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Tracking of a Phase-Locked Loop under high noise.

James F. Campbell
University of Windsor

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UMI
TRACKING OF A PHASE-LOCKED LOOP UNDER HIGH NOISE

by

JAMES F. CAMPBELL Jr.

A Thesis

Submitted to the Faculty of Graduate Studies through the Department of Electrical Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor

Windsor, Ontario

1966
ABSTRACT

Experimental results for the tracking of a second order Phase-Locked Loop are presented. Dynamic lock characteristics are established under high noise conditions. The noise is added to an FM signal modulated by a sinusoid. The characteristics are found from -3dB to -21dB signal to noise ratio at the phase discriminator.

The spectra of the error signal are found for high noise power added to a carrier signal and for a carrier signal frequency modulated by white noise. Spectra of noise and modulation in the error signal are also shown.

This study also gives harmonic maps for the error signal above the resonant frequency of the loop.
ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to Dr. S.N.Kalra, who supervised this work, for his encouragement and advice. Thanks are also due to Mrs. S. Ouellette who did the typing of this manuscript and to Mr. H. Ganczarczyk for his aid in providing hardware for this endeavour.

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CHAPTER I

INTRODUCTION

1.1 The General Problem

This is an extension of the work previously done at the University of Windsor by Mr. Philip H. Alexander\textsuperscript{[1,2]}. He studied the ability of a second order phase-locked loop with a simple RC filter to receive and track an FM signal modulated by a sinusoid. He investigated the maximum dynamic tracking characteristics for high signal to noise ratios.

In the present work these characteristics under high noise (low signal to noise ratios) are considered. White noise is added to the input signal to give an additive white noise condition. The input signal is FM modulated by a sinusoid.

A study of the error signal shows the effects of the nonlinearity. Harmonics and subharmonics of the sinusoidal modulating frequency exist in the error signal from the phase discriminator. The spectrum of the error signal also shows gain changes. The redistribution of energy for noise and modulation are illustrated in the spectra for the error signal for additive white noise and modulation.
1.2 A Review of the Phase-Locked Loop

The basic phase-locked loop consists of a phase discriminator, a low-pass filter, a VCO (voltage controlled oscillator), in a feedback loop. Such a system is illustrated in Fig. 1-1. A list of symbols is given in Appendix I.

A phase discriminator compares the phase of the input signal with the phase of a reference oscillator (the VCO in this case). The output of the phase discriminator is a low frequency voltage proportional to the phase difference between the VCO and the input signal. If the input is an FM signal plus additive white noise, the error signal contains an instantaneous combination of some or all the following voltages:

(a) noise
(b) a replica of the modulating frequency
(c) harmonics of the modulating frequency (since the phase discriminator is nonlinear[3])
(d) a DC voltage (due to tuning error, $\omega_s \neq \omega_c$)

To eliminate or suppress much of the excess noise and harmonics outside the immediate bandwidth of interest, a low pass filter is placed between the phase discriminator and the VCO. This filter gives the loop

![Fig. 1-1 Block Diagram of a Phase-Locked Loop](image)

\[
V_2 \cos[\omega_c t + K_0 K_1 K_2 \frac{F(s)}{s} \sin \phi]
\]
compensation, and narrows the closed loop noise bandwidth.

The VCO is an oscillator with a variable reactance modulator inserted in the resonant circuit. Hence, the VCO is voltage-tunable by the filtered output of the phase discriminator.

In many cases, because of a wideband input, the signal power at the input of the phase discriminator is much lower than the noise power. The narrow banding effect of the loop gives a noticeable improvement in the signal to noise ratio at the output of the VCO. This output is redetectable, and at the same time gives a stabilized signal for reference.

Fig. 1-1 indicates that a phase difference of \( \frac{\pi}{2} \) radians exists between the VCO and the input signal when the VCO is continuously tracking the instantaneous frequency of the input. The error voltage corrects the VCO frequency to coincide with the input frequency.

Several uses for a phase-locked loop are given by T. J. Rey\(^5\). Examples of these uses are satellite tracking, stabilization of oscillators, and FM generation.

A. J. Viterbi\(^4\) studied the dynamic lock characteristics for phase-locked loops using several different filter configurations. In his analysis he used constant and linearly varying input frequencies. He also assumed that the systems are noiseless.

P. H. Alexander\(^1\) studied the dynamic lock characteristics for the reception of noiseless FM signals modulated by a sinusoid. The filters he considered are: a simple RC filter, the filterless case, the ideal integrator, and the proportional plus filter. The experimental verification was for the case of a simple RC filter with low noise power.
1.3 Experimental Technique

The ability of a second order loop to receive and track FM signals under conditions of low signal to noise ratios at the input to the phase detector is studied. The input to the system is an FM signal modulated with a deterministic sinusoid of 100 to 5000 cycles. Noise is added to the input signal by a white noise generator. The output of the VCO is detected to measure the output deviation, and the input deviation is simultaneously monitored. These measurements give a set of tracking characteristics of the VCO for both the input and output signals for decreasing signal to noise ratios at the phase detector.

A spectrum analyzer and oscilloscope are used to monitor the error signal at the output of the phase discriminator. This makes the measurement of the error spectrum possible.
2.1 The General Loop Equation

For the system of Fig. 1-1, assume that adequate filtering is present so that the output of the discriminator is a voltage proportional to the phase difference between the input and the VCO signals. The error voltage can be expressed [1,4] as

\[ \varepsilon = K_0 \sin (\phi - \phi_c) \]

\[ = K_0 \sin (\phi) \] (2-1)

or

\[ \varepsilon = \frac{V_1 V_2}{2} \sin (\phi_s (t) - \omega_c t - K_0 K_1 K_2 \frac{F(s)}{s} \sin \phi) \] (2-2)

From equations (2-1) and (2-2) an expression for \( \phi \) is obtained as

\[ \phi = \phi_s (t) - \omega_c t - K \frac{F(s)}{s} \sin \phi \]
differentiating with respect to time gives

\[ \dot{\phi} + K \frac{F(s)}{s} \sin \phi = \dot{\phi}_s - \omega_c \] (2-3)

where \( K_0 = \frac{V_1 V_2}{2} \)

and \( K = K_0 K_1 K_2 \) expresses the overall loop gain in radians per second.

Equation (2-3) indicates that the low-pass filter determines the order and form of the nonlinear differential equation describing the loop.

2.2 The Error Spectrum for Sinusoidal Modulation

From (2-3) the error spectrum of the discriminator can be derived for the case of sinusoidal modulation [3] of the input signal. The transfer function for a simple RC filter is given as \( F(s) = \frac{\alpha}{s + \alpha} \). Hence, from (2-3)
\[ \ddot{\phi} + \alpha \dot{\phi} + K a \sin \phi = \ddot{\phi}_s + \alpha (\dot{\phi}_s - \omega_c) \]  

(2-4)

For the case under consideration

\[ \phi_s = \omega_s t + \frac{\Delta \omega}{\omega_m} \sin \omega_m t \]  

(2-5)

Hence,

\[ \ddot{\phi} + \alpha \dot{\phi} + K a \sin \phi = \alpha (\omega_s - \omega_c) \]

\[ + \Delta \omega a \cos \omega_m t \]

\[ - \Delta \omega \omega_m \sin \omega_m t \]  

(2-6)

Assuming that \( \omega_s = \omega_c \), and \( \sin \phi \) is approximated by the first two terms of the series expansion

\[ \sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots \ldots \]  

(2-7)

equation (2-6) may be expressed as Duffing's equation in the form

\[ \ddot{\phi} + \alpha \dot{\phi} + K a \phi - \frac{K a}{6} \phi^3 = H \cos \omega_m t - G \sin \omega_m t \]  

(2-8)

where \( H = \Delta \omega a \) and \( G = \Delta \omega \omega_m \).

