A theoretical and experimental study of the forced hydraulic jump.

Edward H. Regts

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/6494

This online database contains the full-text of PhD dissertations and Masters' theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000 ext. 3208.
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI®

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
A THEORETICAL AND EXPERIMENTAL STUDY OF THE FORCED
HYDRAULIC JUMP

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES THROUGH
THE DEPARTMENT OF CIVIL ENGINEERING IN PARTIAL
FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF APPLIED SCIENCE
AT THE UNIVERSITY OF WINDSOR

BY
EDWARD H. REGTS
B. A. Sc., THE UNIVERSITY OF WINDSOR 1966
WINDSOR, ONTARIO
1967
ABSTRACT

A theoretical and experimental analysis of the forced hydraulic jump is presented. The general momentum equation is modified by the inclusion of a baffle force term. The drag coefficient for the baffle is related to the flow in the region of the baffle. The velocity at the baffle is determined by analysing the incoming flow as an expanding jet. The resulting seventh degree equation is solved by computer to give curves of the sequential depth against the Froude number for various values of the drag parameter. For comparison with other investigators, plots of the sequential depths against the baffle position, for definite values of the relative baffle height and Froude number, are plotted. The experimental points show good agreement with the theoretical curves. In addition force measurements on the baffles are compared to the maximum jet force and presented in non-dimensional form.
ACKNOWLEDGEMENTS

The author is deeply indebted to Professor J. A. McCorquodale of the Civil Engineering Department, University of Windsor, under whom this thesis was written. His suggestions and guidance during the investigation and his constructive criticism in the writing of this thesis are gratefully acknowledged.

The financial assistance offered by the National Research Council is greatly appreciated.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iv</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF PHOTOGRAPHS</td>
<td>viii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>FACILITIES AND EQUIPMENT</td>
<td>2</td>
</tr>
<tr>
<td>EXPERIMENTAL PROCEDURE</td>
<td>13</td>
</tr>
<tr>
<td>LITERATURE SURVEY</td>
<td>15</td>
</tr>
<tr>
<td>THEORETICAL ANALYSIS</td>
<td>19</td>
</tr>
<tr>
<td>DISCUSSION OF RESULTS</td>
<td>30</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>55</td>
</tr>
<tr>
<td>FUTURE RESEARCH</td>
<td>56</td>
</tr>
<tr>
<td>REFERENCES AND BIBLIOGRAPHY</td>
<td>59</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>62</td>
</tr>
<tr>
<td>VITA AUCTORIS</td>
<td>64</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
LIST OF TABLES

Table I  Table of Experimental Values for the Forced Hydraulic Jump  49
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Schematic Layout of Project</td>
<td>5</td>
</tr>
<tr>
<td>2. Dimensions of Flume and Headbox</td>
<td>8</td>
</tr>
<tr>
<td>3. Dynamometer</td>
<td>10</td>
</tr>
<tr>
<td>4. Chart Reading</td>
<td>3</td>
</tr>
<tr>
<td>5. Diagramatic Flow Chart</td>
<td>14</td>
</tr>
<tr>
<td>6. Defining Sketch of the Hydraulic Jump</td>
<td>39</td>
</tr>
<tr>
<td>7. Defining Sketch of the Expansion of the Jet</td>
<td>40</td>
</tr>
<tr>
<td>8. Drag Characteristics of Rectangular Baffle Blocks</td>
<td>41</td>
</tr>
<tr>
<td>9. Sequential Depths in the Hydraulic Jump</td>
<td>42</td>
</tr>
<tr>
<td>10. Sequential Depths in the Hydraulic Jump</td>
<td>43</td>
</tr>
<tr>
<td>11. Sequential Depths in the Hydraulic Jump</td>
<td>44</td>
</tr>
<tr>
<td>12. Sequential Depths in the Hydraulic Jump</td>
<td>45</td>
</tr>
<tr>
<td>13. Sequential Depths in the Hydraulic Jump</td>
<td>46</td>
</tr>
<tr>
<td>14. A Comparison of Theoretical and Experimental Sequential Depths of the Forced Hydraulic Jump</td>
<td>47</td>
</tr>
<tr>
<td>15. A Comparison of Theoretical and Experimental Baffle Forces</td>
<td>48</td>
</tr>
</tbody>
</table>
# LIST OF PHOTOGRAPHS

<table>
<thead>
<tr>
<th>Photo</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Side view of Equipment</td>
<td>6</td>
</tr>
<tr>
<td>2.</td>
<td>Upstream view of channel</td>
<td>7</td>
</tr>
<tr>
<td>3.</td>
<td>Position of Baffles, showing false floor</td>
<td>9</td>
</tr>
<tr>
<td>4.</td>
<td>Dynamometer</td>
<td>11</td>
</tr>
<tr>
<td>5.</td>
<td>Brush Amplifier and Recorder</td>
<td>12</td>
</tr>
<tr>
<td>6.</td>
<td>Typical Jump Profile</td>
<td>53</td>
</tr>
<tr>
<td>7.</td>
<td>Typical Jump Profile</td>
<td>53</td>
</tr>
<tr>
<td>8.</td>
<td>Typical Jump Profile</td>
<td>54</td>
</tr>
<tr>
<td>9.</td>
<td>Typical Jump Profile</td>
<td>54</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
INTRODUCTION

The purpose of this project was to determine the effect of appurtenances in the hydraulic jump stilling basin. A theoretical approach was followed whereby a term accounting for the force on the baffles was included in the momentum equation. This necessitated the introduction of a drag coefficient which was related to the flow in the neighbourhood of the baffles, rather than at the entrance section of the jump, as has been suggested by some previous investigators [1, 2, 3].

In the study of the free hydraulic jump i.e., for a horizontal stilling basin without appurtenances, the well known ratio of initial to sequential depth, \( \frac{h_1}{h_i} = \frac{1}{2} \left( \sqrt{1 + \frac{8}{h_i}} - 1 \right) \) is obtained by applying the momentum equation between the beginning and end of the jump, the only assumption being that the force of friction is negligible. In this project it was attempted to determine a theoretical expression for \( \frac{h_2}{h_1} \) for a stilling basin with baffles, by equating the applied force i.e., the force on the baffles and the force due to static pressure, to the rate of change of momentum.

The force on the baffles was measured by a strain gauge dynamometer which was located on one of the baffles. The baffle was allowed to move slightly on ball bearings in order to obtain readings. The deflection of the baffle was kept to a minimum so that the configuration of the baffles remained intact. The dynamometer forces were checked by placing piezometers on the front and rear of one of the baffles and thus calculating a force by summing the pressure forces over the baffle.
FACILITIES AND EQUIPMENT

General Layout

The experimental program was carried out in the University of Windsor Hydraulics Laboratory. The schematic layout is shown in figure 1.

The flow was measured by means of a magnetic flowmeter and then discharged into a rectangular headbox. The flow from the headbox was regulated by an aluminum sluice gate, and thus entered the flume from which it was discharged over a hinged tailgate into the sump and hence re-circulated.

Equipment

A) Flume and Headbox

The flume and headbox were constructed from plywood with a 5 foot plexiglass wall so that photographs could be taken. Photos 1 and 2 show views of the entire set-up.

The structure was placed on blocks and elevated approximately 18 inches from the floor, so that the baffles position could be varied. Three of the four baffles were constructed from plexiglass, with one being hollow so that piezometers could be inserted. The other baffle was constructed from brass and was part of the dynamometer used in recording the forces. The dimensions of the flume and headbox are shown in figure 2. The baffles were attached to a false plexiglass floor (as shown in photo 3) so that adjustments could be made on the baffle position from the entrance.

B) Dynamometer

The device used for measuring the forces on the baffles is shown in figure 3 and photo 4. The device consists of a brass
baffle placed upon ball bearings which were attached to a rigid steel frame. A stainless steel rod extends from the baffle to the bottom of the frame and can be clamped in place. Attached to the rod and the frame is a 3 inch aluminum ring with two strain gauges attached opposite each other on the ring. A deflection of the baffle gives a reading on the gauges which are connected to a Brush amplifier and recorder as shown in photo 5. The test ring, as well as being sensitive to small deflections, increases the natural frequency of the dynamometer and thus reduces the possibility of sympathetic vibrations. The device was calibrated prior to each run and at the end of each run to ensure that the system was operating properly. A tension spring was used in the calibration since weights and pulleys were found to cause some error by creating friction in loading and unloading. Another check of the forces was obtained by the average forces calculated from the piezometer readings. A typical chart reading from the dynamometer is shown in figure 4.

Figure 4. - Chart Reading
Because of the variations in the readings, as shown in figure 4, it was necessary to treat the data statistically in order to find the average forces.
Photo 1. Side View of Equipment
Photo 2. Upstream View of Channel
Figure 2. Dimensions of Flume and Headbox

TOP VIEW

10'-0"  4'-0"

GATE SETTING

PT. GAUGE

SLUICE GATE

3" R.

