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CORRELATION ANALYSIS OF A PHASE LOCKED OSCILLATOR

by

MARAJ AHMAD

A Thesis sumbitted to the Faculty of Graduate Studies through the department of Electrical Engineering in partial fulfilment of the requirements for the degree of Master of Applied Science at the University of Windsor.

Windsor, Ontario

February, 1967

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ABSTRACT

Correlation analysis of a second order phase locked loop is presented. Under stable conditions amplitudes of the fundamental and the third harmonic component of phase, ϕ (radians) are derived for different sinusoidal frequency deviations for which continuous locking is maintained. Verification of these results with theoretical predictions by Dr. S.N. Kalra and P.H. Alexander is carried out.

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ACKNOWLEDGEMENTS

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CHAPTER I.

INTRODUCTION

Correlation functions are related in a quite natural way to time functions which carry information. If a time function carries information and the flow of information is uninterrupted, it is essential that the function be of such a nature that its variations from instant to instant are at least incompletely predictable as far as the receiver is concerned.

One assumption we make is that the time functions are physically of considerable duration so that theoretically they extend from the infinite past to the infinite future. Another assumption is that the statistical properties of these functions are invariant under a shift in the time origin, i.e. the functions are stationary in time. Messages and noise are regarded as stationary random processes and are described and characterised in terms of statistics and probability.

AUTOCORRELATION

For a large number of physical applications, the most useful characteristics of a stationary random process is its autocorrelation function. If $f_1(t)$ represents a member function of an ensemble, which represents the random process, the autocorrelation function is defined as

$$\phi_{11}(T) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t)\, f_1(t + T)dt \qquad (1)$$

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CROSSCORRELATION

The crosscorrelation between two functions $f_1(t)$ and $f_2(t)$ is defined as

$$\emptyset_{12}(T) = \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{+\infty} f_1(t) f_2(t + T)\, dt \qquad (2)$$

The crosscorrelation function for two incoherent functions, for example speech and resistor noise, is a constant or zero.

PERIODIC FUNCTIONS

For periodic functions we need not consider the ensemble average and the duration over which the function is considered need be only one complete cycle of the functions. Also we know from geometric considerations that $\emptyset_{11}(T) = \emptyset_{11}(-T)$ [5], hence for periodic functions

$$\emptyset_{11}(T) = \frac{1}{T_1} \int_0^{T_1} f_1(t)\, f_1(t + T)\, dt \qquad (3)$$

and for periodic functions $f_1(t)$, $f_2(t)$ of the same fundamental frequency crosscorrelation is defined as

$$\emptyset_{12}(T) = \frac{1}{T_1} \int_0^{T_1} f_1(t)\, f_2(t + T)\, dt \qquad (4)$$

where T_1 is the complete period of $f_1(t)$ and $f_2(t)$.

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Phase Locked Oscillator

A phase locked oscillator is a tuneable electronic power source incorporated into a feedback loop. One such oscillator has been developed at the University of Windsor and is used to detect a FM signal in the presence of an appreciable amount of noise power. Phase locking is accomplished by sensing the difference in phase between the signal and the variable frequency local oscillator output and this difference is used to synchronize the controlled phase with the signal phase. A measure of the phase difference is obtained by means of a phase comparator or phase discriminator which produces an output voltage (error function) dependent on the phase difference between the inputs.

It is the purpose of this paper to present correlation data of signal and error functions using the correlation computer. The data is used to analyse the non-linear characteristics of the phase locked oscillator.

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CHAPTER II.

CORRELATOR COMPUTER

The correlator computer performs three operations, namely displacement, multiplication and integration. Fig. 2 is a block diagram of the correlator computer. It consists of a lumped constant delay line with a delay range of 0-400 microseconds in one microsecond steps. This pertains to displacement. Balanced modulators (Fig. 3) in conjunction with the amplifiers (Fig.4) are used in the translation of the information from the signal to the intermediate frequency necessary for multiplier circuit input. The multiplier (Fig. 5) makes use of the properties of 6SA7 pentagrid converter as described by Synder [1] and his colleagues. The integrator and filter unit (Fig. 5) performs integration and eliminates undesired signal in the output. The output of this unit is compared with a d.c. voltage and the desired correlation function can be computed for different delay times.

BALANCED MODULATORS

A normal AM output has the form

$$F_c(t) = K(1 + mf(t)) \cos \omega_c t$$

where ω_c is the carrier frequency and $f(t)$ is the modulating signal.

The corresponding mathematical expression for a DSB output is

$$F_d(t) = Kf(t) \cos \omega_c t$$

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The DSB system differes from normal AM simply by suppressing the carrier term. This means that with no modulating signal applied the output should be zero.

One possible scheme for suppressing or balancing out the carrier term is shown in Fig. 3. This circuit uses 6SA7 pentodes. The carrier is applied in phase to grid 1 while the modulating signal is applied in opposite phase to grid 3 of the 6SA7 pentodes.

The two currents i_1 and i_2 may be written in the form

$$i_1 = K(1 + mf(t)) \cos \omega_c t$$

and

$$i_2 = K(1 - mf(t)) \cos \omega_c t$$

(The other terms generated in the non-linear process are assumed filtered out).

The effective output current is

$$i_e = i_1 - i_2 = Kf(t) \cos \omega_c t$$

Notice that the carrier has been suppressed in the output.

MULTIPLIER

The application of the multigrid vacuum tube in multipliers and squarers has been described in the literature [6]. However, we have employed a multigrid modulator as described by Synder and O'Meara[1]. The operation is dependent on linear characteristic curves, as a

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square law device must be inherently non-linear in its operation. It is basically a four quadrant device in the sense that algebraic sign information is preserved in the output. It is a phasor multiplier in that, given two inputs expressible by e_1 and e_2

$$e_1 = A_i\ e^{j(\omega t + \theta_1)}$$

$$e_2 = A_2\ e^{j(\omega t + \theta_2)}$$

The output is ideally expressible as e_o:

$$e_o = KA_1\ A_2\ e^{j(2\omega t + \theta_1 + \theta_2)} \qquad (5)$$

It is seen that the phase information is preserved in the output.

INTEGRATOR AND FILTER

This unit serves two purposes.

1. The undesired frequencies i.e. frequencies involving carrier frequency, are eliminated. Hence we require a low pass filter.
2. The output of the multiplier is of the form $f_1(t)\ f_2(t)\ \cos^2 \omega_m t$ where ω_m is the modulating angular frequency. The second operation that the unit performs is that of an integrator.

Integrating action can be understood from Fig. 1.

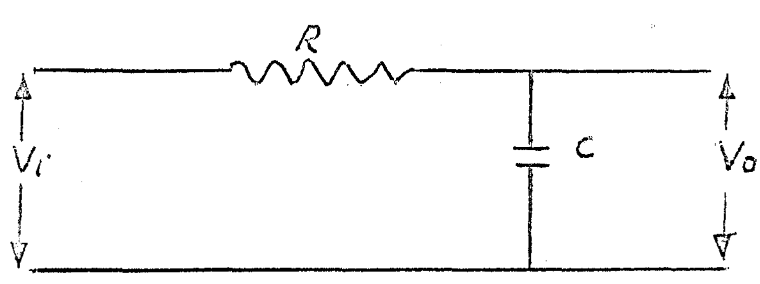

Fig. 1.

