Energy-efficient spectrum sensing approaches for cognitive radio systems

Faroq Ali Awin
University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/5902

This online database contains the full-text of PhD dissertations and Masters' theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000 ext. 3208.
Energy-Efficient Spectrum Sensing Approaches for Cognitive Radio Systems

By

Faroq Ali Awin

A Dissertation
Submitted to the Faculty of Graduate Studies
through the Department of Electrical and Computer Engineering
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy
at the University of Windsor

Windsor, Ontario, Canada

2016

© 2016 Faroq Awin
Energy-Efficient Spectrum Sensing Approaches for Cognitive Radio Systems

by

Faroq Awin

APPROVED BY:

__________________________________________________________________________

W. Hamouda, External Examiner
Concordia University

__________________________________________________________________________

N. Zamani
Department of Mechanical, Automotive and Materials Engineering

__________________________________________________________________________

M. Khalid
Department of Electrical and Computer Engineering

__________________________________________________________________________

K. Tepe
Department of Electrical and Computer Engineering

__________________________________________________________________________

M. Ahmadi, Co-advisor
Department of Electrical and Computer Engineering

__________________________________________________________________________

E. Abdel-Raheem, Advisor
Department of Electrical and Computer Engineering

November 29, 2016
DECLARATION OF CO-AUTHORSHIP AND PREVIOUS PUBLICATION

This dissertation includes five original papers that have been previously submitted/published for publication in peer reviewed journals and conferences, as follows:

<table>
<thead>
<tr>
<th>Dissertation Chapter</th>
<th>Publication Title</th>
<th>Publication Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part of Chapter 3</td>
<td>F. A. Awin, E, Abdel-Raheem, M. Ahmadi, “Agile hierarchical cluster structure-based cooperative spectrum sensing in cognitive radio networks” IEEE 26th International Conference in Microelectronics, ICM, Doha, Qatar (2014): 48-51.</td>
<td>Published</td>
</tr>
</tbody>
</table>
I hereby declare that this dissertation incorporates material that is result of joint research with both Dr. Esam Abdel-Raheem and Dr. Majid Ahmadi and under their supervision. I am aware of the University of Windsor Senate Policy on Authorship, and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained written permission from each of the co-authors to include the above materials in my thesis. I certify that, with the above qualification, this dissertation, and the research to which it refers, is the product of my own work.

I certify that I have obtained a written permission from the copyright owner(s) to include the above published material(s) in my thesis. I certify that the above material describes work completed during my registration as graduate student at the University of Windsor.

I certify that, to the best of my knowledge, my dissertation does not infringe upon anyone’s copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my dissertation, published or otherwise, are fully acknowledged in accordance with the standard referencing practices.

I declare that this is a true copy of my thesis, including any final revisions, as approved by my thesis committee and Graduate Studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.
ABSTRACT

Designing an energy efficient cooperative spectrum sensing for cognitive radio network is our main research objective in this dissertation. Two different approaches are employed to achieve the goal, clustering and minimizing the number of participating cognitive radio users in the cooperative process. First, using clustering technique, a multi-level hierarchical cluster-based structure spectrum sensing algorithm has been proposed to tackle the balance between cooperation gain and cost by combining two different fusion rules and exploiting the tree structure of the cluster. The algorithm considerably minimizes the reporting overhead while satisfying the detection requirements. Second, based on reducing the number of participating cognitive radio users, primary user protection is considered to develop an energy efficient algorithm for cluster-based cooperative spectrum sensing system. An iterative algorithm with low complexity has been proposed to design energy efficient spectrum sensing for cluster-based cooperative systems. Simulation results show that the proposed algorithm can significantly minimize the number of contributing of cognitive radio users in the collaboration process and can compromise the performance gain and the incurred overhead. Moreover, a variable sensing window size is also considered to propose three novel strategies for energy efficient centralized cooperative spectrum sensing system using the three hard decision fusion rules. The results show that strategies remarkably increase the energy efficiency of the cooperative system; furthermore, it is shown optimality of $k$ out of $N$ rule over other two hard decision fusion rules. Finally, joint optimization of transmission power and sensing time for a single cognitive radio is considered. An iterative algorithm with low computational requirements has been proposed to jointly optimize power and sensing time to maximize the energy efficiency metric. Computer results have shown that the proposed algorithm outperforms those existing works in the literature.
DEDICATION

To my beloved parents,
wife, children, all family, and dearest friend Khalid

with love and sincerity.
ACKNOWLEDGEMENTS

All praises and thanks are due to almighty ALLAH, the Lord of the worlds, for his unlimited support in all times of my daily life and for his guidance. I would love to express deep thanks and sincere appreciations to my supervisors Dr. Esam Abdel-Raheem and Dr. Majid Ahmadi for their support, help and supervision throughout my Ph.D. study period. I am thankful to Dr. Kemal Tepe for his valuable comments and discussions about some topics in wireless communications and during my Ph.D. seminars. Special thanks to all my Ph.D. committee members Dr. Mohamed Khalid and Dr. Nader Zamani for their valuable notes, comments and discussion during my Ph.D. seminars. Finally, I would also love to express my gratitude and thanks to those who were behind my success, my beloved parents, wife and children for their numerous patience and support.
### TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declaration of co-authorship and previous publications</td>
<td>iii</td>
</tr>
<tr>
<td>Abstract</td>
<td>v</td>
</tr>
<tr>
<td>Dedication</td>
<td>vi</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>vii</td>
</tr>
<tr>
<td>List of tables</td>
<td>x</td>
</tr>
<tr>
<td>List of figures</td>
<td>xi</td>
</tr>
<tr>
<td>List of appendices</td>
<td>xiii</td>
</tr>
<tr>
<td>List of abbreviations/symbols</td>
<td>xiv</td>
</tr>
<tr>
<td>1 Introduction to cognitive radio</td>
<td>1</td>
</tr>
<tr>
<td>1. Problem statement</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Research objectives</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Research contributions</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Organization of dissertation</td>
<td>6</td>
</tr>
<tr>
<td>2 An Overview of Spectrum Sensing in Cognitive Radio Networks</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Spectrum sensing preliminaries</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Models of cooperative spectrum sensing</td>
<td>10</td>
</tr>
<tr>
<td>2.3.1 Centralized cooperative spectrum sensing</td>
<td>12</td>
</tr>
<tr>
<td>2.3.2 Decentralized cooperative spectrum sensing</td>
<td>14</td>
</tr>
<tr>
<td>2.3.3 Relay assisted cooperative spectrum sensing</td>
<td>15</td>
</tr>
<tr>
<td>2.4 Tradeoff between cooperation gain and cooperation cost</td>
<td>17</td>
</tr>
<tr>
<td>2.4.1 Channel Impairments</td>
<td>18</td>
</tr>
<tr>
<td>2.4.2 Sensing time</td>
<td>20</td>
</tr>
<tr>
<td>2.4.3 Energy efficiency</td>
<td>22</td>
</tr>
<tr>
<td>2.4.4 Mobility</td>
<td>24</td>
</tr>
<tr>
<td>2.5 Cluster based cooperative spectrum sensing</td>
<td>25</td>
</tr>
<tr>
<td>2.5.1 Models of CBSS</td>
<td>26</td>
</tr>
<tr>
<td>2.5.1.1 Performance gain oriented models</td>
<td>27</td>
</tr>
<tr>
<td>2.5.1.2 Overhead reduction oriented models</td>
<td>27</td>
</tr>
<tr>
<td>2.5.1.3 Combined metric based models</td>
<td>28</td>
</tr>
<tr>
<td>2.6 Conclusions</td>
<td>29</td>
</tr>
<tr>
<td>3 Hierarchical cluster based cooperative spectrum sensing in cognitive radio networks</td>
<td>30</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>30</td>
</tr>
<tr>
<td>3.2 System model for proposed algorithm</td>
<td>31</td>
</tr>
<tr>
<td>3.2.1 Existing models</td>
<td>31</td>
</tr>
<tr>
<td>3.2.2 Proposed model</td>
<td>32</td>
</tr>
<tr>
<td>3.2.2.1 Perfect reporting channel</td>
<td>37</td>
</tr>
<tr>
<td>3.2.2.2 Imperfect reporting channel</td>
<td>38</td>
</tr>
<tr>
<td>3.3 Optimization and energy efficiency of the proposed algorithm</td>
<td>41</td>
</tr>
<tr>
<td>3.3.1 Fusion rule optimization of the proposed algorithm</td>
<td>41</td>
</tr>
<tr>
<td>3.3.2 Energy efficiency of the proposed algorithm</td>
<td>44</td>
</tr>
<tr>
<td>3.3.3 Sensing agility and energy consumption</td>
<td>46</td>
</tr>
<tr>
<td>3.4 Simulation results and discussion</td>
<td>47</td>
</tr>
</tbody>
</table>
# 3.5 Conclusions

**4 Novel Energy Efficient Strategies for Cooperative Spectrum Sensing in Cognitive Radio Networks**

4.1 Introduction ........................................................................ 56
4.2 System model for CBSS system ........................................... 57
4.3 Proposed strategies to minimize the number of CRs in centralized CSS system ......................................................... 62
4.3.1 Strategy for OR-Rule ....................................................... 62
4.3.2 Strategy for AND-Rule .................................................... 64
4.3.3 Strategy for k out of N-Rule ............................................ 65
4.4 Simulation results and discussion ......................................... 66
4.5 Conclusions ....................................................................... 75

**5 Optimal Energy Efficient Cluster-Based Spectrum Sensing for Cognitive Radio Networks** ................................................................. 77
5.1 Introduction ....................................................................... 77
5.2 System model ..................................................................... 78
5.3 Formulation of design problem and proposed algorithm ......... 80
5.3.1 The proposed algorithm ............................................... 81
5.4 Simulation results and discussion ........................................ 85
5.5 Conclusions .................................................................... 92

**6 Joint Optimal Transmission Power and Sensing Time for Energy-Efficient Spectrum Sensing in Cognitive Radio Systems** ................................................................. 93
6.1 Introduction ....................................................................... 93
6.2 System model .................................................................... 94
6.3 Problem Formulation ........................................................ 95
6.4 The proposed algorithm .................................................... 96
6.5 Simulation results and discussion ........................................ 103
6.6 Conclusions .................................................................... 111

**7 Dissertation conclusion and future work.**
7.1 Conclusions ..................................................................... 112
7.2 Future works .................................................................... 115

References ........................................................................... 116
Appendices ........................................................................... 125
A Appendix A ....................................................................... 125
B Appendix B ....................................................................... 127
C Appendix C ....................................................................... 129
Vita Auctoris ........................................................................ 130
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Local spectrum sensing (LSS) vs. Cooperative spectrum sensing (CSS)</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Comparison between Hard decision and Soft decision rules</td>
<td>14</td>
</tr>
<tr>
<td>3.1</td>
<td>Results of iterative algorithm for a cluster with 24CRs</td>
<td>47</td>
</tr>
<tr>
<td>3.2</td>
<td>Overhead comparison between the three algorithms, for a cluster with 24 CRs</td>
<td>50</td>
</tr>
<tr>
<td>3.3</td>
<td>Overhead comparison between the three algorithms, for a cluster with 18 CRs</td>
<td>52</td>
</tr>
<tr>
<td>3.4</td>
<td>Reporting overhead and agility gain for different algorithms, for a cluster with 8 CRs</td>
<td>52</td>
</tr>
<tr>
<td>5.1</td>
<td>Comparison between the proposed algorithm and two existing works for a cluster with 10 CRs</td>
<td>92</td>
</tr>
<tr>
<td>6.1</td>
<td>The corresponding $\bar{\theta}_k$ for each $\phi_k$ using the proposed iterative algorithm</td>
<td>102</td>
</tr>
<tr>
<td>6.2</td>
<td>Comparison between the proposed algorithm and some existing algorithms that maximize the energy efficiency for a single CR</td>
<td>110</td>
</tr>
<tr>
<td>6.3</td>
<td>Quantitative comparison between the proposed algorithm and linear search algorithm [107]</td>
<td>111</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Basic cognition cycle</td>
<td>2</td>
</tr>
<tr>
<td>2.1</td>
<td>Models of CSS system, a) centralized model b) decentralized c) relay assisted</td>
<td>11</td>
</tr>
<tr>
<td>2.2</td>
<td>SNR walls vs. number of sensing samples for different noise uncertainty values</td>
<td>19</td>
</tr>
<tr>
<td>2.3</td>
<td>Optimal sensing time in terms of throughput for different cluster sizes</td>
<td>21</td>
</tr>
<tr>
<td>3.1</td>
<td>System model of the conventional CBSS algorithm</td>
<td>32</td>
</tr>
<tr>
<td>3.2</td>
<td>System model of the proposed algorithm</td>
<td>33</td>
</tr>
<tr>
<td>3.3</td>
<td>Block diagram of energy detector</td>
<td>35</td>
</tr>
<tr>
<td>3.4</td>
<td>Basic time frame of the CRN</td>
<td>40</td>
</tr>
<tr>
<td>3.5</td>
<td>Overall probability of detection of the proposed algorithm compared to conventional and MCMG algorithm, for a cluster with 24CRs</td>
<td>48</td>
</tr>
<tr>
<td>3.6</td>
<td>Complementary receiver operating characteristics of the three algorithms, for a cluster with 24CRs</td>
<td>49</td>
</tr>
<tr>
<td>3.7</td>
<td>Overall probability of detection of the three different algorithms, for a cluster with 18CRs</td>
<td>50</td>
</tr>
<tr>
<td>3.8</td>
<td>Complementary receiver operating characteristics of the three algorithms, for a cluster with 18CRs</td>
<td>51</td>
</tr>
<tr>
<td>3.9</td>
<td>Maximum normalized throughput vs. probability of detection constraint for a cluster with 18CRs</td>
<td>51</td>
</tr>
<tr>
<td>3.10</td>
<td>Required number of reporting CRs in the cluster for three different algorithms at $\gamma = 10$ dB</td>
<td>53</td>
</tr>
<tr>
<td>3.11</td>
<td>Data transmission time $T_d$ for the cluster with different numbers of CRs computed by three different algorithms</td>
<td>53</td>
</tr>
<tr>
<td>3.12</td>
<td>Total energy consumption for the three algorithms</td>
<td>54</td>
</tr>
<tr>
<td>4.1</td>
<td>Probability of detection $P_d$ and probability of false alarm $P_f$ for single CR against sensing window size $M_s$</td>
<td>59</td>
</tr>
<tr>
<td>4.2</td>
<td>Basic time frame of the CRN</td>
<td>62</td>
</tr>
<tr>
<td>4.3</td>
<td>Required CRs $N_c$ vs. sensing window size using OR-Rule at different radio conditions</td>
<td>67</td>
</tr>
<tr>
<td>4.4</td>
<td>Global probability of false alarm $Q_{f,OR}$ vs. sensing window size using OR-Rule at different radio conditions</td>
<td>67</td>
</tr>
<tr>
<td>4.5</td>
<td>Energy efficiency $\xi$ vs. sensing window size using OR-Rule at different radio conditions</td>
<td>68</td>
</tr>
<tr>
<td>4.6</td>
<td>Required CRs $N_c$ vs. sensing window size using AND-Rule at different radio conditions</td>
<td>69</td>
</tr>
<tr>
<td>4.7</td>
<td>Global probability of detection $Q_{d,AND}$ vs. sensing window size using AND-Rule at different radio conditions</td>
<td>70</td>
</tr>
<tr>
<td>4.8</td>
<td>Energy efficiency $\xi$ vs. sensing window size using AND-Rule at different radio conditions</td>
<td>70</td>
</tr>
<tr>
<td>4.9</td>
<td>Required CRs $N_c$ vs. sensing window size using $k$ out of $N$-Rule, at $\gamma = -2$dB, at different values of $k$</td>
<td>71</td>
</tr>
</tbody>
</table>
# LIST OF APPENDICES

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix A: Derivations of optimal $k_1$ and $k_2$</td>
<td>125</td>
</tr>
<tr>
<td>Appendix B: Calculation of reporting overhead in cluster with $N$ CRs</td>
<td>127</td>
</tr>
<tr>
<td>Appendix C: Copyright permission</td>
<td>129</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>AMC</td>
<td>Automatic modulation classifier</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white gaussian noise</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary phase shift keying</td>
</tr>
<tr>
<td>BS</td>
<td>Base station</td>
</tr>
<tr>
<td>CBSS</td>
<td>Cluster-based spectrum sensing</td>
</tr>
<tr>
<td>CCSS</td>
<td>Centralized cooperative spectrum sensing</td>
</tr>
<tr>
<td>CH</td>
<td>Cluster head</td>
</tr>
<tr>
<td>CR</td>
<td>Cognitive radio</td>
</tr>
<tr>
<td>CROC</td>
<td>Complementary receiver operating characteristic</td>
</tr>
<tr>
<td>CRN</td>
<td>Cognitive radio network</td>
</tr>
<tr>
<td>CSCG</td>
<td>Circularly symmetric complex gaussian</td>
</tr>
<tr>
<td>CSS</td>
<td>Cooperative spectrum sensing</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal gain combining</td>
</tr>
<tr>
<td>FC</td>
<td>Fusion center</td>
</tr>
<tr>
<td>GH</td>
<td>Group head</td>
</tr>
<tr>
<td>HTP</td>
<td>Hidden terminal problem</td>
</tr>
<tr>
<td>i.i.d</td>
<td>Identical independent distributed</td>
</tr>
<tr>
<td>LSS</td>
<td>Local spectrum sensing</td>
</tr>
<tr>
<td>MANT</td>
<td>Mobile ad hoc network</td>
</tr>
<tr>
<td>MCMG</td>
<td>Multi-cluster multi-group</td>
</tr>
<tr>
<td>MHCBSS</td>
<td>Multi-level hierarchical cluster-based spectrum sensing</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum ratio combining</td>
</tr>
<tr>
<td>OSA</td>
<td>Opportunistic spectrum access</td>
</tr>
<tr>
<td>PU</td>
<td>Primary user</td>
</tr>
<tr>
<td>ROC</td>
<td>Receiver operating characteristic</td>
</tr>
<tr>
<td>RSS</td>
<td>Received signal strength</td>
</tr>
<tr>
<td>SC</td>
<td>Selection combining</td>
</tr>
<tr>
<td>SGH</td>
<td>Subgroup head</td>
</tr>
<tr>
<td>SLC</td>
<td>Square law combining</td>
</tr>
<tr>
<td>SLS</td>
<td>Square law selection</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to noise ratio</td>
</tr>
<tr>
<td>SSDF</td>
<td>Spectrum sensing data falsification</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary user</td>
</tr>
<tr>
<td>TV</td>
<td>Television</td>
</tr>
<tr>
<td>[-]</td>
<td>Ceiling function</td>
</tr>
<tr>
<td>a</td>
<td>Acceleration of a moving CR</td>
</tr>
<tr>
<td>A</td>
<td>Total reporting overhead for a CRN</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Reporting overhead for a conventional CBSS algorithm</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Global probability of detection constraint</td>
</tr>
<tr>
<td>$B_r$</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>$B(m)$</td>
<td>Normalized throughput for an $m$ reporting CRs</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Global probability of false alarm constraint</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Channel capacity at $H_0$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Decision of the $i^{th}$ CR about the existence of the PU</td>
</tr>
</tbody>
</table>
The derived formulas include:

- $d_t$: Final decision of CSS system about the existence of the PU
- $\delta$: Probability of false alarm constraint for a single CR
- $\Delta A$: Agility gain
- $\Delta E$: The percentage of saved total energy consumption
- $\Delta E_r$: The percentage of saved reporting energy
- $\Delta t$: Time elapsed between consecutive measurements
- $\Delta T_r$: The difference in reporting time
- $\varepsilon$: Probability of detection constraint for a single CR
- $\xi$: Control channel gain
- $\xi_{max,j}$: Maximum control channel gain in the $j$th group
- $f(x/H_0)$: Conditional probability density function of $x$ given $H_0$
- $f(x/H_1)$: Conditional probability density function of $x$ given $H_1$
- $\bar{y}$: Average SNR
- $\gamma_i$: SNR for the $i$th CR
- $\gamma_{max,j}$: Maximum SNR in the $j$th subgroup
- $J(T)$: Likelihood ratio test (LRT)
- $H$: Number of CRs in one subgroup
- $h$: Channel gain between TV transmitter and CR receiver
- $H_0$: Hypothesis of channel being idle
- $H_1$: Hypothesis of channel being busy
- $H_j$: Number of CRs in the $j$th subgroup
- $k_1$ and $k_2$: Fusion rule parameters
- $\tilde{k}_1$ and $\tilde{k}_2$: Optimal fusion rule parameters
- $L$: Number of sensing sample or sensing window size
- $L_{max}$: Maximum sensing window size
- $L_{min}$: Minimum sensing window size
- $L(T)$: Logarithmic likelihood ratio test
- $\lambda$: Sensing threshold
- $M$: Number of groups in one cluster
- $m$: Number of reporting CRs
- $M_s$: Number of sensing samples in 1ms
- $\mu_0$: Mean of received signal given $H_0$
- $\mu_1$: Mean of received signal given $H_1$
- $v(i)$: Speed of a moving CR at the $i$th time increment
- $N$: Normal distribution
- $N_r$: Number of required CRs in CRN to meet some criterion
- $N_t$: Total number of CRs in the CRN
- $P$: Received signal strength at the transmitter
- $P(\ )$: Probability
- $\overline{P}_e$: Average error probability for a subgroup
- $P_{e|\xi_{max,j}}$: Error probability rate of BPSK with $\xi_{max,j}$
- $P_d$: Probability of detection for a single CR
- $P_{dSG}$: Probability of detection for a subgroup
- $P_f$: Probability of false alarm for a single CR
$P_f$: Predefined probability of false alarm
$P_{md}$: Probability of missed detection for a single CR
$P_R$: Received signal strength at the receiver
$P(H_0)$: Probability of PU being absent
$P(H_1)$: Probability of PU being present
$P_r(\cdot)$: Denotes probability
$\phi_s, P_s$: Sensing power
$\phi_t, P_t$: Transmission power
$q_e$: Overall error detection rate
$Q(\cdot)$: $Q$-function
$q_d$: Global probability of detection for CRN
$Q_f$: Global probability of false alarm for CRN
$q_m$: Global probability of missed detection for CRN
$q_{dG}$: Probability of detection for a group
$q_{fG}$: Probability of false alarm for a group
$R$: Number of subgroups in one group
$R(m)$: Throughput of an $m$ CRs in a cluster
$\rho$: Exponential decay correlation model
$\sigma_n^2$: Noise power
$\sigma_s^2$: Signal power
$T$: Basic frame time for a CRN
$T_d$: Data transmission time for a CRN
$T_{dc}$: Data transmission time for a CRN using conventional CBSS algorithm
$T_{dp}$: Data transmission time for a CRN using the proposed algorithm
$T_r$: Total reporting time for a CRN
$T_{rc}$: Total reporting time for a CRN using conventional CBSS algorithm
$T_{rp}$: Total reporting time for a CRN using the proposed algorithm
$T_s$: Sensing time period
$\tau_s$: Sensing time
$\tau_r$: Reporting time for a single CR
$\zeta$: Energy efficiency metric
Chapter One: 
*Introduction to Cognitive Radio Networks*

The research on solving spectrum inefficiency dilemma has been intensively rising with the massive growth of wireless communication applications and their users. Opportunistic spectrum access (OSA) has been proposed as an optimal solution to improve the spectrum efficiency by exploiting underutilized frequency bands. Autonomy, adaptation and ability to detect and learn the radio environment conditions were the main criteria to select a cognitive radio (CR) as the best technique to practically implement the OSA. Cognitive radio is considered as an intelligent paradigm that enables unlicensed users to opportunistically exploit the underutilized spectrum bands of licensed users during their inactivity periods on non-interfering basis. The cognition cycle of the CR consists of four phases; spectrum sensing, spectrum management, spectrum sharing and spectrum mobility as depicted in Fig. 1.1. Spectrum sensing is the key function of cognition process of the CR; it is the phase through which the CR can recognize the existence the primary user (PU) (i.e., licensed user). An extensive research has been focused on studying the enabling algorithms for spectrum sensing in CRs; for instance, matched filter technique, feature detection technique, energy detection technique and wavelet technique. Energy detection technique has been widely adopted in narrow band spectrum sensing process for its simple hardware, low computational requirements and as it requires no prior knowledge of the PU signal characteristics [1]-[17].
1. Introduction to Cognitive Radio Networks

The noise uncertainty and destructive radio conditions such as shadowing and multipath propagation fading have an adverse impact on the sensing performance of the energy detector and result in hidden terminal problem (HTP). Collaboration between secondary users (SU) (i.e., unlicensed users) in spectrum sensing process has been proposed to mitigate the deterioration of the detection performance of the individual users and to improve the overall detection performance as a whole system. Cooperative spectrum sensing (CSS) significantly improves the detection performance of the contributing CRs in the system, however, CSS incurs sensing overhead (i.e., complexity, extra energy consumption, bandwidth, and time delay). Therefore, trade-off between improving the performance and minimizing the incurred overhead has drawn a
considerable research works in order to strike a balance between performance and overhead and to find an optimal number of participating CRs in the CSS system. Moreover, the research works have also focused on how to implement the CSS system in a real life. The implementations of CSS can be classified them into three major classes; centralized CSS, decentralized CSS, and relay-assisted CSS [18]-[74].

