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Digital pattern recognition: An investigation of the discriminatory properties exhibited by an information theoretical algorithm when applied to a differential function of pattern.

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DIGITAL PATTERN RECOGNITION: AN INVESTIGATION OF THE DISCRIMINATORY PROPERTIES EXHIBITED BY AN INFORMATION THEORETICAL ALGORITHM WHEN APPLIED TO A DIFFERENTIAL FUNCTION OF PATTERN

BY

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ABSTRACT

Visual pattern recognition is formalized in terms of an information theoretical algorithm which operates upon a contrast function of pattern data. The contrast function facilitates discrimination ability with a slight decrease in decision time. The information theoretical algorithm computes a measure of the additional information which an unknown pattern provides about each of a set of statistically established pattern classes.

An IBM 1620_{II} computer was used to simulate a classification system based upon these concepts. One hundred per cent of the learned patterns and fourty per cent of unlearned random patterns were able to be recognized correctly on the basis of only twelve learned samples of each of thirty-five types of pattern.

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I INTRODUCTION

Recent advances in the scientific technology of digital computer hardware and the coincident development of computer software have enabled researchers to simulate human recognitory processes which would otherwise most certainly have remained philosophical enigmas. As a result, many new mathematical expressions have evolved concerning visual, aural and other neurological concept formation and detection systems. This paper is principally involved in developing a visual pattern classification algorithm based upon the formalized concepts of information theory and upon the view that pattern content is more fundamentally represented by internal contrasts than by an absolute signal structure.

The design of an automaton capable of classifying random patterns is in answer to a demand for faster and more efficient data processing. Medical pictorial diagnosis, automatic radar pattern detection and remote decisive vehicular control (by environmental pattern analysis) are only some of the eventual goals of such an automaton. Before their realization, however, several outstanding problems are yet to be solved.

A. Related Problems

Of chief concern are coding and identification times and memory capacity requirements. A real time system demands an almost instantaneous machine decision capability in order to cope with sequential events. An off line computer, on the other hand, might require a memory of extraordinarily large capacity to contain the background statistics necessary to produce a correct decision. Fortunately, these requirements are being fulfilled, as was mentioned, by a rapid advancement of computer technology.

Noise is indeed a salient problem. In the sense of a loss of data through quantization, noise is unavoidable; yet, the variability of position of pattern data in the field of view is noise of a type which can almost be completely eliminated by proper normalization techniques.[1] Data which is conjunctive of every pattern in the system vocabulary is noise which may easily be programmed out of the decision process (at the expense of off line computing time). Much background and additive interference can possibly be eliminated by operating on a differential function of the pattern, a principle set forth in this thesis. There is often present, too, completely irrelevent data (random or otherwise) which periodically creates unwanted ambiguities. These may be discontinuities in otherwise continuous pattern structures, smudges, fogginess due to an improper focus, glare arising from intensity saturation and otherwise undesireable effects. A particular pattern so adversely affected is often inevitably unrecognizable; but, such effects on the overall statistical impression of a large sample of any particular class of pattern is usually negligible.

Random pattern orientation in the field of view presents an interesting challenge to any recognition scheme. This problem is perhaps the one which is most avoided by pattern recognition researchers. An obvious solution would seem to be an inventive transformation which would render the immediately operative data invariant under translation, rotation and magnification. Unfortunately this problem is more complex than it appears and the result is that no such efficient transformation has yet been discovered.

If the recognition system is an adaptive one, the question of degeneration arises. The addition of recognized random inputs to the overall statistics should increase the discriminatory ability of the decision algorithm.

In a statistically based classification system, one must decide when he has acquired enough initial data about the patterns to be discriminated. He is bounded on one hand by the inefficiency of too much data, and on the other by the cost of ambiguous results due to an insufficient statistical sample. Again, in a digital system, a decision must be made as to the coarseness of segmentation of patterns into elements, and the degree of quantization of the signal energy corresponding to each element. These decisions would not arise in an analogue system; but, the versatility of a digital machine (with the exception of an analogue interface at the input) in both function and memory capacity far outweighs the present speed advantage of analogue recognition systems.

For a more detailed analysis of these and other problems related to pattern recognition research, and how attempts have been made to alleviate them, the reader is invited to refer to a synopsis by Spinard[2] and also to (3).

B. Outline of the Procedure

The classification algorithm under investigation is basically statistical in nature and therefore an amount of prelearning or probabilistic sampling is required. Only after obtaining sufficient background knowledge about a select group of pattern classes can meaningful recognition be accomplished. Hence, there is a need to organize the structural content of patterns into a form most suitable to the techniques employed by the classification algorithm. This involves segmenting the pattern continuum into the minimum number of discrete surface elements which will facilitate an efficient discrimination among the entire set of pattern classes. The number of such elements is critical since there is a proportionate loss of data due to the averaging technique used to measure element signal strength (as in a photo-diode).

Any gradients of light energy over the surface of a single element are therefore extinguished. The resultant average energy of each element is then quantized into one of a predetermined number of signal levels. Thus, patterns are represented by an ordered array of elements and the value of each element belongs to a bounded discrete signal space.

It is important to note that a consistent ordering of these elements be maintained to preserve the very 'essence' of pattern.

To complete the coding process, the above array requires an additional transformation. The reasons for and the effects of this transformation will be discussed in Chapter II. It is sufficient to mention here that a pattern is not merely the communication of a group of parallel signals. Rather, it is meant to convey some concept described by the interrelationship of light energies over the entire pattern field. To realize this idea in terms of the signal valued array, an arbitrary element is first chosen from the array. The value of each element following this one is subtracted in turn from the value of the selected element. The manner in which the remaining elements of the array are selected is also arbitrary. It is important to note, however, that once the starting point and the scheme for exhausting the rest of the elements of the array have been decided, that scheme and no other must absolutely define the transformation. What results is an ordered sequence of contrast valued digits. This process gives rise to a new bounded and discrete signal space. The elements of that space shall be the basis of frequency distributions characterizing individual pattern classes.

If the pattern were segmented into x elements, then the corresponding contrast valued sequence would have x - 1 digits. When accumulating the statistics of a specific class of pattern, each of the x - 1 digits is treated independently. The number of occurrences (over a wide sample of that class) of each value in the contrast valued signal space is then tabulated first for one of the x - 1 digits, and then for another, and so on until x - 1 distributions are formed. Such statistics are accumulated from known patterns and each group of distributions is accordingly labelled.

A very similar process is followed in the recognition of an unknown pattern. After quantizing and transforming the pattern into a contrast value sequence of digits the occurrence of the contrast value in each of the x - 1 digit positions is compared to the frequency of occurrence of the same contrast value in the corresponding digit position of a particular prelearned pattern class. A measure of this correspondence within the same digit position is computed, one for each of the x - 1 digit positions. The measure is derived from Information Theory and will be discussed in detail in Chapter II.

Finally, the x - 1 measures are summed to give an indication of the total additional information which the unknown sample provides about the particular prelearned pattern class. A weighting function may be included in the above summation. Similar totals are computed for each of the remaining prelearned pattern classes and the unknown sample is identified as the pattern class which yields the highest total measure.

This recognition system is adaptive in the sense that the statistics of an identified pattern may be added to those of the identified class.

II THEORY

A. The Effects of Quantizing Patterns In Terms of their Internal Contrast Structure

The content of an optical pattern is perhaps best described by the variations of light energy over the visual plane. These light energy gradients seem more closely related to the concepts which a pattern strives to communicate. It is logical, therefore, to code patterns in a manner which emphasizes their contrast structure.

1. Contrast Coding

Let a pattern be segmented into an array of a X b discrete elements. In addition, let the signal values which an element may assume be limited to a discrete set, S. The elements of S shall consist of zero (0) and the first q positive real integers.

$$S: (s_0, s_1, \ldots\ldots s_q)$$

Now generate a set, C, of contrast values from the elements of the set S, according to the law,

$$c_k = s_i - s_j \qquad (1)$$

$$i = 0,\ 1,\ \ldots\ldots\ q$$

$$j = 0,\ 1,\ \ldots\ldots\ q$$

$$k = -q,\ \ldots\ 0,\ \ldots\ q$$

The elements of C, therefore, include zero (0) and the first q positive and negative real integers.

$$C: (\ c_{-q},\ \ldots\ c_0,\ \ldots\ c_q\ )$$

Just as a pattern can be represented by a matrix of discrete signal values, it can also be represented by a different matrix of contrast values. If the former matrix is denoted as M and the latter is denoted as N, then the matrix N may be generated from the matrix M by the following equation,

$$(n_{i,j}^{u,v}) = (m_{i,j}) - (m^{u,v}) \qquad (2)$$

$$u, v = 1, 1;\ \ldots\ldots\ a, b$$

$$i, j = 1, 1;\ \ldots\ldots\ a, b$$

$n_{i,j}^{u,v}$ are the elements of the matrix, N, while $m_{i,j}$ and $m^{u,v}$ are the elements of the matrix M. Matrix N therefore has $(a \times b)^2$ discrete contrast valued elements. $(n_{i,j}^{u,v})$ represents the contrast between the value of the element of row u and column v of matrix M, and the value of the element of row i and column j of matrix M. Each elemental value, $(m_{i,j})$, of matrix M generates a contrast value between itself and each of the other elements of M forming the i,j^{th} row of matrix N. See Figure 1. for a detailed representation of a generalized signal valued matrix and the resultant generated contrast valued matrix.

The following relationships are given for the N matrix of Figure 1. (See the example in Appendix B)

$$(n_{i,j}^{u,v}) = -(n_{u,v}^{i,j}) \tag{3}$$

$$(n_{i,j}^{i,j}) = (n_{u,v}^{u,v}) = 0 \tag{4}$$

$i,j = 1,1; \; \ldots\ldots \; a,b$

$u,v = 1,1; \; \ldots\ldots \; a,b$

and

$$(n_{i,j}^{u,v}) = (n_{(i,j)-1}^{u,v}) - (n_{(i,j)-1}^{i,j}) \tag{5}$$

$i,j = 1,2; \; \ldots\ldots \; (a,b)-1$

$u,v = (i,j)+1; \; \ldots\ldots \; a,b$

(ie. elements enclosed by the diagonal and the first row of N)

Note: (i, j)-1 denotes the row preceding row i, j in matrix N.

(i, j)+1 denotes the row following row i, j in matrix N .

An example of equation (5) may be found in Appendix B.

Figure 1. Signal Valued Matrix

$$
\begin{array}{llll}
m_{1,1}; & m_{1,2}; & \cdots\cdots & m_{1,b} \\
m_{2,1}; & & & m_{2,b} \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
m_{a,1}; & m_{a,2}; & \cdots\cdots & m_{a,b}
\end{array}
\qquad M
$$

Contrast Valued Matrix (generated from M)

$$
\begin{array}{lllllllll}
n^{1,1}_{1,1}; & n^{1,2}_{1,1}; & . & n^{1,b}_{1,1}; & n^{2,1}_{1,1}; & . & n^{2,b}_{1,1}; & .. & n^{a,b}_{1,1} \\
n^{1,1}_{1,2}; & & & & & & & & n^{a,b}_{1,2} \\
\cdot & & & & & & & & \cdot \\
n^{1,1}_{1,b}; & & & & & & & & n^{a,b}_{1,b} \\
n^{1,1}_{2,1}; & & & & & & & & n^{a,b}_{2,1} \\
\cdot & & & & & & & & \cdot \\
\cdot & & & & & & & & \cdot \\
n^{1,1}_{a,b}; & n^{1,2}_{a,b}; & . & n^{1,b}_{a,b}; & n^{2,1}_{a,b}; & . & n^{2,b}_{a,b}; & . & n^{a,b}_{a,b}
\end{array}
\qquad N
$$

Equation (4) indicates that the diagonal elements of matrix N are always zero and therefore are insignificant in representing the pattern from which the matrix was derived. Equation (3) shows that the elements below the diagonal may be generated by those above the diagonal, and hence only one of these groups of elements is significant in representing the original pattern. From equation (5) it can be shown that the elements above the diagonal (excluding those in the first row of matrix N) may be generated by the aXb -1 elements,

$$n_{1,1}^{1,2}; \dots\dots\dots\dots \; n_{1,1}^{a,b} \qquad (6)$$

Hence, the N matrix in Figure 1. can be uniqly specified by the elements, (6).

Suppose that a pattern is coded by the contrast valued sequence of elements, (6). Its N sequence ((6)) has one less element to consider in the decision process than does the signal valued matrix. An M matrix element could have q + 1 possible signal values; an N sequence digit position may assume any one of 2q + 1 values. That is to say, the 2q + 1 elements of the set, C, represent the contrast valued signal space of an N sequence, while the q + 1 elements of the set, S, represent the signal valued space of an M matrix. This more than doubles the base length of each frequency distribution characterizing a pattern class. The advantage is that a pattern in contrast digit form has a greater dimensionality ($(2q + 1) - (q + 1) = q$) with less time required for classification than in M matrix form. A disadvantage is an increase in the memory capacity required by pattern class frequency distributions. The greater dimensionality is in agreement with the idea that a contrast coded pattern more directly communicates concepts than does a signal coded pattern.

The concept-dimensionality analogy is not to be misinterpreted as a mathematical definition of visual pattern concepts. Nor does it imply that more 'information' can be drawn from a pattern by contrast coding. It is merely a technique which is designed to facilitate pattern discrimination and to meet the more demanding requirements of high speed data processing.

2. <u>The M : N Transformation</u>.

The number of elements of set S is $q + 1$. Then at most $(q + 1)^{ab}$ patterns may be represented in M matrix form. Although the same number of patterns are representable in N sequence form, the $(q + 1)^{ab}$ possible sequences that may be generated are not unique. That is, several different patterns in M matrix form are transformed into identical N sequences. In that sense contrast coding appears to be disadvantageous; yet, it will be demonstrated to be the contrary.

To consider the similarity between patterns which are transformed into the same sequence, first examine the M matrix form in terms of its quantized signal values. Assume that the order of M matrices has been fixed at (a, b) and that the set S has been determined to contain only the elements,

$$s_0, \ldots\ldots s_q$$

The following argument shall indicate how the totality of M matrices can be partitioned into groups, and how one can determine from these groups what M matrices are transformed into identical N sequences.

All of the discrete signal values which make up a particular matrix, Ms_p, can be represented in a set, Ss_p, which is a subset of the set, S. If the largest value, s_q, of S were missing from the set Ss_p, then a unit matrix, M_1 (of order (a, b)), could be added to Ms_p to obtain a new matrix, Ms_{p+1}.

$$Ms_p + M_1 = Ms_{p+1} \qquad (7)$$

If 'p' denotes the type of matrix that can be added, then equation (7) can be written more specifically as,

$$Ms_1 + M_1 = Ms_{1+1} \tag{8}$$

The set, Ss_{1+1}, corresponding to the new matrix, Ms_{1+1}, would also be a subset of the set, S. The number of possible matrices (of the totality of M matrices) whose set Ss_p does not contain s_q is q^{ab}. Similarly there are $(q - 1)^{ab}$ possible M matrices whose set, Ss_p, does not contain $s_q * s_{q-1}$ (* denotes the logical 'and'). To these matrices the duo matrix, M_2, could be added to obtain a new matrix, Ms_{2+2}, whose set, Ss_{2+2}, is a subset of the set, S. In general, if Ss_p (p = 1, ... q) does not contain $s_q * s_{q-1} * \ldots * s_{q-(i-1)}$, then the addition of M_i to Ms_p,

$$Ms_p + M_i = Ms_{p+i} \tag{9}$$

renders a matrix whose set, Ss_{p+i}, is a subset of the set, S. These arguments are condensed in Table I.

Table I M Matrix Grouping

Ss_p Excludes	Distance	No. of Matrices	Group
no element	0	$(q + 1)^{ab}$	Q_{q+1}
s_q	1	q^{ab}	Q_q
$s_q * s_{q-1}$	2	$(q - 1)^{ab}$	Q_{q-1}
....	....	....	....
$s_q * s_{q-1} * \ldots * s_0$	q	1	Q_1

From Table I the following is deduced. There are $q + 1$ groupings of M matrices where in general each member of group Q_i is distant by $(q - (i-1))$ from one member of group Q_{q+1} and is 'distant' by $(q - (i-1)) - 1$ from one member of group Q_q and so on. Each member of group Q_i is a member of all groups of higher order. The significance of this partitioning of the totality of M matrices into such groups is to show that a member of group Q_i and one member of each group 'distant' from the member of Q_i generate the same N sequence. For example, The row matrices,

$$(1, 2, 0, 1); (2, 3, 1, 2); \text{and } (4, 5, 3, 4)$$

all generate the N sequence,

$$(-1, 1, 0),$$

when $s_q = 5$ and when the order of M matrices is (1, 4).

Since every member of group Q_q is distant from Q_{q+1}, then the number of unique N sequences which can be generated by the totality of M matrices is, from Table I,

$$(q + 1)^{ab} - q^{ab}$$

Therefore, the number of redundant N sequences which would be generated by the $(q + 1)^{ab}$ M matrices is,

$$R = q^{ab} \tag{10}$$

M matrices of order (1, 4) with elements under set S: (0, 1, 2) are given in Appendix C in order to exemplify equation (10).

It has been shown, then, that in transforming M matrices to N sequences according to an absolute rule there is no discrimination between patterns which are 'distant'from one another in the sense of Table I. What is lost in the transformation is data of the form represented by matrices M_i , where i denotes the distance. In the author's opinion there is no difference in the visual pattern content between M matrices which are 'distant' by any amount. There is actually a gain, therefore, by avoiding an unnecessary ambiguity which 'distant' patterns would create among the prelearned class frequency distributions when derived from M matrices. Furthermore, the representation of patterns by contrast valued N sequences should greatly improve the discriminatory ability of the proposed classification system.

B. An Information Theoretical Decision Function

In the proposed classification system, visual patterns are to be represented by a sequence of contrast valued digits. The sequence is ordered and of fixed length and the digits take on discrete values belonging to the finite set C. During the learning of a particular class of pattern, frequency distributions are accumulated over the set C for each digit position of the sequence. In identifying a random pattern, therefore, some function is desired of the correlation between the unknown digit values and the corresponding digit value frequencies of the known pattern classes.

Two such functions have been proposed. One treats each digit of the contrast coded sequence with equal weight, while the other stresses values which are more highly correlated with corresponding class digit value frequencies.

1. Decision Function - Model 1

Consider the pattern classes which the system is to recognize as the set of events,

$$Y = (y_i) \qquad i = 1, \ldots\ldots n \tag{11}$$

Let the elements of set C form a set of events,

$$X = (x_j) \quad (12)$$

$$j = 1, \ldots\ldots m \ ; \quad m = 2q + 1$$

Denote the number of samples used in the learning of a particular class y_i as Ni. Let the events x_j be superscripted as, x_j^k, where k refers to a particular digit position of the N sequence. Recall that visual patterns are in the form of contrast valued N sequences. The range of k is,

$$k = 1, \ldots\ldots ab-1$$

Considering an individual digit position k, the additional information which the k^{th} digit of an unknown pattern provides about the k^{th} digit of class y_i, is given by[4],

$$I_k\ (y_i/x_j^k) = \text{Log}\ (\ \Pr\ (y_i/x_j^k)/\Pr\ (y_i)\ ) \quad (13)$$

But,
$$I_k\ (y_i/x_j^k) = I_k\ (x_j^k/y_i)$$

and
$$I_k\ (x_j^k/y_i) = \text{Log}\ (\ \Pr\ (x_j^k/y_i)/\Pr\ (x_j^k)\ ) \quad (14)$$

Therefore
$$I_k\ (y_i/x_j^k) = \text{Log}\ (\ \Pr\ (x_j^k/y_i)/\Pr\ (x_j^k)\ ) \quad (15)$$

$\Pr\ (x_j^k/y_i)$ is the conditional probability that digit position k of pattern class y_i has contrast value x_j. $\Pr\ (x_j^k)$ is the probability that digit position k has contrast value x_j in any class of the set Y. If the frequencies corresponding to $\Pr\ (x_j^k/y_i)$ and $\Pr\ (x_j^k)$ are $Fr\ (x_j^k/y_i)$ and $Fr\ (x_j^k)$

respectively, then,

$$\Pr (x_j^k/y_i) = (\mathrm{Fr}\ (x_j^k/y_i)/Ni) \qquad (16)$$

and

$$\Pr (x_j^k) = \mathrm{Fr}\ (x_j^k)/(\sum_{i=1}^{n} (Ni)\ ) \qquad (17)$$

Therefore,

$$I_k(y_i/x_j^k) = \mathrm{Log} \left[\frac{\mathrm{Fr}(x_j^k/y_i)/Ni}{\mathrm{Fr}(x_j^k)/(\sum_{i=1}^{n} Ni)} \right] \qquad (18)$$

Since $\mathrm{Fr}(x_j^k/y_i)$, $\mathrm{Fr}(x_j^k)$ and Ni (i = 1, ... n) are all known quantities, then, $I_k(y_i/x_j^k)$ can be computed directly. Because each digit value is to be considered of equal importance in the overall decision, then the total additional information which an unknown pattern provides about class y_i, is,

$$I(y_i/x_j) = \sum_{k=1}^{ab-1} I_k(y_i/x_j^k) \qquad (19)$$

2. Decision Function - Model 2

A weighting factor, $\Pr(x_j^k/y_i)$, is included in the decision function represented by equation (19), in order to emphasize the larger values of $I_k(y_i/x_j^k)$ and to de-emphasize the smaller ones. The decision function for a single digit is therefore,

$$I_k^*(y_i/x_j^k) = \frac{Fr(x_j^k/y_i)}{Ni} \cdot \text{Log}\left[\frac{Fr(x_j^k/y_i)/Ni}{Fr(x_j^k)/(\sum_{i=1}^{n} Ni)}\right] \tag{20}$$

The decision function for the total number of digits in the N sequence is,

$$I^*(y_i/x_j) = \sum_{k=1}^{ab-1} I_k^*(y_i/x_j^k) \tag{21}$$

III EXPERIMENTAL PROCEDURE

A. Data Acquisition

1. Source of Patterns

Although the proposed recognition system is sufficiently general in theory to handle any type of visual pattern, it is limited by the unavailability of accurate normalization techniques. It would have difficulty at this stage in discriminating between cyclones and hurricanes, for example. Therefore the twenty-six capital letters of the alphabet and the first nine cardinal numerals were used to test the classification system.

The patterns were hand written in order to create a wide variation in their spatial form. A coarse pen was used so that a medium proportion of the field of view was covered. Note in Figure 2. that the field of view is a one inch square grid of twenty-five segments. Figure 2. gives an example of the type of each pattern used (excluding '10').

2. Normalizing The Pattern

Each sample hand written character was focussed within a paper based grid ten units high and eight units wide. The choice of eighty elements, though arbitrary, was thought to be sufficient

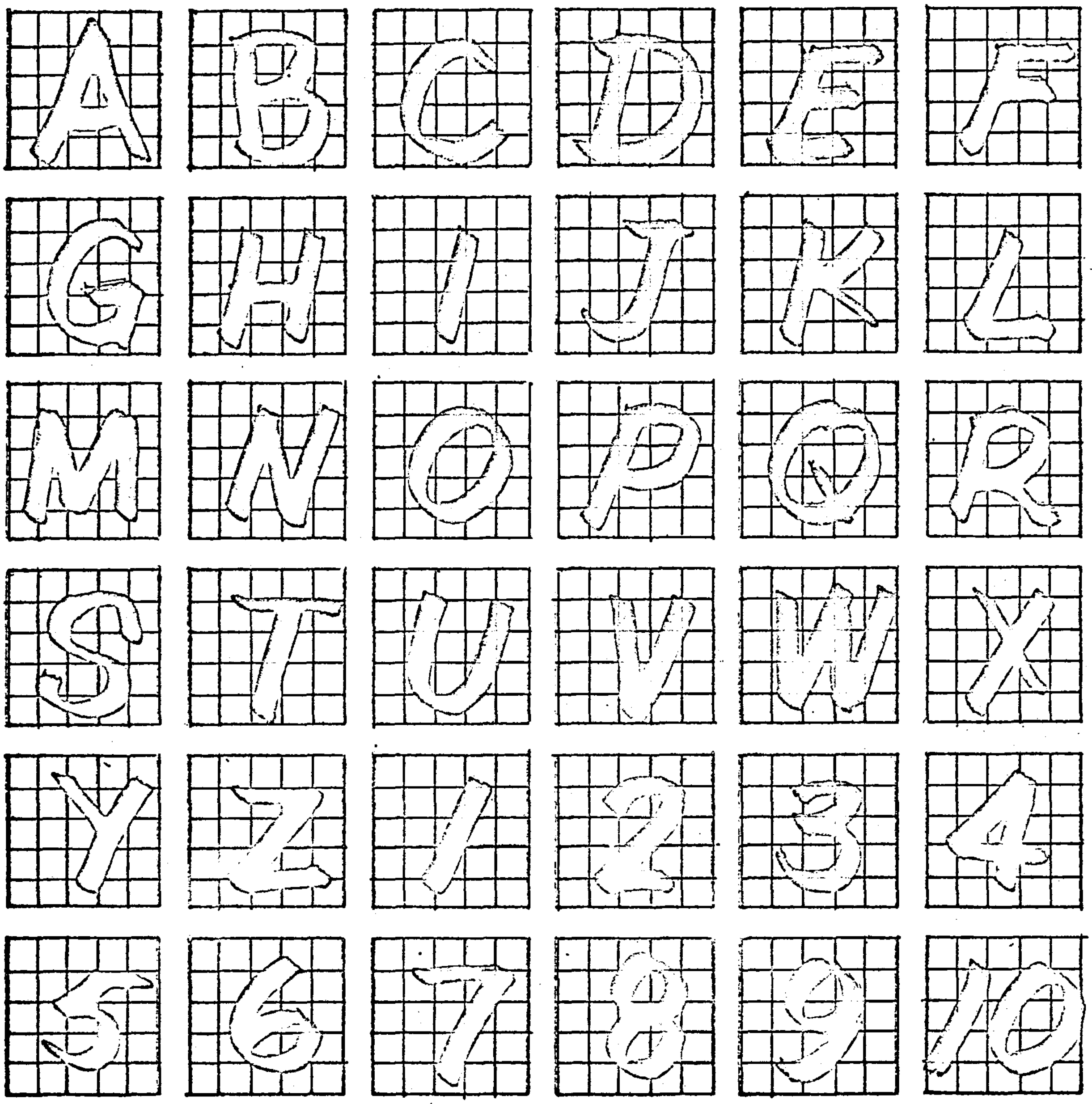


Figure 2. Typical Sample Patterns

for an identification of all thirty-five characters. A rectangular shape was chosen because a large portion of the sample patterns tended to be greater in height than in width. The pattern was positioned so that it was tangent to one pair of opposite grid borders. Then the pattern was centred in the direction of the other pair of borders.

The percentage of each element covered by the projected character was taken as the signal value of that element. Eighty such signal values were computed for each pattern.

3. Signal and Contrast Coding

The eighty pattern signal values were first quantized into discrete values ranging from zero (0) to twenty (20). The choice of that number of contrast values was again an arbitrary one. By subtracting from the value of the first element the value of each of the following elements, row by row, a contrast valued sequence of seventy-nine digits was formed. This was in accordance with equation (2). Therefore, each of the seventy-nine digits had a discrete contrast value in the range (-20 to +20).

For a sample of a signal and contrast value coded pattern of a different size and quantization, see Appendix B. The pattern of Appendix B would be represented by the last eight digits in the first row of matrix N.

B. Computer Simulated Classification Unit

All of the theoretical techniques derived in Chapter II were implemented in several computer programs. It would be too tedious to include all of the program listings and their flow charts here. The major portion of these were to simulate the learning phase of the recognition system in lump form. The first program was written to read all of the data (consisting of 12 samples of each of 35 characters) from prepunched cards[5] into a core memory. It was to pack the data, code it by character and transfer the condensed block to disk storage. Another program was designed to delimit the data already stored on disk, into fields, by flagging the leftmost digit of each value; this enabled the data to be selected and processed individually. The third program quantized the signal data. The fourth generated contrast values for all of the character samples. The fifth program generated frequency distribution tables from the contrast valued data. See Appendix D for a sample of how the distributions were set up in memory and for a comparison of the contrast valued data structure of the thirty-five characters.

The sixth program summed the thirty-five frequency tables into a single table. In the thirty-five tables were stored the values, $Fr(x_j^k/y_i)$, while in the sum table the unconditional frequencies, $Fr(x_j^k)$, were to be found. The purpose of the sum table was to greatly decrease recognition decision time.

The size of these tables necessitated that they be stored in a secondary memory area and be called into core memory only when required. Thus, much time was consumed in seeking and transferring these tables between primary and secondary storage areas.

The machine used was an IBM 1620_{II} computer with 40 K core storage capacity and 1311 disk drive assembly. The programs were written in SPS language and compiled into machine language by a Monitor II System assembler. A basic layout of the recognition unit in terms of the IBM 1620 computer is found in Figure 3.

The final computer program, PCP (Pattern Classification Program - also written in SPS) was designed to handle the data of one pattern at a time and to perform all of the functions of the first four programs. In addition, the decision models were implemented and recognition of an unknown pattern could be attempted by using either decision function 1 (equation (19)) or decision function 2 (equation (21)). It should be noted that the natural logarithm was used in the implementation of both decision functions. PCP was adaptive in the sense that the data of an identified pattern could be added to the system frequency tables and other parameters could be updated to account for the added data. With this facility it could have been determined whether

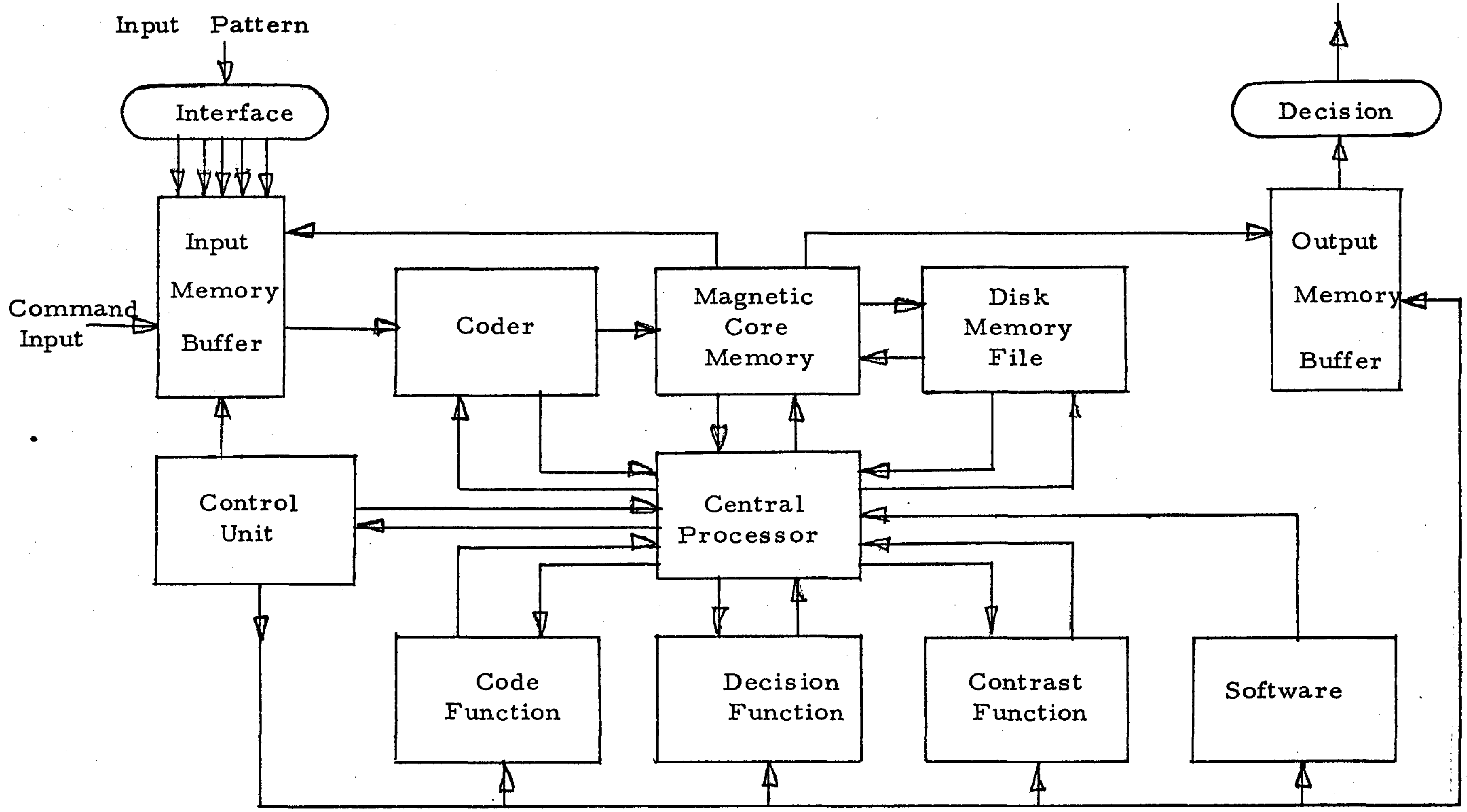


Figure 3. Recognition Unit Block Diagram (as simulated on the 1620_{II} computer)

or not the classification system were degenerative; however, this was not tried.