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$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{1/cs}{R + 1/cs} = \frac{1}{Rcs + 1}$$

Hence for $R \gg 1/cs$, the circuit acts as an integrator.

A simple RC π network as in Fig. 5 performs both these operations.

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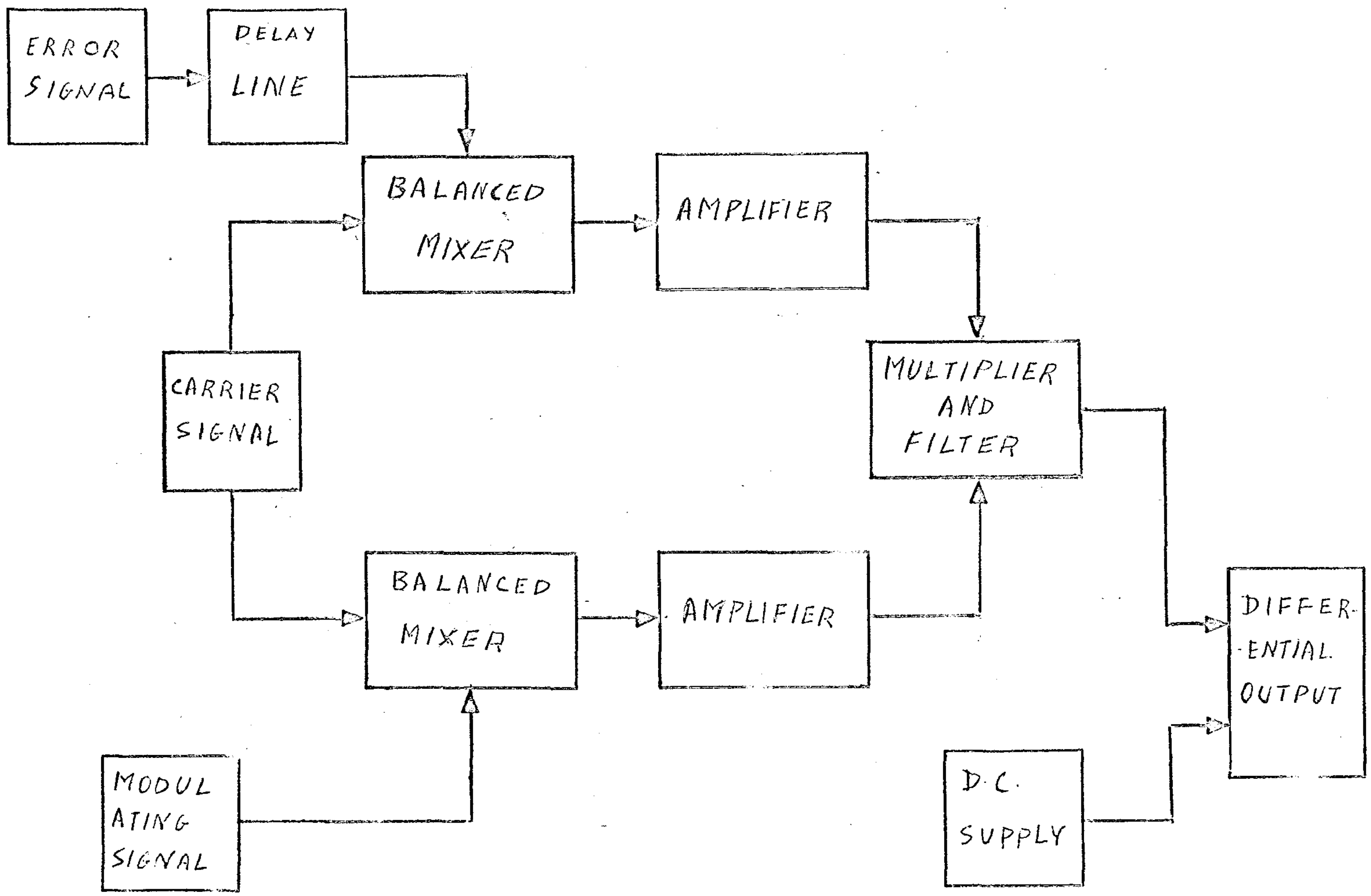

FIG. 2. BLCOK DIAGRAM FOR THE CORRELATOR COMPUTER.