A CSS system results in extra computational burdens and consequently raises the energy consumption and noticeably lessens the system life time; where CRs are battery powered terminals, therefore, research works have focused on designing energy efficient CSS system. Applying clustering techniques on CSS took a place in CSS system for cognitive radio networks (CRN) in order to balance between performance improvement and overhead; research works in this field showed that cluster-based spectrum sensing (CBSS) effectively tackled the trade-off between performance and overhead [75]-[91]. The research trend has been focused on designing optimal CBSS algorithms in many different perspectives, for instance, maximizing the overall throughput [84], maximizing the system energy efficiency [81], and minimizing the total detection error rate [78]. However, designing energy efficient CBSS system with large number of contributing CRs in the cluster is still a critical design issue in CRNs.

1.1 Problem Statement

Applying collaboration principle in spectrum sensing in CRNs remarkably increases the attainable throughput, spectrum efficiency, and global probability of detection as well. However, the attainable improvements come at expense of incurring extra bandwidth requirement, energy consumption and time delay. Therefore, possibility of settling a compromise between improving the performance and reducing the overhead by designing an energy efficient system is considerable target. Number of participating CRs in the CSS, sensing duration and data
transmission duration play vital roles in designing an energy efficient CSS system. In other words, increasing the number of participating CRs in the system leads to increase both consumed energy during CSS process and delay time; moreover, longer sensing time duration increases detection precision, but on the other hand decreases spectrum efficiency and increases the consumed energy during sensing phases (i.e., sensing overhead). In the essence of above mentioned facts, tackling a trade-off between performance improvement and overhead is our main focus research point in this dissertation. The trade-off between the performance and overhead in CSS systems will be tackled in this work by taking the advantage of hierarchical tree structure model of the cluster of CRs and by minimizing the number of contributing CRs in the system considering PU protection.

1.2 Research Objectives

Our overarching aim in this dissertation is to introduce some improvement in field of cooperative spectrum sensing for cognitive radio networks. Striking a balance between improving detection performance and incurring overhead in CSS for a CRN is a key point in designing energy efficient CSS systems in CRNs, which is the main goal in this work. This goal can be achieved by either considering all available CRs in the system or by reducing the number of involved CRs in the system. Therefore, the research work in this dissertation is considering the following objectives.

**First objective:** designing an energy efficient CSS system for a large number of contributing CRs in the CBSS system for CRNs. The advantage of hierarchical tree model of a cluster of CRs has been considered and two different fusion rules have been used in different hierarchical levels in the cluster to tackle the trade-off between performance improvement and overhead in the CBSS system without minimizing the number of participating CRs.
**Second objective:** designing an energy efficient CSS system for CRNs by varying the sensing time duration in the CSS system in order to reduce the number of required CRs and to improve the overall detection performance qualities.

**Third objective:** designing energy efficient CSS systems for CRNs by jointly optimizing the design system parameters (i.e., sensing duration, data transmission duration, transmission power, and the number of contributing CRs in the system). We consider probability of PU resuming activity as quality of service (QoS) metric, in order to reduce the number of participating CRs in the system and to improve the detection performance (i.e., energy efficiency, throughput and global probability of detection) while satisfying the PU protection constraints and spectrum efficiency constraint as well.

### 1.3 Research Contributions

The primary contributions of this dissertation can be summarized as:

- Developed a novel iterative energy efficient CSS algorithm for CBSS system with large number of CRs using the tree structure and by combining two different rules (i.e., polling and most voting) at different hierarchical structural levels in the cluster. The algorithm shows better global detection performance qualities (i.e., global probability of detection and overall achievable throughput).

- Based on varying the sensing window size (i.e., sensing time duration), three novel strategies for energy efficient centralized CSS system using the three hard decision fusion rules were proposed. The strategies showed that increasing the sensing window size considerably minimizes the number of participating CRs in the system, increases the
achievable throughput and consequently increases the energy efficiency of the centralized CSS system.

- Developed a novel algorithm with low computational complexity to jointly optimize the sensing duration, data transmission duration, and number of participating CRs in the CBSS system considering the PU protection constraints and spectrum utilization constraint as well. The algorithm significantly hastens the processing speed and reduces the number of CRs involved in the system while meeting the PU protection and spectrum utilization constraints.

- Developed a novel algorithm with low computational complexity that jointly optimizes the sensing duration, data transmission duration and transmission power for two cases, namely, single CR system and CBSS system for CRNs. The algorithms consider both PU protection and spectrum efficiency constraints.

### 1.4 Organization of Dissertation

Background and literature review about spectrum sensing in CRNs is provided in Chapter two. Chapter three introduces multi-level hierarchical CBSS structure algorithm for CRNs. Chapter four describes the novel strategies of energy efficient centralized CSS using a variable sensing time. How to design an optimal energy efficient CBSS algorithm is explained in Chapter five. Chapter six presents joint optimization of transmission power and sensing time for energy efficient spectrum sensing in CR system. Conclusions and future works are shown in Chapter seven.
Chapter Two:  
*An Overview of Spectrum Sensing in Cognitive Radio Networks*

2.1 Introduction

Recently, spectrum shortage problem aggravates in an apparent manner. The major causes of the spectrum shortage are first, the explosive increase in wireless communication technologies that increases the demands for larger bandwidths and higher sampling rates, especially, by rapid evolution of multimedia technologies and excessive growth of its applications and users; second, the static frequency assignments imposed by the regulatory bodies and the government agencies for owning licences to exploit spectrum band. Many solutions were proposed to solve spectrum usage inefficiency problem, however, most of proposed solutions were complex and incur extra time and cost [3]-[11]. The principle of dynamically exploiting the local vacant spectrum bands of the primary users (i.e., licensed users) during their silence periods by secondary users (i.e., unlicensed users) was among the best proposed solutions for the problem. Cognitive radio (CR) was proposed as a prominent technology to implement dynamic spectrum access (DSA) for its autonomous, agility, and ability of detect the primary user’s signal [6]-[13]. The most important function of the CR is to be able to sense, learn, and be aware of the PU signal’s characteristics. Spectrum sensing is considered as the key function of the CR through which the CR measures and learns the existence of the PU signal [8]-[10].
Many enabling techniques have been proposed to perform local spectrum sensing (LSS) (i.e., a single CR detects the existence of the PU), for instance, energy detector [14], cyclostationary detection [15], matched filter [7], and compressed sensing for wideband spectrum sensing [16]. However, each technique has its advantages, disadvantages and applications. The energy detector is the most widely used technique to perform the spectrum sensing for its simple hardware, low computational requirement, and as it requires no prior knowledge of PU signal’s characteristics, in contrary, both cyclostationary detection technique and matched filter require partial and full prior knowledge of the PU signal, respectively. Moreover, their hardware are complicated and require high computational requirements [7]-[13].

Although the energy detector is the most popular technique for spectrum sensing, this generally faces many challenges, since its performance drastically deteriorates in severe radio conditions, such as deep fading and heavy shadowing. Moreover, noise uncertainty significantly impacts the performance of the energy detector [17]. The performance deterioration leads to hidden terminal problem (HTP) (i.e., CR falsely detects the absence of the PU and decides to transmit its data); therefore, CRs might cause interference to the PU during its transmission. However, the HTP can be solved using cooperative spectrum sensing (CSS) techniques through which an accurate decision about the existence of the PU will be made by fusing a group of CRs’ decisions which are contributing in the CSS system. Furthermore, CSS significantly alleviates the impact of noise uncertainty and drastically improves the detection performance and minimizes the required sensing time [7]-[11]. Before discussing about cooperative spectrum sensing configurations and models; the preliminaries of spectrum sensing process should be discussed to have a global understanding about detection performance of the CR.
2.2 Spectrum sensing preliminaries

Detection of the PU existence is mainly determined based on spectrum sensing hypotheses, which are defined in [7] as:

\[ H_0 (\text{idle channel}) : \quad y(n) = w(n) \]

\[ H_1 (\text{occupied channel}) : \quad y(n) = h(n). x(n) + w(n) \]

where \( H_i \), \( i = 0, 1 \), are the hypotheses for absence and presence of the PU, respectively, while \( y(n) \) is a signal received by the CR receiver, \( w(n) \) is the noise at the receiver end, and \( x(n) \) is the PU transmitted signal, \( n = 1, 2 \ldots, M_s \), where \( M_s \) is the total number of sensing samples, and \( h(n) \) is the channel gain between the sensor and PU. The detection performance is assessed by two criteria. First criterion is for PU protection which is measured by the probability of detection \( P_d \); second criterion is for spectrum efficiency which is measured by the probability of false alarm \( P_f \). The probability of detection and the probability of false alarm are defined in [7] as follows:

\[ P_d = P(\hat{H}_1|H_1), \quad \text{and} \quad P_f = P(\hat{H}_1|H_0), \]

where \( \hat{H}_1 \) is the measured status by the CR for PU being present, while \( H_1 \) and \( H_0 \) represent the actual status for PU being present and absent, respectively. \( P \) denotes the probability.

The CR detects the existence of the PU on periodic basis. Based on the observations, if the sensed channel is idle, the quantity of transmitted data (i.e., throughput) in bit/Hz, \( R \), can be computed in [19] as follows:

\[
R = P_0 C \frac{T-t_s}{T} P(\hat{H}_0|H_0) + P_1 C \frac{T-t_s}{T} P(\hat{H}_0|H_1), \quad (2.1)
\]

[Desired transmission] \quad [Interference caused by the CR]
where $P_0$ and $P_1$ are probabilities of PU being absent and present, respectively, $C$ is the upper bound of channel capacity (i.e., channel capacity is measured in bit/Hz) computed using Shannon theorem [67], $t_s$ is the sensing duration and $T$ is the periodic time, while $\hat{H}_0$ denotes the measured status by the CR for PU being absent.

However, the sensing process incurs a cost (i.e., overhead) in terms of energy consumption, $E$, which is calculated in [19] as follows

$$E = P_s t_s + P_t (T - t_s) \left( P_0 P(\hat{H}_0|H_0) + P_1 P(\hat{H}_0|H_1) \right),$$

(2.2)

where $P_s$ and $P_t$ are the sensing and transmission powers, respectively.

The total detection error rate, $\varphi_e$, is one performance metric which measures the total probability of error in detecting the PU and it is computed in [30] as

$$\varphi_e = P_1 P(\hat{H}_0|H_1) + P_0 P(\hat{H}_1|H_0).$$

(2.3)

The comprehensive performance metric is energy efficiency, $\zeta$, which combines both the throughput and overhead. The energy efficiency in bit/Hz/J can be defined in [30] as:

$$\zeta = \frac{P_0 C \frac{T-t_s}{T} P(\hat{H}_0|H_0) + P_1 C \frac{T-t_s}{T} P(\hat{H}_0|H_1)}{P_s t_s + P_t (T - t_s) \left( P_0 P(\hat{H}_0|H_0) + P_1 P(\hat{H}_0|H_1) \right)},$$

(2.4)

2.3 Models of cooperative spectrum sensing

Cooperation of a group of CRs is very advantageous for the detection performance of every individual CR in the network, where the cooperation tremendously increases both the probability
of detection and throughput while drastically reduces the required sensing time [18]-[23]. Table 2.1 illustrates a comparison between local and cooperative spectrum sensing. From cooperation architecture standpoint, there are three well known models, they are centralized CSS, decentralized CSS and relay assisted CSS as depicted in Fig. 2.1.

Fig. 2.1 Models of CSS system, a) centralized model b) decentralized c) relay assisted.
Table 2.1  Local spectrum sensing (LSS) vs. Cooperative spectrum sensing (CSS)

<table>
<thead>
<tr>
<th>Detection Performance</th>
<th>LSS</th>
<th>CSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PU protection</td>
<td>Less than CSS</td>
<td>Higher than LSS</td>
</tr>
<tr>
<td>Probability of detection</td>
<td>Less than CSS</td>
<td>Higher than LSS</td>
</tr>
<tr>
<td>Probability of false alarm</td>
<td>Higher than CSS</td>
<td>Less than LSS</td>
</tr>
<tr>
<td>Throughput</td>
<td>Less than CSS</td>
<td>Higher than LSS</td>
</tr>
<tr>
<td>Performance in low SNR</td>
<td>Drastically degraded</td>
<td>Significantly improved</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Overhead</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensing time</td>
<td>More than CSS</td>
<td>Less than LSS</td>
</tr>
<tr>
<td>Energy consumption</td>
<td>Less than CSS</td>
<td>Higher than LSS</td>
</tr>
<tr>
<td>Complexity</td>
<td>Less than CSS</td>
<td>More than LSS</td>
</tr>
</tbody>
</table>

2.3.1 Centralized cooperative spectrum sensing

In this scheme of cooperative spectrum sensing, a group of CRs sense the existence of the PU in a specific geographic location and specific frequency channel. In periodic basis, every single CR in the system (i.e., network) measures the signal of the PU independently, based on its local observation, a CR makes its decision about the existence of the PU and then forwards its decision as either one bit form (i.e., hard decision) or as energy form (i.e., soft decision) to the base station (BS). The BS collects the decisions of the collaborating CRs fuses them in order to have a final decision about the presence of the PU and finally the BS informs the CRs in the system [23]-[28]. The data fusion in the BS depends on the form of the received local decisions.
from the CRs in the network, for instance, AND, OR and majority voting are hard decision fusion rule when CRs report their local decisions as one bit (i.e., ‘0’ or ‘1’) [23]-[27], while equal gain combination (EGC), maximal ratio combination (MRC), and square law combination (SLC) [28] when CRs report their local decisions to the BS as energy. The detection performance of centralized CSS using soft decision rules drastically outperforms the performance of the system using hard decision rules [6]-[10], [28]-[29], however, the former techniques requires much more bandwidth than the latter techniques and incur higher overhead than that required by the hard decision techniques. A two-bit hard decision technique was proposed in [28] to minimize both bandwidth and overhead and to attain better detection performance than one-bit hard decision. Table 2.2 shows differences between hard and soft decision rules in terms of performance and overhead.

Generally, the centralized CSS system requires establishing a network backbone (i.e., common control channel or reporting channel) to convey information between the BS and the involved CRs in the network. Though the centralized CSS scheme significantly improves the detection performance (i.e., probability of detection and throughput), however the information exchanges and data fusion lead to incur an extra overhead (i.e., bandwidth, computational requirements, energy consumption, and time). Therefore, the trade-off between improving the performance and incurred overhead should always be considered in designing such a scheme of CSS. The trade-off between performance and overhead attracted the attention of the researchers. Many approaches have been proposed to tackle the trade-off. Some researchers focused on minimizing the number of involved CRs in the network by optimizing the fusion rules [31]-[34], other researchers focused on maximizing the throughput while meeting minimum overhead [21]-
[23], [30], while other researchers focused on maximizing the probability of detection using optimal sensing threshold [35],[36]. Censoring technique using double sensing threshold has also been proposed to minimize the incurred overhead while attaining the required detection performance [37]-[38]. A new trend of research has recently been proposed to maximize the energy efficiency of the CR network (CRN) [39]-[48]. The term energy efficiency is defined as a ratio between the achievable throughput of the CRN and total consumed energy by the CRN. Utilizing the concept of energy efficiency, a designer can employ this criterion to trade-off between the performance and the overhead of the CRN.

Table 2.2 Comparison between Hard decision and Soft decision rules

<table>
<thead>
<tr>
<th></th>
<th>Hard decision</th>
<th>Soft decision</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Detection performance</strong></td>
<td>Moderate performance</td>
<td>Better than hard decision</td>
</tr>
<tr>
<td><strong>Overhead</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardware</td>
<td>Simple</td>
<td>Complicated</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>Processing speed</td>
<td>Fast</td>
<td>Slow</td>
</tr>
<tr>
<td><strong>Rules employed</strong></td>
<td>AND, OR, $k$ out of $N$</td>
<td>EGC, MRC, SLC</td>
</tr>
</tbody>
</table>

### 2.3.2 Decentralized cooperative spectrum sensing

The main feature of this model is that a group of CRs collaborate without central information exchange. In this model of CSS, the CSS process goes through four iterative stages. First, each CR senses the PU signal independently and periodically. Second, every CR establishes communication links with other CRs (i.e., neighbours) having the desired channel characteristics;
afterwards each CR exchanges its local information with its neighbours. Third, every CR combines the collected information with its local observation to make its decision. Finally, consensus among the neighbours about the existence of PU should be achieved, however, if consensus is not met, the first three stages of the process should be iteratively repeated till achieving consensus [49]-[52]. The principle of consensus is inspired from biological phenomena, specifically, the collective animal behaviour in a group making decision the higher level and the individual animals’ communication at the lower level. In [49] and [50], the principle of consensus was applied to the CR mobile ad hoc network (CR MANT) in order to avoid having a network backbone and central entity, and to improve the spectrum sensing data falsification (SSDF). The attained results was compared to OR-Rule and shown the effectiveness of employing the principle to the CSS. A gradient approach was applied in [51] to improve the performance without prior knowledge about the degree of network and network configuration. The results showed that applying a gradient approach led to a good detection performance as attained by the algorithms in [49] and [50] while saving more energy consumption than that consumed by algorithms in [49] and [50]. A network code algorithm was proposed in [52] in to order to reduce the required bandwidth between interacted CRs. The algorithm has better consensus convergence rate than all previously mentioned algorithms and better detection performance (i.e., throughput and probability of detection).

2.3.3 Relay assisted cooperative spectrum sensing

Space diversity motivated the researchers to develop the relay assisted CSS model. The model is considered as a low complexity cooperative diversity that mitigates the influence of fading induced by multipath propagation in wireless network. The principle of the model is to exploit
space diversity available through collaborating CRs relaying signals (i.e., local observations about the PU existence) for one CR to another; final recipient CR is known as cognitive coordinator. In this model, two fixed relaying protocols are the most widely used by cooperating terminals (i.e., CRs) in the system. These protocols are amplify and forward (AF) protocol and decode and forward (DF) protocol [53]-[59]. In the AF protocol, the CR relays its local observation to another CR in the system without any processing, therefore full diversity is achieved, however, all relays have a power constraint [54]-[57]. The efficiency and robustness of transmission for both AF and DF protocols in terms of outage event and its probability of occurrence have been considered in [53], and it was concluded that large power or energy saving are mainly depending on the employing protocol. The work in [54] and [55] focused on taking advantage from the spatial diversity to reduce the sensing time and improve the overall sensing agility. Moreover, the authors of those works have developed a modified AF protocol that guarantees the agility gain for arbitrarily large CRN populations. However, all previously mentioned works did not consider channel impairment (i.e., fading channel) and focused on single relay system. In [56], the authors analyzed the performance of energy detector under Rayleigh fading channel and theoretically derived the mathematical formulae of both average probability of detection and probability of false alarm. Besides, the work was extended to study the case of multiple relay between the PU and the cognitive coordinator. A study of relationship between the detection performance and bit error rate (BER) has been considered in [57]; it was shown that decreasing BER of the CSS leads to increase the probability of detection. Furthermore, the work showed that there is an optimal sensing time at which maximum throughput can be achieved.
From saving energy and bandwidth standpoint, a DF protocol is employed in relay assisted CSS technique. Simply, the general principle of the DF protocol is to consider that all CRs independently and continuously monitor the PU traffic, then one CR, which is in the decodability range of the PU decodes the received signal, re-encodes it, and forwards the newly encoded signal to the cognitive coordinator. However, the DF protocol can only be employed in short range communication due to transmission power constraints. In [58], the CSS using DF protocol has been developed for two cognitive users under Rayleigh fading channel. The detection performance of the DF protocol has also been compared with the AF protocol at the same radio conditions. It has been shown that the DF protocol outperforms the AF protocol in terms of attained probability of detection. Moreover, the work has been extended to consider collaboration among a group of CRs using the DF protocol. However, the possibility that a CR causes interference to the PU was not considered in the previous work. The work in [59] investigated the detection performance of CSS system using both protocols (i.e., AF and DF) under interference constraint and analyzed the performance in terms of probability of detection and probability of interference. The work proved that the detection performance does not only depend on interference level tolerance but also on the employed protocol for the CSS technique.

2.4 Trade-off between cooperation gain and cooperation cost

Investment of cooperation (i.e., spatial diversity) in a group of CRs in spectrum sensing results in an apparent improvement in the detection performance, however, the gained improvement has a penalty of incurring cooperation cost (i.e., extra energy consumption, time, and complexity) which limits attainable cooperation gain (i.e., detection performance improvement). Therefore, balancing between cooperation gain and cooperation cost is very crucial in designing CSS in
CRNs. In order to achieve a compromise between the gain and the cost in designing CSS for CRNs, many factors should be taken in consideration. Some of these important factors are, channel impairment, sensing time, energy efficiency, and mobility.

