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Continuous folded plate structures under uniform load.

Paul P. Fazio

University of Windsor

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CONTINUOUS FOLDED PLATE STRUCTURES
UNDER UNIFORM LOAD

A THESIS
Submitted to the Faculty of Graduate Studies Through the
Department of Civil Engineering in Partial Fulfillment
of the Requirements for the Degree of
Master of Applied Science
at the University of Windsor

by

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Windsor, Ontario, Canada
1964
ABSTRACT

In this thesis continuous folded plate structures with two spans are investigated both analytically and experimentally. The theory used to obtain the analytical solution is developed from the principles outlined by Wilhelm Flügge in his book (8). General equations are developed to find stresses and deflections of the continuous structure when it is uniformly loaded at the edges. A concrete continuous folded plate structure is analyzed experimentally and a general pattern of deflection and stress distribution is established.
ACKNOWLEDGMENTS

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# Table of Contents

**Abstract** .......................... i

**Acknowledgements** ................................. iii

**Notation** ................................. iv

Chapter

I  
**Introduction** ................................. vii

II  
**Historical Discussion** ................................. 1

III  
**Method of Analysis** ................................. 9

IV  
**Analysis of Plate bm Under Uniform Load $S_{bm}$** ................................. 11

V  
**Analysis of Plate am Under Uniform Load $S_{am}$** ................................. 15

VI  
**Analysis of Plate bm Subjected to Unit Moment at B** ................................. 31

VII  
**Analysis of Plate am Subjected to Unit Moment Applied at End B** ................................. 41

VIII  
**Plate m of Span ABC Is Made Continuous** ................................. 55

IX  
**Joint Displacements** ................................. 69

X  
**Analytical Solution of Experimental Prism** ................................. 89

XI  
**Experimental Program** ................................. 92

XII  
**Conclusion** ................................. 114

**Literature Cited** ................................. 119

**Vita Auctoris** ................................. 143

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**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-1</td>
<td>Loads Resolved Into Components Acting in Plane of Plates</td>
<td>31</td>
</tr>
<tr>
<td>10-2</td>
<td>Longitudinal Shear Constants for Section BC</td>
<td>35</td>
</tr>
<tr>
<td>10-3</td>
<td>Longitudinal Shear Constants for Section AB</td>
<td>37</td>
</tr>
<tr>
<td>10-4</td>
<td>Rotation of Plates in Span BC Due to Uniform Load</td>
<td>37</td>
</tr>
<tr>
<td>10-5</td>
<td>Rotation of Plates in Span AB Due to Uniform Load</td>
<td>38</td>
</tr>
<tr>
<td>10-6</td>
<td>Deflections of Plates at $x = \frac{L}{2}$ Due to Uniform Load</td>
<td>38</td>
</tr>
<tr>
<td>10-7</td>
<td>Deflections of Plates at $x = \frac{L}{2}$ Due to Uniform Load</td>
<td>38</td>
</tr>
<tr>
<td>10-8</td>
<td>Shear Forces in lb/ft for Span BC Due to Unit Moments Applied at end B of Different Plates</td>
<td>29</td>
</tr>
<tr>
<td>10-9</td>
<td>Shear Forces in lb/ft for Span AB Due to Unit Moments Applied at End B of Different Plates</td>
<td>29</td>
</tr>
<tr>
<td>10-10</td>
<td>Rotations at B of Plates in Span BC Due to Unit Moments Applied at B</td>
<td>31</td>
</tr>
<tr>
<td>10-11</td>
<td>Rotations at B of Plates in Span AB Due to Unit Moments Applied at B</td>
<td>32</td>
</tr>
<tr>
<td>10-12</td>
<td>Deflections at Midspans Due to Unit Moments Applied at B</td>
<td>33</td>
</tr>
<tr>
<td>10-13</td>
<td>Redundant Moments at Support B</td>
<td>34</td>
</tr>
<tr>
<td>10-14</td>
<td>Deflections at Midspans Due to the Redundant Moments Applied at B</td>
<td>33</td>
</tr>
<tr>
<td>10-15</td>
<td>Deflections at Midspans of the Continuous Structure Under the Uniform Load of 100 lb/ft</td>
<td>33</td>
</tr>
<tr>
<td>10-16</td>
<td>Angles</td>
<td>34</td>
</tr>
<tr>
<td>Table</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-17</td>
<td>Normal Displacements</td>
<td>35</td>
</tr>
<tr>
<td>10-18</td>
<td>Transversal Angles at $x = \frac{L_b}{2}$ and at $x = \frac{L_c}{2}$</td>
<td>36</td>
</tr>
<tr>
<td>10-19</td>
<td>Angles at $x = \frac{L_b}{2}$ and at $x = \frac{L_c}{2}$</td>
<td>36</td>
</tr>
<tr>
<td>10-20</td>
<td>Tangential Loads in the Planes of Four Strips Due to Unit Moment $M_x = 1$ Applied at Edge $b$.</td>
<td>36</td>
</tr>
<tr>
<td>10-21</td>
<td>Tangential Loads in the Planes of Four Strips Due to Unit Moment $M_x = 1$ Applied at Edge $c$.</td>
<td>36</td>
</tr>
<tr>
<td>10-22</td>
<td>Shear Stressed Due to Unit Moments Applied Transversely at Edges $b$ and $c$</td>
<td>36</td>
</tr>
<tr>
<td>10-23</td>
<td>Rotations at $x = C$ of Plates in Span BC Due to $M_x = 1$ at Edge $b$ and $M_x = 1$ at Edge $c$.</td>
<td>36</td>
</tr>
<tr>
<td>10-24</td>
<td>Deflections at $x = \frac{L_b}{2}$ of Plates in Span BC Due to $M_x = 1$ at Edge $b$ and $M_x = 1$ at Edge $c$.</td>
<td>36</td>
</tr>
<tr>
<td>10-25</td>
<td>Redundant Moments $M_r^{(r)}$ Due to Moments $M_x = 1$.</td>
<td>36</td>
</tr>
<tr>
<td>10-26</td>
<td>Deflections at Midspan of BC Due to Redundant Moments $M_r^{(r)}$ Applied to Plates at B</td>
<td>36</td>
</tr>
<tr>
<td>10-27</td>
<td>Deflections of the Plates of the Continuous Structure Due to Uniform Moments $M_x = 1$ Alternately Applied at Edge $b$ and $c$.</td>
<td>36</td>
</tr>
<tr>
<td>10-28</td>
<td>Normal Displacements to Each Plate Strip Due to Moments $M_x = 1$ Applied at Edges $b$ and $c$ Alternately.</td>
<td>36</td>
</tr>
<tr>
<td>10-29</td>
<td>Transversal Rotation of Plate Strips at Midspans of Continuous Structure Due to $M_x = 1$.</td>
<td>36</td>
</tr>
<tr>
<td>10-30</td>
<td>Relative Angle Changes at Joints Due to $M_x = 1$ Alternately Applied at Joints $b$ and $c$.</td>
<td>36</td>
</tr>
<tr>
<td>10-31</td>
<td>Redundant Moments Acting Uniformly Along the Edges $b$ and $c$ of the Continuous Structure.</td>
<td>36</td>
</tr>
<tr>
<td>Table</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>10-8</td>
<td>Final Deflections of the Continuous Structure Due to a Uniform Load of 100 lb/ft Applied at Edges b and c</td>
<td>113</td>
</tr>
<tr>
<td>11-1</td>
<td>Results of Strain Gauges</td>
<td>133-4</td>
</tr>
<tr>
<td>11-2</td>
<td>Reading of Dial Indicators</td>
<td>140</td>
</tr>
<tr>
<td>11-3</td>
<td>Reading of Dial Indicators</td>
<td>141</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1. Continuous Folded Plate Structure ........................................... 2
2. Method of Analysis ..................................................................... 3
3. Uniform Load is Resolved Into the Planes of the Plates .............. 4
4. Plate bm Under Uniform Load S_{bm} ........................................... 5
5. Plate Cross-Section ................................................................. 7
6. Longitudinal Shears on Plate bm ............................................... 7
7. Strip Element of Plate bm ....................................................... 8
8. Moment Area of Cross-Section .................................................. 9
9. Volume Element of Plate bm Under Load S_{am} ............................. 11
10. Plate am Under Uniform Load S_{am} ......................................... 11
11. Longitudinal Shears on Plate am .............................................. 12
12. Strip Element of Plate am ....................................................... 13
13. Plate bm With Unit Moment at B ............................................. 13
14. Longitudinal Shears on Plate am with Unit Moment at B .......... 15
15. Volume Element of Plate bm with Unit Moment at B ................. 15
16. Plate am with Unit Moment at B ............................................. 15
17. Strip Element of Plate am with Unit Moment at B ..................... 17
18. Normal Displacements w_m and w_n ......................................... 70

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Figure

20. Redundant Moment $M_f$ ........................................................... 73
21. Redundant Moment $M_f$ Resolved into Forces Acting on Planes of Plates ........................................................... 74
22. Definition of the Angular Displacements $w_m^p$, $w_m^q$ ........................................................... 73
23. Data for Analytical Problem ........................................................... 72
24. Data for Analytical Problem ........................................................... 73
25. Loads Resolved into Forces Acting on Planes or Plates .............................. 94
26. Longitudinal Shears ........................................................... 86
27. Rotations at B Due to Unit Moment Applied at B ........................................................... 93
28. Angles $\theta_m$ and $\theta_m$ ........................................................... 92
29. Normal Displacements ........................................................... 96
30. Redundant Moment $M_p$ ........................................................... 102
31. Redundant T Moment $M_T$ Resolved into Forces Acting on Planes of Plates ........................................................... 102
32. Numerical Values of $S_b$ ........................................................... 103
33. Numerical values of $S_c$ ........................................................... 104
34-46. Photos ........................................................... 115-127
37. Cross-section of Concrete Specimen ........................................................... 123
40. Strain Gauge Location ........................................................... 129
41. Set Up of Gauge Indicators ........................................................... 130
46. Set Up of Gauge Indicators ........................................................... 131
51. Modulus of Elasticity of Concrete ........................................................... 132

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Figure

32-56. Stress Distribution Diagrams

57. Deflection Diagram

58. Relation of Deflections Due to $S_m$ and $T_m$
NOTATION

Note: A superscript © indicates that the symbol denotes a quantity caused by uniform loads \( S_{am} \) or \( S_{bm} \). If the letters \( a, b, c, d \) appear in the subscript they refer to the four spans shown in Fig. 2. The same letters are used in the superscript to indicate the number of the plate; in the superscript these letters replace the Arabic numbers 1, 2, 3 --- as these numbers could be easily confused with powers. If the symbol hasn’t any of the letters \( a, b, c --- \) in the subscript it refers to the continuous structure.

The first part of the analysis is carried out under the assumption that plates are hinge-connected by piano wires. In the second part of the analysis (joint displacement) the edges are made rigid by applying redundant moments \( M_{mn}^{(r)} \) where the superscript \( r \) denotes the number of the edge to which the moment is applied. These moments are resolved into \( S_{mn}^{(r)} \) loads (Fig. 20, Fig. 21). All resulting symbols carry the superscript \( r \).

\[ h_m \]
Height of plate \( m \).

\[ L_a \]
Length of span AB.

\[ L_b \]
Length of span BC.

\( M_{am}^{(c)} \)
Moment of plate \( am \) (Fig. 2) due to load \( S_{am} \).

\( M_{bm}^{(c)} \)
Moment of plate \( bm \) (Fig. 2) due to load \( S_{bm} \).

\( M_{cm}^{(c)} \)
Moment of plate \( am \) (Fig. 2) due to \( M_{cm} = 2 \) at B.
Moment of plate bm (Fig. 2) due to $N_{om} = 1$ at B.

Moment of plate am due to shears $T_{am}$.

Moment of plate bm due to shears $T_{bm}$.

Moment of plate am due to shears $T_{cm}$.

Moment of plate bm due to shears $T_{dm}$.

Redundant moment (Fig. 2) at B.

Shearing stresses shown in Fig. 1. The superscript o refers to uniform loads while the superscript (i) refers to longitudinal shears $T_{m}$. For subscripts a, b, c, and d the reader may refer to Fig. 2.

Shearing forces due to $s_{am}$ and $s_{bm}$ respectively.

Shearing forces due to unit moment at B.

Uniform load acting on the plane of plate am.

Uniform load acting on the plane of plate bm.

Thickness of plate m.

Longitudinal shearing stress between plate m, and plate (m+1). For the superscripts the reader may refer to the note under Notation.

Shearing constant related to $T_{m}$ by equation (4-18).

Deflection of plate am under the load $s_{am}$.

Deflection of plate bm under the load $s_{bm}$.

Deflection of plate am due to unit moment at B.

Deflection of plate bm due to unit moment at B.
\[
\begin{align*}
\nu_{cm} &= \text{Mom } \nu_{cm} \\
\nu_{dm} &= \text{Mom } \nu_{dm} \\
\gamma_m &= \text{Angles shown in Fig. 3.}
\end{align*}
\]

\[
\begin{align*}
\phi_{am} &= \text{Rotation of plate am due to } S_{am} \cdot \\
\phi_{bm} &= \text{Rotation of plate bm due to } S_{bm} \cdot \\
\phi_{cm} &= \text{Rotation of plate cm due to unit moment at B.} \\
\phi_{dm} &= \text{Rotation of plate bm due to unit moment at B.} \\
\varphi_m &= \text{Angles shown in Fig. 3.}
\end{align*}
\]
CHAPTER I

INTRODUCTION

The prismatic folded plate structure (Fig. 1) is formed by a series of adjoining thin plane slabs mutually supporting each other and rigidly connected along their common edges. They are usually closed off at its ends by integral diaphragms. Even though there are many folded plate structures made out of plywood and some out of metal, most of them are made out of concrete because it can be easily adapted to different forms.

In reinforced concrete the unstressed material is relocated to areas of fuller utilization thus making a substantial reduction in concrete per square foot or surface and making a substantial reduction in dead load. Floor and roof systems can be shaped to accommodate ducts and utility troughs thus obtaining more economy as compared to conventional structure. Folded plates have advantages over curved shells because of the less expensive framework, easier concreting and screeding. Similar advantages exist over more complex structures such as arches and frames. Because of their stiffness, folded plates can be constructed over long spans without an increase in material requirement. For these reasons, in the last decade, folded plate construction has found increasing application for roofs of industrial buildings.
and hangars as well as for the sides and bottoms of elevated bunkers.

The methods of analysis of folded plate structures may fall in the following four categories:

(a) Beam method,
(b) Folded plate theory neglecting relative joint displacement,
(c) Folded plate theory considering relative joint displacement, and
(d) Elasticity method.

Within each of these categories exist many papers using different assumptions and procedures. The term relative joint displacement denotes the displacement of one longitudinal side of a plate with respect to the other longitudinal side.

The basic assumptions in the above methods are the following:

1) The material is homogeneous and linearly elastic.

2) The actual deflections are relatively small as compared to the overall configuration of the structure. Hence, equilibrium conditions for the loaded structure may be developed using the configuration of the undeflected structure.

3) The principle of super-position holds.

4) Longitudinal joints are fully monolithic with slab acting continuously through the joints.

5) Each supporting diaphragm is infinitely stiff parallel to its own plane but is perfectly flexible normal to its plane.

Beam Method.—In this method the structure is assumed to deflect in such a manner that all points in the same cross sections deflect the same amount. Cross sections, however, tend to spread out considerably (Fig. 57)
when the folded plate structure is loaded. The beam theory may be applied when the folded plate structures are stiffened with intermediate transverse ribs spaced and designed so as to maintain the shape of the structure.

Folded Plate Theory (with and without relative joint displacement).--

In both classes of folded plate theory the longitudinal supporting action of each plate is governed by beam theory and the transverse supporting action is that of a continuous one-way slab. When the beam theory in each plate is adopted the following assumptions are implied:

1) Longitudinal stresses in each plate vary linearly across the width of each plate but the transverse rate of variation of stress may be different in the various plates.

2) Membrane shearing stresses in each plate have negligible effect on the deflection of the structure.

3) Normal stresses in each plate in the transverse direction are included in equilibrium considerations but have negligible effect on the deflection of the structure.

By adopting the one-way slab action the following assumptions are implied:

4) Slab bending is essentially one way phenomenon occurring in the transverse direction; the effect of longitudinal slab bending is negligible.

5) Individual plates possess negligible torsional resistance.

Torsional stresses due to twisting of the plates as well as deflections due to these stresses may be ignored.

6) Radial shearing stresses (normal to the circle) have negligible
effect on the deflection of the structure.

For normal span length to plate width ratios assumptions 1 and 2 do not introduce any substantial inaccuracies. As the span-width ratios become smaller, however, the longitudinal stress becomes non-linear and the shear deformation becomes more pronounced. Assumptions 4, 5 and 6 deriving from the use of one-way slab theory are sufficiently accurate for length-width ratios greater than 3 and width-thickness ratios greater than 5 (10).

**Folded Plate Theory Neglecting Relative Joint Displacements.**

In this theory it is assumed that the changes in transverse bending moment and in longitudinal stress due to relative joint displacement are negligible in comparison with the values of these moments and stresses computed on the basis of no relative joint displacements. Investigations (10) have shown, however, that joint displacements are too significant to be neglected and must be considered in analyzing folded plate structures.

**Folded Plate Theory Considering Relative Joint Displacements.**

This theory is based on the general assumptions which have been previously described, and takes into account the effect of relative displacement of the joints on the transverse moments and membrane stresses. Some of the practical methods of analysis are mentioned below.

1) **Vlassow's Method** (26, 27). This method determines the values of the stresses at critical sections in the structure by the solution of a set of simultaneous linear algebraic equations that are established on the basis of equilibrium at the critical section and on the basis of continuity of joints transversely. Longitudinal variation of applied loads and transverse moments...
are approximated on the basis of a Fourier series.

