A modified electron optical system and the gamma-electron angular correlation of gold-198.

Murray Lawrence Trudel
University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/6406

This online database contains the full-text of PhD dissertations and Masters' theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000 ext. 3208.
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.
A MODIFIED ELECTRON OPTICAL SYSTEM
AND THE $\gamma - \alpha^-$ ANGULAR CORRELATION
OF Au$^{198}$

BY

MURRAY LAWRENCE TRUDEL

A Thesis
Submitted to the Faculty of Graduate Studies through the Department of Physics in Partial Fulfillment of the Requirements for the Degree of Master of Science at The University of Windsor

Windsor, Ontario
1965
ABSTRACT

The electron optics used in angular correlation measurements were modified in order to measure both the $A_2$ and the $A_4$ correlation coefficients of the correlation function:

$$W(\alpha) = \sum_K A_{2K} P_{2K} (\cos \alpha)$$

A new experimental method for the study of the electron conversion process involving the $b_2$ particle parameter is outlined.

The modified optics system was employed in a $\gamma - \alpha$ directional correlation experiment for the $2^+ \rightarrow 2^+ \rightarrow 0^+$ cascade in $^{198}\text{Au}$. The results of the experiment were analysed according to the new method above.

From this experiment, the value of the $b_2$ particle parameter obtained was:

$$b_2 = 1.19 \pm 0.10$$

This result agrees with the theoretical calculation of $b_2$ obtained by Biedenharn and Rose (1953) within the limits of experimental error.
ACKNOWLEDGEMENTS

I would like to express my appreciation to Dr. E. E. Habib for his guidance and instruction throughout this work.

My thanks go to Mr. W. Grewe for the part he played in the construction of the modified electron optics system. I would like to thank Miss Linda Cummings for typing the manuscript. I am indebted to the Government of Ontario for financial support in the form of a bursary.
TABLE OF CONTENTS

ABSTRACT

ACKNOWLEDGEMENTS

CHAPTER I ANGULAR CORRELATIONS IN NUCLEAR SPECTROSCOPY

i) Introduction 1
ii) Angular Correlation of Successive Radiations 1
iii) The Purpose of Angular Correlations 2
iv) Theory of Angular Correlations
   a) Introduction 3
   b) The Theoretical Directional Correlation Function 4
v) Particle Parameters 9
vi) The New Technique
   a) Introduction 11
   b) The New Method For Accurate b_2 Determinations 11

CHAPTER II CORRECTIONS FOR FINITE SOLID ANGLE

i) Corrections for the _7^"_ Channel 14
ii) Corrections for the _6^"_ Channel 20
iii) The P_0, P_2, P_4 Baffles 24
iv) A Final Note 27

CHAPTER III THE BAFFLE SYSTEMS

i) The Gerholm Instrument 28
ii) Ring Focus and Baffles 28
iii) Resolution and Transmission 31
iv) The A_2 and A_4 Baffle Systems 32
   a) Special Remarks 32
   b) Choice of Emission Angle 33
v) A_4 Baffle Construction 34
   a) The Camera and Special Baffles 35
   b) The Photography 38
   c) The P_4 Baffle 43

CHAPTER IV PRELIMINARY TESTING OF THE BAFFLES

i) The A_4 Baffle System 44
   a) The Shadow Effect 44
   b) Resolution vs Transmission 44
ii) Testing of the P_4 Baffle 47
CHAPTER V THE Au\textsuperscript{198} EXPERIMENT

i) Introduction 50

ii) Preparation 50

a) Source Preparation 50
b) \( \gamma \)-Probe Centering 53
c) The Spectrometer Settings 53
d) The Experimental Arrangement 53

iii) Experimental Procedure 56

iv) Treatment of Data and Calculations 56

v) Errors 59

vi) Conclusions 59

BIBLIOGRAPHY 62
LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>35</td>
</tr>
<tr>
<td>II</td>
<td>47</td>
</tr>
<tr>
<td>III</td>
<td>57</td>
</tr>
<tr>
<td>IV</td>
<td>60</td>
</tr>
<tr>
<td>V</td>
<td>60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>57</td>
</tr>
<tr>
<td>IV</td>
<td>60</td>
</tr>
<tr>
<td>V</td>
<td>60</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Illustration</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Schematic of a Directional Correlation Measurement</td>
<td>5</td>
</tr>
<tr>
<td>1.2</td>
<td>A γ-γ Cascade</td>
<td>8</td>
</tr>
<tr>
<td>1.3</td>
<td>A Parametric Plot</td>
<td>10</td>
</tr>
<tr>
<td>2.1</td>
<td>A Geometry of Two detectors</td>
<td>15</td>
</tr>
<tr>
<td>2.2</td>
<td>Graphs of J₂ / J₀</td>
<td>18</td>
</tr>
<tr>
<td>2.3</td>
<td>Graphs of J₄ / J₀</td>
<td>19</td>
</tr>
<tr>
<td>2.4</td>
<td>A Geometry of an L⁻ Channel</td>
<td>21</td>
</tr>
<tr>
<td>2.5</td>
<td>Graphs of P₂ (cos α) and P₄ (cos α)</td>
<td>23</td>
</tr>
<tr>
<td>2.6</td>
<td>The P₀, P₂, P₄ Baffles</td>
<td>25, 26</td>
</tr>
<tr>
<td>3.1b</td>
<td>A Schematic of the α-α⁻ Spectrometer</td>
<td>29</td>
</tr>
<tr>
<td>3.1a</td>
<td>A Schematic of the α⁻-γ Spectrometer</td>
<td>29</td>
</tr>
<tr>
<td>3.2</td>
<td>Triangular Field Distribution</td>
<td>30</td>
</tr>
<tr>
<td>3.3</td>
<td>Ring Focus</td>
<td>31</td>
</tr>
<tr>
<td>3.4</td>
<td>A Peak Profile</td>
<td>31</td>
</tr>
<tr>
<td>3.5</td>
<td>The Camera</td>
<td>36</td>
</tr>
<tr>
<td>3.6</td>
<td>Front view of Camera in Spectrometer</td>
<td>36</td>
</tr>
<tr>
<td>3.7</td>
<td>Al Baffle with Annular Slit</td>
<td>37</td>
</tr>
<tr>
<td>3.8</td>
<td>Prints of 4 Photographs</td>
<td>39, 40</td>
</tr>
<tr>
<td>3.9</td>
<td>An Enlarged Print</td>
<td>41</td>
</tr>
<tr>
<td>3.10</td>
<td>A Schematic of the Au⁻ Baffle</td>
<td>42</td>
</tr>
<tr>
<td>4.1a</td>
<td>Profiles of the 662 Kev Conversion Line of Cs¹³⁷</td>
<td>45</td>
</tr>
<tr>
<td>4.1b</td>
<td>Profiles of the 662 Kev Conversion Line of Cs¹³⁷</td>
<td>46</td>
</tr>
<tr>
<td>5.1</td>
<td>The Au¹⁹⁸ Decay Scheme</td>
<td>51</td>
</tr>
<tr>
<td>5.2</td>
<td>The 412 Kev Conversion Line of Au¹⁹⁸</td>
<td>52</td>
</tr>
<tr>
<td>5.3</td>
<td>A Block Diagram of Circuit Layout</td>
<td>54</td>
</tr>
<tr>
<td>5.4</td>
<td>A Two Fold Coincidence Circuit</td>
<td>55</td>
</tr>
<tr>
<td>5.5</td>
<td>The 412 Kev and 679 Kev Photopeaks</td>
<td>58</td>
</tr>
</tbody>
</table>
CHAPTER I

ANGULAR CORRELATIONS IN NUCLEAR SPECTROSCOPY

(i) Introduction

If a nucleus undergoes emission of two radiations in succession (nuclear $\alpha$, $\beta$, or $\gamma$ - radiation, or secondary radiations from the surrounding electron shells), their correlations are an attractive and useful field of study. Such studies can be divided into essentially two kinds: spatial correlations and time correlations. The first ones (angular correlations) constitute the topic of this chapter. For completeness, however, it may be said that time correlations studies concern the time delay between the two transitions (delayed coincidences) and that in many cases, the purpose of these studies is the determination of the coincidence intensity of two kinds of transitions, i.e. the number of transitions of the first kind following (or preceding) the other in the construction of decay schemes. Often, only a very rough knowledge of coincidence intensities is necessary for this last purpose.

