybrid system ($\varepsilon_{1,1}=0.0235$, $c_0=0.50$, $\eta=0.82$, $\lambda=1.2$ and $\gamma=0.91$) 284

Table 7.4 The optimum damper size and the maximum attainable fundamental modal damping ratio of a single damped cable and a symmetric two-cable hybrid system ($\varepsilon_{1,1}=0.0235$, $\psi=0.0$, $\eta=0.82$, $\lambda=1.2$ and $\gamma=0.91$) 288

Table 8.1 Physical properties of the main cables 301

Table 8.2 Material properties of the cross-ties 301

Table 8.3 List of testing cases conducted for the modal behaviour of cable networks 308
Table 8.4  List of testing cases conducted for the modal behaviour of hybrid system A (external damper position =0.55 m (0.065%L))  311
Table 8.5  List of testing cases conducted for the modal behaviour of hybrid system B (external damper in-line with the cross-tie)  312
Table 8.6  Fundamental frequency of a two-cable network with different cross-tie positions (Hz)  319
Table 8.7  Fundamental modal damping ratio of a two-cable network with different cross-tie positions (%)  319
Table 8.8  Material and geometric properties of main cables and cross-tie as non-dimensional parameters  319
Table 8.9  Maximum displacement at each excitation frequency of the shaker in hybrid system A (rigid cross-tie located at 1/4L)  326
Table 8.10 Fundamental frequency of hybrid system A for different positions of cross-tie (damper installed at 0.065%L*) (Hz)  329
Table 8.11 Fundamental modal damping ratio of hybrid system A for different positions of cross-tie (damper installed at 0.065%L*) (%)  329
Table 8.12 Fundamental frequency of hybrid system B for different positions of cross-tie (Hz)  334
Table 8.13 Fundamental modal damping ratio of hybrid system B for different positions of cross-tie (%)  334
CHAPTER 1 Introduction

1.1 Background

The original concept of using cable stays can perhaps be dated back to ancient Egypt. In the construction of sailing ships, inclined ropes hanging from a mast were used to support a basin beam (Troitsky, 1977). Similarly, in some tropical regions, bamboo sticks were used to support pedestrian bridge deck and with the other end of the sticks attached to a tree. Although simple bridges with deck supported by inclined bars or chains were designed in the 17th century (Leonhardt and Zellner, 1991), the first cable-stayed bridge, Roeblings Bridge, was not constructed until the 19th century. However, limited by the availability of high strength materials, analysis methods, and construction techniques, the idea of cable-stayed bridge was abandoned for some time. The rebooming of cable-stayed bridge occurred after the Second World War, when German engineers faced the challenges to replace many bridges destroyed during the war by innovative and inexpensive solutions. In the past two decades, the span length of cable-stayed bridges has been increasing rapidly. The Russky Bridge in Russia, the world’s longest cable-stayed bridge at present, has a central span length of 1104 m with the longest cable being 580 m; whereas the second longest cable-stayed bridge, the Sutong Bridge in China, has a central span length of 1088 m (Weber and Distl, 2015). The world’s tallest bridge, the Millau Viaduct Bridge in southern France, also belongs to the family of cable-stayed bridge. It has an impressive height of 343 m. The growing popularity of cable-stayed bridges is due to its aesthetic, ease of deck erection, economics, small deflection and effectiveness in poor soil condition in comparison to suspension bridges (Bimson, 2007).

However, these encouraging breakthroughs come at a price and present new challenges to engineers. A typical concern is the excessive cable vibrations of bridge stay cables, which are
slender and flexible structural components. In particular, with the growth of bridge span length, stay cables are becoming longer. The longest cable on the Rusky Bridge has a length of 580 m. Further, stay cables are key structural components of cable-stayed bridge. They are subjected to high pre-tension forces. By applying initial tension, the friction force between the wires or strands composing a stay cable changes considerably which significantly reduces the structural damping of the cables (Hard and Holben, 1967; Yamaguchi and Fujino, 1987; Yamaguchi and Nagahawatta, 1995). A study conducted by Hard and Holben (1967) revealed that a significant reduction in the logarithmic decrement of cable oscillation amplitude was observed as the cable tension increased from 20% to 40% of the rated strength and the reduction rate in cable structural damping dropped with the increase of cable tension. Yamaguchi and Adhikari (1995) pointed out that structural damping of a stay cable without initial pretension could be ten times than that of an initially stressed cable. Field data collected from measurements indicated that the intrinsic structural damping ratio of the majority of stay cables was typically less than 0.3% (Meharabi, 2006). Therefore, under the combined effects of low inherent structural damping and long flexible feature, stay cables are prone to dynamic excitations due to various environmental factors, such as wind, wind combined with rain, earthquake and nonlinear coupling between motions of cables, deck and/or pylon (Virlogeux, 1998).

In the past few decades, many violent cable vibration incidents were reported from different bridge sites. The first recorded large amplitude cable vibration was on the Brottonne Bridge in 1977. Later, similar phenomenon was also reported from the Ben Ahin Bridge in Belgium, the Farø Bridge in Denmark and the Glebe Island Bridge in Australia (Virlogeux, 1998). Maximum cable vibration amplitude of 0.6 m was recorded on the Burlington Bridge in the United States (Tabatabai, 2005). A study done by the Rowan Williams Davies & Irwin Inc.
(RWDI) on the Cochrane Bridge in Alabama reported a first modal cable vibration amplitude close to 0.5 m at moderate wind speed of 8 m/s to 10 m/s when accompanied by light rain (Lankin et al., 2000). In a full-scale measurement on the Fred Hartman Bridge conducted by Ozkan et al. (2001), peak-to-peak cable vibration amplitude of 1 m was reported during the passage of a heavy storm. In 2005, almost all the stay cables on the Dubrovnik Bridge in Croatia experienced violent vibration with a disturbing rattling noise (Savor et al., 2006). The cable vibration amplitudes were so large that even the light posts located at a distance 85 cm away were broken down. In Japan, violent cable vibration amplitude of more than 1.5 m occurred on a cable-stayed bridge during passage of a typhoon and the external damper attached to the cable was found to be damaged (Matsumoto et al., 2010). In the case of the Alamillo Bridge in Spain, transverse vibration amplitude of 0.5 m was observed for the longest cable on a rainy day, when the wind speed was at 15 to 20 m/s (Casas and Aparicio, 2010). The stay cables can also be excited due to the motion of stay supports or decks when the global frequency of the decks falls close to the natural frequency of some of the stay cables (Wu et al., 2003; Cao et al., 2012).

1.2 Types of wind-induced cable vibrations

Depending on their mechanisms, the wind-induced cable vibrations mostly related to stay cables can be categorized as the following types:

a) Rain-wind-induced vibration

It is believed that this type of vibration is mainly caused by the formation of water rivulets on the surface of inclined cables. The mechanism of rain-wind-induced vibration has not been fully understood yet and research is still needed. Usually two types of rivulets are formed, one on the top of the cable surface at the windward side and the other on the bottom surface at the leeward side. The position of the rivulets along the cable perimeter would affect the
aerodynamic force on the cable. The presence of the water rivulets would not only change the effective cross-sectional shape of the cable but also oscillates on its surface as it vibrates. The resonance between the motions of the cable and the water rivulets amplifies the vibration amplitude of the cable. The majority of cables that experienced this type of vibration are located on the leeward side of the bridge pylon and geometrically declined in the mean wind direction. The formation of upper water rivulet on the cable surface seems to be a key factor (Yamada et al., 1997). The number of dominant modes generally ranged from 1 to 4, with most of the responses occurred in Mode 2 and Mode 3 (Zuo and Jones, 2010). The frequencies of the dominant modes are distributed over a relatively wide range mostly between 1 and 3 Hz (Main et al., 2001) in moderate to heavy rain within a wind speed range of 5–15 m/s (Phelan et al., 2006; Caetano, 2007). However, some cable vibrations are also reported to occur at wind speeds as high as 40 m/s (Zuo and Jones, 2010). It is worth pointing out that about 95% of the reported stay cable vibration incidents are due to the rain-wind induced vibration (Wagner and Fuzier, 2003).

b) Vortex-induced vibration

When wind blows past a cable, vortex would form and shed alternatively in its wake. This would generate alternating low-pressure zones on the downstream side of the cable. Cable tends to move toward the low-pressure zone and thus would oscillate according to vortex shedding frequency. If the natural frequency of the cable lies in the close proximity of the shedding frequency of the vortices, resonance would occur and results in high amplitude cable vibrations. These vibrations are generally observed in the higher modes typically Mode 5 and up. Although the displacement amplitude of these higher modes is relatively small (20% of the cable diameter), the magnitude of acceleration may be considerable because of the high
oscillation frequencies (Main et al., 2001; Zuo and Jones, 2010). This type of vibration is potentially less damaging, mainly because of its small vibration amplitude, than rain-wind induced or galloping vibration (Mehrabi, 2006). When investigating the cause of the original Tacoma Narrows Bridge failure in 1940, vortex-induced vibration was proposed as one of the possible mechanisms but was dismissed since the frequency of the vortex shedding did not match with that of the bridge.

c) Buffeting

This type of wind-induced cable vibration is due to the velocity fluctuation in the oncoming flow and is directly related to the level of wind speed. Buffeting has not been found to cause serious problems on bridge stay cables. However, this frequent low amplitude vibration could induce fatigue damage at the cable anchorage and thus threat the safety of bridges.

d) Wake Galloping

When a cable is submerged in the wake of other elements, such as towers or other cables, if the vortex shedding frequency of an upstream body is in resonance with the natural frequency of the cable, large amplitude of wake galloping would be excited. The cable oscillates along an elliptical trajectory. Cooper (1985) proposed a stability criterion which could predict the critical value of the wind velocity $U_{crit}$ above which instability could be expected due to wake galloping effects.

e) High-speed vortex excitation

High-speed vortex excitation is directly associated with the formation of the axial flow on the leeward side of an inclined cable. The phenomenon was observed in field and in wind tunnel tests. It occurs at much higher wind velocity ranges than that for regular vortex-induced vibrations. Some studies (e.g. Matsumoto et al., 1990) suggested that shedding of Kármán vortex
interacts with that of the axial vortex, which induced amplified response. Though the mechanism of this phenomenon is not fully understood, observations showed that the frequency seemed to be about one-third of the Kármán vortex shedding frequency. Caetano (2007) showed that its occurrence also depends on some other factors like cable orientation and frequency of cable vibration.

f) Dry inclined cable galloping

Dry inclined cable galloping is an excitation phenomena identified by Saito et al., (1994), Honda et al., (1995) and Cheng et al., (2003) during wind tunnel tests but no formal confirmed field incident has been reported yet. When a single inclined cable is exposed to wind, wind “sees” an elliptical cable cross-section instead of a circular one. When entering into the critical Reynolds number regime, there is a potential to trigger galloping type instability if the level of structural damping in the cable is very low. One of the possible mechanisms is proposed to be linked to the occurrence of negative aerodynamic damping in the critical Reynolds number range (Cheng et al., 2008a; 2008b). Research about this phenomenon and its driven mechanism is still undergoing.

It is worth pointing out that among these different types of wind-induced cable vibrations, rain-wind-induced vibration is the most frequently observed one. Most of the time, it is an in-plane oscillation and occurs at moderate wind speed with the presence of light rain (Lankin et al., 2000).

1.3 Countermeasures

Suppressing vibrations of bridge stay cables is of prime importance since stay cables are the key structural elements of a cable-stayed bridge. Frequent and/or excessive vibrations of stay cables results in connection/anchorage failure, damage/breaking of the cable protection system
and ultimately reducing the life of stay cables (Johnson et al., 2002). Consequently, it would have a considerable impact on the serviceability and life span of the entire bridge.

To control cable vibrations, different countermeasures are adopted, which can be classified as aerodynamic type and mechanical type. The aerodynamic type of countermeasures aims at changing aerodynamic behaviour of stay cables by modifying their surface conditions or cross-sectional shape. Experimental results showed that by wrapping spiral wire around cable surface (Bosdogianni and Olivari, 1996; Zhan et al., 2008) or making dimpled surface (Virlogeux, 1998), rain-wind-induced cable vibrations could be effectively suppressed. The helical wire whirling surface has now become a standard requirement of manufacturing stay cables. Some researchers also recommended wrapping stay cables with viscoelastic damping tapes and providing neoprene rubber bushings (or rings) (Tabatabai and Mehrabi, 2000a). The neoprene rubber bushings do not only contribute to the damping of the vibrating cables but are also effective in reducing bending stresses at the anchorages (Takano et al., 1997). One of the main purposes of these surface treatments is to prevent the formation of water rivulets, which is the main cause of rain-wind-induced vibration. Some of the well-known aerodynamic countermeasure examples, in terms of cable surface treatment, are the axial protuberance installation on the Higashi Kobe Bridge; the dimple distribution on the Tatara Bridge and the helical wire installation on the Normandy Bridge (Matsumoto et al., 2003). However, these surface treatments cannot provide additional damping to the cable and are difficult to be applied to existing structures. In addition, evidence showed that surface treatment might increase drag on stay cables and so it could become more significant in the case of long span cable-stayed bridges (Johnson et al., 2002; Virlogeux, 2005).
On the other hand, mechanical type of countermeasures is directed to enhance either energy dissipation or stiffness of cable(s). External dampers installed near the cable-deck anchorage are used to help dissipate kinetic energy of an oscillating cable and thus increase structural damping of the attached cable (Pacheco et al., 1993; Krenk, 2000; Tabatabai and Mehrabi, 2000b; Zhan et al., 2008; Cheng et al., 2010). External dampers are more effective for stay cables on small to medium-span cable-stayed bridges. However, their efficiency is limited in the case of long-span bridges such as the Normandy Bridge in France and the Sutong Bridge in China due to the longer cables length and constrains on damper installation location. For example, by installing an external viscous damper, if it is expected that the damped cable should achieve a maximum equivalent modal damping ratio of 1% for the fundamental mode of a stay cable with a length of 500 m, the damper would have to be attached at a distance of 10 m from the cable anchorage, which may not be feasible in practice. It is also observed that external dampers installed near the cable-deck anchorage are not activated in case of small amplitude cable vibrations. Besides, it is also important to note that external dampers are delicate devices that require constant maintenance. More recently, researchers are also exploring new methods to mitigate violent cable vibrations. For example, to suppress cable vibrations by the application and removal of constraints dynamically during cable vibrations (Alsahlani et al., 2012).

Cross-tie solution is another mechanical countermeasure. It is becoming more popular in recent years on new bridges (Kangas et al., 2012) and in the rehabilitation of existing ones (Mehrabi et al., 2010). In this solution, a cable which has exhibited or is expected to experience large amplitude vibrations is interconnected with its neighbouring cable(s) through transverse secondary cables, i.e. cross-ties, to form a cable network. It is understood from past studies that a vulnerable cable could be benefited from the cross-tie solution in a number of ways: a) Enhance
in-plane stiffness (Caracoglia and Jones, 2005a) and thus increase the frequency of cables. This cannot only increase the critical onset wind speed of many wind-induced cable vibration phenomena, but also avoid parametric excitation caused by the bridge deck oscillation as was the case for the Normandy Bridge in France (Virlogeux, 1998); b) Introduce additional structural damping (Yamaguchi and Jayawardena, 1992; Yamaguchi and Nagahawatta, 1995; Lankin et al., 2000); c) Redistribute energy into higher modes or other cables in the same network (Ehsan and Scanlan, 1989; Yamaguchi and Nagahawatta, 1995); d) Increase modal mass of the global modes. This would help to increase Scruton number in these lower order modes (Kumarasena et al., 2007). In order to suppress rain-wind-induced cable vibrations, a minimum required Scruton number of 10 is recommended (Kumarasena et al., 2007); e) Help to avoid wake galloping effect in the case of twin-cable networks (Virlogeux, 1999); f) Reduce cable sag variation among stays of different lengths (Gimsing, 1993).

So far, cross-ties have been successfully used on a number of cable-stayed bridges to control cable vibrations. They include the Farø Bridge in Denmark, the Normandy Bridge in France (Virlogeux, 1993), the Yobuko Bridge in Japan (Yamaguchi, 1995), the Fred Hartman Bridge (Caracoglia and Jones, 2005b), the Dames Point Bridge (Kumarasena et al., 2007) and the newly constructed U.S. Grant Bridge (Kangas et al., 2012) in the United States. After installing the cross-ties on the Farø Bridge, cable oscillations were reduced to an acceptable level for all cables in the network except the first cable on each side of the pylon due to special wind condition on these cables (Bloomstine and Stoltzner, 1999). Similarly, no problematic cable vibrations have been reported on the Second Severn Bridge in the United Kingdom after using the cross-tie solution (Stubler et al., 1999). The Texas Department of Transportation (Texas DOT) launched a study to probe cable vibrations on the Fred Hartman and the Veteran Memorial
Bridges after receiving calls from the public about observed excessive cable vibrations (Ramsey, 2005). Based on the study recommendation, a cross-tie solution was proposed for the Fred Hartman Bridge and cable vibrations were reduced significantly. When the cross-tie system was uninstalled for the purpose of maintenance and improvement, excessive cable vibrations appeared again, proving the effectiveness of the cross-tie solutions (Ramsey, 2005).

On the other hand, there are some disadvantages of the cross-tie solution apart from the above mentioned benefits. One of the major drawbacks is the appearance of closely spaced higher order local modes (Caracoglia and Jones, 2005b; Bosch and Park, 2005). These densely populated local modes impose a potential risk to cable network for its sensitivity to dynamic excitations within a narrow frequency band. Due to inherent nature of cable networks, it is almost impossible to eliminate these closely spaced local modes. However, by a careful selection of cross-tie properties, it is possible to shift these local modes to higher order. In addition, it is also important to note that although cross-ties can increase damping of a cable network (Yamaguchi and Jayawardena, 1992; Yamaguchi and Nagahawatta, 1995; Lankin et al., 2000), they are not primary energy dissipating devices and cannot control out-of-plane cable vibrations (Caracoglia and Jones, 2005b).

The main objective of external dampers is to increase structural damping of stay cables, and therefore suppressing undesirable cable vibrations. External dampers are effective as long as a stay cable is not too long and the damper is activated at the proper time. From field observations, it is known that external viscous dampers would not be activated for small amplitude cable vibrations. In the case of cross-tie solution, the main objective is to increase the modal mass and the in-plane stiffness of the network global modes but cannot be used as a direct energy dissipation device. Both of these two vibration control solutions have their respective
merits and limitations. Therefore, some researchers proposed to combine them into a single hybrid device/system. Some of the well-known examples of hybrid system applications include the Normandy Bridge in France (Virlogeux, 1993) and the Fred Hartman Bridge in USA (Caracoglia and Jones, 2005b). On both bridges, it was reported that the hybrid system worked successfully.

Nevertheless, there are only limited numbers of studies available in the literature which has investigated the mechanics of such a hybrid system. Bosch and Park (2005) used finite element simulations to explore the performance of hybrid system. Results showed that the cumulative benefits of both cross-ties and external dampers would not necessarily to earn the same benefits when applying separately. The study by Caracoglia and Jones (2007) revealed that hybrid system was not able to control the formation of local modes. Caracoglia and Zuo (2009) investigated the effectiveness of hybrid system with different configurations. It was found that the configuration of external damper in-line with the cross-tie line was more effective than that of the external dampers installed close the anchorage of the target cable. More recently, Zhou et al. (2015) developed an analytical model of a symmetric two-cable hybrid system, of which the two consisting cables were laid in parallel with each other and connected by a transverse spring. In addition, each of them was attached with a linear viscous damper close to one end. A free vibration analysis was performed to understand the modal behaviour of such a hybrid system, in terms of its in-plane frequency and modal damping associated with the second in-phase and out-of-phase modes when the two main cables were identical. Unfortunately, the hybrid systems discussed in these few existing studies were either based on the cable layout on a particular cable-stayed bridge, or has an idealized symmetric configuration, of which some findings might only be applicable to the corresponding specific system arrangement. Further, all of them were
focused on investigating the in-plane stiffness and damping of the hybrid system, whereas the severity of local mode formation was completely neglected. Therefore, more intensive studies, preferably using analytical approach, are urgently needed to better understand the mechanics of hybrid systems.

1.4 Motivations

In spite of an increasing popularity of the cross-tie application and its proven effectiveness on site, there were numerous cross-tie failure breakage incidents occurred on different bridges. The first two reported cross-tie breakage incidents happened on the Saint-Nazaire Bridge in France and the Zarate Brazo Largo Bridge in Argentina (Virlogeux, 1998). In the case of the Farø Bridge in Denmark, rupture occurred twice on one of the cross-ties and a few others were seriously damaged (Bloomstine and Stoltzner, 1999). Similar incidents were also reported from the Meiko Nishi Bridge and the Yuboko Bridge in Japan (Virlogeux, 1998; Noguchi and Miyauchi 2010); the Burlington Bridge (Zuo and Jones, 2005) and the Fred Hartman Bridge in USA (Ramsey, 2005). All these incidents show that there is a lack of thorough understanding of the mechanics of cable networks.

The majority of the existing analytical studies on cable networks were based on simple network configuration containing a single line of cross-tie(s), whereas in practice, cable networks on real bridges usually possess at least two lines of cross-ties. The addition of another line of cross-tie(s) would considerably increase the complexity of network behaviour and make it very challenging in the analysis. To have a comprehensive understanding of the dynamic behaviour of cable networks, there is a great need to develop an analytical model of a general cable network consisting of multiple main cables interconnected by multiple lines of cross-ties.
Although the most prominent advantage of the cross-tie solution is to enhance the in-plane stiffness of cable network and thus the interconnected vulnerable cable(s), it would also have some help to increase the network damping. However, majority of existing research on cable networks were focused on the increment of in-plane stiffness of cable networks, Yamaguchi and Jayawardena (1992) and Yamaguchi and Nagahawatta (1995) are perhaps the only ones who addressed the damping increments of the cross-tie solutions through experimental studies. There is no analytical model available to verify their findings yet. As pointed out by Ehsan and Scanlan (1989), the cross-tie solution could help to redistribute the energy among different stay cables within a cable network. The energy contained in vulnerable cable(s) could be transferred to its neighbours and thus reduce the vibration amplitude of the problematic cable(s). However, this important feature of the cross-tie solution needs to be further explored.

One of the main drawbacks of the cross-tie solution is the possible generation of closely spaced higher order local modes which do not exist prior to the cross-tie(s) installation (Caracoglia and Jones 2005b; Bosch and Park 2005). Since this kind of local modes are usually difficult to control, how to reduce the number of such local modes also becomes an important issue to be considered in the network design. However, there is no tool/model available to quantitatively measure the global or local nature of a specific network mode. In addition, research on the formation of local mode cluster(s) as well as how the selection of cross-tie properties would affect the appearance and size of local mode cluster(s) are considerably lagging behind the needs of the engineering community.

To comprehend the knowledge of cable network dynamic behaviour, it is crucial to understand the role of different system parameters on the network response. However, the only system parameter that received reasonable attention in the existing literature is the cross-tie
stiffness, whereas the other parameters such as the frequency ratio and the cross-tie position have not been sufficiently explored.

To overcome the respective shortcomings of the damper-only solution and the cross-tie-only solution, the idea of using a hybrid system, i.e. a combined application of cross-tie and external damper, has been explored (Caracoglia and Jones 2007; Caracoglia and Zuo 2009). However, much more intensive research effort is needed to appreciate the behaviour and effectiveness of this novel solution.

1.5 Objectives

The objectives of the current study are as follows:

1. Develop an analytical model to describe the in-plane modal behaviour of a general cable network having a configuration representing those on typical cable-stayed bridges, i.e. consisting of multiple main cables interconnected through multiple lines of cross-ties.

2. Develop an analytical model of cable networks by considering the structural damping of the main cables and the cross-ties in the formulation. Explore how the energy dissipation capacity of an entire cable network is affected by the damping available in different structural components, i.e. main cables and cross-ties.

3. Conduct a parametric study to explore the role of different system parameters in affecting the network response. Provide an insight of the mechanics associated with cable networks and apply this knowledge to practical design.

4. Establish a criterion to quantitatively measure the global nature of a network mode. Investigate the impact of different cross-tie properties on the formation and size of local mode cluster(s). Recommend proper design practice to reduce the number of excited local mode in lower order modes.
5. Formulate an analytical model of a typical hybrid system to explore the possible benefits of combined application of cross-tie(s) and external damper(s).

6. Develop independent finite element simulation models to validate the proposed analytical models of cable network and hybrid system.

7. Perform physical tests to study cable vibration control using cross-tie solution and hybrid system. Investigate the effects of cross-tie installation location and stiffness on cable network modal behaviour. Discuss the assumptions made in the analytical and numerical models on the modal analysis results of conventional and hybrid cable network systems. Evaluate the effectiveness of different hybrid system configurations on suppressing cable vibrations.

8. Propose design tools to facilitate optimum design of conventional and hybrid cable networks.
CHAPTER 2   Literature Review

In this chapter, a review of existing studies addressing the cross-tie solution and the hybrid system will be presented. First, the free vibration of a single cable is addressed and then the discussion is extended to the modal behaviour of cable networks, of which the presentation is categorized according to the major benefits offered by the cross-tie solution along with some drawbacks. Each of the categories is discussed explicitly by reviewing the existing literature. A separate section is dedicated to the state-of-the-art of hybrid system.

2.1 Free vibration of a single cable

Understanding the dynamics of suspended cables has been an interesting topic for a long time. Some of the well-known names, for example, D’Alembert, Euler, Bernoulli and Poisson, all contributed their effort to understand the behaviour of vibrating cables. Among recent studies, Irvine’s theory of free vibration of a suspended cable is simpler and easier to understand. It is reviewed here in detail.

Figure 2.1 shows a horizontally suspended cable studied by Irvine and Caughey (1974). In the figure, $u$ is the longitudinal displacement component and $v$ is the vertical displacement component of cable in-plane motion, $w$ is the transverse horizontal displacement component of cable motion, $l$ is the length of the span and $d$ is the maximum static deflection at cable mid-span. Coordinates of the static profile of the cable are represented by $x$ and $y$. 
The equation of in-plane vertical motion is given by

$$H \frac{\partial^2 \nu}{\partial x^2} + h \frac{d^2 y}{dx^2} = m \frac{\partial^2 \nu}{\partial t^2}. \quad (2-1)$$

where $h$ is the additional horizontal component of cable tension due to cable vibration and is given by

$$h \frac{ds}{dx} \frac{(ds/dx)^3}{E_c A_c} = \frac{\partial u}{\partial x} + \frac{dy}{dx} \frac{\partial v}{\partial x} \quad (2-2)$$

where $A_c$ is the cross-sectional area of the cable, $E_c$ is the elastic modulus of the cable and $ds$ is the length of the cable element considered.

The in-plane vertical modes can be categorized into two types of modes, the symmetric mode and the asymmetric mode. The frequencies of the asymmetric in-plane motion, of which no additional cable tension is developed, can be obtained from Eq. (2-1) as

$$\omega_n = (2n\pi/l)\sqrt{H/m} \quad n = 1, 2, 3, \ldots$$

is the mode number. The vertical modal components are given by

$$v_n(x) = A_n \sin\left(\frac{2n\pi x}{l}\right), \quad n = 1, 2, 3, \ldots \quad (2-3)$$

where $A_n$ is the amplitude of the anti-symmetric vertical component of the $n^{th}$ mode.
In the case of the symmetric in-plane modes, the additional cable tension is non-zero and is treated as a function of time alone. The solution to the eigenvalue problem expressed by Eq. (2-1) leads to the following transcendental equation, from which the natural frequencies of the symmetric in-plane modes may be found (Irvine and Caughey, 1974).

\[ \tan\left(\frac{1}{2\beta l}\right) = \left(\frac{4}{\lambda^2}\right) \left(\frac{4}{\lambda^2}\right)^3 \]  

(2-4)

where \(\beta\) is the non-dimensional in-plane frequency of the cable, \(\lambda^2 = \left(\frac{mg}{H}\right)^2 \frac{l}{(HL_e/E_cA_c)}\) is called the inextensibility or Irvine parameter, and \(L_e = \int_0^l (ds/dx)^3 dx \cong l[1 + 8(d/l)^2]\). The Irvine parameter, \(\lambda^2\), describes the ratio of the elastic to the geometric stiffness of the cable. It governs the natural frequencies and the mode shapes of the cable motion. For very large value of \(\lambda^2\), the cable is theoretically inextensible, the transcendental equation, Eq. (2-4), becomes \(\tan\left(\frac{1}{2\beta l}\right) = \left(\frac{1}{2\beta l}\right)\), which is the same as that derived by Rohrs (1851). On the other hand, for very small value of \(\lambda^2\), the cable behaves like a taut string, the above transcendental equation, Eq. (2-4), becomes \(\tan\left(\frac{1}{2\beta l}\right) = -\infty\) and the first root is \(\beta l = \pi\). Irvine and Caughey (1974) proposed natural frequencies and mode shapes of horizontally suspended uniform cable without sag or with small sag. According to their findings, the natural frequencies of cable asymmetric modes are independent of the Irvine’s parameter \(\lambda^2\). However, in the case of symmetric modes, natural frequencies depend upon \(\lambda^2\). When \(\lambda^2\) is small, natural frequencies of symmetric modes are lower than those of asymmetric modes. With the increase of \(\lambda^2\), natural frequencies of symmetric modes would also increase and approach to the natural frequencies of asymmetric modes. The natural frequency of the 1st symmetric mode coincides with that of the 1st asymmetric mode at \(\lambda^2=4\pi^2\). This phenomenon is known as the modal cross-over. Later, Irvine (1981) extended the solution to an inclined cable.
In the case of small size cables, cable bending stiffness is ignored in the analysis because of high flexibility resulted from small diameter. But due to the growing demand of cable-supported structures, there is a significant increase in the length as well as the diameter of cables. In such case, bending stiffness of cables cannot be neglected. A study done by Ricciardi and Saitta (2008) showed that high bending stiffness in cable could significantly affect frequency of the higher order modes, but not the fundamental mode.

2.2 Modal behaviour of cable networks

It has already been discussed in Chapter 1 that using the cross-tie solution to control stay cable vibrations has both advantages and disadvantages. The existing studies dedicated in understanding these effects will be reviewed in the following subsections.

2.2.1 In-plane frequency

Yamaguchi and Nagahawatta (1995) performed a set of physical tests on a simple two-cable network as shown in Figure 2.2. In their setup, the two main cables were arranged in parallel, with different physical and geometrical properties, and connected through two transverse cross-ties.

![Figure 2.2: Schematic diagram of experimental setup used by Yamaguchi and Nagahawatta (1995)](image-url)
Free vibration analysis was conducted in order to measure the natural frequency and damping ratio of the network fundamental mode. Free vibration was initiated by pulling the top cable from two points of the top cable in the vertical direction. To measure the dynamic displacement, a sensor was placed at the mid-point of the top cable. The natural frequency and damping ratio of the fundamental mode were calculated by using the dynamic displacement data. To explore the effect of cross-tie stiffness on the modal behaviour of this simple cable network, different levels of prestressing force was applied to the cross-ties. The results obtained from experimental tests were compared with finite element simulations and good agreement between the two sets was found. It was observed that the fundamental frequency of the cable network was higher than that of the top individual cable, i.e. connecting cable with its neighbouring ones using cross-ties would enhance its in-plane stiffness. Such an effect was found to be more considerable if the pretension in the cross-tie was higher, i.e. the cross-tie was stiffer.

In 1998, Rowan Williams Davies & Irwin Inc. (RWDI) assessed the potential of wind-induced cable vibrations on the Cochrane Bridge in Alabama (Lankin et al., 2000). Their measurements were recorded during wind combined with light rain event. The wind speed was 13 m/s, with wind gust it was up to 18 m/s. The amplitude of cable vibration in the most severe case was found to be around 1.5 m. On-site free vibration tests were performed for 38 different stay cables out of the 96 cables on the bridge. The tested cable was excited by pulling a rope placed close to its mid-point in order to induce the first modal vibration. The motion of the cable was recorded by an accelerometer. From the measured cable motion time history, damping value of the stay cable could be calculated. The trend showed that the damping values of longer cables were smaller than those of the shorter ones. It was also reported that while most of the cables vibrated in their first mode, some vibrated in the second or higher modes. Test was repeated after
a single line of cross-ties was installed to connect all the cables to form a cable network. Results showed that the fundamental frequency of the cable network increased and the damping required to avoid galloping also reduced to one-quarter of that required prior to cross-tie installation.

A continuum analytical approach was developed by Royer-Carfagni (2003) to understand the modal behaviour of cross-tie cable network on the Normandy Bridge in France. According to his findings, the effect of cross-ties was equivalent to an apparent increase in the pre-tension of the main cables. When an orthogonal configuration of cross-ties was used, a marked increase of network in-plane frequency incurred. However, such a benefit would sharply reduce for inclined cross-ties. The reduction was dependent upon the angle of inclination between main cables and cross-ties.

Caracoglia and Jones (2005a; 2005b) developed an analytical model to study the in-plane free vibration of cable networks. In their model, the taut cable assumption was applied to the main cables whereas the cross-ties were modelled as linear spring connectors. The modal solutions were obtained by solving the eigenvalue problem in the case of simple configuration (Caracoglia and Jones, 2005a). According to their findings, there was a considerable increase in the in-plane stiffness of cable networks. In a special case of a simple two-cable network, cross-tie was extended to the deck. Results showed that the addition of a ground connector, i.e. extension of cross-tie to the deck, would significantly increase the fundamental frequency of the cable network. It was also pointed out that it would be reasonable to simulate cross-ties as rigid connectors (Caracoglia and Jones, 2005b). The approach used in this model was extended to real cable-stayed bridge on the Fred Hartman Bridge. In most of the cases, the cable networks on real cable-stayed bridges are not perfectly orthogonal. In the study, the original general networks on the Fred Hartman Bridge were transformed into an equivalent orthogonal cable network.
(Caracoglia and Jones, 2005b), as shown in Figure 2.3. This study also revealed that two lines of cross-ties could produce better results, in terms of in-plane stiffness and formulation of local modes, than three lines of cross-ties provided that the cross-ties were installed at appropriate locations. In spite of all these findings, this study has the following two limitations; (i) the sag effect and bending stiffness of the main cables, (ii) the inherent damping of stay cables as well as the damping effects of flexible cross-ties were not considered in the model. The formation of local modes in a cable network, which is one of the major drawbacks of the cross-tie solution, was also reported in the study. How to delay or reduce the local mode formation still needs intensive research effort.

Fred Hartman Bridge - Main Span Unit (South Tower) [NET_3C]

(a)
Figure 2.3: Modeling cable network on the Fred Hartman Bridge. (a) Three-dimensional network on real bridge; (b) Equivalent two-dimensional model used by Caracoglia and Jones (2005b)

2.2.2 Modal damping

Yamaguchi and Jayawardena (1992) developed a finite element model for a cable network on a real cable-stayed bridge where fourteen main cables were interconnected through four lines of cross-ties. In their nonlinear finite element analysis, an effort was made to determine the impact of cross-tie installation on the reduction of cable vibration amplitude and the structural damping change of the cable network. They reported that there was a 51% reduction in the vibration amplitude of the outer-most cable while the increase in the modal damping of the same cable was estimated to be 36%. To further increase network damping, it was recommended to use cross-ties possessing higher damping properties.

In a separate study by Yamaguchi and Nagahawatta (1995), a semi-experimental-numerical approach was used to determine the effect of cross-tie stiffness on the damping ratio of the network fundamental mode. The experimental setup was the same as that described in
Section 2.2.1 and shown in Figure 2.2. It was found that the modal damping of this simple cable network was always higher than that of a single isolated main cable. The increment of modal damping was found to be more significant when flexible (soft) cross-ties were used in place of rigid (stiff) ones. It was found that stiffer cross-ties only helped to transfer damping from the bottom cable in the network to the top cable (target cable), but in the case of softer/more flexible cross-ties, the bottom cable and the cross-ties also contributed to the damping increment of the top cable.

A subsequent study by Yamaguchi et al. (2001) experimentally investigated the effect of cross-tie on the modal damping of in-plane and out-of-plane cable vibrations of a simple two-cable network. The setup was a scaled model of a real catwalk system as shown in Figure 2.4, where two sagged cables were connected through a single cross-tie. They used modal synthesis approach as well as conducted experimental work to explore the role of main cables and cross-tie to understand the modal behaviour of cable networks. It was observed that the energy dissipation contributed by the cross-tie was much more than that from the main cables. Among their findings, it is interesting to note that the modal damping increment in the out-of-phase mode is found to be much more than that in the fundamental in-phase mode.
Sun et al. (2007) performed a set of free vibration tests on a scaled model of cable network using three cables connected through a cross-tie as shown in Figure 2.5. Laser instruments were used to measure the dynamic displacement at mid-span and quarter-span of the top cable and quarter-span of the bottom cable. From the measured dynamic displacement data, modal frequency and modal damping of the network were calculated. The experimental results were compared with those obtained from finite element simulation. According to the authors findings, stiff type cross-tie mainly contributed to enhance the in-plane stiffness of a cable network while soft type cross-tie was more effective in increasing system damping.
2.2.3 Energy Distribution

Ehsan and Scanlan (1989) used finite element approach to study the behaviour of a cable network. Based on their findings, the main function of cross-ties in a cable network is to help transfer the energy from a vulnerable cable to its neighbours. Yamaguchi and Alauddin (2003) were perhaps the first who explored the non-linear effect of cross-ties using a simple two-cable network. They carried out a series of forced vibration tests on a network that has the same layout as that in Figure 2.4 where two sagged cables are connected through a single cross-tie. The purpose of this study was to explore the nonlinear effect of cross-tie in the out-of-plane vibration of a simple two-cable network. According to their findings, the energy distribution due to cross-tie non-linearity was one of the important factors that lead to energy redistribution.

Caracoglia and Zuo (2009) applied an analytical model developed by Caracoglia and Jones (2005a; 2005b) to the real cable network on the Fred Hartman Bridge. According to their
study, cross-ties were found effective to suppress cable vibration in some of the lower order network modes. The ineffectiveness of cross-ties in suppressing higher order single cable(s) modes was due to the cross-tie installation position, which happened to be located at the nodal points of those modes.

2.2.4 Local mode formation

One of the main drawbacks of cross-tie solution is the formation of closely-spaced higher order local modes. These modes are difficult to suppress but could be pushed to higher order. Caracoglia and Jones (2005b) explored the modal behaviour of cable networks on a real cable-stayed bridge using an analytical model (Caracoglia and Jones, 2005a). The studied cable network was on the Fred Hartman Bridge in USA and consisted of 12 stay cables. According to their findings, each round of global modes was followed by a number of closely spaced local modes. If these modal frequencies are plotted as a function of the mode number then a clear plateau can be observed for a group of closely-spaced local modes as shown in Figure 2.6. The position of the local mode plateau was influenced by the installation location and number of cross-ties. It was pointed out that with a better placement of cross-ties, less number of cross-ties could be used to achieve better network behaviour in terms of modal frequency and formulation of local modes.
Bosch and Park (2005) simulated the performance of stay cables connected by cross-ties using a finite element model. It was observed that the installation of cross-ties induced local modes which were densely populated over a narrow band of frequency range. Results showed that the number as well as the position of cross-tie(s) play important roles to achieve the desired results. In addition, it was found that although oversized cross-tie could increase the modal frequency of the global modes, the number of excited local modes also increased substantially. In their study, the effect of different system parameters that would influence the modal order and the number of local modes present in the mode plateau were not discussed.

In a technical report prepared for the U.S. Department of Transportation (Kumarasena et al., 2007) to study bridge stay cable vibration and mitigation, it was pointed out that the use of cross-tie would increase the generalized modal mass and the in-plane frequency of the network global modes but excite numerous local modes. Since these local modes are difficult to control, it was recommended to shift these local modes to as higher order as possible.

Figure 2.6: Local mode plateaus for different configuration of cross-ties observed by Caracoglia and Jones (2005b)
2.2.5 Role of different system parameters

The identification of key system parameters and the proper understanding of their respective roles in affecting network behaviour are important for clarifying mechanics of cable networks. Bosch and Park (2006) used a finite element simulation to investigate the role of different system parameters, i.e. the cross-tie stiffness, the number of cross-ties and the cable end conditions, on the performance of cross-tie solution. Their study used a real cable network on the Bill Emerson Memorial Bridge as an example. The cable network contained four lines of cross-ties. Though the modal frequency of global modes benefited, the number of local mode cluster also increased. The modal frequency of the cable network was found to be independent of the end condition of the stay cables. Results showed that the performance of cross-tie solution was sensitive to the frequency of attacking wind and excessive provision of cross-ties would increases the vulnerability of the system to local mode excitation in case of turbulent wind condition.

Sun et al. (2007) performed a set of physical tests on a scaled model of cable network as discussed in Section 2.2.2. In addition to exploring the effect of cross-tie stiffness on the modal behaviour of tested cable network, other factors such as the cross-tie stiffness, the tensioning method and the pretension of the cross-ties were also considered. It was pointed out that the one-time tension method for cross-tie would result in slightly more damping of the cable network than the multi-time tension method. Similarly, it was also found that high initial tension in cross-tie would decrease the network modal damping.

A simplified analytical model was developed by Zhou et al. (2011) to study free vibration of a single cable network. In this model, only a single cable was included in the model and cross-ties were modeled as linear springs as shown in Figure 2.7. A discussion was made on how to
achieve the maximum modal frequency in the cases of single or two springs/cross-ties. According to their findings, the stiffness and location of spring/cross-tie played an important role in increasing the modal frequency of a single cable network.

Giaccu and Caracoglia (2012) investigated the nonlinear behaviour of cross-ties in cable networks by extending a previous model developed by Caracoglia and Jones (2005a). The new model considered the two parallel linear and nonlinear forces in the cross-tie. The cross-tie nonlinearity was described by a cubic stiffness nonlinear spring. The nonlinear spring coefficient was approximated as an equivalent linearized spring coefficient by equating the work done by the linear and the nonlinear springs within the same time duration. This equivalent spring constant (or nonlinear component of cross-tie stiffness) was a function of main cable vibration amplitude and the network frequency. The nonlinear component of cross-tie stiffness would change the net stiffness of the cross-tie which in turn would affect the in-plane frequency of the cable network. According to their findings, larger cable vibration amplitude could result in slacking of cross-tie(s). In a subsequent study (Giaccu and Caracoglia, 2013), the cubic-stiffness model of cross-tie was extended to a generalized power-law stiffness model. In addition, an effort was made (Giaccu et al., 2014) to determine the minimum required initial pretension in the
cross-tie(s) to avoid slackening. A performance coefficient was introduced to measure the severity of malfunction in the cross-tie. It was defined as a function of the main cable vibration amplitude and the cross-tie initial pretension. Based on their results, it was interesting to note that the nonlinear component in the cross-tie stiffness had a negligible effect on the fundamental mode of cable network even in the case of relatively higher vibration amplitude and low initial pretension in cross-tie. On the other hand, for higher order network modes, relatively larger initial pretension in cross-tie was found to help maintain the linearity of cross-tie behaviour even at higher amplitude. Nevertheless, it is worth pointing out that there is no scientific evidence that the proposed generalized power-law stiffness model is close to the actual behaviour of cross-ties on real cable-stayed bridges.

More recently, Giaccu et al. (2015a; 2015b) conducted free vibration analysis of a simple three-cable network by using stochastic approximation algorithm. The model developed earlier by Giaccu and Caracoglia (2013) considered the nonlinear effect of cross-tie which rendered the network frequency become dependent on the cable vibration amplitude. It was combined with the stochastic approximation in the new model, of which the random value of cable vibration amplitude was chosen to illustrate the effect of vibration amplitude uncertainty on the modal behaviour of a three-cable network. The free vibration analysis results yielded from the proposed stochastic approximation approach were compared with those obtained from Monte Carlo simulation.

2.3 Hybrid system

In more recent years, there is a tendency of building longer span cable-stayed bridges due to advancement in building materials, construction techniques and analysis tools. For example, the Sutong Bridge in China, which was constructed in 2012, has a main span of 1088 m and the
longest cable is 577 m in length (Wang et al., 2014). For such kind of long cables, external dampers would not be effective in controlling cable vibrations because of the constraint on the installation location. In addition, external viscous dampers would not be activated if the cable vibration amplitude is relatively small. On the other hand, though cross-ties can help to increase the generalized modal mass and the modal frequency of cable network global modes, they are incapable of dissipating the energy directly (especially when stiffer cross-ties are used). Therefore, using separate external dampers or cross-ties may have limited effect on suppressing excessive cable vibrations. To overcome these limitations, the feasibility of using a hybrid system, which is a combination of both external dampers and cross-ties, to suppress cable vibration was investigated recently. As indicated by Kumarasena et al., (2007) in a technical report prepared for U.S. Department of Transportation, there was a potential of combining cross-ties and external dampers into a single hybrid vibration control system.

Bosch and Park (2005) simulated the performance of stay cables with cross-ties combined with external dampers using the finite element approach. Results showed that the combined use of cross-ties and external dampers would not necessarily earn the cumulative benefits of both when they were applied separately.

Caracoglia and Jones (2007) extended the analytical model developed earlier (Caracoglia and Jones, 2005a) by including external dampers along the cross-tie lines in the formulation. Such a hybrid system was then applied to the stay cables on the Fred Hartman Bridge. Different cable network configurations were studied where external dampers were aligned with the cross-tie lines. It was observed that the installation of external dampers only affected the global modes of the cable network while the local modes remained unaltered. The frequency-damping curves were drawn for the hybrid network under multiple configurations of external dampers. Results
suggested that hybrid cable networks were a preferable configuration to achieve multi-mode optimization when compared to the damper-only solution. This was due to the reason that more than one damper was installed. The formation of large group of local modes was still an issue in hybrid system and they were marginally affected by the installation of external dampers.

Caracoglia and Zuo (2009) used numerical simulations to determine the effectiveness of hybrid system in controlling cable vibrations. The study was based on the network on the Fred Hartman Bridge. The performance of cable networks with various configurations of external dampers was investigated. One of the interesting findings was that the maximum modal damping of a specific hybrid system mode was considerably lower than the maximum achievable modal damping of a single damped cable. It was also pointed out that the hybrid system configuration with dampers in-line with cross-tie lines yielded better performance in terms of the modal damping of the fundamental mode. The option of installing external dampers on every cable was not necessary although the addition of external dampers could help to suppress some of the local modes. It is also important to note from their findings that the combined use of cross-ties and external dampers were not effective in controlling out-of-plane cable vibrations.

2.4 Summary

From the above review, it is clear that dynamic behaviour of cable networks has not been fully understood. The majority of existing studies were based on cable networks with simple configurations and focused mainly on the improvement of its in-plane stiffness, whereas other advantages (e.g. increase in modal damping, energy redistribution) and disadvantages (e.g. formation of closely spaced local modes) resulted from the cross-tie installation are considerably lacking. In addition, clarification of the role played by different system parameters in network vibration would offer a deeper understanding of the mechanics associated with cable network.
However, this part of information is scarce in the literature. The only system parameter that received proper attention is the cross-tie stiffness. Therefore, there is a strong need to develop analytical models, identify key system parameters of cable networks and explore their respective roles. To improve our understanding about the mechanics of hybrid system, thus, there is an urgent need to develop an analytical model to get its physical insights.

The objectives proposed in the present research, as listed in Section 1.5, will address the above identified needs. It will include development of various analytical models, starting from a basic two-cable network system to a more general multi-cable multi-cross-tie network to explore the effect of cross-tie installation on the in-plane frequency and structural damping of cable networks. An effort will be made to understand the role of different system parameters. As discussed earlier, one of the major drawbacks of cross-tie solution is the formation of local modes. The quantification and minimization of local mode formation will be explored. In addition, dynamic behaviour of hybrid systems will be studied analytically to gain deeper insight of the mechanics and effectiveness of this novel cable vibration control means. The current study is not limited to the analytical models and numerical simulations but some of the experimental work will also be performed. These experimental models will explore the modal behaviour of pure cable networks as well as the hybrid systems.
CHAPTER 3 Analytical Study on Modal Behaviour of Cable Networks

Analytical models of two typical cable networks will be presented in this chapter, which would help to better understand the modal behaviour of this type of structural system. As a first step, in Section 3.1, a basic cable network model consisting two main cables and a transverse cross-ties is proposed, where the inherent damping of the main cables and the cross-tie is neglected. This idealized model is extended in Section 3.2 to include the structural damping of all the component members in the cable network in the formulation. Based on this development, an analytical model of a general cable network consisting of a given number of main cables interconnected through multiple lines of cross-ties will be presented next in Chapter 4.

3.1 Undamped Two-Cable Networks

Analytical model of cable networks can play an important role in understanding the behaviour of this type of structural system. However, developing an analytical model that reflects all detailed aspects of an actual system is quite challenging. Therefore, in this section, a relative simple model of a basic two-cable network is considered where the two main cables are connected through a single transverse cross-tie. The intrinsic damping of the main cables and cross-tie is not considered in this model.
3.1.1 System characteristic equation

As portrayed in Figure 3.1, the cable network studied in this section comprises of two unequal length main cables, with \( L_1 \) being the length of the longer cable and \( L_2 \) being that of the shorter one. The longer cable is assumed to be the target cable of which its vibration needs to be controlled. The two cables are connected through a flexible cross-tie, which divides each main cable into two segments. The length of each cable segment and that of the transverse cross-tie are labelled as shown in Figure 3.1. Assume the mass per unit length of cable \( i \) is \( m_i \) and the tension is \( H_i \) \((i=1, 2)\). The position of the cross-tie is \( l_1 \) from the left support of main cable 1. The transverse displacements of the main cables and the axial displacement of the cross-tie are considered positive downward and negative upward. Both main cables are assumed to be fixed at both ends. When formulating the analytical model of a two-cable network with a transverse flexible cross-tie, the main cables are idealized as taut cables with both ends fixed. Only the in-plane transverse motions are considered. Because of taut-cable assumption, the cable sag is ignored and therefore, the additional cable tension due to vibration is neglected. The bending stiffness and structural damping property of the main cables and the cross-tie are not considered.
in this part of the study. The flexible cross-tie is assumed to vibrate only along its axial direction and its behaviour is simulated by a linear spring connector with an equivalent axial stiffness of $K_c$. This equivalent axial stiffness $K_c$ is not only a function of the cross-tie axial stiffness but also its pretension.

The in-plane transverse free vibration of a typical cable segment can be described by Irvine and Caughey (1974)

$$ H \frac{\partial^2 \nu(x,t)}{\partial x^2} = m \frac{\partial^2 \nu(x,t)}{\partial t^2} $$

(3-1)

where $\nu$ is the transverse displacement, $H$ and $m$ are the tension and the unit mass of the taut cable, respectively.

Now denote $\tilde{\nu}(x)$ as the shape function for the cable transverse displacement and $\omega$ as the circular frequency of vibration, by applying the Bernoulli-Fourier method of separation of variables contained in the in-plane transverse displacement $\nu(x,t)$ of a single main cable, i.e. $\nu(x,t) = \tilde{\nu}(x)\sin(\omega t)$, the shape functions $\tilde{\nu}(x)$ for different cable segments of main cables can be expressed as,

$$ \tilde{\nu}_{2i-1}(x_{2i-1}) = B_{2i-1}\sin(\Omega \eta_i x_{2i-1}/L_i) \quad i=1, 2 $$

(3-2a)

$$ \tilde{\nu}_{2i}(x_{2i}) = B_{2i}\sin(\Omega \eta_i x_{2i}/L_i) \quad i=1, 2 $$

(3-2b)

where $B_{2i-1}$ and $B_{2i}$ ($i=1, 2$) are the shape function constants of the four main cable segments shown in Figure 3.1; $\eta_i=f_i/f_1$ is the frequency ratio of the $i^{th}$ ($i=1, 2$) main cable; $f_i$ is the fundamental frequency of the $i^{th}$ ($i=1, 2$) main cable; $\Omega=\pi f/\Omega$ is the non-dimensional frequency of the cable network and $f$ is the corresponding natural frequency of the network. The following boundary, compatibility and equilibrium conditions are applied to this cable network model to determine the shape function constants in Eq. (3-2):
Boundary conditions

\[ \ddot{v}_{2i-1}(0) = 0, \quad \ddot{v}_{2i}(0) = 0 \quad i=1, 2 \]  

(3-3a)

Compatibility conditions

\[ \ddot{v}_{2i-1}(l_{2i-1}) = \ddot{v}_{2i}(l_{2i}) \quad i=1, 2 \]  

(3-3b)

\[ \ddot{V}_3(l_3) - \ddot{V}_1(l_1) = \frac{1}{K_e} \left[ H_1 \left( \frac{\partial \ddot{V}_1}{\partial x_1} \big|_{x_1=l_1} + \frac{\partial \ddot{V}_2}{\partial x_2} \big|_{x_2=L_1-l_1} \right) \right] \]  

(3-3c)

Equilibrium conditions

\[ \sum_{i=1}^{2} \left( \frac{\partial \ddot{V}_{2i-1}}{\partial x_{2i-1}} \bigg|_{x_{2i-1}=l_{2i-1}} + \frac{\partial \ddot{V}_{2i}}{\partial x_{2i}} \bigg|_{x_{2i}=l_{2i}} \right) H_i = 0 \]  

(3-3d)

Implementing Eq. (3-2) and the conditions in Eq. (3-3), and express the resulting equations into a matrix form, yields the following homogeneous system:

\[ [R] \{X\} = \{0\} \]  

(3-4)

where

\[ [R] = \begin{bmatrix}
\sin(\phi_1) & -\sin(\phi_2) & 0 & 0 \\
0 & 0 & \sin(\phi_3) & -\sin(\phi_4) \\
\psi \Omega \cos(\phi_1) + \sin(\phi_1) & \psi \Omega \cos(\phi_2) & -\sin(\phi_3) & 0 \\
\gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4)
\end{bmatrix} \]

is the coefficient matrix, \( \{X\} = [B_1 \ B_2 \ B_3 \ B_4]^T \) is the vector containing all the unknown shape function constants, and \( \{0\} \) is the null vector. In the coefficient matrix \( [R] \), \( \phi_{2i-1} = \Omega \eta_i \epsilon_{2i-1} \) and \( \phi_{2i} = \Omega \eta_i \epsilon_{2i} \) apply respectively to the left and right segment of the \( i^{th} \) main cable \( (i=1, 2) \), \( \eta_i = f_i/f_l \) and \( \gamma_i = \sqrt{H_i m_i/H_1 m_1} \) are respectively the frequency ratio and the mass-tension ratio parameter of the \( i^{th} \) main cable, \( f_i, m_i, H_i \) are respectively the fundamental frequency, the unit mass and the tension of the \( i^{th} \) cable in the network \( (i=1, 2); \psi = H_1/(K_e L_1) \) is the non-dimensional cross-tie flexibility parameter and \( K_e \) is the axial stiffness of the cross-tie.
The non-trivial solution to Eq. (3-4) can be obtained by setting the determinant of the coefficient matrix \([R]\) to zero. After expanding the determinant and making all the trigonometric simplifications, the following equation can be obtained, i.e.

\[
\gamma_1 \sin(\Omega \eta_1) \sin(\delta_3) \sin(\delta_4) + \gamma_2 \sin(\Omega \eta_2) \sin(\delta_1) \sin(\delta_2) + \psi \Omega_1 \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) = 0
\]  

(3-5)

which is the characteristic equation of the studied two-cable network with a flexible cross-tie shown in Figure 3.1. It can be observed from Eq. (3-5) that the left hand side of the equation is the summation of three terms. The first two terms are independent of the cross-tie stiffness and represents the characteristic equation of two-cable network using a rigid cross-tie while the third term represents the impact of cross-tie stiffness on the dynamic behaviour of the studied cable network. If the cross-tie is rigid, i.e. \(K_c = \infty\), the non-dimensional flexibility parameter of the cross-tie \(\psi (= H_1/(K_c L_1))\) would become 0. Thus, the third term in Eq. (3-5) vanishes, and the system characteristic equation becomes the same as that of a basic cable network using a rigid cross-tie.

Equation (3-5) can be applied to a basic cable network having any arbitrary configurations and properties to study its in-plane modal behaviour and to evaluate how the dynamic response of a cable would be altered once it is connected to its neighbours through a flexible cross-tie. Now, the proposed cable network analytical model will be applied to a number of two-cable network systems with different geometric layout and cable properties. As a model validation, a corresponding finite element model will be developed in Abaqus 6.10. The B21 beam element is selected to simulate the main cables, whereas the SPRING2 element is chosen to simulate the flexible cross-tie. The results obtained from the proposed analytical model will be compared with those from the numerical simulations.
3.1.2 Twin-cable network with a flexible cross-tie at arbitrary location

The two main cables in this type of network are twins, i.e. they have the same length, unit mass and tension. Since the position of the cross-tie is arbitrary, it can be assumed that the cross-tie locates at a distance $l_1$ from the left end of main cable 1 and $l_1 \neq L_1/2$ (Figure 3.2).

![Figure 3.2: Schematic diagram of a symmetric twin-cable network with flexible cross-tie at arbitrary point](image)

These conditions give the frequency ratios of $\eta_1 = \eta_2 = 1$, the segment ratios of $\varepsilon_1 = \varepsilon_3 = l_1/L = \varepsilon$ and $\varepsilon_2 = \varepsilon_4 = l_2/L = 1 - \varepsilon$, and the mass-tension ratios of $\gamma_1 = \gamma_2 = 1$. Substitute these non-dimensional system parameter values into Eq. (6), yields

$$2\sin(\Omega)\sin\left(\Omega \frac{l_1}{L}\right)\sin\left(\Omega \frac{l_2}{L}\right) + \psi \Omega (\sin \Omega)^2 = 0$$

or

$$\sin(\Omega)\sin(\Omega \varepsilon) \left\{ 2\sin[\Omega(1 - \varepsilon)] + \frac{\psi \Omega \sin(\Omega)}{\sin(\Omega \varepsilon)} \right\} = 0 \quad (3-6)$$

In the above system characteristic equation, the second term in the curly bracket is a function of the cross-tie flexibility parameter $\psi$. According to the definition, this non-dimensional parameter is related to the axial stiffness of the cross-tie by $\psi = H_1/(K_c L_1)$. Theoretically, the cross-tie axial stiffness varies from 0, for a rigid cross-tie, to $\infty$, for a cross-tie having no axial stiffness, i.e. the cables in the network vibrate independently. However, in practice, this parameter ranges from 0.01 to 1.0 (Caracoglia and Jones, 2005b). In the case of a rigid cross-tie, its axial stiffness
\( K_c = \infty \), the non-dimensional flexibility parameter of the cross-tie is thus \( \psi = 0 \). Therefore, Eq. (3-6) can be reduced to

\[
\sin(\Omega)\sin(\Omega \varepsilon) \sin[\Omega(1 - \varepsilon)] = 0
\]

This equation represents the system characteristic equation of a twin-cable network with rigid cross-tie.

On the other hand, when flexible cross-tie is used in a twin-cable network, the form of its characteristic equation, Eq. (3-6), suggests that three sets of solution are present. The roots for the first set, yielded from \( \sin(\Omega) = 0 \), are responsible for the global modes of the cable network. This set of roots, \( \Omega = n\pi \) \((n=1, 2, 3, \ldots)\), would give symmetric in-phase global modes for odd values of \( n \), and asymmetric in-phase global modes for even values of \( n \). The fundamental frequency of a twin-cable network can be obtained by setting \( n=1 \), which is the same as that of a single main cable in the network. It is interesting to note that the same set of roots also exist in the twin-cable network connected through a transverse rigid cross-tie (Caracoglia and Jones, 2005a). This indicates that the global modes of a twin-cable network is independent of the type of cross-tie, be it rigid or flexible. The second set of roots, determined from \( \sin(\Omega \varepsilon) = 0 \), are the functions of segment ratio \( \varepsilon \). Again, it is worth noting that this set of roots is also present in a twin-cable network connected through a transverse rigid cross-tie, where they are the network local modes dominated by the motions of cable left segments (Caracoglia and Jones 2005a), i.e. the local LS (left segment) modes. The third set of roots can be determined by setting the summation of the two terms within the curly brackets in Eq. (3-6) as zero. It can be written as:

\[
2\sin[\Omega(1 - \varepsilon)] + \frac{\psi(\sin(\Omega))}{\sin(\Omega \varepsilon)} = 0 \tag{3-7}
\]

The first term on the left hand side of Eq. (3-7) is the same as that in a twin-cable network with rigid cross-tie discussed earlier, which describes the local RS (right segment)
modes with the oscillations of cable segments 2 and 4 in Figure 3.2 take the dominance. The second term appears here because of the consideration of cross-tie flexibility. This term is not only a function of the cross-tie type represented by the flexibility parameter \( \psi \), but also a function of the cross-tie position \( \varepsilon \) and includes the contribution of the main cable left segment motion represented by \( \sin(\Omega \varepsilon) \). Depending on the axial stiffness of the cross-tie, the contribution of the second term in Eq. (3-7) varies from 0 (when cross-tie is rigid and hence \( \psi=0 \)) to considerable (when \( \psi \) is large enough to make the first and the second term in Eq. (3-7) comparable to each other). Therefore, compared to the rigid cross-tie case, of which the set of roots yielded from Eq. (3-7) describe the local RS (right segment) modes, when a flexible cross-tie is used in a twin-cable network, not only the modal frequency of the local RS modes will be changed, but more interestingly, their mode shape will also be changed from a local RS mode in a rigid cross-tie network to a global mode in a corresponding flexible cross-tie network where both the left and the right segments of the two main cables are excited.