2) Portland Cement Association Bulletin (24). The analysis contained in this bulletin is similar to that of Vlassow. The typical equations involving the unknown stresses and moments are derived for the case of uniform vertical loading on a simple span structure. The bulletin contains a table of design coefficients for the V-type folded plate structure. Extension of the development to other types of loadings and support conditions is not covered although it is examined in a general way.

3) Gasafar's Method (10). Gasafar used the principle of superposition to consider a folded plate structure actually loaded between and at the ridges as the combination of (a) an identical structure loaded between and at the ridges but supported at the ridges against deflection and (b) an identical structure loaded at the ridges only by the ridge reactions of the structure in (a). In developing the theory for the folded plate structure (b), loaded at the ridges only, he makes the assumption that the elastic curve of deflection due to applied load is the same as the normal deflection curve of the structure (half sine wave for simply supported structure). This assumption is accurate within approximately 2% for longitudinally symmetrical loadings. The half sine wave concept as the deflection curve is in error for loading that is largely anti-symmetric longitudinally.

4) Yitzhail's Method (31). Yitzhail presents several methods of analysis with major emphasis on the so-called method of particular loadings. In this method he uses the principle of superposition used by Gasafar to analyze the structure for the actual load on it, assuming unfolding ridges. The reactions
developed by these assumed but actually non-existent supports are then applied as longitudinal ridge loads which are carried at each ridge by the plates meeting at each ridge. The resulting deflections of the plates are converted, on the basis of geometrical considerations, into relative joint displacements which are associated with transverse bending moments in the slabs. From these, a set of ridge loads, called superfluous loads, necessary to maintain the deflected shape of the structure is obtained. The structure is then subjected in turn to several appropriately chosen, independent ridge loadings. These loadings are each multiplied by an undetermined constant and combined so as to eliminate the preceding superfluous loading at each ridge. In this process n-2 simultaneous equations each containing the n-2 undetermined constants are obtained and solved to evaluate the constants. Here, n equals the number of plates. With the constants known, the controlling stresses and deflections are readily obtainable. The assumptions and approximations used by Yitshaki are the same as those used by Gasfar, previously described. Consequently, Yitshaki's usual procedure is subject to the same limitations on applicability as Gasfar's theory.

Elasticity Method.--Goldberg and Leve developed (13) a solution for the stresses in a folded plate structure by combining the equations of the plate theory for loads normal to the plane of the plates and the elasticity equations defining the plane stress problem for loads in the plane of the plates. Applied loading is approximated by a Fourier series. This method is applied where component plates of the structure are relatively short compared to the width, and where translation of individual joints is completely prevented.
The theory presented in this paper for continuous folded plate structures is developed from Flügge's theory for simply supported folded plate structures which he outlines in his book published in 1962. His theory is particularly attractive since the deflections of plates are given by a differential equation which can be generalised for any loading condition by expressing the load in a Fourier series. Flügge's solution of joint displacement is very much similar to that of Timoshenko. Chapter VIII of this paper presents the method in detail and extends it to continuous structures, while Chapter III describes the method of analysis by which Flügge's theory is extended to a continuous structure. Chapter XI gives the experimental results of a continuous concrete folded plate structure subjected to uniform load.
CHAPTER II

HISTORICAL DISCUSSION

G. Ehlers (6) wrote one of the first papers on folded plate theory in 1930. Ehlers proposed a folded-plate theory based on linear variation of longitudinal stress in each plate but neglected the effect of the relative displacement of the joints. In 1932 E. Gruber (14) took approximate account of joint translations and rotations by using a strip theory approach. A further refinement of the membrane theory was made by H. Craemer (2) and Gruber (15) by considering the actual stress distributions in the planes of the slab. In 1947 the method was introduced in the U.S.A. in a paper by George Winter, F. ASCE and M. Pei (80). Their theory neglected the relative joint displacements but developed a convenient iteration procedure to determine the longitudinal stresses patterned after the moment distribution procedure.

The relative displacement of the joints which was first proposed by Gruber and Gruening in 1932 (14, 17) was considerably simplified by W. Z. Vlassow, 1936 (28) by using linear algebraic equations to calculate the longitudinal stresses and ridge moments at critical sections of the structure.

K. Girkmann and The Portland Cement Association (12, 24) submitted methods similar to Vlassow's. The theory involving relative joint displacements was
also developed by I. Gaafar, M. ASCE and D. Yitzhaki (21, 31). A. Werfel and then J. E. Goldberg and H. L. Leve (29, 13) developed approaches that considered both two-dimensional elasticity theory for determination of membrane stresses and two way slab theory for determination of bending and twisting of the slab. In 1962 Filișe published his book in which he outlined the theory which was used in this paper to develop a method to analyze continuous folded plate structures.
CHAPTER III

METHOD OF ANALYSIS

Plate strip $m$, bounded by edges $m-1$ and $m$ (Fig. 1) and subjected to the resolved uniform loads $S_{am}$ and $S_{bm}$, is analyzed as a continuous beam supported at $A$, $B$, and $C$. The continuous beam $AC$ is then divided into two simply supported beams $AB$ and $BC$ (Fig. 2b). Due to the uniform load $S_{bm}$ plate $BC$ rotates $\theta_{bm}$ degrees at end $B$, while plate $AB$, due to the uniform load $S_{am}$, rotates $-\theta_{am}$ degrees at end $B$ (Fig. 2a). Similarly deflections $v_{bm}$ and $v_{am}$ are produced.

The relative rotation of tangents $a$ and $b$ is $\theta_b - \theta_c$. In the continuous plate $AC$, however, the tangents $a$ and $b$ form a straight line since the deflection curve must be continuous. It is, therefore, necessary to apply a set of moments $M_{om}$ at ends $B$ of plates $AB$ and $BC$ which will produce a relative rotation equal and opposite to $\theta_{bm} - \theta_{am}$, where $r$ in $M_{om}$ goes from $K = 1$ to $K - 1$ and $K$ = number of plates. The stresses and deflections due to the moment $M_{om}$ would then be added to the stresses and deflections produced by the uniform loads $S_{am}$ and $S_{bm}$. The resulting stresses and deflections would be those of the continuous plate $AC$ subjected to loads $S_{am}$ and $S_{bm}$ at spans $AB$ and $BC$ respectively. The moment $M_{om}$, however, is an...
unknown quantity and must be found.

At ends B of plates AB and BC a unit moment is applied which will produce rotations \( \theta_{cm} \), \( \theta_{dm} \) and deflections \( v'_{cm} \) and \( v'_{dm} \) (Fig. 2c). The relative rotation between tangents \( d \) and \( c \) is \( \theta_{dm} - \theta_{cm} \). Hence

\[
\frac{M_{om}}{M_{cm}} = \frac{\theta_{am} - \theta_{bm}}{\theta_{cm} - \theta_{dm}}. \tag{3-1}
\]

The stresses and deflections of the continuous plate can then be computed. Once these stresses and deflections have been obtained joint displacements can be considered as described in Chapter IX.

**Computation of \( S_m \)**

It is assumed that the edge loads are vertical and uniformly distributed in the \( x \) direction. The load at every edge may be different in magnitude but it must be uniformly distributed along \( x \) throughout the whole length of the beam.

Filigge (8) resolves the load \( P_m \) acting on an element of unit length of the \( m \)-th edge into two components in the directions \( y_m \) and \( y_{m+1} \):

\[
s_m^1 = P_m \frac{\cos \phi_m + 1}{\sin \gamma_m}, \quad s_m^2 = P_m \frac{\cos \phi_m}{\sin \gamma_m}. \tag{3-2}
\]

These loads can be carried by the strips \( m \) and \( m + 1 \) respectively (Fig. 8c). The resultant load acting on plate \( m \) would be

\[
s_m = s_m^2 - s_m^{1+1} + s_m^1. \tag{3-3}
\]
FIG. 2. METHOD OF ANALYSIS
FIG. 3. THE UNIFORM LOAD $S_m$ IS RESOLVED INTO FORCES ACTING ON THE PLANES OF PLATES.
CHAPTER IV

ANALYSIS OF PLATE $h_{bm}$ UNDER UNIFORM LOAD $s_{hm}$

Bending Moment Due to $s_{hm}$

The beam shown in Fig. 4 (a) subjected to the uniform load $s_{hm}$ has the bending moment

$$M_{hm}^{(0)} = s_{hm} \frac{Lb^3}{2} - x^2$$  \hspace{1cm} (4-1)

Shearing Forces Due to $s_{hm}$

$$Q_{hm}^{(0)} = s_{hm} \left( \frac{Lb}{2} - x \right).$$  \hspace{1cm} (4-2)

Let us now assume that the plate strip is slender, that is, the height $h_m$ is much smaller than the length $L_b$, then the bending stress $\sigma_x$ and the shear stress $\tau$ may be found from formulas of elementary beam theory. Moreover, let $N_x = \sigma_x t_m$ and $N_{xy} = \tau t_m$ where $t_m$ is the thickness of the plate (Fig. 5). Hence

$$N_{hm}^{(0)} = \tau t_m = \frac{\partial M_{hm}^{(0)}}{\partial x} = \frac{d M_{hm}^{(0)}}{dx} \frac{Ay}{I}$$  \hspace{1cm} (4-3)

$$N_{hm}^{(0)} = \frac{Q_{hm}^{(0)} \left( \frac{h_m}{2} - y_m \right)}{1/2} \left[ y_m + \frac{1}{3} \left( \frac{h_m}{2} - y_m \right) \right] t_m$$  \hspace{1cm} (4-4)

Simplifying the above equation

$$N_{hm}^{(0)} = \frac{6Q_{hm}^{(0)} h_m}{h_m^2} \left( \frac{1}{4} - \frac{y_m^2}{h_m^2} \right).$$  \hspace{1cm} (4-5)
FIG. 4. PLATE bm UNDER UNIFORM LOAD $S_{bm}$
FIG. 5. PLATE CROSS-SECTION
Also the stress

\[ N_{b_{m}}^{(0)} = C_{x} t_{m} = \frac{12 M_{b_{m}}^{(0)} y_{m}}{h_{m}^{3}} \quad (4-6) \]

Longitudinal Shear Stress Equations

At the lower edge of the strip \( m \) \( (y_{m} = \frac{h_{m}}{2}) \) the normal force \( N_{b_{m}}^{(0)} \) produces the strain

\[ \varepsilon_{b_{m}}^{(0)} = \frac{N_{b_{m}}^{(0)}}{E t_{m}} = \frac{6 M_{m}^{(0)}}{E t_{m} h_{m}^{2}} \quad (4-7) \]

And the strain at the upper edge of the adjacent strip is

\[ \varepsilon_{b_{m+1}}^{(0)} = \frac{6 M_{m+1}^{(0)}}{E t_{m+1} h_{m+1}^{2} + 1} \quad (4-8) \]

Since the two adjacent plates are connected to one another these strains must be made equal to each other. When this operation is done the two strips will exert forces upon each other. These additional forces are shearing forces \( T_{m} \) acting on the plane of both strips. These forces are shown in Fig. 7 acting in a positive direction in accordance with the sign convention for \( N_{xy} \) (Fig. 6b).

From Fig. 7 it is seen that the edge shears will add an additional moment \( M_{m}^{(1)} \) to the bending moment \( M_{m}^{(0)} \) of the strip. An axial force \( N_{m}^{(1)} \) is also produced. Applying the equilibrium equations to the strip element shown in Fig. 7 the following relations are obtained.

\[ \frac{dN_{b_{m}}^{(1)}}{dx} = T_{b_{m-1}} + T_{b_{m}} \quad (4-9) \]

\[ \frac{dM_{b_{m}}^{(1)}}{dx} = - \frac{h_{m}}{2} (T_{b_{m-1}} + T_{b_{m}}) \quad (4-10) \]
FIG. 6. LONGITUDINAL SHEARS IN $E_{bm}$ DUE TO UNIFORM LOAD $S_{bm}$
FIG. 7. STRIP ELEMENT OF $E_{b_m}$
The shearing forces $T_{bm}$ must be known in terms of $x$ before (4-9) and (4-10) can be integrated.

The shearing forces $T_m$ produce the force $N_{bm}^{(1)}$ and the moment $M_{bm}^{(1)}$ which in turn produce beam bending stresses from which the normal force

$$N_{bm}^{(1)} = \frac{N_{bm}^{(1)}}{h_m} + \frac{12 M_{bm}^{(1)} y_m}{h_m}$$

(4-11)

and the edge strain

$$\epsilon_{bm}^{(1)} = \frac{N_{bm}^{(1)}}{E_t m h_m} + \frac{M_{bm}^{(1)}}{E_t m h_m^2}$$

(4-12)

In equation (4-12) the second term on the right side is negative if $y = -\frac{h_m}{2}$ and positive if $y = +\frac{h_m}{2}$.

To avoid discrepancy of strains between connecting edges the following condition must exist,

$$\epsilon_{bx m}^{(0)} + \epsilon_{bx m}^{(1)} = \epsilon_{bx(m+1)}^{(0)} + \epsilon_{bx(m+1)}^{(1)}$$

(4-13)

After having substituted expressions for the $\dot{\epsilon}$'s

$$\frac{6 M_{bm}^{(0)}}{E_t m h_m^2} + \frac{N_{bm}^{(1)}}{E_t m h_m} + \frac{6 M_{bm}^{(1)}}{E_t m h_m^2} = -\frac{6 M_{bm}^{(0)}}{E_t m+1 h_m^2} + \frac{N_{bm}^{(1)}}{E_t m+1 h_m} + \frac{6 M_{bm}^{(1)}}{E_t m+1 h_m^2 + 1}$$

(4-14)

Equation (4-14) is the compatibility equation from which an equation for the unknown shearing forces $T_m$ can be derived. After differentiation in respect to $x$, equation (4-14) yields
Equation (4-1) is differentiated in respect to $x$ to yield

$$\frac{d M_{bm}^{(0)}}{dx} = \frac{S_{bm}}{2} (L_b - 2 x). \quad (4-16)$$

Equations (4-16), (4-9), and (4-10) are substituted in equation (4-15). After simplification

$$\frac{1}{t_m h_m} \left( 2T_{b(m-1)} + 4T_{bm} \right) + \frac{1}{t_{m+1} h_{m+1}} \left( 4T_{bm} + 2T_{m+1} \right)$$

$$= \frac{3S_{bm} (L_b - 2 x)}{2 t_m h_m} + \frac{3S_{b(m+1)} (L_b - 2 x)}{5 t_{m+1} h_{m+1}} \quad (4-17)$$

A total of $(K-1)$ equations such as (4-17) may be written for every edge from $m = 1$ to $m = K - 1$ in order to solve $K - 1$ unknown functions $T_m (x)$. Since in equation (4-17) both terms on the right hand side are proportional to $(L_b - 2 x)$, it may be said that

$$T_{bm} = T_{bm} (L_b - 2 x)$$
$$T_{b(m+1)} = T_{b(m+1)} (L_b - 2 x) \quad (4-18)$$

where $T_m$, $T_{m+1}$, . . . . are constants.
Expressions (4-18) are substituted in equations (4-17). The resulting equations are then differentiated in respect to \( x \) and simplified to yield a set of ordinary linear equations for the unknown constants \( T_m \).

\[
\frac{1}{t_m h_m} T_{b(m-1)}' + 2 \left( \frac{1}{t_m h_m} + \frac{1}{t_{m+1} h_{m+1}} \right) T_{bm}' + \frac{1}{t_{m+1} h_{m+1}} T_{b(m+1)}' = \frac{S b(m)}{2} + \frac{S b(m+1)}{2} \quad \frac{t_m h_m}{t_{m+1} h_{m+1}} \tag{4-19}
\]

The Bending Moment Resultant \( M_{bm} \)

The moment of the strip due to the uniform load \( b_{bm} \) and the shearing force \( T_m \) may be expressed as

\[
M_{bm} = M_{bm}^{(0)} + M_{bm}^{(1)} \quad \tag{4-20}
\]

The moment \( M_{bm}^{(1)} \) is found by substituting expressions (4-18) into equation (4-10) and integrating in respect to \( x \). Hence

\[
\frac{d M_{bm}^{(1)}}{dx} = - \frac{h_m}{2} \left( T_{b(m-1)}^1 + T_{bm}^1 \right) (L_b - 2 x) \quad \tag{4-21}
\]

\[
M_{bm}^{(1)} = - \frac{h_m}{2} \left( T_{b(m-1)}^1 + T_{bm}^1 \right) (L_b x - x^2) + C \quad \tag{4-22}
\]

Substituting the boundary conditions

\[
x = 0 \quad M_{bm}^{(1)} = 0
\]

\[
x = L_b \quad M_{bm}^{(1)} = 0
\]

into equation (4-22) it is found that \( C = 0 \). Therefore

\[
M_{bm}^{(1)} = - \frac{h_m}{2} \left( T_{b(m-1)}^1 + T_{bm}^1 \right) (L_b x - x^2)
\]

but

\[
M_{bm} = M_{bm}^{(2)} + M_{bm}^{(1)}
\]
Hence

\[ M_{\text{bm}} = \frac{S_{\text{bm}}}{2} (L_b x - x^2) + \frac{b_m}{2} (T_{m-1}^1 + T_m^1) (x^2 - L_b x) \]  
(4-23)

The Shear Stress Resultant \( q_{\text{bm}} \)

The total shear stress resultant \( q_m = q_m^{(0)} + q_m^{(1)} \). But \( q_m^{(1)} = 0 \) since the shears \( T_m \) don't produce any vertical shear component. Therefore

\[ q_{\text{bm}} = \frac{S_{\text{bm}} L_b}{2} - S_{\text{bm}} x \]

\[ q_{\text{bm}} = S_{\text{bm}} \left( \frac{L_b}{2} - x \right) \]  
(4-24)