Equation (2-8) is treated extensively by J.J. Stoker[8]. The solution of equation (2-8) is found by an iterative method[3]. The method assumes that the forcing function is of the order of magnitude of the nonlinearity \( \frac{K a}{6} \). This is generally true for the loop under locked conditions.

The first solution is assumed to be

\[ \phi_0 = A_1 \cos \omega_m t \]  

(2-9)

Substituting (2-9) into (2-8) and using the trigonometric identity

\[ \cos^3 \omega_m t = \frac{3}{4} \cos \omega_m t + \frac{1}{4} \cos 3 \omega_m t \]

it is found that

\[ [A_1 (aK - \omega_m^2) - \frac{K a}{8} A_1^3] \cos \omega_m t - aA_1 \omega_m \sin \omega_m t = \]

\[ H \cos \omega_m t - G \sin \omega_m t - \frac{K a A_1^3}{24} \cos 3 \omega_m t \]  

(2-10)
The \( \cos 3\omega_m t \) term is neglected and the coefficients of sine and cosine terms on each side of equation (2-10) are equated to give

\[
A_1(aK - \omega_m^2) - \frac{Ka}{8} A_1^3 = H \tag{2-11a}
\]

\[
aA_1 \omega_m = G \tag{2-11b}
\]

If one lets \( H^2 + G^2 = L^2 = \Delta \omega^2 (\alpha^2 + \omega_m^2) \) then (2-11a) and (2-11b) are combined to give

\[
[A_1(aK - \omega_m^2) - \frac{Ka}{8} A_1^3]^2 + (aA_1 \omega_m)^2 = L^2 \tag{2-12}
\]

In general \( L, \omega_m, \alpha \) and \( K \) are parameters known in the experiment. \( A_1 \) can be determined by equation (2-12) and response curves [3] drawn in the \( /A_1/-\omega \) plane.

The next iteration of equation (2-8) is

\[
\phi_1 + \omega_m^2 \phi_1 = -\alpha \phi_o + (\omega_m^2 - Ka) \phi_o + \frac{Ka}{6} \phi_o^3 \\
+ H \cos \omega_m t - G \sin \omega_m t \\
= (\omega_m A_1 - G) \sin \omega_m t \\
+ (H + \omega_m^2 A_1 - Ka A_1 + \frac{Ka A_1^3}{8}) \cos \omega_m t \\
+ \frac{Ka A_1^3}{24} \cos 3\omega_m t \tag{2-13}
\]

To eliminate secular terms (\( t \cos \omega_m t \) or \( t \sin \omega_m t \)) the coefficients of \( \sin \omega_m t \) and \( \cos \omega_m t \) are equated to zero.

Hence

\[
\omega_m = a \omega_m A_1 = G \\
\omega_m^2 = Ka - \frac{Ka}{8} A_1^2 - \frac{H}{A_1} \tag{2-14}
\]

Here \( A_1 \) is assumed to be known and \( \omega_m \) is unknown, but is found by its relation to \( A_1 \) in (2-14). With equation (2-14) satisfied, we then have a solution [8,3] to (2-13) given by
\[ \phi_1(t) = A_1 \cos \omega_m t - \frac{KaA_1^3}{216\omega_m^2} \cos 3\omega_m t \]

\[ = A_1 \cos \omega_m t - A_3 \cos 3\omega_m t \] \hspace{1cm} (2-15)

where \( A_3 = \frac{KaA_1^3}{216\omega_m^2} \).

For \( \omega_s = \omega_c \) it is seen by (2-15) that only odd order harmonics exist in the error.

From (2-1) the error voltage can be expressed\(^3\) as

\[ \varepsilon = K_o \sin (A_1 \cos \omega_m t - A_3 \cos 3\omega_m t) \]

\[ = K_o \{ \sin(A_1 \cos \omega_m t) \cos (A_3 \cos 3\omega_m t) \]

\[ - \cos(A_1 \cos \omega_m t) \sin (A_3 \cos 3\omega_m t) \} \] \hspace{1cm} (2-16)

Equation (2-16) gives rise to Bessel Functions of the first kind and order \( n \). By assuming \( A_3 \) is approximately zero equation (2-16) after simplification becomes the spectrum\(^3\)

\[ \varepsilon = 2K_o \{ J_1(A_1) \cos \omega_m t - J_3(A_1) \cos 3\omega_m t \]

\[ + J_5(A_1) \cos 5\omega_m t + \ldots \ldots \} \] \hspace{1cm} (2-17)

This spectrum contains only odd order harmonics of \( \omega_m \).

When \( \omega_s \neq \omega_c \) even order harmonics may also be present in the spectrum. This is shown qualitatively by treating only the cubic term in the beginning of the iterative method for Duffing's equation (2-18) in simplified form.

\[ \ddot{\phi} + \ddot{\phi} + \phi - \phi^3 = F_o + F_1 \cos \omega_m t \] \hspace{1cm} (2-18)

Assume a solution

\[ \phi_o = C_0 + C_1 \cos \omega_m t \] \hspace{1cm} (2-19)

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The cubic term is given by

\[ \phi^3 = (C_0 + C_1 \cos \omega_m t)^3 \]

\[ = C_0^3 + 3C_0^2 C_1 \cos \omega_m t + 3C_0 C_1^2 \cos^2 \omega_m t \]

\[ + C_1^3 \cos^3 \omega_m t \]

\[ = (C_0^3 + \frac{3C_0 C_1^2}{2}) + (3C_0^2 C_1 + \frac{3C_1^3}{4}) \cos \omega_m t \]

\[ + \frac{3C_0 C_1^2}{2} \cos 2\omega_m t + \frac{C_1^3}{4} \cos 3\omega_m t. \]  

(2-20)

The addition of a sine term in the assumed solution would not effect the resulting term, $3C_0 C_1^2 \cos^2 \omega_m t$, which gives a second harmonic.

2.3 Noise Bandwidth and Spectrum of the Closed Loop System

Viterbi\cite{4} and Laughlin\cite{10} have done work on a linearized closed loop noise bandwidth. These methods will now be used to find the closed loop bandwidth for a system with a simple RC filter. One basic assumption is, that the loop is locked and the tuning error is zero. The system is tracking a noisy carrier signal having no modulation. The noise at the input of the system is white and stationary. The bandwidth of all components in the loop are assumed to be sufficiently large so that the only effective filtering is done by the low-pass filter and VCO.

Assume $\sin \phi = \phi$, so that the equation may be linearized. On this basis the linear model\cite{4,10} of Fig. 2-1 is made.
Fig. 2-1 Linearized Loop

The concept of loop noise bandwidth is based on the fact that for constant input spectral density of $n$ watts per radian at the input of a linear system, a total noise power of $2n B_{nl}$ watts occurs at the output. $B_{nl}$ is the bandwidth of an ideal rectangular filter which produces the same amount of power at the output as $H_1(s)$ (a realizable filter).

The noise bandwidth is found by the integral\(^1\)

\[
\frac{2B_{nl}}{j} = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \left| H_1(j\omega) \right|^2 d\omega \quad (2-21)
\]

The closed loop transfer function\(^[4,10]\) is given by

\[
H_1(s) = \frac{\phi_c(s)}{\phi_s(s)} = \frac{K F(s)}{s + K F(s)} \quad (2-22)
\]

Therefore

\[
H_1(s) = \frac{K \alpha}{s^2 + \alpha s + K\alpha} \quad (2-23)
\]

Substituting (2-23) into (2-21) and letting $2\pi f = \omega$ and $2\pi df = d\omega$ the result is

\[
\frac{2B_{nl}}{j} = \frac{2\pi (K\alpha)^2}{2\pi j} \int_{-\infty}^{\infty} \left| \frac{1}{K\alpha + \alpha 2\pi j f + (2\pi)^2 (jf)^2} \right|^2 df \quad (2-24)
\]
The integral in equation (2-24) can be evaluated as ([6], p. 260)
\[ I_n = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{g_n(x)}{h_n(x) h_n(-x)} \, dx \]

where \( h_n(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n \)
and \( g_n(x) = b_0 x^{2n-2} + b_1 x^{2n-4} + \ldots + b_{n-1} \)

In this case \( n = 2 \), therefore

\[ \frac{2B}{\pi} = I_2 \left[ 2\pi (Kn)^2 \right] \]

\[ b_0 = 0 \quad a_0 = -(2\pi)^2 \]

\[ b_1 = 1 \quad a_1 = 2\pi \alpha j \]

\[ a_2 = Kn \]

\[ I_2 = -b_0 + \frac{a_0 b_1}{2a_0 a_1} \]

the noise bandwidth is thus given as

\[ B_{in} = \frac{K}{4} \text{ C/sec.} \]

or \[ B_{in} = \frac{\pi K}{2} \text{ rad/sec.} \]  \hspace{1cm} (2-25)

The white noise passing through the closed loop modulates the VCO to become phase noise [5]. This phase noise mixes with incoming white noise in the bandwidth of \( 2B_{n1} \). The spectrum of the error voltage at the output of the discriminator will show the closed loop noise superimposed on the white input noise.