The above facts suggest that in the case of twin cable networks, the two cross-tie properties, i.e. the position \( \varepsilon \) and the flexibility \( \psi \), would dictate the type of mode (global or local) and the associated modal behaviour of a twin cable network. The cross-tie flexibility, however, would influence the modal frequencies of the right segment local modes.

**Numerical Example**

To validate the proposed analytical model and the modal solution of a twin-cable network, a numerical example is presented. The twin main cables in this example are assumed to be the same as the type AS14 cable on the Fred Hartman Bridge (Caracoglia and Jones, 2005b). Both main cables have a tension of 1598 kN, a unit mass of 47.9 kg/m, and a length of 67.34 m. The flexibility parameter of the cross-tie is assumed to be \( \psi=1.0 \) and it locates at one-fourth span
from the left end of the main cables. The modal properties of the first ten modes of this twin-
cable network determined from the proposed analytical model and numerical simulations are
given in Table 3.1, with the corresponding mode shapes portrayed in Figure 3.3. For comparison
purpose, the modal properties of the first ten modes of the same twin-cable network but using
rigid cross-tie and their mode shapes are also given in Table 3.1 and Figure 3.3.

Table 3.1: Comparison of in-plane modal properties of a symmetric twin-cable network with
rigid ($\psi = 0$) and flexible ($\psi = 1.0$) cross-tie at quarter span

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Flexible cross-tie ($\psi = 1.0$)</th>
<th>Rigid cross-tie ($\psi = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed Analytical Model</td>
<td>FEA</td>
</tr>
<tr>
<td>1</td>
<td>1.3562</td>
<td>1.3562</td>
</tr>
<tr>
<td>2</td>
<td>1.4647</td>
<td>1.4646</td>
</tr>
<tr>
<td>3</td>
<td>2.7124</td>
<td>2.7118</td>
</tr>
<tr>
<td>5</td>
<td>4.0685</td>
<td>4.0660</td>
</tr>
<tr>
<td>7</td>
<td>5.4247</td>
<td>5.4184</td>
</tr>
<tr>
<td>8</td>
<td>5.4247</td>
<td>5.4184</td>
</tr>
</tbody>
</table>
Figure 3.3: Transformation of first ten modes of a symmetric twin-cable network as flexibility parameter $\Psi$ varies from 0 to 1.0 and cross-tie locates at quarter span.
As can be seen from Table 3.1, the modal properties of the global modes are not affected by the type of cross-tie. By replacing a rigid cross-tie with a flexible one, the modal frequency and the mode shape of both symmetric and asymmetric global modes (modes 1, 3, 5, 7 and 9 in Table 3.1 and Figure 3.3) remain the same. However, in the case of local modes, no matter if it is dominated by vibrations of the left segments (LS modes) or the right segments (RS modes), such a change in the cross-tie stiffness renders them to evolve into global modes, as can be seen from modes 2, 4, 6, 8, and 10 depicted in Figure 3.3. It is interesting to note that in the case of mode 8, which is defined from \( \sin(\Omega t) = 0 \) (the second set of roots), though the mode shape evolves from the local LS mode in the rigid cross-tie network to the global mode if flexible cross-tie is used instead, the frequencies associated with the LS mode and the global mode remain the same and are not affected by the cross-tie flexibility. However, for the local RS modes (mode 2, 4, 6 and 10 in Table 3.1 and Figure 3.3), both mode shapes and modal frequencies are affected. By changing cross-tie type from rigid to flexible, a local RS mode becomes a global mode with lower frequency instead. To have a more clear picture on how the change in cross-tie flexibility would lead to such a mode shape evolution, in Figure 3.4, mode shapes of the first three RS modes (modes 2, 4, 6) and the first LS mode (mode 8) corresponding to \( \psi = 0, 0.01, 0.1 \) and 1.0 are presented.
As can be seen from Figure 3.4, in modes 2 and 4, the left segment of the main cables starts to excite when $\psi$ increases to 0.1. For mode 6, the left cable segments are excited at an even lower $\psi$ value of 0.01. Further reduction in the cross-tie rigidity eventually allows a full development of vibration in the cable left segments and results in a global mode. The decrease of the modal frequency of these three modes with the increase of cross-tie flexibility implies that the in-plane stiffness of a cable network will be increased the most when a rigid cross-tie is used. This analytical finding is consistent with the experimental observations by Sun et al., (2007). In
addition, as can be clearly seen from Table 3.1, the results obtained from the proposed analytical model are found to agree well with those from numerical simulations.

### 3.1.3 Symmetric two-cable network with a flexible cross-tie at mid-span

A symmetric cable network comprises two unequal length main cables connected by a transverse flexible cross-tie is shown in Figure 3.5.

**Figure 3.5**: Symmetric SMT two-cable network with unequal length main cables and flexible cross-tie

Based on the mass-tension ratio parameter of the main cables, the cable network is categorized into two types, the first one is SMT cable network where consisting cables have the same mass-tension ratios ($\gamma_1 = \gamma_2 = 1$), whereas in the DMT cable network, both the main cables have different mass-tension ratio parameter. It is assumed that the two cables have the same tension and unit mass, and the flexible cross-tie locates at the mid-span. The network thus has the following system parameters: the frequency ratios $\eta_1 \neq \eta_2$, the mass-tension ratios $\gamma_1 = \gamma_2$ and the segment ratios $\varepsilon_j = 1/2 (j = 1 \text{ to } 4)$. By defining $\lambda_2 = L_1 / L_2$ as the length ratio parameter of main cable 2, the cable length $L_2$ is chosen such that $\eta_2 \lambda_2 = 1$. Substitute the non-dimensional system parameter values into Eq. (3-5), yields
Equation (3-8) has three sets of solution. The first two sets, yielded respectively from \( \sin(\Omega/2) = 0 \) (originated from \( \sin(\Omega \eta_1/2) = 0 \)) and \( \sin(\Omega \eta_2/2) = 0 \), are responsible for the local modes dominated respectively by main cables 1 and 2. This suggests that in the case of a symmetric SMT cable network, local modes associated with predominant vibration of a single main cable are present for all main cables as far as the cross-tie is placed at the mid-span. The modal properties of these modes are independent of the cross-tie type used in the network system. The third set, derived from Eq. (3-8),

\[
\sin \left( \frac{\Omega}{2} (1 + \eta_2) \right) + 2\psi \Omega \cos \left( \frac{\Omega}{2} \right) \cos \left( \frac{\Omega}{2} \eta_2 \right) = 0
\]

is responsible for the global modes of the flexible cross-tie network. The first term in the above equation, i.e. \( \sin[\Omega (1 + \eta_2)/2] \), is exactly the same as that in the rigid cross-tie case, leaving the second term, \( 2\psi \Omega \cos(\Omega/2)\cos(\Omega \eta_2/2) \), to be responsible for the change in the modal frequency due to the adoption of a flexible cross-tie. Further, the form of the second term reveals that if a flexible cross-tie is used, not only the flexibility of the cross-tie itself, but also the frequency ratio of the neighbouring cable \( \eta_2 \) will play a role in affecting modal frequency of the global modes.

**Numerical Example**

To further validate the proposed analytical model, a numerical example of a symmetric unequal length two-cable network is analyzed. A corresponding finite element model of the network is developed in Abaqus 6.10. The physical properties of the two cables are

Main Cable 1: \( H_1=1598 \text{ kN} \quad m_1=47.9 \text{ kg/m} \quad L_1=67.34 \text{ m} \)
Main Cable 2: \( H_2 = 1598 \text{ kN} \quad m_2 = 47.9 \text{ kg/m} \quad L_2 = 59.52 \text{ m} \)

The flexibility parameter of the cross-tie is assumed to be \( \psi = 1.0 \). The modal analysis results of the first ten modes determined from the proposed analytical model and numerical simulation are listed in Table 3.2, from which a good agreement between the two sets can be clearly seen. Besides, for a better understanding of the impact of cross-tie type on the modal behaviour of such kind of cable network, the modal properties of a corresponding rigid cross-tie system are also given in the table.

Table 3.2: In-plane modal properties of a symmetric unequal length two-cable network with system parameters as frequency ratio \( \eta_2 = 0.88 \), segment ratio \( \varepsilon_j = 1/2 \) (\( j = 1 \) to \( 4 \)), mass-tension ratio \( \gamma_1 = \gamma_2 = 1 \) and flexibility parameter \( \psi = 1.0 \)

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Flexible cross-tie (( \psi = 1.0 ))</th>
<th>Rigid cross-tie (( \psi = 0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed Analytical Model FEA Mode Shape</td>
<td>Modal frequency (Hz) Mode Shape</td>
</tr>
<tr>
<td>1</td>
<td>1.4149 1.4150 GM, 1-Sym., in-phase</td>
<td>1.4397 GM, 1-Sym., in-phase</td>
</tr>
<tr>
<td>2</td>
<td>1.7073 1.7075 GM, 1-Sym., out-of-phase</td>
<td>2.7124 LM, Cable 1, 1-Asym.</td>
</tr>
<tr>
<td>3</td>
<td>2.7124 2.7123 LM, Cable 1, 1-Asym.</td>
<td>2.8796 GM, 1-Sym., out-of-phase</td>
</tr>
<tr>
<td>4</td>
<td>3.0688 3.0687 LM, Cable 2, 1-Asym.</td>
<td>3.0688 LM, Cable 2, 1-Asym.</td>
</tr>
<tr>
<td>5</td>
<td>4.1099 4.1090 GM, 2-Sym., in-phase</td>
<td>4.3193 GM, 2-Sym., in-phase</td>
</tr>
<tr>
<td>6</td>
<td>4.6512 4.6503 GM, 2-Sym., out-of-phase</td>
<td>5.4247 LM, Cable 1, 2-Asym.</td>
</tr>
<tr>
<td>7</td>
<td>5.4247 5.4219 LM, Cable 1, 2-Asym.</td>
<td>5.7592 GM, 2-Sym., out-of-phase</td>
</tr>
<tr>
<td>8</td>
<td>6.1374 6.1345 LM, Cable 2, 2-Asym.</td>
<td>6.1374 LM, Cable 2, 2-Asym.</td>
</tr>
<tr>
<td>10</td>
<td>7.6996 7.6932 GM, 3-Sym., out-of-phase</td>
<td>8.1371 LM, Cable 1, 3-Asym.</td>
</tr>
</tbody>
</table>

A comparison between the modal frequencies and mode shapes of the two cable networks indicate that in the case of a symmetric SMT two-cable network, the type of cross-tie has no
influence on the modal properties of the local modes dominated by motion of a single main cable. For example, the first asymmetric local mode of cable 1 in the rigid cross-tie case (mode 2) remains the same in the flexible cross-tie system, except becomes the third mode. However, it is interesting to note that the frequencies of all the global modes in the rigid cross-tie system listed in Table 3.2 are decreased when a flexible cross-tie is used. The frequency reduction is much more significant in the case of an out-of-phase global mode when compared with an in-phase one. For example, the fundamental mode of a rigid cross-tie network, which is an in-phase global mode, is reduced by 1.7% from 1.4397 Hz to 1.4149 Hz in a flexible cross-tie system; whereas mode 3 in the rigid cross-tie network, which is an out-of-phase global mode with frequency of 2.8796 Hz, is reduced to 1.7073 Hz in the flexible cross-tie case by 41% and becomes the second mode. The mode shape of a few typical modes of this example network is presented in Figure 3.6.

Figure 3.6: A few typical modes of a symmetric SMT cable network with system parameters as frequency ratio \( \eta_2 = 0.883 \), segment ratio \( \varepsilon_j = 1/2 \) (\( j = 1 \) to 4) and flexibility parameter \( \psi = 1.0 \) (GM: global mode, LM: local mode, Sym.: symmetric, Asym.: asymmetric)

3.1.4 Asymmetric two-cable network with a flexible cross-tie at one-third span

In majority of cable networks installed on real cable-stayed bridges, the consisting main cables have different length, unit mass and tension, which results in different mass-tension
(DMT) ratio. Besides, since the spacing between cables is typically closer on the pylon side than on the deck side, the geometric layout of a real cable network is generally asymmetric. Therefore, the proposed analytical model is applied to study the modal behaviour of an asymmetric two-cable network with the cable data taken from a real cable-stayed bridge (Caracoglia and Jones, 2005b). The results obtained from the proposed analytical model are compared with those from the numerical simulations.

The two main cables in the studied asymmetric DMT cable network are rearranged in such a way that the left and the right offsets of the neighbouring cable with respect to the target cable (Figure 3.1) are respectively 3 m and 9 m. The cross-tie is located at one-third span of the target cable from its left support, i.e. \( \varepsilon = 1/3 \). The properties of the two main cables and the cross-tie are:

- **Main Cable 1:** \( L_1 = 72 \text{ m} \quad H_1 = 2200 \text{ kN} \quad m_1 = 50 \text{ kg/m} \)
- **Main Cable 2:** \( L_2 = 60 \text{ m} \quad H_2 = 2400 \text{ kN} \quad m_2 = 42 \text{ kg/m} \)
- **Cross-tie:** \( \varepsilon = 1/3 \quad K_c = 30.54 \text{ kN/m} \quad (\psi = 1.0) \)

By solving Eq. (3-5), the modal properties of the first ten network modes can be calculated. The results are tabulated in Table 3.3, together with those obtained from numerical simulations. A good agreement between the two sets can be clearly seen. In addition, the modal analysis results of a corresponding rigid cross-tie network are also given in the same table for the convenience of comparison. The mode shapes of these ten modes are depicted in Figure 3.7.
Table 3.3: Comparison of modal properties of an asymmetric DMT two-cable network with flexible or rigid cross-tie at \( \varepsilon = 1/3 \)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Flexible cross-tie</th>
<th>Rigid cross-tie</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f (\text{Hz}) )</td>
<td>( f (\text{Hz}) )</td>
</tr>
<tr>
<td></td>
<td>Ana.</td>
<td>FEA</td>
</tr>
<tr>
<td>1</td>
<td>1.53</td>
<td>1.53</td>
</tr>
<tr>
<td>2</td>
<td>2.11</td>
<td>2.11</td>
</tr>
<tr>
<td>3</td>
<td>2.97</td>
<td>2.97</td>
</tr>
<tr>
<td>4</td>
<td>4.04</td>
<td>4.03</td>
</tr>
<tr>
<td>5</td>
<td>4.37</td>
<td>4.37</td>
</tr>
<tr>
<td>6</td>
<td>5.85</td>
<td>5.85</td>
</tr>
<tr>
<td>7</td>
<td>5.98</td>
<td>5.97</td>
</tr>
<tr>
<td>8</td>
<td>7.30</td>
<td>7.29</td>
</tr>
<tr>
<td>9</td>
<td>8.00</td>
<td>7.99</td>
</tr>
<tr>
<td>10</td>
<td>8.74</td>
<td>8.72</td>
</tr>
</tbody>
</table>
The results shown in Table 3.3 indicate that by replacing the rigid cross-tie with a flexible one, the frequency of the network fundamental mode, which is an in-phase global mode, decreases. The target cable has a fundamental frequency of 1.46 Hz. When it is connected with the neighbouring cable using a rigid cross-tie, the modal frequency increases to 1.65 Hz by 13%. However, when main cables are connected through a flexible cross-tie with the non-dimensional cross-tie flexible parameter being $\psi=1.0$, the increase in its fundamental frequency is only 4.8% to 1.53 Hz. The same phenomenon can be observed in Mode 2, which is an out-of-phase global mode. The use of flexible cross-tie also reduces the modal frequency of an out-of-phase global...
mode. It is interesting to note that the drop in modal frequency due to the use of more flexible cross-tie is more considerable in the out-of-phase global mode than in the in-phase global mode. For example, in the studied cable network, the modal frequency of the first mode, which is an in-phase global mode, is reduced by 7% from 1.65 Hz to 1.53 Hz; whereas for the second mode which is an out-of-phase global mode, it is reduced by 17% from 2.55 Hz to 2.11 Hz. In addition, it should be noted that the change in cross-tie stiffness from rigid to more flexible would lead to the excitation of more local modes dominated by one of the main cables. This could be mainly attributed to the increased flexibility in the cross-tie, which offers more freedom to one cable from the constraint of the other so it can oscillate more independently. Among the first ten modes listed in Table 3.3, the number of local modes increases from 2 to 8 when a rigid cross-tie is replaced by a flexible one with $\psi=1.0$. Mode 4 and Mode 9 in the rigid cross-tie case are dominated respectively by the $3^{rd}$ and the $6^{th}$ mode of an isolated target cable. The position of cross-tie at $\varepsilon=1/3$ happens to coincide with the nodal point of these two single cable modes. Thus, the modal properties of these two local modes are not affected by the cross-tie stiffness except they become the $5^{th}$ and the $10^{th}$ modes when a flexible cross-tie is used instead.

3.2 Damped Two-Cable Network

In almost all the existing analytical and numerical models of cable networks, the inherent structural damping of the main cables is ignored. Intrinsic damping in cables is typically in the range of 0.05% to 0.1% for long cables and up to 0.3% for short cables (Stoyanoff et al., 2007). However, on some cable-stayed bridges, for example, the Tatara Bridge, the inherent structural damping of stay cables is relatively high, varying from 0.2% to 2.0% (Bu et al., 2011). Although the level of damping in the main cables is generally low, it could have sizeable effect in reducing the vibration amplitude of the cable network. On the other hand, damping property of the cross-
tie(s) plays an important role in the overall equivalent damping of the cable networks, as pointed out by Yamaguchi and Nagahawatta (1995). Thus, analytical models which ignore the intrinsic damping of main cables and cross-ties are not capable of predicting how the structural damping of a target cable would be affected after it is connected to its neighbours, neither can they adequately infer the optimal cross-tie location. Nevertheless, the optimal design of a cable network should consider the combined effects of cross-tie installation on the network frequency and damping property. A cable network analytical model including the damping property of main cables and cross-ties would not only allow predicting the network in-plane frequency, but the system energy dissipation capacity as well.

In view of the above mentioned research needs, the cable network analytical model developed in Section 3.1 is extended to include the damping property of main cables and cross-tie(s) in the formulation. The network system characteristic equation will be derived analytically and the equivalent modal damping ratio of the cable network will be determined by solving the associated complex eigenvalue problem. The in-plane modal behaviour, including the modal frequency, the mode shape and the modal damping property will be examined.

The damping of a vibrating main cable in a cable network generally comes from two major sources: i) The structural (hysteresis) damping induced by internal friction within the cable material and at its connections with cross-tie and end supports; ii) The fluid (viscous) damping due to fluid-structure interaction resulted from its vibration in air. Since it is impossible to find the exact mathematical expression for each of the energy-dissipation mechanisms in actual structures, damping in actual physical systems is usually represented in a highly idealized form. The equivalent linear viscous damping model, being the simplest form of damping and thus amenable to derive analytical solutions to the system equation of motion, is commonly used to
describe the energy dissipated in an actual structure by various mechanisms. The linear viscous
damping model was first proposed by Rayleigh (1945), of which the damping force is assumed
to be linearly proportional to the motion velocity by a constant of damping coefficient. In the
current study, the damping of the main cables is assumed to have a uniform distribution along
the cable length and of a linear viscous type. The same damping model is applied to the cross-tie.
To find out how structural damping in a target cable will be influenced by the formation of the
cable network, the structural damping of an isolated target cable and that of a networked target
cable need to be compared. The latter equals to the equivalent structural damping of the cable
network.

3.2.1 System characteristic equation

The current model is an extension of the undamped two-cable network model described
in Section 3.1.1, with the consideration of main cables and cross-tie damping. The damping
property of the two main cables and that of the cross-tie are all assumed to be the linear viscous
type and uniformly distributed along the member length. The schematic layout of the proposed
model is shown in Figure 3.3.
The unit mass, tension and structural damping ratio of the two main cables are denoted by \( m_j, H_j \) and \( \zeta_j \) (\( j=1, 2 \)), respectively. The transverse cross-tie is assumed to be located at \( l_1 \) (\( l_1 < l_2 \)) from the left end of the target cable. Its axial stiffness property is represented by a linear spring connector with an associated stiffness constant of \( K_c \), where the subscript “\( c \)” refers to “cross-tie”. The damping of a vibrating main cable in a cable network generally comes from two major sources: i) The structural (hysteresis) damping induced by internal friction within the cable material and at its connections with the cross-tie and end supports; ii) The fluid (viscous) damping due to fluid-structure interaction resulted from its vibration in air. Since it is impossible to find the exact mathematical expression for each of the energy-dissipation mechanisms in actual structures, damping in actual physical systems is usually represented in a highly idealized form. The equivalent linear viscous damping model, being the simplest form of damping and thus amenable to derive analytical solution to the system equation of motion, is commonly used to describe the energy dissipated in an actual structure by various mechanisms. Therefore, in the current study, the damping of the main cables is assumed to have a uniform distribution along the cable length and of a linear viscous type. The equivalent linear viscous damping model
proposed by Rayleigh (1945) assumes the damping force to be linearly proportional to the 
motion velocity. The corresponding damping coefficients are denoted \( C_j \) \((j=1, 2)\) for the \( j^{th} \) main 
cable and \( C_c \) for the cross-tie. The additional tension in the main cables caused by vibration is 
neglected in the proposed model.

When the cable network in Figure 3.8 is excited to vibrate within its plane, all four main 
cable segments oscillate in the transverse direction whereas the cross-tie moves along its 
longitudinal direction. The motion of each main cable segment can be described by the equation 
of motion of a taut cable subjected to in-plane damped free vibration, i.e.

\[
H \frac{\partial^2 \nu(x,t)}{\partial x^2} = m \frac{\partial^2 \nu(x,t)}{\partial t^2} + 2m\xi \omega_o \frac{\partial \nu(x,t)}{\partial t}
\]  
(3-9)

where \( \nu, H, m \) are respectively the transverse displacement, the tension and the unit mass of the 
taut cable, \( C = 2m\xi \omega_o \) is the cable damping coefficient per unit length, \( \xi \) is the cable structural 
damping ratio; and \( \omega_o \) is the undamped circular frequency of the taut cable. Separating the 
temporal and spatial variables contained in the cable transverse displacement \( \nu(x,t) \) using the 
Bernoulli-Fourier method, it can be expressed as \( \nu(x,t) = \bar{\nu}(x)e^{i\omega t} \), where \( \bar{\nu}(x) \) is the shape 
function, and \( \omega \) is the complex circular frequency of cable vibration. Substituting this expression 
into Eq. (3-9), it becomes

\[
\bar{\nu}''(x) + \alpha^2 \bar{\nu}(x) = 0
\]  
(3-10)

and its general solution would be

\[
\bar{\nu}(x) = A\cos(\alpha x) + B\sin(\alpha x)
\]  
(3-11)

where \( A \) and \( B \) are constants determined from the boundary conditions, and \( \alpha \) is a complex wave 
number of the form

\[
\alpha = \sqrt{\frac{m\omega^2 - i2m\xi\omega \omega_o}{H}}
\]  
(3-12)
Since all four main cable segments in Figure 3.8 have one end fixed, i.e. \( \bar{v}(0) = 0 \), constant \( A \) in Eq. (3-11) would be zero. Therefore, their transverse motion shape functions can be reduced to

\[
\bar{v}_{2j-1}(x_{2j-1}) = B_{2j-1} \sin(\alpha_j x_{2j-1}) \quad j = 1, 2 \quad (3-13a)
\]

\[
\bar{v}_{2j}(x_{2j}) = B_{2j} \sin(\alpha_{2j} x_{2j}) \quad j = 1, 2 \quad (3-13b)
\]

where \( \bar{v}_{2j-1} \) and \( \bar{v}_{2j} \) represent respectively the shape function of the left and the right segments of the \( j^{th} \) main cable \( (j = 1, 2) \), and \( B_{2j-1} \) and \( B_{2j} \) are the corresponding shape function constants.

The mass of the cross-tie is not considered in the proposed model since it is usually very small compared to that of the main cables. The behaviour of the damped flexible cross-tie is described by a linear tension/compression reversal spring connector in parallel with a linear viscous damper. When the cross-tie oscillates along its axial direction, the force developed in it can be expressed as

\[
F_c(t) = K_c u(t) + C_c \frac{du}{dt} \quad (3-14)
\]

where \( u(t) \) is the change in cross-tie length, i.e.

\[
u(t) = v_1(l_1, t) - v_3(l_3, t) = [\bar{v}_1(l_1) - \bar{v}_3(l_3)] e^{i\omega t} \quad (3-15)
\]

At point \( N_1 \) where the cross-tie connects with the target cable (Figure 3.8), the equilibrium requires the force exerted by the cross-tie on the target cable equals to the transverse force in the left and the right segments of the target cable induced by its tension, i.e.

\[
\left( \left. \frac{\partial \bar{v}_1}{\partial x_1} \right|_{x_1 = l_1} + \left. \frac{\partial \bar{v}_2}{\partial x_2} \right|_{x_2 = l_2} \right) H_1 e^{i\omega t} = -F_c(t) \quad (3-16)
\]

Plug Eqs. (3-14) and (3-15) into Eq. (3-16), it gives

\[
\alpha_1 H_1 [B_1 \cos(\alpha_1 l_1) + B_2 \cos(\alpha_1 l_2)] = [B_3 \sin(\alpha_2 l_3) - B_1 \sin(\alpha_1 l_1)] [K_c + i\omega C_c] \quad (3-17)
\]

Moreover, longitudinal equilibrium of the isolated cross-tie should be satisfied.
Substitute Eq. (3-13) into Eq. (13-18), the following equation is obtained

\[ \alpha_1 H_1 [B_1 \cos(\alpha_1 l_1) + B_2 \cos(\alpha_1 l_2)] + \alpha_2 H_2 [B_3 \cos(\alpha_2 l_3) + B_4 \cos(\alpha_2 l_4)] = 0 \]  

(3-19)

The transverse displacement compatibility between the left and the right main cable segments at nodes \( N_1 \) and \( N_2 \) gives

\[ \bar{v}_{2j-1}(l_{2j-1}) = \bar{v}_{2j}(l_{2j}) \quad j = 1, 2 \]  

(3-20)

which, by considering Eq. (3-13), yields

\[ B_1 \sin(\alpha_1 l_1) - B_2 \sin(\alpha_1 l_2) = 0 \]  

(3-21a)

\[ B_3 \sin(\alpha_2 l_3) - B_4 \sin(\alpha_2 l_4) = 0 \]  

(3-21b)

Now, writing Eqs. (3-17), (3-19) and (3-21) in a matrix form,

\[ [S] \{X\} = \{0\} \]  

(3-22)

where

\[
[S] = \begin{bmatrix}
\sin(\varphi_1) & -\sin(\varphi_2) & 0 & 0 \\
0 & 0 & \sin(\varphi_3) & -\sin(\varphi_4) \\
\psi R_1 \cos(\varphi_1) + \sin(\varphi_1) & \psi R_1 \cos(\varphi_2) & -\sin(\varphi_3) & 0 \\
\psi R_2 \cos(\varphi_1) & \psi R_2 \cos(\varphi_2) & \psi R_2 \cos(\varphi_3) & \psi R_2 \cos(\varphi_4)
\end{bmatrix}
\]

is the coefficient matrix, \( \{X\} = [B_1 \quad B_2 \quad B_3 \quad B_4]^T \) is a vector containing all four unknown shape function constants, and \( \{0\} \) is the null vector. In the coefficient matrix \([S]\), \( \varphi_{2j-1} = R_j \varepsilon_{2j-1} \) and \( \varphi_{2j} = R_j \varepsilon_{2j} \) applies respectively to the left and the right segment of the \( j^{th} \) main cable (\( j = 1, 2 \)), \( R_j = \alpha_j L_j \) is a complex parameter, \( \alpha_j \) is the complex wave number defined in Eq. (5), \( \varepsilon_{2j-1} = l_{2j-1}/L_j \) and \( \varepsilon_{2j} = l_{2j}/L_j \) are the segment ratio parameters for the left and the right cable segments of the \( j^{th} \) main cable \((j = 1, 2)\), \( \psi \) is the complex mass-tension ratio parameter of the \( j^{th} \) cable which is defined by

60
\[ \gamma_j = \frac{H_j R_j / L_j}{H_1 R_1 / L_1} \quad (3-23) \]

\( \bar{\psi} \) is the non-dimensional complex cross-tie parameter having the form of

\[ \bar{\psi} = \frac{H_1}{L_1 [K_c + i\omega C_c]} \quad (3-24) \]

Define \( \Omega = \pi f / f_1 \) as the non-dimensional complex frequency of the cable network and \( \eta_j = f_i / f_j \) as the frequency ratio of the \( j^{th} \) \((j=1, 2)\) main cable, where \( f \) and \( f_i \) are respectively the complex frequency of the cable network and the undamped fundamental frequency of the \( j^{th} \) \((j=1, 2)\) main cable, the complex parameter \( R_j \) can be rewritten as

\[ R_j = \sqrt{\left( [\Omega \eta_j \eta_j]^2 - i \cdot 2\pi \xi_j \Omega \eta_j \eta_j \right)} \quad j = 1, 2 \quad (3-25) \]

where \( \xi_j \) \((j = 1, 2)\) is the structural damping ratio of the \( j^{th} \) cable.

To find the non-trivial solution to Eq. (3-22), the determinant of the coefficient matrix \( [S] \) should be set to zero. This leads to the characteristic equation of the two-cable network shown in Figure 3.1, which consists of two horizontally laid damped taut main cables interconnected by a transverse damped flexible cross-tie, i.e.

\[ \bar{\gamma}_1 \sin(R_1) \sin(\varnothing_3) \sin(\varnothing_4) + \bar{\gamma}_2 \sin(R_2) \sin(\varnothing_1) \sin(\varnothing_2) + \bar{\psi} R_1 \bar{\gamma}_1 \bar{\gamma}_2 \sin(R_1) \sin(R_2) = 0 \quad (3-26) \]

If we neglect the damping in the two main cables and the cross-tie, the three complex parameters \( R_j, \gamma_j \) \((j = 1, 2)\) and \( \bar{\psi} \) in Eqs. (3-23) to (3-25) would reduce to \( R_j = \Omega \eta_j, \gamma_j = \sqrt{H_j m_j / H_1 m_1} \) and \( \psi_o = H_1 / L_1 K_c \). Therefore, Eq. (3-26) would be the same as the system characteristic equation of
an undamped two-cable network connected through an undamped flexible cross-tie derived in Section 3.1.1.

3.2.2 Application examples

In a real cable network system, structural damping exists in both main cables and cross-ties. The role of a cross-tie in the interaction between a target cable and its neighbours can be separated into two parts: i) The cross-tie serves as a “transparent channel” which allows the target cable and its neighbours to communicate their response without the influence of a third party, i.e. the cross-tie is assumed to have zero damping and infinitely large stiffness and thus can be modeled as an undamped rigid link; ii) When transmitting the response of a target cable and its neighbours between each other, the cross-tie behaves like a “filter” such that the transmitted response would be altered by the damping and stiffness properties of the cross-tie. To properly understand the mechanics of a cable network, it is important to distinguish the respective impact of neighbouring cables and cross-ties on the response of the connected target cable. Therefore, in the current analytical model, the main cables are assumed to be damped taut cables, whereas the cross-tie is modeled as an undamped rigid link. The “filtering” effect of cross-tie will be explored in the upcoming publications. Therefore, this section is virtually divided into two parts, in the first part, the modal behaviour of damped cable networks will be explored using rigid cross-ties and in the second part, flexible damped cross-tie will be employed. Each of the prescribed case will use three different configurations of damped cable networks, i.e. twin-cable network, symmetric DMT cable network and asymmetric cable network.
3.2.2.1 Twin-cable rigid cross-tie network

A twin cable network consists of two main cables having the same physical and mechanical properties. Though this idealized type of cable network does not exist on real cable-stayed bridge, its unique modal behaviour would help us to better understand the mechanics associated with in-plane vibration of cable networks. Under the assumptions of undamped main cables, studies (Caracoglia and Jones, 2005a) showed that three types of modes could be excited, including a global mode and two local modes dominated respectively by either the left or the right part of the network. In particular, if an undamped rigid cross-tie is used, the modal frequencies and the mode shapes of the global modes are the same as those of a single main cable. In the current study, the two main cables are assumed to possess linear viscous type of damping. It is expected to observe the same three types of modes and the equivalent modal damping ratio of the network global mode should equal to that of the corresponding isolated main cable mode. Therefore, as a first validation case, the system characteristic equation of a general orthogonal two-cable network given by Eq. (3-26) is applied to a twin-cable network. By plugging the following conditions into Eq. (3-26),

\[
\begin{align*}
R_1 &= R_2 & \varnothing_1 &= R_1 \varepsilon_1 & \varnothing_3 &= R_2 \varepsilon_3 \\
\gamma_1 &= \gamma_2 & \varnothing_2 &= R_1 \varepsilon_2 & \varnothing_4 &= R_2 \varepsilon_4 \\
\varepsilon_1 &= \varepsilon_3 &= \varepsilon & \varepsilon_2 &= \varepsilon_4 &= 1-\varepsilon
\end{align*}
\]

The system characteristic equation reduces to:

\[
\sin(R_1)\sin(R_1\varepsilon)\sin[R_1(1-\varepsilon)] = 0 \tag{3-27}
\]

It can be clearly seen from the above equation that three sets of roots exist, which corresponds respectively to the global modes, the left segment (LS) local modes and the right segment (RS) local modes. The modal properties of the network global modes, including modal frequency and
modal damping ratio, can be determined from \( \sin(R_1) = 0 \), the roots of which are \( R_1 = n\pi, n = 1, 2, 3 \ldots \). Noticing the definition of the complex parameter \( R_j \) given by Eq. (15), and also expressing the network complex frequency as

\[
\Omega = \Omega_o + i\Omega_{im}
\]

(3-28)

where \( \Omega_{re} = \Omega_o\sqrt{1 - \xi^2_{eq}} \) and \( \Omega_{im} = \Omega_o\xi_{eq} \) are respectively the real and the imaginary parts of \( \Omega \), \( \Omega_o \) is the non-dimensional undamped frequency of the system and \( \xi_{eq} \) is the equivalent damping ratio of the cable network. Substitute Eqs. (3-25) and (3-28) into \( \sin(R_1) = 0 \), it gives

\[
\Omega_{re} = \pi\sqrt{n^2 - \xi^2}
\]

(3-29a)

\[
\Omega_{im} = \pi\xi
\]

(3-29b)

where \( n \) and \( \xi \) are the mode number and the damping ratio of a single main cable, respectively. Therefore, the non-dimensional modal frequency and the modal damping ratio of the \( n^{th} \) global mode are determined to be

\[
\Omega_0 = \sqrt{\Omega_{re}^2 + \Omega_{im}^2} = n\pi \quad \text{n=1, 2, 3,\ldots} \quad (3-30a)
\]

\[
\xi_{eq} = \Omega_{im}/\sqrt{\Omega_{re}^2 + \Omega_{im}^2} = \xi/n \quad \text{n=1, 2, 3,\ldots} \quad (3-30b)
\]

This set of modal property results indicates, as expected, that when a twin-cable network vibrates in global modes, not only the modal frequency but also the modal damping ratio are the same as those of a single isolated cable, i.e. the presence of an undamped rigid cross-tie would not affect the modal properties (modal frequency, modal damping and mode shape) of global modes.

Similarly, the modal properties associated with local LS modes and RS modes can be found by setting \( \sin(R_1\varepsilon) = 0 \) and \( \sin[R_1(1-\varepsilon)] = 0 \), respectively. They are

Local LS modes:

\[
\Omega_0 = n\pi/\varepsilon \quad \text{(3-31a)}
\]

\[
\xi_{eq} = \varepsilon\xi/n \quad \text{(3-31b)}
\]
Local RS modes: $\Omega_0 = n\pi/(1 - \epsilon)$  

(3-32a) $\xi_{eq} = (1 - \epsilon)\xi/n$  

(3-32b)  

It is interesting to note that besides the complementary nature of modal frequencies associated with a local LS mode and a particular local RS mode, as been earlier reported by Caracoglia and Jones (2005a) and also observed in Section 3.1, the modal damping ratio of the same set of local modes also forms a complementary pair. Depending on the cross-tie location represented by the segment ratio $\epsilon$, the modal order of the local LS and RS modes forming a complementary pair varies. Figure 3-9 depicts the impact of cross-tie position on the equivalent modal damping ratio of a twin-cable network, of which the non-dimensional network modal damping ratio $\Xi = \xi_{eq}/\xi_1$ is used for the vertical axis, where $\xi_1$ is the damping ratio of the target cable. It can be seen from the figure, when the cross-tie is placed at a certain location, the modal damping ratio of certain global and local modes are the same, which are represented by the intersection points in Figure 3.9.
Type “a” is associated with the extreme cases of cross-tie location at either end of the main cables, i.e. $\varepsilon=0$ or $\varepsilon=1$. When $\varepsilon=0$, the right segment of the cable has the same length as the cable itself, so the local RS modes are the same as the global modes. Thus, these local modes will not only have the same modal frequency of the corresponding global mode, but the same modal damping ratio as well. The same remark applies to the cases of $\varepsilon=1$, of which the local LS modes have the same modal properties as the corresponding global modes. By referring to Eq. (3-30b), the modal damping ratio is inversely proportional to the mode number. The first symmetric global mode has $\Xi=1.0$. It is not included in Figure 3.9 so the scale would allow a more clear illustration of the modal damping complementary property of the other global and local modes.

Type “b” intersection points correspond to the coexistence of an anti-symmetric global mode and
a pair of complementary local LS and RS modes, whereas the type “c” intersection is associated with the coexistence of a symmetric global mode and a pair of complementary local LS and RS modes. Based on Eqs. 3-30(b), 3-31(b) and 3-32(b), the modal order of these coexisting modes should satisfy

\[ \frac{1}{n_G} = \frac{\varepsilon}{n_{LS}} = \frac{1-\varepsilon}{n_{RS}} \] (3-33)

where \( n_G, n_{LS} \) and \( n_{RS} \) are the mode number of the global mode, the local LS mode and the local RS mode, respectively. Symmetric and anti-symmetrical modes are associated with odd and even order numbers, respectively.

Figure 3.10 depicts the mode shapes of the global and local modes that would have the same modal damping ratio when the cross-tie is located at \( \varepsilon=1/3, 1/2, 1/4 \). The mode shapes of these coexisting modes show that when this phenomenon occurs, not only the cross-tie is located at the nodes of these modes but also the shapes of the local modes are the same as their corresponding parts in the global mode, except the target cable and the neighbouring cable vibrate out-of-phase. In addition, the modal frequencies of these modes are also found to be the same, as can be derived from Eqs. 3-30(a), 3-31(a) and 3-32(a). Therefore, in the case of a twin-cable network, if an undamped transverse rigid cross-tie is used, the modal behaviour of the cable network is governed by the cross-tie position. When the cross-tie is located at/or very close to the node shared by a global mode and a pair of complementary local modes, a slight variation of the cross-tie position would render the switch among these three modes. This phenomenon could happen for two reasons: a) The three modes have the same modal frequency; and b) They have the same modal damping ratio. Thus, no extra energy needs to be absorbed or dissipated in order to shift from one mode to another.
Figure 3.10: Mode shapes (real part) of global and local modes having the same modal frequency and modal damping: a) $\varepsilon=1/3$, b) $\varepsilon=1/2$, c) $\varepsilon=3/4$. (GM: global mode, LM: local mode)

To further verify the validity of the proposed analytical model, an example is presented, where the modal results are compared with those obtained from an independent numerical simulation. A finite element model of the studied twin-cable network is developed using Abaqus 6.10 (2010). The B21 beam element and the RB2D2 rigid body elements are chosen to simulate the behaviour of the main cables and the rigid cross-tie, respectively. The Rayleigh viscous damping model is applied to simulate the structural damping in the main cables.

Both of the twin main cables in the numerical example have a length of 72 m, a unit mass of 50 kg/m, a tension of 2200 kN and a structural damping ratio of 0.5%. The cross-tie connects...
the two cables at their 1/3 span. The natural frequency and the damping ratio of the first 10 modes are listed in Table 3.4, along with the numerical simulation results.

Table 3.4: In-plane modal frequency and modal damping of a twin-cable network with a rigid cross-tie at one-third span

<table>
<thead>
<tr>
<th>Mode</th>
<th>Modal frequency (Hz)</th>
<th>Modal damping ratio (%)</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>FEA</td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
<td>1.457</td>
<td>1.457</td>
<td>0.500</td>
</tr>
<tr>
<td>2</td>
<td>2.185</td>
<td>2.185</td>
<td>0.333</td>
</tr>
<tr>
<td>3</td>
<td>2.913</td>
<td>2.913</td>
<td>0.250</td>
</tr>
<tr>
<td>4</td>
<td>4.370</td>
<td>4.367</td>
<td>0.167</td>
</tr>
<tr>
<td>5</td>
<td>4.370</td>
<td>4.367</td>
<td>0.167</td>
</tr>
<tr>
<td>6</td>
<td>4.370</td>
<td>4.368</td>
<td>0.167</td>
</tr>
<tr>
<td>7</td>
<td>5.827</td>
<td>5.820</td>
<td>0.125</td>
</tr>
<tr>
<td>8</td>
<td>6.555</td>
<td>6.545</td>
<td>0.111</td>
</tr>
<tr>
<td>9</td>
<td>7.283</td>
<td>7.270</td>
<td>0.100</td>
</tr>
<tr>
<td>10</td>
<td>8.740</td>
<td>8.716</td>
<td>0.083</td>
</tr>
</tbody>
</table>
As it can be seen from the table, the modal analysis results based on the proposed analytical model agree well with those obtained from numerical models. The mode shapes (real part) of four typical modes, i.e. the first symmetric global mode, the first anti-symmetric global mode, the first local LS mode and the first local RS mode are shown in Figure 3.11.

![Mode shapes](image)

Figure 3.11: Selected modes (real part) of a twin-cable network with a rigid cross-tie at 1/3 span (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 3.10)

### 3.2.2.2 Symmetric DMT two-cable rigid cross-tie network

For a symmetric DMT two cable network, the two main cables have different mass-tension ratio ($\gamma_1 \neq \gamma_2$) but the offsets on the left and the right ends of main cable 2 are the same. If the cross-tie is located at $l_1$ ($l_1 < O_L + L_2$) from the left end of the target cable, the segment ratios of the four main cable segments can be expressed as

$$
\varepsilon_1 = \frac{l_1}{L_1} = \varepsilon \\
\varepsilon_2 = 1 - \varepsilon \\
\varepsilon_3 = \frac{1}{2} + (\varepsilon - \frac{1}{2}) \lambda_2 \\
\varepsilon_4 = \frac{1}{2} - (\varepsilon - \frac{1}{2}) \lambda_2
$$

where $\lambda_2 = L_1/L_2$ is the length ratio of main cable 2. Noticing $\Phi_{2j-1} = R_j \varepsilon_{2j-1}$ and $\Phi_{2j} = R_j \varepsilon_{2j}$ ($j=1, 2$), the network characteristic equation, Eq. (3-26), can be expressed as
For the special case of cross-tie at the mid-span, $\varepsilon_1=\varepsilon_2=\varepsilon_3=\varepsilon_4 = 1/2$. Thus, Eq. (3-34) can be reduced to

$$
\sin(R_1)\sin\{R_2[1/2 + (\varepsilon - 1/2)\lambda_2]\}\sin\{R_2[1/2 - (\varepsilon - 1/2)\lambda_2]\} + \gamma_2\sin(R_2)\sin(R_1\varepsilon)\sin[R_1(1-\varepsilon)] = 0
$$

or

$$
2 \sin(R_1/2)\sin(R_2/2) \left[ \sin(R_2/2)\cos(R_1/2) + \gamma_2 \sin(R_1/2)\cos(R_2/2) \right] = 0
$$

(3-35)

The pattern of Eq. (3-35) clearly shows that it has three sets of roots, which can be determined by setting respectively the first two sine terms and the term with the square bracket as zero.

The condition of $\sin(R_1/2)=0$ describes the local modes dominated by the target cable (main cable 1), whereas that of $\sin(R_2/2)=0$ gives the local modes dominated by the neighbouring cable (main cable 2). The solution to the above two equations leads to the modal frequency and the modal damping ratio associated with these two types of local modes. They are

Local modes of the target cable:

$$
\Omega_0 = 2n\pi \quad \text{n}=1, 2, 3,\cdots
$$

(3-36a)

$$
\xi_{eq} = \xi_1/(2n) \quad \text{n}=1, 2, 3,\cdots
$$

(3-36b)

Local modes of the neighbouring cable:

$$
\Omega_0 = 2n\pi /\eta_2 \quad \text{n}=1, 2, 3,\cdots
$$

(3-37a)

$$
\xi_{eq} = \xi_2/(2n) \quad \text{n}=1, 2, 3,\cdots
$$

(3-37b)

The modal results given in Eqs. (3-36) and (3-37) suggest that the local modes of the network dominated by either the target or the neighbouring cable have the same modal properties, i.e. modal frequency and modal damping ratio, as those of the anti-symmetric modes of a corresponding single cable. As expected, the modal frequencies of these two types of local modes are the same as those derived earlier for a symmetric DMT two-cable network with
undamped main cables in Section 3.1.3. The modal damping ratio in both cases is inversely proportional to the mode number, which suggests that modal damping ratio would gradually decrease in higher order modes.

The modal properties of the global modes can be obtained by solving
\[
\sin(R_2/2)\cos(R_1/2) + \gamma_2 \sin(R_1/2)\cos(R_2/2) = 0
\] (3-38)

Substitute Eq. (3-25) into Eq. (3-38) and set both the real and the imaginary parts of the resulting equation to zero, it will yield the modal frequency and the modal damping ratio of the global modes. As an example, the modal properties of the following symmetric DMT two-cable network are analyzed and the results are listed in Table 3.5.

Main Cable 1: \( L_1=72 \text{ m} \quad H_1=2200 \text{ kN} \quad m_1=50 \text{ kg/m} \quad \zeta_1=0.5\% \)

Main Cable 2: \( L_2=60 \text{ m} \quad H_2=2400 \text{ kN} \quad m_2=42 \text{ kg/m} \quad \zeta_2=0.8\% \)

Rigid cross-tie: \( \varepsilon=1/2 \)
Table 3.5: In-plane modal frequency and modal damping of a symmetric two-cable network with a rigid cross-tie at mid-span

<table>
<thead>
<tr>
<th>Mode</th>
<th>Modal frequency (Hz)</th>
<th>Modal damping ratio (%)</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>FEA</td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
<td>1.677</td>
<td>1.677</td>
<td>0.608</td>
</tr>
<tr>
<td>2</td>
<td>2.913</td>
<td>2.913</td>
<td>0.250</td>
</tr>
<tr>
<td>3</td>
<td>3.375</td>
<td>3.374</td>
<td>0.342</td>
</tr>
<tr>
<td>4</td>
<td>3.984</td>
<td>3.983</td>
<td>0.400</td>
</tr>
<tr>
<td>5</td>
<td>5.037</td>
<td>5.033</td>
<td>0.208</td>
</tr>
<tr>
<td>6</td>
<td>5.827</td>
<td>5.820</td>
<td>0.125</td>
</tr>
<tr>
<td>7</td>
<td>6.742</td>
<td>6.732</td>
<td>0.166</td>
</tr>
<tr>
<td>8</td>
<td>7.968</td>
<td>7.955</td>
<td>0.200</td>
</tr>
<tr>
<td>9</td>
<td>8.406</td>
<td>8.388</td>
<td>0.129</td>
</tr>
<tr>
<td>10</td>
<td>8.740</td>
<td>8.716</td>
<td>0.083</td>
</tr>
</tbody>
</table>

The modal results obtained from an independent numerical simulation are presented in the same table for comparison. The mode shapes of the first four modes are portrayed in Figure 3.12.
Figure 3.12: First four modes (real part) of a symmetric DMT two-cable system with a rigid cross-tie at mid-span

The results given in Table 3-5 and Figure 3-12 show that for the studied symmetric DMT two-cable network, the fundamental mode is a symmetric global mode with both main cables vibrating in phase. By connecting the target cable to its neighbouring cable which has higher frequency and damping, the frequency of the target cable increases by 15% from 1.475 Hz to 1.677 Hz, and its modal damping ratio jumps from 0.50% to 0.61% by almost 25%. Though the increase in the target cable modal frequency due to cross-tie application has already been observed in previous analytical studies (Caracoglia and Jones 2005a), the increase in its modal damping is observed for the first time here through the analytical approach. The second global mode, which corresponds to mode 3 of the network, is also a symmetric mode, except with the two main cables vibrating out-of-phase (Figure 3.12). It is worth noting that while the second mode of an isolated target cable, with the modal frequency of 2.91 Hz and the modal damping ratio of 0.25%, is an anti-symmetric mode, i.e. the left and the right portions of the cable vibrate out-of-phase, in the presence of a rigid cross-tie, the left and the right segments of the target cable in a network global mode would vibrate in-phase. Compared to an isolated cable, the one in the network has its frequency and damping ratio increased by 16% and 36%, respectively. The 2\textsuperscript{nd} and the 4\textsuperscript{th} network modes are the local modes dominated respectively by the target cable and
the neighbouring cable in the lowest anti-symmetric mode. Both modal frequency and modal damping ratio are the same as those of a single isolated cable vibrating in that mode, which could be attributed to the coincidence of cross-tie position and the node of the mode. Therefore, whether the cable is single or networked, the presence of a rigid cross-tie would not affect the modal behaviour of the local modes, in particular, the modal frequency and the modal damping ratio of these modes will not gain benefit from the cross-tie application.

For the symmetric DMT two-cable network studied in this section, the neighbouring cable is assumed to have higher structural damping than the target one. If we revisit the network system characteristic equation given in Eq. (3-26), and notice the definition of the complex parameter \( R_j \) \((j =1, 2) \) by Eq. (3-25), it can be clearly seen that the modal damping ratio of the cable network depends on the intrinsic damping of each of the main cables and it is also affected by the cross-tie position. To quantify the effect of the intrinsic damping in the neighbouring cable, in particular, its relation with the damping in the target cable, on the damping property of a target cable in the network, a non-dimensional damping relation parameter \( \chi = \xi_2/\xi_1 \) is introduced. The impact of the damping level in the neighbouring cable on the network damping property is thus investigated by varying \( \chi \) from 0 to 2 while keeping the damping ratio of the target cable as 0.5%. Figure 3-13 illustrates the results of this set of analysis, of which the non-dimensional modal damping ratio of the network fundamental mode \( \Xi = \xi_{eq}/\xi_1 \) is plotted against the non-dimensional damping relation parameter \( \chi \).
It can be observed that the network non-dimensional modal damping ratio $\Xi$ has an approximate linear relation with the non-dimensional damping relation parameter $\chi$. When the neighbouring cable has less damping than the target cable ($\chi<1$), the non-dimensional network modal damping ratio $\Xi$ is also less than 1, implying that in such case, the network option would actually decrease the damping level of the target cable. The damping property of the networked target cable will only be improved provided the neighbouring cable has higher damping, i.e. $\chi>1$. The pattern of the $\Xi$-$\chi$ curve suggests that within a cable network, the damping capacity would be “transferred” from the cable with higher damping to that having lower damping. To complete the picture of the impact of the neighbouring cable damping level on the damping property of the cable network, the $\Xi$-$\chi$ relation corresponding to cross-tie positions of $\varepsilon=1/4$ and $\varepsilon=1/3$ are also presented in Figure 3-13. Similar to the case of $\varepsilon=1/2$, an approximate linear relation between the

![Graph showing the relationship between non-dimensional modal damping ratio $\Xi$ and non-dimensional damping relation parameter $\chi$. The graph includes lines for isolated target cable and networked target cables at $\varepsilon=1/4$, $1/3$, and $1/2$. The y-axis represents $\Xi = \xi_2/\xi_1$, and the x-axis represents $\chi = \xi_2/\xi_1$. The lines show the trend of $\Xi$ increasing with $\chi$.](image-url)
non-dimensional network modal damping ratio and the non-dimensional damping relation parameter also exists. In addition, the discrepancy in the slopes of the three $\Xi$-$\chi$ relation curves indicates that if a rigid cross-tie is located closer to the cable mid-span, the damping property of the network would be more sensitive to the intrinsic damping of the neighbouring cable.

Figure 3-14 depicts the impact of cross-tie position on the modal damping ratio of the studied cable network. In a given cable network, the structural damping in the target cable and the neighbouring cable are 0.5% and 0.8%, respectively, which yields a non-dimensional damping relation parameter $\chi=1.6$.

![Graph showing the impact of cross-tie position on modal damping ratio](image)

**Figure 3.14:** Non-dimensional damping ratio of the fundamental mode as a function of cross-tie position for a symmetric DMT two-cable network

The offsets on both ends of the neighbouring cable are 6 m, which is 8.3% of the target cable length. Thus, the extreme position of the rigid cross-tie would be $\varepsilon=0.083$ on the left and $\varepsilon=0.917$ on the right. One interesting phenomenon can be observed in Figure 3.14 is that if the
cross-tie locates within the ranges of $\varepsilon=0.083$ to 0.13 and $\varepsilon=0.87$ to 0.917, the damping level of the target cable would not increase ($\Xi<1$) even after connected to a neighbouring cable with higher damping ($\chi=1.6$), but rather, would decrease. Referring to the layout of this symmetric DMT two-cable network, when the cross-tie position falls within these two regions, the cross-tie actually locates very close to either the left or the right end of the neighbouring cable. Therefore, when vibration occurs, due to its proximity to the fixed end of the neighbouring cable, the transverse movement of the rigid cross-tie would be limited, which would actually make it serve as an additional rigid support for the target cable. The oscillation of the cable network would thus be dominated by the vibration of target cable, with the connection point between the cross-tie and the target cable being the node of this local mode. The modal damping ratio of such a local mode is lower than that of the fundamental mode of an isolated target cable. Once the cross-tie position is beyond these two “end regions”, the cross-tie would have more ability to move along its axial direction which allows the activation of the global mode as the fundamental mode of the cable network. By moving the cross-tie closer to the cable mid-span, the damping level of the target cable gradually increases, until reaches the maximum value of $\xi_{eq}=1.22\xi_1$ at $\varepsilon=1/2$. Thus, for the network studied here, placing the cross-tie closer to the cable mid-span would not only be beneficial to enhance the system damping property but its in-plane stiffness as well.

To better understand the impact of the neighbouring cable damping level on the $\Xi$-$\varepsilon$ curve pattern, another set of analysis has been conducted by reducing the damping ratio of the current neighbouring cable from 0.8% to 0.25% ($\chi$ drops from 1.6 to 0.5) while keeping all the other cable properties of the two main cables remain the same. This set of results of the analysis is also presented in Figure 3.14. It shows that when the neighbouring cable has less damping than
the target one, instead of a convex pattern as in the case of \( \chi=1.6 \), the \( \Xi - \varepsilon \) curve now exhibits a concave form. This implies that if the damping level in the neighbouring cable is lower, the equivalent damping of the cable network will always be less than that of an isolated target cable no matter where the cross-tie is located. The closer it is to the target cable mid-span, the more damping would be “transferred” from the target cable to the neighbouring one, and a more considerable damping reduction in the networked target cable would occur. However, as mentioned earlier, when \( \lambda_2 > 1.0 \) and \( \lambda_2 \eta_2 \leq 1.0 \), placing cross-tie closer to the cable mid-span would enhance the network in-plane stiffness and frequency. Thus, in such a case, the position of the cross-tie should be carefully selected to compromise the gain and/or loss in system damping and stiffness.

The variation of the non-dimensional fundamental frequency and the associated non-dimensional modal damping ratio of the studied symmetric DMT two-cable network against the cross-tie position \( \varepsilon \) are portrayed together in Figure 3.15.

Figure 3.15: Optimum cross-tie position range for a symmetric DMT two-cable network
The range of the optimum cross-tie position can be conveniently pinpointed, within which both the modal frequency and the modal damping ratio of the target cable can be increased to certain level. For example, by connecting the target cable with its neighbour using a rigid cross-tie, it is required to raise both its fundamental frequency and its modal damping ratio by 15%, by referring to Figure 8, the cross-tie should be placed within $\varepsilon = 0.43 - 0.56$ to satisfy the frequency increment requirement, whereas the range of $\varepsilon = 0.33 - 0.67$ is needed for the damping increment requirement. Therefore, to satisfy both, the optimum cross-tie position range is determined to be $\varepsilon = 0.43 - 0.56$.

3.2.2.3 Asymmetric DMT two-cable rigid cross-tie network

In Section 3.2, the system characteristic equation for describing the in-plane free vibration of a general orthogonal two-cable network has been derived analytically and given in Eq. (3-26). While for special cases such as the twin-cable network studied in Section 3.2.2.1, the analytical form solution to Eq. (3-26) is still possible to be derived, for a more general two-cable network of which the two main cables have different physical properties and are arranged asymmetrically, the explicit form of the analytical solution will be challenging to conceive.

For a given cable network, the only unknown in its characteristic equation, Eq. (3-26), is the non-dimensional complex frequency of the cable network, i.e. $\Omega = \Omega_{re} + i \cdot \Omega_{im}$, where $\Omega_{re} = \Omega_o \sqrt{1 - \xi_{eq}^2}$ and $\Omega_{im} = \Omega_o \xi_{eq}$ are respectively the real and the imaginary parts of $\Omega$, $\Omega_o$ is the non-dimensional undamped network frequency and $\xi_{eq}$ is the equivalent modal damping ratio of the cable network, which, upon finding $\Omega$ from Eq. (3-26), can be determined by

$$\xi_{eq} = \frac{\Omega_{im}}{\sqrt{\Omega^2_{re} + \Omega^2_{im}}}$$

(3-39a)

$$\Omega_o = \sqrt{\Omega^2_{re} + \Omega^2_{im}}$$

(3-39b)
In order to solve the system characteristic equation described by Eq. (3-26), the complex parameter \( R_j \) \((j=1, 2)\) is also expressed as

\[
R_j^2 = (R_j^2)_{\text{re}} + i \cdot (R_j^2)_{\text{im}}
\]

where \((R_j^2)_{\text{re}} = \left[ r_j^2(\Omega_j^2 - \Omega_{\text{im}}^2) + 2\pi \xi_j \eta_j \Omega_{\text{im}} \right] \) and \((R_j^2)_{\text{im}} = [2\eta_j^2 \Omega_{\text{re}} \Omega_{\text{im}} - 2\pi \xi_j \eta_j \Omega_{\text{re}}] \) are respectively the real and the imaginary parts of \( R_j^2 \).

Alternatively, the complex parameter \( R_j \) can also be expressed in polar form as follows,

\[
R_j = \sqrt{r_j} \cdot [\cos (\theta_j/2) + i \cdot \sin(\theta_j/2)] \quad j = 1, 2 \tag{3-40}
\]

where \( r_j = \sqrt{[(R_j^2)_{\text{re}}]^2 + [(R_j^2)_{\text{im}}]^2} \) and \( \theta_j = \tan^{-1}\left[(R_j^2)_{\text{im}}/(R_j^2)_{\text{re}}\right] \).

Denoting \( a_j = \sqrt{r_j} \cdot \cos (\theta_j/2) \) and \( b_j = \sqrt{r_j} \cdot \sin (\theta_j/2) \), Eq. (30) can further be written as

\[
R_j = a_j + i \cdot b_j \quad j = 1, 2 \tag{3-41}
\]

The definition of mass-tension ratio parameter given by Eq. (3-24) implies \( \gamma_1 = 1 \), whereas \( \gamma_2 = (H_2 L_1 R_2)/(H_1 L_2 R_1) \) is complex. Substitute \( R_1 \) and \( R_2 \) into \( \gamma_2 \), it yields

\[
\gamma_2 = P_2 + i \cdot Q_2 \tag{3-42}
\]

where

\[
P_2 = \frac{H_2 L_1}{H_1 L_2} \sqrt{\frac{r_2}{r_1}} \cos \left(\frac{\theta_2-\theta_1}{2}\right)
\]

\[
Q_2 = \frac{H_2 L_1}{H_1 L_2} \sqrt{\frac{r_2}{r_1}} \sin \left(\frac{\theta_2-\theta_1}{2}\right)
\]

Plug Eqs. (3-41) and (3-42) into Eq. (3-26) and set both the real and the imaginary parts of the system characteristic equation as zero, it leads to

\[
A_1 C_2 E_2 - A_1 D_2 F_2 - B_1 C_2 F_2 - B_1 D_2 E_2 + P_2(A_2 C_1 E_1 - A_2 D_1 F_1 - B_2 C_1 F_1 - B_2 D_1 E_1) - Q_2(A_2 C_1 F_1 + A_2 D_1 E_1 + B_2 C_1 E_1 - B_2 D_1 F_1)=0 \tag{33a}
\]
\[ A_1C_2F_2 + A_1D_2E_2 + B_1C_2E_2 - B_1D_2F_2 + P_2(A_2C_1F_1 + A_2D_1E_1 + B_2C_1E_1 + B_2D_1F_1) - Q_2(A_2C_1E_1 - A_2D_1F_1 - B_2C_1F_1 - B_2D_1E_1) = 0 \]  

where, \( A_j, B_j, C_j, D_j, E_j, F_j, P_j \) and \( Q_j \) \((j = 1, 2)\) are the cable constants which have the general forms of

\[
A_j = \sin(a_j)\cosh(b_j) \\
B_j = \cos(a_j)\sinh(b_j) \\
C_j = \sin(\varepsilon_2\cdot a_j)\cosh(\varepsilon_2\cdot b_j) \\
D_j = \cos(\varepsilon_2\cdot a_j)\sinh(\varepsilon_2\cdot b_j) \\
E_j = \sin(\varepsilon_2\cdot a_j)\cosh(\varepsilon_2\cdot b_j) \\
F_j = \cos(\varepsilon_2\cdot a_j)\sinh(\varepsilon_2\cdot b_j)
\]

The real part \( \Omega_{\text{re}} \) and the imaginary part \( \Omega_{\text{im}} \) of the non-dimensional complex frequency \( \Omega \) of the cable network can be determined by solving Eq. (3-43). This is achieved using the Newton-Raphson method implemented in MatLab 7.0.

