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An experimental determination of settling length for turbulent flow of air in annular ducts.

R. K. Goel
University of Windsor

AN EXPERIMENTAL DETERMINATION OF SETTLING LENGTH FOR TURBULENT FLOW OF AIR IN ANNULAR DUCTS

A THESIS

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R. K. Goel

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APPROVED BY:

W.G. Colborne

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ABSTRACT

The development of velocity profiles for turbulent flow of air in the entrance region of annular ducts has been studied. The diameter ratio ranged from 0.2 to 0.7 and the Reynolds number range covered was between 4,000 and 70,000.

The general purpose of the work undertaken was to determine the behaviour of fluids flowing in smooth concentric annuli. The specific problem was that of determining settling length as a function of diameter ratio and Reynolds number for isothermal flow of air at room temperature. The study was based on the assumption that two successive identical velocity profiles constituted settled flow. This emphasized the need for exact concentricity and uniformity in diameter throughout the annulus. The eccentricity involved in these experiments was less than 3% and the results of Ivey (10), a graduate student at the University of Windsor, who carried out research on annular flow, indicated that these small order eccentricities had no significant effect on velocity profile. The variance in the tube diameter along the length was, of course, minimized by strict selection of tubes and also by using an aluminum tube as a core which had a tighter tolerance than the acrylic tubes

used in previous test runs.

The results of the present investigation indicated that the settling length increased rapidly with an increase in Reynolds number up to approximately 22,000, but this strong influence of Reynolds number was greatly reduced with any further increase in Reynolds number. The settling length was then dependant mainly on the diameter ratio. The settling length decreased with an increase in diameter ratio for the same Reynolds number.

The settling length for flow in an annulus of diameter ratio close to 1 was found to be about the same as that for flow between parallel plates. However, for an annulus having a diameter ratio approaching zero the settling length deviated significantly from the corresponding value for flow in pipes. This is similar to Sparrow's (27) predictions for developing laminar flow in the entrance region of annular ducts. The friction factor data obtained agreed closely with Davis' (6) results, but was found to be higher by about 10% than the average of other experimental results.

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Free use of published literature has been made on this subject, the bibliography of which is appended. The author wishes to acknowledge the assistance received from these sources, in compiling this work.

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NOTATION

D_1	Outer diameter of the inner tube, L
D_2	Inner diameter of the outer tube, L
De	Equivalent diameter of the annulus $(D_2 - D_1)$, L
dw	Diameter of a tube, L
f	Friction factor, dimensionless
g_c	Gravitational constant, mL^2/Ft^2
Le	Settling length for fully developed velocity profile, L
P_f	Pressure decrease equivalent to energy loss due to friction, F/L^2
Re	Reynolds number based on equivalent diameter, $\frac{U\ De}{\nu}$ dimensionless
r_1	Outer radius of the inner tube, L
r_2	Inner radius of outer tube, L
r	Radial location, L
r_w	Radius of a tube, L
U	Average velocity for cross-section, L/t
U_{max}	The maximum velocity in the fully developed flow region, L/t
u	Velocity at radius r, L/t
u_{max}	Maximum velocity in the cross section, L/t
x	A linear distance, L
τ_1	Shear stress at the inner wall, F/L^2

τ_2	Shear stress at the outer wall, F/L^2
ν	Kinematic viscosity, L^2/t
δ	Thickness of hydrodynamic boundary Layer, L
ρ	Density of air, M/L^3

SYSTEM OF DIMENSIONS USED ABOVE :-

F	Force, pounds
L	Length, feet
M	Mass, pounds
t	Time, seconds

CHAPTER I

INTRODUCTION

Numerous experimental studies have been made regarding the determination of settling length or entrance length for the development of velocity profiles in a conduit or between parallel planes, but there seems to be a complete absence of data on settling length in an annulus. Moreover, most of the work carried out on the developing flow in the entrance region of conduit or parallel planes is limited to the laminar region.

Annular flow is encountered in the case of heat exchangers, axial flow pumps and compressors. A thorough knowledge of settling length and velocity distribution is a prerequisite to axial flow turbo machinery studies and annular heat exchanger design. The annulus is also of academic interest because of its unique cross-section geometry and forms a bridge between pipe and parallel planes which are its limiting cases. This and many other applications justify the need to accurately determine the settling length for flow in an annulus.

Due to a lack of previous work, it was recognized that the present investigation may be only of preliminary value in the study of developing flow in the entrance region of annular spaces. Four annuli of diameter ratios

0.2, 0.3, 0.5 and 0.7 were tested between the Reynolds number range of 4,000 to 70,000. Static pressure measurements were also taken for three combinations of diameter ratio 0.30, 0.33 and 0.5 to study the behaviour of shear stress and friction factor with change in Reynolds number and diameter ratio.

CHAPTER 2

THEORY AND EXISTING LITERATURE

A review of past literature reveals that although a considerable amount of work has been done on annular flow, most investigators have been concerned with the structure of fully developed velocity profiles, and little work has been done on developing flow in the entrance region of annular ducts.

When a fluid such as air enters a pipe, the velocity at the entry will be practically uniform over the cross-section. The velocity at the wall is however, zero, so that an infinitely thin boundary layer is formed around the walls of the pipe, the thickness of which increases further downstream. The fluid outside the boundary layer is called the core. Since the fluid adjacent to the wall is retarded by friction, it is necessary that the flow in the core be accelerated in order that the same mass flow passes through every cross section of the duct. The velocity boundary layer grows in thickness along the length of the duct until it reaches the centre line where it meets the boundary layer from other wall of the duct. At this point, fully developed flow is obtained, and assuming no disturbances, this condition will be maintained throughout the remainder of the pipe.

Of main interest now, is the determination of the length required before two successive, identical, velocity profiles are obtained. This length to produce the fully developed profile is termed "settling length" or "entrance length". Rothfus (21) called it "calming length" while some others have used "inlet length".

Tubes and parallel plates are effectively limiting cases of annuli as the radius ratio progresses from zero to unity. It is, therefore, reasonable to assume that a general method of correlating annular flow settling lengths might be developed which would include tubes and parallel plates. In view of the absence of past work, and to get a clear insight into the problem of settling length in an annulus, it seems useful to review the existing literature on the subject for tubes and parallel plates.

A. SETTLING LENGTH CORRELATIONS

(a) TUBES

The flow conditions at the entrance of a conduit greatly influence the length required for a fully developed velocity profile to form. The entrance to a conduit may be either a sudden expansion or contraction in the cross-sectional area of the flow, and for this reason, the configuration of the entrance is an important consideration in studying the flow downstream. When the fully developed flow in a tube is laminar, the boundary layer will be laminar and as such, the entrance configuration will only have a slight effect on the entrance length. However, for turbulent flow, the entrance configuration becomes of prime importance in determining the dynamics of flow downstream. If the entrance is abrupt, the boundary layer will probably be turbulent from the beginning. If the entrance is rounded, the boundary layer may be laminar immediately beyond the entrance and then become turbulent some distance downstream. In considering flow in the entrance section of the conduit, two entrance lengths are involved, the length required for the velocity profile to develop and the length required for the shear stress at the wall to reach its fully developed value. As the fluid enters the conduit, the velocity gradient at the wall is theoretically infinite. In a relatively short distance, the velocity profile adjacent to the wall will have developed to a steady value. Formation of the fully developed velocity profile, however, requires considerable length.

Laminar Flow: - For laminar flow in pipes, Schiller (24) predicted theoretically that

$$\frac{Le}{dw} = 0.02875 \text{ Re} \tag{1}$$

Prandtl and Tietjans (20) indicated that the velocity profile developed in a length given by

$$\frac{Le}{dw} = 0.05 \text{ Re} \tag{2}$$

Langhaar (14) by solving the momentum equation showed that,

$$\frac{Le}{dw} = 0.0575 \text{ Re} \tag{3}$$

Where Le is the entrance length required for the centre line velocity, to reach 99% of the fully developed value.

Turbulent Flow: - For turbulent flow, the main factors contributing to the settling length are Reynolds number and equivalent diameter. However, the entrance configuration and turbulence of the entering stream also influence the settling length to a lesser extent. Since little work has been done on developing flow in tubes, no relationship exists for the prediction of the settling length from the parameters such as Reynolds number and equivalent diameter.

Schiller and Kirsten (23) studied the development of the velocity profile for turbulent flow in pipes for both rounded and sharp edged entrances. For Reynolds numbers ranging from 10,000 to 50,000, they observed that an entrance length of 50 to 100 equivalent diameters, characterized by $U/U_{max} = 0.8$, was necessary for the formation of a fully developed velocity profile. They also noted that for a rounded entrance, the settling length decreased with an increase of Reynolds number, but for a sharp edged entrance, the opposite was true.

They explained this on the basis of the disturbance caused in the flow by the sharp edged entrance. Their results indicated that the entrance length required was five times greater than that given by Latzko's (15) theory. Schiller and Kirsten suggested that Latzko's theory should be modified to obtain a closer agreement with the experimental results.

Deissler (7) studied the variation of velocity along a tube at the centre and at $r/rw = 0.9$ for both a rounded and a right-angle-edge entrance. He found that the development of the velocity profile was more rapid for the right-angle-edge entrance than for the rounded entrance; with the right angle edge entrance fully developed flow was obtained after 45 tube diameters from the entrance, but with the rounded entrance the distribution was still developing slightly at 100 tube diameters from the entrance.

It should be noted that for turbulent flow, the distance required for the local friction factor to equal the fully developed friction factor is considerably less than that required for the development of the velocity profile. Deissler's (8) theoretical results showed a length of 6 diameters was required for the local friction factor to become constant and consequently for the velocity profile adjacent to the wall to become steady.

Latzko (15) predicted the entrance length required for the friction factor to become constant as,

$$Le/dw = 0.623\ (Re)^{1/4} \qquad (4)$$

which appears to agree with Deissler's (8) theoretical results.

(b) PARALLEL PLANES

Laminar flow - Sparrow (26) studied laminar flow in the inlet section of two parallel planes and obtained the following relation:

$$Le/2b = 0.00648\ Re \qquad (5)$$

where 2b represents the equivalent diameter, b being the distance between the two planes.

There is no reliable data available for settling length for the fully developed velocity profile in the turbulent region. However, it is generally believed that the distance required for the formation of a fully developed velocity profile between parallel planes is about 50 equivalent diameters from the entrance.

(c) ANNULI

The most recent and to the best of the author's knowledge, the only work done for developing flow in the entrance region of an annulus is that published by Sparrow and Lin (27) in 1964. They solved the momentum and continuity equations in their excellent analysis for developing laminar flow in annular ducts. Sparrow and Lin found that the flow development in annular ducts with a radius ratio substantially less than unity (0.4) was quite similar to that in parallel plate channels ($r_1/r_2 \rightarrow 1$). On the other hand, it was interesting to note that the results for an annulus with radius ratio as small as 0.001 deviated substantially from those for a circular tube ($r_1/r_2 \rightarrow 0$). It was also noted that the settling length, measured as a multiple of equivalent diameter, increased as the duct ratio was decreased at a fixed Reynolds number.

Rothfus, Monrad, Sikchi and Heideger (22) plotted their results for shear stress on the outer wall of the concentric annulus $\frac{(\tau_2)_{x = x}}{(\tau_2)_{x = \infty}}$ versus $x/(d_2-d_1)$. They noted that this ratio does not approach unity until $x/(d_2-d_1) = 200$ or above is reached. The investigation was based on the assumption that the position of maximum velocity remains the same in the turbulent flow, as it is in fully viscous flow in the same annulus. The assumption does not seem to be true according to more recent work in this field. Brighton and Jones (4) and Nicol and Medwell (17) have found the position of maximum velocity in turbulent flow to differ from that for laminar flow.

B. FRICTION FACTORS IN ANNULI

The friction factor for an annulus is expressed by the equation

$$f = \frac{(D_2-D_1)}{2\rho U^2} \cdot g_c \frac{-dP_f}{dx} \qquad (6)$$

The Reynolds number for an annulus is based on the equivalent diameter $(D_2 - D_1)$.

Davis (6) studied annular friction factor data and proposed the following equation

$$f = 0.055\ (Re)^{-0.2} \left[1 - \frac{D_1}{D_2}\right]^{0.1} \qquad (7)$$

His equation for turbulent friction factors involves the diameter ratio D_1/D_2. It also shows that the effect of diameter ratio is small at values of D_1/D_2 less than 0.3.

Experimental correlations of most investigators express friction factor data in terms of Reynolds number alone. The average of various experimental results can be represented by the following equation;

$$f = 0.076\ (Re)^{-0.25}$$

and the experimental data deviate as much as 35% from the above equation.

Brighton and Jones (4) found that the friction factor depended only on the Reynolds number and is independent of diameter ratio, at least for D_1/D_2- values of 0.625 and higher.