### 2.4.1 Channel impairments

The impact of the channel impairments on the performance of the CSS technique mainly depends on the employed detection techniques and underlying transmission technique. In more details, sensing performance of the energy detector technique severely deteriorates in weak radio conditions such as low SNR scenarios, noise uncertainty, heavy shadowing and deep multipath fading. Moreover, noise uncertainty in very low SNR prevents energy detector from detecting the existence of the PU. The effect of noise uncertainty is called SNR wall (i.e., a SNR value beyond which the detector cannot sense the PU signal no matter how long sensing time is) [60]-[61]. The location of the SNR wall can be determined using (2.5) as proven in [61]

\[
M \approx \frac{(Q^{-1}(\bar{P}_f)-Q^{-1}(\bar{P}_d))^2}{(SNR-(U^{-1/3}U))^2} \tag{2.5}
\]

where \(M\) is the sensing samples, \(\bar{P}_f\) and \(\bar{P}_d\) are the target probability of false alarm and the target probability of detection, \(Q(\cdot)\) denotes \(Q\)-function, and \(U\) is the noise uncertainty (i.e., \(U = \{0.01, 0.1 \text{ and } 1\} \text{ dBs}\)).
Fig 2.2: SNR walls vs. number of sensing samples for different noise uncertainty values.

Figure 2.2 displays SNR walls locations at different values of noise uncertainty. Clearly form the figure, the higher value of noise uncertainty causes the earlier energy detector measurement barrier. In contrary, feature detection has advantage of ability to measure the PU existence in very low SNR environments, further, this type of detection technique is able extract the PU signal from interference and classified as anti-interference detection technique [6]-[10],[15].

From other viewpoint, scattering and reflections of the PU signal causes multipath fading effect which weakens the received signals at the CRs, while presence of natural or man-made obstacles between the PU transmitter and CRs attenuate the PU signal too, the effect is called shadowing effect. The impacts of both effects on the detection performance of the CSS system have been widely considered in the most research works. In [62], closed form expressions of the
probability of detection and the probability of false alarm using energy detector over additive white gaussian noise (AWGN) and different fading channels (i.e., Rayleigh, Nakagami and Rician) have been developed for a single sensing unit employing energy detector. Moreover, the work also focused on the impact of Rayleigh fading channels on the CSS system employing some soft decision fusion rules such as equal gain combining (EGC), selection combining (SC), and switch and stay combining (SSC). The work has been extended in [63] to study comprehensively other soft decision fusion rules such square law combining (SLC) and square law selection (SLS).

The effect of shadowing on the performance of the CSS system was considered thoroughly in [64], an analytical framework for design CSS system for CRs using energy detector over correlated Log-Normal shadowing channel was also proposed in [64]. It was shown that the proposed framework can overcome the detrimental impact of the correlated shadow fading. Furthermore, the impact of shadowing and fading channel on the performance of a single CR has been investigated in [18], further, the work showed that the CSS system using OR-Rule significantly improves the performance and combats the effect of both shadowing and Rayleigh fading.

### 2.4.2 Sensing time

In a centralized CSS system, all CRs in the system detect the PU periodically in a synchronized manner and then report their local observations to the BS. The observations are collected using time division multiplexing (TDM) technique. Selecting a proper sensing time for CSS system is a challenge, since long sensing time improves detection efficiency, however, it reduces
transmission data duration which possibly results in decreasing the throughput. In other words, increasing sensing time can effectively reduce the spectrum efficiency [6]-[13]. Therefore, selecting an optimal sensing time is essential design factor for CSS systems. Extensive research works have been done to determine the optimal sensing time with different criteria. The authors in [23] proved that the overall throughput is a non-convex function in sensing time; moreover, they developed a novel framework for the CSS system using energy detectors to determine an optimal sensing time in order to maximize the achievable throughput under a constraint of protecting the PU sufficiently. Using (2.1), the throughput can be displayed in Fig. 2.3. The figure shows that the attainable throughput is a concave function in sensing time. Furthermore, the figure shows that there is an optimal sensing time that maximizes the achievable throughput of a cluster with different sizes.

![Fig 2.3: Throughput vs. optimal sensing time for different cluster size.](image-url)
Cross layer optimization has been employed in [65] to tackle the trade-off between maximizing the achievable throughput and minimizing the interference to the PU signal. The impacts of sensing time and power adaptation have also been considered in the work. It was shown that optimizing both sensing time and power effectively improve the performance of the CRNs. Joint optimal sensing time and power allocations design has been proposed in [66]. Two strategies have been developed in order to maximize the ergodic throughput of the wideband spectrum performance of CRNs. Moreover, the work in [66] considered a discussion of the impacts of the average transmit power and the average tolerable interference power on the optimal sensing time.

### 2.4.3 Energy efficiency

Energy efficiency is a criterion that compromises between the performance (i.e., throughput) and overhead (i.e., consumed energy). The criterion is defined as the ratio between attainable throughput and incurred energy consumption. In other words, energy efficiency can be defined as the total number of bits delivered per joule of consumed energy. For a non-cooperative spectrum sensing system, selecting a joint optimal sensing and transmission durations that maximize the achievable energy efficiency of the system in terms spectrum exploitations of idle PU spectrum band was studied in details in [67]. The study considered three spectrum exploitation scenarios which are fully utilizing the idle PU band with sufficient average power capacity, partially utilizing the idle band with limit power capacity, and interim between two previous scenarios. However, the work has not considered the PU traffic (i.e., possibility that PU reoccupies the idle band during CR transmission duration). The PU traffic or activity has been considered in [68]. Based on PU activity model, authors established a mathematical formula for
designing energy efficient CR system in terms of sensing and transmission durations under constraints of interference to the PU (i.e., probability of detection and PU reoccupation probability). The work showed that there is a joint optimal sensing and transmission duration that maximizes the energy efficiency; an iterative suboptimal algorithm was proposed to determine both optimal sensing and transmission durations.

On the other hand, energy efficient cooperative spectrum sensing systems were considered in [43], [46], [69]-[74] with different design perspectives. Two different setups have been proposed in [46] in order to determine the optimal number of CRs required performing the centralized CSS process. The first setup is energy efficiency setup through which the number of CRs in the system is reduced for the $k$-out-$N$ fusion rule under constraints of having sufficient PU protection and sufficient spectrum utilization. The second setup is throughput optimization that maximizes the throughput while sufficiently protecting the PU. However, the work did not consider the energy consumed by CRs during CSS process and life time of CR’s battery. It is known that CR is a battery powered sensor terminal; this issue imposes a critical constraint on CSS system design. Greedy heuristic approach has been employed in [69] to optimize the schedule order and minimize terminal switching time. The work in [69] proposed a joint spectrum sensing and scheduling optimization using the Greedy algorithm in order to considerably prolong the life time of the CSS system. Optimizing the fusion rule is another technique to improve the energy efficiency of the CSS system. In [70], $k$ out of $N$ fusion rule has been optimized to maximize both the throughput and detection accuracy; moreover, closed form expressions of optimal $k$ and $N$ have been derived. A novel approach has been proposed in [43] to reduce the consumed energy during the CSS process and consequently improves the energy
efficiency. The approach aims to limit the number of participating CRs in the process. Based on the distances between the CRs and the BS each, CR estimates the expected energy consumed, compares it with a predefined threshold, and then decides to participate or not. The proposed approach greatly saved energy and significantly improved the energy efficiency.

A novel frame structure for a CRN has been proposed in [71] in order to jointly improve spectrum efficiency and energy efficiency. The proposed frame structure allows a CR to sense the PU in other CRs’ reporting times. A novel multi-mini slots CSS scheme has also been proposed to consider time varying channel in the CSS system. The proposed algorithm in [71] showed a significant improvement in both spectrum efficiency and energy efficiency. In [72], the impacts of different types of fading channels on both the optimal number of participating CRs in the CSS system and the energy efficiency have been considered. Energy efficiency optimization strategy has also been proposed to determine the final decision threshold in order to significantly improve the energy efficiency of the CSS system over different fading channels (e.g., AWGN, Rayleigh, and Nakagami fading channels). Furthermore, it was proven in [72] that there is an optimal number of participating CRs in the CSS that maximizes the energy efficiency, and that optimal number varies according to the type of fading channel.

2.4.4 Mobility

Mobility is an inherent characteristic of modern wireless communication systems. The mobility has direct impact on network capacity [73], coverage, connectivity and routing [10]. Nevertheless, few research works has considered the movement of CR terminal or PU movement. The impact of mobility on spectrum sensing performance for a single CR terminal
An Overview of Spectrum Sensing in Cognitive Radio Networks

with energy detector has been investigated in [74]. It was proven in the work that increasing the moving speed of the CR terminal on both urban and suburban environments drastically enhances the spatio-temporal diversity and consequently the sensing observation of the CR get faster uncorrelated which leads to significantly improvement in spectrum sensing performance. It is shown in [74] that higher terminal mobility can effectively improve the sensing performance of the terminal with low received signal strength (RSS) in received PU signal. Moreover, the work has considered the effect of mobility on sensing scheduling and the required number of CRs to performance CSS process. It was proven that increasing the speed of the terminal considerably reduces both the number of required CRs in the CSS system and the cooperation cost.

2.5 Cluster-based cooperative spectrum sensing

Sensing performance of the conventional CSS systems degrades remarkably in the case of a large number of participating CRs, especially, those CRs are randomly scattered in different radio circumstances, since reporting errors might increase and becomes inevitable. Moreover, the global decision made by the BS might not include all CRs; in addition, reporting process of a large number of CRs incurs high energy consumption, and control and transmission overhead. Therefore, clustering techniques were proposed in to order to minimize the energy consumption and to reduce the control and the transmission overhead [10]-[12], [75]. The principle of cluster-based spectrum sensing technique (CBSS) is to perform the sensing process through two hierarchical level of CRs collaboration in order to support cooperative sensing tasks with efficient network organization and to minimize the communication load burden and energy consumption of the BS.
In CBSS technique, CRs are divided into small groups (i.e., clusters) according to geographical locations and spectral circumstances. The BS collects the IDs and some radio information of all participating CRs, determines the clusters according to some criterion, and then assign a cluster head (CH) for each cluster. The remaining CRs in the cluster are called cluster members (CMs) [75]-[84]. The CH is usually selected as a CR with the largest reporting channel gain in the cluster. The CH coordinates the sensing process inside its cluster and links the other CRs in the cluster with the BS. In more details, after clustering, CH receives the commands from the BS about which PU channel to be detected, the CH informs its cluster members to start sensing the required channel. Each CM in the cluster detects the PU channel and then relays its local observation to the CH to make a cluster decision based on its local observation and aggregated observations from CMs in the cluster (i.e., the lowest cooperation level). The CHs forward their clusters’ decisions to the BS to come up with the final decision about the PU existence (i.e., the highest cooperation level). Employing clustering techniques in the CSS process results in improving the sensing performance, reducing the sensing overhead, and prolonging the life time of both the BS and CRs.

### 2.5.1 Models of CBSS

Many approaches and algorithms have been proposed to form clusters for CBSS scheme [75]-[77]. However, the CBSS system can be categorized into three major categories, namely, performance gain oriented models, overhead reduction oriented models, and combined metrics based models [75].
2.5.1.1 Performance gain oriented models

The authors in [78] were the pioneer in proposing a conventional CBSS scheme. Both decision and energy fusion rules (i.e., hard and soft decisions, respectively) were studied. Further, the work showed that CBSS scheme significantly outperforms the conventional CSS scheme. A novel clustering strategy was proposed in [79] to tackle the trade-off between sensing performance and sensing overhead. The strategy has two phases; CH selection phase and cluster formation phase. Polling fusion rule was also proposed in this work. A closed form expression of probability of detection based on polling rule was also derived. An optimized multi-cluster multi-group (MCMG) algorithm was proposed in [80], in order to improve the sensing performance of the CBSS scheme. The idea of the algorithm is to divide the CRs in a one cluster into identical group (i.e., all groups in the cluster has the same number of CRs). Each group has a group head (GH) which polls the maximum received signal PU strengths of its group members, makes a group decision, and then forwards to the CH of the cluster which extracts the final cluster decision about the PU existence using $k$ out of $N$ rule. Moreover, the optimal fusion parameters of the majority rule were derived in terms of minimum global detection error rate.

2.5.1.2 Overhead reduction oriented models

These models of CBSS technique are focusing on reducing the incurred overhead in CBSS technique. The works in [81]-[84] tackled the overhead reduction in different standpoints. From energy efficiency viewpoint, the work in [81] considered frequency distances to develop an energy efficient CBSS algorithm. Clustering technique was employed to save energy during both reporting and information exchange phases.
Bandwidth limitation has been considered in [82]-[84], however, each work has its own tactic in tackling the limitation. In [82], location information (i.e., received SNR at each CR) as a censoring method played an important role in decreasing the CRs participating in the cluster, therefore, the average of sensing bit is reduced and consequently the required bandwidth and energy are also reduced. Moreover, closed form mathematical expression has been derived for both optimal number of cluster and number of CRs in a cluster. The minimal dominating set (MDS) approach has been employed in [83] in order to determine the minimal set of cluster that maintains the network connected, therefore, the bandwidth requirement of the reporting channel was consequently reduced. On the other hand, the location of the CH in the cluster is selected using graphic theory. Furthermore, it was shown in [83] that throughput of the CBSS using MDS approach outperforms the conventional CBSS approach especially in the case of imperfect reporting channel.

### 2.5.1.3 Combined metric based models

Striking a balance between the improvement of sensing performance and the reduction of incurred sensing overhead is a target of this kind of CBSS schemes. In [84], frequency division based parallel reporting mechanism was proposed to reduce reporting time, energy consumption and required reporting bandwidth as well, while meeting sensing performance constraint. Moreover, the CH in [84] is selected based on sensing data reliability and the final decision about the PU existence is extracted at the BS using Chair-Varshney rule [85]. Another approach to balance sensing performance and energy consumption was provided by [86] and was called clustered hybrid energy aware cooperative spectrum sensing (CHESS) algorithm which has three consecutive phases, namely, training, clustering, and activity. In the training phase, each CR in
the system measures its reliability in detecting the PU existence, while in the clustering phase, the cluster is formed and the CH is selected. Finally, in the activity phase the CRs in the cluster forward their local observations to the CH, which combines the observations to extract cluster decision and then report it to the BS. Simulation results showed that CHESS algorithms guarantee reliable performance and prolong the life time of the system.

2.6 Conclusions

This chapter presented a brief introduction to spectrum sensing in cognitive radio networks and its preliminaries. Some types of enabling sensing algorithms, detection performance criteria, and overhead criterion have been discussed. Cooperative spectrum sensing models, fusion rules, and cooperation gain and cost have been introduced in this chapter. Moreover, a comparison between local spectrum sensing and cooperative spectrum sensing has been presented. Trade-off between the detection performance of the CSS system and incurred overhead has been discussed in some details. Finally, a discussion about the models of cluster-based spectrum sensing algorithms has also been provided.
Chapter Three:  
*Hierarchical Cluster-Based Cooperative Spectrum Sensing for Cognitive Radio Networks*

### 3.1 Introduction

The objective of this chapter is to design energy efficient cluster-based cooperative spectrum sensing (CBSS) system in cognitive radio networks (CRNs) that strikes a balance between the detection performance and the incurred overhead. All participating cognitive radios (CRs) in the cluster are considered. The employment of two different fusions rules (i.e., polling [75] and $k$ out of $N$) and exploitation of the hierarchical structure of the cluster lead to significant reduction of the reporting overhead and consequently the energy efficiency of the CBSS system increases. An iterative energy efficient algorithm is proposed based on hierarchical structure model of the cluster. The algorithm aims to minimize the reporting channel overhead, and to increase both throughput and energy efficiency, while meeting reliable detection requirements. Moreover, derivations of the optimal sensing threshold, optimal decision threshold (i.e., $k$) for the fusion rule, and energy efficiency are provided in this chapter. The calculation of the required reporting CRs in the system, sensing agility, and consumed energy are also presented in this chapter.

This chapter is organized as follows. Section 3.2 introduces the system model for the proposed algorithm. Fusion rule optimization and energy efficiency of the proposed algorithm
are presented in Section 3.3. Simulation results are shown and discussed in Section 3.4, while summary is provided in Section 3.5.

3.2 System model for proposed algorithm

Assume that there are some CRs scattered around the primary user (PU) transmitter; the CRs are grouped into clusters according to their geographical locations as depicted in figure 3.1. Each CR in a cluster uses energy detector to sense the existence of the PU independently, and then relays its local observations as a hard decision (i.e., one bit). To have a comprehensive understanding about the proposed model, a brief explanation about existing models is introduced.

3.2.1 Existing models

In the conventional CBSS, the CR in a cluster with the largest reporting gain channel is assigned as CH, while other CRs report their local observations to the CH to fuse all observations, and then the CH sends the cluster’s decision to the FC. The FC comes up with the final decision about the PU existence, and then resends it to the CH in order to inform its CMs in the cluster. Figure 3.1 depicts the system model of the conventional CBSS. The CH can fuse the aggregated observations from its CMs using hard decision voting rules (e.g., OR, AND, or \( k \) out of \( N \)). Actually, OR and AND rules are special cases of \( k \)-out-of-\( N \) rule (i.e., \( k = 1 \), for OR, and \( k = N \), for AND). In [35], CBSS has been performed using OR-rule; while in [78], CBSS has been performed using \( k \) out of \( N \) rule. However, OR-rule increases drastically both global probabilities of detection \( Q_d \) and false alarm \( Q_f \). Although OR-rule increases the PU protection by increasing \( Q_d \), it decreases significantly the spectrum efficiency by increasing \( Q_f \). It is proven in [35] that \( k \) out of \( N \) fusion rule compromises between the PU protection and the spectrum efficiency. For multi-cluster multi-group (MCMG) algorithm [80], each cluster is divided into \( J \)
groups, all groups have \( W \) CRs. Each group has a group head (GH) and group members (GM). The GH is selected to have the largest reporting channel gain among others in the group. The GH extracts the group decision and then reports it to the CH which combines its local observation with the observations of the GHs in the cluster using \( k \) out of \( N \) rule, after fusion phase the CH reports cluster decision to FC. Finally, FC collects the decisions from the clusters, and extracts the final decision about the existence of the PU.

Figure 3.1: System model of the conventional CBSS algorithm.

### 3.2.2 Proposed model

In the proposed model, one static PU is assumed (i.e., PU is not moving); a cluster with \( N_e \) CRs is divided into \( M \) groups. Each group is also divided into \( R \) subgroups. Each subgroup has \( H \)
CRs. Every group and subgroup have group head GH and subgroup head SGH, respectively. The model is illustrated in Figure 3.2. The heads are selected to have the largest reporting channel gain among their neighbors. Note that the nearest CR to FC has the largest reporting channel gain [77], [89], and the FC is responsible for clustering and determining the CH.

Figure 3.2: System model of the proposed algorithm.

In each subgroup, $H^1$ of CRs are located close to each other within a small area, where the variation of received signal strength is low, where all CRs experience almost the same radio

\[^1\] $H^1$ is practically determined according to geographical separations between CRs in a cluster. In the simulation, same number of CRs in any subgroup in all groups is assumed in order to simplify the comparison between the proposed algorithm and other algorithms as shown in Section 3.4.
environmental conditions (i.e., path loss, fading, and shadowing). Therefore, polling is selected as the fusion rule for subgroups to reduce the number of reporting CRs and to save reporting energy, where no CR within any subgroup reports its observation to the SGH. Only, SGH extracts its local decision by polling the received signal of its subgroup members, and then relays the local decision to its group’s head. In each group, the GH collects the aggregated observations from SGHs, fuses them with its local observation using $k$ out of $N$ fusion rule, and then reports group’s decision to the CH. Subsequently, the CH combines the collected decisions from GHs, and then extracts the cluster decision using $k$ out of $N$ rule. Finally, the FC collects the decisions from CHs, and combines them using $k$ out of $N$ rule to have final decision about the existence of the PU.

Detecting the existence of the PU is mainly based on spectrum sensing hypotheses, which are defined as:

$$H_0$$ (idle channel) : \hspace{0.5cm} y(n) = w(n)

$$H_1$$ (occupied channel): \hspace{0.5cm} y(n) = h(n). x(n) + w(n)

where $H_i$, $i = 0, 1$, are the hypotheses for absence and presence of the PU, respectively, while $y(n)$ is a signal received by the sensor, and $x(n)$ is the PU transmitted signal, $n = 1, 2, \ldots, M_s$, where $M_s$ is the total number of sensing samples, $w(n)$ is the received noise and $h$ is the channel gain between the sensor and PU. Energy detector block diagram is depicted in figure 3.3.

The test statistic of the energy detector $M(y)$ is determined as follows

$$M(y) = \sum_{n=1}^{M_s} |y(n)|^2$$
The probability of detection, $P_d$, and the probability of false alarm, $P_f$, are defined as follows:

$$ P_d = P(M(y) > \lambda | H_1) \quad \text{and} \quad P_f = P(M(y) > \lambda | H_0) $$

where $\lambda$ is the sensing threshold, and $P$ denotes the probability.

In [23], it is assumed that $x(n)$ is a complex-valued phase shift keying (PSK) signal, and $w(n)$ is an identical independent distributed (i.i.d) circularly symmetric complex Gaussian (CSCG) noise; therefore, the probability of detection and probability of false alarm for the $i$-th CR can be approximated as

$$ P_d = Q\left(\frac{\lambda}{\sigma_n^2} - \gamma_i - 1\right)\sqrt{\frac{M_s}{2\gamma_i+1}} \tag{3.1} $$

$$ P_f = Q\left(\frac{\lambda}{\sigma_n^2} - 1\right)\sqrt{M_s} \tag{3.2} $$

where $\sigma_n^2$ is noise variance, $\gamma_i$ is signal to noise ratio (SNR) at the $i$-th sensor, and $Q(\cdot)$ is $Q$-function [23]. Practically, the sensing channels between the CRs and the PU may experience shadowing and multipath propagation fading due to the presence of some obstacles (i.e.,
buildings, trees... etc.) [7]. For simplicity, it is assumed that sensing channels are subjected to multipath propagation fading which can be modeled as a Rayleigh fading, since the distribution of the amplitudes of the received SNR in the space can be considered as Rayleigh distribution [80]. Over Rayleigh fading channel, the SNR $\gamma_i$ of the received signal varies exponentially as follows

$$f(\gamma_i) = \left(\frac{1}{\bar{\gamma}_i}\right) e^{-\left(\frac{\gamma_i}{\bar{\gamma}_i}\right)}$$  \hspace{1cm} (3.3)

where $\bar{\gamma}_i$ is the average SNR at the i-th sensor [62].

For the j-th subgroup, with $H_j$ CRs, the fading probability density function is given as [23]

$$f(\gamma_{max,j}) = \frac{H_j}{\bar{\gamma}} e^{-\frac{\gamma_{max,j}}{\bar{\gamma}}} \left(1 - e^{-\frac{\gamma_{max,j}}{\bar{\gamma}}}\right)^{H_j-1}$$  \hspace{1cm} (3.4)

where $\gamma_{max,j}^2$ is the largest received SNR in the j-th subgroup.

To ease the mathematical analysis of the model, we simply assume that all CRs in the cluster use the same threshold which is computed based on predefined probability of false alarm, $P_f$, SNR of all CRs is constant or varies gradually, and all CRs are closed enough such that transmission errors can be neglected.