T. GATE

2'-0"

FALSE FLOOR

BAFFLE UNIT

3/4"
Photo 3. Position of Baffles, showing False Floor
Photo 5. Brush Amplifier and Recorder
EXPERIMENTAL PROCEDURE

For each run the following procedure was used:

1. Set and record the position of the baffles;
2. Set and record the sluice gate opening;
3. Set and record the discharge;
4. Set and record the tailwater level;
5. Measure the dynamic load by recorder;
6. Take piezometric readings at (a) baffles, (b) flume, (c) headbox;
7. Measure length of jump and temperature of water;
8. Mark maximum and minimum water levels of the jump;
9. Photograph the jump.

Four horizontal baffle settings at 26.5, 40.5, 52.0, and 65.0 centimeters were examined. For each setting, at least 5 gate openings at 2, 4, 6, 8, and 10 centimeters were used. For each gate opening various flows from 0.5 C.F.S. to 5.5 C.F.S. were examined. Finally, for each flow several tailwater levels, high, standard and minimum were examined. For each standard tailwater level the steps (1) to (9) were carried out. The entire procedure is best summarized by the diagramatic flow chart in figure 5.
Figure 5. Diagramatic Flow Chart
Although the hydraulic jump has been the subject of investigation by hydraulic engineers since the work of Bidone \( \square \) in 1820, this phenomena is still open to a great deal of further study.

The theory of the hydraulic jump was summarized by B. A. Bakhmeteff and A. E. Matzke \( ] \) in 1936, with studies conducted at Columbia University. In addition to studying and summarizing previous results on the vertical elements of the jump, such as the relationship between the initial and sequential depths, their studies revealed the important predictable characteristics of the longitudinal elements as well. The results were presented for the first time in dimensionless form and related to the initial Froude number. The theory developed by Bakhmeteff and Matzke is based on the "momentum principle" and contains several assumptions, mainly that the rate of change of momentum equals the difference in hydrostatic pressure between the initial and sequential depths. The assumptions are valid as long as friction is negligible and then the well-known ratio \( \frac{H_x}{Y_x} = \frac{1}{2} \left( \sqrt{1 + \frac{F_x}{H_x}} - 1 \right) \) is developed.

Rouse, Siao and Nagaratnam \( 6 \), in 1958, made an important contribution to the understanding of the role of turbulence in the hydraulic jump.

Since the hydraulic jump is a useful method of dissipating energy, engineers have found it advantageous to place baffles and
sills in the stilling basin, to increase the dissipation of energy and reduce the length of the basin which is required to prevent erosion and scouring in a natural channel.

The effect of sills on the formation of the hydraulic jump was discussed by John W. Forster and Raymond A. Skrinde [7], in 1950. A great deal of work in the design of stilling basins and dimensioning of sills, baffles and piers has been done by the U. S. Corps of Engineers at the Waterways Experimental Station at Vicksburg, Mississippi, the University of Minnesota (on the St. Anthony Falls Project) and the U. S. Bureau of Reclamation.

J. N. Bradley and A. J. Peturka, reporting the work of the U. S. B.R. in 1957, published a comprehensive paper on the hydraulic design of stilling basins, dams and outlet structures under various flow conditions [8].

The 1964 progress report of the ASCE Task Force on Energy Dissipators and Outlet Works [9], contains almost 500 references that deal with energy dissipators. This report points out the fact that no standard practice exists among hydraulic engineers for the design of stilling basins with appurtenances. Basin lengths may vary from 6 times to as low as 2 times the sequential depth, depending on the type of appurtenances and the reliance placed on them [7, 8, 10, 11].

Forster and Skrinde, who studied an abrupt rise in the channel and Hsu, who investigated the sudden drop, proposed to modify the momentum equation such that it included the force on the obstruction. This necessitated the use of a drag coefficient
which was to behave in the same manner as the coefficient of a flat plate. Thus Forster and Skrinde stated [7] "In any rigorous analysis $C_D$ must be considered a drag coefficient applied to the approach velocity head, not a lump correction factor to absorb the shortcomings of the equation. It is the form of the equation not the nature of the coefficient that must be modified to changing flow conditions."

This type of analysis whereby the drag coefficient is related to the incoming flow has been advocated by both Rand and Rajaratnam [2, 3]. These investigators related $C_D$ to the initial Froude number as well as the shape and spacing of baffles; however, Harleman [12] has pointed out that the velocity is a function of the distance to the baffles and this seems to indicate the necessity of an analysis in which the drag force is related to the flow conditions in the neighbourhood of the baffle, rather than to those at the entrance section of the stilling basin. A number of investigators have contributed to the analytical treatment of stilling basins with baffles and/or sills, but there appears to be shortcomings in each approach.

The work of Forster and Skrinde has already been mentioned. In discussing the work of Forster and Skrinde, R. M. Weaver [7] mentions the measurements of forces on sills in the hydraulic jump. Weaver found that the sill force decreases as the baffles are moved downstream from the beginning of the jump until a minimum is obtained after which the force increases slightly. Harleman [12], although he did not make a general theoretical analysis
of the problem, reported that the forces on a differential transformer force measuring device indicated that:

1) The baffle force decreases with increasing distance from the start of the jump.

2) This force can be related through the momentum equation to the tailwater depth.

3) The effectiveness of the baffles increases with increasing Froude numbers.

In his analysis of the forced hydraulic jump, Rajaratman [3] (1964), includes the effects of sills by introducing a drag coefficient which depends on the sill location, shape, height and spacing; the entrance depth and the initial Froude number. The drag coefficient becomes in fact a lump correction factor which accounts for the decrease in $\frac{f^2}{c^2}$ of the forced jump. This type of coefficient provides a satisfactory means of correlating experimental data but as previously stated a more meaningful and realistic approach would be to relate the baffle drag force to the flow conditions at the baffle rather than at the beginning of the jump.

More recently, Rand [12], in 1965 and 1966, has made a dimensional analysis of the effects of end sills and baffle blocks on the forced hydraulic jump; however, a mathematical treatment of the forced jump is not presented.

Thus it appears that an analysis, whereby the effect of baffles on the hydraulic jump is given a mathematical treatment and which relates the baffle drag coefficient to flow conditions in the region of the baffles, should be the next step in the investigation of the forced hydraulic jump.
The Reynolds equations of motion for a turbulent liquid are commonly written in the tensor form as:

\[
\frac{\partial}{\partial x_j} \left( \bar{u}_i \cdot \bar{u}_j \right) + \frac{\partial}{\partial x_j} \left( \bar{u}_i \bar{u}_j \right) = -\frac{1}{\rho} \frac{\partial}{\partial x_i} (\bar{P}) + X_i + \frac{\mu}{\rho} \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j} \tag{1}
\]

which may be integrated by using Gauss's Divergence Theorem to obtain:

\[
\int_f \rho \bar{u}_i \bar{u}_j \frac{\partial x_i}{\partial \eta} df + \int_f \rho \bar{u}_i \bar{u}_j \frac{\partial x_j}{\partial \eta} df = -\int_f \bar{P} \frac{\partial x_i}{\partial \eta} df + \int_f C_i df + \int_f \mu \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial x_j}{\partial \eta} df \tag{2}
\]

where the terms are from left to right \([13, 14]\):

(i) the net flux of momentum through the boundary \(f\) of the mean flow,
(ii) the net flux of momentum through the boundary \(f\) due to turbulence,
(iii) the mean normal force exerted on the boundary \(f\),
(iv) the weight of the fluid in the region \(f\),
(v) the mean tangential force exerted on the boundary of the region \(f\).

In the analysis of the forced hydraulic jump the following assumptions are made:

(i) The jump forms on a wide horizontal apron;
(ii) The shear stresses at the free surface are negligible.