As a numerical example, the modal analysis of an asymmetric two-cable network is conducted. The two main cables in this example network are the same as those in the symmetric DMT two-cable network studied in Section 3.2.2.2. The cross-tie is placed at the three-quarter span from the left end of the target cable. The offset of the neighbouring cable with respect to the target cable is 3 m on the left side and 9 m on the right side. The modal properties of the first ten modes of this cable network are listed in Table 3.6, along with those obtained from the numerical simulation. A good agreement between the two sets of results is evident. Figure 3.16 portrays the mode shape of the first ten modes.
Table 3.6: In-plane modal frequency and modal damping of an asymmetric two-cable network with a rigid cross-tie at three-quarter span

<table>
<thead>
<tr>
<th>Mode</th>
<th>Modal frequency (Hz)</th>
<th>Modal damping (%)</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>FEA</td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
<td>1.685</td>
<td>1.685</td>
<td>0.531</td>
</tr>
<tr>
<td>2</td>
<td>2.187</td>
<td>2.187</td>
<td>0.632</td>
</tr>
<tr>
<td>3</td>
<td>3.363</td>
<td>3.362</td>
<td>0.287</td>
</tr>
<tr>
<td>4</td>
<td>4.229</td>
<td>4.227</td>
<td>0.254</td>
</tr>
<tr>
<td>5</td>
<td>5.017</td>
<td>5.014</td>
<td>0.243</td>
</tr>
<tr>
<td>6</td>
<td>5.827</td>
<td>5.820</td>
<td>0.125</td>
</tr>
<tr>
<td>7</td>
<td>6.708</td>
<td>6.699</td>
<td>0.195</td>
</tr>
<tr>
<td>8</td>
<td>7.482</td>
<td>7.468</td>
<td>0.133</td>
</tr>
<tr>
<td>9</td>
<td>8.426</td>
<td>8.406</td>
<td>0.128</td>
</tr>
<tr>
<td>10</td>
<td>9.529</td>
<td>9.503</td>
<td>0.130</td>
</tr>
</tbody>
</table>
The first distinct phenomenon which can be observed from Table 3.6 and Figure 3.16 is that from the first ten modes of the cable network, only one local mode, mode 6, is identified, whereas for a symmetric cable network formed by the same two main cables and a rigid cross-tie at mid-span, five out of the first ten modes are local (Table 3.5). Compared to an isolated target cable, by forming a cable network as the one studied here, its fundamental frequency can be increased by 16% from 1.457 Hz to 1.685 Hz, whereas the associated modal damping ratio increases by 6% from 0.50% to 0.53%.
Another noteworthy fact is the modal damping ratio of the second mode, which appears to be higher than that of the fundamental mode and exhibits a modal damping reversal behaviour. Compared to the network fundamental mode, of which the frequency and the modal damping ratio of the networked target cable is respectively 16% and 6% higher than those of the isolated one, in the case of mode 2, the modal damping ratio of the target cable in the network has increased by 26%. As learned from Eqs. 3-36(b) and 3-37(b), the modal damping ratio of a single cable is inversely proportional to the mode number and decreases in higher order modes. However, the results shown in Table 3.6 indicate that it might not be the case for the network global modes. It can be seen from Figure 3.16 that the two main cables in the first two modes both vibrate in approximately a half-sine shape, except they are in-phase in the fundamental mode but out-of-phase in the second mode. In addition, compared to the fundamental mode, the oscillation of the neighbouring cable in mode 2 is much more considerable. It seems that the excitation of a neighbouring cable with higher damping level would help to “transfer” damping into the target cable and increase its modal damping ratio. By properly controlling the vibration amplitude of the neighbouring cable, this phenomenon could be useful for a more effective cross-tie design. Further parametric study is needed before any firm conclusion of the modal damping reversal phenomenon can be reached. Figure 3.17 shows the impact of cross-tie position on the non-dimensional fundamental modal frequency and the non-dimensional fundamental damping ratio of the studied asymmetric two-cable network.
A combined $\Omega-\varepsilon$ and $\Xi-\varepsilon$ relation graph will greatly assist in selecting the optimum cross-tie position to maximize the efficiency of the designed cable network. For instance, assume the requirement is such that an increase of the frequency and the modal damping ratio of the target cable fundamental mode both by 15% for the design of a cable network. From Figure 3.17, it can be observed that to increase the fundamental modal frequency by 15%, the cross-tie should be placed within the range of $\varepsilon=0.46-0.81$, whereas to increase the fundamental modal damping ratio by 15%, the cross-tie should be installed between $\varepsilon=0.31$ and 0.67. Therefore, to satisfy both requirements, the cross-tie should be placed between $\varepsilon=0.46$ and 0.67. It is worth pointing out that this optimum range of cross-tie position is obtained based on the rigid cross-tie assumption, of which the effect of cross-tie stiffness, the intrinsic damping of the cross-tie, and the non-linear interaction between the cross-tie and the main cables are not considered.
3.2.2.4 Twin-cable damped flexible cross-tie network

Since the two main cables in a twin-cable network have the same length, unit mass, tension and damping, it gives $\phi_1=\phi_3$, $\phi_2=\phi_4$, $R_1=R_2$, and $\gamma_1=\gamma_2=1$. By inserting these conditions into Eq. (3-26), the system characteristic equation can be reduced to

$$\sin(R_1)\left[2\sin(\phi_1)\sin(\phi_2) + \ddot{\psi}R_1\sin(R_1)\right] = 0 \quad (3-44)$$

Three sets of roots can be determined from Eq. (3-44). The first set, yielded from $\sin(R_1)=0$, describes network global modes and is independent of cross-tie properties. Since the damping of main cables and cross-tie are considered in the current study, the network complex frequency can be expressed as

$$\Omega = \Omega_{re} + i\Omega_{im} \quad (3-45)$$

where $\Omega_{re} = \Omega_o\sqrt{1-\xi_{eq}^2}$ and $\Omega_{im} = \Omega_o\xi_{eq}$ are respectively the real and the imaginary parts of $\Omega$, $\Omega_o$ is the non-dimensional undamped network frequency and $\xi_{eq}$ is the equivalent damping ratio of the system. The real part of the complex frequency describes network vibration frequency whereas the imaginary part gives the system energy dissipation capacity. Substitute Eq. (3-45) into $\sin(R_1)=0$, we obtain $\Omega_{re} = n\pi\sqrt{1-(\xi/n)^2}$ and $\Omega_{im} = \pi\xi$, where $n$ and $\xi$ are respectively the mode number and damping ratio of an isolated single main cable. The non-dimensional modal frequency $\Omega_0$ and modal damping ratio $\xi_{eq}$ of the corresponding network global mode can thus be computed from

$$\Omega_0 = \sqrt{\Omega_{re}^2 + \Omega_{im}^2} = n\pi \quad n=1, 2, 3, \ldots \quad (3-46a)$$

$$\xi_{eq} = \frac{\Omega_{im}}{\sqrt{\Omega_{re}^2 + \Omega_{im}^2}} = \frac{\xi}{n} \quad n=1, 2, 3, \ldots \quad (3-46b)$$
This set of network modal property is exactly the same as those of an isolated single cable, which suggests that in this kind of global modes, the two main cables would oscillate in-phase with the same shape and the modal properties are not affected by the presence of cross-tie.

The other two sets of roots can be obtained by setting the summation of the two terms in the square bracket of Eq. (3-44) to zero, i.e.

\[ 2\sin(\phi_1)\sin(\phi_2) + \psi R_1 \sin(R_1) = 0 \quad (3-47) \]

It is important to note that should a rigid cross-tie be used in a twin-cable network, i.e. \( \psi = 0 \), the second term in Eq. (3-47) would vanish. Therefore, the two remaining sets of roots of Eq. (3-44) can be directly obtained from \( \sin(\phi_1) = 0 \) and \( \sin(\phi_2) = 0 \), which represent respectively the network local modes dominated by its left segments (LS) or right segments (RS). However, if the cross-tie has certain flexibility and damping, the second term in Eq. (3-47) would reflect the effect of cross-tie properties (stiffness, damping and position) on the network local modes. Not only their modal frequencies and damping would be “modified”, but also their mode shapes would evolve from that dominated by the oscillations of either the network left or right segments to the out-of-phase global modes. This agrees with the earlier findings discussed in Section 3.1 when studying two identical taut main cables interconnected by an undamped flexible cross-tie.

When installing a cross-tie, it can be placed either at the nodal point of a specific main cable mode or off the nodal point. Noticing \( R_1 = \phi_1/\varepsilon_1 = \phi_2/\varepsilon_2 \), Eq. (3-47) can also be expressed as

\[ 2\sin(\phi_1)\sin(\phi_2) + \psi R_1 \sin \left[ \frac{1}{m} \frac{\phi_1}{\varepsilon_1} + \left( 1 - \frac{1}{m} \right) \frac{\phi_2}{\varepsilon_2} \right] = 0 \quad (3-48) \]

where \( m \) is a positive integer or fraction which would satisfy \( m\varepsilon_1 = 1 \) and thus \( 1 - 1/m = \varepsilon_2 \) when the cross-tie happens to be placed at a nodal point of a particular main cable mode. Therefore, Eq. (3-48) can be further simplified as
\[
\sin(\varnothing_1) \left\{2\sin(\varnothing_2) + \psi R_1 \left[ \cos(\varnothing_2) + \cos(\varnothing_1) \frac{\sin(\varnothing_2)}{\sin(\varnothing_1)} \right] \right\} = 0
\] (3-49)

In Eq. (3-49), the condition of \(\sin(\varnothing_1) = 0\) describes the counterpart of LS modes in a rigid cross-tie network, the modal frequency and damping ratio of which are

\[
\Omega_0 = n\pi/\varepsilon \\
\xi_{eq} = \varepsilon \xi/n
\] (3-50a) (3-50b)

This implies that if a damped flexible cross-tie is placed at the nodal point of a particular main cable mode, although the change in cross-tie properties would lead to evolution of mode shapes into out-of-phase globe modes, the modal frequency and damping of these modes will not be affected.

The roots obtained by setting the term enclosed by the curly bracket in Eq. (3-49) to zero reflects how modal properties of local RS modes in a rigid cross-tie case would be influenced by the flexibility and damping of a cross-tie. They would not only contribute to reduce modal frequency and increase modal damping, but also excite more sizable oscillations of the left segments. Therefore, a network local RS mode would evolve into a global one should a rigid cross-tie be replaced by a damped flexible one. The same mode evolution phenomenon is already discussed in Section 3.1.2 for a twin-cable network with an undamped flexible cross-tie.

From the above discussion, it is clear that in a twin-cable network, the in-phase global modes are independent of the cross-tie position, stiffness and damping. However, modal properties of local RS modes in the rigid cross-tie case would be “modified” by the cross-tie stiffness and damping and evolve into global modes. The impact of cross-tie stiffness and damping on the modal properties of local LS modes depends on the cross-tie installation location. If the cross-tie is located at the main cable nodal point, the frequency and damping of the LS modes would be independent of cross-tie stiffness and damping. However, if the cross-tie
is not placed at the nodal point, the presence of cross-tie stiffness and damping would alter both the modal frequency and the damping of the LS modes. In both cases, such a change in cross-tie properties would render a local LS mode evolve to an out-of-phase global mode.

**Numerical example:**

To validate the proposed cable network analytical model and further discuss the modal characteristics associated with twin-cable networks, a numerical example is presented. Both main cables are assumed to have a length of 72 m, a unit mass of 50 kg/m, a tension of 2200 kN and a structural damping ratio of 0.5%. The stiffness coefficient of cross-tie is assumed to be $K_c = 30.54$ kN/m, and its damping coefficient being $C_c = 1.0$ kN·s/m. Two cross-tie installation locations of $\varepsilon = 1/3$ and $\varepsilon = 2/5$ are considered in the example.

a) Cross-tie installed at $\varepsilon = 1/3$

In this case, a flexible damped cross-tie is placed at the one-third span of the two main cables, which happens to be the nodal point of the $3^{rd}$ mode of an isolated single main cable. The modal properties of the first ten network modes obtained from the proposed analytical model and finite element simulation are listed in Table 3.7, and the mode shapes are depicted in Figure 3.18.
Table 3.7: Comparison of modal properties of a twin-cable network with damped flexible or rigid cross-tie at $\varepsilon=1/3$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f$ (Hz)</th>
<th>$\xi_{eq}$ (%)</th>
<th>Mode Shape</th>
<th>$f$ (Hz)</th>
<th>$\xi_{eq}$ (%)</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>FEA</td>
<td>Analytical</td>
<td>FEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.46</td>
<td>1.46</td>
<td>0.50</td>
<td>0.50</td>
<td>GM, in-phase</td>
<td>1.46</td>
</tr>
<tr>
<td>2</td>
<td>1.63</td>
<td>1.63</td>
<td>3.34</td>
<td>3.35</td>
<td>GM, out-of-phase</td>
<td>2.18</td>
</tr>
<tr>
<td>3</td>
<td>2.91</td>
<td>2.91</td>
<td>0.25</td>
<td>0.25</td>
<td>GM, in-phase</td>
<td>2.91</td>
</tr>
<tr>
<td>4</td>
<td>3.02</td>
<td>3.02</td>
<td>2.49</td>
<td>2.49</td>
<td>GM, out-of-phase</td>
<td>4.37</td>
</tr>
<tr>
<td>5</td>
<td>4.37</td>
<td>4.37</td>
<td>0.17</td>
<td>0.17</td>
<td>GM, in-phase</td>
<td>4.37</td>
</tr>
<tr>
<td>6</td>
<td>4.37</td>
<td>4.37</td>
<td>0.17</td>
<td>0.17</td>
<td>GM, out-of-phase</td>
<td>4.37</td>
</tr>
<tr>
<td>7</td>
<td>5.83</td>
<td>5.82</td>
<td>0.13</td>
<td>0.13</td>
<td>GM, in-phase</td>
<td>5.83</td>
</tr>
<tr>
<td>8</td>
<td>5.88</td>
<td>5.87</td>
<td>1.19</td>
<td>1.19</td>
<td>GM, out-of-phase</td>
<td>6.56</td>
</tr>
<tr>
<td>9</td>
<td>7.28</td>
<td>7.27</td>
<td>0.10</td>
<td>0.10</td>
<td>GM, in-phase</td>
<td>7.28</td>
</tr>
<tr>
<td>10</td>
<td>7.33</td>
<td>7.31</td>
<td>1.03</td>
<td>1.03</td>
<td>GM, out-of-phase</td>
<td>8.74</td>
</tr>
</tbody>
</table>

(GM: global mode, LM: local mode, LS: left segment mode, RS: right segment mode.)
Mode 1 (GM, in-phase), $\Omega = 1.0\pi$, $\xi_{eq} = 0.50\%$

Mode 2 (GM, out-of-phase), $\Omega = 1.12\pi$, $\xi_{eq} = 3.34\%$

Mode 3 (GM, in-phase), $\Omega = 2.0\pi$, $\xi_{eq} = 0.25\%$

Mode 4 (GM, out-of-phase), $\Omega = 2.08\pi$, $\xi_{eq} = 2.49\%$

Mode 5 (GM, in-phase), $\Omega = 3.0\pi$, $\xi_{eq} = 0.17\%$

Mode 6 (GM, out-of-phase), $\Omega = 3.0\pi$, $\xi_{eq} = 0.17\%$

Mode 7 (GM, in-phase), $\Omega = 4.0\pi$, $\xi_{eq} = 0.13\%$

Mode 8 (GM, out-of-phase), $\Omega = 4.04\pi$, $\xi_{eq} = 1.19\%$

Mode 9 (GM, in-phase), $\Omega = 5.0\pi$, $\xi_{eq} = 0.10\%$

Mode 10 (GM, out-of-phase), $\Omega = 5.03\pi$, $\xi_{eq} = 1.03\%$

Figure 3.18: First ten modes of a twin-cable network with a damped flexible cross-tie ($K_c=30.54$ kN/m, $C_c=1.0$ kN·s/m) at $\epsilon=1/3$

The two sets of results are found to agree well. Given also in the same table are the modal properties of the corresponding rigid cross-tie twin-cable network. It can be seen from the table that for all five in-phase global modes, i.e. modes 1, 3, 5, 7, and 9, their modal frequencies, modal damping ratios and mode shapes are not only independent of the cross-tie flexibility and damping but are also not affected by the presence of cross-tie. The properties of these modes remain the same as those of an isolated single cable. Since the cross-tie is placed at the main cable one-third span, the modal frequency and damping of the first local LS mode, Mode 6,
remain the same when the rigid cross-tie is replaced by a damped flexible one, but the mode
shape evolves to an out-of-phase global mode. Moreover, this particular cross-tie position also
renders the modal frequency and modal damping ratio of Mode 6 to be the same as those of
Mode 5, which is the 3rd in-phase network global mode. In the case of modes 2, 4, 8, and 10,
which are pure local RS modes in the rigid cross-tie case, the adoption of a flexible damped
cross-tie is found to not only considerably affect their modal frequencies and damping ratios, but
also alter their mode shapes. For example, in the case of Mode 2, such a change in the cross-tie
properties would excite the left segments of the network so that the mode shape becomes an out-
of-phase global mode. The modal frequency is reduced by 25% from 2.18 Hz to 1.63 Hz,
whereas the modal damping ratio increases substantially from 0.33% to 3.34% by roughly ten
times. One possible reason for such a drastic increment in the network modal damping ratio
could be the relatively high damping coefficient \( C_c=1.0 \text{ kN} \cdot \text{s/m} \) and relatively low stiffness
coefficient \( K_c=30.54 \text{ kN/m} \) assumed in this example. Besides, it is also important to note that
since linear viscous type of damping model is used for the cross-tie, the energy dissipation due to
damped cross-tie would only occur when the two ends of the cross-tie oscillate at different
velocities. Therefore, in the case of network in-phase global modes (modes 1, 3, 5, 7 and 9) of
which the twin cables vibrate with the same shape, the oscillating velocities at the cross-tie two
ends are the same so that a flexible damped cross-tie is not capable of dissipating energy. Thus,
all the network in-phase global modes have the same modal damping ratio as that of an isolated
single cable vibrating in the same mode. On the other hand, when a network out-of-phase global
mode is excited, velocities at the two ends of the cross-tie are equal but opposite in direction, so
the flexible damped cross-tie would manifest the maximum possible energy dissipation capacity.
Similar pattern, i.e. decrease in the modal frequency and significant increase in the modal damping, can also be found in higher order out-of-phase global modes (modes 4, 8, 10).

b) Cross-tie installed at ɛ=2/5

Modal analysis of the same twin-cable network is conducted by relocating the cross-tie position to ɛ=2/5. Table 3.8 summarizes the modal properties of the first ten network modes, and the mode shapes are portrayed in Figure 3.19.

Table 3.8: Comparison of modal properties of a twin-cable network with damped flexible or rigid cross-tie at ɛ=2/5

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical f (Hz)</th>
<th>Analytical ξeq (%)</th>
<th>FEA f (Hz)</th>
<th>FEA ξeq (%)</th>
<th>Mode Shape</th>
<th>Analytical f (Hz)</th>
<th>Analytical ξeq (%)</th>
<th>FEA f (Hz)</th>
<th>FEA ξeq (%)</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.46</td>
<td>0.50</td>
<td>1.46</td>
<td>0.50</td>
<td>GM, in-phase</td>
<td>1.46</td>
<td>0.50</td>
<td>GM, in-phase</td>
<td>1.46</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>1.68</td>
<td>4.11</td>
<td>1.68</td>
<td>4.12</td>
<td>GM, out-of-phase</td>
<td>2.43</td>
<td>0.30</td>
<td>LM, RS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.91</td>
<td>0.25</td>
<td>2.91</td>
<td>0.25</td>
<td>GM, in-phase</td>
<td>2.91</td>
<td>0.25</td>
<td>GM, in-phase</td>
<td>2.91</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>2.96</td>
<td>1.30</td>
<td>2.96</td>
<td>1.30</td>
<td>GM, out-of-phase</td>
<td>3.64</td>
<td>0.20</td>
<td>LM, LS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.37</td>
<td>0.17</td>
<td>4.37</td>
<td>0.17</td>
<td>GM, in-phase</td>
<td>4.37</td>
<td>0.17</td>
<td>GM, in-phase</td>
<td>4.37</td>
<td>0.17</td>
</tr>
<tr>
<td>6</td>
<td>4.40</td>
<td>0.82</td>
<td>4.40</td>
<td>0.82</td>
<td>GM, out-of-phase</td>
<td>4.86</td>
<td>0.15</td>
<td>LM, RS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5.83</td>
<td>0.13</td>
<td>5.83</td>
<td>0.13</td>
<td>GM, in-phase</td>
<td>5.83</td>
<td>0.13</td>
<td>GM, in-phase</td>
<td>5.83</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>5.89</td>
<td>1.49</td>
<td>5.89</td>
<td>1.49</td>
<td>GM, out-of-phase</td>
<td>7.28</td>
<td>0.10</td>
<td>LM, RS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7.28</td>
<td>0.10</td>
<td>7.27</td>
<td>0.10</td>
<td>GM, in-phase</td>
<td>7.28</td>
<td>0.10</td>
<td>GM, in-phase</td>
<td>7.28</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>7.28</td>
<td>0.10</td>
<td>7.27</td>
<td>0.10</td>
<td>GM, out-of-phase</td>
<td>7.28</td>
<td>0.10</td>
<td>LM, LS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mode 1 (GM, in-phase), $\Omega=1.0\pi$, $\xi_{eq}=0.50\%$

Mode 2 (GM, out-of-phase), $\Omega=1.15\pi$, $\xi_{eq}=4.11\%$

Mode 3 (GM, in-phase), $\Omega=2.0\pi$, $\xi_{eq}=0.25\%$

Mode 4 (GM, out-of-phase), $\Omega=2.04\pi$, $\xi_{eq}=1.30\%$

Mode 5 (GM, in-phase), $\Omega=3.0\pi$, $\xi_{eq}=0.17\%$

Mode 6 (GM, out-of-phase), $\Omega=3.02\pi$, $\xi_{eq}=0.82\%$

Mode 7 (GM, in-phase), $\Omega=4.0\pi$, $\xi_{eq}=0.13\%$

Mode 8 (GM, out-of-phase), $\Omega=4.05\pi$, $\xi_{eq}=1.49\%$

Mode 9 (GM, in-phase), $\Omega=5.0\pi$, $\xi_{eq}=0.10\%$

Mode 10 (GM, out-of-phase), $\Omega=5.0\pi$, $\xi_{eq}=0.10\%$

Figure 3.19: First ten modes of a twin-cable network with a damped flexible cross-tie ($K_c=30.54$ kN/m, $C_c=1.0$ kN·s/m) at $\varepsilon=2/5$

A good agreement between the modal results determined by the proposed analytical model and finite element simulations can be clearly observed from Table 3.8. For convenience of the comparison, the modal properties of the corresponding rigid cross-tie network are also listed in Table 3.8. Results show that similar to the previous case of $\varepsilon=1/3$, the modal characteristics (modal frequency, modal damping ratio and mode shape) of the in-phase global modes, i.e. modes 1, 3, 5, 7, and 9, are not affected by the presence of cross-tie. They remain the same as the flexibility and damping of the cross-tie change. If using a rigid cross-tie network as a reference
base, by increasing cross-tie flexibility and damping, modal frequency of local RS modes decreases whereas modal damping ratio increases. In addition, their mode shapes evolve to out-of-phase global modes. Take Mode 2 as an example, if replacing the rigid cross-tie by a flexible damped one with $K_c=30.54$ kN/m and $C_c=1.0$ kN·s/m, its frequency drops from 2.43 Hz to 1.68 Hz by 31%, but the associated modal damping ratio increases approximately 12.5 times from 0.33% to 4.11%, and the oscillation extends from the right segments to the entire network. The same phenomenon can be observed in Mode 6 and Mode 8. In the case of Mode 4 and Mode 10, both of which are local LS modes in the rigid cross-tie network, a cross-tie position of $\epsilon=2/5$ is off the nodal point of the 1\textsuperscript{st} anti-symmetric mode of an isolated cable in Mode 4, but happens to be the nodal point of the 3\textsuperscript{rd} symmetric mode of an isolated cable in Mode 10. Thus, for Mode 4, the change in cross-tie properties would not only cause evolution of its mode shape into an out-of-phase global mode, but also alters its modal frequency and damping ratio. Whereas for Mode 10, its frequency and damping ratio remain the same as those of the rigid cross-tie case, although the mode evolution phenomenon occurs.

### 3.2.2.5 Symmetric DMT damped flexible cross-tie Cable Network

In the majority of cable networks on real cable-stayed bridges, the consisting main cables have different length, unit mass and tension, which results in different mass-tension (DMT) ratio. Besides, since the spacing between cables is typically closer on the pylon side than on the deck side, the geometric layout of a real cable network is generally asymmetric. However, it is observed in Section 3.1 that if rigid or flexible undamped cross-ties are used in a symmetric cable network, pure local modes dominated by oscillations of individual main cables could form. Therefore, before analyzing a more realistic asymmetric DMT cable network, the impact of
using flexible damped cross-ties on the modal response of a DMT cable network having symmetric layout is studied first.

When the cable network in Figure 3.8 has a symmetric layout, the left and the right offsets of main cable 2 are the same, i.e. $O_L=O_R$, and the flexible damped cross-tie locates at the mid-span of the two main cables. Therefore, the segment parameters would satisfy $\phi_1=\phi_2=R_1/2$ and $\phi_3=\phi_4=R_2/2$. Substitute these relations into Eq. (3-26), the system characteristic equation of a symmetric DMT two-cable network can be expressed as

$$\sin\left(\frac{R_1}{2}\right)\sin\left(\frac{R_2}{2}\right)\left[\cos\left(\frac{R_1}{2}\right)\sin\left(\frac{R_2}{2}\right) + \gamma_2\sin\left(\frac{R_1}{2}\right)\cos\left(\frac{R_2}{2}\right) + 2\psi R_1\gamma_2\sin\left(\frac{R_1}{2}\right)\right] = 0 \quad (3-51)$$

By setting each of the three terms on the left side of Eq. (3-34) to zero, i.e. the two sine terms and the one enclosed by the square bracket, three sets of roots can be determined. The first two sets, yielded respectively from $\sin(R_1/2) = 0$ and $\sin(R_2/2) = 0$, describe the local modes dominated by the target or the neighbouring cable. They are

Local modes of the target cable:

$$\Omega_n = 2n\pi \quad n=1, 2, 3,\ldots \quad (3-52a)$$

$$\xi_{eq} = \xi_1/(2n) \quad n=1, 2, 3,\ldots \quad (3-52b)$$

Local modes of the neighboring cable:

$$\Omega_n = 2n\pi / \eta_2 \quad n=1, 2, 3,\ldots \quad (3-53a)$$

$$\xi_{eq} = \xi_2/(2n) \quad n=1, 2, 3,\ldots \quad (3-53b)$$

First of all, it is interesting to note that, the form of Eqs. (3-52) and (3-53) implies that these two types of network local modes have the same modal properties as the respective isolated single cable anti-symmetric modes. Clearly, since the cross-tie is placed at the nodal point of the main cables, the network local modes dominated by an individual main cable would not be affected by the stiffness and damping of the cross-tie.
The third set of roots, describing the modal properties of network global modes, can be found by setting the term in the square bracket of Eq. (3-51) to zero. While the first two terms inside the square bracket shows the interaction between the two main cables and the coupling in their motions, the third term reflects the role of cross-tie properties in “modifying” the frequency and damping of network global modes. Should a rigid cross-tie be used, this term would vanish, suggesting that the properties of network global modes would only be affected by the main cable properties. This set of solution can be determined by separating the real and imaginary terms and using the same procedure explained in Section 3.2.2.3.

**Numerical example:**

Consider a symmetric DMT cable network with the following properties:

- **Main Cable 1:** \( L_1 = 72 \text{ m} \) \( H_1 = 2200 \text{ kN} \) \( m_1 = 50 \text{ kg/m} \) \( \xi_1 = 0.5\% \)
- **Main Cable 2:** \( L_2 = 60 \text{ m} \) \( H_2 = 2400 \text{ kN} \) \( m_2 = 42 \text{ kg/m} \) \( \xi_2 = 0.8\% \)
- **Cross-tie:** \( \varepsilon = 1/2 \) \( K_c = 30.54 \text{ kN/m} \) \( C_c = 1.0 \text{ kN⋅s/m} \)

Modal properties of the first ten modes, obtained from the proposed analytical model and finite element simulation, are given in Table 3.9. The corresponding mode shapes are illustrated in Figure 3.20.
Table 3.9: Comparison of modal properties of a symmetric DMT two-cable network with damped flexible or rigid cross-tie at $\varepsilon=1/2$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Damped flexible cross-tie</th>
<th>Rigid cross-tie</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(K_c=30.54 \text{ kN/m}, \ C_c=1.0 \text{ kN\cdot s/m})$</td>
<td>$(K_c=\infty, \ C_c=0)$</td>
</tr>
<tr>
<td>Mode</td>
<td>$f$ (Hz)</td>
<td>$\xi_{eq}$ (%)</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>----------------</td>
</tr>
<tr>
<td>Ana.</td>
<td>FEA</td>
<td>Ana.</td>
</tr>
<tr>
<td>1</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>2</td>
<td>2.15</td>
<td>2.15</td>
</tr>
<tr>
<td>3</td>
<td>2.91</td>
<td>2.91</td>
</tr>
<tr>
<td>4</td>
<td>3.98</td>
<td>3.98</td>
</tr>
<tr>
<td>5</td>
<td>4.42</td>
<td>4.41</td>
</tr>
<tr>
<td>6</td>
<td>5.83</td>
<td>5.82</td>
</tr>
<tr>
<td>7</td>
<td>6.03</td>
<td>6.02</td>
</tr>
<tr>
<td>8</td>
<td>7.31</td>
<td>7.30</td>
</tr>
<tr>
<td>9</td>
<td>7.97</td>
<td>7.96</td>
</tr>
<tr>
<td>10</td>
<td>8.74</td>
<td>8.72</td>
</tr>
</tbody>
</table>
Mode 1 (GM, 1-in-phase), $\Omega = 1.07\pi$, $\zeta_{eq} = 1.71\%$

Mode 2 (GM, out-of-phase), $\Omega = 1.48\pi$, $\zeta_{eq} = 4.01\%$

Mode 3 (LM, Cable 1), $\Omega = 2.0\pi$, $\zeta_{eq} = 0.25\%$

Mode 4 (LM, Cable 2), $\Omega = 2.74\pi$, $\zeta_{eq} = 0.40\%$

Mode 5 (LM, Cable 1), $\Omega = 3.03\pi$, $\zeta_{eq} = 1.11\%$

Mode 6 (LM, Cable 1), $\Omega = 4.0\pi$, $\zeta_{eq} = 0.12\%$

Mode 7 (LM, Cable 2), $\Omega = 4.14\pi$, $\zeta_{eq} = 1.29\%$

Mode 8 (LM, Cable 1), $\Omega = 5.02\pi$, $\zeta_{eq} = 0.72\%$

Mode 9 (LM, Cable 2), $\Omega = 5.47\pi$, $\zeta_{eq} = 0.20\%$

Mode 10 (LM, Cable 1), $\Omega = 6.0\pi$, $\zeta_{eq} = 0.08\%$

Figure 3.20: First ten modes of a symmetric DMT two-cable network with a damped flexible cross-tie ($K_c = 30.54\text{ kN/m}$, $C_c = 1.0\text{ kN}\cdot\text{s/m}$) at $\varepsilon = 1/2$

Again, the two sets of results are found to agree well with each other. To assess the impact of cross-tie stiffness and damping on the modal behaviour of the studied symmetric DMT network, modal response of the same network but using rigid cross-tie are also listed in the same table. Noticing that the fundamental frequency of the target cable is 1.46 Hz, and the associated modal damping ratio is 0.5%, results in Table 3.9 show that when the target cable is connected to the neighbouring one using a rigid cross-tie, its fundamental frequency increases from 1.46 Hz to 1.68 Hz by 15% and the modal damping ratio from 0.5% to 0.61% by 22%. However, if a flexible damped cross-tie with a stiffness coefficient of $K_c = 30.54\text{ kN/m}$ and damping coefficient of $C_c = 1.0\text{ kN}\cdot\text{s/m}$ is used instead, the fundamental frequency of the target cable would be
increased by 6.2% to 1.55Hz, whereas the modal damping ratio to 1.71% by 3.4 times. These suggest that using more rigid cross-tie would further enhance the in-plane stiffness of a cable network and thus the target cable, which agrees with the experimental observations by Yamaguchi and Nagahawatta (1995) and Sun et al. (2007). Although the network fundamental mode is an in-phase global mode, the velocities at the cross-tie two ends are different due to the difference in the dynamic properties of the two main cables. Thus, unlike the twin-cable network case, damping existed in the cross-tie would offer non-zero damping force and help to dissipate more energy during oscillation.

In the case of the first out-of-phase global mode, the flexibility in the cross-tie reduces its modal frequency, so it is advanced from the third network mode \( (f=3.375 \text{ Hz}) \) in the rigid cross-tie case to the second network mode \( (f=2.153 \text{ Hz}) \) should a damped flexible cross-tie be used. Besides, since the relative velocity between the two cross-tie ends reaches its maxima in this oscillation mode, the large damping offered by the cross-tie leads to a drastic increment of its modal damping ratio from 0.34% in the rigid cross-tie case to 4.01% for a flexible damped cross-tie case.

The modal properties of the network local modes dominated by either the target cable or its neighbouring one are found to be independent of the cross-tie stiffness and damping (modes 3, 4, 6, 9 and 10). In these cases, one of the main cables vibrates in an anti-symmetric shape. Thus, the cross-tie happens to locate at the nodal point of the mode shape and would not have a role in altering modal properties. However, it is interesting to note that although the frequency of the 5\(^{th}\) network mode, 4.42 Hz, is very close to the third modal frequency of the isolated target cable, which is 4.38 Hz \( (\Omega_o=3.0\pi) \), the modal damping ratio jumps by roughly 7 times from 0.16% \( (\xi_{eq} = \xi_1/3) \) for a single cable to 1.11% when it is networked. As can be seen from
Figure 3.20, when the network oscillates in this mode, the target cable vibrates in its 3rd mode and is dominant. The cross-tie is located at the maximum deformation location of the target cable whereas the neighbouring cable is almost at rest. Therefore, in terms of energy dissipation, the damped cross-tie acts like a dashpot damper installed at the mid-span of the target cable and “rigidly” supported by the neighbouring cable. Similar phenomenon is also observed in the 7th and 8th modes of the cable network, of which the motion is mainly dominated by one cable with the other cable almost at rest.

3.2.2.6 Asymmetric DMT damped flexible cross-tie Cable Network

The same two main cables in the symmetric DMT cable network of Section 3.2.2.5 are rearranged in this section such that the left and the right offsets of the neighbouring cable with respect to the target cable (Figure 3.8) are respectively 3 m and 9 m. In addition, the cross-tie is relocated to one-third span of the target cable from its left support, i.e. $\varepsilon=1/3$. These changes in the layout lead to an asymmetric DMT cable network. Table 3.10 lists the modal properties of the first ten network modes obtained from the proposed analytical model and numerical simulation. A good agreement between the two sets can be clearly seen. In addition, the modal analysis results of a corresponding rigid cross-tie network are also given in the same table for the convenience of comparison. The mode shapes of these ten modes are depicted in Figure 3.21.
Table 3.10: Comparison of modal properties of an asymmetric DMT two-cable network with damped flexible or rigid cross-tie at $\varepsilon=1/3$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f$ (Hz)</th>
<th>$\xi_{eq}$ (%)</th>
<th>Mode Shape</th>
<th>$f$ (Hz)</th>
<th>$\xi_{eq}$ (%)</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ana.</td>
<td>FEA</td>
<td>Ana.</td>
<td>FEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.53</td>
<td>1.53</td>
<td>1.49</td>
<td>1.49</td>
<td>GM, in-phase</td>
<td>1.65</td>
</tr>
<tr>
<td>2</td>
<td>2.11</td>
<td>2.11</td>
<td>2.88</td>
<td>2.89</td>
<td>GM, out-of-phase</td>
<td>2.55</td>
</tr>
<tr>
<td>3</td>
<td>2.97</td>
<td>2.97</td>
<td>1.42</td>
<td>1.42</td>
<td>LM, Cable 1</td>
<td>3.49</td>
</tr>
<tr>
<td>4</td>
<td>4.04</td>
<td>4.03</td>
<td>1.48</td>
<td>1.48</td>
<td>LM, Cable 2</td>
<td>4.37</td>
</tr>
<tr>
<td>5</td>
<td>4.37</td>
<td>4.37</td>
<td>0.17</td>
<td>0.17</td>
<td>LM, Cable 1</td>
<td>5.00</td>
</tr>
<tr>
<td>6</td>
<td>5.85</td>
<td>5.85</td>
<td>0.66</td>
<td>0.66</td>
<td>LM, Cable 1</td>
<td>5.97</td>
</tr>
<tr>
<td>7</td>
<td>5.98</td>
<td>5.97</td>
<td>0.30</td>
<td>0.30</td>
<td>LM, Cable 2</td>
<td>6.43</td>
</tr>
<tr>
<td>8</td>
<td>7.30</td>
<td>7.29</td>
<td>0.51</td>
<td>0.51</td>
<td>LM, Cable 1</td>
<td>7.54</td>
</tr>
<tr>
<td>9</td>
<td>8.00</td>
<td>7.99</td>
<td>0.94</td>
<td>0.94</td>
<td>LM, Cable 2</td>
<td>8.74</td>
</tr>
<tr>
<td>10</td>
<td>8.74</td>
<td>8.72</td>
<td>0.08</td>
<td>0.08</td>
<td>LM, Cable 1</td>
<td>9.05</td>
</tr>
</tbody>
</table>
Results in Table 3.10 indicate that by replacing the rigid cross-tie with a damped flexible one, the frequency of the network fundamental mode, which is an in-phase global mode, decreases whereas its modal damping ratio increases drastically. The target cable has a fundamental frequency of 1.46 Hz and a modal damping ratio of 0.50%. When it is connected with the neighbouring cable using a rigid cross-tie, the modal frequency increased to 1.65 Hz by 13% and the damping ratio to 0.58% by 16%. However, if the cross-tie has properties of $K_c=30.54$ kN/m and $C_c=1.0$ kN·s/m, the increment of its fundamental frequency and damping ratio becomes 4.8% to 1.53 Hz and approximately three times to 1.49%, respectively. The same
phenomenon can be observed in Mode 2, which is an out-of-phase global mode, i.e. although using a damped flexible cross-tie would reduce the gain in network stiffness to some extent, it could greatly improve the energy dissipation capacity of the formed cable network. This is consistent with observation reported in (e.g. Yamaguchi and Nagahawatta, 1995; Sun et al., 2007). In addition, it should be noted that similar to the symmetric layout case in Section 3.2.2.5, such a change in cross-tie properties would lead to excitation of more local modes dominated by one of the main cables. This could be mainly attributed to the increased flexibility in cross-tie, which offers more freedom to one cable from the constraint of the other so it can oscillate more independently. Among the first ten modes listed in Table 3.10, the number of local modes increases from 2 to 8. Mode 4 and Mode 9 in the rigid cross-tie case are dominated respectively by the $3^{rd}$ and the $6^{th}$ mode of an isolated target cable. The position of cross-tie at $\varepsilon=1/3$ happens to coincide with the nodal point of these single cable modes. Thus, the modal properties of these two local modes are not affected by the cross-tie stiffness and damping except they become the $5^{th}$ and the $10^{th}$ modes when a damped flexible cross-tie is used instead.

Besides, a parametric study is conducted for this asymmetric DMT cable network to better understand the effect of cross-tie stiffness and damping on the modal frequency and damping ratio of the network global modes. Figure 3.22 depicts the modal property variation of the lowest network in-phase global mode and out-of-phase global mode with respect to the undamped cross-tie stiffness parameter $\psi_0$. 
Figure 3.22: Effect of undamped cross-tie stiffness parameter $\psi_o$ on modal frequency and modal damping ratio of the lowest in-phase and out-of-phase global modes of an asymmetric DMT cable network

In the analysis, the cross-tie damping coefficient is assumed to be $C_c=1.0$ kN·s/m, whereas $\psi_o$ varies from 0 (rigid) to 1.0, which is a typical range of cross-tie stiffness on real cable-stayed bridges (Caracoglia and Jones, 2005b). It can be seen from Figure 3.22 that overall, the modal properties of the out-of-phase global mode are more sensitive to the cross-tie stiffness. As expected, the frequencies of both global modes decrease monotonically with the increase of cross-tie flexibility. Within the studied range of $\psi_o$, the frequency of the in-phase global mode decreases by 7% while that of the out-of-phase global mode drops roughly by 18%. In terms of modal damping ratio, since the linear viscous damping model is used for describing cross-tie damping property, a more flexible cross-tie would result in higher relative motion velocity between the cross-tie two ends and thus more contribution to energy dissipation of the oscillating main cables in the network. It is also interesting to note from the figure that while the damping
ratio increment rate of the in-phase global mode is more steady when $\psi_o$ increases from 0 to 1, that of the out-of-phase global mode appears to gradually decrease as the cross-tie becomes more and more flexible. In general, the patterns of the $\psi_o$-$\Omega$ and $\psi_o$-$\xi_{eq}$ curves in Figure 3.22 imply that although using a more flexible cross-tie would cause some loss in network in-plane stiffness, the energy dissipation capacity could be greatly improved, which is beneficial for cable vibration control.

The influence of cross-tie damping level on the modal properties of the two lowest network global modes is shown in Figure 3.23.

![Figure 3.23: Effect of cross-tie damping coefficient $C$ on modal frequency and modal damping ratio of the lowest in-phase and out-of-phase global modes of an asymmetric DMT cable network](image)

The modal frequencies of the two global modes are independent of the cross-tie damping level and remain as constants, whereas their modal damping ratios increase almost linearly with
the increase of cross-tie damping. Again, the out-of-phase global mode is found to be more sensitive to change in cross-tie damping. By increasing $C_c$ from 0 to 1.0 kN·s/m, the modal damping ratio of the lowest out-of-phase global mode increases from 0.44% to 2.88% by roughly 6.5 times, whereas that of the in-phase global mode increases almost three times from 0.58% to 1.49%.

3.3 Summary

In this chapter, analytical models have been developed to observe the modal behaviour of basic two-cable networks with and without the consideration of intrinsic damping of main cables and cross-tie. The conclusions of this chapter can be summarized as follows:

1) In twin-cable networks with rigid cross-tie, the in-plane stiffness and the modal damping of the system remain unchanged. They are the same as those of a single cable present in the network. Therefore, it is recommended that the vulnerable (target) cable should be connected to the stiffer and more damped neighbouring cable to enhance its in-plane stiffness and the damping.

2) The fundamental frequency of a cable network increases monotonically with the frequency ratio of the neighbouring cable. This effect is more pronounce if a more rigid cross-tie is used and it is placed closer to the mid-span of the target cable.

3) The cross-tie flexibility plays an important role in influencing the in-plane frequency of global modes. Results obtained from the studied two-cable networks suggest that a damped flexible cross-tie would affect both the frequency and the damping of different cable network modes. It is observed that the out-of-phase global modes are more sensitive to the change of cross-tie stiffness and damping than the in-phase ones.
4) The modal damping of the target cable will increase if it is connected with the neighbouring cable possessing higher damping, whereas the damping in the neighbouring cable would reduce at the same time. This arrangement of cross-tie solution results in “transfer” of damping from a more damped cable to the less damped one present in the cable network. In such a case, placing a rigid cross-tie closer to the cable mid-span will be beneficial for the increase of network damping property.
CHAPTER 4  Modal Analysis of General Cable Network

In Chapter 3, the basic cable networks consisting of two main cables connected through a single flexible cross-tie were analysed, with and without the consideration of main cable and cross-tie damping property. Although these simple models are helpful in covering the basic mechanics associated with cable network modal behaviour, it would be necessary to generalise the formulation to include the modal behaviour of cable networks on real cable-stayed bridges, which typically contains multiple main cables and cross-tie lines. The present chapter deals with the development an analytical model of a general cable network consisting of a given number of main cables connected through multiple lines of flexible cross-ties.

4.1  Formulation of analytical model

Consider a general orthogonal cable network depicted in Figure 4.1. It consists of $n_c$ main cables and $n_t$ lines of transverse flexible cross-ties. Each cable is divided into $(n_t + 1)$ segments by cross-ties. It is assumed that the cables are fixed at both ends and their behaviour can be described by the taut cable theory. The unit mass and tension of the $i^{th}$ main cable is denoted by $m_i$ and $H_i$, respectively. The additional tension due to cable vibrations is neglected. The mass of the cross-ties is generally small when compared to that of the main cables and is thus neglected in the current formulation.
The oscillations of the cross-ties are assumed to be restricted along their respective axial direction, and are simulated by linear reversible tension/compression connectors. The stiffness of the cross-ties along the $j^{th}$ ($j=1, 2, \ldots, n_t$) cross-tie line is assumed to be a constant and represented by an equivalent spring stiffness constant $K_j$. For the cable network shown in Figure 3.5, the presence of $n_t$ lines of cross-ties leads to a total of $n_c(n_t + 1)$ cable segments. A local coordinate system is defined and attached to each cable segment left node, except for the last segment of each cable on the right end, of which the attachment is at the right node. Take a cable segment $(i, j)$ as an example (the $j^{th}$ segment of the $i^{th}$ cable), the local coordinate system $x_{i,j}$ and $y_{i,j}$ has its origin at the left end of cable segment. The transverse displacement of the cable segment along $y_{i,j}$ axis is denoted as $v_{i,j}$ and positive downward. The direction of $x_{i,j}$ is positive towards right except for the last segment of each cable, which is positive towards left. The length of the cable
segment is $L_{i,j}$. The in-plane transverse free vibration of a typical cable segment $(i, j)$ can be described by Irvine’s linear theory of free vibrations (1981).

$$H_i \frac{\partial^2 v_{ij}(x_{i,j}, t)}{\partial x_{i,j}^2} = m_i \frac{\partial v_{ij}(x_{i,j}, t)}{\partial t^2} \quad (4-1)$$

where $H_i$ and $m_i$ are respectively the tension and the unit mass of the $i^{th}$ main cable, $v_{ij}$ is the transverse displacement of cable segment $(i, j)$. The Bernoulli-Fourier method is used to separate the temporal and spatial variables in the cable transverse displacement $v_{ij}$, i.e. $v_{ij}(x_{i,j}, t) = \tilde{v}_{i,j}(x_{i,j})\sin(\omega t)$, where $\tilde{v}_{i,j}(x_{i,j})$ is the shape function and $\omega$ is the circular frequency of vibration. By substituting the expression into Eq. (4-1), it yields

$$\tilde{v}_{i,j}(x_{i,j}) = A_{i,j}\cos(\alpha_l x_{i,j}) + B_{i,j}\sin(\alpha_l x_{i,j}) \quad (4-2)$$

where $\alpha_l = \sqrt{m_i \omega^2/H_i}$ is the wave number, $A_{i,j}$ and $B_{i,j}$ are constants which can be determined from boundary conditions, as well as equilibrium and compatibility conditions. A total of $2n_c(n_t + 1)$ equations are needed to find all the $A_{i,j}$ and $B_{i,j}$ ($i=1, 2, \ldots n_c$, $j=1, 2, \ldots n_t+1$) for the cable segments in the studied cable network. They include

a) The boundary conditions at two ends of each main cable:

$$\tilde{v}_{i,1}(0) = \tilde{v}_{i,n_t+1}(0) = 0 \quad i=1, 2, \ldots n_c \quad (4-3a)$$

b) The transverse displacement compatibility of the two adjacent cable segments in $i^{th}$ cable ($i=1, 2, \ldots n_c$) at the sharing node:

$$\tilde{v}_{i,j}(L_{i,j}) = \begin{cases} 
\tilde{v}_{i,j+1}(0) & \text{for } j = 1, 2, \ldots n_t - 1 \\
\tilde{v}_{i,j+1}(L_{i,j+1}) & \text{for } j = n_t 
\end{cases} \quad (4-3b)$$

c) The compatibility between the axial deformation of a cross-tie element along the $j^{th}$ line which connects the $i^{th}$ and the $(i+1)^{th}$ main cables ($i=1, 2, \ldots n_c-1$) and the difference between the transverse displacements of the two connecting cable nodes, which is given by
In addition, the equilibrium of the isolated $j^{th}$ cross-tie line along its axial direction requires
\[ \sum_{l=1}^{n_c} \left[ H_l \left( \frac{\partial \bar{v}_{l,j}}{\partial x_{l,j}} \bigg|_{x_{l,j}=L_{l,j}} - \frac{\partial \bar{v}_{l,j+1}}{\partial x_{l,j+1}} \bigg|_{x_{l,j+1}=0} \right) \right] = 0 \quad \text{for} \ j = 1, \ldots n_t - 1 \quad (4-5a) \]
\[ \sum_{l=1}^{n_c} \left[ H_l \left( \frac{\partial \bar{v}_{l,j}}{\partial x_{l,j}} \bigg|_{x_{l,j}=L_{l,j}} + \frac{\partial \bar{v}_{l,j+1}}{\partial x_{l,j+1}} \bigg|_{x_{l,j+1}=L_{l,j+1}} \right) \right] = 0 \quad \text{for} \ j = n_t \quad (4-5b) \]

In total, there are $2n_c$ and $n_c n_t$ equations resulted from Eqs. (4-3a) and (4-3b), respectively; $(n_t - 1)n_t$ equations yielded from the compatibility conditions described by Eq. (4-4); and Eq. (4-5), which defines the requirement to maintain longitudinal equilibrium of an isolated cross-tie line, would give $n_t$ equations. Thus, by combining Eqs. (4-3) to (4-5), a total of $2n_c(n_t + 1)$ equations can be obtained to determine all the unknown constants $A_{i,j}$ and $B_{i,j}$ ($i=1, 2 \cdots n_c; j=1, 2, \cdots n_t + 1$) in Eq. (4-2). Substitute Eq. (4-2) into Eqs. (4-3) to (4-5), it yields:

\[ A_{i,1} = A_{i,n_t+1} = 0 \quad \text{for} \ i=1, 2, \ldots n_c \quad (4-6a) \]
\[ A_{i,j} \cos(\Phi_{i,j}) + B_{i,j} \sin(\Phi_{i,j}) - A_{i,j+1} = 0 \quad \text{for} \ i=1, 2, \ldots n_c; j=1, 2, \ldots n_t - 1 \quad (4-6b) \]
\[ A_{i,n_t} \cos(\Phi_{i,n_t}) + B_{i,n_t} \sin(\Phi_{i,n_t}) - A_{i,n_t+1} \cos(\Phi_{i,n_t+1}) - B_{i,n_t+1} \sin(\Phi_{i,n_t+1}) = 0 \quad (4-6c) \]
\[ A_{i+1,j+1} - A_{i,j+1} = \psi_j \Omega \sum_{p=1}^{n_c} \gamma_p \left[ -A_{p,j} \sin(\Phi_{p,j}) + B_{p,j} \cos(\Phi_{p,j}) - B_{p,j+1} \right] \quad \text{for} \ j=1, \ldots n_t - 1 \quad (4-6d) \]
\[ A_{i+1,n_t+1} \cos(\Phi_{i+1,n_t+1}) + B_{i+1,n_t+1} \sin(\Phi_{i+1,n_t+1}) - A_{i,n_t+1} \cos(\Phi_{i,n_t+1}) - B_{i,n_t+1} \sin(\Phi_{i,n_t+1}) = \psi_{n_t} \Omega \sum_{p=1}^{n_c} \gamma_p \left[ -A_{p,n_t} \sin(\Phi_{p,n_t}) + B_{p,n_t} \cos(\Phi_{p,n_t}) - A_{p,n_t+1} \sin(\Phi_{p,n_t+1}) + B_{p,n_t+1} \cos(\Phi_{p,n_t+1}) \right] \quad (4-6e) \]
\[ \sum_{i=1}^{n_c} \left[ -A_{i,j} \gamma_i \sin(\Phi_{i,j}) + B_{i,j} \gamma_i \cos(\Phi_{i,j}) - \gamma_i B_{i,j+1} \right] = 0 \quad \text{for} \ j=1, \ldots n_t - 1 \quad (4-6f) \]
\[ \sum_{i=1}^{n_c} \left[ -A_{i,n_t} \gamma_i \sin(\Phi_{i,n_t}) + B_{i,n_t} \gamma_i \cos(\Phi_{i,n_t}) - A_{i,n_t+1} \gamma_i \sin(\Phi_{i,n_t+1}) + B_{i,n_t+1} \gamma_i \cos(\Phi_{i,n_t+1}) \right] = 0 \quad (4-6g) \]
In Eq. (4-6), $\Omega_{i,j} = \Omega \eta_i \varepsilon_{i,j}$ is the physical property of cable segment $(i,j)$, $\Omega = \pi f / f_1$ is the non-dimensional frequency of the cable network, $\eta_i = f_1 / f_i$ and $\gamma_i = \sqrt{H_i m_i / H_1 m_1}$ are respectively the frequency ratio and the mass-tension ratio of the $i^{th}$ main cable, $\varepsilon_{i,j} = L_{i,j} / L_i$ is the segment ratio of cable segment $(i,j)$, $f_i$, $m_i$, $H_i$, $L_i$ are respectively the fundamental frequency, the unit mass, the tension and the length of the $i^{th}$ cable, $L_{i,j}$ is the length of cable segment $(i,j)$; $\psi_j = H_1 / (K_j L_1)$ is the non-dimensional flexibility parameter of the $j^{th}$ line of cross-tie with $K_j$ being the axial stiffness of the $j^{th}$ line of cross-tie. Eqs. (4-6a) to (4-6g) can be written in the matrix form as follows

$$[R][X] = [0] \quad \text{(4-7)}$$

where $[R]$ is square coefficient matrix with the order of $2n_c(n_t + 1)$; $[X] = [A_{1,1} \ B_{1,1} \ A_{1,2} \ B_{1,2} \ \cdots \ A_{n_c,n_t+1} \ B_{n_c,n_t+1}]^T$ is a $2n_c(n_t + 1)$ vector containing all the unknown coefficients $A_{i,j}$ and $B_{i,j}$ $(i=1, 2, \cdots n_c; j=1, 2, \cdots n_t + 1)$; and $[0]$ is null vector. To find non-trivial solutions to Eq. (4-7), the determinant of the coefficient matrix $[R]$ should be equated to zero, based on which the non-dimensional frequency $\Omega$ can be determined. A MatLab script is written to solve the characteristic polynomial. Substitute $\Omega$ into Eq. (4-7), then all the unknown coefficients in $[X]$ can be found. Therefore, the transverse motion of each cable segment as defined by Eq. (4-2) can be obtained, which, when put together would give a complete picture of in-plane transverse vibration of the entire cable network.

4.2 Application to cable networks with real configurations

The model developed in this chapter is for general cable network with any given number of main cables connected through multiple lines of cross-ties. In order to prove the validity of proposed analytical mode, it is necessary that cable networks with real configuration should be
analysed. Therefore, the modal behaviour of two cable networks with real configuration will be explored in next two sections.

4.2.1 Five Cable Network

This example cable network consists of five unequal length parallel main cables interconnected by two transverse lines of cross-ties. The five main cables are assumed to have the same properties as those of the type AS17 to AS21 cables on the Fred Hartman Bridge Caracoglia and Jones (2005b). The length, tension, unit mass and left support offset of these five cables are

Main Cable 1: \( L_1 = 154.08 \text{ m} \) \hspace{1cm} \( H_1 = 3831 \text{ kN} \) \hspace{1cm} \( m_1 = 70.1 \text{ kg/m} \) \hspace{1cm} \( O_{1,L} = 0.00 \text{ m} \)

Main Cable 2: \( L_2 = 139.70 \text{ m} \) \hspace{1cm} \( H_2 = 3351 \text{ kN} \) \hspace{1cm} \( m_2 = 70.1 \text{ kg/m} \) \hspace{1cm} \( O_{2,L} = 0.88 \text{ m} \)

Main Cable 3: \( L_3 = 125.78 \text{ m} \) \hspace{1cm} \( H_3 = 3204 \text{ kN} \) \hspace{1cm} \( m_3 = 65.2 \text{ kg/m} \) \hspace{1cm} \( O_{3,L} = 1.70 \text{ m} \)

Main Cable 4: \( L_4 = 112.28 \text{ m} \) \hspace{1cm} \( H_4 = 2732 \text{ kN} \) \hspace{1cm} \( m_4 = 52.9 \text{ kg/m} \) \hspace{1cm} \( O_{4,L} = 3.50 \text{ m} \)

Main Cable 5: \( L_5 = 99.38 \text{ m} \) \hspace{1cm} \( H_5 = 2394 \text{ kN} \) \hspace{1cm} \( m_5 = 52.9 \text{ kg/m} \) \hspace{1cm} \( O_{5,L} = 3.80 \text{ m} \)

Main cable 1 is assumed to be the target cable. In addition, it is assumed that the two lines of cross-ties have the same axial stiffness with the non-dimensional flexibility parameter being \( \psi_1 = \psi_2 = 0.01 \). The two lines of cross-ties are evenly installed along main cable 1 (\( \varepsilon_{1,1} = \varepsilon_{1,2} = 1/3 \)).

Table 4.1 lists the modal properties of the first ten modes of the studied cable network predicted by the proposed analytical model. The associated mode shapes are shown in Figure 4.2. Results show that while the fundamental frequency of an isolated single target cable (main cable 1) is 0.64 Hz, the formation of cable network increases it significantly by 34\% to 1.02 Hz. An inspection of the mode shapes, shown in Figure 4.2, reveals that the first two modes are global, whereas Modes 3 and 4 are transition modes manifesting a shift in the network oscillation from a global to a more localized one. They are followed by a group of six closely spaced local
modes from Mode 5 to Mode 10. The frequency range of this group of local modes is between \( \Omega = 2.79\pi \) and \( 3.01\pi \), with an average frequency increment between the two consecutive modes being roughly 1.3%, which is very narrow. This kind of “local mode cluster” phenomenon, also observed by other researchers (Caracoglia and Jones, 2005b; Bosch and Park, 2005), is due to the installation of cross-ties. The presence of cross-ties would divide the main cables into shorter segments and excite local vibration modes.

In order to prove the validity of the proposed analytical model, an independent numerical simulation is conducted using the finite element analysis software Abaqus 6.10 (2010). In the finite element model, the linear two-node beam element B21 and the SPRING2 element from the Abaqus element library are chosen to simulate the behaviour of the main cables and the cross-ties, respectively. The initial axial stress is introduced into the B21 beam element to model the pretension in the main cables. The in-plane frequency and the mode shapes of first ten modes of the studied cable network obtained from the numerical simulation are also listed in Table 4.1, which are found to agree well with those yielded from the proposed analytical model.
Mode 1 (GM, Sym.), $\Omega = 1.34\pi$

Mode 2 (GM, Asym.), $\Omega = 2.37\pi$

Mode 3 (TM), $\Omega = 2.61\pi$

Mode 4 (TM), $\Omega = 2.74\pi$

Mode 5 (LM), $\Omega = 2.79\pi$

Mode 6 (LM), $\Omega = 2.83\pi$

Mode 7 (LM), $\Omega = 2.87\pi$

Mode 8 (LM), $\Omega = 2.89\pi$

Mode 9 (LM), $\Omega = 2.92\pi$

Mode 10 (LM), $\Omega = 3.01\pi$

Figure 4.2: First ten modes of five-cable network with two lines of flexible cross-ties ($\psi_1 = \psi_2 = 0.01$) at a position of $\varepsilon_{1,1} = \varepsilon_{1,2} = 1/3$ (GM: global mode, LM: local mode, TM: transition mode, Sym.: symmetric, Asym.: asymmetric)
Table 4.1: Modal properties of a general five-cable network with two lines of flexible cross-ties $(\psi_1=\psi_2=0.01)$ evenly installed along the target cable $(\epsilon_{1,1}=\epsilon_{1,2}=1/3)$

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Modal frequency (Hz)</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical (Proposed)</td>
<td>FEA</td>
</tr>
<tr>
<td>1</td>
<td>1.02</td>
<td>1.03</td>
</tr>
<tr>
<td>2</td>
<td>1.80</td>
<td>1.81</td>
</tr>
<tr>
<td>3</td>
<td>1.98</td>
<td>2.01</td>
</tr>
<tr>
<td>4</td>
<td>2.08</td>
<td>2.09</td>
</tr>
<tr>
<td>5</td>
<td>2.12</td>
<td>2.13</td>
</tr>
<tr>
<td>6</td>
<td>2.15</td>
<td>2.16</td>
</tr>
<tr>
<td>7</td>
<td>2.18</td>
<td>2.19</td>
</tr>
<tr>
<td>8</td>
<td>2.19</td>
<td>2.21</td>
</tr>
<tr>
<td>9</td>
<td>2.22</td>
<td>2.23</td>
</tr>
<tr>
<td>10</td>
<td>2.28</td>
<td>2.28</td>
</tr>
</tbody>
</table>

(GM: global mode, LM: local mode, TM: transition mode, Sym.: symmetric, Asym.: asymmetric)

4.2.2 Cable Network on the Fred Hartman Bridge

In this Section, the real cable network on the Fred Hartman Bridge (Caracoglia and Jones, 2005b) is chosen for observing its modal behaviour. The Fred Hartman Bridge is a twin-deck cable-stayed bridge over the Houston ship channel, with a maximum span of 380 m. The bridge deck is supported by 192 cables in four inclined planes with four units in each plane. The ‘‘AS-line’’ unit on the south tower of the Fred Hartman Bridge, as shown in Figure 4.3, will be used to understand its modal behaviour. The physical properties of the consisting cables are provided in Table 4.2. The original configuration of the cable network at the south line of the Fred Hartman Bridge has three lines of cross-ties placed almost symmetric along the longest cable (AS24) in the network. Therefore, the same configuration of cross-tie layout $(\epsilon_{1,1}=\epsilon_{1,2}=\epsilon_{1,3}=1/3)$ is assumed in this part of the study with the cross-tie flexibility parameter being $\psi_1=\psi_2=\psi_3=0.01$. 