CHAPTER 3

THE APPARATUS

In general, the experimental apparatus consisted of horizontal conduits through which a measured quantity of room air could be made to flow under steady isothermal conditions. The air was drawn in by a fan through the 18 foot long annulus. The test sections were located at 2 foot intervals along the annulus except the first and second test sections which were located at 16 and 34 inches respectively from the entrance. Additional test sections were made wherever necessary. Respective distances of the various test sections from the entrance of the annulus are as shown in table-1.

The experimental equipment consisted of a large number of component parts and accessories, the most important of which are described below:

A. THE FLOW SYSTEM

(a) Fan: The air was drawn through the annular space by an American Blower Corporation, Type "1V", industrial centrifugal fan driven by a 208 volt, 3 phase, 3 h.p. induction motor.

(b) Flow Control: The air flow through the annulus was controlled by a butterfly valve placed at the inlet of the fan.

(c) The Annuli: Six foot lengths of cast and extruded Acrylic tubing were connected to form the inner and outer tubes of the annuli. The entire length of the annulus was supported on a plywood table by rectangular plywood supports, 0.75 inch thick at approximately 2 feet intervals (See appendix B - Photograph A). The outer tubes were joined together by aluminum couplings on the outside. The same outer pipe was used for all four annuli; the internal diameter of this outer pipe was 2.5 inches and the wall thickness was 1/8 inch. Micrometer measurements at various points on the periphery and along the pipe length indicated that the maximum deviation in internal diameter was 0.010 inch. The inner pipes or cores consisted of three seamless acrylic tubes having outside diameters of 0.5, 0.75 and 1.75 inches and an aluminum pipe having an outside diameter of 1.25 inches. The micrometer measurements at various points showed variations of 0.005 to 0.020 inch in those diameters. The inner tubes were held together with aluminum connectors pushed inside the adjoining ends of the tubes. The ends of the tubes were faced carefully to ensure that they formed a smooth continuous joint when connected together. Such a joint offered negligible discontinuity in the main flow. Appropriate sizes of rubber hose were used to connect the annulus to the blower section.

(d) Core Supports: Core supports were found necessary at about two foot intervals to prevent any appreciable sagging of the core. Consequently, eight supports were installed at distances of 2, 4, 6, 8, 10, 12, 14 and 16 feet from the entrance. Each set of supports consisted of a pair of drill rods of 0.04 inch diameter inserted at right angles to each other in the annulus. The test sections were located 21 inches downstream of every core support and at 45° to the drill rods, so that the flow was free from disturbances caused by the preceding core support.

(e) Static Pressure Taps: Holes (Bradshaw 2) 0.04 inch diameter were drilled for the static pressure openings in the outer pipes. Internal burrs were carefully removed with emery cloth. One inch sections of 0.25 inch outer diameter acrylic tube were glued to the outer tubes over the 0.04 inch static pressure holes.

Tygon tubing was then used to connect the acrylic rod to the micromanometer. Diametrically opposite to the static pressure holes, small slots were cut in the outer tube to allow entry of the pitot tube at each test section.

(f) Core Plugs: Finely machined wooden plugs of hemispherical shape were pushed into the two open ends of the core to prevent air leakage to the inner tube.

B. EQUIPMENT FOR VELOCITY MEASUREMENT

(a) Impact Tube: The impact tube selected was made from Stainless Steel hypodermic needle tubing having 0.025 inch outside diameter and 0.012 inch inside diameter.

In his apparatus, Rothfus (21) used two impact tubes, one was a straight tube with a slot for measurement of small velocities near the wall, and the other was of a more common design for measurements in the mainflow away from the wall. His straight tube was 0.042 inch diameter and had a slot of 0.015 inch wide and 0.029 inch long. The end of the tube was closed by a cap of aluminum foil 0.001 inch thick and 0.042 inch in diameter. It required calibration.

(b) The Impact Tube Traversing Mechanism: The impact tube was precisely located in the annular section by means of the traversing mechanism (See appendix B - Photograph B). The traversing mechanism was installed on stands mounted on a heavy base plate to guard against any vibrations of the table. A 2 inch micrometer head was mounted on a housing which contained the pitot tube assembly under spring tension. This arrangement enabled the pitot tube location to be determined to an accuracy of 0.001 of an inch. A threaded shaft between the housing and the base plate provided the vertical adjustment of the impact tube.

(c) The Micromanometer: The dynamic pressures and static pressure differences were measured by a Merium Model A-750 micromanometer. Tygon tube of 0.125 inch inside diameter connected the pitot tube assembly to the micromanometer.

CHAPTER 4

EXPERIMENTAL PROCEDURE

A set of four different annuli were selected with equivalent diameters ranging from 0.75" to 2". The diameter ratios ranged from 0.2 to 0.7.

The mass flow rate of the air through each annulus was varied by changing the cross sectional area of the inlet section of the blower, by regulating the butterfly valve. The atmospheric pressure was measured to the nearest 0.01" of Hg at the beginning of each individual test run with a barometer. The air temperature variation (Rothfus 21) between the entrance and the exit of the annular space is of the order of 1° F, and consequently, the air temperature throughout the annular space was assumed constant. A mercury thermometer was employed to record the room air temperature.

Velocity pressures at each location were indicated by the micromanometer which measured the difference between the total pressure incident on the impact tube and the static pressure on the outer tube wall at the point corresponding to the impact tube position. The sensitivity of the manometer was 0.001" water gauge. The zero position of the pitot tube was selected as being the point where it initially came into contact with the inner surface of the outer wall. Contact of the pitot tube with the tube wall was judged using a magnifying glass and velocity traverses were always made in the direction of decreasing radius. In order to obtain a complete velocity profile, point velocities were measured at seven to eleven different positions in the annular space for each test section, depending upon the size of the gap. Velocity traverses were made for each test section until two successive velocity profiles completely overlapped. This indicated the length required for fully developed flow. Data were obtained over as large a Reynolds number range as possible, the upper limit of this range being established by the blower capacity, and the lower limit being determined by the response of the micromanometer. The Reynolds number ranged from approximately 4,000 to 70,000. The test runs used in the determination of the settling length are summarized in Table- 2 (See appendix A). The fluctuations in the micromanometer readings became very predominant at high mass flow rates and limited the upper end of the working range of Reynolds number. These fluctuations were probably due to the pulsating action of the fan, and the low frequency turbulence in the flow, the magnitude of which increased at high Reynolds number. It was noticed that the fluctuations increased for the measurements taken away from the wall. This was to be expected since the local velocity increased considerably towards the centre of the annular gap. Entrance effects were apparent in the velocity measurements made at the test sections close to the annulus inlet. Severe fluctuations were noted near the entrance but these seemed to be damped out downstream.

The average velocity was determined by integrating the velocity profile. The mass flow and the corresponding Reynolds number were then calculated using this average velocity.

The friction factor data were obtained for three sizes of annuli by taking static pressure readings in the fully developed flow region, and details of test runs concerning friction factor data are tabulated in Table-3 (See appendix A).

CHAPTER 5

DATA PROCESSING

The experimental data were processed by an IBM 1620 Computer. The density of air was determined from a knowledge of the air temperature and barometric pressure. The input to the computer consisted of air temperature, barometric pressure, corresponding density of water, kinematic viscosity of air and local dynamic pressure readings. The computer printed the point velocities throughout the cross-section. Further, the ratio of $\frac{u}{u_{max}}$ was obtained corresponding to the ratio $\frac{r - r_1}{r_2 - r_1}$.

Using the statistical method of linear regression the computer located the lines of best fit below and above the critical Reynolds number of 22,000. In both cases the settling length was a function of Reynolds number and diameter ratio. The regression lines were of the form:-

$$\text{Log} (Le/De) = \log a + m \log Re + n \log \frac{D_1}{D_2}$$

where a, m and n are constants.

Correlation coefficients in both cases indicated the "goodness of fit" of the regression lines.

The friction factors were obtained for each Reynolds number and diameter ratio by using the measured static pressure readings.

CHAPTER 6

RESULTS AND DISCUSSION

Figures 2 to 15 show the results of the present investigation. Figures 2, 3, 4 and 5 illustrate the development of the velocity profiles along the annulus. The ordinate u/U_{max} represents the non-dimensional ratio of the axial point velocity to the fully developed maximum velocity, while the abscissa $(r - r_1)/(r_2 - r_1)$ represents the location of the velocity measurement in the form of a fraction of the total annular space. The velocity traverse data made at various test sections along the annulus for the same test run are plotted such that the velocity distribution profiles superimposed on one another. By inspection of the figures, it is seen that for any duct diameter ratio and Reynolds number, the flow undergoes a velocity development during its course of flow through a certain initial length of the annulus. Beyond this initial length, the velocity profile achieves a fully developed shape which does not change with further increase in downstream distances. This position where two consecutive velocity profiles coincide with each other gives the measure of entrance length. Obviously, the ratio u/U_{max} approaches unity for fully developed flow. The velocity profile is flat in the middle with a steep velocity gradient at the sides as the result of high shear stress at the wall near the entrance section and it gradually attains a parabolic shape skewed towards the core as the flow develops downstream.

The development of the maximum velocity corresponding to different Reynolds numbers for each annulus is shown in figures 6, 7, 8 and 9. The non-dimensional ratio u_{max}/U_{max}, which represents the maximum axial velocity at a test section in the developing flow region, as a fraction of fully developed maximum velocity for the same test run, is plotted against settling length expressed in terms of equivalent diameter. The flow gradually develops as the ratio u_{max}/U_{max} approaches unity along the length of the annulus. The Reynolds number is based on the equivalent diameter De, and an average velocity which is obtained by integrating the individual fully developed velocity profiles. The different curves obtained for the same annulus illustrate the effect of varying Reynolds number on the velocity development. It is apparent that the length required for the formation of a fully developed velocity profile increased with an increase in Reynolds number for the same annulus. However, the curves for higher Reynolds number seem to merge into each other indicating that the settling length tends to become independent of Reynolds number in this region.

Figure 10 shows a family of curves obtained by plotting settling length versus Reynolds number for annuli having different diameter ratios. It gives a more complete picture of the entire data and is the combination of previous curves.

Figures 10 and 11 are essentially the same except that logarithmic scales are used in the latter case. It is noticed from this figure that there appears to be a definite correlation between settling length, equivalent diameter, Reynolds number and diameter ratio. The higher the diameter ratio the lower is the length required for the development of the velocity profile. This is similar to Sparrow's (27) predictions for developing laminar flow in an annulus.

The Reynolds number has a varied influence on the settling length. Two distinct regions can be seen, above and below the transition Reynolds number of about 22,000. Below this Reynolds number the settling length has a marked dependence on Reynolds number. Above a Reynolds number of 22,000, the influence of Reynolds number is greatly reduced and the settling length is seen to depend mainly on diameter ratio. An explanation of the shape of the settling length versus Reynolds number curves can be obtained by considering the development of the boundary layer. The thickness of the boundary layer is assumed to be of the form

$$\delta = f\left[\frac{1}{(Re)^{1/m}}\right].x \tag{8}$$

which is similar to the expression for the boundary layer on a flat plate.

The boundary layer thickness at a distance x can be expressed as follows:

$$\delta_1 = k\frac{1}{(Re_1)^{1/m}}.x \tag{9}$$

where k and m are constants, and δ_1 is the boundary layer thickness corresponding to the Reynolds number Re_1.

Now if the Reynolds number is increased to Re_2, and δ_2 represents the new boundary layer thickness then,

$$\delta_2 = k\frac{1}{(Re_2)^{1/m}}.\ x \tag{10}$$

or

$$\frac{\delta_2}{\delta_1} = \left(\frac{Re_2}{Re_1}\right)^{-1/m} \tag{11}$$

Assigning different values to the ratio $\frac{Re_2}{Re_1}$ the corresponding values of $\frac{\delta_2}{\delta_1}$ can be obtained and finally, the percentage reduction in boundary layer thickness can be calculated.

A few calculated values of percentage reduction in boundary layer thickness for corresponding increases in Reynolds number assuming a value of m equal to 10, are tabulated in Table 4 - (See Appendix - A) for illustration purposes.

A curve plotted between percentage decrease in boundary layer thickness and the ratio $\frac{Re_2}{Re_1}$, shown in figure 15, has a rising trend in the beginning but soon flattens out with further increase in Reynolds numbers ratio. The proportional reduction in boundary layer thickness becomes smaller as higher values of Reynolds number are attained. This appears to be the reason why Reynolds numbers of over 22,000 have a relatively smaller effect on settling length.