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BALANCED MIXERS

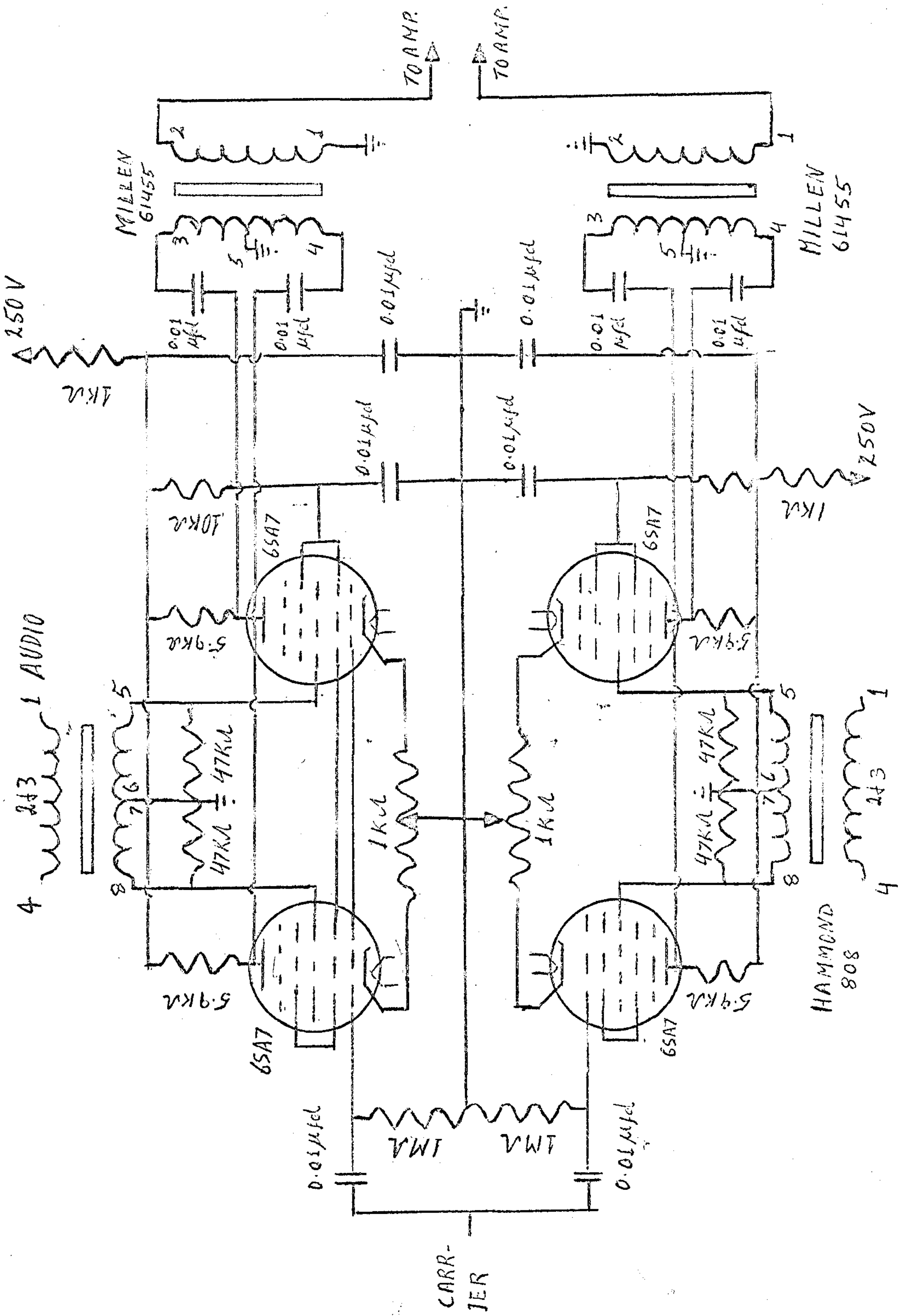

FIG. 3. BALANCED MIXERS

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AMPLIFIER

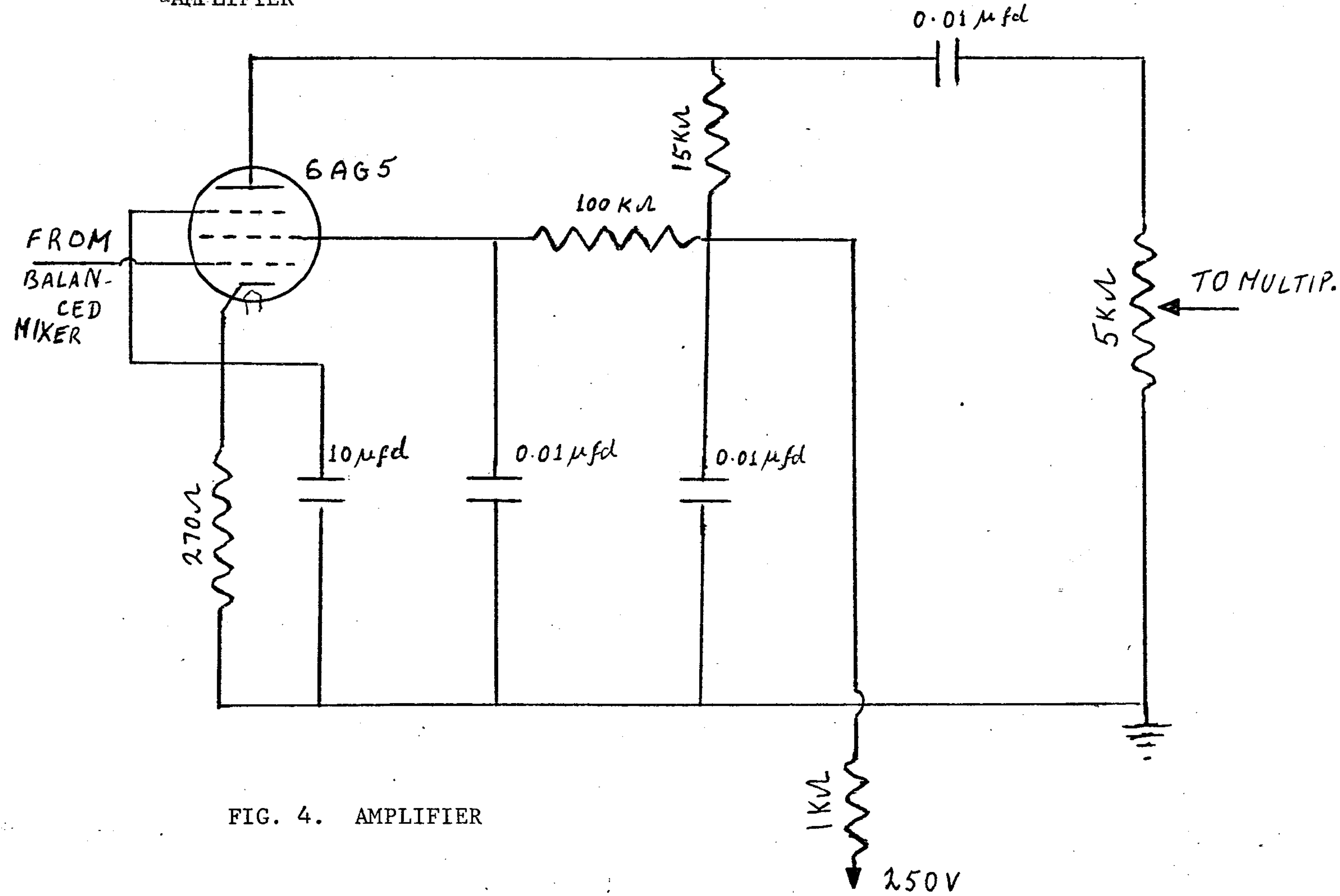

FIG. 4. AMPLIFIER

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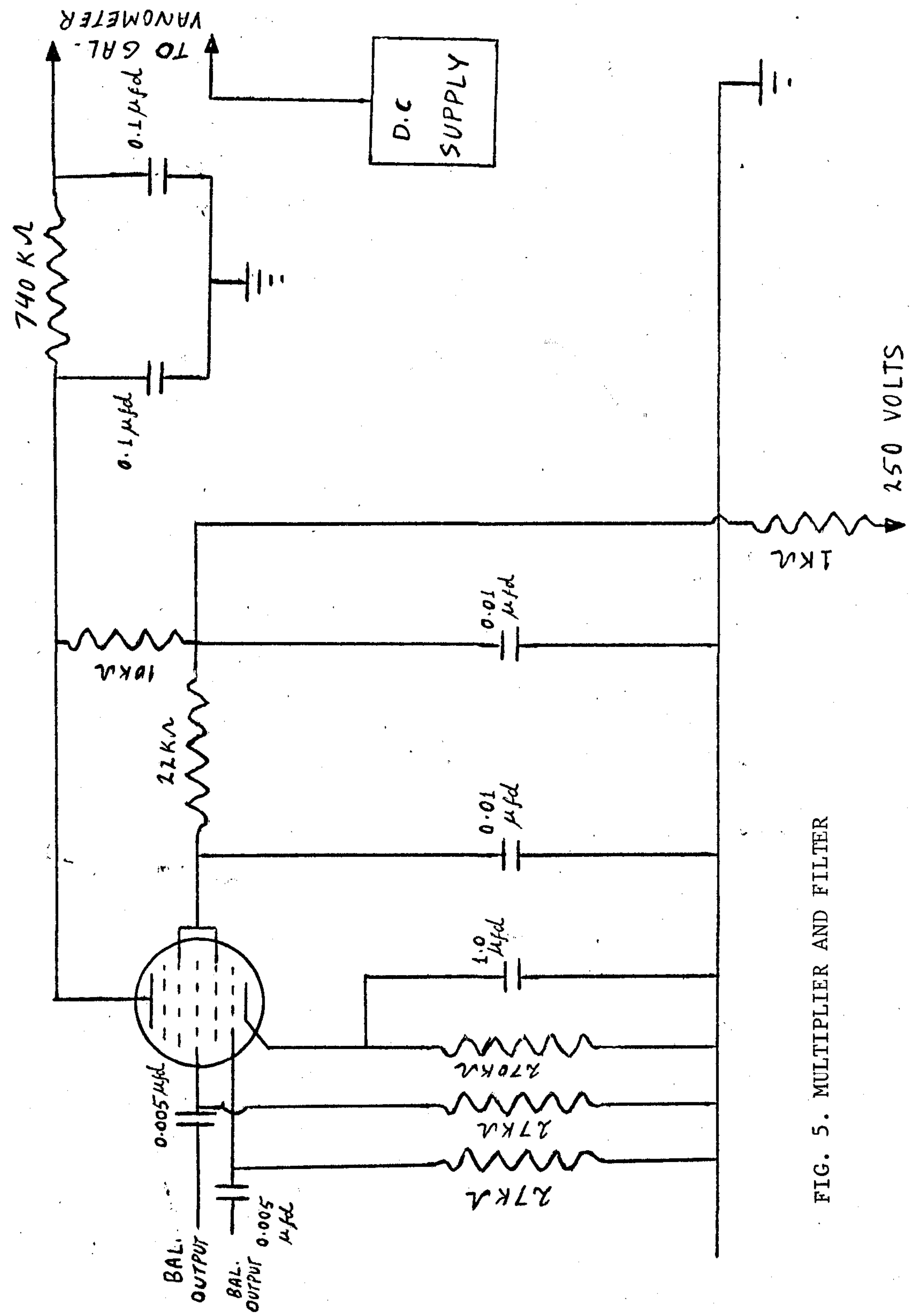