Of the total number of sample patterns tested by PCP, 140 learned patterns were tested by function model 1, 105 learned patterns were tested by function model 2 and thirty-five unlearned samples were tested by function model 1. No unlearned samples were tested by function model 2.

A complete listing of the PCP program may be found in Appendix A. The comments adjacent to the listing provide an explanation of the functional flow of the program.

C. Effective Statistical Zero

Since the number of samples used to learn the pattern classes was very small and due to the digital nature of operation, many of the frequencies used by the decision algorithms were of value zero (0). Log(0) being indeterminate, an effective zero value had to be established. It was decided arbitrarily that the occurrence of a zero represented the least possible information and, therefore, a 'zero' was chosen to give an information measure which was slightly more negative than the value that could have resulted in the most negative case.

IV RESULTS

Of the prelearned sample patterns, one hundred per cent were correctly identified by the function model 1. Only seventy-two per cent of the prelearned patterns were correctly identified by the function model 2. Fourty per cent of the unknown patterns were correctly identified by the function model 1; function model 2 was not applied to the unlearned samples.

The time required for the complete identification of a single pattern sample was approximately three minutes. This time could have been substantially reduced by employing a faster computer.

Figures 4. - 8. represent typical results using function model 1 on learned samples. Figures 9. - 13. represent the the information measures obtained by function model 2 from the learned samples. Figures 14. - 27. are typical results obtained by applying function model 1 to unlearned patterns.

Figure 4. Identification Trial (mod 1) - A (learned)

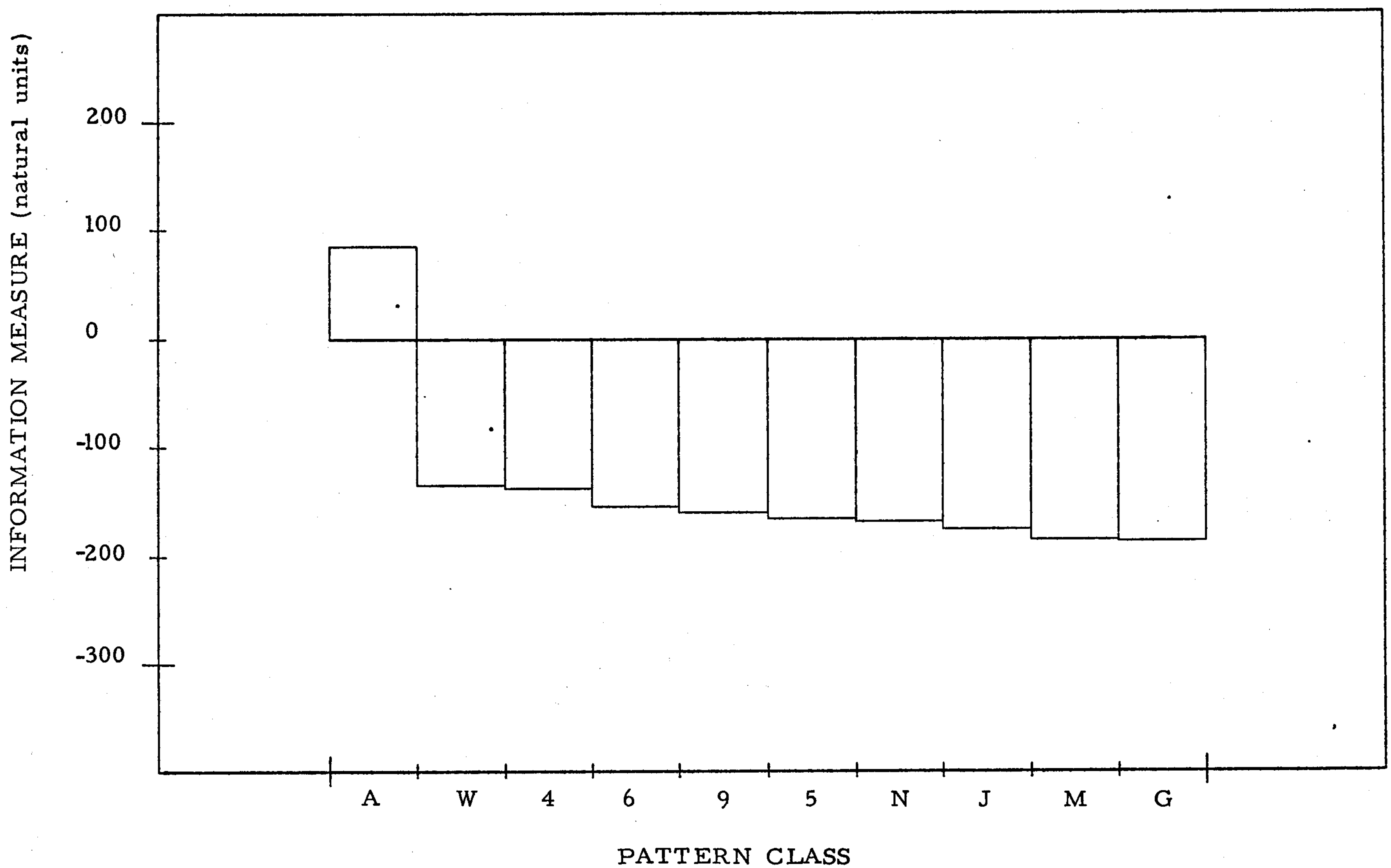


Figure 5. Identification Trial (mod 1) - B (learned)

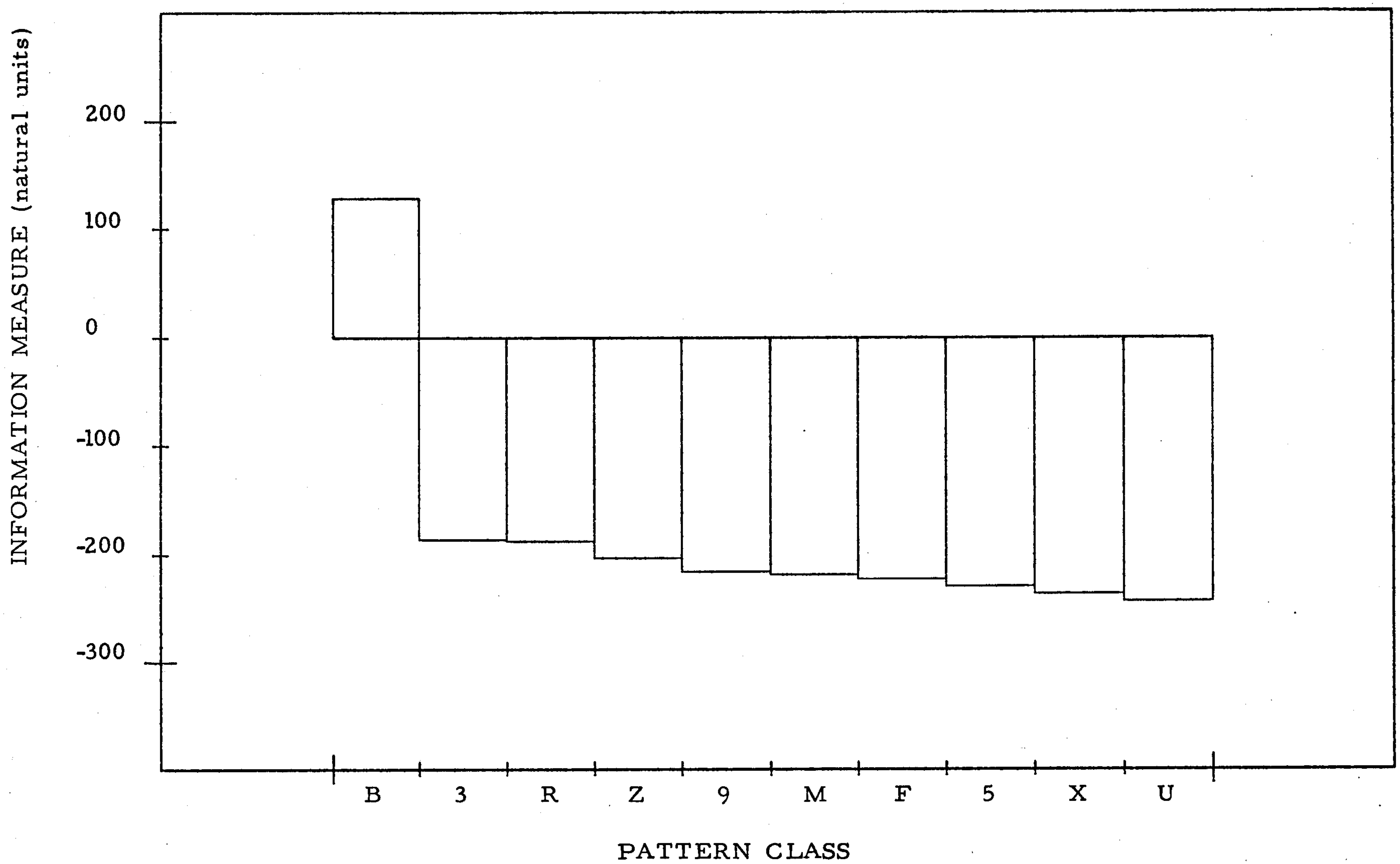


Figure 6. Identification Trial (mod 1) - C (learned)

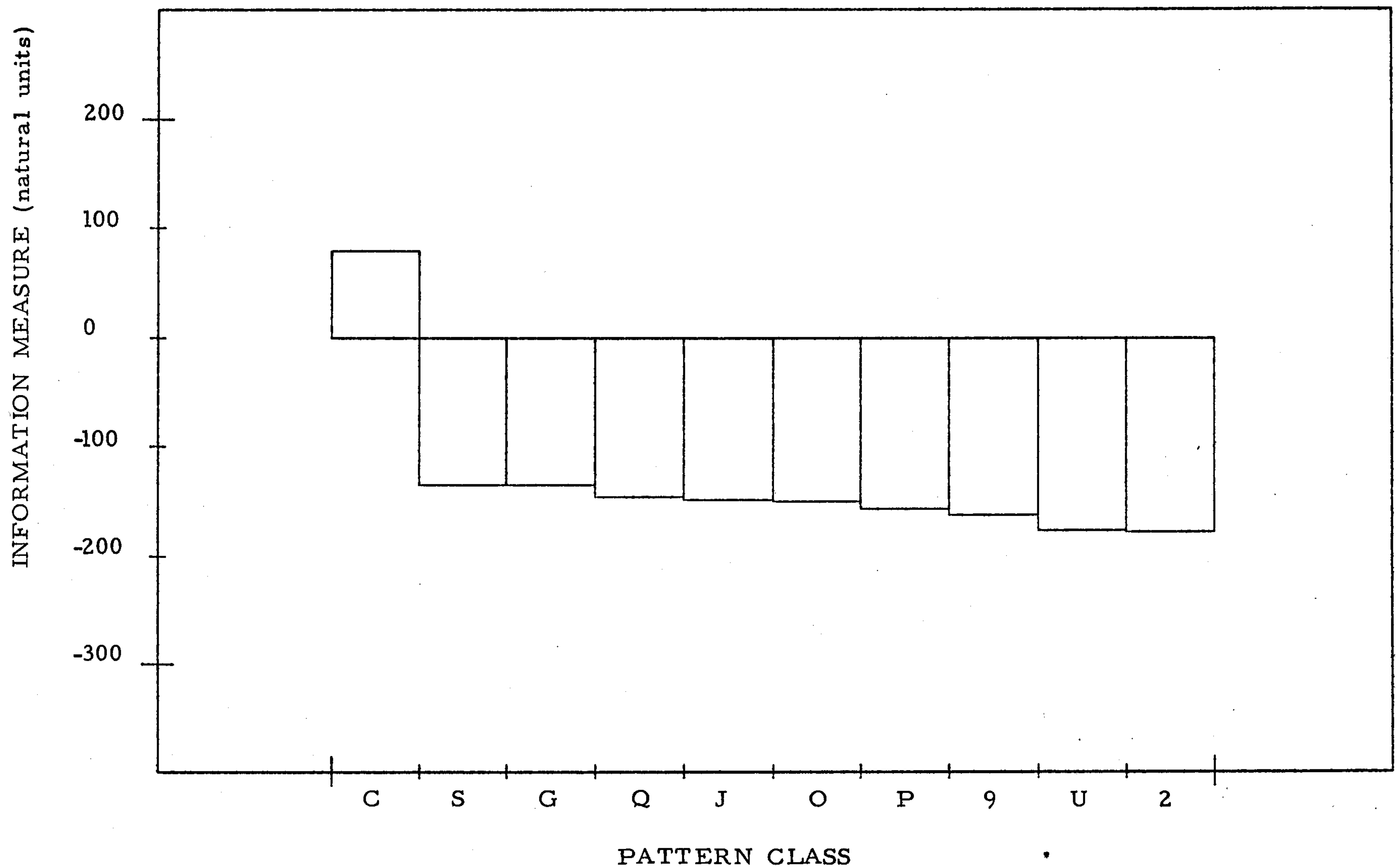


Figure 7. Identification Trial (mod 1) - D (learned)

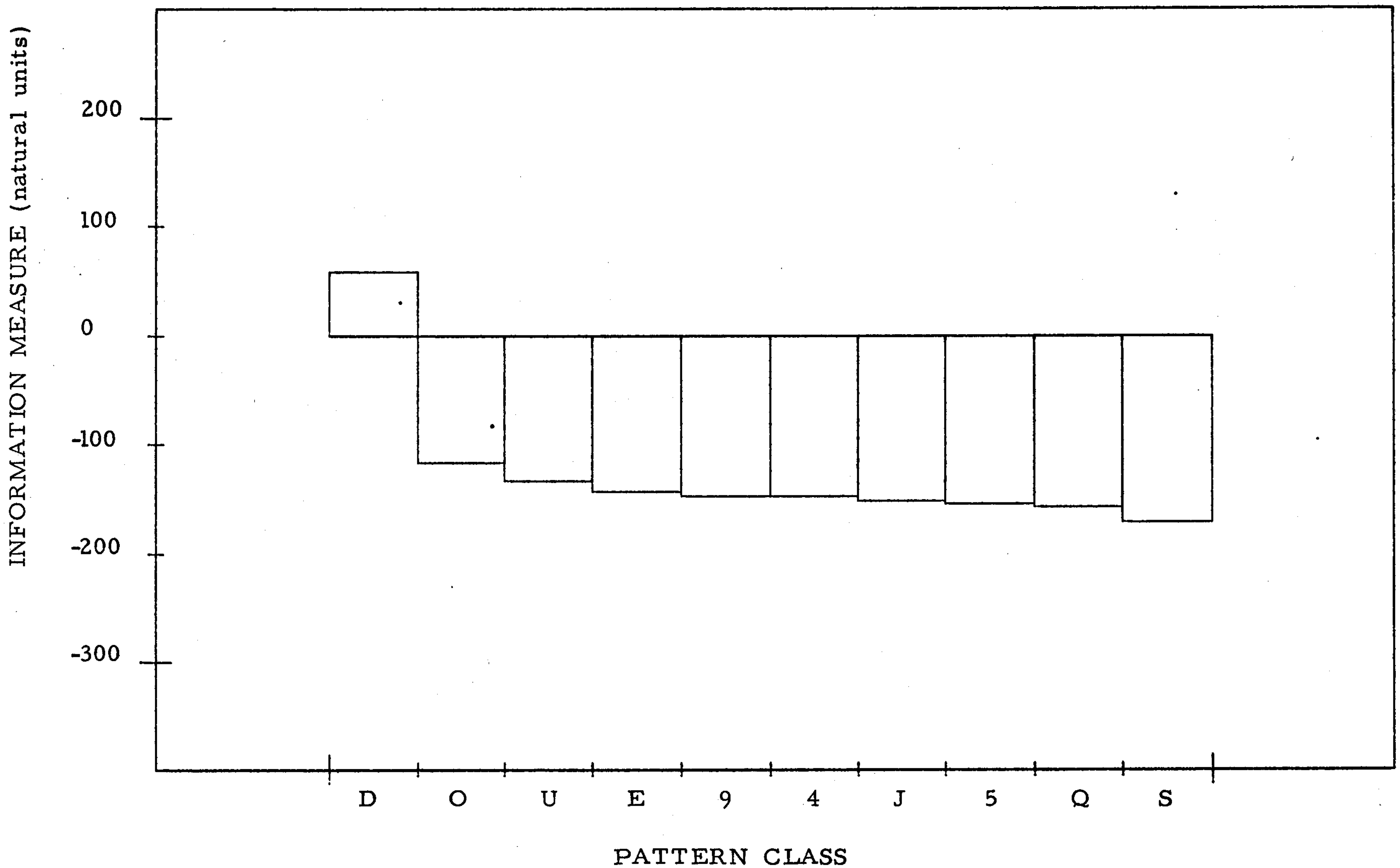


Figure 8. Identification Trial (mod 1) - E (learned)

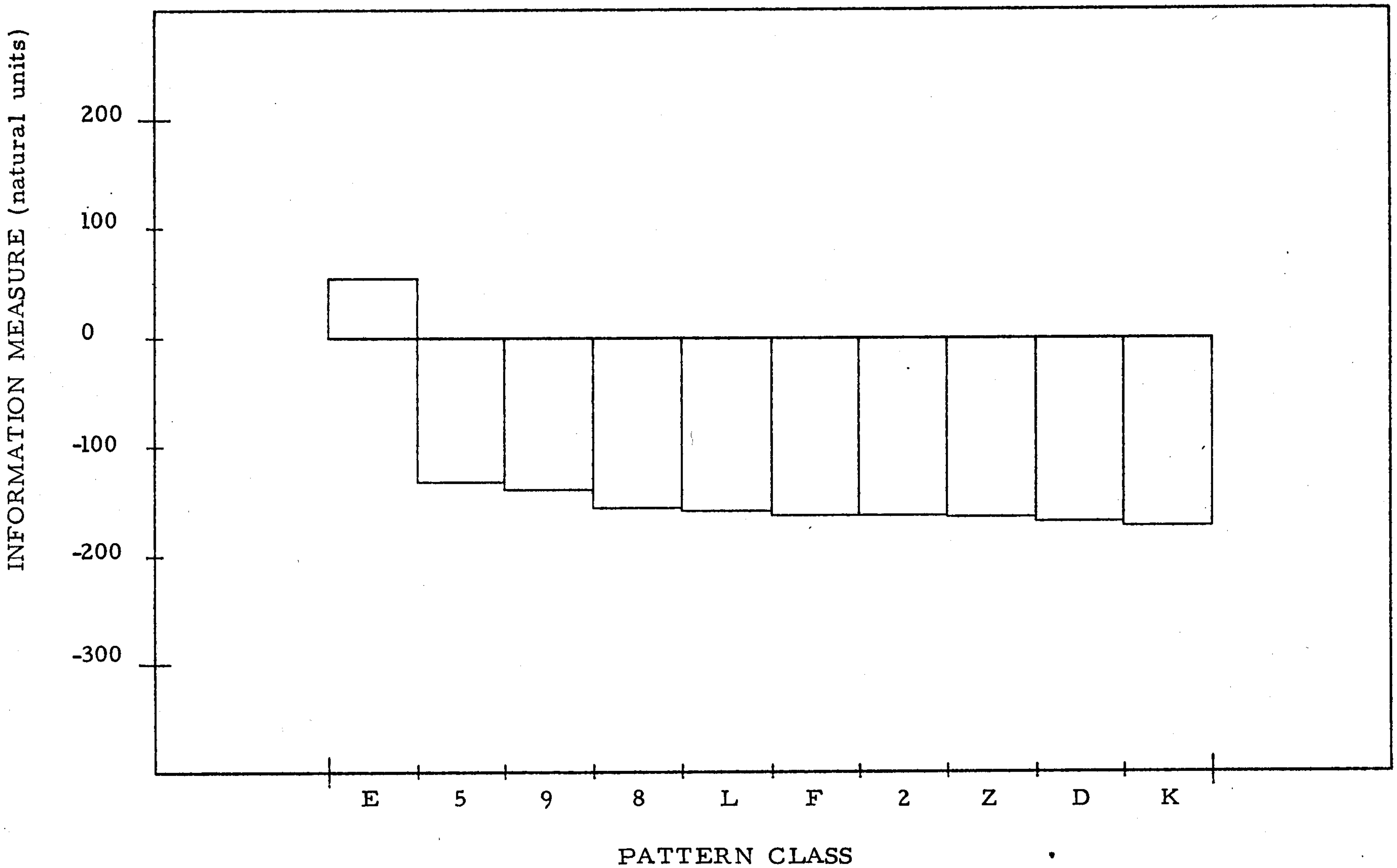


Figure 9. Identification Trial (mod 2) - A (learned)

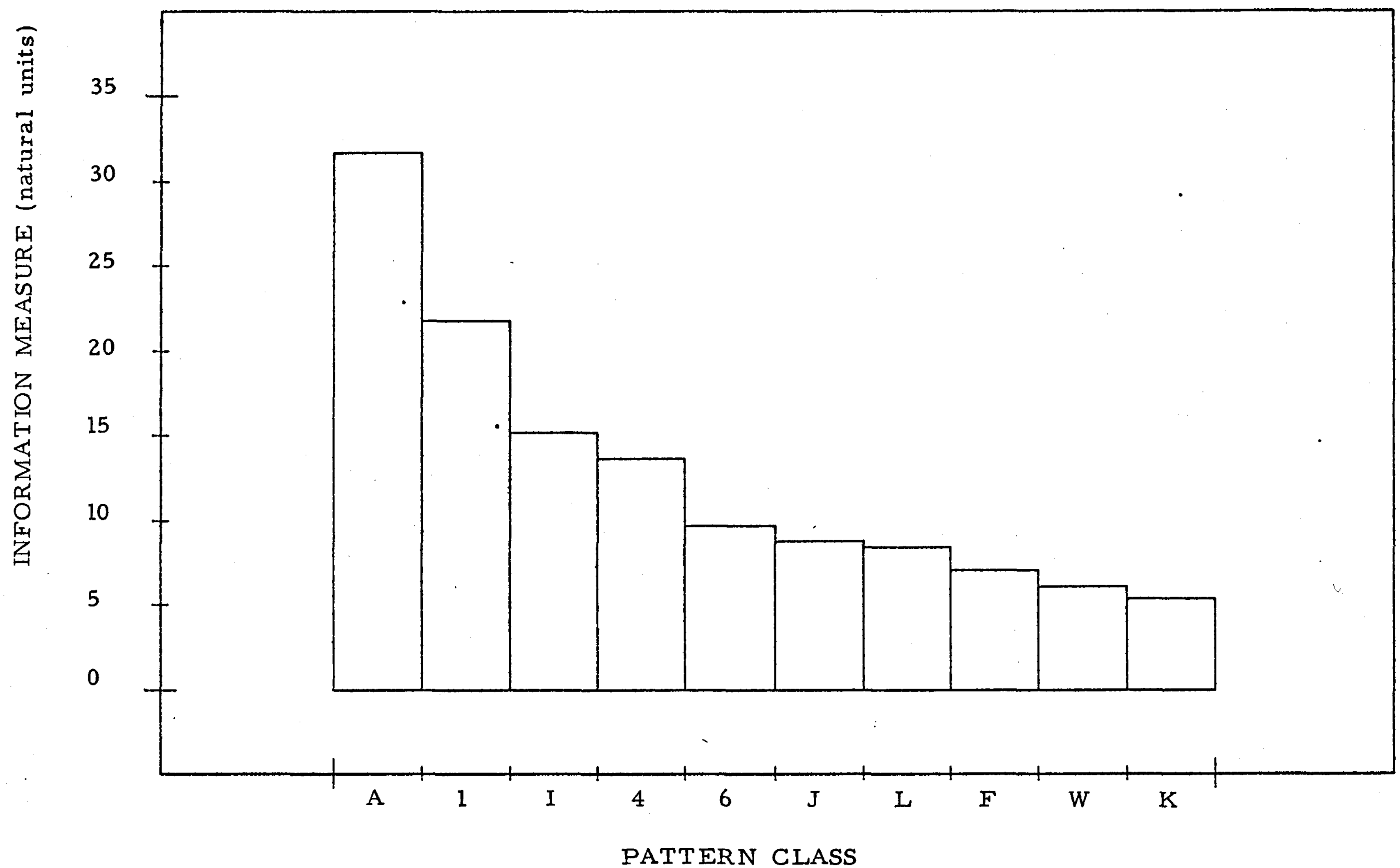


Figure 10. Identification Trial (mod 2) - B (learned)

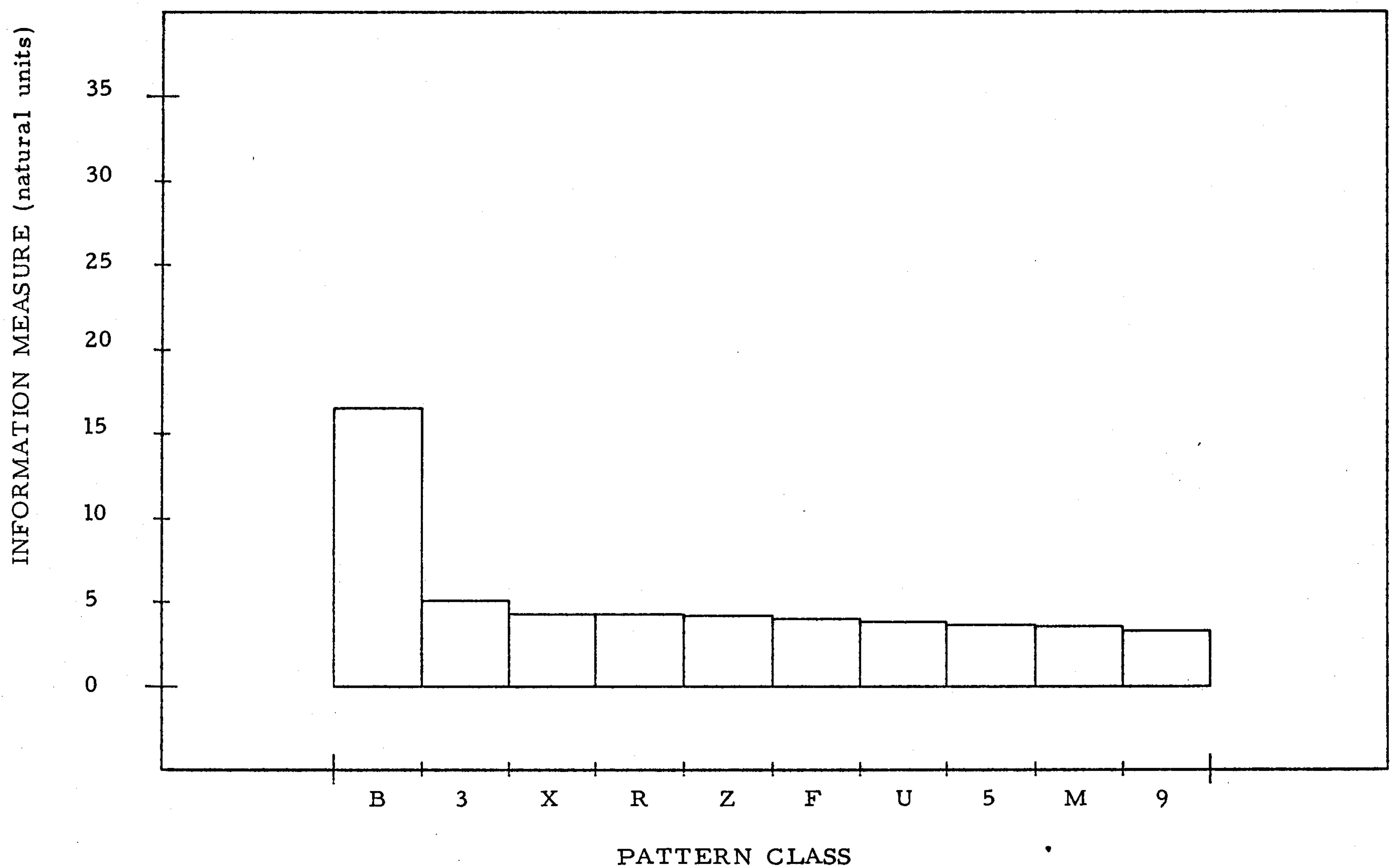


Figure 11. Identification Trial (mod 2) - C (learned)

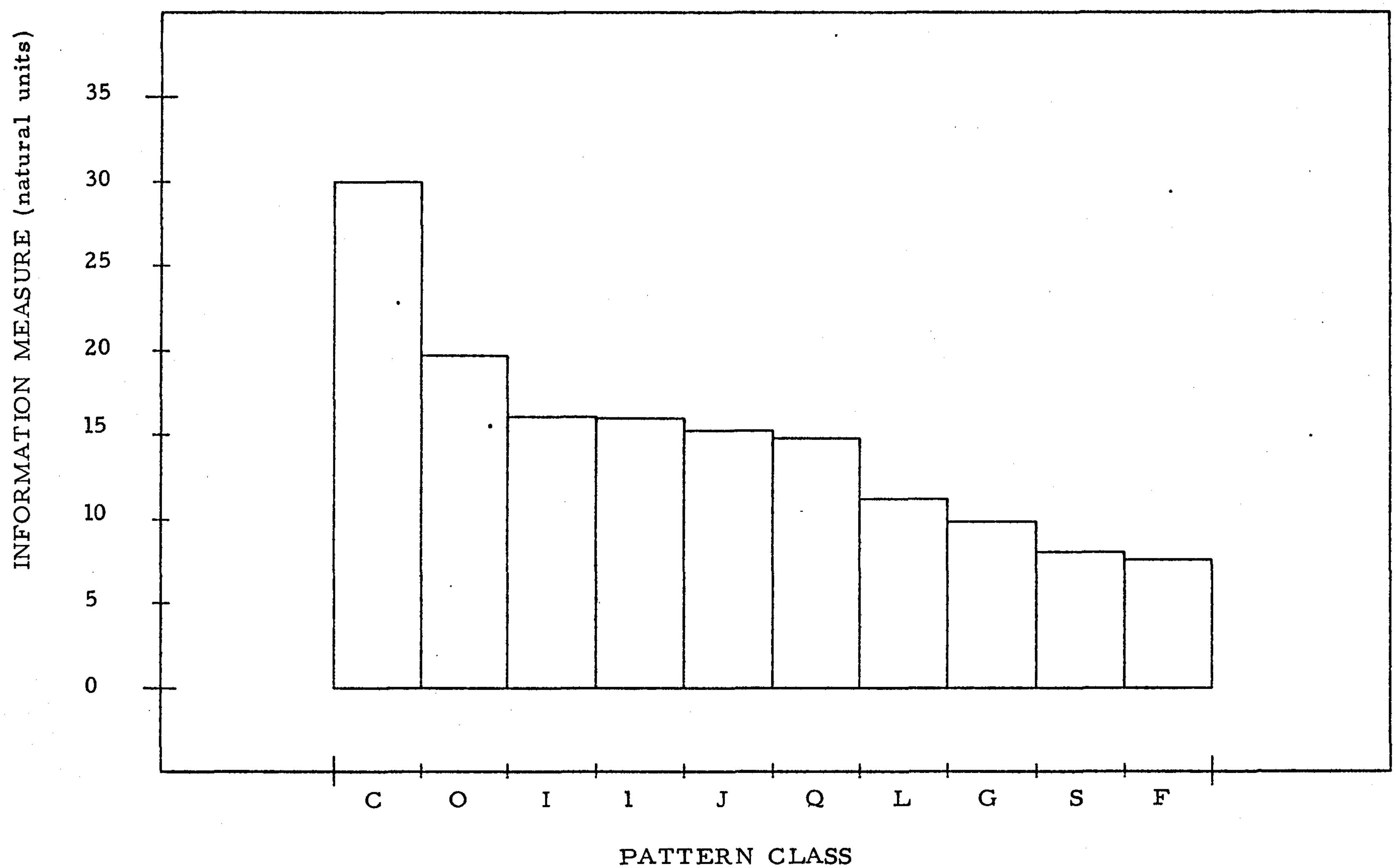


Figure 12. Identification Trial (mod 2) - D (learned)