Equation (2) is treated one-dimensionally since the net changes in the vertical and transverse directions will be zero.
Applying equation (2) to a basin of width $W$ and $N$ baffles, with area $A_b$ normal to the direction of flow, (see defining diag. figure 6), gives:

\[-\int_0^{W_y} \left( \phi \bar{u}_1 \right) \frac{d^2}{dy^2} d(yW) + \int_0^{W_y} \left( \phi \bar{u}_2 \right) \frac{d^2}{dy^2} d(yW) - \int_0^{W_y} \left( \phi \bar{u}_1 \right) \frac{d^2}{dy^2} d(yW) + \int_0^{W_y} \left( \phi \bar{u}_2 \right) \frac{d^2}{dy^2} d(yW) \]

\[= -\int_0^{W_y} \left( \phi \bar{u}_1 \right) \frac{d^2}{dy^2} d(yW) + \int_0^{W_y} \left( \phi \bar{u}_2 \right) \frac{d^2}{dy^2} d(yW) - \int_0^{W_y} \left( \phi \bar{u}_1 \right) \frac{d^2}{dy^2} d(yW) + \int_0^{W_y} \left( \phi \bar{u}_2 \right) \frac{d^2}{dy^2} d(yW) \]

Harleman's notation[12] leads to:

\[\beta \frac{U_2}{g} y_2 - \beta \frac{U_2}{g} y_1 + I \frac{U_2}{g} y_2 - I \frac{U_2}{g} y_1 = \alpha \frac{U_1}{2} - \alpha \frac{U_1}{2} - \int_0^{W_bh_1} \frac{\bar{p}_1 - \bar{p}_2}{\gamma} d(yW) \]

with $U_1$ and $U_2$ representing the average velocities at the respective cross-sections and

\[\beta = \int_0^y \frac{(\frac{\phi'}{\phi}) \bar{u}^2}{u^2} dy \]

\[I = \int_0^y \frac{(\frac{\phi'}{\phi}) (\bar{u}')^2}{u^2} dy \]

\[\alpha' = \int_0^y \frac{(\bar{u})}{\frac{1}{2} y^2} dy \]

where $\rho'$ = density of the air-water mixture.
The baffle force term,

\[ \int_0^{NW_b} \frac{\Delta P_b}{\delta W} d(yW_bN) \]

constant

\[ \int_0^{h_b} C_D \left( \frac{W_bN}{W} \right) \left( \frac{y}{f} \right) \bar{u}_b^2 dy = S \int_0^{h_b} \left( \frac{y}{f} \right) \bar{u}_b^2 dy \]

with the assumptions that;

\[ C_D = \frac{\Delta P_b}{\bar{u}_b^2} = \text{constant} \]

\[ S = \frac{W_bN}{W} \]

Rewriting Equation (4)

\[ \beta \frac{U_z^2}{g} y_2 - \beta \frac{U_t^2}{g} y_1 + I_2 \frac{U_z^2}{g} y_2 - I_1 \frac{U_t^2}{g} y_1 \]

\[ = \alpha_1 \frac{y_1^2}{2} - \alpha_2 \frac{y_2^2}{2} - S \int_0^{h_b} \left( \frac{y}{f} \right) (\bar{u}_b)^2 dy - \int_0^{\gamma} \frac{f}{k} dx \]

The work of Rouse et al [6] indicates that the net turbulent momentum flux from a free jump is negligible; although this may not be true of the forced jump, the \( I_1 \) and \( I_2 \) terms in equation (11) will be neglected for the present investigation.

Also assuming that \( \beta = \beta = \alpha_2 = \alpha_1 = \alpha = 0 \), and neglecting the boundary shear force, equation (11) becomes,

\[ \frac{U_z^2}{g} y_2 - \frac{U_t^2}{g} y_1 + U_z^2 y_2 - U_t^2 y_1 = \frac{y_1^2}{2} - \frac{y_2^2}{2} \]

\[ - S \int_0^{h_b} \left( \frac{y}{f} \right) (\bar{u}_b)^2 dy - \int_0^{\gamma} \frac{f}{k} dx \]

--- (12)
The term \( \overline{u_2} \cdot \overline{u_2} \, dy \) is proportional to the momentum flux through an elemental area in the neighbourhood of the baffle; thus \( \int_0^{h_2} \overline{u_2}^2 \, dy \) would represent to some scale the momentum flux in the neighbourhood of the baffle. Evaluation of this term requires a knowledge of the velocity distribution in the neighbourhood of the baffles; however an approximation of the momentum flux upstream from the baffle can be obtained by treating the incoming flow as an expanding jet. The expansion of the jet is caused by the adverse pressure gradient of the primary roller (figure 7) and the boundary shear force.

The velocity distribution in the jet varies with increasing \( x_b \) from a uniform distribution to a distribution similar to that sketched in figure 7. However in order to estimate the average velocity acting on the baffle, it is assumed that the velocity distribution is nearly uniform everywhere upstream from the baffle.

**Expansion of the Jet**

Consider an element of length of the jump in figure 7, within the range \( 0 \leq x \leq x_b \), and assume that the net momentum in the \( x \)-direction is concentrated in the jet, Newton's second law may now be applied for a unit width of channel, so that:

\[
\text{FORCE} = \text{RATE OF CHANGE OF MOMENTUM}
\]

Assuming that the shear force will be relatively small compared to the adverse pressure gradient of the primary roller,

\[
\frac{\partial}{\partial x_b} ( \overline{u_2} ) = - \frac{dF}{dF} \quad - - - - 13a
\]
where $U = \text{average velocity}$, and $j$ refers to the jet.

By continuity

$$\frac{dU_j}{dy_j} = -\frac{dU_i}{dy_i},$$

but

$$U_j \cdot y_j = U_i \cdot y_i,$$

thus

$$\frac{dU_j}{dy_j} = -\frac{dU_i}{dy_i} U_j \cdot y_j,$$

Now assume

$$j = (y_1 + y_2 \frac{X}{X_f})$$

where

$$X_f \approx X_b + y_2^2.$$

Differentiating

$$\frac{dy}{dy^2} = \frac{X}{X_f} \frac{dX}{dX} = \frac{X}{X_f}$$

The force term is

$$F = \frac{1}{2} \rho \frac{v^2}{dX}$$

$$\therefore \quad dF = \rho \frac{v^2}{dX} dy$$

Substituting in equation (13d) from (13b) and (13c),

$$dF = \rho \left( y_1 + y_2 \frac{X}{X_f} \right) \left( \frac{dy}{dy^2} \right) \frac{dX}{dX}$$

Combining (13) and (14)

$$\int \left( \frac{dy}{dy^2} \right) U_i \cdot y_i = \int \left( y_1 + y_2 \frac{X}{X_f} \right) \frac{dX}{dX}$$

Integrating from $y_1$ to $y_b$ and $X = 0$ to $X = X_f$,

$$\frac{1}{2} \left( \frac{1}{y_1} - \frac{1}{y_b} \right) = \frac{y_2}{X_f} \left( y_1 \frac{X_b}{X_f} + \frac{y_1^2}{X_f} \right)$$

$$\text{from which}$$

$$\frac{y_1}{y_b} = \frac{\frac{y_2}{X_f} \left( X_b + \frac{y_1^2}{X_f} \right)}{\frac{y_2}{X_f} \left( y_1 \frac{X_b}{X_f} + \frac{y_1^2}{X_f} \right)}$$

Returning to the term $\int \left( \frac{dX}{dX} \right) \frac{dX}{dX} dy$ in equation (12) and substituting $U_b = U_i y_2^2 \frac{X_b}{X_f}$ we obtain;

$$\int \left( \frac{dX}{dX} \right) \frac{dX}{dX} dy$$

where $h_b^* = h_b \text{ or } \frac{y_2}{X_f}$ whichever is smaller.
If it is assumed that the entrained air is concentrated in the roller then \( J' = \alpha \) and the baffle force term in equation (12) becomes:

\[
\frac{C_s h_b^* U^2}{2\theta} \left( \frac{y_1}{y_2} \right)^2
\]

Substituting for \( \frac{y_1}{y_2} \) and returning to equation (12) one obtains:

\[
2F^2 \left( \frac{y_2}{y_1} - 1 \right) = 1 - \left( \frac{y_1}{y_2} \right)^2 \frac{C_p}{F_1} \frac{\alpha}{\theta} F^2 \left[ F - y_2 (\frac{X_b}{y_2}) - \frac{1}{2} \left( \frac{y_1}{y_2} \right)^2 (\frac{X_b}{y_2})^2 \right]^2
\]

where, \( a = \) effective area of baffle = \( SL_b^* \)

and \( S = \frac{W_b}{(W_{st} + W_b)} \)

Let \( \theta = \frac{y_2}{y_1} \), \( \alpha = \frac{\alpha}{\theta} \), \( \lambda = \frac{F_1}{2} \)

thus

\[
2\theta (1 - \theta) - \theta - \theta^2 - C_p \alpha \theta \lambda - \theta R - \frac{1}{2} (\omega R)^2
\]

Expanding:

\[
\left( \frac{C_p \alpha \theta}{\lambda y_1} \right) \theta^5 + \left( \frac{C_p \alpha \theta^3}{\lambda y_1} \right) \theta^4 - \left( \frac{C_p \alpha \theta^2}{\lambda y_1} \right) (\lambda - 1) \theta - \left( \frac{C_p \alpha \theta}{\lambda y_1} \right) (2\theta^4 - 2\theta^3 + 2\theta^2 - 2\theta) = 0 \quad (16)
\]

Let \( B = \frac{X}{y_1} \), thus \( R \) becomes \( \frac{X}{y_1} - \frac{B}{B + \theta} \)