(ii) Angular Correlation of Successive Radiations

The photons emitted by a sample in which a large number of nuclei are undergoing identical $\gamma$ - ray transitions will be isotropic in the laboratory coordinates. There is no preferred direction of emission for the $\gamma$ - ray photon from the individual transition $I_a \rightarrow I_b$ because the atoms and nuclei are oriented at random. The same is true for $\alpha$ - ray, $\beta$ - ray and conversion electron emission. If the transition $I_a \rightarrow I_b$ is followed by a second transition $I_b \rightarrow I_c$, the individual radiations from the second transition are likewise isotropic in the laboratory coordinates.
However, in a two step cascade transition, such as
\[ I_a \xrightarrow{\gamma} I_b \xrightarrow{\gamma'} I_c \] there is often an angular correlation between the directions of emission of two successive \( \gamma \)-ray photons \( \gamma_1 \) and \( \gamma_2 \), which are emitted from the same nucleus. Often there are similar angular correlations for other pairs of successive radiations, such as \( \alpha-\gamma \), \( \beta-\gamma \), \( \beta-\alpha^- \) (where \( \alpha^- \) means a conversion electron), \( \gamma-\alpha^- \), .......

The existence of an angular correlation arises because the direction of the first radiation is related to the orientation of the angular momentum \( I_B \) of the intermediate level. This orientation can be expressed in terms of the magnetic angular momentum quantum number \( m_B \) with respect to some laboratory direction such as that of the first radiation. If \( I_B \) is not zero, and if the lifetime of the intermediate level is short enough so that the orientation of \( I_B \) persists, then the direction of emission of the second radiation will be related to the direction of \( I_B \) and hence to that of the first radiation.

(iii) The Purpose of Angular Correlations

The angular correlation of radiations emitted or absorbed in nuclear processes has been one of the principal tools available for the nuclear spectroscopy of excited states.

The information that can be obtained from angular correlation work depends on the type of radiation observed (\( \alpha \), \( \beta \), \( \gamma \), \( \alpha^- \)), on the properties that are singled out by the experiment (direction, polarization, energy), and on the extranuclear fields acting on the nucleus.

If we assume that the decaying nuclei are free, i.e. that no extranuclear fields act on the nucleus and disturb its orientation in the intermediate state, then, to the extent that this can be realized in practice, angular correlation measurements provide information about the properties of the nuclear levels involved and about the angular momenta carried away by the

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
radiations. To be more precise, $\alpha - \gamma$ and $\gamma - \gamma$ directional correlations yield the spins of the nuclear levels, but not the parities. The relative parities can be determined, however, if one observes in addition to the direction, the polarization of the $\gamma$-rays, or if one measures the directional correlation between conversion electrons ($\alpha^\prime$). $\alpha - \gamma$ correlations provide new information concerning the details of nuclear structure, viz, the newly discovered "nuclear penetration matrix elements" [Church and Weneser, (1956)]. Angular correlations that involve either the polarization of $\beta$-particles or the circular polarization of $\gamma$-rays, yield information on the interaction in $\beta$-decay.

The information that can be obtained from the influence of extranuclear fields on the nuclear angular correlation is also multifold. Very often, one can determine the quadrupole coupling from the change of the correlation due to extranuclear fields. In many cases, one can measure the $g$-factor of an excited nuclear state by observing the directional correlation as a function of an external magnetic field. From the $g$-factor one gets the magnetic moment if the spin of the nuclear state is known, e.g. from the unperturbed directional correlation.

The above specified information by no means exhausts the vast amount of knowledge obtainable from angular correlations. Further elaboration, however, is beyond the scope of this work.

(iv) Theory of Angular Correlations

(a) Introduction

The theory of angular correlation is highly developed and general expressions can be written down for even very complex situations. However, the general formulation of the theory is very complicated and is somewhat lengthy. Very good formulations of the theory are present in the literature.
[Biedenharn and Rose, (1953)]. We will therefore present in this section a simple treatment of directional correlation that, while restricted in its usefulness, provides some insight into the correlation mechanism.

The common feature of all correlation problems is to be found in the fact that they involve an initial nuclear state of sharp angular momentum $I_1$ and parity that undergoes successive transformations either emitting or absorbing radiations through intermediate nuclear states of sharp angular momenta $I_a, I_b, \ldots$ with sharp parity and terminating as a nucleus with sharp angular momentum $I_2$ and parity. This assumption of sharp angular momenta and parity is restrictive and distinguishes the correlation problem from say the closely related problem of the angular distribution of nuclear reactions which show coherent mixing of the intermediate nuclear states.

(b) The Theoretical Directional Correlation Function

The schematic picture of a directional correlation measurement is shown in Figure 1.1. A nuclear cascade involving states $a, b, c$ with spins $I_a, I_b, I_c$ occurs through the successive emission of particles (or photons) $R_1$ and $R_2$. (It is assumed here that the spins and parities of the states $a, b, c$ are well defined).

The most convenient form in which to express the correlation function $W(\Theta)$ is:

$$W(\Theta) = \sum_{K=-\infty}^{\infty} A_K P_K(\cos \Theta)$$  \hspace{1cm} (1.1)

The functions $P_K$ are Legendre polynomials. The coefficients $A_K$ contain all the physical information. Usually one chooses $A_0 = 1$, so that the correlation function integrated over all angles is unity:

$$\int W(\Theta) \, d\Omega = 1$$  \hspace{1cm} (1.2)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 1.1 Schematic of a Directional Correlation Measurement

(a) Nuclear cascade

(b) $\mathcal{W}(\phi)\,d\omega$ is the relative probability that the radiation $R_1$ is emitted into the solid angle $d\omega$ at an angle $\phi$ with respect to $R_2$.

(c) The two counters 1 and 2 subtend an angle $\phi$ at the source. From the coincidence count rate as a function of angle, $C(\phi)$, one obtains after suitable correction the correlation function $\mathcal{W}(\phi)$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
At this point we introduce the following nomenclature for a double cascade, viz., \( I_a (L_1) I_b (L_2) I_c \) where \( I_a, I_b \) and \( I_c \) denote the angular momenta of the first, intermediate and final nuclear levels, and \( L_1 \) and \( L_2 \) are the angular momenta of the two successive radiations.

Three general rules for the coefficients \( A_\chi \) hold for all types of particles:

\[
K \text{ integer, even} \hspace{1cm} (1)
\]
\[
0 \leq K_{\text{max}}. \leq \min \left( 2I_b, 2L_1, 2L_2 \right) \hspace{1cm} (2)
\]
\[
A_\chi = A_\chi (a,b) A_\chi (b,c) \hspace{1cm} (3)
\]

Rule 1 expressing the fact that only even Legendre polynomials appear, holds as long as one measures only directions and linear polarizations of the radiations. The observation of the circular polarization, however, introduces also odd integers.

Rule 2 states that the highest term \( K_{\text{max}} \) in the expansion is smaller than, or equal to, the smallest of the three numbers \( 2I_b, 2L_1, 2L_2 \) where \( L_1 \) and \( L_2 \) are the angular momenta carried away by \( R_1 \) and \( R_2 \). This rule implies isotropic correlation if the angular momentum of either of the radiations or of the intermediate state equals 0 or \( 1/2 \).

Rule 3 expresses the fact that the coefficients \( A_\chi \) for a cascade can be broken up into two factors, each factor depending on only one transition of the cascade. The second factor \( A_\chi (b,c) \) for instance is entirely determined by the properties of the levels \( b \) and \( c \) of the radiation \( R_2 \).

Equation 1.1 is very general indeed, and with the proper evaluation of \( A_\chi \) it applies to all two step cascades \( \alpha - \gamma, \beta - \gamma, \gamma - \alpha^- \), \( \alpha^- - \alpha^- \), etc. as well as to nuclear scattering experiments and nuclear disintegrations.
Numerical values for the factors $A_k(a,b)$ for given properties of the levels (a) and (b) and of the radiation $R$ have been calculated and tabulated for most cases of interest.