The Normal Stress Resultant \( N_{\text{bm}} \)

The total normal stress resultant \( N_{\text{bm}} = N_{\text{bm}}^{(0)} + N_{\text{bm}}^{(1)} \). But \( N_{\text{bm}}^{(0)} = 0 \) since the uniform load \( S_{\text{bm}} \) does not produce any net force in the \( x \)-direction at a given cross section. The \( N_{\text{bm}}^{(1)} = N_{\text{bm}}^{(1)} \). Equation (3-10) states that

\[ \frac{d}{dx} N_{\text{bm}}^{(1)} = T_{b(m-1)} - T_{\text{bm}} \]

\[ \frac{d}{dx} N_{\text{bm}}^{(1)} = (T_{b(m-1)}^1 - T_{\text{bm}}^1) (L_b - 2x) \]

\[ N_{\text{bm}}^{(1)} = (T_{b(m-1)}^1 - T_{\text{bm}}^1) (L_b x - x^2) + C \]  
(4-25)

at \( x = 0 \) and at \( x = L_b \) \( N_{\text{bm}}^{(1)} = 0 \). Therefore \( C = 0 \).

\[ N_{\text{bm}} = (T_{b(m-1)}^1 - T_{\text{bm}}^1) (L_b x - x^2) \]  
(4-26)

The Stress Resultant \( N_{\text{bx}} \)

From elementary theory

\[ c_x = \frac{P}{A} \pm \frac{M_y}{l} \]
Then

\[ \sigma_{bx} = \frac{T_{b(m-1)}^1 - T_{bm}^1 - (L_2 x - x_2^2)}{t_m h_m} + \frac{12 y_m}{t_m h_m} \frac{s_{bm}}{2} (L_2 x - x_2^2) + \left( T_{b(m-1)}^1 + T_{bm}^1 \right) \left( \frac{h_m}{2} \right) \]

(4-27)

\[ \left( x^2 - L_b x \right) \]

but \[ N_{bx} = t_m \sigma_{bx} \]

(4-28)

Shear Stress Resultant \( N_{bxy} \)

\[ N_{bxy} = N_{bxy}^{(0)} + N_{bxy}^{(1)} \] \hspace{1cm} (4-29)

\[ N_{bxy}^{(0)} = -t_m \frac{d M_{bm}^{(0)}}{dx} \]

\[ I = \frac{1}{12} t_m h_m^3 \]

\[ c = A \bar{y} = t_m \frac{6}{8} (h_m^2 - 4 y_m^2) \]

\[ N_{bxy}^{(0)} = \frac{3}{4} \frac{s_{bm}}{h_m^2} (L_b - 2 x) (h_m^2 - 4 y_m^2) \]

(4-30)

To find \( N_{bxy}^{(1)} \) the element shown in Fig. 9 is considered.

\[ \tau_{b} \frac{t_m}{dx} = T_{bm}^1 (L_b - 2 x) \]

\( \int d \frac{N_{bxy}^{(1)}}{y_m} \)

(4-31)

\[ N_{bxy}^{(1)} = \frac{N_{bxy}^{(1)}}{h_m} + \frac{12 M_{bm}^{(1)}}{h_m^3} \]

Differentiating \( N_{bxy}^{(1)} \) in respect to \( x \) and substituting for \( \frac{d N_{bxy}}{dx} \) and \( \frac{d M_{bm}}{dx} \),

the following equation is obtained.
FIG. 8. MOMENT AREA OF CROSS-SECTION
FIG. 9. VOLUME ELEMENT OF PLATE $b_m$

UNDER LOAD $S_{b_m}$
Equation (4-32) is substituted into equation (4-31), which is integrated in respect to y and simplified.

\[ N_{bxy} = \frac{T_{bm} (L_b - 2x)}{2h_m} (2y_m - h_m)(6y_m - h_m)(2y_m - h_m)(6y_m + h_m) \]  

Equations (4-33) and (4-30) are substituted into equation (4-20).

\[ N_{bxy} = \frac{3}{4} \frac{5_{bm}}{h_m} (L_b - 2x)(h_m^2 - 4y_m^2) + \frac{T_{bm} (L_b - 1)}{4h_m} (L_b - 2x)(y_m - h_m)(6y_m + h_m) \\
+ \frac{T_{bm} (L_b - 2x)}{4h_m} (6y_m - h_m) \]  

The normal force \( N_y \) in the direction of the \( y_m \) axis shown in Fig. 6(b) is rather small and there is no need to compute it.

**Rotation of Plate BC**

The deflection of plate BC is subject to the differential equation

\[ \frac{d^2 v_{bm}}{dx^2} = -\frac{12}{E t_m h_m^3} M_{bm} \]  

The moment \( M_{bm} \) is given in terms of \( T_{m}^1 \) in equation (3-23) which is substituted in equation (4-35) to yield

\[ \frac{d^2 v_{bm}}{dx^2} = \frac{12}{E t_m h_m^3} \left[ \frac{b_m}{2} (L_b x - x^2) + \frac{h_m}{2} (T_{m-1}^1 + T_{m}^1) (x^2 - L_b x) \right] \]
if integrated once in respect to \( x \), equation (4-36) will yield the slope of the elastic curve.

\[
\frac{d v_{bx}}{dx} = \epsilon_{bx} = \frac{12 \left[ \frac{s_{bm} L_b x^2}{4} - \frac{E_{bm} x^3}{6} + \frac{h_m}{2} \left( T_{b(m-1)}^1 + T_{bm}^1 \right) \left( \frac{x^3}{3} - \frac{L_b x^2}{2} \right) \right]}{E t_m h_m} + C
\]

(4-37)

at \( x = \frac{L_b}{2} \) \( \theta = 0 \). (4-38)

The boundary condition (4-38) is substituted in equation (4-37). After simplification

\[
C = -\frac{12}{E t_m h_m} \left[ \frac{k_{bm} L_b^3}{24} - \frac{h_m}{2} \left( T_{b(m-1)}^1 + T_{bm}^1 \right) \left( \frac{x^3}{3} - \frac{L_b x^2}{2} \right) \right]
\]

(4-39)

The integration constant (4-39) is substituted into equation (4-37) to obtain the equation of the slope of the elastic curve.

\[
\theta_{bx} = \frac{12}{E t_m h_m} \left[ \frac{s_{bm} L_b x^3}{4} - \frac{s_{bm} x^3}{6} + \frac{h_m (T_{b(m-1)}^1 + T_{bm}^1)}{2} \left( \frac{x^3}{3} - \frac{L_b x^2}{2} \right) \right]
\]

(4-40)

\[
\theta_{bx=0} = -\frac{12}{E t_m h_m} \left[ \frac{s_{bm} L_b^3}{24} - \frac{h_m}{2} \left( T_{b(m-1)}^1 + T_{bm}^1 \right) \frac{L_b^3}{12} \right]
\]

(4-41)

**Deflection of Plate BC**

In order to find the equation of the deflection curve of plate BC, equation (4-36) must be integrated twice in respect to \( x \).

\[
\omega_{bx} = \frac{12}{E t_m h_m} \left[ \frac{s_{bm} L_b x^2}{12} - \frac{s_{bm} x^3}{6} + \frac{h_m \left( T_{b(m-1)}^1 + T_{bm}^1 \right)}{2} \left( \frac{x^3}{3} - \frac{L_b x^2}{2} \right) \right] + C x + D
\]

(4-42)
The deflection $\nu_{bhm} = 0$ at $x = 0$ and at $x = L_b$. Applying these conditions to equation (4-42) it is found that the integration constant $D = 0$ and the integration constant $C$ is the same as that given by equation (4-39). The values of $C$ and $D$ are substituted back into (4-42). After simplification

$$\nu_{bhm} = -\frac{1}{2 \frac{E_t}{h_m} L_b} \left[ (x^4 - 2 L_b x^3 + L_b^3 x) \left( \delta_{bh} - h_m \left< T_{b_m-1}^1 + T_{b_m}^1 \right> \right) \right] \quad (4-43)$$
CHAPTER V

ANALYSIS OF PLATE am UNDER UNIFORM LOAD Sam

Plate am shown in Fig. 10(a), is analyzed in the same way as plate bm. Most of the resulting equations are correspondingly the same as those shown for plate bm while others may show some differences in negative and positive signs. For this reason explanations and derivations of equations have been omitted in this section. The reader may refer back to plate bm for more details.

Bending Moment Due to Sam

\[ M_{am}^0 = -S_{am} \left( \frac{L_a}{2} x + x^2 \right) \]  \hspace{1cm} (5-1)

Shear Forces Due to Sam

The reader may refer to equations (4-2) to (4-6) for more details.

\[ q_{am}^{(0)} = -S_{am} \left( \frac{L_a}{2} + x \right) \]  \hspace{1cm} (5-2)

\[ N_{axy}^{(0)} = \frac{6}{b_m} q_{am}^{(0)} \left( \frac{1}{4} - \frac{y_m}{b_m} \right) \]  \hspace{1cm} (5-3)

\[ N_{ax}^{(0)} = \frac{12}{b_m} M_{am}^{(0)} y_m \]  \hspace{1cm} (5-4)

*Longitudinal Shear Stress Equations*

The main equations used to develop the shear stress equations are
FIG. 10. PLATE cm UNDER UNIFORM LOAD $S_{am}$
shown below. The reader may refer to equations (4-7) to (4-19) for details.

\[ \varepsilon^{(0)}_{\text{axm}} = \frac{6 M^{(0)}_{\text{am}}}{\kappa t_m h_m^2} \]  
\[ \varepsilon^{(0)}_{\text{ax(m+1)}} = -\frac{6 M^{(0)}_{\text{am(m+1)}}}{E t_m h_m + 1} \]  
\[ \frac{d N_{\text{am}}^{(1)}}{dx} = T_{\text{am(m-1)}} - T_{\text{am}} \]  
\[ \frac{d M_{\text{am}}^{(1)}}{dx} = -\frac{h}{2} (T_{\text{am(m-1)}} + T_{\text{am}}) \]  
\[ N_{\text{axm}}^{(1)} = \frac{N_{\text{am}}^{(1)}}{h_m} + \frac{12 M_{\text{am}} V_m}{h_m^3} \]  
\[ \varepsilon^{(1)}_{\text{axm}} = \frac{N_{\text{am}}^{(1)}}{E t_m h_m} + \frac{6 M_{\text{am}}}{E t_m h_m} \]  
\[ \varepsilon^{(0)}_{\text{axm}} + \varepsilon^{(1)}_{\text{axm}} = \varepsilon^{(0)}_{\text{ax(m+1)}} + \varepsilon^{(1)}_{\text{ax(m+1)}} \]  

\[ \frac{6 M^{(0)}_{\text{am}}}{\kappa t_m h_m^2} + \frac{N_{\text{am}}^{(1)}}{E t_m h_m} + \frac{6 M^{(0)}_{\text{am}}}{\kappa t_m h_m^2} = \]  
\[ \frac{6 M^{(0)}_{\text{am(m+1)}}}{\kappa t_m h_m^2} + \frac{N_{\text{am(m+1)}}^{(1)}}{E t_m h_m + 1} - \frac{6 M^{(1)}_{\text{am(m+1)}}}{\kappa t_m h_m^2} \]  

\[ \frac{6 M_{\text{am}}^{(1)}}{dx} = \frac{d M_{\text{am}}^{(0)}}{dx} + \frac{d M_{\text{am}}^{(1)}}{dx} \]  
\[ \frac{d N_{\text{am}}^{(1)}}{dx} = \frac{d N_{\text{am}}^{(0)}}{dx} + \frac{d N_{\text{am}}^{(1)}}{dx} \]  
\[ \frac{d M_{\text{am+1}}^{(1)}}{dx} = \frac{d M_{\text{am+1}}^{(0)}}{dx} + \frac{d M_{\text{am+1}}^{(1)}}{dx} \]  

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The Bending Rontent Riesultant $M_{am}$

The reader may refer to equations (4-20) to (4-23) for details.

\[
M_{am} = M_{am}^{(0)} + M_{am}^{(1)}
\]

\[
\frac{d M_{am}^{(1)}}{dx} = - \frac{h_m}{2} (T_{am}^1 + T_{am}^1) (L_a + 2x)
\]

\[
\frac{d M_{am}^{(1)}}{dx} = - \frac{h_m}{2} (T_{am}^1 + T_{am}^1) (L_a + 2x)
\]

\[
M_{am}^{(1)} = - \frac{h_m}{2} (T_{am}^1 + T_{am}^1) (L_a + 2x) + C
\]

The moment $M_{am}^{(1)} = 0$ at $x = 0$ and at $x = -L_a$. Hence $C = 0$

\[
M_{am}^{(1)} = - \frac{h_m}{2} (T_{am}^1 + T_{am}^1) (L_a + x^2)
\]
The Shear Stress Resultant $\tau_{am}$

The reader may refer to equation (4-24) for details.

\[
\tau_{am} = \frac{S_m}{2} (L_a x + x^2) - \frac{h_m}{2} (T_{am}^{1} (m-1) + T_{am}^{1}) (L_a x + x^2)
\]  

(5-22)

\[
\begin{align*}
\tau_{am}^{(0)} &= \tau_{am}^{(1)} + \tau_{am}^{(2)} \\
\tau_{am}^{(0)} &= \frac{S_m L_a}{2} - S_m (L_a + x) \\
\tau_{am}^{(1)} &= 0 \\
\tau_{am}^{(2)} &= -S_m \left( \frac{L_a^2}{2} + x \right)
\end{align*}
\]  

(5-23)  

(5-24)  

(5-25)  

(5-26)

The Normal Stress Resultant $N_{am}$

The reader may refer to equations (4-25) to (4-26) for explanations.

\[
N_{am} = N_{am}^{(0)} + N_{am}^{(1)}
\]  

(5-27)

\[
N_{am}^{(0)} = 0
\]  

(5-28)

\[
\frac{d N_{am}^{(1)}}{dx} = (T_{am}^{1} (m-1) - T_{am}^{1}) (L_a + 2 x)
\]  

(5-29)

\[
N_{am}^{(1)} = (T_{am}^{1} (m-1) - T_{am}^{1}) (L_a x + x^2) + C
\]  

(5-30)

The stress $N_{am}^{(1)} = 0$ at $x = 0$ and at $x = -L_a$. Therefore, $C = 0$.

\[
N_{am} = (T_{am}^{1} (m-1) - T_{am}^{1}) (L_a x + x^2)
\]  

(5-31)

The Stress Resultant $N_{ax}$

The reader may refer to equations (4-27) to (4-28) for explanations.
\[ C_{axm} = \frac{(T_{a(m-1)}^1 - T_{am}^1) (L_a x + x^2) + 12 y_m}{h_m} \left[ \frac{C_{am} (L_a x + x^2)}{2} \right. \]

\[ - \frac{h_m}{2} (T_{a(m-1)}^1 + T_{am}^1) (L_a x + x^2) \]

\[ N_{axm} = t_m C_{axm} \]  

\[ N_{axm} = \frac{L_a x + x^2}{h_m} \left[ T_{a(m-1)}^1 - T_{am}^1 + \frac{6 h_m y_m}{h_m^2} \right. \]

\[ \frac{g y_m}{h_m} (T_{a(m-1)}^1 + T_{am}^1) \]  

---

**Shear Stress Resultant \( N_{axm} \)**

The reader may refer to equations \( (4-29) \) to \( (4-34) \) for explanations.

\[ N_{axm} = N_{axm}^{(0)} + N_{axm}^{(1)} \]  

\[ N_{axm}^{(0)} = \frac{d M_{am}^{(0)}}{dx} \left. \right|_{am} \]  

\[ q = \frac{t_m}{8} \left( h_m^2 - 4 y_m^2 \right) \]  

\[ N_{axm}^{(0)} = \frac{3}{4} \frac{S_m}{h_m^3} (L_a + 2 x) \left( h_m^2 - 4 y_m^2 \right) \]  

\[ N_{axm}^{(1)} = \frac{t_m}{h_m} \left( L_a + 2 x \right) dx + \int_{y_m}^{h_m} \frac{dx}{y_m} d N_{axm} \]  

\[ N_{axm}^{(1)} = \frac{N_{axm}^{(1)}}{h_m} + \frac{12 M_{am}^{(1)}}{h_m^3} \]  

\[ \frac{d N_{axm}^{(1)}}{dx} = \frac{1}{h_m} \left( T_{a(m-1)}^1 - T_{am}^1 \right) dx - \frac{G(L_a + 2 x)}{h_m^2} \left( T_{a(m-1)}^1 - T_{am}^1 \right) y dx \]
\[
N_{axym} = \frac{3}{4} \frac{S_{am}}{h_m^3} (L_a + 2x) \left(h_m^2 - 4y_m^2\right) + \frac{T_{am}^1 (L_a + 2x)}{4 h_m^2} \left(2y_m + h_m\right) \left(6y_m - h_m\right)
\]

\[
\frac{d^2 v_{am}}{dx^2} = -\frac{12}{E t_m h_m^3} \left[-\frac{S_{am}}{2} (L_a x + x^2) - \frac{h_m}{2} (T_{am}^1 + T_{am}^1) (L_a x + x^2)\right]
\]

\[
\frac{dv_{am}}{dx} = E_{exm} = \frac{12}{E t_m h_m} \left[-\frac{S_{am}}{2} \left(\frac{L_a x^2}{2} + \frac{x^3}{3}\right) - \frac{h_m}{2} (T_{am}^1 + T_{am}^1) \left(\frac{L_a x^2}{2} + \frac{x^3}{3}\right)\right] + C
\]

at \(x = \frac{L_a}{2}\), \(\theta = 0\)

\[
C = \frac{1}{2 E t_m h_m^3} \left[ S_{m} \frac{3}{L_a} + h_m (T_{am}^1 + T_{am}^1) \frac{3}{L_a}\right]
\]
\[
\Phi_{axm} = \frac{12}{E_t m h_m^3} \left[ -\frac{s_{am}}{2} \left( \frac{L_a x^2}{2} + \frac{x^4}{3} \right) - \frac{h_m}{2} (T_{am}^1 \frac{f_{a} x^2}{2} + \frac{x^3}{3}) \right]
\]

\[
+ \frac{1}{2E_t m h_m} \left[ s_m L_a^2 + h_m (T_{a(m-1)}^1 + T_{am}^1) L_a^3 \right] \quad (5-48)
\]

\[
\Phi_{a(x\rightarrow 0)m} = \frac{1}{2E_t m h_m} \left[ s_m L_a^2 + h_m (T_{a(m-1)}^1 + T_{am}^1) L_a^3 \right] \quad (5-49)
\]