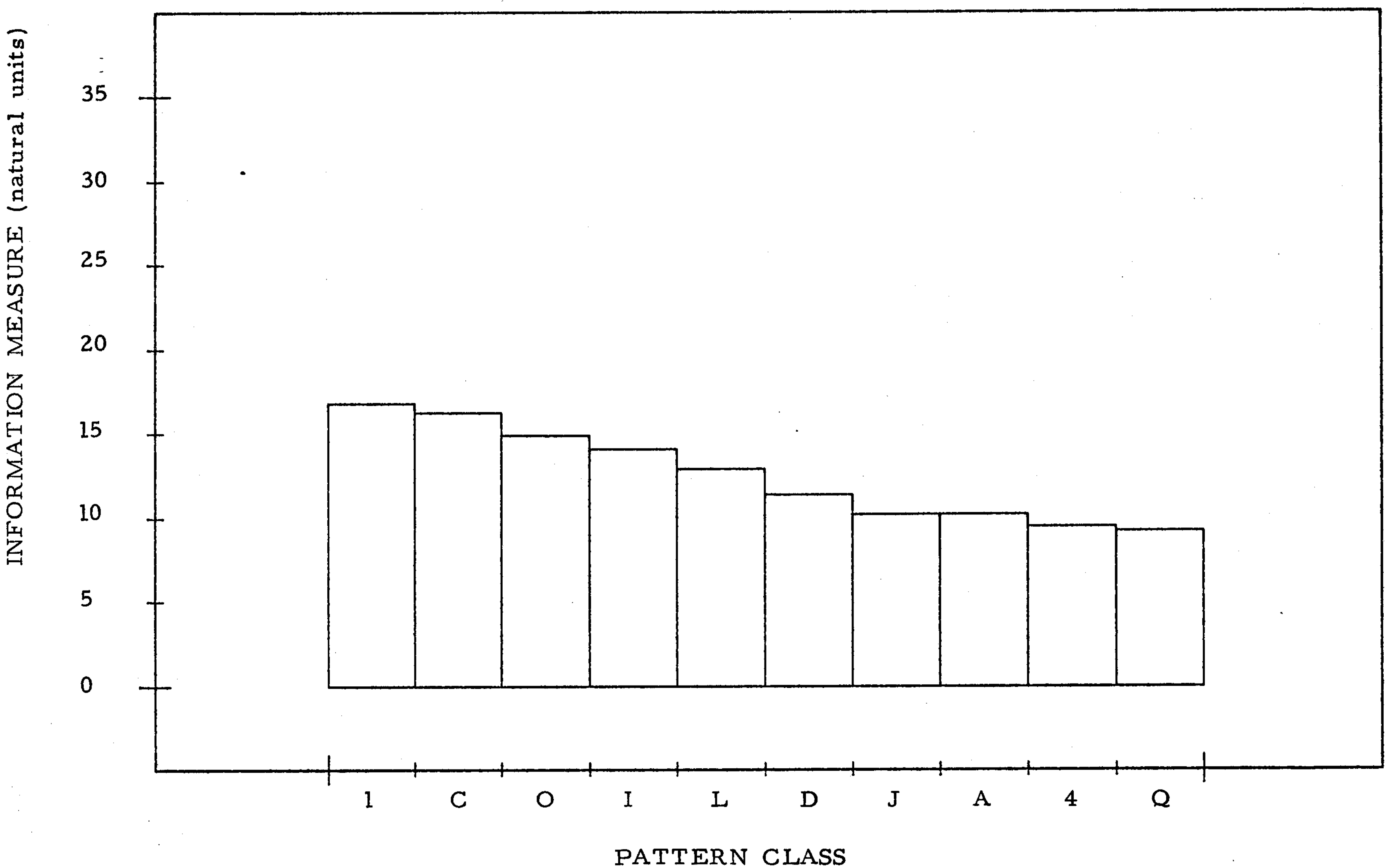


Figure 13. Identification Trial (mod 2) - E (learned)

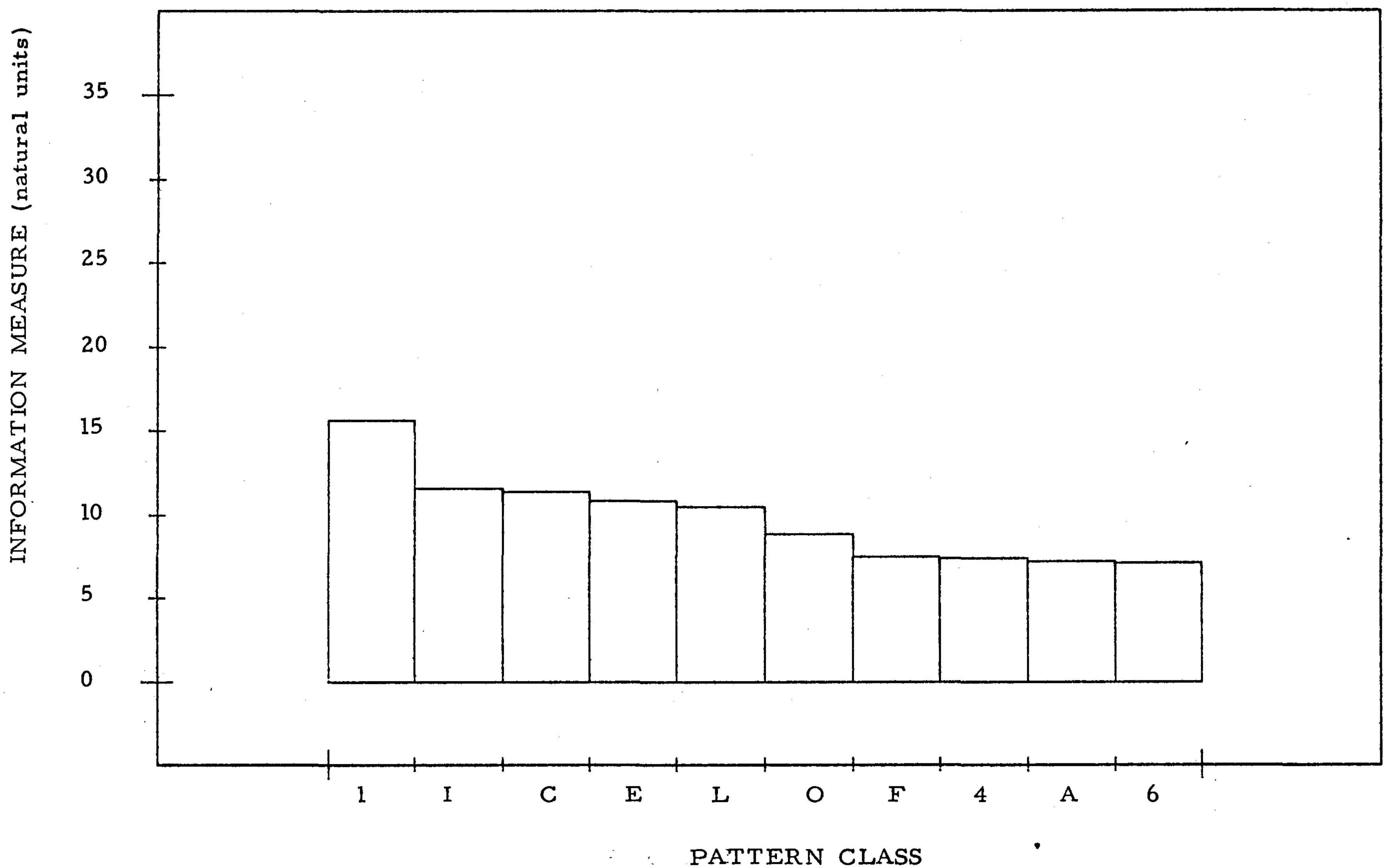


Figure 14. Identification Trial (mod 1) - A (unlearned)

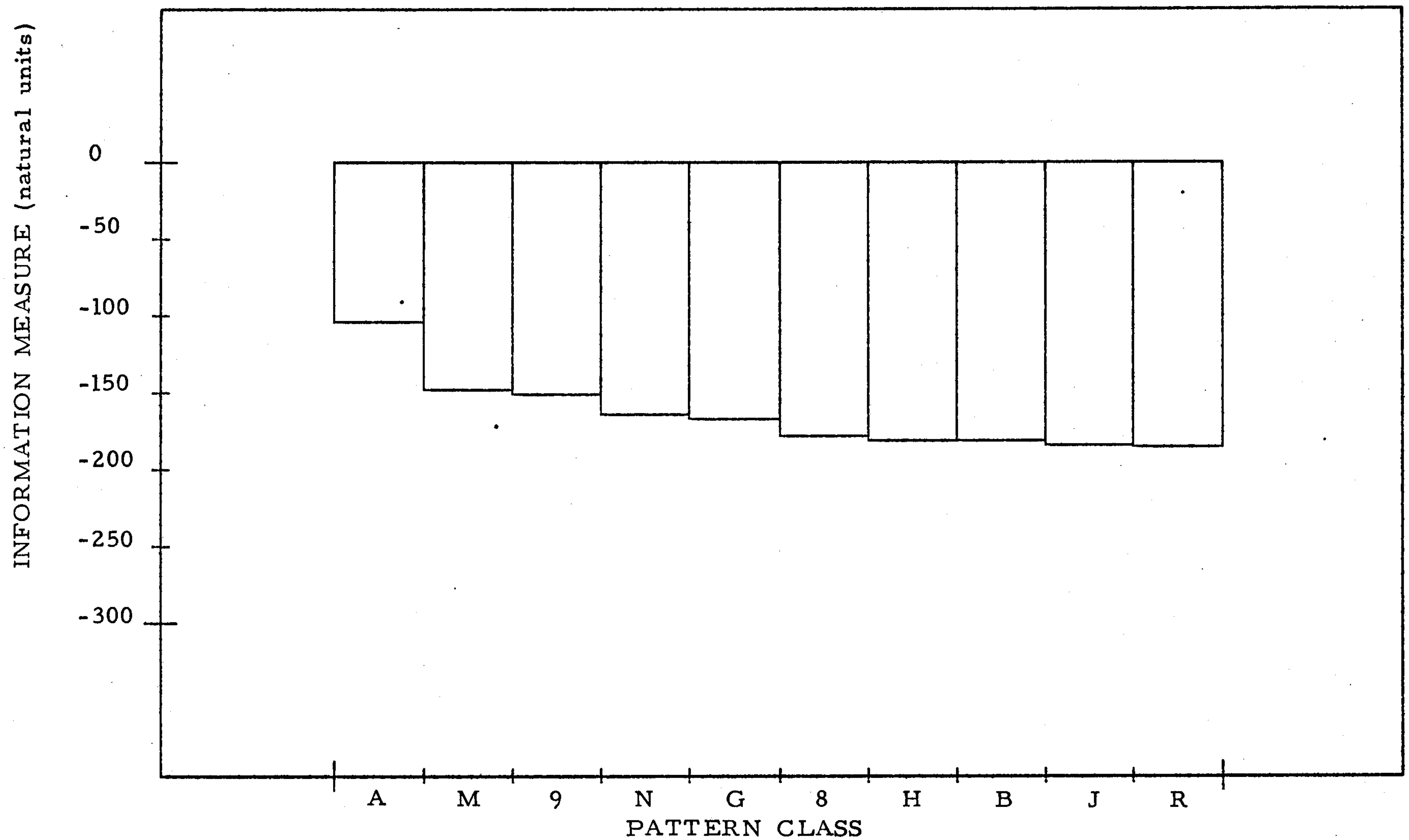


Figure 15. Identification Trial (mod 1) - B (unlearned)

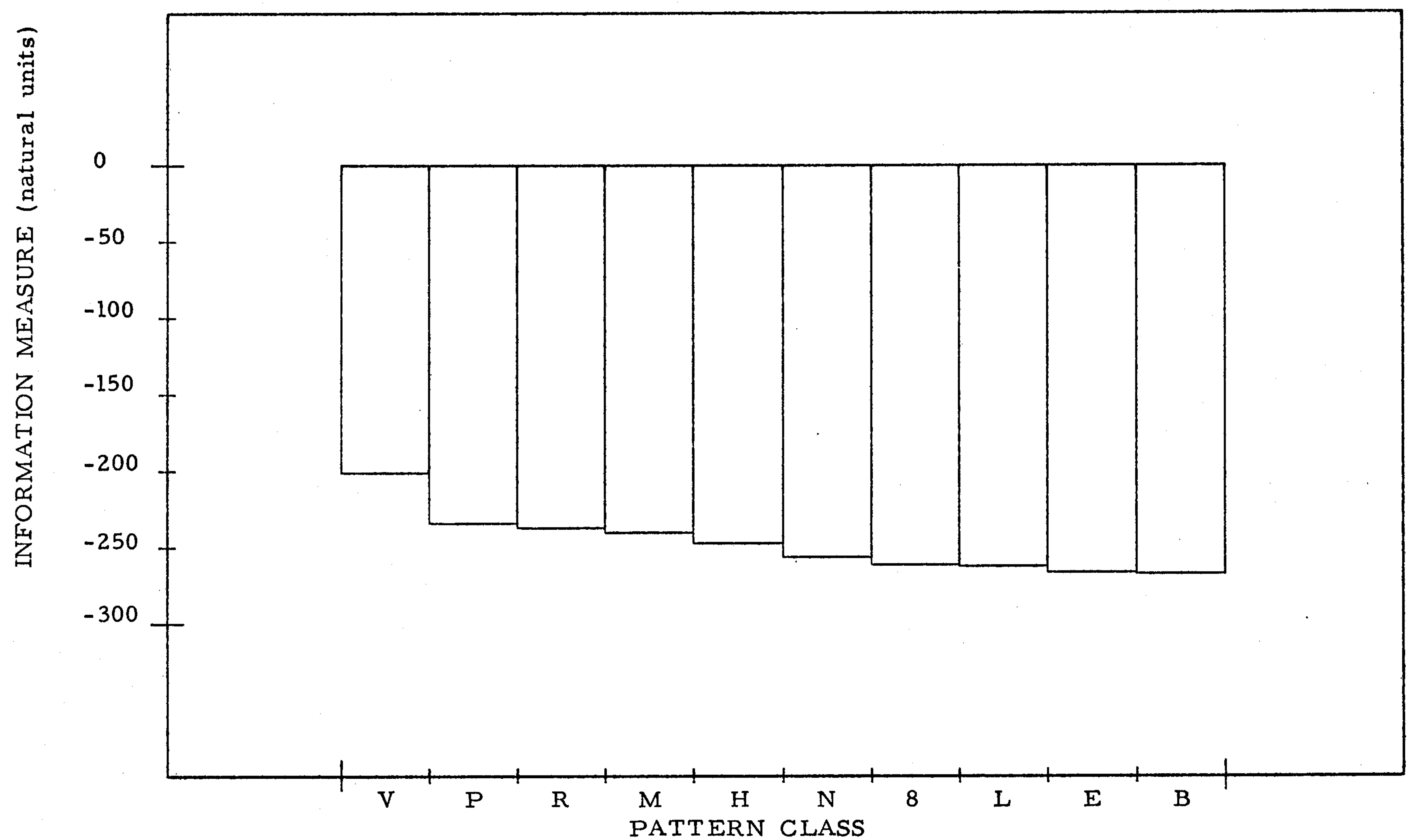


Figure 16. Identification Trial (mod 1) - C (unlearned)

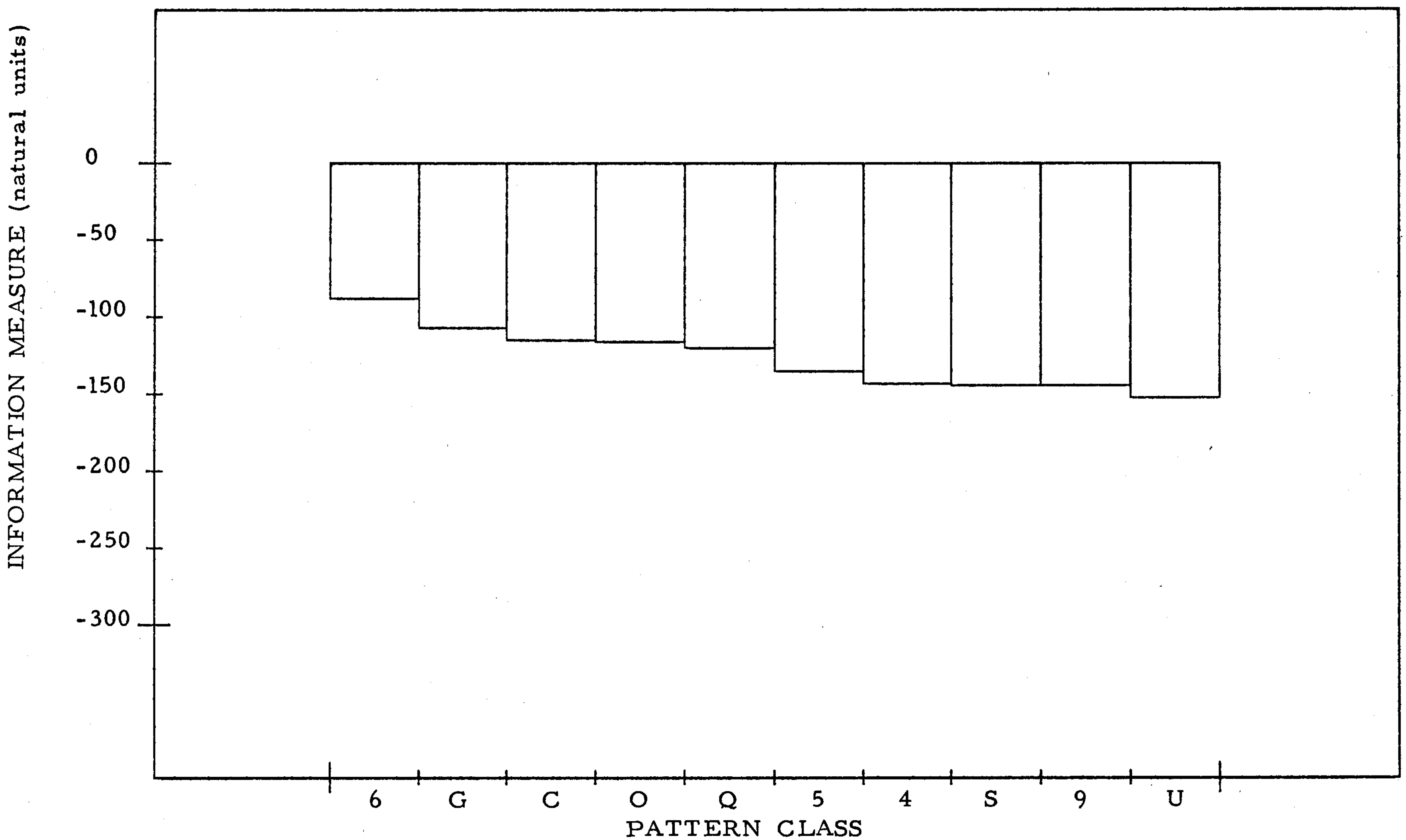


Figure 17. Identification Trial (mod 1) - E (unlearned)

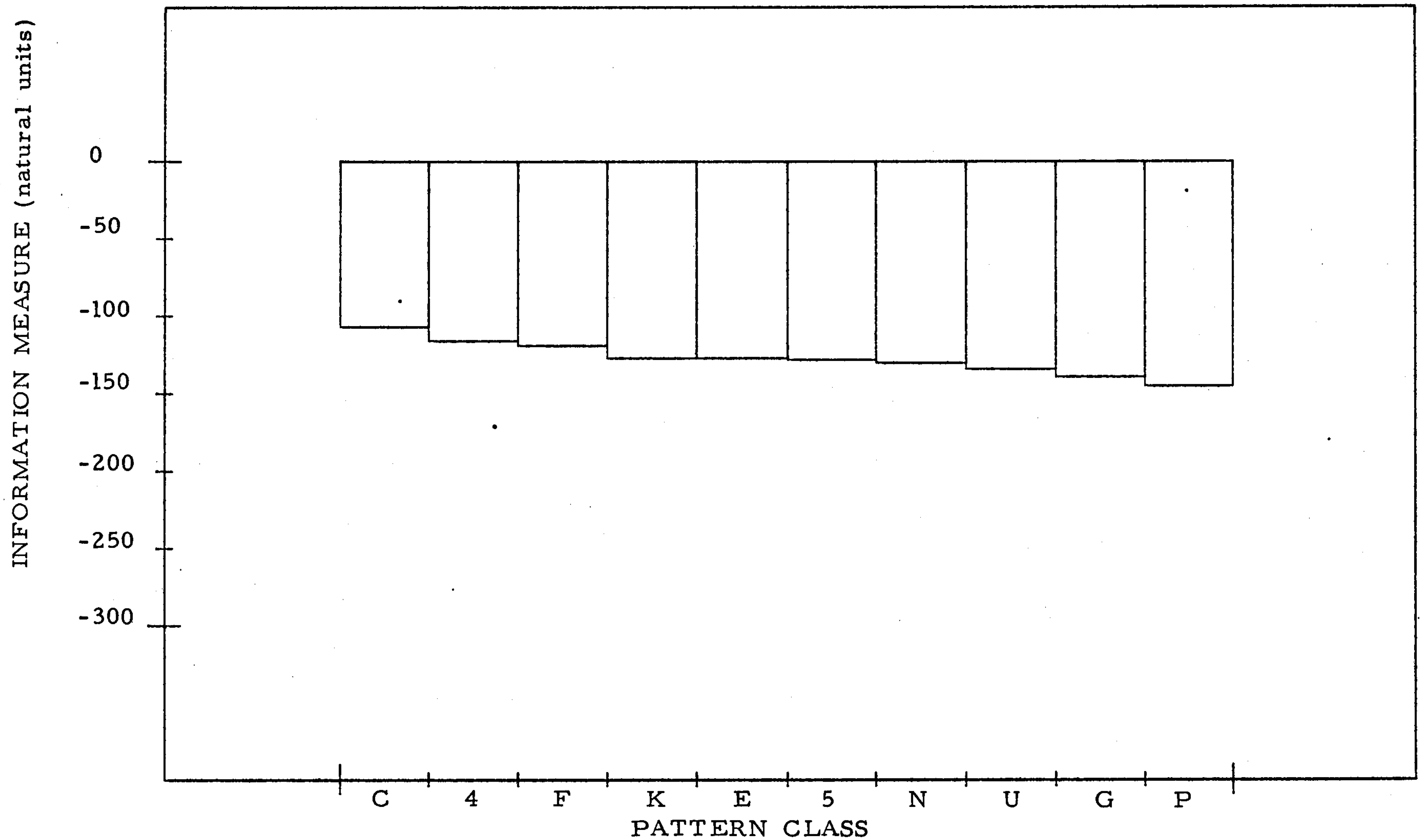


Figure 18. Identification Trial (mod 1) - F (unlearned)

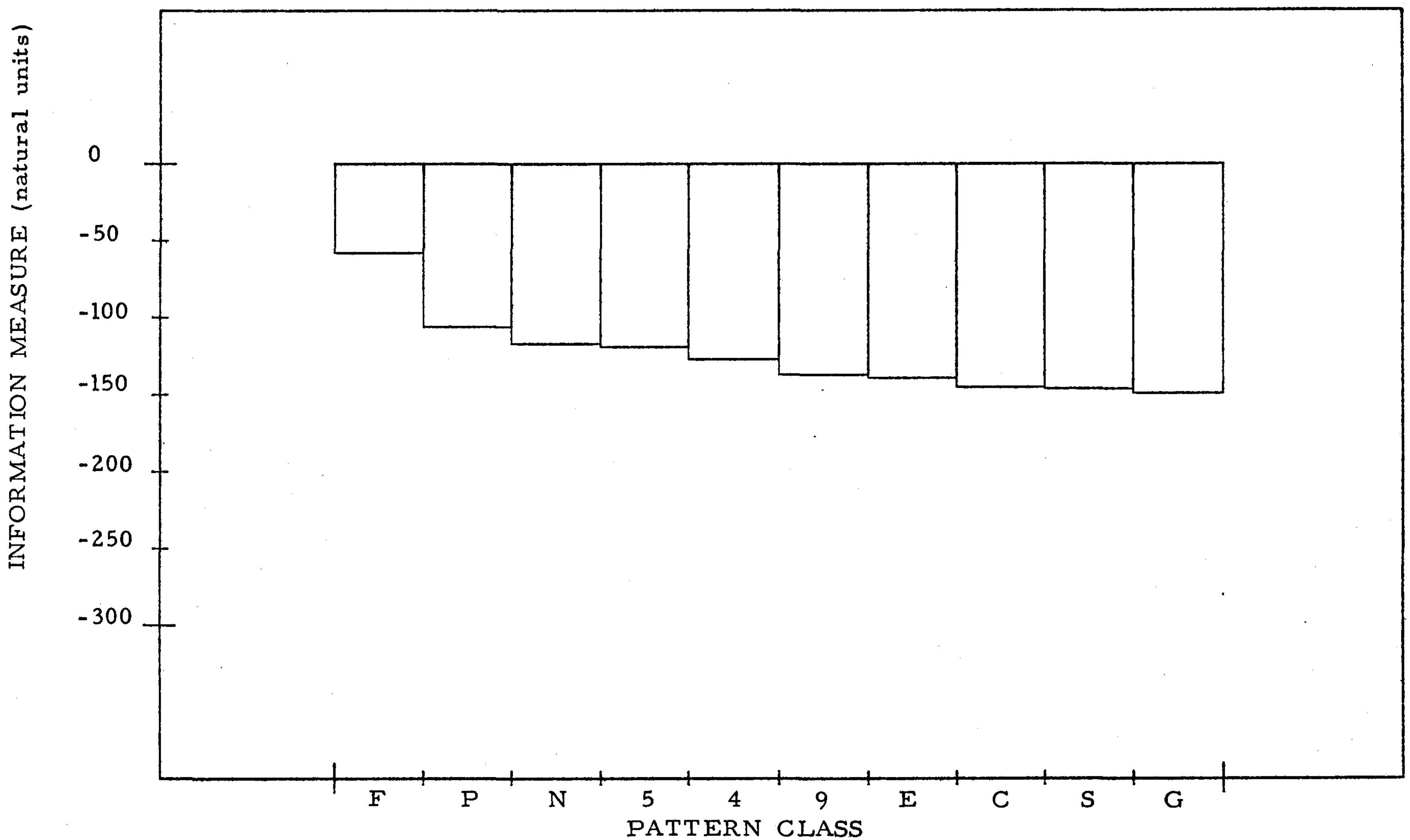


Figure 19. Identification Trial (mod 1) - G (unlearned)

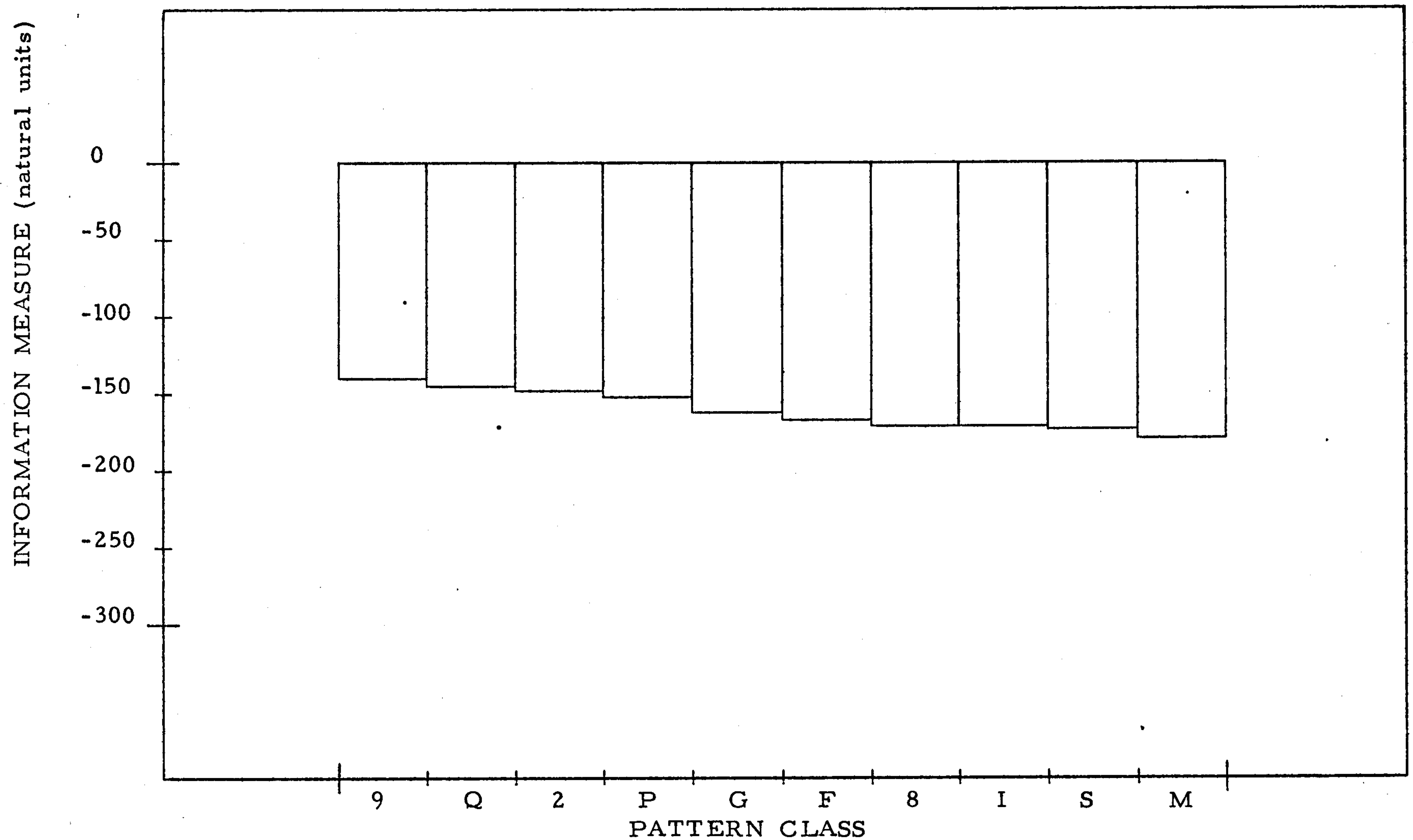


Figure 20. Identification Trial (mod 1) - H (unlearned)