118
The modal properties of the first 25 modes of the studied cable network predicted by the proposed analytical model for the general cable network are tabulated in Table 4.3. Figure 4.4 depicts the associated mode shapes of the first ten modes. The non-dimensional fundamental
frequency of the network is $\Omega = 1.43\pi$, indicating a 43% increase in the fundamental frequency of the longest cable (AS24) within the network. Similarly, the fundamental frequency of the other main cables, e.g. AS23 and AS22, is increased by 36% and 32%, respectively. Five of the twelve main cables, i.e. AS24, AS23, AS22, AS21 and AS20, in the studied network are benefited from the cross-tie installation to increase their fundamental frequencies.

The overall view of the network modes presented in Figure 4.4 clearly shows the active participation of all main cables in the first few modes but the dominance by fewer cables and/or cable segments in the higher order modes. This observation suggests that these network modes can be categorized into three different types. The first type is classified as the global modes. They not only exhibit even distribution of energy among different cable segments of the cable network, but the active oscillation of all cable segments, with more or less the same amplitude, also results in a significant increase in their modal mass. Another important feature of the global modes is that the frequencies of the adjacent global modes are relatively far from each other. In the studied cable network, mode 1 to mode 3 are the global modes. The frequency of mode 2 is 60% higher than mode 1, whereas that of mode 3 is 16% higher than mode 2.
Figure 4.4: First ten modes of the cable network on the Fred Hartman Bridge with three lines of cross-ties ($\psi_1 = \psi_2 = \psi_3 = 0.01$) at a position of $\varepsilon_{1,1} = \varepsilon_{1,2} = \varepsilon_{1,3} = 1/3$ (GM: global mode, LM: local mode, TM: transition mode, Sym.: symmetric, Asym.: asymmetric)
Table 4.3: Modal properties of “A-line” cable network at Fred Hartman Bridge with three lines of flexible cross-ties ($\psi_1=\psi_2=\psi_3=0.01$) evenly installed along the target cable ($\varepsilon_{1,2}=\varepsilon_{1,3}=1/3$)

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Modal frequency (Hz)</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical (Proposed)</td>
<td>FEA</td>
</tr>
<tr>
<td>1</td>
<td>0.88</td>
<td>0.91</td>
</tr>
<tr>
<td>2</td>
<td>1.42</td>
<td>1.42</td>
</tr>
<tr>
<td>3</td>
<td>1.65</td>
<td>1.66</td>
</tr>
<tr>
<td>4</td>
<td>1.83</td>
<td>1.87</td>
</tr>
<tr>
<td>5</td>
<td>2.03</td>
<td>2.01</td>
</tr>
<tr>
<td>6</td>
<td>2.09</td>
<td>2.06</td>
</tr>
<tr>
<td>7</td>
<td>2.15</td>
<td>2.10</td>
</tr>
<tr>
<td>8</td>
<td>2.15</td>
<td>2.11</td>
</tr>
<tr>
<td>9</td>
<td>2.16</td>
<td>2.16</td>
</tr>
<tr>
<td>10</td>
<td>2.21</td>
<td>2.17</td>
</tr>
<tr>
<td>11</td>
<td>2.23</td>
<td>2.18</td>
</tr>
<tr>
<td>12</td>
<td>2.26</td>
<td>2.22</td>
</tr>
<tr>
<td>13</td>
<td>2.29</td>
<td>2.24</td>
</tr>
<tr>
<td>14</td>
<td>2.35</td>
<td>2.28</td>
</tr>
<tr>
<td>15</td>
<td>2.38</td>
<td>2.29</td>
</tr>
<tr>
<td>16</td>
<td>2.40</td>
<td>2.32</td>
</tr>
<tr>
<td>17</td>
<td>2.41</td>
<td>2.35</td>
</tr>
<tr>
<td>18</td>
<td>2.41</td>
<td>2.38</td>
</tr>
<tr>
<td>19</td>
<td>2.47</td>
<td>2.41</td>
</tr>
<tr>
<td>20</td>
<td>2.51</td>
<td>2.42</td>
</tr>
<tr>
<td>21</td>
<td>2.68</td>
<td>2.73</td>
</tr>
<tr>
<td>22</td>
<td>2.92</td>
<td>2.99</td>
</tr>
<tr>
<td>23</td>
<td>3.06</td>
<td>3.02</td>
</tr>
<tr>
<td>24</td>
<td>3.22</td>
<td>3.30</td>
</tr>
<tr>
<td>25</td>
<td>3.37</td>
<td>3.36</td>
</tr>
</tbody>
</table>

(GM: global mode, LM: local mode, TM: transition mode, Sym.: symmetric, Asym.: asymmetric)
In the second type of network modes, the kinetic energy of the cable network is confined in certain parts of the system and does not propagate to the rest of the cable network. Therefore, this kind of mode is dominated by localized oscillation and is thus termed as local mode. For example, modes 5 to 10 in Figure 4.4 are all local modes. In modes 5 to 10, only the central segments of the cable network are actively vibrating while their outer counterparts (cable segments near the pylon and the deck side) are almost at rest. The vibration amplitude of the central segments is much larger than those of their outer counterparts. In particular, in mode 10, almost all the cable network segments are at rest except the central segments of three main cables. Unlike the global modes, these local modes have two unique features: kinetic energy in these modes is confined in a few cables and/or cable segments and they occur within a narrow frequency band. For example, confinement of energy in mode 9 and mode 10 can be clearly seen from their mode shapes in Figure 4.4 and the relative frequency difference between them is only 2%. Besides, from the modal frequencies of mode 5 to mode 20 tabulated in Table 4.3, it is observed that the relatively frequency difference between the two adjacent modes is very small and varies from 0.2% to 3%.

The mode(s) lying in between the group of global modes and local modes can be classified as the third type, which are the transition mode(s). The occurrence of transition mode is an indication for the end of either the global or the local modes and the beginning of the other type of modes. The major characteristics of these modes are similar from their predecessors (e.g. global modes) but they exhibit some features of their successors (e.g. local modes). For example, in this particular network example, the first transition mode appears at mode 4. Mode 4 exhibits the major attributes of the global modes, i.e. the modal frequency is relatively far from its adjacent modes and the majority of the cable segments are in oscillation. However, the features
of the local modes in the form of uneven distribution of energy can also be observed by comparing the vibration amplitude of the central segments and their left counterparts. Mode 4 is followed by a group of local modes, i.e. mode 5 to mode 20, as given in Table 4.3. The second group of global modes starts at mode 22 with mode 21 being the second transition mode.

This practical example of a general cable network on a real cable-stayed bridge clearly illustrates the advantages and the disadvantages of the cross-tie solution. The in-plane stiffness of the more vulnerable cables in a constituted cable network can be enhanced considerably, with the maximum in-plane frequency increment of almost 42% in this example network. However, a large number of local modes are formed. In the design of cable networks, an effort should be made to reduce the number of local modes contained in a group/cluster of local modes and its occurrence should be delayed to higher order modes. More details about the formation of local modes will be discussed in Chapter 5.

4.3 Summary

The approach discussed in this chapter can be used to explore the modal behaviour of a general cable network with multiple number of main cables and multiple lines of cross-ties. In addition to a five-cable network, a real twelve-cable cable network on the Fred Hartman Bridge has also been studied. The observations from the modal behaviour of these two cable networks suggest that the use of cross-tie(s) significantly increase the frequency of network global modes. The observed system modes can be categorized into three types. The first type is the global modes where almost an even distribution of energy is present among different cable segments. The unique feature of global modes is that their modal frequencies are relatively far apart from each other. The second category of modes is the local modes, where the kinetic energy of the cable network is confined in certain parts of the network. Unlike to the global modes, the local
modes are closely-spaced over a narrow band of frequency range. In the studied cable networks, the relative frequency difference between the adjacent local modes varies from 0.2% to 3%. The modes lying in between the groups of the global modes and the local modes are classified as the the third type of network modes, which are the transition modes. They are found to have the characteristics of the local modes as well as the global modes.
CHAPTER 5  Formation of Local Modes

Mode localization phenomenon was first predicted by Anderson (1958) in solid-state physics, who shared the 1977 Physics Nobel Prize for his work. Although the theory of mode localization was well understood in solid-state physics, it did not receive enough attention in structural engineering until early 1980s. In structural dynamics, mode localization is defined as a phenomenon of which the kinetic energy induced by structural vibration due to excitation by an external source cannot be propagated a larger distance and is confined to a specific region close to the excitation source. As a result, certain parts of the structure have larger vibration amplitude than the rest. The confinement of energy within specific region of a structure may damage the structure and ultimately shorten its life span. The presence of subsystems with similar geometric and physical properties is the major source of mode localization for a structure. Some of the well known engineering examples/structures exhibiting the phenomenon of mode localization are multi-span beams, bladed-disks in turbomachines (Whitehead, 1966; Yang and Griffin, 1997), space antennae (Ghosh and Ghanem, 2012), cooling towers of nuclear power plants (Kim and Lee, 2000), power transmission line system (Poovarodom and Yamaguchi, 1999).

Theoretically, a serious consequence of mode localization is the absence of regular features of global modes (nodal spacing, regular mode shape) associated with the accumulation of oscillation energy in specific part(s) of a structure which may lead to localized damage and early fatigue failure of the structure. On the other hand, mode localization can be used to estimate fatigue life and peak dynamic stresses of a structure. Mode localization can also be useful in cases where spreading of vibration energy throughout the structure is not desired (Ghosh and Ghanem, 2012).
Ozono and Maeda (1999) conducted a study to explore the effect of support stiffness on the closely-spaced modes of two-span cables. Poovarodom and Yamaguchi (1999) explained the role of flexible supports on the mode localization of in-plane and out-of-plane modes of power transmission line system. In the case of multi-span beams and systems consisting of identical substructures, the theory of mode localization was applied to study the behaviour of structures formed by connecting identical substructures using couplers (Pierre et al., 1987; Pierre and Dowell, 1987; Lust et al., 1993; Kim and Lee, 1998; 2000). In addition to cable-supported structures and multi-span beams, there is also a reasonable amount of work available in literature to study the phenomenon of mode localization in turbine disks (Valero and Bendiksen, 1986), strings (Hodges and Woodhouse, 1985), rods (Luongo, 1992) and space structures (Bendiksen, 1987; Cornwell and Bendiksen, 1987).

5.1 Mode localization in cable-stayed bridges

The built-in nature of cable-stayed bridge makes it ideal candidate for the occurrence of mode localization (Abdel-Ghaffar and Khalifa, 1991). In existing literature very little has been done to characterize the mode localization in cable-stayed bridges. The available ones (e.g. Gattulli and Lepidi, 2007) are mostly related to cable-deck interactions in cable-stayed bridges. To the best knowledge of the author, no study has been done to study the formation of closely-spaced local modes in cross-tied cable networks. It is reported in the literature (Caracoglia and Jones, 2005b; Kumarasena, 2007) that one of the major drawbacks of cross-tied cable network is the formation of closely-spaced local modes and effort should be made to suppress their formation. Therefore, this chapter will be dedicated to understand the formation of closely-spaced local modes in the cross-tied cable networks. It was observed by Caracoglia and Jones, (2005b) that central segments of the main cables in a cross-tied cable network were more
dominant in vibration than their outer counterparts. The studies on mode localization in the case of multi-span beams (Lust et al., 1993; Kim and Lee, 2000) showed that stiffness of coupler/springs (used to couple two adjacent spans of multi-span beams) played an important role in the formation of local modes. These indicate that in order to properly understand the role of central cable segments and stiffness of cross-tie/coupler on the formation of closely-spaced local modes, it is necessary to develop an analytical model with multiple lines of cross-ties in order to explore the role of cross-ties properties on the mode localization of cable networks. In addition, besides identifying the occurrence of local modes, it would also be helpful to quantify the severity of mode localization of a specific mode. Therefore, in Section 5.2, a tool will be introduced to quantify the degree of mode localization of a network mode and discuss the role of different system parameters on the degree of mode localization. Section 5.3 will be dedicated to explore different measures/tools in order to minimize the formation of local modes.

5.2 Degree of mode localization and system parameters

In this section, analytical model of a two-cable network consisting two transverse flexible cross-ties will be developed and a numerical example will be presented to quantify the global and the local nature of the first ten network modes. Figure 5.1 depicts the layout of this cable network.
Figure 5.1: Schematic diagram of two-cable network with two transverse cross-ties

The two-cable network in Figure 5.1 contains six cable segments. All the assumptions, symbols, boundary and compatibility conditions are kept the same as those discussed earlier in Chapter 4 for a general cable network. In the case of in-plane transverse free vibration of a typical cable segment \((i, j)\), the shape function of cable motion can be described by Eq. (4-2). Therefore, a total of twelve shape function constants \(A_{ij}\) and \(B_{ij}\) \((i=1, 2; j=1, 2, 3)\) need to be determined to describe the in-plane transverse free vibration of the entire network. By applying the boundary conditions at two fixed ends of each main cable, four of the twelve shape function constants becomes zeros, i.e. \(A_{ij}=0\) \((i=1, 2; j=1, 3)\). The compatibility conditions at the nodal points and the equilibrium conditions of isolated cross-ties will lead to another eight equations, which can be expressed in the matrix form as

\[
[R]{X}={0}
\]  

(5-1)
where \([R]\) is a square coefficient matrix with a size of 8; 
\([X]=\begin{bmatrix} B_{1,1} & A_{1,2} & B_{1,3} & B_{2,1} & A_{2,2} & B_{2,2} & B_{2,3} \end{bmatrix}^T\) is a vector containing all 8 unknown shape function constants; and \(\{0\}\) is the null vector. The non-trivial solution to Eq. (5-1) can be obtained by setting determinant of \([R]\) to 0. After expanding the determinant and making all the trigonometric simplifications, the following equation can be obtained, i.e.

\[
\begin{align*}
\gamma_1^2 \sin(\Omega \eta_1) \sin(\phi_{2,1}) \sin(\phi_{2,2}) \sin(\phi_{3,2}) &+ \gamma_2^2 \sin(\Omega \eta_2) \sin(\phi_{1,1}) \sin(\phi_{1,2}) \sin(\phi_{1,3}) + \gamma_1 \gamma_2 \sin(\phi_{1,1} + \phi_{1,2}) \sin(\phi_{2,1} + \phi_{2,2}) \sin(\phi_{2,3}) \\
-2 \gamma_1 \gamma_2 \sin(\phi_{1,1}) \sin(\phi_{1,3}) \sin(\phi_{2,1}) \sin(\phi_{2,3}) &+ \gamma_1^2 \Omega \sin(\Omega \eta_1) [\psi_1 \gamma_2 \sin(\phi_{2,1} + \phi_{2,2}) \sin(\phi_{2,3})] \\
+ \psi_1 \gamma_2 \sin(\phi_{2,1} + \phi_{2,2}) \sin(\phi_{1,1}) &+ \gamma_1^2 \Omega^2 \psi_1 \psi_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) = 0
\end{align*}
\] (5-2)

Equation (5-2) is the characteristic equation describing the in-plane transverse free vibration of a cable network consisting of two cables interconnected by two transverse flexible cross-ties.

Three types of terms appear in the equation. The first type, which includes the first two terms on the left hand side of the equation, show the interaction between one of the main cables with the segments of the other cable. Terms 3 to 5 describe the interaction between different segments of the two main cables and belong to the second type. While these two types of terms are independent of the cross-tie stiffness, the third type, from term 6 to term 8, reflects the effect of cross-tie flexibility on the network modal behaviour. In Chapter 3, the system characteristic equation of a two-cable network containing a single transverse flexible cross-tie has been derived, which is

\[
\gamma_1 \sin(\Omega \eta_1) \sin(\phi_3) \sin(\phi_4) + \gamma_2 \sin(\Omega \eta_2) \sin(\phi_1) \sin(\phi_2) + \psi \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) = 0
\] (3-5)

A quick comparison between Eqs. (5-2) and (3-5) clearly reveals that even in the simplest formation of a cable network consisting only two main cables, the addition of an extra cross-tie
would considerably increase the complexity of the system characteristic equation, and thus the network modal behaviour. The numerous Sine terms associated with different cable segments in Eq. (5-2) represent the existence of a large group of local modes dominated by the vibrations of these segments. It is worth noting that $\varphi_{i,j}$ in these Sine terms are defined as $\varphi_{i,j} = \Omega \varepsilon_{i,j}$. This suggests that the frequency ratio of the main cables and the cross-tie position (represented by the segment ratio $\varepsilon_{i,j}$) would control the excitation of local modes. By properly choosing these two parameters, it is possible to “push” the local modes to higher order and reduce the number of local modes excited within the first ten or twenty modes.

A numerical example is presented to explore the mode localization behaviour of a cable network with multiple cross-ties. The two main cables are assumed to have the same properties as type AS24 and AS22 cables on the Fred Hartman Bridge (Caracoglia and Jones 2005b) and interconnected by two transverse flexible cross-ties. The properties of the two cables are

Main Cable 1 (target cable): $H_1=4530$ kN $m_1=76.0$ kg/m $L_1=197.85$ m

Main Cable 2 (neighbouring cable): $H_2=3547$ kN $m_2=70.1$ kg/m $L_2=168.40$ m

The two cables are arranged parallel to each other, with cable 2 has an offset of 7 m on the left end and 22.45 m on the right end with respect to cable 1. The two cross-ties are placed respectively at $0.35L_1$ and $0.60L_1$ from the left end of cable 1. It is assumed that they have the same axial stiffness with the non-dimensional flexibility parameter being $\psi_1=\psi_2=0.1$.

The modal properties of the first ten modes of the studied cable network are given in Table 5.1 and the mode shapes are illustrated in Figure 5.2. Results show that compared to the fundamental frequency of an isolated target cable, which is 0.617 Hz, the formation of the current cable network could help to increase the frequency by 3.9% to 0.641 Hz. Also, as it can be seen from Figure 5.3, besides the global modes, a number of local modes dominated by
certain cable segments are excited. Modes 1, 2 and 3 are all global modes with the two main cables vibrating in-phase or out-of-phase in either symmetric or asymmetric pattern. Mode 4 appears as a transition from the first three global modes to the two subsequent local modes, i.e. modes 5 and 6. While mode 5 is dominated by vibrations of cable 1 and the right segment of cable 2, in mode 6, the oscillation of cable 2 is dominant. The same pattern is repeated for the rest of the four modes, i.e. mode 7 is a global mode, mode 8 and mode 9 are transition modes and mode 10 is a local mode dominated by vibration of cable 2. This kind of modal order pattern is consistent with the site observation (e.g. Caracoglia and Jones, 2005b), of which plateaus of numerous closely spaced local modes exist between the global modes.
Figure 5.2: First ten modes of two-cable network with two lines of flexible cross-ties ($\psi_1=\psi_2=0.1$) at a position of $\varepsilon_1=0.35$ and $\varepsilon_2=0.25$ (GM: global mode, LM: local mode, TM: transition mode, Sym.: symmetric, Asym.: asymmetric)
Table 5.1: Modal properties of a general two-cable network with two lines of flexible cross-ties at a position of $\varepsilon_1=0.35$ and $\varepsilon_2=0.25$ ($\psi_1=\psi_2=0.1$)

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Modal frequency $f$ (Hz)</th>
<th>Mode Shape</th>
<th>DML</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed Analytical Model</td>
<td>FEA</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.641</td>
<td>0.643</td>
<td>GM, Sym., in-phase</td>
</tr>
<tr>
<td>2</td>
<td>1.254</td>
<td>1.258</td>
<td>GM, Asym., in-phase</td>
</tr>
<tr>
<td>3</td>
<td>1.451</td>
<td>1.459</td>
<td>GM, out-of-phase</td>
</tr>
<tr>
<td>4</td>
<td>1.604</td>
<td>1.612</td>
<td>TM</td>
</tr>
<tr>
<td>5</td>
<td>1.931</td>
<td>1.937</td>
<td>LM, Cable 1 dominate</td>
</tr>
<tr>
<td>6</td>
<td>2.035</td>
<td>2.041</td>
<td>LM, Cable 2 dominate</td>
</tr>
<tr>
<td>7</td>
<td>2.544</td>
<td>2.552</td>
<td>GM, Asym., in-phase</td>
</tr>
<tr>
<td>8</td>
<td>2.979</td>
<td>2.989</td>
<td>TM</td>
</tr>
<tr>
<td>9</td>
<td>3.195</td>
<td>3.207</td>
<td>TM</td>
</tr>
<tr>
<td>10</td>
<td>3.431</td>
<td>3.445</td>
<td>LM, Cable 2 dominate</td>
</tr>
</tbody>
</table>

To validate the proposed analytical model and modal analysis results, an independent FEA simulation is conducted using the commercial software Abaqus 6.10 (2010). In the finite element model, the B21 beam element and the SPRING2 element from the Abaqus element library are chosen to simulate the main cables and the cross-ties, respectively. The initial axial stress is introduced in the B21 beam element to model the pretension in the main cable. The modal responses of the studied cable network obtained from the numerical simulation are also listed in Table 5.1, which agree well with the analytical results from the proposed analytical model.
5.2.1 Degree of mode localization

The phenomenon of local modes has been reported and discussed in different studies and technical reports (Caracoglia and Jones, 2005b; Kumarasena et al., 2007), the tools to quantitatively measure the degree of mode localization (DML) of a specific cable network mode is still lacking. Kim and Lee (2000) introduced such tool during the study of mode localization in multi-span beams in order to quantify the “global” or “local” nature of different modes. In the current study, this coefficient will be used to evaluate the degree of localization of a specific cable network mode. It is defined as

\[
DML = \frac{n_c(n_t+1) - n_v}{n_c(n_t+1) - 1} \tag{5-3}
\]

where \(n_c\) and \(n_t\) are respectively the number of main cables and number of cross-ties in the cable network. Therefore, \(n_c(n_t + 1)\) is the total number of cable segments in the network and \(n_v\) denotes the number of vibrating cable segments in the studied cable network mode, which can be determined from

\[
n_v = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^{n_t+1} \bar{v}_{i,j}^2}{\sum_{i=1}^{n_c} \sum_{j=1}^{n_t+1} \bar{v}_{i,j}^2} \tag{5-4}
\]

where \(\bar{v}_{i,j}\) is the absolute value of the maximum vibration amplitude of cable segment \((i, j)\) in the studied mode. The definition of DML given in Eq. (5-3) suggests that in the case of global mode of which all cable segments are vibrating with the same amplitude, DML=0; whereas for an extremely localized mode within which only one cable segment oscillates, DML=1. Therefore, the closer the DML coefficient approaches to 1, the more “local” the mode is. By revisiting the numerical example presented before in this section, the DML values of the first ten cable network modes are also given in Table 5.1. The three types of modes can be clearly identified from their respective DML coefficient. For the global mode (modes 1, 2, 3 and 7), DML is no
more than 0.1; for the transition mode (modes 4, 8 and 9), DML is roughly around 0.2; whereas for the local mode (modes 5, 6 and 10), DML is always higher than 0.3.

5.2.2 Role of system parameters on degree of mode localization

To evaluate the impact of cross-tie properties, i.e. stiffness, position and number of cross-ties, on the mode localization of a cable network, a parametric study will be conducted in this section. Three cable networks will be studied, the networks consist of two main cables and respectively one, two and three transverse flexible cross-ties to connect the target and the neighbouring cables. The properties and geometric layout of the two main cables are the same as those used in the previous numerical example. The cross-tie(s) in these three networks are assumed to space evenly along the target cable, as depicted in Figure 5.3.
It can be observed from Eq. (5-2) that the frequency ratio $\eta$, the mass-tension ratio $\gamma$, the non-dimensional cross-tie flexibility parameter $\psi$, and the segment ratio $\varepsilon$ (in $\emptyset_{i,j} = \Omega_i \varepsilon_{i,j}$) are the key system parameters governing the cable network behaviour. When more than one cross-tie is used in a cable network, such as Networks B and C in Figure 5.3, it is challenging to vary the position of different cross-ties simultaneously in the study. Therefore, the parametric study...
presented below would be conducted for the first three system parameters, focusing on how the addition of an extra cross-tie would affect modal behaviour of two-cable networks with different system properties.

### 5.2.2.1 Frequency ratio

The frequency ratio parameter, \( \eta_i = f_1/f_i \), represents the flexibility of the target cable versus that of the \( i^{th} \) cable in the network. If the \( i^{th} \) cable is stiffer than the target cable, then \( \eta_i < 1 \). To isolate the effect of the frequency ratio on the network dynamic response, free vibration analysis of Networks A, B and C is performed by varying the frequency ratio \( \eta_2 \) from 0 to 1.0 while keeping the rest of the system parameters unchanged. The two limiting values of \( \eta_2 = 0 \) and 1 represent respectively the special cases of a rigid neighbouring cable and a neighbouring cable having the same frequency as that of the target cable.

The impact of the frequency ratio on the fundamental frequency and degree of mode localization of the three cable networks are shown in Figures 5.4(a) and 5.4(b), with the cross-ties being either very rigid (\( \psi=0.01 \)) or flexible (\( \psi=1.0 \)).
Figure 5.4(a): Non-dimensional fundamental frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Networks A, B and C with non-dimensional cross-tie flexibility parameter $\Psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.4(b): Non-dimensional fundamental frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Networks A, B and C with non-dimensional cross-tie flexibility parameter $\Psi=1.0$ and mass-tension ratio parameter $\gamma_2=0.85$. 

139
It can be observed from Figure 5.4(a) that the fundamental frequency of all three networks decreases monotonically with larger frequency ratio. A more sizable drop of fundamental frequency occurs if more cross-ties are present in a network. For example, by increasing $\eta_2$ from 0.2 to 0.4, the fundamental frequency of Networks A, B and C reduces by 13%, 20% and 23%, respectively. An increase of $\eta_2$ corresponds to a more flexible neighbouring cable, as if the target cable is connected to softer foundation through cross-ties. Thus, the stiffening effect provided by the cross-tie solution would be less, and such a reduction would be more obvious if the network consists of more of such cross-ties. A comparison of the three $\Omega_1/\pi$ vs $\eta_2$ curves in Figure 5.4(a) suggests that depending on the number of cross-ties in the original cable network, the addition of an extra cross-tie would result in different stiffness enhancing effect. Take the case of $\eta_2 = 0.4$ as an example, by introducing a second cross-tie (Network B), the fundamental frequency of Network A increases by 8% from $1.38\pi$ to $1.49\pi$ in, whereas when a third cross-tie (network C) is added the frequency increased by 3% from $1.49\pi$ to $1.54\pi$. These results imply that the stiffening effect is not cumulative and it gradually decays if a number of cross-ties is reached in the original system. These observations are consistent with the findings reported by Bosch and Park (2005). In the extreme case of $\eta_2 = 1.0$, i.e. when the two cables have the same frequency, the in-plane stiffness of the network will not be benefitted from adding more lines of cross-ties. Similar phenomena can be seen from Figure 5.4(b) when more flexible cross-ties are used in these three networks, except the increment of the network fundamental frequency is much less.

The results yielded from the DML analysis suggest that when very rigid cross-ties are used (Figure 5.4(a)), the fundamental mode of Network A remains global over the entire studied frequency range. In the case of Networks B and C, by increasing the frequency ratio, the stiffness
of the neighbouring cable gradually decreases and approaches to that of the target one and thus renders the energy distribution more even between the different cable segments within the cable network. Therefore, the mode globalization will be increased until it becomes a pure global mode at \( \eta_2 = 1.0 \). When flexible cross-ties are used, however, as indicated by the DML results shown in Figure 5.4(b), the decrease of frequency ratio would have a more considerable influence on mode localization, in particular, when fewer lines of cross-ties are used in the cable network. Therefore, when the neighbouring cable is stiffer, using stiffer but less lines of cross-ties, or flexible but more lines of cross-ties would ensure a more even energy distribution among different cable segments and reduce localized oscillation in the network fundamental mode.

Figure 5.5(a): Non-dimensional frequency, \( \Omega/\pi \), and its degree of mode localization, as a function of frequency ratio parameter for Modes 2, 3 and 4 in Network A with non-dimensional flexibility parameter \( \Psi=0.01 \) and mass-tension ratio parameter \( \gamma_2=0.85 \)
Figure 5.5(b): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Modes 2, 3 and 4 in Network B with non-dimensional flexibility parameter $\Psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$.

Figure 5.5(c): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of frequency ratio parameter for Modes 2, 3 and 4 in Network C with non-dimensional flexibility parameter $\Psi=0.01$ and mass-tension ratio parameter $\gamma_2=0.85$. 

142
Figures 5.5(a), 5.5(b) and 5.5(c) depict the influence of frequency ratio on the modal frequency and degree of mode localization of Mode 2 to Mode 4 in Networks A, B and C, respectively. The stiffness of cross-ties in all three cases is assumed to be very rigid, i.e. $\psi=0.01$, to better reflect the frequency ratio effect. An interesting mode cross-over phenomenon has been observed, of which at certain frequency ratio, a pair of a lower order local mode and a higher order global mode would switch their order. Similar kind of phenomenon was reported earlier by Irvine and Caughey (1974) when studying the modal behaviour of a horizontally suspended elastic cable having small sag. Take Network A as an example, when a rigid neighbouring cable is present in the network, Modes 2 and 4 are the local modes with DML of 0.67 for both cases, and Mode 3 is a pure global mode with its DML approximately equals to 0. With a gradual increase of the frequency ratio, while the frequency of Mode 3 decreases, whereas Mode 2 is not affected. When the frequency ratio reaches $\eta_2 = 0.90$, the frequency of these two modes becomes the same. Further increase of $\eta_2$ makes the frequency of Mode 3 lower than that of Mode 2, and thus switches their original modal order so that Mode 2 becomes a more global mode while Mode 3 is now a local mode. This is also reflected in the pattern of their respective DML variation. When $\eta_2 < 0.90$, the DML of Mode 2 remains at 0.67. It then drops suddenly to 0.38 at $\eta_2 = 0.90$, implying that at this frequency ratio, Mode 2 changes from a pure local mode to a more global mode. Whereas for Mode 3, its DML remains roughly zero till $\eta_2 = 0.46$, and then gradually increases with the frequency ratio, indicating the energy distribution among different cable segments in Mode 3 becomes less even and more localized oscillation appears. This phenomenon lasts until the frequency ratio reaches 0.90, when the mode is dominated by vibrations of the target cable and becomes a pure local mode. Figure 5.6 illustrates the change in the shape of Mode 2 and Mode 3 of Network A when the frequency ratio takes the values of $\eta_2$. 
=0.85, 0.90 and 0.92, respectively. The impact of frequency ratio on Mode 4 is similar to that of Mode 2. It remains as a pure local mode until a mode cross-over occurs at \( \eta_2 = 0.44 \), when Mode 4 switches its order with Mode 5 and becomes a more globalized mode. The modal frequency then decreases monotonically with the increase of frequency ratio. Correspondingly, the magnitude of DML associated with Mode 4 drops suddenly from 0.67 to 0.35 at this frequency ratio.

![Diagram](image)

**Figure 5.6:** Mode cross-over behaviour of Mode 2 and Mode 3 in Network A with cross-tie non-dimensional flexibility parameter \( \Psi = 0.01 \) and mass-tension ratio parameter \( \gamma_2 = 0.85 \)

The same mode cross-over phenomenon can also be observed from Figs. 5.5(b) and 5.5(c). In Network B (Figure 5.5(b)), it occurs between Mode 3 (local mode dominated by target cable) and Mode 4 (global mode) at frequency ratio of 0.81, when DML of Mode 3 drops from 0.6 to 0.42, and that of Mode 4 jumps from 0.42 to 0.6. Further, the original Mode 3, which now becomes Mode 4 at \( \eta_2 = 0.81 \), switches its order again with Mode 5 (not shown in the figure) at \( \eta_2 = 0.94 \) with more evenly distributed energy within the system and thus increased mode
globalization, as indicated by a reduction of DML from 0.6 to 0.52. In Network C (Figure 5.5(c)), Mode 2 and Mode 3 are both global modes and the impact of the frequency ratio on their frequency and DML are similar to Mode 1 (Figure 5.4(a)). At \( \eta_2 = 0.77 \), mode cross-over occurs between Modes 4 and 5 (Mode 5 is not shown in the figure), which makes Mode 4 to change from a local mode to a more global one. Also, the results in Figure 5.5 show that the mode cross-over phenomenon starts to appear respectively in Mode 2, Mode 3 and Mode 4 for Networks A, B and C. This could be due to the fact that when more cross-ties are used in a cable network, cable segments tend to be shorter and thus stiffer, so localized oscillation is excited at higher frequency.

5.2.2 Mass-tension ratio

In this section, the influence of the mass-tension ratio parameter on the degree of mode localization of cable networks having multiple cross-ties will be explored. The modal responses of Networks A, B and C are analyzed by keeping the frequency ratio \( \eta_2 = 0.92 \), the non-dimensional cross-tie flexibility parameter \( \psi = 0.01 \), while varying the mass-tension ratio parameter \( \gamma_2 \) from 0.4 to 1.2. This \( \gamma_2 \) range is deduced from the cable-stayed bridge database compiled by Tabatabai et al., (1998).
Figure 5.7: Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Networks A, B and C with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility $\Psi=0.01$

Figure 5.7 shows how the fundamental frequency and DML of the studied networks are affected by the mass-tension ratio parameter. In all three cases, a monotonic increase of the network fundamental frequency with respect to the mass-tension ratio parameter is observed. Over the entire studied range, the increase of the fundamental frequency is approximately 2% regardless the number of cross-ties used to interconnect the two cables. In all three networks, the magnitude of DML indicates that the fundamental mode remains as a pure global mode, so the energy distribution among different cable segments is even and not affected by the variation of the mass-tension ratio parameter. For Modes 2, 3 and 4, the mass-tension ratio effect on the modal frequency and the corresponding DML for Networks A to C is illustrated in Figs. 5.8(a) to 5.8(c), respectively. In the case of Network A, the mass-tension ratio $\gamma_2=0.40$, all the three modes are local, as reflected by the DML values of 0.69, 0.67 and 0.59, respectively.
Figure 5.8(a): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Modes 2, 3 and 4 in Network A with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility parameter $\Psi=0.01$

Figure 5.8(b): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Modes 2, 3 and 4 in Network B with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility parameter $\Psi=0.01$
Figure 5.8(c): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of mass-tension ratio parameter for Modes 2, 3 and 4 in Network C with frequency ratio $\eta_2=0.92$ and cross-tie non-dimensional flexibility parameter $\Psi=0.01$

It is observed from their mode shapes that the vibrations in Modes 2 and 4 are dominated respectively by the left and the right cable segments, whereas by the target cable in Mode 3. The gradual increase of mass-tension ratio, though hardly affects the frequencies associated with these three modes, seems to considerably change the energy distribution in Modes 2 and 4 and make it more even over all the cable segments. Whereas for Mode 3, energy remains to be confined only in the target cable. Similar phenomenon can be seen in Mode 3 and Mode 4 of Network B from Figure 5.8(b) and Mode 4 of Network C from Figure 5.8(c). In addition, Mode 2 in Network B, as well as Modes 2 and 3 in Network C, are pure global modes. They are subjected to the same mass-tension ratio effect as that of the fundamental mode (Figure 5.7). It can be concluded from Figs. 5.7 and 5.8 that regardless of the cross-tie number in a network, the mass-tension ratio has negligible effect on the modal properties of network global modes, except a slight variation of the fundamental frequency. If a local mode is dominated by oscillations of
certain segment(s) of the main cable(s), increasing the mass-tension ratio would contribute to a more even energy distribution and thus globalization of the mode without sacrificing the network in-plane stiffness. However, if a local mode is dominated by vibrations of a single main cable, the modal response would be independent of this parameter.

5.2.2.3 Cross-tie stiffness

To study the influence of cross-tie stiffness on the modal behaviour of Networks A to C, the non-dimensional cross-tie flexibility parameter $\psi$ is varied from 0 to 1.0 (Caracoglia and Jones, 2005a), representing transition from rigid to flexible cross-tie condition. The frequency ratio and the mass-tension ratio of these three networks are maintained at $\eta_2 = 0.92$ and $\gamma_2 = 0.85$.

The results shown in Figure 5.9 describe how the network fundamental mode is affected by the cross-tie stiffness. It can be clearly seen from the figure that using more number of stiffer cross-tie is beneficial in increasing the network fundamental frequency. This is consistent with the earlier observation in the frequency ratio effect. The variation of the DML associated with the fundamental mode of Networks A to C suggests that if the cross-ties are relatively flexible, adding an extra cross-tie would not only help to further enhance the network in-plane stiffness, but has also the advantage to promote the globalization of the mode by distributing energy more evenly within the network.

The cross-tie stiffness effect on Modes 2, 3 and 4 of Networks A, B and C is depicted respectively in the three subplots of Figure 5.10. Overall, the variation of cross-tie stiffness seems to have a more significant impact on these three modes than on the fundamental mode. In particular, the way how the energy is distributed among the different cable segments in Networks B and C is highly dependent on the cross-tie stiffness, as can be seen from Figs. 5.10 (b) and (c). In the case of Network A in Figure 5.10(a), when a rigid cross-tie is used ($\psi = 0$), Modes 2, 3 and...
4 are all local. Mode 2 is dominated by vibrations of the left cable segments in the network, whereas Mode 4 by the right cable segments. Although in Mode 2, with the decrease of cross-tie stiffness (or increase of $\psi$), the energy contained in the left cable segments of the network gradually transfers to the right cable segments such that the local mode evolves to an out-of-phase global mode at $\psi=0.1$, in the case of Mode 4, such a change in the cross-tie stiffness renders the network frequency become the same as that of the second mode of an isolated neighbouring cable. Thus, Mode 4 remains a local mode when $\psi\geq0.1$ but now dominated by the oscillations of the neighbouring cable. However, the modal property of Mode 3, represented in terms of its frequency and DML, seems to be independent of the cross-tie stiffness. It remains as a network local mode governed by the isolated target cable vibrating in its second mode over the entire studied cross-tie stiffness range.

Figure 5.9: Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Networks A, B and C with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$.
Figure 5.10(a): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Modes 2, 3 and 4 in Network A with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$

Figure 5.10(b): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Modes 2, 3 and 4 in Network B with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$
Figure 5.10(c): Non-dimensional frequency, $\Omega/\pi$, and its degree of mode localization, as a function of cross-tie non-dimensional flexibility parameter for Modes 2, 3 and 4 in Network C with frequency ratio $\eta_2=0.92$ and mass-tension ratio $\gamma_2=0.85$

In Figs. 5.10(b) and (c), localized peaks are observed in the DML curves, implying that the energy is confined to limited parts of the cable network at certain cross-tie stiffness, and resulted in localized oscillations in these particular regions. Take Mode 2 of Network B as an example, when $\psi < 0.11$, it is a pure global mode with the non-dimensional modal frequency at $2.06\pi$ and DML value close to 0. As $\psi$ increases to 0.14, the frequency of this mode reduces slightly to $2\pi$, which agrees with the second modal frequency of the isolated target cable, the mode shape is anti-symmetric with a node at the mid-span. However, since the target cable in Network B is constrained by the two cross-ties at 1/3 and 2/3 of the span (Figure 5.3), the excitation of this target-cable-dominated local mode is prohibited. Instead, the energy contained in the left part of the target cable seems to be transferred to the left part of the neighbouring cable through the cross-ties, so this local mode is “reshaped” in such a way that its motion is now
dominated by the two right segments of the target cable and the two left segments of the neighbouring cable, as shown in Figure 5.11. A further increase of $\psi$ renders the network frequency to deviate from this local mode and thus Mode 2 of the network becomes a global mode again. These changes of energy distribution pattern associated with Mode 2 is clearly reflected by the localized peak in the corresponding DML curve in the vicinity of $\psi = 0.14$. It is formed by a sudden jump of DML from 0.06 at $\psi = 0.11$ to 0.38 at $\psi = 0.14$, followed by a sharp dip to 0.14 at $\psi = 0.20$. The same “local mode reshape” phenomenon is also observed in the third mode of Network B at $\psi = 0.11$, when the change in the cross-tie stiffness resulted in a network frequency approach to $2.15\pi$, i.e. the second mode of an isolated target cable. The corresponding DML jumps to 0.35. In the case of Network C, the local mode “reshape” phenomenon occurs at $\psi = 0.23$ in Mode 2, $\psi = 0.19$ in Mode 3, and $\psi = 0.11$ in Mode 4. These $\psi$ values yield network frequencies close respectively to the second mode of an isolated target cable, the second mode of an isolated neighbouring cable, and the third mode of an isolated target cable.

![Mode-2](image)

Figure 5.11: Mode “reshaping” of Mode 2 in Network B with frequency ratio parameter $\eta_2=0.92$ and mass-tension ratio parameter $\gamma_2=0.85
The above observations indicate that although the use of softer cross-ties would reduce the network in-plane stiffness, it is beneficial for an even distribution of system energy and thus globalization of the network modes, in particular, when more number of softer cross-ties are used. Further, it is important to ensure that in the cross-tie design, the selected cross-tie stiffness should avoid to yield network frequencies in the proximity of the isolated cable frequencies. Otherwise, highly localized oscillation would govern the network response.

5.3 Local mode clusters

In the previous section, the role of cross-tie properties, along with a few other system parameters, on the degree of mode localization of a specific mode has been discussed. In a cable network, not only numerous local modes are excited but there also exists clusters of closely-spaced local modes which need to be suppressed. The modal behaviour of the original and the modified cable networks in the central and side spans of the Fred Hartman Bridge (Caracoglia and Jones, 2005b) suggests that the stiffness and the position of cross-tie may affect the position and size of local modes clusters.

Bosch and Park (2005) numerically simulated the response of a group of stay cables on the Bill Emerson Memorial Bridge with different cross-tie layouts. Results showed that the effectiveness of cross-tie solution was dependent on the deployment geometry, the quantity, the size and the anchorage condition of cross-ties. Although occurrence of mode localization phenomenon and existence of cluster of local modes were reported in these few studies, no further research has been conducted to eliminate/minimize the formation of local modes. Therefore, this section will be dedicated to gain deeper insight of clusters of local modes by developing a tool to measure the severity of local mode cluster (LMC). The effects of the position, the stiffness and the number of cross-ties on the fundamental frequency and local mode
excitation of a cable network will be extensively explored. It is worth mentioning that in the current study, it is assumed that each cross-tie line would connect all the main cables in the network. To properly identify a cluster of closely spaced local modes, it is necessary to establish the associated criteria.

The modal properties of cable networks on the Fred Hartman Bridge are well documented by Caracoglia and Jones (2005b) and Kumarasena et al. (2007). By evaluating the modal frequencies of the first five modes in the first group of closely spaced local modes of the central-span cable network in the former (2005b), it is found that the frequency difference between the two adjacent modes varies from 0.55% to 3.93%. In the latter study (Kumarasena et al., 2007), the modal frequencies of all twenty-six narrowly spaced local modes (mode 4 to mode 29 inclusively) were reported. A frequency difference of 0.2% to 3.28% between the two consecutive modes can be noted. However, with the exception of these two sets, modal frequencies of local modes in cable networks are rarely available in literature. Based on these reports, the maximum relative frequency difference between two adjacent modes in a local mode cluster is less than 4%.

Meanwhile, it is to be recognized that defining LMC solely based on the relative frequency difference between two adjacent modes may not be adequate. Other possible components of the LMC criteria could be associated with modal properties such as generalized modal mass, similarity in mode shapes and the degree of mode localization (evaluate the extent of how global or local a network mode is). When exploring the possibility of including generalized modal mass as one possible component of the LMC criteria, it was found that the two adjacent modes having the same or very closely spaced frequencies do not necessarily have similar generalized modal masses. For example, in the case of an idealized twin cable network,
assuming the two identical cables interconnected by a rigid transverse cross-tie at 1/3 span. At a frequency of $3\pi$, there are three co-existing modes, i.e. a symmetric global mode, two local modes dominated respectively by the oscillations of the left or the right cable segments as reported. However, the generalized modal masses are very different for these three modes despite that they have exactly the same modal frequencies. For the same reason, it would not be appropriate to include similar pattern of mode shapes as part of the LMC criteria.

The concept of degree of mode localization (DML) to measure the global or local nature of specific mode is already proposed in Section 5.2.1. Based on this definition, the DML value for any mode varies from 0 (pure global mode of which the distribution of modal amplitudes is the same for all the segments in the cable network) to 1.0 (100% local mode of which energy is confined in one of the segments in the cable network). The analysis conducted in Section 5.2.1 suggested that any mode had a DML value higher than or equal to 0.30 could be considered a local mode. Thus, it seems to be reasonable to include DML value of a network mode in the LMC identification.

Therefore, the formation of a local mode cluster (LMC) is proposed to be defined as a combination of two criteria: (i) Three or more consecutive modes with DML coefficient higher than or equal to 0.30; and (ii) The relative frequency difference between any adjacent two modes is no more than 3%. The modal number of the first mode in a cluster is defined as the position of the LMC, whereas the total number of local modes within a cluster is defined as the size of the LMC. These two criteria will be adopted in the current study for identifying the presence of LMCs. If only two consecutive modes can satisfy the requirement of relative frequency difference and LMC value, they are defined as local mode pair.
5.3.1 Cross-tie position effect

The installation location of cross-ties, represented by the segment ratio $\varepsilon$, is an important design parameter for a cable network. This parameter would not only affect the network fundamental frequency, but also influence the position of LMCs (Caracoglia and Jones, 2005b). The effect of cross-tie position on the modal behaviour of cable networks will be investigated using two different configurations. In both cases, the networks consist of five main cables with the same properties and layout as those in the numerical example in Section 4.2.1 except Configuration A has one line of cross-ties, whereas Configuration B has two lines. The sample layout of cable network Configuration A is shown in Figure 5.12(a).

(a) Layout of Configuration A ($\varepsilon_{1,1}=0.1, 0.3, 0.5, 0.65$ for Network A1, A2, A3 and A4, respectively)
In Configuration A, a single line of cross-ties is assumed to be installed transverse to the main cables at respectively four different locations, i.e. $\varepsilon_{3,1} = 0.10$, 0.30, 0.50 and 0.65, where $\varepsilon_{3,1}$ is the segment ratio representing the position of the cross-tie on the target cable from its left support. The corresponding networks are referred to as Network A1, A2, A3 and A4. The flexibility parameter of the cross-ties are taken as $\psi = 0.01$. It is worth mentioning that when $\varepsilon_{3,1}$
= 0.65, the position of the cross-tie line is very close to the right support of the shortest cable (main cable 5) in the network. The modal properties of the first twenty modes of these four networks are analyzed using the proposed analytical model. The non-dimensional modal frequencies are listed in Table 5.2, and plotted against the mode number in Figure 5.13.
Table 5.2: Non-dimensional modal frequencies of cable networks A1 to A4  
(Single line of cross-ties, $\psi=0.01$)

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Network A1 ($\varepsilon_{1,1}=0.10$)</th>
<th>Network A2 ($\varepsilon_{1,1}=0.30$)</th>
<th>Network A3 ($\varepsilon_{1,1}=0.50$)</th>
<th>Network A4 ($\varepsilon_{1,1}=0.65$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.054</td>
<td>1.114</td>
<td>1.16</td>
<td>1.267</td>
</tr>
<tr>
<td>2</td>
<td>1.132</td>
<td>1.462</td>
<td>1.848 *</td>
<td>1.435 *</td>
</tr>
<tr>
<td>3</td>
<td>1.269</td>
<td>1.643</td>
<td>1.901 *</td>
<td>1.464 *</td>
</tr>
<tr>
<td>4</td>
<td>1.447</td>
<td>1.930</td>
<td>1.917 *</td>
<td>1.495 *</td>
</tr>
<tr>
<td>5</td>
<td>1.559</td>
<td>2.245</td>
<td>1.998 $\Delta$</td>
<td>1.531 *</td>
</tr>
<tr>
<td>6</td>
<td>2.115</td>
<td>2.477</td>
<td>2.000 $\Delta$</td>
<td>2.322</td>
</tr>
<tr>
<td>7</td>
<td>2.264</td>
<td>2.861</td>
<td>2.196</td>
<td>2.835 *</td>
</tr>
<tr>
<td>8</td>
<td>2.533</td>
<td>3.052 *</td>
<td>2.646</td>
<td>2.877 *</td>
</tr>
<tr>
<td>9</td>
<td>2.872</td>
<td>3.124 *</td>
<td>3.145</td>
<td>2.929 *</td>
</tr>
<tr>
<td>10</td>
<td>3.099</td>
<td>3.211 *</td>
<td>3.640</td>
<td>3.024 $\Delta$</td>
</tr>
<tr>
<td>11</td>
<td>3.214</td>
<td>3.234 *</td>
<td>3.802 *</td>
<td>3.061 $\Delta$</td>
</tr>
<tr>
<td>12</td>
<td>3.399</td>
<td>3.411</td>
<td>3.813 *</td>
<td>3.368</td>
</tr>
<tr>
<td>13</td>
<td>3.788</td>
<td>3.557</td>
<td>3.850 *</td>
<td>4.049</td>
</tr>
<tr>
<td>14</td>
<td>4.169</td>
<td>4.087</td>
<td>4.001 $\Delta$</td>
<td>4.307 *</td>
</tr>
<tr>
<td>15</td>
<td>4.391</td>
<td>4.219</td>
<td>4.029 $\Delta$</td>
<td>4.384 *</td>
</tr>
<tr>
<td>16</td>
<td>4.536 $\Delta$</td>
<td>4.474</td>
<td>4.390</td>
<td>4.485 *</td>
</tr>
<tr>
<td>17</td>
<td>4.672 $\Delta$</td>
<td>4.707</td>
<td>4.776</td>
<td>4.585</td>
</tr>
<tr>
<td>18</td>
<td>5.026</td>
<td>5.095</td>
<td>5.259</td>
<td>4.927</td>
</tr>
<tr>
<td>19</td>
<td>5.324</td>
<td>5.442</td>
<td>5.633</td>
<td>5.379</td>
</tr>
<tr>
<td>20</td>
<td>5.619</td>
<td>5.754</td>
<td>5.691</td>
<td>5.709</td>
</tr>
</tbody>
</table>

* Local mode in a LMC.  
$\Delta$ Local mode in a closely-spaced pair.
Figure 5.13: Effect of cross-tie position on the modal frequency of Networks A1 to A4 (Single line of cross-ties, $\psi=0.01$)

(a) Modes 1-10

(b) Modes 11-20
The results show that as $\varepsilon_{1,1}$ increases, the network fundamental frequency increases as well. Compared to the fundamental frequency of a single isolated target cable, the frequency in Network A1 ($\varepsilon_{1,1} = 0.10$) increased by 5.4% ($\Omega=1.054$), while that in Network A4 ($\varepsilon_{1,1} = 0.65$) increased by 26.7% ($\Omega=1.267$), implying that as the cross-ties move closer to the mid-span of the main cables, the in-plane stiffness of the cable network can be further strengthened. Based on the proposed criteria for LMC, it can be seen from Table 5.2 that when $\varepsilon_{1,1} = 0.10$, no LMC is formed within the first twenty modes, although there is a pair of closely-spaced local modes, i.e. modes 16 and 17. A LMC appears at mode 8 for $\varepsilon_{1,1} = 0.30$, with a size of 4. When the cross-ties are located at the center of the target cable ($\varepsilon_{1,1}= 0.50$), besides two LMCs, three pairs of closely-spaced local modes are also excited, which are modes 5 and 6, modes 14 and 15, and modes 19 and 20. The two LMCs appear respectively at mode 2 and mode 11, both have a size of 3. By another “push” of cross-tie position to $\varepsilon_{1,1} = 0.65$, a total of 3 LMCs are identified within the first twenty modes, along with a pair of closely spaced local modes consisting modes 10 and 11. The positions of these three LMCs are mode 2, mode 7, and mode 14, respectively, with a size of 4, 3 and 3. The LMCs of networks A2 to A4 can be clearly observed in Figure 5.14 in terms of plateaus on the $\Omega$-mode number curves. These observations indicate that by moving cross-ties towards mid-span of the target cable, i.e. increasing $\varepsilon_{1,1}$, although the in-plane stiffness of the network can be improved, it is at the expense of advancing LMC formation and increasing its size and number. A compromise between the gain and drawback suggests that among the four studied cross-tie positions, $\varepsilon_{1,1} = 0.30$ would be the optimum choice. It would not only increase the network fundamental frequency by 11.4% ($\Omega_1=1.114$), but also keep the excitation of LMC reasonable. Within the first twenty modes, only one LMC of size 4 appeared at mode 8.
To have a better picture on how the cross-tie position would affect the modal behaviour of a network with Configuration A, a modal analysis was conducted by gradually changing the cross-tie position from $\varepsilon_{1,1} = 0.10$ to $\varepsilon_{1,1} = 0.65$. The variation of the network fundamental frequency and the position of the first LMC are plotted against the cross-tie position in Figure 5.14. It can be seen that as the cross-tie moves towards the cable mid-span, the network fundamental frequency increases monotonically. However, the position of the first LMC manifests a very different pattern. Though overall, by placing cross-ties closer to the target cable mid-span would advance the appearance of the first LMC to lower order mode, when $\varepsilon_{1,1} < 0.49$, the position of the first LMC could vary considerably with a slight change in the cross-tie location. Within a cross-tie position range of $\varepsilon_{1,1} = 0.10$ to 0.17 and at $\varepsilon_{1,1} = 0.26, 0.27$ and 0.29, there is no formation of LMC within the first twenty modes. This is believed to be associated with the breakage of the formed LMC at these few cross-tie positions, of which the frequency
difference between certain consecutive modes in the original LMC exceeds 3%. It is interesting to note that once $\varepsilon_{1,1}$ reaches 0.49, the first LMC will appear at the second mode of the network and remains there with further increase of $\varepsilon_{1,1}$. This observation is consistent with earlier findings reported by Caracoglia and Jones (2005b) that local modes would be excited if cross-ties are placed at the mid-span of the target cable.

The modal behaviour of five cable networks in Configuration B is also studied. All of them have two lines of cross-ties. It was indicated by Caracoglia and Jones (2005a) that the typical range of the cross-tie non-dimensional flexibility parameter $\psi$ is between 0.01 and 1, a transition corresponding to close to rigid connector and soft cross-tie, respectively. In the current example, the stiffness of all the cross-ties is taken as $\psi = 0.01$. It is assumed that the position of these two cross-tie lines would evenly divide one of the main cables in each case, and the so obtained five networks are named Network B1 to B5, respectively. For example, in Network B2, the two cross-tie lines are installed evenly along the main cable 2 as illustrated in Figure 5.12(b). Therefore, the location of the two cross-tie lines in these five cable networks can be defined by their positions on main cable 1, i.e. $\varepsilon_{1,1}$ and $\varepsilon_{1,2}$, as follows

Network B1: $\varepsilon_{1,1}=1/3$ $\varepsilon_{1,2}=1/3$
Network B2: $\varepsilon_{1,1}=0.31$ $\varepsilon_{1,2}=0.30$
Network B3: $\varepsilon_{1,1}=0.28$ $\varepsilon_{1,2}=0.27$
Network B4: $\varepsilon_{1,1}=0.27$ $\varepsilon_{1,2}=0.24$
Network B5: $\varepsilon_{1,1}=0.24$ $\varepsilon_{1,2}=0.22$

The modal analysis results of networks B1 to B5 are illustrated in Figure 5.15, where the frequencies of the first twenty modes are plotted against the mode number.
Figure 5.15: Effect of cross-ties position on the modal frequency of Network B1 to B5 (two lines of cross-ties, $\psi=0.01$)

In addition, for a more convenient comparison, the fundamental network frequency, the characteristics of the LMCs and the number of closely-spaced local mode pair in these five networks are summarized in Table 5.3.

Table 5.3: Summary of local mode cluster and pairs of local modes for cable networks B1 to B5

<table>
<thead>
<tr>
<th>Network</th>
<th>$\Omega_1$</th>
<th>1st LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>1.336</td>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>B2</td>
<td>1.217</td>
<td>5</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>B3</td>
<td>1.180</td>
<td>6</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>B4</td>
<td>1.165</td>
<td>7</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>B5</td>
<td>1.151</td>
<td>9</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

The results shown in Table 5.3 indicate that if cross-ties are installed evenly along a shorter main cable, the gain in the network in-plane stiffness would be less than that obtained by
placing the cross-tie lines evenly along the longest cable, as reflected by the relatively lower fundamental frequency. For example, the formation of a network with the layout of type B1 would increase the fundamental frequency of an isolated target cable (main cable 1) by 33.6% ($\Omega_1=1.336$), whereas the increase is only 15.1%($\Omega_1=1.151$) for layout of B5. However, installing cross-ties evenly on shorter main cables would be beneficial for suppressing local modes. As it can be seen from Table 5.3 and Figure 5.15, though the size of the first LMC in networks B1 to B5 remains almost the same, the position of the first LMC gradually advances from mode 4 in Network B1 to mode 9 in Network B5, i.e. the first LMC appears at a higher mode. In addition, the number of LMC within the first twenty modes is reduced from two in networks B1 and B2 to one for the other three networks. The formation and shift of the first LMC in the five studied networks are clearly reflected by the modal frequency plateaus on the frequency-mode number curves in Figure 5.16. By evaluating the modal response characteristics of these five networks, it is reasonable to propose that using cross-tie position in Network B3, which places the two lines of cross-ties evenly along the main cable 3, would be a better choice. This would allow achieving the combined benefits of increasing the modal frequency and reducing the formation of local modes. It should be noted that the main cable 3 is the intermediate cable in the network. This recommendation also agrees well with the findings by Caracoglia and Jones (2005b) when studying modal behaviour of cable networks on the Fred Hartman Bridge, which was subsequently included in a technical report by Kumarasena et al. (2007). It was pointed out that a symmetric placement of cross-ties on intermediate cables would be preferable to shift the modal frequency plateau to higher order modes.

It is also worth mentioning that compared to the modal properties of networks in Configuration A, although the addition of an extra line of cross-ties in Configuration B would
further enhance the in-plane frequency of the formed cable network, it would considerably increase the size of LMC and excite more local modes.

5.3.2 Cross-tie stiffness effect

The stiffness of cross-ties, represented by the non-dimensional stiffness parameter $\psi$, is another important system parameter that needs to be properly selected in designing cable networks. It is already observed in Chapter 3 and 4 that cross-tie stiffness would considerably affect the in-plane stiffness and the damping property of a cable network. In this subsection, the impact of cross-tie stiffness on the modal behaviour of three general cable networks C1, C2, and C3 will be studied, focusing on its influence on the excitation of local modes. All three networks contain five main cables with the same properties and layouts as those in the numerical example of Section 2, and there are respectively one, two and three lines of transverse cross-ties in those three networks. As discussed in the previous section, installing cross-ties evenly along the intermediate cable in a network could gain the combined benefits of increasing the network in-plane stiffness and suppress local mode excitation. Therefore, it is assumed that the cross-ties are positioned evenly along the main cable 3 in networks C1 to C3, i.e. the single line of cross-ties in Network C1 is installed at the mid-span of main cable 3, the two lines of cross-ties in Network C2 are installed at the 1/3 and 2/3 span of main cable 3, whereas the three lines of cross-ties in Network C3 are installed at 1/4, 1/2, and 3/4 span of main cable 3. Figure 5.12(c) portrays the layout of Network C3. Modal analyses of these three networks are performed for four different levels of cross-tie stiffness, i.e. $\psi = 0.00, 0.01, 0.10, 1.00$, representing a transition from rigid to flexible cross-tie cases.
(a) Network C1 (Single line of cross-tie evenly installed along main cable 3)

(b) Network C2 (Two lines of cross-tie evenly installed along main cable 3)
The modal frequencies of the first twenty modes of networks C1 to C3 are plotted against the corresponding mode number in Figures 5.16(a) to 5.16(c), respectively. The associated modal characteristics are summarized in Tables 5.4 to 5.6. The results clearly show that by using more flexible cross-ties (larger $\psi$ value), not only the formation of the first LMC can be pushed to higher mode or even eliminated within the first twenty modes, but also the size of LMC can be greatly reduced. For example, in the case of Network C1, if the five main cables are interconnected by a single line of rigid cross-ties located at the mid-span of main cable 3, the first LMC will appear at mode 4 with a size of 5 (Table 5.4); whereas by increasing the cross-tie flexibility to $\psi=1.00$, the first LMC will not form until mode 15, and the size is reduced to 3. For Network C3, where three lines of cross-ties are installed at even spacing along main cable 3, by reducing the cross-tie stiffness from rigid ($\psi=0$) to more flexible ($\psi=1.00$), the formation of the first LMC is shifted from mode 7 to mode 14, accompanied by a significant reduction in size from 12 to 4, as given in Table 5.6. It is also interesting to note that when certain cross-tie
stiffness is selected, though within the first twenty modes, there exists a few pairs of closely-spaced local modes, no LMC is actually formed. For example, in Network C2 which has two lines of cross-ties installed respectively at 1/3 and 2/3 span of main cable 3, when the cross-tie stiffness is selected to be $\psi=0.10$ and 1.00, there exists three pairs of closely spaced local modes but no LMC is formed (Table 5.5). In addition, the number of LMCs formed within the first twenty modes can be reduced by using less stiff cross-ties (choose larger $\psi$ value). This phenomenon is more visible in networks C1 and C2. By reducing the cross-tie stiffness from $\psi=0$ to $\psi=1.00$, the number of LMCs decreases from 2 to 1 in the former, and from 1 to 0 in the latter. In Network C1, though the formation of LMC is suppressed in the first twenty modes for $\psi=0.10$, there still exists four pairs of closely spaced local modes. They are modes 6 and 7, 10 and 11, 15 and 16, and 17 and 18. Once $\psi$ increases to 1.00, modes 15 to 17 regroup into a LMC. Though such a grouping phenomenon is not observed in networks C2 and C3, it is quite possible that the phenomenon could occur by further increasing of cross-tie flexibility to $\psi>1.00$, in particular for Network C2 of which there exists three pairs of closely spaced local modes at $\psi=0.10$ and 1.00. The presence of LMCs is also clearly reflected by the frequency plateaus in Figure 5.16.

