An analysis of steady state void fraction distribution and steam void response to sinusoidal power modulation in a boiling water coolant channel.

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AN ANALYSIS OF STEADY STATE VOID FRACTION DISTRIBUTION
AND STEAM VOID RESPONSE TO SINUSOIDAL POWER
MODULATION IN A BOILING WATER
COOLANT CHANNEL

A Thesis
Submitted to the Faculty of Graduate Studies through the
Department of Chemical Engineering in Partial Fulfilment
of the Requirements for the Degree of
Master of Applied Science at
University of Windsor

by
Afamefuna E.A. Egbonwhe

Windsor, Ontario
1967
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ABSTRACT

In this theoretical analysis, various models and correlations were tested for their ability to predict correctly the steady state cross-sectional average void fractions in a boiling water channel. A modified Bowring's model is proposed and then employed to predict the steam weight fraction distribution in the subcooled boiling region. This modified model is used also in the theoretical prediction of steam void response to small power perturbation in a boiling water channel.

Bowring's model [1], was employed to predict the inception of the subcooled boiling region. In this subcooled boiling region, a parameter ε was introduced to relate the heating component to the evaporative component of the total heat flux. The parameter ε was assumed to vary linearly throughout the subcooled boiling region. In the bulk boiling region, ε equals zero, and the steam weight fraction distribution is calculated from a thermal energy balance.

Various theoretical models and correlations were tested against available experimental data [17,29,36] for their ability to predict accurately the steam void fraction for both the subcooled boiling region and the bulk boiling region. Satisfactory agreement was obtained with the majority of the models over a wide range of channel geometries, heat fluxes, pressure and inlet flow rates. Neal's model [7] was found to give the best overall agreement. The range of operating conditions considered were:

<table>
<thead>
<tr>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>140-1000 psia</td>
</tr>
</tbody>
</table>
Flow Area
rectangle, annular, and circular

Inlet Liquid Velocity
above 2 ft/sec.

Subcooling
1.1°F to 21.6°F

Heat Flux
22,800-157,000 Btu/hr-ft²

In the unsteady state analysis, the equations of change were written in the one-dimensional macroscopic form, differenced in the height variable and solved employing a stagewise complex integration procedure. Improved agreement with experimental data (36) was obtained employing the modified Bowring's model in the transient analysis.
ACKNOWLEDGEMENTS

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CONTENTS

ABSTRACT
ACKNOWLEDGEMENTS
LIST OF FIGURES
CHAPTER I  INTRODUCTION
CHAPTER II LITERATURE SURVEY
   A. Cross-sectional Average Void Fraction
      1. Bulk Boiling Region
      2. Subcooled Boiling Region
   B. Theoretical Models for the Dynamics of
      Coolant Channels
   C. Power to Void Transfer Function Measurement
CHAPTER III PRESENT ANALYSIS
   A. Bowring's Model
   B. Proposed Modification of Bowring's Model
   C. Unsteady State Analysis
CHAPTER IV MACROSCOPIC EQUATIONS OF CHANGE FOR A TWO-PHASE
   FLOWING SYSTEM
   A. Basic Equations
      1. Macroscopic Mass Balance
      2. Macroscopic Momentum Balance
      3. Macroscopic Energy Balance
      4. Channel Wall Energy Balance
   B. Dimensionless form of Equations of Change
   C. Steady State Equations
      1. Non-boiling Region
      2. Subcooled Boiling Region
      3. Bulk Boiling Region
      4. Boiling Region Transitions
   D. Unsteady State Equations of Change
      1. Non-boiling Region
      2. Subcooled Boiling Region
      3. Bulk Boiling Region
CHAPTER V DISCUSSION
   A. Steady State Analysis
   B. Unsteady State Analysis

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<table>
<thead>
<tr>
<th>Chapter/Apdx</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER IV CONCLUSIONS</td>
<td>39</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>55</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>59</td>
</tr>
<tr>
<td>APPENDIX I</td>
<td>63</td>
</tr>
<tr>
<td>APPENDIX II</td>
<td>66</td>
</tr>
<tr>
<td>APPENDIX III</td>
<td>70</td>
</tr>
<tr>
<td>APPENDIX IV</td>
<td>74</td>
</tr>
<tr>
<td>VITA AUCTORIS</td>
<td>79</td>
</tr>
</tbody>
</table>

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LIST OF FIGURES

FIGURES

1 Void Fraction Prediction Using the Modified Bowring's Model and Bowring's Model with Experimental Data (Ref. 17).
2 Void Fraction Prediction Using the Modified Bowring's Model and Bowring's Model with Experimental Data (Ref. 17).
3 Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model Run 9.
4 Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model Run 10.
5 Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model Run 12.
6 Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model Run 16.
7 Comparison of St. Pierre's Experimental Data (Ref. 36) with Values Predicted by Modified Bowring's Model and constant ε Run 12.
8 Comparison of St. Pierre's Experimental Data (Ref. 36) with Values Predicted by Modified Bowring's Model and constant ε Run 6.
9 Comparison of St. Pierre's Experimental Data (Ref. 36) with Values Predicted by Modified Bowring's Model and constant ε Run 6.
10 Comparison of St. Pierre's Experimental Data (Ref. 36) with Values Predicted by Modified Bowring's Model and constant ε Run 12.
11 Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by an assumed Linear Variation of Slip Ratio with axial distance in the Subcooled Boiling Region, Run 10.
12 Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by an assumed Linear Variation of Slip Ratio with axial distance in the Subcooled Boiling Region, Run 12.
13 Comparison of Power-to-void transfer Data (Ref. 36) with Values Predicted by the Modified Bowring's Model and ε = 1.3.
In a boiling water reactor, there is an inherent interaction of reactor power output with the volume void fraction present in the core. Consequently, a knowledge of the vapour volume distribution along the channel axial length is of importance for reactor design since this interaction can result in boiling water reactor instabilities. Instabilities can arise from reactivity feedback, causing divergent power oscillations, or from hydrodynamic instability which may occur in natural circulation boiling systems at constant power.

The steady state behaviour and the dynamic response of a reactor are strongly dependent on the vapour volume distribution present in the core. Theoretical predictions of the vapour volume distributions in a channel are often complicated, due to the presence of three boiling regions: a non-boiling region, a subcooled boiling or non-equilibrium region, and a bulk boiling region. In addition, the boundaries between these regions must be considered.

Several attempts have been made by different authors to formulate theoretical models or semi-empirical correlations to predict vapour volume distribution in a boiling water channel. At present there is no model or correlation which can be used for all of the possible operating conditions in present day reactors. In general, the semi-empirical correlations employ dimensionless groups which the authors believe to contain all the significant variables of the problem. However,
predictions employing these correlations cannot be extrapolated with any
degree of confidence beyond the experimental data used to obtain the
correlations.

Authors in deriving the theoretical models have employed many
assumptions because of the complexity of the problem. Often these
assumptions are valid only for specific flow regimes in two phase flow.
In addition, experimental data are normally analysed to obtain values
of semi-empirical constants required for the theoretical models. Once
more extension of the models beyond a particular flow regime or range
of experimental data cannot be done with confidence.

Bowring's model(1) was one of the first to treat the three distinct
boiling regions. His physical model can be used to predict the inception
of subcooled boiling and the axial quality distribution for the non­
equilibrium region. Bowring's model, with various correlations and
theoretical models relating quality and void fraction, has been used to
predict void fractions at reactor operating conditions(2). However, a
discontinuity in the slope of vapour volume fraction versus steam
quality curve was found to exist at the bulk boiling boundary.
Physically this behaviour is not expected.

The present work is concerned with a modification of Bowring's
physical model to give a smooth curve of vapour volume fraction versus
quality over the whole channel length. It is hoped that better
agreement between experimental data and void fractions predicted
employing the modified Bowring's model will be obtained. In addition,
the ability of several theoretical models and correlations to
predict accurately the cross-sectional average vapour volume fraction is
tested by comparison with experimental data.

Finally a study of the dynamic response of a boiling water channel to power modulation is considered employing this modified Bowring's model. The predictions are compared with available data.
II LITERATURE SURVEY

A. Cross-sectional Average Void Fraction

The majority of the empirical correlations and void fraction models relating quality and void fraction are valid only in the bulk boiling region.

1. Bulk Boiling Region

Several of the models employed in predicting the steady state cross-sectional average void fraction distribution, are summarized below.

Bankoff (3) formulated a theoretical model, in which the contribution of the local slip effect on the two phase system was neglected. The two phase mixture was conceived as a single fluid with radial density dependency. A power law distribution was assumed to hold for both the liquid phase and the vapour phase. A flow parameter was derived and its numerical value determined by analyzing available experimental data. In order to eliminate the discontinuity in Bankoff's expression for the slip velocity ratio, Jones (4) modified the value of this flow parameter. Later Neal and Zivi (5) claimed a better agreement with experimental data, employing a value of the flow parameter obtained by analyzing Marcheterre's data (6).

Neal (7) derived a theoretical model relating quality and void fraction by considering two dominant effects on the two phase system. One effect results from the relative velocity or local slip between the
two phases. The second effect is the radial variation of void fraction and velocity. Values for the two parameters which account for the local slip effect and the distributional (radial variation of void fraction and velocity) effect were obtained by analyzing previous theories and experimental data for specific flow regimes (8,9,10,11).

Zuber and Findlay (12), formulated a general expression for the void fraction claimed to be applicable to any two phase flow regime. Incorporated in the theoretical model were expressions which accounted for both the distributional and the local slip effects. These expressions are dependent on the flow regime.

Marcheterre and Hoglund (13) obtained a semi-empirical correlation relating quality and void fraction. Dimensional analysis was employed to correlate vapour volume data using the Froude number, the velocity ratio of the vapour and liquid phases, and the ratio of the volumetric flow rates as the significant dimensionless groups.

Primeau and Roger have obtained an empirical correlation (14) which is a modification of the Collier-Sher correlation (15).

2. Subcooled Boiling Region

Theoretical models to predict axial distribution of void fractions in the subcooled boiling region are scarce because of the complexity of the problem.

A physical model for the calculation of void fraction in the subcooled boiling region was formulated by Bowring (1). The subcooled boiling region was subdivided into a region of high subcooling, in which the wall voidage is important; and a region of slight subcooling, in which the void fraction increases rapidly. To account for the rapid rise in
void fraction in the slightly subcooled region the concept of bubble detachment was introduced. At low subcooling bubbles growing on the heated surface detach and are swept downstream, condensing as they travel through the subcooled boiling region.

Two new parameters were introduced. The first related the subcooling at which a rapid rise in void fraction occurred, to the inlet liquid velocity and the heat flux. The second parameter gave the ratio of the latent heat flux to the agitative heat flux. Values of both parameters were obtained by analyzing experimental data.

Zuber, Staub and Bijwaard (16) developed a theoretical model by considering the effects of velocity, vapour concentration, and temperature profiles, together with the effects of local slip. A temperature distribution believed to be realistic for the subcooled boiling region was assumed. Employing a steady state energy equation in the Lagrangian form (neglecting kinetic energy and potential energy effects) an expression was derived for the void fraction distribution. A distribution parameter whose value is dependent on the flow regime was introduced. Values of the distribution parameter were obtained by analyzing experimental data.

Extensive experimental work has been performed by Rouhani (17) on void measurements in the region of subcooled boiling. Void fraction measurements were carried out in an annular test section employing a gamma radiation technique. These data were used to test the predictions from the modified Bowring's model.
B. Theoretical Models for the Dynamics of Coolant Channels

In order to predict accurately the dynamic performance of a boiling water reactor (BWR) in the design stage, it is necessary to understand the transient behaviour of steam bubbles in the reactor core as the heat production in the fuel varies. This constitutes the so called power-to-void transfer function which expresses in a concise manner the dynamic relationship between a power disturbance and the ensuing steam-void response. Two main types of approach have been attempted in stability or dynamic analysis of boiling water reactor coolant channels. The first employed the three conservation laws of mass, momentum, and energy in addition to the equation of state. The second (18,19,20,21) employed the pressure drop versus flow rate curve as the controlling phenomenon and studied the influence of other parameters of the system on this relationship.