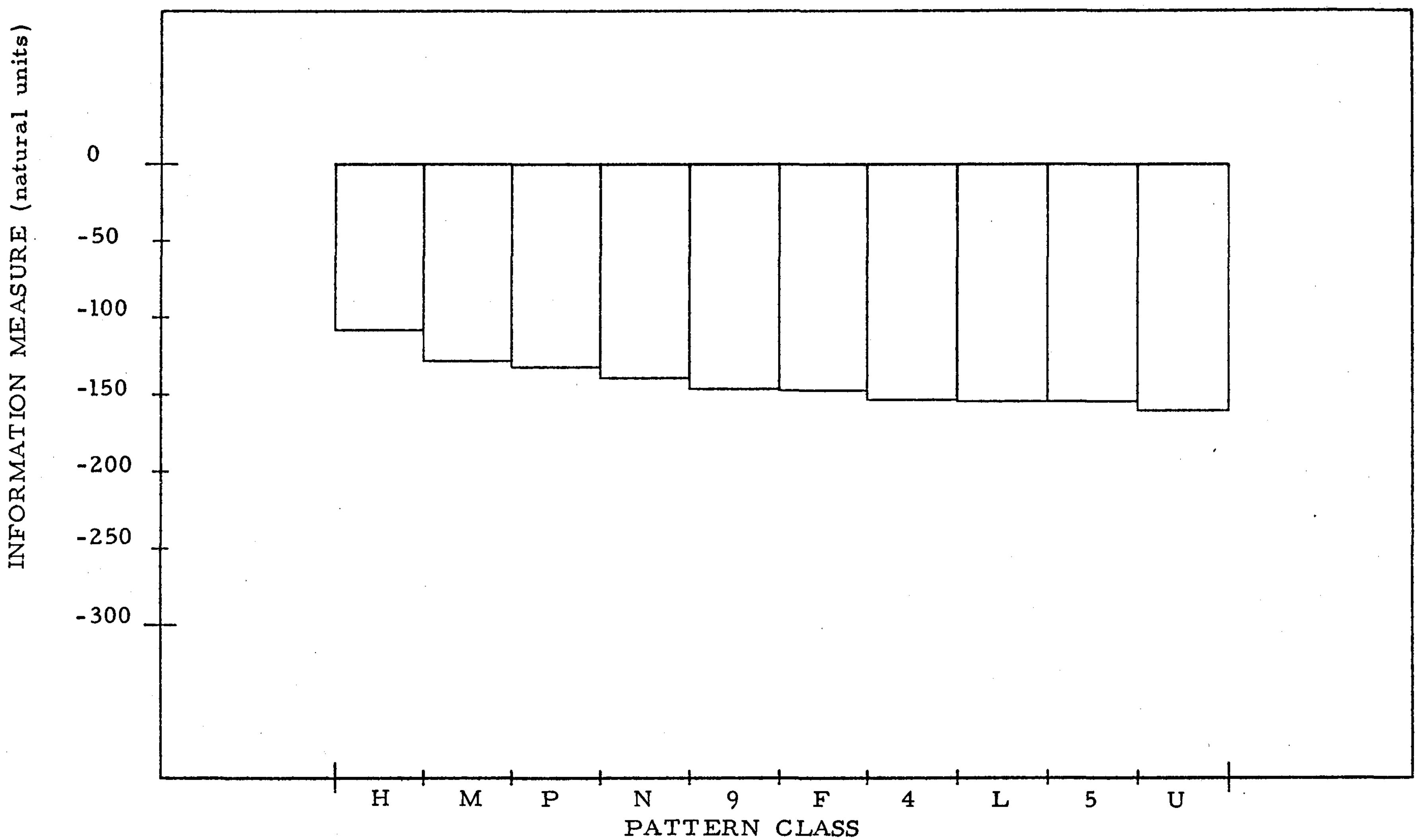


Figure 21. Identification Trial (mod 1) - I (unlearned)

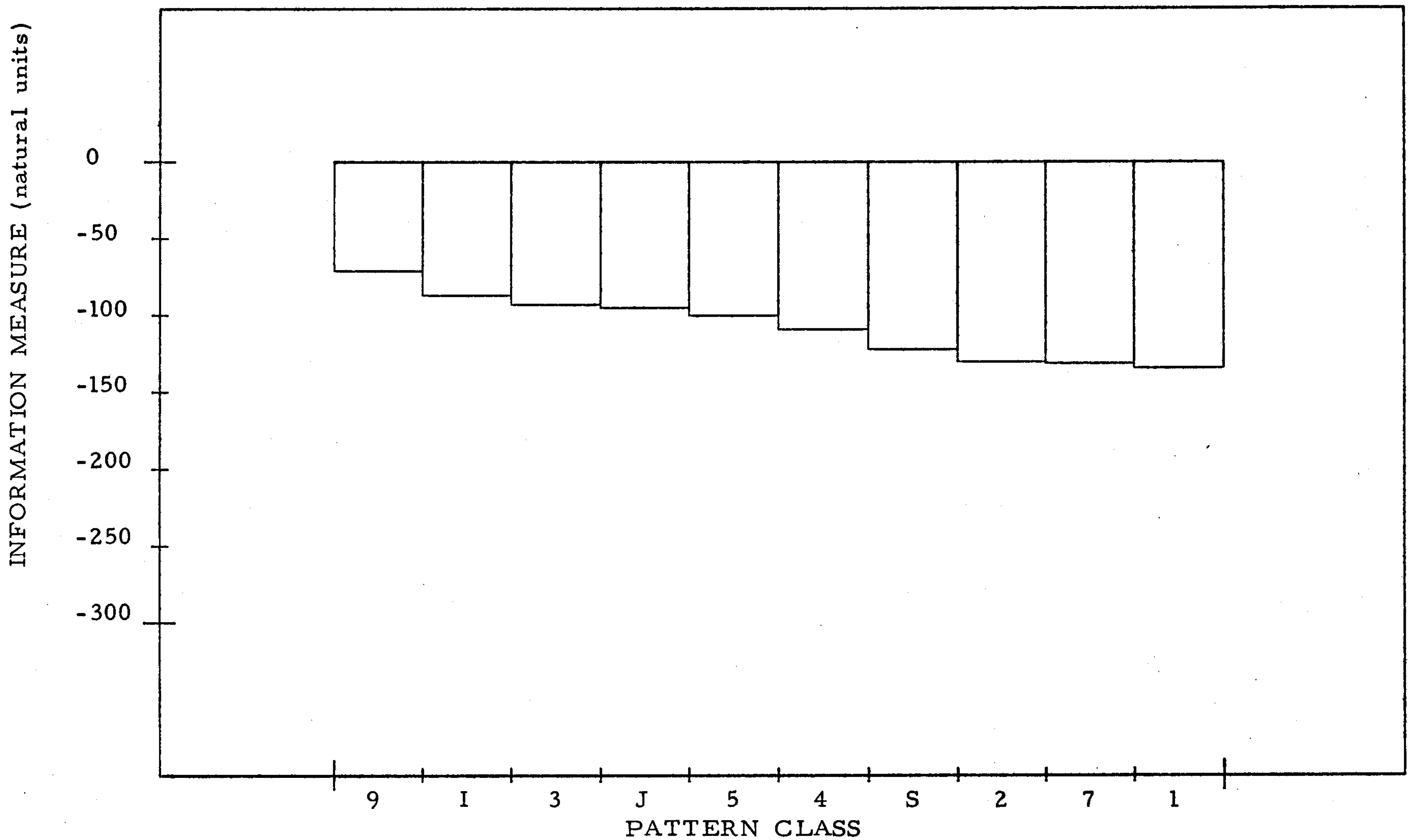


Figure 22. Identification Trial (mod 1) - J (unlearned)

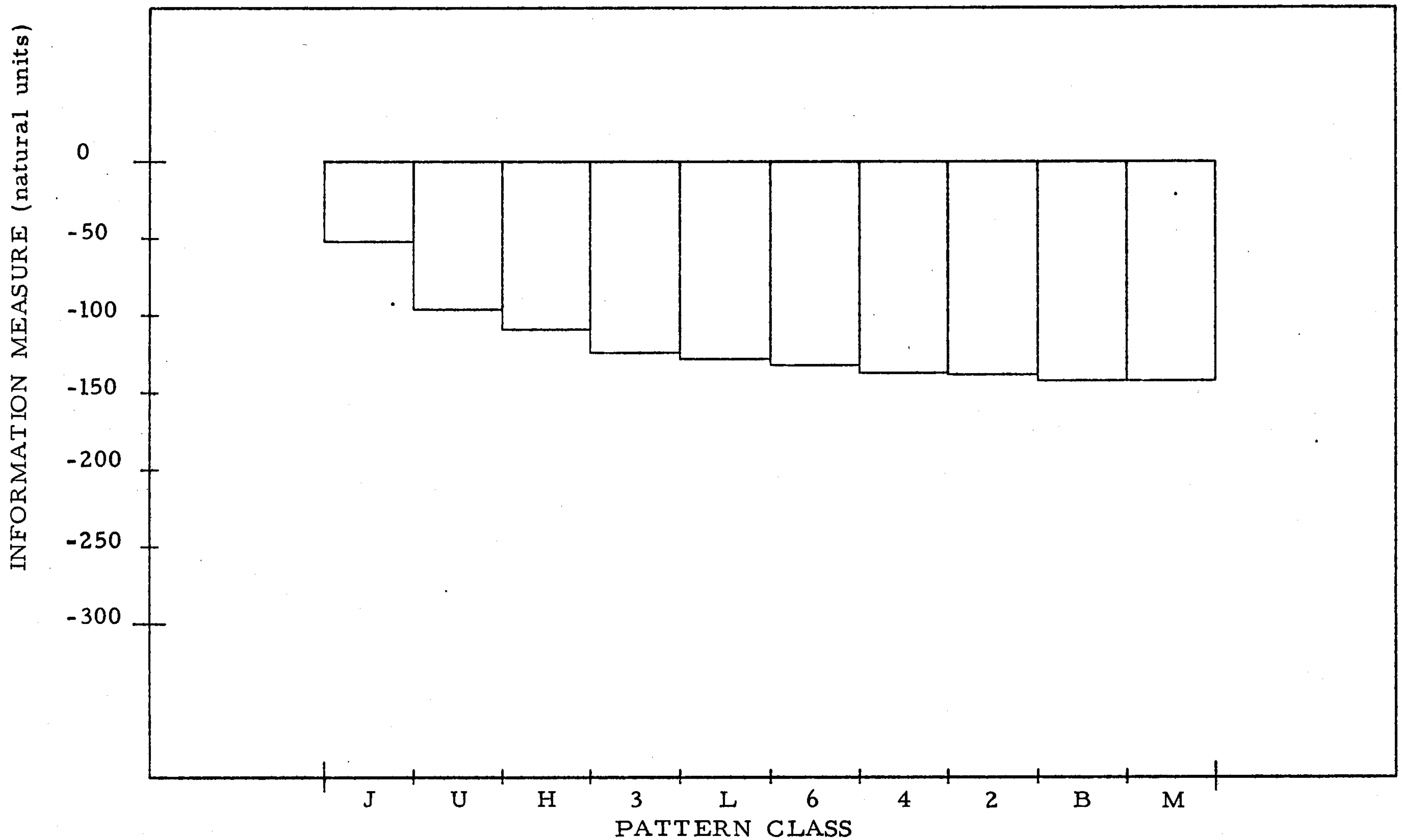


Figure 23. Identification Trial (mod 1) - K (unlearned)

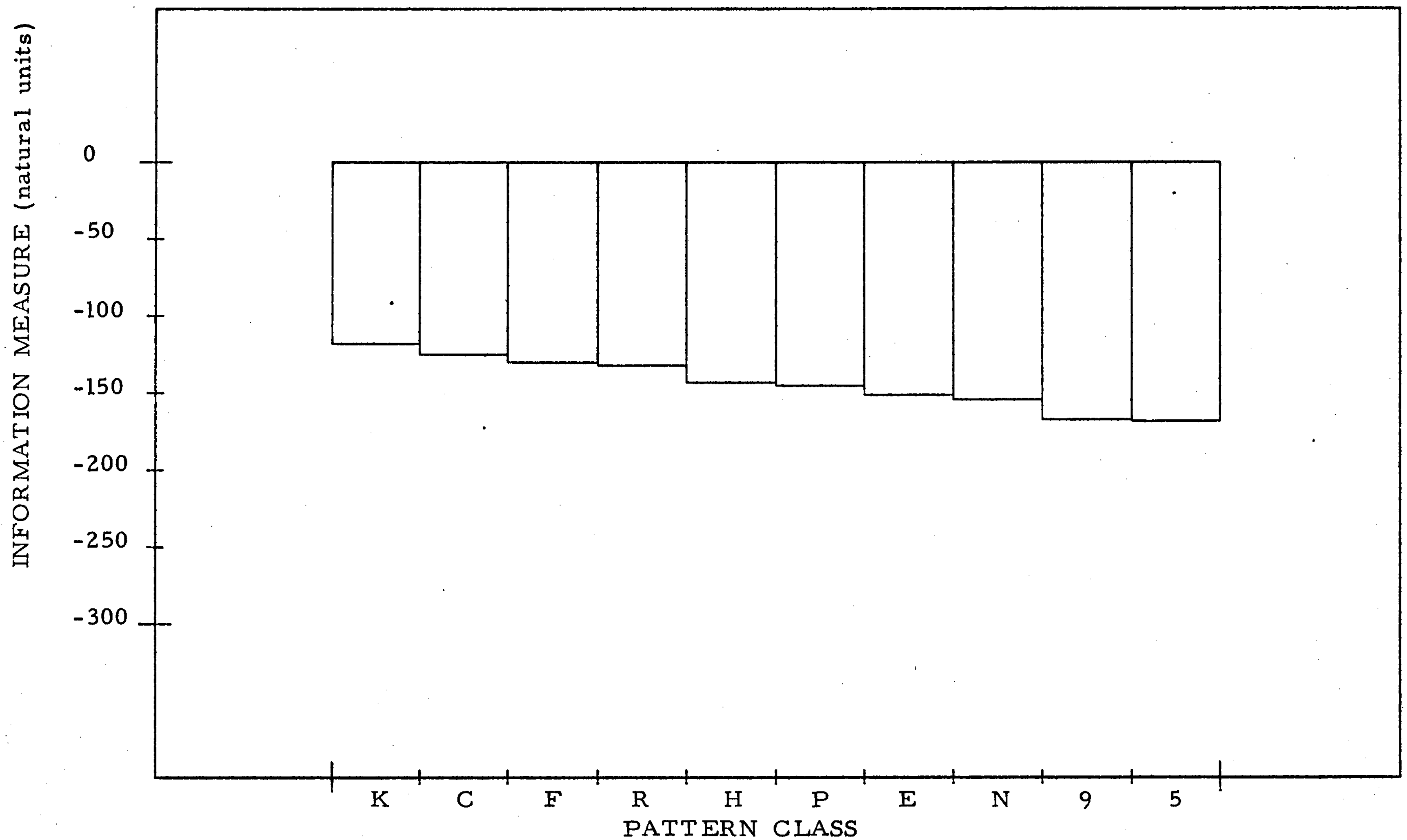


Figure 24. Identification Trial (mod 1) - L (unlearned)

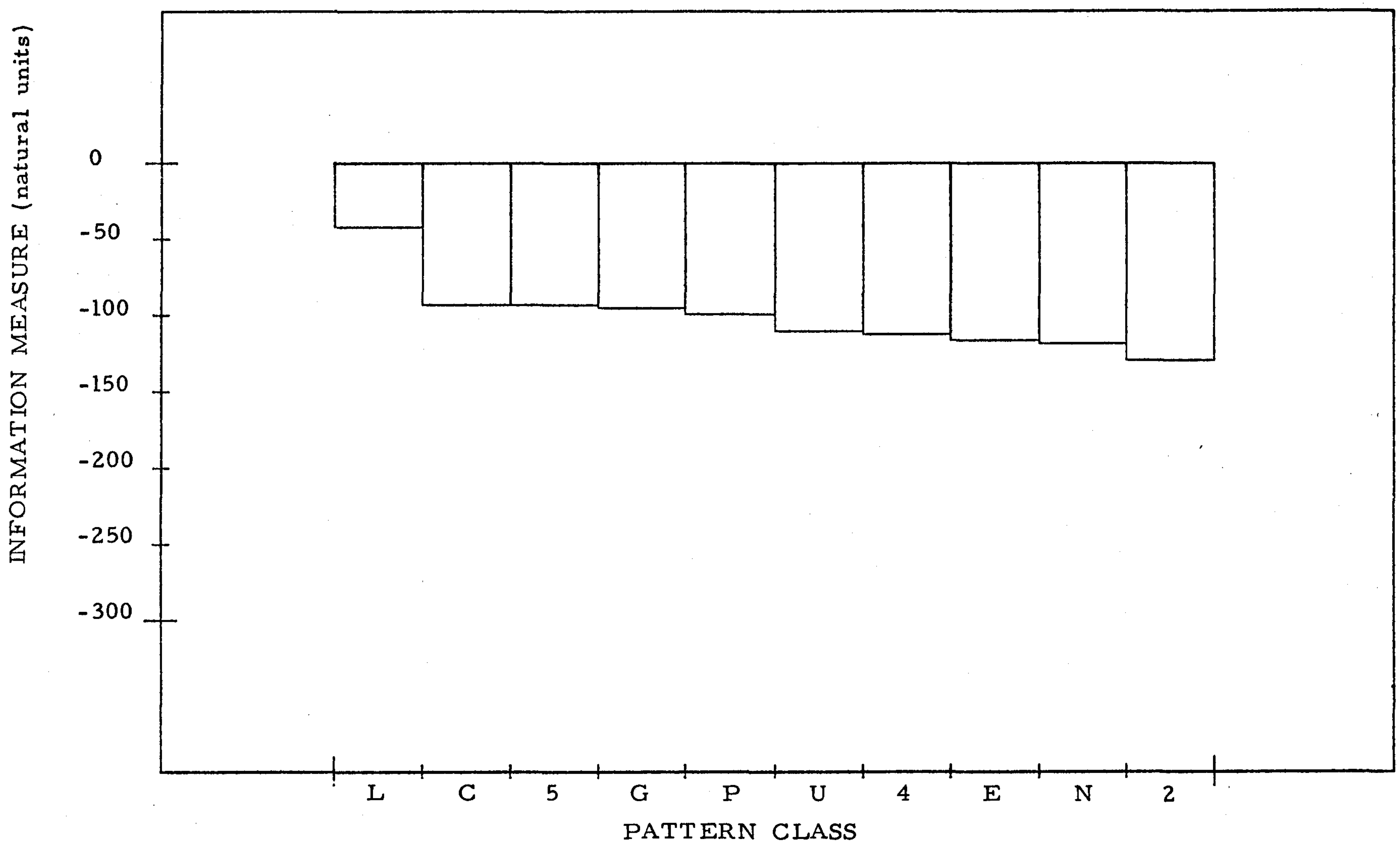


Figure 25. Identification Trial (mod 1) - M (unlearned)

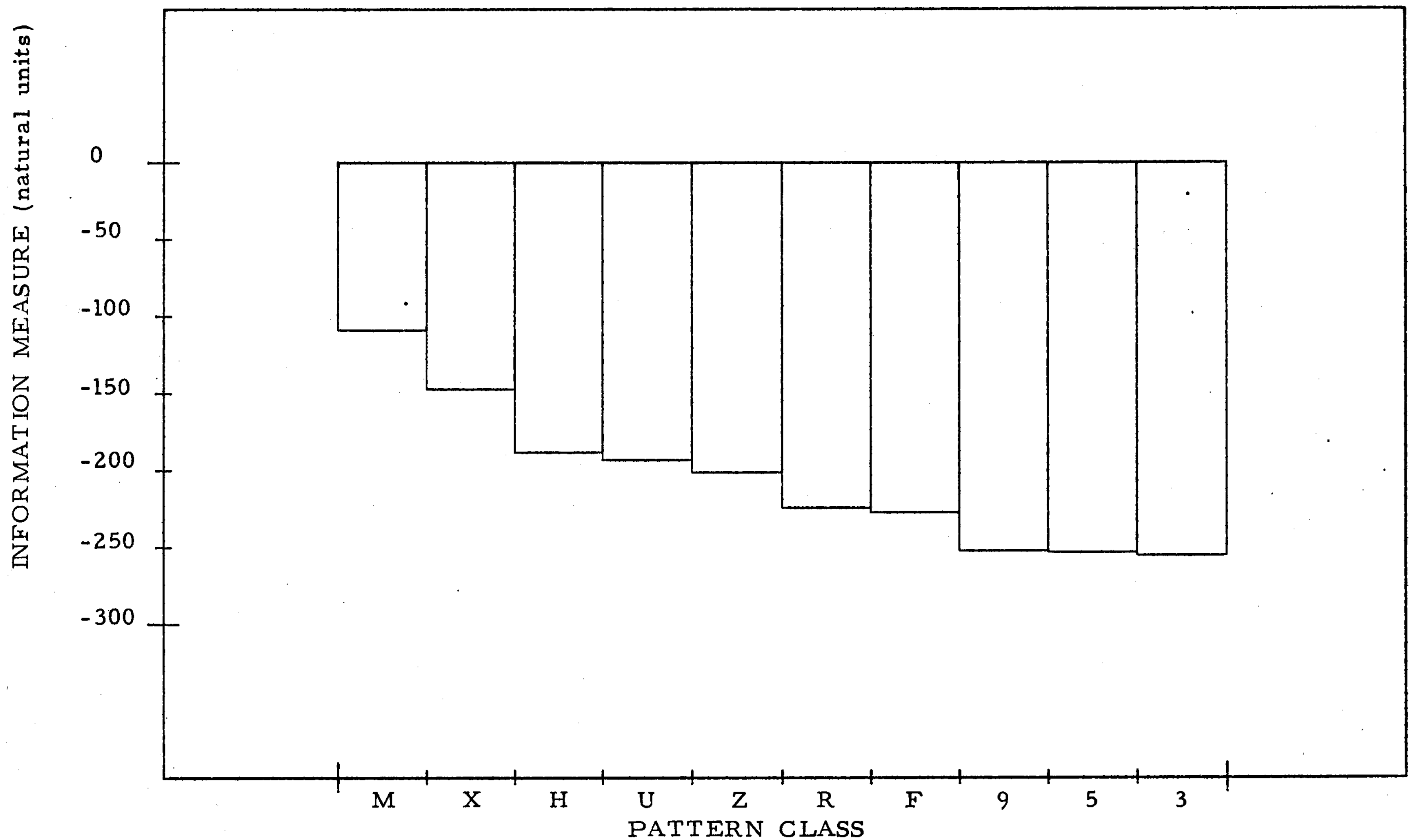


Figure 26. Identification Trial (mod 1) - N (unlearned)

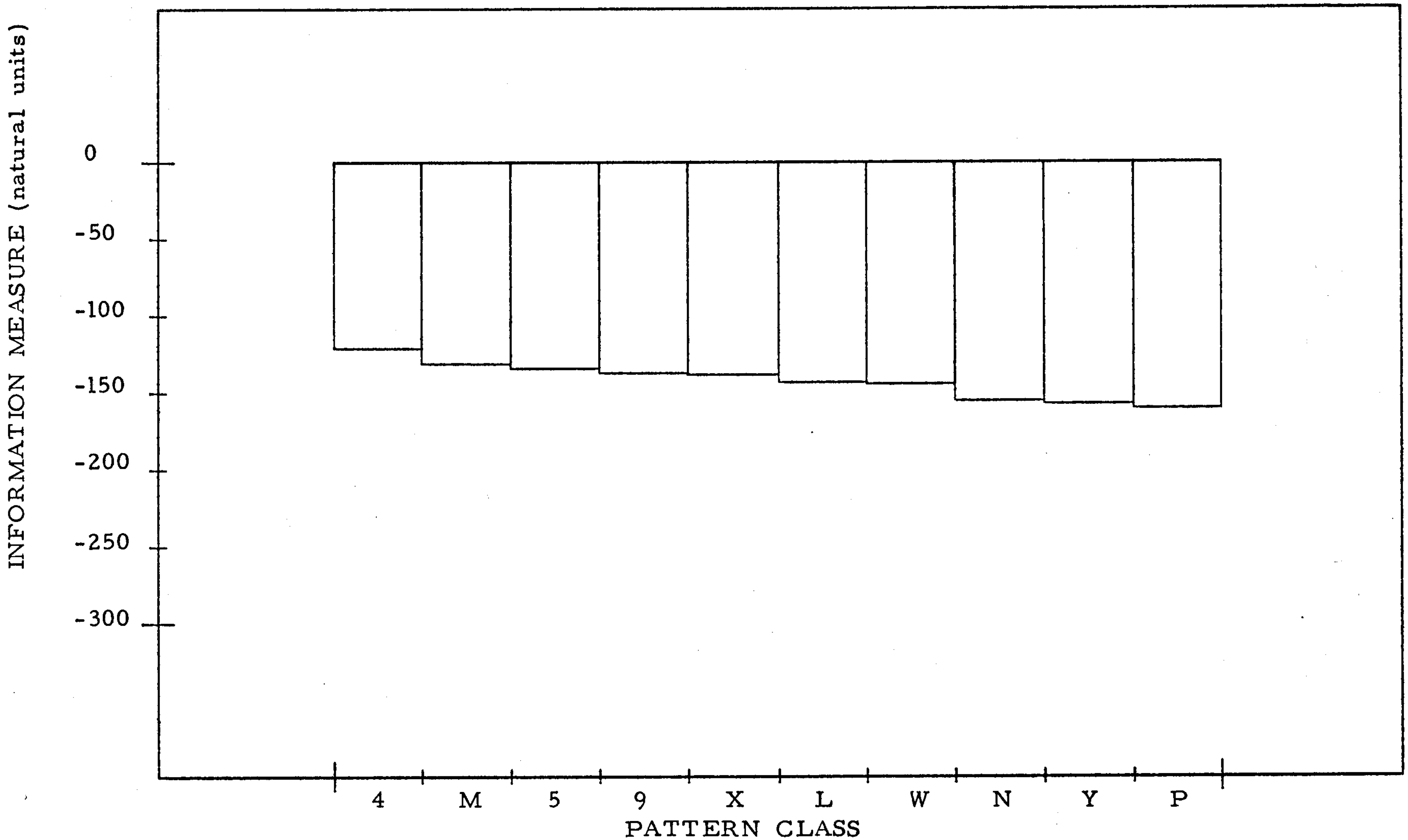
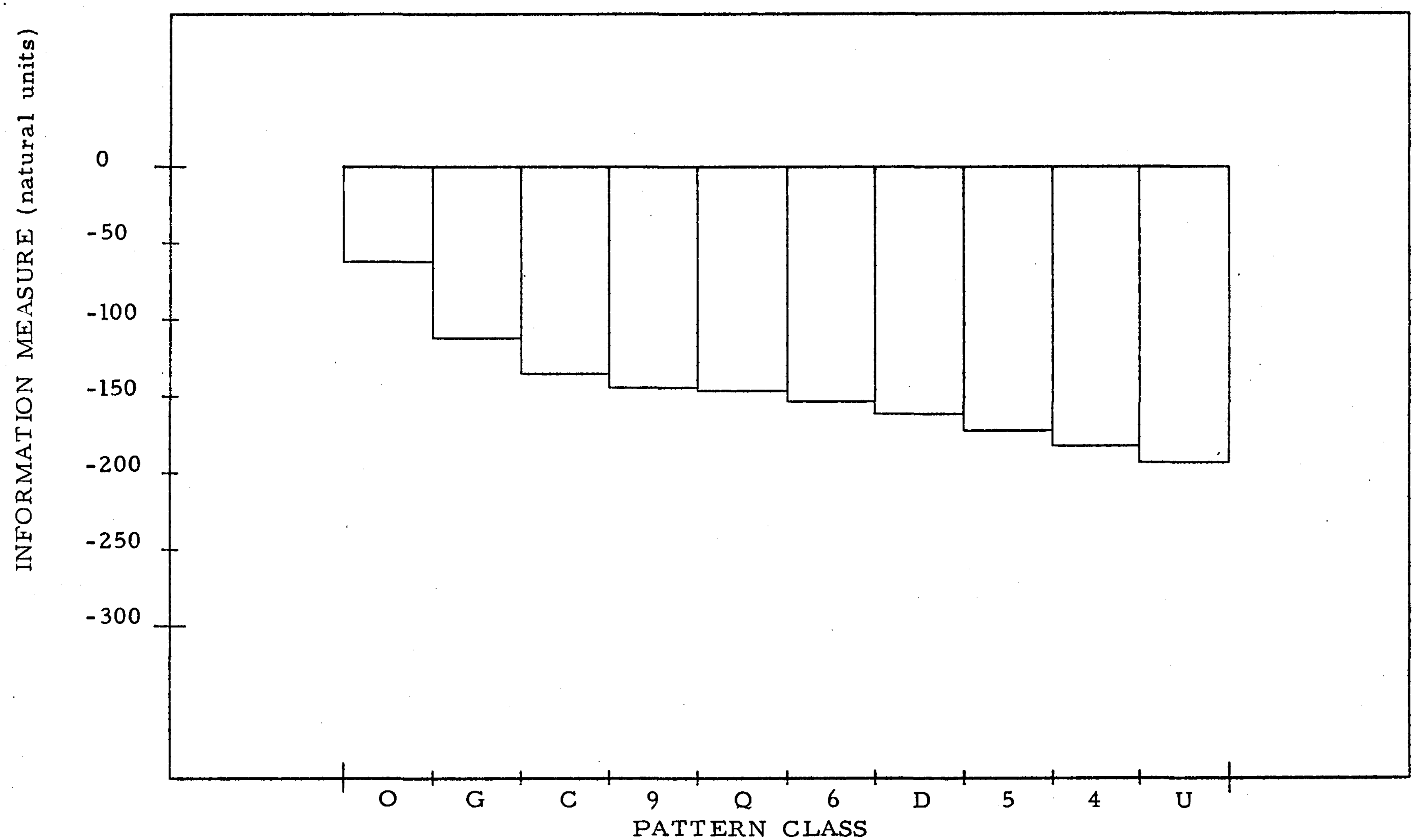


Figure 27. Identification Trial (mod 1) - O (unlearned)



V CONCLUSIONS AND RECOMMENDATIONS

Function model 1 is definitely a successful algorithm. Function model 2, however, is inadequate in its ability to weight the component measures of function model 1. A proper weighting function has yet to be formulated.

The advantages of the contrast transformation are not fully utilized in the simulation program (PCP). The decision time is shorter than it would be for signal coded patterns; but, the small number of samples as well as the method of data collection limit the opportunity for the contrast transformation to be fully effective. It would be interesting to compare contrast coding and signal coding based upon a larger sample of patterns.

The problem of an effective statistical zero has by no means been solved. Further study is required to find out its effect upon a digital pattern recognition system.

The choice of grid size and the degree of signal quantization are variables that should be investigated further.