Table 5.4: Summary of local mode cluster and pairs of local modes for cable network C1

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\Omega_1$</th>
<th>1st LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>1.145</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>0.01</td>
<td>1.140</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>0.10</td>
<td>1.105</td>
<td>&gt;20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.00</td>
<td>1.035</td>
<td>15</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 5.5: Summary of local mode cluster and pairs of local modes for cable network C2

<table>
<thead>
<tr>
<th>ψ</th>
<th>Ω_1</th>
<th>1st LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>1.188</td>
<td>5</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>0.01</td>
<td>1.180</td>
<td>6</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>0.10</td>
<td>1.137</td>
<td>&gt;20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.00</td>
<td>1.049</td>
<td>&gt;20</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.6: Summary of local mode cluster and pairs of local modes for cable network C3

<table>
<thead>
<tr>
<th>ψ</th>
<th>Ω_1</th>
<th>1st LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>1.247</td>
<td>7</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>0.01</td>
<td>1.228</td>
<td>8</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>0.10</td>
<td>1.162</td>
<td>12</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1.00</td>
<td>1.061</td>
<td>14</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

For a better characterisation of the cross-tie stiffness influence on the network fundamental frequency and formation of LMC, the fundamental frequency and position of the first LMC of networks C1 to C3 are plotted with respect to the variation of cross-tie flexibility parameter ψ in Figures 5.16(a) to 5.16(c), respectively. The results show that, irrespective of the number of cross-tie lines used in a network, the selection of more flexible cross-ties would reduce the stiffening effect of a cross-tie solution. Also, for all the three studied networks, there exist some cross-tie stiffness ranges that no LMC is formed within the first twenty modes. These ranges are not continuous, implying that the formation of LMC is sensitive to the selected cross-tie stiffness. This phenomenon is more obvious for Network C3. As can be seen from Figure
5.17(c), the formation of LMC is generally suppressed when \( \psi \) is between 0.07 and 0.62, with exception of the cases \( \psi = 0.10 \) and 0.18, the first LMC is formed at mode 12 and mode 8, respectively. Another important observation is that the formation of LMC can even be suppressed at very low \( \psi \) levels, for example, when \( \psi \) slightly exceeds 0.03, 0.08, and 0.07, respectively in networks C1 to C3. This indicates that to suppress LMC, it is not necessary to use very flexible cross-ties to sacrifice the gain in network in-plane stiffness, but rather, it is possible to select relatively rigid cross-ties in the design to achieve benefits in both network stiffness enhancement and local mode suppression.

(a) Network C1 (Single line of cross-ties evenly installed along cable 3)
(b) Network C2 (Two lines of cross-ties evenly installed along cable 3)

(c) Network C3 (Three lines of cross-ties evenly installed along cable 3)

Figure 5.17: Effect of cross-tie stiffness on the fundamental frequency and position of the 1st LMC of Network C1 to C3
5.3.3 Effect of number of cross-tie lines

To evaluate the impact of number of cross-ties on the modal behaviour of cable networks, the modal analysis results of networks C1 to C3 are studied again in this section, along with those of Network C4. The layouts of networks C1 to C3 have been explained in Section 3.2, while Network C4 contains four lines of cross-ties installed evenly along main cable 3 ($\varepsilon_{3,j}=0.20$, $j=1$ to 5). The non-dimensional network frequencies of these four networks are plotted against the mode number in Figure 5.18, with the two subplots corresponding to cross-tie stiffness of $\psi=0.01$ and 0.10, respectively. The pattern of the four frequency-mode number curves in Figure 5.18(a) suggests that connecting the main cables using more lines of cross-ties would considerably improve the in-plane stiffness of a cable network, which is in agreement with reported experience (Bosch and Park, 2005). However, although the formation of LMC can be shifted to higher mode, its size would increase greatly at the same time. For the four networks studied here, the first LMC appears at mode 4, 6, 8, and 9, respectively, with the corresponding size being 4, 7, 11, and 15. The general trend of the four frequency-mode number curves in Figure 5.18(b) is similar, except that the formation of LMC in networks C1 to C4 is suppressed with the adoption of cross-ties with higher flexibility and these LMCs are broken down into “local mode pairs”. These observations imply that by using more lines of cross-ties to connect all the main cables in the network, not only the in-plane stiffness of a cable network would be enhanced, but the appearance of the 1st LMC would be delayed. Although the major drawback is the significant increase of the LMC size, the use of more flexible cross-ties may help to break the LMCs into local mode pairs.
Figure 5.18: Effect of number of cross-ties on modal frequency of first 20 modes (Cross-ties evenly installed along cable 3)

From the above discussion, the major findings regarding local mode formation in a cable network can be summarized. As a general trend, the use of more flexible cross-ties is beneficial in reducing the formation of local modes but has significant adverse effect on decreasing in-
plane stiffness of the network. However, a careful selection of less rigid cross-tie(s) would reduce the formation of local modes without a considerable compromise on the losing the in-plane stiffness of a cable network. Increasing the mass-tension ratio parameter could effectively improve the globalization of a network mode through a more even energy distribution over the entire network without sacrificing the in-plane stiffness of the system. The severity of local mode excitation in a cable network design can be evaluated based on the position, the size and the number of formed local mode cluster (LMC) within certain range of low order network modes. An effort should be made to properly choose cross-tie installation location, stiffness and number to not only enhance the network in-plane rigidity, but also delay the formation of LMC and reduce its size. The position of cross-tie(s) plays an important role on the formation of local modes. The placement of cross-tie(s) evenly distributed along the intermediate cable(s) would allow achieving combine benefits of network in-plane stiffness and reducing the formation of local modes. Choosing less stiff cross-ties would be helpful to suppress low order local modes while retain the advantages of improving network in-plane stiffness and delay the formation of LMC.

5.4 Summary

The major disadvantage of cross-tie solution is the formation of closely-spaced local modes. Therefore, this chapter is dedicated to understand the mode localization in cross-tied cable networks. Two criteria are introduced to quantify the local mode formation. The first criterion is the DML coefficient used to measure the global nature of an individual network mode, whereas the severity of a group of closely-spaced local modes is measured by the local mode cluster (LMC). The effect of different system parameters on the DML coefficient and the LMC is
examined by using different network configurations. The conclusions drawn from this chapter are summarized as follows:

1) Modal behaviour of a two-cable network with multiple lines of cross-ties suggests that a neighbouring cable with lower frequency ratio would effectively increase the in-plane stiffness of the network fundamental mode. However this would deteriorate its global nature in case multiple cross-ties are used.

2) The use of less rigid cross-ties lowers the modal frequency of global modes and thus helps to shift the system local modes to the higher order.

3) The mass-tension ratio parameter has an imperceptible effect on the global nature of a two-cable network fundamental mode regardless of the number of cross-tie lines being used. However, increasing the mass-tension ratio of the neighbouring cable would improve the global nature of local modes a dominated by a single cable.

4) In the case of a single cross-tie line, installing it close to the mid-span of the target cable could improve the in-plane stiffness of a cable network, but lead to an early appearance of the first LMC. Thus, it is recommended to seek a balance between the in-plane stiffness and the early appearance of the first LMC in cable networks with a single line of cross-tie.

5) In case of multiple lines of cross-ties, it is recommended to install cross-ties evenly along one of the intermediate cables present in the network. This arrangement would achieve the combined benefits of enhancing the in-plane stiffness and delaying the formation of the first LMC.

6) Using stiffer cross-ties are found to be effective in enhancing the in-plane stiffness of cable networks, but would result in early appearance of the first local mode cluster. However, in
the studied five-cable network, there exist some high cross-tie stiffness ranges where no LMC is formed within the first twenty network modes.

7) In the studied five-cable network where multiple cross-tie lines interconnecting all the main cables present in the network, it is observed that installing more number of cross-lines would considerably increase the system in-plane stiffness and also effectively delay the formation of local mode clusters. However, the sizes of these clusters are found to be significantly increased.
CHAPTER 6  Effect of System Parameters on Modal Behaviour of Cable Networks

System properties of a cable network play an important role in affecting its structural behaviour. In an optimized cable network design, system properties should be chosen in such a way that the in-plane stiffness and the modal damping of the network is maximized while the formation of local modes is kept as low as possible. However, the majority of existing studies are dedicated to the influence of various system properties on the in-plane stiffness of a cable network, with the focus either on the effect of a specific system property (e.g. Yamaguchi and Nagahawatta, 1995; Sun et al., 2007; Giaccu and Caracoglia, 2013) or limited to networks on a particular cable-stayed bridge (e.g. Caracoglia and Jones, 2005b; Bosch and Pagenkopf, 2013). Therefore, to better comprehend how system property variation would affect the in-plane stiffness, the modal damping and the local mode formation of a cable network, a more comprehensive parametric study needs to be conducted. The current chapter will be dedicated to this matter.

In order to identify the important system parameters associated with a cable network, the characteristic equation of a typical cable network with two horizontally suspended cables connected through a transverse cross-tie can be used. The characteristic equation of such a cable network has been derived in Section 3.1.1 and is reproduced below for the convenience of discussion.

\[ y_1 \sin(\Omega \eta_1) \sin(\theta_3) \sin(\theta_4) + y_2 \sin(\Omega \eta_2) \sin(\theta_1) \sin(\theta_2) + \psi \Omega y_1 y_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) \]  

Refer to the definition of \( \phi_{2i-1} \) and \( \phi_{2i} \) (i=1, 2), i.e. \( \phi_{2i-1} = \Omega \eta_i \varepsilon_{2i-1}, \phi_{2i} = \Omega \eta_i \varepsilon_{2i} \), they apply respectively to the left and the right segments of the \( i^{th} \) cable. The segment ratios \( \varepsilon_{2i-1} = l_{2i-1}/L_i \) and \( \varepsilon_{2i} = l_{2i}/L_i \) are defined based on the position of the cross-tie connection on the \( i^{th} \) cable. If
representing the position of the cross-tie on main cable 1 as the non-dimensional parameter $\varepsilon$, then the segment ratios of the main cables can be expressed as,

\[
\varepsilon_{2i-1} = \varepsilon \lambda_i - O_{li}/L_i \quad \quad \varepsilon_{2i} = 1 - \varepsilon \lambda_i + O_{li}/L_i
\]

where $\lambda_i = L_1/L_i$ and $O_{li}$ are, respectively, the length ratio and the left support offset of the $i^{th}$ cable, as shown in Figure 3.1. Besides the parameters representing the mechanical and material properties of the main cables, i.e. the frequency ratio $\eta$ and the mass-tension ratio $\gamma$, another important parameter can be identified from the third term in Eq. (3-5), i.e. the cross-tie flexibility parameter $\psi$. It is important to note that Eq. (3-5) represents the characteristic equation of a two-cable network with a single cross-tie but the number of cross-tie lines in real cable networks generally varies from one to four. Therefore, number of cross-tie lines, $n_t$, should also be considered as one of the key system parameters. Based on the above discussion, it is proposed that the key system parameters which would dictate the in-plane dynamic behaviour of a cable network are: a) length ratio $\lambda_i = L_1/L_i$; b) frequency ratio $\eta_i = f_1/f_i$; c) mass-tension ratio $\gamma_i = \sqrt{H_1 m_i H_1 m_i}$; d) cross-tie position $\varepsilon = l_1/L_1$; e) cross-tie flexibility $\psi = H_1/(K_c L_1)$; and f) number of cross-tie lines $n_t$.

In general, the main cable properties are determined at the stage of the bridge design based on the load resistance requirements, which makes the selection of cross-tie installation location, stiffness and the number of lines being the main design task for the cross-tie solution. This leads as to divide the above identified cable network system parameters into two categories. In the first category, the system parameters are associated with the mechanical and material properties of the two main cables, whereas the system parameters representing the geometric and material properties of the cross-ties are considered as the second category. Therefore, Section 6.1 to Section 6.3 will discuss the effect of system parameters in the first category, including the
length ratio $\lambda$, the frequency ratio $\eta$ and the mass-tension ratio $\gamma$ parameters, on the modal behaviour of relatively simple cable networks. The effect of system parameters in the second category, i.e. the cross-tie position $\varepsilon$, cross-tie flexibility $\psi$ and the number of cross-tie lines $n_t$, on the network modal behaviour will be discussed in Section 6.4 to Section 6.6 using the configuration of a general cable network on a real cable-stayed bridge.

The performance of a cable network with a set of particular system parameters will be evaluated based on three design indicators, i.e. the in-plane frequency, the modal damping and the formation of local modes. To measure the global nature of any mode, the concept of degree of mode localization (DML) has been proposed in Chapter 5. The DML coefficient of a particular mode is an indicator of its global nature and varies from 0 to 1, with a value of 0 representing a pure global mode while a value of 1 for a pure local mode. The DML coefficient for any mode can be calculated by

$$DML = \frac{n_c(n_t+1)-n_v}{n_c(n_t+1)-1}$$  \hspace{1cm} (5-3)

where $n_c$ and $n_t$ are respectively the number of main cables and the number of cross-ties in a cable network. Therefore, $n_c(n_t+1)$ is the total number of cable segments in the network and $n_v$ denotes the number of vibrating cable segments in the studied cable network mode, which can be determined from

$$n_v = \frac{(\sum_{i=1}^{n_c} \sum_{j=1}^{n_t+1} \bar{v}_{i,j})^2}{\sum_{i=1}^{n_c} \sum_{j=1}^{n_t+1} \bar{v}_{i,j}^2}$$  \hspace{1cm} (5-4)

### 6.1 Length ratio

The discussion of the effect of length ratio parameter on the modal behaviour of cable networks is divided into two parts. In the first part of this section, the length ratio effect is
studied based on a typical rigid cross-tie ($\psi=0$) case while in the later part of this section, different levels of cross-tie stiffness will be considered for one specific position of cross-tie to evaluate how the length ratio effect will be influenced by the flexibility of cross-tie.

To investigate the effect of length ratio with a rigid cross-tie ($\psi=0$), three two-cable networks with symmetric layout, one SMT (same mass-tension ratio, i.e. $\gamma_1=\gamma_2$) system and two DMT (different mass-tension ratio, i.e. $\gamma_1 \neq \gamma_2$) systems, are studied. The frequency ratio $\eta_2$ of the neighbouring cable is taken as 0.67 while the mass-tension ratio parameters for the three networks are

Network-1 (DMT): $\gamma_1=1.0$, $\gamma_2=0.67$

Network-2 (SMT): $\gamma_1=1.0$, $\gamma_2=1.0$

Network-3 (DMT): $\gamma_1=1.0$, $\gamma_2=1.5$

By varying the length ratio, $\lambda_2$, in all three networks from 1.0 to 2.0, their fundamental frequencies and the associated DML coefficients corresponding to the cross-tie position of $\varepsilon=1/4$, 1/3 and 1/2 are portrayed in Figures 6.1(a) to 6.1(c), respectively. The black curves represent the non-dimensional fundamental frequency while their associated DML coefficients are shown in gray. It can be observed in Figure 6.1 that for all three cable networks, when a rigid cross-tie is placed at the mid-span, i.e. $\varepsilon=1/2$, the non-dimensional fundamental frequency $\Omega$ of the network is independent of the length ratio $\lambda_2$. It remains at $1.16\pi$, $1.20\pi$, $1.25\pi$ for networks 1 to 3, respectively. In addition, if the main cable 2 in a network has a higher mass-tension ratio $\gamma_2$, the network will have a higher fundamental frequency.

For the cross-tie position other than $1/2$, the system fundamental frequency shows monotonic increase with respect to the length ratio $\lambda_2$. If the mass property and the pretension of main cable 2 remain unchanged, a higher length ratio $\lambda_2$ represents physically a more stiff cable.
This implies that connecting the target cable (main cable 1) to a stiffer neighbouring cable (main cable 2) would enhance the in-plane stiffness of the resulting cable network. Comparisons of the \( \Omega - \lambda_2 \) relation curves corresponding to the three studied segment ratios show that if the cross-tie is located away from the mid-span, i.e. varies from \( \varepsilon=1/2 \) to \( 1/4 \), the system fundamental frequency becomes more and more sensitive to the length ratio between the target and the neighbouring cable. For example, in the case of Network-2, which is a SMT network, it can be seen in Figure 6.1(b) that the non-dimensional system fundamental frequency \( \Omega \) corresponding to the two extreme length ratio values \( \lambda_2=1.0 \) and \( \lambda_2=2.0 \) are \( 1.17\pi \) and \( 1.25\pi \) respectively when \( \varepsilon=1/3 \), whereas they are \( 1.14\pi \) and \( 1.33\pi \) respectively when \( \varepsilon=1/4 \), i.e. increased by \( 6.4\% \) in the former and \( 17\% \) in the latter. The same phenomenon can also be observed from Figures 6.1(a) and 6.1(c), of which both are DMT networks. For Network-1(\( \gamma_2=0.67 \)), by placing the cross-tie at \( 1/3 \) or \( 1/4 \) of the span, a variation of length ratio \( \lambda_2 \) from 1.0 to 2.0 will cause \( 6.1\% \) and \( 20\% \) increment in the system fundamental frequency, whereas it is \( 6.3\% \) or \( 13.9\% \) in the case of Network-3 (\( \gamma_2=1.5 \)).
Figure 6.1(a): Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the length ratio parameter, $\lambda$, for three different cross-tie positions (Network-1)

Figure 6.1(b): Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the length ratio parameter, $\lambda$, for three different cross-tie positions (Network-2)
Figure 6.1(c): Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the length ratio parameter, $\lambda$, for three different cross-tie positions (Network-3)

The most interesting finding from Figure 6.1 is that in the case of a SMT network (Figure 6.1(b)), a common intersection point for the three $\Omega-\lambda_2$ curves associated with different segment ratios can be identified. A closer inspection of this point reveals that once the system parameters $\eta_2$ and $\lambda_2$ of a symmetric SMT two-cable network satisfy the condition of $\eta_2 \cdot \lambda_2 = 1$, the change in the rigid cross-tie ($\psi=0$) position will not affect the system fundamental frequency. The magnitude of the length ratio $\lambda_2$ corresponding to this intersection point is defined as the critical length ratio $\lambda_c$, of which the fundamental frequency of a symmetric SMT two-cable network will be independent of the cross-tie position. In the sub-critical length ratio range ($\lambda_2 < \lambda_c$ or $\eta_2 \cdot \lambda_2 < 1$), a higher fundamental system frequency can be achieved by moving the rigid cross-tie closer to the mid-span. However, if $\lambda_2 > \lambda_c$ or $\eta_2 \cdot \lambda_2 > 1$, i.e. in the super-critical length ratio range, the rigid cross-tie should be moved further away from the mid-span in order to achieve a higher
frequency. In the case of the two DMT networks shown in Figures 6.1(a) and 6.1(c), no common intersection point of the three $\Omega$-$\lambda_2$ curves exists, but rather, each pair of two curves intersect at a different value of length ratio $\lambda_2$. For example, in Network-1 (Figure 6.1(a)), if $\lambda_2 = 1.42$, then placing the cross-tie at 1/3 and 1/4 span will yield the same fundamental frequency of the system. However, $\lambda_2$ should be 1.44 for $\varepsilon=1/2$ and $\varepsilon=1/3$ to have the same frequency, and 1.43 for $\varepsilon=1/2$ or $\varepsilon=1/4$ to have the same frequency.

The effect of the length ratio parameter on the DML coefficient of the fundamental mode can be observed through gray curves in Figure 6.1. It can be clearly seen that in all three networks, the DML coefficients of the fundamental mode is almost 0 (pure global mode) as long as the cross-tie is placed at the mid-span of the target cable. But the role of the length ratio parameter $\lambda_2$ on the DML coefficient becomes pronounced if the cross-tie is placed at $\varepsilon=1/3$ and 1/4. In these two cases, the DML coefficient increases monotonically with the length ratio parameter. With the increase of the length ratio parameter $\lambda_2$, the length of the neighbouring cable becomes shorter, which thus shifts the left anchorage of the neighbouring cable closer to the cross-tie position so that segment length becomes more uneven. This pattern suggests that due to the unequal lengths of the main cable segments, the energy distribution among the different cable segments becomes uneven, especially when the neighbouring cable has larger length ratio parameter. The sensitivity of the DML coefficient to the length ratio parameter $\lambda_2$ increases as the cross-tie moves away from the mid-span of the target cable. For example, in a SMT network, it can be seen in Figure 6.1(b) that when $\varepsilon=1/4$, the DML coefficient corresponding to the two extreme values of the length ratio parameter $\lambda_2 = 1.0$ and 2.0 are 11% and 66%, respectively, whereas they are 4% and 13% for $\varepsilon=1/3$. 

186
The cross-tie flexibility in cable networks on real cable-stayed bridges is typically not purely rigid ($\psi=0$). Therefore, it is worth to explore the non-rigid cross-tie cases ($\psi\neq 0$) to examine the role of the length ratio parameter $\lambda_2$ on the modal properties of cable network by considering different levels of cross-tie stiffness. The cable network used for this part of the study is Network-2, i.e. a SMT network with cross-tie installed at the mid-span of the target cable. Four different levels of cross-tie flexibility, $\psi=0$ (rigid), 0.01, 0.1 and 1.0 (more flexible), are used. The effect of the length ratio parameter $\lambda_2$ on the network non-dimensional fundamental frequency and its associated DML coefficient corresponding to the four levels of cross-tie stiffness is depicted in Figure 6.2. It can be seen in Figure 6.2 that for the studied cable network ($\eta_2\cdot \lambda_2 = 1$), its fundamental frequency as well as the degree of mode localization are independent of the length ratio parameter. While the use of a more rigid cross-tie ($\psi=0.0$, 0.01 and 0.10) would lead to a higher fundamental frequency of the network and also keeps its DML coefficient close to 0, the installation of a more flexible cross-tie ($\psi=1.0$) would not only considerably reduce the system fundamental frequency to $\Omega=1.07\pi$, but also changes the global nature of the fundamental mode to become a local mode with DML=34%.
By summarizing the effect of length ratio parameter on the network fundamental frequency and its associated DML coefficient, it is found that although moving the cross-tie closer to the cable end, for example $\varepsilon=1/4$ in Figure 6.1, and connecting the target cable with a shorter neighbouring cable (larger $\lambda_2$ value) would be beneficial for increasing the network fundamental frequency, such an advantage is at the cost of resulting strongly localized fundamental mode (refer to the DML curves on the right side of Figure 6.1). Therefore, such a configuration will not be recommended for the design. On the other hand, as can be seen in Figure 6.2, the length ratio parameter $\lambda_2$ has almost no effect on the fundamental frequency as well as its associated DML coefficient in case a cross-tie is placed at the mid-span of the target cable.
6.2 Frequency ratio

To isolate the effect of the frequency ratio of the neighbouring cable(s) on the dynamic behaviour of the target cable and the entire cable network, the discussion in this section will be based on four symmetric cable networks with, respectively, two to five main cables having the following properties:

- **Frequency ratio:** \( \eta_1 \neq \eta_2 = \eta_3 = \cdots = \eta_n \)
- **Mass-tension ratio:** \( \gamma_1 = \gamma_2 = \gamma_3 = \cdots = \gamma_n = 1 \)
- **Length ratio:** \( \lambda_1 \neq \lambda_2 = \lambda_3 = \cdots = \lambda_n = 1.2 \)

where \( n=2, 3, 4, 5 \).

The four studied cable networks used in this section are all SMT networks where the main cables are interconnected by a rigid cross-tie (\( \psi=0 \)) at a cross-tie position \( \varepsilon=1/2 \) or \( 1/3 \) from the left end of the target cable. In the first scenario the cross-tie is placed at the mid-span of the target cable and the effect of the frequency ratio, \( \eta_2 \), on the fundamental frequency and the associated DML coefficients of the four SMT cable networks is shown in Figure 6.3(a). It can be seen from the figure that in order to increase the network fundamental frequency, the target cable should be connected to neighbouring cable(s) having lower frequency ratio. If the connected neighbouring cable(s) has/have the same frequency ratio as the target cable, the fundamental frequency of the cable network remains the same as that of the single target cable and is irrelevant to the total number of connected neighbouring cables. This is reflected by the rightmost point in Figure 6.3(a), of which as the frequency ratio approaches to 1, the fundamental frequencies of the four cable networks, with the total number of cables varying from 2 to 5, all converge to the fundamental frequency of the target cable. On the other hand, the leftmost point of the figure implies that when the frequency ratio approaches to zero, i.e. the target cable is connected to
extremely rigid neighbouring cable(s), its fundamental frequency can be doubled. This phenomenon is also independent of the number of rigid neighbouring cables being connected. Observations from these two extreme cases suggest that when the neighbouring cables are all rigid or all have the same frequency ratio as that of the target cable, including more cables in the network will not help to further increase the fundamental frequency of the system. Between these two extreme cases, results show that connecting a target cable with more neighbouring cables would be beneficial for enhancing its in-plane stiffness. For example, if all the neighbouring cable(s) has/have a frequency ratio of 0.6, by connecting the target cable to one, two, three or four of such neighbouring cable(s), its fundamental frequency is found to increase by 25%, 35%, 41% and 45%, respectively.

Figure 6.3(a): Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the frequency ratio parameter, $\eta_2$, for four SMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at the mid-span ($\varepsilon=1/2$)
Figure 6.3(b): Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the frequency ratio parameter, $\eta_2$, for four SMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at the one-third span ($\varepsilon=1/3$).

The effect of the frequency ratio on the global nature of the network fundamental mode can be explored through the DML curves in Figure 6.3(a). It can be clearly seen that if the frequency ratio is high enough, i.e. $\eta_2 > 0.7$, the fundamental mode of all four studied cable networks is a pure global mode. The effect of frequency ratio parameter on the DML coefficient becomes pronounced as more neighbouring cables with a low frequency ratio are added into the cable network. For example, the fundamental mode DML coefficient of a three-cable network at a frequency ratio $\eta_2 = 0.2$ being 9% but subsequently jumps to 22% and 32% for the four- and the five-cable networks, respectively. This is mainly caused by the uneven distribution of energy between the target and the neighbouring cables. Since the low frequency ratio neighbouring
cables are stiffer than the target cable, more kinetic energy would be stored in the target cable if the frequency ratio of the neighbouring cables becomes lower.

In another scenario, the cross-tie position was chosen to be at 1/3 span length from the left end of the target cable. The effect of frequency ratio on the modal frequency and the DML coefficient of the fundamental mode of the four studied cable networks is depicted in Figure 6.3(b). Similar phenomenon, i.e. the neighbouring cable with low frequency ratio results in increased fundamental frequency as well as its DML coefficient, as observed in Figure 6(a) for the case of \( \varepsilon = 1/2 \) can also be seen in Figure 6(b). The only difference is that as the cross-tie moves from \( \varepsilon = 1/2 \) to \( \varepsilon = 1/3 \), the fundamental system frequency reduces and the associated DML coefficient increases, i.e. by moving the cross-tie away from the target cable mid-span, with the increase of the neighbouring cable stiffness, the network fundamental mode gradually becomes a local mode of higher frequency.

To evaluate the frequency ratio effect on the network modal properties with a non-rigid cross-tie, a two-cable network with flexible cross-tie installed at the mid-span of the target cable will be studied for four different levels of cross-tie flexibility, i.e. \( \psi = 0, 0.01, 0.1 \) and 1.0. The length ratio of the neighbouring cable (main cable 2) is assumed to be \( \lambda_2 = 1.2 \). Figure 6.4 gives the non-dimensional network fundamental frequency \( \Omega \) and its associated DML coefficient as a function of the frequency ratio \( \eta_2 \) at various levels of cross-tie stiffness when it is placed at the mid-span of the target cable (main cable 1). It can be seen that at all studied levels of cross-tie stiffness, \( \Omega \) decreases monotonically with larger value of \( \eta_2 \), and the reduction in the network fundamental frequency is found to be much considerable if a more rigid cross-tie is adopted. For instance, by replacing a rigid neighbouring cable (\( \eta_2 = 0 \)) with a one that has the same frequency ratio as the target cable (\( \eta_2 = 1.0 \)), if a rigid cross-tie connects the two main cables, the system
fundamental frequency would be reduced by half, whereas if a more flexible cross-tie ($\psi=1.0$) is used, the system frequency reduction will be only 9.2%.

Figure 6.4 Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the frequency ratio parameter $\eta_2$ for a symmetric SMT two-cable network with a flexible cross-tie installed at mid-span ($\epsilon=1/2$, $\lambda_2=1.2$)

On the other hand, the effect of the frequency ratio parameter, $\eta_2$, on the global nature of the system fundamental mode is almost negligible if very stiff cross-tie (e.g. $\psi=0$ and 0.01) is used in the network. However, when a more flexible cross-tie ($\psi=1.0$) is installed, the modal nature of the network fundamental mode is sensitive to the variation of frequency ratio parameter and could be changed at a relatively high frequency ratio. For example, when $\psi=1.0$, the DML coefficient of the fundamental mode is 30% (DML coefficient above 30% is an indicator of the local mode) at $\eta_2=0.72$. This suggests that for this particular symmetric SMT cable network, the fundamental mode would remain global as long as the frequency ratio parameter $\eta_2 \geq 0.72$. In addition, it is worth noting that in the case of a symmetric SMT two-cable network, if the two
main cables have the same frequency ratio, i.e. \( \eta_1 = \eta_2 = 1.0 \), the network fundamental frequency and the associated DML coefficient are independent of the cross-tie flexibility. The fundamental frequency remains the same as that of a single target cable and the fundamental mode is a pure global mode. This fact is reflected by the rightmost point in the Figure 6.4, where both the \( \Omega-\eta_2 \) curves and the DML-\( \eta_2 \) curves corresponding respectively to four different levels of cross-tie flexibility converge to the same point at \( \eta_2 = 1.0 \).

6.3 Mass-tension ratio

To investigate how the mass-tension ratio parameter would affect the modal behaviour of a cable network, four symmetric different mass-tension ratio parameter (DMT) cable networks are studied in this section, with the number of main cables in each network varies from 2 to 5. To better reveal the role of this system parameter, the frequency ratio and the length ratio of all the cables in each network are taken as

Frequency ratio: \( \eta_1 \neq \eta_2 = \eta_3 = \cdots = \eta_n = 0.83 \)

Length ratio: \( \lambda_1 \neq \lambda_2 = \lambda_3 = \cdots = \lambda_n = 1.2 \)

Mass-tension ratio: \( \gamma_1 \neq \gamma_2 = \gamma_3 = \cdots = \gamma_n = 0 \sim 2 \)

The impact of the mass-tension ratio parameter on the fundamental frequency and the associated DML coefficients of the above four cable networks are examined for two cross-tie positions of \( \varepsilon = 1/2 \) and \( \varepsilon = 1/3 \), with the results portrayed in Figures 6.5(a) and 6.5(b), respectively. Results show that when a rigid cross-tie is placed at mid-span (Figure 6.5(a)), the fundamental frequencies of all four studied networks increase monotonically with the mass-tension ratio parameter. By connecting more neighbouring cables to the target cable (main cable 1) results an increase in the system frequency. This implies that by connecting the target cable to more neighbouring cables with higher mass-tension ratio will be beneficial to improve the in-plane
stiffness, and thus the modal frequency of the target cable. Further, a comparison between Figures 6.5(a) and 6.5(b) suggests that the relation between the system fundamental frequency and the mass-tension ratio parameter is hardly influenced by the cross-tie position. The $\Omega - \gamma$ curves in the two figures not only have the same pattern, but also almost the same values.

Figure 6.5(a): Non-dimensional modified frequency, $\Omega/\pi$, and DML coefficient as a function of the mass-tension parameter, $\gamma$, for four symmetric DMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at mid-span ($\varepsilon=1/2$)
Figure 6.5(b): Non-dimensional modified frequency, $\Omega/\pi$, and DML coefficient as a function of the mass-tension parameter, $\gamma$, for four symmetric DMT cable networks with different number of main cables ($n=2, 3, 4, 5$) and a rigid cross-tie installed at mid-span ($\varepsilon=1/3$).

The pattern of the DML curves in Figure 6.5(a) suggests that the DML coefficient of the fundamental mode is independent of the mass-tension ratio parameter of the neighbouring cables. The fundamental mode remains global over the selected range of mass-tension ratio parameter $\gamma = 0 \sim 2$. It is also interesting to note that even the number of consisting main cables hardly have any influence on the global nature of the studied network fundamental mode. A slight increase of the fundamental mode DML coefficient can be observed in Figure 6.5(b) of which a rigid cross-tie is installed at one-third span of the target cable. However, this small increment of the DML coefficient hardly affects the global nature of the system fundamental mode.
To investigate how the mass-tension ratio parameter would affect the modal properties of a cable network with a flexible cross-tie, a symmetric DMT two-cable network with a flexible cross-tie installed at the mid-span of the target cable is used as an example. A frequency ratio \( \eta_2 = 0.833 \) and a length ratio \( \lambda_2 = 1.2 \) are assumed for the neighbouring cable in the studied network.

The impact of the mass-tension ratio parameter \( \gamma_2 \) on the network fundamental frequency \( \Omega \) and its associated DML coefficient under the condition of different cross-tie flexibility is portrayed in Figures 6.6. Similar to the rigid cross-tie case, the network fundamental frequency is found to increase monotonically with the mass-tension ratio parameter \( \gamma_2 \). This increment becomes more considerable with the increase of cross-tie stiffness. The results in Figure 6.6 show that by increasing \( \gamma_2 \) from 1.0 to 2.0, corresponding to the four levels of cross-tie flexibility 0, 0.01, 0.1 and 1.0, the network fundamental frequency is found to be increased by 3.4%, 3.3%, 3.2% and 1.3%, respectively.
Figure 6.6: Non-dimensional fundamental frequency, $\Omega/\pi$, and DML coefficient as a function of the mass-tension ratio parameter $\gamma$ for a symmetric DMT two-cable network with a flexible cross-tie installed at mid-span ($\varepsilon=1/2, \eta_2=0.833$ and $\lambda_2=1.2$)

On the other hand, the pattern of the fundamental mode DML curves in Figure 6.6 shows that the global nature of the fundamental mode would be affected if a more flexible ($\psi=1.0$) cross-tie is used, especially when the mass-tension ratio parameter is large. Using a more flexible ($\psi=1.0$) cross-tie would result in vibration dominance of the target cable (main cable 1). Therefore, the DML coefficient of the fundamental mode increases from 17% to 30% for the respective mass-tension ratio parameter of 1.0 and 2.0.

Results obtained from this section indicates that to achieve a system fundamental mode with a higher frequency and more global nature, it is recommended to connect the target cable with a neighbouring cable having higher mass-tension ratio through a more rigid cross-tie.
6.4 Cross-tie position

In the design of a cross-tie solution, the main focus is directed towards the selection of the material and geometrical properties of the cross-tie rather than these of the main cables. Therefore, the effects of cross-tie properties, i.e. the installation location, the stiffness, as well as the number of cross-ties lines on the performance of a cable network will be explored in Sections 6.4 to 6.6. The discussion will be based on a cable network with a more general layout. It consists of twelve main cables and multiple lines of cross-ties. The performance of the network will be evaluated by comparing the fundamental frequency of the system with that of the longest main cable in the cable network. The severity of local modes formation will be evaluated using the ‘position’ and the ‘size’ attributes of the local mode cluster (LMC) defined in Chapter 5. These two attributes of LMC will be used to determine the best possible cross-tied solution for different cross-tie installation positions. The material properties and physical layout of the main cables in the network are the same as those used in a general cable network discussed in Section 4.2.1.

In this section, the effect of the cross-tie installation position on the modal behaviour of cable networks will be investigated using four different network configurations. Configuration A has one line of cross-tie, whereas Configurations B, C and D have two, three and four lines of cross-ties, respectively, as shown in Figure 6.7. The properties of the main cables and their offsets with respect to the longest cable in the network are listed in Table 4.2.
(a) Layout of Network A1 in Configuration A
(b) Layout of Network B4 in Configuration B
(c) Layout of Network C6 in Configuration C
To simulate the fan type stay cable arrangement on real cable-stayed bridges, the left offset of the main cables in the studied networks is less than the right one, which leads to an asymmetric cable network layout. In Configuration A, a single line of cross-ties is installed (Figure 6.7(a)). The possible range of the cross-tie position is within 0.10 to 0.38 of the length of main cable 1 from its left end in order to connect all the main cables in the network. The flexibility parameter of the cross-ties is taken as \( \psi = 0.01 \) as this value represents the stiffness of cross-ties used on the studied bridge (Caracoglia and Jones, 2005b).

The modal properties of the first 20 modes of the network in Configuration A are analyzed using the analytical model proposed in Section 4.2.1. The network fundamental frequency and the position of the first local mode cluster (LMC) are plotted in Figure 6.8 as a
function of the cross-tie position $\epsilon_{1,1}$. It can be seen from Figure 6.8 that the fundamental frequency of Network A increases as the single cross-tie line moves from the left end (pylon side) of main cable 1 towards its mid-span. At the cross-tie position of $\epsilon_{1,1}=0.38$, the maximum achievable non-dimensional fundamental frequency is $\Omega=1.29\pi$. However, such a change in the cross-tie line position advances the formation of local modes. There is no formation of LMC within the first twenty modes for a cross-tie position range of $\epsilon_{1,1} = 0.10$ to 0.20. But for cross-tie location $\epsilon_{1,1}>0.24$, the position of the first LMC keeps on advancing towards the lower order mode. On the other hand, it can be seen from Figure 6.8 that the system fundamental frequency is relatively high when a single line of cross-tie is placed between $0.30 < \epsilon_{1,1} \leq 0.38$. Therefore, a reasonable compromise for achieving a higher fundamental frequency and delaying an early appearance of the first local mode cluster can be achieved by placing the cross-tie within the range of $\epsilon_{1,1} = 0.30$ to 0.38. It is noticed from Figure 6.8 that within this $\epsilon_{1,1}$ range, there are two cross-tie positions, $\epsilon_{1,1} = 0.33$ and 0.34, which would yield the same modal position of the first local mode cluster but different fundamental frequency. The cross-tie position $\epsilon_{1,1} = 0.34$ can be chosen as the optimum one by keeping the balance between the cable network fundamental frequency and the formation of local modes, i.e. $\Omega=1.24\pi$ and the modal position of the first LMC being the 9th mode. This position of cross-tie ($\epsilon_{1,1} = 0.34$) is the same as to install a single cross-tie line at the mid-span of the sixth longest cable in the studied twelve-cable network. It is thus labelled as Network A6 for later discussion.
The above discussion suggests that using a single cross-tie line is not adequate to achieve a significant gain in the system fundamental frequency while suppressing the early formation of local modes at the same time. Therefore, the modal behaviour of cable networks with different layout is also studied using Configuration B of which two lines of cross-ties are installed. Six different network layouts are considered in Configuration B with each network named as Bi (i=1 to 6), where i represents the \(i^{th}\) longest cable in the twelve-cable network. For example, Network B4 represents a layout of which two lines of cross-ties installed evenly along the 4\(^{th}\) longest cable (i.e. Cable AS21 (Caracoglia and Jones, 2005b)) in the studied cable network, as shown in Figure 6.7(b). Therefore, the location of the two cross-tie lines in these six cable networks can be defined by their positions on main cable 1, i.e. \(\varepsilon_{1,1}\) and \(\varepsilon_{1,2}\), as follows:

- Network B1: \(\varepsilon_{1,1}=1/3\) \(\varepsilon_{1,2}=1/3\)
- Network B2: \(\varepsilon_{1,1}=0.31\) \(\varepsilon_{1,2}=0.31\)
- Network B3: \(\varepsilon_{1,1}=0.29\) \(\varepsilon_{1,2}=0.28\)
Network B4: \( \varepsilon_{1,1}=0.27 \quad \varepsilon_{1,2}=0.26 \)

Network B5: \( \varepsilon_{1,1}=0.25 \quad \varepsilon_{1,2}=0.24 \)

Network B6: \( \varepsilon_{1,1}=0.23 \quad \varepsilon_{1,2}=0.21 \)

The modal analysis results, in terms of the fundamental frequency and the position of the first LMC of networks B1 to B6 are illustrated in Figure 6.9.

In addition, for a more convenient comparison, the fundamental network frequency, the characteristics of the first LMC and the number of closely-spaced local mode pairs in these six networks are summarized in Table 6.1.
Table 6.1: Summary of fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks B1 to B6

<table>
<thead>
<tr>
<th>Network</th>
<th>$\Omega_1/\pi$</th>
<th>1$^{st}$ LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>1.343</td>
<td>4</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>B2</td>
<td>1.371</td>
<td>4</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>B3</td>
<td>1.393</td>
<td>5</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>B4</td>
<td>1.476</td>
<td>8</td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>B5</td>
<td>1.351</td>
<td>8</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>B6</td>
<td>1.497</td>
<td>12</td>
<td>19</td>
<td>1</td>
</tr>
</tbody>
</table>

The results in Table 6.1 indicate that by varying the installation location of the two cross-tie lines according to the layouts of Networks B1 to B6, the system fundamental frequency would be slightly affected except for Networks B4 and B6, however, it has a significant impact on the formation of local modes. The relatively higher fundamental frequency in the case of Networks B4 and B6, i.e. $\Omega_1=1.476\pi$ and $\Omega_1=1.497\pi$, respectively, is due to the position of the 2$^{nd}$ cross-tie close to the cable anchorage of either the 8$^{th}$ cable (AS17) or the 10$^{th}$ cable (AS15) in these two layouts. Moving cross-tie lines towards the left end of the main cables (pylon side) may result in drop of the network in in-plane frequency. However, this arrangement/layout would allow the second line of cross-ties to interconnect more number of main cables in the network to increase the fundamental frequency. It is the cumulative effect of these two cross-tie lines which determines the change in the fundamental frequency of cable networks with different layouts. For example, in Network B1, the two cross-tie lines are installed evenly along main cable 1, and the second cross-tie line connects five longest cables in the network. Whereas in the case of Network B2, of which the two cross-tie lines are moved towards the cable left anchorage
to be installed evenly along main cable 2, the second cross-tie line connects six longest cables in the Network. The cumulative effect leads to a slight increase in the fundamental frequency of Network B2. The close proximity of the second cross-tie line to certain main cable right anchorage further enhances this commutative effect in Networks B4 and B6. For example, the formation of a network with the layout of B1 would increase the fundamental frequency of the isolated longest main cable by 34.3% ($\Omega_1=1.343\pi$), whereas a 47.6% ($\Omega_1=1.476\pi$) is achieved with the layout of B4. Besides, using layout of B4 can also delay the formation of the first local mode cluster. As it can be seen from Table 6.1 and Figure 6.9, though the size of the first LMC in networks B1 and B4 only changes slightly, the position of the first LMC jumps from mode 4 in Network B1 to mode 8 in Network B4. In addition, the number of LMC within the first thirty modes is reduced from two in networks B1 to B3 to one in the remaining three networks (B4 to B6).

Caracoglia and Jones (2005b) assumed different cable network configurations based on the Fred Hartman Bridge in order to optimize the cable network performance. Their NET_2SC configuration, which is the similar to the Network B3 configuration here, was considered to be better than the original network configuration on the Fred Hartman Bridge. Results from the current study reveal that Network B4 can achieve even better performance than Network B3 in terms of increasing the system fundamental frequency and delaying the formation of the first local mode cluster. The formation and shifting of the local mode clusters (LMCs) can be clearly identified from the mode-frequency evolution curves. Therefore, mode-frequency evolution curves for the three network layouts under Configurations B, i.e. B1 (the original network configuration on the Fred Hartman Bridge), B3 (NET_2SC configuration recommended by
Caracoglia and Jones (2005b)) and B4 (recommended by the current study) are depicted in Figure 6.10.

The pattern of mode-frequency evolution curves in Figure 6.10 clearly shows an early appearance of the first local mode cluster at mode 4 in the Network B1. Networks B3 and B4 are not only effective in delaying the formation of the first local mode cluster without compromising the modal frequency of first group of global modes but also increases the frequency of the local modes contained in the first LMC. However, the modal frequency of the second group of global modes in Network B4 is less than the corresponding modes in Networks B1 and B3.

![Figure 6.10: Mode-frequency evolution curves for Networks B1, B3 and B4 ($\psi=0.01$)](image)

Therefore, by evaluating the modal response characteristics of networks B1 to B6, it is reasonable to propose that using configuration of Network B4, which places the two lines of cross-ties evenly along main cable 4, would be a better choice in terms of achieving the dual benefits of increasing the network fundamental frequency and delaying the formation of the first
LMC. It should be noted that main cable 4 is one of the intermediate cables in the studied network. This recommendation also agrees well with the findings in a technical report by Kumarasena et al. (2007). It was pointed out that a symmetric placement of cross-ties on the intermediate cables would be preferable to shift the modal frequency plateau to higher order modes.

To have a better comprehension on how the cross-tie position parameter would influence the network fundamental frequency and the formation of local mode clusters, conducting modal analysis for cable networks with three or even four lines of cross-ties will be helpful. Again, six different layouts are chosen for the three and four cross-tie lines in Configurations C and D, with each network, respectively, Ci and Di (i=1 to 6), as shown in Figures 6.7(c) and 6.7(d), respectively. For example, Networks C6 and D6 represent a layout of which three and four lines of cross-ties installed evenly along the 6th longest cable in the studied cable network, as shown in Figures 6.7(c) and 6.7(d), respectively. The modal analysis results, in terms of the system fundamental frequency and the position of the first LMC, of networks with Configurations C and D, are illustrated in Figures 6.11 and 6.12, respectively. The fundamental network frequency, the characteristics of the first LMC and the number of closely-spaced local mode pairs of Networks Ci and Di (i=1 to 6) are summarized in Tables 6.2 and 6.3, respectively.
Figure 6.11: Effect of cross-tie position on the fundamental frequency and position of the first local mode cluster of Network Ci (i=1 to 6)

Figure 6.12: Effect of cross-tie position on the fundamental frequency and position of the first local mode cluster of Network Di (i=1 to 6)
Table 6.2: Summary of fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks C1 to C6

<table>
<thead>
<tr>
<th>Network</th>
<th>$\Omega_1/\pi$</th>
<th>1st LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>1.407</td>
<td>6</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>C2</td>
<td>1.452</td>
<td>6</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>C3</td>
<td>1.530</td>
<td>5</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>C4</td>
<td>1.470</td>
<td>9</td>
<td>22</td>
<td>1</td>
</tr>
<tr>
<td>C5</td>
<td>1.448</td>
<td>10</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>C6</td>
<td>1.422</td>
<td>14</td>
<td>17</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.3: Summary of fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks D1 to D6

<table>
<thead>
<tr>
<th>Network</th>
<th>$\Omega_1/\pi$</th>
<th>1st LMC</th>
<th>Number of LMC</th>
<th>Pairs of LM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>1.425</td>
<td>8</td>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>D2</td>
<td>1.511</td>
<td>6</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>D3</td>
<td>1.496</td>
<td>11</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>D4</td>
<td>1.495</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>D5</td>
<td>1.479</td>
<td>13</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>D6</td>
<td>1.435</td>
<td>16</td>
<td>15</td>
<td>1</td>
</tr>
</tbody>
</table>

The results in Table 6.2 reveal that the layout of Network C6 gives the best performance among all the six analysed networks in configuration C. Network C3 has slightly higher fundamental frequency ($\Omega=1.53\pi$) than that of Network C6, but the first LMC forms at mode 5 which is relatively early. The first LMC does not appear in Network C6 until mode 14.

Similarly, in the case of Configuration D where four lines of cross-ties are used (Network D1 to D6), the even installation of these four lines of cross-ties along the 6th cable (Network D6)
would shift the position of the first LMC to mode 16 compare to mode 8 in the layout of Network D1. Further, although the fundamental frequency of Network D2 is slightly higher ($\Omega=1.511\pi$) than that of Network D6 ($\Omega=1.435\pi$), the early formation of the first LMC at mode 6 is the major drawback of this layout. The mode-frequency evolution curves for networks with three different layouts, i.e. the first is to evenly install the cross-tie lines along the longest cable, the second is to generate the highest system fundamental frequency and the third which is considered as the optimum layout are depicted in Figures 6.13 and 6.14, respectively. They correspond to layouts C1, C3 and C6 in Configuration C and layouts D1, D2 and D6 in Configuration D.

Figure 6.13: Mode-frequency evolution curves for selected cable networks (three lines of cross-ties, $\psi=0.01$)
In Figure 6.13, it can be seen that network layout C6 is effective in delaying the formation of the first local mode cluster while a long local mode plateau exists in the layouts C1 and C3. In the case of Network C3, 25 local modes are present in the first local mode cluster. Similar pattern of mode-frequency curves in Figure 6.14 can be observed. Although Network D2 has relatively higher fundamental frequency, an early appearance of the first local mode cluster is its major drawback. Network D6 seems to be an effective layout in delaying the formation of the first local mode cluster as well as enhancing the fundamental frequency.

From the above discussion, it is clear that a symmetric placement of cross-ties along the longest cable in a network, as it is the case on the Fred Hartman Bridge, is not a good choice. Instead, moving cross-tie(s) towards the pylon side would lead to better network performance in reducing the formation of closely-spaced local modes without compromising the system in-plane stiffness. The performance of Networks C6 and D6 with three and four lines of cross-ties, respectively, has not improved over Network B4 in terms of network fundamental frequency.
However, Networks C6 and D6 seem to be effective in delaying the formation of the first LMC. A more extensive comparison between the performance of Networks B4, C6 and D6 will be covered when discussing the role of the number of cross-tie lines in Section 6.6.

6.5 Cross-tie stiffness

Although it is understood from existing studies (Yamaguchi and Nagahawatta, 1995; Lankin et al., 2000; Caracoglia and Jones, 2005a) that the use of more flexible cross-ties would reduce the in-plane frequency associated with the global modes of cable networks, its effect on the formation of local modes is still not clear. In Chapter 5, when studying the mode localization, two concepts: the degree of mode localization (DML) and the local mode cluster (LMC), were introduced. The DML coefficient is used to measure the global nature of an individual mode while a group of closely-spaced local modes are evaluated through the local mode cluster. A local mode cluster (LMC) is considered to be formed if it satisfies the following two criteria: (i) Three or more consecutive modes with their respective DML coefficient higher than or equal to 0.30; and (ii) The relative frequency difference between any two adjacent modes is no more than 3%. Some discussions were made in Sections 6.1 to 6.3 based on a two-cable network that sheds light on the role of cross-tie flexibility on the in-plane frequency as well as the global nature of the network fundamental mode. In Figures 6.2, 6.4 and 6.6, one can clearly see that the use of a more flexible cross-tie not only reduces the in-plane stiffness but also the global nature of the fundamental mode. However, the analysis of modal behaviour of a five-cable network in Section 5.3.2 reveals that flexible cross-ties are effective in controlling the formation of local modes. The formation of the first local mode cluster can be delayed if relatively flexible cross-ties are used. Nevertheless, it is worth mentioning that in that particular five-cable network, the cross-ties interconnect all the main cables in the network. However, for cable networks on real cable-
stayed bridges, the cross-ties installed near the pylon side would connect all the stay cables in the network while the cross-ties close to the deck side would more likely to connect only the few longer cables. Therefore, it is also necessary to examine the role of the cross-tie flexibility on the modal behaviour of this kind of cable networks. The same twelve-cable network as discussed in Section 6.4 will be analyzed for this purpose.

In Section 6.4, an effort was made to find the position of cross-tie which could maximize the network in-plane stiffness and also suppress the formation of local modes. The cable network layouts B4, C6 and D6 are identified as the optimum ones in the configuration of using two, three and four lines of cross-ties, respectively, based on a cross-tie flexibility parameter $\psi =0.01$ used on the Fred Hartman Bridge. In this section, the effect of the cross-tie stiffness will be examined over the full practical range of the cross-tie flexibility parameter $\psi$ from 0 to 1.0 based on these three optimum network layouts. The mode-frequency evolution curves of networks B4, C6 and D6 are depicted in Figures 6.15(a), (b) and (c), respectively.

![Mode-frequency evolution curves](image-url)
Figure 6.15: Mode-frequency evolution curves of the twelve-cable network in different layouts
The non-dimensional fundamental frequency as well as the position and the size attributes of the first LMC of these three cable networks (B4, C6 and D6) at four different levels of cross-tie flexibility, i.e. \( \psi = 0.01, 0.05, 0.10 \) and 1.0 are tabulated in Table 6.4.

Table 6.4: Summary of the fundamental frequency, the first local mode cluster and pairs of local modes within the first 30 modes of cable networks B4, C6 and D6

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>Network B4</th>
<th>Network C6</th>
<th>Network D6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_1/\pi )</td>
<td>First LMC</td>
<td>( \Omega_1/\pi )</td>
<td>First LMC</td>
</tr>
<tr>
<td></td>
<td>Position</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>1.476</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>0.05</td>
<td>1.217</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>0.10</td>
<td>1.154</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>1.0</td>
<td>1.049</td>
<td>19</td>
<td>4</td>
</tr>
</tbody>
</table>

The mode-frequency evolution curves in Figure 6.15 clearly shows that using stiffer cross-ties (e.g. \( \psi = 0.01 \)) has a significant effect on increasing the modal frequency of the first group of global modes and enlarge the size attribute of the first local mode cluster (LMC), whereas the use of more flexible cross-ties (\( \psi = 1.0 \)), though would reduce the modal frequency of the global modes, could considerably suppress the formation local modes. Results in Table 6.4 suggest that when more flexible cross-ties are used, not only the size of the first local mode cluster is significantly reduced, its occurrence is also delayed to higher modes, in particular when a higher level of cross-tie flexibility (e.g. \( \psi = 1.0 \)) is used. For example, in Network B4 (refer to Table 6.4), if cross-ties with flexibility parameter \( \psi = 0.01 \) are used, the network fundamental frequency is \( \Omega_1 = 1.476\pi \) and the first LMC appears at Mode 8 with a size of 14. However, if increasing the cross-tie flexibility to \( \psi = 0.05 \), the network fundamental frequency drops by 18% to \( \Omega_1 = 1.217\pi \). Although the first LMC still appears at the same modal position, i.e. Mode 8, but its size reduces almost by half to 8. Similarly, further increase in cross-tie flexibility parameter to \( \psi = 0.1 \) may
push the position of the 1\textsuperscript{st} LMC to further high at Mode 11 and even jumped to Mode 19 by using a cross-tie flexibility parameter $\psi = 1.0$. The reduction of the size of the first LMC with the increase of cross-tie flexibility implying that using more flexible cross-ties would help to break the cluster of local modes.

The same local mode cluster breaking down phenomenon can also be observed in Networks C6 and D6. In Network C6, increasing the cross-tie flexibility parameter $\psi$ from 0.01 to 0.05 seems to be effective in breaking the first LMC, of which its size is reduced by more than a half from 25 to 12. But such a cross-tie flexibility increment does not change the size of the first LMC appeared in a stiffer Network D6 (because four lines of cross-ties are installed). For Network D6, further increment of cross-tie flexibility to $\psi = 0.10$ is required to break the first LMC such that the size drops from 34 to 10. It addition, it can be seen from Table 6.4 that once the cross-tie flexibility increases to $\psi = 1.0$, not only the size of the first LMC is reduced but its position is also delayed.

It is reasonable to conclude from the above discussions that using flexible cross-ties would reduce the in-plane frequency of the network global modes. The increase of the cross-ties flexibility would first help to break down the size of the local mode cluster, then to push its formation to higher order modes. Further, the impact of cross-tie flexibility on modifying the local mode cluster(s) attributes depends on the stiffness of the cable network itself. Relative stiffer cable network, for example Network D6 which has four lines of cross-ties, requires higher level of cross-tie flexibility, i.e. $\psi = 1.0$, to achieve this.

The primary role of the cross-ties is not to dissipate energy directly unless damped flexible cross-ties are used. Therefore, it is worth to examine how damping property of a cable network be affected when damped flexible cross-ties are used. The modal behaviour of such type
of cable network, in terms of the system in-plane frequency, the damping ratio and the impact of cross-tie flexibility, have been studied in Chapter 3. For completeness, the effect of cross-tie flexibility on the formation of local modes in this kind of network should also be examined. An asymmetric DMT two-cable network with a damped flexible cross-tie, as discussed in Section 3.2.2.6, is considered. Modal analysis of this cable network was conducted in Section 3.2.2.6 to determine the in-plane frequency and the modal damping ratio. Here, the degree of mode localization (DML) of the network fundamental mode is calculated using Eq. (5-3) in Section 5.2.1. The variation of the in-plane frequency, the damping ratio and the DML coefficient associated with the fundamental mode of this two-cable network is depicted in Figure 6.16 as a function of the cross-tie flexibility parameter ψ.

In the analysis, the cross-tie damping coefficient is assumed to be \( C_c = 1.0 \) kN·s/m, whereas its flexibility parameter \( \psi \) varies from 0 (rigid) to 1.0. It can be seen from Figure 6.16 that, as expected, the system fundamental frequency decreases with the increase of the cross-tie flexibility and such a change in the cross-tie flexibility results in higher value of modal damping ratio. In addition, it is observed that the use of a more flexible cross-tie also reduces the global nature of the fundamental mode. Within the studied range of \( \psi \), the fundamental frequency decreases only by 7% while there is a considerable increase in the fundamental modal damping ratio. It increases from 0.58% to 1.49% by almost three times. In the case of a rigid cross-tie (\( \psi = 0 \)), the fundamental mode is a pure global mode with the DML coefficient being 2.6%. However, if a more flexible cross-tie with \( \psi = 1.0 \) is used, its DML coefficient jumps to 34.5% (a mode with the DML coefficient greater than 30% is regarded as a local mode), so the fundamental mode would be dictated by more localized oscillation.
6.6 Number of cross-tie lines

In Section 6.4, cable networks A6, B4, C6 and D6 are identified as the optimum ones in the network configurations using respectively one to four lines of cross-ties. In this section, an effort will be made to understand how the addition of a new cross-tie line would affect the cable network modal behaviour based on these four networks. The non-dimensional fundamental frequency as well as the position and the size of the first local mode cluster of these four cable networks are tabulated in Table 6.5 for two levels of cross-tie flexibility parameter $\psi=0.01$ and 0.10. The mode-frequency evolution curves are depicted in Figure 6.17. Results in Figure 6.17 suggest that the addition of a new cross-tie line would help increasing the network modal frequency, especially for the higher order modes. The impact on local mode formation can be
better understood based on the position and the size attributes of the first LMC listed in Table 6.5. The addition of a new cross-tie line may delay the formation of local modes by shifting the modal position of the first LMC to higher order modes. However, its associated size increases significantly at the same time, in particular if the cross-ties are stiffer.

It is also important to note that by adding a new cross-tie line, the benefit of increasing the in-plane frequency is not cumulative. For example, the non-dimensional fundamental frequency of Network A6 increases significantly from $1.241\pi$ to $1.476\pi$ with the addition of the second cross-tie line having a cross-tie flexibility parameter $\psi=0.01$. However, the addition of the third and the fourth cross-tie line seems to slightly reduce the fundamental frequency by 3.6% and 2.7%, respectively. The reason behind this is that the position of the existing cross-tie line(s) would be rearranged after a new cross-tie line is added, to keep an even spacing among them. The new cross-tie line (the cross-tie line on the deck side is assumed to be the new one) pushes the existing cross-tie lines towards the pylon side. Moving the cross-tie lines toward the left end of main cables (pylon side) results in reducing the in-plane frequency of global modes. The increment in network in-plane frequency due to the additional cross-tie line depends upon the new position/layout of cross-tie lines. As can be seen from Figures 6.7(b) and (c), the two cross-tie lines in Network B4 are pushed towards the left end (the pylon side) when a third cross-tie line is installed to form Network C6. As moving the cross-tie lines towards the pylon side results in reducing the in-plane frequency of global modes, therefore, a new arrangement of cross-tie lines in Network C6 could not bring any improvement in the fundamental frequency of Network B4. Similar comparisons can be drawn between Networks C6 and D6 where the fundamental frequency of two cable networks are very close, even Network D6 has four lines of cross-ties oppose to the three cross-tie lines in Network C6. Therefore, the main advantage of
using more lines of cross-ties, as can be seen in Figures 6.17, is to delay the formation of the first local mode cluster. However, it is at the cost of a significant increase of its size. Even in case of cable networks with more cross-tie lines, it becomes very difficult to affect the attributes of local mode clusters until relatively higher values of flexibility is introduced in cross-ties as can be seen in the last row of Table 6.5 (Network D6).