Horning and Corben (22) employed a one-point model. One differential equation was written for the total steam volume which accounted for the heat capacity of the fuel. A constant fractional rate of removal of steam voids was assumed.

Iriarte (23) employed the one point model also but characterized the power-to-void transfer function by time constants and a time delay.

Beckford (24), by dividing the fuel plate into several regions, derived three time constants but omitted any contributions due to the non-boiling region.

Fleck and Huseby (25), and Fleck (26) obtained solutions based on linearity of void fraction, water velocity and steam velocity with channel length. The nuclear radiation heat deposited directly in the
coolant was considered also. Akcazu (27) recognized that a perturbation in the mass flow rate of steam could cause the local void fraction changes to be less than would be expected, as the perturbation may not propagate along the boiling section with a velocity equal to the steam velocity.

Zivi and Wright's theoretical model (28) considered the effects of longitudinal displacement of axial void fraction distribution as the power was modulated. The transient void response was obtained by accounting for the displacement of the non-boiling length by superimposing the effects of power, flow and pressure variations on the steady state void fraction curve. The phase lag was obtained as the sum of the moving boundary phase lag plus that due to propagation (away from the boundary) of the void disturbance travelling with the liquid velocity. The void amplitude was obtained by multiplying the magnitude of the boundary disturbance and the total derivative of the zero-frequency void fraction with respect to axial distance.

Christensen (29) in his treatment of the power-to-void transfer function considered the pressure changes in the channel arising from the power variations. He derived the power-to-void transfer function, which fitted his experimental data, employing Laplace transform theory.

Bankoff, Hudson and Atit (30) employed the macroscopic continuity, energy and momentum balances. These were differenced in the height variable and then linearized around steady state. Bankoff's variable density model (3) was used to relate quality and void fraction.

Jones (31) and Jones and Night (32) included an approximate
representation of the subcooled boiling region in their analysis and
the transient energy in the fuel element. A modified Bankoff slip ratio
correlation was used and the effects of the moving boundary between
subcooled boiling and bulk boiling regions were considered in addition
to bubble recondensation effects. Solberg (33) and Schjetne (34)
developed and tested respectively the Kjeller model. Conservation
laws were employed in the non-boiling region, subcooled and bulk
boiling regions and an analysis of the subcooled region was based on
Bowring's work (1).

Bijwaard, Staub and Zuber (35) recently formulated a transformed
continuity equation in terms of kinematic waves describing the transient
behaviour of the vapour concentration in a two-phase flow system.

A complex variable method was used by St. Pierre (36) to solve the
differential equations employing a stage-wise integration procedure.
The transitional boundaries between the subcooled and the bulk boiling
regions were treated in a manner similar to that in the Kjeller model.
Treatment of the bulk boiling region was based upon the work of Hudson
et al. (30)

C. Power to Void Transfer Function Measurement

A detailed account of the early experimental work on boiling water
reactors up to 1958 has been given by Kramer (37). Early experimental
research on the stability of boiling water reactors was done by employ-
ing a harmonically oscillated reactivity input and studying the resul-
tant reactor power output. Eriksen (38) measured void volume by their
reactivity worth. This investigation and Daavettilas (39) revealed
that a reactor was a poor detector of voids for studies of dynamic
boiling phenomena.

Deshong and Lipinski (40) measured the transfer functions of the reactor at several low powers, employing a rod oscillator technique. Experimental data were compared with those theoretically predicted using an analog computer. Transfer functions for higher powers were then obtained by extrapolation, and these predicted accurately, instability points at higher power levels.

Zivi and Wright (28) measured transfer functions using an electrically heated aluminium channel. For low subcooling tests, the channel power was modulated about the mean level at a given frequency with small amplitude (usually 10%). The amplitude and phase of the void response were obtained over a frequency range of zero to 10 cps. The void response linearity for a frequency of 1/4 cps. extended up to power modulation of 20% corresponding to a void amplitude of 17%.

Christensen (29) extended the experimental range of the parameters employed by Zivi and Wright (28). Power modulation around a steady state level was limited to 10% peak-to-peak and measurements of the amplitude and phase of the void response were made employing a cross correlation technique. Void response was found to fall off at a much lower frequency (0.30-0.60 cps) than that predicted by theoretically derived power-to-void transfer function. A sharp node and an undulation in the phase measurements occurred at the node frequency.

St. Pierre (36) measured power-to-void transfer function using an electrically heated rectangular test section. Notches at frequencies ranging from 1.2 to 3.7 cps were obtained for all the void amplitude data plotted. The notch frequency decreased with the downstream axial
position attaining a maximum value at the inlet. Maximum notch frequency at the centre of the channel was obtained for the transverse measurements. The phase measurements had an undulation close to the notch frequency. The range of linearity of the void response to power modulation extended to amplitudes of 20% of the average power level for the runs at 200, 300, and 400 psia. Decreased inlet subcooling increased the zero frequency amplitude and the local void propagation velocity was found to be a function of the distance from the wall at a fixed axial position.

Bijwaard, Staub and Zuher (35) measured power-to-void transfer functions in a vertical, stainless steel tube employing a forced circulation loop. Power modulation ranged from 12.5 to 40% of the average power level. Within the range of frequencies (0.05 to 1) studied, the void phase lag increased linearly with frequency, and decreased as the flow rate was increased and with downstream axial distance. The void amplitude response decreased at the higher frequencies. Zero frequency amplitudes at high flow rates, high heat flux and low inlet subcooling were less than those predicted from steady state data.
III PRESENT ANALYSIS

The present analysis is concerned first with an accurate prediction of the steady state cross-sectional average void fractions in a two phase flowing system. In addition, theoretical predictions of the dynamic response of vapour volume void fraction in a boiling channel to small power modulation is considered.

A. Bowring's Model

Bowring (1) formulated a physical model based upon bubble detachment for calculating steady state cross-sectional average void fractions in the subcooled boiling region. A parameter \( \epsilon \), which is the ratio of the heating component, \( q_a \), to the evaporative portion, \( q_e \), of the total heat flux was introduced. Experimental data (33) were analysed to obtain a value of \( \epsilon \) equal to 1.3, constant in the subcooled boiling region for the pressure range of 9 to 50 atmospheres. In the bulk boiling region, \( q_a \) is zero and thus the value of \( \epsilon \) is zero. For this reason, a step change is obtained in the quality at the bulk boiling boundary, \( Z_B \).

In the slightly subcooled region Bowring divided the total heat flux, \( q \), into four components:

\[
q = q_e + q_a + q_c + q_{sp} \quad (3.1)
\]

Here, \( q_e \) represents the evaporative heat flux in the form of the latent heat content of the bubble; \( q_a \) is the convective heat flux arising from bubble agitation of the boundary layer; \( q_{sp} \) is the single phase heat...
transfer between patches of bubbles and $q_c$ is the heat loss by condensation while the bubble is still attached to the surface. The terms $q_e$, $q_a$ and $q_c$ were considered by Forster and Greif (41) who showed that $q_c$ was negligible when compared to $q_a$ at atmospheric pressure. Assuming $q_c$ and $q_{sp}$ to be negligible at high pressures and heat fluxes Eq. (3.1) reduces to:

$$q = q_e + q_a$$  \hspace{1cm} (3.2)

A parameter $\varepsilon$ was introduced by Bowring for the subcooled boiling region as:

$$\varepsilon = \frac{q_a}{q_e} = (\frac{\delta c p}{\rho g \lambda}) \theta$$  \hspace{1cm} (3.3)

where $\theta$ is an effective temperature difference through which an amount of liquid (equal to the volume of the detached bubble) is raised as it is drawn to the heated wall and pushed out again by the bubble. Here $\theta$ is a function of the bulk temperature, the degree of superheat near the wall and the efficiency with which the bubble circulates the liquid.

B. Proposed Modification of Bowring's Model

At the onset of subcooled boiling the evaporative heat flux, $q_e$, is very small and thus $\varepsilon$ should tend to be large at the transition boundary $Z_T$. At the bulk boiling boundary, $Z_B$, $q_a$ tends to zero and thus $\varepsilon$ approaches zero. Consequently, $\varepsilon$ varies from some large value at the point of bubble detachment, $Z_T$, to zero at the onset of bulk boiling.

The parameter $\varepsilon$ is a function of the degree of liquid superheat near the wall, the bulk fluid temperature and the efficiency of circulation according to Bowring (1). The degree of wall superheat is
approximately constant throughout the subcooled boiling region (42). The degree of liquid superheat near the wall is reasoned also to vary slightly throughout the subcooled boiling region. Downstream of the transition boundary the increased void fraction should tend to increase the efficiency of circulation. As the circulation increases the residence time of the liquid near the wall decreases which would tend to decrease the effective temperature difference, \( \theta \). This increased bubble circulation would tend to decrease \( \varepsilon \) as the bulk boiling boundary is approached. In addition the bulk fluid temperature increases approximately linearly through the subcooled boiling region which would also tend to decrease linearly the value of \( \varepsilon \) as the bulk boiling boundary is approached.

A linear variation of \( \varepsilon \) with axial distance in the subcooled boiling region is assumed as a modification of Bowring's model (1), with \( \varepsilon \) decreasing linearly from an initial value of 2.6 at the transitional boundary to zero at the bulk boiling boundary. The constant value of 1.3 for \( \varepsilon \) as used by Bowring is thus taken as an average value over the subcooled boiling region. This assumed modification should give lower values of steam quality near the transition boundary and hence smaller steam voidage values than were predicted with a constant \( \varepsilon \). As the bulk boiling region is approached, higher values of steam qualities and higher steam volume fractions will be obtained than those predicted employing a constant \( \varepsilon \). This modified Bowring's model, which eliminates the discontinuity at \( Z_B \) is tested employing available steady state void fraction data (17, 29, 36) for both the subcooled and bulk boiling regions.
C. Unsteady State Analysis

One method which has been employed (36) to solve the linearized equations of change describing a two phase flowing system uses a stage-wise integration procedure which employs a complex variable theory. This procedure which will be used in this present analysis results in a set of complex difference equations with coefficients determined from the steady state values of the problem variables. Consequently, accurate predictions of the steady state values of void fraction, quality, slip ratio and fluid temperature are of importance in the dynamic response analysis.

The modified Bowring's model is employed here in the dynamic analysis to investigate any possible improvements on predictions (36) made with the original Bowring's model.
In this chapter the conservation equations for mass, energy, and momentum are derived for the flowing two phase fluid mixture. These equations, with various theoretical models (3,4,5,7) and correlations (13,14) relating quality and void fraction are employed to calculate both the steady state cross-sectional average axial void distribution and the dynamic response of the volume void fraction to power modulation. Bowring's modified model is employed in the subcooled boiling region. The bulk boiling region is treated in a similar manner to that of Hudson et al (30).

The transition boundaries between the subcooled and the non-boiling regions, and the bulk boiling and the subcooled boiling regions are treated in a manner similar to that of St. Pierre (36). For the dynamic analysis, the effect of the movement of the bulk boiling boundary on the transient void response is assumed to be distributed over the entire length of the subcooled boiling region. This assumption eliminated the discontinuity at the bulk boiling boundary, \( Z_B \), in the transient void amplitude versus axial length curve. The Kjeller model (33,34) had previously considered this effect to be concentrated at the bulk boiling boundary, \( Z_B \).

The channel length is sectioned into \( N \) regions, and linearized around steady state to obtain solutions of the conservation equations.
By employing a step-by-step integration procedure which uses complex variables, the void response, the temperature response, and the flow response to power modulation are computed.

A. Basic Equations

The conservation equations are derived in macroscopic form for a two-phase flow by writing a balance (mass, energy, momentum) over a stationary volume element $A\Delta z$ in a vertical channel. A thermal energy balance is made relating the heat content of the wall to the heat transferred to the flowing two-phase stream.