**Deflection of Plate BC**

The reader may refer to equations (4-42) to (4-43) for explanations.

\[
v_{axm} = \frac{1}{2E_t m h_m^3} \left[ -s_{am} (2 L_a x^3 + x^4) - h_m (T_{a(m-1)}^1 + T_{am}^1) \right]
\]

\[
\left. \left( 2L_a x^3 + x^4 \right) \right] + Cx + D \quad (5-50)
\]

At \( x=0 \) and at \( x=L_a \), \( v_{axm} = 0 \). Hence \( D = 0 \) and \( C \) has the same value as shown in equations (3-10).

\[
v_{axm} = -\frac{1}{2E_t m h_m} \left[ \left( x^4 + 2 L_a x^3 - L_a^2 x \right) (s_{am} + h_m \left( T_{a(m-1)}^1 + T_{am}^1 \right)) \right]. \quad (3-51)
\]

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FIG. 12. STRIP ELEMENT OF $2 \text{cm}$ UNDER UNIFORM LOAD $s_{\text{um}}$
CHAPTER VI

ANALYSIS OF PLATE \( b_m \) SUBJECTED TO A UNIT MOMENT AT B

Bending Moment Due to \( M_{om}^{1} \)

A unit moment \( M_{om}^{1} \) is applied at the bend of plate \( m \), span BC, as shown in Fig. 13 (a). The bending moment in the plate

\[
M_{dm}^{(0)} = 1 + \frac{x}{L_b}
\]  

(6-1)

Where the subscript \( d \) in \( M_{dm}^{(0)} \) is used to distinguish this moment from \( M_{am}^{(0)} \), \( M_{bm}^{(0)} \), and \( M_{cm}^{(0)} \) which will be introduced later. The superscript \( r \) indicates the plate to which the unit moment is applied.

Shearing Forces Due to \( M_{om}^{1} \)

\[
\tau_{dym}^{(0)} = -\frac{1}{L}
\]

(6-2)

Since, as mentioned in Chapter IV, \( h_m \) is much smaller than \( L_b \), the bending stress \( \tau_{dym} \) and the shear stress \( \tau_{dym}^{(0)} \) may be found from elementary beam theory, and so may their products with \( t_m \).

\[
N_{dym}^{(0)} = \tau_{dym}^{(0)} t_m = \frac{d M_{dym}^{(0)}}{ck} \frac{Ay}{t_m}
\]

\[
N_{dym}^{(0)} = \frac{Q_{dym}^{(0)} \left( \frac{h_m}{2} - y_m \right) \left[ y_m + \frac{1}{2} \left( \frac{h_m}{2} - y_m \right) \right] t_m}{\frac{1}{12} t_m h_m^3}
\]

(6-3)
FIG. 15. PLATE BM WITH
UNIT MOMENT AT B
Simplifying (6-3)

\[
N_{dxm}^{(0)} = \frac{6 \cdot M_{dm}^{(0)} y_m}{h_m} \left( \frac{1}{4} - \frac{y_m^{(2)}}{h_m^2} \right)
\]  

(6-4)

Also, the stress

\[
N_{dxm}^{(0)r} = \frac{12 \cdot M_{dm}^{(0)r} y_m}{h_m^3}
\]  

(6-5)

**Longitudinal shear stress equations**

At the lower edge of the plate \(y_m = +\frac{h_m}{2}\) the normal force \(N_{dxm}^{(0)}\) produces the strain

\[
\varepsilon_{dxm}^{(0)} = \frac{N_{dxm}^{(0)}}{E t_m} = \frac{6 \cdot M_{dm}^{(0)}}{E t_m h_m^3}
\]  

(6-6)

and the strain at the upper edge \(y_{m+1} = -\frac{h_m}{2}\) of the adjacent strip is

\[
\varepsilon_{dx(m+1)}^{(0)} = -\frac{6 \cdot M_{dm}^{(0)} y_{m+1}}{E t_{m+1}^2 h_{m+1}^3}
\]  

(6-7)

Since the strips are connected to each other, these strains ought to be equal.

By making the strains equal the two strips will exert forces upon each other. These additional forces are shearing forces \(T_{dm}^r\) acting on the plane of both strips. These forces are shown in Fig. 14 acting in a positive direction in accordance with the sign convention for \(N_{xy}\) (Fig. 6 b).

From Fig. 14 it is seen that the moment caused by the longitudinal shears is balanced by the moment caused by the vertical shears. Hence the moment \(M_{dm}^{(1)r}\) is 0. Since the unit moment is applied only at one plate, say...
FIG. 14. LONGITUDINAL SHEAR ON "am
WITH UNIT MOMENT AT END B
FIG. 15. LONGITUDINAL SHEAR ON 2 cm WITH UNIT MOMENT AT END B
at \( r = m \), then \( \frac{M_m(0)}{d(m+1)} = M_m(0) = C \). From Fig 14

\[
\frac{d N^m_{dm}}{dx} = T_{d(m-1)} - T_{dm} \tag{6-8}
\]

\[
V_{d} = T_{d(m-1)} \frac{h_m}{2} dx + d M_c^m + T_{dm} \frac{h_m}{2} dx
\]

\[
d M_c^m = V_{dx} - \frac{h_m}{2} (T_{d(m-1)} + T_{dm}) dx
\]

\[
d M_c^m = \frac{h_m}{2} (T_{dm} + T_{d(m-1)}) dx - \frac{h_m}{2} T_{d(m-1)} + T_{dm}) dx
\]

\[
d M_{dm} = 0 \tag{6-9}
\]

The shearing forces \( T_{dm} \) must be known in terms of \( x \) before equation (6-8) can be integrated.

The shearing forces \( T_{dm} \) produce the force \( N^m_{dm} \) from which the normal force

\[
N^m_{dm} = \frac{N^m_{dm}}{h_m} \tag{6-10}
\]

and the edge strain

\[
\varepsilon_{dm} = \frac{N^m_{dm}}{E \cdot t_m \cdot h_m} \tag{6-11}
\]

The strains between the two strips \( m \) and \( m + 1 \) are made equal by the following relation.

\[
\varepsilon_{dm}^{(0)} + \varepsilon_{dm}^{(1)} = \varepsilon_{dm+1}^{(0)} + \varepsilon_{dm+1}^{(1)} \tag{6-12}
\]

where \( m \) goes from 1 to \( K \). After having substituted expressions for the \( \varepsilon_{m} \) the following set of equations is obtained.
A set of equations (6-13) are differentiated in respect to $s$.

Equations (6-13) must be written for every plate from $K = 1$ to $K - 1$ where a unit moment may be applied.

\[
\begin{align*}
N_{(0)}r & = 6 M_{(0)}r + \frac{N_{(0)}h}{E_i} \frac{d}{dx} (N_{(0)}h) + \frac{N_{(0)}h}{E_i} \frac{d}{dx} (N_{(0)}h) \\
N_{(m-1)r} & = 6 M_{(m-1)r} + \frac{N_{(m-1)r}}{E_i} \frac{d}{dx} (N_{(m-1)r}) + \frac{N_{(m-1)r}}{E_i} \frac{d}{dx} (N_{(m-1)r}) \\
N_{(m)r} & = 6 M_{(m)r} + \frac{N_{(m)r}}{E_i} \frac{d}{dx} (N_{(m)r}) + \frac{N_{(m)r}}{E_i} \frac{d}{dx} (N_{(m)r}) \\
N_{(m+1)r} & = 6 M_{(m+1)r} + \frac{N_{(m+1)r}}{E_i} \frac{d}{dx} (N_{(m+1)r}) + \frac{N_{(m+1)r}}{E_i} \frac{d}{dx} (N_{(m+1)r}) \\
N_{(m+2)r} & = 6 M_{(m+2)r} + \frac{N_{(m+2)r}}{E_i} \frac{d}{dx} (N_{(m+2)r}) + \frac{N_{(m+2)r}}{E_i} \frac{d}{dx} (N_{(m+2)r}) \\
\end{align*}
\]
From equations (6-1), (6-9), (6-9) and (6-14)

\[
\frac{dN^{(1)}_{(m+1)}}{dx} = \frac{dN^{(1)}_{(m+2)}}{dx}
\]

\[
\sum_{t_{(m+1)} h_{(m+1)}} = \sum_{t_{(m+2)} h_{(m+2)}}
\]

Equations (6-15) give the distribution of shears \( T_{dm} \) throughout the structure due to a unit moment applied at the end \( B \) of plate \( m \) in span \( BC \). Similar sets of equations result when the unit moment is applied at different plates. Hence \( K-1 \) sets of equations (6-15) may be written for a particular folded plate structure.

It may be seen from equations (6-15) that the absolute term \( \frac{6/L_0}{h_{m} h_{m+1}} \) is independent of \( x \). Hence the shear stress \( T_{dm} \) are also independent of \( x \) and are constant quantities.

The Bending Moment Resultant \( M_{dm} \)

The bending moment resultant of strips \( m, m-1, m \) --- in
span BC due to a unit moment applied at plate r = m may be expressed as

\[
M_{d(m-1)}^F = M_{d(m-1)}^{(0)} + M_{d(m-1)}^{(1)} \\
M_{dm}^F = M_{dm}^{(0)} + M_{dm}^{(1)} \\
M_{d(m+1)}^F = M_{d(m+1)}^{(0)} + M_{d(m+1)}^{(1)}
\]

(6-16)

Since the terms on the right hand side of equation (6-16) are all equal to zero except for \( M_{dm}^{(0)} \) then equations (6-16) may be reduced to

\[
M_{dm}^F = M_{dm}^{(0)}
\]

(6-17)

where \( r = m \). Hence

\[
M_{dm}^F = -1 + \frac{r}{L_b}
\]

(6-18)

**The Shear Resultant \( q_{dm} \)**

The total shears stress resultant

\[
q_{dm} = q_{dm}^{(0)} + q_{dm}^{(1)}
\]

(6-19)

But \( q_{dm}^{(1)} = 0 \) since the vertical shears cancel each other. Hence, substituting

equation (6-2) into equation (6-19)

\[
q_{dm}^F = \frac{1}{L_b}
\]

(6-20)

where \( r = m \).

**The Normal Stress Resultant \( N_{dm} \)**

The total normal stress resultant
\[ N_{d\text{m}} = N_{d\text{m}}^{(0)} + N_{d\text{m}}^{(1)} \quad (6-21) \]

But \( N_{d\text{m}}^{(0)} = C \) since the applied unit moment \( B \) does not produce any net force in the \( x \)-direction at a given cross section. Equation (6-21) is differentiated,

\[ \frac{d N_{d\text{m}}}{dx} = \frac{d N_{d\text{m}}^{(1)}}{dx} \quad (6-22) \]

Equation (6-22) is substituted into equation (6-22) and the result is integrated.

\[ \frac{d N_{d\text{m}}}{dx} = T_d(m-1) - T_{d\text{m}} \quad (6-23) \]

\[ N_{d\text{m}} = (T_d(m-1) - T_{d\text{m}}) x + C \]

At \( x = L_b \)

\[ N_{d\text{m}} = C \quad (\text{Fig. } 15) \]

\[ C = - (T_d(m-1) - T_{d\text{m}}) L_b \]

\[ N_{d\text{m}} = (T_d(m-1) - T_{d\text{m}}) (x - L_b) \quad (6-24) \]

The set of equations for the normal stress resultant \( N_{d\text{m}}^{r} \) due to a unit moment applied at plate \( r = m \) is the following.

\[
\begin{align*}
N_{d(m-2)}^{r} &= (T_{d(m-3)}^{r} - T_{d(m-2)}^{r}) (x - L_b) \\
N_{d(m-1)}^{r} &= (T_{d(m-2)}^{r} - T_{d(m-1)}^{r}) (x - L_b) \\
N_{d\text{m}}^{r} &= (T_{d(m-1)}^{r} - T_{d\text{m}}^{r}) (x - L_b) \\
N_{d(m+1)}^{r} &= (T_{d\text{m}}^{r} - T_{d(m+1)}^{r}) (x - L_b)
\end{align*}
\]

(6-25)
A set of such equations may be written for the unit moment applied at each different plate from plate $K = 1$ to $K - 1$.

The Stress Resultant $N_{dx}$

From elementary theory

$$\sigma_x = \frac{F}{A} + \frac{Mv}{I}$$

Then

$$\sigma_{dzm} = \frac{N_{dm}}{t_m h_m} + \frac{y_m M_{dm}}{t_m^2 b_m^3} \quad (6-26)$$

$$\sigma_{dzm} = \frac{(T_{dm-1} - T_{dm}) (x - L_B)}{t_m h_m} = \frac{12 y_m}{t_m^3 h_m} (-1 + \frac{x}{L_B}) \quad (6-27)$$

$$N_{dzm} = \sigma_{dzm} t_m$$

Hence we may write the set of equations for the stress resultant $N_{dzm}$ due to a unit moment applied at plate $m$ in span $BC$.

$$N_{dx} = \begin{cases} \frac{(T_{r}^{(r)} - T_{r}^{(r)}) (x - L_B)}{d(m-1)} & \text{for } m \leq m-1 \\ \frac{(T_{r}^{(r)} - T_{r}^{(r)}) (x - L_B)}{d(m-1)} + \frac{12 y_m}{t_m^3 h_m} (-1 + \frac{x}{L_B}) & \text{for } m = m \\ \frac{(T_{r}^{(r)} - T_{r}^{(r)}) (x - L_B)}{d(m+1)} & \text{for } m+1 \end{cases} \quad (6-28)$$

A total number of $K-1$ of such sets may be written as the unit moment is transferred from plate to plate.
Shear Stress Resultant \( N_{dy}^{x} \)

\[
N_{dy}^{(r)} = N_{dy}^{(0)} + N_{dy}^{(1)} \quad (6-29)
\]

\[
N_{dy}^{(0)} = \int (0) t_m \, \frac{d M_{dy}^{(0)}}{dx} \, \frac{Q}{t_m} \quad (6-30)
\]

\[
I_m = \frac{1}{12} t_m b_m^3 \quad (6-31)
\]

\[
Q = A \bar{y} = \frac{t_m}{8} (b_m^2 - 4y_m^2) \quad (6-32)
\]

\[
\frac{d M_{dy}^{(0)}}{dx} = \frac{1}{L_b} \quad (6-33)
\]

Equations (6-31) to (6-33) are substituted into equation (6-30)

\[
N_{dy}^{(0)} = \frac{t_m}{8} \left( \frac{b_m^2 - 4y_m^2}{L_b b_m^2} \right) \quad (6-34)
\]

Fig. 16 is analyzed next to determine \( N_{dy}^{(1)} \).

\[
\int \int_{dm} t_m \, d_x = \int_{dm} d_x \, \int y_m \, d N_{dy}^{(1)} \, dy \quad (6-35)
\]

But

\[
N_{dy}^{(1)} = \frac{N_{dy}^{(1)}}{h_m} + \frac{12 b_{dm}^{(1)} y_m}{h_m^{3}} \quad (6-36)
\]

Equations (6-6) and (6-9) are substituted in equation (6-36).

\[
N_{dy}^{(1)} = \frac{1}{k_m} (T^r_{dm} \, d(m-1) - T^r_{dm}) \quad (6-37)
\]

Equation (6-37) is differentiated in respect to \( x \) and then substituted in equation (6-35). The term \( dx \) is common and is therefore dropped.
FIG. 16. VOLUME ELEMENT OF $R \ d m$

WITH UNIT MOMENT AT END $B$
Equations (6-34) and (6-39) are substituted in equation (6-29) to give the following set of equations

\[
\begin{align*}
N^{(l)}_{\text{dxy}(m-1)} &= T^{(r)}_{\text{d}(m-1)} - \frac{1}{h_{m-1}} \left( T^{(r)}_{\text{d}(m-2)} - T^{(r)}_{\text{d}(m-1)} \right) (y - \frac{h_{m-1}}{2}) \\
N^{(r)}_{\text{dxy}} &= \frac{3}{2} \left( \frac{h_{m}^2 - 4 y_{m}^2}{12 h_{m}^3} \right) + T^{(r)}_{\text{d}} - \frac{1}{h_{m}} \left( T^{(r)}_{\text{d}(m-1)} - T^{(r)}_{\text{d}(m+1)} \right) (y - \frac{h_{m+1}}{2}) \\
N^{(l)}_{\text{dxy}(m+1)} &= T^{(r)}_{\text{d}(m+1)} - \frac{1}{h_{m+1}} \left( T^{(r)}_{\text{d}(m)} - T^{(r)}_{\text{d}(m+1)} \right) (y - \frac{h_{m+1}}{2})
\end{align*}
\]

The set of equations (6-40) determine the distribution of the shear stress resultant \(N^{(r)}_{\text{dxy}}\) throughout the structure due to a unit moment applied at end B of plate \(m\) in span BC. A total number of \(K-1\) sets of such equations may be written as the unit moment is moved from plate \(K = 1\) to plate \(K = 1\).