Table 6.5: Summary of non-dimensional system frequency and the first local mode cluster attributes for four optimized cable networks in term of positions of cross-ties within the first 50 modes

<table>
<thead>
<tr>
<th>Cross-tie lines</th>
<th>$\psi=0.01$</th>
<th>$\psi=0.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Omega_1/\pi$</td>
<td>$\Omega_1/\pi$</td>
</tr>
<tr>
<td></td>
<td>First LMC</td>
<td>First LMC</td>
</tr>
<tr>
<td>One (Network A6)</td>
<td>1.241</td>
<td>1.097</td>
</tr>
<tr>
<td>Two (Network B4)</td>
<td>1.476</td>
<td>1.154</td>
</tr>
<tr>
<td>Three (Network C6)</td>
<td>1.422</td>
<td>1.169</td>
</tr>
<tr>
<td>Four (Network D6)</td>
<td>1.435</td>
<td>1.189</td>
</tr>
</tbody>
</table>
Figure 6.17: Effect of number of cross-tie lines on the modal frequency of the first 50 modes of cable networks A6, B4, C6 and D6
6.7 Summary

From the discussion in preceding sections, the role of three important system parameters, i.e. the cross-tie position, the cross-tie flexibility and the number of cross-tie lines, on the in-plane stiffness and the local mode formation of a cable network can be concluded. The position of cross-tie line is one of the most important system parameters to influence the in-plane frequency and the local mode formation of cable networks. As the installation of cross-tie lines evenly along one of the intermediate cables is effective in delaying the formation of the first local mode cluster, but this arrangement of cross-tie lines could reduce the in-plane stiffness of the cable network. However, carefully chosen values of cross-tie position could effectively suppress the formation of local modes without compromising the in-plane stiffness of a cable network. Another important system parameter is the cross-tie flexibility that also has its influence on the in-plane stiffness as well as the local mode formation of cable networks. The more stiff (e.g. $\psi=0.01$) cross-ties are effective in enhancing the in-plane stiffness of the cable network, however their adverse effect is the significant increase in the size of local mode cluster, particularly when more number of cross-tie lines are used. However, though the more flexible cross-ties are effective in breaking the size of local mode cluster(s), it does at the cost of in-plane stiffness of cable network. As far as the third parameter, the number of cross-tie lines, is concerned, its major advantage is to delay the formation of the first LMC. The main disadvantage of adding a new cross-tie line is a significant increase in the size of the local mode cluster(s), especially with a more stiff cross-tie (e.g. $\psi=0.01$). The effect of the number of cross-tie lines on the in-plane stiffness of cable network is not cumulative and a significant increase/effect can be achieved only by installing the first few lines of cross-ties and any subsequent addition of cross-tie line would lead to a marginal change in the in-plane stiffness of
the cable network. Therefore, it is recommended that the cross-tie position and its flexibility should be considered as the key design parameters to optimize the cable network performance while adjusting the number of cross-tie lines should be a secondary choice.
CHAPTER 7  Hybrid System

7.1 Introduction

In the earlier chapters, an effort has been made to understand the in-plane modal behaviour of cable networks. It has been observed that cross-ties are effective in enhancing the in-plane stiffness of vulnerable cables in a cable network. However, results detailed in Chapter 3 indicate that cross-tie is not a direct energy dissipating device unless highly damped flexible cross-tie is used. This observation is supported by the findings of earlier researchers (e.g. Yamaguchi and Nagahawatta, 1995). In addition to this, the cross-tie solution is found to be a mechanism which would cause the flow or transfer of structural damping from more damped cables to the less damped ones. A detailed discussion about this observation was given in Section 3.2.2. On the other hand, external damper is an effective tool to suppress cable vibration by providing supplemental damping to main cables which have low intrinsic damping. The major drawback of the external damper solution is its installation location, which is typically restricted to be very close to the cable-deck anchorage and thus cannot provide enough supplemental damping, especially in the case of relatively long cables. Lan et al. (2010) examined the influence of different cable parameters on the effective damping of a cable equipped with a viscous damper using design data of the Sutong Bridge. It was found that the maximum attainable damping of a damped cable would drop significantly if the damper was attached to a very long cable.

To address the deficiencies associated with these two commonly used countermeasures while still retain their respective merits, the idea of combining external dampers and cross-ties into a hybrid system for cable vibration control was proposed and successfully implemented on a number of cable-stayed bridges, such as the Normandy Bridge in France (Virlogeux, 1993) and
the Fred Hartman Bridge in USA (Caracoglia and Jones, 2005b; Kumarasena et al., 2007). However, only limited number of studies is available in literature that discussed the behaviour of this kind of structural system. Bosch and Park (2005) studied the performance of hybrid systems subjected to wind load using finite element simulations. It was assumed that the hybrid system consisted a group of stay cables on the Bill Emerson Memorial Bridge which were interconnected by four lines of transverse cross-ties. In addition, each cable was assumed to be attached by a linear viscous damper. Results suggested that combining dampers and cross-ties into a hybrid system would not necessarily gain the cumulative benefits of applying the two solutions separately. Using the Fred Hartman Bridge as the background, Caracoglia and Jones (2007) extended the analytical model of cable networks developed earlier (2005b) to study the dynamic response of a hybrid system by connecting each cross-tie line to bridge deck through a damper. It was observed that the addition of external dampers to existing cross-tie lines would be more effective in suppressing global modes whereas local modes remained unaltered. In addition, compared to the damper-only solution, hybrid system was found to be a preferable configuration to achieve optimum cable vibration control for multiple modes. Following this, Caracoglia and Zuo (2009) evaluated the effectiveness of hybrid systems having various different configurations in mitigating cable vibrations for the same bridge. Results showed that the configuration of a hybrid system with a damper attached in-line with each cross-tie line would yield more satisfactory vibration suppression effect in terms of the modal damping of the fundamental mode. It was also indicated that it was not necessary to equip each stay cable with a damper although it could help to suppress some of the local modes. It is also important to note from their findings that the combined use of cross-ties and external dampers would not be effective in controlling out-of-plane cable vibrations. More recently, Zhou et al. (2015)
developed an analytical model of a symmetric two-cable hybrid system, of which the two consisting cables were laid in parallel to each other and connected by a transverse spring. In addition, each cable was attached to a linear viscous damper close to one end. A free vibration analysis was performed to understand the modal behaviour of such a hybrid system, in terms of its in-plane frequency and modal damping associated with the second in-phase and out-of-phase modes when the two main cables were identical.

From the above discussion, it can be seen that few studies exist in literature to explore the modal behaviour of hybrid systems. Out of these few studies, only the work of Zhou et al. (2015) was based on an analytical approach while the rest of the studies were conducted using numerical simulations. Compared to the numerical simulations, the analytical study has the merit to offer deeper insight into the physics associated with system behaviour and reveal the role of various system parameters in the system response. The availability of an analytical model for hybrid systems having general layout would greatly assist in better apprehension of mechanics and performance associated with this type of structural system, which would ultimately help to improve the current design practice.

It is usually challenging to develop analytical models, not mentioning that various possible configurations can be used in a hybrid system, and even a minor change in the layout could result in redeveloping a new analytical model. Therefore, it is a million dollar question whether or not it is worth to develop a characteristic equation for every possible change in the system configuration. It is noticed that there are several examples existed in literature (e.g. Krenk, 2000; Main and Jones, 2003; Caracoglia and Jones, 2007) where analytical models for a cable equipped with damper(s) and/or a spring in different configurations are originated from a parent model of a single taut cable. This motivates to develop a generalized approach which can
be used to analytically formulate characteristic equation of a system after its configuration is slightly modified from a parent system, the characteristic equation of which is either already available in literature or requires minimal effort to develop. This would not only greatly save time and effort in developing analytical models for hybrid systems having different configurations, but also offer the opportunity and freedom to extensively explore and better appreciate the impact of various configurations on the performance of a hybrid system and optimize its design.

It was also observed that when studying the damping property of hybrid systems, most of previous studies were dedicated to find the influence of damper position and damper capacity on the system but little work was devoted to investigate the impact of cross-tie position and its flexibility on the damping property of a hybrid system. Further, all of the existing studies were focused on investigating the in-plane stiffness and damping of the hybrid system, whereas the severity of local mode formation was not discussed. Nevertheless, it is already known that the installation of cross-tie would result in formation of numerous closely-spaced local modes which are hard to suppress (Caracoglia and Jones, 2005b) and effort should be made in system configuration design to minimize their formation (Kumarasena et al., 2007).

Damping of a single damped cable is strongly influenced by the position and capacity of the external viscous damper attached to it (Pacheco et al., 1993). It was observed by Krenk and Nielsen (2002) that the presence of low amplitude regions/zones near the damper end would reduce the damping of a single damped cable. Results in Chapter 5, e.g. Figure 5.2, showed that installation of cross-ties may induce such low amplitude regions/zones in a cable network. These low-amplitude regions or zones are resulted from cross-tie installation and influenced by the position and the stiffness of the cross-tie. Therefore, the cross-tie properties could also affect the
damping level in a hybrid system. It is also pointed out by Main and Jones (2003) that installation of a spring near an external linear viscous damper reduces the damping of damped cable. In a hybrid system, the cross-tie is assumed as a reversible tension/compression spring and its presence near an external damper may influence the damping property of an isolated damped cable after it is connected with its neighbouring ones through the cross-tie. All these observations indicate that system parameters like cross-tie position and its flexibility may affect not only the in-plane frequency but also the modal damping of a hybrid system. To be more comprehensive, it is necessary to explore the impact of these system parameters on the in-plane frequency, the damping ratio and the degree of mode localization of the hybrid system natural modes. There is also a strong need to establish a methodology or guideline to optimize the design of a hybrid system, so that not only the increase of in-plane stiffness and modal damping of the system can be maximized but the formation of local modes can be minimized.

In the case of a single cable equipped with an external damper, its maximum attainable damping and optimum damper capacity can be predicted by using the damping estimating curve available in the literature (e.g. Pacheco et al., 1993). However, it is not reasonable to use the same damping estimation curve to predict the damping property of a hybrid system (which is the same as the damping of a damped cable in the hybrid system) whose system parameters could be very different from those of a single damped cable. A significant amount of work is available in the literature that discuss the modal behaviour of single cables attached with passive linear viscous damper. Kovacs (1982) identified the existence of an optimal size for a transverse viscous damper when attached to a cable at a certain location. Yoneda and Maeda (1989) and Uno et al. (1991) have conducted numerical studies on the optimum damper size and showed that the maximum attainable modal damping is directly proportional to the distance between the
damper and the near cable anchorage. Pacheco et al. (1993) proposed a universal damping estimation curve to predict the modal damping of a single taut cable equipped with an external linear viscous damper. Later, this universal damping estimation curve was modified by several researchers to consider the effect of bending stiffness, cable sag, damper stiffness and damper support stiffness (Tabatabai and Mehrabi, 2000; Krenk and Nielsen, 2002; Hoang and Fujino, 2007; Fournier and Cheng, 2014). For a given damper location, the damper size associated with the maximum attainable damping is known as the optimum damper size and it may be influenced by cable properties such as the in-plane stiffness, the sag, the bending stiffness etc. A slight deviation from the optimum damper size may result in a rapid reduction in the maximum attainable damping (Pacheco et al., 1993).

When a single damped cable becomes part of a hybrid system, some of its properties, like the in-plane stiffness and the sag, would be modified with the development of low amplitude zones/regions near the cable ends. These changes in cable properties would in turn affect the maximum attainable damping of the damped cable. Besides, a “damping transfer” phenomenon takes place if the main cables in a hybrid system have different levels of damping. The “damping transfer” phenomenon among the main cables of a network has been explained in Section 3.2 that damping would be transferred/flew from a more damped cable to a less damped one depending on the vibration amplitude of these cables. In a typical hybrid system, not every cable is equipped with a damper. Thus stay cables in a cable network would have different levels of structural damping which would result in damping flow/transfer among these cables. When studying the behaviour of the hybrid system on the Fred Hartman Bridge, it was observed that the maximum modal damping of a specific hybrid system mode was considerably lower than the maximum achievable modal damping of a single damped cable (Caracoglia and Zuo, 2009).
These findings indicate that it is not appropriate to directly apply the damping estimation curves developed for a single taut damped cable to predict the damping of a hybrid system. Therefore, there is also a strong need to revisit these damping estimation curves and refine them in the context of hybrid systems.

The above discussions clearly show that there are still a number of outstanding issues remain obstacles to fully understand the dynamic behaviour of hybrid systems. Therefore, it is worth to explore all these important yet unknown dimensions of hybrid systems. This helps in setting the objectives of this chapter. First, an analytical model of a two-cable hybrid system, which consists a vulnerable cable connected with a neighboring one by a transverse linear flexible cross-tie and also equipped with a linear viscous damper close to one supporting end, will be developed in Section 7.2. Section 7.3 will be dedicated to validate the proposed analytical model where the proposed analytical model will be applied to analyze modal behaviour of a hybrid system experimentally studied by Sandanam (2015). Further, an independent numerical simulation of the same hybrid system will be conducted. The modal properties of the hybrid system obtained from the proposed analytical model will be compared with existing experimental data (Sandanam, 2015) and numerical results. In Section 7.4, the hybrid system characteristic equation developed in Section 7.2 will be extended to a generalized form for formulating characteristic equation of hybrid systems having different configurations, the validity of which will be examined by comparing with the analytical models associated with hybrid system having various configurations available in the literature. Parametric study will be conducted in Section 7.5 to explore the impact of main design parameters on the performance of a hybrid system in terms of the in-plane frequency, the damping and the degree of mode localization of the system fundamental mode. The design optimization for hybrid system will be
discussed in Section 7.6, where the concept of isoquant curve will be utilized to examine the effect of simultaneous variation of main design parameters on the modal behaviour of a hybrid system. Section 7.7 will be dedicated to explore the effect of the two key system parameters, i.e. the cross-tie flexibility and the cross-tie position, on the optimum damper size and the corresponding maximum attainable damping of a two-cable hybrid system. An effort will be made to establish an approximate relationship between the optimum damper size and the corresponding maximum attainable damping as a function of the core system parameters for two-cable hybrid systems.

7.2 Formulation of the Analytical Model

The idea of forming a hybrid system was proposed with the objective to exploit the advantages of the damper solution and the cross-tie solution in suppressing stay cable vibrations while overcoming their respective drawbacks. Thus, a vulnerable cable (referred to as the “target cable” in the rest of the chapter) would be equipped with an external damper to supplement energy dissipation and also connected to its neighboring cable(s) through cross-ties to enhance its in-plane stiffness. Figure 7.1 shows a typical configuration of a hybrid system which consists of two horizontally laid main cables. Both cables are assumed to be taut cables fixed at the two ends.
The length of the two cables are denoted $L_1$ and $L_2$ ($L_1 \geq L_2$), respectively. The longer cable (main cable 1) is assumed to be the target cable which is vulnerable to dynamic excitations. The left and the right offset of main cable 2 (referred to as the “neighbouring cable” in the rest of the chapter) with respect to the target cable are denoted by “$O_L$” and “$O_R$”, respectively. The mass per unit length and tension of the two main cables are denoted by $m_k$ and $H_k$ ($k=1, 2$), respectively. A linear viscous damper is attached transversely to the target cable at a distance $l_{1,1}$ from the cable left support $A$. The capacity of the damper is denoted by its damping coefficient $c$. Besides, a transverse flexible cross-tie is installed to interconnect the target cable with the neighbouring one. The spacing between the damper and the cross-tie installation location on the target cable is $l_{1,2}$. Only the in-plane transverse motion of the system is considered in the analytical model formulation. The additional cable tension due to vibration is neglected, and the cross-tie is assumed to vibrate only along its axial direction with behaviour simulated by a linear spring having a stiffness of $K_c$. As it can be seen from Figure 1, the presence of the damper and the cross-tie divides the target cable into three segments, and the neighboring cable into two.
segments. The transverse oscillation of each cable segment can be described by (Irvine and Caughey, 1974).

\[ H \frac{\partial^2 v(x,t)}{\partial x^2} = m \frac{\partial^2 v(x,t)}{\partial t^2} \]  

(7-1)

where \( v, H, m \) are respectively the transverse displacement, the tension and the mass per unit length of the associated cable segment. By separating the temporal and spatial variables in the cable transverse displacement, i.e. \( v(x, t) = \tilde{v}(x)e^{i\omega t} \), where \( \tilde{v}(x) \) is the shape function and \( \omega \) is the circular frequency of vibration (Krenk, 2000). By solving Eq. (7-1) and applying the boundary conditions of different cable segments, their transverse motion shape functions can be expressed as

\[
\tilde{v}_{1,j}(x_{1,j}) = B_{1,j} \sin(\alpha_1 x_{1,j}) \quad j = 1, 3 \quad (7-2a)
\]

\[
\tilde{v}_{1,2}(x_{1,2}) = A_{1,2} \cos(\alpha_1 x_{1,2}) + B_{1,2} \sin(\alpha_1 x_{1,2}) \quad (7-2b)
\]

\[
\tilde{v}_{2,j}(x_{2,j}) = B_{2,j} \sin(\alpha_2 x_{2,j}) \quad j = 1, 2 \quad (7-2c)
\]

where \( \alpha_k = \sqrt{\frac{m \omega^2}{H_k}} \) \( (k = 1, 2) \) is a complex wave number associated with cable \( k \), \( A_{1,2}, B_{k,j} (k=1, j=1-3; k=2, j=1, 2) \) are the shape function constants.

At nodes \( N_1, N_2 \) and \( N_3 \), where the cables are connected with either the damper or the cross-tie, the compatibility of cable segment transverse displacement at the left and the right side of the node requires:

\[
\tilde{v}_{1,1}(l_{1,1}) - \tilde{v}_{1,2}(0) = 0 \quad (7-3a)
\]

\[
\tilde{v}_{1,2}(l_{1,2}) - \tilde{v}_{1,3}(l_{1,3}) = 0 \quad (7-3b)
\]

\[
\tilde{v}_{2,1}(l_{2,1}) - \tilde{v}_{2,2}(l_{2,2}) = 0 \quad (7-3b)
\]

The force equilibrium along cable transverse direction at nodes \( N_1 \) and \( N_2 \) leads to
By isolating the cross-tie, its longitudinal equilibrium should satisfy

\[
\left(\frac{\partial \bar{v}_{1,2}}{\partial x_{1,2}} \bigg|_{l_{1,2}} + \frac{\partial \bar{v}_{1,3}}{\partial x_{1,3}} \bigg|_{l_{1,3}}\right) H_1 + \bar{v}_{1,3}(l_{1,3}) - \bar{v}_{2,2}(l_{2,2}) K_c = 0
\]  

Substituting Eq. (7-2) into Eqs. (7-3) to (7-5), it yields

\[
B_{1,1}\sin(\alpha_1 l_{1,1}) - A_{1,2} = 0
\]  

\[
A_{1,2}\cos(\alpha_1 l_{1,2}) + B_{1,2}\sin(\alpha_1 l_{1,2}) - B_{1,3}\sin(\alpha_1 l_{1,3}) = 0
\]  

\[
B_{2,1}\sin(\alpha_2 l_{2,1}) - B_{2,2}\sin(\alpha_2 l_{2,2}) = 0
\]  

\[
\alpha_1 H_1 [B_{1,1}\cos(\alpha_1 l_{1,1}) - B_{1,2}] + A_{1,2}i\omega c = 0
\]  

\[
\alpha_1 H_1 [-A_{1,2}\sin(\alpha_1 l_{1,2}) + B_{1,2}\cos(\alpha_1 l_{1,2}) + B_{1,3}\cos(\alpha_1 l_{1,3})] + \left[B_{1,3}\sin(\alpha_1 l_{1,3}) - B_{2,2}\sin(\alpha_2 l_{2,2})\right] K_c = 0
\]  

\[
\alpha_1 H_1 [-A_{1,2}\sin(\alpha_1 l_{1,2}) + B_{1,2}\cos(\alpha_1 l_{1,2}) + B_{1,3}\cos(\alpha_1 l_{1,3})] + \alpha_2 H_2 [B_{2,1}\cos(\alpha_2 l_{2,1}) + B_{2,2}\cos(\alpha_2 l_{2,2})] = 0
\]  

Eq. (6) can also be rewritten in a matrix form, i.e.

\[
[S]{\{X\}} = \{0\}
\]  

where

\[
[S] = 
\begin{bmatrix}
\sin(\phi_{1,1}) & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & \cos(\phi_{1,2}) & \sin(\phi_{1,2}) & -\sin(\phi_{1,3}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sin(\phi_{2,1}) & -\sin(\phi_{2,2}) & 0 \\
\cos(\phi_{1,1}) & i\mu & -1 & 0 & 0 & 0 & 0 \\
0 & -\psi\Omega\sin(\phi_{1,2}) & \psi\Omega\cos(\phi_{1,2}) & \psi\Omega\cos(\phi_{1,3}) \sin(\phi_{1,3}) & 0 & -\sin(\phi_{2,2}) & 0 \\
0 & -\gamma_1\sin(\phi_{1,2}) & \gamma_1\cos(\phi_{1,2}) & \gamma_1\cos(\phi_{1,3}) & \gamma_2\cos(\phi_{2,1}) & \gamma_2\cos(\phi_{2,2}) & 0 \\
\end{bmatrix}
\]
In the coefficient matrix \([S]\), \(i = \sqrt{-1} ; \phi_{k,j} = \Omega \eta_k \varepsilon_{k,j} \; (k=1, j=1–3; k=2, j=1, 2)\) applies to the \(j^{th}\) segment of cable \(k; \varepsilon_{k,j} = l_{k,j}/l_k\) is the segment ratio parameter representing the non-dimensional length of the \(j^{th}\) segment of cable \(k; \eta_k = f_1/f_k\) and \(\gamma_k = \sqrt{H_k m_k/H_1 m_1} \; (k=1, 2)\) are respectively the frequency ratio and the mass-tension ratio parameters of the \(k^{th}\) main cable; \(\psi = H_1/(K_c L_1)\) is the non-dimensional cross-tie flexibility parameter and \(\mu = c/\sqrt{H_1 m_1}\) is the non-dimensional damping parameter; \(\Omega = \pi f/f_1\) is the non-dimensional complex frequency of the hybrid system; and \(f\) is the complex system frequency.

The non-trivial solution to Eq. (7-7) can be found by setting the determinant of the coefficient matrix \([S]\) to zero, from which the characteristic equation of the hybrid system shown in Figure 7.1 can be derived. It has the form of

\[
\gamma_1 \sin(\Omega \eta_1) \sin(\Omega \eta_2) + \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) + \gamma \sin(\Omega \eta_1) \sin(\Omega \eta_2) \\
+ i \cdot \mu \sin(\Omega \eta_1) [\gamma_1 \sin(\Omega \eta_2) \sin(\Omega \eta_2) + \gamma_2 \sin(\Omega \eta_2) \sin(\Omega \eta_2)] = 0
\]

(7-8)

The form of Eq. (7-8) clearly reveals the contribution of different structural components in a hybrid system to its dynamic behaviour. The first two terms in Eq. (7-8) describe the interaction between cable segments in a main cable with the other main cable. For example, the first term represents the interaction between the two segments of the neighboring cable with the target cable, and vice versa for the second term. The third term reflects the influence of the cross-tie in terms of its flexibility \(\psi\) on the system behaviour; whereas the rest of the terms within the square bracket and multiplied by a common factor "\(i \cdot \mu \sin(\Omega \eta_1)\)" (the “damper term”) shows the
impact of the external damper on the response of the system. If a rigid cross-tie ($\psi=0$) is used in
the studied hybrid system and the damper capacity is zero ($\mu=0$), then the left hand side of Eq.
(7-8) is reduced to the first two terms, which is the same as the system characteristic equation of
a two-cable network with a transverse rigid cross-tie derived in Section 3.1.1. If the cross-tie is
flexible whereas the damper capacity is zero ($\mu=0$), the first three terms on the left hand side of
Eq. (7-8) will be retained, and Eq. (7-8) becomes the same as the system characteristic equation
of a two-cable network with a flexible cross-tie represented by Eq. (3-5).

In Eq. (7-8), the non-dimensional complex frequency $\Omega$ of the hybrid system is the only
unknown and can be determined by solving the characteristic equation using the approach
already discussed in Section 3.2. By setting both the real and the imaginary parts of the hybrid
system characteristic equation to zero, Eq. (7-8) can be transformed into two non-linear
equations. The real and the imaginary parts of the complex system frequency $\Omega$ can be found by
solving these two non-linear equations, based on which the in-plane frequency and the damping
ratio of the hybrid system can be determined.

7.3 Model validation

This section is aimed to validate the proposed hybrid system analytical model. The model
will be applied to analyze modal response of a hybrid system experimentally studied by
Sandanam (2015). The results are compared with the findings reported by Sandanam (2015) and
also from an independent numerical simulation.

The hybrid system in the experimental study by Sandanam (2015) consists two main
cables, a transverse cross-tie and a linear viscous damper. The two main cables were arranged in
parallel, both inclined at 13° with respect to horizontal, as illustrated in Figure 7.2.
They were rigidly supported between two vertical steel columns. The bottom cable was assumed to be the target cable (main cable 1) and the top one as the neighboring cable (main cable 2). A linear viscous damper was attached to the target cable at a distance 0.55 m from its lower support. A transverse cross-tie was used to connect the main cables and was installed at 2.83 m from the lower end of the target cable. The geometrical and physical properties of the main elements in this hybrid system are (Sandanam, 2015):

- **Target cable (bottom)**: \( L_1 = 8.5 \text{ m} \quad H_1 = 2500 \text{ N} \quad m_1 = 0.213 \text{ kg/m} 
- **Neighboring cable (top)**: \( L_2 = 8.5 \text{ m} \quad H_2 = 3600 \text{ N} \quad m_2 = 0.195 \text{ kg/m} 
- **Cross-tie**: \( l_{1,1} + l_{1,2} = 2.83 \text{ m} \quad K_c = 1210 \text{ kN/m} 
- **Damper**: \( l_{1,1} = 0.55 \text{ m} \quad c = 19.1 \text{ N} \cdot \text{s/m} 

The frequency and the associated damping ratio of the hybrid system fundamental mode were obtained from forced vibration test and given in Table 7.1 (Sandanam, 2015).
Table 7.1: Fundamental modal frequency and damping ratio of isolated cables and hybrid system with damper position $l_{1,1}=0.55\text{m}$ ($\varepsilon_1=0.065$) and cross-tie position $\varepsilon=1/3$ ($c=19.1\text{ N} \cdot \text{s/m}$, $K_c=1210\text{ kN/m}$)

<table>
<thead>
<tr>
<th>System</th>
<th>Frequency (Hz)</th>
<th>Damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid</td>
<td>6.75</td>
<td>1.06</td>
</tr>
<tr>
<td>Damped target cable</td>
<td>6.36</td>
<td>1.80</td>
</tr>
<tr>
<td>Undamped target cable</td>
<td>6.40</td>
<td>0.30</td>
</tr>
<tr>
<td>Undamped neighboring cable</td>
<td>7.80</td>
<td>0.10</td>
</tr>
</tbody>
</table>

The fundamental modal properties of the isolated target cable and neighboring cable are also listed in the same table. Based on the properties of the main cables, the cross-tie and the damper, the corresponding non-dimensional system parameters are: $\varepsilon_{1,1}=0.065$, $\varepsilon_{1,2}=0.268$, $\psi=0.00024$ and $\mu=0.83$. By substituting these numbers into the hybrid system characteristic equation, Eq. (7-8), the system modal frequency and the associated modal damping can be determined. The modal properties of the first ten modes of the studied hybrid system are given in Table 7.2 and the associated mode shapes are portrayed in Figure 7.3.

The fundamental frequency of the hybrid system yielded from the analytical model is 7.09 Hz, which agrees well with the experimental result of 6.75 Hz. However, the analytically determined fundamental modal damping ratio 0.5% is only one half of the experimentally obtained 1.06%. This discrepancy could be attributed to a number of factors.
Table 7.2: Modal properties of the two-cable hybrid system in the numerical example

<table>
<thead>
<tr>
<th>Mode</th>
<th>Modal frequency (Hz)</th>
<th>Damping ratio (%)</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>FEA</td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
<td>7.09</td>
<td>7.10</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>10.57</td>
<td>10.59</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>14.49</td>
<td>14.52</td>
<td>1.28</td>
</tr>
<tr>
<td>4</td>
<td>19.12</td>
<td>19.16</td>
<td>2.17</td>
</tr>
<tr>
<td>6</td>
<td>23.98</td>
<td>24.02</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>27.71</td>
<td>27.79</td>
<td>0.34</td>
</tr>
<tr>
<td>8</td>
<td>31.91</td>
<td>31.98</td>
<td>1.53</td>
</tr>
<tr>
<td>9</td>
<td>36.70</td>
<td>36.80</td>
<td>0.56</td>
</tr>
<tr>
<td>10</td>
<td>38.24</td>
<td>38.33</td>
<td>6.55</td>
</tr>
</tbody>
</table>

(GM: global mode, LM: local mode, LS: left segment mode, RS: right segment mode.)
First of all, the intrinsic damping of the two main cables is not considered in the formulation of the analytical model. However, it was present in the physical test, being respectively 0.3% and 0.1% for the target and the neighboring cables. Secondly, in the experimental test, when the system vibrates, part of the energy was also dissipated through the friction at the cross-tie connections. These would all contribute to the overall damping of the system but are not taken into account in the analytical model.
To assess the impact of the above listed factors on the system damping and to validate the modal results of higher order modes, an independent numerical simulation is also conducted using the commercial finite element analysis software Abaqus 6.10. The behaviour of the two main cables, the cross-tie and the damper are simulated by the B21 beam element, the SPRING2 element and the DASHPOT1 element, respectively. The modal frequency and modal damping of the first ten modes of the studied hybrid system obtained from finite element simulation are also listed in Table 7.2, which are found in good agreement with those determined by the proposed analytical model. In particular, to be more consistent with the assumptions made for the analytical model, the intrinsic damping of the two main cables and the cross-tie are assumed to be zero in the numerical model. The system fundamental modal damping ratio resulted from finite element simulation is 0.48%, which is very close to the analytical result of 0.5%. This implies the intrinsic damping of the main cables and the cross-tie, as well as the frictions at the cross-tie connections could have a significant contribution to the system damping property.

Based on the modal properties listed in Table 7.2 and the mode shapes illustrated in Figure 3, it can be seen that among the first ten modes of the studied hybrid system, six of them are global modes, i.e. modes 1, 3, 5, 7, 8 and 9, whereas the other four are local modes dominated by oscillations of certain cable and/or cable segment(s). Comparing the fundamental modal properties of the hybrid system with those of the isolated damped target cable and neighboring cable listed in Table 1, it can be seen that the formation of a hybrid system would help to “even out” the stiffness and damping of the consisting cables, i.e. the in-plane stiffness of the target cable is enhanced and the energy dissipation capacity of the neighboring cable is improved. The fundamental modal frequency of the damped target cable increases by 11.5% from 6.36 Hz to 7.09 Hz, whereas the modal damping ratio of the neighboring cable jumps from
0.1% to 0.5% by five times. The same phenomenon of damping “transfer” from an element possessing higher energy dissipating capacity (the damped target cable) to a lightly damped one (neighboring cable) is also observed in Section 3.2 and by Yamaguchi and Ito (1997). In this sense, if a damper is installed on a specific cable, by connecting this damped cable with neighboring one(s) through a cross-tie, the “excessive” amount of damping can be “transferred” to other consisting cable(s). Therefore, forming a hybrid system would not only help increasing in-plane stiffness, but also improve the energy dissipation capacity of lightly damped consisting cables.

On the other hand, it is noticed that the modal damping ratio associated with two local modes, i.e. mode 2 and mode 6, are very low, which is 0.15% and 0, respectively. An inspection on their respective mode shapes reveals that the low damping is associated with the active level of oscillation of the cable segment attached with the damper. Mode 2 is dominated by the oscillation of the right cable segments of the two main cables, whereas the left segments are relatively “calm”. Therefore, the efficiency of the external damper is restricted by the small amplitude cable motion at the damper location. In mode 6, only the neighboring cable is in oscillation and the target cable is at rest. Thus, the damper is not in function. Since the intrinsic damping of the main cables and cross-tie is neglected in the analytical model, the damping ratio of this mode becomes 0. This suggests that local modes dominated by lightly damped cable in a hybrid system will not be benefited for its modal damping by connecting it with a high damped cable. In the case of the other two local modes which are dominated by vibrations of the damped target cable, i.e. mode 4 and mode 10, the neighboring cable is at rest. Therefore, no damping is “transferred” from the damped target cable into the neighboring one, so the resulted system model damping ratio is relatively high.
This set of modal analysis results and the above discussion clearly indicate that to maximize the effectiveness of an external damper in a hybrid system, it should be installed at a location where cable segment is active in oscillation. Further, a lightly damped cable will be benefited by damping transferred from a high damped cable only when itself is active in oscillation. Therefore, a great care is needed when selecting appropriate damper installation location to avoid adverse effect resulted from formation of a hybrid system.

7.4 Generalized approach

In this section, an effort will be made to develop a generalized approach to formulate the characteristic equation of a given hybrid system. For this purpose, the characteristic equation, Eq. (7-8) of a typical hybrid system shown in Figure 7.1, is revisited. It is interesting to note that the form of the three terms within the square bracket in Eq. (7-8) is very similar to the first three terms, except the effect of the left segment of the target cable, represented by \( \sin(\phi_{1,1}) \), is brought out of the bracket and becomes a common factor for the three terms in the square bracket. This observation implies that the formation of the hybrid system shown in Figure 7.1 can be considered as a two-cable flexible cross-tie network with a transverse external damper attached to the target cable. Thus, the system characteristic equation of the resulted hybrid system can be developed based on that of the base system, i.e. the two-cable flexible cross-tie network as represented by the first three terms on the left hand side of Eq. (7-8). The effect of the external damper can be added by using the modified form of the base system equation, which is shown as the damper term in Eq. (7-8).

Enlightened by the observations from Eq. (7-8), it motivates to seek a generalized approach to formulate characteristic equation for hybrid systems with different configurations. As discussed in Section 7.2, if we define a base or a “parent” system in a hybrid system and
considers an external damper (or a cross-tie) as the additional connector for the parent system, the characteristic equation of a hybrid system can be derived by summing up that for the parent system and the effect of the additional connector, with the latter being the product of the connector property and the modified form of the parent system equation. Denote "Λ" as the connector property, if the additional connector is a transverse damper having a non-dimensional damping parameter μ, then the connector property is Λ = i · μ, whereas if the additional connector is a transverse cross-tie having a non-dimensional flexibility parameter of ψ, then the connector property is represented by Λ = 1/(ψΩ). The presence of the additional connector would divide the attached original cable segment into two parts. Thus, the Sine term of the original cable segment would split into two sub-Sine terms, with one of them serve as the common factor in the additional connector term. Take the hybrid system shown in Figure 7.1 as an example. In this case, the two-cable network with a flexible cross-tie can be considered as the parent system, whereas the external damper as the additional connector which “splits” the cable segment AN₂ into two segments of AN₁ and N₁N₂. Therefore, sin(Ωη₁) in the first term of Eq. (7-8), i.e. γ₁sin(Ωη₁)sin(φ₂₁)sin(φ₂₂), becomes sin(φ₁₁)sin(φ₁₂ + φ₁₃), which is in the first damper term i · μsin(φ₁₁)γ₁sin(φ₁₂ + φ₁₃)sin(φ₂₁)sin(φ₂₂) of the Eq. (7-8) (Note: sin(Ωη₁) = sin(φ₁ + φ₂ + φ₃)). The same applies to the second and the third terms associated with the parent system and their correspondence in the damper term. Therefore, it is proposed that the characteristic equation of a hybrid system can be expressed as

\[ \text{Parent term + connector term} = 0 \]  

(7-9)

where the parent term is the characteristic equation of the parent system, and the connector term has the form of

\[ \text{Connector term} = Λ \cdot \left( \text{parent term in split form} \right) \]  

(7-10)
where $\Lambda$ is the connector property.

In the existing literature, characteristic equation of a number of different configurations cable systems have been derived analytically. The proposed generalized approach will be applied to form characteristic equations of these same systems in the next few subsections, and compare with those in literature.

**a) A single taut cable attached with a transverse linear viscous damper**

Many researchers studied the effectiveness of external dampers in suppressing cable vibrations using an idealized fixed-fixed single taut cable equipped with a transverse linear viscous damper, as shown in Figure 7.4(a). The characteristic equation of this cable-damper system has been derived analytically by Krenk (2000). To apply the proposed generalized formulation approach, the fixed-fixed taut cable itself is considered as the parent system, and the damper is treated as the additional connector which is installed at a distance $l_1$ from the left cable support. Assume the cable has a length $L$, a mass per unit length $m$, a tension $H$, the damping coefficient of the linear viscous damper is $c$, and the non-dimensional damping parameter is $\mu = c/\sqrt{mH}$.

The parent term of the cable-damper system is $\sin(\Omega)$ or $\sin(\phi_1 + \phi_2)$, where $\phi_k = \Omega \varepsilon_k$, $\varepsilon_k = l_k/L$ ($k=1, 2$), the connector property is $\Lambda = i \cdot \mu$. The installation of the damper divides the
original cable into two segments of length $l_1$ and $l_2$, respectively. Thus, the split form of the
parent term would be $\sin(\varphi_1)\sin(\varphi_2)$. Therefore, the characteristic equation of the damped taut
cable in Figure 7.4(a) written as

$$
\sin(\Omega) + i \cdot \mu \sin(\varphi_1)\sin(\varphi_2) = 0
$$

Equation (7-11), which is derived using the proposed generalized approach, is the same as the
Eq. (10) used by Krenk (2000), which was derived analytically.

b) A single taut cable with a transverse linear viscous damper and a transverse linear
    spring

Figure 7.4(b) shows schematically a fixed-fixed taut cable attached transversely with a linear
spring of stiffness $k$ and a linear viscous damper of damping coefficient $c$. This model was used
by Main and Jones (2004) to access the influence of installing neoprene rubber bushings on the
performance of a damped stay cable. These bushings are typically mounted inside the steel guide
pipe of a cable near its anchorage for reducing bending stress within the cable. To apply the
generalized approach to form the characteristic equation of the system in Figure 7.4(b), the fixed-
fixed taut cable with a transverse linear viscous damper in Figure 7.4(a) can be considered as the
parent system, and the transverse linear spring can be assumed as the additional connector which
splits the cable segment on the left side of the damper into two.

![Figure 7.4(b): A taut cable attached with transverse linear spring and viscous damper](image)
The left hand side of Eq. (7-11) now becomes the parent term of the current system, whereas the connector property is \( \Lambda = 1/(\psi \Omega) \), where \( \psi = H/(KL) \) is the non-dimensional flexibility parameter of the spring connector. \( \sin(\Omega) \) and \( \sin(\varphi_1) \) in the parent term would be replaced by their respective split form of \( \sin(\varphi_1) \sin(\varphi_2 + \varphi_3) \) and \( \sin(\varphi_1)\sin(\varphi_2) \). Therefore, the characteristic equation of the system in Figure 7.4(b) is
\[
\sin(\Omega) + i \cdot \mu \sin(\varphi_1 + \varphi_2)\sin(\varphi_3) + \frac{1}{\psi \Omega} \sin(\varphi_1)\sin(\varphi_2 + \varphi_3) + i \cdot \mu \sin(\varphi_2)\sin(\varphi_3) = 0 \quad (7-12)
\]
The above equation is the same as that analytically derived by Main and Jones (2003), i.e. Eq. (10).

c) A single taut cable with two transverse linear viscous dampers

Due to geometric constraint, when an external damper is used to suppress stay cable vibrations, it is commonly installed very close to the cable anchorage on bridge deck, which limited its contribution to supplemental damping. To obtain a better system performance, Caracoglia and Jones (2007) explored the behaviour of a fixed-fixed taut cable attached with two discrete transverse linear viscous dampers at arbitrary locations, as illustrated in Figure 7.4(c). The taut cable has a length \( L \), a unit length mass \( m \) and a tension \( H \). The two dampers are installed respectively at a distance of \( l_1 \) towards the left cable support and \( l_3 \) towards the right cable support. Their capacities are denoted by damping coefficients \( c_1 \) and \( c_2 \), or non-dimensional damping parameters of \( \mu_k = c_k/\sqrt{mH} \) (k=1, 2), respectively.

When applying the proposed generalized approach, the taut cable with a single damper (damper 1) system in Figure 7.4(a) is taken as the parent system, whereas damper 2 is considered as the additional connector with its property being \( \Lambda = i \cdot \mu_2 \). The addition of damper 2 divides the original cable segment on the right side of damper 1 into two, with their lengths being \( l_2 \) and \( l_3 \), respectively.
Therefore, the characteristic equation of the current system can be formulated as
\[
\sin(\Omega) + i \cdot \mu_1 \sin(\theta_1) \sin(\theta_2 + \theta_3) + i \cdot \mu_2 \sin(\theta_3) [\sin(\theta_1 + \theta_2) + i \cdot \mu_1 \sin(\theta_1) \sin(\theta_2)] = 0 \quad (7-13)
\]
where the summation of first two terms on the left hand side of Eq. (7-13) is the parent term, and the remaining being the connector term. The split form of \(\sin(\Omega)\) (or \(\sin(\theta_1 + \theta_2 + \theta_3)\)) and \(\sin(\theta_2 + \theta_3)\) in the parent term is given as \(\sin(\theta_1 + \theta_2) \sin(\theta_3)\) and \(\sin(\theta_2) \sin(\theta_3)\) in the connector term, respectively. It is interesting to note that the left hand side of Eq. (7-13) is the same as the system characteristic polynomial derived analytically by Caracoglia and Jones (2007).

d) A two-cable network with connection to ground

The generalized approach proposed in the current study is not restricted to formulate the characteristic equation of a single cable when it is equipped with additional connector(s) but can also be applied to a cable network supplemented by extra damper and/or cross-tie. In the latter case, the non-dimensional damper parameter \(\mu\) and the non-dimensional cross-tie flexibility parameter \(\psi\) are obtained by normalizing the damping coefficient \(c\) of the additional damper and the flexibility of the additional cross-tie with respect to the property of the vulnerable cable (cable 1) in the network. The connector property \(\Lambda\) is defined by \(\Lambda = i \cdot \mu/\gamma_k\) and \(\Lambda = 1/(\gamma_k \psi \Omega)\), respectively, if the external damper or cross-tie/spring is installed on the \(k^{th}\) cable in the network, where \(\gamma_k\) is the mass-tension ratio of the \(k^{th}\) cable. In the case of a two-cable network
with connection to ground, a vulnerable cable (cable 1) and a neighboring cable (cable 2) are laid in parallel with each other and interconnected by a transverse flexible cross-tie. A linear viscous damper or another flexible cross-tie is installed in-line with the existing cross-tie and connects it with ground (or bridge deck). We could consider a more general case in formulating the system characteristic equation, of which it is assumed that a linear viscous damper is attached transversely to the neighboring cable at a distance $l_{2,1}$ from its left support, as shown in Figure 7.4(d). $L_k$, $m_k$ and $H_k$ denotes, respectively, the length, the mass per unit length and the tension of cable $k$ ($k=1,2$).

The damper capacity is given as a damping coefficient $c$, whereas the flexibility of the cross-tie is represented by a linear spring constant $K_c$. The parent system here is the two-cable network with a transverse flexible cross-tie, whereas the damper can be viewed as the additional connector. The characteristic equation of the parent system has been derived earlier in Eq. (3-5), which has the form of

$$\gamma_1 \sin(\Omega_1) \sin(\phi_{2,1} + \phi_{2,3}) + \gamma_2 \sin(\Omega_2) \sin(\phi_{1,1}) \sin(\phi_{1,2}) + \psi \Omega_1 \gamma_2 \sin(\Omega_1) \sin(\Omega_2) = 0$$  \hspace{1cm} (7-14)
Since the additional damper is installed on cable 2, the connector property is \( \Lambda = i \cdot \mu \gamma_2 \), where 
\( \mu = c / \sqrt{m_1 H_1} \) is the non-dimensional damping parameter. The installation of the additional damper divides the cable segment on the left side of the cross-tie in cable 2 into two parts, with their length being \( l_{2,1} \) and \( l_{2,2} \), respectively. Thus, the split form of the associated terms in the parent system, i.e. \( \sin(\varphi_{2,1} + \varphi_{2,2}) \) and \( \sin(\Omega \eta_2) \) (or \( \sin(\varphi_{2,1} + \varphi_{2,2} + \varphi_{2,3}) \) on the left hand side of Eq. (7-14), are \( \sin(\varphi_{2,1}) \sin(\varphi_{2,2}) \) and \( \sin(\varphi_{2,1}) \sin(\varphi_{2,2} + \varphi_{2,3}) \), respectively. Therefore, based on Eqs. (7-9) and (7-10), the characteristic equation of the hybrid system shown in Figure 7.4(d) can be expressed as

\[
\gamma_1 \sin(\Omega \eta_1) \sin(\varphi_{2,1} + \varphi_{2,2}) \sin(\varphi_{2,3}) + \gamma_2 \sin(\Omega \eta_2) \sin(\varphi_{1,1}) \sin(\varphi_{1,2}) \\
+ \psi \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) + \ i \cdot (\mu / \gamma_2) \sin(\varphi_{2,1}) \gamma_1 \sin(\Omega \eta_1) \sin(\varphi_{2,2}) \sin(\varphi_{2,3}) \\
+ \gamma_2 \sin(\varphi_{2,2} + \varphi_{2,3}) \sin(\varphi_{1,1}) \sin(\varphi_{1,2}) + \psi \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \sin(\varphi_{2,2} + \varphi_{2,3}) \right] = 0 \quad (7-15)
\]

Installing an external damper in-line with the cross-tie in a two-cable network can be considered as a special case of the system in Figure 7.4(d), of which \( l_{2,2} \) and \( l_{2,3} \) in Eq. (7-15) are replaced by 0 and \( l_{2,2} \), respectively, which gives

\[
\gamma_1 \sin(\Omega \eta_1) \sin(\varphi_{2,1}) \sin(\varphi_{2,2}) + \gamma_2 \sin(\Omega \eta_2) \sin(\varphi_{1,1}) \sin(\varphi_{1,2}) + \psi \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) \\
+ \ i \cdot (\mu / \gamma_2) \sin(\varphi_{2,1}) \sin(\varphi_{2,2}) \left[ \gamma_2 \sin(\varphi_{1,1}) \sin(\varphi_{1,2}) + \psi \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \right] = 0 \quad (7-16)
\]

Similarly, if the additional damper is now replaced by a cross-tie which has a stiffness \( K_g \) and is installed in-line with the existing cross-tie and anchored to the bridge deck, the associated system characteristic equation can be obtained by replacing the connector property \( i \cdot \mu \) in Eq. (7-16) with \( 1 / (\psi_g \Omega) \), where \( \psi_g = H_1 / (K_g L_1) \) is the non-dimensional flexibility parameter of the cross-tie connecting to the deck. Now if we assume this hybrid system has a symmetric layout with \( l_{1,1} = l_{1,2} \) and \( l_{2,1} = l_{2,2} \), and the two cables have the same mass-tension ratio, its characteristic
equation can be obtained by substituting $\gamma_1 = \gamma_2 = 1, \eta_1 = 1, \varphi_{1,1} = \varphi_{1,2} = \Omega \eta_1 / 2$ and 
$\theta_{2,1} = \theta_{2,2} = \Omega \eta_2 / 2$ into Eq. (7-16), which gives

$$
2 \sin \left( \frac{\alpha}{2} \right) \sin \left( \frac{\alpha}{2} \eta_2 \right) \left\{ \cos \left( \frac{\alpha}{2} \right) \sin \left( \frac{\alpha}{2} \eta_2 \right) + \sin \left( \frac{\alpha}{2} \right) \cos \left( \frac{\alpha}{2} \eta_2 \right) \right. \\
+ 2 \psi \Omega \cos \left( \frac{\alpha}{2} \right) \cos \left( \frac{\alpha}{2} \eta_2 \right) + \frac{1}{\psi \Omega} \left[ \frac{1}{2} \sin \left( \frac{\alpha}{2} \right) \sin \left( \frac{\alpha}{2} \eta_2 \right) + \psi \Omega \cos \left( \frac{\alpha}{2} \right) \sin \left( \frac{\alpha}{2} \eta_2 \right) \right] \right\} = 0 \quad (7-17)
$$

Caracoglia and Jones (2005a) derived the characteristic polynomial of a symmetric SMT (same mass-tension ratio) two-cable network system. The cross-tie in the network was extended to the ground by a connector having non-dimensional flexibility parameter of $d_G$. This characteristic polynomial was expressed as a product of two terms given by Eqs. (28a) and (28b) in the reference (Caracoglia and Jones, 2005a). The solution to the former represents the local modes that were not influenced by the presence of the cross-tie and the ground connector, i.e. the anti-symmetric local modes derived by oscillations either isolated main cable 1 or isolated main cable 2, whereas the solution to the latter was associated with the global modes of cable network, the modal properties of which were affected by the presence of the two transverse connectors. A closer look at the form of the current Eq. (7-17) and Eqs. (28a) and (28b) as used by Caracoglia and Jones (2005a) reveals that the former, which is derived by the generalized formulation approach, is the same as their characteristic polynomial. Applying trigonometric conversion to the term outside the curly brackets on the left hand side of Eq. (7-17), i.e.

$$
2 \sin \left( \frac{\alpha}{2} \right) \sin \left( \frac{\alpha}{2} \eta_2 \right) = \cos \left[ \frac{\alpha}{2} (1 - \eta_2) \right] - \cos \left[ \frac{\alpha}{2} (1 + \eta_2) \right] \quad (7-18)
$$

Noticing the non-dimensional system frequency $\Omega$ in this study equals to “$\alpha \pi$” as used by (Caracoglia and Jones, 2005a), this rewritten form is the same as the first part of the characteristic polynomial in Eq. (28a) (please note, equation (28a) has a typo error, “$\alpha \pi$” in the equation should be “$\alpha \pi / 2$” instead, please refer to Eq. (26) in the same reference for the correct
form). For the term inside the curly bracket of the Eq. (7-17), if it is divided by \( \cos\left(\frac{\Omega}{2}\right) \cos\left(\frac{\Omega}{2} \eta_2\right) \), then it should become

\[
\tan\left(\frac{\Omega}{2} \eta_2\right) + \tan\left(\frac{\Omega}{2}\right) + 2\psi \Omega + \frac{1}{2\psi_g \Omega} \tan\left(\frac{\Omega}{2}\right) \tan\left(\frac{\Omega}{2} \eta_2\right) + \frac{\psi}{\psi_g} \tan\left(\frac{\Omega}{2} \eta_2\right) = 0 \quad (7-19)
\]

which is the same as the second part of the system characteristic polynomial, i.e. Eq. (28b) (Caracoglia and Jones, 2005a). Therefore, by rewriting the term outside the curly bracket of Eq. (7-17) according to Eq. (7-18) and dividing both sides of Eq. (7-17) by \( \cos\left(\frac{\Omega}{2}\right) \cos\left(\frac{\Omega}{2} \eta_2\right) \), the system characteristic equation, Eq. (7-17), which is formulated by the proposed generalized approach, would have the same form as that analytically derived by Caracoglia and Jones (2005a), with the substitution of \( \Omega, \eta_2, \psi \) and \( \psi_g \) in the current Eq. (7-17) to the symbols, \( \alpha \pi, f, d_k \) and \( d_G \) used by Caracoglia and Jones (2005a).

e) A two-cable network with transverse linear viscous dampers installed on both cables

A hybrid system consisting of two horizontally laid main cables interconnected by a transverse cross-tie, with each cable equipped with a linear viscous damper close to cable anchorage is portrayed in Figure 7.4(e). The modal behaviour of this system under a special condition of parallel main cables and symmetric layout was discussed by Zhou et al. (2015).
If assume the hybrid system in Figure 7.1 as the parent system, and the damper installed on cable 2 as the additional connector, by applying the proposed generalized formulation approach, the parent term in the system characteristic equation would be the left hand side of Eq. (7-8), with $i \cdot \mu$ replaced by $i \cdot \mu_1 / \gamma_1$, $\phi_{2,1}$ by $(\phi_{2,1} + \phi_{2,2})$ and $\phi_{2,2}$ by $\phi_{2,3}$, i.e.

$$
y_1 \sin(\Omega \eta_1) \sin(\phi_{2,1} + \phi_{2,2}) \sin(\phi_{2,3}) + y_2 \sin(\Omega \eta_2) \sin(\phi_{1,1} + \phi_{1,2}) \sin(\phi_{1,3}) + \psi \Omega \gamma_1 y_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2)
$$

$$+ \frac{i \cdot \mu_1}{\gamma_1} \sin(\phi_{1,1}) \sin(\phi_{1,2} + \phi_{1,3}) \sin(\phi_{2,1} + \phi_{2,2}) \sin(\phi_{2,3}) + y_2 \sin(\Omega \eta_2) \sin(\phi_{1,2}) \sin(\phi_{1,3}) + \psi \Omega \gamma_1 y_2 \sin(\phi_{1,2} + \phi_{1,3}) \sin(\Omega \eta_2)$$

(7-20)

The connector property would be $\Lambda = i \cdot \mu_2 / \gamma_2$. The addition of a damper to cable 2 divides the cable segment on the left of the cross-tie into two. Thus, the split form of the associated terms in Eq. (7-20), i.e. $\sin(\phi_{2,1} + \phi_{2,2})$ and $\sin(\Omega \eta_2)$ (note: $\sin(\Omega \eta_2) = \sin(\phi_{2,1} + \phi_{2,2} + \phi_{2,3})$), becomes $\sin(\phi_{2,1}) \sin(\phi_{2,2})$ and $\sin(\phi_{2,1}) \sin(\phi_{2,2} + \phi_{2,3})$, respectively. So the connector term in the system characteristic equation can be written as

$$(i \cdot \frac{\mu_2}{\gamma_2}) \sin(\phi_{2,1} \{y_1 \sin(\Omega \eta_1) \sin(\phi_{2,2}) \sin(\phi_{2,3}) + y_2 \sin(\phi_{2,2} + \phi_{2,3}) \sin(\phi_{1,1} + \phi_{1,2}) \sin(\phi_{1,3})}$$
Finally, the characteristic equation of the hybrid system in Figure 7.4(e) can be obtained by setting the algebraic sum of Eqs. (7-20) and (7-21) to zero. The so obtained characteristic equation can also be expressed in a similar form by separating the “flexible cross-tie” and the “main cable” terms. The flexible cross-tie term (all the terms in Eqs. (7-20) and (7-21) which contain $\psi \Omega$) can be written as

$$
\psi \Omega y_1 y_2 \left[ \sin(\Omega \eta_1) \sin(\Omega \eta_2) + i \cdot \frac{\mu_1}{y_1} \sin(\phi_{1,1}) \sin(\phi_{1,2} + \phi_{1,3}) \sin(\Omega \eta_1) \sin(\Omega \eta_2) + i \cdot \frac{\mu_2}{y_2} \sin(\phi_{2,1}) \sin(\phi_{2,2} + \phi_{2,3}) \right]
$$

Or in a summation form as

$$
\psi \Omega y_1 y_2 \prod_{j=1,2} \left[ \sin(\Omega \eta_1) + i \cdot \frac{\mu_1}{y_1} \sin(\phi_{j,1}) \sin(\phi_{j,2} + \phi_{j,3}) \right]
$$

Similarly, the “main cable” term (all the terms in Eqs. (7-20) and (7-21) which don’t contain $\psi \Omega$) can be written in a descending order of $\sin(\phi_{j,3})$ ($j=1,2$) as

$$
\gamma_1 \sin(\phi_{2,3}) \left[ \sin(\Omega \eta_1) \sin(\phi_{2,1} + \phi_{2,2}) + i \cdot \frac{\mu_1}{y_1} \sin(\phi_{1,1}) \sin(\phi_{1,2} + \phi_{1,3}) \sin(\phi_{2,1} + \phi_{2,2}) + i \cdot \frac{\mu_2}{y_2} \sin(\phi_{2,1}) \sin(\phi_{2,2}) \right]
$$

Again, by rearranging and factorization, Eq. (7-24) can be written as
or in a summation form

\[
\sum_{j=1,2} \gamma_j \sin(\phi_{3-j,3}) \left[ \sin(\Omega \eta_j) + i \cdot \frac{\mu_j}{\gamma_j} \sin(\phi_{j,1}) \sin(\phi_{j,2} + \phi_{j,3}) \right] \\
\sin(\phi_{3-j,1} + \phi_{3-j,2}) + i \cdot \frac{\mu_{3-j}}{\gamma_{3-j}} \sin(\phi_{3-j,1}) \sin(\phi_{3-j,2}) \right] \tag{7-26}
\]

Now the characteristic equation of the hybrid system shown in Figure 7.4(e) becomes the summation of Eqs. (7-23) and (7-26) equals to zero. Divide both sides of the obtained equation by \(\psi \Omega \gamma_1 \gamma_2\), it yields

\[
\prod_{j=1,2} \left[ \sin(\Omega \eta_j) + i \cdot \frac{\mu_j}{\gamma_j} \sin(\phi_{j,1}) \sin(\phi_{j,2} + \phi_{j,3}) \right] \\
+ \frac{1}{\psi \Omega} \sum_{j=1,2} \frac{1}{\gamma_{3-j}} \sin(\phi_{3-j,3}) \left[ \sin(\Omega \eta_j) + i \cdot \frac{\mu_j}{\gamma_j} \sin(\phi_{j,1}) \sin(\phi_{j,2} + \phi_{j,3}) \right] \\
\sin(\phi_{3-j,1} + \phi_{3-j,2}) + i \cdot \frac{\mu_{3-j}}{\gamma_{3-j}} \sin(\phi_{3-j,1}) \sin(\phi_{3-j,2}) \right] = 0 \tag{7-27}
\]

Zhou et al. (2015) derived the characteristic equation of the same hybrid system by using the analytical approach, which is Eq. (3) in their formulation, except it was expressed in the form of hyperbolic function and also different symbols were used for system parameters. It is worth pointing out that as indicated by Main and Jones (2002), the assumed solution to Eq. (7-1), which is the equation for the transverse oscillation of each cable segment, can be expressed either in the form of \(v(x, t) = \tilde{v}(x)e^{\omega t}\), where \(\tilde{v}(x)\) is the shape function and \(\omega\) is a dimensional less complex eigenvalue, or in the form of \(v(x, t) = \tilde{v}(x)e^{i\omega t}\). The former form would render the eigenfunctions to be expressed in the form of hyperbolic functions, whereas the latter would yield trigonometric expression for the eigenfunctions. These two forms of eigenfunction expressions are equivalent. Zhou et al. (2015) assumed the cable motion solution in the form of
\[ v(x,t) = \tilde{v}(x)e^{\omega t} \] while the current study assumed it in the form of \[ v(x,t) = \tilde{v}(x)e^{i\omega t}. \] This difference resulted the final form of the system characteristic equation by the current study remains in the Sine function form, while that by Zhou et al. (2015) was expressed as hyperbolic Sine function. By comparing the definition of the system parameters used by Zhou et al. (2015) and the current study, the following equivalence in symbols are found, i.e. \( \Omega \eta_j = \Gamma_j, \varnothing_{j,k} = \Gamma_{j,k}, \mu_j = \eta_j, \gamma_j = 1/\nu_j, \psi = 1/(\pi\gamma_1) \) and \( \Lambda = \Omega/\pi. \) Substitute these into Eq. (7-27), the characteristic equation of the hybrid system in Figure 7.2(e), derived by applying the proposed generalized formulation approach turns to be the same as that by Zhou et al. (2015).

The five different cases presented above clearly shows the validity of the proposed generalized approach in formulating the characteristic equation for a single cable or a cable network equipped with external damper(s) and/or cross-tie(s). In particular, it allows to conveniently develop characteristic equation of hybrid systems with various configurations, which will greatly assist in appreciating the unique behaviour associated with different hybrid systems and evaluating their respective effectiveness in suppressing cable vibrations.