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APPENDIX A

PCP

PATTERN CLASSIFICATION PROGRAM

An SPS computor program originally written
for an IBM 1620 MK II machine with,

MONITOR II RESIDENT SYSTEM

INDEX REGISTERS

40K CORE MEMORY

1311 DISK STORAGE DRIVES

1622 CARD READ PUNCH

The program requires 25, 630 core positions
and is operated in conjunction with 36 data
tables stored on disk pack memory
(satellite - drive #1)

```
*
*       Declarative Statements
*
Output  Das     26
        Dac     1,@
A1      Das     80
        Dac     1,@
A2      Das     80
        Dac     1,@
A3      Das     80
        Dac     1,@
A4      Das     80
        Dac     1,@
A5      Das     80
        Dac     1,@
A6      Das     80,,                        *    Card
        Dac     1,@,,                            Output
A7      Das     80,,                             Block
        Dac     1,@
A8      Das     80
        Dac     1,@
A9      Das     80
        Dac     1,@
A10     Das     80
        Dac     1,@
A11     Das     80
        Dac     1,@
A12     Das     80
        Dac     1,@
Blank   Dac     50,                                                ,
        Dsc     50,000000000000000
        Dsc     11,0000000000@,,            *    Card Output Zero
None    Das     80,,                        *    Blank Card
        Dac     1,@
Mod     Dac     38,Switch 1 - on for Mod1, off for Mod2@
Note    Dac     44,Switch 3 - on to adjust the System Tables.@
Calmes  Dac     36,Switch 4 - on for Further Samples.@
Badnum  Dac     15, Zero Numerator@
Badden  Dac     17, Zero Denomenator@
Inf     Dac     11,1(  ) = @
Two     Dac     3,  @
Dec     Dac     2,.@
Exp     Dac     2,E@
Plus    Dac     2,+@
Empty   Dac     27,                              @
Item    Dac     24,System Tables Adjusted.@
```

```
Stop     Dac    25, Identification Complete. @
Cardin   Das    80, ,                            *   Card Input Area
Shrink   Dss    240, ,                           *   Reduced Input
*                                                    Data
Area     Dss    160, ,                           *   Quantized Data
*                                                    Area
Dummy    Dda    , 0, 0, 0, 0
Tabar    Dss    10000, ,                         *   Frequency Table
*                                                    Core Area
Flarea   Dss    790, ,                           *   Conditional Freq
*                                                    Area
Flarl    Dss    790, ,                           *   Unconditional
*                                                    Freq Area
Sort     Dss    525, ,                           *   Sort Area
Out      Dss    350, ,                           *   Information Area
Mul      Ds     5, ,                             *   Product Areas
Result   Ds     5
Prodct   Ds     5
Quot     Ds     2
Rem      Ds     2, ,                             *   Remainder
Sub      Ds     2, ,                             *   Difference Area
Zero     Dc     8, 0, ,                          *   Fixed Point Zeros
Zr       Dc     7, 0
Fltzer   Ds     10, ,                            *   Floating Pt Zero
Cntrst   Ds     2, ,                             *   Contrast Data
Smtbcn   Ds     2, ,                                 Areas
Id1      Ds     1, ,                             *   Indicators
Id2      Ds     1
Setind   Ds     1
Compar   Ds     10, ,                            *   Compare Area
Sum      Ds     10, ,                            *   Accumulator
Smallf   Ds     10, ,                            *   Effective
*                                                    Statistical Zero
Denu     Ds     2, ,                             *   Number of
Denomu   Ds     10, ,                                Samples / Class
Denl     Ds     3, ,                             *   Total Number
Denoml   Ds     10, ,                                of Samples
Patcls   Ds     5, ,                             *   Identified Class
*                                                    Coded Area
Hold     Ds     15, ,                            *   Temporary
Flot     Ds     11, ,                                Locations
Bang     Ds     10
Bangf    Ds     10
Tempf    Ds     10
Wait     Ds     10
Save     Ds     10
```

```
Sap      Ds     10
Can      Ds     5
Still    Ds     5
Tmpi     Ds     3
Sav      Ds     2
Tempi    Ds     2
Holdl    Ds     2
Table    Dsa    Tabar
*                                                  *  Disk Control Flds
Tab      Dda    ,3,0,65,Tabar
         Dc     1,@
Sumtab   Dda    ,3,09600,98,Tabar
         Dc     1,@
A        Dc     5,02456,,                          *  Constants
Three    Dc     2,35
Seven    Dc     2,79
*
*        Input Unknown Data
*
Start    H
         Cf     Setind,,,                          *  Reset Modl-2 Indr
         Tdm    Setind,0
         Rcty
         Waty   Mod
         Rcty
         H
         Bncl   Noflag
         Sf     Setind,,,                          *  Set Modl-2 Indr
Noflag   B      *+12
         Bsba   *+12
         Tf     Denu,Zr-5,,                        *  Zero Sample Totls
         Tf     Denl,Zr-4
         Tfm    Tab+5,06000
Count    Sk     Tab
         Rdn    Tab
         Cdn    Tab
         A      Denl,Tabar+3,,                     *  Tally Totl Sampls
         Am     Tab+5,100,,                           From Disk-Stored
         Cm     Tab+5,09500,,                         Freq Tables
         Bnz    Count
         Blxm   *+12,-240(A1)
         Racd   Cardin,,,                          *  Read Data Cards
         Racd   Cardin
         Racd   Cardin
Rutine   Td     Shrink+240(A1),Cardin+028,,           Compact Input Dat
         Td     Shrink+241(A1),Cardin+032
```

```
        Td      Shrink+242(A1), Cardin+034
        Td      Shrink+243(A1), Cardin+040
        Td      Shrink+244(A1), Cardin+044
        Td      Shrink+245(A1), Cardin+046
        Td      Shrink+246(A1), Cardin+052
        Td      Shrink+247(A1), Cardin+056
        Td      Shrink+248(A1), Cardin+058
        Td      Shrink+249(A1),Cardin+064
        Td      Shrink+250(A1), Cardin+068
        Td      Shrink+251(A1), Cardin+070
        Td      Shrink+252(A1), Cardin+076
        Td      Shrink+253(A1), Cardin+080
        Td      Shrink+254(A1), Cardin+082
        Td      Shrink+255(A1), Cardin+088
        Td      Shrink+256(A1), Cardin+092
        Td      Shrink+257(A1), Cardin+094
        Td      Shrink+258(A1), Cardin+100
        Td      Shrink+259(A1), Cardin+104
        Td      Shrink+260(A1), Cardin+106
        Td      Shrink+261(A1), Cardin+112
        Td      Shrink+262(A1), Cardin+116
        Td      Shrink+263(A1), Cardin+118
        Bcxm    Rutine-12, 24(A1)
        Racd    Cardin
        Racd    Cardin
        Blxm    *+12, -240(A1)
        Blxm    *+12, -160(A2)
Setflg  Sf      Shrink+240(A1), , ,            *   Flg Compcted Data
Quant   Mm      Shrink+242(A1), 2, 10,         *   Quantize Data
        Tf      Mul, 00099, ,                      Into 21 Levels
        Sf      Mul-2
        Tf      Quot, Mul-1
        Td      Rem, Mul
        Tdm     Rem-1, 0, 11
        Cm      Rem, 5
        Bl      Noadd
        Am      Quot, 1, 10
Noadd   Tf      Area+161(A2), Quot
        Bxm     *+12, 2(A2)
        Bcxm    Setflg, 3(A1)
        Blxm    *+12, -158(A1)
        Tf      Hold1, Area+1
Subtr   Tf      Sub, Hold1, ,                  *   Generate Contrast
        S       Sub, Area+161(A1), ,               Value Sequence
        Tf      Area+161(A1), Sub
        Bcxm    Subtr, 2(A1)
```

```
*
*       Calculate Mutual Informations
*
        Tf      Smallf-2, Zero,,                 *   Set Up
        Tfm     Smallf, 2, 10,                       Statistical Zero
        Tdm     Smallf-9, 1, 11
        Sf      Smallf
        Tf      Fltzer-2, Zero,,                 *   Set Up Floating
        Tfm     Fltzer, 99, 10,                      Pt Zero
        Sf      Fltzer
        Blxm    *+12, -790(A1)
Back    Tfl     Flarea+799(A1), Fltzer,,         *   Zero Cond and
*                                                    Uncond Freq Areas
        Bcxm    Back, 10(A1)
        Blxm    *+12, -790(A1)
Bac     Tfl     Flarl+799(A1), Fltzer
        Bcxm    Bac, 10(A1)
        Blxm    *+12, -350(A5)
        Blxm    *+12, -35(A2)
Nxtcls  Tf      Sav, Three,,                     *   Find Total Number
        Mf      00313, 00310,,                       of Samples for
        A       Sav, 00314,,                         Class Under Test
        Mf      00310, 00313
        Mm      Sav, 100, 9
        Bxm     *+12, 1(A2)
        Tf      Prodct, 00099
        Am      Prodct, 06000
        Tf      Tab+5, Prodct,,                  *   Set Up Sctr Addrs
        Sk      Tab,,,                           *   Bring in Cond
        Rdn     Tab,,,                               Freq Table
        Cdn     Tab
        Tf      Denu, Tabar+3
        Blxm    *+12, -158(A4)
        Blxm    *+12, -790(A1)
        Blxm    *+12, -79(A3)
Bbls79  Tf      Sav, Seven,,                     *   Set Up Cond Freq
        Mf      00318, 00315,,                       Table Addrs
        A       Sav, 00319
        Mf      00315, 00318
        Mm      Sav, 82, 9
        Bxm     *+12, 1(A3)
        Tf      Result, 00099,,                  *   Add Digit Posn
        Tf      Cntrst, Area+161(A4),,               To Addrs
        Am      Cntrst, 21, 10
        Mm      Cntrst, 2, 9
        Tf      Prodct, 00099
```

```
        A       Result,Prodct,,              *   Add Cntrst Value
        A       Result,Table,,                   Posn to Addrs
        Am      Result,3
        Tf      Tempi,Result,11,             *   Extract Freq
        Tfl     Tempf,Fltzer,,                   From Table
        Cm      Tempi,9,,                    *   Freq to Flt Pt
        Bh      Alter
        Td      Tempf-9,Tempi
        Sf      Tempf-9
        Tfm     Tempf,1,10
        C       Tempf-8,Fltzer-8
        Bnz     L1
        Tfl     Tempf,Fltzer
        B7      L1
Alter   Tf      Tempf-8,Tempi
        Tfm     Tempf,2,10
L1      Tfl     Flarea+799(A1),Tempf,,       *   Flt Pt Value to
*                                                Cond Freq Area
        Bxm     *+12,10(A1)
        Bcxm    Bbls79,2(A4)
        Sk      Sumtab,,,                    *   Cond Freq Table
        Rdn     Sumtab,,,                        To Core
        Cdn     Sumtab
        Blxm    *+12,-790(A1)
        Blxm    *+12,-79(A3)
        Blxm    *+12,-158(A4)
Bls79   Tf      Sav,Seven,,                  *   Set Up Uncond
        Mf      00318,00315,,                    Freq Addrs
        A       Sav,00319
        Mf      00315,00318
        Mm      Sav,123,9
        Bxm     *+12,1(A3)
        Tf      Result,00099,,               *   Add Digit Posn
        Tf      Smtbcn,Area+161(A4),,            to Addrs
        Am      Smtbcn,21,10
        Mm      Smtbcn,3,9
        Tf      Prodct,00099
        A       Result,Prodct,,              *   Add Cntrst Value
        A       Result,Table,,                   Posn to Addrs
        Am      Result,3
        Tf      Tmpi,Result,11,              *   Extract Freq
        Tfl     Tempf,Fltzer,,                   from Table
        Cm      Tmpi,99,,                    *   Freq to Flt Pt
        Bnh     Twoo
        Tf      Tempf-7,Tmpi
        Tfm     Tempf,3,10
```

```
        B7      M1
Twoo    Cm      Tmpi, 9
        Bnh     One
        Mf      Tmpi-1, Tmpi-2
        Tf      Tempf-8, Tmpi
        Tfm     Tempf, 2, 10
        B7      M1
One     Td      Tempf-9, Tmpi
        Sf      Tempf-9
        Tfm     Tempf, 1, 10
        C       Tempf-8, Fltzer-8
        Bnz     M1
        Tfl     Tempf, Fltzer
M1      Tfl     Flarl+799(A1), Tempf, ,       *   Flt Pt Value to
*                                                 Uncond Freq Area
        Bxm     *+12, 10(A1)
        Bcxm    Bls79, 2(A4)
        Tf      Tempf, Fltzer
        Blxm    *+12, -790(A1)
Comput  Tfl     Sum, Fltzer, ,                *   Zero Inftn Acc
        Tfl     Tempf, Fltzer
        Tfm     Denomu, 2, 10,                *   Sample Totls to
        Tf      Denomu-2, Zero, ,                 Floating Pt Form
        Tf      Denomu-8, Denu
        Tfm     Denoml, 3, 10
        Tf      Denoml-2, Zero
        Tf      Denoml-7, Denl
Cycle   Bnf     Mod2, Setind, ,               *   To Selected Mod
Modl    Tfl     Compar, Flarea+799(A1)
        C       Compar-8, Fltzer-8, ,         *   Test Cond Freq
        Bnz     Nosame
        Tfl     Compar, Flarl+799(A1)
        C       Compar-8, Fltzer-8, ,         *   Test Uncond Freq
        Bnz     Fad
        B       Mak, , ,                      *   No Information
Fad     Tfl     Bangf, Smallf, ,              *   Substitute Stat
        Fdiv    Bangf, Denomu, ,                  Zero and
        Fdiv    Bangf, Flarl+799(A1), ,           Compute Measure
        Fmul    Bangf, Denoml
        Fln     Bang, Bangf
        Fadd    Sum, Bang, ,                  *   Add Effective Inf
        Bnc2    Away
        Rcty
        Waty    Badnum
        Rcty
        B       Away
```

```
Mak     Bncl    Away
        Rcty
        Waty    Badden
        Rcty
        B       Away
Nosame  Fmul    Flarea+799(Al), Denoml, ,            *    Compute Measure
        Fmul    Flarl+799(Al), Denomu
        Fdiv    Flarea+799(Al), Flarl+799(Al)
        Tfl     Tempf, Flarea+799(Al)
        Fln     Bang, Tempf
        Fadd    Sum, Bang, ,                         *    Add Information
        B       Away
Mod2    Tfl     Compar, Flarea+799(Al)
        C       Compar-8, Fltzer-8, ,                *    Test Cond Freq
        Bnz     Nosam
        Tfl     Compar, Flarl+799(Al)
        C       Compar-8, Fltzer-8, ,                *    Test Uncond Freq
        Bnz     Fadd
        B       Kam, , ,                             *    No Information
Fadd    Tfl     Tempf, Smallf, ,                     *    Substitute Stat
        Fdiv    Tempf, Denomu, ,                          Zero and
        Fdiv    Tempf, Flarl+799(Al), ,                   Compute Measure
        Fmul    Tempf, Denoml
        Fln     Flarl+799(Al), Tempf
        Tfl     Tempf, Smallf
        Fdiv    Tempf, Denomu
        Fmul    Flarl+799(Al), Tempf
        Fadd    Sum, Flarl+799(Al), ,                *    Add Effective Inf
        Bnc2    Away
        Rcty
        Waty    Badnum
        Rcty
        B       Away
Kam     Bncl    Away
        Rcty
        Waty    Badden
        Rcty
        B       Away
Nosam   Tfl     Sap, Flarea+799(Al), ,               *    Compute Measure
        Fdiv    Sap, Denomu
        Fmul    Flarea+799(Al), Denoml
        Fmul    Flarl+799(Al), Denomu
        Fdiv    Flarea+799(Al), Flarl+799(Al)
        Tfl     Tempf, Flarea+799(Al)
        Fln     Bang, Tempf
        Fmul    Bang, Sap
```

```
        Fadd   Sum, Bang, ,                          *   Add Information
Away    Bcxm   Cycle, 10(A1)
        Tfl    Out+359(A5), Sum
        Bcxm   Nxtcls, 10(A5)
        Rcty
        Waty   Stop
        Rcty
*
*       Output Mutual Informations
*
        Tdm    Id1, 0, ,                             *   Zero Column Indrs
        Tdm    Id2, 0
        Trnm   Output-1, Empty-1, ,                  *   Set Up Alpha Flds
        Trnm   Output-1, Inf-1
        Trnm   Output+23, Dec-1
        Trnm   Output+39, Exp-1
        Trnm   Output+41, Plus-1
        Trnm   None-1, Blank-1
        Blxm   *+12, -350(A3)
        Blxm   *+12, -525(A1)
        Tf     Can, Zr-2
Trnsfr  Tfl    Sort+539(A1), Out+359(A3), ,              Index Cls Measurs
        Am     Can, 1
        Tf     Sort+529(A1), Can
        Bxm    *+12, 10(A3)
        Bcxm   Trnsfr, 15(A1)
        Bsbb   *+12
        Blxm   *+12, 525(B3)
        Blxm   *+12, 34(B1)
Gobak   Bsx    *+12, 00354(B1)
        Tfl    Save, Sort-1(B3), ,                   *   Sort Measures
        Fsub   Sort-1(B3), Sort-16(B3)
        C      Sort-3(B3), Zero
        Bnh    Remain
        Tfl    Sort-1(B3), Save
        Tfl    Hold, Sort-1(B3)
        Tf     Hold-10, Sort-11(B3)
        Tfl    Sort-1(B3), Sort-16(B3)
        Tf     Sort-11(B3), Sort-26(B3)
        Tfl    Sort-16(B3), Hold
        Tf     Sort-26(B3), Hold-10
        B      Remain+12
Remain  Tfl    Sort-1(B3), Save
        Bxm    *+12, -15(B3)
        Bcxm   Gobak+12, -(B2)
        Blxm   *+12, 525(B3)
```

```
        Bcxm   Gobak, -1(B1)
        Bsba   *+12
        Blxm   *+12, -525(A1)
        Blxm   *+12, 13(A5)
        Blxm   *+12, 13(A6)
        Blxm   *+12, 12(A7)
        Tf     Still, A
        Cf     Id1, , ,                      *   Reset Indicators
        Cf     Id2
Return  Tfl    Flot, Sort+539(A1), ,         *   Sorted Measures
        Sm     Flot, 1, 10,                      to Data Out Area
        Mf     Flot-10, Flot-9
        Tnf    Output+22, Flot-9, ,          *   Data Left One Dig
        Tdm    Output+20, 0
        Tdm    Output+19, 0
        Sf     Flot-8
        C      Flot-2, Zr, ,                 *   Test Mantisa Sign
        Bnl    Go
        Cf     Flot-2
        Tdm    Output+19, 2, ,               *   Mantisa Negative
        Tdm    Output+20, 0
        B7     Go+24
Go      Tdm    Output+19, 1, ,               *   Mantisa Positive
        Tdm    Output+20, 0
        Tnf    Output+38, Flot-2, ,          *   Mant to Out Area
        Cm     Flot, 0, 10                   *   Test Expon Sign
        Bnl    Notlow
        Cf     Flot
        Tdm    Output+41, 2, ,               *   Expon Negative
        Tdm    Output+42, 0
        B7     Notlow+24
Notlow  Tdm    Output+41, 1, ,               *   Expon Positive
        Tdm    Output+42, 0
        Tnf    Output+46, Flot, ,            *   Expon to Out Area
        Mf     Sort+528(A1), Sort+525(A1)
        Tnf    Output+6, Sort+529(A1), ,     *   Index to Out Area
        Bnf    *+24, Id1, ,                  *   Column 1 Check
        B      Yes2-36
        Bcxm   No1, -1(A5)
Yes1    Sf     Id1, , ,                      *   Column 1 Complete
        Tf     Still, A
        Am     Still, 56
        Bnf    *+24, Id2, ,                  *   Column 2 Check
        B      Yes3-12
        Bcxm   No2, -1(A6)
Yes2    Sf     Id2, , ,                      *   Column 2 Complete
```

```
        Tf      Still, A
        Am      Still, 108
        Bcxm    No3, -(A7)
Yes3    Trnm    Still, Empty-1, 6,          *   Column 3 Complete
        Wacd    A1, , ,                     *   Punch Information
        Wacd    A2, , ,                         Measures In
        Wacd    A3, , ,                         Descending Order
        Wacd    A4, , ,                         of Magnitude
        Wacd    A5
        Wacd    A6
        Wacd    A7
        Wacd    A8
        Wacd    A9
        Wacd    A10
        Wacd    A11
        Wacd    A12
        Wacd    Cardin, , ,                 *   Punch Sample
        Wacd    Cardin, , ,                     Coded Cards
        B7      Art
No3     Trnm    Still, Output-1, 6,         *   Columnator Rutine
        Am      Still, 162
        B7      Gas
No2     Trnm    Still, Output-1, 6
        Am      Still, 162
        B7      Gas
No1     Trnm    Still, Two-1, 6
        Am      Still, 4
        Trnm    Still, Output-1, 6
        Am      Still, 158
Gas     Bxm     Return, 15(A1)
*
*       Frequency Table Adjustment Option
*
Art     Rcty
        Waty    Note
        Rcty
        H
        Bnc3    Call
        Tf      Patcls, Sort+4, ,           *   Get Class Code
        Mf      Patcls-1, Patcls-4, ,           of Max Measure
        Mm      Patcls, 100, 9
        Tf      Prodct, 00099
        Am      Prodct, 05900
        Tf      Tab+5, Prodct, ,            *   Set Up Cond Freq
*                                               Table Addrs
        Sk      Tab, , ,                    *   Table to Core
```

```
        Rdn     Tab
        Cdn     Tab
        Am      Tabar+3, 1, 10          *   Update Sample Qty
        Blxm    *+12, -158(A1)
        Blxm    *+12, -79(A3)
Again   Tf      Sav, Seven
        Mf      00318, 00315
        A       Sav, 00319
        Mf      00315, 00318
        Mm      Sav, 82, 9
        Bxm     *+12, 1(A3)
        Tf      Result, 00099, ,        *   Add Digit Posn
        Tf      Cntrst, Area+161(A1), ,     to Addrs
        Am      Cntrst, 21, 10
        Mm      Cntrst, 2, 9
        Tf      Prodct, 00099
        A       Result, Prodct, ,       *   Add Cntrst Value
        A       Result, Table, ,            Posn to Addrs
        Am      Result, 3
        Am      Result, 1, 610,         *   Update Cond Freq
        Bcxm    Again, 2(A1)
        Sk      Tab, ,                  *   Rewrite Table
        Wdn     Tab, , ,                    Onto Disk
        Cdn     Tab
        Sk      Sumtab, , ,             *   Uncond Freq
        Rdn     Sumtab, , ,                 Table to Core
        Cdn     Sumtab
        Blxm    *+12, -158(A1)
        Blxm    *+12, -79(A3)
Contin  Tf      Sav, Seven, ,           *   Set Up Uncond
        Mf      00318, 00315                Freq Addrs
        A       Sav, 00319
        Mf      00315, 00318
        Mm      Sav, 123, 9
        Bxm     *+12, 1(A3)
        Tf      Result, 00099, ,        *   Add Digit Posn
        Tf      Smtbcn, Area+161(A1), ,     to Addrs
        Am      Smtbcn, 21, 10
        Mm      Smtbcn, 3, 9
        Tf      Prodct, 00099
        A       Result, Prodct, ,       *   Add Cntrst Value
        A       Result, Table, ,            Posn to Addrs
        Am      Result, 3
        Am      Result, 1, 69,          *   Updat Uncond Freq
        Bcxm    Contin, 2(A1)
        Sk      Sumtab, , ,             *   Rewrite Table
```

```
        Wdn     Sumtab,,,                  Onto Disk
        Cdn     Sumtab
        Rcty
        Waty    Item
        Rcty
Call    Waty    Calmes
        Rcty
        H
        Bnc4    End,,,                  *  Recycle Option
        B       Start
End     Call    Exit
        Dend    Start
```

APPENDIX B

EXAMPLE OF EQUATION (5)

Consider a pattern which is segmented into nine discrete elements. Let the set S of signal values be,

S: (0, 1, 2, 3)

Assume the following M matrix,

3	1	2
2	2	0
2	0	1

Using equation (2), the following N matrix is generated,

0	2*	1	1	1	3	1	3	2''
-2	0	-1	-1	-1	1	-1	1	0'
-1	1	0	0	0	2	0	2	1
-1	1	0	0	0	2	0	2	1
-1	1	0	0	0	2	0	2	1
-3	-1	-2	-2	-2	0	-2	0	-1
-1	1	0	0	0	2	0	2	1
-3	-1	-2	-2	-2	0	-2	0	-1
-2	0	-1	-1	-1	1	-1	1	0

(* - $n_{1,1}^{1,2}$; '' - $n_{1,1}^{3,3}$; ' - $n_{1,2}^{3,3}$)

According to equation (5),

$$(n^{3,3}_{1,2}) = (n^{3,3}_{(1,2)-1}) - (n^{1,2}_{(1,2)-1})$$

$$= (n^{3,3}_{1,1}) - (n^{1,2}_{1,1})$$

$$= 2 - 2 = 0 = (n^{3,3}_{1,2})$$

APPENDIX C

EXAMPLE OF EQUATION (10)

Equation (10) gives the number of redundant N sequences which would be generated by the transformation in Chapter II. Consider M matrices of order (1, 4),

$$a = 1$$

$$b = 4$$

Define the set S,

$$S: (0,\ 1,\ 2)$$

$$s_q = 2$$

Let the order of the N sequence generation be row by row starting with $m_{1,1}$. Then the total number of possible M matrices is,

$$(q + 1)^{ab} = 3^4 = 81$$

The number of redundant matrices according to equation (10) is,

$$R = q^{ab} = 2^4 = 16$$

The number of unique N sequences is,

$$(q + 1)^{ab} - q^{ab} = 81 - 16 = 65$$

From Table II on the following page, the number of unique N sequences (those without an *) is seen to be 65, in agreement with the general formula,

$$U = (q+1)^{ab} - q^{ab} \tag{22}$$

Therefore the number of redundant matrices is,

$$R = 81 - 65 = 16,$$

the number predicted by equation (10).

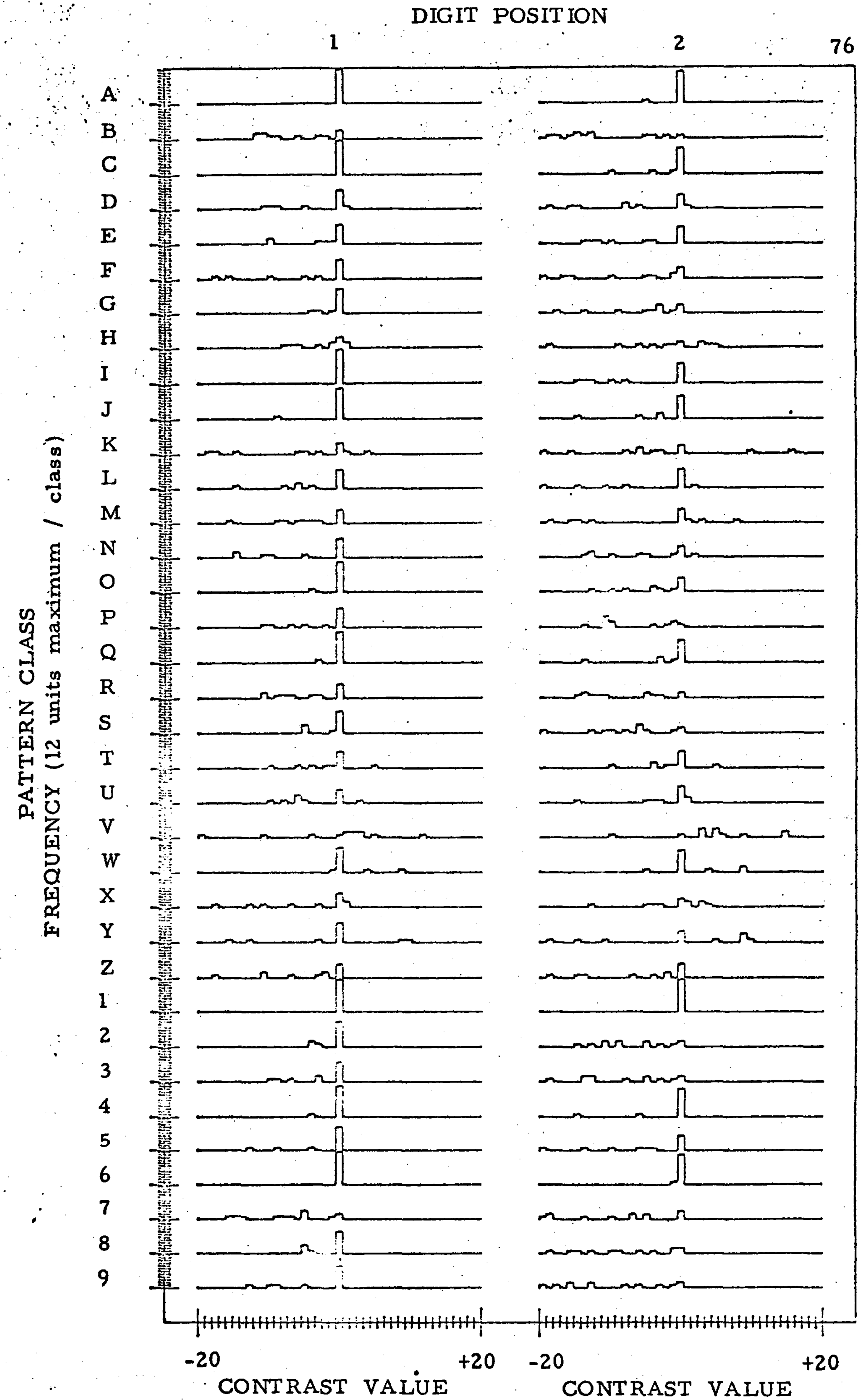
Table II All Possible M and N Matrices

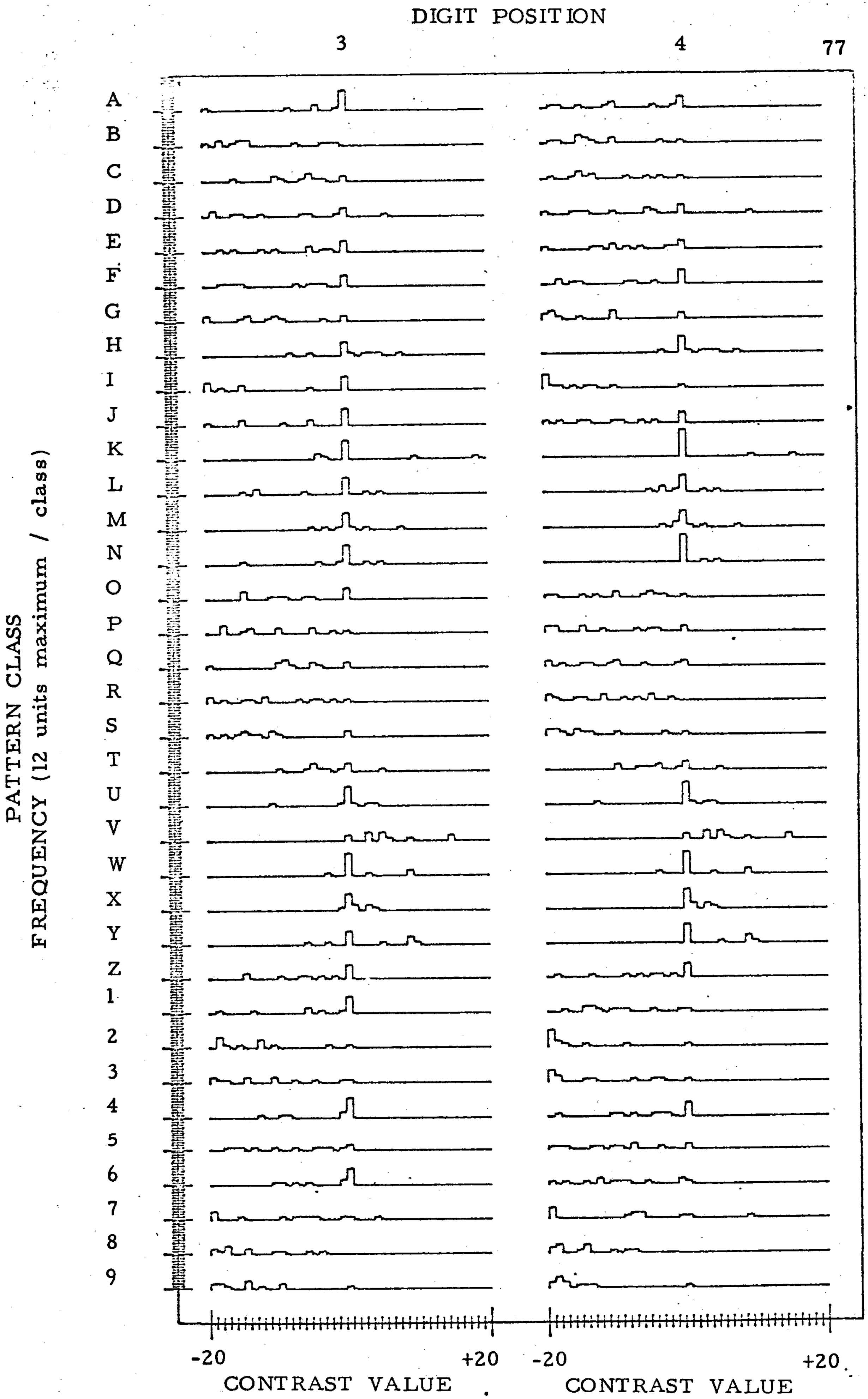
M	N	M	N	M	N
0000	000	*0001	00-1	0002	00-2
1000	111	*1001	110	1002	11-1
2000	222	2001	221	2002	220
0100	-100	*0101	-10-1	0102	-10-2
1100	011	*1101	010	1102	01-1
2100	122	2101	121	2102	120
0200	-200	0201	-20-1	0202	-20-2
1200	-111	1201	-110	1202	-11-1
2200	022	2201	021	2202	020
0010	0-10	*0011	0-1-1	0012	0-1-2
1010	101	*1011	100	1012	10-1
2010	212	2011	211	2012	210
0110	-1-10	*0111	-1-1-1	0112	-1-1-2
1110	001	*1111	000	1112	00-1
2110	112	*2111	111	2112	110
0210	-2-10	0211	-2-1-1	0212	-2-1-2
1210	-101	*1211	-100	1212	-10-1
2210	012	*2211	011	2212	010
0020	0-20	0021	0-2-1	0022	0-2-2
1020	1-11	1021	1-10	1022	1-1-1
2020	202	2021	201	2022	200
0120	-1-20	0121	-1-2-1	0122	-1-2-2
1120	0-11	*1121	0-10	1122	0-1-1
2120	102	*2121	101	2122	100
0220	-2-20	0221	-2-2-1	0222	-2-2-2
1220	-1-11	*1221	-1-10	1222	-1-1-1
2220	002	*2221	001	*2222	000