To determine the shape of the closed loop noise spectrum, consider the second-order lag filter expressed by equation (2-23). The noise at the input to the closed loop filter \( H_1(s) \) is white, with constant spectral density \( n \) watts per radian for both positive and negative frequencies.
Let the linearized transfer function of (2-23) be
\[
H_1(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]
(2-26)

where \( K_n = \omega_n^2 \) and \( \alpha = 2\zeta\omega_n \).

The output spectral-density ([11] p. 333) for a linear system is given as
\[
\phi_{oo}(\omega) = \phi_{ii}(\omega) |H_1(j\omega)|^2
\]
(2-27)

where \( \phi_{ii}(\omega) = n \) watts per radian.

The output spectral-density becomes
\[
\phi_{oo}(\omega) = n \left| \frac{1}{(j\omega_n^2 + j2\zeta\omega_n + 1)^2} \right|^2
\]
(2-28)

A plot of Equation (2-28) is given in Fig. 2-2.

![Fig. 2-2 Output Power Spectral-Density of H_1(s)](image-url)
Since the system has a fixed value of $\alpha$, the overall loop gain $K$ controls the natural resonant frequency $\omega_n$ and the amplitude of the noise near $\omega_n$ (i.e., $\zeta = \frac{1}{2} \sqrt{\frac{\alpha}{K}}$).

### 2.4 The Effect of Modulation and Noise on Gain

The error voltage is given by equation (2-1). For sinusoidal modulation without noise the phase difference is sinusoidally varying. The input to the discriminator is defined as

$$x_i = \phi \cos \omega_m t$$

when $\omega_s = \omega_c$.

The output $x_o$ is related to the input by the constant of proportionality $K_o$ volts/rad. Approximating $\sin \phi$ by $\phi - \frac{\phi^3}{3!}$,

$$x_o = K_o \left( \frac{x_i^3}{3!} \right) = K_o \left( \phi \cos \omega_m t \frac{-3\phi^3}{24} \cos \omega_mt \frac{-\phi^3}{24} \cos 3\omega_mt \right)$$

Neglecting the term $\cos 3\omega_mt$

$$x_o = K_o \left( 1 - \frac{\phi^2}{8} \right) \phi \cos \omega_m t$$

The equivalent gain of the phase detector is defined by the describing function technique ([6] pp. 170 to 174 and 157 to 165) as

$$K_{eq} = \frac{x_o}{x_i} = K_o \left( 1 - \frac{\phi^2}{8} \right) \quad \text{(2-29)}$$

The new overall loop gain $K'$ is now defined as

$$K' = K_{eq} K_1 K_2 \quad \text{(2-30)}$$

From (2-29) $K'$ can be determined to be $0.693K$ at $\phi = \frac{\pi}{2}$, since $K = K_o K_1 K_2$. 

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The qualitative effect of noise on gain is determined by replacing the detector characteristic by an ideal limiter as shown in Fig. 2-3 by the solid lines. The dashed line indicates the characteristic of the actual phase detector.

Consider the effect of the noise when the tuning error is zero (the nonlinearity is symmetric). The solution is found by the random describing function method ([6] pp. 234 to 236). To apply this analysis the following simplifications are assumed: (i) the amplitude of the input phase to the limiter is assumed to have a Gaussian distribution, (ii) the output is related to the input by \( K_0 \) volts/rad., (iii) the signal is assumed to be stationary so that the ensemble average is equal to the time average.

If \( f(x_i) \) is the nonlinearity ([6] p. 232)
The variance of the random input is $\sigma_i^2$.

Since $x_i$ is normally distributed

$$p(x_i) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp \left( -\frac{x_i^2}{2\sigma_i^2} \right);$$

$f(x_i)$ is given by Fig. 2-3 or

$$f(x_i) = \begin{cases} K_o a & x \geq a \\ K_o x & -a < x < a \\ -K_o a & x \leq -a \end{cases}$$

Therefore,

$$K_{eq} = K_o \int_{-\infty}^{\infty} f(x_i) p(x_i) \, dx_i$$

and

$$\int_{-\infty}^{\infty} f(x_i) p(x_i) \, dx_i$$

$$= \int_{-\infty}^{-a} x_i p(x_i) \, dx_i + \int_{a}^{\infty} x_i p(x_i) \, dx_i$$

$$+ \int_{-a}^{a} x_i p(x_i) \, dx_i \quad (2-33)$$

Evaluating (2-33) and substituting the result into (2-32) gives

$$K_{eq} = \frac{2K_o}{\sqrt{\pi}} \int_{\sigma_i}^{\infty} \frac{a}{\sqrt{2}} e^{-y^2} \, dy$$

where $y$ comes from a change of variables in the evaluation of (2-33)

$$\left\{ \text{i.e. } y = \frac{x_i}{\sqrt{2} \sigma_i} \right\}.$$
The expression (2-34) is in terms of the error function

\[ \text{erf}(z) = \int_{0}^{z} e^{-y^2} dy \]

where \( z = \frac{a}{\sqrt{2} \sigma_i} \).

Therefore,

\[ K_{eq} = K_0 \text{ erf} \left( \frac{a}{\sqrt{2} \sigma_i} \right) \]  \hspace{1cm} (2-35)

Figure 2-4 shows the effect of \( \sigma_i \) on \( K_{eq} \). The curve is plotted for \( K_0 = 1 \) volt/radian. It is seen that \( K_{eq} \) decreases with increasing input \( \sigma_i \). For low values of random amplitude the variation of \( K_{eq} \) is small so that the effect of nonlinearity is not pronounced. However, for large values of \( \sigma_i \) there is a rapid decrease of \( K_{eq} \) and hence \( K' \). Qualitatively this would be expected because of the nonlinear effects being more pronounced in this region.
CHAPTER III

ANALOGUE COMPUTER ANALYSIS OF THE LOOP

3.1 The Dynamic Tracking Range

For convenience equation (2-6) is rewritten as

\[ \frac{d^2\phi}{dt^2} + \alpha \frac{d\phi}{dt} + K\alpha \sin \phi = \alpha (\omega_s - \omega_c) \]

\[ + \Delta \omega \sqrt{\alpha^2 + \omega_m^2} \cos \left(\omega_m t - \tan^{-1} \frac{\omega_m}{\alpha}\right) \quad (3-1) \]

Equation (3-1) is that of a simple pendulum with constant damping. The forcing function is a constant force plus sinusoid.