1. Macroscopic Mass Balance

A macroscopic mass balance over the stationary volume element gave:

$$\frac{\partial}{\partial t} \left[ \alpha \rho_g + (1-\alpha) \rho_\ell \right] = -\frac{1}{A \Delta z} \left( W_\ell + W_g \right). \tag{4.1}$$

where

$$\frac{1}{A} \left( W_\ell + W_g \right) = (1-\alpha) \rho_\ell V_\ell + \rho_g V_g.$$  

The term on the left side of Eq. (4.1) represents the time rate of accumulation of mass within a unit volume element $A\Delta z$. The right side gives the net mass flow rate into the volume element.

2. Macroscopic Momentum Balance

From a macroscopic momentum balance the following equation is obtained:

$$\frac{\partial}{\partial t} \left[ W_\ell + W_g \right] \Delta z = \frac{A}{\Delta} \left[ \frac{W_\ell^2}{\rho_\ell (1-\alpha)} + \frac{W_g^2}{\rho_g} \right] - A\Delta \rho g - \tau_c \Delta z g_c.$$
The left side gives the time rate of accumulation of momentum within a unit volume element. The four terms on the right side represent respectively, the net rate of momentum flux due to the bulk fluid motion; the pressure differential across the control volume element; the net force of the solid surfaces on the fluid and the gravitational force on the total mass of the fluid in the control volume.

3. Macroscopic Energy Balance

The macroscopic energy equation is derived as:

\[
\frac{\partial}{\partial t} \left[ \alpha \rho g + \alpha \rho g C_c (T - T_o) + (1-\alpha) \rho \lambda C_l (T - T_o) \right] + z(\rho g + (1-\alpha)\rho \lambda) \rho g_c + \frac{1}{2A^2 \rho g_c} \left[ \frac{w^2}{\rho} + \frac{w^2}{\rho g_c} \right] = -\Delta \left[ \frac{w_0}{g} + \frac{w_1}{\rho g_c} (T - T_o) \right] \frac{w_0}{\rho g_c} \frac{w_1}{\rho g_c} (T - T_o)
\]

\[
= -\Delta \left[ \frac{w_0}{g} + \frac{w_1}{\rho g_c} (T - T_o) \right] \frac{w_0}{\rho g_c} \frac{w_1}{\rho g_c} (T - T_o)
\]

\[
- \Delta \left[ \frac{w_0}{\rho g_c} + (1-\alpha)\rho \lambda \right] \frac{w_0}{\rho g_c} \frac{w_1}{\rho g_c} (T - T_o)
\]

\[
- \frac{\rho g_c}{\rho} \Delta (z(W_g + W_\lambda)) + p_c h(T_w - T) \Delta z
\]
represents the potential energy of the system, \( \phi_{\text{tot}} \).

The first four terms on the right side of Eq. (4.3) are, respectively, the spatial difference of the internal energy, the expansion work due to pressure variation, the kinetic energy, and the potential energy. The last expression gives the thermal energy transferred from the heated surface to the bulk fluid.

4. Channel Wall Energy Balance

An energy balance on the channel wall gave:

\[
P_c \rho C_w \frac{\partial T}{\partial t} + \rho C \frac{h(T_c - T)}{T_0} = \frac{Q_{\text{gen}}}{T_0} \tag{4.4}
\]

B. Dimensionless Form of Equations of Change

Introducing the following dimensionless variables:

\[
W^* = \frac{W}{W_{\text{g},0}}; \quad W_L^* = \frac{W_L}{W_{\text{g},0}}; \quad T^* = \frac{C_{\text{g},0} C}{g H} T; \quad \rho^* = \frac{\rho C}{W_{\text{g},0}}; \quad p^* = \frac{\rho C}{W_{\text{g},0}}; \quad t^* = \frac{W_{\text{g},0}}{\rho AH} t; \quad Q^* = \frac{Q_{\text{gen}}}{\rho g H}; \quad z^* = \frac{Z}{H}; \quad \tau^* = \frac{A_{\text{c}}^2 C_{\text{c}}^2 \rho L}{W_{\text{g},0}^2} \tau.
\]

equations (4.1) to (4.4) can be written as:

\[
(1-\gamma) \frac{\partial a^*}{\partial t^*} = \frac{\Delta}{\Delta z^*} (W_{\text{g}}^* + W_L^*); \tag{4.5}
\]

\[
\frac{\partial}{\partial t^*} (W_L^* + W_{\text{g}}^*) = - \tau^* - \frac{\Delta p^*}{\Delta z^*} - a_1 (1-\alpha + \alpha \gamma) - \frac{\Delta}{\Delta z^*} \left( \frac{W_{\text{g}}^2}{1-\alpha} + \frac{W_L^2}{\alpha \gamma} \right) \tag{4.6}
\]
\[
\frac{\partial}{\partial t} \left\{ \alpha \gamma \left[ 1 + a_4(T^* - T_o^*) + a_3z^* + 0.5a_5 \left( \frac{\omega}{\omega^*} \right)^2 \right] \right\} \\
\quad + (1-\alpha) \left\{ a_3(T^* - T_o^*) + a_3z^* + 0.5a_5 \left( \frac{\omega}{\omega^*} \right)^2 \right\} \\
= - \frac{\Delta}{\Delta z^*} \left\{ \frac{a_5p^*}{1-\alpha + \alpha \gamma} \right\} \left[ \frac{\Delta}{\Delta z^*} \left\{ a_3(T^* - T_o^*) + a_3z^* + 0.5a_5 \left( \frac{\omega}{\omega^*} \right)^2 \right\} \\
\quad + \frac{a_5p^*}{1-\alpha + \alpha \gamma} \right\} + a_6(T_w^* - T^*)
\]

\[
\frac{a_3T_w^*}{a_3T_w^*} + a_7(T_w^* - T^*) = a_8 Q^*
\]

where

\[
\begin{align*}
a_1 &= \left( \frac{\alpha}{\omega_{L,0}} \right)^2 g_H \; ; \; a_2 = \frac{a_1}{g_H} \; ; \; a_3 = g_L^2 \; ; \; a_4 = \frac{a_2C_L}{C_L} \; ; \; a_5 = \frac{a_3}{a_1} \; ; \\
\end{align*}
\]

\[
\begin{align*}
a_6 &= \frac{p_c h_c h_w}{C_L \omega_{L,0} \lambda g_c} \; ; \; a_7 = \frac{\alpha p_c h_w}{\rho_c C_L \omega_{L,0} \omega_{w,0}} \; ; \; a_8 = \frac{\rho_c C_L \omega_{L,0} \omega_{w,0}}{p_c C_L \omega_{L,0} \omega_{w,0}}
\end{align*}
\]

In deriving equations \((4.5)\) to \((4.8)\) the following assumptions have been made: the physical properties of the liquid and vapour phases are constant in time and space and are referenced to the total system pressure; at any axial position the bulk fluid and the channel wall temperatures are uniform and all heat generated enters the coolant; axial conduction of energy in the channel wall and the bulk fluid is neglected; the two phase flow is one dimensional and radial variation of velocity, vapour concentration are neglected. A single valued
velocity may be associated with each phase and both the void fraction and phase velocities are smooth functions of time and space.

C. Steady State Equations of Change

The equations of mass, momentum, and energy transport are obtained by setting to zero the time dependent terms in Eqs. (4.1) to (4.8). In predicting the vapour fraction distribution along the channel axial length, three distinct regions are considered: the non-boiling region, where the void fraction is zero; the subcooled boiling region, where there is a rapid rise in void fraction; and the bulk boiling or thermodynamic equilibrium region.

The steady state macroscopic mass and energy balance are obtained from Eq. (4.5) and Eq. (4.7), respectively:

\[ \frac{\Delta}{\Delta z^*} (\bar{\omega}_g^* + \bar{\omega}_l^*) = 0 \]  \hspace{1cm} (4.9)

\[ \frac{d}{dz^*} \left\{ \bar{\omega}_g^* \left[ 1 + a_4(\bar{\tau}^* - \bar{\tau}_o^*) + a_3z^* + 0.5a_5 \left( \bar{\omega}_g^* \right)^2 + \frac{a_6^p}{1-a^* + \gamma a^*} \right] \right\} \\
+ \bar{\omega}_l^* \left[ a_3(\bar{\tau}^* - \bar{\tau}_o^*) + a_3z^* + 0.5a_5 \left( \bar{\omega}_l^* \right)^2 + \frac{a_6^p}{1-a^* + \gamma a^*} \right] \\
+ a_6(\bar{\tau}_w^* - \bar{\tau}^*) = 0 \]  \hspace{1cm} (4.10)

Also, the energy balance on the channel wall gave:

\[ (\bar{\tau}_w^* - \bar{\tau}^*) = \frac{a_8}{a_7} \]  \hspace{1cm} (4.11)

1. Non-boiling Region

In the non-boiling region the steam voidage \( \bar{\alpha} \) is zero and
Eq. (4.9) simplifies to:

$$\frac{\Delta}{\Delta z}(\bar{\bar{W}}^*_L) = 0 \quad (4.12)$$

and \( \bar{\bar{W}}^*_L = \bar{\bar{W}}^*_{L,0} = 1 ; \bar{\bar{W}}^*_{g} = 0 \).

The values of \( \bar{\bar{W}}^*_L \) and \( \bar{\bar{W}}^*_g \) are substituted into the macroscopic energy balance, Eq. (4.10) to obtain:

$$dz' \left[ a_3(T^* - T_0^*) + a_6(T_w^* - T^*) \right] = 0 \quad (4.13)$$

In obtaining Eq. (4.13) from Eq. (4.10), the pressure term was neglected as it was found to be very small compared to the other terms in the expression.

The expression for \( (T_w^* - T^*) \), Eq. (4.11) is substituted into Eq. (4.13) and the resulting equation integrated with the boundary condition at \( z = 0; T = T_0^* \), to give:

$$T^* = \frac{a_6a_7}{a_3a_7} z^* + T_0^* \quad (4.14)$$

Eq. (4.14) represents the expression for the bulk fluid temperature distribution in the non-boiling region.

2. Subcooled Boiling Region

The boundary conditions at the end of the non-boiling region, give the conditions at the inception of the subcooled boiling region. Thus, at the inception of subcooled boiling, \( \bar{\bar{W}}^*_L = 1 \), and \( \bar{\bar{W}}^*_g = 0 \).

The steady state macroscopic mass balance gave:
\[
\frac{\Delta}{\Delta z} (\bar{W}_g^* + \bar{W}_l^*) = 0 \tag{4.15}
\]

Eq. (4.15) is integrated and the boundary conditions \( \bar{W}_l^* = 1, \bar{W}_g^* = 0 \); at the transitional axial position, \( Z_T \), substituted to give:

\[
\bar{W}_g^* + \bar{W}_l^* = 1 \tag{4.16}
\]

where

\[
\bar{W}_g^* = \frac{(\bar{L} \bar{W} \bar{u}_g)}{\rho_g} \tag{4.17}
\]

and

\[
\bar{W}_l^* = \frac{(\bar{L} \bar{W} (1-\bar{a}) \rho_l)}{\rho_l} \tag{4.18}
\]

The slip ratio is defined as:

\[
S = \begin{bmatrix} \bar{V}_g \\ \bar{V}_l \end{bmatrix} \tag{4.19}
\]

The expression for \( \bar{V}_g \) and \( \bar{V}_l \) from Eqs. (4.17) and (4.18) are substituted into Eq. (4.19) to give:

\[
S = \begin{bmatrix} \bar{W}_g^* \\ \bar{W}_l^* \end{bmatrix} \left( \begin{array}{c} \rho_g \\ \rho_l \end{array} \right) \left( \begin{array}{c} \frac{1-\bar{a}}{\bar{a}} \end{array} \right) \tag{4.20}
\]

Also the steam weight fraction or quality, \( X \), is defined as:

\[
X = \bar{W}_g^* = \frac{A \bar{a} \bar{D}_g \bar{V} \rho_g}{A \bar{V} \rho_l} \tag{4.21}
\]