FIG. 5. MULTIPLIER AND FILTER

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CHAPTER III.

Application of Generalized Harmonic Analysis to Phase Locked Oscillator

Consider the loop initially in lock with zero phase and zero frequency error as measured at the phase comparator output. If the frequency of the input signal is increased, the increase in frequency at the input is detected as an increase in phase difference between the IF signal and IF reference. The error function is a voltage and its magnitude depends upon the deviation from the reference frequency.

A meaningful correlation analysis of the phase locked loop involves Fourier representation of the input namely the modulating and error signals. We consider the expansions

$$f_1(t) = \frac{a_{10}}{2} + \sum_{n=1}^{\infty} (a_{1n} \cos n\,\omega_1 t + b_{1n} \sin n\,\omega_1 t) \qquad (6)$$

$$f_2(t) = \frac{a_{20}}{2} + \sum_{n=1}^{\infty} (a_{2n} \cos n\,\omega_1 t + b_{1n} \sin n\,\omega_1 t) \qquad (7)$$

Another representation of these functions using elementary Fourier analysis is

$$f_1(t) = \sum_{n=\infty}^{n=\infty} F_1(n)\, e^{jn\omega_1 t} \qquad (8)$$

$$f_2(t) = \sum_{n=\infty}^{n=\infty} F_2(n)\, e^{jn\omega_1 t} \qquad (9)$$

where

$$F_{1n} = \frac{1}{2}(a_{1n} - j\, b_{1n}) \qquad (10)$$

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$$F_{2n} = \frac{1}{2}(a_{2n} - j\, b_{2n}) \tag{11}$$

and

$$F_{1n} = \frac{1}{T_1}\int_{-T_{1/2}}^{+T_{1/2}} f_1(t)\, e^{-jn\omega_1 t}\, dt \tag{12}$$

$$F_{2n} = \frac{1}{T_1}\int_{-T_{1/2}}^{+T_{1/2}} f_2(t) e^{-jn\omega_1 t}\, dt \tag{13}$$

where $n = 0, \pm 1, \pm 2 \ldots.$

we have therefore the crosscorrelation function as

$$\phi_{12}(T) = \frac{1}{T_1}\int_{-T_{1/2}}^{+T_{1/2}} f_1(t)\, f_2(t+T)\, dt$$

$$= \sum_{n=\infty}^{n=\infty} F_2(n) e^{jn\omega_1 T}\, \frac{1}{T_1}\int_{-T_{1/2}}^{+T_{1/2}} f_1(t) e^{jn\omega_1 t}\, dt \tag{14}$$

where $F_1(n)$ and $F_2(n)$ are given by (10) and (11), if for either $f_1(t)$ and $f_2(t)$, we let

$$C_n = \sqrt{a_n^2 + b_n^2} \tag{15}$$

$$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) \tag{16}$$

$$F_n = \frac{C_n}{2}\, e^{j\theta_n} \quad . \tag{17}$$

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We can show $\theta_{-n} = \theta_n$ by first proving $a_{-n} = a_n$ and $b_{-n} = - b_n$.

Making use of these details in reduction of (14) to the required form, we have

$$\phi_{12}(T) = \sum_{n_2 - \infty}^{+\infty} \frac{C_{1n} C_{2n}}{4} e^{j(n\omega_1 + \theta_{2n} - \theta_{1n})} \tag{18}$$

or

$$\phi_{12}(T) = \frac{a_{10}\, a_{20}}{4} + \frac{1}{2} \sum_{n=1}^{\infty} C_{1n} C_{2n} \cos(n\omega_1 T + \theta_{2n} - \theta_{1n}) \tag{19}$$

Two comments concerning this result would be useful for our purpose. First, the constant and harmonic coefficients in the cross-correlation function appear as products of the corresponding quantities in the given functions $f_1(t)$ and $f_2(t)$. Consequently, if the constant or any harmonic is absent in either $f_1(t)$ or $f_2(t)$, the constant term of the corresponding harmonic term will be absent in the cross-correlation functions. Second a notable difference between autocorrelation and cross-correlation is that, whereas autocorrelation discards all phase information in the given function, cross-correlation retains all the phase differences of the harmonics which are present in both periodic functions.

An expression for the autocorrelation function in terms of the coefficients of the Fourier expansion of a given periodic function follows from (19) and is given by

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$$\phi_{11}(T) = \frac{a_{10}^{2}}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_{1n}^{2} + b_{1n}^{2}) \cos n\omega_1 T \tag{20}$$

$$= \sum_{n=0}^{\infty} C_{1n}^{2} \cos n\omega_1 T$$

where $C_{10}^{2} = \frac{a_{10}^{2}}{4}$ and $C_{1n}^{2} = \frac{a_{1n}^{2} + b_{1n}^{2}}{2}$

Correlation Analysis

The general block diagram of a phase locked looped as considered by S.N. Kalra and P.H. Alexander [4] is reproduced below.

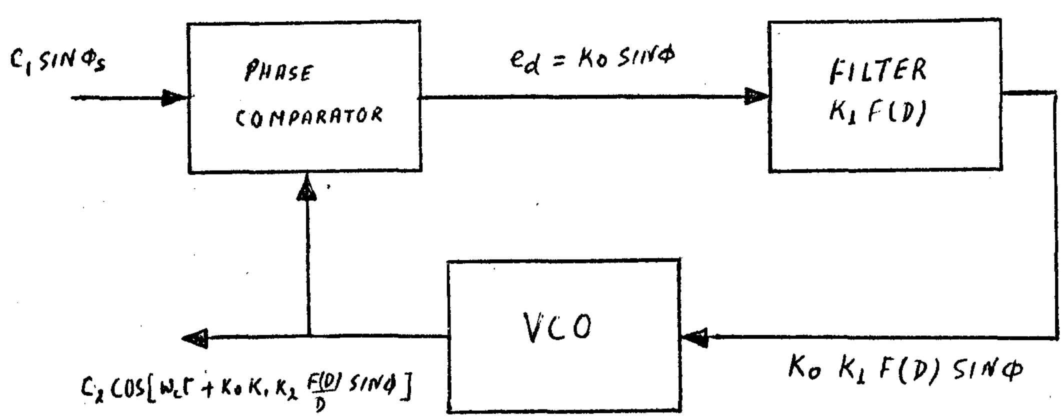

Fig. 6. BLOCK DIAGRAM OF THE PHASE LOCKED LOOP

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A simple case of a sinusoidally varying reference or input frequency with an integrator in the loop is considered. An R-C loop filter which has the transfer function of the type $\frac{\alpha}{P+\alpha}$ is used, hence a second order system is obtained.