Substituting and multiplying by \( (B + \theta)^2 \) leads to the seventh degree equation:

\[
\theta^7 + 4 B \theta^6 + \sum_{i=1}^{6} \frac{C_p \alpha \theta^i}{\lambda y_1} \left( \frac{B_1 + B^2 - (\lambda - 1) B^2 - 2\lambda B + \lambda^2}{} + 6B^2 \theta + (\lambda - 1) \theta \right) \theta^5
\]

\[
+ \sum_{i=1}^{5} \frac{C_p \alpha \theta^i}{\lambda y_1} \left( B^2 - 2(\lambda - 1) B^2 - 6\lambda B + 6\lambda^2 \right) + 4B^3 - 4(\lambda - 1) B^2 + 2 \theta \theta^4
\]

\[
+ \sum_{i=1}^{3} \frac{C_p \alpha \theta^i}{\lambda y_1} \left( -B^2 (\lambda - 1) - 6\lambda B + 6\lambda^2 \right) + B^2 - (\lambda - 1) B^3 + 6\lambda B \theta \theta^3
\]

\[
+ \sum_{i=1}^{2} \frac{C_p \alpha \theta^i}{\lambda y_1} \left( -2\lambda B + \lambda^2 \right) - 4(\lambda - 1) B^3 + 12\lambda B \theta \theta^2
\]

\[
+ \sum_{i=1}^{1} \frac{C_p \alpha \theta^i}{\lambda y_1} \left( -2\lambda B + \lambda^2 \right) - 4(\lambda - 1) B^3 + 12\lambda B \theta \theta^2 \theta + 2\lambda B^4 = 0 \quad (17)
\]
Equation (17) can best be solved with the aid of a digital computer, as has been the procedure for this project. The results of these equations are discussed in detail in the results of this paper.

Approximations to equation (17) may be obtained by considering several special cases and by having some knowledge of the expansion of the jet; thus the general seventh degree equation may be reduced to a simpler cubic equation.

Consider the term \( \frac{\epsilon}{y} \) from equation (15), letting \( \frac{\epsilon}{y} \) = \( m \), the slope of the roller.

and

\[
A = \frac{m}{y} \frac{dx}{dy} (1 + \frac{m}{y} \frac{dx}{dy})
\]

Equation (15) becomes

\[
y_1/y_0 = (1 - A)
\]

Equations (11) and (12) become

\[
\frac{\partial^2}{\partial x^2} \frac{y}{y_0} - \frac{\partial^2}{\partial x^2} \frac{y}{y_0} = \frac{\alpha}{y_0} \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial y^2} \frac{y}{y_0} - \frac{C_0}{y_0} \frac{y}{y_0} (1 - A) \frac{y}{y_0} - (19)
\]

By continuity \( u_0 y_0 = u_0 y_0 \), thus:

\[
\Theta + \int \frac{C_0}{F} \frac{y}{y_0} (1 - A) \frac{y}{y_0} - 2 \frac{F}{y_0} - \frac{1}{F} \Theta + 2 \frac{F}{y_0} \frac{y}{y_0} = 0
\]
The Determination of $C_D$ and $m$

In order to solve equation (19) for \( \frac{y_2}{y_1} \), mathematical expressions must be found for $C_D$ and $m$, the average slope of the primary roller.

The drag coefficient for baffle blocks depends primarily on the baffle shape ($W_b/h_b$), the baffle height relative to the impinging jet ($h_b/y_b$), the baffle spacing ($S = W_b/(W_b + W_o)$) and the Reynolds number. If the spacing and shape are fixed and the Reynolds number is sufficiently high to obtain a stable separation pattern ($R_n = 2000$ for a sharp-edged plate) then $C_D$ is a function of $y_b/h_b$, the height of the impinging jet relative to the baffle height. For $y_b/h_b \gg 1$, $C_D$ can be obtained from value given in standard texts on fluid mechanics [14]. For the U. S. B. R. standard baffles with $W_b/h_b = 0.75$, an approximate value of $C_D$ (neglecting the spacing effect) would be 1.16. On the other hand for $y_b/h_b \ll 1$ (say for $W_b/h_b = 0.75$), an estimate of $C_D$ can be obtained by considering the momentum change of the impinging jet. If the depth of the impinging jet is small compared to the baffle height and width the component of momentum normal to the baffle face would largely be lost on impact. Therefore for a rectangular baffle \[ T = C_D \frac{y_b h_b^2}{2 g} \]

or \[ C_D \rightarrow 2.0 \text{ as } (y_b/h_b \text{ and } y_b/W_b) \rightarrow 0.0 \]

Using a dynamometer and several piezometers connected to a baffle block (U. S. B. R. type Baffle) a few values of $C_D$ were
determined experimentally for, $0 < \frac{y_b}{h_b} < 1.0$, and rectangular blocks. The data is shown in figure (8). The curve

$$C_D = 1.16 + 0.84 e^{-\left(2\frac{y_b}{h_b}\right)^3}$$

was fitted to the experimental and 'theoretical' data by trial and error.

In order to obtain an expression for the slope of the primary roller of a forced jump the writer examined the water surface profiles presented by a number of investigators [2, 3, 8]. This led to the following empirical equation for the average slope of the primary roller

$$m = \frac{1}{2} \left( \frac{h_b + y_1 + y_2}{x_b+\frac{y_2}{2}} \right)$$

which has a lower limit of applicability of about 0.20, i.e., approximately the slope of the free jump roller. For simplification in the seventh degree equation the slope was estimated as

$$m = \frac{y_2}{x_b + y_2}$$

Solution of the General Equation for the Forced Jump

Some Special Cases

There are three cases of equation (19) that are of interest:

1) $C_D = 0.0$ (the free hydraulic jump);

ii) $\Delta = 0.0$ and $C_D \neq 0.0$ (imminent sweep-out of the forced jump);

iii) $(1-\Delta) = \frac{y_1}{y_2}$ and $C_D \neq 0.0$ (baffles at the downstream end of the forced jump).
Case (i): The Free Hydraulic Jump

If $C_D = 0.0$ equation (19) becomes

$$\theta^3 + \left[ -2 \left( \frac{\theta}{C_D} \right)^2 - \alpha_1' \right] \theta + 2 \frac{\theta^2}{\alpha_2} \frac{C_D}{\theta} = 0 \quad ---(22)$$

and for $\alpha_1' = \alpha_2' = \beta_1 = \beta_2 = 1.0$, this gives

$$\theta = \left( \frac{y_2}{y_1} \right) = \frac{1}{2} \left( \sqrt{1 + \beta_1^2} - 1 \right) \quad ---(23)$$

However, if $\alpha_1, \beta_1, \text{etc.}$ do not equal unity then equation could be solved analytically as will be done with case (ii).

Case (ii): The Forced Hydraulic Jump near Sweep-Out

For $\Delta = 0.0$ equation (19) becomes

$$\theta^3 + \left[ \pm C_D \frac{C_D}{\theta} \right] - 2 \left( \frac{\theta}{\beta_1} \right)^2 - \alpha_1' \right] \theta = \frac{\alpha_1'}{\alpha_2'} \quad ---(24)$$

Let $a = \left[ \pm C_D \frac{C_D}{\theta} \right] - 2 \left( \frac{\theta}{\beta_1} \right)^2 - \alpha_1' \right] / \alpha_2' \quad ---(25)$

$$b = 2 \frac{\theta^2}{\alpha_2} \quad ---(26)$$

and $D = \left( \frac{b}{2} \right)^2 + \left( \frac{a}{3} \right)^3 \quad ---(27)$

In general we are interested in the case of $D \leq 0$ which yields the roots

$$\theta = 2 \sqrt{-\frac{a}{3}} \cos \left( \frac{\theta}{3} + \frac{\pi}{3} \right) \quad ---(28)$$

and

$$2 \sqrt{-\frac{a}{3}} \cos \left( \frac{\theta}{3} + \frac{\pi}{3} \right) \quad ---(29)$$

where

$$\theta = \cos^{-1} \left( -\frac{b}{2} / \sqrt{-\left( \frac{a}{3} \right)^3} \right) \quad ---(29)$$
Case (iii): Baffle Piers at the Downstream End of the Jump

At the downstream end of the jump the depth of the jet occupies the entire depth of flow $y_2$, thus the average velocity at the downstream baffles would be $U_i \frac{y_i}{y_2}$.

In equation 19 this is represented by $(1 - \Delta) = \frac{y_i}{y_2}$.