To get a better understanding of the operation of the system, the study of a simple pendulum is instructive. The equation for an undamped pendulum is

\[ \ddot{\phi} + \frac{g}{l} \sin \phi = 0 \quad (3-2) \]

Let \( y = \frac{dy}{dx} = \phi \), \( x = \phi \) and \( \frac{g}{l} = 1 \)

\[ y = - \frac{\sin x}{\frac{dy}{dx}} \quad (3-3) \]

The relation (3-3) can be represented in the phase-plane \((y - x \text{ plane})\) as isoclines for constant slope \( \frac{dy}{dx} \). Since the system is periodic in \( 2\pi \) the isoclines are drawn from \( -\pi \) to \( \pi \) in Fig. 3-1.
Simple rules will help to draw the trajectories in the phase plane. Trajectories always intersect a particular isocline at the slope for which the isocline is drawn. In the upper half plane trajectories move from left to right and in the lower half plane they move from right to left. The trajectories have a phase portrait as shown in Fig. 3-2.

Fig. 3-1 Isoclines of a Simple Undamped Pendulum

Fig. 3-2 Trajectories of an Undamped Pendulum
The trajectories passing through the points $-\pi$ and $\pi$ are called sepratrices. Inside the sepratrices trajectories have periodic motion and outside the sepratrices they have aperiodic motion. The stable centers of Fig. 3-2 are defined by $y = 0$ and $x = \pm 2\pi$ rad. The unstable saddle points are defined by $y = 0$ and $x = \pi \pm 2\pi$ rad.

Consider now the equation for a simple pendulum with positive damping

$$\ddot{\phi} + k\dot{\phi} + \sin \phi = 0$$  \hspace{1cm} (3-4)

where $k$ is the positive damping constant. The isoclines for (3-4) are defined as

$$y = \frac{-\sin x}{\frac{dx}{dy} + k}$$  \hspace{1cm} (3-5)

where $\phi = \frac{dy}{dx}$, $y = \dot{\phi}$ and $x = \phi$.

When $x = 0$, $\frac{dx}{dy} = -k$. Closed trajectories now become spirals. Trajectories coming from infinity will spiral into some stable focus. The singularities are still defined as for the undamped case. Fig. 3-3 is a phase portrait for the system with damping. Trajectories are the dashed lines.

![Fig. 3-3 Phase Portrait of a Damped Pendulum](image)
The time taken by the system to traverse from a to b in Fig. 3-3 is given by
\[ t_{a-b} = \int_{a}^{b} \frac{dx}{y} \quad \text{(3-6)} \]

When the undamped pendulum is subjected to a constant force \( F_o \), the equation is
\[ \ddot{\phi} + \sin \phi = F_o \quad \text{(3-7)} \]

The system is now nonautonomous, and trajectories must now be studied individually. Each trajectory is plotted by hand ([9] pp.105 to 112 and pp.165 to 171) or by an analogue computer [4]. However, it is possible to study the singularities.

Let \( \frac{dy}{dx} = \dot{\phi} \) and \( x = \phi \)
\[ y = \frac{F_o - \sin x}{\frac{dx}{dy}} \quad \text{(3-8)} \]

The singularities are defined by:

(a) \( y = 0, x = \arcsin(F_o) \pm 2n\pi \) radians, are stable singularities.
(b) \( y = 0, \sin x = \pi - \arcsin(F_o) \pm 2n\pi \) radians are unstable singularities.

Let \( \phi_{ss} = \arcsin(F_o) \). The phase plane singularities are shown in Fig. 3-4. For \( \phi_{ss} = \frac{\pi}{2} \) rad, the stable and unstable singularities coincide. In this situation the pendulum is on the verge of instability. When damping is present the singularities are treated in the same manner. They are found to follow the same pattern as the undamped case under constant force.
Fig. 3-4 Phase Plane Singularities for an Undamped Pendulum

Under Constant Force

Equation (3-1) is put into standard form by letting $\alpha = 2\zeta \omega_n$ and

$$K\alpha = \omega_n^2$$

$$\frac{d^2\phi}{dt^2} + 2\zeta \frac{\omega_n}{n} \frac{d\phi}{dt} + \omega_n^2 \sin \phi = \alpha(\omega_s - \omega_c) + \Delta \omega \sqrt{\alpha^2 + \frac{\omega_n^2}{m}} \cos(\omega_m t - \tan \frac{\omega_m}{\alpha})$$

Time is normalized by letting

$$t = \frac{\tau}{2\zeta \omega_n}$$

Now

$$\dot{\phi} = \frac{d\phi}{dt} = \frac{1}{2\zeta \omega_n} \frac{d\phi}{dt}$$

or

$$\frac{d\phi}{dt} = 2\zeta \omega_n \phi$$

Similarly

$$\frac{d^2\phi}{dt^2} = 4\zeta^2 \omega_n \phi$$

Equation (3-9) takes the form

$$4\zeta^2 \omega_n^2 \phi + 4\zeta^2 \omega_n^2 \dot{\phi} + \omega_n^2 \sin \phi = \alpha(\omega_s - \omega_c) + \Delta \omega \sqrt{\alpha^2 + \frac{\omega_n^2}{m}} \cos(\omega_m t - \tan \frac{\omega_m}{\alpha})$$
or
\[
\phi = \alpha (\omega_s - \omega_c) + \Delta \omega \frac{\sqrt{\alpha^2 + \omega_m^2}}{} \cos \left( \omega_m t - \tan \frac{\omega_m}{\alpha} \right) \frac{-1}{4 \zeta^2 \omega_n^2}
\]

\[
\frac{-\omega_m^2 \sin \phi}{4 \zeta^2 \omega_n^2} = -\phi
\]

Let \( y \frac{dv}{dx} = \dot{y}, y = \dot{\phi} \) and \( x = \phi \)

Equation (3-9) becomes
\[
\frac{dv}{dx} = \frac{\Delta \omega}{\omega_n^2} \frac{\sqrt{\alpha^2 + \omega_m^2}}{} \cos \left( \omega_m t - \tan \frac{\omega_m}{\alpha} \right) \frac{-1}{4 \zeta^2 \omega_n} \]

\[
\frac{a}{\omega_n^2} (\omega_s - \omega_c) - \sin x
\]

\[
+ \frac{\omega_s - \omega_c}{4 \zeta^2 \omega_n} - 1
\]

(3-11)

The phase plane singularities occur for indeterminate slope.

Therefore, the stable singularities are defined by \( y = 0 \) and

\[
x = \arcsin \left[ \frac{\Delta \omega}{\omega_n^2} \frac{\sqrt{\alpha^2 + \omega_m^2}}{} \cos \left( \omega_m t - \arctan \frac{\omega_m}{\alpha} \right) \right]
\]

\[
+ \frac{a}{\omega_n^2} (\omega_s - \omega_c) \right] \pm 2n \pi.
\]

(3-12a)

The unstable singularities are at \( y = 0 \) and

\[
x = \pi - \arcsin \left[ \frac{\Delta \omega}{\omega_n^2} \frac{\sqrt{\alpha^2 + \omega_m^2}}{} \cos \left( \omega_m t - \arctan \frac{\omega_m}{\alpha} \right) \right]
\]

\[
+ \frac{a}{\omega_n^2} (\omega_s - \omega_c) \right] \pm 2n \pi.
\]

(3-12b)

Then the stable singularity is defined in terms of

\[
\left[ \frac{a}{\omega_n^2} (\omega_s - \omega_c) + \frac{\Delta \omega}{\omega_n^2} \sqrt{\alpha^2 + \omega_m^2} \right] \leq 1
\]

or

\[
\omega_n^2 \geq \Delta \omega \sqrt{\alpha^2 + \omega_m^2} + \omega_s - \omega_c
\]
The system is continuously in lock if
\[ \frac{\Delta \omega}{K - \frac{\omega_s - \omega_c}{\omega_c^2 + \omega_m^2}} \leq \frac{\alpha}{\sqrt{\omega_s^2 + \omega_m^2}} \]  
(3-13)

With tuning errors the frequency deviation must decrease for inequality (3-13) to hold.