Combining Eqs. (4.21), (4.20), and (4.16) one obtains:

\[
S = \begin{bmatrix} X \\ 1-X \end{bmatrix} \left[ \begin{array}{c} \frac{1-\bar{a}}{\bar{a}} \\ \frac{1}{\gamma} \end{array} \right] \tag{4.22}
\]

where

\[
\gamma = \frac{\rho_p}{\rho_l}.
\]
A thermal energy balance on an infinitesimal axial length, \( dz \), for vapour generation gave:

\[
\frac{q_p}{\rho c} \, dz = \frac{q_p \, dz}{\lambda} \quad (4.23)
\]

Eq. (3.2) gives the total heat flux, \( q \), removed from the channel surface in the subcooled boiling region. Combining Eqs. (3.2) and (3.3) we obtain:

\[
q_a = \frac{\epsilon q}{1 + \epsilon} \quad (4.24)
\]

\[
q_e = \frac{q}{1 + \epsilon} \quad (4.25)
\]

A linear variation of \( \epsilon \) with dimensionless axial distance, \( z^* \), is assumed as a modification of Bowring's model (1):

\[
\epsilon = a(z^* - z_{T*}) + 2.6 \quad (4.26)
\]

A change of variable from \( z \) to \( \epsilon \) is introduced first by employing Eq. (4.26) and then Eq. (4.25) is substituted into Eq. (4.23). The resulting expression is integrated assuming \( q \) to be uniform, and applying the following boundary conditions:

For \( z^* = z_{T*} \), \( \dot{w}_g = X_b = 0 \), and \( \epsilon = 2.6 \), an expression for the steam quality, \( X_b \), at an axial position, \( z \), in the subcooled boiling region is thus obtained:

\[
X_b = \frac{q_p (z_{T*} - z_{B*}) \ln(1+\epsilon) - \ln(3.6))}{A \rho \epsilon \lambda} \quad (4.27)
\]

With the steam quality, \( X_b \), calculated from Eq. (4.27), the cross-sect-
ioned average void fraction, $\tilde{a}_b$, at any axial position can be calculated using Eq. (4.22), if the slip ratio, $S$, is known.

If the wall voidage, $\tilde{a}_w$, is significant, the total void fraction at any axial position, $z$, is equal to:

$$\tilde{a} = \frac{\tilde{a}_w}{\tilde{a}_w} + \tilde{a}_b$$

(4.28)

An expression for estimating the wall voidage is given in reference (1):

$$\frac{\delta}{\tilde{a}} = \frac{p\delta}{c}$$

(4.29)

where $\delta$ is the effective thickness of the vapour film, and in (1), $\delta$ is the lesser of:

$$\delta = \frac{0.066R}{d}$$

(4.30)

$$\delta = \frac{(Pr)kV_{in}}{1.07n\epsilon^2}$$

(4.31)

3. Bulk Boiling Region

In the bulk boiling region, the total heat flux, $q$, generated from the heated surfaces is converted into the latent heat of evaporation. Proceeding in the same manner as was done for the subcooled boiling region, the steam quality at an axial position, $z$, in the bulk boiling region is expressed as:

$$\chi = \chi_B + \frac{p\chi(z - Z_B)}{AV_{in}\lambda p \xi}$$

(4.32)

where $\chi_B$ is the steam quality calculated from Eq. (4.27) by setting $\epsilon$ equal to zero. Employing Eq. (4.22) and Eq. (4.32), the cross-sectional
average void fraction at an axial position, \( z \), in the bulk boiling region can be calculated if the slip ratio, \( S \), is known.

4. Boiling Region Transitions

The transitional axial position, \( Z_T \), represents the point at which steam bubbles begin to detach from the channel wall, and a rapid rise in the void fraction begins. The axial position \( Z_T \), is determined by the subcooling, \( \Delta T_r \), at the point of bubble detachment, which was given in (1) to be the least of:

\[
\Delta T_{in} = T_{sat} - T_{in} \tag{4.33}
\]

\[
\Delta T_{scb} = \frac{q}{h} - \beta q^{0.25} \tag{4.34}
\]

\[
\Delta T_{d} = \frac{\eta q}{V_{in}} \tag{4.35}
\]

where \( \Delta T \) represents the subcooling at the axial position considered; the subscripts \( r, in, scb, \) and \( d \) are explained in the nomenclature; \( \eta \) is the subcooled void parameter, and the value was obtained by analyzing experimental subcooled void fraction data; \( h \) is the single phase heat transfer co-efficient, and \( \beta \) has the value \((0.359)e^{(-0.196)(p-1)}\), (1). For the pressure range of 11 to 136 atmospheres, \( \eta \) has the value of \((8.62)(14.0 + (0.1)p)10^{-5}\), (1).

The value of \( Z_T \) corresponding to the subcooling \( \Delta T_r \) is determined from a heat balance equation:

\[
Z_T = \frac{\bar{V}_{in} A_p e_c (\Delta T_{in} - \Delta T_r)}{q_p e} \tag{4.36}
\]
Z may be calculated from the expression, Eq. (4.14), for the bulk fluid temperature profile in the non-boiling region, by setting $T^* = T_{zt}^*$, where $T_{zt}^*$ is given by:

$$T_{zt}^* = Tsat - C_l \frac{\Delta T}{H}$$  \hspace{1cm} (4.37)

At the subcooled boiling region and bulk boiling region boundary, $Z_B$, the bulk fluid temperature is equal to the saturation temperature. The steady state macroscopic liquid energy balance is obtained from Eq. (4.10):

$$\frac{d}{dz} \left\{ -a_3 (T^* - T_0^*) + a_3 Z^* + 0.5 a_3 \left( \frac{p_{lw}^*}{1-a} \right)^2 + \frac{a_5 p^*}{1-a + \gamma a} \right\} = a_{12} (\tilde{T}_w^* - \tilde{T}_l^*)$$  \hspace{1cm} (4.38)

where $a_{12} = \frac{p_c h_b g H^2 \varepsilon}{c_l \lambda_g c (1 + \varepsilon)}$.

The mechanical energy terms (potential and kinetic), and the pressure term in Eq. (4.38) were found to be small when compared with the sensible heat term. Hence Eq. (4.38) simplifies to:

$$\frac{d}{dz} \left\{ -a_3 (T^* - T_0^*) \right\} = \frac{p_c h_b g H^2 \varepsilon (\tilde{T}_w^* - \tilde{T}_l^*)}{c_l \lambda_g c (1 + \varepsilon)}$$  \hspace{1cm} (4.39)

Upon substituting for $(\tilde{T}_w^* - \tilde{T}_l^*)$ from Eq. (4.11) the following is obtained:

$$\frac{d}{dz} \left\{ -a_3 (T^* - T_0^*) \right\} = \frac{p_c h_b g H^2 a \varepsilon}{c_l \lambda_g c a_7 (1 + \varepsilon)}$$  \hspace{1cm} (4.40)

Since $\tilde{w}_l^* = 1 - X_b$, and is approximately equal to unity in the subcooled boiling region Eq. (4.40) is expanded into:
\[
\frac{dT^*}{dz^*} = \left[ \frac{p c h_b h^2 a_8 \epsilon}{C_k \tau^*_1, o \lambda a_3 \gamma (1 + \epsilon)} + \frac{(T^* - T_o^*)}{dz^*} \right] \quad (4.41)
\]

A change of variable from \( z^* \) to \( \epsilon \) is made, and Eq. (4.46) is integrated by applying the appropriate boundary conditions at \( Z_T^* \), and \( Z_B^* \) to obtain an expression for the bulk boundary axial position, \( Z_B^* \):

\[
\frac{Z_B^* - Z_T^*}{A} = 1.85 a_4 a_1 a_3 C_k \lambda W_z, o \quad (4.42)
\]

where

\[
a_4 = \frac{A \rho h^b}{C_t W_z, o \rho_w}.
\]

An approximate temperature distribution in the subcooled boiling region is obtained by integrating Eq. (4.41), neglecting the last term in the bracket in Eq. (4.41):

\[
T^* = T_{sat} - \frac{A \rho h^b}{C_t W_z, o \lambda 2.6} (\gamma - 1.4191 - \log (1 + \epsilon))
\]

Alternatively, the bulk fluid temperature at any axial position, \( z \), is calculated from equation (4.41) by employing Runge Kutta's method of integration.

D. Unsteady State Equations of Change

Equations (4.5) to (4.8) will now be solved with the following assumptions: a constant pressure drop exists across the coolant channel; variations of pressure within the channel are negligible; the sensible heat terms are negligible with respect to the latent heat term; the inlet bulk fluid temperature and inlet velocity are constant.

Three distinct regions of the channel will be considered: the non-
boiling region, the subcooled boiling or non-equilibrium region, and the bulk boiling region. The superscript * in Eqs. (4.5) to (4.8) will be dropped from the dimensionless variables, and a very small power modulation will be considered. Hence, the spatial derivative in Eqs. (4.5) to (4.8) will be eliminated by employing the method of central differences and dividing the channel into N parts over which the stream variables are assumed to vary linearly. The equations obtained are integrated over one section employing the trapezoidal rule for evaluation of an integral.

The transient value of a function F will be defined as:

$$ F = \bar{F} + F' $$

(4.44)

where \( \bar{F} \) is the steady state value of the function, F, and \( F' \) is equal to \( \text{Re}(F^c e^{i\omega t}) \) and is the deviation of the transient value, F, from the steady state value. Here, \( F^c \) is a complex number whose real and imaginary parts yield the amplitude and the phase lag of F. Since small disturbances of the stream variables are considered, the value of a stream variable will be calculated employing a Taylor series expansion around the steady state value of the variable.

The unsteady state equations of change are developed in Appendix (1-3). The final forms of the equations for the three distinct regions of the channel are summarized here.

1. **Non-boiling Region**

In the non-boiling region, there are no free steam bubbles, and the problem variables are the liquid temperature, \( T(t,z) \), and the wall temperature, \( T_w(t,z) \). The channel wall and the bulk fluid tempera-
ture responses at a given spatial node, are given respectively by:

\[
\tau_{w,N}^c = a_0^c + \frac{a_7^c (a_7 - i\omega)}{(a_7^2 + \omega^2)} + a_2^c,
\]

\[
T_{N+1}^c = \left\{ \begin{array}{l}
0.5i\omega + \frac{1}{a_3} \frac{a_7(a_7^2 + \omega^2)}{(a_7^2 + \omega^2) - 1} \\
+ a_1 a_0^c(a_7 - i\omega) \frac{(a_7^2 + \omega^2)}{a_7^2 + \omega^2} - 1
\end{array} \right\}
\]

\[
T_N^c \left[ \frac{1}{\Delta z} - 0.5i\omega \right] + a_1 a_0^c(a_7 - i\omega) \frac{(a_7^2 + \omega^2)}{a_7^2 + \omega^2}.
\]

The value of \( T_N^c \) at each spatial node is calculated applying the appropriate boundary conditions at the inlet to the channel: \( z = 0, N = 0, \) and \( T_N^c = 0 \). From the real and imaginary parts of \( T_N^c \), the amplitude and the phase lag of the temperature responses are calculated.