Substituting $(1 - \Delta) = \frac{y_i}{y_2}$ into equation (19) simplifying one obtains

$$\Theta'' + \left[ - \frac{5}{11} - \frac{2}{\alpha_i} \right] \Theta + 2 \frac{C_S}{\alpha_i} \frac{h_i}{\alpha_i} \Theta + \frac{C_D F_i^2}{\alpha_i} \frac{h_i}{y_i} = 0 - - (30)$$

This equation can be solved by trial and error.
DISCUSSION OF RESULTS

Analytical Results

The seventh degree equation (17) was solved by computer for approximately the same baffle positions and gate openings as in the experimental portion of the project. The charts were plotted to give even values of $(aC_D)$ from 0.0 to 1.2 and $\frac{y}{y_1}$ values of $x_b$ of 0.0, 2.5, 5.0, 10.0, and 16.0. The resulting plots of $y_2$ against $F_1$, for various values of $(aC_D)$ are presented in figures (9 to 13). These curves are valid for all values of $\frac{y}{y_b} > \frac{h_b}{h}$, however, when the height of the jet is less than the height of the baffle, the effective area of the baffle is reduced and it becomes necessary to solve the equation for:

$$S \left( \frac{h_b}{y_b} \right) \frac{C_D}{y_1} \left( \frac{y_1}{y} \right) \left( \frac{y}{y_1} \right)$$

The introduction of the effective height necessitates an expression for the expansion of the jet. Equation (15) was solved for different values of $F_1$ and $\frac{y_2}{y_1}$ to give the plots of $\frac{y_2}{y_1}$ vs $\frac{y_b}{y_1}$ for various Froude numbers, as presented on the left of figures (9 to 13).

The procedure for using the charts is as follows:
EXAMPLE

\[ \begin{align*}
1.0 \quad & \left( \frac{x_b}{y_1} \right) = 5.0 \\
& F_1 = 6.0 \\
& \left( \frac{h_b}{y_1} \right) = 1.0
\end{align*} \]

Assume \( a_{CD} = 0.80 \)

from figure 11 \( \theta = 6.6 \)

from figure 11(b) \( \left( \frac{y_b}{y_1} \right) = 1.22 \quad \left( \frac{h_b}{y_1} \right) \)

\( \left( \frac{y_b}{h_b} \right) = 1.22 \quad C_D = 1.16 \)

\[ \text{thus } a_{CD} = \frac{1}{2} \times 1.22 \times 1.16 = 0.71 \]

Enter figure 11 \( \theta = 6.8 \)

from figure 11(b) \( \left( \frac{y_b}{y_1} \right) = 1.23 \quad C_D = 1.16 \)

\( \text{thus } a_{CD} = \frac{1}{2} \times 1.23 \times 1.16 = 0.71 \)

Therefore use \( \left( \frac{y_2}{y_1} \right) = 6.8 \)

Example 1.0 shows that a value for \( a_{CD} \) may be assumed

and that after one trial the method quickly converges to the correct value.
Generally, the charts verify that \( \frac{y_2}{y_1} \) is reduced by:

1. reducing \( \frac{X_b}{y_1} \),
2. increasing \( a_{CD} \)

**Experimental Results**

The results of the experimental study are summarized in Table 1.0, which shows the observed and calculated values under conditions of flow from 1.0 to 5.5 C.F.S., at baffle positions of 26.5, 40.5, 52.0, and 65 centimeters. The calculated values shown in Table 1.0 are derived from the observed values.

a) **Ratio of Depths**

For comparison with other investigators, plots were made of \( \frac{y_2}{y_1} \) vs. \( \frac{X_b}{y_1} \) for Froude numbers of 4, 6, 8, and 10, at values of \( \frac{h_b}{y_1} \) of 0.0, 1.0, 1.5 and 4.0. Even Froude numbers were obtained by observing the trend of the family of curves of \( a_{CD} \) and extending the observed data along this trend to the nearest even Froude number. The results are plotted in figure (14). Generally, using the values of other investigators, the agreement of experimental results with the theory presented in this paper is good. Considering the scatter of data, the results obtained at the University of Windsor show fair agreement with the theory (eq'n. 17).
Figure 14 indicates that the effectiveness of the baffles increases with increasing $\left(\frac{h}{y_1}\right)$ and $F_1$. At the point where the curves approach the curve for the free jump, the effect of the baffles may be assumed to be negligible. Using $X_b = 1.5 \left(\frac{y_2}{y_1}\right)$ as recommended by the ASCE Task Force on Energy Dissipators, and using $\left(\frac{y_2}{y_1}\right)$ from the free jump, the baffle distance is well within the effective range.

Harleman [12] found that the reduction in depth due to baffles is negligible. His findings may have been influenced by the fact that only $\left(\frac{h}{y_1}\right) = 1.0$ (the upper curves in figure 14) was considered, and also high values of $X_b$ were examined. Thus his results would fall very near to the free jump where the curves begin to approach each other. This indicates that perhaps values of $\left(\frac{h}{y_1}\right)$ greater than 1.0 should be considered in the design of stilling basins. However, in the present study no limits of $\left(\frac{X_b}{y_1}\right)$ and $\left(\frac{h}{y_1}\right)$ were evaluated and no study of the important effect of cavitation on the baffles was introduced.
b) Force Measurements

The results of the force measurements are presented in figure (15). The theoretical curves were developed by considering the ratio of the baffle force to the jet force of the incoming flow, as follows:

Drag Force

\[ \frac{F_B}{\eta} = \delta \frac{h_b^*}{\eta} \frac{W C_D y_b^2}{\eta^2} (1-\Delta)^2 \quad - - (31) \]

Maximum Jet Force

\[ \frac{F_J}{\eta} = \frac{\frac{1}{2} \frac{h_b}{\eta} C_D (1-\Delta)^2}{\frac{1}{2} \frac{y_b}{\eta}} \quad - - (32) \]

Thus

\[ \frac{F_B}{F_J} = \frac{1}{2} \frac{h_b}{\eta} C_D (1-\Delta)^2 \quad - - (33) \]

The force ratio can thus be calculated by considering the term \((1-\Delta)^2\) or \(\frac{y_b}{y_1}\) from the charts in figure (9 to 13), and multiplying by \(\frac{1}{2} \frac{h_b}{\eta} C_D\).

**EXAMPLE 2.**

\[ \left( \frac{X_B}{y_1} \right) = 10.0, \quad F_1 = 6.0, \quad \left( \frac{h_b}{y_1} \right) = 1.0, \]

From figure 12 \( \left( \frac{y_2}{y_1} \right) = 7.25 \)

thus \( \left( \frac{y_b}{y_1} \right) = 1.55 \) and \( \left( \frac{h_b}{y_1} \right) = 1.55 \)

From figure 8 \( C_D = 1.16 \)

\[ F_b = \frac{1}{2} \times 1.0 \times 1.16 \times \left( \frac{1}{1.55} \right)^2 = .241 \]

The experimental results (using the piezometric forces) are shown as the plotted points in figure 15. The force measurements and the resulting plots are of an exploratory nature and further
investigation should be carried out before any definite conclusions are drawn.

In the present analysis, the force measurements for given values of \( \frac{h_b}{y_1} \) and \( F_1 \) are limited to small ranges of \( \frac{X_b}{y_1} \). In order to fit an experimental curve to the data, force measurements should be made over a wider range of \( \frac{X_b}{y_1} \) for specified \( \frac{h_b}{y_1} \) and \( F_1 \) values.

Again, from the experimental data available, the general trend, suggested by the theoretical curves, is followed.

**Dynamic Load**

The dynamic loads, as measured by the dynamometer, are shown in Table 1.0. Generally, the forces from the dynamometer are slightly smaller than those calculated from the piezometric readings. Because of the variation of the data, as shown in figure 4, the data was treated statistically and the standard deviation was calculated by considering, that for any given number of observations \( n \), an estimate of the standard deviation \( S.D. \) of the underlying population can be obtained from the mean value of the sample range \( R \) [28].

\[
S.D. \approx R x d
\]

The values of "d" may be obtained from tables in any elementary test of statistics.

Thus for each value of the dynamic load, 15 readings of the range were recorded, the average was calculated, and this was multiplied by "d" to obtain the standard deviation.

When more data is available figure 15 should facilitate the
determination of the force on the baffle, for use in design
when $C_D/h_b$, $F_1$ and $X_b$ are known. It should be pointed out
that figure 15 of $F_B/y_b$ depends on the particular plot of
$C_D/y_b$ and thus different curves of $F_B/y_b$ would be obtained
for different curves of $C_D/y_b$; however, the charts in figures
9 to 13 of $y_2$ vs. $F_1$, for various values of $(aC_D)$ are independent
of the plot of $C_D/y_b$ i.e., the curves themselves do not change
for different plots of $C_D/y_b$.

The curve of $C_D/y_b$ which was used in the determination of
figures 14 and 15 could be modified for a different type of baffle
block to give slightly different plots in those figures.

Length of Jump Measurements

Length measurements are presented in Table 1.0 for the sake
of completeness. No plots of a non-dimensional length ratio vs
Froude number were made. The photos, taken for each of the
measurements shown in Table 1.0, suggest that a more reasonable
criteria for the length of the jump, might be to consider where
the air bubbles leave the bottom of the channel. This point
indicates that large scale turbulence in the jump no longer reaches the bottom of the channel. This length is clearly indicated for each baffle position (see photos 6, 7, 8 and 9), and measurements can be made from the photos.