\footnote{Note that all CRs in the cluster have almost the same channel quality and experience almost the same path loss [89], therefore, $\gamma_{max,j}$ can be considered as the average $\gamma$ received at any CR in j-th subgroup, especially that CRs in any subgroup are very close to each other.}
For the j-th subgroup with $H_j$ CRs, the SGH polls the received signals of its subgroup members and makes its decision based on the largest one among them, $\gamma_{\text{max},j}$. The probability of detection of the SGH $P_{\text{dSG}}$, is computed as:

$$P_{\text{dSG}} = \int_0^\infty P_d(\gamma_{\text{max},j}) f(\gamma_{\text{max},j}) d\gamma_{\text{max},j}$$

(3.5)

$P_{\text{dSG}}$ is numerically calculated using adaptive Gauss-Kronrod Quadrature technique [90]. Upon SGH determines its decision, it relays its observation to the GH of its group over a control channel. Perfect reporting channel and imperfect reporting channel are two scenarios to be considered.

### 3.2.2.1 Perfect reporting channel (error free):

For the n-th group with $R$ subgroups, using $k$ out of $N$ rule, the probability of detection, $q_{dG}$, and the probability of false alarm, $q_{fG}$, are calculated as [35]:

$$q_{dG} = q_{dG}^{R,k_1} = \sum_{k=k_1}^R \binom{R}{k} P_{\text{dSG}}^k (1 - P_{\text{dSG}})^{R-k}$$

(3.6)

and

$$q_{fG} = q_{fG}^{R,k_1} = \sum_{k=k_1}^R \binom{R}{k} P_f^k (1 - P_f)^{R-k}$$

(3.7)

where $k_1 = \left\lceil \frac{R}{2} \right\rceil$ for majority rule.

For a cluster with $M$ groups, the overall probability of detection, $Q_d$, and the overall probability of false alarm, $Q_f$, using $k$ out of $N$ rule, are as follows

$$Q_d = Q_d^{M,k_2} = \sum_{m=k_2}^M \binom{M}{m} q_{dG}^m (1 - q_{dG})^{M-m}$$

(3.8)

and
Hierarchical Cluster-Based Cooperative Spectrum Sensing for Cognitive Radio Networks

\[ Q_f = Q_f^{M,k_2} = \sum_{m=k_2}^{M} \binom{M}{m} q_{fG}^m \left(1 - q_{fG}\right)^{M-m} \]  

(3.9)

where \( k_2 = \left\lceil \frac{M}{2} \right\rceil \) for majority rule.

3.2.2.2 Imperfect reporting channel:

In this case, it is assumed that transmission experience errors over Rayleigh fading channel with probability density function given in (3.4). For simplicity, assume that SGHs report their observations as binary phase shift keying (BPSK) signal with error probability rate of

\[ P_{e|\xi_{max,j}} = Q\left(\sqrt{2\xi_{max,j}}\right) \] where \( \xi_{max,j} \) is the maximum control channel gain in the j-th group.

Therefore, the average error probability over Rayleigh fading channel is given as [62]:

\[ \bar{P}_e = \int_0^{\xi_{max,j}} P_{e|\xi_{max,j}} f\left(\xi_{max,j}\right) d\xi_{max,j} \]

\[ = \sum_{v=0}^{H_j-1} \binom{H_j}{v} (-1)^{H_j-v-1} \frac{H_j}{2(H_j-v)} \left(1 - \sqrt{\frac{\bar{\xi}_j}{H_j-v+\bar{\xi}_j}}\right) \]  

(3.10)

where \( H_j \) denotes the number of CRs in j-th group.

For the n-th group with \( R \) subgroups, using \( k \) out of \( N \) rule, the probability of detection, \( q_{dG} \), and the probability of false alarm, \( q_{fG} \), are calculated as given in (3.6) and (3.7), respectively.

For a cluster with \( M \) groups, the overall probability of detection, \( Q_d \), and the overall probability of false alarm, \( Q_f \), using \( k \) out of \( N \) rule, are as follows
\[ Q_d = Q_d^{M,k_2} = \sum_{m=k_2}^{M} \binom{M}{m} \beta_d^m (1 - \beta_d)^{M-m} \]  

(3.11)

and

\[ Q_f = Q_f^{M,k_2} = \sum_{m=k_2}^{M} \binom{M}{m} \beta_f^m (1 - \beta_f)^{M-m} \]  

(3.12)

where \( k_2 = \left\lceil \frac{M}{2} \right\rceil \) for majority rule, while \( \beta_d \) and \( \beta_f \) are given as [78]:

\[
\beta_d = (1 - q_{dG}) \bar{P}_e + q_{dG} (1 - \bar{P}_e),
\]

and

\[
\beta_f = (1 - q_{fG}) \bar{P}_e + q_{fG} (1 - \bar{P}_e)
\]

After fusing the observations, the CH relays its cluster’s decision to the FC to find out the final decision about the existence of the PU.

The cluster detection performance can be improved by either one of two ways. First, minimizing the overall detection error rate (i.e., detection error constraint), \( \varphi_e \) which can be minimized by optimizing the fusion rule. Second, maximizing the throughput. The throughput can be increased by optimizing the fusion rule [31], the reporting time and/or sensing time.

Generally, the time frame structure \( T \) of each CRN is divided into two slots; sensing slot \( T_s \) and data transmission slot \( T_d \) as shown in figure 3.4. Sensing slot \( T_s \) consists of sensing duration \( \tau_s \) and total reporting time \( T_r \). During data transmission slot \( T_d \), a CR transmits its data when the channel is idle. The frame time \( T = T_s + T_d \), where \( T_s = \tau_s + T_r \).
Note that each reporting CR requires time $\tau_r$ to relay its local observations; the total reporting time $T_r$ for $m$ reporting CRs is obtained as $T_r = m \tau_r$. Generally, upon sensing phase ends, SGHs in each group report their observations consecutively. The GHs combine SGHs observations and then report their observations in consecutive manner as well. The overhead control channel for a single CR is determined by time-bandwidth product $B_r \tau_r$ where $B_r$ is the bandwidth required for a CR to relay its data through reporting channel during reporting time $\tau_r$ [91]. Therefore, total overhead of $m$ reporting CRs is determined as $m B_r \tau_r$. Obviously, as $m$ increases the overhead increases as well. However, transmission time $T_d$ decreases. Therefore, the throughput decreases consequently [23].

![Basic time frame of the CRN.](image)

The total average throughput, $R(m)$, in terms of $m$ reporting CRs is calculated in [32] as

$$R(m) = C_0 \left( \frac{T - \tau_s - m \tau_r}{T} \right) \left( 1 - Q_f(m) \right) P(H_0)$$

(3.13)

where $P(H_0)$ is the probability of absence of the PU, and $C_0$ is an average channel capacity per time unit per unit frequency when having idle channel, while $\tau_s$ is the sensing time which is assumed to be constant in this work. The normalized average throughput $B(m)$ is given in [32] as
3. Hierarchical Cluster-Based Cooperative Spectrum Sensing for Cognitive Radio Networks

\[ B(m) = (T - \tau_s - m\tau_r) \left(1 - Q_f(m)\right) \]  \hfill (3.14)

### 3.3 Optimization and Energy Efficiency of the Proposed Algorithm

#### 3.3.1 Fusion rule optimization of the proposed algorithm

The overall cluster detection error rate \( \varphi_e \) is calculated as

\[ \varphi_e = P(H_1)Q_m + P(H_0)Q_f \] \hfill (3.15)

where \( P(H_1) \) is the probability of presence of the PU and \( Q_m \) is the missed-detection probability, which is defined as \( Q_m = 1 - Q_d \).

The optimization problem can be formulated as

\[ \min_{M,R,k_1,k_2} (\varphi_e) \] \hfill (3.16)

Subject to \( R \geq 1, M \geq 1, k_1 \geq 1, k_2 \geq 1 \)

For given \( M \) and \( R \), the optimal \( k_i \) can be obtained as

\[ \frac{\partial \varphi_e}{\partial k_i} \bigg|_{k_i=\hat{k}_i} = 0 \] \hfill (3.17)

where \( i = 1,2 \)

Therefore,

\[ \hat{k}_2 = \frac{\ln\left(\frac{P(H_0)}{P(H_1)}\right) + M\ln\left(\frac{1-\beta_f}{1-\beta_d}\right)}{\ln\left(\frac{\beta_d(1-\beta_f)}{\beta_f(1-\beta_d)}\right)} \] \hfill (3.18)

and

\[ \hat{k}_1 = \frac{\ln\left(\frac{C_0}{C_1}\right) + R\ln\left(\frac{1-P_f}{1-P_{dSG}}\right)}{\ln\left(\frac{P_{dSG}(1-P_f)}{P_f(1-P_{dSG})}\right)} \] \hfill (3.19)
where $C_0 = P(H_0)\beta_f^{k_2}(1 - \beta_f)^{M-k_2}$, and $C_1 = P(H_1)\beta_d^{k_2}(1 - \beta_d)^{M-k_2}$

More details about deriving $\hat{k}_2$ and $\hat{k}_1$ are provided in Appendix (A). To obtain optimal $M$, solve

$$\frac{\partial \varphi_e}{\partial M} = P(H_0)\frac{\partial Q_f}{\partial M} - P(H_1)\frac{\partial Q_d}{\partial M} = 0$$  \hspace{1cm} (3.20)

Note that (3.20) is a transcendental equation, and it is very hard to find out a closed form expression for optimal $M$ [31]. Moreover, it is also too hard to obtain optimal $R$. Therefore, the optimization problem is modified to have suboptimal $M$ and $R$ as follows

$$\max_{M,R,k_1,k_2} (Q_d)$$  \hspace{1cm} (3.21)

Subject to $R \geq 1$, $M \geq 1$

$$k_1 \geq 1, k_2 \geq 1$$

$$Q_f \leq \beta$$

Note that $\beta$ reflects spectrum utilization and it is usually that $\beta \leq 0.1$. Intuitively, if two conditions (i.e., maximizing $Q_d$ and keeping $Q_f$ below certain level $\beta$) are met, consequently, the $\varphi_e$ will be minimized. Here, an iterative algorithm to determine suboptimal $M$ and $R$ is proposed as follows:

1. $N_t$ is determined by clustering.
2. Specify $\beta$.
3. Specify $\bar{P}_f$, then $\lambda = \sigma_n^2 \left( \frac{1}{\sqrt{M_s}} Q^{-1}(\bar{P}_f) + 1 \right)$
4. Find factorization of $N$ and determine its divisors \{a_1, a_2, \cdots, a_m\}
5. Let $M_1 = \{1, a_1, a_2, \cdots, a_m\}$, $R_1 = \{1, a_1, a_2, \cdots, a_m\}$
6. $k = 1$
7. FOR $i = 1: m$
Hierarchical Cluster-Based Cooperative Spectrum Sensing for Cognitive Radio Networks

M = M_1(i)
FOR j = 1:m
R = R_1(j)
b = M \times R
IF b > N_t
Continue
ELSE
H = \frac{N_c}{b}
k_1 = \left\lceil \frac{R}{2} \right\rceil
k_2 = \left\lceil \frac{M}{2} \right\rceil
Compute P_{dSG}, P_{dG}, Q_d, P_{fG} and Q_f
IF Q_f \leq \beta
M_2(k,:,:) = [Q_d \ M \ R \ H]
k = k + 1
ELSE
Continue
END
END
END

8. \([i_1,i_2] = \max(M_2(:,1))\)

9. \(M = M_2(i_2,2), R = M_2(i_2,3), H = M_2(i_2,4)\)

Note in step 8, \(i_1 = \max(Q_d)\) and \(i_2\) is index of \(\max(Q_d)\).

For the special case when \(N_t\) is a prime, \(N_t\) can be divided into two groups as \(N_t = p + n_1\), where \(n_1 = m_1 \times r_1 \times h_1\) and \(p\) is a smaller prime (e.g., 17 =1+16). The \(m_1, r_1, h_1\) can be
found using above the algorithm. The overall detection performance in this case is found by fusing the decisions from the two groups.

### 3.3.2 Energy efficiency of the proposed algorithm

From Figure 3.4, the time frame, $T$, can be computed as follows

$$ T = \tau_s + T_r + T_d $$

(3.22)

For the conventional algorithm [78], considering a cluster with $N_t$ CRs, the number of CRs that report to the CH is $(N_t - 1)$CRs as described in Appendix (B). Therefore, the data transmission period for the conventional algorithm, $T_{dc}$ is computed as follows

$$ T_{dc} = T - \tau_s - (N_t - 1) \tau_r = T - \tau_s - T_{rc} $$

(3.23)

While the number of reporting CRs, in the proposed algorithm, is $(MR - 1)$ as also shown in Appendix (B), where $R$ is the number of subgroups in any group and $M$ is the number of groups in the cluster. Therefore, the data transmission period for the proposed algorithm, $T_{dp}$, is computed as

$$ T_{dp} = T - \tau_s - (MR - 1) \tau_r = T - \tau_s - T_{rp} $$

(3.24)

However, $N = M \times R \times H$, where $H$ is the number of CRs in any subgroup; therefore, (3.24) can be rewritten as

$$ T_{dp} = T - \tau_s - \left( \frac{N_t}{H} - 1 \right) \tau_r $$

(3.25)

Clearly, $\left( \frac{N_t}{H} - 1 \right) < (N_t - 1)$. Except for $H = 1$, (i.e., the conventional algorithm) $T_{dp} = T_{dc}$ and $T_{rc} = T_{rp}$.

Otherwise, $T_{dp} > T_{dc}$, and $T_{rc} > T_{rp}$. The difference in reporting time $\Delta T_r$ can be computed as follows:
\[ \Delta T_r = T_{rc} - T_{rp} = (N_t - 1)\tau_r - \left(\frac{N_t}{H} - 1\right)\tau_r \]

\[ \Delta T_r = N_t \left(\frac{H-1}{H}\right)\tau_r \quad (3.26) \]

Thereby, transmission time of the proposed algorithm increases as

\[ T_{dp} = T_{dc} + \Delta T_r \quad (3.27) \]

The attained reporting time difference, \( \Delta T_r \), consequently leads to increase the throughput as will be shown in Section 3.4. Moreover, from energy efficiency perspective, the proposed algorithm leads to save reporting energy (i.e., decreases overhead). It is known that, total consumed energy \( E \) during CSS process is the sum of sensing energy, \( E_s \), reporting energy, \( E_r \), and transmission energy, \( E_d \). Assuming that all CRs are identical, in other words, all CRs consume same sensing power, \( P_s \), reporting power, \( P_r \), and transmission power, \( P_d \). Hence,

\[ E = E_s + E_r + E_d = \tau_s P_s + m\tau_r P_r + T_d P_d \quad (3.28) \]

Therefore, reporting energy of the conventional algorithm, \( E_{rc} \), is calculated as \( E_{rc} = (N_t - 1)\tau_r P_r \), while the reporting energy of the proposed algorithm is \( E_{rp} = \left(\frac{N_t}{H} - 1\right)\tau_r P_r \).

The percentage of saved reporting energy, \( \Delta E_r \), can be calculated as

\[ \Delta E_r = \frac{E_{rc} - E_{rp}}{E_{rc}} \times 100 = \frac{N_t(H-1)}{H(N_t-1)} \times 100 \quad (3.29) \]
3.3.3 Sensing agility and energy consumption

For each CR, the reporting overhead is determined as time-bandwidth product $\tau_r B_r$, where $B_r$ is the minimum required bandwidth to report sensing data from a CR to the GH or the CH [96]. Therefore, the total reporting overhead $A$ in cluster is determined as follows

$$A = T_r B_r = m \times \tau_r B_r$$

(3.30)

It can be noted that more reporting CRs leads to more sensing overhead. It can be observed that sensing agility increases as reporting overhead decreases and vice versa. The agility gain $\Delta A$ is defined in [96] as

$$\Delta A = \frac{A_c}{A_t}$$

(3.31)

where, $A_c$ is the reporting overhead of the conventional algorithm [35], and $A_t$ is the reporting overhead of another algorithm. In the proposed algorithm, reporting channel is split into two sub-channels (i.e., first sub-channel is between SGHs in a group and its GH, and second sub-channel is between GHs and CH) such that each sub-channel bandwidth requirement is smaller than that required for one reporting channel as in the conventional cluster scheme. The proposed algorithm effectively decreases the reporting overhead and increases the agility gain as shown in Table 3.4.

The total consumed energy $E_T$ during both sensing and reporting phases, for all CRs in the cluster can be computed as:

$$E_T = N_t E_s + m E_r$$

(3.32)

where $E_s$ and $E_r$ are the required energy for a single CR during sensing and reporting phases.
Note that $N_t E_s = T_s P_s$ and $m E_r = T_r P_r$, where $P_s$ and $P_r$ are the consumed power for a single CR during sensing and reporting phases, respectively. The percentage of saved energy consumption, $\Delta_E$, can be calculated as

$$\Delta_E = \frac{E_T|_{\text{conventional}} - E_T|_{\text{proposed}}}{E_T|_{\text{conventional}}} \times 100\% \quad (3.33)$$

### 3.4 Simulation results and discussion

In this section, the simulation results of the proposed algorithm are presented. The simulation results illustrate detection performances for different scenarios. Throughput and reporting overhead are considered. The simulation results of the proposed algorithm are compared with two algorithms, namely, the conventional CBSS using majority rule in [78], and the MCMG algorithm in [80]. Based on [92] and [44], we set sampling frequency $f_s = 6$ MHz, bit rate $R_b = 250$ kbps, and time frame $T = 10$ ms. The iterative algorithm was performed to select the suboptimal number of groups and subgroups under some radio conditions and design parameters (i.e., $\bar{P}_f$ and $\beta$) as shown in Table 3.1.

<table>
<thead>
<tr>
<th>$i_2$</th>
<th>$Q_d$</th>
<th>$Q_f$</th>
<th>$M$</th>
<th>$R$</th>
<th>$H$</th>
<th>$k_1$</th>
<th>$k_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9859</td>
<td>0.0334</td>
<td>1</td>
<td>1</td>
<td>12</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.9877</td>
<td>0.0120</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.9967</td>
<td>0.0066</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0.9989</td>
<td>0.0229</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0.9997</td>
<td>0.0192</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

$M$: Number of groups.
$R$: Number of subgroups.
$H$: Number of CRs in a subgroup.
$k_1$: Fusion rule parameters at the level of group.
$k_2$: Fusion rule parameters at the level of cluster.
Table 3.1 illustrates the results of performing the iterative algorithm for a cluster with 24CRs at SNR of 3dB (i.e., $\gamma = 3$dB) and reporting channel gain $\xi = 1$dB. The design parameters are selected as $\beta = 0.01$ and $\overline{P}_f = 0.1$. It can be noted that the third combination in Table 3.1 meets the requirements.

Figure 3.5: Overall probability of detection of the proposed algorithm compared to conventional and MCMG algorithm, for a cluster with 24CRs.

Overall probabilities of detection for the three available algorithms are illustrated in figure 3.5. It is shown that the proposed algorithm outperforms both the MCMG and the conventional algorithms in terms of overall probability of detection, especially, in low SNR scenarios, while it has almost the same total probability of detection of both algorithms in higher SNR (i.e., $\gamma \geq 2$dB ).
For a comprehensive comparison, complementary receiver operating characteristics (CROC) (i.e., the relationship between $Q_m$ vs. $Q_f$ of the three available algorithms are illustrated in figure 3.6. The figure shows that the proposed algorithm has better sensing performance than both the MCMG and the conventional algorithms in the range $Q_f \geq 2 \times 10^{-3}$. Furthermore, from energy efficiency perspective, both the proposed and MCMG algorithms save about 69.5% of reporting energy compared to conventional algorithm [78] as illustrated in Table 3.2.

![Figure 3.6: Complementary receiver operating characteristics (CROC) of the three algorithms, for a cluster with 24CRs.](image)

For a cluster with 18 CRs, by assuming that $\bar{P}_f = 0.05$, and using the iterative algorithm, it is found that the values for $M$ and $R$ are 3 and 2, respectively. The overall probability of detection and CROC of the three algorithms are displayed in figures 3.7 and 3.8, respectively. It is shown in both figures that the proposed algorithm provides better sensing performance
compared to the MCMG and the conventional algorithms. Moreover, the proposed algorithm grants about 70% saving in reporting energy compared to the conventional algorithm [78] as shown in Table 3.3, while the MCMG algorithm [80] provides about 53% saving in reporting energy compared the conventional algorithm [78]. Furthermore, the proposed algorithm significantly reduces the reporting overhead.

Table 3.2: Overhead comparison between the three algorithms, for a cluster with 24 CRs.

<table>
<thead>
<tr>
<th></th>
<th>Reporting CRs</th>
<th>Reporting overhead</th>
<th>Energy saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional [78]</td>
<td>23</td>
<td>$23B \tau_r$</td>
<td>0%</td>
</tr>
<tr>
<td>MCMG [80]</td>
<td>5</td>
<td>$5B \tau_r$</td>
<td>69.5%</td>
</tr>
<tr>
<td>Proposed</td>
<td>5</td>
<td>$5B \tau_r$</td>
<td>69.5%</td>
</tr>
</tbody>
</table>

Figure 3.7: Overall probability of detection of the three different algorithms, for a cluster with 18 CRs.
Figure 3.8: CROC of the three algorithms, for a cluster with 18CRs.

Figure 3.9: Normalized average throughput vs. probability of detection constraint for a cluster with 18CRs.
Maximum normalized throughputs of the three algorithms are illustrated in figure 3.9. It is clear that the proposed algorithm outperforms both the conventional and the MCMG algorithms. The proposed algorithm grants better throughput over the conventional and the MCMG algorithms by approximately 5% and 1.5%, respectively. This improvement was a result of reducing the number of reporting CRs attained by the proposed algorithm.

**Table 3.3:** Overhead comparison between the three algorithms, for a cluster with 18 CRs.

<table>
<thead>
<tr>
<th></th>
<th>Reporting CRs</th>
<th>Reporting overhead</th>
<th>Energy saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional [78]</td>
<td>17</td>
<td>$17B \tau_r$</td>
<td>0%</td>
</tr>
<tr>
<td>MCMG [80]</td>
<td>8</td>
<td>$8B \tau_r$</td>
<td>53%</td>
</tr>
<tr>
<td>Proposed</td>
<td>5</td>
<td>$5B \tau_r$</td>
<td>70%</td>
</tr>
</tbody>
</table>

**Table 3.4:** Reporting overhead and agility gain for different algorithms, for a cluster with 8 CRs.

<table>
<thead>
<tr>
<th></th>
<th>Reporting overhead</th>
<th>Agility gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional [78]</td>
<td>$7t_r B_r$</td>
<td>1</td>
</tr>
<tr>
<td>MCMG [80]</td>
<td>$3t_r B_r$</td>
<td>2.3</td>
</tr>
<tr>
<td>Proposed</td>
<td>$t_r B_r$</td>
<td>7*</td>
</tr>
<tr>
<td></td>
<td>$t_r B_r$</td>
<td>7**</td>
</tr>
</tbody>
</table>

*For sub-channel between SGHs and GH.