It is also noticed from figure 11 that the three lines corresponding to diameter ratios of 0.2, 0.3 and 0.5 show a similar trend while the last, corresponding to 0.7 deviates somewhat. It appears this may be due to the inclusion of the point for a low Reynolds number of 4,620, which may still be in the transition range. The other reason may be the lack of information very near to the entrance, which was due to the rather short settling length in this case. Additional test sections closer to the entrance were not feasible because of the increasing fluctuations in the flow due to disturbances caused by the sharp edged entrance.

Figure 12 shows the correlation obtained, using the statistical method of linear regression for the region Re $<$ 22,000. The correlation coefficient of .9908 indicates the "goodness of fit" and the equation of the best line is

$$Le/De = 0.795 \ (Re)^{0.375} (D_1/D_2)^{-0.6} \qquad (12)$$

The reason for the scatter in the first point corresponding to a Reynolds number of 4,620 for an annulus of diameter ratio 0.7, is the same as that offered in the preceding paragraph. A few extra points corresponding to Reynolds numbers somewhat higher than 22,000 are also included in the graph, and they appear as scatter.

The relation obtained for Re $>$ 22,000, from the experimental data drawn in figure 13 is:

$$Le/De = 15.96 \ (Re)^{0.077} (D_1/D_2)^{-0.624} \qquad (13)$$

Two extrapolated points corresponding to a Reynolds number of 50,000 for annuli having diameter ratios 0.5 and 0.7 are also shown in the above figure. These points are identified by a subscript e. The correlation coefficient obtained for this case is 0.987.

The power of (D_1/D_2) in the two correlations obtained, indicate that the effect of diameter ratio is almost the same in both cases. It would therefore seem intuitive that the power of the diameter ratio (D_1/D_2), should remain the same throughout the whole Reynolds number range, and if an average power of 0.61 is chosen then both correlations can be shown on one graph. This is shown in figure 14.

At diameter ratios close to 1, the annulus tends to approach the case of flow between parallel planes. The results indicate that a length of about 40-45 equivalent diameters is needed for the formation of the fully developed velocity distribution in the case of an annulus having a diameter ratio of 0.7. This figure appears to be somewhat lower than the 50 equivalent diameters generally quoted as the entrance length for turbulent flow of air between parallel planes. Probably, this is because the highest Reynolds number tested on this annulus was limited to about 24,000.

In their recent study of developing laminar flow in the entrance region of annular ducts, Sparrow and Lin (27) concluded that the flow development in annular ducts with radius ratios substantially less than unity is quite similar to that in parallel plate channels ($r_1/r_2 \rightarrow 1$). On the other hand, the results for an annular duct with diameter ratio as small as .001 depart significantly from those for a circular tube ($r_1/r_2 \rightarrow 0$).

The results of the present investigation for turbulent flow in the entrance region indicate a similarity to Sparrow and Lin's conclusions for laminar flow. It may be noted that for an annulus of diameter ratio of 0.2, a length of 100 equivalent diameters is required for the formation of fully developed flow. This differs significantly from that of the generally accepted figure of about 50 equivalent diameters (7) for the tube.

Presented in figure 15, are the friction factor data obtained for three different annuli having diameter ratios of 0.3, 0.333, and 0.5. The friction factors show good agreement with Davis' results given by equation $f = 0.055 \ Re^{-0.2} \left[\frac{D_2 - D_1}{D_2}\right]^{0.1}$, but are found to be about 10% higher than those obtained from the relation $f = 0.076 \ Re^{-0.25}$, which is the average of a number of experimental results.

Critical Comparison of Settling Length used by Previous Investigators

For investigations concerned with the structure of fully developed flow, the test section must be located beyond the settling length required for the prevailing Reynolds number and geometry, to ensure that fully developed flow exists. Due to the lack of specific data on settling length in annuli, most past investigators have more or less used judgement in the location of their test section. They seem to have used a fixed entrance length irrespective of the diameter ratios of the annuli and the range of Reynolds number tested.

Rothfus (21) located his test section at approximately 20 feet downstream from the inlet of the annulus during his entire investigation in which he tested two annuli with diameter ratios of 0.162 and 0.65. This length corresponds to 93 and 223 equivalent diameters respectively. The Reynolds number range covered was 1,000 to 21,000. The results of the present investigation indicate that for a Reynolds number of 21,000, a settling length of approximately 100 equivalent diameters is necessary for the annulus of diameter ratio 0.162, which is fairly close to the figure of 93 equivalent diameters actually used. For the annulus having a diameter ratio of 0.65, the settling length provided was more than adequate.

Okiishi and Serovy (19) used an entrance length of 197 inches in their work, and the dimensions of the test annuli and Reynolds number covered are shown in Table 5 (See Appendix A). Computations based on the two correlations obtained in the present study show that the settling length used in their investigation for annuli having diameter ratios of 0.531 and 0.781, was adequate for fully developed flow. However, a length of 75 equivalent diameters is needed for the annulus of diameter ratio 0.344, and this is slightly more than the figure of 68 used by them.

Brighton and Jones (4) found the entrance length for their outer pipe alone, and then used the same entrance length for the various annuli combinations using this outer pipe. They argued that "if the flow was fully developed for a pipe alone for a fixed value of Uo (The maximum time average velocity), it would be fully developed with a centre pipe in place for the same Uo." This appears to be reasonable since the equivalent diameter would be smaller when a centre pipe is inserted and as such the inlet length allowed expressed in terms of the equivalent diameters would be greater. Also the corresponding Reynolds number would be less than in the case of the outer pipe alone and thus would require a relatively shorter settling length.

To ensure a fully developed flow at the measuring station of the apparatus used by Brighton and Jones (4), the inside surface of the outer pipe was artificially roughened by bonding sand grains to the wall of the tube for a length of 3 feet from the inlet. Also a screen of concentric circular (1/32 inch diameter) wire rings was placed at the inlet of the test channel in order to support the inner pipe and to assist in producing an artificially thickened boundary layer. He emphasized that fully developed flow was obtained in an entrance length of 34.5 diameters only with the aid of the inlet roughness elements and screen and without these, the required inlet length was much greater. This is surprising in view of the data of Rothfus (22) for "low" and "high" turbulence. Neither the turbulence level, nor the Reynolds number (at least for $5,000 < Re < 45,000$) had much effect away from the entrance. Hart and Lawther (Discussion of 4) found that sand-roughening the pipe wall at the inlet had no effect on the development length and screens were similarly ineffective. This can be explained as follows by the work of Barbin & Jones (1). Early in the developing flow there is a deficiency in turbulent energy production in the wall layers, and this leads to an acceleration of the core region past its fully developed value. The later stages of development therefore depend on radial transfer of momentum towards the wall regions to restore equilibrium, and will not be materially affected by the production of excess turbulent energy near the entrance, since this energy is dissipated in maintaining a higher shearing rate near the wall. A screen increases turbulence, but the scale is small and the energy is rapidly dissipated. A honeycomb may be more effective.

CHAPTER 7

CONCLUSIONS

Based on the results of the present investigation the following conclusions were made:

(a) For annuli of similar geometry to those tested, it was found that settling length could be expressed as a function of Reynolds number, diameter ratio and equivalent diameter.

(i) Settling length increased with an increase in Reynolds number and a decrease in diameter ratio for turbulent flow.

(ii) Two distinct settling length regions above and below a Reynolds number of 22,000 were observed.

(iii) For $Re < 22{,}000$, the Reynolds number had a marked influence on settling length and the relation obtained was;
$Le/De = 0.795\ (Re)^{0.375}(D_1/D_2)^{-0.6}$

(iv) In the region of $Re > 22{,}000$, the settling length was only slightly affected by Reynolds number. The following relation fitted the data:
$Le/De = 15.96\ (Re)^{0.077}(D_1/D_2)^{-0.624}$

(v) For an annulus the influence of diameter ratio on settling length was almost the same throughout the entire range of Reynolds numbers tested.

(b) For annuli having diameter ratios close to unity the results approached those for flow between parallel plates. On the other hand for annuli of diameter ratios close to zero, the results deviated significantly from those for pipe flow. This was similar to Sparrow's predictions for laminar flow in annular ducts.

(c) Friction factor data agreed closely with Davis' results, but were about 10% higher than the values calculated from the equation $f = 0.075\ Re^{-0.25}$, the average of most experimental results.

RECOMMENDATIONS FOR FUTURE WORK

It is recommended that the following investigations be undertaken to expand the research programme described herein. All the recommended investigations should be carried out with the present apparatus provided the suggested modifications to the equipment are made.

1. The Reynolds number range covered in the present study should be extended. More data should be collected for Reynolds number greater than 100,000 and less than 10,000 for all the radius ratios available. A blower with a higher capacity should be installed to meet the larger mass flow rates required at the higher Reynolds numbers.
2. A surge tank should be included to reduce the distrubances in the flow encountered at high Reynolds number.
3. The effect of entrance shape on the velocity profile development should be studied in detail; the present annulus inlet section could be modified to satisfy the desired entrance conditions.
4. Annuli having diameter ratios in the vicinity of 0.8 or greater if possible, as well as in

the vicinity of 0.01 should be studied to simulate more clearly the flat duct and pipe cases.

5. The present study should be duplicated at a constant diameter ratio but at varying pipe and core dimensions.
6. The effect of pipe wall roughening, and screens, on inlet length should be studied in detail.

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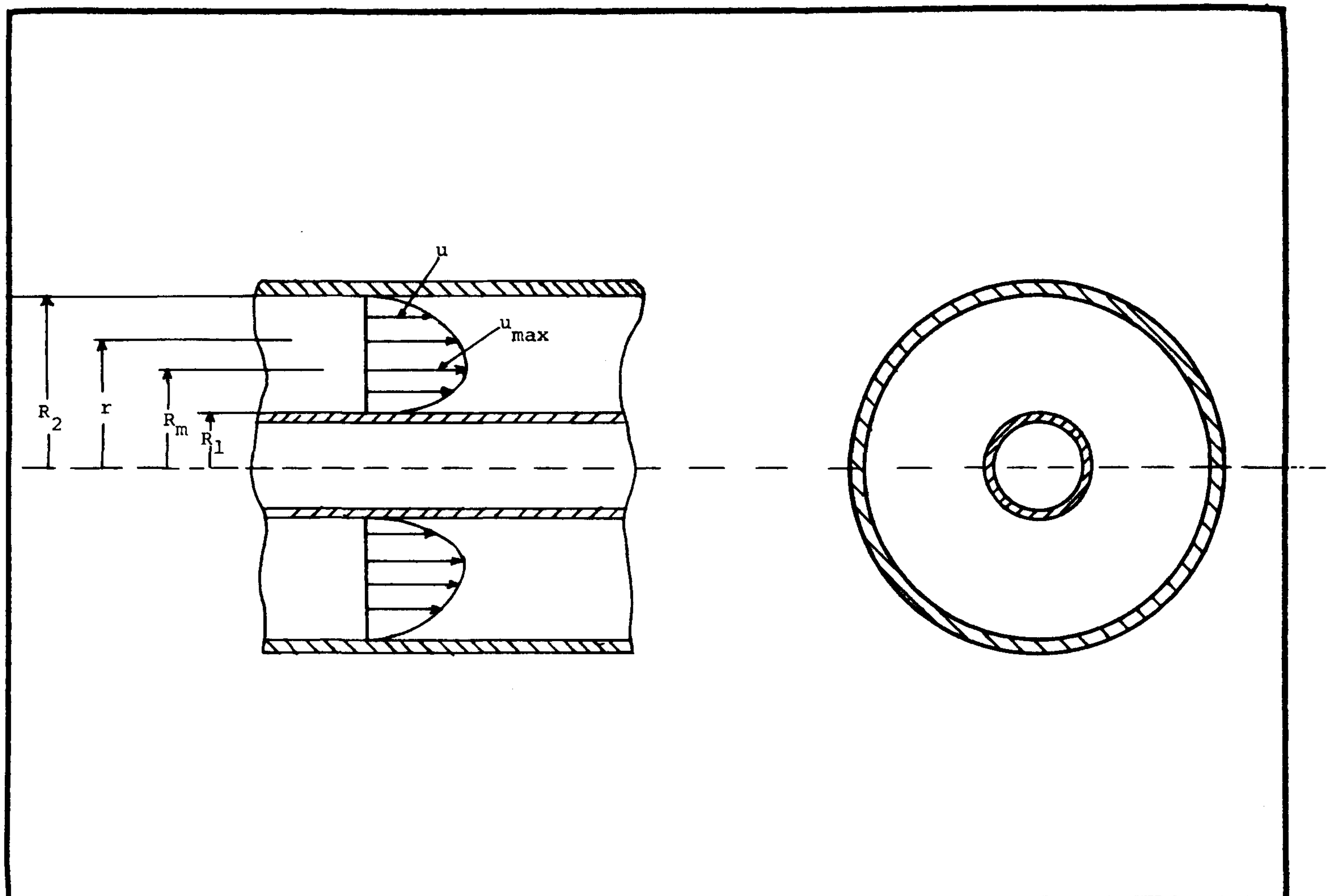