For \( \omega_s = \omega_c \), the maximum dynamic lock range\(^1\) without noise is given by

\[ \Delta \omega_{\text{max}} = K \frac{\alpha}{\sqrt{\omega_s^2 + \omega_m^2}}. \]  
(3-14)

For \( \omega_m = 0 \), \( K = \Delta \omega_{\text{max}} \). \( K \) is also defined as \( \omega_{dc} \), the static tracking range. It is established\(^5\) that \( K = \left| \frac{\omega_s - \omega_c}{K} \right| \), if the damping is sufficiently large (i.e., \( \sqrt{\frac{\alpha}{K}} > 0.11 \)). When \( \omega_s \neq \omega_c \), the steady state phase error is \( \phi_{ss} = \arcsin \left( \frac{\omega_s - \omega_c}{K} \right) \).  
(3-15)

In the next section equation (3-1) is analyzed in the above manner and the trajectories are plotted with the help of an analogue computer simulated on a digital computer.

3.2 Analogue Simulation

Since equation (3-1) is nonautonomous, the trajectories of the system are simulated by Pactolis Digital Analogue Simulator Program. The program is incorporated into an IBM 1620 digital computer and the trajectories are plotted by a CALCOMP on-line plotter.

Equation (3-1) is simulated directly without normalization. For modulating frequencies of 1000 cycles or less accuracy is good. Since timing is done in discrete samples of machine time, at higher modulation frequencies the accuracy obtained is not good.

The analogue block diagram in Fig.3-5 is made universal by the fact that different inputs can be studied by changing the inputs to block 1. A typical program is given in Appendix II.
Fig. 3-5 Block Diagram for Analog Simulation

\[
\phi \sin
\]

\[
\phi \cos
\]

\[
\phi (\theta)
\]

\[
\phi (\theta)
\]

\[
\phi \sin (x + \phi)
\]

\[
\phi \cos (x + \phi)
\]

\[
\phi \tan
\]

\[
\phi \cot
\]

\[
\phi \sec
\]

\[
\phi \csc
\]

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Figures 3-6 to 3-8 show the results for sinusoidal modulation of 1000 radians, with $K = 6000$ and $\alpha = 1030$ (radians).

Figures 3-7 to 3-8 show the effects of tuning errors. Step response for medium and high damping are shown in Figures 3-9 and 3-10 respectively. The integration time for all curves is 0.00004 seconds of machine time.

In Fig. 3-6 trajectory 1 shows that the time to lock is about one cycle of the modulating frequency. The time for the limit cycle to complete one revolution of the phase plane is given by equation (3-6).

By applying equation (3-14) to trajectory 2 in Fig. 3-6, it is seen that the system is unstable.

The limit cycle in Fig. 3-7 is highly distorted. Since a tuning error is present it is expected that even and odd harmonics will exist in the phase-time plot. By application of inequality (3-13) it can be shown that the system is on the verge of instability. Also by inspecting the maximum phase $\phi$ and equations (3-12a) and (3-12b) it is seen that the singular points corresponding to stable and unstable operation almost coincide. Trajectory 2 of Fig. 3-7 is a good example of the violation of inequality (3-13).

Figure 3-8 shows that distortion of the limit cycle is less pronounced for smaller tuning errors. Again the time to lock is just greater than one cycle of the modulating frequency. The time for the limit cycle to traverse the closed path is given by (3-6).

In the step response curves the steady state phase error is given by equation (3-15). The time to lock for medium damping is greater than that for high damping. Although, $\phi_{ss}$ is not such that the stable and unstable singularities coincide for trajectories 4 and 5 of figure (3-9) the transient has not yet died out sufficiently to allow locking. The significance of

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\( \frac{\omega_s - \omega_c}{K} > 1 \) in both figures 3-9 and 3-10 is seen by equation (3-15) and Fig. 3-4. Hence the stable and unstable singularities overlap and these trajectories will continue to be aperiodic.

Of interest is the fact that in Fig. 3-10 all values are reduced by \( \frac{1}{6} \) of those in Fig. 3-9. This is done to make it possible to compare some of the results in both figures. For example, in Fig. 3-9 when \( \phi_{ss} = 0.95 \) rad. the system did not lock. However, locking occurred for \( \phi_{ss} = 0.95 \) rad. in Fig. 3-10.
\( K = 6000 \text{ rad/sec.} \)

\( \alpha = 1030 \text{ rad/sec}. \)

\( \omega_m = 1000 \text{ rad/sec}. \)

trajectory 1 = \( \Delta \omega = 2800 \text{ rad/sec}. \)

trajectory 2 = \( \Delta \omega = 5000 \text{ rad/sec}. \)

Fig. 3-6 Trajectories of Modulation

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trajectory 1 = $\Delta \omega = 2800$ rad/sec,

$\omega_s - \omega_c = 2000$ rad/sec.

trajectory 2 = $\Delta \omega = 2800$ rad/sec,

$\omega_s - \omega_c = 3000$ rad/sec.
$K = 6000 \text{ rad/sec.}$

$\alpha = 1030 \text{ rad/sec.}$

$\omega_s - \omega_c = 1000 \text{ rad/sec.}$

$\omega_m = 1000 \text{ rad/sec.}$

$\Delta \omega = 2800 \text{ rad/sec.}$

Fig. 3-8 Trajectory of Medium Tuning Error and Modulation
Fig. 3-9 Step Response With Medium Damping
\[ K = 500 \text{ rad/sec.} \]
\[ \zeta = 0.707 \]
\[ \alpha = 1030 \text{ rad/sec.} \]
\[ \phi(0) = -1 \text{ rad.} \]
\[ \dot{\phi}(0) = 333 \text{ rad/sec.} \]

<table>
<thead>
<tr>
<th>trajectory #</th>
<th>( \phi_{ss} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.703 rad.</td>
</tr>
<tr>
<td>3</td>
<td>0.95 rad.</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{515}{500} &gt; 1 )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{647}{500} &gt; 1 )</td>
</tr>
</tbody>
</table>

Fig. 3-10 Step Response With High Damping

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CHAPTER IV

EXPERIMENTAL CONDITIONS

4.1 Experimental Loop

A block diagram of the complete experimental loop is illustrated in Fig. 4-1. The actual loop is modified from the one shown in Fig. 1-1. To control the bandwidth of the input to the discriminator and the loop gain. An IF amplifier with a 30Mc center frequency is provided. This amplifier has a maximum variable gain of 95dB and a variable bandwidth of from 150kc to 11Mc. To prevent power line noise the heaters of the vacuum tubes are supplied by a D.C. source. In order to translate the 100Mc input frequency to the IF center frequency a balanced crystal mixer is placed between the VCO and the input signal. The result of these modifications required that a reference oscillator be provided.

The VCO is a tunnel diode oscillator with a varicap reactance modulator. It is followed by a 70Mc transistor amplifier. The simple RC filter is made up of the components in the varicap bias supply, the discriminator output, and the varicap capacitance. This filter at the operating point of the VCO (70Mc) has a cutoff frequency of 1030 cycles.

The input signal is generated by a Marconi AM/FM Signal Generator, Type TF 995A/2m. This generator has a calibrated deviation meter to measure the input deviation. A 30Mc reference input signal is provided by a General Radio, Type 1211-B, Unit Oscillator.

White noise is generated by an Airborn Instrument Laboratory Diode Noise Generator, AIL Type 7006. The generator is capable of generating

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Fig. 4-1 Block Diagram of the Complete Experimental Loop
3 turns of no. 18 gauge tinned copper
bus 1/4" diameter x 1/2" long, tapped one
turn from ground

3 turns of no. 18 gauge tinned copper
bus 3/8" diameter x 3/4" long

Q₁: RCA 2N2708 n-p-n
C₁, C₂, C₆: 10 - 100 pf.
C₃, C₄, C₅, C₈, C₉: 1000 pf.
C₇: 4-40 pf.
R₁: 10 Kohms
R₂: 1.5 Kohms
R₃: 2.2 Kohms

Fig. 4-2 Schematic Diagram of 100 Mc Transistor Amplifier
excess noise in the 10 to 250 Mc range. The AIL Type 7112 Diode Noise Generator Power Supply is calibrated in noise figures from 0 to 16db.