The total temperature response of the bulk fluid at the transition axial position, \( Z_T \), is obtained by summing the temperature response calculated by a stagewise integration of Eq. (4.46) up to the transition boundary, \( Z_T \), and the bubble detachment effect which is equal to:

\[
\tau_T^c = \frac{C_{\text{gen}}^c Y_{\text{in}}^c}{H p V_{\text{in}}^c} \]

2. **Subcooled Boiling Region**

In the subcooled boiling region, the problem variables are: the void fraction, \( \alpha(t,z) \); the vapour phase velocity, \( V_g(t,z) \); the liquid phase velocity, \( V_L(t,z) \); the bulk fluid temperature, \( T(t,z) \); and the channel wall temperature, \( T_w(t,z) \).
The dimensionless macroscopic mass, vapour energy, and liquid energy balance equations are respectively:

\[
\left[0.5(1 - \gamma)\right]_{N+1/2} \frac{d}{dt} \left\{ \alpha'_{N+1} + \alpha'_N \right\} = W'_{g,N+1} + W'_{g,N+1} - W'_{g,N} - W'_{g,N},
\]

\( (4.48) \)

\[
C_{N+1/2} \frac{d}{dt} \left\{ \alpha'_{N+1} + \alpha'_N \right\} + C_{N+1/2} \frac{d}{dt} \left\{ \nu'_{g,N+1} + \nu'_{g,N} \right\}
\]

\[- CG_{N+1} \nu'_{g,N+1} + CG_{N} \nu'_{g,N} + CC_{N+1} \alpha'_{N+1} - CC_{N} \alpha'_N
\]

\[+ 0.5 \left\{ \left( a_{13}(T'_w - T'_c) \right)_{N+1} + \left( a_{13}(T'_w - T'_c) \right)_{N} \right\} \]

\( (4.49) \)

where \( a_{13} = \frac{p_c g H^2 h_b}{C_k \bar{W}_k, \lambda (1 + \varepsilon) g_c} \).

\[
C_{C_k} \frac{d}{dt} \left\{ \alpha'_{N+1} + \alpha'_N \right\} - C_{C_k} \frac{d}{dt} \left\{ \nu'_{g,N+1} + \nu'_{g,N} \right\} - C_{C_k} \frac{d}{dt} \left\{ \nu'_{g,N+1} \right\}
\]

\[+ W'_{ks,N} \right\}
\]

\[- CS_{N+1} \nu'_{g,N+1} - CS_{N} \nu'_{g,N} + CV_{N+1} \nu'_{g,N+1} - CV_{N} \nu'_{g,N} - CV \alpha'_{N} + CV_{N+1} \alpha'_{N+1}
\]

\[+ 0.5 \left\{ \left( a_{12}(T'_w - T'_c) \right)_{N+1} + \left( a_{12}(T'_w - T'_c) \right)_{N} \right\} \]

\( (4.50) \)

where \( a_{12} = \frac{p_c g H^2 h_b \varepsilon}{C_k \bar{W}_k, \lambda (1 + \varepsilon) g_c} \).

The energy storage equation in the channel wall simplifies to:

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\[
\frac{dT_w}{dt} + a_{14}(T_w - T) = a_8 Q
\]  
(4.51)

where \( a_{14} = \frac{A_\delta b H}{\rho C_W W_f W_i \ell, o} \).

The quality is related to the void fraction by:

\[
\frac{W}{g, N} = N \frac{W}{t, N} + \frac{M}{N} \frac{\alpha}{N}
\]  
(4.52)

where \( N \) and \( M \) are obtained from any of the theoretical model or correlation relating steady state average cross-sectional void fraction to steam quality.

From Eqs. (4.48) to (4.52), the void response, the flow response, and the temperature response are calculated by employing a stagewise integration method, and the appropriate boundary conditions at the transition axial position \( Z_T \). There are two possible cases of the transition position as determined by using Bowring’s criterion (Eqs. (4.33) to (4.35)). If there is no non-boiling length, the boundary conditions obtained are: for \( N = 0, z = 0, \alpha'_o(t) = \frac{W}{g, o}(t) = T'_o(t) = 0 \).

If the non-boiling length exists, then the boundary conditions are:

for \( N = N_{zt}, z = Z_T, \) and \( T_{zt} \) is the total temperature response of the bulk fluid at the transition boundary and is discussed in Appendix I.

The void response \( \alpha'_{zt}(t) \), and the flow response \( \frac{W}{z, zt}(t) \) are obtained by calculating the quality change \( \frac{dX}{dz} \) due to the movement of the transition axial position, \( Z_T \), and then employing a quality-void fraction correlation to calculate \( \alpha'_{zt}(t) \) and \( \frac{W}{z, zt}(t) \). An expression for \( Z_T \) is given in Appendix I, Eq. (A.1.8).

An additional component to the void and flow responses results...
from the movement of the bulk boiling boundary, $Z_B$. This contribution to the flow and void responses at an axial position, $z_{N+1}$, in the sub-cooled boiling region was given in reference (36):

$$W_{B,N+1} = \frac{dX}{dz} Z_{B,N+1}$$

$$Z_{B,N+1} = \left( \frac{dz}{dT} \right) (T'_{N+1} - T'_N)$$

where $\frac{dX}{dz}$ and $\frac{dz}{dT}$ are obtained from equations (4.23) and (4.41) respectively. In Eq. (4.54), $Z_{B,N+1}$ is taken to be in phase with $T'_{N+1}$. The temperature responses, $T'_{N+1}$, and $T'_N$, are calculated using Eqs. (4.48) to (4.52). A quality-void fraction correlation is then employed to calculate the additional component to the void response arising from the movement of $Z_B$. The total void response is then obtained by summing the effect of the movement of $Z_B$, and that calculated using Eqs. (4.48) to (4.52).

3. Bulk Boiling Region

In the bulk boiling region, the bulk fluid temperature is equal to the saturation temperature. If the pressure variation within the channel is neglected, the saturation bulk fluid temperature is constant through the entire length of the bulk boiling region, and is referenced to the system operating pressure. Hence, the problem variables reduce to four: the void fraction, $\alpha(t,z)$; the liquid phase velocity, $V_L(t,z)$; the vapour phase velocity, $V_g(t,z)$; and the channel wall temperature, $T_w(t,z)$.

The dimensionless macroscopic mass, and energy balance equations
are, respectively:

\[ 0.5(1 - \gamma) \frac{d}{dt} \left( \alpha_{N+1} + \alpha_N \right) = \frac{W_{g,N+1}}{\left( n+1 \right)} + \frac{W_{e,N+1}}{\left( n+1 \right)} - \frac{W_{g,N}}{\left( n+1 \right)} - \frac{W_{e,N}}{\left( n+1 \right)} \]  

\[ - \frac{P_{N+1}}{\left( n+1 \right)} + \frac{q_{N}}{\left( n+1 \right)} + r_{N} - s_{N+1} + a_{N} + a_{N+1} + a_{N+1}^{2} = 0 \]  

(4.55)

(4.56)

In the bulk boiling region, the channel wall temperature response at any spatial node gave:

\[ T_{w,N}^{c} = \frac{a_{8}Q_{o}^{c}(a_{7} - i\omega)}{a_{8}^{2} + \omega^{2}} \]  

(4.57)

Employing Eqs. (4.55) to (4.57) and an expression Eq. (4.52) relating quality and void fraction, the flow response, and the void response at any spatial node can be calculated employing a stagewise integration procedure, and the boundary conditions at Z_g. Since St. Pierre's experimental void amplitude versus axial distance curve was smooth and continuous, the values of the void and the flow responses calculated at Z_g using the subcooled boiling region equations, Eqs. (4.48) to (4.52), constitute the boundary condition for N = 0, for the bulk boiling region.
V DISCUSSION

A. Steady State Analysis

For a set of data on heat flux, pressure, inlet subcooling, inlet water velocity, and channel cross-section, the values of steam quality in the subcooled boiling region are calculated using Eq. (4.27). In the bulk boiling region, Eq. (4.32) is employed to predict the axial distribution of the steam quality.

Bowring's model (1) was employed also to predict the axial distribution of steam quality for the same reactor operating conditions. Employing the predicted values of steam quality from Eqs. (4.27) and (4.32), together with one of the theoretical models (4,5,7) or correlations (13,14) relating quality and void fraction, the steady state cross-sectional void fractions were predicted.

The modified Bowring's model with \( \varepsilon \) varying linearly with axial distance in the subcooled boiling region, predicted larger values of steam quality at the bulk boiling boundary than Bowring's model (1). In the bulk boiling region, this resulted in larger steam qualities, and hence, greater void fractions than those obtained with Bowring's model.

Comparisons of the predicted cross-sectional average void fractions and steam qualities in the bulk boiling region with the experimental void fraction and steam quality data of Rouhani (17) are shown in Figs. 1 and 2. A better agreement between experimental data and the predicted values was obtained with the modified Bowring's model. Of the models and correlations employed to relate quality and void frac-
tion, Neal's model (7) gave the closest agreement with the experimental data (see Figs. 1 and 2). The agreement obtained with Rankoff's model (4) and Marchaterre-Hoglund's correlation (13) was approximately the same.

The heating component, $q_a$, of the total heat flux was used to calculate the axial position $\bar{z}_{B}$ at which the bulk fluid temperature equals the saturation temperature. The bulk boiling boundary, $\bar{z}_{B}$, was located using Eq. (4.42). This boundary was found to lie downstream of the bulk boiling boundary computed by employing a constant value of $\varepsilon$ equal to 1.3 in the subcooled boiling region. This may not be expected since the average value of the variable $\varepsilon$ in the subcooled boiling region was 1.3, the same value as was assumed constant throughout the subcooled boiling region by Bowring. However, it is the value of the heating component, $q_a$, and not $\varepsilon$, that determines $\bar{z}_{B}$.

The ratio of the average value of $q_a$ in the subcooled boiling region for the two cases of $\varepsilon$ equal to 1.3 and $\varepsilon$ given by Eq. (4.26) was found to be 1.02:1 respectively. The bulk boiling boundary computed with Eq. (4.42) is labelled $\bar{Z}_B'$ in all Figs. 3 to 12. The boundary denoted as $\bar{Z}_B'$ is that computed by employing Bowring's model (1) for some available experimental data (29,36).

Bowring's physical model (1) was employed in the subcooled boiling region by St. Pierre (2) to compare with available steady state void fraction data (2). The predicted void fraction versus axial distance profiles had a discontinuous change in slope at the bulk boiling boundary, $\bar{Z}_B'$. For the present analysis, the modified Bowring's model employing a linear variation of $\varepsilon$ with dimensionless axial distance and $\varepsilon$ equal to 1.3 in the subcooled boiling region were employed together with theoretical models and correlations relating quality and void fraction to compare with the experimental data of Christensen (29) and St. Pierre (36), (see Figs. 3 to 10).
It is apparent that the predicted void fraction profiles employing the modified Bowring's model are smooth and continuous in contrast to those predicted with a constant $\varepsilon$, which have a discontinuous change in slope at the bulk boiling boundary, $Z_B$.

All of the theoretical models and correlations were developed for the bulk boiling or thermodynamic boiling region excepting Ref. 16. However, these models and correlations were employed to relate quality and void fraction in the subcooled boiling region, and the agreement between the predicted values of void fraction and experimental data was good (see Figs. 3 to 10). In Figs. (4, 5, 6, 8, 9), the subcooled boiling region is appreciable. Here, the agreement might be considered fortuitous since the theoretical models and correlations are not strictly valid in the subcooled boiling region.

The use of a constant slip ratio of unity in the subcooled boiling region was attempted. This no-slip condition was employed together with Eq. (4.27) to predict void fraction profiles. A discontinuous change in the slope of the void profile at the bulk boiling boundary was obtained and for this reason the void profiles were not included in the figures.

A linear variation of slip with axial distance (in the subcooled boiling region) was assumed also. An initial value of 0.8 for slip at the transition boundary, $Z_T$, was employed based on Gunther's data (43). The value of slip at the bulk boiling boundary was determined by employing one of the theoretical models or correlations. Comparisons of the predicted cross-sectional average void fraction with experimental void data of Christensen (29) are shown in Figs. 11 to 12.

Finally, a theoretical model proposed by Zuber, Staub and Bijwaard (16) valid for the subcooled boiling region was tested. The predicted void
fraction profiles were compared with those of the modified model (see Figs. 3 to 10).

B. Unsteady State Analysis

Of all the theoretical models and correlations relating quality and void fraction employed together with the modified Bowring's model to predict steady state void fractions, Neal's model (7) and Bankoff's (4) gave the best agreement with the available experimental void data. Hence, these two models were employed for the unsteady state analysis to relate quality and void fraction. The predicted void amplitudes and phase lags employing these models were approximately the same.

The modified model was employed for the stage-wise integration of the void, flow, and temperature response equations, Eqs. (4.48) to (4.52), in the subcooled boiling region. The predicted void amplitudes and phase lags employing the modified model and those obtained using a constant value of $\varepsilon$ equal to 1.3 are shown in Fig. 13 together with the experimental frequency response data of St. Pierre (36).