**For sub-channel between GHs and CH.

From Table 3.4, it is obvious that the proposed algorithm shows higher agility gain over the two other algorithms and incurs less reporting overhead compared to the two other algorithms.
3. Hierarchical Cluster-Based Cooperative Spectrum Sensing for Cognitive Radio Networks

Figure 3.10: Required number of reporting CRs in the cluster for three different algorithms at $\gamma = 10$ dB.

Figure 3.11: Data transmission time $T_d$ for the cluster with different numbers of CRs computed by three different algorithms.
Figure 3.10 shows that the proposed algorithm requires less number of reporting CRs compared with both conventional and MCMG algorithms, therefore, the data transmission time $T_d$ of the proposed algorithm is greater than that required for both the conventional and the MCMG algorithm as illustrated in figure 3.11. This matter makes the proposed algorithm shows better throughput over the conventional [35] and the MCMG [80] algorithms.

The total energy consumptions of the three algorithms are illustrated in figure 3.12. It can be seen from the figure that the proposed algorithm can save energy by about average percentage of 30% and 12% compared to the conventional and MCMG algorithms, respectively. Obviously, the proposed algorithm saves more energy as the number of CRs in the cluster increases. This is because the proposed algorithm requires less reporting CRs than that required for the two other algorithms as shown in figure 3.10.

![Figure 3.12: Total energy consumption for the three algorithms.](image-url)
3.5 Conclusions

Trade-off between sensing performance and control channel overhead has been considered in a CRN system design. A multi-level hierarchical cluster-based algorithm has been proposed to tackle the trade-off. An iterative algorithm has been developed to determine suboptimal number of groups and subgroups. Optimal threshold, optimal fusion rule, and energy efficiency have been derived for sensing performance. Simulation results have shown that the proposed algorithm provides higher PU protection than MCMG and conventional algorithms. In addition, it was shown that the proposed algorithm reduces the reporting energy by 70% of that required by the conventional algorithm [78]. Furthermore, using double fusion stages in the proposed algorithm have reduced the reporting overhead in the cluster. Besides, the simulation results have also shown that the proposed algorithm can increase sensing agility, and can reduce the number of reporting CRs. Finally, it was shown that the proposed algorithm can save energy consumption by average percentage about 30% and 12% compared to the conventional [78] and the MCMG [80] algorithms, respectively. However, the improvement attained by the proposed algorithm comes at the expense of increasing the computational requirement.
Chapter Four

Novel Energy Efficient Strategies for Cluster-Based Cooperative Spectrum Sensing in Cognitive Radio Networks

4.1 Introduction

Many research works focused on obtaining the optimal number of collaborating CRs in the CSS that balances the performance and overhead. Many strategies have been applied to minimize the required number of CRs, such as optimizing $k$ out of $N$ rule to maximize energy efficiency [70], and optimizing the fusion rules to maximize the throughput [45]. This chapter tackles the balance between the performance and overhead of the cluster-based spectrum sensing (CBSS) in cognitive radio networks (CRNs) by minimizing the number of the contributing cognitive radios (CRs) in the system.

In chapter three, the number of participating CRs in the cluster was not minimized, and sensing time period was assumed constant. In this chapter, three strategies using a variable sensing window size are proposed to minimize the required number of participating CRs in the CBSS system using the three hard decision fusion rules (i.e., AND, OR and $k$ out of $N$).

This chapter is organized as follows, Section 4.2 describes the system model for CBSS system, Section 4.3 presents the proposed strategies to minimize the number of CRs in the
system, and Section 4.4 shows the simulation results while the conclusions are presented in Section 4.5.

4.2 System model for CBSS system

Assume that there is a CRN with $N_t$ CRs. Each CR employs energy detector to sense the existence of the PU. After detection phase, each CR makes its decision $d_i$ as one bit, and forwards it to the BS, which collects the decisions from the CRs and fuses them using hard decision fusion rule to have a final decision $d_t$ about the presence of the PU signal.

Detecting the existence of the PU is mainly based on spectrum sensing hypotheses, as mentioned in section 2.2. Note that the test statistic $M(y)$ follows Gaussian distribution corresponding to the above hypothesis as follows [8]:

$$M(y) \sim \begin{cases} 
\mathcal{N} \left( \sigma_n^2, \frac{2}{M_s} \sigma_n^4 \right) & \text{for } H_0 \\
\mathcal{N} \left( (1 + \gamma) \sigma_n^2, \frac{2}{M_s} (1 + 2\gamma) \sigma_n^4 \right) & \text{for } H_1
\end{cases}$$

(4.1)

where $\sigma_n^2$ denotes the noise variance, $\gamma$ is the signal to noise ratio and defined as $\gamma = \frac{\sigma_x^2}{\sigma_n^2}$ and $\sigma_x^2$ is the PU signal variance.

The detection performance of the CBSS system is assessed by the global probability of detection, $Q_d$, and the global probability of false alarm $Q_f$ of collaborating CRs in the system. The $Q_d$ indicates to how much the PU is protected from CR’s interference, while the $Q_f$ reflects the spectrum efficiency of the system.
For the \( i \)-th CR, the probability of detection \( P_{d,i} \) and the probability of false alarm \( P_{f,i} \) are defined as provided in [61]

\[
P_{d,i} = P(M(y) > \lambda | H_1) = Q \left( \frac{\lambda - (1 + \gamma) \sigma_n^2}{\sigma_n^2 \frac{2}{Ms}(1 + 2 \gamma)} \right) \tag{4.2}
\]

\[
P_{f,i} = P(M(y) > \lambda | H_0) = Q \left( \frac{\lambda - \sigma_n^2}{\sigma_n^2 \frac{2}{Ms}} \right) \tag{4.3}
\]

where \( \lambda \) is the sensing threshold, \( P \) denotes the probability, and \( i = 1, 2, \ldots, N_t \).

It can be noted that \( P_{d,i} \) is an increasing function in \( M_s \) and \( \gamma \) as shown in Figure 4.1. In contrary, \( P_{f,i} \) is a decreasing function in \( M_s \).

For simplicity, assume all CRs are grouped as clusters. For a cluster, we can assume that all CRs in the cluster have identical detection performance \( P_{d,i} = P_d \) and \( P_{f,i} = P_f \) for \( i \); because all CRs experience the same average \( \gamma \). Therefore, the global probability of detection \( Q_d \) and the global probability of false alarm \( Q_f \) of the CSS system can be determined according to the employed fusion rule, as illustrated below

**OR-Rule**

The final decision \( d_t \) is determined as in [36]

\[
d_t = \begin{cases} 
  \text{idle} & \text{if } \sum_{i=1}^{N_t} d_i = 0 \\
  \text{busy} & \text{otherwise}
\end{cases}
\]
Figure 4.1: probability of detection $P_d$ and probability of false alarm $P_f$ for a single CR against sensing window size $M_s$.

The global probability of detection and the global probability of false alarm are computed as shown in [37]

$$Q_{d,OR} = 1 - (1 - P_d)^{N_t} \quad (4.4)$$

$$Q_{f,OR} = 1 - (1 - P_f)^{N_t} \quad (4.5)$$

Clearly, it can be noted that both of $Q_{d,OR}$ and $Q_{f,OR}$ are increasing by increasing $N_t$. In other words, increasing the $Q_{d,OR}$ leads to enhance the PU protection from CR’s interference, while increasing the $Q_{f,OR}$ reduces spectrum efficiency (i.e., poorer spectrum efficiency).

**AND-Rule**

The final decision $d_t$ is determined as in [37]
\[ d_t = \begin{cases} 
\text{busy} & \text{if } \sum_{i=1}^{N_t} d_i = N_t \\
\text{idle} & \text{otherwise}
\end{cases} \]

The global probability of detection and the global probability of false alarm are computed as shown in [36]

\[ Q_{d,\text{AND}} = P_d^{N_t} \quad (4.6) \]

\[ Q_{f,\text{AND}} = P_f^{N_t} \quad (4.7) \]

Obviously, \( Q_{f,\text{AND}} \) decreases by increasing \( N_t \) which means better spectrum efficiency, however, \( Q_{d,\text{AND}} \) also decreases by increasing \( N_t \) which means poorer PU protection.

**k out of N-Rule**

The final decision \( d_t \) is determined as in [37]

\[ d_t = \begin{cases} 
\text{busy} & \text{if } \sum_{i=1}^{N_t} d_i \geq k \\
\text{idle} & \text{otherwise}
\end{cases} \]

The global probability of detection and the global probability of false alarm are computed as provided in [36]

\[ Q_{d,k,\text{out}_N} = \sum_{i=k}^{N_t} \binom{N_t}{i} P_d^i (1 - P_d)^{N_t-i} \quad (4.8) \]

and

\[ Q_{f,k,\text{out}_N} = \sum_{i=k}^{N_t} \binom{N_t}{i} P_f^i (1 - P_f)^{N_t-i} \quad (4.9) \]
The $k$ out of $N$ rule compromises between improving the PU protection and the spectrum efficiency. Moreover, $k$ out of $N$ rule is the optimal fusion rule in terms of the total detection error rate $\varphi_e$ [35]. The total detection error rate, $\varphi_e$, evaluates the detection accuracy in one metric, and is calculated as follows

$$\varphi_e = P_1(1 - Q_d) + P_0 Q_f$$

(4.10)

where $P_1$ and $P_0$ are the probabilities of the PU channel is being busy and idle, respectively.

Energy efficiency, $\xi$, is another important detection performance metric for the CSS system. Recall that the energy efficiency $\xi$ is defined as a ratio of system throughput $R$ and total energy consumption $\bar{E}$ of the system. The energy efficiency $\xi$ is computed as [40]

$$\xi = \frac{R}{\bar{E}} = \frac{C_0 t_d (1 - Q_f) P_0}{N t_s P_s + N t_r P_t + P_{idle} t_d P_t}$$

(4.11)

where $C_0$ is the data rate, $t_s$ is the sensing duration, $t_r$ is the reporting duration, $P_s$ is the sensing power for a single CR, $P_t$ is the transmission power, $t_d$ is the data transmission data, and $P_{idle}$ is the probability of perfectly having idle channel and computed as $P_{idle} = P_0 (1 - Q_f) + P_1 (1 - Q_d)$.

Figure 4.2: Basic time frame of the CRN.
The basic frame time $T$ of the CRN is shown in Figure 4.2, where, $T$ is composed of sensing period, reporting period and data transmission period. Mathematically, the frame time is determined as $T = t_s + N_t t_r + t_d$.

It can be noted that increasing the number of collaborating CRs in the CBSS system drastically increases the consumed energy (i.e., sensing energy and reporting energy) which means higher incurred overhead. Moreover, increasing the number of CRs not necessarily increases the throughput. In other words, increasing the number of CRs may decrease both the energy efficiency [70], and the throughput [35]. The impact of the number of CRs in the CSS system depends on used fusion rule. This matter motivated us to propose the following strategies to minimize the required number of CRs in the system $N_c$ by changing the window size $M_s$. The strategies basically depend on the employed fusion rule.

4.3 Proposed strategies to reduce the number of CRs in the CBSS system

4.3.1 Strategy for OR-Rule

It is known for OR-Rule that both of the $Q_{d,OR}$ and the $Q_{f,OR}$ drastically increase as the number of CRs in the CSS system increases, therefore, we have to find the minimum number of CRs such that the system meets the target performance (i.e., $Q_{d,OR} \geq \alpha$), while reduces the $Q_{f,OR}$.
\[ \min N_c \]  

Subject to  

\[ M_{s_{\text{min}}} \leq M_s \leq M_{s_{\text{max}}} \]

\[ Q_{d,\text{OR}} \geq \alpha \]

\[ P_f \leq \delta \]

where \( \alpha \) is the global probability detection constraint and \( \delta \) is the probability of false alarm constraint for a single CR, while \( M_{s_{\text{min}}} \) and \( M_{s_{\text{max}}} \) are the minimum and the maximum sensing window sizes, respectively.

To keep \( Q_{f,\text{OR}} \) as minimum as possible, the sensing threshold is computed using (4.3) in terms of \( \delta \) as

\[ \lambda = \sigma_n^2 \left( \sqrt{\frac{2}{M_s}} Q^{-1}(\delta) + 1 \right) \]  

The required number of CRs, \( N_1 \) is determined using (4.4) and substituting \( Q_{f,\text{OR}} = \alpha \), as follows

\[ N_1 = \left\lceil \frac{\log(1-\alpha)}{\log(1-P_d)} \right\rceil \]  

where \( \lceil \cdot \rceil \) is ceiling function. The minimum required number of CRs using OR-Rule is \( N_{c_{\text{OR}}} = \min(N_1, N_c) \).

From (4.14), \( N_1 \) is function of \( P_d \) which mainly depends on both \( \gamma \) and \( M_s \). Based on this fact, \( N_1 \) varies as \( M_s \) varies, and clearly it is shown in (4.14) that \( N_1 \) is a decreasing function in \( M_s \), in other words, our strategy is to enlarge \( L \) in order to reduce \( N_1 \) and vice versa as shown in Section 4.4.
The values of \( M_{s_{max}} \) and \( M_{s_{min}} \) can be determined by solving (4.2) and (4.3) and using the performance constraints, as follows

\[
M_{s_{min}} = \frac{2(Q^{-1}(\delta_{min}) - Q^{-1}(\varepsilon_{min})\sqrt{1+2\gamma})^2}{\gamma^2} \quad (4.15)
\]

\[
M_{s_{max}} = \frac{2(Q^{-1}(\delta) - Q^{-1}(\varepsilon)\sqrt{1+2\gamma})^2}{\gamma^2} \quad (4.16)
\]

where \( \varepsilon \) is the probability of detection constraint for a single CR, while \( \delta_{min} \) and \( \varepsilon_{min} \) are the minimum probability of false alarm and probability of detection, respectively.

### 4.3.2 Strategy for AND-Rule

For AND-Rule, both \( Q_{d,AND} \) and \( Q_{f,AND} \) drastically decrease as more CRs are involved in the CSS system, therefore, we have to find the minimum number of CRs such that the system meets the target performance (i.e., \( Q_{f,AND} \leq \beta \)), while enlarges \( Q_{d,AND} \)

\[
\min N_c \quad (4.17)
\]

Subject to \( M_{s_{min}} \leq M_s \leq M_{s_{max}} \)

\[
Q_{f,AND} \leq \beta
\]

\[
P_d \geq \varepsilon
\]

where \( \beta \) is the global probability of false alarm constraint.

To keep \( Q_{d,AND} \) as large as possible, sensing threshold is computed in terms of \( \varepsilon \) as

\[
\lambda = \sigma_n^2 \left( \frac{2}{M_s} (1 + 2\gamma) Q^{-1}(\varepsilon) + 1 + \gamma \right) \quad (4.18)
\]
The required number of CRs $N_2$ to meet target performance $\beta$, is determined using (4.7) as follows

$$N_2 = \left\lceil \frac{\log(\beta)}{\log(P_f)} \right\rceil$$

(4.19)

The minimum required number of CRs using AND-Rule is $N_c^{AND} = \min(N_2, N_t)$.

Obviously from (4.19), $N_2$ is a function of $P_f$ which mainly depends on $M_s$. Based on this fact, $N_2$ varies as $M_s$ varies, and clearly it is shown in (4.19) that $N_2$ is decreasing function in $M_s$, in other words, a designer can vary $M_s$ in order to control $N_2$ as shown in Section 4.4.

### 4.3.3 Strategy for $k$ out of $N$-Rule

The $k$ out of $N$ Rule is an optimal fusion rule [35], since it balances between enlarging $Q_{d,k_{out},N}$ and reducing $Q_{f,k_{out},N}$. To find the minimum number of CRs such that the system meets the target performance (i.e., $Q_{d,k_{out},N} \geq \alpha$), we have

$$\min N_c$$

Subject to $M_{s_{min}} \leq M_s \leq M_{s_{max}}$

$$Q_{d,k_{out},N} \geq \alpha$$

$$P_f \leq \delta$$

To keep $Q_{f,k_{out},N}$ as minimum as possible, the sensing threshold is computed in terms of $\delta$ as given in (4.13).
To find the minimum number of CRs that meets the performance using (4.8), we use the Demoiver-Laplace theorem [102] which is approximation of a normal distribution to the binomial distribution. Therefore, (4.8) can be approximated as

\[ Q_{d,k,\text{out},N} \approx Q\left(\frac{k-0.5-NP_{d}}{\sqrt{NP_{d}(1-P_{d})}}\right) \quad (4.21) \]

The \( N_{3} \) that satisfies \( Q_{d,k,\text{out},N} \geq \alpha \) is determined by solving quadratic equation as follows

\[ N_{3} = \frac{(2k-1+W)\pm Q^{-1}(\alpha)\sqrt{(1-P_{d})(4k-2+W)}}{2P_{d}} \quad (4.22) \]

where \( W = \left(Q^{-1}(\alpha)\right)^{2}(1-P_{d}) \).

The minimum required number of CRs using \( k \) out of \( N \)-Rule is \( N_{c}^{k,\text{out}N} = \min(N_{3}, N_{t}) \). Clearly that \( N_{3} \) is a function of \( k, \gamma \) and \( M_{s} \). Section 4.5 shows the impact of varying \( M_{s} \) on \( N_{3} \).

### 4.4 Simulation results and discussion

In this section, the simulation results of the proposed strategies are presented. The simulation results illustrate the required number of CRs to collaborate and detection performances for the proposed strategies. Energy efficiency for each case is also considered. Based on [92], we set sampling frequency \( f_{s} = \) 6 MHz, data rate \( R = \) 100 kbps, time frame \( T = \) 100 ms, sensing power \( P_{s} = 0.3W \), transmitting power \( P_{t} = 1W \), and \( P_{0} = 0.8 \). Moreover, the detection performance constraints, \( \varepsilon = 0.9, \delta = 0.1, \alpha = 0.95, \) and \( \beta = 0.05 \), while \( \delta_{\min} \) and \( \varepsilon_{\min} \) are equal to 0.5.
Figure 4.3: Required CRs $N_c$ vs. sensing window size using OR-Rule at different radio conditions.

Figure 4.4: Global probability of false alarm $Q_{f,OR}$ vs. sensing window size using OR-Rule at different radio conditions.
For OR-Rule, the impact of varying the window size $M_s$ on the required number of CRs, $N_c$, the global probability of false alarm, $Q_{f,OR}$, and the energy efficiency, $\xi$, for different radio conditions are displayed in the three following figures. From the figures, it can be noticed that $N_c$ decreases by increasing $M_s$, and consequently the $Q_{f,OR}$ decreases as well. Therefore, the spectrum efficiency consequently increases. Moreover, the decrease of both $N_c$ and $Q_{f,OR}$ results in improving $\xi$ with increasing $M_s$ as shown in Figure 4.5. Although increasing of $M_s$ leads to increase the sensing energy (i.e., $t_s P_s$), however, the impact of decreasing both $N_c$ and $Q_{f,OR}$ on $\xi$ is greater than of the impact of increasing $M_s$. Furthermore, it can be seen from the figures that better radio conditions (i.e., higher $\gamma$) require less cooperating CRs, $N_c$, and enhances both the
global probability of false alarm and the energy efficiency; these results corroborate the veracity of the proposed strategy, since better radio condition enhances the detection performance [35].

![Figure 4.6: Required CRs $N_c$ vs. sensing window size using AND-Rule at different radio conditions.](image)

The impact of changing the window size $M_s$ on the number of CRs and the detection performance metrics of the CRN using AND-Rule are illustrated in Figures 4.6,7,8. From figure 4.6, it is noticeable that as the window size $M_s$ increases the number of CRs required to satisfy detection performance (i.e., $Q_{f,AND} = 0.05$) decreases, and consequently the energy consumption decreases too. Moreover, decreasing $N_c$ results in improving both the total probability of detection $Q_{d,AND}$ and the energy efficiency $\xi$ as shown in figures 4.7 and 8, respectively. Further, better radio conditions significantly improve the detection performance (i.e., $Q_{d,AND}$, and $\xi$).
4. Novel Energy Efficient Strategies for Cluster-Based Cooperative Spectrum Sensing in CRNs

Figure 4.7: Global probability of detection $Q_{d,\text{AND}}$ vs. sensing window size using AND-Rule at different radio conditions.

Figure 4.8: Energy efficiency $\xi$ vs. sensing window size using AND-Rule at different radio conditions.
It can be realized from comparing the results attained from the two first strategies (i.e., AND-Rule and OR-Rule) that the required number of CRs $N_c$ for OR-Rule is less than that required for AND-Rule, at the same radio condition (i.e., $\gamma$) and same sensing window size $M_s$. Besides, the OR-Rule has faster energy efficiency improvement than that for AND-Rule, however, the AND-Rule outperforms the OR-Rule in terms of global probability of false alarm, where the minimum global probability of false alarm achieved by OR-Rule is 0.1 which greater than the probability of false alarm constraint for the AND-Rule. In other words, AND-Rule provides better spectrum efficiency than that provided by the OR-Rule.

Figure 4.9: Required CRs $N_c$ vs. sensing window size using $k$ out of $N$-Rule, at $\gamma = -2$dB, at different values of $k$. 
4. Novel Energy Efficient Strategies for Cluster-Based Cooperative Spectrum Sensing in CRNs

Figure 4.10: Global probability of false alarm $Q_{f,k \text{ out } N}$ vs. sensing window size using $k$ out of $N$-Rule, at $\gamma = -2\text{dB}$.

Figure 4.11: Energy efficiency $\xi$ vs. sensing window size using $k$ out of $N$-Rule, at $\gamma = -2\text{dB}$, at different values of $k$. 
The simulation results of the third strategy applied to $k$ out of $N$-Rule are shown in figures 4.9, 10, and 11. For different values of $k$, and at $\gamma = -2$dB. The required number of CRs $N_c$ is computed and illustrated in figure 4.9, it can be seen from the figure that $N_c$ increases as the value of $k$ increases. Moreover, a higher value of $k$ requires larger window size to minimize $N_c$. Figure 4.10 shows that $Q_{f,k \text{ out } N}$ considerably improved by increasing $M_s$. Furthermore, as the value of $k$ increases, $Q_{f,k \text{ out } N}$ diminishes faster. Besides, we can realize from the figure that $k$ out of $N$-Rule provides better $Q_f$ compared with that provided by both OR-Rule and AND-Rule.

The energy efficiency $\xi$ attained by $k$ out of $N$-Rule is displayed in figure 4.11. Obviously, the energy efficiency $\xi$ improves by increasing both the window size $M_s$ and the value of $k$. However, the number of CRs $N_c$ drastically reduced by increasing $M_s$ and decreasing $k$ as shown in the figure 4.9 and consequently the consumed energy lessens and the efficiency improves. The energy efficiency of the third strategy has faster rate of change with respect to the two other strategies. Therefore, the third strategy can be considered as the optimal strategy among the three proposed strategies to improve the performance of the centralized CSS in CRNs.

In order to determine which strategy is the best, a comparison between the three proposed strategies in terms of both energy efficiency and global probability of false alarm at the same radio condition (i.e., $\gamma = -4$dB) is performed and depicted in figures 4.12 and 13.
Figure 4.12: Comparison in energy efficiency $\xi$ between the three proposed strategies, at $\gamma = -4$dB.