Low level output of the noise generator made it necessary to design and build the 100Mc RF transistor amplifier illustrated in Fig. 4-2. This amplifier has a 3dB bandwidth of 6Mc and a power gain of about 20db.

Noise is added to the signal through a Hewlett-Packard Dual Directional Coupler, Model 764D. The coupler attenuates the 100Mc signal by 27db. Noise is allowed to pass directly to the RF 100Mc amplifier.

The output of the VCO is monitored by an AM/FM Eddystone Receiver whose audio output was calibrated for measurement of frequency.

The error signal is monitored by a Hewlett-Packard Model 140A oscilloscope. The spectrum of the error signal is obtained by a Nelson-Ross PSA-032 spectrum analyzer operating in a Hewlett-Packard 140A oscilloscope.

4.2 Noise Figure and Signal to Noise Ratio

The open loop noise figure is measured at the output of the IF amplifier by the use of the white noise generator and a 30Mc AM receiver. All equipment is turned on and the 100Mc signal generator is turned as low as possible. Noise coming from this source is seen in this system through a 27d attenuation.

An open loop noise figure of 15.7dB is measured as referred to the input of the system. The noise bandwidth for this measurement is 260kc. This bandwidth is assumed to remain constant throughout all experiments performed.

The signal to noise ratio at the input to the phase discriminator of the closed loop system is calculated by the expression
\[
\frac{S_d}{N_d} = \frac{1}{F_{\text{loop}}} \frac{S_i}{N_i} \quad (4-1)
\]

where \( F_{\text{loop}} = 15.7 \text{dB} \).

4.3 Loop Gain and Input Level

Under low signal to noise ratio conditions loop gain, \( K \), is of considerable importance since it controls noise bandwidth and transient response. The best range was found to be between 5 and 20kc; the damping was then sufficient to allow good tracking and oscillator frequency drift had very little effect.

The IF amplifier was found to saturate at an input of 1mv for the values of IF gain used in the experiments. To prevent limiting by the amplifier inputs were held below 100\( \mu \)v.
CHAPTER V

RESULTS

5.1 Noise Spectra of the Error Signal

Spectra of the error signal under the conditions of high noise power added to a low power unmodulated carrier and for white noise frequency modulating a carrier are shown in Fig. 5-1. These spectra are taken under closed loop conditions.

Fig. 5-1(a) is the error spectrum for additive white noise at the input to the system. It is found that the spectrum outside the region shown in Fig. 5-1(a) has a constant amplitude to at least 40 Kc. Increased amplitude of the noise results from noise modulating the VCO in a bandwidth of $2B_{n1}$.

Figs. 5-1(b) to Fig. 5-1(h) are the error signal spectra for the condition of the input carrier frequency modulated by white noise.

Fig. 5-1(e) is a typical unlock error spectrum for noise.

The other spectra show the effects of increasing random input on $K'$ and hence on the closed loop filter. Equation (2-35) and Fig. 2-4 show that the detector gain, $K_{eq}'$, decreases with increasing random input and hence $K'$. Since the noise amplitude and the natural resonant frequency are dependent upon the overall loop gain, for a fixed value of $a$ equation (2-23) no longer gives a good approximation of the closed loop transfer function under high noise conditions. By replacing the gain block in Fig. 2-1 by $K' = K_{eq} K_1 K_2$ a modified transfer function is found to be

$$H_1(s) = \frac{K'a}{s^2 + as + K'a}$$ (5-1)

where $\zeta' = \sqrt{\frac{a}{K'}}$ and $\omega_n' = \sqrt{\frac{a}{K'}}$. 

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Fig. 5-1  Noise Spectra of the Error Signal

(a) $K = 6.5 \text{ Kc}$,  \hspace{1cm} $\frac{S_d}{N_d} = -10\text{dB}$

(b) $K = 6.5 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 1.0 \text{ Kc}

(c) $K = 6.5 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 3.0 \text{ Kc}

(d) $K = 6.5 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 5.0 \text{ Kc}

(e) $K = 6.5 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 5.0 \text{ Kc}

(f) $K = 6.5 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 6.5 \text{ Kc}

(g) $K = 6.5 \text{ Kc}$  \hspace{1cm} \text{random deviation} = 8.0 \text{ Kc}

(h) $K = 6.5 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 14.0 \text{ Kc}

(i) $K = 19.4 \text{ Kc}$,  \hspace{1cm} \text{random deviation} = 2.75 \text{ Kc}
Fig. 5-1 a, b, c

Fig. 5-1 d, e, f

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Fig. 5-1 g,h

Fig. 5-1 i
From equation (2-24) and (2-30) \( \omega_n^r = \sqrt{K(1 - \frac{\phi^2}{4})} \alpha \) for sinusoidal modulating at \( \phi = \frac{\pi}{2} \)

\[
\omega_n^r = \sqrt{0.693} \frac{K\alpha}{2}
\] (5-2)

The results of such modifications as shown in (5-1) are seen directly in the spectra. The natural resonant frequency and the amplitude of the noise at the resonant frequency will decrease with increasing input. Comparing these spectra with Fig. 2-2, \( \zeta' \) increases with decreasing \( K' \) resulting in less overshoot at \( \omega_n' \).

Application of integral (2-21) to (5-1) gives

\[
B_{n1} = \frac{K'}{4} \text{ cycles}
\] (5-3)

Thus (5-3) shows a decreasing noise bandwidth for increasing random input.

Fig. 5-1(i) is the output of the VCO for a white noise frequency modulated input carrier. Peaks at natural resonance frequency of the loop are clearly seen. No compensation is used for the receiver's audio and deemphasis characteristics.

It is not possible at this time to explain Fig. 5-1(g) and Fig. 5-1(h) as the effects under very high noise are not covered by the theory developed in Chapter II.

5.2 Spectra of Error Signal for System with Modulated Signal and Additive White Noise Input

It is seen in Fig. 5-2(a), (b) and (c) that the amplitude of the modulating frequency in the error signal is different at different modulating frequencies. In this case the input deviation is the same for all three figures. The noise part of the spectra changes shape with the
**Fig. 5-2** Spectra of the Error for Additive White Noise and Modulation

<table>
<thead>
<tr>
<th>K</th>
<th>( \frac{S_d}{N_d} )</th>
<th>( f_m )</th>
<th>( \Delta f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 8.0 Kc</td>
<td>-10 dB</td>
<td>1.0 Kc</td>
<td>1.0 Kc</td>
</tr>
<tr>
<td>(b) 8.0 Kc</td>
<td>-10 dB</td>
<td>2.5 Kc</td>
<td>1.0 Kc</td>
</tr>
<tr>
<td>(c) 8.0 Kc</td>
<td>-10 dB</td>
<td>4.0 Kc</td>
<td>1.0 Kc</td>
</tr>
<tr>
<td>(d) 8.0 Kc</td>
<td>-10 dB</td>
<td>Noise Spectrum for Additive White Noise at the Input</td>
<td></td>
</tr>
<tr>
<td>(e) 8.0 Kc</td>
<td>-10 dB</td>
<td>1.0 Kc</td>
<td>2.5 Kc</td>
</tr>
<tr>
<td>(f) 8.0 Kc</td>
<td>-10 dB</td>
<td>4.0 Kc</td>
<td>4.0 Kc</td>
</tr>
<tr>
<td>(g) 7.33Kc</td>
<td>-21.5dB</td>
<td>5.0 Kc</td>
<td>1.0 Kc</td>
</tr>
<tr>
<td>(h) 7.33Kc</td>
<td>-21.5dB</td>
<td>5.0 Kc</td>
<td>5.8 Kc</td>
</tr>
<tr>
<td>(i) 6.25Kc</td>
<td>-3 dB</td>
<td>2.5 Kc</td>
<td>2.0 Kc</td>
</tr>
<tr>
<td>(j) 9 Kc</td>
<td>0 dB</td>
<td>3.0 Kc</td>
<td>3.0 Kc</td>
</tr>
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<td>(k) 7.50Kc</td>
<td>-1 dB</td>
<td>2.5 Kc</td>
<td>2.5 Kc</td>
</tr>
<tr>
<td>(l) 16 Kc</td>
<td>10 dB</td>
<td>4.0 Kc</td>
<td>4.0 Kc</td>
</tr>
</tbody>
</table>
Fig. 5-2 k, l
value of modulating frequency. It is also noted that the amplitude of the noise peak is decreased for modulating frequencies outside the natural resonant frequency as shown in Fig. 5-2(c). Fig.5-2(g) and (h) show more clearly what happens. As the input deviation increases the noise peak tends to move toward zero frequency and is less pronounced. This redistribution of energy for high modulating frequencies (i.e. \( \omega_m > \omega_n \)) is a point of interest. An answer to this phenomena may be that it is due to changes in gain. As the input increases the loop gain \( K' \), decreases and hence the noise bandwidth of the filter given by equation (5-2) decreases allowing less noise to modulate the VCO.