As an example of the above, consider the directional correlation between successive gamma rays in the cascade shown in Figure 1.2(a) ($I_a = 2^+$, $I_b = 2^+$, $I_c = 0^+$). The second gamma ray in this cascade must be pure quadrupole, and from the references in the literature, we find the corresponding directional correlation factors $A_2(b,c) = -0.5976$, $A_4(b,c) = -1.069$. If we assume the first transition to be pure also, for instance dipole radiation ($L_1 = 1$), we find immediately $A_2(a,b) = -0.4183$ and $A_4(a,b) = 0$ and hence $A_2 = 0.2500$ and $A_4 = 0$. These values have to be compared with experiment.

Generally, however, the first transition in a $2^+ ightarrow 2^+ ightarrow 0^+$ cascade will be mixed, i.e. the radiation carried away involves more than one single value of angular momentum. Angular momentum selection rules allow $L_1 = 1, 2, 3$ or 4. As a rule only the two lowest multiples, $L_1 = 1$, and $L_1' = 2$, occur with measurable intensity [cf. Figure 1.2(b)]. The coefficients $A_2(a,b)$ and $A_4(a,b)$ and hence also $A_2$ and $A_4$ now become continuous functions of the mixing ratio $\xi$, which in this case indicates the ratio of amplitude of the quadrupole to that of the dipole contribution ($\xi^2$ is defined as the ratio of the total intensity of the $L_1$ pole to that of the $L$ pole).

The most convenient representation of the dependence of $A_2$ and $A_4$ on $\xi$ is a parametric plot [Coleman, (1958)]; one draws the curve $A_2 = A_2(\xi)$, $A_4 = A_4(\xi)$ in an $A_2 - A_4$ plane [The functions $A_2(\xi)$ and $A_4(\xi)$ are calculated from the formulas and coefficients by Frauenfelder (1955), Biedenharn and Rose (1953), Ferentz and Rosenzweig...
Figure 1.2 A $\gamma-\gamma$ cascade.

(a) $2^+ \rightarrow 2^+ \rightarrow 0^+$ cascade (where the superscript $+$ denotes even parity), occurring frequently in even-even nuclei.

(b) The first gamma ray $\gamma_1$ is usually mixed in this case; the two dominant angular momenta being $L_1 = 1$, $L'_1 = 2$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
A parametric plot for the cascade $2^+ \rightarrow 2^+ \rightarrow 0^+$ is shown in Figure 1.3. The experimental points with coordinates $A_2, \text{exp.}$ and $A_0, \text{exp.}$ must lie on this curve and thus yield values for sign and magnitude of the mixing ratio $\delta$.

(v) Particle Parameters

At this point we recall the form of the $A_K$ coefficients, viz, $A_K = A_K(a,b)A_K(b,c)$. From the theory of angular correlations it may be shown [Biedenharn and Rose, (1953)] that the individual factors $A_K(a,b)$ and $A_K(b,c)$ also factor into two separate factors namely into a "nuclear factor" which depends on specific nuclear properties, and a "geometrical factor" which describes the geometry of the problem.

It is convenient to adopt the $\gamma-\gamma$ angular correlation function as the standard representation of the "geometry" of the correlation process and to express the angular correlation for different radiations by defining a multiplicative particle parameter $b_K$ to effect the necessary change from the standard correlation. Of course, the choice of the $\gamma-\gamma$ correlation as the standard one is arbitrary. We could have chosen the $\alpha-\alpha$ correlation which, from the point of view of analytical simplicity and availability of numerical results, would have been almost equally suitable. However, the fact that the parity restriction precludes the mixing of even and odd $L$ militates against this choice. For $\gamma$-rays the parity for given $L$ depends on the character of the radiation (electric or magnetic) and mixtures of all $L, L'$ pairs (consistent with angular momentum conservation) are permissible in principle.

Particle parameters $b_K$ may be introduced to express the angular correlation between any two successive radiations. The detailed information obtainable from the use of these parameters is beyond the scope of this work.
Figure 1.3  A parametric plot.

Parametric plot of the directional correlation coefficients $A_2$ and $A_4$ for the cascade $2^+ \rightarrow 2^+ \rightarrow 0^+$. $\delta$ is the mixing ratio (quadrupole to dipole radiation) in the first transition. The curve is labeled with the values of the parameter $\delta' = \frac{\delta}{|\delta|}$. Two experimental points are shown.
Of particular interest to us here is the $\gamma - e^-$ angular correlation since the experiment performed in this thesis was the 679 Kev $\gamma - 412$ Kev $e^-$ angular correlation in Au$^{198}$. A new experimental technique involving the $b_K$ particle parameters was employed.

(vi) The New Technique

(a) Introduction

A new experimental technique for the study of the internal conversion process has been developed. The new method implies an indirect and accurate determination of the particle parameter $b_2$ from an experimental measurement of the $b_4$ particle parameter. It is shown that the accuracy in the value for $b_2$ thus obtained is considerably higher than would be possible on the basis of a direct measurement of $b_2$. On the basis of the theory of Green and Rose (1958), the parameter $b_2$ when accurately determined, may be used to derive experimental values for any dynamic contributions to the conversion process if they are present. However, to be decisive for this analysis $b_2$ has to be known to within a few percent. An experimental accuracy of this order has so far not been possible to achieve.

(b) The New Method For Accurate $b_2$ Determinations

The usual expansion of the correlation function $W(\phi)$ in terms of Legendre polynomials, reads, in the case of gamma - gamma directional correlations:

$$W(\phi, \gamma \gamma) = \sum_{K} A_{2K} (\gamma \gamma) P_{2K} (\cos \phi) \quad (1.3)$$

For pure gamma - electron ($\gamma - e^-$) correlations a similar expansion applies:

$$W(\phi, \gamma e^-) = \sum_{K} b_{2K} A_{2K} (\gamma e^-) P_{2K} (\cos \phi) \quad (1.4)$$
Thus from a study of the gamma - electron and gamma - gamma correlations measured with the same source and under identical experimental conditions we obtain:

\[
b_2 = \frac{A_2 (\gamma^2)}{A_2 (\gamma \gamma)} = b_2^* \\
b_4 = \frac{A_4 (\gamma^2)}{A_4 (\gamma \gamma)} = b_4^*
\]

(1.5)  

(1.6)

This comparison method has the additional advantage that the particle parameters determined by (1.5) and (1.6) are unaffected by all attenuations caused by extra-nuclear fields since the attenuation coefficients if present disappear in the ratios.

The value for \( b_2 \) thus obtained cannot be sufficiently accurately determined to be of significance for analysis of dynamic effects. However, the two particle parameters are not mutually independent. As shown by Biedenharn and Rose (1953) \( b_2 \) and \( b_4 \) are inter-related by means of a recursion formula. In this particular case, with the numerical constants introduced, the recursion formula reads:

\[
b_2 = 1.4 - \frac{b_4}{2.5}
\]

(1.7)

Consequently we may use our experimentally determined \( b_4 \) value to derive a corresponding value for \( b_2 \). Moreover, because of the appearance of the numerical factors 1.4 and the 2.5 in the denominator the relative error in \( b_2 \) is considerably smaller than the corresponding error in \( b_4 \). Thus, provided \( b_4 \) can be measured with a reasonable degree of accuracy, the corresponding value for \( b_2 \) will be quite precisely determined. This condition is fulfilled whenever \( A_4 (\gamma \gamma) \) is sufficiently large which
is the case for the 679 Kev - 412 Kev gamma gamma cascade in Au$^{198}$.

This indirect way to arrive at a $b_2$ value from a measurement of $b_4$, results in an accuracy which is considerably higher than is possible to achieve in any direct determination of $b_2$ from the measured $A_2(\xi^2)$ and $A_2(\xi^4)$.
CHAPTER II
CORRECTIONS FOR FINITE SOLID ANGLE

(i) Corrections for the Gamma Channel

The coincidence counting rate per unit solid angle \( \sin \theta \, d\theta \, d\phi \) (where the angles represent the relative orientation of the propagation vectors of the two radiations) for an angular correlation measurement is proportional to:

\[
W(\theta) = \sum_{K,\phi} A_{2K} P_{2K} (\cos \theta)
\]

The measured correlation, however, is less than the true correlation because of the finite solid angle that the two detectors must subtend in any feasible experiment.