### 7.5 Parametric Study

To better understand the dynamic behaviour of the hybrid system shown in Figure 7.1, the influence of different system parameters on its response needs to be comprehended. The characteristic equation of this hybrid system is given in Eq. (7-8), from which two types of system parameters can be identified. The first type is related to the mechanical and material properties of the two main cables, i.e. the frequency ratio \( \eta \) and the mass-tension ratio \( \gamma \), whereas the second type is associated with the properties of the damper and the cross-tie, i.e. the non-dimensional damping parameter \( \mu \), the non-dimensional cross-tie flexibility parameter \( \psi \), and the segment ratio \( \varepsilon \) which defines the installation location of the damper, the cross-tie, and the
spacing between them. In general, the properties of the main cables are determined at the stage of bridge design based on load resistance requirement. As for the damper, its installation location is dictated by geometric restriction and its capacity could be chosen based on damping optimization of certain selected mode(s) using the universal damping estimation curve developed by Pacheco et al. (1993). This would leave the selection of cross-tie stiffness and its installation location, or the spacing between the cross-tie and the damper, to be the main task in the hybrid system design. Therefore, the parametric study conducted in this section would focus on the impact of cross-tie stiffness and the spacing between its installation location and the damper position on the performance of the hybrid system in Figure 7.1, in terms of the in-plane frequency, the modal damping and the global nature of the system fundamental mode. In Chapter 5, the author proposed a new concept, i.e. the “degree of mode localization (DML)” to evaluate the global nature of a cable network mode. The DML coefficient of a network mode is defined as

\[
DML = \frac{n_c(n_t+1)-n_v}{n_c(n_t+1)-1}
\]  

(5-3)

where \(n_c\) and \(n_t\) are respectively the number of main cables and the number of cross-ties in the cable network, \(n_c(n_t+1)\) is the total number of cable segments in the network, and \(n_v\) represents the number of vibrating cable segments in the evaluated network mode, which is determined from

\[
n_v = \frac{\sum_{k=1}^{n_c} \sum_{j=1}^{n_t+1} \bar{v}_{k,j}^2 \bar{v}_{k,j}^2}{\sum_{k=1}^{n_c} \sum_{j=1}^{n_t+1} \bar{v}_{k,j}^2}
\]  

(5-4)

where \(\bar{v}_{k,j}\) is the absolute value of the modal amplitude associated with the \(j^{th}\) segment in the \(k^{th}\) cable. Based on the above definition, the DML value of any mode would vary between 0 and 1, with the former represents a pure global mode of which all cable segments in a network have the same modal amplitude distribution pattern, and the latter stands for a pure local mode with
energy confined in one of the segments in a cable network. The same concept will be adopted in the current study to evaluate the global nature of the natural modes in a hybrid system.

The two-cable hybrid system used in the parametric study has the same layout as the one shown in Figure 7.1, except the two cables are assumed to have the same length and there is no offset on the left and the right end of the neighboring cable. This configuration is chosen with the objective to study the effect of cross-tie when it is placed between the damper and the cable left support. Besides, a frequency ratio parameter of $\eta_2 = 0.80$ and a mass-tension ratio parameter of $\gamma_2 = 1.15$ are assumed for the two consisting cables. A damper having a non-dimensional damper parameter of $\mu = 0.83$ is assumed to attach to the target cable at 6.5% of its length from the left support.

### 7.5.1 Cross-tie stiffness

In order to explore the influence of cross-tie stiffness on the modal behaviour of the hybrid system fundamental mode, the range of the non-dimensional cross-tie flexibility parameter $\psi$ is taken as 0 to 1.0 (Caracoglia and Jones, 2005a), which represents the condition of rigid to very flexible cross-tie. In addition, four different cross-tie installation locations, i.e. $\varepsilon_c = 1/4, 1/3, 1/2$ and $3/4$, where $\varepsilon_c = \varepsilon_{1,1} + \varepsilon_{1,2}$ (Figure 7.1) are chosen in this study. The effect of cross-tie stiffness on the fundamental frequency of a two-cable network with either symmetric or asymmetric layout is studied in Chapter 6. It was found that as the cross-tie flexibility increases, the fundamental frequency of the cable network would gradually decrease and approach to that of the isolated target cable (assume the neighboring cable has higher in-plane stiffness). By adding an external damper to the target cable close to one of its end supports to form the current hybrid system, the effect of the cross-tie stiffness on the fundamental frequency of the hybrid system is expected to remain the same as that of the pure cable network.
Figure 7.5 illustrates how the variation in cross-tie stiffness would affect the modal damping ratio of the hybrid system fundamental mode. Besides the reference case of the isolated damped target cable which is shown in a thick solid line, each of the rest four curves represents the $\xi_1$-$\psi$ relation at a specific cross-tie position $\varepsilon_c$.

It can be clearly observed from Figure 7.5 that all four $\xi_1$-$\psi$ curves have the same pattern, i.e. with the gradual increases of cross-tie flexibility, damping ratio of the hybrid system fundamental mode would increase accordingly. This implies that choosing “softer” cross-tie would help the hybrid system to dissipate more energy, which agrees with the experience from a number of experimental studies on cable networks (Yamaguchi and Nagahawatta, 1995; Sun et al., 2007). When a more flexible cross-tie is used, the motion of the damped target cable would
be less constrained by the presence of the neighboring cable. Thus, the damper would provide higher amount of damping to the system due to more “active” oscillation of the target cable at the damper installation location. This modal damping increment is more obvious if the cross-tie is located closer to the damper. For example, if the cross-tie is installed at the quarter span of the target cable, i.e. $\varepsilon_c =1/4$, by varying the cross-tie flexibility from $\psi =0$ to $\psi =1.0$, the fundamental modal damping ratio of the hybrid system would increase from 0.43% to 0.87%, which is doubled. On the other hand, by moving the cross-tie to $\varepsilon_c =3/4$, the same change in cross-tie flexibility would lead to a 21% increase in the modal damping ratio from 0.91% to 1.1%. It is interesting to note that in the latter case, once the cross-tie flexibility reaches $\psi =0.6$, further increase in $\psi$ would have a negligible effect on the system modal damping ratio. This is due to the effect of cross-tie on constraining the motion of cable segment at the damper installation location would become less as it is placed further away from the damper. Therefore, when the cross-tie is flexible enough, the target cable hardly “feels” the presence of the cross-tie and its behaviour would not be affected by further reducing the cross-tie stiffness. The above observations suggest that in order to achieve higher modal damping in a hybrid system, the constraint on the external damper should be lifted by either using more flexible cross-tie or installing cross-tie away from the damper. If a cross-tie has to be placed close to the damper, flexible cross-tie would be a preferable choice.

The effect of cross-tie stiffness on the global nature of the hybrid system fundamental mode is shown in Figure 7.6, of which the relation between the non-dimensional cross-tie flexibility parameter and the DML coefficient of the fundamental mode is plotted for the cross-tie locations of $\varepsilon_c =1/4, 1/3, 1/2$ and 3/4.
Figure 7.6: Effect of cross-tie flexibility parameter $\psi$ on the DML coefficient of the fundamental mode of a two-cable hybrid system ($\eta_2=0.80$)

It is worth noting that because of the symmetric layout of the two main cables and close proximity of the damper to cable support, installing cross-tie at $\varepsilon_c=1/4$ and $3/4$ would yield almost the same DML coefficient for the system fundamental mode. Results show that for all four studied cross-tie locations, the DML coefficient of the fundamental mode increases monotonically with larger $\psi$ value, which means that the use of stiffer cross-tie would result in a more “global” fundamental mode. When a more rigid cross-tie is used, motions of the two main cables are more affected by each other, and their respective amplitudes are more or less at the same level. Thus, energy would be distributed more evenly over different cable segments, which results in a more global mode. As the cross-tie becomes softer, oscillation of the target cable
would be less constrained by the neighboring cable and manifests more significant motion than the neighboring one. This makes more energy to be confined within the target cable and its motion becomes more dominant in the fundamental mode. Therefore, the fundamental mode begins to have more “localized” feature. Besides, it is also noticed from Figure 7.6 that placing a cross-tie closer to the mid-span would yield a lower value of DML coefficient for the fundamental mode. Installing a cross-tie closer to the cable mid-span would result in cable segments of more even length, which allows more uniform distribution of energy during system vibration and thus a more “global” fundamental mode. By comparing the variation of the fundamental mode DML coefficient over the $\psi$ range of 0 to 1.0 at the four cross-tie locations, it is found that the cross-tie flexibility has a more considerable impact on the global nature of a hybrid system fundamental mode when the cross-tie is installed closer to the cable support. For example, when $\varepsilon_c = 1/4$ (or $\varepsilon_c = 3/4$), as $\psi$ changes from 0 to 1.0, the DML coefficient increases gradually from 5% to 40% and becomes a local mode; whereas if move the cross-tie to the cable mid-span, i.e. $\varepsilon_c = 1/2$, the same change in the cross-tie flexibility would render the DML coefficient increase from 0 to 24%.

Overall, the above results show that the change in cross-tie flexibility would have different effect on the modal frequency, the modal damping and the DML coefficient associated with the fundamental mode of a hybrid system. While choosing a more rigid cross-tie would help to enhance the in-plane stiffness of a hybrid system and promote the global nature of the mode, a more flexible cross-tie would provide more “freedom” to damper operation and thus assist in dissipating more energy from the oscillating system. Therefore, a careful balance between the advantages and the disadvantages should be made in selecting cross-tie stiffness.
7.5.2 Spacing between damper and cross-tie

It was observed in Figure 7.5 that the studied hybrid system would have higher modal damping if the cross-tie is placed far from the damper so that the oscillation of the cable segment at the damper location would be less constrained by the presence of the cross-tie. The effect of the spacing between the damper and the cross-tie on the modal properties of the hybrid system fundamental mode will be further explored in this section. This spacing is denoted by the non-dimensional spacing parameter \( \varepsilon_{1,2} = l_{1,2}/L_1 \) (Figure 7.1). In the analysis, it is assumed that the installation location of the cross-tie could vary from the left support to the right support of the target cable. Thus, the range of \( \varepsilon_{1,2} \) varies from -0.065 to 0.935 in the parametric study. The negative value of the spacing parameter represents that the cross-tie is placed between the left cable support and the damper whereas \( \varepsilon_{1,2} = 0 \) stands for the case cross-tie installed in-line with the damper. The cross-tie is assumed to have four different levels of stiffness represented by \( \varphi = 0.0, 0.02, 0.10, 1.0 \). In the current hybrid system configuration, due to the close proximity of the external damper to the target cable left end, the presence of the damper is as if the effective length of the target cable is slightly reduced, in particular for the rigid cross-tie case. Besides, as discussed in the previous section, since installing a damper close to target cable support would have negligible effect on the system frequency, the effect of the spacing parameter \( \varepsilon_{1,2} \) on the system in-plane stiffness would thus be similar to that of the cross-tie installation location, \( \varepsilon_c = \varepsilon_{1,2} + \varepsilon_{2,2} \), on the modal frequency of the corresponding two-cable network. The latter has been already investigated in Chapter 5, the results of which indicated that placing cross-tie closer to cable mid-span (or larger spacing between damper and cross-tie in the current case) would be beneficial to increase system frequency associated with in-plane oscillation.
The influence of the damper-cross-tie spacing on the fundamental modal damping ratio of the studied hybrid system is portrayed in Figure 7.7.

Figure 7.7: Effect of damper-cross-tie spacing parameter $\varepsilon_{1,2}$ on the fundamental modal damping ratio of a two-cable hybrid system ($\eta_2=0.80$)

The thick solid line in the figure represents the fundamental modal damping ratio of the isolated damped target cable. It is given here as a reference base. Each of the rest four $\xi_{1,-}$ curves corresponds to a specific level of cross-tie flexibility. In general, the variation of $\xi_1$ with respect to the spacing parameter $\varepsilon_{1,2}$ has the same pattern for all four investigated cross-tie flexibility levels. The two extreme cases of $\varepsilon_{1,2} = -0.065$ and 0.935 represent the cross-tie is installed at either the left or the right cable support, which would not affect the response of the hybrid system. Thus, irrelevant to the cross-tie flexibility, the system fundamental modal damping ratio of all four cases would be the same as that of the isolated damped target cable. Other than these
two extreme positions, as expected, using more flexible cross-tie would yield higher system modal damping ratio except when the cross-tie is installed very close to the right cable support (far end of the damper).

Besides, it is interesting to note that each $\xi_1$-$\varepsilon_{1,2}$ curve has three zones. The first zone corresponds to the case of cross-tie installed between the cable left end and the location where the system modal damping ratio drops to the minimum. In the case of rigid cross-tie, this range covers $\varepsilon_{1,2}=-0.065$ to 0, i.e. from cable left end to the damper location. It would extend slightly beyond the damper location should the cross-tie become more flexible. Within this zone, a sharp drop of $\xi_1$ can be seen when moving the cross-tie from the cable left support towards the damper location. This reduction is more considerable in the case of rigid cross-tie. Similar phenomenon was reported by Takano et al. (1997) and Main and Jones (2003) in two independent studies conducted to explore the effect of neoprene rubber bushings on the modal damping of a single cable equipped with an external viscous damper. The neoprene rubber bushings are installed near the cable anchorage to strength the in-plane stiffness of a stay cable. Its behaviour can be simulated as a linear transverse spring installed between the cable anchorage and the damper. Thus, its presence would reduce the effective damper length (Main and Jones, 2003) and render the damper to be closer to a constraint, so the damper would be less active and results in reduced modal damping.

The second zone is the largest, which covers a $\varepsilon_{1,2}$ range from where the system fundamental modal damping ratio is the minimum to where it becomes the same as that of the isolated damped target cable. Within this zone, $\xi_1$ increases monotonically with $\varepsilon_{1,2}$. This is because by gradually moving the cross-tie away from the damper, its constraint on the motion of the cable segment at the damper location would be “lifted”. Therefore, the damper would operate
more actively to provide supplemental damping to the system. When the cross-tie is far enough from the damper, its presence would no longer affect damper performance. Therefore, the fundamental modal damping ratio of the hybrid system would equal to the case of an isolated target cable attached with a damper. Depending on the cross-tie flexibility, this spacing would require to be larger for more rigid cross-tie. For instance, a rigid cross-tie ($\psi=0$) should be installed at a distance of $\varepsilon_{1,2}=0.80$ from the damper location to yield the same modal damping as the isolated target cable, whereas a flexible cross-tie with $\psi=1.0$ only needs to be moved to $\varepsilon_{1,2}=0.58$ to achieve this.

The third zone is between the intersection of the $\xi_1$ curve and the solid base line for the isolated damped target cable and the cable right support. For example, when $\psi=0.02$, zone 3 is located between $\varepsilon_{1,2}=0.78$ and 0.935. More interestingly, it is observed that within zone 3, a hybrid system with a more rigid cross-tie could achieve a slightly higher fundamental modal damping, which is different from the existing experience of which using more flexible cross-tie is found to be beneficial for dissipating energy. To explain this phenomenon, mode shapes of the system fundamental mode corresponding to the spacing parameter $\varepsilon_{1,2}$ when $\xi_1$ becomes the same as that of the isolated damped target cable and when $\xi_1$ reaches its maximum are plotted for two cross-tie flexibility levels of $\psi=0$ and $\psi=1.0$ in Figure 7.8. Based on the observation from Figure 7.8, it is believed that the unique behaviour of the hybrid system in the third zone could be associated with the development of a “barrier” effect at the cross-tie location.
Compared with the case of $\psi=1.0$, when a rigid ($\psi=0$) cross-tie is used, the installation location of the cross-tie corresponding to the maximum fundamental modal damping ratio of the hybrid system results in a much shorter right segment in the target cable and the neighboring cable is almost at rest, as can be seen from Figure 7.8. In this case, the rigid cross-tie behaves more like a support for the target cable. It is as if, a “barrier” exists at the cross-tie location to prohibit the propagation of cable motion to the right segment so the short right cable segment becomes less active in oscillation (compare to $\varepsilon_{1,2}=0.78$, $\psi=0.0$ case on the left of Figure 7.8). Therefore, the presence of a rigid cross-tie would shorten the effective length of the target cable, which would consequently increase the non-dimensional damper location $\varepsilon_{1,1}$ which is the ratio between $l_{1,1}$ and the effective cable length. This would lead to an increase in the equivalent damping of the damped target cable and thus the system damping level. A further displacement of the cross-tie towards cable right support would increase the target cable effective length. Therefore, the non-dimensional damper location $\varepsilon_{1,1}$ would be reduced, resulting in a decrease in the equivalent damping of the damped target cable and thus system damping level, as it can be observed from Figure 7.7. However, this “barrier” effect would be less prominent should a more flexible cross-tie be used (refer to the $\varepsilon_{1,2}=0.72$, $\psi=1.0$ case in Figure 7.8). This explains why in zone 3 of the
$\varepsilon_{1,2}-\xi_1$ curve, using a more rigid cross-tie would yield a slightly higher modal damping ratio. In addition, it is noticed from Figure 6 that irrespective of the cross-tie flexibility, there always exists a specific cross-tie location, of which the spacing between the damper and the cross-tie would maximize the “barrier” effect and lead to the highest system fundamental modal damping ratio, i.e. $\varepsilon_{1,2}=0.86, 0.86, 0.82$ and 0.72 for the cross-tie flexibility of $\psi=0, 0.02, 0.10$ and 1.0, respectively. Beyond this point, this “barrier” effect gradually diminishes and would disappear completely when the cross-tie is at the cable right support where the system fundamental modal damping ratio becomes the same as that of isolated damped target cable again.

It is worth pointing out that although the results in Figure 7.7 suggest that by increasing the spacing between the damper and the cross-tie and installing the latter close to the cable right support would help to obtain higher system damping, which agrees with the recent finding by Zhou et al. (2015), such a configuration would reduce the global nature of the fundamental mode and render it to be a local mode. This is clearly reflected in Figure 7.9, of which the DML coefficient of the hybrid system is plotted against the variation of the spacing parameter $\varepsilon_{1,2}$.
Figure 7.9: Effect of damper-cross-tie spacing parameter $\varepsilon_{1,2}$ on the DML coefficient of the fundamental mode of a two-cable hybrid system ($\eta_2=0.80$)

As a matter of fact, since the damper is installed very close to the left end of the target cable, the effect of the spacing parameter $\varepsilon_{1,2}$ on the DML coefficient of the hybrid system fundamental mode is almost the same as the effect of the cross-tie location $\varepsilon_c$ on the fundamental modal nature of the corresponding two-cable network. The four DML-$\varepsilon_{1,2}$ curves in Figure 7.9 are approximately symmetric about the cable mid-span where $\varepsilon_{1,2}=0.435$ (or $\varepsilon_c=0.50$). The mode would be more “global” if the cross-tie is placed closer to the cable mid-span, whereas it would become a local mode if the cross-tie is installed towards either end of the cable. In the former case, the presence of cross-tie would divide the main cables into more even segments which lead to more uniform energy distribution within the system, whereas in the latter, the energy will be confined within certain longer and more flexible cable segments (refer to Figure 7.8). The motion of these cable segments would dominate the mode so it becomes “local”. Therefore, although by placing the cross-tie far from the damper and close to the cable right support could
yield a higher system modal damping, the modal nature of the fundamental mode would be changed. In addition, the fundamental modal frequency would be less if the cross-tie is installed closer to the cable ends. Thus, the range of the spacing parameter $\varepsilon_{1,2}$ corresponding to zone 3 in Figure 7.7 would not be a preferable design choice. Overall, by considering the effect of the spacing parameter on the frequency, damping ratio and global nature of the system fundamental mode, in the current case, placing the cross-tie within the range of $\varepsilon_{1,2} = 0.4~0.6$ would be an optimized design solution.

### 7.6 Design Optimization

When external damper(s) and cross-tie(s) are implemented together to form a hybrid system to control excessive vibration of a vulnerable cable, the effectiveness of the hybrid system design could be evaluated based on three indices, i.e. the in-plane stiffness and damping property of the system, as well as the severity of local mode formulation. To optimize the design of a hybrid system, effort should be made to maximize its in-plane frequency and damping while minimize the formation of local modes. It is understood from the parametric study conducted in the previous section that variation of system parameters could have different impact on these three indices. For example, using a more rigid cross-tie would help to enhance the system in-plane stiffness and retain the global nature of system fundamental mode, but would not be favorable for increasing system damping. Although placing a cross-tie far away from the damper or close to the cable support on the other end could help to obtain higher system damping, it would yield a mode dictated by the oscillation of certain parts of the system and is also limited in increasing system in-plane frequency. Besides, different system parameters could vary simultaneously in practice. This would make the resulting impact on system response even more
complicated. Therefore, it is imperative to have a tool for assessing the influence of multiple system parameter variations on the modal behaviour of a hybrid system.

In the current study, it is proposed to apply the concept of “isoquant curve” to evaluate how simultaneous change in cross-tie stiffness and installation location (in terms of the spacing between damper and cross-tie) would affect the in-plane frequency, the damping ratio and the modal nature of the hybrid system fundamental mode and find out the associated parameter ranges to optimize hybrid system design. An isoquant curve is a contour line which shows all possible combinations of two or more inputs which would result in the same output (Chiang, 1984). For the purpose of the current study, the isoquant curves of the frequency, the damping ratio and the DML coefficient of the hybrid system fundamental mode under different combinations of non-dimensional cross-tie flexibility parameter $\psi$ and non-dimensional damper-cross-tie spacing parameter $\varepsilon_{1,2}$ will be plotted in the same figure. Figure 7.10 shows such a sample plot. In Figure 7.10, the same ranges of non-dimensional cross-tie flexibility parameter $\psi$ and damper-cross-tie spacing parameter $\varepsilon_{1,2}$ as used in the parametric study, i.e. $\psi=0$ to 1.0 and $\varepsilon_{1,2} = -0.065$ to 0.935, are adopted to generate the isoquant curves of the non-dimensional modal frequency $\Omega_1$, the modal damping ratio $\xi_1$ and the DML coefficient associated with the hybrid system fundamental mode.
Figure 7.10: Isoquant curves representing in-plane frequency, modal damping ratio and DML of a two-cable hybrid system fundamental mode ($\eta_2=0.80$)

Three isoquant curves of the non-dimensional system fundamental frequency $\Omega_1=1.07\pi$, $1.10\pi$ and $1.11\pi$, as well as three isoquant curves of the system fundamental modal damping ratio $\xi_1=0.5\%$, 0.7\% and 0.85\% are plotted in Figure 7.10. Besides, the isoquant curve corresponding to DML coefficient of 30\% is also shown in the figure with plus (+) and minus (-) sign on the two sides. Defined in Chapter 5, the DML coefficient of a global mode is less than 0.3, whereas that of a local mode is greater than 0.3. Thus, in Figure 7.10, the isoquant curve of DML=0.3 distinguishes the modal nature of the system fundamental mode, with these fall on the minus (-) sign side to be global, and those on the plus (+) sign side to be local. The two zones on the plus (+) sign side of the isoquant curve DML=30\% located at the top and the bottom of Figure 7.10 and extend over the entire studied $\psi$ range. This implies that as far as the cross-tie is...
installed at a distance $\varepsilon_{1,2}$ from the damper within these two zones, then regardless of the cross-tie stiffness, it would always yield a fundamental mode dominated by localized oscillation. Thus, these cross-tie positions should not be considered in design. The same phenomenon can also be observed from Figure 7.9, where the fundamental mode would be dictated by the oscillation of certain part(s) of the hybrid system if the cross-tie is installed too close or too far from the damper.

Other phenomena observed from conventional parametric study in the previous section are also reflected in the isoquant curve plot. As clearly shown in Figure 7.10, when a cross-tie is placed at a specific non-dimensional distance $\varepsilon_{1,2}$ from the existing damper, although choosing a more flexible cross-tie would enhance the energy dissipation capacity of the system, the system modal frequency would reduce and the fundamental mode would become a more localized. This is consistent with the results observed in Figures 7.7 and 7.8. On the other hand, when the cross-tie stiffness is determined, in the majority of cases, there exists two cross-tie positions in terms of the spacing parameter $\varepsilon_{1,2}$ which would yield either the same system fundamental frequency or the same damping ratio. The latter can also be observed from Figure 7.7. However, an isoquant curve plot has the merit to allow a comparison between the impacts of these two cross-tie positions on the other modal properties of the resulted hybrid system to make a better design choice. For example, when the non-dimensional cross-tie flexibility parameter $\psi$ is 0.4, the system fundamental modal damping ratio $\xi_1$ can reach 0.85% when the cross-tie is installed at a non-dimensional distance $\varepsilon_{1,2} = -0.0036$ or $\varepsilon_{1,2} = 0.45$ from the existing damper. When comparing the system modal frequency and modal nature associated with these two cross-tie positions, it can be readily seen from the isoquant curve plot in Figure 7.10 that installing cross-tie at $\varepsilon_{1,2} = -0.0036$ would not only result in a lower system frequency, but also a system fundamental mode
dictated by more localized oscillation with the DML coefficient exceeding 30%. Therefore, to achieve better overall system performance, the cross-tie should be installed at $\varepsilon_{1,2} = 0.45$.

As stated earlier, the isoquant curve plot is very useful in understanding the influence of simultaneous variation of multiple parameters on the system response. Take the bottom left region of Figure 7.10 as an example, when approaching this region, it represents physically that a more stiff cross-tie is installed very close to the external damper, which would not only result in reduced modal frequency and modal damping, but also a more localized modal nature. Thus, a combination of cross-tie stiffness and installation location within this region should be avoided.

If an isoquant curve plot such as Figure 7.10 is available for designing a hybrid system, it would conveniently narrow down the selection of cross-tie stiffness and installation location to be within the region where the modal nature would remain global. In Figure 7.10, that is the region bounded by the isoquant curve of DML=30% on the minus (-) sign side. Then, based on the requirement on how the performance of the vulnerable cable is expected to improve, possible combinations of cross-tie stiffness and installation location can be identified. This will be elucidated using the design example given below. It is worth pointing out that although field incidents show that many large amplitude cable vibrations are dominated by the second, the third or even higher order modes, the example below is based on suppression of cable vibrations dominated by the fundamental mode due to the focus of the current work. However, the same approach as illustrated in the example below can be applied to design a hybrid system for controlling cable vibrations dictated by any other mode(s). Further, by developing isoquant curves for different modes of interest and choosing the cross-tie stiffness and installation location based on overall effectiveness of vibration mitigation, an optimized design choice can be made to achieve multi-mode vibration control for a hybrid system.
Numerical example:
It is assumed that the longest cable on an existing cable-stayed bridge (Yoneda and Maeda, 1989) exhibits undesired vibrations dominated by the fundamental mode. Preliminary analysis results show that to mitigate undesired oscillation, it is required to increase the fundamental modal frequency of the cable by 8% and the associated modal damping ratio should be no less than 0.80%. After installing a linear viscous damper at a distance of 2.35% cable length from its lower anchorage with an optimum capacity of $c=255$ kN·s/m (Pacheco et al., 1993), the fundamental modal damping ratio of the damped cable increases to 1.22%. However, the fundamental modal frequency is hardly changed. To increase the cable modal frequency while keeping its modal damping ratio reasonably high, it is proposed to add a cross-tie to connect the vulnerable cable with its neighboring one to form a hybrid system. Therefore, the resulted hybrid system should help increasing the fundamental frequency of the vulnerable (target) cable by 8% and retain the fundamental modal damping ratio to be no less than 0.8%, i.e. to achieve $\Omega_1=1.08\pi$ and $\xi_1=0.8\%$. In addition, it is required to maintain the global nature of the hybrid system fundamental mode, preferably to keep its DML coefficient less than 20%. Thus, the main design task is to find an appropriate combination of cross-tie stiffness and installation location to satisfy these requirements. The properties of the two cables are:

<table>
<thead>
<tr>
<th>Cable Type</th>
<th>Length $L$ (m)</th>
<th>Tension $H$ (kN)</th>
<th>Mass $m$ (kg/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target (vulnerable)</td>
<td>$L_1=215.11$</td>
<td>$H_1=3690$</td>
<td>$m_1=98.6$</td>
</tr>
<tr>
<td>Neighboring</td>
<td>$L_2=180$</td>
<td>$H_2=3400$</td>
<td>$m_2=88.0$</td>
</tr>
</tbody>
</table>

The two cables are arranged in parallel with each other, with the neighboring cable having an offset of 10 m with respect to the target cable on the left end. Thus, the range of cross-tie position in terms of its spacing against the existing damper would be $\varepsilon_{1,2} = 0.023$ to 0.860 ($\varepsilon_c = 0.046$ to 0.883). The range of non-dimensional cross-tie flexibility parameter $\psi$ is taken as 0 to
The isoquant curves associated with the required design outcomes, i.e. $\Omega_1=1.08\pi$, $\xi_1=0.8\%$ and DML=20% are depicted in Figure 7.11.

![Figure 7.11: Isoquant curves representing in-plane frequency, modal damping and DML on $\psi$-$\varepsilon_{1,2}$ plane of a two-cable hybrid system in the numerical example](image)

To have a better insight, isoquant curves corresponding to $\Omega_1=1.07\pi$ and 1.09$\pi$, $\xi_1=0.6\%$ and 0.9%, as well as DML=30% are also portrayed in the same figure. Based on these isoquant curves, the combined $\psi$-$\varepsilon_{1,2}$ region which would yield $\Omega_1\geq1.08\pi$, $\xi_1\geq0.8\%$ and DML $\leq 20\%$ can be easily identified, which is the shaded area in Figure 7.11. Any combination of cross-tie stiffness and installation location within this shaded region would result in a hybrid system satisfying the specified cable vibration suppression requirement. For example, a combination of $\psi=0.2$ and $\varepsilon_{1,2}=0.576$ ($\varepsilon_c=0.60$) within the shaded region would yield a hybrid system with $\Omega_1=1.095\pi$, $\xi_1=0.83\%$ and DML coefficient of the fundamental mode being 1.6%.
7.7 Damping estimation curves for hybrid systems

Pacheco et al. (1993) developed a universal damping estimation curve to predict the optimum damper size and the corresponding maximum attainable damping for a taut cable equipped with an external linear viscous damper. As the maximum attainable damping of a damped cable is a function of cable properties such as its bending stiffness and sag, as well as the stiffness of the damper itself and the damper support, numerous studies (e.g. Tabatabai and Mehrabi, 2000; Krenk and Nielsen, 2002; Hoang and Fujino, 2007; Fournier and Cheng, 2014) were conducted to refine this universal damping estimation curve by considering the above factors. When a damped cable is connected with adjacent cables through a transverse cross-tie to constitute a hybrid system, its in-plane stiffness and sag would be changed, which would affect the optimum damper size and the corresponding maximum attainable damping. A study by Krenk and Nielsen (2002) pointed out that during cable vibration, a single damped cable may exhibit low amplitude regions near its anchorages. If an external damper happens to be installed in these regions, its efficiency would be considerably reduced. These low amplitude regions commonly exist in pure cable networks or hybrid systems, as can be observed in Figure 5.2. Main and Jones (2004) reported that the maximum attainable damping of a single damped cable would be reduced if a spring is installed in between the cable anchorage and the external damper. In a hybrid system, the cross-tie is assumed to behave like a reversible tension/compression spring. Therefore, its stiffness may affect the optimum damper capacity and the corresponding maximum attainable damping. The role of cross-tie position in affecting the modal behaviour of a hybrid system has been explained in Section 7.5.2. It was observed that the presence of a cross-tie might not always reduce the modal damping of a hybrid system. At certain positions, it might increase the damping of a hybrid system to be higher than that of a single damped cable. This
observation suggests that the cross-tie position is another important system parameter affecting the damping of a single damped cable after it is connected with its neighbours. On top of that, the “damping transfer” phenomenon takes place if main cables with different levels of structural damping are connected through a transverse cross-tie, i.e. damping would transfer/flow from a more damped cable to a less damped one depending on their respective level of participation in the network vibration. It is also worth mentioning that Caracoglia and Zuo (2009) pointed out that the maximum modal damping of a specific hybrid system mode was considerably lower than the maximum achievable modal damping of a single damped cable. This is caused by damping contained in the damped cable being transferred to other connected neighbouring cables which have lower damping.

The available universal damping estimation curve developed by Pacheco et al. (1993) is a good tool to predict the optimum damper capacity and the corresponding maximum attainable damping of a single damped cable unless there is a considerable change in the cable properties. However, it might not be appropriate to apply the same damping estimation curve to predict the damping of a hybrid system (which is the same as the damping of a damped cable in the hybrid system) since the properties of a damped cable would be changed once connected to other cables through cross-tie(s). Therefore, there is a strong need to examine the applicability of the universal damping estimation curve to predict the damping of a hybrid system.

Due to the variety and complexity of hybrid system configurations, discussion in this section will be based on a typical two-cable hybrid system of which the two main cables with a symmetric layout are connected through a transverse cross-tie while the external damper is installed on main cable 1, as shown in Figure 7.1. The key system parameters of such a two-cable hybrid system include the damper capacity, the damper position, the cross-tie position, the
cross-tie flexibility, the frequency ratio, the length ratio and the mass-tension ratio of the neighbouring cable. They can be categorized into two types, the first type relates to the mechanical and geometrical properties of the damper and the cross-tie while the properties of main cables can be considered as the second category. Although the system parameters in the second category may also influence the optimum damper size and the corresponding maximum attainable system damping, cable properties are generally selected at the stage of bridge design based on load resisting requirements. On the other hand, the first category system parameters are dictated by the design of hybrid system. The damper position $e_{1,1}$, the cross-tie position $e_c$ and the non-dimensional cross-tie flexibility parameter $\psi$ have a critical role in affecting the optimum damper size and the corresponding maximum attainable damping of a damped cable in a hybrid system. The impact of an external damper position is well understood in existing literature (e.g. Pacheco et al., 1993; Krenk, 2000; Hoang and Fujino, 2007; and Cheng et al., 2010) but the role of cross-tie position $e_c$ and the non-dimensional cross-tie flexibility parameter $\psi$ on the optimum damper size and the corresponding maximum attainable damping of the hybrid system is yet to be explored. The effects of the cross-tie stiffness and position on the damping of a two-cable hybrid system have been discussed in Section 7.5.1 and Section 7.5.2, respectively. However, it is still worth to explore the influence of these two system parameters on the optimum damper size and the corresponding maximum attainable fundamental modal damping ratio of an isolated damped cable after it becomes part of a hybrid system. Also, an effort should be made to develop a tool to predict the optimum damper size and the corresponding maximum attainable fundamental modal damping ratio of a damped cable in a typical two-cable hybrid system. A practical range of these parameters will be considered in the following discussion. The installation location of an external linear viscous damper is generally less than 8% of the damped
cable length while the cross-tie flexibility parameter used on real cable-stayed bridges varies from $\psi=0$ (pure rigid) to $\psi=1.0$ (more flexible one). There is no restriction on the position of cross-tie but generally it is evenly installed along one of the main cables in the hybrid system. The frequency ratio, the length ratio and the mass-tension ratio parameters of the neighbouring cable in the studied two-cable hybrid system will cover the practical ranges used on real cable-stayed bridge.

### 7.7.1 Effect of cross-tie stiffness on the damping estimation curve

In this section, the effect of the cross-tie stiffness on the modal damping of a damped cable in a two-cable hybrid system will be explored. In order to observe the effect of the cross-tie flexibility parameter on the optimum damper size and the corresponding maximum attainable damping of the fundamental mode, the same hybrid system as discussed in the numerical example of Section 7.6 is studied with the cross-tie installed at the mid-span of the damped cable, i.e. $\varepsilon=1/2$. The selection of the cross-tie installation location is consistent with the field practice of which cross-ties are in general evenly installed along one of the main cables. The practical range of cross-tie flexibility $\psi$ covers 0 to 1.0. To observe the effect of using very flexible cross-tie on the damping property of the studied hybrid system, $\psi=2.0$ is also included in the analysis. Therefore, five different levels of cross-ties stiffness, i.e. $\psi=0.0, 0.1, 0.5, 1.0$ and 2.0, are chosen for discussion.

The damping estimation curves for the fundamental mode of a two-cable hybrid system are plotted in Figure 7.12. The non-dimensional parameter $\sigma = [c/(m_1 L_1 \omega_1)] \varepsilon_{1,1}$ is taken as the abscissa while the ordinate is the ratio between the system fundamental modal damping ratio and the damper position, $\xi/\varepsilon_{1,1}$. They are consistent with those used in the universal damping estimation curve for a single damped cable developed by Pacheco et al. (1993). The thick solid
curve in Figure 7.12 represents the damping estimation curve of an isolated damped cable while the rest five curves represent those of the studied hybrid system at five different levels of cross-tie flexibility. The non-dimensional parameters $\sigma$ and $\xi/e_{1,1}$ corresponding to the apex of each curve in Figure 7.12 are associated with, respectively, the optimum damper size and the maximum attainable fundamental modal damping ratio of a two-cable hybrid system. They are tabulated in Table 7.3 along with the optimum damper size and the corresponding maximum attainable system fundamental modal damping ratio.

Figure 7.12: Damping estimation curves for a single damped cable and a symmetric two-cable hybrid system ($e_{1,1}=0.0235$, $e_c=0.50$, $\eta=0.82$, $\lambda=1.2$ and $\gamma=0.91$)
Table 7.3: The optimum damper size and the maximum attainable fundamental modal damping ratio of a single damped cable and a symmetric two-cable hybrid system (ε₁,₁=0.0235, ε₁=0.50, η=0.82, λ=1.2 and γ=0.91)

<table>
<thead>
<tr>
<th>Case</th>
<th>σ_{opt}</th>
<th>ξ_{1,max}/ε₁,₁</th>
<th>c_{opt} (kN·s/m)</th>
<th>ξ_{1,max} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single damped cable</td>
<td>0.100</td>
<td>0.520</td>
<td>255</td>
<td>1.22</td>
</tr>
<tr>
<td>ψ=0.0</td>
<td>0.093</td>
<td>0.288</td>
<td>236</td>
<td>0.68</td>
</tr>
<tr>
<td>ψ=0.1</td>
<td>0.093</td>
<td>0.310</td>
<td>236</td>
<td>0.73</td>
</tr>
<tr>
<td>ψ=0.5</td>
<td>0.094</td>
<td>0.378</td>
<td>240</td>
<td>0.89</td>
</tr>
<tr>
<td>ψ=1.0</td>
<td>0.095</td>
<td>0.429</td>
<td>242</td>
<td>1.01</td>
</tr>
<tr>
<td>ψ=2.0</td>
<td>0.097</td>
<td>0.471</td>
<td>248</td>
<td>1.11</td>
</tr>
</tbody>
</table>

The results in Table 7.3 show that both the optimum damper size and its corresponding maximum attainable fundamental modal damping ratio are influenced by the stiffness of cross-tie. The optimum damper size and the associated maximum attainable fundamental modal damping ratio of a single damped cable based on the universal damping estimation curve by Pacheco et al. (1993) are, respectively, 255 kN·s/m and 1.22%. When it becomes part of a hybrid system, the optimum damper size is reduced slightly, especially in the case of a more stiff cross-tie, i.e. e.g. ψ=0.0. On the other hand, the associated maximum attainable fundamental modal damping ratio of an isolated damped cable would drop more considerably when connected with other cables through a transverse cross-tie. This reduction becomes more significant if a more rigid cross-tie is used. With the increase of cross-tie flexibility, the maximum attainable fundamental modal damping ratio of a hybrid system approaches to that of an isolated single damped cable, which can be clearly seen in Figure 7.12. For example, in the current case, by connecting the single damped cable to its neighbour using a flexible cross-tie with ψ=1.0 at the mid-span, the maximum attainable fundamental modal damping ratio of the damped cable
reduces from 1.22% to 1.01% by 14%. However, the reduction of the fundamental modal damping ratio jumps to 43%, i.e. from 1.22% to 0.68%, if a rigid cross-tie with \( \psi = 0.0 \) is used instead, as can be seen in Table 7.3.

The above results suggest that once an isolated damped cable is connected with its neighbour(s) through a transverse cross-tie, the maximum attainable modal damping would drop. This could be mainly due to the “damping transfer” phenomenon. In the current case, the damped cable (main cable 1) in the hybrid system has higher damping than the neighbouring cable (main cable 2). The “extra” damping would “transfer/flow” from the damped cable to the neighbouring one during system vibration as reflected in the modal analysis results. The fundamental modal damping ratio of the damped cable decreases once it becomes part of a hybrid system, whereas that of the neighbouring cable in the system increases sizably. In the current case, if a rigid cross-tie is used, the fundamental modal damping ratio of the damped cable reduces from 1.2% to 0.68%, whereas that of the neighbouring cable increases from 0 to 0.68%. It implies that cross-tie(s) can be used as a tool to “transfer” structural damping from a more damped cable to a less damped one in a hybrid system.

### 7.7.2 Effect of cross-tie position on the damping estimation curve

In Section 7.7.1, the effect of cross-tie flexibility on the optimum damper size and the associated maximum attainable fundamental modal damping ratio of a two-cable hybrid system is explored. This section will evaluate the impact of another important parameter, i.e. the cross-tie position, on the optimum damper size and the associated fundamental modal damping. The damping estimation curves are depicted in Figure 7.13 for the cases of a two-cable hybrid system with a rigid cross-tie (\( \psi = 0.0 \)) installed at five different locations, i.e. \( \varepsilon_c = 0.25, 0.33, 0.50, 0.67 \) and 0.75, from the left end of the damped cable (the cable end close to the external damper). The
thick solid curve in the figure represents the damping estimation curve of an isolated damped cable. The optimum damper size and the corresponding maximum attainable fundamental modal damping ratio of an isolated damped cable and the two-cable hybrid system are listed in Table 7.4.

The general pattern regarding the effect of the cross-tie position on the optimum damper size and the corresponding maximum attainable fundamental modal damping ratio is almost the same as that observed in Section 7.7.1. The optimum damper size of an isolated damped cable is slightly reduced after connecting with its neighbour and forming a hybrid system. However, the difference in the maximum attainable fundamental modal damping ratio between an isolated damped cable and a two-cable hybrid system increases if the cross-tie is installed closer to the external damper. The possible reasons for this increasing difference are well explained in Section 7.5.2. When a cross-tie is installed away from an external damper, its constraint on the operation of the external damper is released, which would yield a higher modal damping for the hybrid system. For example, the maximum attainable fundamental modal damping ratio of a two-cable hybrid system could increase 83% from 0.48% to 0.88% by just relocating a rigid cross-tie from the quarter span ($\varepsilon_c=0.25$) to the three-quarter span ($\varepsilon_c=0.75$).
Figure 7.13: Damping estimation curves for a single damped cable and a symmetric two-cable hybrid system (ε_{1,1}=0.0235, ψ=0.0, η=0.82, λ=1.2 and γ=0.91)
Table 7.4: The optimum damper size and the maximum attainable fundamental modal damping ratio of a single damped cable and a symmetric two-cable hybrid system ($\varepsilon_{1,1}=0.0235$, $\psi=0.0$, $\eta=0.82$, $\lambda=1.2$ and $\gamma=0.91$)

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{\text{opt}}$</th>
<th>$\xi_{1,\text{max}}/\varepsilon_{1,1}$</th>
<th>$\xi_{1,\text{opt}}$ (kN·s/m)</th>
<th>$\xi_{1,\text{max}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single damped cable</td>
<td>0.100</td>
<td>0.520</td>
<td>255</td>
<td>1.22</td>
</tr>
<tr>
<td>$\varepsilon_c=0.25$</td>
<td>0.096</td>
<td>0.204</td>
<td>244</td>
<td>0.48</td>
</tr>
<tr>
<td>$\varepsilon_c=0.33$</td>
<td>0.094</td>
<td>0.239</td>
<td>240</td>
<td>0.56</td>
</tr>
<tr>
<td>Hybrid system</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_c=0.50$</td>
<td>0.093</td>
<td>0.288</td>
<td>236</td>
<td>0.68</td>
</tr>
<tr>
<td>$\varepsilon_c=0.67$</td>
<td>0.091</td>
<td>0.337</td>
<td>232</td>
<td>0.79</td>
</tr>
<tr>
<td>$\varepsilon_c=0.75$</td>
<td>0.090</td>
<td>0.375</td>
<td>230</td>
<td>0.88</td>
</tr>
</tbody>
</table>

7.7.3 Approximate relation equation for estimating damping in a two-cable hybrid system

The impact of cross-tie flexibility and installation location on the optimum damper size and the corresponding maximum attainable fundamental modal damping ratio of a two-cable hybrid system are summarized in Table 7.3 and Table 7.4, respectively. The results clearly indicate that these two system parameters have a significant influence on the maximum attainable fundamental modal damping ratio of a typical two-cable hybrid system.

The damping estimation curves in Figure 7.12 and Figure 7.14 can be used to predict the optimum damper size and its corresponding maximum attainable fundamental modal damping ratio of a two-cable hybrid system with similar configuration as that studied in Sections 7.7.1 and 7.7.2. However, they are limited to a specific combination of system parameters. In order to establish a more general relation between the optimum damper size, the maximum attainable fundamental modal damping ratio and the key parameters of a hybrid system, an effort will be made in this section to develop a set of approximate relation equation to estimate the optimum damper size and the maximum attainable fundamental modal damping ratio for symmetric two-
cable hybrid systems. These approximate relation equations will be developed based on typical values of the frequency ratio, the length ratio and the mass-tension ratio parameters of constituting stay cables in real cable networks on site, which are \( \eta = 0.82 \), \( \lambda = 1.2 \) and \( \gamma = 0.91 \), respectively (Caracoglia and Jones, 2005b). The external damper position \( \varepsilon_{1,1} \), the cross-tie position \( \varepsilon_c = \varepsilon_{1,1} + \varepsilon_{1,2} \) and the cross-tie flexibility parameter \( \psi \) take the values as follows:

External damper position (\( \varepsilon_{1,1} \)): 0.02, 0.04, 0.06, 0.08

Cross-tie position (\( \varepsilon_c \)): 0.25, 0.33, 0.5, 0.67, 0.75

Cross-tie flexibility parameter (\( \psi \)): 0.0, 0.02, 0.05, 0.10, 0.2, 0.5, 0.8, 1.0, 1.5, 2.0

Combinations based on these selected values of system parameters give a total of 200 possible configurations of a two-cable hybrid system. The optimum damper size, in terms of the non-dimensional damper parameter \( \mu_{\text{opt}} = c_{\text{opt}}/\sqrt{mH} \) and the corresponding maximum attainable fundamental modal damping ratio \( \xi_{1,\text{max}} \), are calculated for each of these configurations. A regression analysis is then performed to obtain the following two approximate relation equations to predict the optimum damper size and the associated maximum attainable fundamental modal damping ratio of a two-cable hybrid system with a symmetric layout. The coefficients of determination of these two approximate relation equations are 0.99993 and 0.9967, respectively.

\[
\mu_{1,\text{opt}} = 0.4974809 + (0.2934065 + 0.0081868\psi)/\varepsilon_{1,1} - \varepsilon_c
\]

\[
\xi_{1,\text{max}} = \varepsilon_{1,1}(0.2312520 + 0.2442251\varepsilon_c^2 + 0.2309834\psi - 0.0686518\psi^2)
\]

With an error of less than 0.1\%, these two approximate relation equations can be further simplified for the convenience of application:

\[
\mu_{1,\text{opt}} = 0.4975 + (0.2934 + 0.0082\psi)/\varepsilon_{1,1} - \varepsilon_c \quad (7-28)
\]

\[
\xi_{1,\text{max}} = \varepsilon_{1,1}(0.2312 + 0.2442\varepsilon_c^2 + 0.231\psi - 0.0686\psi^2) \quad (7-29)
\]
Equation (7-29) can be used to predict the maximum attainable fundamental modal damping ratio of a two-cable hybrid system. Similar expression was proposed by Pacheco et al. (1993) for a single damped cable i.e. $\xi_{1,\text{max}} = 0.52\varepsilon_{1,1}$. It is interesting to see that when an isolated damped cable becomes part of a two-cable hybrid system, the coefficient of 0.52 in the expression by Pacheco et al. (1993) is replaced by the terms within the brackets in Eq. (7-29). This suggests that the location of an external damper has a critical role in affecting the maximum attainable fundamental modal damping ratio of the isolated damped cable as well as the two-cable hybrid system. It can be clearly seen in Eq. (7-29) that the maximum attainable fundamental modal damping ratio of a two-cable hybrid system would increase as the damper moves towards the mid-span of the target cable.

The influence of the other two system parameters, i.e. the cross-tie location $\varepsilon_c$ and the cross-tie flexibility $\psi$, on the maximum attainable fundamental modal damping ratio can be judged from their respective coefficient and exponent. The coefficient, both its sign and magnitude, of a system parameter in Eq. (7-29) can be used to determine its impact on the maximum attainable fundamental modal damping ratio of the hybrid system, while its exponent is a measure of the sensitivity of system damping to the parameter. For example, in a two-cable hybrid system, moving a rigid ($\psi=0.0$) cross-tie from a position of $\varepsilon_c=0.50$ to $\varepsilon_c=0.75$ would yield an increase in the maximum attainable fundamental modal damping ratio from $0.29\varepsilon_{1,1}$ to $0.37\varepsilon_{1,1}$ by 28%. The sensitivity of the maximum attainable fundamental modal damping ratio to a particular system parameter can be explained from the exponent of the parameter. In Eq. (7-29), it can be seen that the maximum attainable fundamental modal damping ratio is a quadratic function of $\varepsilon_c$ and $\psi$. For any quadratic function, the dependent variable is more sensitive to the independent variable(s) in the upper range. As the sign of the coefficient associated with the
quadratic term of $\varepsilon_c$ in Eq. (7-29) is positive, increasing $\varepsilon_c$ in its upper range, e.g. $\varepsilon_c=0.75$, would have more impact on the maximum attainable fundamental modal damping ratio than by having the same increment in the lower range of $\varepsilon_c$, e.g. $\varepsilon_c=0.25$. On the other hand, the coefficient of the quadratic term associated with $\psi$ has a negative sign. Therefore, the same increment of $\psi$ in the lower range would have more impact on the maximum attainable fundamental modal damping ratio than that in its higher range. For example, in a two-cable hybrid system with $\varepsilon_{1,1}=0.0235$ and $\varepsilon_c=0.6$, an increment of 0.2 for $\psi$ in its lower range from 0.1 to 0.3 would yield an increase of $\xi_{1,max}$ from 0.8% to 0.9% which is 12%, while the same increment of 0.2 in the higher range of $\psi$ from 1.0 to 1.2 would only render $\xi_{1,max}$ to increase from 1.13% to 1.17% which is 3.5%.

Similarly, the effect of the non-dimensional damper location parameter $\varepsilon_{1,1}$, the cross-tie location parameter $\varepsilon_c$ and the non-dimensional cross-tie flexibility parameter $\psi$ on the optimum damper size can be explained based on their respective coefficients and exponents in Eq. (7-28). The damper location parameter $\varepsilon_{1,1}$ is present in the denominator of the second term in Eq. (7-28), which indicates that the optimum damper size decreases as the damper moves towards the mid-span of the target cable. This is consistent with the case of a single damped cable (Fournier and Cheng, 2014). The coefficient of the cross-tie flexibility parameter $\psi$ is much smaller than that of $\varepsilon_{1,1}$ and $\varepsilon_c$, suggesting that $\psi$ has a minor effect on the optimum damper size. Nevertheless, the positive sign of this coefficient indicates that using more flexible cross-tie would slightly increase the optimum damper size, as can be seen in Table 7.3. The role of the cross-tie location parameter $\varepsilon_c$ in affecting the optimum damper size is interesting. From its form in Eq. (7-28), it suggests that mathematically the optimum damper size in a two-cable hybrid system would reduce by the same amount as the magnitude of $\varepsilon_c$ itself. Since the cross-tie
location parameter $\varepsilon_c$ is always less than 1, therefore, its influence on the optimum damper size of a two-cable hybrid system is also very small. The same phenomenon, i.e. the cross-tie stiffness and location have trivial impact on the optimum damper size of a two-cable hybrid system, has been observed in Sections 7.7.1 and 7.7.2.

From the above discussion, it is observed that although the optimum damper size in a two-cable hybrid system is not sensitive to the cross-tie location parameter $\varepsilon_c$ and the cross-tie flexibility parameter $\psi$, the associated maximum attainable fundamental modal damping ratio is significantly influenced by these two parameters. Therefore, it is worth to further explore the combined effect of these two parameters on the maximum attainable fundamental modal damping ratio. When design a hybrid system, two scenarios can be commonly encountered. In the first scenario, an external damper has been used to mitigate vibration of a vulnerable cable. To enhance its in-plane stiffness, cross-tie will be added to connect the cable with its neighbor(s). In such a case, the main focus would be to choose appropriate cross-tie properties in terms of the cross-tie flexibility $\psi$ and the cross-tie location $\varepsilon_c$, so their combined effect could maximize the system fundamental modal damping ratio. The form of Eq. (7-29) and the pattern of the curves in Figures 7.13 and 7.14 imply that installing a more flexible cross-tie away from the damper would yield a maximum attainable system fundamental modal damping ratio closer to that of an isolated damped cable. However, using a more flexible cross-tie (larger $\psi$) has an adverse effect on the in-plane stiffness of a hybrid system. Therefore, it is recommended to first choose the cross-tie location. Assume a relative stiff cross-tie (e.g. $\psi=0.01$), place it far enough from the external damper to ensure it satisfies the damping requirement. Then gradually reduce the cross-tie stiffness such that it will meet the in-plane stiffness requirement of the hybrid system while increasing its damping. In the second scenario, cross-tie has been used to enhance
the in-plane stiffness of the vulnerable cable. To increase its damping level, external damper needs to be installed. Thus, the design task is to determine the optimum size and the position of the damper to satisfy the specified damping requirement. This can be achieved by directly applying Eq. (7-29) to compute the damper location $\varepsilon_{1,1}$ and then substitute $\psi$, $\varepsilon_c$ and $\varepsilon_{1,1}$ into Eq. (7-28) to determine the optimum damper size.

In order to prove the validity of the two approximate relations in Eqs. (7-28) and (7-29), the numerical example of a two-cable hybrid system discussed in Section 7.6 is revisited. In this example, it is required to determine the cross-tie position (in terms of the damper-spacing parameter $\varepsilon_{1,2}$) and its flexibility parameter so that the modal damping of the system fundamental mode should be no less than $\xi_1=0.8\%$. Upon using the optimization isoquant curves developed based on analytical results, the damper-spacing parameter $\varepsilon_{1,2}$ and its flexibility parameter $\psi$ were selected as $\varepsilon_{1,2}=0.576$ and $\psi=0.2$ to achieve a fundamental modal damping of $\xi_1=0.83\%$ along with the corresponding non-dimensional in-plane frequency $\Omega_1=1.095\pi$ and the DML coefficient being 1.6\%. To validate the proposed approximate relations, the system non-dimensional parameters, $\varepsilon_{1,1}=0.0235$, $\varepsilon_c=0.6$ ($\varepsilon_c=\varepsilon_{1,1}+\varepsilon_{1,2}$) and $\psi=0.2$, are plugged into Eq. (7-29), which gives a maximum attainable fundamental modal damping ratio of $\xi_{1,\text{max}}=0.85\%$. It agrees well with the analytically determined value of $\xi_1=0.83\%$ with an error of 2\%. Similarly, the non-dimensional optimum damper size can be estimated by substituting these system parameters ($\varepsilon_{1,1}=0.0235$, $\varepsilon_c=0.6$ and $\psi=0.2$) into Eq. (7-28), which yields $\mu_{\text{opt}}=12.45$ or $c_{\text{opt}}=237$ kN·s/m. These results indicate that an external damper with a damper coefficient of 237 kN·s/m installed at a location of 2.35\% of the damped cable length from cable end is required to achieve a fundamental modal damping ratio of $\xi_{1,\text{max}}=0.85\%$ for the hybrid system discussed in the numerical example of Section 7.6. It is important to note that the optimum damper size in the
hybrid system example is 7% smaller than that of a single damped cable which is 255 kN·s/m, and the maximum attainable fundamental modal damping ratio is 0.85% as compare to 1.22% in a single damped cable (Pacheco et al., 1993).

7.8 Summary

A two-cable hybrid system is used to study its modal behaviour in terms of the in-plane frequency, the modal damping and the degree of mode localization. The role of the key system parameters, i.e. the cross-tie stiffness and the spacing between the external damper and the cross-tie, on the in-plane frequency and the damping ratio of the hybrid system fundamental mode are explored. The findings of this chapter are summarized below:

1) Using a more flexible cross-tie would lift the constraint on the operation of an external damper and thereby provide higher modal damping to the hybrid system. However, this would reduce the system in-plane stiffness.

2) The cross-tie position has two unique features depending on if it is close to the near end or the far end of an external damper. In the case of the near end installation, the role of the cross-tie is similar as the neoprene rubber bushings which would result in reducing the hybrid system modal damping. In the case of the far end installation, the damping ratio of the hybrid system could become more than that of a single damped cable for certain ranges of cross-tie position. Nevertheless, this would reduce the global nature of the system fundamental mode.

3) A concept of isoquant curve is introduced in order to optimize the performance of a selected hybrid system mode. The same approach can be extended to multi-mode optimization of hybrid systems.

4) A state-of-the-art generalized approach is proposed to develop the analytical models of the more complex conventional and hybrid cable networks from a simpler parent system.
5) The applicability of the universal damping estimation curve of a single damped cable proposed by Pacheco et al. (1993) to a hybrid system is discussed. Approximate relation equations are developed to predict the optimum damper size and the maximum attainable damping of a basic two-cable hybrid system.
CHAPTER 8  Experimental Study on the In-plane Modal Behaviour of Pure Cable Networks and Hybrid Systems

The objective of the present chapter is to understand the mechanics associated with two cable vibration control solutions, i.e. the cross-tie-only solution (pure cable network) and the cross-tie(s) combined with external viscous damper(s) solution (hybrid system), by using an experimental approach. The in-plane modal behaviour of pure cable networks and hybrid systems have been studied in previous chapters using analytical and numerical approaches. Although the results yielded from these two approaches are in good agreement, they are based on certain simplifying hypothesis. A number of assumptions have been made in the formulation of the analytical models to reduce the level of complexity. They include ignoring the cable sag, the cable bending stiffness and the intrinsic damping of main cables and cross-ties. Therefore, to understand the impact of these assumptions on the modal analysis results of cable networks and hybrid systems, it is necessary to conduct an experimental study and the results will be compared with those obtained from the analytical and the numerical approaches. In addition, this part of the study will provide modal data, i.e. the in-plane frequency and the damping ratio, of cable networks and hybrid systems obtained from three different approaches, i.e. the analytical, the numerical and the experimental one, which is lacking in existing literature. The co-existence of these three sets of modal data would be critical to understand the impact of the assumptions made in the analytical and the numerical approaches on the modal analysis results of cable networks and hybrid systems. On top of that, comparison between the modal behaviour of two hybrid systems having different configurations is scarce in literature. The modal response of two different hybrid systems, with an external damper installed either near the cable anchorage or in-line with a cross-tie will be studied. All experimental tests were carried out in the Structures
Laboratory at the University of Windsor. The experimental setup and instrumentations used in the experimental study are explained in the following section.

8.1 Experimental setup

The cable networks discussed in the previous chapters can be categorized into two main types based on the use of cross-tie(s) and passive viscous damper(s). The first type is pure cable networks where main cables are connected through transverse cross-tie(s) in order to increase the in-plane stiffness of vulnerable cable(s). The second type combines the use of cross-tie(s) and external viscous damper(s) and is regarded as hybrid systems. In hybrid systems, the main cables are connected by transverse cross-ties and some of them are equipped with external linear viscous damper in order to enhance the damping property of the hybrid system. Based on the installation location of an external damper, the hybrid systems studied in Chapter 7 are further classified into hybrid system A and hybrid system B. In hybrid system A, the external viscous damper is attached with the target cable and installed near the cable anchorage, whereas in hybrid system B the external damper is installed in line with the cross-tie.

The experimental setups used to study the modal behaviour of the pure cable network and the two types of hybrid systems are portrayed in Figure 8.1. The two main cables were arranged in parallel, both inclined at 13º with respect to horizontal, as illustrated in Figure 8.1. They were rigidly supported between two vertical steel columns. The cables were fixed at both ends with the upper and the lower end of the cable simulating the anchorage points at the pylon and the deck on a real cable-stayed bridge, respectively. The bottom cable was assumed to be the target cable (main cable 1) and the top one as the neighboring cable (main cable 2). A transverse cross-tie was used to connect the main cables. Its position is measured from the lower end of the target cable.
(a) Cable network

(b) Hybrid system A
The test setup of the pure cable network is sketched in Figure 8.1(a). Free vibration tests were conducted to study its modal behaviour using cross-tie of different stiffness and installation locations. In hybrid system A, a linear viscous damper was attached to the target cable at a distance of 0.55 m from its lower support whereas it was installed in-line with the cross-tie in hybrid system B. The typical layouts of hybrid system A and hybrid system B used in the experimental study are shown in Figure 8.1(b) and Figure 8.1(c), respectively. As the presence of an external damper would cause rapid decay of system response through free vibration test, therefore, forced vibration test was conducted instead to investigate the modal behaviour of hybrid systems. An electronic dynamic smart shaker was used to excite the hybrid system in-plane vibration. It was installed at 5% of cable length near the top end of the target cable (bottom cable), as shown in Figures 8.1(b) and 8.1(c).

In the following sub sections, a description of each instrumentation and equipment used in the experimental study will be presented.
8.1.1 Main cables

The main cables used in the experimental study had a clear span length of 8.5 m, a nominal diameter of 4.65 mm, and a unit mass of 0.095 kg/m. One end of each of the main cables was attached to a hydraulic pump to apply pretension force while the other end was connected to a load cell for measuring the pretension in the main cables. The inextensibility parameter $\lambda^2$, as proposed by Irvin (1981) and discussed in Section 2.1, is used to differentiate the taut-cable and the sagged one. The stay cables used on real cable-stayed bridges are highly prestressed with an inextensibility parameter $\lambda^2$ less than 1 (Tabatabai and Mehrabi, 2000b). Therefore, the two cables used in the experimental tests were simulated as taut cables of which $\lambda^2$ approaches to 0. In order for the main cables used in the experimental study to achieve dynamic properties more agreeable with those of the real stay cables on cable-stayed bridge, small mass blocks with a weight of 50 gram each were attached evenly along the length of the two main cables. A total of 20 and 17 mass blocks were attached, respectively, to the target and the neighbouring cable, which yielded an equivalent unit mass of 0.213 kg/m and 0.195 kg/m. Huang (2011) found that when the pretension force in the two cables varied between 2500 N to 4000 N, the inextensibility parameter $\lambda^2$ of the two cables would be in the range of 0.0009 – 0.0033, which would satisfy the taut cable assumption. Thus, the in-plane frequency of the target and the neighbouring cables can be calculated as $f = \frac{1}{2L} \sqrt{\frac{H}{m}}$. The physical properties of the two main cables are listed in Table 8.1.
Table 8.1: Physical properties of the main cables

<table>
<thead>
<tr>
<th>Main cable</th>
<th>Length (m)</th>
<th>Unit mass (kg/m)</th>
<th>Tension (N)</th>
<th>Fundamental mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target (Bottom)</td>
<td>8.5</td>
<td>0.213</td>
<td>2580</td>
<td>6.47</td>
</tr>
<tr>
<td>Neighbouring (Top)</td>
<td>8.5</td>
<td>0.195</td>
<td>3750</td>
<td>8.16</td>
</tr>
</tbody>
</table>

8.1.2 Cross-ties

Two types of cross-ties were used. The rigid type was made of steel wire, whereas the flexible type was made of rubber material. Their respective stiffness coefficients are listed in Table 8.2.