Figure 4.12 shows that the achievable energy efficiency of the first and the third proposed strategies (i.e., OR and $k$ out of $N$-Rule, respectively) are almost the same over almost all sensing window size. In contrary, the achievable energy efficiency of the AND-Rule has the poorest attained energy efficiency because of three factors. First, constraining the global probability of false alarm (i.e., $Q_{f, AND} \leq 0.05$) as shown in figure 4.13. Second, AND-Rule strategy requires more CRs comparing with those required by the two other fusion rules, especially for small sensing window sizes. Third, AND-Rule attains low global probability of detection compared with the two other strategies. All these factors lead consequently to increase the consumed energy, and decrease the energy efficiency considerably.
4. Novel Energy Efficient Strategies for Cluster-Based Cooperative Spectrum Sensing in CRNs

Figure 4.13: Comparison in global probability of detection between the three proposed strategies, at $\gamma = -4$dB.

It is obvious from figure 4.13, that $k$ out of $N$-Rule with two values of $k$ outperforms the OR-Rule in terms of the global probability of false alarm in all sensing windows sizes. However, the AND-Rule outperforms the $k$ out of $N$ in case of small sensing window sizes (i.e., $M_s < 60$, for $k = 2$ and $M_s < 10$, for $k = 4$) where the strategy of AND-Rule constrains the global probability of false alarm (i.e., $Q_{f,AND} \leq 0.05$) in all sensing window range. Therefore, $k$ out of $N$-Rule can be considered as the best proposed strategy.

4.5 Conclusions

The impact of the number of required CRs in the CBSS system on the overall detection performance of CRN has been considered. Three novel energy efficient strategies have been
proposed for the CBSS system using the three different hard decision fusion rules. The simulation results have shown that increasing the window size effectively minimizes the required number of CRs and drastically improves the energy efficiency while meeting the target detection performance. Furthermore, the simulation results have also show that the $k$ out of $N$-Rule is the best strategy among the proposed strategies.
Chapter Five:

**Optimal Energy Efficient Cluster-Based Spectrum Sensing for Cognitive Radio Networks**

5.1 Introduction

In the previous chapter, the impact of varying sensing time period on the required participating CRs in a cluster using different hard combination rules has been considered. The optimization of sensing time or required contributing CRs that maximizes the attainable energy efficiency was not considered. In this chapter, the impact of transmitting power on the probability of false alarm for a single cognitive (CR) is investigated. Second; joint optimal design parameters (i.e., sensing time, transmission time and the required number of participating CR users) for an energy efficient cluster-based spectrum sensing (CBSS) system is considered. The formulation of design problem as a function that depends only on two variables is presented. The first variable of the design function considers joint sensing time and data transmission time while the second variable considers the number of CR users. The objective function is the energy efficiency metric subjected to probability of false alarm constraint. An iterative algorithm with low computational complexity is proposed to determine the optimal design parameters for energy efficient CBSS system.

The chapter is organized as follows, Section 5.2 describes the system model, Section 5.3 presents the formulation of design problem while the proposed algorithm is presented in Section
5. Optimal Energy Efficient Cluster-Based Spectrum Sensing for Cognitive Radio Networks

5.4, and Section 5.5 shows the simulation results while the conclusions are presented in Section 5.6.

5.2 System model

It is assumed that there are some CRs randomly located around one primary user (PU). The CRs are grouped into clusters according to their geographical locations. The CRs operate in a periodic basis with a constant time frame $T$ which consists of sensing time, $t_s$, reporting time $Nt_r$, and transmission time, $t_d$ (i.e., $T = t_s + t_d + Nt_r$), where $N$ is the number of CRs in the cluster, and $t_r$ is the reporting time duration for each CR. The CRs in the cluster independently detect the presence of the PU in some spectrum band during $t_s$. Based on their local observations, each CR forwards its decision to the cluster head (CH) which combines the observations using most voting rule (i.e., $k$ out of $N$, $k \geq \frac{N}{2}$) and then the CH decides either to transmit their data during $t_d$ if the PU is idle, or to sleep if the PU still occupies the band. However, the PU might resume its activity during CR’s transmission period $t_d$, resulting in an expected interference to the PU. Therefore, PU traffic must be considered. Generally, PU activity is modeled as Poisson arrival processes with two states “ZERO” and “ONE” for idle and busy states, respectively. The duration of both states are assumed to be exponentially distributed with means $\alpha_0$ and $\alpha_1$, respectively [99]. The probability density function (PDF) of idle state duration is given as [68]

$$f_0(t) = \alpha_0^{-1} e^{-t/\alpha_0} u(t) \quad (5.1)$$

where $u(t)$ is step function. The PDF of the of the busy state period can similarly be determined as in (5.1). The probability of PU being idle, $P_0$, is determined as $P_0 = \alpha_0 / (\alpha_0 + \alpha_1)$, similarly, the probability of PU being busy is $P_1 = \alpha_1 / (\alpha_0 + \alpha_1)$. 


The probability that PU might resume its activity when CRs transmit their data over the band during $t_d$ can be determined as in [68]:

$$q_I(T_d) = \int_0^{T_d} f_0(t) dt = 1 - e^{-\frac{T_d}{\alpha_0}}$$  \hspace{1cm} (5.2)

The detection performance of a CR system is assessed by the probability of detection $P_d$ and the probability of false alarm $P_f$, which are defined as [23]

$$P_d = Q\left(\frac{\lambda}{\sigma_n^2} - \gamma - 1\right) \sqrt{\frac{t_s f_s}{(1+2\gamma)}}$$  \hspace{1cm} (5.3)

$$P_f = Q\left(\frac{\lambda}{\sigma_n^2} - 1\right) \sqrt{t_s f_s}$$  \hspace{1cm} (5.4)

where $\lambda$ is the sensing threshold, $\gamma$ is the signal to noise ratio, $\sigma_n^2$ is the noise power, and $t_s$ is the sensing time, while $f_s$ is the sampling frequency, and $Q(\cdot)$ is the Q-function [23].

To guarantee more PU protection, the probability of detection of each CR user should at least be greater than some constraint (i.e., $P_d \geq \bar{P}_d$). Therefore, the probability of false alarm can be computed as

$$P_f = Q\left(\sqrt{2\gamma + 1} Q^{-1}(\bar{P}_d) + \gamma \sqrt{t_s f_s}\right)$$  \hspace{1cm} (5.5)

The global probability of detection $Q_d$ and global probability of false alarm $Q_f$ of the CSS system using $k$ out of $N$-Rule are as shown in [36]

$$Q_d = Q_d^{n,k} = \sum_{i=k}^{n} \binom{n}{i} P_d^i (1 - P_d)^{n-i}$$  \hspace{1cm} (5.6)
\[ Q_f = Q_f^{n,k} = \sum_{i=k}^{n} \binom{n}{i} P_f^i (1 - P_f)^{n-i} \] (5.7)

Simply, CRs will not transmit their data in the following two cases; (a) if CRs successfully detect the presence of the PU with probability of \( P_1 Q_d \) where \( Q_d \) is the global probability of detection of the system, (b) if the CRs falsely detect the absence of the PU with probability of \( P_0 Q_f \), where \( Q_f \) is the global probability of false alarm of the system. Data transmission occurs if CRs successfully detect the absence of the PU with probability of \( P_0 (1 - Q_f) \) or CRs fail in correctly detecting the existence of the PU with probability of \( P_1 (1 - Q_d) \).

### 5.3 Formulation of design problem and proposed algorithm

The objective is to jointly optimize the design parameters, namely, sensing time and transmission time and the required number of contributing CRs in sensing process in the cluster, to maximize the average quantity of transmitted data per energy unit subjected to PU protection constraints and probability of false alarm constraint which directly affects the spectrum efficiency of the system. To setup the design problem, first we consider the average achievable transmission data quantity, \( R \) (i.e., throughput) which is determined [68]

\[ R(t_s, t_d, n) = P_0 R_0 t_d \left( 1 - Q_f^{n,k}(t_s) \right) \left( 1 - P_1(t_d) \right) \] (5.8)

where \( R_0 \) is the upper bound of the channel capacity that can be transmitted over the period \( t_d \).

In the next step, we consider the total incurred energy consumption, \( E(t_s, t_d, n) \) for the two transmission cases which is calculated as \( E(t_s, t_d, n) = n t_s \phi_s + n t_r \phi_r + E_t \), where \( \phi_s \) is the
sensing power, $\phi_t$ is the transmitting power while $E_t$ is the data transmission energy and it is computed as

$$E_t = \left( P_0 \left( 1 - Q_f^{n,k}(t_s) \right) + P_1 \left( 1 - Q_d^{n,k}(t_s) \right) \right) n \phi_t t_d$$

(5.9)

The general design problem is to maximize the energy efficiency by varying sensing time, transmission time and the number of participating CRs in the cooperation. The energy efficiency is determined as

$$\zeta(t_s, t_d, n) = \frac{R(t_s, t_d, n)}{E(t_s, t_d, n)}$$

(5.10)

Therefore, the general design problem can be formulated as

$$\text{max } \zeta(t_s, t_d, n)$$

(5.11)

Subject to

$$P_d \geq \bar{P}_d$$

$$P_I(t_d) \leq \alpha_I$$

$$\lambda, t_s, t_d \geq 0$$

$$N \geq n \geq 1$$

where $\alpha_I$ is the maximum allowable interference level to the PU, $\bar{P}_d$ is the target probability of detection, and $\lambda$ is the sensing threshold.

5.4 The proposed algorithm

Assume that each CR in the cluster employs energy detector to sense the existence of the PU, furthermore, all CRs are identical (i.e., all have the same $\phi_s$, $\phi_t$, and data transmission rate $r_b$). We start analyzing the objective function which is subjected to the given constraints, by
assuming $t_s = \theta T$. Practically, $T \gg t_r$, where $t_r$ is the reciprocal of $r_b$ which is in the order of Kbit/s. Therefore, $t_d \approx (1 - \theta)T$. Moreover, to satisfy the first constraint of PU protection, the probability of false alarm for each CR can be determined as

$$P_f(\theta) = Q\left(\sqrt{2\gamma + 1}Q^{-1}(\bar{P}_d) + \sqrt{f_s T \theta}\right),$$

where $\gamma$ is the average signal to noise ratio (SNR) of the PU signal over the cluster, and $f_s$ is a sampling frequency, while $Q(\cdot)$ is $Q$-function [23].

**Proposition 1**: For a single CR, $P_f(\theta)$ is a convex function of $\theta$

**Proof**: Let $\dot{P}_f(\theta)$ is the first derivative of $P_f(\theta)$ w.r.t. $\theta$, and is calculated as

$$\dot{P}_f(\theta) = \frac{-B}{\sqrt{8\pi\theta}} \exp\left(\frac{-1}{2}(A + B\sqrt{\theta})^2\right)$$ (5.12)

where $A = \sqrt{2\gamma + 1}Q^{-1}(\bar{P}_d)$ and $B = \gamma\sqrt{f_s T}$.

Obviously from (5.12) that $\dot{P}_f(\theta) < 0$, $\forall \theta$, which indicates that $P_f(\theta)$ is a decreasing function of $\theta$. However, $\theta$ should be limited to be within some interval $[\theta_{min}, \theta_{max}]$. The lower bound $\theta_{min}$ is determined by letting $P_f(\theta) < 0.5$, therefore, $\theta_{min} = \left(\frac{\sqrt{2\gamma + 1}Q^{-1}(\bar{P}_d)}{\gamma\sqrt{f_s T}}\right)^2$, while the upper bound $\theta_{max}$ is readily determined using the second constraint in (4.10) as $\theta_{max} = 1 + \frac{a_0}{T} \ln(1 - \alpha_t)$. The second derivative of $P_f(\theta)$ w.r.t. $\theta$ is calculated as

$$\ddot{P}_f(\theta) = \frac{B}{4\sqrt{2\pi\theta}} \exp\left(\frac{-1}{2}(A + B\sqrt{\theta})^2\right)\left(\frac{A + B\sqrt{\theta}}{\sqrt{\theta}} + \theta^{-1}\right)$$ (5.13)

It is very clear that $\ddot{P}_f(\theta) > 0$, $\forall \theta$. Therefore $P_f(\theta)$ is a convex function of $\theta$.

The first constraint always grants that $P_d(\theta)$ of any CR in the cluster equals to $\bar{P}_d$ at any radio condition. Simply, in case of collaboration of CRs, the term $\left(1 - Q_d^{n,k}(\theta)\right)$ tends to null.
5. Optimal Energy Efficient Cluster-Based Spectrum Sensing for Cognitive Radio Networks

for \( \forall \theta, n \geq 2 \). Therefore, the approximated total energy consumption, \( \hat{E} \), is computed as

\[
\hat{E}(\theta,n) = n\theta T\phi_s + n\phi_t t_r + n\phi P_0 \left(1 - Q_f^{n,k}(\theta) \right) (1 - \theta)T.
\]

The throughput can be rewritten in terms of \( \theta \) as

\[
R(\theta, n) = P_0R_0 (1 - \theta)T \left(1 - Q_f^{n,k}(\theta) \right) \exp \left(-\frac{(1-\theta)T}{\alpha_0} \right)
\]

(5.14)

Therefore, the approximated energy efficiency \( \hat{\zeta} \) is computed as \( \hat{\zeta}(\theta, n) = \frac{R(\theta, n)}{\hat{E}(\theta, n)} \).

Consequently, the problem function in (5.11) can be rewritten as

\[
\text{max } \hat{\zeta}(\theta, n) \quad \text{Subject to } \theta_{\text{max}} > \theta \geq \theta_{\text{min}}
\]

\[
N \geq n \geq 1 , n \in \mathbb{Z}
\]

\[
Q_f^{n,k} \leq \beta
\]

(5.15)

Note that the third constraint in (5.15) is added to guarantee sufficient spectrum efficiency, where \( \beta \) is the probability of false alarm constraint.

**Proposition 2:** For any given \( n \) CRs, there is an optimal \( \bar{\theta} \) that maximize the \( \hat{\zeta}(\theta, n) \), where

\( \bar{\theta} \in [ \theta_{\text{min}}, \theta_{\text{max}} ] \).

**Proof:** obviously, \( \hat{\zeta} \) is a continuous function of \( \theta \) in all given interval, therefore, the first partial derivate of \( \hat{\zeta} \) w.r.t. \( \theta \) is \( \hat{\zeta} = \frac{\dot{E}R - R\dot{E}}{E^2} \), where \( \dot{R} \) and \( \dot{E} \) are the first partial derivate w.r.t. \( \theta \). The \( \dot{E} \) is calculated as \( \dot{E} = nT\phi_s - n\phi_t P_0 T \left(1 - Q_f^{n,k}(\theta) \right) + (1 - \theta)\dot{Q}_f^{n,k}(\theta) \) and \( Q_f^{n,k}(\theta) = \)
\( n \hat{P}_f(\theta) \Delta Q_f \), where \( \Delta Q_f = Q_f^{n-1,k-1} - Q_f^{n-1,k} > 0 \) as in [4]. Note that \( \dot{Q}_f^{n,k}(\theta) > \left( 1 - Q_f^{n,k}(\theta) \right) \) and \( \hat{P}_f(\theta) < 0 \), \( \forall \theta \). Therefore, \( \hat{E} > 0 \), which means that \( \hat{E} \) is an increasing function of \( \theta \).

Similarly, \( \hat{R} = F_1 - F_2 + F_3 \), where

\[
F_1 = \frac{-R}{(1-\theta)}, \quad F_3 = \frac{TR}{\alpha_0}
\]

and

\[
F_2 = -P_0 R_0 (1-\theta) T \dot{Q}_f^{n,k}(\theta) \exp\left( -\frac{(1-\theta)T}{\alpha_0} \right).
\]

Obviously, \( F_2, F_3 > 0 \), while \( F_1 < 0 \). However, \( |F_3| > |F_1| \), therefore, \( \hat{R} > 0 \), and consequently, \( \dot{\zeta} > 0 \). Similarly, taking second partial derivate w.r.t. \( \theta \), it is readily proven that \( \ddot{R} < 0 \), \( \ddot{E} > 0 \), and \( \ddot{\zeta} < 0 \). Therefore, \( \dot{\zeta}(\theta,n) \) is a concave unimodal function of \( \theta \), and \( \exists \bar{\theta} \) such that \( \dot{\zeta}(\theta,n) \bigg|_{\theta=\bar{\theta}} = 0 \). Golden section search approach [99] can be employed to determine \( \bar{\theta} \).

Note also that \( n \) is an integer number, which means \( \dot{\zeta}(\theta,n) \) is a non-continuous function of \( n \); therefore, it is not feasible to derive an optimal \( n \) maximizes \( \dot{\zeta}(\theta,n) \); moreover, it is also not feasible to derive an optimal \( k \), therefore we select \( k = \left\lceil \frac{n}{2} \right\rceil \). The matter motivated the proposed suboptimal iterative algorithm to jointly optimize \( \theta \) and \( n \) which maximize \( \dot{\zeta}(\theta,n) \).

**Proposed Iterative Algorithm**

Given \( T, t_r, N, \gamma, \bar{P}_d, \beta, \alpha, \alpha_0, \alpha_1, f_s, \epsilon, \phi_s, \phi_t \)

1. Compute \( \theta_{min}, \theta_{max}, P_0, P_1 \)
2. Let \( n = 1 \)
3. \( k = \left\lceil \frac{n}{2} \right\rceil \), using Golden section search method, determine \( \bar{\theta}_n \) which maximizes \( \dot{\zeta}(\theta,n) \).
4. If \( \dot{Q}_f^{n,k} \leq \beta \) then
   \( n_{opt} = n, \theta_{opt} = \bar{\theta}_n \) and \( \zeta_{max} = \dot{\zeta}(\theta_{opt}, n_{opt}) \)
   Else \( n = n + 1 \)
\[ \theta_1 = \bar{\theta}_{n-1} + \epsilon \]

If \( \zeta(\theta_n, n) \geq \zeta(\bar{\theta}_{n-1}, n) \)

\[ \theta_{\text{min}} = \bar{\theta}_{n-1} \]

Else

\[ \theta_{\text{max}} = \bar{\theta}_{n-1} \]

End

Go to 3.

End

5.5 Simulation results and discussion

Based on [67]-[68], the set of simulation parameters are \( \bar{P}_d = 0.9, T = 150 \text{ ms}, N = 10 \text{ CRs}, \)

\( \beta = 0.1, \alpha_t = 0.1, \alpha_0 = 0.65 \text{ s}, \alpha_1 = 0.352 \text{ s}, f_s = 6 \text{ MHz}, \phi_s = 0.2 \text{ W}, \) and \( \phi_t = 0.3 \text{ W}. \) The

transmission rate is 100 kbps and the channel capacity, \( R_0 \) is computed according to [68]. The

simulation started with the energy efficiency of a single CR system at different radio

environmental conditions as depicted in Fig. 5.1.

Figure 5.1 shows the energy efficiency of a single CR at different conditions (i.e., \( \gamma = -16, -18 \) and \(-20 \text{ dB}) \) and the corresponding optimal \( \theta \) for each radio condition. It can be seen

from the figure that the optimal \( \theta \) increases as \( \gamma \) decreases because CR needs more sensing time

as radio condition gets worse in order to meet the target probability of detection \( \bar{P}_d \) [23]. The

corresponding probabilities of false alarm of the single CR are illustrated in Fig. 5.2.
5. Optimal Energy Efficient Cluster-Based Spectrum Sensing for Cognitive Radio Networks

Fig. 5.1 Energy efficiency for single CR vs. $\theta$, for different radio conditions.

Fig. 5.2 The probability of false alarm for single CR vs. $\theta$, for different radio conditions.
Besides, it is shown in Fig. 5.2 that the values of the probability of false alarm in all cases do not satisfy the probability of false alarm target of 0.1, which violates the detection performance constraints.

![Graph showing probability of false alarm vs. $\phi_r$ for single CR, at $\gamma = -18$ dB.]

Fig. 5.3 The probability of false alarm vs. $\phi_r$ for single CR, at $\gamma = -18$ dB.

The work in [67] and [68] suggested to increase the transmitting power $\phi_r$ in order to increase the sensing time $t_s$ (i.e., $\theta T$); therefore, the probability of false alarm accordingly decreases; however increasing $\phi_r$ is an ineffective approach as shown in Fig. 5.3. Moreover, increasing $\phi_r$ leads to almost linearly increase in energy consumption as shown in Fig. 5.4, and consequently the energy efficiency decreases remarkably [68]. Furthermore, from Fig. 5.3, though $\phi_r$ is increased till 2W, yet this high cost of transmission power cannot even satisfy the probability of false alarm target (i.e., $\beta = 0.1$).
Fig. 5.4 The consumed energy $\hat{E}$ vs. $\phi_r$ for single CR, at $\gamma = -18$ dB.

Fig. 5.5 The energy efficiency $\hat{\zeta}$ vs. $\theta$, for different number of CRs.
Fig. 5.6 Maximum energy efficiency vs. Number of CRs.

Fig. 5.7 Optimal $\bar{\theta}$ for each number of CRs.
Fig. 5.8 The global probability of false alarm $Q_f$ vs. Number of CRs.

Fig. 5.9 Consumed energy $\hat{E}$ vs. number of CRs.
The energy efficiencies of the proposed algorithm for each \( n \) are illustrated in Fig. 5.5, while Fig. 5.6 presents the maximum values of the \( \hat{\zeta}(\hat{\theta}_n, n) \), \( \forall n \) in the cluster, at \( \gamma = -18 \text{ dB} \).

From Fig. 5.7, it can be seen that the interval to which \( \bar{\theta} \) belongs is significantly shortened, for instance, \( \bar{\theta}_1 \in [\theta_{\text{min}}, \theta_{\text{max}}) \), \( \bar{\theta}_2 \in [\bar{\theta}_1, \theta_{\text{max}}) \) while \( \bar{\theta}_3 \in [\bar{\theta}_1, \bar{\theta}_2] \), etc. The shrinking of the interval remarkably increases the processing speed of the iterative algorithm.

Figure 5.8 presents the global probability of false alarm for all possible number of participating CRs in a cluster of 10 CRs, at \( \gamma = -18 \text{ dB} \). It is obvious that the optimal number of CRs is seven CRs (i.e., \( n_{\text{opt}}=7 \) CRs), which meets the probability of false alarm target. Moreover, the corresponding maximum energy efficiency is very close to the highest among the possibilities as depicted in Figs. 5.6, only 0.31% loss compared to the case with 9 CRs. Furthermore, the incurred energy consumption by 7 CRs is less than that consumed by 9 CRs as illustrated in Fig. 5.9.