When the detectors for the two radiations subtend finite solid angles \( \alpha \) and \( \beta \) at the source, it is advisable to modify the theoretical correlation and compare this smeared correlation with the measured one. The geometry envisaged is shown in Figure 2.1. The detectors (scintillation counters) are assumed to be crystals cut in the form of right circular cylinders with the base oriented towards the source. The source, at the origin, is on the intersection of the axes of the cylinders. In this case, as the following shows, the form of the correlation function is unchanged and each coefficient \( A_{2K} \) becomes multiplied by an attenuation factor for which one can obtain an exact and very simple expression. Taking into account the absorption of the radiation in each crystal, we introduce the following notation. The distance from the source to the front face of each crystal is \( h \), the thickness is \( t \), and \( r \) is the radius of the crystal. If \( x(B) \) is the distance traversed by the radiation incident on the crystal at an angle \( B \) with the axis, the absorption is proportional to \( (1 - e^{-\gamma x(B)}) \) where \( \gamma \) is the absorption coefficient. Then the measured correlation

\[14\]

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 2.1  A geometry of two $\gamma$ detectors.

Geometry for finite resolution in angular momentum. The notation applies for singles counts as in angular distribution measurements. The azimuth angles $\phi_1$ and $\phi_2$ are measured with respect to the cylinder axes $A_1$ and $A_2$ respectively.
function would be:

\[
W(\theta) = \frac{\int d\omega_1 \, d\omega_2 \, W(\theta') \left(1 - x_1 - \tau x_1 \right) \left(1 - x_2 - \tau x_2 \right)}{\int d\omega_1 \, d\omega_2 \, \left(1 - x_1 - \tau x_1 \right) \left(1 - x_2 - \tau x_2 \right)} \tag{2.2}
\]

where \( x_1 \) and \( x_2 \) refer to the two crystals, \( d\omega_1 \) and \( d\omega_2 \) are the solid angle elements for each radiation, and \( \theta' \) is the angle between their propagation vectors, while \( \theta \) is the angle between the cylinder axes.

The required integrals are of the form:

\[
I_\ell = \int d\omega_1 \, d\omega_2 \, P_\ell \left(\cos \theta' \right) \left(1 - x_1 - \tau x_1 \right) \left(1 - x_2 - \tau x_2 \right) \tag{2.3}
\]

which by application of the Legendre polynomial addition theorem reduces to:

\[
I_\ell = 4\pi^2 P_\ell \left(\cos \theta \right) J_\ell \left(1 \right) J_\ell \left(2 \right) \tag{2.4}
\]

where

\[
J_\ell = \int_0^\phi P_\ell \left(\cos \theta \right) \left(1 - x_1 - \tau x_1 \right) \sin \theta \, d\theta \tag{2.5}
\]

\( \phi \) being the half angle subtended on the front face, and \( (1) \) and \( (2) \) refer to the respective crystals. We have to evaluate \( J_\ell \) to obtain the required correction factors. Finally, we have:

\[
W(\theta) = 1 + \frac{\left( J_1 \left(1 \right) \right) \left( J_1 \left(2 \right) \right) A_2 \, P_2 \left(\cos \theta \right)}{\left( J_0 \left(1 \right) \right) \left( J_0 \left(2 \right) \right)}
\]

\[
+ \frac{\left( J_4 \left(1 \right) \right) \left( J_4 \left(2 \right) \right) A_4 \, P_4 \left(\cos \theta \right)}{\left( J_0 \left(1 \right) \right) \left( J_0 \left(2 \right) \right)} \tag{2.6}
\]
The attenuation factor is:

\[ \frac{\mathcal{F}_2(\xi)}{\mathcal{F}_1(\xi)} = \left( \frac{J_2}{J_0} \right)^2 \]  

(2.7)

for similar detectors, and for \( \gamma \)-rays with similar absorption coefficients.

For a single detector as would be used in an angular distribution measure, with the data represented by equation 1.1, the attenuation factor would be simply:

\[ \frac{\mathcal{F}_2(\xi)}{\mathcal{F}_1(\xi)} = \frac{J_2}{J_0} \]  

(2.8)

Values for \( J_2/J_0 \) and \( J_4/J_0 \) have been calculated (Figures 2.2 and 2.3) by West (1959) for crystal sizes of 1 1/2 inches in diameter by 1 inch long, 1 3/4" x 2", 2" x 2", and 3" x 3" for source to crystal face distances of 3, 5, 7 and 10 cm.

In the \( \gamma - \alpha \) correlation experiment to be described (Chapter V) a 2" x 2" crystal and a 1 3/4" x 2" crystal were utilized at a distance of 10 cm. from the source. The graphs (Figures 2.2 and 2.3) then yield the following values for the correction factors:

\[ \frac{\mathcal{F}_2(\xi)}{\mathcal{F}_1(\xi)} = \frac{J_2}{J_0} = 0.965 \]  

for 2" x 2" crystal

\[ \frac{\mathcal{F}_4(\xi)}{\mathcal{F}_1(\xi)} = \frac{J_4}{J_0} = 0.886 \]

\[ \frac{\mathcal{F}_6(\xi)}{\mathcal{F}_1(\xi)} = \frac{J_6}{J_0} = 0.973 \]  

for 1 3/4" x 2" crystal

\[ \frac{\mathcal{F}_4(\xi)}{\mathcal{F}_1(\xi)} = \frac{J_4}{J_0} = 0.912 \]
(ii) Corrections for the Electron Channel

Apart from the \( \left( I_x / I_e \right) \) correction factors for the \( \gamma \) - channel discussed above, we have in the case of an electron correlation, also to apply due corrections for the smearing out of the directional correlation owing to the finite aperture angle subtended by the accepted cone of electrons in the beta spectrometer.

In this case, the form of the \( \alpha^- \) correction factor, denoted \( F_a(x^-) \), is similar to the previously discussed \( \gamma \)-correction factor \( F_a(\gamma) \) since the geometrical arrangements of detector and source in both cases are similar (compare Figures 2.4 and 2.1).

Then, for the \( \alpha^- \)-channel the appropriate expression for \( F_a(x^-) \) is given by:

\[
F_a(x^-) = \frac{\left( P_2(\cos \alpha) Q(\alpha) \sin \alpha \right)}{\int_0^{\alpha_1} Q(\alpha) \sin \alpha \, d\alpha} \left( \sin \frac{\pi}{2} \right)
\]

where \( \alpha_1 \) and \( \alpha_2 \) represent the minimum and maximum electron take-off angles (Figure 2.4) and \( Q(\alpha) \) is the detector efficiency for the emission angle \( \alpha \).

Equation (2.9) is to be compared with the expression for \( F_a(\gamma) \), namely:

\[
F_a(\gamma) = \frac{\left( P_2(\cos B) \left( 1 - x^{-\gamma X(B)} \right) \sin B \right)}{\frac{\pi}{2}} \int_0^\frac{\pi}{2} \left( 1 - x^{-\gamma X(B)} \right) \sin B \, dB
\]

It is evident from a comparison of equations (2.9) and (2.10) and of Figures 2.1 and 2.4 that the expression \( Q(\alpha) \sin \alpha \) in the \( \alpha^- \)-case is equivalent to the expression \( \left( 1 - x^{-\gamma X(d)} \right) \sin B \) in the \( \gamma \)-case. Further, it is seen that the integration over \( \alpha \) from \( \alpha_1 \) to \( \alpha_2 \) in the \( \alpha^- \)-case is
Figure 2.4  A geometry of an \$\alpha\$-channel.
equivalent to the integration over $B$ from $0 \rightarrow \theta$ in the $\Omega$-case.

In general, then, the expression for the experimentally measured directional correlation becomes:

$$W(\theta) = 1 + f_2 A_2 P_2 (\cos \theta) + f_4 A_4 P_4 (\cos \theta)$$

(2.11)

where $f_2$ and $f_4$ are always $\leq 1$ and represent the total smearing out of the angular correlation pattern due to the finite apertures in the two channels.

In particular, for an $\Omega$-$\chi$ correlation:

$$f_2 = f_2 (\Omega) f_2 (\chi) \leq 1$$

$$f_4 = f_4 (\Omega) f_4 (\chi) \leq 1$$

(2.12)

Returning to equation (2.9), it may be noted that for small transmission settings where $\chi$ is very nearly equal to $\chi_0$, we obtain:

$$f_2 (\chi) \simeq p_2 (\cos \chi)$$

i.e.

$$f_2 (\chi) \simeq p_2 (\cos \chi)$$

(2.13)

$$f_4 (\chi) \simeq p_4 (\cos \chi)$$

(2.14)

These functions (i.e. $P_2 (\cos \chi)$ and $P_4 (\cos \chi)$) are reproduced in Figure 2.5.