The following nomenclature is used

NOMENCLATURE

A_1 = Amplitude of the fundamental component of phase (radians)

A_3 = Amplitude of the third harmonic component of phase (radians)

C_1 = Reference signal amplitude (volts)

C_2 = VCO output amplitude

e_d = Phase comparator output (error signal) in volts

$F(p)$ = Filter transfer function

K_0 = $K_p C C$ (volts/rad)

K_1 = Filter D.C. Gain

K_2 = VCO gain (rad./volt)

K = Loop gain (rad./sec)

P = Laplace operator

ϕ_c = VCO phase

ϕ_s = Reference signal phase

ϕ = Phase difference between reference signal and VCO

ω_c = VCO center frequency

ω_m = Modulation frequency

ω_s = Reference signal frequency

The system equation for Fig. 5 may be written as [4]

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$$e_d = K_0 \sin \phi = \frac{C_1 C_2}{2} \left| \sin \ \phi_s(t) - \omega_c t - \frac{K_0 K_1 K_2 F(p)}{p} \sin \phi \right|$$

It has been shown by S.N. Kalra and P.H. Alexander[4] that the solution for the phase ϕ of the error signal is given by

$$\phi(t) = A_1 \cos \omega_m t - A_3 \cos 3\omega_m t \tag{21}$$

A very simplified case of interest under conditions when $A_3 \sim 0$ offers a simple solution for the spectrum of the error signal and is given by equation (22) below.

$$e_d = 2K_0 [J_1(A_1) \cos \omega_m t - J_3(A_3) \cos 3\omega_m t + J_5(A_1) \cos 5\omega_m t \tag{22}$$

Only odd harmonics are contained in equation (22). The correlation of error and modulating signals is of interest.

We note that if the fundamental frequency of our modulating signal and the error signal is the same, then the cross-correlation contains only the fundamental terms, the other terms being absent.

The amplitudes of the harmonic components A_1, A_3, A_5 etc. are a function of deviation from the centre frequency of voltage controlled oscillator and tend to increase in amplitude as the deviation increases.

$$\phi_{10}(T) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t) \, f_0(t + T) \, dt \tag{23}$$

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Since the system is non-linear, we write

$$f_0(t + T) = \text{linear terms} + \text{non-linear terms}$$

In the measured cross-correlation the non-linear terms do not appear, but the amplitude of the output function is dependent upon the deviation from the center frequency $(\Delta\omega)$, hence

$$\phi_{10}(T) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t)dt \;\; A(\Delta\omega) \int_{-\infty}^{+\infty} h(\nu) f_1(t+T-\nu)dt \qquad (24)$$

where $h(\nu)$ is the unit impulse response.

We have introduced the superposition integral for $f_0(t + T)$ above. By inverting the order of integration in (24) above, we have

$$\phi_{10}(T) = A(\Delta\omega) \int_{-\infty}^{+\infty} h(\nu)d \;\; \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t)f_1(t+T-\nu)dt \qquad (25)$$

But

$$\phi_{11}(T-\nu) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t)f_1(t+T-\nu)dt \qquad (26)$$

$$\therefore \;\; \phi_{10}(T) = A(\Delta\omega) \int_{-\infty}^{+\infty} h(\nu)\, \phi_{11}(T-\nu)d\nu \qquad (27)$$

We have shown that the input-output cross-correlation of our system is the convolution of the unit impulse response $h(t)$ and the input autocorrelation. Its value changes with the change in deviation from center frequency. When we take into consideration the effect of the noise, the problem becomes very complex[3]. A simplified approach is

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presented below.

Phase ϕ of the error signal is given by equation (21)

$$\phi(t) = A_1 \cos \omega_m t - A_3 \cos 3\omega_m t \quad .$$

The error signal can be approximated by the first two terms of the sine series, i.e.,

$$e_d = K_0 \sin \phi$$
$$\simeq K_0 \left(\phi - \frac{\phi^3}{3!}\right)$$

This is justified due to the limitation that the phase difference between the reference signal and the VCO signals must be less than $\pi/2$ radians. Experimentally this is a good approximation and creates no problems.

Hence

$$e_d = K_0(A_1 \cos \theta - A_3 \cos 3\theta) - \frac{K_0}{6} (A_1 \cos \theta - A_3 \cos 3\theta)^3 \quad (28)$$

where $\theta = \omega t$

$$\cos^3\theta = \frac{1}{4} (3\cos \theta + \cos 3\theta)$$

$$\cos^3 3\theta = \frac{1}{4} (3\cos 3\theta + \cos 9\theta)$$

$$\cos^2\theta \cos 3\theta = \frac{\cos 3\theta}{2} + \frac{1}{4} \cos \theta + \cos 5\theta \quad (29)$$

$$\cos \theta \cos^2 3\theta = \frac{\cos \theta}{2} + \frac{\cos 5\theta}{4} + \frac{\cos 7\theta}{4}$$

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using the relations above it follows that

$$\sin \phi = \cos \theta \, [A_1 - \frac{A_1^3}{8} + \frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{4}]$$

$$+ \cos 3\theta \, [- A_3 - \frac{A_1^3}{24} + \frac{A_3^3}{8} - \frac{A_1^2 A_3}{4}] \qquad (30)$$

$$+ \cos 5\theta \, [\frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{8}]$$

$$+ \cos 7\theta \, [- \frac{A_1^2 A_3^2}{8}] + \frac{A_3^3}{24} \cos 9\theta$$

If the modulating signal is given by $f_2 = A \cos(\omega t + \theta)$, then the error signal and modulating signal cross-correlation is

$$\phi_{12}(T) = \frac{1}{2} A K_0 \left[A_1 - \frac{A_1^3}{8} + \frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{4} \right] \cos(\omega_m T - \theta) \qquad (31)$$

where only the terms of fundamental frequency contribute.

The amplitude is given by the product of the amplitude of the modulating signal and one half the amplitude of the error voltage. Note that we have not considered the phase of the error signal. In general the phase difference between the modulating signal and the error is π radians.

The expression for the autocorrelation of error functions is given by

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$$\phi_{11}(T) = \frac{a_{10}^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} (a_{1n}^2 + b_{1n}^2)\cos n\omega_m T \qquad (32)$$

where the coefficients of the error signal are given by (30), S.N. Kalra and P.H. Alexander have calculated the values of A_1, A_3, A_5 etc. From their predicted values a good approximation of the autocorrelation function is derived below

$$a_{10} \doteq 0$$

$$a_{11} = A_1 - \frac{A_1^3}{8} + \frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{4}$$

$$a_{13} = - A_3 - \frac{A_1^3}{24} + \frac{A_3^3}{8} - \frac{A_1^2 A_3}{4}$$

∴ Using (32) the autocorrelation is given by (33)

$$\phi_{11}(T) = \frac{1}{2}\left[A_1 - \frac{A_1^3}{8} + \frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{4} \right]^2 \cos \omega T$$
$$+ \frac{1}{2}\left[- A_3 - \frac{A_1^3}{24} + \frac{A_3^3}{8} - \frac{A_1^2 A_3}{4} \right]^2 \cos 3\omega T \qquad (33)$$

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CHAPTER IV.