For modulating frequencies near the natural resonant frequency, there seems to be an increase in the amount of noise present. Also there is an increase in the amplitude of the modulating frequency. The increase in amplitude of the modulating frequency may be due to the shape of the closed loop filter. The increase in noise may be due to noise mixing with the input signal giving rise to a large signal times noise term, which will appear as noise. At a certain value of modulating frequency and deviation there appears to be a complex entrainment with noise at the natural resonant frequency. Such entrainment is found to take place within 500 cycles on either side of the natural resonant frequency for high noise as shown in Fig.5-2(i), (j) and (k). Since the signal to noise ratio is 10dB in the case of Fig.5-2(l) it is felt that this spectrum is a true entrainment of \( \frac{1}{2} \) order subharmonics. To obtain this photograph an intentional tuning error is produced.

For modulating frequencies inside the natural resonant frequency, Fig. 5-2(a) and (e) show little change in the noise part of the spectrum for increasing input deviation.
5.3 The Tracking Characteristics

Table 5-1 is an average of 9 readings for low signal to noise ratio inputs with white noise added to a modulated input signal. In view of the spectra previously shown, it is best to divide the characteristics in Fig. 5-3 into 3 sections. These sections are $\omega_m < \omega_n$, $\omega_m$ near $\omega_n$ and $\omega_m > \omega_n$. The spectra show that modulating frequencies below 1000 cycles have little effect on the noise. As the noise power increases the relative frequency deviation decreases in this region. When the modulating frequency approaches the natural resonant frequency there is a complex entrainment with noise. The VCO in this case appears to have a large output deviation. It is most likely that the system is tracking a large amount of noise which cannot be taken into account without more precise measurements of the noise present.

An interesting region is for modulating frequencies outside of the natural resonant frequency. Here possible gain changes in the system seem to play an important role.

From the point of view of FM reception the input characteristic and the VCO characteristic coincide very well up to the cutoff frequency of the low pass filter (1030 cycles). In the region of the natural resonant frequency there is a definite narrowbanding of the input with respect to the VCO. As $\omega_m$ increases beyond $\omega_n$ this is rapidly reversed. For signal to noise ratio less than -21 dB, the output of the VCO is poorly defined. There is also definite difficulty in the tracking of low modulating frequencies in high noise when the signal to noise ratio is below -11 dB. In Fig.5-1, the characteristics for low loop gain are as illustrated. Explanation of these curves is very difficult at this time. A tentative
### TABLE 5-1

Relative Deviation $\Delta f_{\text{max}} / K$ for VCO Output Under High Noise

<table>
<thead>
<tr>
<th>$f_m$ (cps)</th>
<th>$S_d / N_d$</th>
<th>-3.00 dB</th>
<th>-11.1 dB</th>
<th>-15.2 dB</th>
<th>-17.2 dB</th>
<th>-19.2 dB</th>
<th>-21.2 dB</th>
<th>-23.2 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.937</td>
<td>0.937</td>
<td>0.911</td>
<td>0.885</td>
<td>0.833</td>
<td>0.782</td>
<td>0.676</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>0.937</td>
<td>0.937</td>
<td>0.937</td>
<td>0.885</td>
<td>0.833</td>
<td>0.754</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.885</td>
<td>0.833</td>
<td>0.822</td>
<td>0.782</td>
<td>0.782</td>
<td>0.770</td>
<td>0.730</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0.885</td>
<td>0.820</td>
<td>0.790</td>
<td>0.770</td>
<td>0.730</td>
<td>0.698</td>
<td>0.650</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.776</td>
<td>0.708</td>
<td>0.666</td>
<td>0.650</td>
<td>0.670</td>
<td>0.615</td>
<td>0.615</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>0.680</td>
<td>0.650</td>
<td>0.650</td>
<td>0.640</td>
<td>0.625</td>
<td>0.600</td>
<td>0.515</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.603</td>
<td>0.578</td>
<td>0.557</td>
<td>0.516</td>
<td>0.498</td>
<td>0.485</td>
<td>0.400</td>
<td></td>
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<tr>
<td>1500</td>
<td>0.490</td>
<td>0.476</td>
<td>0.470</td>
<td>0.465</td>
<td>0.455</td>
<td>0.450</td>
<td>0.435</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.455</td>
<td>0.448</td>
<td>0.435</td>
<td>0.437</td>
<td>0.411</td>
<td>0.405</td>
<td>0.335</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.383</td>
<td>0.364</td>
<td>0.330</td>
<td>0.312</td>
<td>0.312</td>
<td>0.270</td>
<td></td>
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</tr>
<tr>
<td>5000</td>
<td>0.223</td>
<td>0.222</td>
<td>0.180</td>
<td>0.165</td>
<td>0.173</td>
<td>0.157</td>
<td>0.098</td>
<td></td>
</tr>
</tbody>
</table>

$K_{\text{av}} = 7.33$ Kc

This table is an average of 9 readings for $K$ between 7 and 8 Kc.
Fig. 5-3 High Noise Characteristics

- Average Input Characteristic
- Theoretical noise-less curve

K_{av.} = 7.33 Kc

\( \Delta f_{\text{max}} \) relative deviation

\( \frac{S_d}{N_d} (dB) \)

- \( \times -3.0 \)
- \( \Theta -11.1 \)
- \( \Delta -17.2 \)
- \( \square -21.2 \)
- \( \ast -23.2 \)
Fig. 5-4 Low Gain Characteristics

- theoretical noiseless curve
- input characteristic
- VCO characteristic

\[
\frac{S_d}{N_d} = -1 \text{ dB}
\]

\[
K = 2.0Kc
\]
Fig. 5-5 VCO Characteristics for Medium Noise Power
Fig. 5-6 Input Characteristics
\[ \frac{\Delta f_{\text{max}}}{K} \text{ relative deviation} \]

\[ K = 40.5 \text{ Kc} \]

\[ f_n = \sqrt{K\alpha} = 6.48 \text{ Kc} \]

\[ f'_n = \sqrt{0.693K\alpha} = 5.35 \text{ Kc} \]

**Fig. 5-7** Enlarged View of High Gain Input Characteristic
hypothesis connects the variation of shape of the curves with variation in $\omega_n$.

Fig. 5-5 shows the VCO characteristics for medium noise from 10 to 0 dB signal to noise ratio. The most noticeable effect of noise is a decrease in the maximum tracking range at high modulating frequencies.

In Fig. 5-6 a set of input characteristics for different values of loop gain are shown. An enlarged view of a high gain input characteristic is illustrated in Fig. 5-7. It is possible that these characteristics have some connection with the gain of the system and the closed loop filter. The expression $\sqrt{0.693 K \alpha}$ (equation 5-2) predicts the low point in these characteristics within 20% or better.