At higher transmission settings the approximate relations (2.13) and (2.14) no longer apply. Both factors decrease, $f_4$ more rapidly than $f_2$, since the transmission increases with $\chi$, and larger $\chi$-values therefore dominate the contributions to the $f_2$ and $f_4$ integrals, an effect which is more pronounced for the $f_4$ factor than it is for the $f_2$ factor. For this reason the experimentally measured $f_2$ and $f_4$ factors at higher transmission settings no longer correspond to the geometrical mean value for $\chi$. We use, of course, our experimental values for $f_2$ and $f_4$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 2.5 Graphs of $P_2 (\cos \alpha)$ and $P_4 (\cos \alpha)$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Three special baffles, the so-called P₀, P₂ and P₄ baffles (Figure 2.6), are used to evaluate equations (2.8) and (2.9) experimentally.

The P₂ and P₄ baffles have been made such that they simulate a P₂ and a P₄ distribution respectively. The reduced counting rate therefore becomes a direct measure of the smearing out effects. The P₀ baffle is used, of course, for normalization. The P₄ baffle and the P₂ baffle have openings cut in them each of which has an area given by:

\[
\begin{align*}
A_4 &= K \int_{\alpha_1}^{\alpha_2} P_4(\cos \alpha) \sin \alpha \, d\alpha \\
A_2 &= K \int_{\alpha_1}^{\alpha_2} P_2(\cos \alpha) \sin \alpha \, d\alpha
\end{align*}
\]  

(2.15)  

(2.16)

respectively, where \( K = \text{constant} \), while the P₀ baffle has openings of area:

\[
\begin{align*}
A_0 &= K \int_{\alpha_1}^{\alpha_2} \sin \alpha \, d\alpha
\end{align*}
\]  

(2.17)

If the P₂ or P₄, and the P₀ baffles are now placed in turn at the spectrometer entrance, the count rates obtained will be proportional to the areas of the baffle openings. Hence:

\[
\frac{N(P_2)}{N(P_0)} = \frac{\int_{\alpha_1}^{\alpha_2} Q(\alpha) P_2(\cos \alpha) \sin \alpha \, d\alpha}{\int_{\alpha_1}^{\alpha_2} Q(\alpha) \sin \alpha \, d\alpha} = \frac{C(\alpha)}{f_2(\alpha)}
\]

(2.18)

120922
and

\[
\frac{N(P_{4})}{N(P_{0})} = \int_{a_{1}}^{a_{2}} \frac{Q(x) \, P_{4}(\cos \alpha) \, \sin \alpha \, dx}{\int_{a_{1}}^{a_{2}} Q(x) \, \sin \alpha \, dx} = \int_{x}^{x} (2.19)
\]

where \(N(P_{2}), N(P_{4}), N(P_{0})\) are the respective count rates with the \(P_{2}, P_{4}\) and \(P_{0}\) baffles in place.

(iv) A Final Note

It is noted here that the angular correlation function is often written as follows:

\[
W(\Theta) = 1 + \sum_{2} G_{2} A_{2} P_{2}(\cos \Theta) + \sum_{4} G_{4} A_{4} P_{4}(\cos \Theta) \quad (2.20)
\]

where \(G_{2}\) and \(G_{4}\) are attenuation factors. In the case of \(\mathcal{F} - \mathcal{L}^{-}\) correlations there are three possible kinds of attenuation-mechanisms, namely, after-effects of \(K\)-hole formation, electron straggling and static quadrupole interaction [Gerholm, Holmberg, Petterson, (unpublished)]. For the experiment in \(\text{Au}^{198}\) to be described, it was found by the above authors that \(G_{2}\) and \(G_{4}\) are essentially unity.
CHAPTER III

THE BAFFLE SYSTEMS

(i) The Gerholm Instrument

The spectrometer used in this experiment is due to T. R. Gerholm (1961). Magnetic momentum selection is provided in the electron channel which consists of an iron incapsuled long lens beta ray spectrometer with "triangular field" focussing. The instrument is a modification of the electron-electron coincidence spectrometer earlier developed by Gerholm.

Figure 3.1 (a) is a schematic of the $\alpha-\alpha$ spectrometer, whereas Figure 3.1 (b) is a schematic of the $\alpha-\gamma$ spectrometer.

The "triangular field" distribution is depicted in Figure 3.2. Magnetic field measurements [Gerholm, (1961)] show that the source is in a field free region and that the strength of the magnetic field increases roughly proportional to the distance from the source, when measured along the axis of symmetry.

(ii) Ring Focus and Baffles

In a lens spectrometer, the electrons leaving the source at an entrance angle $\alpha$ to the axis eventually return to the axis again to form an image. The spherical aberration of this image is large for many field shapes. A position of minimum spherical aberration generally occurs before the electrons cross the axis at the so-called ring focus (Figure 3.3). A great improvement in performance is obtained by placing the defining baffles at this focus rather than at the detector [Nichols, (1954)]. Ring focussing is employed in the Gerholm instrument.
Figure 3.1(a) Electron - Electron Spectrometer

Figure 3.1(b) Angular Correlation Spectrometer
Figure 3.2  Triangular field distribution.
(iii) Resolution and Transmission

The resolution of a spectrometer is usually determined by observing its response to a source of mono-energetic electrons. When the counting rate is plotted as a function of the magnetic field $B$, or the momentum $p$, one obtains response curves of the type shown below (Figure 3.4):

![Diagram of spectrometer with source, spectrometer axis, detector, baffle, and ring focus.]

The resolution $R$ is defined as $(\Delta p / p_0)$ where $\Delta p$ is the momentum spread at half maximum and $p_0$ is the momentum at the peak counting rate. $R$ is independent of $p_0$. It should be noted that each instrument has its own characteristic line profile determined by its own particular focussing properties, but this definition of $R$ can be used for all of them.

The transmission $\omega$ of a spectrometer is defined as the...
fraction of the electrons of momentum $p$ emitted from the source which
arrive at the detector when the instrument is focussed on $p$. The
expression $(d\omega / 4\pi)$, where $d\omega$ is equal to the solid angle subtended
by the baffle system, is a good approximation of $\omega$.

(iv) The $A_2$ and $A_4$ Baffle Systems

(a) Special Remarks

The resolution of a magnetic spectrometer is essentially
determined by three factors: the width of the source, the spherical
aberration, and the width of the defining exit slit (generally the
annular ring focus exit). If we neglect the width of the source and the
spherical aberration the resolving power is determined solely by the width
of the exit slit. Under these approximations the transmission increases
proportional to $\sin \alpha$, $\alpha$ being the mean emission angle. Thus, the
figures of merit of a magnetic spectrometer would seem to increase with
increasing emission angle. The gain in transmission at a given resolution
is, however, counterbalanced by an increased spherical aberration. If the
spherical aberration is taken into account, the transmission $\omega(\alpha)$
considered as a function of the emission angle $\alpha$ has a maximum for a
certain value of $\alpha$ which, of course, should be taken as the most favourable
choice for the emission angle. It may be shown [DuMond, (1957)] that for
a uniform field, $\omega(\alpha)$ has a broad maximum around $40^\circ$ to $45^\circ$. Both
theoretical and experimental studies of the "triangular field" show that
its spherical aberration is somewhat larger than that of the uniform field
and therefore $\omega(\alpha)$ in this case will have its maximum at a lower $\alpha$-value.
$30^\circ$ should be close to the optimum value.

Thus, the net effect of the spherical aberration is to limit
the mean emission angle and thereby the transmission (at a given resolution)
to a certain maximum value. The shape of the field determines the most
favourable emission angle. The point to be noted then, is that the
transmission (at a given resolution) is not a proper figure of merit for
a directional correlation instrument. With increasing emission angle
there will be a smearing out of the directional correlation owing to
the finite aperture angle subtended by the accepted cone of electrons in
the spectrometer.

Under normal circumstances the $\xi$-correction factors $f_2(\xi)$
and $f_4(\xi)$ are fairly close to unity and minor errors in these correct ($\xi$)
factors, therefore, do not seriously affect the experimental results.