The resulting effect of the movement of the bulk boiling boundary, $Z_B$, on the void response in the subcooled boiling region was accounted for by considering the effect to spread out through the subcooled boiling region. This procedure gave a smooth continuous void amplitude versus axial distance curves.
IV CONCLUSIONS

A linear variation (Eq. 4.26) of the parameter ε based on a qualitative consideration of the factors affecting ε was assumed to relate the heating component, $q_a$, to the evaporative component, $q_e$, of the total heat flux, $q$. Physically the parameter ε should vary from some large value at the transition boundary, $Z_T$, to zero at the bulk boiling boundary. Employing this assumed expression of ε, it was found that:

1. The predicted void fraction profiles were smooth and continuous.

2. For the steady state analysis, the predicted cross-sectional average void fractions were in close agreement with the available void fraction experimental data.

3. Of all the theoretical models and correlations employed to relate quality and void fraction, Neal's (7) and Bankoff's (4) gave the best agreement with the available experimental void data for the subcooled boiling region and the bulk boiling region.

4. For the dynamic analysis, the agreement between the predicted void amplitude, the phase lag and experimental data was good. Thus the use of steady state values of void fraction, bulk fluid temperature and the parameter ε in a small perturbation analysis is satisfactory.

For all the available reactor conditions tested the theoretical model proposed by Zuber et al (16) predicted much lower values of void fraction than the experimental data.
The effect of the inlet subcooling, $\Delta T_{in}$, on the predicted void fraction is shown in Figs. 3 and 4, and one can draw the following conclusions:

1. As the inlet subcooling is decreased, the subcooled boiling length decreases.

2. The steady state vapour generation increases with decrease of the inlet subcooling.

The effect of the system pressure is shown in Figs. 7 and 8. In Fig. 7 the subcooling is less than that of Fig. 8, and thus the exit void fraction should be higher than that of Fig. 8. However, the exit void fraction in Fig. 8 is higher than that of Fig. 7. This indicates that as the system pressure increases, the volumetric rate of steam generation decreases.

Concluding, it should be noted that a number of assumptions were made to simplify both the steady state and the unsteady state analysis. Some of these assumptions are:

1. The effect of bubble condensation as the bubble is swept downstream by the subcooled liquid was not included in this present analysis. For very low inlet liquid flow rate, this effect may be important in determining the void fraction in the subcooled boiling region and should be accounted for. The experimental void data tested were obtained for appreciably high liquid velocities. Consequently, this effect was assumed negligible.

2. For the dynamic analysis an expression for the single phase heat transfer coefficient that accounts for the void fraction distribution must be employed. This variation of the heat transfer coefficient as a function of the void fraction will result in a different rate of heat...
transfer and hence energy storage in the channel wall.

Although the agreement between the predicted cross-sectional average void fractions and the experimental void data was good, it is apparent that the present expressions employed to account for such variables as wall voidage, and the slip ratio in the subcooled boiling region are not satisfactory. The overall result will be much improved if the exact forms of the wall voidage, the parameter $\epsilon$ and the slip ratio in the subcooled boiling region are known. These require extensive accurately measured void fraction, quality, liquid phase velocity and vapour phase velocity data in the subcooled boiling region which are not available to date.
Fig. 1. Void Fraction Predictions Using the Modified Bowring's Model and Bowring's Model with Experimental Data (Ref. 17).
Fi, 2. Void Fraction Prediction Using the Modified Bowring's Model and Bowring's Model with Experimental Data (Ref. 17).
Run 0
\[ q = 6.76 \times 10^4 \text{ Btu/hr-ft}^2 \]
\[ p = 400 \text{ psia} \]
\[ V_{\text{in}} = 2.526 \text{ ft/sec.} \]
\[ \Delta T_{\text{in}} = 5.20 \text{ F} \]

Model
A Rogers
B Neal
D March-Hoglund
E Bankoff (Jones)
F Bankoff (Neal & Zivi)
G Zuber, Staub & Bilwaard

Figure 3. Comparison of Christensen's Experimental Data (Ref. 20) with Values Predicted by Modified Bowering's Model.
Fig. 4. Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model.
**Fig. 5.** Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model.
Parameters
\( p = 1000\text{psia,} \)
\( \eta = 15.70 \times 10^4 \text{ Btu/hr-ft}^2 \)
\( V_{in} = 3.77 \text{ ft/sec.} \)
\( \Delta T_{in} = 21.8 \text{ F} \)

Model
A Rogers
B Neal
C Rankoff (Neal & Zivi)
D March.-Hoglund
E Rankoff (Jones)
F Zuher, Staub & Ritwaard

Fig. 6. Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by Modified Bowring's Model.

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Parameters
\[ \begin{align*}
  p &= 800 \text{psia} \\
  q &= 4.56 \times 10^4 \text{ Btu/hr-ft}^2 \\
  V_{in} &= 2.52 \text{ ft/sec.} \\
  \Delta T_{in} &= 4.9 \text{ } ^\circ\text{F}
\end{align*} \]

Model
\[ \begin{align*}
  &\text{A Rogers} \\
  &\text{B Neal} \\
  &\text{C Bankoff(Jones) constant } \epsilon \\
  &\text{D March.-Hoglund} \\
  &\text{E Bankoff(Jones)} \\
  &\text{F Bankoff(Neal & Zivi)} \\
  &\text{G Zuber,Staub & Bilwaard}
\end{align*} \]

Fig. 7. Comparison of St.Pierre's Experimental Data (Ref. 36) with
Values Predicted by Modified Bowring's Model and constant \( \epsilon \).
Parameters

- $p = 400$ psia
- $q = 4.56 \times 10^4$ Btu/hr-ft$^2$
- $V_{in} = 2.52$ ft/sec
- $\Delta T_{in} = 0.5^\circ F$

Model

- A Rogers
- B Neal
- C Rankoff (Jones) constant $\epsilon$
- D March.-Hoglund
- E Bankoff (Jones)
- F Bankoff (Neal & Zivi)
- G Zuher, Staub & Rijwaard

Fig. 8. Comparison of St. Pierre's Experimental Data (Ref. 36) with Values Predicted by Modified Bowring's Model and constant $\epsilon$. 
$\rho = 400 \text{ psia}$
$q = 6.4 \times 10^4 \text{ Btu/hr-ft}^2$
$V_{in} = 3.78 \text{ ft/sec}$
$\Delta T_{in} = 11.1 \degree F$

Parameters

Model
A Rogers
B Neal
C Bankoff(Jones) constant $\varepsilon$
D March.-Hoglund
E Bankoff(Jones)
F Bankoff(Neal & Zivi)
G Zuber,Staub & Bilvaard

Fig. 9. Comparison of St. Pierre's Experimental Data(Ref. 36) with Values Predicted by Modified Bowring's Model and constant $\varepsilon$. 

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Fig. 10. Comparison of St. Pierre's Experimental Data (Ref. 36) with Values Predicted by Modified Bowring's Model and constant $\epsilon$. 

Parameters
- $p = 300$ psia
- $q = 2.28 \times 10^5$ Rtu/hr-ft$^2$
- $V_{in} = 2.52$ ft/sec
- $\Delta T_{in} = 1.8$ $^\circ$F

Model
- A Rogers
- B Neal
- C Bankoff(Jones) constant $\epsilon$
- D March-Hoglund
- E Bankoff(Jones)
- F Bankoff(Neal & Zivi)
- G Zuber,Stauh & Bijwaard

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Fig. 11. Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by an assumed Linear Variation of Slip Ratio with axial distance in the Subcooled Boiling Region.
Fig. 12. Comparison of Christensen's Experimental Data (Ref. 29) with Values Predicted by an assumed Linear Variation of Slip Ratio with axial length in the Subcooled Boiling Region.

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**NOMENCLATURE**

Dimensions are given in terms of mass (M), length (L), time (t), and temperature (T).

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
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<tr>
<td>A</td>
<td>Cross-sectional area of channel</td>
<td>$L^2$</td>
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<tr>
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<td>Constant defined for Eq. (4.26)</td>
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<td>Constant defined for Eq. (4.49)</td>
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<td>Constant defined for Eq. (4.42)</td>
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<td>C</td>
<td>Heat capacity at constant pressure per unit mass</td>
<td>$L^2/t^2T$</td>
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<td>g</td>
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<td>Description</td>
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<tr>
<td>$g_c$</td>
<td>Gravitational conversion factor</td>
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<tr>
<td>$H$</td>
<td>Total channel length</td>
<td>L</td>
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<td>$h$</td>
<td>Single phase heat transfer coefficient</td>
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<td>$k$</td>
<td>Kinetic energy</td>
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<td>$k$</td>
<td>Thermal conductivity</td>
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<td>$p$</td>
<td>Fluid pressure</td>
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<td>Perimeter of channel</td>
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<td>Rate of energy generation in test section walls per unit length</td>
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<td>$Q_o$</td>
<td>Fractional power variation</td>
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<td>$q$</td>
<td>Total heat flux</td>
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<td>Agitative or heating component of $q$</td>
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<tr>
<td>$q_e$</td>
<td>Evaporative component of $q$</td>
<td>$M/t^3$</td>
</tr>
<tr>
<td>$q_c$</td>
<td>Heat loss by condensation</td>
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<td>Constant defined for Eq. (A.3.5)</td>
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<td>$q_{sp}$</td>
<td>Single phase heat component of $q$</td>
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<td>$R_d$</td>
<td>Vapour bubble diameter</td>
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<td>Slip ratio</td>
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<td>$T$</td>
<td>Temperature</td>
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<td>$T_{sat}$</td>
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<td>T</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>t</td>
</tr>
<tr>
<td>$t_w$</td>
<td>Thickness of test section wall</td>
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<tr>
<td>$U$</td>
<td>Internal energy</td>
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<td>$V$</td>
<td>Fluid velocity</td>
<td>$L/t$</td>
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<td>$W$</td>
<td>Mass flow rate</td>
<td>$M/t$</td>
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<td>$X$</td>
<td>Steam weight fraction or quality</td>
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<td>$Z_T$</td>
<td>Transition boundary</td>
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<tr>
<td>$Z_p$</td>
<td>Bulk boiling boundary</td>
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### Greek Symbols

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<th>Description</th>
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<td>Cross-sectional average void fraction</td>
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<td>Constant defined for Eq. (4.34)</td>
<td>$t^{3/4}T/M^{1/4}$</td>
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<tr>
<td>$\delta$</td>
<td>Vapour film thickness</td>
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<td>$\epsilon$</td>
<td>Ratio of the agitative to evaporative heat fluxes</td>
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<td>Constant defined for Eq. (4.35)</td>
<td>$TLt^2/M$</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<td>$\tau$</td>
<td>Shear stress tensor</td>
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<td>$\phi$</td>
<td>Potential energy</td>
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<td>$\lambda$</td>
<td>Latent heat of evaporation</td>
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<tr>
<td>$\gamma$</td>
<td>Ratio of the vapour phase to the liquid phase densities</td>
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### Superscripts

- $^*$: Reduced with respect to some characteristic-dimension variable
- $'$: Denotes deviation of a quantity from the steady state value
- $-$: Denotes steady state value of a quantity
- $c$: Denotes a complex number whose real and imaginary parts yield the transient response of that variable

### Subscripts

- $b$: Denotes subcooled boiling region
- $g$: Denotes vapour phase
- $L$: Denotes liquid phase
- $in,o$: Denotes inlet of test section
- $p$: Denotes constant pressure
- $tot$: Denotes total quantity in macroscopic system
- $w$: Denotes test-section wall
- $zt,T$: Denotes transition boundary
s Denotes the thermodynamic point for the condition with no subcooled boiling

r Denotes the point of rapid rise in subcooled void fraction

sch Denotes subcooled boiling

d Denotes bubble detachment condition

B Denotes bulk boiling boundary
REFERENCES


APPENDIX I

Unsteady State Equations of Change in the Non-boiling Region

When an external disturbance, in the form of a power modulation, is applied to the test section, there is a dynamic interchange of heat between the subcooled bulk fluid and the channel wall. The bulk fluid temperature response is obtained by solving the bulk fluid energy equation, and the energy balance equation on the channel wall. In the non-boiling region, \( W_b = 1 \), and \( W_g = 0 \), since no free steam bubbles exist. Therefore, the energy equation, Eq. (4.7), reduces to:

\[
\frac{\partial T'}{\partial t} + \frac{\partial T'}{\partial z} = \frac{a_{10}(T'_w - T')}{a_3} \tag{A.1.1}
\]

where \( a_{10} = \frac{\rho c_b H^2 g}{C_w L_0 \lambda g_c} \).