FIG. 1 GENERAL FLOW DESCRIPTION

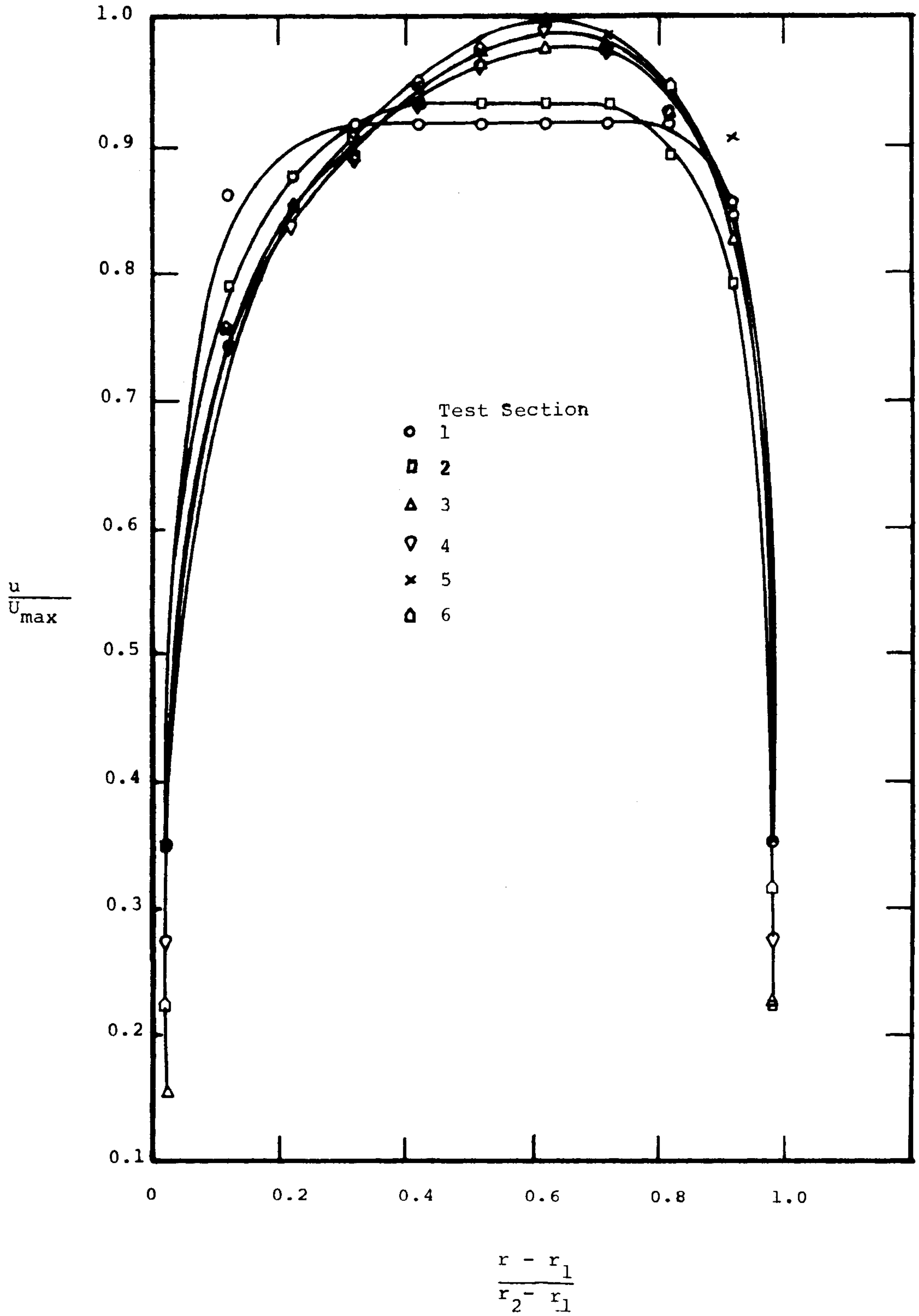


FIG. 2 DEVELOPMENT OF NON DIMENSIONAL VELOCITY PROFILE FOR ANNULUS 1 AT Re =11,700

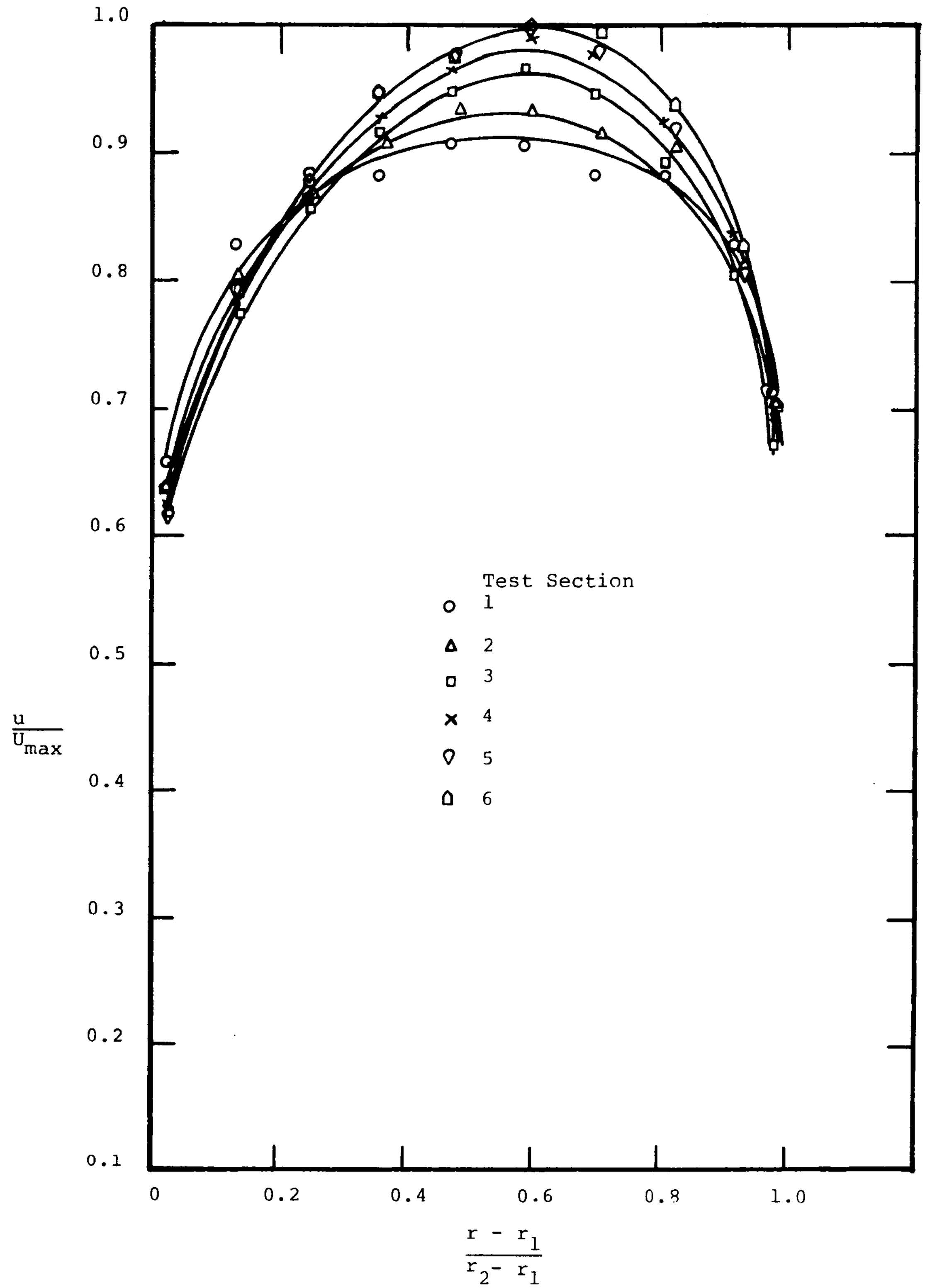


FIG. 3 DEVELOPMENT OF NON DIMENSIONAL VELOCITY PROFILE FOR ANNULUS 2, AT Re = 30,400

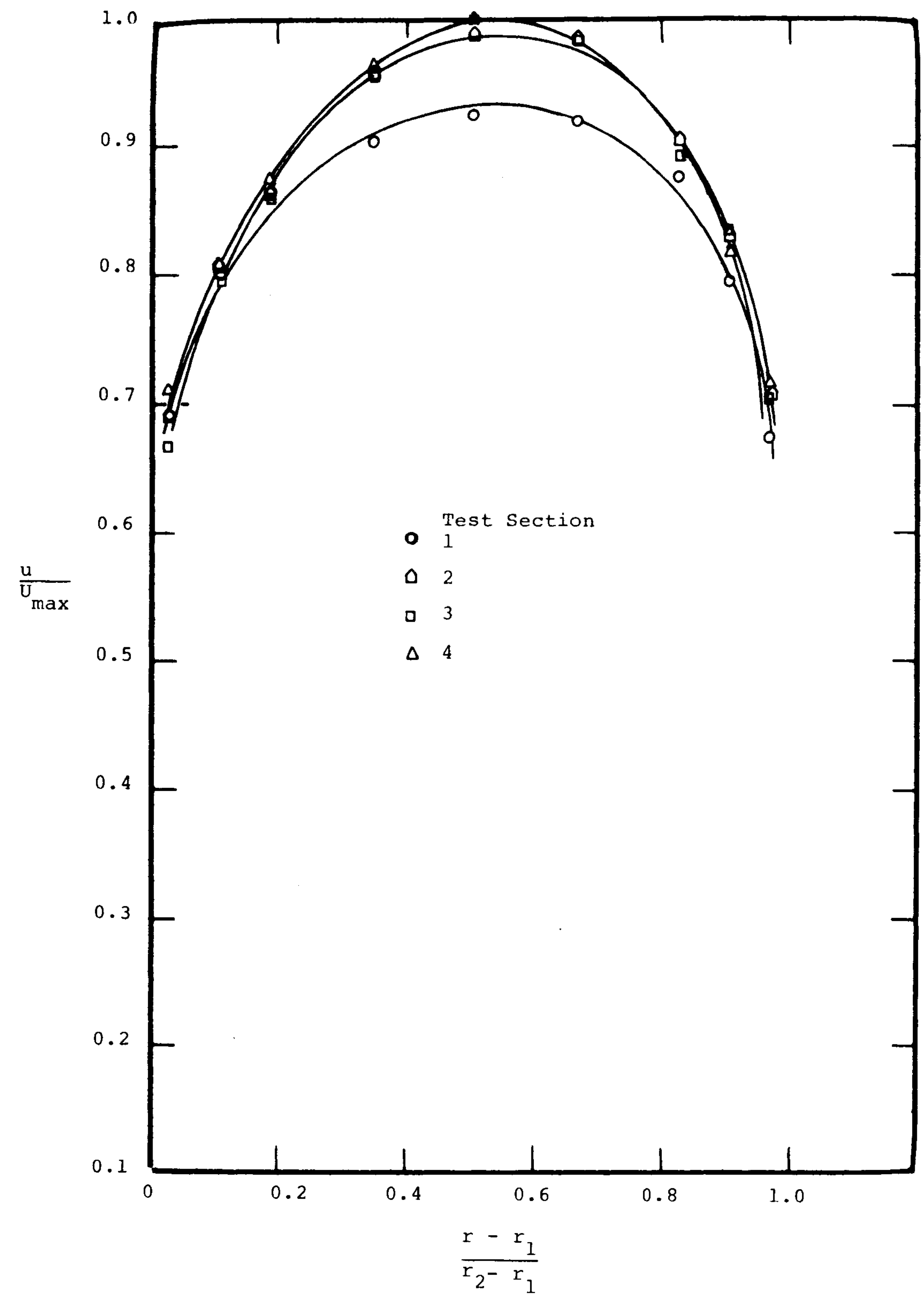


FIG. 4 DEVELOPMENT OF NON DIMENSIONAL VELOCITY PROFILE FOR ANNULUS 3, AT Re = 21,000

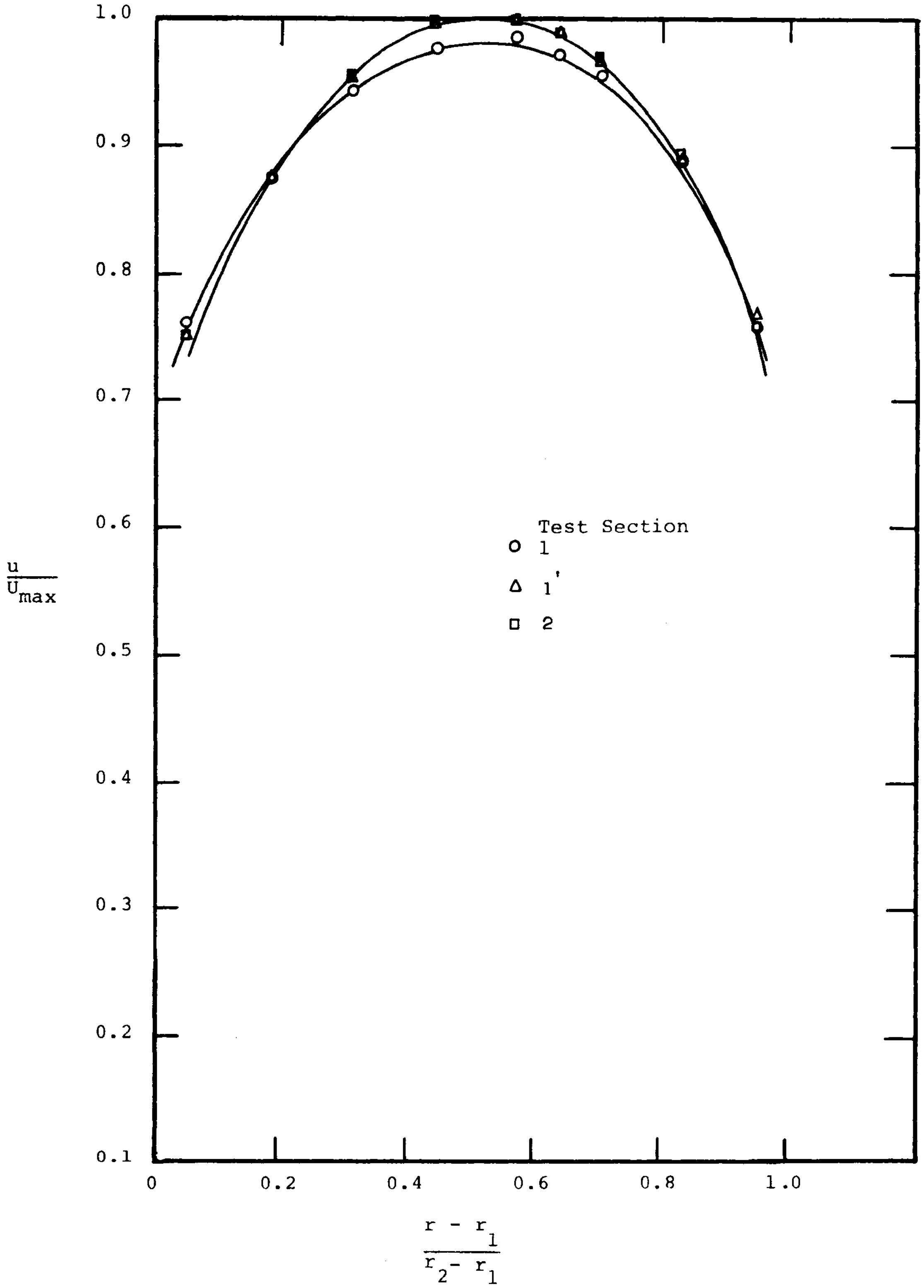


FIG. 5 DEVELOPMENT OF NON DIMENSIONAL VELOCITY PROFILE FOR ANNULUS 4, AT Re = 13,750

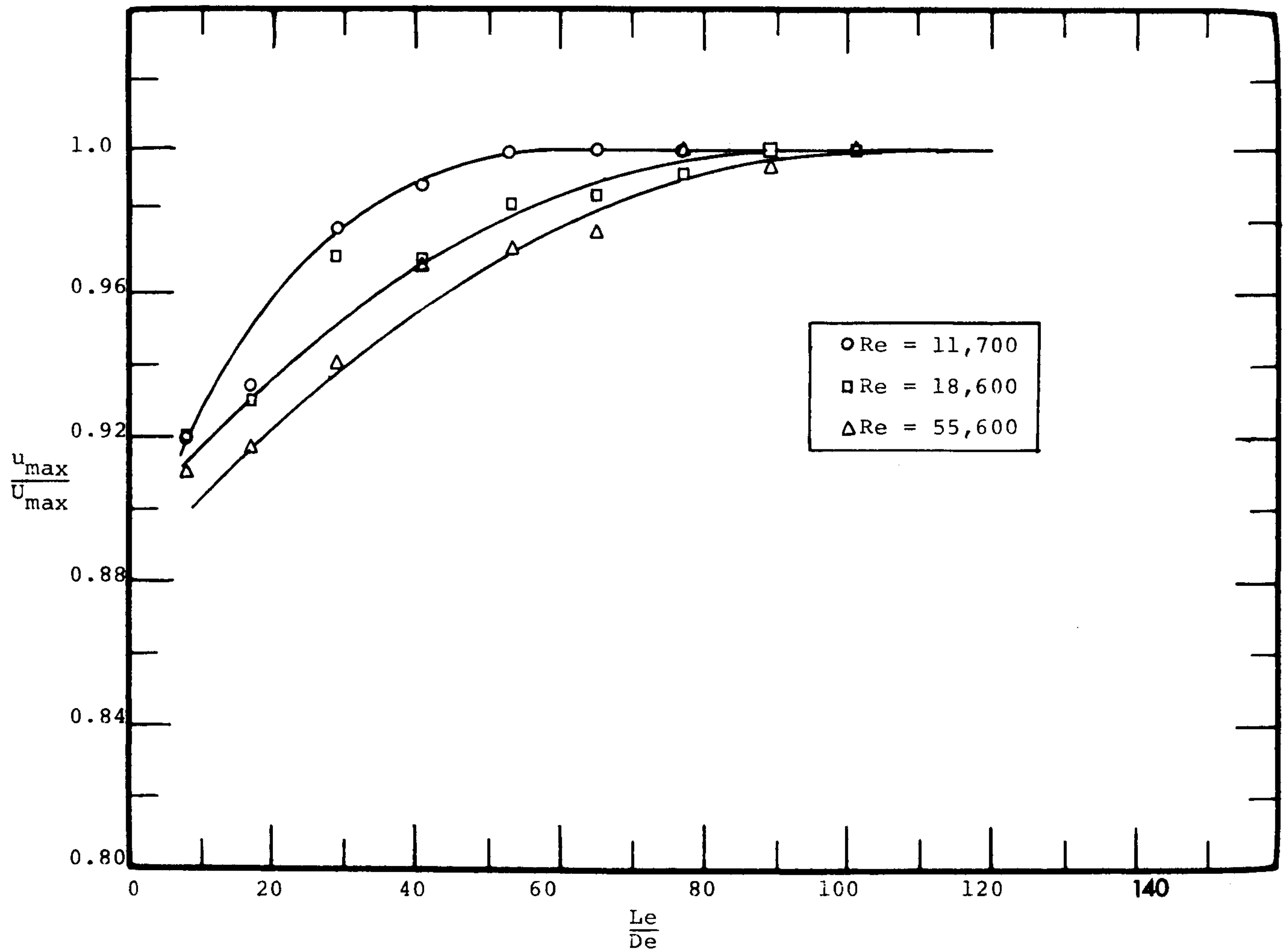


FIG. 6 PLOT OF u_{max}/U_{max} VERSUS Le/De FOR DIFFERENT REYNOLDS NUMBER, ANNULUS 1

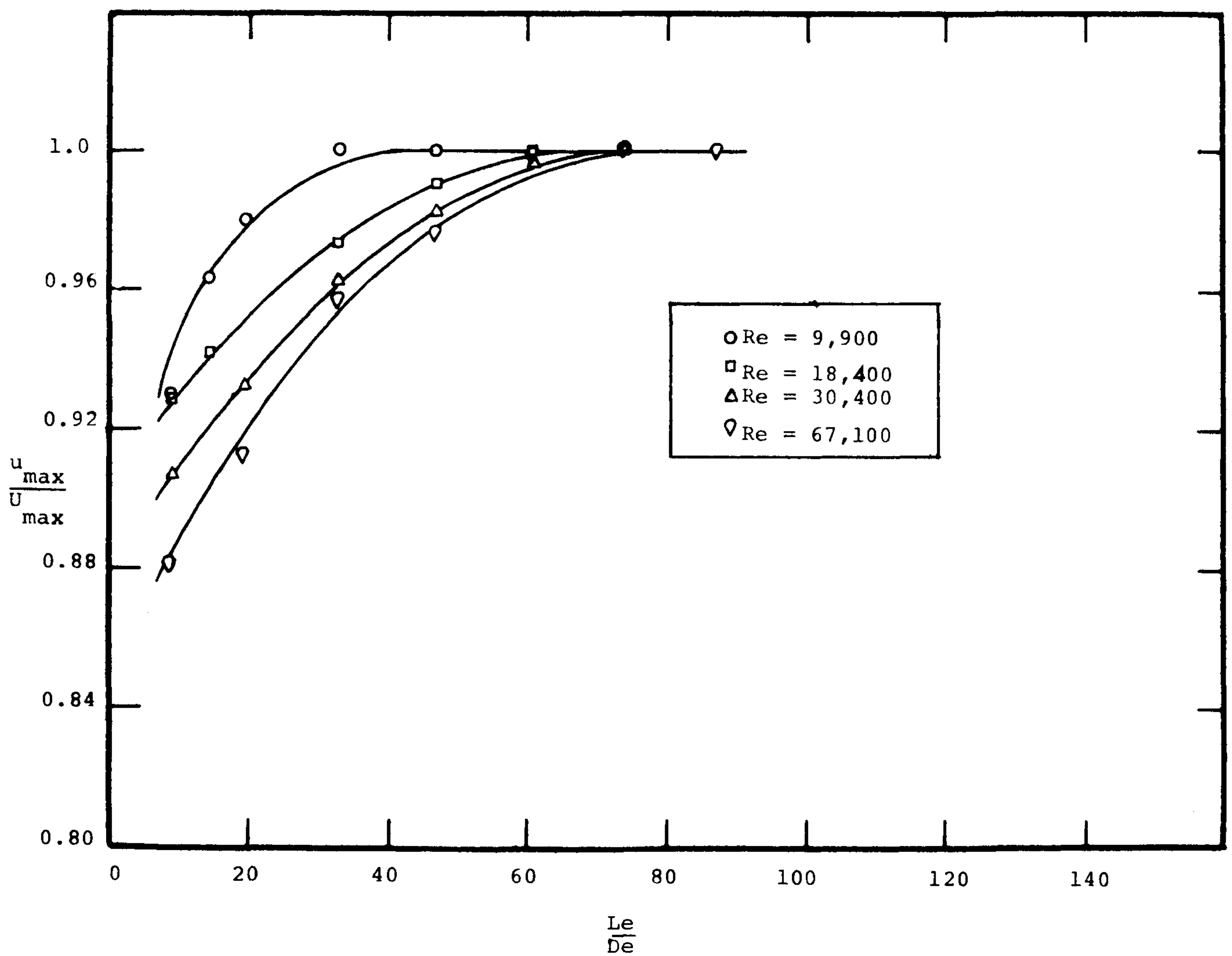


FIG. 7 PLOT OF u_{max}/U_{max} VERSUS Le/De FOR DIFFERENT REYNOLDS NUMBER, ANNULUS 2

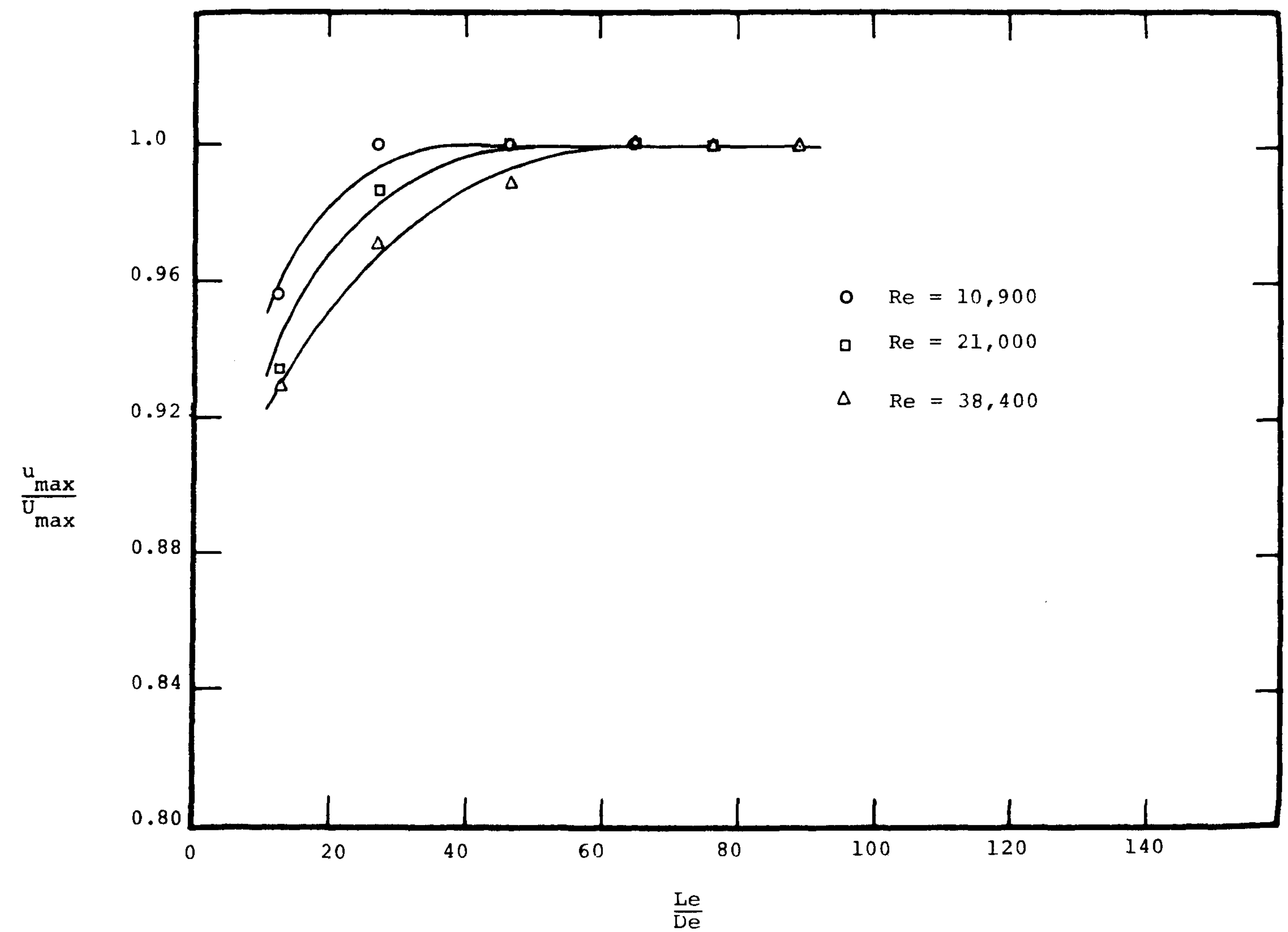