**Rotation and Deflection of Plate bm**

The deflection of plate \(bm\) is subject to the differential equation

\[
\frac{d^2 v^{(r)}_{bm}}{dx^2} = -\frac{12}{E t_{m} h_{m}^3} M_{dm}
\]

Equation (6-18) is substituted into equation (6-41)

\[
\frac{d^2 v^{(r)}_{bm}}{dx^2} = -\frac{12}{E t_{m} h_{m}^3} \left( -1 + \frac{x}{L_{b}} \right)
\]
If integrated once in respect to \( x \), equation (6-42) will yield the slope of the elastic curve of plate \( m \) in span BC due to a unit moment applied at \( r = m \).

\[
\frac{d v}{dr} = \frac{12}{E t_m h_m^3} \left( -x + \frac{x^2}{2} \right) + C
\]  

(6-43)

If equation (6-42) is integrated twice in respect to \( x \), then the equation of the elastic curve is obtained,

\[
v(r) = \frac{12}{E t_m h_m^3} \left( -x + \frac{x^2}{2} + \frac{x^3}{6 L_b} \right) + C_x + D
\]  

(6-44)

The deflection \( v(r) = C \) at \( x = 0 \) and at \( x = L_b \). These conditions are substituted in equation (6-44) to find

\[
D = 0
\]  

(6-45)

\[
C = \frac{4 L_b}{E t_m h_m^3}
\]  

(6-46)

Equation (6-46) is substituted in equation (6-43) to find the rotation of plate \( m \).

\[
\frac{d \phi}{dr} = \frac{12}{E t_m h_m^3} \left( -x + \frac{x^2}{2} \right) + \frac{4 L_b}{E t_m h_m^3}
\]  

(6-47)

\[
\phi(x=0) = \frac{4 L_b}{E t_m h_m^3}
\]  

(6-48)

Equations (6-45) and (6-46) are substituted in equation (6-43) to yield the equation of the deflection curve

\[
v(r) = \frac{12}{E t_m h_m^3} \left( -x + \frac{x^2}{2} + \frac{L_b x}{5} \right)
\]  

(6-49)
CHAPTER VII

ANALYSIS OF PLATE A in SUBJECT TO UNIT MOMENT APPLIED AT END B

Plate A in (Fig. 17) is analyzed in the same way as plate B in Chapter VI. Most of the equations in this chapter are the same as those appearing in Chapter VI. The only differences may be in signs. For this reason explanations have been omitted in this section. The reader may refer back to Chapter VI for more details.

Bending Moment Due to Unit Moment Applied at B,

\[ M_{cm}^{(0)r} = -1 - \frac{x}{L_a} \quad (7-1) \]

Shearing Forces Due to \( M_{cm}^{(0)r} \)

\[ q_{cm}^{(0)r} = - \frac{1}{L_a} \]

\[ N_{cxsym}^{(0)r} = \frac{d}{cm} \frac{M_{cm}^{(0)r}}{dx} \frac{\Delta y}{I_{yy}} \quad (7-2) \]

\[ N_{cxsym}^{(0)} = \frac{d q_{cm}}{dx} \left( \frac{b_m}{2} - y_m \right) \left[ y_m + \frac{1}{2} \left( \frac{b_m}{2} - y_m \right) \right] \frac{t_m}{\frac{1}{12} t_m b_m^3} \quad (7-3) \]

Simplifying (7-3)

\[ N_{cxsym}^{(0)} = \frac{6 q_{cm}}{b_m^3} \left( \frac{1}{3} - \frac{y_m^2}{b_m^2} \right) \quad (7-4) \]

\[ N_{cxsym}^{(0)} = \frac{12 M_{cm}}{b_m^3} \frac{y_m}{t_m} \quad (7-5) \]

56

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Fig. 17. Plate am with unit moment at B.
Longitudinal shear stress Equations

\[ y_m = -\frac{h_m}{2} \]

\[ \varepsilon^{(0)}_{cm} = \frac{F^{(0)}_r}{E t_m} = \frac{6 M_{cm}^{(0)} r}{E t_m h_m^3} \quad (7-6) \]

at \( y_{m+1} = -\frac{h_{m+1}}{2} \)

\[ \varepsilon^{(0)}_{cm+1} = -\frac{6 M_{cm+1}^{(0)}}{E t_{m+1} h_{m+1}^3} \quad (7-7) \]

Since the unit moment is applied only at one plate, say at \( r = m \), then

\[ M_{cm}^{(0)} = M_{cm+1}^{(0)} = 0. \]

From Fig. 18

\[ d M_{cm}^{(1)} = (T_{cm-1} + T_{cm}) \frac{h_m}{2} \, dx - V_{cm} \quad (7-8) \]

\[ d M_{cm}^{(1)} = \frac{h_m}{2} (T_{cm-1} + T_{cm}) \, dx - \frac{h_m}{2} (T_{cm-1} + T_{cm}) \]

\[ d M_{cm}^{(1)} = 0 \]

\[ \frac{d N_{cm}^{(1)}}{dx} = -T_{cm-1} + T_{cm} \quad (7-9) \]

\[ N_{cm}^{(1)} = \frac{N_{cm}}{h_m} \quad (7-10) \]

\[ \varepsilon^{(1)}_{cm} = \frac{N_{cm}}{E t_m h_m} \quad (7-11) \]

The equation of compatibility is

\[ \varepsilon^{(0)}_{cm} + \varepsilon^{(1)}_{cm} = \varepsilon^{(0)}_{cm(m+1)} + \varepsilon^{(1)}_{cm(m+1)} \quad (7-12) \]

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from which

\[
\frac{N^{(1)r}_{c(m-2)}}{E t_{m-2} h_{m-2}} = \frac{N^{(1)r}_{c(m-1)}}{E t_{m-1} h_{m-1}}
\]

\[
\frac{N^{(1)r}_{c(m-1)}}{E t_{m-1} h_{m-1}} = \frac{6 M^{(0)r}_{cm}}{E t_m h_m^2} + \frac{N^{(1)r}_{cm}}{E t_m h_m}
\]

\[
\frac{6 M^{(0)r}_{cm}}{E t_m h_m^2} + \frac{N^{(1)r}_{m}}{E t_m h_m} = \frac{(1)r_{m+1}}{E t_{m+1} h_{m+1}}
\]

\[
\frac{(1)r_{m+1}}{E t_{m+1} h_{m+1}} = \frac{N^{(1)r}_{m+2}}{E t_{m+2} h_{m+2}}
\]

(7-13)

The shears \( T_{cm} \) are substituted in the differentiated form of equations (7-13).

\[
- \frac{T^{Fr}_{c(m-3)} + T^{Fr}_{c(m-2)}}{E t_{m-2} h_{m-2}} = - \frac{T^{Fr}_{c(m-2)} + T^{Fr}_{c(m-1)}}{E t_{m-1} h_{m-1}}
\]

\[
- \frac{T^{Fr}_{c(m-2)} + T^{Fr}_{c(m-1)}}{E t_{m-1} h_{m-1}} = \frac{1/L_a}{E t_m h_m^2} + \frac{T^{Fr}_{c(m-1)} + T^{Fr}_{cm}}{E t_m h_m}
\]

\[
\frac{-1/L_a}{E t_m h_m^2} + \frac{-T^{Fr}_{c(m-1)} + T^{Fr}_{cm}}{E t_m h_m} = \frac{-T^{Fr}_{cm} + T^{Fr}_{c(m+1)}}{E t_{m+1} h_{m+1}}
\]

\[
- \frac{T^{Fr}_{cm} + T^{Fr}_{c(m+1)}}{E t_{m+1} h_{m+1}} = \frac{T^{Fr}_{c(m+1)} + T^{Fr}_{c(m-2)}}{E t_{m+2} h_{m+2}}
\]

(7-14)
FIG. 18. STRIP ELEMENT OF R cm
WITH UNIT MOMENT AT B
The Bending Moment $M_{cm}$

\[
M_{cm}^{(0)} = M_{cm}^{(0)(m-1)} + M_{cm}^{(1)(m-1)} \\
M_{cm}^{(1)} = M_{cm}^{(0)(m)} + M_{cm}^{(1)(m)} \\
M_{cm}^{(0)(m+1)} = M_{cm}^{(0)(m+1)} + M_{cm}^{(1)(m+1)}
\] (7-15)

\[
M_{cm} = M_{cm}^{(0)} \\
M_{cm}^{(r)} = -1 - \frac{x}{L_a}
\] (7-16) (7-17)

The Shear Resultant $Q_{cm}$

\[
Q_{cm}^{(0)} = Q_{cm}^{(0)} + Q_{cm}^{(1)} \\
Q_{cm}^{(r)} = -\frac{1}{L_a}
\] (7-18) (7-19)

The Normal Stress Resultant $N_{cm}$

\[
N_{cm}^{(r)} = N_{cm}^{(0)} + N_{cm}^{(1)} \\
\frac{d}{dx} N_{cm}^{(0)} = \frac{d}{dx} N_{cm}^{(1)} \\
\frac{d}{dx} N_{cm}^{(r)} = -T_{cm}^{(m-1)} + T_{cm}^{(m)} \\
N_{cm}^{(r)} = (-T_{cm}^{(m-1)} + T_{cm}^{(m)}) x + C
\] (7-20) (7-21)
At $x = -L_a \quad N_{cm}^F = 0$

\[
\sigma_{cm}^F = \left( -T_{c(m-1)}^F + T_{cm}^F \right) \left( -L_a \right)
\]

\[
N_{c(m-2)}^F = \left( -T_{c(m-3)}^F + T_{c(m-2)}^F \right) (x - L_a)
\]

\[
N_{c(m-1)}^F = \left( -T_{c(m-2)}^F + T_{c(m-1)}^F \right) (x - L_a)
\]

\[
N_{cm}^F = \left( -T_{c(m-1)}^F + T_{cm}^F \right) (x - L_a)
\]

\[
N_{c(m+1)}^{(r)} = \left( -T_{cm}^F + T_{c(m+1)}^F \right) (x - L_a)
\]

The Stress Resultant $N_{dx}$

\[
\sigma_{cm}^F = \frac{N_{cm}^F}{t_m b_m} + \frac{y_m M_{cm}^F}{12 t_m b_m^3}
\]

\[
\sigma_{cm}^F = \frac{\left( -T_{c(m-1)}^F + T_{cm}^F \right) (x - L_a)}{t_m b_m} + \frac{12 y_m}{t_m b_m^3} \left( -1 - \frac{x}{L_a} \right)
\]

\[
N_{cm}^F = \sigma_{cm}^F t_m
\]

\[
N_{c(m-1)}^F = \frac{\left( -T_{c(m-2)}^F + T_{cm}^F \right) (x - L_a)}{h_m}
\]

\[
N_{c(m-1)}^F = \frac{\left( -T_{c(m-1)}^F + T_{cm}^F \right) (x - L_a)}{h_m} + \frac{12 y_m}{t_m b_m^3} \left( -1 - \frac{x}{L_a} \right)
\]

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\[ N_{cx(m+1)}^r = \left\{ \frac{-T_{cm}^r + T_{cm}^r (x - L_m)}{h_m} \right\} \] (7-26)

\textbf{Shear Stress Resultant } N_{cxym}^r \textbf{ }

\[ N_{cxym}^r = N_{cxym}^{(0)r} + N_{cxym}^{(1)r} \] (7-27)

\[ N_{cxym}^{(0)r} = \left( \frac{d M_{cm}^{(0)r}}{d x} \right) \frac{t_m}{I_m} \] (7-28)

\[ I_m = \frac{1}{12} t_m h_m^3 \]

\[ Q = \Delta \bar{y} = \frac{t_m}{6} \left( b_m^2 - 4 y_m^2 \right) \] (7-29)

\[ \frac{d M_{cm}^{(0)r}}{d x} = -\frac{1}{l_a} \] (7-30)

\[ N_{cxym}^{(1)r} = -\frac{3}{2} \left( \frac{b_m^2 - 4 y_m^2}{L_a h_m^3} \right) \] (7-31)

\[ \left( \frac{1}{c_m} \right) _m dx = T_{cm}^r dx + \int_{y_m}^{h_m/2} d N_{cxym}^{(1)r} dy \] (7-32)

\[ N_{cxym}^{(1)r} = \frac{N_{cm}^{(1)r}}{h_m} + \frac{12 M_{cm}^{(1)r} y_m}{h_m^3} \]

\[ N_{cxym}^{(1)r} = \frac{1}{h_m} \left( -T_{cm}^r (x - L_m) + T_{cm}^r \right) \] (7-33)
\[ v = - \frac{12 - x^2}{2.76} \]

\[ \frac{dv}{du} = 12 \left( x - \frac{x^3}{2} \right) \]

\[ \frac{d^2v}{du^2} = 12 \left( 1 - \frac{x^2}{3} \right) \]

\[ N_x y_{y+1} = 2 \left( b_m - \frac{4y_m}{3} \right) \left( T_{c(m-1)} - \frac{T_{c(m+1)}}{2} \right) + \frac{T_{c(m+1)}}{b_m} \left( b_m - \frac{4y_m}{3} \right) \]

\[ N_x y_{y+1} = 2 \left( b_m - \frac{4y_m}{3} \right) \left( T_{c(m-1)} - \frac{T_{c(m+1)}}{2} \right) + \frac{T_{c(m+1)}}{b_m} \left( b_m - \frac{4y_m}{3} \right) \]

\[ \frac{d}{dx} \left( \frac{N_x}{y_{y+1}} \right) = 2 \left( b_m - \frac{4y_m}{3} \right) \left( T_{c(m-1)} - \frac{T_{c(m+1)}}{2} \right) + \frac{T_{c(m+1)}}{b_m} \left( b_m - \frac{4y_m}{3} \right) \]
\[ C = -\frac{4 L_a}{E t_m h_m^3} \]  

(7-42)

\[ \Theta_c = \frac{12}{E t_m h_m^3} \left( -x - \frac{x^2}{2 L_a} \right) - \frac{4 L_a}{E t_m h_m^3} \]  

(7-43)

at \( x = 0 \)

\[ \Theta_c = -\frac{4 L_a}{E t_m h_m^3} \]  

(7-44)

\[ v_{cm}^1 = \frac{12}{E t_m h_m} \left( -x^2 - \frac{x^3}{6 L_a} \right) - \frac{4 L_a x}{E t_m h_m^3} \]

(7-45)
CHAPTER VIII

PLATE m OF SPAN ABC IS MALE CONTINUOUS

Equation (3-1) states

\[ M_{om} = \frac{\varphi_{am} - \varphi_{bm}}{\varphi_{cm} - \varphi_{dm}} \]  

(3-1)

Now

\[ \varepsilon_d = \frac{-4 L_d}{EI_{am} h_{am}^3} - \frac{4 L_b}{EI_{bm} h_{bm}^3} \]  

(3-1)

\[ \varepsilon_a - \varepsilon_b = \frac{-1}{2 Et_{am} h_{am}^3} \left[ \frac{S_{am} L_a^3 + b_{am} (T_{a(m-1)}^1 + T_{am}^1) L_a^3}{L_a^3} \right] \]  

(3-2)

\[ + \frac{1}{2 Et_{bm} h_{bm}^3} \left[ \frac{S_{bm} L_b^3 - b_{bm} (T_{b(m-1)}^1 + T_{bm}^1) L_b^3}{L_b^3} \right] \]

Equations (3-1) and (3-2) are substituted in equation (3-1) and the redundant moment \( M_{om} \) is found.

\[ M_{om} = \frac{\varphi_{am} - \varphi_{bm}}{\varphi_{cm} - \varphi_{dm}} \left[ \frac{1}{2 Et_{am} h_{am}^3} \left[ \frac{S_{am} L_a^3 + b_{am} (T_{a(m-1)}^1 + T_{am}^1) L_a^3}{L_a^3} \right] \right] \]

\[ + \frac{4 L_d}{EI_{am} h_{am}^3} + \frac{4 L_b}{EI_{bm} h_{bm}^3} \]

\[ + \frac{1}{2 Et_{bm} h_{bm}^3} \left[ \frac{S_{bm} L_b^3 - b_{bm} (T_{b(m-1)}^1 + T_{bm}^1) L_b^3}{L_b^3} \right] \]  

(3-3)

\[ + \frac{3 L_a}{EI_{am} h_{am}^3} + \frac{3 L_b}{EI_{bm} h_{bm}^3} \]  

66
Since $v_{dm}$ is the deflection due to one unit moment applied at $B$, then

$$v_{dm} = v_{dm} M_0$$  \hspace{1cm} (5-4)$$

$$v_{dm} = M_{om} \frac{12}{E t_{bm} h_{bm}^3} \left( \frac{x^3}{6 L_b} - \frac{x^2}{2} + \frac{L_b x}{3} \right)$$  \hspace{1cm} (5-5)$$

Similarly

$$v_{cm} = M_{om} \frac{12}{E t_{am} h_{am}^3} \left( \frac{x^3}{6 L_a} - \frac{x^2}{2} + \frac{L_a x}{3} \right)$$  \hspace{1cm} (5-6)$$

The deflection of the continuous plate $m$ for $x$ larger than zero would consist of the sum of $v_{bm}$ and $v_{dm}$. While for $x$ smaller than zero the deflection would consist of the sum of $v_{am}$ and $v_{cm}$. Hence

for $x > 0$ \hspace{1cm} $v_m = v_{bm} + v_{dm}$ \hspace{1cm} (5-7)$$

$$v_m = \frac{1}{2 E t_{bm} h_{bm}^3} \left[ \left( x^4 - 2 L_b x^3 + L_b^3 x \right) \left( S_m - h_m \left< T^{1}_{b(m-1)} + T^{1}_{bm} \right> \right) \right]$$

$$+ M_0 \frac{12}{E t_{dm} h_{dm}^3} \left( \frac{x^3}{6 L_b} - \frac{x^2}{2} + \frac{L_b x}{3} \right)$$

for $x < 0$ \hspace{1cm} $v_m = v_{am} + v_{cm}$ \hspace{1cm} (5-8)$$

$$v_m = \frac{1}{2 E t_{am} h_{am}^3} \left[ \left( x^4 + 2 L_a x^3 - L_a^3 x \right) \left( S_m + h_m \left< T^{1}_{a(m-1)} + T^{1}_{am} \right> \right) \right]$$

$$+ M_{om} \frac{12}{E t_{am} h_{am}^3} \left( \frac{x^3}{6 L_a} + \frac{x^2}{2} + \frac{L_a x}{3} \right)$$

Similarly stress equations could be written for the continuous plate $m$.