The energy balance on the channel wall gave:

\[
\frac{\partial T'_w}{\partial t} + a_7(T'_w - T') = a_8 Q' \tag{A.1.2}
\]

where \( Q' \), the power modulation is expressed as \( \text{Re}(Q'_o e^{i\omega t}) \). An expression for \( T'_{w,N} \) is derived from Eq. (A.1.2):

\[
T'_{w,N} = a_8 Q'_o + \frac{a_7 T'_N(a_7 - i\omega)}{a_7^2 + \omega^2} \tag{4.45}
\]

A difference-differential approximation method is employed to obtain a solution to Eq. (A.1.1):

\[ \text{K3} \]
As a first approximation, \((T'_w - T'_T)^{N+1/2}\) may be written as:

\[
(T'_w - T'_T)^{N+1/2} = 0.5 \left\{ (T'_{w, N+1} - T'_{N+1}) + (T'_{w, N} - T'_{N}) \right\}
\]  

(A.1.4)

After Eq. (A.1.3) is substituted into Eq. (4.45), the resulting expression is simplified to give:

\[
(T'_w - T'_c)^{N+1/2} = 0.5 \left[ \frac{a_7(a_7 - i\omega)}{a_7 + \omega^2} \right] - \frac{1}{1} \left( T'_c + T'_N \right) + \frac{a_8q_c(a_7 - i\omega)}{a_7^2 + \omega^2}
\]

(A.1.5)

Combining Eqs. (A.1.3) and (A.1.5), an expression is derived for \(T'_c\):

\[
T'_c\{N+1\} = \left[ \frac{0.5i\omega + 1}{1 - \frac{a_10}{a_3} \frac{a_7(a_7 - i\omega)}{a_7 + \omega^2} - 1} \right] \left[ T'_c \left[ \frac{1}{1} \right] - 0.5i\omega \right]
\]

\[
+ \frac{a_10}{a_3} \left[ \frac{a_7(a_7 - i\omega)}{a_7^2 + \omega^2} - 1 \right] + \frac{a_10a_8q_c(a_7 - i\omega)}{a_3(a_7^2 + \omega^2)}
\]

(4.46)

The variation in position of the transition boundary, \(Z_T\), introduces an additional component to the amplitude and phase lag of the temperature response at the transition point. The steady state transition point, \(Z_T\), is determined from the bubble detachment criterion, Eqs. (4.35), which in dimensionless form, may be written as:

\[
T_{sat} - T_{zt} = \frac{C_p \bar{n} \delta \cot \varphi_c}{H_{cp} \bar{V}_{in} \delta}
\]

(A.1.6)
By expanding Eq. (A.1.6) in a Taylor series expansion around steady state, we obtain:

\[
T_{zt}^c = \frac{C_{\eta} \bar{Q}_{\text{gen}} Q_o^c}{H P_{\text{in}}} 
\]  

(A.1.7)

Eq. (A.1.7), represents the bubble detachment contribution to the total temperature response, at the transition point resulting from a power modulation.

The unsteady state value of the transition boundary, $Z_T^c$, is obtained from a Taylor series expansion around the steady state value:

\[
Z_T = \bar{Z}_T + \left( \frac{\partial Z}{\partial T} \right)_{zt} T' 
\]

(A.1.8)

where $T_{zt}$, is the total temperature response at $Z_T$, and $\left( \frac{\partial Z}{\partial T} \right)_{zt}$, is obtained from Eq. (4.14). The total temperature response at $Z_T$ is obtained by adding the effect (Eq. (A.1.7)) due to the variation in position of the transition boundary, to that obtained by a stagewise integration of Eq. (4.46) up to $Z_T$. 

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APPENDIX II

Unsteady State Equation of Change in the Subcooled Boiling Region

The treatment of the subcooled boiling region, will be based on the modified Bowring's model for the steady state prediction of the vapour void fraction distribution. Pressure variation within the channel will be assumed negligible. This eliminated the compression term in the energy equation, Eq. (4.7), making it possible for Eq. (4.7) to be separated into a liquid energy equation, and a vapour energy equation. Thus, the liquid energy equation, and the vapour energy equation, are respectively:

\[ \frac{2}{\alpha y} \left[ 1 + a_3 z + 0.5a_5 \left( \frac{y}{\alpha y} \right)^2 \right] = - \frac{A}{l^2} \left[ \frac{\dot{W}_g}{\alpha y} \right] \left[ a_3 (T - T_o) + a_3 z + 0.5a_5 \left( \frac{y}{\alpha y} \right)^2 \right] + a_{12} (T_w - T) \quad (A.2.1) \]

\[ \frac{2}{\alpha y} \left[ 1 + a_3 z + 0.5a_5 \left( \frac{y}{\alpha y} \right)^2 \right] = - \frac{A}{l^2} \left[ \frac{\dot{W}_g}{\alpha y} \right] \left[ 1 + a_3 z + 0.5a_5 \left( \frac{y}{\alpha y} \right)^2 \right] \]

\[ + a_{13} (T_w - T) \quad (A.2.2) \]

When sectioned in the height variable, and linearized around steady state a difference-differential approximation method is applied to Eqs.
(A.2.1) and (A.2.2) to give respectively:

\[
\left(0.5a_3 T + a_3 0.5z - 0.25a_5 \left(\frac{\bar{w}_g}{1-\alpha}\right)^2\right)_{N+1/2} \frac{\partial}{\partial t}\left(\alpha_{N+1} - \alpha_N\right)
\]

\[+
\left(0.5a_5 \left(\frac{\bar{w}_g}{1-\alpha}\right)\right)_{N+1/2} \frac{\partial}{\partial t}\left(\bar{w}_g, N+1 + \bar{w}_g, N + \frac{a_3}{2}(1 - \alpha)\right)_{N+1/2} \frac{\partial}{\partial t}\left[T_{N+1} + T_N\right]
\]

\[= \frac{1}{\alpha\bar{a}} a_3 T + a_3 z + 1.5a_5 \left(\frac{\bar{w}_g}{1-\alpha}\right)^2 \left[\bar{w}_g, N+1 - \bar{w}_g, N\right] - \frac{\bar{w}_g a_3}{\alpha} \left[T_{N+1} - T_N\right]
\]

\[+ \left(0.5a_5 \left[\frac{\bar{w}_g}{1-\alpha}\right]^3\right)\left(\alpha_{N+1}^\prime - \alpha_N^\prime\right)
\]

\[+ 0.5 \left(a_{12}(T_w' - T')\right)_{N+1} + \left(a_{12}(T_w' - T')\right)_{N}\]

(A.2.3)

\[
\left[0.5\left(\gamma + a_3 z\gamma - 0.5a_5 \left[\frac{\bar{w}_g}{\alpha\gamma}\right]^2\right)\right]_{N+1/2} \frac{\partial}{\partial t}\left(\alpha_{N+1}^\prime + \alpha_N^\prime\right)
\]

\[+ \left[\frac{a_5\bar{w}_g}{2\alpha\gamma}\right]_{N+1/2} \frac{\partial}{\partial t}\left(\bar{w}_g, N+1 + \bar{w}_g, N\right)
\]

\[= \frac{1}{\alpha\bar{a}} \left[1 + a_3 z + 1.5a_5 \left(\frac{\bar{w}_g}{\alpha\gamma}\right)^2\right] \left[\bar{w}_g, N+1 - \bar{w}_g, N\right] - \frac{\bar{w}_g a_3}{\alpha\gamma}
\]

\[+ \left[\frac{a_5}{\alpha\gamma^2} \frac{\bar{w}_g}{\alpha}\right]^3\left(\alpha_{N+1}^\prime - \alpha_N^\prime\right)
\]

\[+ 0.5 \left(a_{13}(T_w' - T')\right)_{N+1} + \left(a_{13}(T_w' - T')\right)_{N}
\]

(A.2.4)

Eqs. (A.2.3) and (A.2.4) can be written as:
\[
\begin{align*}
& C_{\text{CC}} N^{1/2} \frac{\partial}{\partial t} (-\alpha'_{N+1} - \alpha'_N) + C_{\text{CL}} N^{1/2} \frac{\partial}{\partial t} \left\{ T'_N + T'_N \right\} \\
& + C_{\text{C }} N^{1/2} \frac{\partial}{\partial t} \left\{ \frac{\alpha'}{N^2}, N+1 + W'_{N, N} \right\} \\
= & - C_{\text{S}} N^{1/2} W'_{N, N+1} + C S_{N} W'_{N, N} - C V_{N+1} T'_{N+1} \\
& + C V_{N} T'_{N} + C X_{N} \alpha'_{N} - C X_{N+1} \alpha'_{N+1} + 0.5 \left\{ (a_{12}(T'_{w} - T')_{N+1} + (a_{12}(T'_{w} - T')_{N} \right) \\
& \quad + 0.5 \left\{ (a_{13}(T'_{w} - T')_{N+1} + (a_{13}(T'_{w} - T')_{N} \right) \right\} \tag{A.2.5}
\end{align*}
\]

\[
\begin{align*}
& C_{\text{J}} N^{1/2} \frac{\partial}{\partial t} \left\{ \alpha'_{N} + \alpha'_N \right\} + C D_{N} N^{1/2} \frac{\partial}{\partial t} \left\{ G'_{N, N+1} + W'_{g, N} \right\} \\
= & - C G_{N+1} W'_{g, N+1} + C G_{N} W'_{g, N} + C C_{N+1} \alpha'_{N+1} - C C_{N} \alpha'_N \\
& + 0.5 \left\{ (a_{13}(T'_{w} - T')_{N+1} + (a_{13}(T'_{w} - T')_{N} \right) \right\} \tag{A.2.6}
\end{align*}
\]

where

\[
\begin{align*}
& C_{\text{CC}} N^{1/2} = \left[ 0.5 a_3 T + a_3 z - 0.5 a_5 \left\{ \frac{W'_{g}}{1-\alpha} \right\} \right]_{N+1/2} ; \\
& C_{\text{CL}} N^{1/2} = \left[ 0.5 a_3 (1-\alpha) \right]_{N+1/2} ; \\
& C_{\text{S}} N = \left\{ \frac{a_5}{b z} \left[ a_3 \frac{W'_{g}}{1-\alpha} \right] \right\}_{N} ; \\
& C_{\text{V}} N = \left\{ \frac{a_3 W'_{g}}{b z} \right\}_{N} ; \\
& C_{\text{X}} N = \left\{ \frac{a_5 W'_{g}}{b z (1-\alpha)} \right\}_{N} ; \\
& C_{\text{J}} N = \left\{ \frac{a_5}{b z} \left[ 1 + a_3 z + 1.5 a_5 \frac{W'_{g}}{1-\alpha} \right] \right\}_{N+1/2} ; \\
& C_{\text{C}} N = \left\{ \frac{a_3 W'_{g}}{b z (1-\alpha)} \right\}_{N} ; \\
& C_{\text{C}} N = \frac{1}{b z} \left[ 1 + a_3 z + 1.5 a_5 \frac{W'_{g}}{1-\alpha} \right]_{N} ; \\
& C_{\text{C}} N = \frac{1}{b z} \left\{ \frac{a_5 \frac{W'_{g}}{1-\alpha}}{a \gamma} \right\}_{N} .
\end{align*}
\]

A dimensionless macroscopic mass balance gave:
Employing the difference-difference method on the interval $\Delta z = z_{N+1} - z_N$, Eq. (A.2.7) is reduced to:

\[
(0.5(1-\gamma))^{N+1/2} \frac{\partial}{\partial t}(a_{N+1}^t + a_N^t) = \frac{\Delta}{\Delta z} (W_g^t + W_k^t) - \frac{W_g^t, N - W_k^t, N}{(N+1)}
\]  

(A.2.8)