FIG. 8 PLOT OF u_{max}/U_{max} VERSUS Le/De FOR DIFFERENT REYNOLDS NUMBER, ANNULUS 3

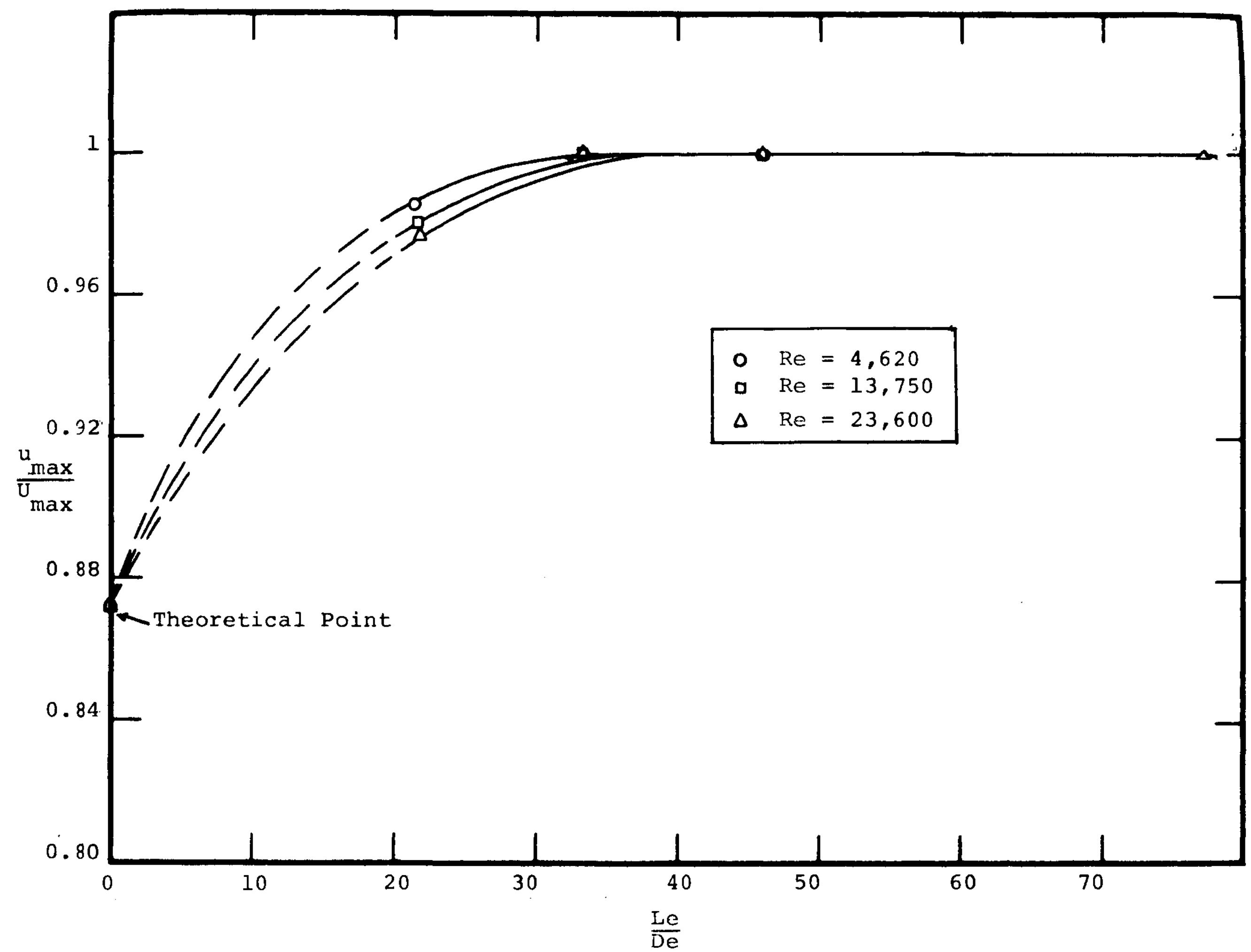


FIG. 9 PLOT OF u_{max}/U_{max} VERSUS Le/De FOR DIFFERENT REYNOLDS NUMBER, ANNULUS 4

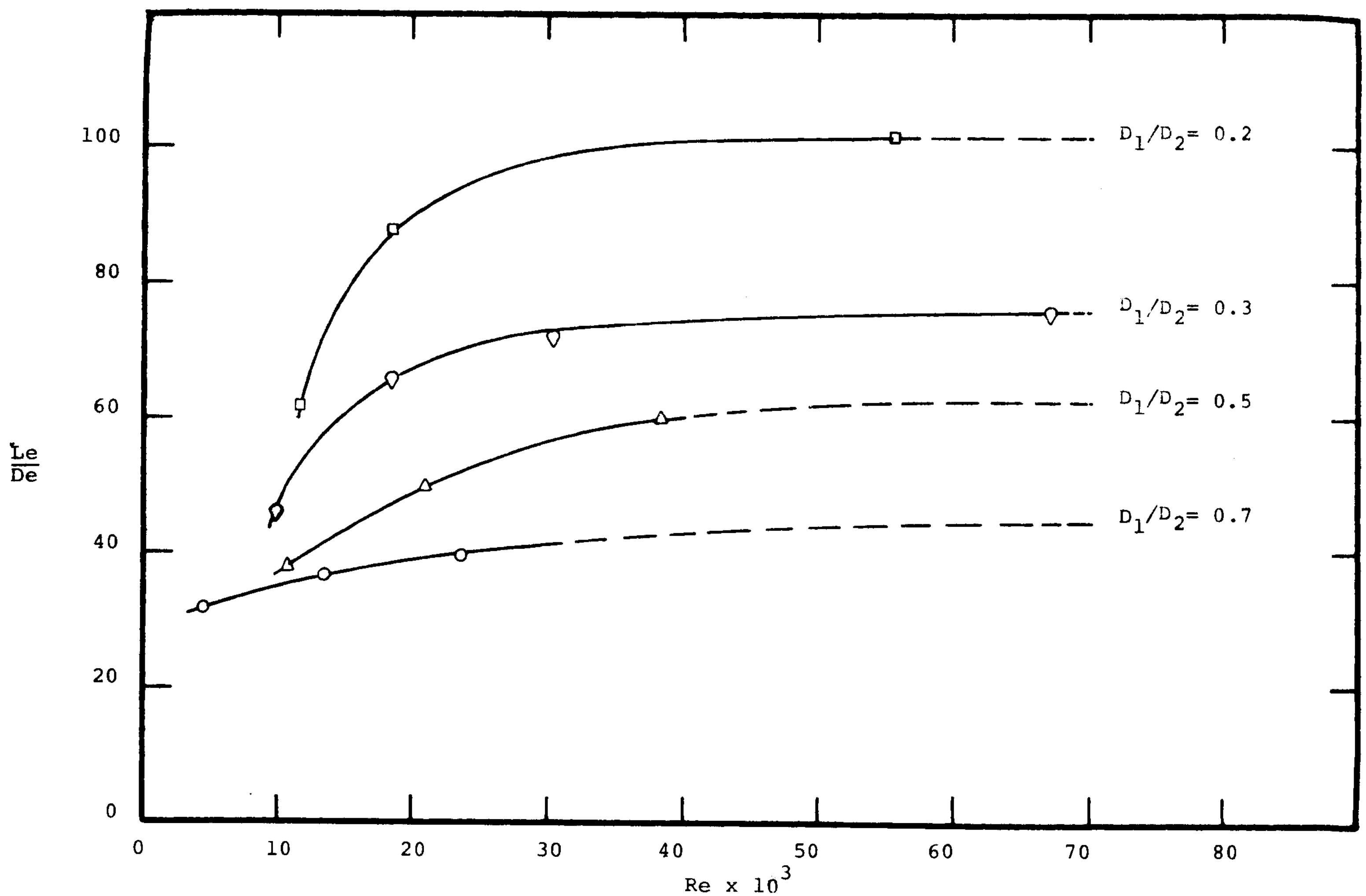


FIG. 10 VARIATION OF Le/De WITH Re FOR ANNULI OF DIFFERENT DIAMETER RATIOS

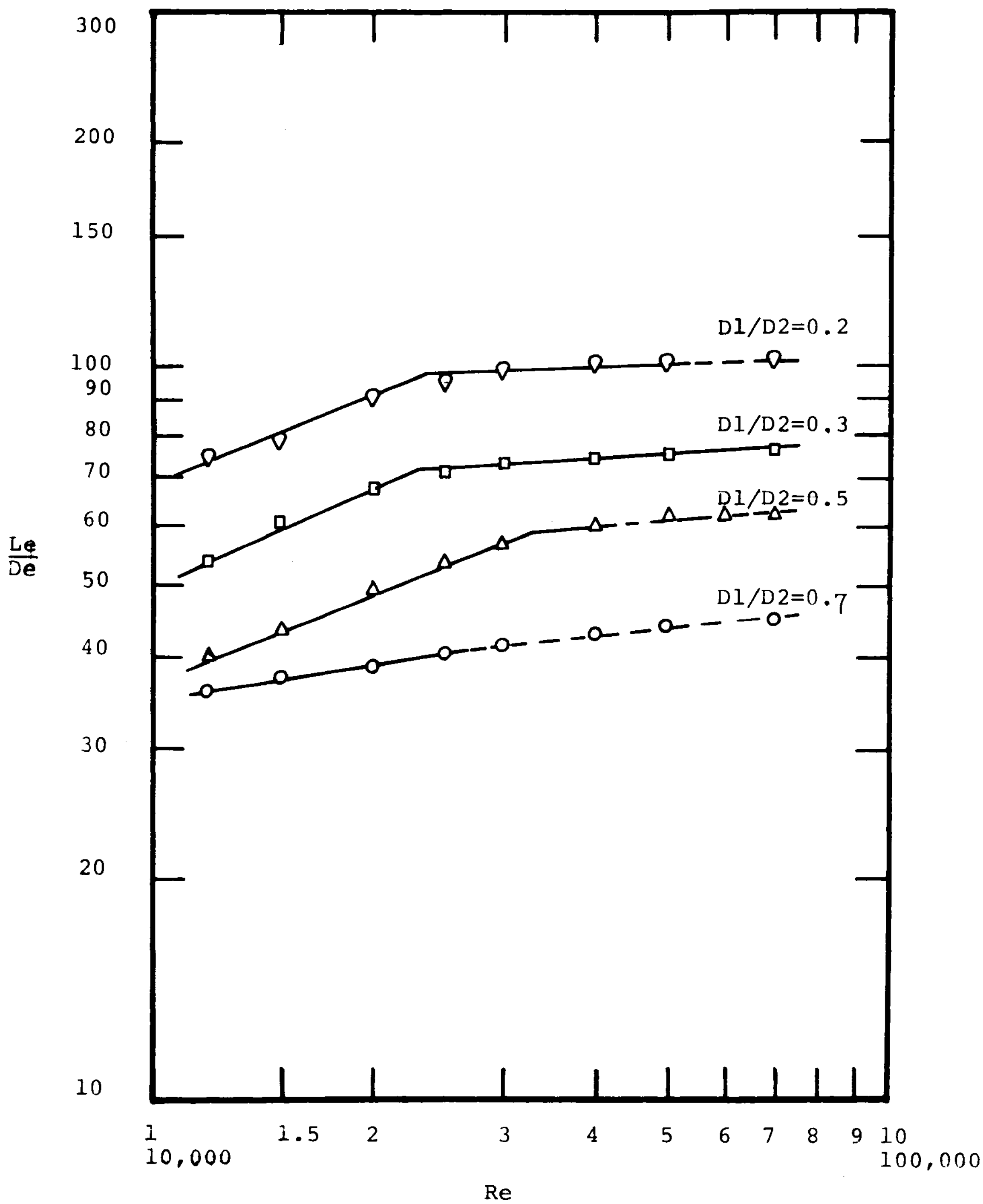


FIG. 11 LOGARITHIMIC PLOT OF Le/De AND Re FOR ANNULI OF DIFFERENT DIAMETER RATIOS

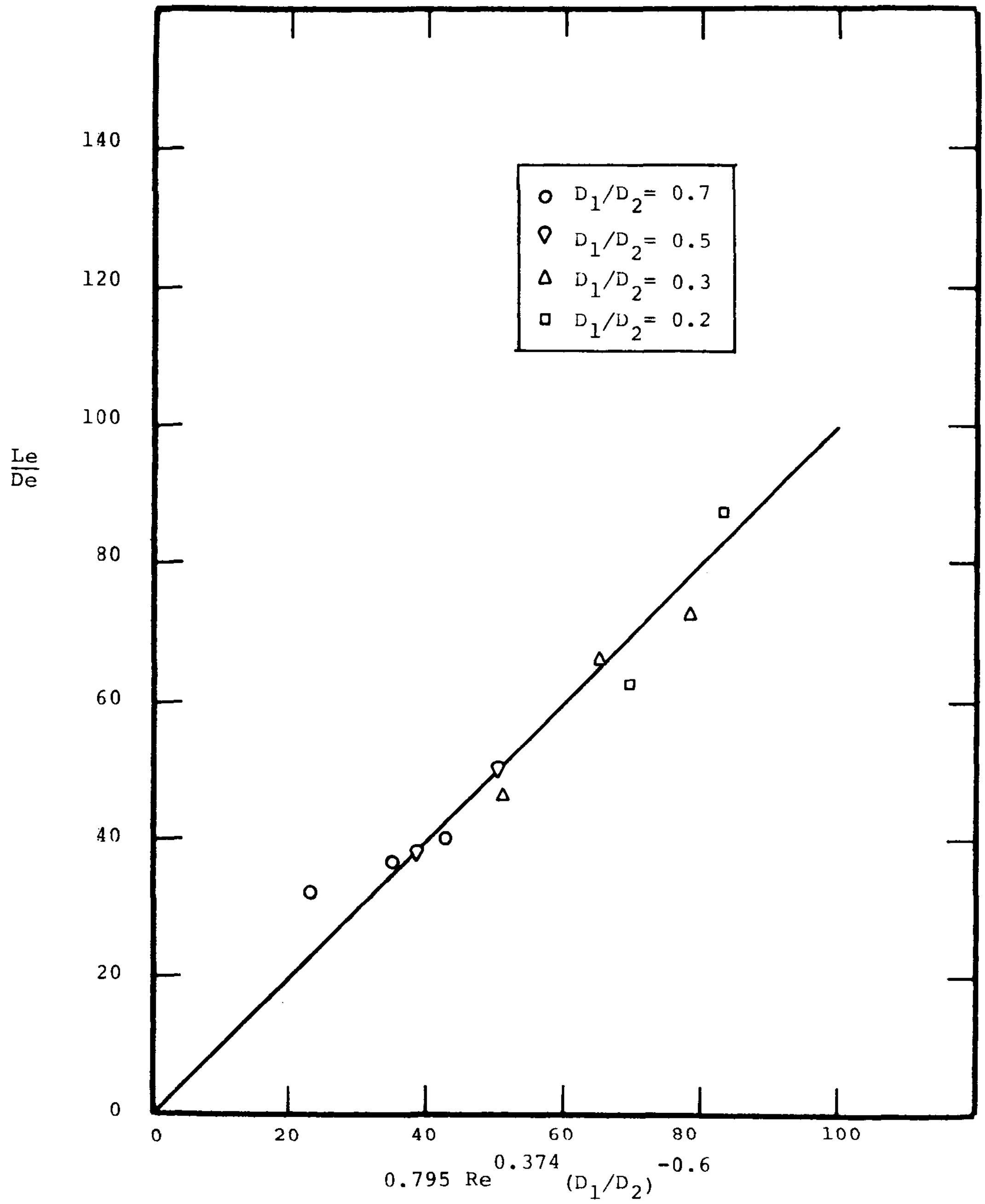


FIG. 12 EXPERIMENTAL DATA SHOWN ON REGRESSION LINE FOR Re < 22,000

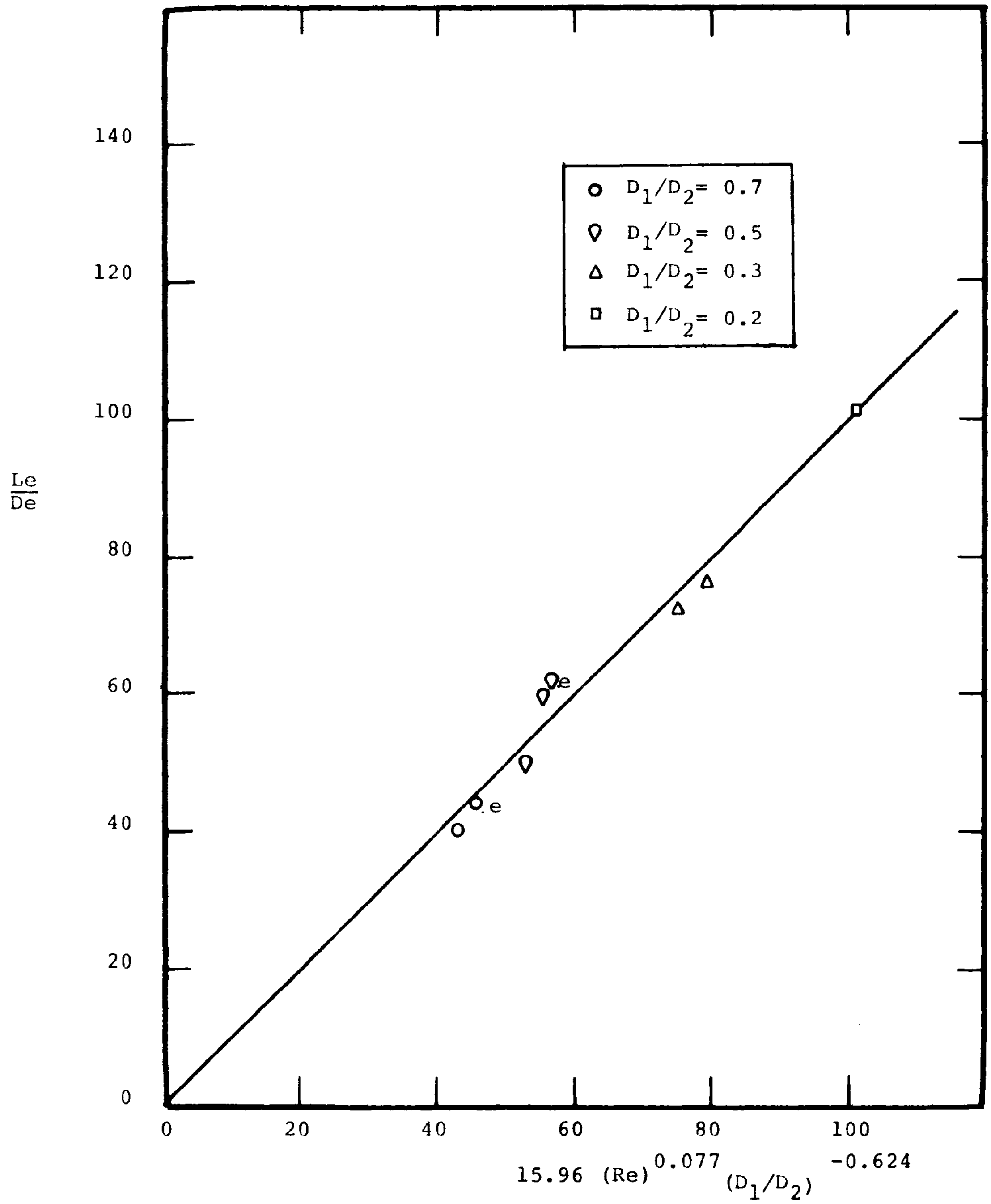


FIG. 13 EXPERIMENTAL DATA SHOWN ON REGRESSION LINE FOR Re > 22,000

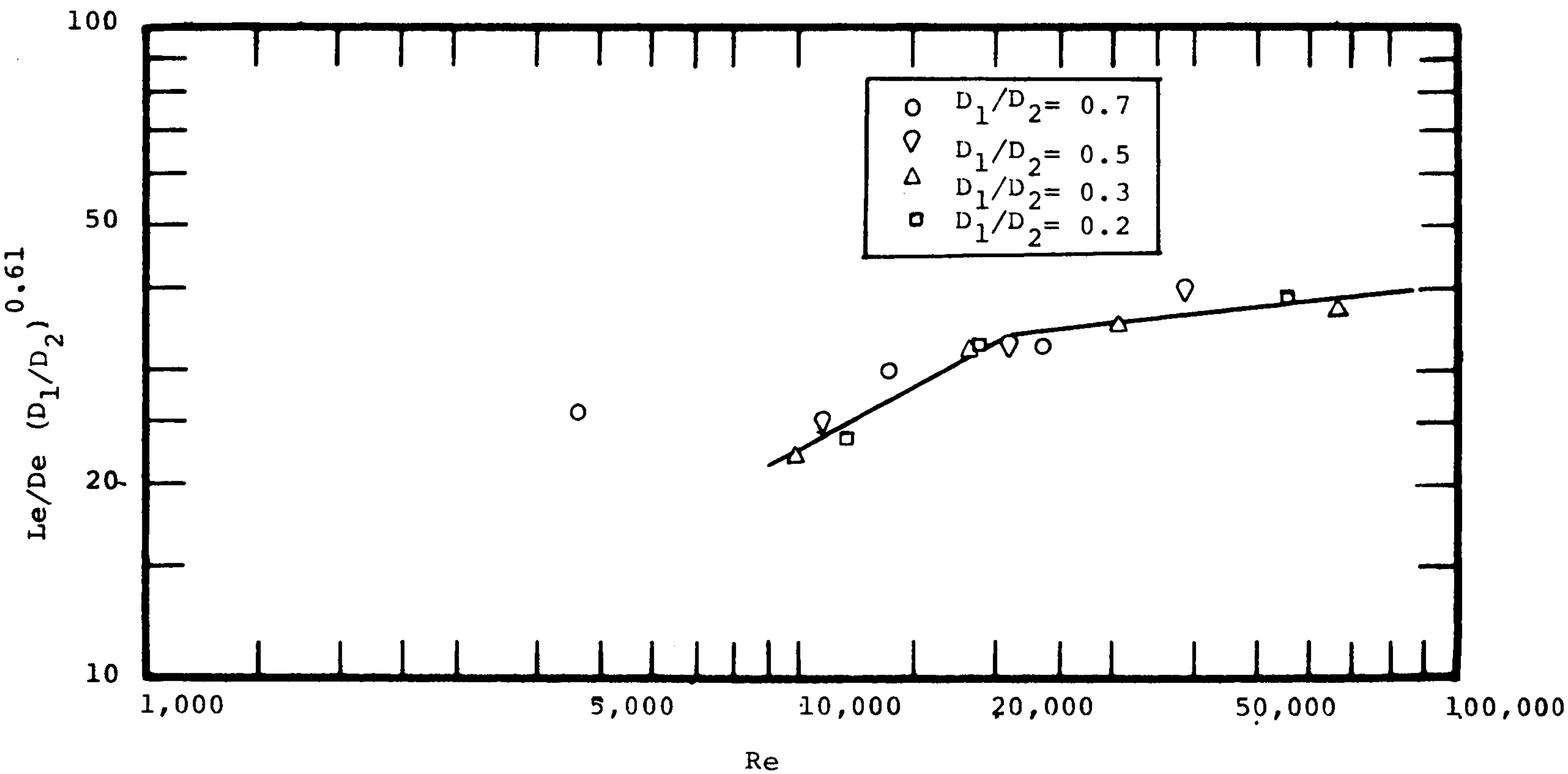


FIG. 14 LOGARITHIMIC PLOT OF $Le/De\ (D_1/D_2)^{0.61}$ VERSUS Re

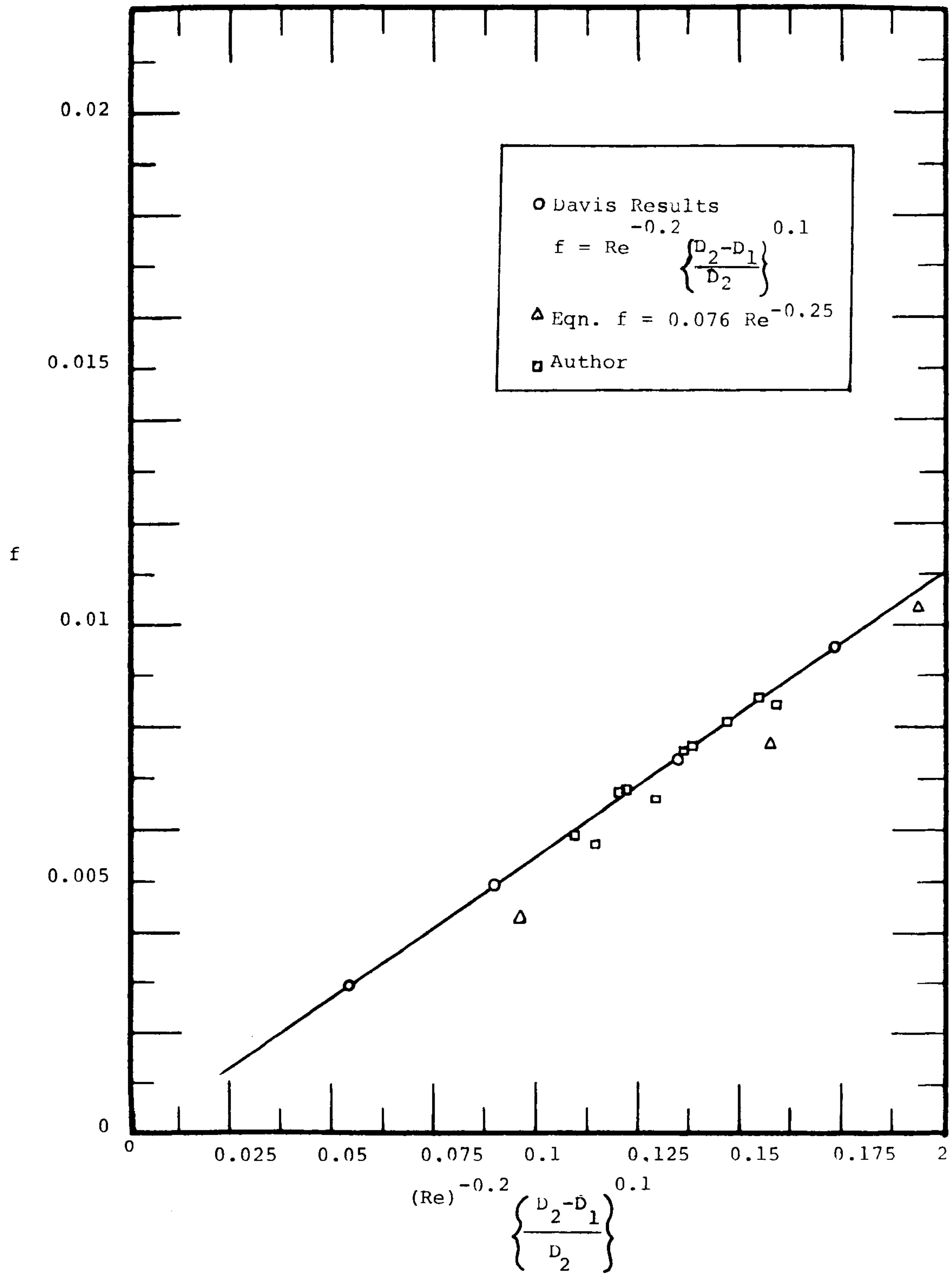