Suggested Revisions to Equipment

Future investigations may be facilitated by regarding some of the following suggestions.

1) The radius of the guide plate attached to the sluice gate should be increased so that more nearly parallel flow lines will be obtained.

2) Careful measurements should be made of $y_1$, after the contraction at the sluice gate.

3) The piezometers should be improved by providing more damping in the lines, so that a better estimate of the average forces may be obtained.

4) The dynamometer could be improved in several ways:
   (i) Friction may be reduced by improving the bearings system
   (ii) The force measurements would give more accurate readings of the horizontal forces if tilting of the baffle block is further reduced. This is a particularly difficult problem since the point of application of the average force depends on the flow conditions. Possibly the tilting of the baffle may be kept to a minimum by hollowing the baffle block and placing the proving ring inside the opening. This would
also reduce leakage around the baffle. Perhaps the best method of measuring the horizontal forces is to place several transducers on the front and rear of one of the baffles and calculating the resulting force.
Figure 6. Defining Sketch of the Hydraulic Jump

\[ \gamma' = \text{Specific Weight of Water + Air} \]
Figure 7. Defining Sketch of the Expansion of the Jet

$y = y_i + mx$

AVERAGE SURFACE OF PRIMARY ROLLER

EFFECT OF AIR ENTRAINMENT (MOST OF AIR IN ROLLER)

SCHEMATIC OF ACTUAL VELOCITY DISTRIBUTION

PRESSURE DISTRIBUTIONS ON ELEMENT.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 8. Drag Characteristics of Rectangular Baffle Blocks

\[ S = \frac{1}{2} \]

\[ \frac{Y_b}{h_b} = 0.75 \]

\[ C_0 = 1.16 + 0.84 \left( \frac{2Y_b}{h_b} \right)^3 \]

**EXPLANATION**

- EXPERIMENTAL DATA
- THEORETICAL LIMITS
Figure 9. Sequential Depths in the Hydraulic Jump

Curves of \( \frac{h_b^*}{y_1} = 0 \) to 1.2

\( x_b / y_1 = 0 \)

Lower Limit
Figure 10. Sequential Depths in the Hydraulic Jump

\[ \theta = \frac{y_2}{y_1} \]

\[ \frac{X_b}{y_1} = 2.5 \]

Curves of \( S C_{b} \left( \frac{y_b}{y_1} \right) = 0 - 12 \)

\( F = 8 \)
\( F = 6 \)
\( F = 4 \)
\( F = 3 \)

\[ \frac{y_b}{y_1} \]

\( F_1 \)

43

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 11. Sequential Depths in the Hydraulic Jump
Figure 12. Sequential Depths in the Hydraulic Jump
Figure 13. Sequential Depths in the Hydraulic Jump

CURVES OF $SCD \left( \frac{h_p^*}{y_1} \right) = 0 - 1.2$

$X_D / y_1 = 16.0$

$\theta = \frac{y_2}{y_1}$

$F_1 = 12$

$F_1 = 10$

$F_1 = 8$

$F_1 = 6$

$y_0 / y_1$

$F_1$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 14. A Comparison of Theoretical and Experimental Sequential Depths of the Forced Hydraulic Jump

Explanation:
- UNIVERSITY OF WINDSOR DATA
- Approximated from RAND
- Approximated from WEIDKNECHT
- Approximated from HARLEMAN & UNNY
  - \( \frac{h_b}{y_1} \) (theory)
  - \( \frac{h_b}{y_1} = 0 \)
  - \( \frac{h_b}{y_1} = 1.0 \)
  - \( \frac{h_b}{y_1} = 1.5 \)
  - \( \frac{h_b}{y_1} = 4.0 \) (curved)

U.S. BUREAU OF RECLAMATION SWEEP-OUT DATA

\[ \frac{X_D}{y_1} \]
Figure 15. A Comparison of Theoretical and Experimental Baffle Forces
Table I — Observed and Calculated Values of the Forced Hydraulic Jump

<p>| Q (CFS) | H08 (CM) | Xb (CM) | y1 = dC | y2 (CM) | Lr (CM) | Lj (CM) | F_b(LB) | F_b(LB) | S.D. | F1 | Cc | Cv | ( \frac{Y_b}{Y} ) | ( \frac{hb}{Y} ) | Xb (CM) | ( \frac{F_b}{F} ) |
|--------|----------|---------|--------|--------|--------|--------|---------|---------|------|----|----|----|----|-----------|----------|--------|-----------|
| 0.78   | 27.0     | 26.5    | 2.12   | 9.67   | 65.5   | 0.50   | 4.87    | 1.0     | 4.57 | 2.5 | 1.0 | 1.0 | 1.0 | 4.2       | 12.5     | 7.50   |
| 1.23   | 55.0     | 26.5    | 2.12   | 16.70  | 66.5   | 0.50   | 7.06    | 1.0     | 7.87 | 4.2 | 1.0 | 1.0 | 1.0 | 4.2       | 12.5     | 5.58   |
| 1.67   | 91.0     | 26.5    | 2.12   | 21.90  | 84.0   | 0.50   | 9.15    | 1.0     | 10.32 | 4.2 | 1.0 | 1.0 | 1.0 | 4.2       | 12.5     | 5.87   |
| 1.89   | 120.0    | 26.5    | 2.12   | 27.10  | 106.0  | 0.50   | 10.53   | 1.0     | 12.75 | 4.2 | 1.0 | 1.0 | 1.0 | 4.2       | 12.5     | 7.64   |
| 1.23   | 24.5     | 26.5    | 3.95   | 11.80  | 59.5   | 0.20   | 3.16    | .96     | .99  | 2.95| 2.25| .99| .99| 6.72      | 6.72     | 6.56   |
| 1.67   | 32.5     | 26.5    | 3.95   | 16.65  | 64.0   | 0.20   | 3.74    | .96     | .99  | 4.22| 2.25| .99| .99| 6.72      | 6.72     | 4.94   |
| 2.23   | 51.5     | 26.5    | 3.95   | 21.40  | 67.0   | 0.20   | 4.85    | .99     | .99  | 5.08| 2.25| .99| .99| 6.72      | 6.72     | 1.72   |
| 2.87   | 81.5     | 26.5    | 3.95   | 28.10  | 69.0   | 0.20   | 6.37    | .99     | .99  | 7.12| 2.25| .99| .99| 6.72      | 6.72     | 6.00   |
| 3.34   | 113.5    | 26.5    | 3.95   | 33.10  | 77.0   | 0.20   | 7.63    | .99     | .99  | 8.38| 2.25| .99| .99| 6.72      | 6.72     | 6.55   |
| 1.67   | 24.5     | 26.5    | 5.47   | 14.70  | 70.0   | 0.20   | 2.57    | .90     | .98  | 2.69| 1.63| .98| .98| 4.85      | 5.10     | 4.85   |
| 2.23   | 32.0     | 26.5    | 5.47   | 18.30  | 71.0   | 0.20   | 3.15    | .90     | .98  | 3.34| 1.63| .98| .98| 4.85      | 5.10     | 1.60   |
| 3.12   | 55.0     | 26.5    | 5.47   | 26.10  | 98.0   | 0.20   | 4.17    | .90     | .98  | 4.77| 1.63| .98| .98| 4.85      | 5.82     | 4.85   |
| 4.02   | 82.0     | 26.5    | 5.47   | 32.50  | 100.0  | 0.20   | 4.84    | .90     | .98  | 5.95| 1.63| .98| .98| 4.85      | 5.99     | 4.85   |
| 4.47   | 105.5    | 26.5    | 5.47   | 37.10  | 105.0  | 0.20   | 5.95    | .90     | .98  | 6.80| 1.63| .98| .98| 4.85      | 6.13     | 6.13   |
| 2.84   | 34.8     | 26.5    | 7.0    | 19.5   | 98.0   | 0.20   | 2.87    | .87     | .98  | 2.78| 1.27| .87| .98| 3.78      | 4.52     | 3.78   |
| 3.78   | 57.5     | 26.5    | 7.0    | 28.10  | 118.0  | 0.20   | 3.71    | .87     | .98  | 4.02| 1.27| .87| .98| 3.78      | 4.67     | 3.78   |
| 4.90   | 82.5     | 26.5    | 7.0    | 36.4   | 136.0  | 0.20   | 4.53    | .87     | .98  | 5.20| 1.27| .87| .98| 3.78      | 4.26     | 3.78   |
| 3.34   | 40.5     | 26.5    | 8.35   | 25.8   | 105.0  | 0.16   | 2.58    | .83     | .98  | 3.07| 1.07| .83| .98| 3.18      | 3.43     | 3.43   |
| 4.32   | 57.0     | 26.5    | 8.35   | 32.5   | 138.0  | 0.20   | 3.20    | .83     | .98  | 3.92| 1.07| .83| .98| 3.18      | 3.82     | 3.82   |
| 5.52   | 75.0     | 26.5    | 8.35   | 35.5   | 170.0  | 0.20   | 3.70    | .83     | .98  | 4.28| 1.07| .83| .98| 3.18      | 3.88     | 3.88   |</p>
<table>
<thead>
<tr>
<th>Q</th>
<th>HDD</th>
<th>$X_b$</th>
<th>$Y_1 = dC_c$</th>
<th>$Y_2$</th>
<th>$L_T$</th>
<th>$L_J$</th>
<th>$F_b (LB)$</th>
<th>$F_b (LB)$</th>
<th>S.D.</th>
<th>$F_i$</th>
<th>$C_c$</th>
<th>$C_V$</th>
<th>$Y_2 / Y_1$</th>
<th>$h_b / y_i$</th>
<th>$X_b$</th>
<th>$F_b / F_J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CFS)</td>
<td>(CM)</td>
<td>(CM)</td>
<td>(CM)</td>
<td>(CM)</td>
<td>(CM)</td>
<td>(CM)</td>
<td>PIEZ.</td>
<td>DYN.</td>
<td>(LB)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.11</td>
<td>41.0</td>
<td>65.0</td>
<td>2.12</td>
<td>16.4</td>
<td>64</td>
<td>79</td>
<td>2.31</td>
<td>2.6</td>
<td>0.35</td>
<td>6.13</td>
<td>1.0</td>
<td>1.0</td>
<td>7.73</td>
<td>4.2</td>
<td>30.6</td>
<td>.32</td>
</tr>
<tr>
<td>1.73</td>
<td>108.5</td>
<td>65.0</td>
<td>2.12</td>
<td>24.8</td>
<td>66</td>
<td>80</td>
<td>8.92</td>
<td>6.8</td>
<td>1.10</td>
<td>10.80</td>
<td>1.0</td>
<td>1.0</td>
<td>11.70</td>
<td>4.2</td>
<td>30.6</td>
<td>.35</td>
</tr>
<tr>
<td>3.14</td>
<td>121.5</td>
<td>65.0</td>
<td>2.12</td>
<td>26.7</td>
<td>68</td>
<td>81</td>
<td>11.00</td>
<td>8.2</td>
<td>1.24</td>
<td>8.00</td>
<td>1.0</td>
<td>1.0</td>
<td>12.60</td>
<td>4.2</td>
<td>30.6</td>
<td>.35</td>
</tr>
</tbody>
</table>