Table 8.2: Material properties of the cross-ties

<table>
<thead>
<tr>
<th>Cross-tie type</th>
<th>Stiffness, $K_c$ (N/m)</th>
<th>Flexibility parameter, $\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>$2.1 \times 10^5$</td>
<td>0.0003</td>
</tr>
<tr>
<td>Flexible</td>
<td>205</td>
<td>1.5</td>
</tr>
</tbody>
</table>

8.1.3 Passive linear viscous damper

A passive linear viscous damper was designed by Huang (2011) and Fournier (2012) and modified by Sandanam (2015), as shown in Figure 8.2, was used in this experimental study. The linear viscous damper consisted of a plastic container with an inner diameter of 100 mm. The viscous fluid Synfluid PAO 100 had a kinematic viscosity of 1250 centistokes (cSt) at 40°C. An acrylic block, 48 mm x 48 mm x 39 mm, was used as a piston for this damper. It had four symmetrically laid orifices for the purpose of increasing the contact surface area with the viscous fluid to increase the damping coefficient. The damping force of this damper was generated by the
movement of the piston through the viscous fluid. The calibrated damping coefficient of the damper was 19.1 N·s/m (Sandanam, 2015).

![Damper stick](image)

Figure 8.2: Passive linear viscous damper

### 8.1.4 Load cells

In order to measure the applied pretension in the two cables, two Universal Flat Load Cells (model number FL25U-2SG) were mounted at the bottom ends of the cables, as shown in Figure 8.3. The maximum capacity of the load cells is 110 kN (25,000 lb). Calibration of the load cells was performed by using a universal tensile machine which yielded a calibration constant of 5.481 kN/mV and 5.585 kN/mV for the bottom and the top cable, respectively. These calibration constants were applied to the recorded voltages by the two load cells to obtain the corresponding tension in the two cables.
The hydraulic pumps were used to introduce the pretension force into the main cables. The range of pretension force varied from 2500 N to 3800 N in the current test. The top ends of the two cables were connected to manually-operated hydraulic pump (model number PH-84). Each unit had a loading capacity of 69 MPa (10,000 psi). The setup of the hydraulic pumps is shown in Figure 8.4.
In the case of the forced vibration tests, the target cable was excited by an electronic dynamic smart shaker (model number K2007E01) manufactured by Modal Shop Inc. The unit is capable of providing up to 31 N (7 lb) of peak sine force and a testing frequency range of 1–9000 Hz. The shaker was installed at a distance of 5% of the target cable length from the top end of the cable and placed on a supporting tripod, as shown in Figure 8.5.
8.1.7 Signal generator

An HP signal generator (model number 33120A), as shown in Figure 8.6, was used to generate dynamic excitation functions for controlling the dynamic shaker. It has the ability to generate output functions with various forms including sine, square, triangle, and ramp functions. The sinusoidal function with a frequency range of 1 – 15 Hz was used in the current experimental study as the form of the excitation force.
8.1.8 Accelerometer

A miniature lightweight ceramic shear ICP© accelerometer (model number 352A24) manufactured by PCB Piezotronics was used to record the acceleration response of the cable network and the hybrid system. The accelerometer was placed on the top surface of the target cable at the mid-span in order to measure its transverse in-plane response. This unit has a frequency range of 1 – 8000 Hz, whereas the testing range used in the experimental study was 5 – 10 Hz.

8.1.9 Data acquisition (DAQ) system

The data acquisition system, AstroDAQ Xe, was used to collect all the real time data. The AstroLINK Xe software was supplied along with the PC-based data acquisition system shown in Figure 8.7. This unit has eight input channels. In the current tests, Channels 7 and 8 were connected with the two load cells and Channel 3 was connected with the accelerometer placed on the target cable. The input signals could be monitored and recorded in the Real time mode while the captured signals could be reviewed later in the Review mode. Each channel is capable of
recording signals with a sampling frequency up to 200 kHz. A sampling frequency of 1000 Hz was used in the current test.

Figure 8.7: PC-based AstroDAQ Xe data acquisition system

8.2 Test procedures

This section explains the procedures used in the free vibration and the forced vibration tests of the cable network and the hybrid systems.

8.2.1 Free vibration test

Free vibration tests were conducted to determine the in-plane frequency and the damping ratio associated with the fundamental mode of the cable network. A total of eight different testing cases were conducted, four for the rigid cross-tie and four for the flexible one. They are listed in Table 8.3.
Table 8.3: List of testing cases conducted for the modal behaviour of cable networks

<table>
<thead>
<tr>
<th>Testing case</th>
<th>Cross-tie position*</th>
<th>Cross-tie type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/4L</td>
<td>Rigid</td>
</tr>
<tr>
<td>2</td>
<td>1/3L</td>
<td>Rigid</td>
</tr>
<tr>
<td>3</td>
<td>1/2L</td>
<td>Rigid</td>
</tr>
<tr>
<td>4</td>
<td>3/4L</td>
<td>Rigid</td>
</tr>
<tr>
<td>5</td>
<td>1/4L</td>
<td>Flexible</td>
</tr>
<tr>
<td>6</td>
<td>1/3L</td>
<td>Flexible</td>
</tr>
<tr>
<td>7</td>
<td>1/2L</td>
<td>Flexible</td>
</tr>
<tr>
<td>8</td>
<td>3/4L</td>
<td>Flexible</td>
</tr>
</tbody>
</table>

* L is the length of the target cable.

After the two main cables were setup as shown in Figure 8.1(a), the steps listed below were followed to obtain modal response of the studied system.

i. Apply the required pretension force to the two main cables using manually operated hydraulic pump. The applied pretension force was measured by load cells and collected by the DAQ system in the Real time mode of the AstroLINK Xe software.

ii. Connect the two main cables through a cross-tie at desired location to form a cable network. Two clamp connectors were used to connect the cross-tie at its two ends with the main cables. The connection details between the cross-tie and the main cables, both for rigid and flexible one, are shown in Figure 8.8.
iii. Mount/place an accelerometer on the top surface of the target cable at the mid-span, and then connect it to the DAQ system.

iv. Create a file name for the current test and select the sampling frequency as 1000 Hz and sampling time as 5 seconds in the AstroLink Xe software.

v. Excite the target cable by attaching a mass block through a string at the cable mid-span, and then burn the string to initiate free vibration of the system. Observe the acceleration response of the target cable in the AstroLink Xe software to ensure proper functioning of the accelerometer.

vi. Monitor and collect the acceleration response of the cable network in the transverse in-plane direction by the DAQ system in the Real time mode.
vii. Repeat steps iv to vi to collect multiple sets of data for the same cable network configuration.

Once the acceleration response of the cable network was collected, the in-plane frequency and the damping ratio of the cable network fundamental mode were evaluated according to the following procedures.

The power spectral density (PSD) of a signal describes the strength of energy present in a signal as a function of frequency. The ‘pwelch’ function in the MatLab gives the Welch power spectral density (PSD) estimate of the input signal (Biran and Breiner, 1996). MatLab 7.01 was used to estimate the PSD via the ‘pwelch’ functions which accepts acceleration response data as the input parameter and its PSD graph as output. The fundamental frequency of a two-cable network can be determined from the first peak of Power Spectral Density (PSD) graph.

Once the fundamental frequency of the cable network was found, the raw signal was filtered to isolate the first modal response. The Butterworth filter was designed to retain the signals within a certain frequency range. In the current study, this frequency range was chosen to be from \((f_1 - 0.5)\) Hz to \((f_1 + 0.5)\) Hz, where \(f_1\) is the fundamental frequency of the cable network. The ‘filter’ function in the MatLab was used to isolate the acceleration response of the cable network fundamental mode. Then the acceleration time history of the network fundamental mode was transferred from the time domain to the frequency domain by applying a Fourier Transform. The acceleration data in the frequency domain was divided by \((2\pi f_1)^2\) to yield the corresponding displacement data in the frequency domain. By applying the inverse Fourier Transform, the displacement time history of the cable network could be obtained. Once the displacement time history was available, the damping ratio of fundamental mode could be calculated by using the logarithmic decrement approach (Chopra, 2007) given in Eq. (8-1):
\[ \delta = \frac{1}{m} \ln \frac{y_n}{y_{n+m}} = 2\pi \xi \]  

(8-1)

where \( \delta \) is the logarithmic decrement, \( y_n \) and \( y_{n+m} \) are the amplitude of the \( n \)th and the \( (n+m) \)th cycle (the \( n \)th and the \( (n+m) \)th cycle are \( m \) cycles apart), respectively, and \( \xi \) is the damping ratio.

### 8.2.2 Forced vibration test

Forced vibration test was conducted to determine the in-plane frequency and the damping ratio of hybrid system fundamental mode. For hybrid system A, eight different cases were tested based on the position and the stiffness of the cross-tie. In hybrid system B where an external damper is installed in-line with the cross-tie, six cases were tested. The testing cases for hybrid system A and hybrid system B are listed in Tables 8.4 and 8.5, respectively.

Table 8.4: List of testing cases conducted for the modal behaviour of hybrid system A (external damper position = 0.55 m (0.065%L))

<table>
<thead>
<tr>
<th>Testing case</th>
<th>Cross-tie position*</th>
<th>Cross-tie type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/4L</td>
<td>Rigid</td>
</tr>
<tr>
<td>2</td>
<td>1/3L</td>
<td>Rigid</td>
</tr>
<tr>
<td>3</td>
<td>1/2L</td>
<td>Rigid</td>
</tr>
<tr>
<td>4</td>
<td>3/4L</td>
<td>Rigid</td>
</tr>
<tr>
<td>5</td>
<td>1/4L</td>
<td>Flexible</td>
</tr>
<tr>
<td>6</td>
<td>1/3L</td>
<td>Flexible</td>
</tr>
<tr>
<td>7</td>
<td>1/2L</td>
<td>Flexible</td>
</tr>
<tr>
<td>8</td>
<td>3/4L</td>
<td>Flexible</td>
</tr>
</tbody>
</table>

* L is the length of the target cable.
Table 8.5: List of testing cases conducted for the modal behaviour of hybrid system B (external damper in-line with the cross-tie)

<table>
<thead>
<tr>
<th>Testing case</th>
<th>Cross-tie position*</th>
<th>Cross-tie type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/4L</td>
<td>Rigid</td>
</tr>
<tr>
<td>2</td>
<td>1/3L</td>
<td>Rigid</td>
</tr>
<tr>
<td>3</td>
<td>1/2L</td>
<td>Rigid</td>
</tr>
<tr>
<td>4</td>
<td>1/4L</td>
<td>Flexible</td>
</tr>
<tr>
<td>5</td>
<td>1/3L</td>
<td>Flexible</td>
</tr>
<tr>
<td>6</td>
<td>1/2L</td>
<td>Flexible</td>
</tr>
</tbody>
</table>

* L is the length of the target cable.

The steps required for performing a forced vibration test and data analysis are outlined as follows:

i. Complete steps i to iv in Section 8.2.1 to set up the desired configuration for the hybrid system, install accelerometer, select sampling time and frequency.

ii. Install an external viscous damper at a location according to Tables 8.4 and 8.5.

iii. Install a dynamic shaker at a distance 5% of the target cable length from its upper end. Connect the dynamic shaker to the signal generator and adjust the excitation frequency.

iv. Determine the approximate fundamental frequency of the hybrid system by gradually changing the excitation frequency of the shaker. The excitation frequency corresponding to the peak response of the hybrid system gives the approximate fundamental frequency of the hybrid system.

v. Capture the transverse in-plane acceleration time history of the hybrid system over the excitation frequency range of \( f_1 - 0.5 \) Hz to \( f_1 + 0.5 \) Hz with an interval of 0.05 Hz, where \( f_1 \) is the approximate fundamental frequency of the hybrid system identified in step iv.
vi. Design a Butterworth filter to remove the response associated with higher modes and retain only the fundamental modal response of the hybrid system. The frequency range used in the Butterworth filter is \((f_1 - 0.5)\) Hz to \((f_1 + 0.5)\) Hz.

vii. Convert the acceleration response data to the corresponding displacement response data by applying a Fourier Transform and then an Inverse Fourier Transform as described in Section 8.2.1. Determine the maximum amplitude of vibration at each excitation frequency.

viii. Plot the frequency-response curve of the hybrid system as shown in Figure 8.9. The frequency associated with the peak of the frequency-response curve, \(D_{\text{max}}\), is the fundamental frequency of the hybrid system whereas the damping ratio can be determined by using the half-power method (Paz and Leigh, 2004), as given by Eq. (8-2).

\[
\xi = \frac{R_2 - R_1}{R_2 + R_1} \quad (8-2)
\]

where \(R_1\) and \(R_2\) represent respectively the two different excitation frequencies on each side of the peak displacement that correspond to the same half-power amplitude \(D_{\text{max}}/\sqrt{2}\).
Figure 8.9: Half-power method used to calculate the damping ratio

8.3 Experimental results and discussion

As discussed earlier in this chapter, the objective of the experimental study is to evaluate the in-plane frequency and the damping ratio associated with the fundamental mode of cable network and hybrid system, and then compare the experimental results with those yielded from analytical models and numerical simulations. The comparison of data among these three different approaches, i.e. the analytical, the numerical and the experimental, will help us to understand the impact of the different assumptions made in the analytical and numerical approaches on the modal response of cable networks and hybrid systems. A list of testing cases for cable network and hybrid system is presented in Tables 8.3 to Table 8.5, respectively.
8.3.1 Cable network

This section is dedicated in understanding the modal behaviour of a two-cable network shown in Figure 8.1(a). The eight testing cases used to study the modal behaviour of the cable network are listed in Table 8.3. The properties of the main cables and the cross-ties are listed in Tables 8.1 and 8.2, respectively.

The dynamic properties of the consisting cables in a cable network play an important role in affecting the modal behaviour of the formed system. Therefore, the in-plane frequency and the damping ratio associated with the fundamental mode of the isolated target and neighbouring cables were determined first by conducting free vibration tests. They are listed in Table 8.1.

A sample case of cable network, i.e. testing case 1 in Table 8.3, of which a rigid cross-tie was installed at the quarter-span of the target cable, is selected to illustrate the data analysis procedure. The acceleration response history of the network in the sample case was collected by following the procedures outlined in Section 8.2.1. Figure 8.10 portrays a fraction of the recorded acceleration time history of the target cable. The modal frequency of the cable network can be obtained by applying the power spectrum analysis. The power spectral density curve, obtained by using the ‘pwelch’ function in MatLab 7.01, is portrayed in Figure 8.11. Five peaks can be seen in the PSD curve. The first peak occurs at very low frequency and represents the background noise contained in the signal. The rest of the four peaks represent the modal frequency of the lowest four network modes as labelled in Figure 8.11. Once the fundamental frequency was found, the raw data was filtered to isolate the response of the network fundamental mode. Fourier Transform was then applied to the network fundamental modal acceleration response, as discussed in Section 8.2.1, to obtain the displacement time history of the network fundamental mode. It is depicted in Figure 8.12. Once the displacement time
response was known, the damping ratio of the fundamental mode was calculated by applying the logarithmic decrement method. Based on Eq. (8-1), the fundamental modal damping ratio of the studied cable network is 0.2%.

Figure 8.10: A fraction of sample acceleration time history raw data of the cable network (rigid cross-tie located at 1/4L)
Figure 8.11: Power spectral density curve of the cable network (rigid cross-tie located at $1/4L$)

Figure 8.12: Extracted fundamental modal displacement time history of the cable network (rigid cross-tie located at $1/4L$)
Similarly, the in-plane frequency and the damping ratio associated with the network fundamental mode in the rest of the testing cases were determined and the results are summarized in Tables 8.6 and 8.7, respectively. In order to verify the validity of the experimental results, the physical testing results were compared with those obtained from the analytical model and the numerical simulation. The analytical model of a two-cable network considering damping in the main cables was developed and validated in Section 3.2. The characteristic equation of such a two-cable network, Eq. (3-26), can be used to determine the in-plane frequency and the damping ratio of the network mode. In Eq. (3-26), the material and geometric properties of the main cables and the cross-tie are represented as non-dimensional parameters. The non-dimensional form of the properties of the main cables and the cross-ties in the studied cable network in are listed in Table 8.8. Numerical simulation of the modal behaviour of the cable network was conducted by using the commercial finite element software Abaqus 6.10 (SIMULIA, 2010). The results obtained from the analytical and the numerical approaches are also listed in Tables 8.6 and 8.7 for the convenience of comparison.

Table 8.6: Fundamental frequency of a two-cable network with different cross-tie positions (Hz)

<table>
<thead>
<tr>
<th>Cross-tie position ($e_c$)</th>
<th>Rigid cross-tie ($\psi=0.0003$)</th>
<th>Flexible cross-tie ($\psi=1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Analytical</td>
</tr>
<tr>
<td>1/4</td>
<td>7.00</td>
<td>7.13</td>
</tr>
<tr>
<td>1/3</td>
<td>7.07</td>
<td>7.22</td>
</tr>
<tr>
<td>1/2</td>
<td>7.15</td>
<td>7.28</td>
</tr>
<tr>
<td>3/4</td>
<td>7.00</td>
<td>7.13</td>
</tr>
</tbody>
</table>
Table 8.7: Fundamental modal damping ratio of a two-cable network with different cross-tie positions (%)

<table>
<thead>
<tr>
<th>Cross-tie position (εc)</th>
<th>Rigid cross-tie (ψ=0.0003)</th>
<th>Flexible cross-tie (ψ=1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Analytical</td>
</tr>
<tr>
<td>1/4</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>1/3</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>1/2</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>3/4</td>
<td>0.22</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 8.8: Material and geometric properties of main cables and cross-tie as non-dimensional parameters

<table>
<thead>
<tr>
<th>Element</th>
<th>Frequency ratio (η)</th>
<th>Mass-tension ratio (γ)</th>
<th>Cross-tie position (εc)</th>
<th>Cross-tie flexibility (ψ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target cable</td>
<td>1.0</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Neighbouring cable</td>
<td>0.79</td>
<td>1.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cross-tie</td>
<td>-</td>
<td>-</td>
<td>0.25, 0.33, 0.5, 0.75</td>
<td>0.0003, 1.5</td>
</tr>
</tbody>
</table>

As can be seen from Table 8.6, the in-plane fundamental frequency of the cable network with a rigid cross-tie obtained from the experimental approach is slightly lower, in general 2%, than those yielded from the other two approaches; whereas in the flexible cross-tie cases, the difference between all three approaches is negligible. In terms of the network fundamental modal damping ratio, in the case of a rigid cross-tie, the experimental results are in good agreement with those obtained from the analytical and the numerical approaches. In the flexible cross-tie case, the experimentally obtained fundamental modal damping ratio is much higher than those yielded from the other two approaches. The same phenomenon was observed earlier by Yamaguchi and Nagahawatta (1995). This should be attributed to the reason that although
damping of the cross-tie is physically present, it was not considered in the formulation of the analytical and the numerical models. In the case of flexible cross-tie, damping comes from not only the axial oscillation of the flexible cross-tie but also from its bending, of which the latter is not the case if a rigid cross-tie is used (Yamaguchi and Alauddin, 2001).

The impact of the assumptions made in the analytical and the numerical models, e.g. assuming idealized taut-cables and ignoring the intrinsic damping in cross-tie, on the modal response of cable networks can be evaluated from the above discussed results. For the fundamental in-plane frequency, the results obtained from these three different approaches are generally in good agreement. These results suggest that idealizing main cables as taut cables and neglecting the damping of cross-tie has an unsizable effect on the in-plane frequency of a pure cable network. In the studied experimental cases, the inextensibility parameter $\lambda^2$ of main cables is well below 1, therefore, it is reasonable to conclude that a taut cable assumption in an analytical or numerical model for a cable network is acceptable as long as the inextensibility parameter $\lambda^2$ is under 1. On the other hand, the influence of these assumptions on the network damping ratio seems negligible as long as the cross-tie is very stiff and with very low damping. However, ignoring damping in a flexible damped cross-tie could considerably underestimate the system damping since a substantial portion of the network damping comes from the damped flexible cross-tie.

The results in Tables 8.6 and 8.7 are portrayed in Figures 8.13 and 8.14, respectively. There are three curves in each figure, two of them represent, respectively, the rigid and the flexible cross-tie case while the third curve gives the modal properties of an isolated target cable as a reference base.
In-plane fundamental frequency of a two-cable network as a function of cross-tie position $\varepsilon_c$ ($\eta=0.79$)

Fundamental modal damping ratio of a two-cable network as a function of cross-tie position $\varepsilon_c$ ($\eta=0.79$)
It was pointed out in Chapter 6 that cross-tie position and cross-tie flexibility are the two important system parameters in the design of a cable network. The effect of these parameters on the in-plane frequency and the damping ratio of the studied two-cable network can be observed from Figure 8.13 and Figure 8.14. As can be seen in Figure 8.13, the in-plane fundamental frequency of the cable network reaches maximum when the cross-tie is placed at the mid-span of the target cable in both the rigid and the flexible cross-tie cases. The same phenomenon has been observed in Section 6.5. Besides, the results in Figure 8.13 also suggest that the use of a rigid ($\psi=0.0003$) or stiffer cross-tie would yield considerably higher fundamental frequency than using a flexible cross-tie ($\psi=1.5$). In one particular case when the cross-tie was placed at the mid-span of the target cable, there was a 10% increase in the fundamental frequency of the target cable by using a rigid cross-tie of $\psi=0.0003$, whereas only 5% increase is observed for the flexible cross-tie case ($\psi=1.5$).

However, the results in Figure 8.14 show that using a more flexible cross-tie is beneficial to the system damping. The fundamental modal damping ratio of the network becomes three times more in a more flexible cross-tie ($\psi=1.5$) case than in a rigid one ($\psi=0.0003$). This implies that a large portion of the damping present in a cable network comes from the energy dissipation through the oscillation of flexible cross-tie(s). This phenomenon was observed in Section 3.2.2.6 and Section 6.4 where the modal behaviour of a two-cable network with a damped flexible cross-tie was studied by the analytical approach. Further, similar behaviour was reported by Yamaguchi and Ito (1997) and Sun et al. (2007). It is important to note that connecting a target cable with a neighbouring one using a rigid cross-tie would not only give a much lower modal damping ratio than using a flexible cross-tie, but it could also result in a network damping ratio lower than that of an isolated target cable, as seen in Figure 8.14. This reveals that in a cable
network, damping in a more damped main cable would “transfer” or “flow” into the low damped ones. In the studied cable network, the damping ratio of the isolated target cable and the neighbouring cable are, respectively, 0.3% and 0.1%. After connecting the target cable with its neighbouring one through a rigid cross-tie placed at the mid-span, damping in the target cable transfers to the neighbouring cable and yields the equivalent damping of the formed network to be 0.2%.

The influence of cross-tie position on the in-plane fundamental frequency of a two-cable network can also be evaluated from Figure 8.13. The results in the figure reveal that the maximum in-plane fundamental frequency, both for the rigid as well as the flexible cross-tie, can be achieved by installing the cross-tie at the mid-span of the target cable. However, the fundamental modal damping ratio of a pure cable network was found to be slightly less when the cross-tie locates at the cable mid-span than at the quarter-span. This will be associated with the oscillation amplitude of the neighbouring cable, which is considerable at its mid-span than at the quarter-span. Since the neighbouring cable has a lower damping ratio than the target one, when cross-tie is installed at cable mid-span, the more active vibration of the less damped neighbouring cable would result in a larger “transfer” of damping from the more damped target cable and lead to a reduction of the overall/net system damping.

The above discussions reveal that both rigid and flexible cross-ties have their pros and cons on the performance of cable networks. Therefore, a reasonable compromise between the in-plane frequency and the damping ratio should be considered while selecting the stiffness of cross-tie.
8.3.2 Hybrid systems

When both cross-tie and external damper are used to control vibration of a target cable, a hybrid cable network, or simply a hybrid system, is formed. Typical configuration of hybrid systems A and B used in the current experimental study are shown in Figure 8.1(b) and Figure 8.1(c), respectively. Based on the cross-tie position and the stiffness, eight testing cases were conducted for hybrid system A. For hybrid system B, as the two cross-tie positions (1/4 L and 3/4 L) would render the same system layout, therefore, only three cross-tie positions of 1/4 L, 1/3 L and 1/2 L were experimentally tested for the rigid and the flexible cross-tie cases. The testing cases of hybrid system A and hybrid system B are listed in Tables 8.4 and 8.5, respectively. The material and geometrical properties of the main cables and the cross-ties used in the hybrid systems are given in Tables 8.1 and 8.2, respectively. The damping coefficient of the external viscous damper used in the experimental study was 19.1N·s/m. In the case of hybrid system A, the damper was attached to the target cable at a distance of 0.55 m from its lower support, as shown in Figure 8.1(b). The external damper was installed in-line with the cross-tie in the case of hybrid system B, as shown in Figure 8.1(c).

Due to the presence of the external damper in the hybrid system, free vibration response decays very quickly. Therefore, forced vibration test was used to determine the modal response of hybrid systems A and B. A dynamic shaker was installed at a distance 5% of the target cable length from its top end, as shown in Figures 8.1(b) and 8.1(c), with details shown in Figure 8.5. The shaker was placed on a supporting tripod to excite the hybrid system.

The procedures to determine the modal properties of both hybrid systems A and B are the same. A sample case of hybrid system A is selected below for illustration. In the sample system, a rigid cross-tie ($\psi=0.0003$) is placed at the quarter-span of the target cable. An approximate
identification of the system fundamental frequency was achieved by adjusting the excitation
frequency of the shaker through the signal generator and identifying the excitation frequency
which would yield the largest amplitude of the target cable response. It was found to be 7.0 Hz.
The acceleration response time history of the target cable in hybrid system A was captured at
different excitation frequencies between 6.5 Hz and 7.5 Hz, i.e. 0.5 Hz below and above the
approximate system fundamental frequency.

Once the system acceleration time history was captured, a Butterworth filter was
designed, as explained in step vi of Section 8.2.2, to retain only the fundamental modal response.
The filtered fundamental modal acceleration response data was converted to the corresponding
displacement time response data by applying a Fourier Transform and then an Inverse Fourier
Transform as described in Section 8.2.1. The maximum displacement at each excitation
frequency of the shaker is tabulated in Table 8.9. The frequency-response curve for this sample
case is shown in Figure 8.15.
Table 8.9: Maximum displacement at each excitation frequency of the shaker in hybrid system A (rigid cross-tie located at 1/4L)

<table>
<thead>
<tr>
<th>Excitation frequency (Hz)</th>
<th>Maximum displacement (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.50</td>
<td>0.270</td>
</tr>
<tr>
<td>6.55</td>
<td>0.302</td>
</tr>
<tr>
<td>6.60</td>
<td>0.352</td>
</tr>
<tr>
<td>6.65</td>
<td>0.395</td>
</tr>
<tr>
<td>6.70</td>
<td>0.453</td>
</tr>
<tr>
<td>6.75</td>
<td>0.540</td>
</tr>
<tr>
<td>6.80</td>
<td>0.639</td>
</tr>
<tr>
<td>6.85</td>
<td>0.784</td>
</tr>
<tr>
<td>6.90</td>
<td>1.004</td>
</tr>
<tr>
<td>6.95</td>
<td>1.316</td>
</tr>
<tr>
<td>7.00</td>
<td>1.640</td>
</tr>
<tr>
<td>7.05</td>
<td>1.243</td>
</tr>
<tr>
<td>7.10</td>
<td>0.843</td>
</tr>
<tr>
<td>7.15</td>
<td>0.644</td>
</tr>
<tr>
<td>7.20</td>
<td>0.515</td>
</tr>
<tr>
<td>7.25</td>
<td>0.434</td>
</tr>
<tr>
<td>7.30</td>
<td>0.359</td>
</tr>
<tr>
<td>7.35</td>
<td>0.313</td>
</tr>
<tr>
<td>7.40</td>
<td>0.272</td>
</tr>
<tr>
<td>7.45</td>
<td>0.242</td>
</tr>
<tr>
<td>7.50</td>
<td>0.218</td>
</tr>
</tbody>
</table>
Figure 8.15: Frequency-response curve of hybrid system A (rigid cross-tie located at 1/4L)

The fundamental frequency of the system is the frequency corresponding to the peak of the frequency-response curve, which is 7.00 Hz in Figure 8.15. The damping ratio of the system can be calculated using the half-power method (Paz and Leigh, 2004). The peak displacement of the frequency-response curve in Figure 8.15 is \( D_{\text{max}} = 1.64 \) cm. Thus, the displacement corresponding to the half-power point is \( \frac{D_{\text{max}}}{\sqrt{2}} = \frac{1.64}{\sqrt{2}} = 1.16 \) cm. There exist two excitation frequencies \( R_1 \) and \( R_2 \), which would give the same displacement amplitude of 1.16 cm, i.e. 6.916 Hz and 7.066 Hz, respectively. The system fundamental modal damping ratio can thus be determined using Eq. (8-2) as

\[
\xi_1 = \frac{R_2 - R_1}{R_2 + R_1} = \frac{7.066 - 6.916}{6.916 + 7.066} = 1.07\% 
\]
The in-plane frequency and the damping ratio associated with the fundamental mode of the sample hybrid system A, along with the other cases, are listed in Tables 8.10 and 8.11, respectively.

In order to compare the experimental results with the analytical and numerical ones, the analytical model developed in Section 7.2 was used to determine the in-plane fundamental frequency and the fundamental modal damping ratio of the sample hybrid system A. The properties of the main cables, the cross-tie and the external damper used in the system characteristic equation, Eq. (7-8), are in the non-dimensional form. Therefore, the non-dimensional properties of the main cables and the cross-tie are listed in Table 8.8 while the non-dimensional damper position parameter and the damper capacity parameter are $\varepsilon_{1,1}=0.065$ and $\mu=0.81$, respectively. By substituting these non-dimensional system parameters into Eq. (7-8) and using the approach discussed in Section 7.3, the analytical model can be used to determine the in-plane frequency and the damping ratio of the fundamental mode of hybrid system A. Numerical simulation was performed by using the finite element commercial package Abaqus 6.10. The results obtained from the analytical and numerical approaches are also listed in Tables 8.10 and 8.11 for the convenience to compare with the experimental results.

From the results in Table 8.10, one can see that the fundamental frequency of the hybrid system A is almost the same as that of the corresponding pure cable network listed in Table 8.6, which suggests that attaching an external damper of the current size to the target cable has negligible effect on the system in-plane stiffness. On the other hand, compared to the pure cable network, the damping ratio of the hybrid system fundamental mode increases significantly even in the rigid cross-tie case. For example, in a sample case of rigid cross-tie installed at the mid-span of the target cable, hybrid system A yields a fundamental modal damping ratio of 1.15% as
compare to 0.2% in the corresponding pure cable network. This indicates that even in a rigid cross-tie hybrid system, a sufficient amount of damping can be achieved by adding an external damper with appropriate capacity.

Table 8.10: Fundamental frequency of hybrid system A for different positions of cross-tie (damper installed at 0.065%L*) (Hz)

<table>
<thead>
<tr>
<th>Cross-tie position ($\varepsilon_c$)</th>
<th>Rigid cross-tie ($\psi=0.0003$)</th>
<th>Flexible cross-tie ($\psi=1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Analytical</td>
</tr>
<tr>
<td>1/4</td>
<td>7.00</td>
<td>7.14</td>
</tr>
<tr>
<td>1/3</td>
<td>7.08</td>
<td>7.22</td>
</tr>
<tr>
<td>1/2</td>
<td>7.15</td>
<td>7.28</td>
</tr>
<tr>
<td>3/4</td>
<td>7.00</td>
<td>7.14</td>
</tr>
</tbody>
</table>

* L is the length of the target cable.

Table 8.11: Fundamental modal damping ratio of hybrid system A for different positions of cross-tie (damper installed at 0.065%L*) (%)

<table>
<thead>
<tr>
<th>Cross-tie position ($\varepsilon_c$)</th>
<th>Rigid cross-tie ($\psi=0.0003$)</th>
<th>Flexible cross-tie ($\psi=1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Analytical</td>
</tr>
<tr>
<td>1/4</td>
<td>1.07</td>
<td>0.42</td>
</tr>
<tr>
<td>1/3</td>
<td>1.10</td>
<td>0.49</td>
</tr>
<tr>
<td>1/2</td>
<td>1.15</td>
<td>0.61</td>
</tr>
<tr>
<td>3/4</td>
<td>1.40</td>
<td>0.91</td>
</tr>
</tbody>
</table>

* L is the length of the target cable.

The results in Table 8.10 suggest that the in-plane fundamental frequency of hybrid system A obtained from the three different approaches is almost the same as was observed in the pure cable network cases. The discrepancy between the three approaches is negligible for the flexible cross-tie cases, whereas a maximum difference of 2% is observed in the rigid cross-tie cases. However, the experimentally obtained fundamental modal damping ratio of hybrid system
A is much higher than those obtained from the analytical and the numerical approaches, as can be seen from Table 8.11. It is worth mentioning that the fundamental modal damping ratio yielded from the analytical and the numerical approaches are in good agreement with each other. Many factors could be attributed to the higher system damping ratio obtained in the experimental test. There are numerous energy dissipating mechanisms that co-exist in hybrid system A. They include the external damper, the intrinsic damping of the main cables, the damping resulted from axial as well as flexural oscillation of the cross-tie, the frictional damping at cable anchorage and connection points between the main cable and the cross-tie, and the aerodynamic damping. In the analytical and the numerical model, the only source of damping considered is provided by the external damper, whereas the rest of the physically present energy dissipating mechanisms were not included in the formulation. Further, the discrepancy between the experimental study and the analytical or the numerical approach is found to be more in the rigid cross-tie cases than the flexible cross-tie ones. The reason behind this is the influence of cross-tie flexibility on different sources of energy dissipating mechanisms. As some of these sources, such as the aerodynamic damping, the intrinsic damping of the main cables and the friction damping at the connection points, are almost independent of cross-tie flexibility, whereas the damping provided by the external damper (the only source of damping mechanism in the analytical and the numerical model) is strongly influenced by the cross-tie flexibility. Using a more flexible cross-tie would impose less constraint on the cable motion at the damper attaching point and thus on damper operation. Therefore, in the experimental tests with flexible cross-tie, the contribution of an external damper towards the overall damping of the hybrid system is relatively high as compare to the damping provided by other mechanisms. This would reduce the discrepancy between the experimental results and the analytical or the numerical results of hybrid system fundamental
damping ratio. For the same reasons, the damping provided by an external damper would increase when a rigid cross-tie is moved away from an external damper and consequently increase the contribution of damping provided by the external damper to the overall damping of the hybrid system. Therefore, again, the discrepancy between the results obtained from the experimental tests and the analytical or the numerical models is reduced as the installation location of a rigid cross-tie moves away from the external damper, as can be seen from Table 8.11.

The above comparisons between the modal response of hybrid system A obtained from three different approaches helps to evaluate the impact of different assumptions made in the formulation of the analytical and the numerical models. In general, the in-plane fundamental frequency of hybrid system A obtained from these three different approaches agrees well, which suggests that idealizing the main cables as taut cables and ignoring the physically present energy dissipating mechanisms (e.g. the aerodynamic damping, the intrinsic damping of main cables and the friction damping at connection point) would have marginal impact on its in-plane fundamental frequency as long as the inextensibility parameter $\lambda^2$ of main cables is less than 1. However, ignoring the various damping mechanisms other than the external damper in the analytical and the numerical models could considerably underestimate the damping in a hybrid system A.

To better visualize the effect of the cross-tie position parameter, $\varepsilon_c$, and the cross-tie flexibility parameter, $\psi$, on the fundamental frequency and modal damping ratio of hybrid system A, the results in Table 8.10 and Table 8.11 are also depicted in Figure 8.16 and Figure 8.17, respectively. The solid line in both figures represents the modal property of an isolated target cable. It is given as a reference base. The impact of $\varepsilon_c$ and $\psi$ on the system in-plane
frequency is the same as those observed in the pure cable network case in Section 8.3.1, i.e. the maximum in-plane stiffness of hybrid system A can be achieved by placing a rigid cross-tie at the mid-span of the target cable. On the other hand, the effects of $\varepsilon_c$ and $\psi$ on the fundamental modal damping ratio of hybrid system A are different from those in the pure cable networks. Figure 8.17 shows that the fundamental modal damping ratio of hybrid system A increases monotonically with the increase of cross-tie position parameter $\varepsilon_c$. This monotonic increment could be attributed to the level of constraint the cross-tie has on the operation of an external viscous damper. By moving the cross-tie away from the external damper, its constraint on the operation of an external damper would be gradually lifted and results in higher system modal damping. This damping increment is more considerable when a rigid cross-tie is used and relocated from the mid-span to the three-quarter span of the target cable. The same behaviour of hybrid system A was observed in Section 7.5.2 where the spacing between the external damper and the cross-tie position was taken as one of the key system parameters.

![Figure 8.16: In-plane fundamental frequency of a two-cable hybrid system A as a function of cross-tie position $\varepsilon_c$ ($\eta=0.79$)](image-url)
Figure 8.17: Fundamental modal damping ratio of a two-cable hybrid system A as a function of cross-tie position $\varepsilon_c$ ($\eta=0.79$)

In terms of hybrid system B of which an external damper is installed in-line with the cross-tie, the testing cases are listed in Table 8.5. By conducting forced vibration test, the experimentally obtained in-plane fundamental frequency and modal damping ratio of all the cases are tabulated in Tables 8.12 and 8.13, respectively.

To obtain analytical results, the characteristic equation of hybrid system B, i.e. Eq. (7-16) developed in Section 7.3, can be used to determine its modal properties. Numerical results were obtained by developing corresponding models using the finite element commercial software Abaqus 6.10. The in-plane fundamental frequency and the fundamental modal damping ratio of the studied hybrid system B obtained from these three different approaches are given in Tables 8.12 and 8.13.
Table 8.12: Fundamental frequency of hybrid system B for different positions of cross-tie (Hz)

<table>
<thead>
<tr>
<th>Cross-tie position ($\varepsilon_c$)</th>
<th>Rigid cross-tie ($\psi=0.0003$)</th>
<th>Flexible cross-tie ($\psi=1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Analytical</td>
</tr>
<tr>
<td>1/4</td>
<td>7.00</td>
<td>7.19</td>
</tr>
<tr>
<td>1/3</td>
<td>6.80</td>
<td>7.29</td>
</tr>
<tr>
<td>1/2</td>
<td>6.60</td>
<td>7.33</td>
</tr>
</tbody>
</table>

Table 8.13: Fundamental modal damping ratio of hybrid system B for different positions of cross-tie (%)

<table>
<thead>
<tr>
<th>Cross-tie position ($\varepsilon_c$)</th>
<th>Rigid cross-tie ($\psi=0.0003$)</th>
<th>Flexible cross-tie ($\psi=1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Analytical</td>
</tr>
<tr>
<td>1/4</td>
<td>5.55</td>
<td>5.00</td>
</tr>
<tr>
<td>1/3</td>
<td>9.00</td>
<td>8.30</td>
</tr>
<tr>
<td>1/2</td>
<td>12.55</td>
<td>11.71</td>
</tr>
</tbody>
</table>

The results in Table 8.12 show that the experimentally obtained in-plane fundamental frequency of hybrid system B are slightly lower than the results obtained from the analytical and the numerical models in both the rigid and the flexible cross-tie cases. This discrepancy becomes more considerable as the damped cross-tie moves towards the mid-span of the target cable. Further, unlike the cable network or the hybrid system A cases, even in the case of a flexible cross-tie, the discrepancy is noticeable. These testing cases were repeated several times to minimize potential measurement error. One possible reason for this discrepancy could be that when a damper is installed in-line with a cross-tie, the in-plane vibration of hybrid system B could be coupled with the out-of-plane motion. This was clearly observed during the tests, in particular, when a damped flexible cross-tie was installed at the mid-span of the target cable. As in hybrid system B, the damper is always installed in-line with the cross-tie and this
configuration significantly increases the damping of the in-plane modes, especially when the damped cross-tie is installed relatively far from the cable anchorage. In addition to this, the cross-tie is also installed in the cable plane to enhance the in-plane stiffness of the hybrid system. However, the cross-ties are ineffective in controlling out-of-plane cable vibrations and there is no other constraint present for the system motion in the out-of-plane direction. Therefore, when external energy is imparted into the hybrid system to excite its in-plane motion, since the in-plane motion is constrained by the damped cross-tie, a part of the energy “flows” to the out-of-plane mode. This “transfer” of energy would excite the out-of-plane mode, resulting in a coupling between the in-plane and the out-of-plane modes of the hybrid system B and thus a transfer of stiffness from the relative stiffness in-plane mode to the less stiff out-of-plane mode. This would cause a reduction of the in-plane frequency of the hybrid system B, as observed in Table 8.12.

The impact of this coupled oscillation can also be observed from the fundamental modal damping ratio of hybrid system B in Table 8.13. Since the major source of damping in hybrid system B comes from the external damper, which is installed along the in-plane motion direction, therefore, its out-of-plane vibration modes are less damped than the in-plane ones. When the out-of-plane mode couples with the in-plane mode, damping contained in the more damped in-plane mode would be transferred to the less damped out-of-plane mode. This damping transfer would reduce the damping ratio of the in-plane mode, which can clearly be seen from the experimental results corresponding to a damped flexible cross-tie located at the one-third and the mid-span of the target cable listed in Table 8.13.

The experimentally obtained modal results of hybrid system B given in Tables 8.12 and 8.13 are depicted in Figures 8.18 and 8.19, respectively. As it can be seen in Figure 8.18 that
moving the damped cross-tie towards the mid-span of the target cable results in reducing the system in-plane stiffness. This reduction of the in-plane stiffness is only observed in the experimental results, whereas both analytical and numerical models show increment in in-plane frequency instead. As explained earlier, the reduction of the in-plane stiffness observed in the experimental study is caused by the coupling between the in-plane and the out-of-plane vibration modes, of which the system stiffness “transfers” from a more stiffen in-plane mode to the less stiffen out-of-plane mode. On the other hand, the fundamental modal damping ratio of hybrid system B increases as the damped cross-tie moves towards the mid-span of the target cable. The results in Table 8.13, especially for the flexible cross-tie cases, indicate that the discrepancy between the experimental results and the analytical or the numerical ones increases as the damped cross-tie moves towards the mid-span of the target cable. This discrepancy could be attributed to the “transfer” of damping from the more damped in-plane mode to the less damped out-of-plane during their coupled vibration.

The major difference between hybrid systems A and B is the significant increase in the fundamental modal damping ratio in the latter case. This is mainly due to the difference in the damper installation location. In hybrid system A, an external damper was installed relatively close to the cable anchorage ($\varepsilon_{1,1} = 0.065$) whereas in hybrid system B, the damper installation location is the same as the cross-tie position, of which $\varepsilon_{1,1}$ varies from 0.25 to 0.5, far from the cable anchorage. This would allow the damper operation to be less constrained by the proximity to the cable anchorage and help to dissipate more energy from the system. In terms of the in-plane fundamental frequency, an external damper of the current size would not cause any substantial change in the two studied hybrid system configurations.
Figure 8.18: In-plane fundamental frequency of a two-cable hybrid system $B$ as a function of cross-tie position $\varepsilon_c$ ($\eta=0.79$)

Figure 8.19: Fundamental modal damping ratio of a two-cable hybrid system $B$ as a function of cross-tie position $\varepsilon_c$ ($\eta=0.79$)
8.4 Summary

In this chapter, an effort has been made to understand the modal behaviour, in terms of the in-plane fundamental frequency and the fundamental modal damping ratio, of two-cable networks and hybrid systems with two different configurations in experimental study. The results obtained from the experimental study are validated by comparing with those obtained from analytical and numerical approaches. In the case of pure cable network and hybrid system A, the in-plane fundamental frequency obtained from the three different approaches generally agrees well, whereas for hybrid system B, the in-plane fundamental frequency obtained from the experimental tests is found to be lower than that obtained from the analytical model or numerical simulation. This could be caused by coupling between the in-plane and the out-of-plane vibration modes. This set of comparison suggests that the taut cable assumptions made in the analytical and the numerical models would have a marginal effect on predicting the in-plane fundamental frequency of pure cable networks and hybrid systems consisting of main cables with low inextensibility parameter $\lambda^2$ (e.g. $\lambda^2 < 1$). However, the modal damping obtained from the experimental tests is found to be always higher than that obtained from the analytical and the numerical models, whereas the later two are consistently in good agreement. Since only damping supplied by the external damper is included in the analytical and the numerical model formulation, neglecting other physically present damping mechanisms, such as the aerodynamic damping, the intrinsic damping of main cables and cross-tie and the friction damping at connection point, in these two approaches would considerably underestimate the actual damping especially when a damped flexible cross-tie is used. A comparison between the modal response of two different configurations of hybrid system reveals that hybrid system B is much more effective in producing higher modal damping. The considerably higher damping ratio of hybrid
system B is mainly associated with the location of the external damper in this configuration, which is generally far from the cable anchorage.
9.1 Conclusions

The current dissertation is dedicated to the study of the in-plane modal behaviour of conventional cable networks (cross-tie only networks) and hybrid cable networks (combined use of cross-ties and external dampers). In order to fully understand and cover the different aspects of the research topic, a comprehensive literature review is conducted in Chapter 2, which is not only limited to the pure cable network and the hybrid system solutions, but the other related topics such as cable dynamics; the external dampers installed on isolated cables; the understanding of damping; the mode localization in cable-stayed bridges, beams and other similar structures were also extensively reviewed.

In Chapter 3, analytical models were developed for two-cable networks with and without consideration of the damping in main cables and cross-ties. One of the main purposes of developing analytical model of damped cable networks was to gain the physical insights of cable network damping mechanism and to observe the transfer of structural damping among the main cables. The approach used in developing basic two-cable networks was extended to develop an analytical model of a generalized cable network consisting of a given number of main cables with multiple lines of cross-ties in Chapter 4.

The formation of local modes is one of the major drawbacks of the cross-tie solution. Therefore, Chapter 5 was dedicated to quantifying the degree of mode localization in cross-tied cable networks. The concept of the “degree of mode localization” (DML) was proposed to quantitatively assess the global nature of an individual network mode, whereas the local mode cluster (LMC) was proposed to evaluate the severity of local mode excitation in a cable network design based on the position, size and number of local modes formed within certain range of low
order network modes. These two indexes, DML and LMC, can be used to measure the severity of local modes present in a cable network. The roles of different system parameters were also explored to reduce the severity of local mode formation. In Chapter 6, key system parameters which would affect the modal behaviour of cable networks were identified. The effect of these system parameters on the in-plane stiffness as well as the modal damping and the formation of local modes were discussed. In addition to this, a cable network on a real cable-stayed bridge is chosen to evaluate the role of different system parameters in optimizing its performance in terms of the in-plane stiffness and the local mode formation.

Besides the study of conventional cable networks, a significant contribution of the current work is dedicated to understand the modal behaviour of the hybrid cable network systems. In Chapter 7, analytical model of hybrid cable network system was developed, based on which the system in-plane stiffness, the modal damping, and also the degree of mode localization were studied. A concept of ‘isoquant curve’ (similar to isobar or contour) was introduced to optimize the design of a hybrid system. A state of the art ‘generalized approach’ was developed to formulate the characteristic equation of a more complex hybrid system based on that of a relatively simple parent system. On top of these, the universal damping estimation curve for a single damped cable developed by Pacheco et al. (1993) was revisited and an extensive discussion was made on how the damping of a single damped cable would be modified once it became part of a hybrid system. Approximation equations were developed for a basic two-cable hybrid system in order to predict its optimum damper size and maximum attainable fundamental modal damping ratio.

All the analytical models developed for conventional and hybrid cable networks in Chapters 3 to 7 were validated by respective numerical simulations using a commercial finite element
software package Abaqus 6.10. Further to this, experimental tests were conducted for the two-cable networks and the hybrid systems with different configurations. This part of experimental work was presented in Chapter 8. The main purpose of the experimental study was to evaluate the impact of different assumptions made in formulating the analytical and the numerical models on the modal response of the studied cross-tied and hybrid cable networks.

The remaining part of this chapter is divided into two parts. The first part is devoted to conclusions drawn at different phases of this study while future recommendations are provided in the second part. The in-plane stiffness, the damping increment and the local mode formation are the three main design indexes of the conventional and hybrid cable network systems. Therefore, conclusions drawn from the current study are summarized below in terms of these three design indexes in three separate sub-sections and Section 9.1.4 is dedicated to conclusions drawn for hybrid systems.

9.1.1 In-plane stiffness

In exploring the role of various system parameters, it was observed that there were four important system parameters which would significantly affect the in-plane stiffness of a cable network. They are the frequency ratio $\eta$ of the neighbouring cable(s), the cross-tie position parameter $\varepsilon$, the cross-tie flexibility parameter $\psi$ and the number of cross-tie lines.

The in-plane stiffness of a cable network can be increased by connecting the more vulnerable cable(s) to the neighbouring ones having lower frequency ratio (i.e. more stiff neighbouring cables). In case all the cables in the network have the same frequency ratio (e.g. twin-cable network), the in-plane stiffness of the cable network would not improve. The in-plane stiffness of a cable network would increase if more number of main cables are included in the
cable network. This would have the same effect as reducing the frequency ratio parameter of the neighbouring cables.

The frequency ratio is a function of the physical and geometrical properties of main cables, which are determined by the actual loading conditions and other design requirements of a bridge. Therefore, most of the time, cross-tie designers have no choice on the frequency ratio parameter, whereas selecting the position and the stiffness of cross-tie(s) are their main design tasks. In the case of an ideal symmetric layout of cable network, cross-tie should be placed at the mid-span of the target cable (i.e. the longest cable in the network) to maximize the in-plane stiffness of the cable network. However, in general, cable networks on site have asymmetric layout of main cables. The fan or the semi-fan configurations of cable layout on real cable-stayed bridges are examples of an asymmetric cable network. Although installing the cross-tie in the vicinity of the longest cable mid-span would yield higher in-plane stiffness, it is at the cost of inducing local mode formation. When study the role of the cross-tie position on the in-plane stiffness and the formation of local modes, it is observed that installing the cross-tie lines evenly along one of the intermediate cables in the cable network could help reducing its local mode formation without compromising its in-plane stiffness. It has also been noticed that the in-plane stiffness of a cable network would also be increased if a cross-tie line is installed near the cable anchorage of any of the main cables present in a cable network.

The cross-tie flexibility is another important system parameter which should be chosen carefully. A cross-tie having a lower value of cross-tie flexibility parameter $\psi$ (i.e. a stiffer cross-tie) is preferable to maximize the in-plane stiffness of a cable network. However, it comes at the cost of increasing the size of local mode clusters. On the other hand, using a cross-tie of higher value of $\psi$ (i.e. a more flexible cross-tie) is beneficial to the increase of system damping and the
suppression of local mode formation, whereas it could adversely affect the in-plane stiffness of the cable network.

The number of cross-tie lines is also found to be an important system parameter in affecting the cable network in-plane stiffness. However, the present study shows that the effect is not cumulative. In the studied cable networks, installing the first two lines of cross-ties seem to be adequate to enhance the network in-plane stiffness and suppress the local mode formation. The subsequent addition of cross-tie line is found to have a marginal effect on enhancing the system in-plane stiffness. In addition, it is important to note that adding a new line of cross-tie is beneficial for delaying the appearance of the first local mode cluster, but at the cost of increasing its size.

9.1.2 Damping increment

Cross-tie solution does not only help enhancing the in-plane stiffness of a target cable, but also affects its damping property. Energy dissipation within cable network comes from various mechanisms including the aerodynamic damping, the friction damping at the cable and/or the cross-tie connection and anchorage points, the intrinsic damping in the main cables and the cross-ties. In the current study, the damping property of a cable network is explored by considering the intrinsic damping of the main cables as well as the cross-tie in the formulation of the analytical and the numerical models, whereas the other energy dissipation mechanisms were not included. Results show that the modal damping of a networked target cable will only increase provided it is connected with a neighbouring cable possessing higher damping. In such a case, placing a rigid cross-tie closer to the cable mid-span will be beneficial for increasing the network damping property. Adopting a damped flexible cross-tie would decrease the frequencies of network global modes but considerably increase their modal damping ratio. It is also found that
compared to the network in-phase global modes, modal properties associated with the out-of-phase global modes are more sensitive to the change of cross-tie stiffness and damping. Therefore, a careful balance between the loss in network in-plane stiffness and the gain in energy dissipation capacity should be kept when selecting cross-tie stiffness and damping in the network design.

9.1.3 Formation of local modes

A typical cable network design includes the selection of system parameters such as the cross-tie position, the cross-tie flexibility and the number of consisting main cables, in such a way that the combined effects of these should maximize the in-plane stiffness and energy dissipation of the cable network while minimizing the number of excited local modes. It is recommended to place cross-tie lines evenly along one of the intermediate cables to achieve the combined benefits of enhancing network in-plane stiffness and reducing the formation of local modes. As a general trend, using more rigid cross-ties would help improving the in-plane stiffness of a cable network, but it is at the expense of advancing the appearance of LMC and increasing its size considerably. Installing more number of cross-tie lines in a cable network would considerably increase its in-plane stiffness and push LMC to be formed at higher order modes but the size of LMC would greatly increase. In exploring the role of different system parameters in reducing the formation of local modes, a cable network on real cable-stayed bridge is studied. The results indicate that delaying early appearance of the first LMC and reducing its size can be achieved by reducing the number of cross-tie lines and installing them evenly along one of the intermediate cables without compromising the in-plane stiffness of the cable network.
9.1.4 Hybrid system

Results obtained from the modal behaviour of hybrid systems reveal the role of some of the important system parameters, such as the cross-tie position and the cross-tie flexibility, on influencing the in-plane stiffness and modal damping of hybrid system with different configurations. The use of more flexible cross-tie(s) would lift the constraints on the operation of external viscous damper and, therefore, results in higher system modal damping. But, on the other hand, the use of more flexible cross-tie(s) would reduce the in-plane frequency of system global modes and the vibration of target cable would become more dominant in the system fundamental mode which results in higher DML coefficient for the hybrid system fundamental mode.

The position of cross-tie has two distinct features depending on whether it is installed close to the near end or the far end of the external viscous damper. In the case of near end installation (install the cross-tie between the external damper and the cable support on the damper side), the role of the cross-tie is similar to that of the neoprene rubber bushings. It would result in a reduction of the hybrid system modal damping. In the case of the far end installation (install the cross-tie close to cable support not on the damper side), the damping ratio of the hybrid system can achieve higher than that of a single damped target cable. However, it is at the cost of losing the global nature of the system fundamental mode.

The effect of key system parameters, such as the cross-tie position and the cross-tie flexibility, on the hybrid system design indexes, i.e. the in-plane stiffness, the modal damping and the local mode formation, is different. A change in one system parameter may improve the performance of one design index but could have an adverse effect on the other design index(s). Therefore, a proposed concept of isoquant curve is recommended to optimize the response of a
selected hybrid system mode. In addition to this, the proposed generalized approach is an excellent tool to develop analytical models of more complex conventional and hybrid cable networks from a much simpler parent model.

In the current study, it is also observed that the optimum damper size and the maximum attainable fundamental modal damping ratio of a single damped cable would be altered once being connected with the neighbouring cable(s) and becomes part of a hybrid system. The effect of different hybrid system parameters, such as the cross-tie position and the cross-tie flexibility, on the optimum damper size is marginal. However, the maximum attainable fundamental modal damping ratio would be considerably influenced. The form of the approximation equations developed in Chapter 7 for predicting the optimum damper size and the maximum achievable damping in a hybrid system indicate that increasing the cross-tie position in its upper range (e.g. $\varepsilon_c=0.75$) would have more impact on the maximum attainable fundamental modal damping ratio than having the same increment in its lower range (e.g. $\varepsilon_c=0.25$). On the other hand, the same increment of cross-tie flexibility in its lower range would have more impact on the maximum attainable fundamental modal damping ratio than the one in its higher range.

9.2 Recommendations

An effort has been made in the current study to cover the most demanding needs in properly understanding the dynamic behaviour of conventional and hybrid cable network solutions. However, there are still numerous aspects that need to be further explored in future studies.

1) In the present study, the bending stiffness of main cables is ignored, which is an acceptable assumption for cables on small to medium size cable-stayed bridges. However, in the case of recent long-span cable-stayed bridges, ignoring the bending stiffness of main cables would not be a reasonable assumption. Therefore, it is recommended that the modal behaviour of
pure cable networks and hybrid systems should be further studied by considering the bending stiffness of main cables. Similarly, majority of stay cables on real cable-stayed bridge fulfil the criteria of taut cable assumption, however, in the case of very long stay cables (e.g. the longest cable being 580 m on the Russky Bridge in Russia is 580 m), there is a significant amount of cable sag which should not be ignored. Therefore, sag in the main cables should also be considered when developing refined analytical model for conventional and hybrid cable networks.

2) Linear behaviour is assumed in the current study for the main cables as well as the cross-tie(s). The non-linearity in the cross-ties and in the main cables should be considered to refine the analytical models proposed in the current study.

3) In existing literature, the cross-tie is assumed to oscillate along its axial direction and its out-of-plane vibration is ignored. Consider all possible vibration motions of cross-tie (e.g. the out-of-plane motion and the bending behaviour) will be a challenging task for future researchers. However, this kind of research will be beneficial to further understand the mechanism of the cross-tie solution.

4) In the cross-tie solution, only the in-plane vibration of cable network is considered, whereas research on cable dynamics indicates that external force acting in the in-plane direction of the cable might lead to coupling between the in-plane and the out-of-plane vibration modes. Therefore, it is recommended that future research should also consider the out-of-plane vibrations of main cables during the development of conventional and hybrid cable network models.
5) The concept of isoquant curves introduced in the current study is used to optimize the fundamental mode of hybrid system. The same approach can also be extended to multi-mode optimization of hybrid systems.
Appendix A: Copyright permission

Request for the reproduce of the copyrighted material and the response of the author:

To: Javaid Ahmad
From: Luca Caracoglia
Date: 6/12/2016 1:46 PM
Subject: Re: Permission for Figs used in your published papers

Sure no problem, thanks

----- Original Message -----  
From: Javaid Ahmad  
To: Luca Caracoglia  
Sent: 6/12/2016 1:26 PM  
Subject: Permission for Figs used in your published papers

Dear Dr. Caracoglia,

My dissertation needs a reference to following figures from two of your published papers. I really appreciate you if you give me permission to use these figures for academic purpose only.

1: Figure 4 and Fig 11 ("In-plane dynamic behaviour of cable networks. Part 2: prototype prediction and validation" published in Journal of Sound and Vibration 279 (2005).
2: Figure 3 ("Effectiveness of cable networks of various configurations in suppressing stay-cable vibration" published in Engineering Structures, 31 (2009).

Thanks so much for your support and kindness

with best regards,

Javaid Ahmad  
(PhD candidate at University of Windsor, ON Canada)
Appendix B: Copyright permission

Request for the reproduce of the copyrighted material and the response of the author:

To: Javaid Ahmad
From: Michael Haijun Zhou
Date: 6/13/2016 1:35 AM
Subject: Re: Permission for Figs used in your published papers

Dear Javaid Ahmad:

It is OK, you can use the figure for academic purpose.

Best
Dr. Michael Haijun Zhou

Associate Professor, Associate Head
Department of Civil Engineering, College of Civil Engineering, Shenzhen University;
Guangdong Provincial Key Laboratory of Durability for Marine Civil Engineering (Shenzhen University).

----- Original Message -----
From: Javaid Ahmad
To: Michael Haijun Zhou
Sent: 6/12/2016 1:38 PM
Subject: Permission for Figs used in your published papers

Dear Dr. Zhou,

My dissertation needs a reference to following figures from two of your published papers. I really appreciate you if you give me permission to use these figures for academic purpose only.


Thanks so much for your support and kindness

with best regards,

Javaid Ahmad
(PhD candidate at University of Windsor, ON Canada)
REFERENCES


VITA AUCTORIS

NAME: Javaid Ahmad

PLACE OF BIRTH: D.G.Khan, Pakistan

YEAR OF BIRTH: 1964

EDUCATION:
University of Engineering and Technology
Lahore, Pakistan 1984-1989
B.Sc. Civil Engineering

University of Windsor
Windsor, Ontario 2009-2010
MEng. Civil Engineering

University of Windsor
Windsor, Ontario 2010-2012
M.A.Sc. Civil Engineering

University of Windsor
Windsor, Ontario 2012-2016
Ph.D. Civil Engineering