The bulk boiling boundary, $Z_B$, represents the axial position at which the bulk fluid temperature is equal to the saturation temperature. When the pressure variation within the channel is negligible, then for a boiling two-phase system flowing in a channel with a constant inlet feed velocity, the point $Z_B$ depends on the heat flux, and the inlet subcooling. Hence, for a constant inlet subcooling, and for a modulated power input, $Z_B$ is temperature dependent, and moves with time. For a small power disturbance, the transient value of the bulk boundary can be estimated by a Taylor series expansion around the steady state value, $\bar{Z}_B$:

\[
Z_B = \bar{Z}_B + \left\{\frac{\partial z}{\partial T}\right\}_T T'
\]

(A.2.9)

where $\left\{\frac{\partial z}{\partial T}\right\}$ is obtained from Eq. (4.41), and $T'$, the liquid temperature response at $Z_B$ which is calculated by a step-by-step integration of the expression for $T_N^C$ derived for the subcooled boiling region.
APPENDIX III

Unsteady State Equations of Change in the Bulk Boiling Region

The unsteady state equations of change in the bulk boiling region were derived by Hudson et al (30), and St. Pierre (36). The two base macroscopic energy balance equation, Eq. (4,7), gave:

\[
\begin{align*}
\rho \delta z + 0.5a_5 \left( \frac{W_k}{1-\alpha} \right)^2 \frac{\partial}{\partial t} (\alpha) + \left[ \gamma + a_3z + 0.5a_5 \left( \frac{W_k}{\alpha y} \right)^2 \right] \frac{\partial \alpha}{\partial t} \\
+ (1-\alpha)0.5a_5 \left( \frac{2W_k}{(1-\alpha)^2} \right) \frac{\partial W_k}{\partial t} - \left( \frac{2W_k}{(1-\alpha)^2} \right) \frac{\partial (\alpha)}{\partial t} \\
+ 0.5a_5 \left[ \frac{2W_k}{\alpha^2 \gamma^2} \frac{\partial W_k}{\partial t} - \frac{2W_k}{\alpha^2 \gamma^2} \frac{\partial \alpha}{\partial t} \right] \\
= - \left[ 1 + a_3z + 0.5a_5 \left( \frac{W_k}{\alpha y} \right)^2 \right] \frac{\Delta W_k}{\Delta z} - \left[ a_3z + 0.5a_5 \left( \frac{W_k}{\alpha y} \right)^2 \right] \frac{\Delta W_k}{\Delta z} \\
- 0.5a_5W_k \left( \frac{2W_k}{(1-\alpha)^2} \right) \frac{\Delta W_k}{\Delta z} - \left( \frac{2W_k}{(1-\alpha)^2} \right) \frac{\Delta (\alpha)}{\Delta z} \\
- 0.5a_5W_k \left( \frac{2W_k}{\alpha^2 \gamma^2} \frac{\Delta W_k}{\Delta z} - \frac{2W_k}{\alpha^2 \gamma^2} \frac{\Delta \alpha}{\Delta z} \right) + a_6 (T_w - T) \\
\end{align*}
\]

Eq. (A.3.1) is further simplified to give:

\[
\begin{align*}
\gamma + a_3z \gamma - 0.5a_5 \gamma \left( \frac{W_k}{\alpha y} \right)^2 - a_3z + 0.5a_5 \left( \frac{W_k}{1-\alpha} \right)^2 \frac{\partial \alpha}{\partial t} + a_5 \left( \frac{W_k}{1-\alpha} \right) \frac{\partial W_k}{\partial t}
\end{align*}
\]
A difference-differential approximation method is now employed to solve Eq. (A.3.2), and employing Eq. (4.52) to eliminate the terms, $W^N_N$ and $W^N_{g,N+1}$, the resulting expression obtained is:

\[
\begin{align*}
&\left[0.5\gamma + 0.5a_3\gamma - 0.25a_5\left(\frac{\tilde{W}_N}{\alpha\gamma}\right) - 0.5a_3z + 0.25a_5\left(\frac{\tilde{W}_N}{1-\alpha}\right) + 0.5a_5\left(\frac{\tilde{W}_N}{\alpha\gamma}\right)^M\right]_{N+1/2} \\
\frac{3}{\partial t}(a_N^' + a_N^' + 0.5a_5\left\{\frac{\tilde{W}_N}{1-\alpha}\right\} + a_5\left\{\frac{\tilde{W}_N}{\alpha\gamma}\right\})_{N+1/2} + \frac{3}{\partial t}[W^N_{g,N+1} + W^N_{g,N}] \\
&= -\frac{\Delta W^N_{g}}{\Delta z}\left(1 + a_3z + 1.5a_5\left\{\frac{\tilde{W}_N}{\alpha\gamma}\right\}^2\right) + a_3z + 1.5a_5\left\{\frac{\tilde{W}_N}{1-\alpha}\right\}^2 \\
&- \frac{\Delta a^'_{N}}{\Delta z}\left(0.5a_5\left[\frac{\tilde{W}_N}{1-\alpha}\right] - 0.5a_5\left\{\frac{\tilde{W}_N}{\alpha\gamma}\right\}^3\right) + 1 + a_3z + \left[1.5a_5\left\{\frac{\tilde{W}_N}{\alpha\gamma}\right\}^2\right]_M \\
&+ a_6T^N_{w} \\
\end{align*}
\]

Eq. (A.3.3) may be written as:

\[
B_{N+1/2} + D_{N+1/2}M_{N+1/2} + \frac{3}{\partial t}(a_{N+1}^' + a_N^') + C_{N+1/2} + D_{N+1/2}M_{N+1/2} + \frac{3}{\partial t}[W^N_{\alpha,N+1}]
\]

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\[ \begin{align*}
&= - C_{N+1} W_{N+1} + F_{N+1} W_{N+1} + C_N W_N + F_N W_N \\
&\quad - E_{N+1} + G_{N+1} \alpha_{N+1} + F_N + G_N \alpha_N + a_6 T_w
\end{align*} \]

(A.3.4)

where

\[ B_{N+1/2} = \left[ 0.5 \gamma - a_3 z (1-\gamma) + 0.5 a_5 \left( \frac{2}{1-\alpha} \right)^2 - 0.5 a_5 \left( \frac{2}{1-\alpha} \right)^2 \right]^{N+1/2}; \]

\[ D_{N+1/2} = \left[ 0.5 a_5 \frac{a_5}{\alpha y} \right]^{N+1/2}; \quad C_{N+1/2} = \left[ 0.5 a_5 \frac{a_5}{1-\alpha} \right]^{N+1/2}; \]

\[ G_N = \frac{1}{\Delta z} \left[ 1 + a_3 z + 1.5 a_5 \left( \frac{\bar{W}_e}{\alpha} \right)^2 \right]^{N}; \quad F_N = \frac{1}{\Delta z} \left[ a_3 z + 1.5 a_5 \left( \frac{\bar{W}_e}{1-\alpha} \right)^2 \right]^{N}; \]

\[ E_N = 0.5 a_5 \frac{1}{\Delta z} \left[ \left( \frac{\bar{W}_e}{1-\alpha} \right)^3 - \left( \frac{\bar{W}_e}{\alpha} \right)^3 \right]^{N+1/2}. \]

The coefficient \( C_{N+1/2} + D_{N+1/2} \) was found to be very small and was therefore left out in Eq. (A.3.4). In the bulk boiling region, the channel wall energy balance equation, Eq. (4.8) is solved to obtain an expression for the channel wall temperature response at any spatial node:

\[ T_{C_w,N} = \frac{a_8 h_{C}(a_7 - i\omega)}{a_7^2 + \omega^2} \]  

(4.57)

Combining Eqs. (A.3.4) and (4.57), we obtain:

\[ - P_{N+1} W_{N+1} + q_{N,N+1} W_{N+1} + r_N \alpha_N - s_{N+1} \alpha_{N+1} + \frac{a_6 a_8 h_{C}(a_7 - i\omega)}{a_7^2 + \omega^2} = 0. \]  

(A.3.5)

where \( P_{N+1} = B_{N+1/2} + D_{N+1/2} W_{N+1/2} \left[ \frac{2(1+L_{N+1})}{(1-\gamma)\Delta z} \right] + F_{N+1} + G_{N+1} W_{N+1}; \)
\[ q_N = B_{N+1/2} + D_{N+1/2} M_{N+1/2} \left[ \frac{2(1 + L_N)}{(1 - \gamma) \Delta z} \right] + F_N + G_N^N; \]

\[ r_N = B_{N+1/2} + D_{N+1/2} M_{N+1/2} \left[ \frac{2 \kappa_N}{(1 - \gamma) \Delta z} \right] + F_N + G_N^M; \]

\[ s_{N+1} = B_{N+1/2} + D_{N+1/2} M_{N+1/2} \left[ \frac{2 \kappa_N}{-(1 - \gamma) \Delta z} \right] + F_{N+1} + G_{N+1}^M; \]
APPENDIX IV

General Description of Diabatic Two Phase Flow

In a boiling vertical channel, the vapour void fraction increases along the length of the channel. Consequently, the boiling process produces at least two or three flow patterns existing over varying lengths of the channel. Figure IV. I. represents a typical flow regime distribution in a boiling vertical channel.

Fig. IV. I Flow Regimes in a Boiling Vertical Water Channel.
In the bubbly flow regime, the controlling mechanism is nucleate boiling. As more of the liquid evaporates, the slug flow and the annular flow regimes are formed respectively as shown in Fig. IV.1. In the annular flow regime, the liquid adheres to the channel wall, the remainder being entrained in the vapour.

As one proceeds along the channel length, three distinct boiling regions are observed as shown in Fig. IV.2.

Fig. IV.2 A Typical Void Fraction Profile as a Function of the Length of a Boiling Vertical Channel.

The first region is known as the non-boiling region. Here, there are no free steam bubbles and the void fraction is zero. The next region is the subcooled boiling region and in this region, vapour bubbles nucleate on the heated surface, grow bigger in size and finally detach from the heated surface. This region is sometimes referred to as the non-equilibrium region since the bulk fluid temperature is less than the saturation.
temperature. Here, the liquid phase and the vapour phase enthalpies vary along the axial length of the channel.

In the bulk boiling region, the bulk fluid temperature is equal to the saturation temperature and the total heat flux, \( q \), is converted to the latent heat content of the bubbles.

The steam quality in the bulk boiling region is calculated from a thermal energy balance. However, in the subcooled boiling region a direct calculation of the steam quality from an energy balance is not easy because of the difficulty in estimating the actual fraction of the total heat flux, \( q \), converted to the latent heat content of the bubbles. The total heat flux, \( q \), was divided into four components in the subcooled boiling region:

\[
q = q_e + q_a + q_{sp} + q_c.
\]

The mechanism giving rise to these modes of heat transfer are summarized below.

A. The Agitative Heat Flux, \( q_a \)

In the figure below, a volume of the hot liquid is pushed into the bulk fluid.
In the figure below, the vapour bubble is detached from the heated surface and a volume of cold liquid replaces the bubble at the heated surface.

**B: The Latent Heat content of the Bubble, $q_e$**

The component $q_e$, represents the latent heat content of the steam bubbles or the part of the total heat flux converted into the heat of evaporation.

**C. The Single Phase Heat Flux, $q_{sp}$**

The heat flux, $q_{sp}$, represents the single phase heat transfer from the heated surface to the bulk fluid and occurs through the spaces between the bubbles as shown in the figure below.
C. Heat Loss by Condensation, $q_c$

As heat flows into the individual bubble near the superheated base at the heating surface, latent heat of evaporation is absorbed at the vapour liquid interface. At the top of the bubble, vapour condenses while giving off the latent heat of evaporation to the subcooled liquid. This represents the heat loss by condensation, $q_c$.

The relative magnitude of each of the four components was considered, and for our analysis it was concluded that $q_a$ and $q_e$ comprised the major portion of the total heat flux, $q$. With the steam quality calculated for both the subcooled boiling and the bulk boiling regions the steam void fractions are easily calculated using theoretical models and correlations which relate the steam quality to the steam void fraction.
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