RELATED DATA

S.N. Kalra and P.H. Alexander solved the non-linear loop equation by reducing it to Duffing's equation. Under conditions that the phase difference between the reference signal and the VCO signal must be less than $\pi/2$ radians, they derived the value of the amplitudes of the fundamental and the third harmonic component of phase, ϕ(radians). Their theoretical values are listed below under specified conditions.

Modulating frequency = 5KC/s

SIGNAL POWER / NOISE POWER db	LOOP GAIN KC/s	FREQUENCY DEVIATION KC/s	AMPLITUDE A1 radians	AMPLITUDE A3 radians
40	16	2.0	0.8955	0.00219
40	16	3.0	1.21157	0.00542
40	16	4.0	1.4657	0.0096
40	16	5.0	1.6775	0.0155
65	8	2.0	0.5718	0.00028
65	8	3.0	0.8401	0.0009
65	8	5.0	1.3247	0.00354

Results of correlation measurements

(a) A phase locked loop is basically very unstable for high frequency deviation and low loop gain. In general when the frequency deviation is larger than modulating frequency the lock becomes very unstable.

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(b) Since a phase locked loop is inherently a non-linear device, correlation analysis of noise is very complex. This work is, therefore, restricted to the case of high signal to noise ratio.

(c) Due to the fact that we employ a lumped constant delay line with a delay range of 0 to 400 microseconds in one microsecond step, we are restricted in the range of practical modulation frequencies. Data is presented at 5KC only.

(d) Experimentally the output of the filter for different delay times was measured. The autocorrelation data in arbitrary units has been normalized and Fourier series coefficients found in order to compare with theoretical results.

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TABLE I

Autocorrelation of Error Signal

Observed output data of error signal in arbitrary units

Modulation frequency f_m = 5KC/s

Loop gain K = 16KC/s

S/N Signal Power/Noise Power = 40db

Frequency deviation :Δ f

Differential output of filter in arbitrary units:Δ E

Delay μ secs.	Δ f = 2KC/s ΔE (a.u)	Δf = 3KC/s ΔE (a.u)	Δ f = 4KC/s Δ E (a.u)	Δ f = 5KC/s ΔE (a.u)
0	1.8	2.4	1.8	1.2
20	1.1	1.8	1.2	0.7
40	-1.0	-0.6	-0.6	-1.2
60	-3.4	-3.4	-2.8	-3.3
80	-4.6	-5.8	-4.7	-4.6
100	-5.0	-6.5	-5.6	-4.8
120	-4.2	-5.6	-4.8	-3.4
140	-2.5	-3.2	-2.85	-.24
160	-0.6	-0.5	-0.7	-1.0
180	1.0	1.6	1.1	0.4
200	1.8	2.4	1.8	1.2

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TABLE II

Autocorrelation of Error Signal

Observed output data of error signal in arbitrary units

Modulation frequency f_m = 5KC

Loop gain K = 8KC/s

Signal to noise ratio: S/N = 65db

Delay μ secs.	Diff.Output of filter (arb.units) Freq.Dev. = 2KC/s	Diff.Output of filter (arb.units) Freq.Dev. = 3KC/s	Diff.Output of filter (arb.units) Freq.Dev. = 5KC/s
0	1.0	1.0	1.0
20	0.6363	0.5128	0.5833
40	-0.2727	-0.5128	-1.0
60	-1.4090	-1.6666	-2.75
80	-2.409	-2.6666	-3.8333
100	-2.7727	-2.9743	-4.0
120	-2.3636	-2.6666	-2.8333
140	-1.3636	-1.6666	-2.0
160	-0.2727	-0.4615	-1.16666
180	0.6363	0.6486	-0.3333
200	1.0	1.0	1.0

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TABLE III

Crosscorrelation of Error and Modulation Signals

Modulation frequency f_m = 5KC/s

Loop gain K = 16KC/s

Signal to Noise ratio = 40db

Frequency deviation : Δf

Arbitrary units: (a.u)

Delay μ secs.	Diff.Output of filter (a.u) Δf = 2KC/s	Diff.Output of filter (a.u) Δf = 3KC/s	Diff.Output of filter (a.u) Δf = 4KC/s	Diff.Output filter (a.u) Δf = 5KC/s
0	-5.9	-5.8	-6.2	-6.6
33	-3.2	-3.2	-3.2	-3.2
66	0.0	-0.1	0.8	1.0
100	0.8	1.2	1.4	1.6
133	-2.0	-1.8	-1.8	-1.8
166	-5.4	-5.2	-5.6	-5.8
200	-5.9	-5.8	-6.2	-6.6

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TABLE IV

Crosscorrelation of Error and Modulation Signals

Modulation frequency f_m = 5KC/s

Loop gain K = 8KC/s

Frequency deviation: Δf

Arbitrary units : (a.u)

Delay µ secs.	Diff.Output of filter (a.u) Δf = 2KC/s	Diff.Output of filter (a.u) Δf = 3KC/s	Diff.Output of filter (a.u) Δf = 5KC/s
0	-2.8	-2.8	-2.6
33	-1.2	-9.8	-0.8
66	+1.2	+2.2	+2.4
100	+1.8	+3.0	+1.4
133	-2.0	-1.45	-1.6
200	-2.8	-2.8	-2.6

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Fourier series coefficients.

Tables V and VI give the Fourier series coefficients determined from data in Tables I and II respectively. A computer program was written for this purpose specifically. The results were confirmed using a standard IMB program[7].

TABLE V

Modulation frequency f_m = 5KC/s

Loop gain K = 16KC/s

Fourier Series Coefficients	Frequency Deviation Δf = 2KC/s	Frequency Deviation Δf = 3KC/s	Frequency Deviation Δf = 4KC/s	Frequency Deviation Δf = 5KC/s
a_0	-3.98	-3.96	-3.59	-3.76
a_1	3.387	4.5196	3.658	2.864
a_2	1.54×10^{-1}	-6.854×10^{-2}	-8.04×10^{-2}	1.965×10^{-1}
a_3	-0.4116×10^{-3}	-2.46×10^{-2}	4.687×10^{-2}	1.3599×10^{-1}
a_4	-1.944×10^{-2}	-1.459×10^{-3}	-2.454×10^{-2}	-1.165×10^{-1}
a_5	4.0002×10^{-2}	-7.999×10^{-2}	-2.454×10^{-2}	1.86×10^{-6}
b_1	-2.8254×10^{-1}	-5.70616×10^{-2}	5.204×10^{-1}	4.146×10^{-1}
b_2	1.538×10^{-4}	8.784×10^{-2}	5.877×10^{-3}	4.146×10^{-1}
b_3	9.576×10^{-2}	3.5267×10^{-2}	2.04×10^{-2}	-8.89×10^{-2}
b_4	-3.632×10^{-2}	2.8×10^{-2}	-9.51×10^{-3}	-3.289×10^{-2}
b_5	-6.239×10^{-8}	-1.047×10^{-7}	-7.319×10^{-8}	-4.799×10^{-8}