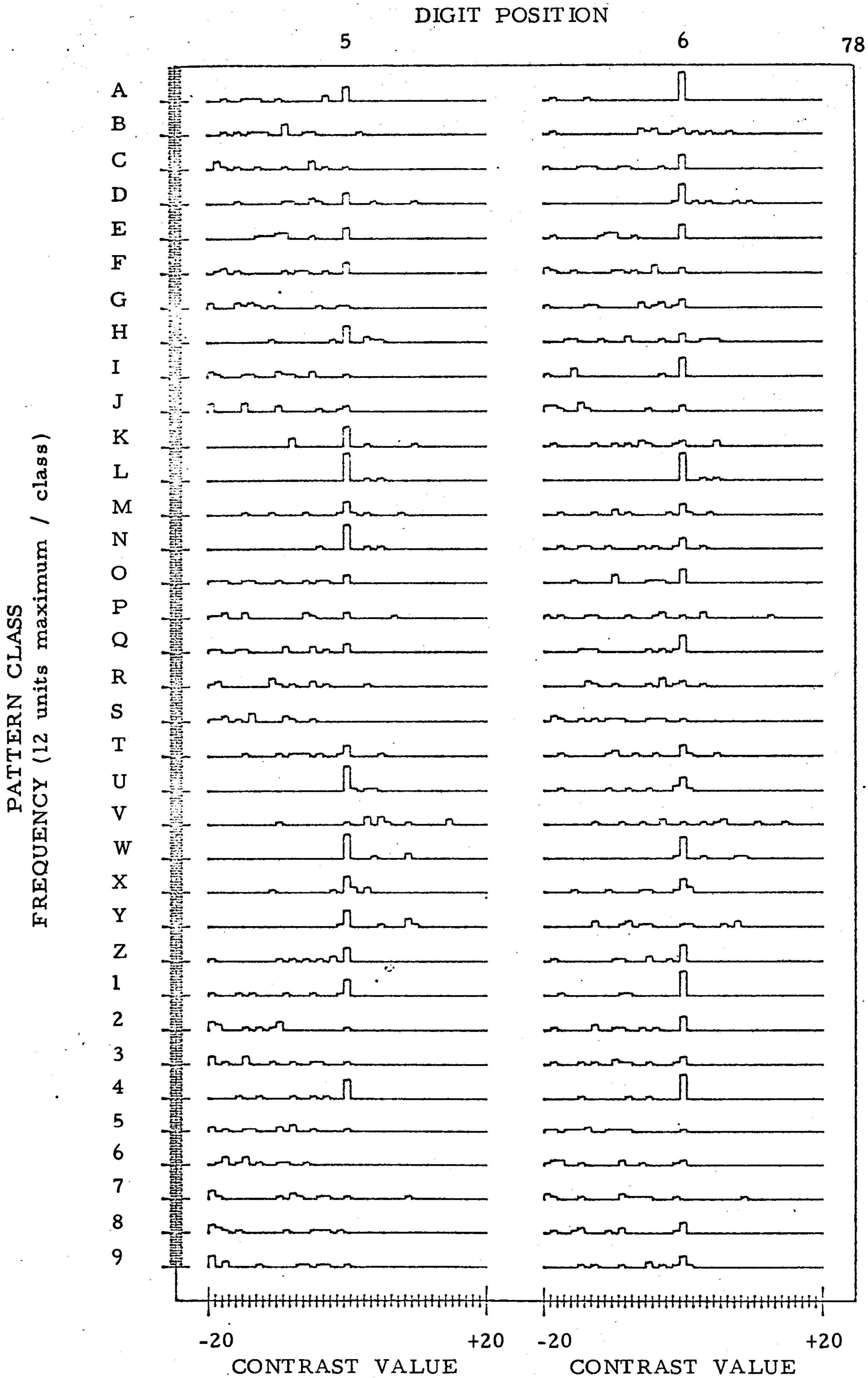
APPENDIX D

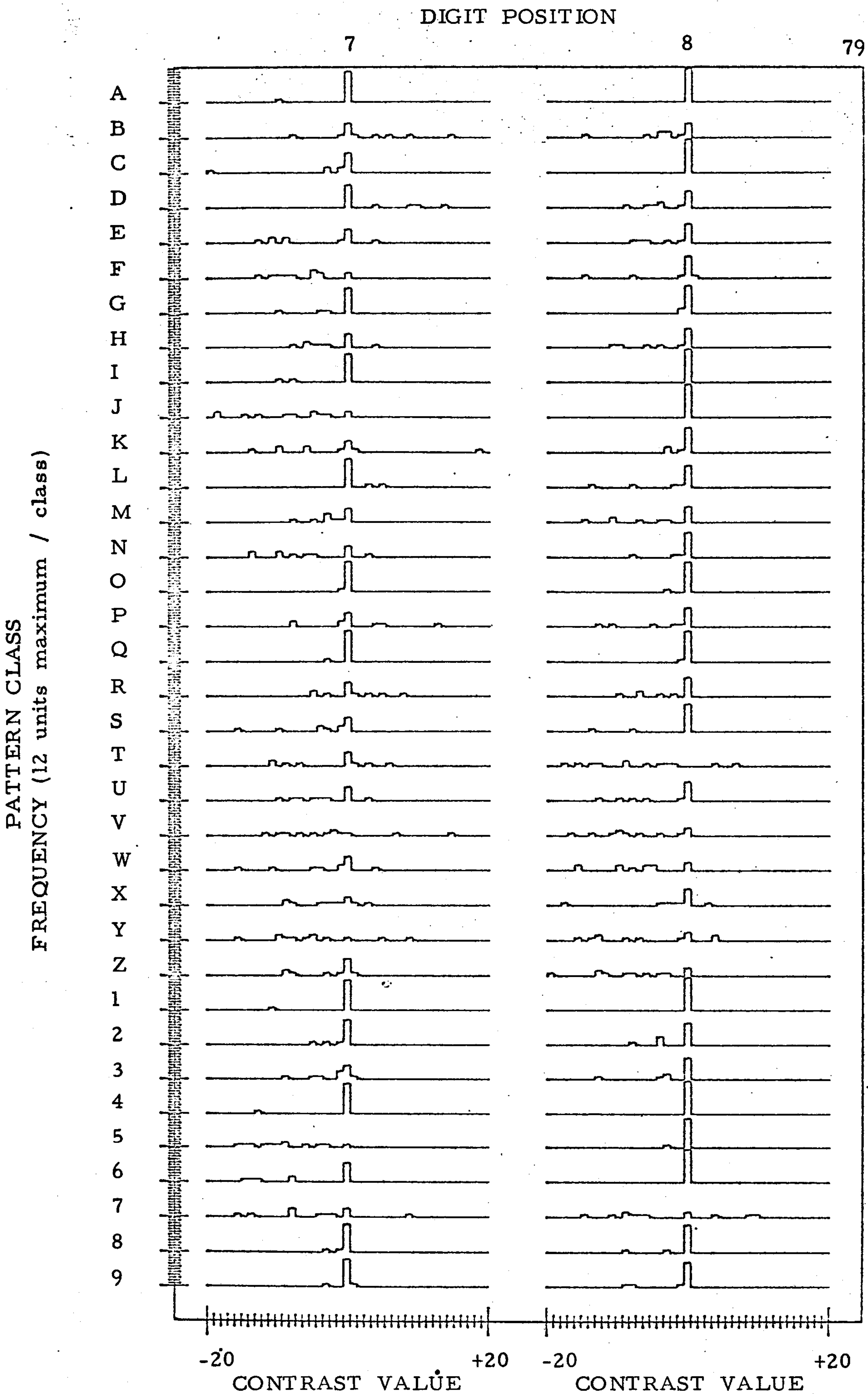
PATTERN CLASS FREQUENCY DISTRIBUTION

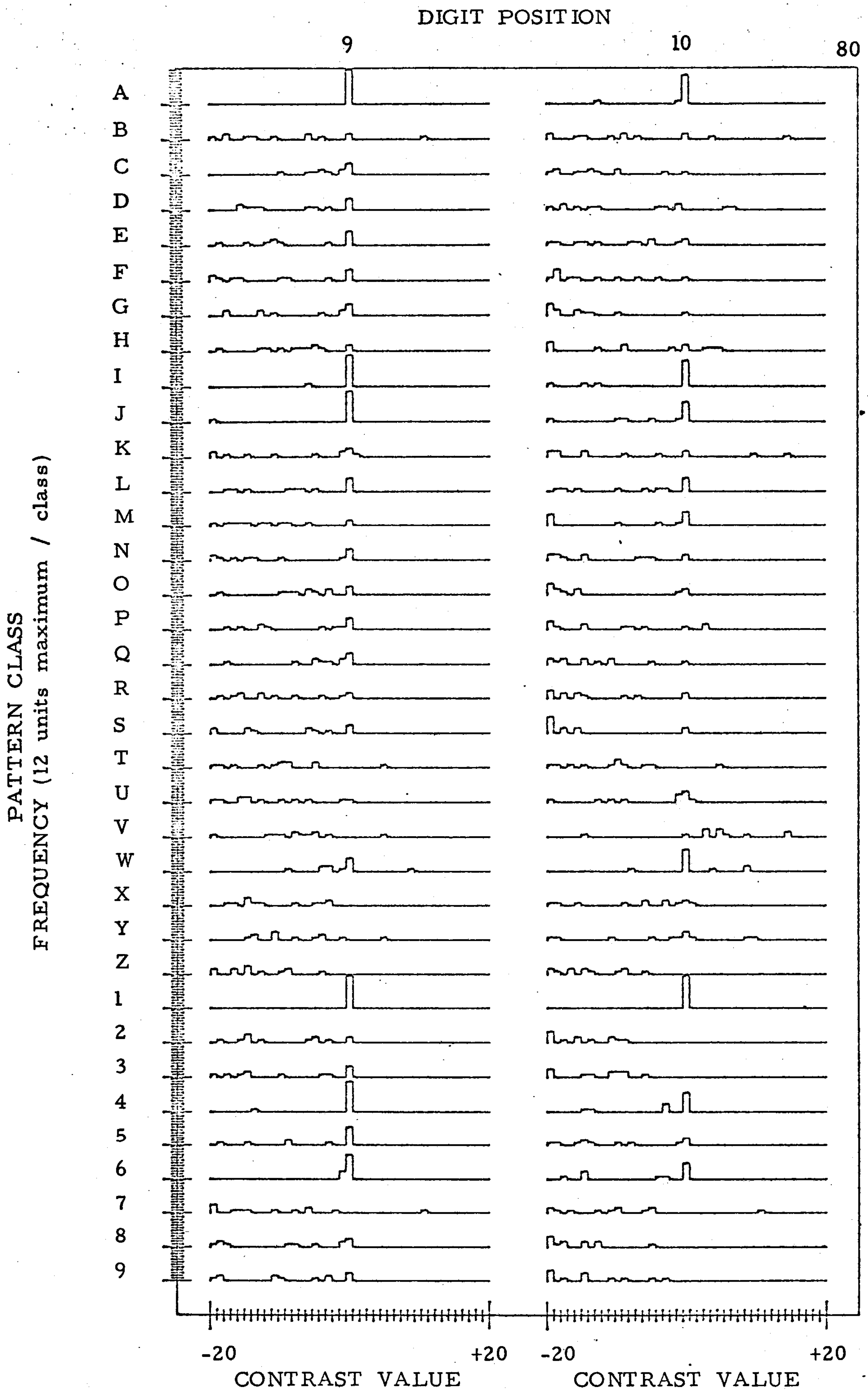
The curves on the following pages represent the statistics gathered from twelve samples of each of thirty-five pattern classes. Each column corresponds to a particular digit position. The frequencies of occurence of forty-one contrast values (ranging from -20 to +20) are represented in each curve. Of the seventy-nine digit positions which define an N - sequence, fifty-two are shown.