The proposed algorithm employs only 7 CRs out of 10 CRs in the cluster to maximize the EE, while the three other algorithms involve all CRs in the cluster; this matter provides the proposed algorithm an advantage of considerably reducing the energy consumption. Moreover, it is shown in Table 5.1 that the proposed algorithm outperforms the fixed threshold algorithms in [23] and [106] in terms of \( Q_f \) and \( Q_d \), respectively. In addition, the proposed algorithm outperforms the optimal threshold algorithm [6] in terms of \( Q_f \) and \( Q_d \). As a conclusion, the proposed algorithm balances between \( Q_f \) and \( Q_d \), while the three other algorithms cannot trade-off between \( Q_f \) and \( Q_d \).
Table 5.1: Comparison between the proposed algorithm and two existing works for a cluster with 10 CRs, at $\gamma = -18$ dB, $\alpha_0 = 0.65$ s, $\alpha_1 = 0.352$ s, $\phi_s = 0.2$ W, $\phi_r = 0.3$ W, and $T = 0.1$ s.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$n_{opt}$</th>
<th>$k_{opt}$</th>
<th>$Q_f$</th>
<th>$Q_d$</th>
<th>E (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed sensing threshold [23]</td>
<td>10</td>
<td>7</td>
<td>0.4643</td>
<td>0.9984</td>
<td>0.0338</td>
</tr>
<tr>
<td>Fixed sensing threshold [106]</td>
<td>10</td>
<td>2</td>
<td>0.0702</td>
<td>0.7282</td>
<td>0.0399</td>
</tr>
<tr>
<td>Optimal sensing threshold [106]</td>
<td>10</td>
<td>7</td>
<td>0.1719</td>
<td>0.9802</td>
<td>0.0399</td>
</tr>
<tr>
<td>Proposed</td>
<td>7</td>
<td>4</td>
<td>0.0954</td>
<td>0.9973</td>
<td>0.0289</td>
</tr>
</tbody>
</table>

5.6 Conclusions

The impact of varying the transmission power on the probability of false alarm of single CR has been investigated. Investigations have shown that increasing transmission power is not always effective to meet probability of false alarm target. To meet the target, designing an optimal energy efficient CBSS that satisfies the sensing accuracy metrics has been considered in this chapter. The problem of design has been formulated and analysis has also been provided. An iterative algorithm with low computational complexity has been proposed to jointly determine the optimal design parameters of CBSS system that maximize the energy efficiency while satisfying all detection accuracy metrics. Simulation results have shown that proposed algorithm significantly outperforms the previous existing works.

---

1 The work in [23], the sensing threshold is computed in terms of probability of false alarm constraint. In [106], two algorithms have been proposed, first, the sensing threshold is computed in terms of probability of detection constraint, and then authors of [106] optimized the sensing threshold to maximize the attainable energy efficiency.
Chapter Six: 
Joint Optimal Transmission Power and Sensing Time for Energy-Efficient Spectrum Sensing in Cognitive Radio System

6.1 Introduction

The previous chapters have focused on cluster-based spectrum sensing (CBSS) algorithm. In this chapter, joint optimal transmission power and sensing time (i.e., optimal operating point) that maximizes the energy efficiency of the spectrum sensing (SS) of cognitive radio (CR) is considered. The joint optimization problem is formulated as one problem with two variables. Energy efficiency of the CR is selected as objective function subjected to two PU protection criteria. An iterative algorithm which remarkably reduces the complexity of solving the optimization problem is proposed. Simulation results of the proposed algorithm are provided, evaluated and compared with those existing works in the literature. Moreover, the probability of false alarm at the optimal operating point is investigated.

This chapter is organized as follows. Section 6.2 introduces the system model. Problem formulation is presented in Section 6.3. Details about the proposed algorithm are provided in Section 6.4. Simulation results are shown and discussed in Section 6.5, while conclusions are provided in Section 6.6.
6.2 System Model

Assuming that there is a system consists of only one PU and only one CR. Every constant frame time $T$, the CR detects the existence of the PU over some frequency band. The time frame $T$ consists of two sensing durations which are sensing time, $t_s$ and data transmission time, $t_d$ (i.e., $T = t_s + t_d$). Based on the CR’s observation, the CR decides either to start transmitting its data during $t_d$ if the PU is idle, or to be silent if the PU still occupies the band. However, the PU might recommence its activity during CR’s data transmission time $t_d$ leading to an unexpected interference to the PU. Therefore, PU traffic must be considered. It is assumed that periods of PU’s states (i.e., busy ‘1’ and idle ‘0’) are exponentially distributed with mean values of $\alpha_0$ and $\alpha_1$, respectively. The probability density functions (PDF) for busy and idle states are as follows:

$$f_1(t) = \frac{1}{\alpha_1} \exp\left(-\frac{t}{\alpha_1}\right), \quad \text{and} \quad f_0(t) = \frac{1}{\alpha_0} \exp\left(-\frac{t}{\alpha_0}\right),$$

where $f_1(t)$ and $f_0(t)$ are the p.d.f.s of the busy and idle PU state periods, respectively, and $t \geq 0$.

Accordingly, the probabilities of the PU being idle, $P_0$, or busy, $P_1$, can easily be computed. The probability that PU might recommence its activity when CR transmits its data over the band during $t_d$ can be determined as in [68]:

$$P_1(t_d) = \int_{t=0}^{t_d} f_1(t) dt = 1 - e^{-\frac{t_d}{\alpha_0}}. \quad (6.1)$$

In general, the CR starts transmitting its data if CR successfully detects the absence of the PU with probability of $P_0(1 - P_f)$ or if it fails in correctly detecting the presence of the PU with probability of $P_1(1 - P_d)$, where $P_f$ is the probability of false alarm, and $P_d$ is the probability of detection of the system. Otherwise, the CR sleeps if it successfully detects the existence of the PU with probability of $P_1P_d$ or if it falsely detects the absence of the PU with probability of $P_0P_f$.

6.3 Problem Formulation

The goal is to jointly optimize the transmission power and sensing time that maximize the energy efficiency subjected to PU protection constraints. To setup the design problem, we consider throughput which is determined in [68] as

$$R(t_s, \phi_t) = P_0R_0 t_d \left(1 - P_f(t_s)\right) \left(1 - P_t(t_d)\right),$$  \hfill (6.2)

where $\phi_t$ is the transmission power while $R_0$ is the volume of transmitted data that the CR can transmit over the period $t_d$. The data volume is calculated as $R_0 = B \log_2 \left(1 + \frac{\phi_s}{\Gamma}\right)$, where $B$ is the channel bandwidth and $\Gamma$ is the noise power over the channel bandwidth. The total incurred energy consumption, $E(t_s, \phi_t)$, is determined as

$$E(t_s, \phi_t) = t_s \phi_s + \left(P_0(1 - P_f) + P_1(1 - P_d)\right) \phi_t t_d,$$  \hfill (6.3)

where $\phi_s$ is the sensing power. The objective function is the energy efficiency metric which is determined as

$$\zeta(t_s, \phi_t) = \frac{R(t_s, \phi_t)}{E(t_s, \phi_t)}.$$

95
The general design problem aims to maximize the energy efficiency by varying sensing time, and the transmission power. Therefore, the design problem can be formulated as

\[
\max \zeta(t_s, \phi_t) \quad \quad (6.5)
\]

subject to \( \phi_{\text{max}} \geq \phi_t \geq \phi_{\text{min}} \)

\[ T \geq t_s \geq 0 \]

\[ P_I(t_d) \leq \alpha_I \]

\[ P_d \geq \bar{P_d}, \]

where \( \alpha_I \) is the maximum allowable interference level to the PU, and \( \bar{P_d} \) is the probability of detection target, while \( \phi_{\text{min}} \) and \( \phi_{\text{max}} \) are the minimum and maximum transmission powers, respectively.

### 6.4 The Proposed Algorithm

Assuming that the CR adopts energy detector to sense the existence of the PU. Consider the objective function subjected to the given constraints, by assuming that \( t_s = \theta T \), and \( t_d = (1 - \theta)T \), generally \( \theta \in [0,1] \). Note that \( \theta \) represents the ratio \( t_s \) to \( T \) (i.e., duty cycle) This assumption is very helpful from two folds; first, to enhance the convergence rate to determine the optimal operating point since this assumption significantly reduces the time interval which the optimal \( t_s \) belongs to, as it will be shown below. Second, to reduce the complexity of solving the optimization problem by minimizing the number of constraints imposed to the objective function, therefore, it makes the proposed algorithm simpler in terms of computational requirements than those existing.
The probability of false alarm can be determined in terms of $\theta$ as

$$P_f(\theta) = Q\left(\sqrt{2\gamma + 1Q^{-1}(P_d)} + \sqrt{f_s T \theta}\right),$$

where $\gamma$ is the average signal to noise ratio (SNR) of the PU signal over the channel, and $f_s$ is the sampling frequency, while $Q(\cdot)$ is Q-function. It can be easily proven that $P_f(\theta)$ is a convex function of $\theta$, (i.e., $\dot{P}_f(\theta) < 0, \ddot{P}_f(\theta) > 0$) as in [67], where $\dot{P}_f$ and $\ddot{P}_f$ are the first and second partial derivatives of $P_f$ w.r.t. $\theta$, respectively.

The integration interval of $\theta$ (i.e., $\theta \in [\theta_{min}, \theta_{max}]$) can be determined as follow; the lower bound $\theta_{min}$ is determined by letting $P_f(\theta) < 0.5$, therefore, $\theta_{min} = \left(\frac{\sqrt{2\gamma + 1Q^{-1}(P_d)}}{\sqrt{f_s T \theta}}\right)^2$, while the upper bound $\theta_{max}$ is readily determined using the second constraint in (6.5) as $\theta_{max} = 1 + \frac{\alpha_0}{\tau} \ln(1 - \alpha_I)$. Moreover, to satisfy the detection performance constraints, the first constraint always grants that $P_d(\theta)$ of the CR equals to $\bar{P}_d$ at any radio condition. Simply, the term $(1 - \bar{P}_d)$ can be neglected, because the difference is negligible. Therefore, the energy consumption, $\hat{E}$, can be computed in terms of $\theta$, and $\phi_\tau$ as follows:

$$\hat{E}(\theta, \phi_\tau) = \theta T \phi_s + \phi_\tau P_0 \left(1 - P_f(\theta)\right) (1 - \theta) T, \quad (6.6)$$

while the throughput can be determined in terms of $\theta$ and $\phi_\tau$ as follows:

$$R(\theta, \phi_\tau) = P_0 R_0 (1 - \theta) T \left(1 - P_f(\theta)\right) \exp\left(-\frac{(1 - \theta) T}{\alpha_0}\right), \quad (6.7)$$

where $R_0 = B \log_2 \left(1 + \frac{\phi_\tau}{\tau}\right)$, and $B$ is the channel bandwidth.

Therefore, the approximated energy efficiency $\zeta$ is computed as $\zeta(\theta, \phi_\tau) = \frac{R(\theta, \phi_\tau)}{\hat{E}(\theta, \phi_\tau)}$. Consequently, the optimization problem in (6.5) can be rewritten as
\[
\max \hat{\zeta}(\theta, \phi_t) \tag{6.8}
\]
subject to \( \theta_{\text{max}} > \theta \geq \theta_{\text{min}} \)

\[
\phi_{\text{max}} \geq \phi_t \geq \phi_{\text{min}}.
\]

From (6.8), it is seen that employing \( \theta \) reduces the complexity of solving the optimization problem in (6.5) by minimizing the number of constraints in (6.5) from four to only two in (6.8), which will significantly make the proposed algorithm simpler to implement. More details are provided below.

**Proposition 1**: For given \( \phi_t \) and \( \gamma \), there is an optimal \( \theta \) (i.e., \( \bar{\theta} \)) that maximizes \( \hat{\zeta}(\theta, \phi_t) \).

**Proof**: obviously, \( \hat{\zeta} \) is a continuous function of \( \theta \) in all given interval; therefore, the first partial derivate of \( \hat{\zeta} \) w.r.t. \( \theta \) is

\[
\hat{\zeta} = \frac{\dot{R} - R\dot{E}}{E^2},
\]

where \( \dot{R} \) and \( \dot{E} \) are the first partial derivate of \( R \) and \( E \), respectively, w.r.t. \( \theta \). The \( \dot{E} \) is determined as:

\[
\dot{E} = T\phi_s - \phi_t P_0 T \left( (1 - P_f(\theta)) + (1 - \theta)\dot{P}_f(\theta) \right).
\]

Note that \( \dot{P}_f(\theta) < 0, \forall \theta \). Therefore, \( \dot{E} > 0 \), which means that \( \dot{E} \) is an increasing function of \( \theta \).

Similarly, \( \dot{R} = R_1 + R_2 + R_3 \), where

\[
R_1 = \frac{-R}{(1 - \theta)}, \quad R_3 = \frac{TR}{a_0}
\]

and

\[
R_2 = -P_0R_0(1 - \theta)T\dot{P}_f(\theta)\exp\left(-\frac{(1 - \theta)T}{a_0}\right).
\]

It is obvious that \( R_2, R_3 > 0 \), while \( R_1 < 0 \).

However, \( |R_3| > |R_1| \), therefore, \( \dot{R} > 0 \), and consequently, \( \dot{\zeta} > 0 \). Similarly, if we take the second partial derivate of \( \zeta \) w.r.t. \( \theta \), it is readily proven that \( \ddot{R} < 0, \ddot{E} > 0 \), and \( \ddot{\zeta} < 0 \). Therefore, \( \hat{\zeta}(\theta, n) \) is a concave function of \( \theta \), and \( \exists \bar{\theta} \) such that \( \hat{\zeta}(\theta, n) \big|_{\theta=\bar{\theta}} = 0 \). Bisection method can be employed to determine \( \bar{\theta} \).
Proposition 2: There is an optimal transmission power $\phi_t$ (i.e., $\phi_o$) that maximizes the energy efficiency.

Proof: Let $\zeta(\theta, \phi_t) = \frac{c \log_2 (1 + \frac{\phi_t}{\theta})}{a_1 + a_2 \phi_t} = \frac{c (a_1 + a_2 \phi_t)}{(\frac{1}{\phi_t} + 1) \ln 2} - a_2 \log_2 (1 + \frac{\phi_t}{\theta})$, where $a_1 = \theta T \phi_s$, $a_2 = P_0 (1 - P_f(\theta)) (1 - \theta) T$, and $C = a_2 B \exp \left( - \frac{(1-\theta)T}{\alpha_0} \right)$. The first partial derivative of $\zeta(\theta, \phi_t)$ w.r.t $\phi_t$ is calculated as

$$\varphi(\theta, \phi_t) = \frac{c \zeta(\theta, \phi_t)}{\zeta(\theta, \phi_t)^2} = \frac{c (a_1 + a_2 \phi_t)}{(\frac{1}{\phi_t} + 1) \ln 2} - a_2 \log_2 (1 + \frac{\phi_t}{\theta}) \left( \frac{1}{a_1 + a_2 \phi_t} \right)^2. \quad (6.9)$$

It is difficult to derive mathematical formula for the optimal $\phi_t$. However, $\varphi(\theta, \phi_t = 0) = \frac{c}{a_1 \ln 2} > 0$, and $\lim_{\phi_t \to \infty} \varphi(\theta, \phi_t) = -\infty$. These two facts confirm that there is an optimal $\phi_t$ that maximizes $\zeta(\theta, \phi_t)$, such that $\phi_t \in (0, \infty)$. Therefore, three cases can be considered.

Case 1: If $\varphi(\theta, \phi_{min}) = 0$ or $\varphi(\theta, \phi_{max}) = 0$, therefore, $\phi_o = \phi_{min}$ or $\phi_o = \phi_{max}$, respectively.

Case 2: If $\varphi(\theta, \phi_{min}) > 0$ and $\varphi(\theta, \phi_{max}) < 0$, therefore, $\phi_o \in [\phi_{min}, \phi_{max}]$.

Case 3: If $\varphi(\theta, \phi_{min}) > 0$ and $\varphi(\theta, \phi_{max}) > 0$, therefore, $\phi_o \not\in [\phi_{min}, \phi_{max}]$, which is clearly impractical.

We propose an iterative algorithm to determine the joint optimal $\phi_t$ and $\theta$ (i.e., $\phi_o$ and $\bar{\theta}$) that maximizes $\zeta(\theta, \phi_t)$. Bisection method is used to numerically determine $\bar{\theta}$ that maximizes $\zeta(\theta, \phi_t)$ at any given $\phi_t$, and it is denoted as bisect($f(t), t_1, t_2$) in the proposed algorithm, where $f(t)$ is the function that we like to determine its root, while $t_1$ and $t_2$ are the lower and upper bounds of the time interval to which the root belongs.

**Proposed iterative Algorithm**

<table>
<thead>
<tr>
<th>Initialization</th>
<th>( T, \gamma, \bar{P}<em>d, \alpha_1, \alpha_0, \alpha_1, f_s, \varepsilon, \phi_s, \phi</em>{max}, \phi_{min} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compute</td>
<td>( \theta_{min}, \theta_{max}, P_o, P_1 )</td>
</tr>
<tr>
<td>Let</td>
<td>( \phi_1 = \phi_{max} )</td>
</tr>
<tr>
<td>Compute</td>
<td>( \bar{\theta}<em>1 = \text{bisection}\left(\hat{\zeta}(\theta, \phi_1), \theta</em>{min}, \theta_{max}\right))</td>
</tr>
<tr>
<td>Let</td>
<td>( \theta_{max} = \bar{\theta}_1 )</td>
</tr>
<tr>
<td>Let</td>
<td>( \phi_2 = \phi_{min} )</td>
</tr>
<tr>
<td>Compute</td>
<td>( \bar{\theta}<em>2 = \text{bisection}\left(\hat{\zeta}(\theta, \phi_2), \theta</em>{min}, \theta_{max}\right))</td>
</tr>
<tr>
<td>Let</td>
<td>( \theta_{min} = \bar{\theta}_2 )</td>
</tr>
<tr>
<td>Let</td>
<td>( k = 2 )</td>
</tr>
<tr>
<td>1. while (</td>
<td>\hat{\zeta}(\bar{\theta}_k, \phi_k)</td>
</tr>
<tr>
<td>2. ( k = k + 1 )</td>
<td></td>
</tr>
<tr>
<td>3. ( \phi_k = \frac{\phi_{max} + \phi_{min}}{2} )</td>
<td></td>
</tr>
<tr>
<td>4. ( \bar{\theta}<em>k = \text{bisection}\left(\hat{\zeta}(\theta, \phi_k), \theta</em>{min}, \theta_{max}\right))</td>
<td></td>
</tr>
<tr>
<td>5. If ( \varphi(\bar{\theta}_k, \phi_k) &lt; 0 )</td>
<td></td>
</tr>
<tr>
<td>6. ( \phi_{max} = \phi_k )</td>
<td></td>
</tr>
<tr>
<td>7. ( \theta_{max} = \bar{\theta}_k )</td>
<td></td>
</tr>
<tr>
<td>8. Else |</td>
<td></td>
</tr>
<tr>
<td>9. ( \phi_{min} = \phi_k )</td>
<td></td>
</tr>
<tr>
<td>10. ( \theta_{min} = \bar{\theta}_k )</td>
<td></td>
</tr>
<tr>
<td>11. end if |</td>
<td></td>
</tr>
<tr>
<td>12. end while |</td>
<td></td>
</tr>
<tr>
<td>13. Return ( \phi_o = \phi_k, \theta_o = \bar{\theta}<em>k ) and ( \zeta</em>{max} = \hat{\zeta}(\theta_o, \theta_o) )</td>
<td></td>
</tr>
</tbody>
</table>

It is clear from the proposed iterative algorithm that the algorithm significantly reduces the interval which \( \bar{\theta} \) belongs to; this matter provides an advantage of speeding up the convergence rate. The algorithm combines both the principle half interval search and bisection method in order to jointly determine the \( \phi_o \) and \( \theta_o \). This combination effectively reduces the computational complexity of the algorithm and increases the processing speed of finding the optimal operating point (i.e., \( (\phi_o, \theta_o) \)). The algorithm initially reduces the upper and lower bounds of the time interval by determining \( \bar{\theta}_1 \) and \( \bar{\theta}_2 \), for \( \phi_{max} \), and \( \phi_{min} \), respectively.
Moreover, the interval to which $\bar{\theta}_k$ belongs, is reduced at every iteration of $k$ as illustrated in Table 6.1. The proposed algorithm is designed based on the fact that optimal sensing time increases as the transmission power $\phi_t$ increases as in [67]-[68]. The relationship between $\bar{\theta}$ and $\phi_t$ is demonstrated in Fig. 6.1. It is seen from the figure that increasing $\phi_t$ results in an increase of the value of corresponding $\bar{\theta}$ which corroborates the veracity of the above fact and supports the validity of the proposed algorithm.

The results of the first six runs of the proposed iterative algorithm are displayed in Table 6.1. The table shows effectiveness of the proposed algorithm to significantly reduce the intervals which $\phi_o$, and $\bar{\theta}$ belong to, in each run of $k$. This provides the proposed algorithm advantages of drastically reducing the computational complexity and having a faster convergence rate compared with the existing works [68] and [107]. Half interval search method is employed to
Joint Optimal Transmission Power and Sensing Time for Energy-Efficient SS in CR System

determine $\phi_o$ which has a computational complexity of $O(Log(M))$ if we divide the range of $\phi_t$ into $M$ points. A bisection method is employed to determine $\bar{\theta}_k$ for value of $\phi_k$. The method has computational complexity of $O\left(\log\left(\frac{\theta_{max}-\theta_{min}}{\varepsilon_\theta}\right)\right)$ in the worst case scenario, where $\varepsilon_\theta$ is the allowable tolerance. Therefore, the worst case complexity for the proposed algorithm is $O\left(\log_2(M)\log_2\left(\frac{\theta_{max}-\theta_{min}}{\varepsilon_\theta}\right)\right)$.

Table 6.1: The corresponding $\bar{\theta}_k$ for each $\phi_k$ using the proposed iterative algorithm.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\phi_k (W)$</th>
<th>$\bar{\theta}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\phi_{max} = 4$</td>
<td>$\bar{\theta}<em>1 \in [\theta</em>{min}, \theta_{max}]$</td>
</tr>
<tr>
<td>2</td>
<td>$\phi_{min} = 0.1$</td>
<td>$\bar{\theta}<em>2 \in [\theta</em>{min}, \bar{\theta}_1]$</td>
</tr>
<tr>
<td>3</td>
<td>$\phi_3 = 0.5(\phi_{max} + \phi_{min}) = 2.05$</td>
<td>$\bar{\theta}_3 \in [\bar{\theta}_2, \bar{\theta}_1]$</td>
</tr>
<tr>
<td>4</td>
<td>$\phi_4 = 0.5(\phi_3 + \phi_{min}) = 1.075$</td>
<td>$\bar{\theta}_4 \in [\bar{\theta}_3, \bar{\theta}_2]$</td>
</tr>
<tr>
<td>5</td>
<td>$\phi_5 = 0.5(\phi_3 + \phi_4) = 1.5625$</td>
<td>$\bar{\theta}_5 \in [\bar{\theta}_4, \bar{\theta}_3]$</td>
</tr>
<tr>
<td>6</td>
<td>$\phi_6 = 0.5(\phi_5 + \phi_4) = 1.3188$</td>
<td>$\bar{\theta}_6 \in [\bar{\theta}_5, \bar{\theta}_4]$</td>
</tr>
</tbody>
</table>

In order to compare the proposed algorithm with the existing works in terms of computational complexity, assume that all algorithms will have the same simulation parameters (i.e., step size). Let $\varepsilon_\theta$ be the step size; therefore, the number of possible steps is $N = \frac{\theta_{max}-\theta_{min}}{\varepsilon_\theta}$ to determine $\bar{\theta}_i$ and $1 \leq i \leq N$. Similarly, let $\varepsilon_p$ be the step size, therefore, the number of possible steps is $M = \frac{\phi_{max}-\phi_{min}}{\varepsilon_\phi}$ to determine $\phi_o$. Therefore, the complexity of exhaustive search method is $O(MN)$ while the complexity of linear search method is $O\left(N\log_2\left(\frac{\theta_{max}-\theta_{min}}{\varepsilon_\theta}\right)\right) + O\left(M\log_2\left(\frac{\phi_{max}-\phi_{min}}{\varepsilon_\phi}\right)\right)$, as shown in [107]. For large values of $N$ and $M$, it is obvious that the proposed algorithm has less complexity than both linear search and exhaustive methods.
Moreover, in the proposed algorithm the interval bounds are reduced in each iteration of the algorithm (i.e., Table 6.1) which remarkably reduces the number of runs for the bisection method which consequently reduces the computational complexity of the proposed algorithm.