**Table I (cont'd.)**
Table I (cont’d.)

<table>
<thead>
<tr>
<th>Q</th>
<th>Hdb (CM)</th>
<th>Xdb (CM)</th>
<th>y1 = dC</th>
<th>y2 (CM)</th>
<th>Lr (CM)</th>
<th>Lj (CM)</th>
<th>Fb (LB)</th>
<th>Fb (LB)</th>
<th>S.D.</th>
<th>y2/y1</th>
<th>Cc</th>
<th>Cv</th>
<th>h0/y1</th>
<th>Xdb</th>
<th>Fb/F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>33.0</td>
<td>40.5</td>
<td>2.12</td>
<td>14.0</td>
<td>41</td>
<td>51</td>
<td>2.95</td>
<td>3.2</td>
<td>0.27</td>
<td>5.50</td>
<td>1.0</td>
<td>1.0</td>
<td>6.6</td>
<td>4.2</td>
<td>19.1</td>
</tr>
<tr>
<td>1.67</td>
<td>78.0</td>
<td>40.5</td>
<td>2.12</td>
<td>22.0</td>
<td>41</td>
<td>52</td>
<td>9.48</td>
<td>7.6</td>
<td>0.68</td>
<td>8.45</td>
<td>1.0</td>
<td>1.0</td>
<td>10.35</td>
<td>4.2</td>
<td>19.1</td>
</tr>
<tr>
<td>2.01</td>
<td>109.5</td>
<td>40.5</td>
<td>2.12</td>
<td>25.2</td>
<td>41</td>
<td>53</td>
<td>18.32</td>
<td>13.0</td>
<td>1.12</td>
<td>10.15</td>
<td>1.0</td>
<td>1.0</td>
<td>11.9</td>
<td>4.2</td>
<td>19.1</td>
</tr>
<tr>
<td>1.78</td>
<td>37.0</td>
<td>40.5</td>
<td>3.95</td>
<td>19.0</td>
<td>42</td>
<td>55</td>
<td>6.14</td>
<td>4.2</td>
<td>0.52</td>
<td>4.06</td>
<td>.96</td>
<td>.99</td>
<td>4.82</td>
<td>2.25</td>
<td>10.25</td>
</tr>
<tr>
<td>2.78</td>
<td>80.0</td>
<td>40.5</td>
<td>3.95</td>
<td>27.6</td>
<td>46</td>
<td>65</td>
<td>18.72</td>
<td>12.0</td>
<td>1.32</td>
<td>6.15</td>
<td>.96</td>
<td>.99</td>
<td>7.00</td>
<td>2.25</td>
<td>10.25</td>
</tr>
<tr>
<td>3.23</td>
<td>104.5</td>
<td>40.5</td>
<td>3.95</td>
<td>29.9</td>
<td>50</td>
<td>71</td>
<td>31.20</td>
<td>25.2</td>
<td>1.23</td>
<td>7.08</td>
<td>.96</td>
<td>.99</td>
<td>7.57</td>
<td>2.25</td>
<td>10.25</td>
</tr>
<tr>
<td>3.56</td>
<td>119.5</td>
<td>40.5</td>
<td>3.95</td>
<td>32.4</td>
<td>53</td>
<td>72</td>
<td>34.68</td>
<td>27.2</td>
<td>1.61</td>
<td>7.58</td>
<td>.96</td>
<td>.99</td>
<td>8.21</td>
<td>2.25</td>
<td>10.25</td>
</tr>
<tr>
<td>1.34</td>
<td>26.0</td>
<td>40.5</td>
<td>3.95</td>
<td>12.5</td>
<td>35</td>
<td>51</td>
<td>4.30</td>
<td>4.4</td>
<td>0.35</td>
<td>3.32</td>
<td>.96</td>
<td>.99</td>
<td>3.16</td>
<td>2.25</td>
<td>10.25</td>
</tr>
<tr>
<td>2.23</td>
<td>22.5</td>
<td>40.5</td>
<td>5.47</td>
<td>20.9</td>
<td>41</td>
<td>60</td>
<td>6.80</td>
<td>5.2</td>
<td>0.63</td>
<td>2.42</td>
<td>.90</td>
<td>.98</td>
<td>3.82</td>
<td>1.63</td>
<td>7.41</td>
</tr>
<tr>
<td>3.34</td>
<td>65.0</td>
<td>40.5</td>
<td>5.47</td>
<td>29.0</td>
<td>51</td>
<td>67</td>
<td>18.08</td>
<td>16.0</td>
<td>1.12</td>
<td>4.58</td>
<td>.90</td>
<td>.98</td>
<td>5.32</td>
<td>1.63</td>
<td>7.41</td>
</tr>
<tr>
<td>4.22</td>
<td>97.5</td>
<td>40.5</td>
<td>5.47</td>
<td>35.0</td>
<td>65</td>
<td>88</td>
<td>31.52</td>
<td>21.6</td>
<td>1.84</td>
<td>5.72</td>
<td>.90</td>
<td>.98</td>
<td>6.42</td>
<td>1.63</td>
<td>7.41</td>
</tr>
<tr>
<td>2.34</td>
<td>24.5</td>
<td>40.5</td>
<td>8.35</td>
<td>17.0</td>
<td>47</td>
<td>77</td>
<td>12.92</td>
<td>7.8</td>
<td>0.63</td>
<td>1.83</td>
<td>.83</td>
<td>.98</td>
<td>2.04</td>
<td>1.07</td>
<td>4.86</td>
</tr>
<tr>
<td>3.67</td>
<td>38.5</td>
<td>40.5</td>
<td>8.35</td>
<td>25.4</td>
<td>47</td>
<td>105</td>
<td>19.12</td>
<td>18.8</td>
<td>0.68</td>
<td>2.57</td>
<td>.83</td>
<td>.98</td>
<td>3.04</td>
<td>1.07</td>
<td>4.86</td>
</tr>
<tr>
<td>5.47</td>
<td>54.5</td>
<td>40.5</td>
<td>8.35</td>
<td>28.5</td>
<td>52</td>
<td>105</td>
<td>3.20</td>
<td>8.3</td>
<td>.98</td>
<td>3.41</td>
<td>.83</td>
<td>.98</td>
<td>3.41</td>
<td>1.07</td>
<td>4.86</td>
</tr>
<tr>
<td>Value</td>
<td>Calculated Values</td>
<td>Forcés</td>
<td>Values</td>
<td>Observed Values</td>
<td>$X_b$</td>
<td>$X_2/L$</td>
<td>${F}_b$</td>
<td>$F_2/L$</td>
<td>$F_{bc}$</td>
<td>$F_c$</td>
<td>$F_{bc}/F_c$</td>
<td>$b_{bc}/b_c$</td>
<td>$y_1$</td>
<td>$y_2$</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------------------</td>
<td>---------</td>
<td>-------</td>
<td>----------------</td>
<td>-------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>-------</td>
<td>-------------</td>
<td>-------------</td>
<td>------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>1.23</td>
<td>52.0</td>
<td>2.112</td>
<td>26.4</td>
<td>53</td>
<td>4.45</td>
<td>0.14</td>
<td>10.64</td>
<td>2.45</td>
<td>1.91</td>
<td>3.88</td>
<td>4.06</td>
<td>1.26</td>
<td>19.5</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>1.82</td>
<td>52.0</td>
<td>2.112</td>
<td>23.2</td>
<td>52</td>
<td>1.91</td>
<td>0.14</td>
<td>10.64</td>
<td>2.45</td>
<td>1.91</td>
<td>3.88</td>
<td>4.06</td>
<td>1.26</td>
<td>19.5</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>2.01</td>
<td>52.0</td>
<td>2.112</td>
<td>19.7</td>
<td>52</td>
<td>1.91</td>
<td>0.14</td>
<td>10.64</td>
<td>2.45</td>
<td>1.91</td>
<td>3.88</td>
<td>4.06</td>
<td>1.26</td>
<td>19.5</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>3.61</td>
<td>52.0</td>
<td>2.112</td>
<td>16.9</td>
<td>52</td>
<td>1.91</td>
<td>0.14</td>
<td>10.64</td>
<td>2.45</td>
<td>1.91</td>
<td>3.88</td>
<td>4.06</td>
<td>1.26</td>
<td>19.5</td>
<td>24.5</td>
<td></td>
</tr>
</tbody>
</table>