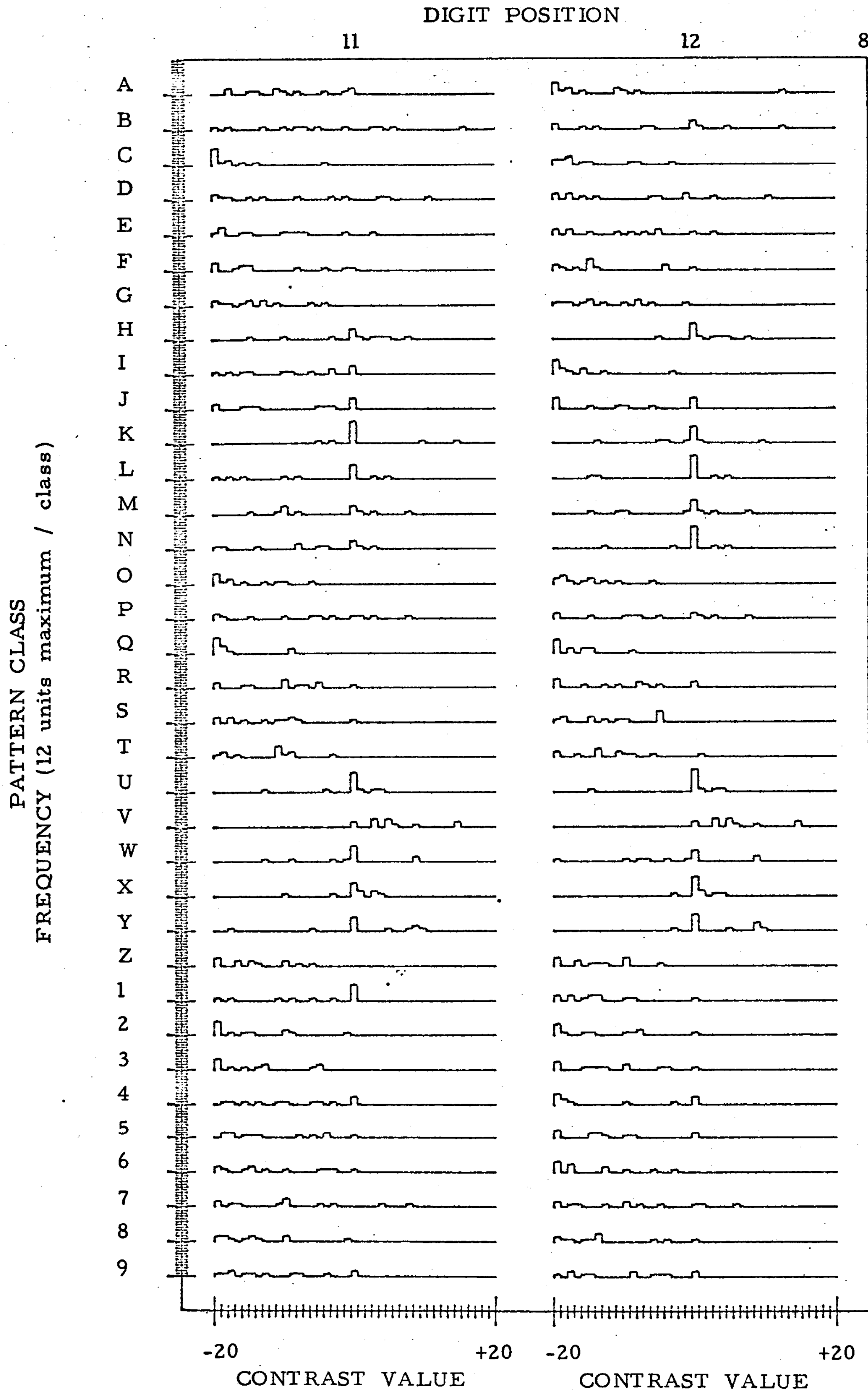

DIGIT POSITION
1
2
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE

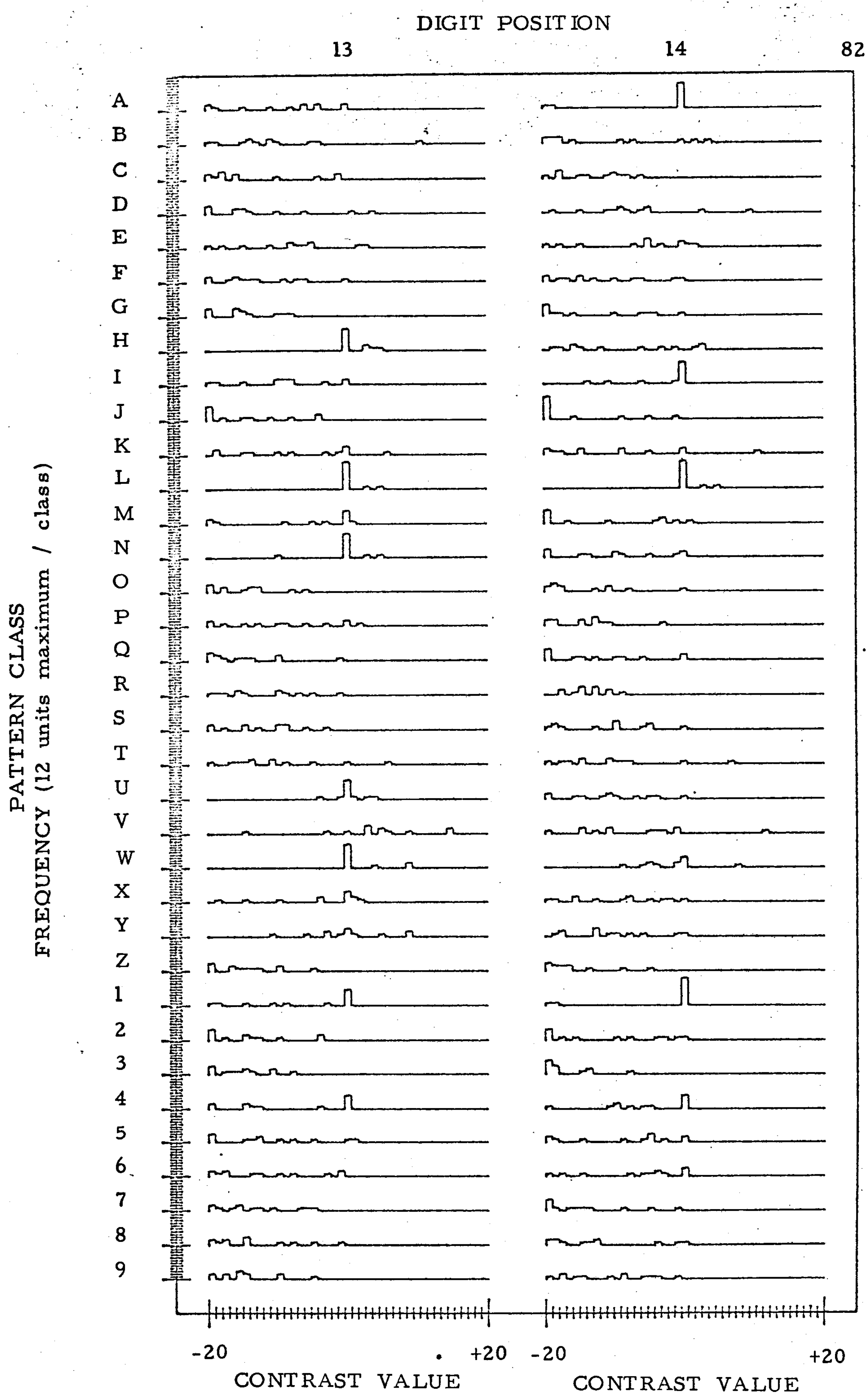

DIGIT POSITION
3
4
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE

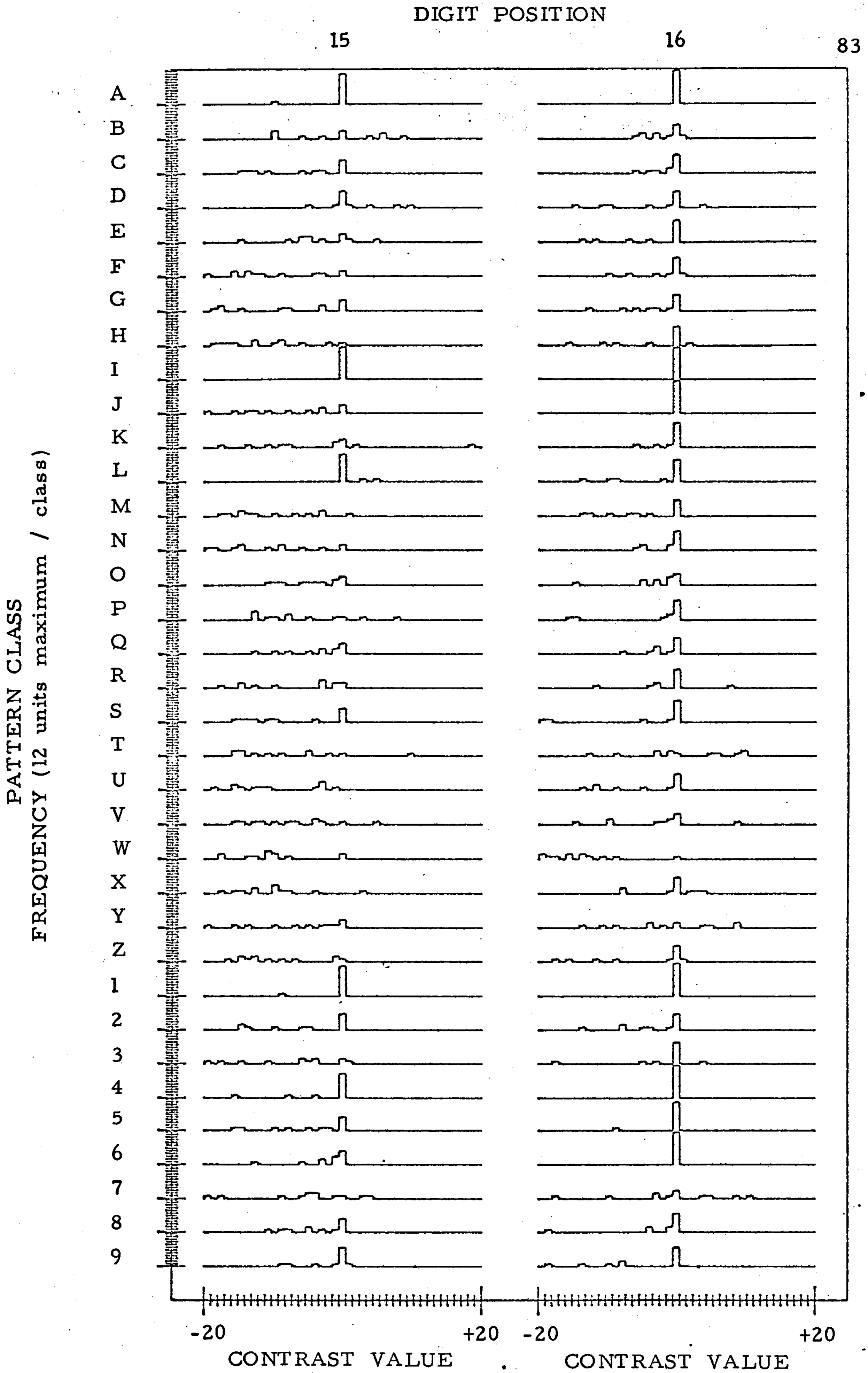

DIGIT POSITION
5
6
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE


DIGIT POSITION
7
8
79
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
CONTRAST VALUE
-20
+20
CONTRAST VALUE


DIGIT POSITION
9
10
80
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
CONTRAST VALUE
-20
+20
CONTRAST VALUE


DIGIT POSITION
11
12
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE


DIGIT POSITION
13
14
82
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE


DIGIT POSITION
15
16
PATTERN CLASS
FREQUENCY (12 units maximum / class)
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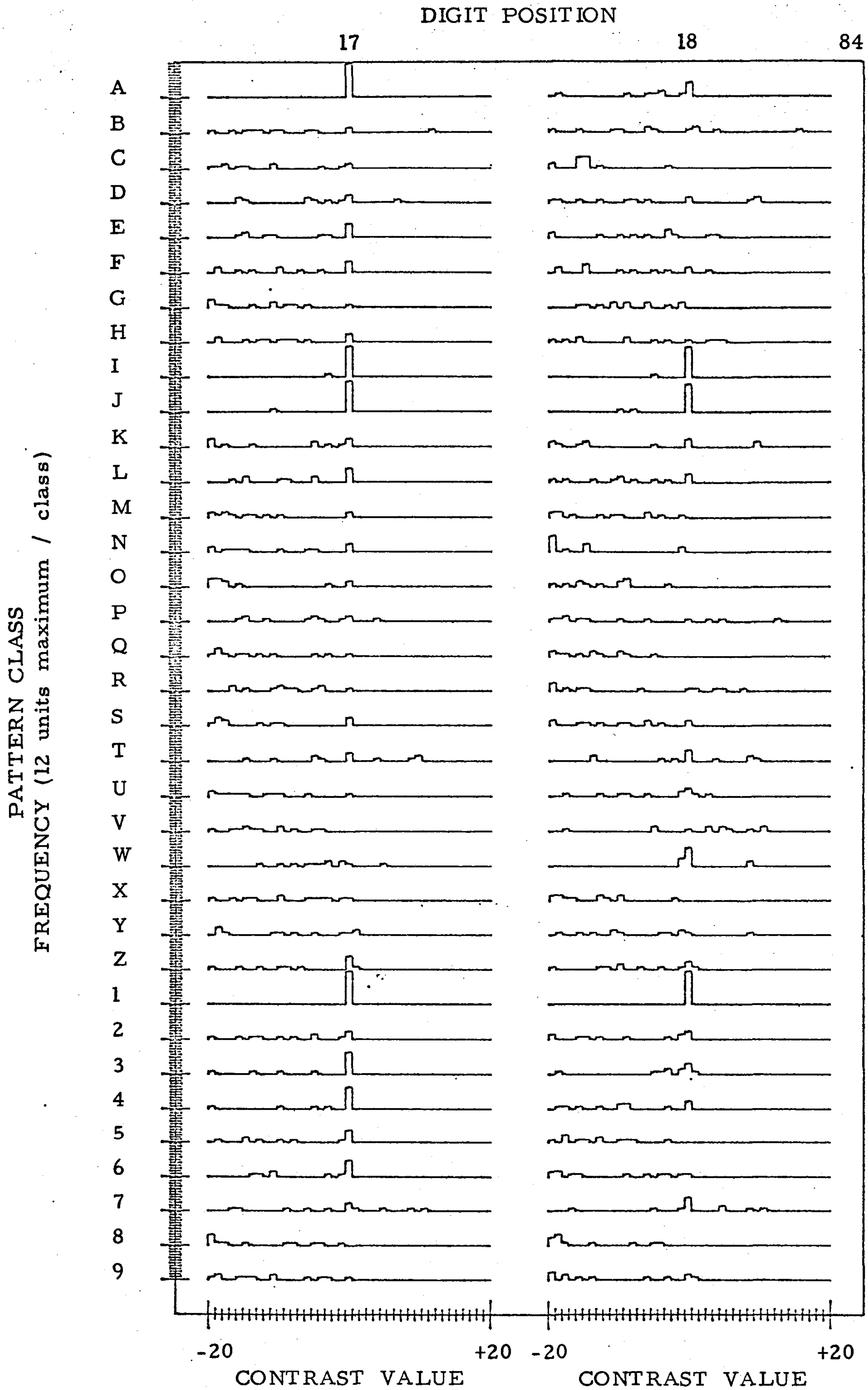
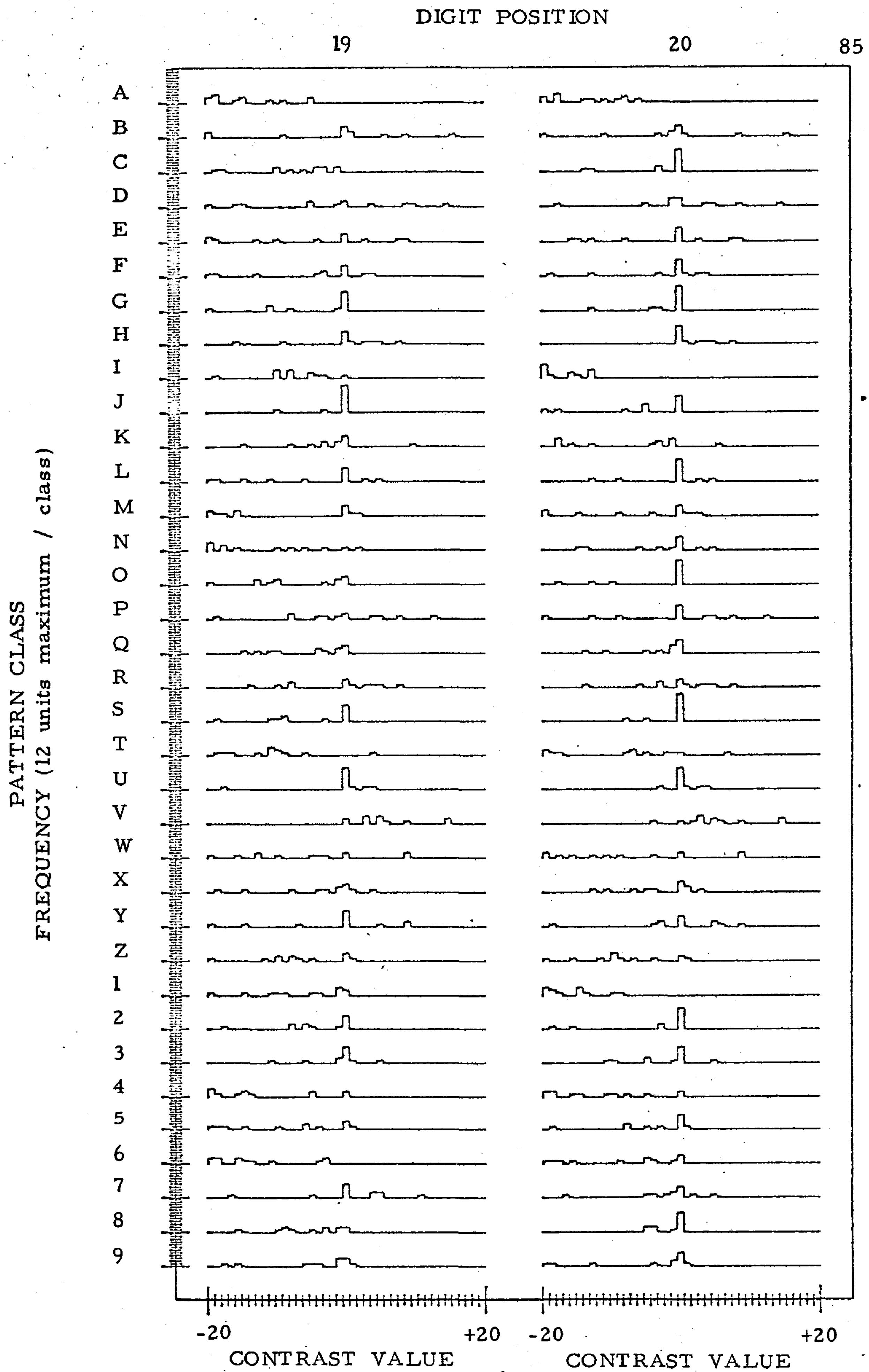
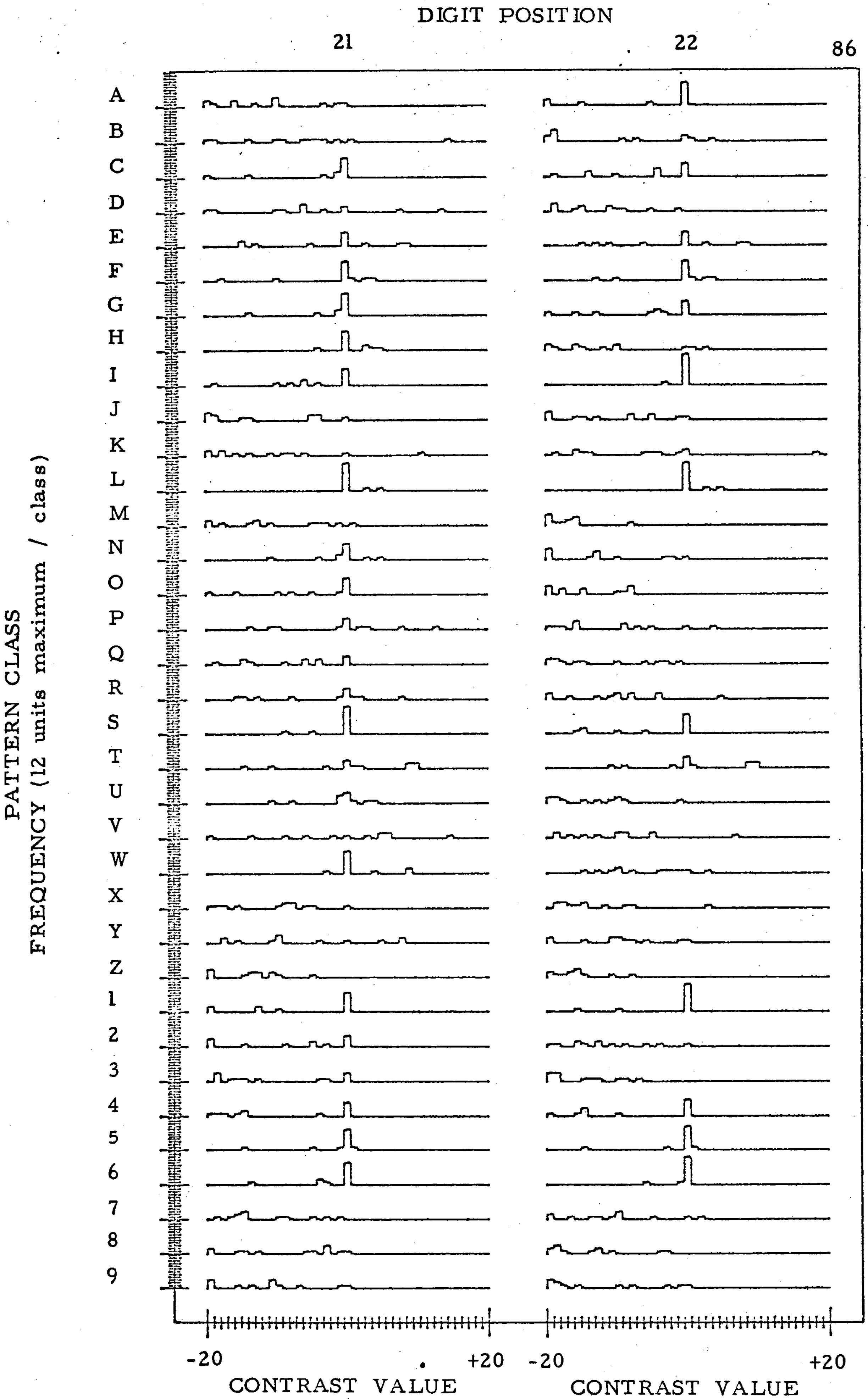
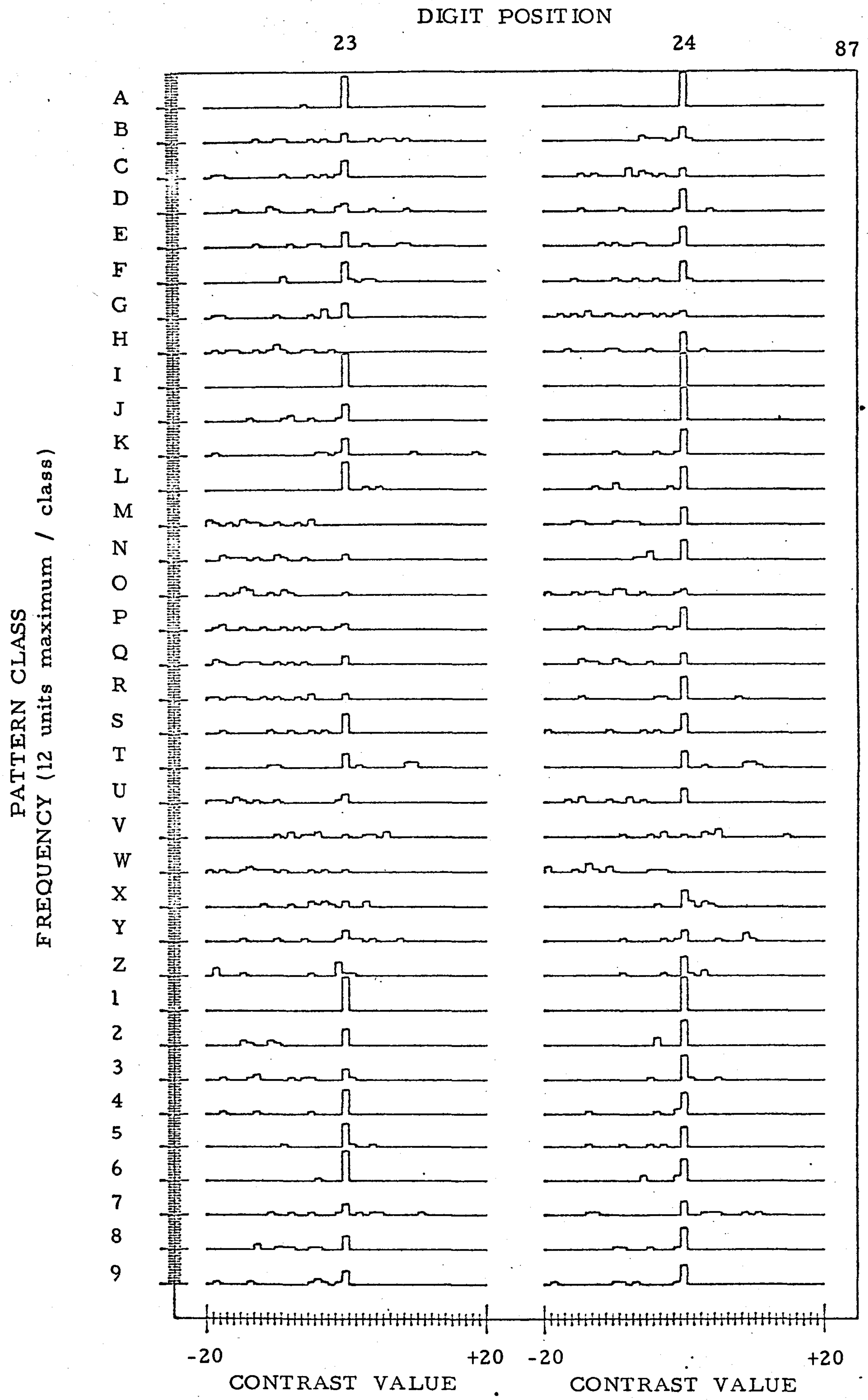
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE

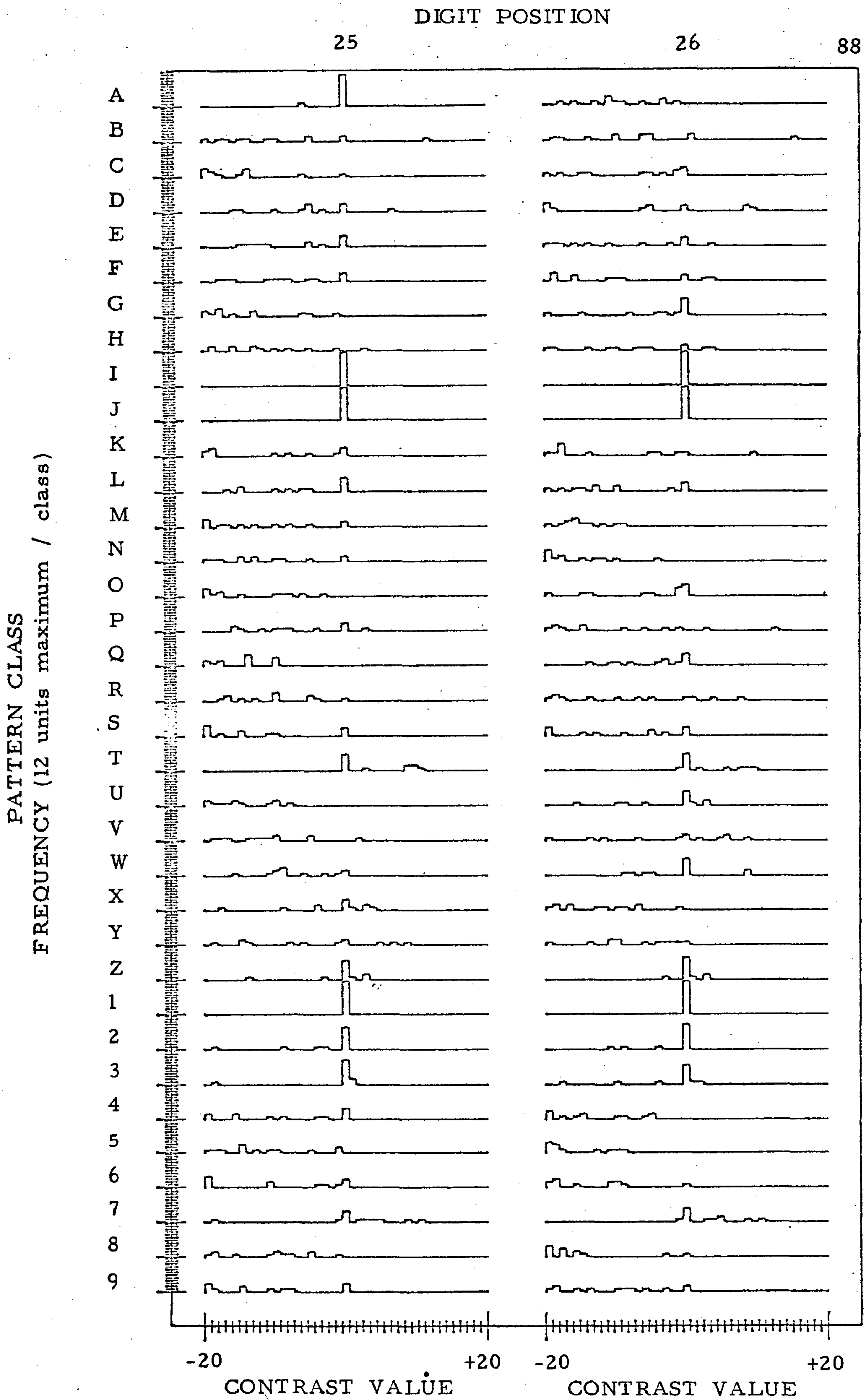


DIGIT POSITION
17
18
84
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE

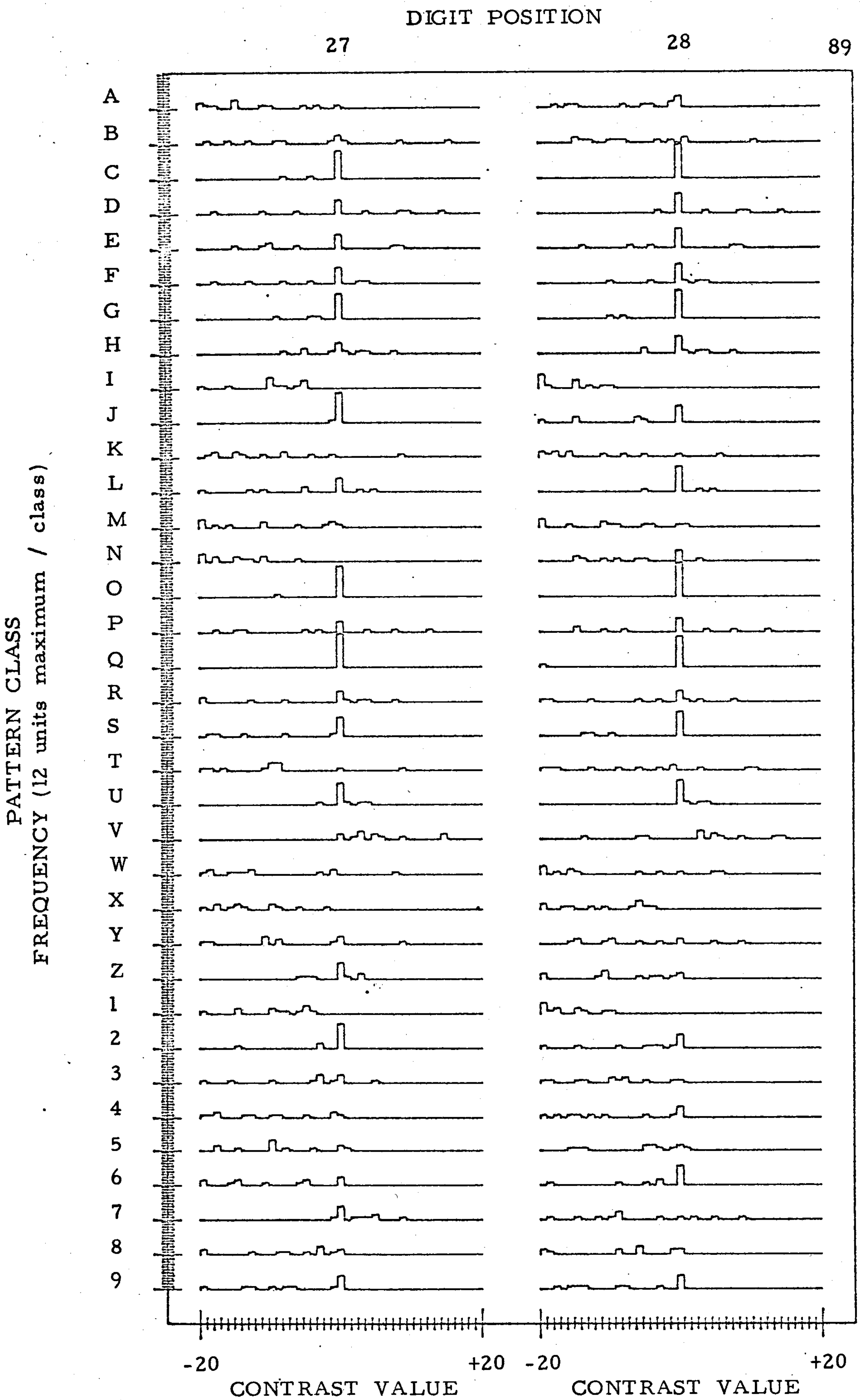

DIGIT POSITION
19
20
85
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
CONTRAST VALUE
-20
+20
CONTRAST VALUE


DIGIT POSITION
21
22
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE


DIGIT POSITION
23
24
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
-20
+20
-20
+20
CONTRAST VALUE
CONTRAST VALUE


DIGIT POSITION
25
26
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
1
2
3
4
5
6
7
8
9
PATTERN CLASS
FREQUENCY (12 units maximum / class)
-20
+20
CONTRAST VALUE
-20
+20
CONTRAST VALUE

89



DIGIT POSITION
27
28
PATTERN CLASS
FREQUENCY (12 units maximum / class)
A
B
C
D
E
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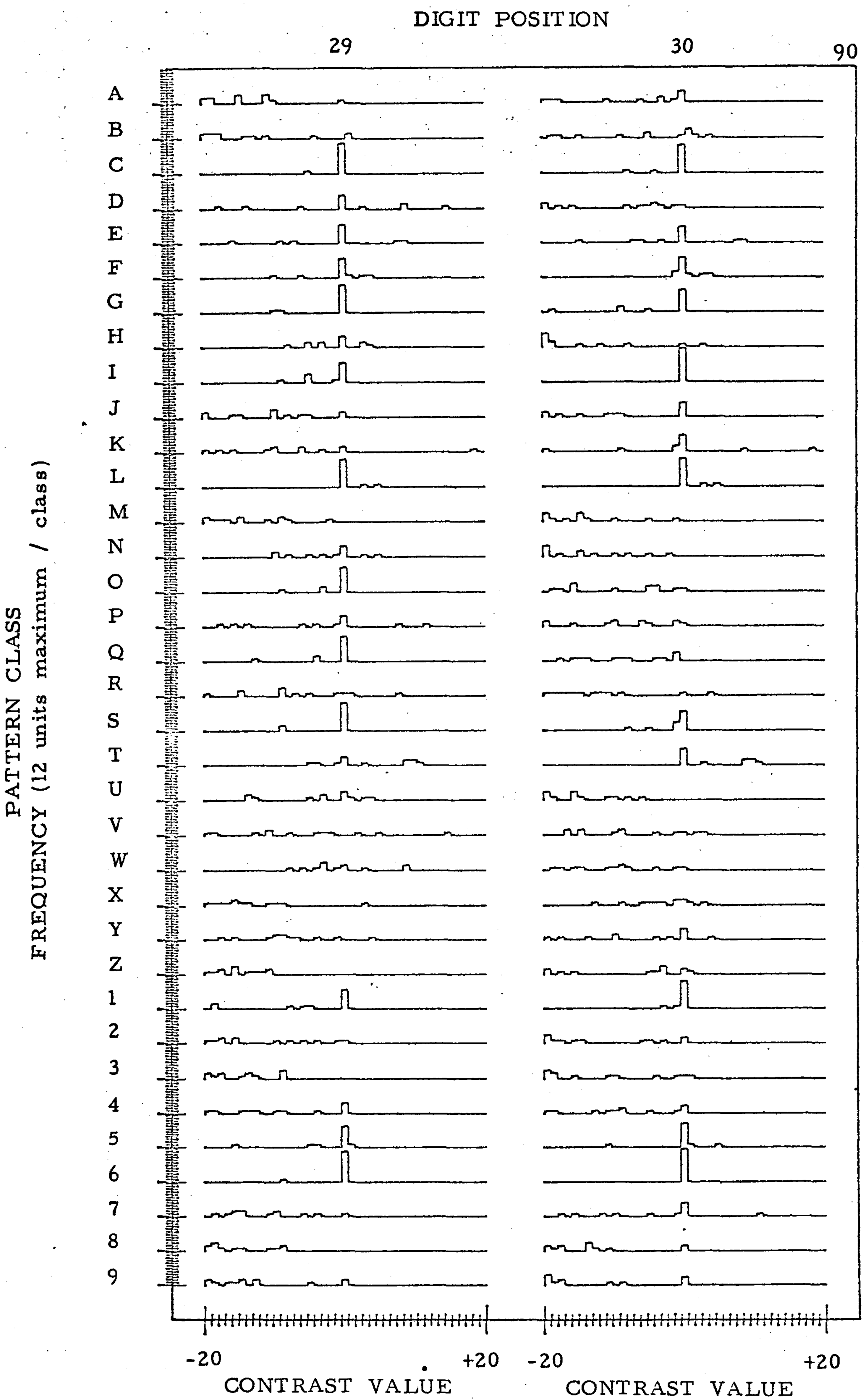
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29
30
90
PATTERN CLASS
FREQUENCY (12 units maximum / class)
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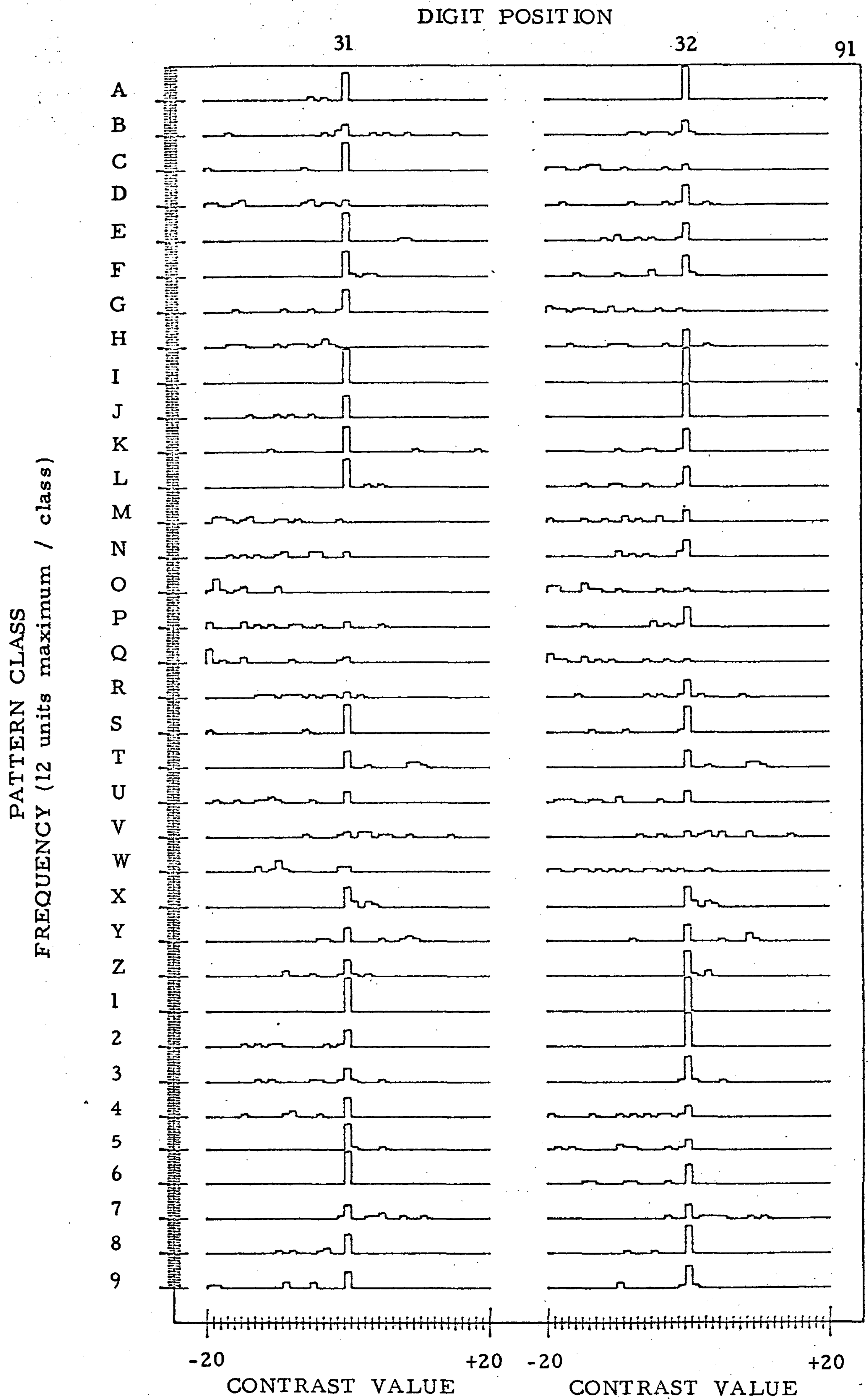
DIGIT POSITION
31
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91
PATTERN CLASS
FREQUENCY (12 units maximum / class)
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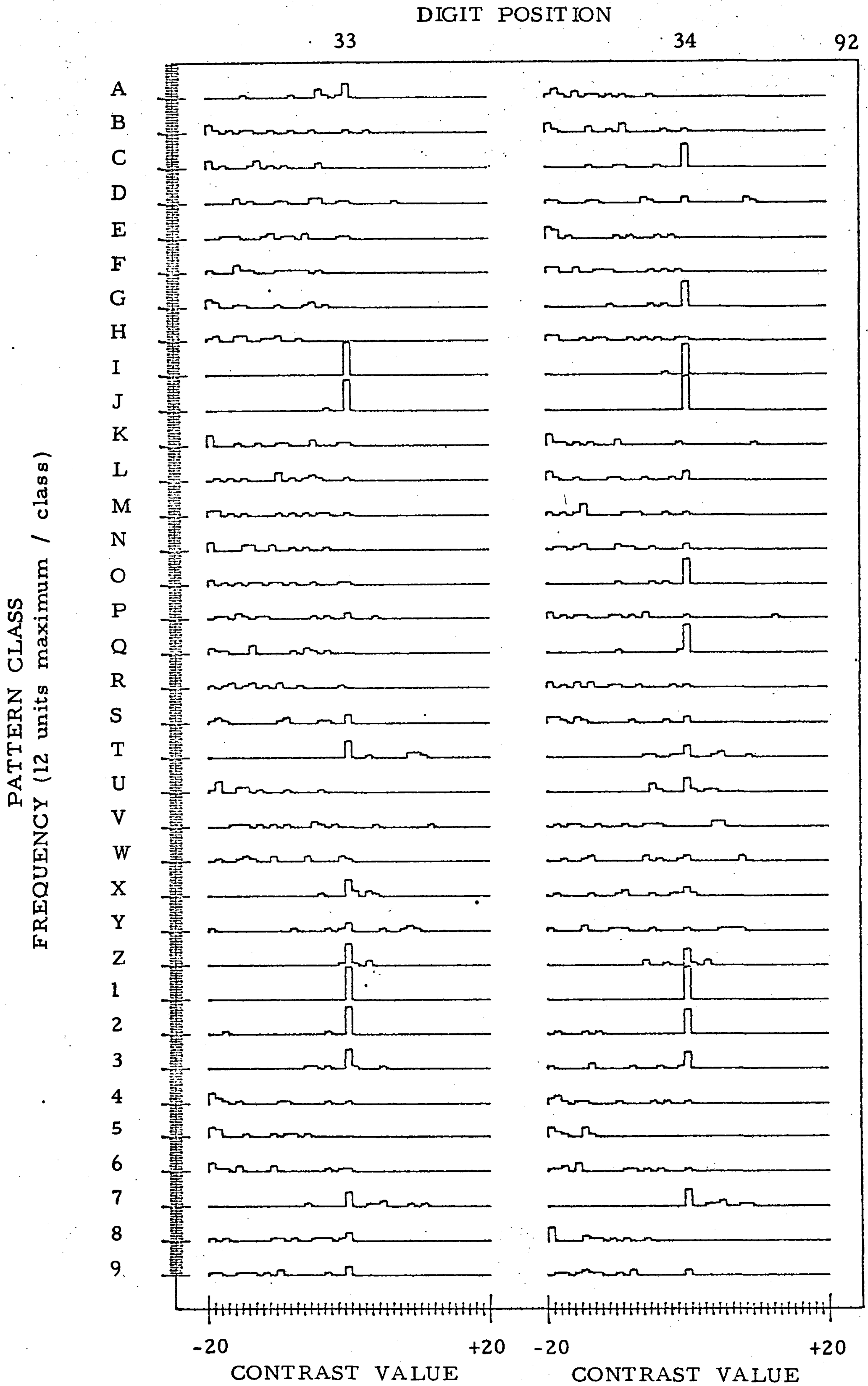
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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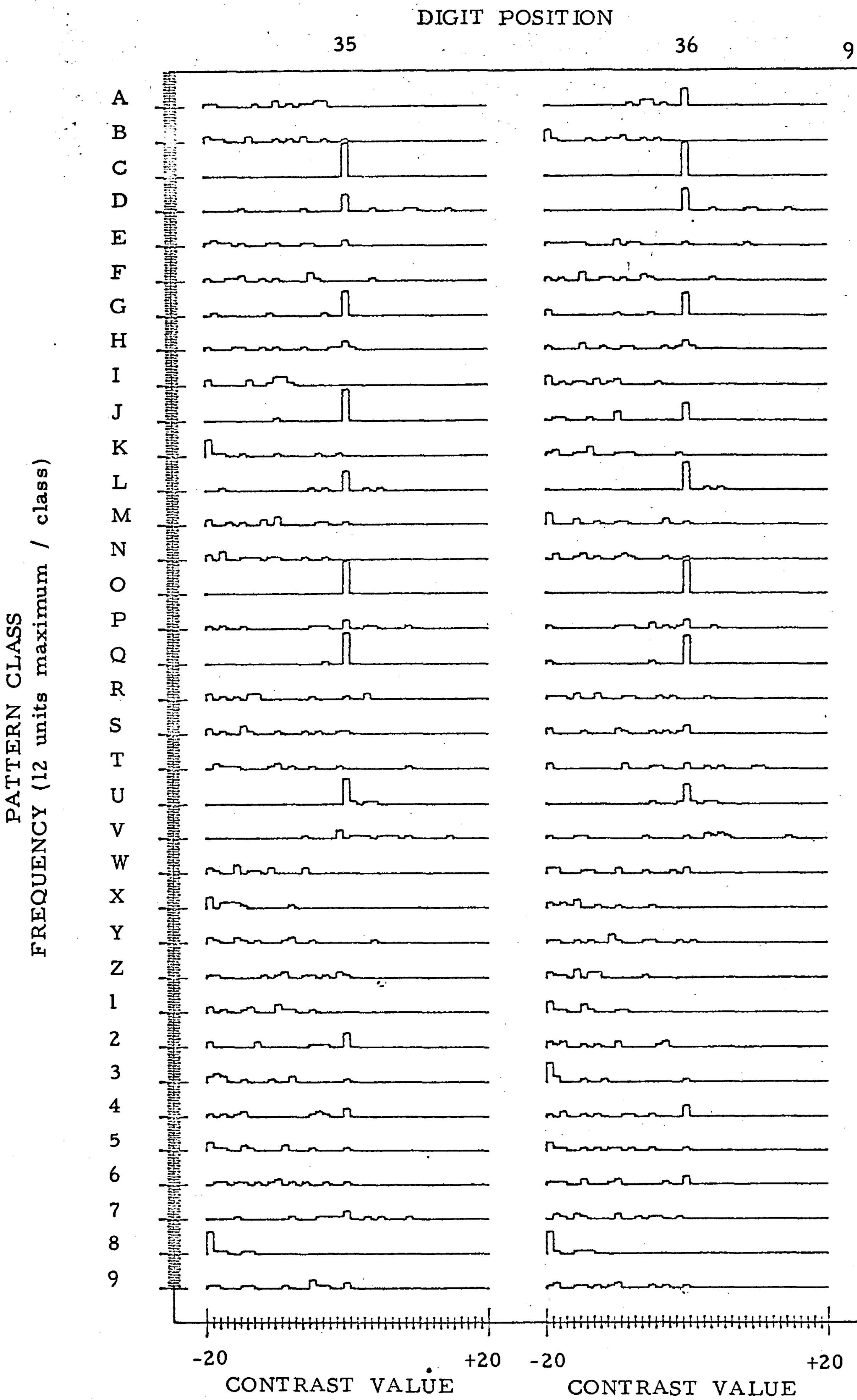
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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CONTRAST VALUE
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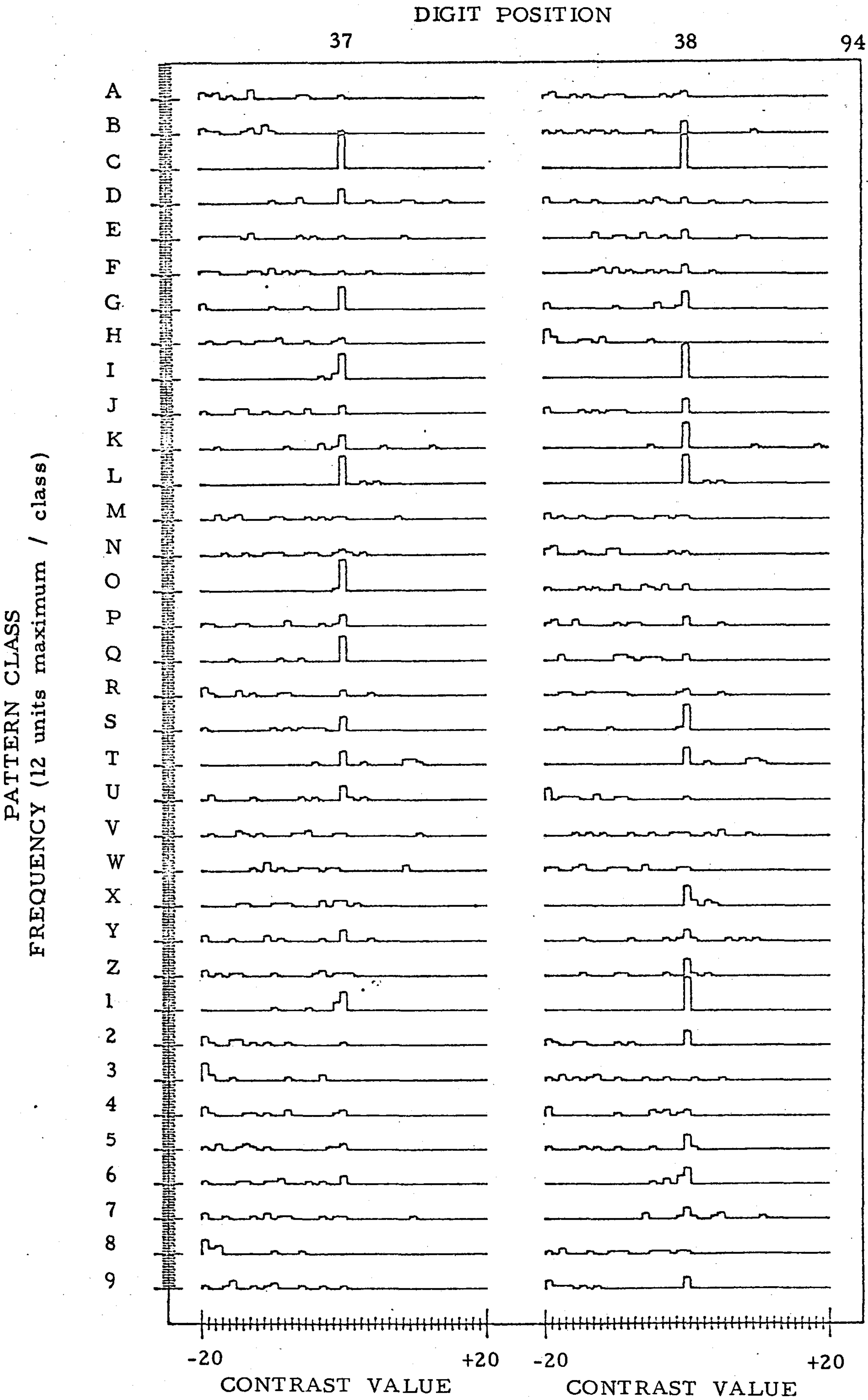
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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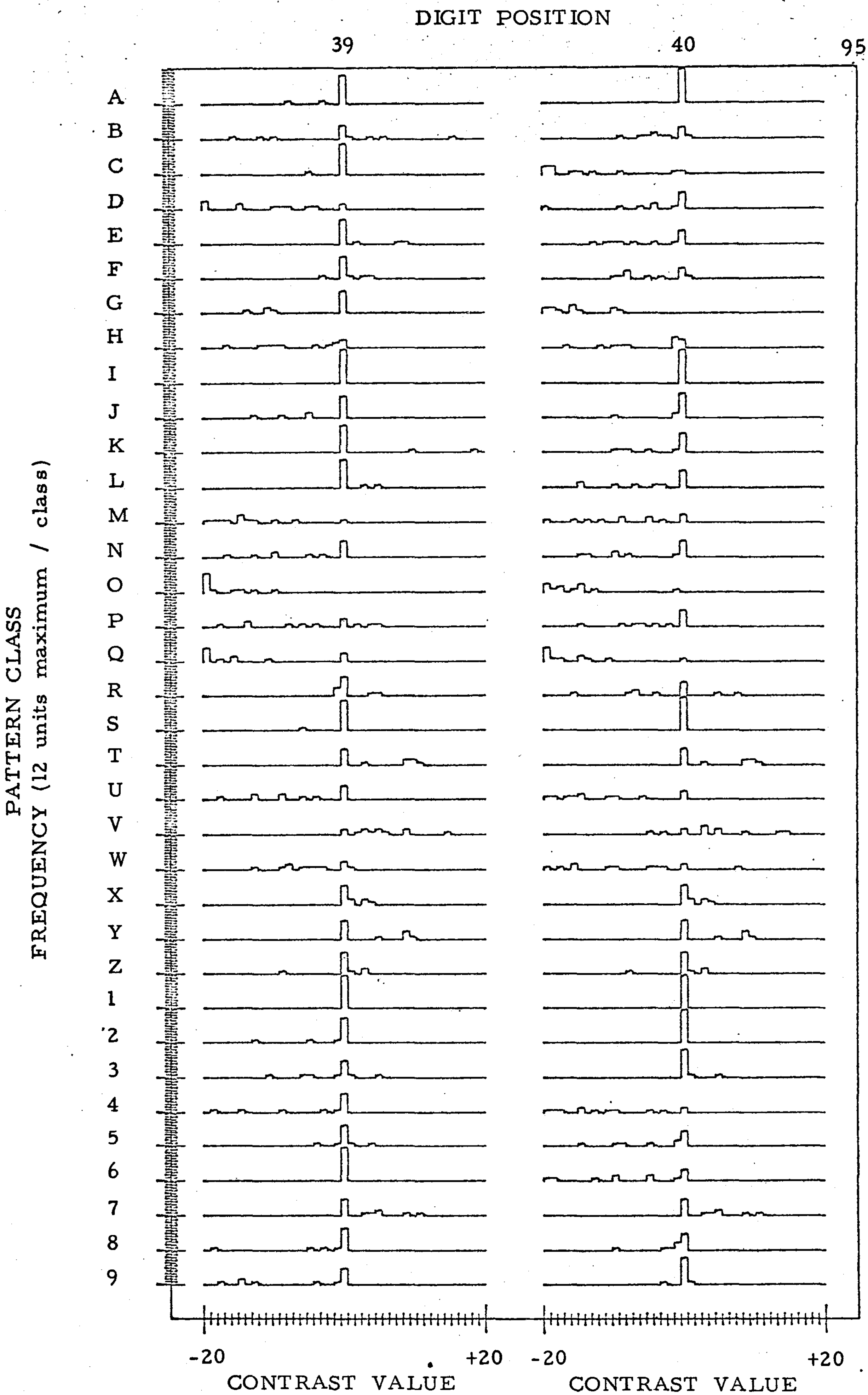
CONTRAST VALUE


DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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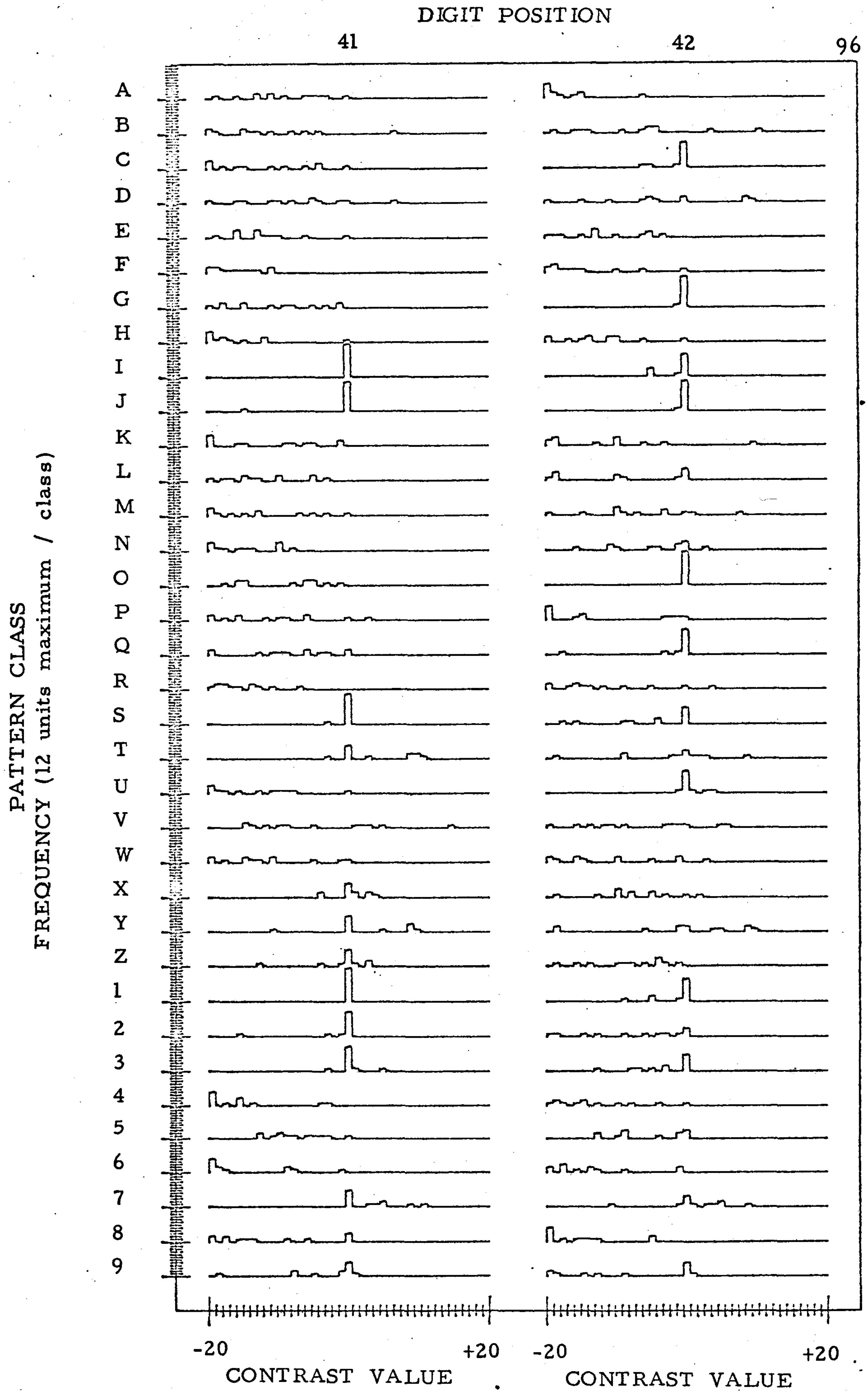
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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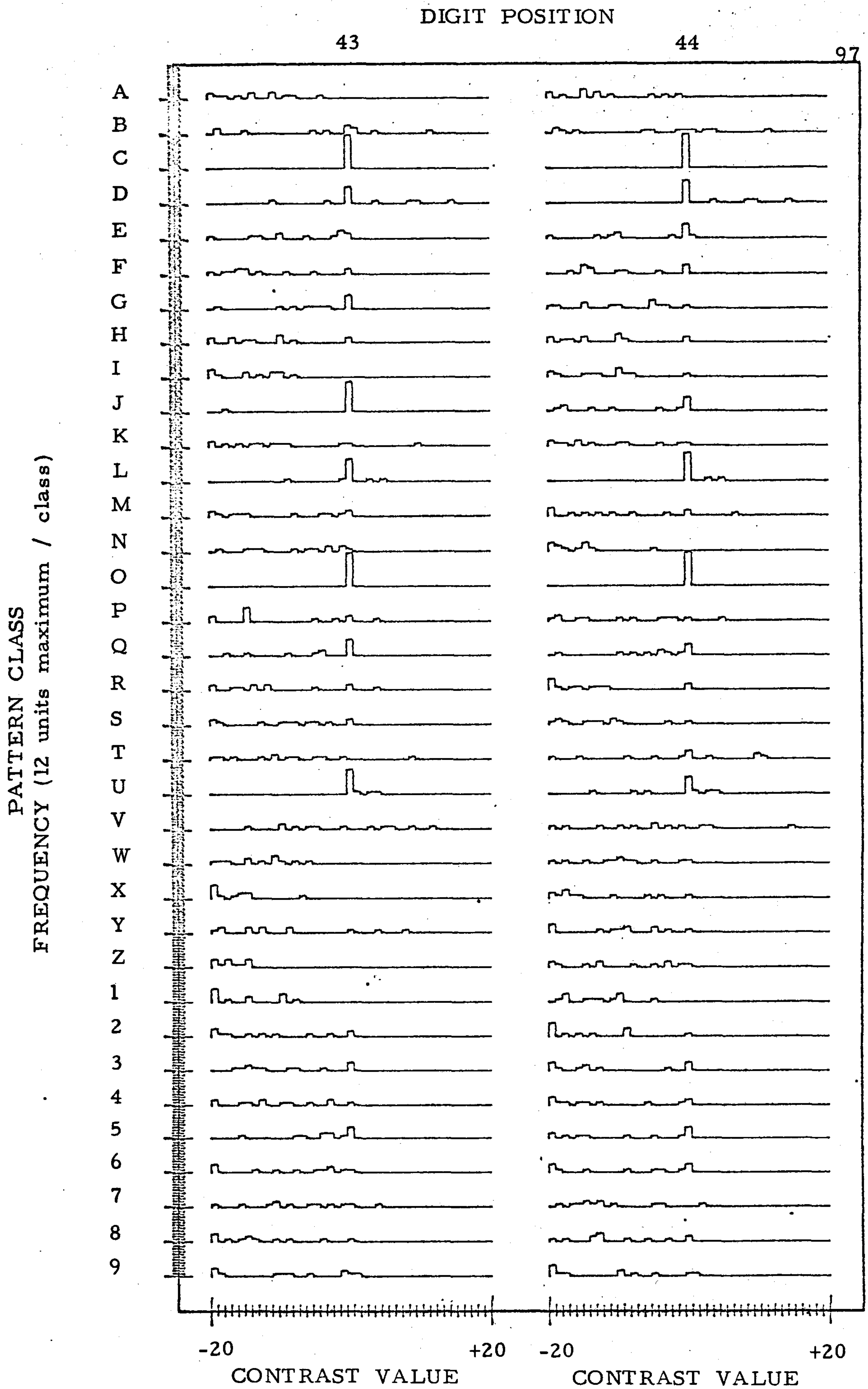
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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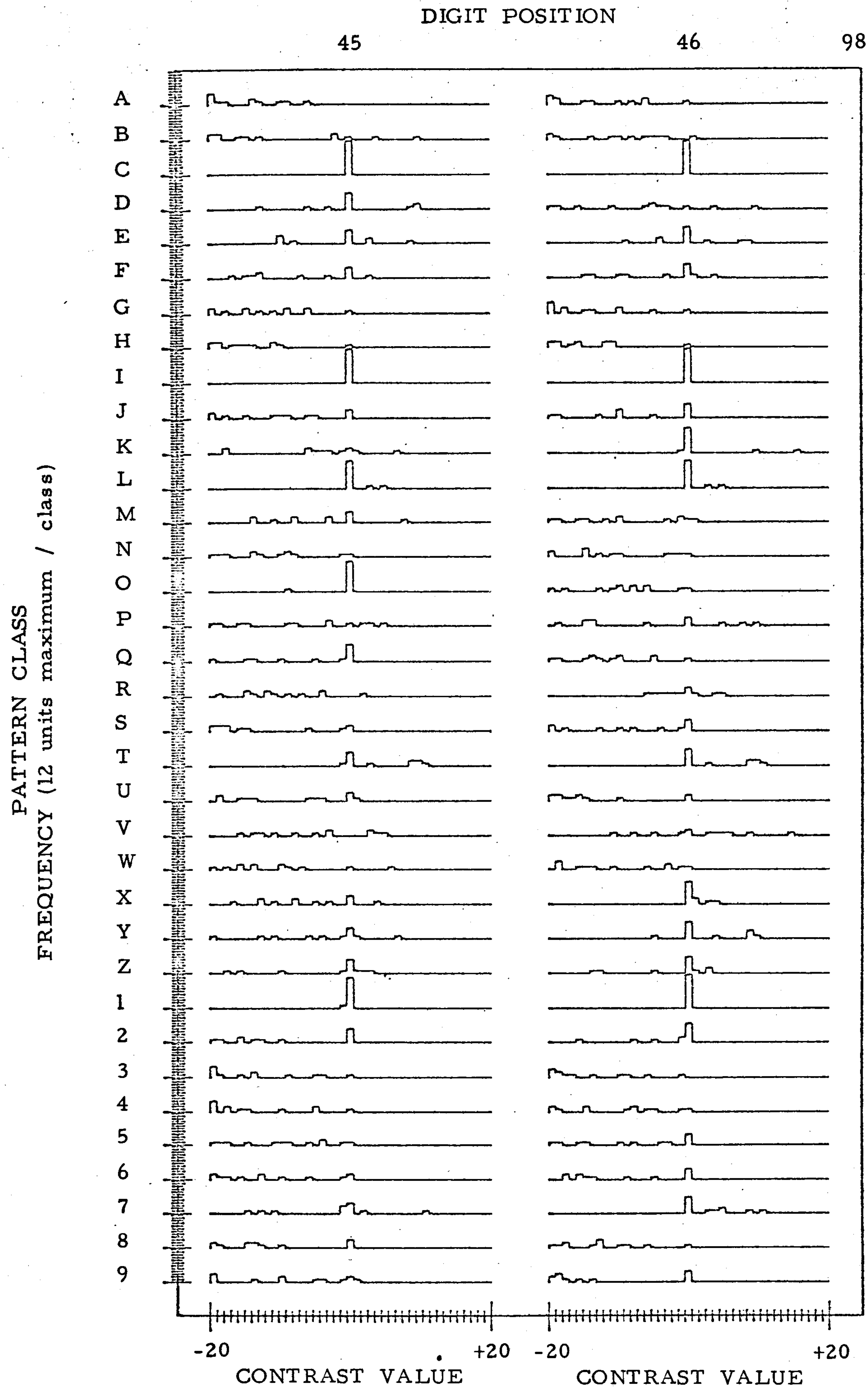
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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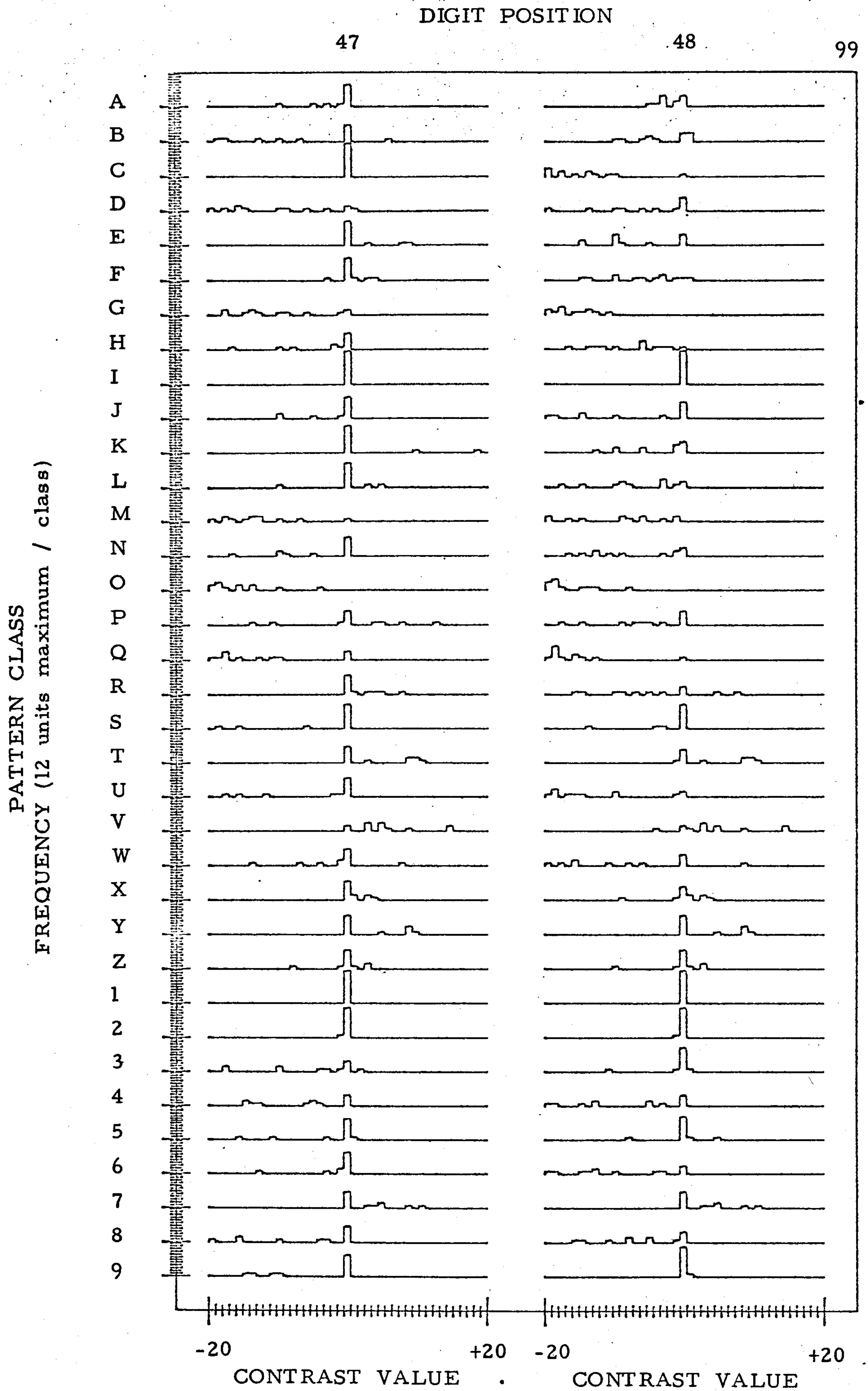
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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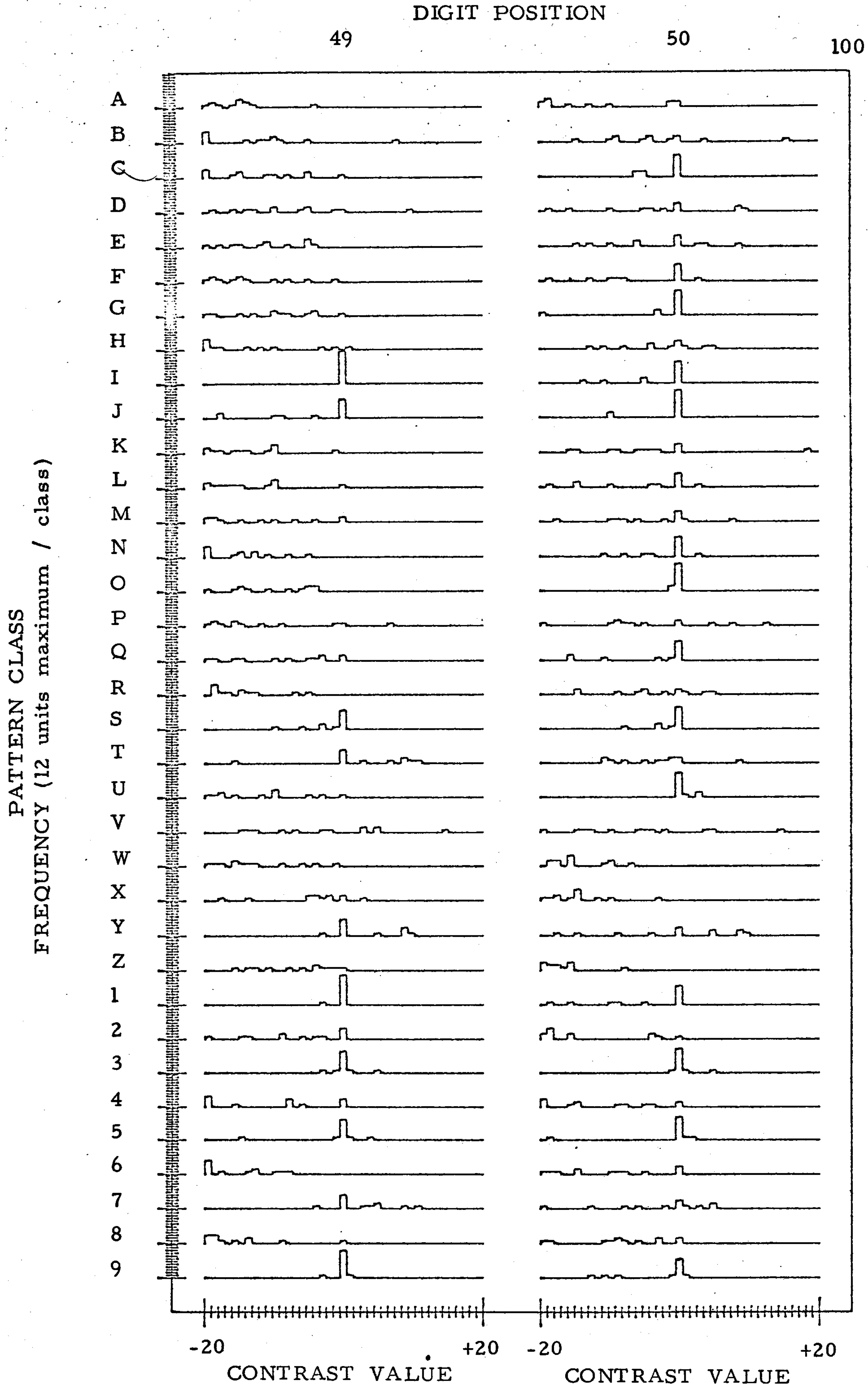
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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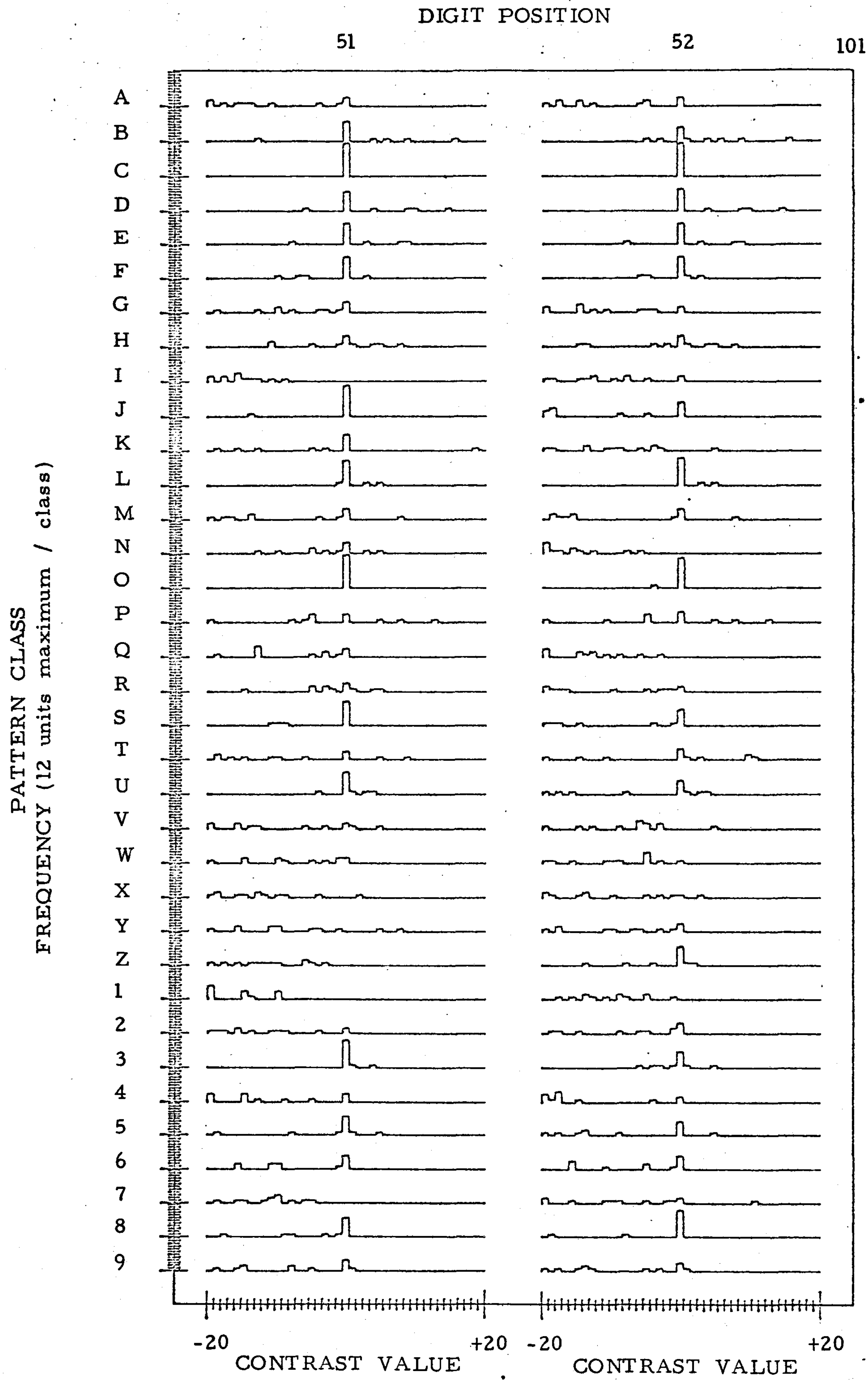
DIGIT POSITION
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PATTERN CLASS
FREQUENCY (12 units maximum / class)
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CONTRAST VALUE
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CONTRAST VALUE

VITA AUCTORIS

1943	Born on February 25, in Belleville, Ontario.
1956	Graduated from Central Public School, Windsor, Ontario.
1961	Completed Senior Matriculation at Hon. W. C. Kennedy Collegiate Institute, Windsor, Ontario.
1965	Graduated from the University of Windsor, Windsor, Ontario; B.A.Sc. in Engineering Science.
1966	Candidate for the degree of M.A.Sc. in Electrical Engineering (Interdisciplinary Studies in Communications) at the University of Windsor, Windsor, Ontario.