### 6.5 Simulation Results and Discussion

The set of simulation parameters are selected as $\bar{P}_d = 0.9$, $T = 150$ ms, $\alpha_t = 0.1$, $\alpha_0 = 0.65$ s, $\alpha_1 = 0.352$ s, $\gamma = -18$ dB, $f_s = 6$ MHz, and $\phi_s = 0.2$ W, while $\phi_{\text{min}} = 0.1$ W and $\phi_{\text{max}} = 4$ W. The noise power $\Gamma$ is computed according to [68], and probability of false alarm constraint $\beta = 0.1$.

The 3-dimensional plot of $\zeta(\theta, \phi_t)$ and its contour are illustrated in Figs. 6.2 a and b, respectively. It is shown that there is a unique global operating point (i.e., joint optimal $\phi_o$ and $\theta_o$) that maximizes the energy efficiency. Quantitatively, $\phi_o = 1.283$ W, $\theta_o = 0.1467$, and $\zeta_{\text{max}} = 2.96$ Mbit/ Hz/ J.

A comparison in the maximum magnitude and the corresponding optimal $\theta$ between energy efficiency at three different values of $\phi_t$ (i.e., $\phi_1 = 0.5\phi_o$, $\phi_2 = \phi_o$, and $\phi_3 = 2\phi_o$) is demonstrated in Fig. 6.3. It can be seen from the figure that the highest peak of energy efficiency, $\zeta_{\text{max}}$, can be attained at $\phi_2$ which is the optimal operating point (i.e., $\bar{\theta} = \theta_2$, and $\phi_t = \phi_2 = \phi_o$). Moreover, the corresponding optimal $\theta$ (i.e. $\theta_1, \theta_2, \theta_3$) increases as $\phi_t$ increases (i.e., $\theta_3 > \theta_2 > \theta_1$). These simulation results also support the validity of above mentioned fact.
Fig. 6.2 (a) Energy efficiency $\hat{\zeta}$ vs. $\phi_r$ and $\theta$ (b) Contour of energy efficiency $\hat{\zeta}$
Fig. 6.3. Energy efficiency $\zeta$ vs. $\phi_\tau$.

Fig. 6.4. Probability of false alarm vs. $\phi_\tau$. 
The impact of varying $\phi_1$ on the probability of false alarm at the corresponding optimal $\theta$ is displayed in Fig. 6.4. From Fig. 6.3, it is clearly shown that the probability of false alarm $P_{f2}$ at the optimal operating point $\left(\theta_2, \phi_2\right)$ cannot meet the probability of false alarm target which violates the detection performance constraints. The target cannot be met in case of $P_{f1}$ which represents the probability of false alarm at the point $\left(\theta_1, \phi_1\right)$; however, the probability of false alarm target is satisfied at the point $\left(\theta_3, \phi_3\right)$ (i.e., $P_{f3}$). In other words, increasing $\phi_1$ is an ineffective approach to achieve the probability of false alarm target; because increasing $\phi_1$ leads to more energy consumption and consequently minimizing the attainable energy efficiency as shown in Fig. 6.3.

To achieve the target, there are two possible approaches. First, by imposing the probability of false alarm target as an additional constraint to the optimization problem; however, this approach leads to the loss of the optimal operating point; because imposing this constraint will result to have only one value for the sensing time (i.e., $\theta = \frac{\left(q^{-1}(\beta) - \sqrt{2\gamma + 1}q^{-1}(\bar{P}_d)\right)^2}{\gamma^2Tfs} \approx \theta_3$); and hence, the maximum energy efficiency cannot be achieved as illustrated in Fig. 6 (i.e., $\zeta_2(\theta_3, \phi_2)$). Second, by reducing the probability of detection target; the requirement can be satisfied and the corresponding optimal $\theta$ (i.e., $\theta_4$ in Figs. 6.5 and 6) will be considerably reduced; because lower value of $\bar{P}_d$ requires less $\theta$ than that required by larger values of $\bar{P}_d$.

For a comprehensive illustration, the impact of varying $\bar{P}_d$ on the energy efficiency, corresponding consumed energy and probability of false alarm are demonstrated in Figs. 6.5, 6, and 7, respectively.
It can be noted from Fig. 6.5 that reducing the $\overline{P}_d$ leads to decrease the energy efficiency; this is because reducing the value of $\overline{P}_d$ incurs more energy consumption than that incurred in case of $\overline{P}_d = 0.9$, as displayed in Fig. 6.6, since the data transmission time (i.e., $(1 - \theta_4)T$) will considerably increase. Moreover, it is noticed that reducing $\overline{P}_d$ leads to minimize the optimal $\theta$ (i.e., $\theta_4$) as illustrated in Fig. 6.5. However, the reduction in the value of $\overline{P}_d$ provides an advantage of lowering the probability of false alarm such that it can satisfy the target requirement as shown in Fig. 6.7 (i.e., $P_{f4}$). However, lowering the PU protection is the drawback of this approach (i.e., imperfect spectrum sensing).

Fig. 6.6. Consumed energy vs. $\theta$, for two different $\bar{P}_d$.

Fig. 6.7. Probability of false alarm vs. $\theta$, for two different $\bar{P}_d$. 

$E_f(\theta_1, \phi_1) \neq E_f(\theta_2, \phi_2)$
For further discussion, a comparison between the proposed algorithm and four existing algorithms is shown in Table 6.2. It is clear from the table that the proposed algorithm outperforms algorithms [67] and [68] in terms of its capability of determining a joint optimal transmission power, $\phi_t$, and sensing time, $t_s$, while outperforms the algorithms in [107] in terms of low complexity and ability to reach the optimal solution.

![Comparison in achievable energy efficiency of the proposed algorithm and algorithm [107]](image)

Fig. 6.8. Comparison in achievable energy efficiency of the proposed algorithm and algorithm [107], at $\gamma = -18$ dB and $P_d = 0.9$.

Energy efficiencies of the proposed algorithm, $\zeta_1$, and algorithm in [107], $\zeta_3$, are illustrated in Fig. 6.8. Clearly, the proposed algorithm outperforms the algorithm in [107] in terms of energy efficiency metric; the proposed algorithm attains higher maximum energy efficiency $\zeta_1(\theta_0, \phi_0)$ by about 6% than that attained by the algorithm [107] $\zeta_3(\theta_5, \phi_0)$. 
Moreover, the proposed algorithm reaches its peak energy efficiency faster than the algorithm [107] does (i.e., $\theta_5 > \theta_0$, and $\theta = \frac{t_s}{T}$); however, the corresponding probability of false alarm attained by the algorithm [107] at its maximum energy efficiency is better than that attained by the proposed algorithm at its maximum energy efficiency (i.e., $P_f(\theta_2) > P_f(\theta_5)$). Moreover, the quantitative comparison in Table 6.3 between the two algorithms shows that the proposed algorithm requires less computational requirements than that required by the linear search algorithm in [107] for all values of $M, N, \epsilon_\phi$, and $\epsilon_\theta$.

Table 6.2 A comparison between the proposed algorithm and some existing algorithms that maximize the energy efficiency for a single CR.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Optimized parameter</th>
<th>QoS metric</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm in [67]</td>
<td>$t_s$</td>
<td>Average power capacity.</td>
<td>Exhaustive search is used to determine optimal $t_s$.</td>
</tr>
<tr>
<td>Algorithm in [68]</td>
<td>$t_s$</td>
<td>Probability of PU reoccupation.</td>
<td>Computational complexity is better than exhaustive search, but did not consider optimizing $\phi_r$.</td>
</tr>
<tr>
<td>Linear search algorithm in [107]</td>
<td>$t_s$ and $\phi_r$</td>
<td>Outage probability of transmission.</td>
<td>Computational complexity is better than exhaustive search but still high.</td>
</tr>
<tr>
<td>Iterative-based algorithm in [107]</td>
<td>$t_s$ and $\phi_r$</td>
<td>Outage probability of transmission.</td>
<td>Optimal solution cannot be guaranteed to obtain [107].</td>
</tr>
<tr>
<td>Proposed algorithm</td>
<td>$t_s$ and $\phi_r$</td>
<td>Probability of PU reoccupation.</td>
<td>Low computational requirement and optimal solution is guaranteed to obtain.</td>
</tr>
</tbody>
</table>
Table 6.3 Quantitative comparison between the proposed algorithm and linear search algorithm [107]

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$\widehat{z}_{\text{max}}$ (Mbit/Hz/J)</th>
<th>$t_s$ (ms)</th>
<th>$P_f$</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear search algorithm [107]</td>
<td>2.8283</td>
<td>26.2</td>
<td>0.099</td>
<td>$O \left( N \log_2 \left( \frac{\theta_{\text{max}} - \theta_{\text{min}}}{\varepsilon_\theta} \right) \right)$ + $O \left( M \log_2 \left( \frac{\phi_{\text{max}} - \phi_{\text{min}}}{\varepsilon_\phi} \right) \right)$</td>
</tr>
<tr>
<td>Proposed algorithm</td>
<td>2.996</td>
<td>22</td>
<td>1.75</td>
<td>$O \left( \log_2(M) \log_2 \left( \frac{\theta_{\text{max}} - \theta_{\text{min}}}{\varepsilon_\theta} \right) \right)$</td>
</tr>
</tbody>
</table>

6.6 Conclusions

Joint optimal transmission power and sensing time that maximizes energy efficiency for a single CR system has been considered. The design problem has also been formulated. An iterative suboptimal algorithm was proposed to determine a joint optimal transmission power and sensing time that maximizes the energy efficiency of the CR system. The proposed algorithm has shown better results over existing works in terms of computational complexity and convergence rate. The impact of variation of the transmission power on the optimal sensing time was also considered. Furthermore, simulation results have been conducted to present the impact of varying the power on the corresponding probability of false alarm. The results have shown that the spectrum efficiency condition cannot be satisfied even at the optimal operating point; two solutions have been suggested and discussed to solve this issue. Moreover, the simulation results have shown also that the proposed algorithm outperforms the existing work in terms of maximum energy efficiency.
Chapter Seven: 
*Dissertation Conclusions and Future Works*

7.1 Conclusions

Trade-off between cooperation gain and its cost in CBSS system was tackled in this dissertation using two different strategies. The first strategy considers all available participating CRs in the system as described in chapter three. The second strategy reduces the number of participating CRs in the system by either varying sensing window size as illustrated in chapter four or by considering PU protection constraints as shown in chapter five. Moreover, designing an energy efficient spectrum sensing CR system is provided in chapter six. The achieved research results are summarized below.

A multi-level hierarchical cluster-based algorithm has been proposed to compromise between cooperation gain and cost. An iterative algorithm has also been developed to determine suboptimal number of hierarchical structure levels. Moreover, optimal sensing threshold, optimal fusion rule parameters and energy efficiency were derived for sensing performance. Simulation results showed that the proposed algorithm provides higher PU protection than MCMG and conventional algorithms. In addition, the proposed algorithm’s simulation results showed that it can reduce the reporting energy by 70% of that required by the conventional algorithm [78]. Furthermore, combining two different fusion rules and exploiting hierarchical structure model of the cluster in the proposed algorithm has shown that it remarkably minimizes the reporting overhead in the cluster. Besides, the simulation results also have shown that the proposed
algorithm increases sensing agility and reduces the number of reporting CRs. Finally, it was shown that the proposed algorithm can save energy consumption by average percentage about 30% and 12% compared to the conventional [78] and the MCMG [80] algorithms, respectively.

Energy efficiency of CBSS system was tackled in this work using variable sensing window size in order to minimize the number of participating CRs in the system. Three novel energy efficient strategies were proposed for the CBSS system using the three different hard decision fusion rules, namely, OR, AND, and $k$ out of $N$-Rules. The simulation results show that increasing the window size effectively minimizes the required number of CRs and drastically improves the energy efficiency while meeting the target detection performance. Furthermore, the simulation results also corroborates the optimality of the $k$ out of $N$-Rule over the other two hard decision fusion rules as proven in the previously existing works.

Suboptimal design of energy efficient CBSS in CRNs was considered. An iterative suboptimal algorithm that maximizes the energy efficiency of the CBSS system by jointly optimizing sensing duration, data transmission duration and number of contributing CRs in the system while satisfying both PU protection spectrum efficiency constraints was proposed in this dissertation. The problem of design has been formulated as only two variable function and analysis have also been provided in details. The simulation results of the proposed algorithm have shown the effectiveness of the proposed algorithm in determining the optimal sensing time, data transmission and number of contributing CRs in the system that maximize the energy efficiency while satisfying all performance requirements for CBSS system. Moreover, for single CR, the impacts of changing the transmitting power on the probability of false alarm and consumed energy have also been investigated.
The impact of variation of the transmission power on both optimal sensing and corresponding probability of false alarm of single CR has been investigated. Investigations have shown that increasing transmission power is not always effective to meet probability of false alarm target. To meet the target, designing suboptimal energy efficient CBSS that satisfies the sensing accuracy metrics has been considered in this dissertation. The problem of design has been formulated and analysis has also been provided. An iterative algorithm with low computational complexity has been proposed to jointly determine the optimal design parameters of CBSS system that maximize the energy efficiency while satisfying all detection accuracy metrics. Simulation results have shown that proposed algorithm significantly outperforms the previous existing works.

Joint optimal transmission power and sensing time that maximizes energy efficiency for a single CR system has been considered. The design problem has also been formulated. An iterative suboptimal algorithm was proposed to determine a joint optimal transmission power and sensing time that maximizes the energy efficiency of the CR system. The proposed algorithm has shown better results over existing works in terms of computational complexity and convergence rate. The impact of variation of the transmission power on the optimal sensing time was also considered. Furthermore, simulation results have been conducted to present the impact of varying the power on the corresponding probability of false alarm. The results have shown that the spectrum efficiency condition cannot be satisfied even at the optimal operating point; two solutions have been suggested and discussed to solve this issue. Moreover, the simulation results have shown also that the proposed algorithm outperforms the existing work in terms of maximum energy efficiency.
7.2 Future works

The research work in the field of CSS is versatile area; numerous research works can be conducted. Here, I suggest considering the following points in future

The impact of correlated multipath fading and shadowing on the detection performance of all proposed algorithm can be conducted. Moreover, designing joint energy efficient spectrum sensing and automatic modulation classification (AMC) techniques in order to jointly improve both spectrum sensing and sharing processes for CR system can also be considered.

The impact of mobility of CRs on the detection performance in above algorithms is a valid point for future research. Furthermore, the impact of joint channel estimation and equalization on the detection performance of a moving CR can be investigated. Besides, different radio environmental conditions such as Rayleigh fading, and shadowing can be considered to have a comprehensive study.

Designing energy detector with adaptive sensing window size in order to improve spectrum efficiency of a single CR is very interesting research point. Markov process can be employed to predict the PU status in next sensing step. The principle of double threshold can also be employed to increase energy efficiency of the system.

Practical implementations of all proposed algorithms using GNU radio software platform to investigate the detection performance of the proposed algorithms in real time world can be considered.
References


References


Appendices

Appendix (A)

Derivations of optimal \( k_1 \) and \( k_2 \)

According to [31] and [35]

\[
\frac{\partial Q}{\partial k} = \left(\frac{M}{k}\right) P^k (1 - P)^{M-k} \quad (A.1)
\]

For given \( M \) and \( R \), the optimal \( k_2 \) is determined at the level of cluster as

\[
\frac{\partial \varphi_e}{\partial k_2} |_{k_2=\hat{k}_2} = 0 = P(H_0) \frac{\partial Q_f}{\partial k_2} - P(H_1) \frac{\partial Q_d}{\partial k_2} \quad (A.2)
\]

\[
P(H_0) \left(\frac{M}{\hat{k}_2}\right) \beta_f^{\hat{k}_2} (1 - \beta_f)^{M-\hat{k}_2} = P(H_1) \left(\frac{M}{\hat{k}_2}\right) \beta_d^{\hat{k}_2} (1 - \beta_d)^{M-\hat{k}_2}
\]

\[
\hat{k}_2 = \frac{\ln(P(H_0)P(H_1)) + M \ln\left(\frac{1-\beta_f}{1-\beta_d}\right)}{\ln\left(\frac{\beta_d(1-\beta_f)}{\beta_f(1-\beta_d)}\right)} \quad (A.3)
\]

For given \( M, R \) and \( k_2 \), the optimal \( \hat{k}_1 \) is determined at the level of group

\[
\frac{\partial \varphi_e}{\partial k_1} |_{k_1=\hat{k}_1} = 0 = P(H_0) \frac{\partial Q_f}{\partial k_1} - P(H_1) \frac{\partial Q_d}{\partial k_1} \quad (A.4)
\]

\[
P(H_0) \beta'_f \left(\frac{M}{\hat{k}_2}\right) \beta_f^{\hat{k}_2} (1 - \beta_f)^{M-\hat{k}_2} = P(H_1) \beta'_d \left(\frac{M}{\hat{k}_2}\right) \beta_d^{\hat{k}_2} (1 - \beta_d)^{M-\hat{k}_2}
\]

Where \( \beta'_d = q'_d(1 - 2 \bar{P}_e) \),

\[
\beta'_f = q'_f(1 - 2 \bar{P}_e).
\]

\[
q'_d = \frac{\partial q_d}{\partial k_1} = \left(\frac{R}{k_1}\right) p_{d \mid k_1} (1 - P_{d \mid k_1})^{R-k_1}, \quad \text{and} \quad q'_f = \frac{\partial q_f}{\partial k_1} = \left(\frac{R}{k_1}\right) p_{f \mid k_1} (1 - P_f)^{R-k_1}
\]

Therefore
\[ P(H_0) P_f^{k_1} (1 - P_f) R^{-k_1} \beta_f^{k_2} (1 - \beta_f)^{M-k_2} = P(H_1) P_{dSG}^{k_1} (1 - P_{dSG}) R^{-k_1} \beta_d^{k_2} (1 - \beta_d)^{M-k_2} \]

Let \( C_0 = P(H_0) \beta_f^{k_2} (1 - \beta_f)^{M-k_2} \), and \( C_1 = P(H_1) \beta_d^{k_2} (1 - \beta_d)^{M-k_2} \)

Therefore

\[ C_0 P_f^{k_1} (1 - P_f) R^{-k_1} = C_1 P_{dSG}^{k_1} (1 - P_{dSG}) R^{-k_1} \]

\[ \hat{k}_1 = \frac{\ln \left( \frac{C_0}{C_1} \right) + R \ln \left( \frac{1 - P_f}{1 - P_{dSG}} \right)}{\ln \left( \frac{P_{dSG} (1 - P_f)}{P_f (1 - P_{dSG})} \right)} \] (A.5)
Appendix (B)

Calculation of reporting overhead in cluster with N CRs

Assume that $B$ is the required bandwidth to transmit one observation; and $\tau_r$ is reporting time. From [91] and [96], the overhead is computed in terms of time-bandwidth product (i.e., $\tau_r B$) as follows

\[
\text{Reporting overhead} = K \times B \tau_r \quad \text{(B.1)}
\]

where $K$ is the number of reporting CRs.

For conventional algorithm [78]:

One CR among others in the cluster is selected as CH, all CRs senses the PU, and then report their observation to the CH, therefore, the number of reporting CRs is $N - 1$.

For MCMG algorithm [80]:

A cluster with $N$ CRs is divided into $M_1$ groups. Each group has $H_1$ CRs, which are assumed to be very close to each other, and one among them is GH, which polls the received signals of its group members and makes its decision based on the largest SNR among them, $\gamma_{\text{max},j}$ [79]-[80]. The GHs report their observations to the CH, which is one of the GHs, therefore, the number of reporting CRs is $M_1 - 1$.

For the proposed algorithm:

A cluster with $N$ CRs is divided into $M$ groups. Each group is also divided into $R$ subgroups, each with $H$ CRs.

In subgroup level, $H$ of CRs are assumed to be very close to each other; therefore they are subjected to almost the same radio environment condition (i.e., SNR, and fading), therefore,
polling [79]-[80] is used to extract subgroup’s observation, which means that only one CR (i.e. SGH) will report to the GH in its group.

In group level, each group has $R$ of SGHs; one SGH among other SGHs in the group is selected to be a GH. Therefore, the number of reporting SGH is $R - 1$ in a group. As a first fusion stage, a GH combines its local observation and received observations from other SGHs in a group to have group’s decision and then forward the decision to the CH in the cluster. From bandwidth perspective, the required bandwidth for the channel between SGHs and a GH is only $(R - 1)B$.

In cluster level, each cluster has $M$ GHs; one of the GHs is selected to be a CH, therefore, the number of reporting GHs $= M - 1$. As a second fusion stage, a CH gathers aggregated decisions from other GHs in the cluster and then combines them with its decision in order to come up with cluster final decision. Furthermore, the required bandwidth for the channel between GHs and a CH is only $(M - 1)B$.

Finally, the total reporting CRs in the proposed algorithm is $(M - 1) + M(R - 1) = MR - 1$. 
Appendix (C)

Copyright permission

Dear Dr. Abdel-Raheem and Dr. Ahmadi

Hope you are doing well. I am emailing in regards to having your permission to refer our publications in my Ph.D. dissertation. According to graduate studies regulations, though the thesis includes the statement of declaration of co-authorship and previous publication; your permission should be appended in the thesis.

Regards,
Faroq

Hi Faroq
No problem with me.
M.Ahmadi

Dear Faroq:

I have neither an issue nor an objection for considering your publications into your Dissertation.

Regards

Esam Abdel-Raheem PhD, PEng
Professor
Vita Auctoris

Faroq Awin, born in 1973, Libya. He got his bachelor degree (B.Sc.) in electrical engineering in 1996 from University of Tripoli- Libyan. In 2004, he got his master degree (M.Sc.) in field of precise instrument and machinery from Beihang University, Beijing, China. He got his doctor of philosophy (Ph.D.) degree in field of spectrum sensing for cognitive radio networks from University of Windsor, Canada, in 2016. He worked as researcher in Electronic Research Centre, Tripoli, Libya, from 1997 to 2008. Moreover, he worked as a faculty member in electrical and electronic engineering department, University of Tripoli, Libya, from 2008 to 2010, and as a research assistant from 2011 to 2016 in University of Windsor. Furthermore, he served as a reviewer in several referred international journals and international conferences. Additionally, he published several articles and conference papers in well-known international journals and conferences. His research interest includes cooperative spectrum sensing, cognitive radio, wireless communications and signal processing in communications.