FIG. 15 COMPARISON OF EXPERIMENTAL FRICTION FACTOR DATA WITH RESULTS OF DAVIS AND OTHERS

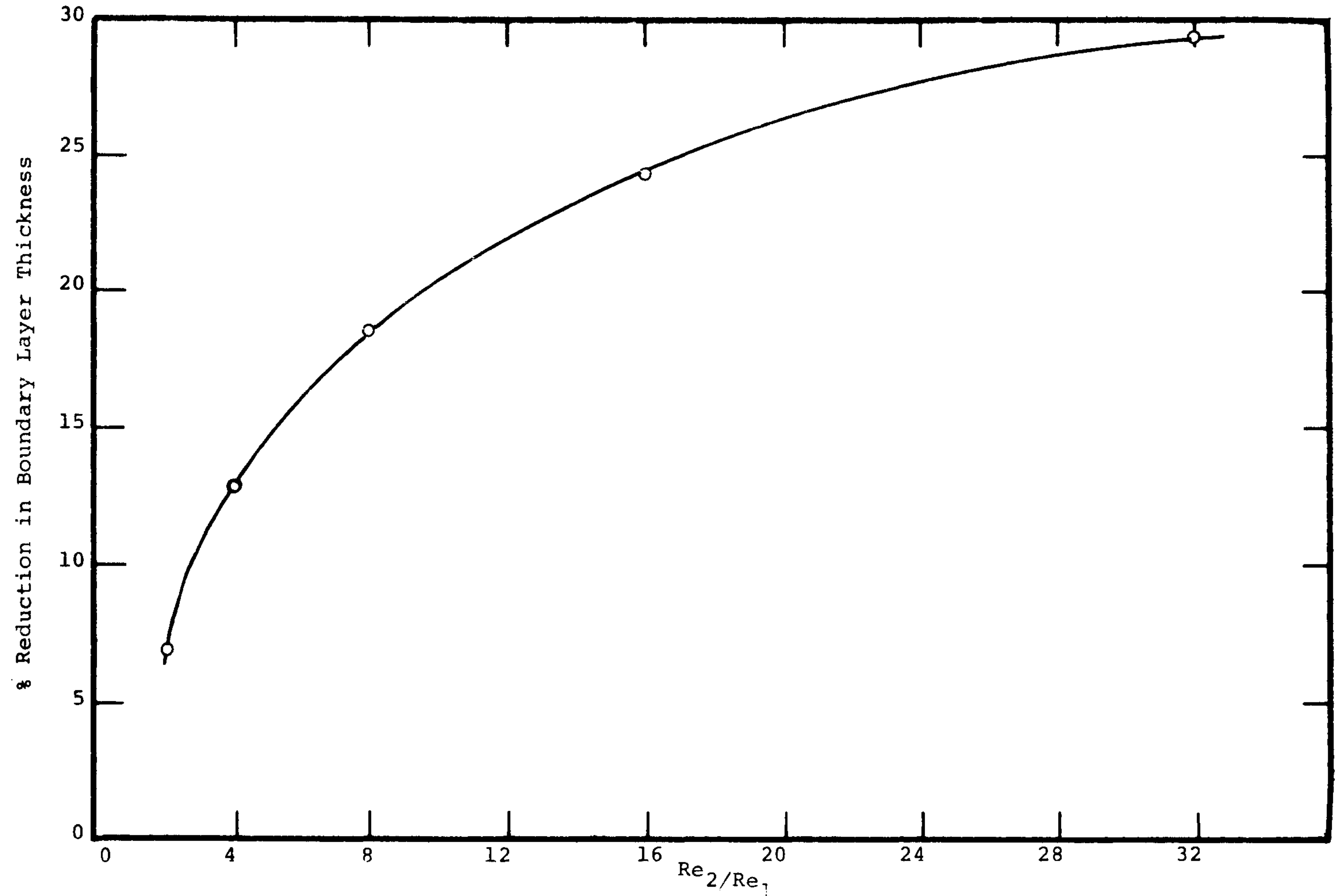


FIG. 16 VARIATION OF % REDUCTION IN BOUNDARY LAYER THICKNESS WITH INCREASE IN REYNOLDS NUMBER

APPENDICES

APPENDIX A

TABLE 1. Location of Test Sections From the Entrance

Test Section	Distance From Entrance, Inches
1	16
1'	25
2	34
3	58
4	82
4'	94
5	106
6	130
7	154
8	178
9	202

TABLE 2. A Summary of Test Runs in Determination of Settling Length

Annulus Designation	Inner Tube Outside Diameter Inches	Outer Tube Inside Diameter Inches	Diameter Ratio	Reynolds number based on equivalent diameter $(D_2 - D_1)$
1	0.5	2.5	0.2	11,700 18,600 55,600
2	0.75	2.5	0.3	9,900 18,400 30,400 67,100
3	1.25	2.5	0.5	10,900 21,000 38,400
4	1.75	2.5	0.7	4,620 13,750 23,600

TABLE 3. Details of The Test Runs for Friction Factor Data