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TABLE VI

Fourier Series Coefficients

Modulation frequency f_m = 5KC/s

Loop gain K = 8KC/s

Fourier Series Co-efficients	Frequency Deviation Δf = 2KC/s	Frequency Deviation Δf = 3KC/s	Frequency Deviation Δf = 5KC/s
a_0	-1.718	-2.0907	-3.266
a_1	1.87	1.9914	2.278
a_2	-3.398×10^{-2}	4.4276×10^{-2}	1.225×10^{-1}
a_3	2.055×10^{-2}	1.179×10^{-2}	1.545×10^{-1}
a_4	6.708×10^{-3}	1.3933×10^{-2}	1.0758×10^{-2}
a_5	-0.058×10^{-3}	-3.2278×10^{-2}	1.3335×10^{-2}
b_1	-1.397×10^{-2}	-2.5721×10^{-2}	-1.2077×10^{-1}
b_2	1.397×10^{-2}	-3.1861×10^{-2}	4.72311×10^{-1}
b_3	-3.298×10^{-3}	-1.98×10^{-2}	5.2719×10^{-2}
b_4	-3.298×10^{-3}	-6.206×10^{-2}	5.0962×10^{-2}
b_5	-4.2185×10^{-8}	-9.957×10^{-8}	8.0047×10^{-9}

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CHAPTER V

COMPARISON OF RESULTS

We have shown earlier that correlation results are given by equations(31) and (33), theoretically A_1 has been found to be roughly 100 times greater than A_3. We follow the scheme enunciated below to compare experimental autocorrelation results with those obtained on the basis of theoretical findings.

The autocorrelation obtained in equation (33) is reproduced for convenience.

$$\phi_{11}(T) = \frac{1}{2}\,(\text{coeff }1)^2 \cos \omega T + \frac{1}{2}\,(\text{coeff }3)^2 \cos 3\omega T + \ldots$$

where $\text{coeff }1 = A_1 - \frac{A_1^3}{8} + \frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{4}$

$$\text{coeff }3 = A_3 - \frac{A_1^3}{24} + \frac{A_3^3}{8} - \frac{A_1^2 A_3}{4} \, .$$

Let $$\beta_1 = \frac{(\text{coeff }1)^2}{2}, \qquad \beta_3 = \frac{(\text{coeff }3)^2}{2} \qquad (34)$$

β_1 and β_3 obtained from theoretical results are listed in Table VIII. Experimental results must be normalized for a fruitful comparison. The normalization process is indicated below.

$$f(t) = \frac{a_o}{2} + \sum_{n=1}^{5} a_{1n} \cos n\omega t \; b_{1n} \sin n\omega t$$

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$$g(t) = f(t) - \frac{a_o}{2} = \sum_{n=1}^{5} a_{1n} \cos n\omega t + b_{1n} \sin n\omega t$$

$$m(t) = \frac{g(t)}{g(0)} = \sum_{n=1}^{5} P_{1n} \cos n\omega t + q_n \sin n\omega t \qquad (35)$$

Example

The results in Table I for frequency deviation of 2KC/s are used to explain the normalization process above.

TABLE VII

Normalized Fourier Series Coefficients

Frequency deviation = 2KC/s

Loop gain K = 16KC/s

$$\frac{a_o}{2} = -1.74$$

$f(t)$	$g(t) = f(t) - \frac{a_o}{2}$	$m(t) = \frac{g(t)}{g(0)}$	Fourier Series Co-efficients	Normalized Fourier Series Coefficients
1.8	3.54	1.0	$a_1 = 3.38$	$p_1 = 0.055$
1.1	2.84	0.803	$a_2 = 0.159$	$p_2 = 0.045$
-1.0	-0.74	-0.262	$a_3 = -0.0094$	$p_3 = -0.0026$
-3.44	-1.66	-0.469	$a_4 = -0.0194$	$p_4 = -0.00548$
-4.6	-2.80	-0.79	$a_5 = 0.004$	$p_5 = 0.00113$
-5.0	-3.26	-0.92	$b_1 = -0.2825$	$q_1 = -0.0807$
-4.2	-2.46	-0.695	$b_2 = -0.153$	$q_2 = -0.045$
-2.5	-0.76	-0.215	$b_3 = -0.0957$	$q_3 = -0.0271$
-0.6	1.14	+0.322	$b_4 = -0.0363$	$q_4 = -0.01$
1.0	2.74	+0.775	$b_5 = 0.0000$	$q_5 = 0.0000$

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Following the normalization scheme mentioned above, the results from our data and theoretical values obtained earlier are listed below in Tables VIII (a) and (b). We observe that $\left| p_3/p_1 \right|$ is in general of the order of β_3/β_1. We do not expect β_1 and $|p_1|$ and β_3 and $|p_3|$ to be comparable, since these have been obtained in an

TABLE VIII (a)

Loop gain K = 16KC/s

S/N = 40db

Frequency Deviation KC/s	THEORETICAL RESULTS			EXPERIMENTAL RESULTS		
	β_1 arb.units	β_3 arb.units	β_3/β_1	$\lvert p_1\rvert$ arb.units	$\lvert p_3\rvert$ arb.units	$\lvert p_3/p_1\rvert$
2	0.32	0.5 E-03	1.56 E-03	0.955	2.06 E-03	2.78 E-03
3	0.5	0.27 E-02	0.54 E-02	1.02	0.97 E-02	0.96 E-02
4	0.57	0.9 E-02	1.58 E-02	1.03	1.42 E-02	1.4 E-02
5	0.59	0.2 E-01	3.4 E-02	1.05	0.53 E-01	5.0 E-02

TABLE VIII (b)

Loop gain K = 8KC/s

S/N = 65db

Frequency Deviation KC/s	THEORETICAL RESULTS			EXPERIMENTAL RESULTS		
	β_1 arb.units	β_3 arb.units	β_3/β_1	$\lvert p_1\rvert$ arb.units	$\lvert p_3\rvert$ arb.units	$\lvert p_3/p_1\rvert$
2	0.15	0.32 E-04	2.13×10^{-4}	0.99	1.07 E-03	1.098 E-03
3	0.293	3.225xE-04	1.1 E-03	0.97	5.75 E-03	5.92 E-03
5	0.555	4.9 E-02	9.15 E-02	0.86	5.87 E-02	6.78 E-02

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altogether different manner, only the ratios of these can be compared. Note that the normalization process in no way effects these ratios. A certain amount of divergence in results is expected and occurs because of the following reasons.

(a) The divergence is higher for lower loop gain due to greater instability of the system.

(b) The experimental correlation results are observed to two decimal places due to practical limitations. This produces an error in the determination of Fourier coefficients and the error is more relevant for higher Fourier coefficients which are small in our case. In order to clarify this point further, consider the problem of fitting a finite trigonometric sum to a set of observed values (x_i, y_i). Let the set of observed values

$$(x_o, y_o),\ (x_1, y_1)\ \ldots\ (x_{2n-1}, y_{2n-1}),\ (x_{2n}, y_{2n}),\ \ldots$$

be such that the values of y start repeating with y_{2n} (i.e. $y_{2n} = y_o$). Assume x_i are equally spaced and $x_o = 0$ and that $x_{2n} = 2\pi$.