The correction factors for the electron channel, however,
decrease rapidly with increasing emission angle. This implies, in the
first place, that the experimentally measured effect becomes smaller and
therefore more difficult to determine. The gain in statistical accuracy
obtained as a consequence of higher transmission is in this way
counterbalanced. Obviously, one should choose a value for the emission
angle $\alpha$ which is the most favourable compromise between statistical
accuracy ($\propto \sqrt{\omega(\alpha)}$) and smearing out ($\propto f_2(\alpha^+, \alpha)$)
. Secondly, the choice of a fairly large emission angle implies that $f_2(\alpha^-)$ and
$f_4(\alpha^-)$ will become rather small and the results, therefore, more
sensitive to the uncertainty in $f_2(\alpha^-)$ and $f_4(\alpha^-)$.

(b) Choice of Emission Angle

The problem consists in finding the value for $\alpha$ which
maximizes the expression $[\omega''(\alpha) f_2(\alpha^-, \alpha)]$. This expression rather
than the transmission should be considered as the proper figure of merit
for an electron-gamma directional correlation spectrometer. If we neglect
the spherical aberration and the broadening caused by the width of the
source, the transmission may be written [ $\omega(\alpha) = \text{const.} \times \sin \alpha$]. If,
further, we restrict ourselves to a small angular interval $\delta \alpha$ around the

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
mean emission angle $\alpha$, the appropriate expression for $f_2(x^-,\alpha)$ becomes:

$$f_2(x^-,\alpha) = P_2(\cos\alpha) = \frac{3\cos^2\alpha - 1}{2}$$  \hspace{1cm} (3.1)

Consequently, one has:

$$\omega^{1/2} f_2 = \text{const.} \left( 3\cos^2\alpha - 1 \right) \sqrt{\sin\alpha}$$  \hspace{1cm} (3.2)

which has a broad maximum around $\alpha = 21.4^\circ$.

Similar considerations apply for the $f_4(x^-)$ correction factor. This factor, of course, decreases more rapidly with $\omega$ and calls for an even smaller emission angle.

In the construction of an $A_2$ baffle system it is preferable to choose the emission angle so large that the $P_4$ contribution is more or less completely wiped out, i.e. $f_4(x^-) \approx 0$. In this way the $A_2$ coefficients can be accurately determined directly from anisotropy measurements. The $A_4$ contribution will give only a negligible contribution to the anisotropy. If necessary a minor correction can be applied [Petterson, Thun, and Gerholm, (1961)].

If one wishes, however, to determine the $A_4$ coefficient, use of the above $A_2$ baffle system is of no help whatsoever because of the large $\alpha$ value. Optimum conditions of operation for $A_4$ coefficient determinations are obtained for $\alpha = 12^\circ$ maximizing the expression:

$$\omega^{1/2} f_4 = \text{const.} \left( 35 \cos^4\alpha - 30 \cos^2\alpha + 3 \right) \sqrt{\sin\alpha}$$  \hspace{1cm} (3.3)

In the following pages, we will describe the actual construction of the $A_4$ baffle system used in this experiment.

(v) $A_4$ Baffle Construction

In the case of non-homogeneous fields where the electron trajectories cannot be calculated, the correct design parameters of the
baffle system must be determined by photographic ray tracing, for a
detailed knowledge of the electron trajectories is important.
(a) The Camera and Special Baffles

A special camera (Figure 3.5) was built in order to take
photographs of the electron trajectories inside the spectrometer chamber.
This camera consists of a brass plate, with edges A and B accurately
parallel to each other, and four Al rods inserted in it. These rods are
placed at each end on either side of a centre line and perpendicular to
the plate. A plastic film holder (Polaroid Slide Mount #633) is inserted
in place between the respective pairs of Al rods.

When the camera is properly placed, the edges A and B rest
firmly in the vacuum chamber, and the axis of the spectrometer passes
through the film (i.e. the film is mounted in an axial plane (Figure 3.6)).

In taking an actual photograph, X-ray film was carefully cut
and mounted in the film frame. Proper fiducial marks were made in order
to determine its exact location in the spectrometer chamber.

In addition to the camera, 4 special Al baffles were
constructed (Figure 3.7). These baffles have annular slits defining the
electron emission angles to the range 12° to 18.5°. The mean emission
angle and the transmission of each baffle are indicated in Table I.

<table>
<thead>
<tr>
<th>Baffle</th>
<th>Mean Emission Angle</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>13.0°</td>
<td>0.5%</td>
</tr>
<tr>
<td>II</td>
<td>15.3°</td>
<td>0.5%</td>
</tr>
<tr>
<td>III</td>
<td>17.5°</td>
<td>0.5%</td>
</tr>
<tr>
<td>IV</td>
<td>15.1°</td>
<td>1.5%</td>
</tr>
</tbody>
</table>
Figure 3.5 The Camera

Figure 3.6 A front view of camera in spectrometer.
Figure 3.7  Al baffle with annular slit.
(b) The Photography

A $^{137}$Cs source was used in the ray tracing experiments. Using baffle IV (for maximum transmission) the 662 Kev conversion line of $^{137}$Cs was focussed in the spectrometer. The current setting for focussing was recorded and all photographs were taken at this current setting. In using any particular baffle, such baffle was placed perpendicular to the axis at the entrance of the spectrometer.

Photographs of the electron trajectories were taken with the entire series of baffles. Prints of these photographs are shown in Figure 3.8. Since the electrons describe spiroidal paths in the magnetic field they impinge at the sensitive surface of the film. After exposure and processing the film gives a picture of the electron trajectories as they would appear in a meridional plane revoling with the electrons around the axis of symmetry. From these photographs it was decided that a mean emission angle much less than $15^\circ$ would make the construction of the $A_4$ baffle system impractical. Hence a mean emission angle of approximately $15^\circ$ was adopted for the $A_4$ baffle system rather than the optimum angle of $12^\circ$.

In Figure 3.9 we have an enlarged print of a photograph taken with baffle IV in place. A similar photograph was used to determine the dimensions of the $A_4$ baffle system. The ring focus is located in the region of maximum intensity on the photograph.

A superposition of photographs I, II and III was made in order to determine the upper and lower defining edges of the exit slit. Upon analysis of the photographs, the $A_4$ baffle system was constructed to fit the profile of the K-line. A schematic of the baffle is shown in Figure 3.10.
Figure 3.9  An enlarged print.
Figure 3.10 A schematic (side view) of the Al Al baffle system.
(c) The P₄ Baffle

The construction of the A₄ baffle system necessitated the construction of a P₄ baffle as well, in order to determine the correction factor $f_A(x^-)$. Since this baffle had to be normalized relative to the P₀ baffle, a point by point plot of the P₄ baffle was imperative. This was accomplished by writing up a suitable computer program for the description of the P₄ baffle. This program was fed into the 1620 - II I. B. M. computer at the University.

The results of the program were utilized to plot the profile (c.f. Figure 2.6) of the P₄ baffle. It was found that the P₄ baffle profile corresponded exactly to the arc of a circle of radius 1.725". This latter fact enabled the construction of the P₄ baffle to proceed with relative ease.
CHAPTER IV
PRELIMINARY TESTING OF THE BAFFLES

(i) The $A_4$ Baffle System

(a) Shadow Effect

The central part of the baffle system is supported by six legs. Three of them in the entrance end and three placed near the ring focus. It is possible to adjust the angular setting of the entrance and exit legs such that their shadows overlap. Thus there are effectively only three legs.

In this experiment the $A_4$ baffle system was tested for the shadow effect. This was accomplished as follows: the Cs$^{137}$ 662 Kev conversion line was scanned in the beta-spectrometer with the entrance baffle set at various angles of rotation with respect to the exit baffle which was held fixed. The relationship between peak counting rate and angle setting was observed. The entrance baffle was then permanently fixed in that position which was found to correspond to the maximum peak counting rate.

(b) Resolution vs Transmission

The resolution and transmission of the $A_4$ baffle system for different "turns open" of the end ring were determined with the 662 Kev conversion line of Cs$^{137}$. The profiles obtained at different "turns open" are indicated in Figures 4.1 (a) and 4.1 (b).

From these profiles, values of resolution vs transmission were obtained. These values are listed in the following table:
Figure 4.1(a) Profiles of the 662 keV conversion line of Cs$^{137}$. 

Counts/10 sec.

Spectrometer Current Setting

5 - 6 Turns Open
6 - 7 Turns Open
7 - 8 Turns Open
8 - 9 Turns Open
9 - Full Open
Figure 4.1(b) Profiles of the 662 keV conversion line of Cs$^{137}$. 