These equations would be too complicated as it is seen in the deflection curve.
equations above. For computation it is easier to solve the equations for the
simply supported plates and then regroup the numerical answers. Hence it is
unnecessary to give the stress equations for the continuous plates in this chapter.
CHAPTER IX

JOINT DISPLACEMENTS

The membrane theory outlined in the previous chapters is rather imperfect. Hence, a bending theory is outlined in the following pages in order to get a more realistic stress analysis.

The procedure used is identical with the usual analysis of statically indeterminate structures. The rigid connections of the plate strips are replaced by piano hinges which can transmit the shear $T_m$, but which cannot transmit plate bending moments $M_y$ from one strip to the next. These moments are later chosen as the redundant quantities. The resulting hinged system is statically indeterminate because of the edge shears. The superscript $(0)$ is going to be used to indicate the load action in the piano hinged system.

Fig. 19 shows a cross section ($x = $ constant) of the structure with plate strips $m-1$, $m$, $m+1$. These plates are considered to be connected to each other by hinges; a study of their deformation is made. The deflection curve equations for these plates have been developed in Chapters IV to VII for simply supported plates and in Chapter VIII for continuous plates.

The corner $m$ in Fig. 19 is at the same time a point of two strips. According to plate $m$ point $m$ has to undergo the displacement $v_m$ while the strip $m+1$ requires that the same point have displacement $v_{m+1}$. To satisfy
both requirements arbitrary displacements normal to each plate strip are added at point \( m \). These normal displacements are justified only if the cross section remains straight and the plates are thin and long enough not to offer any substantial resistance to lengthwise bending and twisting. Hence we add to \( v m \) the normal displacement \( w_m' \) at the corner \( m \) and the displacement \( w_m'' \) at the corner \( m-1 \). Similarly to \( v_{m+1} \) we add the normal displacement \( w_m'' \) at corner \( m \) and \( w_{m+1}' \) at point \( m+1 \).

By simple trigonometry we find the following relations:

\[
\begin{align*}
w_m' \sin \psi_m &= v_{m+1} - v_m \cos \psi_m, \\
\frac{w_m''}{h_m} \sin \psi_m &= v_{m+1} \cos \psi_m - v_m.
\end{align*}
\] (9-1)

The bending moments \( M_x \) and \( M_y \) are small enough to be neglected. On the other hand, the bending moment \( M_y \) is rather large and it must be solved.

Because of the displacements \( w_m'' \) and \( w_m' \) of its ends, the straight line \( m-1, m \) rotates clockwise by the angle

\[
\theta_m = \frac{w_m' - w_m''}{h_m},
\] (9-2)

and the line \( m, m+1 \) rotates by a similar angle \( \theta_{m+1} \). The difference of the two angles \( \theta_{m+1} \) and \( \theta_m \) is the angle \( \psi_m \). Hence

\[
\psi_m = \theta_{m+1} - \theta_m = \frac{w_{m+1}' - w_m''}{h_{m+1}} - \frac{w_m' - w_{m-1}''}{h_m}.
\] (9-3)

Equation (9-1) may be used to express \( w', w'' \) in terms of the deflection \( v \) of the continuous plate. Then equations (9-7) and (9-8) may be used to express...
FIG. 20. REDUNDANT MOMENT $M_r$
v in terms of the loads S and the edge shears T that go with them in the hinged structure. We shall designate the ensuing deformation in eqs. (3-7), (3-8), (3-2), (3-3) by the superscript (0), i.e., \( y_m^{(0)}, \theta_m^{(0)} \). They are the deformations of the principal system under the given load.

In the actual structure the strips are not connected by plane hinges but are so fixed that a relative rotation \( \gamma_m \) cannot take place. It is prevented by bending moments, which deform the straight cross sections shown in Fig. 19 into gentle curves whose tangents meet at the same angles \( \gamma_m \) as do the strips in the unstressed structure.

The moment \( M_r \) transmitted across the edge \( m = r \) from the strip \( r \) to the strip \( r + 1 \) is denoted by \( M_r \) and it depends on \( m \).

We now have to study the internal force system set up by applying the moment \( M_r = 1 \) as an external unit load to the hinged system. The moment shown in Fig. 20(a) is replaced by the two loads in Figs. 20 b, c. The forces and moments shown in Fig. 20(b) are in local equilibrium.

The forces in Fig. 20(c) are applied as loads to the entire structure.

Taking moments about \( r \)

\[
P_{r-1} \left( h_r \cos \theta_r \right) = 1 \quad (3-4)
\]

\[
P_{r+1} \left( h_{r+2} \cos \theta_{r+2} \right) = 1 \quad (3-5)
\]

Hence

\[
P_{r-1} = \frac{1}{h_r \cos \theta_r} \quad (3-6)
\]

\[
P_{r+1} = \frac{1}{h_{r+1} \cos \theta_{r+1}} \quad (3-7)
\]
\begin{align*}
&\textbf{(a)} \\
&\textbf{(b)}
\end{align*}

\textbf{FIG. 21. REDUNDANT MOMENT } M_r \text{ \textbf{RESOLVED} INTO FORCES ACTING IN PLANES OF PLATES}
The vertical load $P$ can be resolved into loads $S$ acting in the planes of the corresponding plate strips. From Fig. 21(a)

$$
S_r^{(r)} = - \frac{P}{\sin \gamma_{r-1}}
$$

$$
S_r^{(r-1)} = - \frac{1}{h_r \sin \gamma_{r-1}}
$$

$$
S_{r+1}^{(r)} = - \frac{P}{\sin \gamma_{r+1}}
$$

The superscript $(r)$ indicates the edge where the moment $M_r = 1$ is applied.

From Fig. 21(b) we obtain the following:

$$
\frac{S_r^{(r)}}{\sin (90^\circ - \Theta_{r+1})} = \frac{P_{r-1} + P_{r+1}}{\sin \gamma_r}
$$

$$
S_r^{(r-1)} = \left( \frac{P_{r-1} + P_{r+1}}{\sin \gamma_r} \right) \frac{\cos \gamma_{r+1}}{\sin \gamma_r}
$$

The expressions for $P_{r-1}$ and $P_{r+1}$ are substituted into equation (9-10). After simplification

$$
S_r^{(r)} = - \frac{\cos \gamma_{r+1}}{h_r \cos \Theta_r \sin \gamma_r} - \frac{1}{h_{r+1} \sin \gamma_r}
$$

$$
S_r^{(r-1)} = S_r^{(r)}
$$

Equations (9-9) and (9-11) are substituted into (9-12).

$$
S_r^{(r)} = \frac{1}{h_{r+1} \sin \gamma_r} + \frac{1}{h_r \cos \Theta_r \sin \gamma_r} \left( \frac{\cos \gamma_{r-1}}{\sin \gamma_{r-1}} + \frac{\cos \gamma_{r+1}}{\sin \gamma_r} \right)
$$
Fig. 21(b) is used again to solve for $s_{r+1}^{(r)}$

$$- \frac{s_{r+1}^{(r)}}{\sin (90^0 + \varphi_r)} = \frac{p_{r-1} + p_{r+1}}{\sin y_r}$$

$$s_{r+1}^{(r)} = - \left( \frac{1}{h_r \cos \varphi_r} + \frac{1}{h_{r+1} \cos \varphi_{r+1}} \right) \frac{\cos \varphi_r}{\sin y_r}$$

$$s_{r+1}^{(r+1)} = \cos \varphi_r \sin y_r - \frac{h_{r+1} \cos \varphi_{r+1} \sin y_r}{h_{r+1} \cos \varphi_{r+1} \sin y_r}$$

(3-14)

From Fig. 21(c)

$$\frac{s_{r+1}^{(r)}}{\sin (90^0 + \varphi_{r+2})} = \frac{p_{r+1}}{\sin y_{r+1}}$$

$$s_{r+1}^{(r+1)} = \frac{-1}{h_{r+1} \cos \varphi_{r+1}} \frac{\cos \varphi_{r+2}}{\sin y_{r+1}}$$

(3-15)

Combining equations (3-14) and (3-15)

$$s_{r+1}^{(r)} = - \frac{1}{h_r \sin y_r} - \frac{1}{h_{r+1} \cos \varphi_{r+1}} \left( \frac{\cos \varphi_r}{\sin y_r} + \frac{\cos \varphi_{r+2}}{\sin y_{r+1}} \right)$$

(3-16)

Again from Fig. 21(c)

$$\frac{s_{r+2}^{(r)}}{\sin (90^0 - \varphi_{r+1})} = \frac{p_{r+1}}{\sin y_{r+1}}$$

$$s_{r+2}^{(r)} = \frac{1}{h_{r+1} \sin y_{r+1}}$$

(3-17)

The resolved loads $s_{r-1}^{(r)}$, $s_{r}^{(r)}$, $s_{r+1}^{(r)}$, $s_{r+2}^{(r)}$ acting on the plane of their respective plate strips are the result of the force system shown in Fig. 21(c).

They may be summarized in the following way.
\[ S_{r-1}^{(r)} = -\frac{1}{h_r \sin \gamma_{r-1}} \]
\[ S_r^{(r)} = \frac{1}{h_{r+1} \sin \gamma_r} + \frac{1}{h_r \cos \gamma_r} \left( \frac{\cos \phi_{r-1}}{\sin \gamma_{r-1}} + \frac{\cos \phi_{r+1}}{\sin \gamma_{r+1}} \right) \]
\[ S_{r+1}^{(r)} = -\frac{1}{h_r \sin \gamma_r} - \frac{1}{h_{r+1} \cos \gamma_{r+1}} \left( \frac{\cos \phi_{r}}{\sin \gamma_{r}} + \frac{\cos \phi_{r+2}}{\sin \gamma_{r+2}} \right) \]
\[ S_{r+2}^{(r)} = -\frac{1}{h_{r+1} \sin \gamma_{r+1}} \]

Equations (9-18) are substituted into equation (4-10), which will then yield the set of edge shears \( T_{m}^{(r)} \) that goes with the unit load \( M_r = 1 \) when section BC is considered simply supported. Hence

\[ \frac{4 T_r^{(r)} + 2 T_{r+1}^{(r)}}{T_{r-1}^{(r)} + 2 T_{r-2}^{(r)}} + \frac{3 T_r^{(r)} + 4 T_{r-1}^{(r)}}{3 T_{r-1}^{(r)} - 3 S_{r-1}^{(r)}} = \frac{t_r h_r}{h_{r-1}} \]
\[ \frac{4 T_r^{(r)} + 2 T_{r+1}^{(r)}}{T_{r+1}^{(r)} + 2 T_{r+2}^{(r)}} + \frac{3 T_r^{(r)} + 4 T_{r+1}^{(r)}}{3 T_{r+1}^{(r)} - 3 S_{r+1}^{(r)}} = \frac{t_r h_r}{h_{r+1}} \]

\[ T_r^{(r)} \text{ is the shear caused along different edges due to the unit moment } M_y \text{ applied at edge } r. \]

The loads \( S_r^{(r)} \) and the shears \( T_r^{(r)} \) must now be introduced into equations (4-51) and (4-52) in order to find the set of angles \( \theta_{b(x=0)} \) and
the set of deflections $v^{(r)}_{0(x=\frac{L}{2})m}$.

At this point adjustments for continuity must be made. Use equations (6-47) and (6-48) to find the rotations $\Theta_{0(x=0)m}$ and the deflections $v_{0(x=\frac{L}{2})m}$ due to a unit moment $M^{(r)}_{0(x=0)m}$. To solve $M^{(r)}_{0(x=0)m}$ equation (3-1) is used as follows

$$M^{(r)}_{0(x=0)m} = \Theta^{(r)}_{0(x=0)m} - \Theta^{(r)}_{0(x=0)m}$$

(3-20)

Now the deflections $v^{(r)}_{m}$ of the continuous plate in supported at A, B, C can be calculated by using equations (8-7) and (8-9). With these values of $v^{(r)}_{m}$ and equations (9-1), (9-2), (9-3) we find the angles $\psi^{(r)}_{m}$ by which the strips rotate with respect to each other in the hinges due to a unit moment $M^{(r)}_{r} = 1$ applied at edge $r$. All these computations must be made separately for every $r$ from $r = 1$ to $r = k-1$.

The angle $\psi^{(r)}_{m}$ represents only the deformation pertaining to the forces shown in Fig. 20(c). The forces and moments of Fig. 20(b) cause additional movements in the hinges. Since this load is in local equilibrium it only causes bending of the strips $r$ and $r+1$, leading to the rotations of the end tangents shown in Fig. 22. Instead of first writing the rotations caused by $M^{(r)}_{r}$ at different hinges and then adding the effects of different such moments on the rotation at the hinge $m$, we may at once write the angle $w^{(r)}_{m} = w^{(r)}_{m} + w^{(r)}_{m}$ caused by the action of moments $M^{(r)}_{m-1}$, $M^{(r)}_{m1}$, $M^{(r)}_{m+1}$. Hence, from a well known beam formula we can write
FIG. 22. DEFINITION OF THE
ANGULAR DISPLACEMENTS \( w' \) & \( w'' \)
where \( K_m \) is the bending stiffness of the \( m \)th strip and is calculated from the equation

\[
K_m = \frac{E \cdot t_m}{12(1 - \nu^2)}
\]  
(9-22)

Now we may collect all the contributions to the relative rotation taking place at the hinge \( m \). The given loads yield the value \( \gamma_m \) obtained from equation (9-3) in the way already described. The moment \( M_r \) at any arbitrary hinge \( r \) makes of contribution \( \gamma_m^r \cdot M_r \), and we have to write the sum of all these contributions from \( r = 1 \) to \( r = K - 1 \). Lastly, there is the contribution \( w_m \) of equation (9-21). The sum of all these is the relative rotation of the strips \( m \) and \((m+1) \) in the hinge \( m \), and since there is no hinge in the actual structure, the rotation must equal zero.

\[
\sum_{r=1}^{K-1} \gamma_m^r \cdot M_r + \frac{h_m}{6K_m} M_{m-1} + \frac{1}{3} \left( \frac{h_m}{K_m} + \frac{h_{m+1}}{K_{m+1}} \right) M_m + \frac{h_{m+1}}{6K_{m+1}} m_{m+1} + \gamma_m^j = 0
\]  
(9-23)

There are \((K-1)\) such equations for the edges \( m = 1, 2 \ldots (K-1) \), and these are just enough linear equations for the \((K-1)\) unknown moments \( M_r \). When these equations have been solved, we may easily retrace our steps and calculate all the stress resultants and displacements we desire.
CHAPTER X

ANALYTICAL SOLUTION OF EXPERIMENTAL PRISM

In this chapter a plate structure (Fig. 23) 24 feet long and supported at diaphragms A, B, C (Fig. 24) is analyzed for stresses and maximum deflections when the structure is subjected to a uniform load of 100 lb/ft applied at the upper edges b and c.

We consider the structure to be cut at the middle support thus consisting of two simply supported structures symmetrical about point B. Section BC is first analyzed. The vertical loads are resolved into components acting on the planes of the plate strips (Fig. 25). The longitudinal shears are calculated by substituting numerical values into equation (4-19). After simplification the

<table>
<thead>
<tr>
<th>m</th>
<th>( P_m ) lb/ft</th>
<th>( \theta_m )</th>
<th>( y_m = \theta_{m+1} - \theta_m )</th>
<th>( \cos \theta_m )</th>
<th>( \sin \theta_m )</th>
<th>( S'_m )</th>
<th>( S''_m )</th>
<th>( S = S'_m + S''_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>270</td>
<td>60</td>
<td>0</td>
<td>( \frac{600}{6} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>100</td>
<td>330</td>
<td>30</td>
<td>( \frac{600}{6} )</td>
<td>( \frac{500}{6} )</td>
<td>-200</td>
<td>+173.2</td>
<td>-200</td>
</tr>
<tr>
<td>c</td>
<td>100</td>
<td>360</td>
<td>30</td>
<td>1.00</td>
<td>( \frac{500}{6} )</td>
<td>-173.2</td>
<td>+200</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>( \frac{600}{6} )</td>
<td>( \frac{360}{6} )</td>
<td>0</td>
<td>0</td>
<td>+200</td>
</tr>
<tr>
<td>e</td>
<td>0</td>
<td>90</td>
<td>--</td>
<td>0</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE 10-1
FIG. 25. LOADS RESOLVED INTO FORCES ACTING IN THE PLANES OF THE PLATES
four simultaneous shear equations are

\[
\begin{align*}
32.62 \ T^1_{b1} & + 5.88 \ T^1_{b2} & = -1727.4 \\
5.88 \ T^1_{b1} & + 23.76 \ T^1_{b2} & + 5.88 \ T^1_{b3} & + 25.76 \ T^1_{b2} & + 5.88 \ T^1_{b4} & = 1727.4 \\
6 \ T^1_{b2} & + 25.76 \ T^1_{b3} & + 5.88 \ T^1_{b4} & = 1727.4
\end{align*}
\]

The shear constants \( T^1 \) produced by the vertical uniform load of 100 lb/ft applied at edges \( b \) and \( c \) are given in Table 10-2.