5.4 Subharmonics in the Error Signal

Under low noise conditions the system was found to entrain subharmonics of the order 1/2 and 1/3. J. J. Stoker\textsuperscript{[8]} studies the entrainment of the 1/3 subharmonics for Duffing's equation. The method he uses is similar to that used in section 2.2 for the harmonic solutions. The addition of a tuning error gives 1/2 subharmonics. The restriction on the 1/3 subharmonic in this case is that $\omega_m < 3 \omega_n$ because of the saturating nonlinearity. The damping must also be small (high loop gain).

The 1/2 subharmonic spectrum is shown in Fig. 5-2 (1).

Fig. 5-8 illustrates the spectrum of the 1/3 subharmonic in the error signal. Experimental conditions are as follows:

\[ \Delta f = 20.0 \text{ Kc} \]
\[ f_m = 21.0 \text{ Kc} \]
\[ K = 100 \text{ Kc} \]

The above entrainment is established when oscillation at the resonant frequency is not present in the error signal.
Fig. 5-8 and 5-9
Fig. 5-10 Approximate Harmonic Map 1

\[
K = 10^5 \text{ cycles}
\]

\[
f_n = \sqrt{1.03 \times 10^8} \text{ cycles}
\]

\[
\frac{S}{N} > 30 \text{ db}
\]

\[
\Delta f = \text{Deviation in } \text{Kc}
\]

Normalized Frequency \( \frac{\omega_m}{\omega_n} \)

Unstable and Harmonic

Very Unstable

Unstable and Harmonic

1/2

2nd

Mostly fundamental

3rd

Unstable

1/2
Fig. 5-11 Approximate Harmonic Map 2

\[ K = 10^5 \text{ cycles} \]
\[ f_n = \sqrt{1.03 \times 10^8} \text{ cycles} \]
\[ S/N > 30 \text{ db} \]

Unstable and Harmonic

Mostly Fundamental

Normalized Frequency \( \frac{\omega_m}{\omega_n} \)

Entrainment with Natural Resonance Present.

Subharmonics
\[ \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2} \]
Entrainment is also possible for \( \omega_m > 3\omega_n \) if oscillation at resonance is present in the error signal before modulation is added. Fig. 5-9 illustrates the spectrum of such a resonance in the error signal at \( K = 100 \text{ Kc} \).

Fig. 5-10 and 5-11 are approximate harmonic maps taken for the system with modulating frequencies in steps of 1000 cycles.

Work is continuing on the evaluation of the magnitude of the components present in the subharmonic spectrum.

5.5 Change of Static Gain for Additive White Noise

![Graph showing the effect of additive white noise on static gain.]

Fig. 5-12 The Effect of Additive White Noise on Static Gain

Fig. 5-12 is obtained for an unmodulated carrier with additive white noise at the input. The signal to noise ratio is decreased at the input and the static tracking range is determined for different values of signal to noise ratio.

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The change of static loop gain for white noise in Fig. 5-12 is similar in appearance to that shown for the ideal limiter for $K_{eq}$ in Fig. 2-1. It is difficult to determine the static gain for signal to noise ratios below -21 dB.
CHAPTER VI

DISCUSSION

A linear model of the loop does not explain the conditions under high noise and high input frequency deviations. To get some idea of the effects of the nonlinearity, describing function techniques are used.

The characteristics are found for loop gains between 5 and 20 Kc. The theory developed here predicts many of the experimental results. High noise characteristics for very low and very high loop gains require further study.

Correlation between the modulating frequency and the error signal are required to establish the relationship between input and the error signal.

Further study is required for the analysis of the subharmonics. More harmonic mapping is needed to give the complete picture of this phenomena.

Two interesting aspects of the results found here are the entrainment near \( \omega_n \) for high noise conditions and the effects of modulating frequencies greater than the natural resonant frequency.

Equation (5-2) predicts the low point in the input characteristics within 20% or better. The best predictions are for loop gains of 7.33 and 15 Kc. This is the range in which many of the results are taken.

The noise spectrum under low loop gains may give some answers as to what happens for low loop gains.

The error spectrum for a carrier frequency modulated by white noise has many aspects which may make it possible to measure the actual value of the nonlinearity and the transfer function of the system. This endeavour has yet to be done. The correlation computer built at the University of Windsor will help.
For the tracking characteristics it is found that a signal to noise ratio of -21 dB at the input to the discriminator is about the minimum for predictable tracking with loop gains of 7 to 8 Kc.
REFERENCES


APPENDIX I

Glossary of Symbols

\( V_1 \) = Amplitude of the input (volts)

\( V_2 \) = Amplitude of the VCO output (volts)

\( \phi_s \) = Input phase (radians)

\( \varepsilon \) = Error output of the discriminator (volts)

\( \alpha \) = Cut-off frequency of the low-pass filter (rad/sec)

\( \omega_c \) = VCO center frequency (rad/sec)

\( \omega_s \) = Input frequency (rad/sec)

\( F(s) \) = Transfer function of filter

\( s \) = Laplace, operator

\( \cdot \) = \( \frac{d}{dt} \)

\( \phi \) = Error phase (radians)

\( \phi_c \) = VCO output phase (radians)

\( K_o \) = Phase comparator, gain (volts/rad.)

\( K_1 \) = DC gain of loop filter

\( K_2 \) = VCO gain (rad./sec/volt)

\( \Delta \omega \) = Frequency deviation (rad./sec)

\( \omega_m \) = Modulating frequency (rad./sec)

\( \phi_{ss} \) = Steady state tuning phase (radians)

\( H \) = Amplitude of the in-phase component of the forcing function

\( G \) = Amplitude of the quadrature component of the forcing function

\( J_n \) = Bessel functions of the first kind and order \( n \)
\( F_0 \) = Steady Forcing function

\( F_1 \) = Amplitude of cosine forcing function

\( C_0 \) = Steady force of assumed solution

\( C_1 \) = Amplitude of assumed solution

\( A_1 \) = Amplitude of the fundamental component of phase (radians)

\( A_3 \) = Amplitude of the third harmonic component of phase (radians)

\( \phi_0 \) = First approximation of phase for iteration of Duffing's equation

\( \phi_1 \) = First iteration of Duffing's equation

\( L \) = Amplitude of forcing function

\( k \) = Pendulum damping

\( F_0 \text{loop} \) = Open loop noise figure

\( n \) = Spectral density of white noise (watts/rad.)

\( K^1 \) = Overall equivalent loop gain (radians)

\( \omega_n \) = Natural resonant frequency of the loop (radians)

\( \zeta \) = Damping factor

\( S_i \) = Signal power at the input of the 100 mc amplifier (watts)

\( N_i \) = Noise power at the input of the 100 mc amplifier (watts)

\( S_d \) = Signal power at the input to discriminator (watts)

\( N_d \) = Noise power at the input to discriminator (watts)

\( \sigma_i^2 \) = variance of the random input

\( K_{eq} \) = equivalent gain of the discriminator (volts/rad)

\( \phi_{ii}(\omega) \) = input spectral density

\( \phi_{oo}(\omega) \) = output spectral density

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### APPENDIX II

**PACTOLUS DIGITAL ANALOG SIMULATOR PROGRAM**

**CONFIGURATION SPECIFICATION**

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### INITIAL CONDITIONS AND PARAMETERS

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<td>4000000.00000</td>
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### PACTOLUS SYMBOLS

- **I**: integral
- **S**: sine
- **W**: weighted summer
- **C**: cosine
- **-**: inverter
- **A**: arctangent
- **K**: constant potentiometer
- **76**: time generator
VITA AUCTORIS

1937  Born on August 30, in Carnarvon Township, Manitoulin Island, Ontario.

1952  Completed elementary education at Little Current Public School, Little Current, Ontario.

1958  Completed Grade XIII at Little Current High School, Little Current, Ontario.

1961  Completed first year Engineering at Laurentian University, Sudbury, Ontario.

1964  Graduated from the University of Windsor, Windsor, Ontario, with the degree of B.A.Sc. in Electrical Engineering.

1966  Candidate for the degree of M.A.Sc. in Electrical Engineering at the University of Windsor.