Spectrometer Current Setting

Counts/10 sec.
The values of the transmission were found as follows:

with the baffle system "full open" the transmission, defined as $d\omega / 4\pi$

is 1.5%. This value of $\omega$ is determined by the annular slit width of the
entrance baffle. Since the transmission is proportional to the solid
angle $d\Omega$ subtended at the exit end of the baffle, and since the
counting rate is also proportional to $d\Omega$, $\omega$ was calculated in the
following way:

$$\omega = \frac{\text{Peak counting rate at a given "turns open"}}{\text{Peak counting rate at "full open"}} \times 1.5 \quad (4.1)$$

Upon examining the above profiles and resolution-transmission
values, it was decided to conduct the Au$^{198}$ experiment at 6 "turns open".

(ii) Testing of the P$_4$ Baffle

Inasmuch as there was very little published information
pertaining to the construction of the P$_4$ baffle, it was imperative that it
be tested as soon as possible, and its performance compared to theoretical
calculation. In the Au$^{198}$ experiment to be described in the next chapter,
the 412 kev K - conversion line of Au$^{198}$ was observed in the beta
spectrometer, and the ratios $[N(P_2) / N(P_0)]$ and $[N(P_4) / N(P_0)]$ were
determined experimentally. The results were:

\[
\frac{N(P_2)}{N(P_0)} = 0.908 \pm 0.013
\]

\[
\frac{N(P_4)}{N(P_0)} = 0.686 \pm 0.010
\]

Since the transmission was \(< 1\%\) the experimental values of \(f_2(x^-)\) and \(f_4(x^-)\) should have approximated very closely the \(P_2\) and \(P_4\) Legendre polynomials respectively, (cf. Chapter II) calculated at the mean emission angle of \(15.1^\circ\), i.e.

\[
\begin{align*}
\frac{f_2(x)}{f_4(x)} &= P_2(\cos \alpha) \\
\frac{f_4(x)}{f_4(x)} &= P_4(\cos \alpha)
\end{align*}
\]

\[
\alpha = 15.1^\circ
\]

A calculation of \(P_2(\cos 15.1^\circ)\) and \(P_4(\cos 15.1^\circ)\) yields:

\[
P_2(\cos 15.1^\circ) = 0.900
\]

\[
P_4(\cos 15.1^\circ) = 0.682
\]

The experimentally determined values of \(f_2(x^-)\) and \(f_4(x^-)\) therefore correspond to the theoretical values within the limits of error. This result was indeed expected for \(\frac{f_2(x)}{f_4(x)} = \frac{N(P_2)}{N(P_0)}\) since the performance of the \(P_2\) and \(P_0\) baffles has long been established in this laboratory. The correspondence between \(f_2(x^-)_{exp.}\) and \(f_4(x^-)_{theor.}\), however, proves conclusively that the \(P_4\) baffle was properly constructed and that it is a reliable piece of equipment.

It is noted here that an error in the \(f_2(x^-)\) and \(f_4(x^-)\) determinations may be introduced as a result of the shadow of the three \(x\)-axis legs. The most correct method to measure \(f_2(x^-)\) and \(f_4(x^-)\) is:
\[ S_2(z^-) = \frac{\int N_{p_2}(\phi) \, d\phi}{\int N_{p_0}(\phi) \, d\phi} \tag{4.1} \]

\[ S_4(z^-) = \frac{\int N_{p_4}(\phi) \, d\phi}{\int N_{p_0}(\phi) \, d\phi} \tag{4.2} \]

where \( \phi \) represents the angle of rotation of the \( P_0 \), \( P_2 \), and \( P_4 \) baffles with respect to the fixed entrance baffle, and \( N_{p_0}(\phi) \), \( N_{p_2}(\phi) \) and \( N_{p_4}(\phi) \) represent the peak counting rates at the angle \( \phi \). This is, however, a tedious procedure and according to Gerholm (1961) the formulae given above are sufficiently accurate provided the \( P_0 \), \( P_2 \) and \( P_4 \) baffles are placed such that there is no shadow of the rear legs in the three "windows". Due to the rotation of the electron trajectories, this means that the windows should be rotated somewhat with respect to the entrance legs. Provided this is done the angular setting of the baffles is not critical. The above procedure was followed in the experimental determination of \( S_2(z^-) \) and \( S_4(z^-) \).
CHAPTER V

THE $^{198}$Au $^{79}$ EXPERIMENT

(i) Introduction

Figure 5.1 shows the main modes of decay of the Au$^{198}$ isotope. It has only one gamma - gamma cascade, viz., the 679 KeV $\gamma$ - 412 KeV $\gamma$ cascade. In this chapter a gamma-electron directional angular correlation experiment to obtain the $A_4 (\gamma$ $\gamma$) and $A_2 (\gamma$ $\gamma$) correlation coefficients of the 679 KeV $\gamma$ - 412 KeV $\gamma$ cascade is described.

The $b_2$ and $b_4$ particle parameters are then calculated from these values and the tabulated values of $A_2 (\gamma$ $\gamma$) and $A_4 (\gamma$ $\gamma$) [Biedenharn and Rose, (1953)].

This experiment has been performed before [Gerholm, (unpublished)]. However, the main objective of the experiment is to indicate a new experimental technique for the study of the internal conversion process, and to assess the performance of the newly constructed $A_4$ baffle system.

(ii) Preparation

(a) Source Preparation

The source material was obtained in the form of irradiated gold foil. The gold was irradiated at McMaster University. It had an initial specific activity of 50 mc. / mg. A 5 mm. in diameter strip of this gold was glued onto a 0.00035" Al backing. This in turn was glued to an Al ring constructed so as to fit into the instrument source holder. This was then mounted in the Gerholm instrument. The source was centered by focussing the 412 KeV K - conversion line of Au$^{198}$ (Figure 5.2) in the beta spectrometer.

(b) Gamma Probe Centering

Two gamma probes utilizing NaI crystals were used independently in the experiment. These probes were centered by comparing the "$\gamma$-singles" count rates at various angles of the probe. The first probe, denoted $\gamma$ -1 ,
Figure 5.1 The Au$^{198}$ Decay Scheme.
counts/10 sec.

412 Kev K-Line

Figure 5.2  412 Kev conversion line of Au$^{198}$. Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
was set at angles of 180°, 225° and 270°; the second probe, denoted \( \gamma_2 \), was set at angles of 90°, 135° and 180°.

(c) The Spectrometer Settings

The gamma spectrometers were finally fixed in position so that the face of each scintillator crystal was 10 cm. from the source. The \( \gamma_1 \) probe utilized a 2 x 2" crystal whereas the \( \gamma_2 \) probe utilized a 1 3/4 x 2" crystal.

The A\(_4\) baffle opening was set at 6.0 turns open. A plastic scintillator in the beta spectrometer was used as the conversion electron detector.

(d) The Experimental Arrangement

Figure 5.3 shows a block diagram of the circuit layout used for the \( \gamma_1 \)-\( \gamma_2 \)-angular correlation experiment.

The outputs of the detectors are divided into a slow and fast channel. The slow channels using linear amplifiers contain the pulse height information. In the fast channels, only the very first part of each pulse is used. This leading edge is shaped into a very short pulse \(( \approx 10 \text{ ms} \)\) with delay lines. The width of these pulses essentially determines the resolving time of the circuit. The fast circuit gives an output for \( \gamma_1 \)-\( \gamma_2 \) and \( \gamma_2 \)-\( \gamma_2 \) coincidences, but not for \( \gamma_1 \)-\( \gamma_2 \) coincidences (Figure 5.4). The outputs of the two slow channels and of the fast coincidence circuit are fed into two slow triple coincidence circuits. The outputs of these slow coincidence circuits and of the single channel analysers are recorded in scalers.

The gamma-electron directional correlation was fully automated [Young, (1965)]. A detailed description of the electronics described above has been given by Colclough (1963) and Young (1965), and may be referred to for further information.
Figure 5.3 A Block Diagram of Circuit Layout
Figure 5.4 A Two Fold Fast Coincidence Circuit

[Diagram of a circuit with resistors, capacitors, and transistors labeled 2N274, 2N828, 2N1308, etc., connected to power supply (+25V) and output nodes (OUT 1, OUT 2).]
(iii) Experimental Procedure

The $f_2(\beta^-)$ and $f_4(\beta^-)$ correction factors were determined experimentally using the 412 Kev K - conversion line of Au$^{198}$. The results were:

$$f_2(\beta^-) = 0.908 \pm 0.013$$
$$f_4(\beta^-) = 0.686 \pm 0.010$$

The $\gamma$-spectrometers were then set to accept the 679 Kev gamma photopeak (Figure 5.5). This energy setting, however, was located on the tail of the strong 412 Kev gamma photopeak (Figure 5.5) and so $\beta^{-}$-412 Kev gamma coincidences appeared as background.