Annulus Designation	Inner Tube Outside diameter Inches	Outer Tube Inside diameter Inches	Diameter Ratio	Reynolds number based on Equivalent Diameter $(D_2 - D_1)$		
1	0.75	2.5	0.3	9,900	18,400	30,400
2	0.5	1.5	0.33	7,500	15,500	30,300
3	1.25	2.5	0.5	10,200	20,000	35,700

TABLE 4. Variation in Boundary Layer Thickness with Increase in Reynolds Number

For m = 10

Re_2/Re_1	δ_2/δ_1	% Reduction in boundary layer thickness
1	1	--
2	0.932	6.8
4	0.872	12.8
8	0.815	18.5
16	0.757	24.3
32	0.707	29.3

TABLE 5. Pertinent Dimensions of The Test Annuli

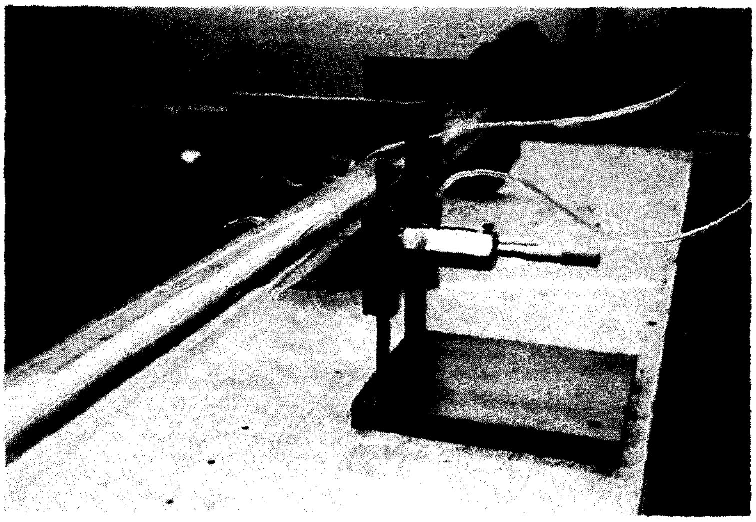
Used by Okishi And Serovy

Annulus	Inside Tube Outside Diameter Inches	Outer Tube Inside Diameter Inches	Diameter Ratio	Approximate Range of Reynolds Number	Hydraulic Diameters Between Entrance & Traverse Location
1	1.375	4.00	0.344	28,000 to 100,000	68
2	2.125	4.00	0.531	24,000 to 70,000	95
3	3.125	4.00	0.781	12,000 to 40,000	200

APPENDIX B

Photograph A

The Annulus



Photograph B

Impact Tube Traversing Mechanism

VITA AUCTORIS

1940	Born in Khandwa, India on December 24.
1955	Graduated from the Bal-Vinay-Mandir High School, Indore, India in May.
1957	Completed Intermediate in Science at the Holkar College, Indore, India in May.
1961	Received the Degree of Bachelor of Mechanical Engineering (Honours) from the University of Jabalpur, Jabalpur, India.
1962	Received the Degree of Master of Mechanical Engineering from the University of Roorkee, Roorkee, India.
1964	Served as Mechanical Engineer at the Heavy Engineering Corporation Ltd., Ranchi, India.
1965	Candidate for Degree of Master of Applied Science at the University of Windsor, Windsor, Ontario, Canada.
1966	Currently employed as a Manufacturing Engineer with Canadian Westinghouse Co. Ltd., Hamilton, Ontario.