The trigonometric polynomial

$$y = P_o + \sum_{k=1}^{n-1} P_k \cos kx + \sum_{k=1}^{n} \theta_k \sin kx \tag{36}$$

Contains the 2n unknown constants

$$P_o,\ P_1, P_2,\ \ldots,\ P_{n-1},\ \theta_1,\ \theta_2,\ \ldots,\ \theta_n\ ,$$

which can be determined so that equation (36) will pass through the 2n given points (x_i, y_i) by solving the 2n simultaneous equations

$$y_i = P_o + \sum_{k=1}^{n-1} P_k \cos k\, x_i + \sum_{k=1}^{n} \theta_k \sin k\, x_i \tag{37}$$

$$i = 0, 1, \ldots, 2n-1\ .$$

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It can be shown [8] that Fourier coefficients are given by

$$nP_j = \sum_{i=0}^{2n-1} y_i \cos \frac{ij\pi}{n} \qquad j = 1,2,\ldots,n-1 \qquad (38)$$

and

$$n\theta_j = \sum_{i=0}^{2n-1} y_i \sin \frac{ij\pi}{n} \qquad j = 1,2,\ldots,n-1 \qquad (39)$$

From equation (38) and (39), we observe that when P_j and θ_j are small, an accurate determination of these coefficients depends heavily on the degree of accuracy to which y_i are known. A rough estimate of error can be obtained by introducing a random error function ϵ_i such that

$$n(P_j + \delta P_j) = \sum_{i=0}^{2n-1} (y_i + \epsilon_i) \cos \frac{ij\pi}{n} \qquad j = 1,2,\ldots,n-1 \qquad (40)$$

$$n(\theta_j + \delta \theta_j) = \sum_{i=0}^{2n-1} (y_1 + \epsilon_i) \sin \frac{ij\pi}{n} \qquad j = 1,2,\ldots,n-1 \qquad (41)$$

where δP_j and $\delta\theta_j$ are the error Fourier coefficients. Since in our case $2n = 10$ and we are interested in 3rd Fourier coefficient, the maximum error in terms of average error ϵ is given by

$$\delta P_3 = \frac{1}{5} \sum_{i=0}^{9} \epsilon_i \cos \frac{ij\pi}{5} \sim \frac{1}{5} \bar{\epsilon}$$

$\bar{\epsilon} \sim 0.005$ (Follows from the output accuracy considerations)

hence $\delta P_3 \sim 0.001 \sim 10^{-3}$.

This puts an upper limit to the degree to which the coefficients can be compared.

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The crosscorrelation results are used to obtain a rough estimate of the amplitude of the fundamental and the third harmonic component of phase. ϕ (radians). We assume the following

(a) Noise has insignificant effect on the differential output

(b) the differential output of the filter is directly proportional to the crosscorrelation when the two inputs are the modulation and the error signals.

These assumptions are true under balanced conditions, i.e. when signal to noise ratio is high and on interchanging the two inputs the output shows no appreciable change.

The autocorrelation of a sinusiodal signal is given by

$$\phi_{11}(T) = \frac{A^2}{2} \cos \omega T \qquad (42)$$

where A is the amplitude of frequency ω and T is the delay. If f = 5KC/s and T = 100 μ secs, then it follows that

$$\phi_{ii}(0) - \phi_{ii}(100) = GA^2 \qquad (43)$$

where G is a constant of proportionality.

With a known signal amplitude we have observed autocorrelation results of a 5KC/s sinusoid and found G to be 25.

From equation (31) the crosscorrelation results follow.

$$\phi_{ie}(T) = 1/2\ AK_o \left| \text{coeff } 1 \right| \cos \omega T \qquad (44)$$

where $\text{coeff } 1 = A_1 - \frac{A_1^3}{8} + \frac{A_1^2 A_3}{8} - \frac{A_1 A_3^2}{4}$

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$$\therefore \quad \phi_{ie}(0) - \phi_{ie}(100) = GA\ (\text{coeff } 1) \qquad (45)$$

$\omega T = \pi$ when $T = 100\ \mu$ secs.

The amplitude A has been maintained constant and is 0.3, K_o was of order unit, hence it does not appear in (45). From Tables III and IV, we can easily obtain coeff 1 as follows.

$$\text{coeff } 1 = \frac{\phi_{ie}(0) - \phi_{ie}(100)}{(G \times A)}$$

$$= \frac{\phi_{ie}(0) - \phi_{ie}(100)}{7.5}$$

Considering $A_1 \gg A_3$, a rough estimate of A_1 is obtained from experimentally obtained coeff 1. Since

$$\text{coeff } 1 \sim A_1 - \frac{A_1^3}{8}$$

we can easily find A_1. Results appear in Table IX.

TABLE IX .

Comparison of Results

Loop gain K KC/s	Frequency Deviation ω KC/s	THEORETICAL			EXPERIMENTAL	
		A_1 rad	A_3 rad	coeff 1	A_1 rad	coeff 1
16	2	0.8955	0.00219	0.893	0.89	0.8058
16	3	1.21157	0.00542	0.9333	1.2	0.99024
16	4	1.4657	0.00960	0.986	1.45	1.07484
16	5	1.67767	0.0144	1.095	1.62	1.09241
8	2	0.5718	0.00028	0.619	0.55	0.54844
8	3	0.84011	0.0009	0.772	0.845	0.76607
8	5	1.3247	0.000354	0.8	1.3	1.03489

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A rough estimate of A_3 can be obtained by using the values of coeff 1 determined experimentally and values of A_1 obtained from theory. We know

$$\text{coeff } 1 \sim A_1 - \frac{A_1^2 A_3}{8} + \frac{A_1 A_3^2}{8} - \frac{A_1^3}{8}$$

By choosing trial values we can easily obtain a rough estimate of A_3. Results using this technique appear in Table X.

TABLE X

Comparison of Results

Loop Gain K KC/s	Frequency Deviation KC/s	THEORETICAL		EXPERIMENTAL	
		A_1 rad	A_3 rad	A_3 rad	coeff 1
16	2	0.8955	0.00219	0.0021	0.8059
16	3	1.21157	0.00542	0.0052	0.99024
16	4	1.4657	0.0096	0.011	1.07484
16	5	1.67767	0.0144	0.015	1.09241
8	2	0.5718	0.00028	0.0002	0.5844
8	3	0.84011	0.0009	0.001	0.76607
8	5	1.3247	0.00035	0.0003	1.03489

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CHAPTER VI

CONCLUSIONS

It has been found that the correlation analysis gives results which compare favourably with those derived from the theoretically predicted values by Dr. S.N. Kalra and P.H. Alexander.

In Tables VIII (a) and (b) we have obtained $\left|\frac{P_3}{P_1}\right|$ from our auto-correlation results and these are reasonably close to β_3/β_1 obtained from theory we have chosen to compare the ratios of normalized Fourier series coefficients due to the fact that experimentally correlation results are in the form of a differential output.

We have derived coeff 1 and A_1 from crosscorrelation output which is in agreement with coeff 1 and A_1 calculated from theoretical results. The results are listed in Table IX.

In general we have found that under stable conditions, i.e. high loop gain, high signal to noise ratio and frequency deviation less than the modulating frequency the results obtained are in close agreement with theory.

When these conditions are not met, the loop remains stable for short durations, also a certain amount of drift ensues, the data observed shows a considerable change if repeated.

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