<table>
<thead>
<tr>
<th>Long shear Constants</th>
<th>lb/ft²</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T^1_{b1} )</td>
<td>-37.79</td>
</tr>
<tr>
<td>( T^1_{b2} )</td>
<td>-34.75</td>
</tr>
<tr>
<td>( T^1_{b3} )</td>
<td>+34.75</td>
</tr>
<tr>
<td>( T^1_{b4} )</td>
<td>+37.79</td>
</tr>
</tbody>
</table>

**TABLE 10-2**

Longitudinal Shear Constants for Section BC

Because of symmetry the longitudinal shear constants \( T^1_a \) for section \( AB \) are equal and opposite to the shear constants \( T^1_b \) (Fig. 26a).
FIG. 26. LONGITUDINAL SHEARS

- \( T_{b3} = 84.75 \frac{\text{lb}}{\text{ft}^2} \)
- \( T_{b4} = 37.79 \frac{\text{lb}}{\text{ft}^2} \)
- \( T_{d3} = +0.323 \frac{\text{lb}}{\text{ft}^2} \)
- \( T_{d4} = -0.206 \frac{\text{lb}}{\text{ft}^2} \)

- \( M_0 = 1 \)
TABLE 10-3

Longitudinal shear Constants for Section AB

Next, equation (4-51) is used to calculate the rotations of simply supported plates due to the uniform load of 100 lb/ft acting at edges b and c.

Equation (4-51) yields the results shown in Table 10-4.

<table>
<thead>
<tr>
<th>Rotation of Plates</th>
<th>Radii ans</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{b1}$</td>
<td>-.001463</td>
</tr>
<tr>
<td>$\theta_{b2}$</td>
<td>-.000194</td>
</tr>
<tr>
<td>$\theta_{b3}$</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_{b4}$</td>
<td>+.000194</td>
</tr>
<tr>
<td>$\theta_{b5}$</td>
<td>+.001463</td>
</tr>
</tbody>
</table>

TABLE 10-4

Rotation of Plates in Span BC Due to Uniform Load

Because of symmetry the rotations at $x = c$ for span AB are equal and opposite to those shown in Table 10-4.
**TABLE 10-5**

Rotation of Plates in Span AB Due to Uniform Load

Equation (4-53) is used to find the deflections of plates between B and C. The results are shown in Table 10-6.

C. The results are shown in Table 10-6.

<table>
<thead>
<tr>
<th>Rotation of Plates</th>
<th>Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{a1}$</td>
<td>+.001403</td>
</tr>
<tr>
<td>$\theta_{a2}$</td>
<td>+.000194</td>
</tr>
<tr>
<td>$\theta_{a3}$</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_{a4}$</td>
<td>-.000194</td>
</tr>
<tr>
<td>$\theta_{a5}$</td>
<td>-.001403</td>
</tr>
</tbody>
</table>

**TABLE 10-6**

Deflections of Plates at $x = \frac{L_p}{2}$ Due to Uniform Load

Because of symmetry about the y-axis, the deflections of plates at $x = \frac{L_p}{2}$ are equal to those shown in Table 10-6.

<table>
<thead>
<tr>
<th>Deflections of Plates at $x = \frac{L_p}{2}$</th>
<th>Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{b1}$</td>
<td>-.003075</td>
</tr>
<tr>
<td>$v_{b2}$</td>
<td>-.009736</td>
</tr>
<tr>
<td>$v_{b3}$</td>
<td>0</td>
</tr>
<tr>
<td>$v_{b4}$</td>
<td>+.009736</td>
</tr>
<tr>
<td>$v_{b5}$</td>
<td>+.003075</td>
</tr>
</tbody>
</table>
The two 12-foot sections (i.e. AB and BC) are now analyzed for shears, rotations, and deflections due to unit moments applied at end B of each different plate. When the unit moment is applied at end B of plate a, equations (6-15) will yield the following four simultaneous equations.

\[
\begin{align*}
16.26 T_{b1}^a &- 5.88 T_{b2}^a = 6.73 \\
5.88 T_{b1}^a &- 11.33 T_{b2}^a + 6 T_{b3}^a = 0 \\
6 T_{b2}^a &- 11.33 T_{b3}^a + 5.88 T_{b4}^a = 0 \\
5.33 T_{b3}^a &- 16.26 T_{b4}^a = 0
\end{align*}
\]

Similarly, for a unit moment applied at end B of plate b.

\[
\begin{align*}
16.36 T_{b1}^b &- 5.83 T_{b2}^b = 1.44 \\
5.83 T_{b1}^b &- 11.33 T_{b2}^b + 6 T_{b3}^b = -1.44 \\
6 T_{b2}^b &- 11.33 T_{b3}^b + 5.83 T_{b4}^b = 0 \\
5.33 T_{b3}^b &- 16.36 T_{b4}^b = 0
\end{align*}
\]
For a unit moment applied at end B of plate C

\[
16.26 \ T_{b1}^c - 5.88 \ T_{b2}^c = 0 \\
5.88 \ T_{b1}^c - 11.33 \ T_{b2}^c + 6 \ T_{b3}^c = -1.50 \\
6 \ T_{b2}^c - 11.68 \ T_{b3}^c + 5.88 \ T_{b4}^c = -1.50 \\
5.88 \ T_{b3}^c - 16.26 \ T_{b4}^c = 0. 
\]

For a unit moment applied at end B of plate d

\[
16.26 \ T_{b1}^d - 5.88 \ T_{b2}^d = 0 \\
5.88 \ T_{b1}^d - 11.33 \ T_{b2}^d + 6 \ T_{b3}^d = 0 \\
6 \ T_{b2}^d - 11.68 \ T_{b3}^d + 5.88 \ T_{b4}^d = -1.44 \\
6.33 \ T_{b3}^d - 16.26 \ T_{b4}^d = -1.44. 
\]

For a unit moment applied at end B of plate e

\[
16.26 \ T_{b1}^e - 5.88 \ T_{b2}^e = 0 \\
5.88 \ T_{b1}^e - 11.33 \ T_{b2}^e + 6 \ T_{b3}^e = 0 \\
6 \ T_{b2}^e - 11.68 \ T_{b3}^e + 5.88 \ T_{b4}^e = 0 \\
5.88 \ T_{b3}^e - 16.26 \ T_{b4}^e = -6.73. 
\]

The results of the above five sets of shear equations are tabulated in Table 10-8.

<table>
<thead>
<tr>
<th>Unit M applied at B of plate</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{b1} )</td>
<td>.559</td>
<td>.306</td>
<td>.145</td>
<td>.672</td>
<td>.089</td>
</tr>
<tr>
<td>( T_{b2} )</td>
<td>.401</td>
<td>.323</td>
<td>.400</td>
<td>.199</td>
<td>.247</td>
</tr>
<tr>
<td>( T_{b3} )</td>
<td>.247</td>
<td>.189</td>
<td>.400</td>
<td>.323</td>
<td>.401</td>
</tr>
<tr>
<td>( T_{b4} )</td>
<td>.039</td>
<td>.072</td>
<td>.145</td>
<td>.298</td>
<td>.559</td>
</tr>
</tbody>
</table>

**TABLE 10-8**

Shear Forces in lb/ft for span BC Due to Unit Moments Applied at end B of Different Plates
Because of symmetry about the $y_m$ axis the shears in the simply supported structure AB are equal and opposite [Fig. 26(a)] to those tabulated in Table 10-8.

<table>
<thead>
<tr>
<th>Unit Moment Applied at End B of Plate</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{a1}$</td>
<td>-0.559</td>
<td>-0.206</td>
<td>-0.165</td>
<td>-0.072</td>
<td>-0.089</td>
</tr>
<tr>
<td>$T_{a2}$</td>
<td>-0.401</td>
<td>-0.323</td>
<td>-0.400</td>
<td>-0.199</td>
<td>-0.247</td>
</tr>
<tr>
<td>$T_{a3}$</td>
<td>-0.247</td>
<td>-0.199</td>
<td>-0.400</td>
<td>-0.333</td>
<td>-0.401</td>
</tr>
<tr>
<td>$T_{a4}$</td>
<td>-0.089</td>
<td>-0.072</td>
<td>-0.145</td>
<td>-0.206</td>
<td>-0.559</td>
</tr>
</tbody>
</table>

**TABLE 10-9**

Shear Forces in $\frac{\text{lb/ft}}{\text{in}}$ for Span AB Due to Unit Moments Applied at End B of Different Plates

When a unit moment is applied at the end of a plate it causes the plate to rotate thus giving rise to deflections. The rotation of plates in span BC at B are found by substituting numerical values in Equation (6-47) and are tabulated in Table (10-10).

<table>
<thead>
<tr>
<th>Angles at x=0 in Span BC</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{d1}$</td>
<td>2.673 x 10^{-6}</td>
</tr>
<tr>
<td>$\theta_{d2}$</td>
<td>2.159 x 10^{-7}</td>
</tr>
<tr>
<td>$\theta_{d3}$</td>
<td>2.296 x 10^{-7}</td>
</tr>
<tr>
<td>$\theta_{d4}$</td>
<td>2.158 x 10^{-7}</td>
</tr>
<tr>
<td>$\theta_{d5}$</td>
<td>2.673 x 10^{-6}</td>
</tr>
</tbody>
</table>

**TABLE 10-10**

Deflections at B of Plates in Span BC Due to Unit Moments Applied at B
Because of symmetry the angles of corresponding plates in the simply supported structure in span AB are the same in magnitude but opposite in sign as those shown in Table 10-10. The set of angles $\Theta_c(x=0)$ is found by using equation (7-44).

<table>
<thead>
<tr>
<th>Angles at $x=0$ in Span AB</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_{c1}$</td>
<td>$-2.673 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Theta_{c2}$</td>
<td>$-2.158 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Theta_{c3}$</td>
<td>$-2.296 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Theta_{c4}$</td>
<td>$-2.168 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Theta_{c5}$</td>
<td>$-2.673 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 10-11
Rotations at B of Plates in Span AB Due to Unit Moments Applied at B.

The deflections at $x = +\frac{L}{2}$ due to unit moment at $x=0$ are calculated by equation (6-48). The deflections at $x = -\frac{a}{2}$ can be calculated by equation (7-45). Because of symmetry about the $y_m$ axis, however, the deflections are the same at both point $x = \frac{L}{2}$ and point $x = -\frac{a}{2}$. Hence both results are tabulated in Table 10-12. At this point we must make adjustments for continuity.

The redundant Moment $M_o$ is found by substituting numerical values into equation (31). The results are shown in Table 10-13.
FIG. 27. ROTATIONS AT B DUE TO UNIT MOMENT AT B
TABLE 10-12

Deflections at Midspans Due to Unit Moments Applied at B

<table>
<thead>
<tr>
<th>Redundant Moments</th>
<th>ft-lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{o,1}$</td>
<td>-524.21</td>
</tr>
<tr>
<td>$M_{o,2}$</td>
<td>-901.19</td>
</tr>
<tr>
<td>$M_{o,3}$</td>
<td>0</td>
</tr>
<tr>
<td>$M_{o,4}$</td>
<td>+901.19</td>
</tr>
<tr>
<td>$M_{o,5}$</td>
<td>+524.81</td>
</tr>
</tbody>
</table>

TABLE 10-13

Redundant Moments at Support B.

The deflections due to unit moments shown in Table 10-13 are now multiplied by the respective $M_{o}$ (i.e., $v_{c1} = y_{c1} x M_{o,1}$ etc.). The results are given in Table 10-14.
To find the deflections at midspans for the continuous system due to the uniform load of 100 lb/ft the deflections of Table 10-14 are subtracted from those of Table 10-6. The results are shown in Table 10-15.

<table>
<thead>
<tr>
<th>Deflections at x = -L/3</th>
<th>Deflections at x = +L/3</th>
<th>Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>v₁</td>
<td>v₀</td>
<td>0.02521</td>
</tr>
<tr>
<td>v₂</td>
<td>v₁</td>
<td>-0.03227</td>
</tr>
<tr>
<td>v₃</td>
<td>v₂</td>
<td>0</td>
</tr>
<tr>
<td>v₄</td>
<td>v₃</td>
<td>0.03227</td>
</tr>
<tr>
<td>v₅</td>
<td>v₄</td>
<td>0.02521</td>
</tr>
</tbody>
</table>

**TABLE 10-15**

Deflections at Midspans of the Continuous Structure Under the Uniform Load of 100 lb/ft.
It must be remembered at this point that the deflections of Table 10-15 have been found under the assumption that the plates were connected to one another by piano hinges. We know, however, that the edges are rigidly fixed. Hence the theory of Chapter IX must be applied here to find the redundant moments $M_y$ and study their effect.

Let us examine the deformation of the hinged system at $x = \frac{L}{2}$. Equations (3-1) are used to find the quantities $w'_m, w''_m$. From Fig. 28 we find the value $\gamma'_m$.

| $\gamma'_1$ | 60°  |
| $\gamma'_2$ | 30°  |
| $\gamma'_3$ | 30°  |
| $\gamma'_4$ | 60°  |

**TABLE 10-16**

The values of $w'_m$ and $w''_m$ are tabulated below.

<table>
<thead>
<tr>
<th>Displacements</th>
<th>Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w'_1$</td>
<td>+.01052</td>
</tr>
<tr>
<td>$w'_2$</td>
<td>+.02710</td>
</tr>
<tr>
<td>$w'_3$</td>
<td>+.00604</td>
</tr>
<tr>
<td>$w''_2$</td>
<td>+.06974</td>
</tr>
<tr>
<td>$w'_3$</td>
<td>+.06974</td>
</tr>
<tr>
<td>$w'_4$</td>
<td>+.00604</td>
</tr>
<tr>
<td>$w''_3$</td>
<td>+.02710</td>
</tr>
<tr>
<td>$w''_4$</td>
<td>+.01052</td>
</tr>
</tbody>
</table>

**TABLE 10-17**

Normal Displacements (see Fig. 28)
Equation (9-2) is used to calculate the angles $\Theta_m$. These values are tabulated in Table 10-18.

<table>
<thead>
<tr>
<th>Angles</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_1$</td>
<td>$-8.5959 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Theta_2$</td>
<td>$-8.5959 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Theta_3$</td>
<td>0</td>
</tr>
<tr>
<td>$\Theta_4$</td>
<td>$+8.5959 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Theta_5$</td>
<td>$+8.5959 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

**TABLE 10-18**

Transversal angles at $x = \pm \frac{L}{2}$ and at $x = -\frac{L}{2}$

The angles $\psi_m$ are calculated from equation (9-3) and are given in Table (10-19).

| $\psi_1$ | 0                |
| $\psi_2$ | $+8.5959 \times 10^{-4}$ rad |
| $\psi_3$ | $+8.5959 \times 10^{-4}$ rad |
| $\psi_4$ | 0                |

**TABLE 10-19**

Angles at $x = \pm \frac{L}{2}$ and at $x = -\frac{L}{2}$ (Fig. 29).

Since the strips are not connected by piano hinges in the actual structure, the relative rotation $\psi_m$ cannot take place; it is prevented by the bending moments $M_y$. Let the moment $M_y$ transmitted across the edge $m = x$ from the strip $r$ to
the strip \( r + 1 \) be denoted by \( M_r \). Now, we will study the internal force system set up by applying this moment with \( M_r = 1 \).

When moment \( M_r = 1 \) is applied at edge \( b \) of the structure (Fig. 30) equations (9-18) yield the results shown in Table 10-20. The superscript \( b \) above the \( S \) denotes the edge where the moments \( M_r = 1 \) is applied.

<table>
<thead>
<tr>
<th>Tangential Loads</th>
<th>1b/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>-5.8656</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>+2.1212</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>-2.7117</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>+1</td>
</tr>
<tr>
<td>( S_5 )</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 10-20**

Tangential Loads in the Planes of Four Strips (Fig. 30) Due to Unit Moment \( M_r = 1 \) Applied at Edge \( b \).

When moment \( M_r = 1 \) is applied at edge \( c \) of the structure (Fig. 31), equations (9-13) yield the results shown in Table 10-21.