The beta spectrometer was set on the peak of the 412 Kev K-line (Figure 5.2) and coincidences were recorded with the gamma spectrometers at the angles $180^\circ$, $225^\circ$, $270^\circ$ for $\gamma$-$\gamma$, and $90^\circ$, $135^\circ$, $270^\circ$ for $\gamma$-$\beta$.

The beta spectrometer was then set off the peak of the 412 Kev K-line and the background coincidences were recorded at the above angles.

The chance (accidental) coincidence counting rate was determined under each set of conditions. This was accomplished by delaying the $\beta^{-}$ pulses from the beta spectrometer by means of a delay line cable.

(iv) Treatment of Data and Calculations

The data was corrected for chance and decay of the source, and normalized to the single channel counting rates. The genuine coincidence rate at each angle was then determined by subtracting the corrected "OFF PEAK" data from the corrected "ON PEAK" data. A summary of the results are shown in Table III for each $\gamma$-$\beta$-correlation angle.

The uncorrected values of $A_2 (\gamma \beta^-)$ and $A_4 (\gamma \beta^-)$ were obtained by solving the equation:
<table>
<thead>
<tr>
<th>Angle</th>
<th>On Peak Counts</th>
<th>Off Peak Counts</th>
<th>Genuine Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>3765 ± 61</td>
<td>2317 ± 48</td>
<td>1448 ± 78</td>
</tr>
<tr>
<td>225°</td>
<td>4478 ± 67</td>
<td>2368 ± 49</td>
<td>2110 ± 83</td>
</tr>
<tr>
<td>270°</td>
<td>4565 ± 68</td>
<td>2128 ± 46</td>
<td>2437 ± 82</td>
</tr>
</tbody>
</table>

\[ A_2 (\gamma \ell^-) = -0.27 ± 0.03 \] Uncorrected for Finite Solid Angle

\[ A_4 (\gamma \ell^-) = +0.07 ± 0.04 \] Solid Angle

<table>
<thead>
<tr>
<th>Angle</th>
<th>On Peak Counts</th>
<th>Off Peak Counts</th>
<th>Genuine Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>6110 ± 78</td>
<td>4452 ± 67</td>
<td>1658 ± 103</td>
</tr>
<tr>
<td>135°</td>
<td>6383 ± 80</td>
<td>4402 ± 66</td>
<td>1901 ± 104</td>
</tr>
<tr>
<td>90°</td>
<td>6782 ± 82</td>
<td>4269 ± 65</td>
<td>2513 ± 105</td>
</tr>
</tbody>
</table>

\[ A (\gamma \ell^-) = -0.29 ± 0.04 \] Uncorrected for Finite Solid Angle

\[ A (\gamma \ell^-) = +0.06 ± 0.04 \] Solid Angle
Figure 5.5  The 412 Kev and 679 Kev photopeaks.
\[ W(\Theta) = 1 + A_2 (\gamma \rightarrow \pi^-) P_2 (\cos \Theta) + A_4 (\gamma \rightarrow \pi^-) P_4 (\cos \Theta) \quad (5.1) \]

at all angles for each gamma probe \( \gamma^1 \) and \( \gamma^2 \). These values are tabulated in Table III. The values of \( A_2 (\gamma \rightarrow \pi^-) \) and \( A_4 (\gamma \rightarrow \pi^-) \) corrected for finite solid angle along with the values of \( S^2 \) and \( S^4 \) are tabulated in Table IV.

The mean values obtained were:

\[ A_2 (\gamma \rightarrow \pi^-) = -0.32 \pm 0.02 \]
\[ A_4 (\gamma \rightarrow \pi^-) = +0.10 \pm 0.04 \]

The values of the \( A_2 \) and \( A_4 \) coefficients and of the \( b_2 \)
particle parameter obtained by Gerholm (unpublished), which are in agreement with the theoretical values of Biedenharn and Rose (1953), are listed in Table V.

Utilizing the values of \( A_2 (\gamma \gamma^-) \) and \( A_4 (\gamma \gamma^-) \) from Table V and our experimental values of \( A_2 (\gamma \rightarrow \pi^-) \) and \( A_4 (\gamma \rightarrow \pi^-) \) we obtain:

\[ b_2 = \frac{A_2 (\gamma \rightarrow \pi^-)}{A_2 (\gamma \gamma^-)} = 1.19 \pm 0.15 \]
\[ b_4 = \frac{A_4 (\gamma \rightarrow \pi^-)}{A_4 (\gamma \gamma^-)} = 0.53 \pm 0.26 \]

Using the recursion formula for \( b_2 \) (Chapter I) we obtain:

\[ b_2 = 1.40 - b_4 = 1.19 \pm 0.10 \]

\[ \frac{2.5}{(v)} \text{Errors} \]

The dominant experimental error was statistical in nature and was due primarily to the low specific activity of the course.

\[ (vi) \text{Conclusions} \]

The values obtained for the \( b_2 \) particle parameter indicate that the new experimental method for the study of the internal conversion

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
### TABLE IV

<table>
<thead>
<tr>
<th>Probe</th>
<th>( f_2 )</th>
<th>( f_4 )</th>
<th>( A_2 (\gamma \Lambda^-) ) [Corrected]</th>
<th>( A_4 (\gamma \Lambda^-) ) [Corrected]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathfrak{D} ) -1</td>
<td>0.875</td>
<td>0.608</td>
<td>( -0.31 \pm 0.03 )</td>
<td>( +0.11 \pm 0.06 )</td>
</tr>
<tr>
<td>( \mathfrak{D} ) -2</td>
<td>0.883</td>
<td>0.626</td>
<td>( -0.33 \pm 0.04 )</td>
<td>( +0.10 \pm 0.07 )</td>
</tr>
</tbody>
</table>

Mean Values: \( A_2 (\gamma \Lambda^-) = -0.32 \pm 0.02 \)  
\( A_4 (\gamma \Lambda^-) = +0.10 \pm 0.04 \)

**Experimental Results**

### TABLE V

<table>
<thead>
<tr>
<th>( k )</th>
<th>( A_k (\gamma \Lambda^-) )</th>
<th>( A_k (\gamma \phi) )</th>
<th>( b_k [A_k (\gamma \Lambda^-) / A_k (\gamma \phi)] )</th>
<th>( b_k ) [From Formula]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( -0.38 \pm 0.02 )</td>
<td>( -0.27 \pm 0.02 )</td>
<td>( 1.41 \pm 0.15 )</td>
<td>( 1.32 \pm 0.04 )</td>
</tr>
<tr>
<td>4</td>
<td>( +0.04 \pm 0.02 )</td>
<td>( +0.19 \pm 0.02 )</td>
<td>( 0.20 \pm 0.10 )</td>
<td></td>
</tr>
</tbody>
</table>

**Gershon's Results**
process yields an improved accuracy in the $b_2$ determination. A further merit of the method is seen in the fact that a reasonable value of $b_2$ is obtained from the recursion formula despite a somewhat large error in $A_4$.

Furthermore, it is concluded that the $A_4$ baffle system was properly constructed since the experimental determinations of the $f_2^{*}(x^-)$ and $f_4^{*}(x^-)$ correction factors agreed with the theoretical values. This agreement is contingent upon the proper construction of the $A_4$ system.

In future work involving the determination of particle parameters as well as of conversion coefficients the use of stronger sources seems strongly indicated.
BIBLIOGRAPHY


Frauenfelder, H.F. 1955. in "Beta and Gamma Spectroscopy" (K. Siegbahn, ed.). p. 531, Interscience, N.Y.


1941 - born in Windsor, Ontario on December 8.

1960 - graduated from Corpus Christi High., Windsor, Ontario.

1963 - received Bachelor of Science in Physics, University of Windsor.
- entered the Faculty of Graduate Studies at the University of Windsor to proceed toward M.Sc. degree in Physics.