Table I (cont.)
Photo 6. Typical Jump Profile \((F_1 = 3.7)\)

Photo 7. Typical Jump Profile \((F_1 = 6.4)\)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Photo 8. Typical Jump Profile ($F_1 = 7.6$)

Photo 9. Typical Jump Profile ($F_1 = 4.0$)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
CONCLUSIONS

A theoretical analysis of the forced hydraulic jump has been presented and partially verified by experimental investigation. The general momentum equation has been modified by the introduction of a baffle force term which is related to the flow in the neighbourhood of the baffles. The resulting seventh degree equation (17) was solved to give dimensionless plots of $\frac{y_2}{y_1}$ vs $F_1$ for various values of $(Sh_bC_D)$. These charts can be used for design purposes when $F_1$, $x$, $C_D$, $X_b$, $S$ and $h_b$ are known (as is generally the case). An approximate curve of $\frac{C_D}{h_b}$ vs $\frac{y_b}{y_1}$, indicating the drag characteristics of the baffles, was also presented.

Force measurements of the baffle force term are also presented. Theoretical curves of the baffle force to the maximum jet force $\frac{F_b}{F_j}$ are plotted against $\frac{X_b}{y_1}$ to indicate the behavior of the forces under various $\frac{h_b}{y_1}$ ratios as well as varying Froude numbers. The resulting plot if further verified by experimental data would facilitate the determination of the forces on baffles in stilling basins.
For future research the following areas are suggested:

(1) Further study of the factors that affect $C_D$ (e.g., $S$ and $y_b/h_b$) is needed.

(2) The effect of the tailwater level on the baffle force should be investigated. In the present study it was found that the force on the baffle was increased on increasing or decreasing the tailwater level from the standard position. In this connection the slope of the primary roller and its effect on the baffle force term should also be investigated.

(3) A comparison of the length of the jump, as suggested previously (i.e., downstream to the point where the air bubbles leave the floor), to the length of the jump and roller as found by other investigators, is also suggested.

(4) Varying the geometry of the stilling basin by varying:
   
   (i) the spacing of the baffles,
   (ii) the dimensions of the baffles,
   (iii) the number of rows of baffles,
   (iv) the slope of the channel,

   should also be included in future research.

(5) In the analytical portion of the study, future research might include the introduction of a friction
term in the solution of the general equation. A friction force was not included in the present study because the inclusion of a friction force term is offset by a decrease of the baffle force term due to the apparent increased jet expansion when friction is included in the analysis. This step certainly warrants further investigation.

(6) Another assumption in the theoretical analysis which should be considered is the fact that the net turbulent momentum flux was assumed to be negligible. This was found to be true for the free jump 6 , but no studies seem to be available for the forced hydraulic jump.

(7) The term $\Theta$, the momentum coefficient, and $\Lambda$, the pressure coefficient, are assumed to be 1.0, i.e., the flow is considered to be parallel and the pressure distribution is hydrostatic.

(8) Air entrainment was also neglected for the present study. Inclusion of the previously mentioned terms in the general equation may have only slight effects on the previous solutions; however, their inclusion is suggested for the sake of completeness.

(9) The effect of cavitation on the baffle blocks is suggested as a further study.
The limits of the ratios $Y_b$ and $Y_b$ might also be determined.


20. Waterways Experiment Station, Corps of Engineers, "Laboratory Development of Cavitation Free Baffle Piers, Bluestone Dam, New River, West Virginia."


LIST OF SYMBOLS

a = Shb (effective area).
b = subscript indicating baffle.
Cc = contraction coefficient.
Cd = baffle drag coefficient.
Cv = velocity coefficient.
D2 = free jump sequent depth.
e = base of natural logarithms.
F = Froude number.
FB = force on the baffle.
Fj = force of the jet.
g = acceleration due to gravity.
hb = baffle height.
hb* = effective baffle height.
I = momentum coefficient (turbulent).
Lj = length of jump.
Lr = length of roller.
m = average slope of the primary roller.
N = number of baffles.
p = average pressure at a point.
q = discharge per unit width.
S = (wb/(wb+ws)), a spacing parameter.
u = (u+u') = instantaneous point velocity.
u = average velocity at a point.
u' = turbulent velocity fluctuation.
\( U \) = average velocity in a cross-section (x-direction).
\( W \) = Volume.
\( W_b \) = basin width.
\( w_b \) = baffle width.
\( w_s \) = space between the baffles.
\( X, x \) = horizontal coordinate in the flow direction.
\( X_b \) = distance from the start of the jump to the baffle.
\( X_r \) = distance to the roller.
\( y \) = vertical coordinate
\( y_b \) = depth of the jet just upstream of the baffle.
\( \alpha' \) = pressure coefficient.
\( \beta \) = momentum coefficient.
\( \gamma' \) = specific weight of the air-water mixture at a point.
\( \gamma \) = specific weight of water.
\( \Delta \) = jet expansion parameter.
\( \mathcal{S} \) = surface of integration.
\( \eta \) = positive normal to \( \mathcal{S} \).
\( \theta \) = \( y_2/y_1 \), i.e., sequent depth ratio.
\( \rho \) = density of the water.
\( \mu \) = viscosity of the water.
\( \tau_0 \) = boundary shear stress.
\( 1 \) = a subscript indicating the section at the start of the jump.
\( 2 \) = a subscript indicating the section at the end of the jump.
Since the drag coefficient has been related to the known incoming flow conditions, by other investigators, some explanation is needed to justify relating the drag force to the velocity in the region of the baffle.

When the drag coefficient is related to the initial flow conditions, the expansion of the jet is ignored and the coefficient, which must be determined experimentally, becomes a lump correction factor to compensate for the shortcomings of the equation. By relating the drag coefficient to the velocity in the region of the baffle, the coefficient, which again is determined experimentally is more realistically related to the geometric properties of the baffle (height, width, shape and spacing) relative to the expanding jet.

In effect, in the latter case, the lump correction factor has been broken down into its components which are the geometric properties of the baffle and the expansion of the jet. The expansion of the jet has been treated theoretically in this study.
VITA AUCTORIS

1942 Edward Hans Regts was born in Oranjewoud, Friesland, Netherlands, on April 3, 1942.

1948 In September, 1948, started his elementary education at Heerenveen, Friesland, Netherlands.

1957 In September, 1957, entered Tecumseh Secondary School in Chatham, Ontario, Canada.

1962 In September, 1962, enrolled in Civil Engineering at the University of Windsor, Windsor, Ontario.

1966 In June, 1966, graduated from the University of Windsor with the degree of Bachelor of Applied Science in Civil Engineering.

1966 In September, 1966, enrolled at the University of Windsor in a programme leading to the degree of Master of Applied Science in Civil Engineering.