<table>
<thead>
<tr>
<th>Tangential Loads</th>
<th>1b/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>0</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>-1.3000</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>+2.7117</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>-2.1312</td>
</tr>
<tr>
<td>( S_5 )</td>
<td>+.8656</td>
</tr>
</tbody>
</table>

**TABLE 10-21**

Tangential Loads in the Planes of Four Strips (Fig. 31) Due to Unit Moment \( M_r = 1 \) Applied at Edge \( c \).
FIG. 30. REDUNDANT MOMENT $L_1 = 1$

APPLIED AT EDGE $b$
FIG. 31. REDUNDANT MOMENT $M = 1$ AT $c$
RESOLVED INTO FORCES ACTING IN
THE PLANES OF PLATES
FIG. 32. NUMERICAL VALUES OF LOADS $S^b$
$S_3 = 2.717 \text{ lb/ft}$

$S_4 = -2.132 \text{ lb/ft}$

$S_5 = 0.565G$

FIG. 33. NUMERICAL VALUES OF $S^c$
Next we may calculate shears $T$ caused by unit moments $M_x = 1$ applied separately at edges b and c. For $M_x = 1$ (Fig. 32) applied at edge b, equations (4-19) yield the following simultaneous equations.

$$
32.5116 T_1^{(b)} + 5.8775 T_2^{(b)} = -4.440
$$

$$
5.8775 T_1^{(b)} + 23.7550 T_2^{(b)} + 6 T_3^{(b)} = -6.0
$$

$$
6.0000 T_2^{(b)} + 23.7550 T_3^{(b)} + 5.8775 T_4^{(b)} = -15.769
$$

$$
5.8775 T_3^{(b)} + 32.5116 T_4^{(b)} = 3.6364
$$

For $M_x = 1$ (Fig. 33) applied at edge c, equations (4-19) yield the following simultaneous equations.

$$
32.5116 T_1^{(c)} + 5.3775 T_2^{(c)} = -5.6364
$$

$$
5.8775 T_1^{(c)} + 23.7550 T_2^{(c)} + 6.000 T_3^{(c)} = +15.7689
$$

$$
6.0000 T_2^{(c)} + 23.7550 T_3^{(c)} + 5.8775 T_4^{(c)} = +6.0
$$

$$
5.8775 T_3^{(c)} + 32.5116 T_4^{(c)} = 4.440
$$

The shear stresses due to unit moments applied transversely at edges b and c are tabulated in Table 10-22.

| $T_1^{(b)}$ | -13.13 |
| $T_2^{(b)}$ | -6.291 |
| $T_3^{(b)}$ | -7.560 |
| $T_4^{(b)}$ | -0.4023 |
| $T_1^{(c)}$ | -4.023 |
| $T_2^{(c)}$ | +7.560 |
| $T_3^{(c)}$ | +6.291 |
| $T_4^{(c)}$ | +13.14 |

**TABLE 10-22**

Shear Stresses Due to Unit Moments Applied Transversely at Edges b and c.
The loads \( S^{(c)} \) and the edge shears \( T^{(c)} \) must now be introduced into equations (4-51) and (4-53) to find rotations at \( x = 0 \) and deflections \( z = \frac{L_b}{2} \). The results are shown in Tables (10-23 and 10-24) respectively. The structure at this point still consists of two separate simply supported sections.

\[
\begin{array}{|c|c|}
\hline
\phi_{b1}^{(c)} & +2.2345 \times 10^{-5} \text{ rad.} \\
\phi_{b2}^{(c)} & -0.95510 \times 10^{-5} \\
\phi_{b3}^{(c)} & +0.47169 \times 10^{-5} \\
\phi_{b4}^{(c)} & -0.66893 \times 10^{-5} \\
\phi_{b5}^{(c)} & +1.49213 \times 10^{-5} \\
\hline
\end{array}
\]

**TABLE 10-23**

Rotations at \( x = 0 \) of plates in Span BC Due to \( M_y = 1 \) 
Applied at Edge b and \( M_y = 1 \) Applied at Edge c

Because of symmetry about the \( y_m \) axis the angles of rotation at \( x = 0 \) of plates in span AB due to the moment \( M_y = 1 \) separately applied at edges b and c are equal and opposite to the values appearing in Table 10-23.

Because of symmetry about the \( y_m \) axis the deflections at \( x = \frac{L_b}{2} \) of plates in span AB due to the moment \( M_y = 1 \) separately applied at edges b and c are equal to the values of Table 10-24.
\[
\begin{array}{|c|c|}
\hline
v_{b1} & +.001005 \text{ inches} \\
\hline
v_{b2} & -.0004293 \\
\hline
v_{b3} & +.0003123 \\
\hline
v_{b4} & -.0003010 \\
\hline
v_{b5} & +.0006714 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
v_{b1} & -.000714 \text{ inches} \\
\hline
v_{b2} & +.0003010 \\
\hline
v_{b3} & -.0002123 \\
\hline
v_{b4} & +.0004298 \\
\hline
v_{b5} & -.001005 \\
\hline
\end{array}
\]

**TABLE 10-24**

Deflections at \( x = \frac{L}{2} \) of Plates in Span BC Due to

\( M_y = 1 \) Applied at Edge \( b \) and \( M_y = 1 \) Applied at Edge \( c \)

We can now make adjustments for continuity. Equation (3-1) becomes

\[
M_{o(x=0)m} = \frac{e_{cm}^{(r)} - e_{cm}^{(c)}}{e_{cm} - e_{cm}^{(r)}}.
\]

The angles \( e_{cm}^{(r)} \) and \( e_{cm}^{(c)} \) are given in Table 10-22, while the angles \( e_{cm} \) and \( e_{cm}^{(c)} \) are given in Tables 10-11 and 10-10 respectively. The resulting values of the redundant moments \( M_{o(x=0)m} \) are tabulated in Table 10-25.
The deflections in Table 10-12 are multiplied by the redundant moments in Table 10-25. The results are given in Table 10-26.

The deflections of the continuous beams ABC due to the moments $M_r$ are now calculated. The deflections of Table 10-12 are subtracted from the deflections of Table 10-24. The results are the deflections of the plates of the continuous structure due to uniform moments $M_r = 1$ separately applied at edges $b$ and $c$.

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<th>Moments at $x=0$</th>
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<td>$M_{0,1}$</td>
<td>$+8.360$ ft-lb</td>
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<tr>
<td>$M_{0,3}$</td>
<td>$+20.544$</td>
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<td>$M_{0,4}$</td>
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<td>$+44.259$</td>
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<tr>
<td>$M_{0,5}$</td>
<td>$-8.360$</td>
</tr>
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</table>

**TABLE 10-25**

Redundant Moments $M_0(x=0)$ Due to Moments $M_r = 1$
### TABLE 10-26

Deflections at Midspan of BC Due to Redundant Moments $M_{o}(r)$

Applied to Plates at B

The deflections at midspan of AD are the same as those shown in Table 10-26.

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<tr>
<th>Deflections at $X = rac{1}{2}b/2$</th>
<th>Inches</th>
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</thead>
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<tr>
<td>$v_{d_1}$</td>
<td>+6.0342 x $10^{-4}$</td>
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<td>$v_{d_2}$</td>
<td>-2.5790 x $10^{-4}$</td>
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<td>$v_{d_3}$</td>
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<td>$v_{d_5}$</td>
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### TABLE 10-27

Deflections of the Plates of the Continuous Structure Due to

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<th>Deflections</th>
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<td>$v_{e}$</td>
<td>+0.0008629</td>
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The deflections of Table 10-27 are substituted in equations (9-1) and the resulting \( w^2 \) are shown in Table 10-28.

| \( w^1 \) (b) | \( 0.0009185 \) in. |
| \( w^1 \) (b) | 0.0004814 |
| \( w^2 \) (b) | 0.0004677 |
| \( w^2 \) (b) | 0.0004910 |
| \( w^3 \) (b) | -0.0003880 |
| \( w^3 \) (b) | -0.0003785 |
| \( w^4 \) (b) | 0.0003795 |
| \( w^4 \) (b) | 0.0003795 |
| \( w^5 \) (b) | 0.0002940 |
| \( w^5 \) (b) | 0.0002940 |
| \( w^6 \) (b) | 0.0003795 |
| \( w^6 \) (b) | 0.0003795 |
| \( w^7 \) (b) | -0.0003785 |
| \( w^7 \) (b) | -0.0003785 |
| \( w^8 \) (b) | 0.0004910 |
| \( w^8 \) (b) | 0.0004910 |
| \( w^9 \) (b) | 0.0004814 |
| \( w^9 \) (b) | 0.0004677 |

**TABLE 10-28**

Normal Displacements to Each Plate Strip Due to Moments \( M_T=1 \) Applied at Edges b and c alternately.
The values in Table 10-28 are substituted in equation (9-2) and the results are given in Table 10-29.

| \( \theta_1^{(b)} \) | \(-5.592 \times 10^{-7}\) rad. |
| \( \theta_2^{(b)} \) | \(-5.592 \times 10^{-7}\) |
| \( \theta_3^{(b)} \) | \(-3.6625 \times 10^{-5}\) |
| \( \theta_4^{(b)} \) | \(3.0939 \times 10^{-5}\) |
| \( \theta_5^{(b)} \) | \(3.0939 \times 10^{-5}\) |
| \( \theta_1^{(c)} \) | \(-3.0939 \times 10^{-5}\) |
| \( \theta_2^{(c)} \) | \(-3.0939 \times 10^{-5}\) |
| \( \theta_3^{(c)} \) | \(3.6625 \times 10^{-5}\) |
| \( \theta_4^{(c)} \) | \(3.5918 \times 10^{-7}\) |
| \( \theta_5^{(c)} \) | \(5.5918 \times 10^{-7}\) |

**TABLE 10-29**

*Transversal Rotation of Plate Strips at Midspans of Continuous Structure Due to \( m_e = 1 \)*

The relative change of angle at the joints is given by equation (9-3). Values from Table 10-29 are substituted in equations 9-3 and the results are given in Table 10-30.

The values from Table 10-30 are substituted in equation (9-22) to find the redundant moments \( M_r \). The results are given in Table 10-31.
TABLE 16-30

Relative Angle Changes at Joints Due to $M_r = 1$
Alternately Applied at Joints b and c

| $\chi_{(a)}$ | $\chi_{(b)}$ | $\chi_{(c)}$ | $\chi_{(d)}$ |
| 1 | $-2.6066 \times 10^{-5}$ rad. | $+6.7564 \times 10^{-5}$ | 0 |
| 2 | $6.7564 \times 10^{-5}$ | $-3.6066 \times 10^{-5}$ | 0 |
| 3 | $-3.6066 \times 10^{-5}$ | 0 | 0 |
| 4 | 0 | 0 | 0 |

TABLE 16-31

Redundant Moments Acting Uniformly along the Edges b and c of the continuous structure

| $M_2$ | $M_3$ |
| -11.0594 ft-lb | -11.0594 ft-lb |

The deflections of Table 16-27 are multiplied by the redundant moments of Table 16-31 and the results are subtracted from the deflections given in Table 16-15.

Hence the final deflections of the structure are given in Table 16-32.
\begin{array}{|c|}
\hline
v_1 &= -0.003011\ \text{in.} \\
v_2 &= -0.002873\ \text{in.} \\
v_3 &= 0 \\
v_4 &= +0.002873\ \text{in.} \\
v_5 &= +0.0016011 \\
\hline
\end{array}

**TABLE 10-3:**

Final Deflections of the Continuous Structure Due to a Uniform Load of 100 lb/ft Applied at Edge b and c.
Fig. 34 Three-lead wires connect the gages to the equipment. Electronic gages are connected to Budd instruments while the load cells are connected to Baldwin Lima instruments.
CHAPTER XI

EXPERIMENTAL PROGRAM

In order to learn the actual pattern of deflections and of stress distribution of continuous folded plate structures a simple prismatic structure was made out of concrete ($f_C = 6,000$ psi). The maximum size aggregate used in the concrete was $3/8"$. Reinforcing steel was used as shown in Fig. 47. The two spans were $13'-6"$ each from center of supports. The thickness of the three top plates was $1"$ while the two vertical plates containing $3/8"$ $\phi$ bars were $1 1/2"$ thick (Fig. 47). To avoid concentration of stresses in lifting, the whole structure was constructed right on the load cells through the use of a platform which was removed after 28 days from the day of pouring. Concrete cylinders were tested after 28 days and the results are given in Fig. 51.

Strain gages (SR4, type-C6-141-B with gage factor 2.05 ± 1/2%) were applied on the steel as well as on the concrete surface (Fig. 43). The results are given in Tables 11-1 and in Figs. 52-56. Gauge indicators were set up as shown in Figs. 49-50 and the results are given in Tables 11-2, 11-3 and in Fig. 57.

Selected pictures are introduced in the following pages to avoid a long and tedious description of the experiment.

114
Fig. 36. Some of the strain gages are applied on the steel imbedded in the concrete, others are applied directly on the concrete surface. SR4 gages were used.
Fig. 36. Electronic equipment used to read strains of the strain gages.
Fig. 37. Strain gage readings and dial indicator readings are taken simultaneously.
Fig. 38. Dial indicators are placed symmetrically around sections A and B to check whether the deflections of the symmetrical structure were the same on both sides. The results are given in Table 11-2.
Fig. 39. Dial indicators are set up at midspans A and B to study the deflection pattern of the structure. Because of symmetry the dial indicators are placed only on one side of the structure. The results are given in Table 11-3.
Fig. 40. The dial indicators are applied on stiff frames made out of 2" x 2" x 1/4" steel angles.
Fig. 41. Concrete cylinders are tested to determine the modulus of elasticity $E_c$. The resulting curve is given in Fig. 51. The types of steel (3/3 in, 3/16 in) which were used in the structure are also shown in these pictures.
Fig. 42. Pieces of pig iron (each weighing about 40 lbs.) are loaded on a wooden platform. The uniform load is transferred to edges b and c of the structure through 1" x 2" wood strips.
Fig. 43. The picture below shows pressure cylinders which may be used to apply concentrated loads, at edge b and c of the structure.
Fig. 44. A simply supported folded plate structure is loaded to failure.
Fig. 46. Typical Pattern of cracks in folded plate structures.
Fig. 46. Typical cracks along line of longitudinal shear.
FIG. 46. LOCATION OF STRAIN GAUGES
FIG. 50. SETUP OF GAUGE INDICATORS
AT MIDSPAN SECTIONS A & B
(TABLE 11-3)
\[ E = 2,178,000 \text{ for } 0 < \sigma < 2831 \text{ psi} \]

\[ E = 860,000 \text{ for } 2831 < \sigma < 6016 \text{ psi} \]
## RESULTS OF STRAIN GAUGES (strains in microinches)

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<th>394 lb/ft</th>
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### Table 11-1

Strains in microinches

(continued)

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**TABLE 11-1**

Strains in **Microinches**

Inch

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SR4 gauges applied on the reinforcing steel.

SR4 gauges applied on the concrete surface.

FIG. 52. STRESS DISTRIBUTION IN psi AT SECTION A-A (FIG. 48)
DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED
AT EDGES 2 & 3 OF BOTH SPANS.
SR4 gauges applied on the reinforcing steel
SR4 gauges applied on the concrete surface

Fig. 53. Stress distribution in psi at section A-A (Fig. 48) due to uniform loads of 500 lbs/ft applied at edges 2 & 3 of both spans.
FIG. 54. STRESS DISTRIBUTION IN psi AT THE MIDDLE SUPPORT SECTION DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED AT EDGES 2 & 3 OF BOTH SPANS. THE STRESS READINGS ABOVE ARE FROM SR4 GAUGES APPLIED ON THE CONCRETE SURFACE.
FIG. 55. STRESS DISTRIBUTION (psi) IN THE REINFORCING STEEL AT SECTION C-C AND D-D (FIG. 48) DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED AT EDGES 263 OF BOTH SPANS.
SR4 gauges applied on the reinforcing steel
SR4 gauges applied on the concrete surface

FIG. 56. STRESS DISTRIBUTION (psi) AT SECTION B-B(Fig. 46)
DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED
AT EDGES 283 OF BOTH SPANS
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**TABLE II-3**
FIG. 57. EXPERIMENTAL DEFLECTIONS AT SECTION A-A

(Fig. 48) due to uniform loads of 1000 lbs/ft

Applied at edges 2 & 3 of both spans.

The readings are in \( \frac{1}{10000} \) " (Table 11-3)
CHAPTER XII

CONCLUSION

The experimental program described in Chapter XI yields relatively consistent results. The concrete prismatic structure was subjected to repeated identical tests in order to obtain a realistic pattern of deflection and stress distribution. The results are shown in Tables 1-1, 1-2, 4-3, and Figs. 52 to 57.

No comparison can be made between the experimental results and analytical results obtained from equations developed from Filigge’s theory.

In the deflection equation (3-7) we have the factor \( S_m - h_m \left( T_{b(m-1)}' + T_{b(n)}' \right) \), where \( S_m \) is the uniform load and \( T_{b(m-1)}' \) as well as \( T_{b(n)}' \) are constants related to the longitudinal shears. When equation (3-7) is solved the uniform load \( S_m \) produces certain deflection in the direction in which it itself acts (Fig. 58) while the shear constants tend to produce a deflection opposite to that of \( S_m \).

The sum of the two deflections should be smaller than and in the same direction as the one produced by \( S_m \). These shearing constants from Filigge’s theory, however, seem to be too large. As a result the term \( h_m \left( T_{b(m-1)}' + T_{b(n)}' \right) \) larger than \( S_m \) thus upsetting the direction and numerical value of the deflection.

If the shearing constants could logically be made smaller the whole theory would seem to conform with experimental results. It is therefore necessary to carry on further studies on the longitudinal shearing stresses in
Filippe's theory and then apply the results to the theory for continuous folded plates developed in this paper. After the revision has been made the same theory may be used for any kind of loading condition by expressing these loads in Fourier series.
deflection due to uniform load

plate

longitudinal shear

deflection due to uniform load

+ 

deflection due to longitudinal shear

resulting deflection

FIG. 58. RELATION OF DEFLECTIONS DUE TO $s_m$ AND $t_m$


Roberts, B. C. "Folded Plate Structures." Thesis presented to the University of Western Australia, Australia, in January, 1960, in partial fulfillment of the requirements for the degree of Honorary Bachelor of Engineering.


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In May, 1963, he was graduated from "Le Scuole Medie Di San Nicola" in Alvito, Frosinone, Italy. In December, 1953, he came to Canada.

In May, 1955, he completed Canadian elementary education at De La Salle School in Windsor, Ontario, Canada. In September of the same year he enrolled at Assumption High School, Windsor, Ontario, where he obtained his secondary education.

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In May, 1963, he was graduated from Assumption University of Windsor with a Bachelor of Applied Science degree. In September, 1963, he continued his studies at the University of Windsor in order to obtain the degree of Master of Applied Science in Civil Engineering.